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TECHNICAL REPORT ARBRL-TR-2029

MEASUREMENTS OF ANGULAR MOMENTUM TRANSFER IN LIQUID-FILLED PROJECTILES

Andrew Mark

November 1977

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER (14) TECHNICAL REPORT ARBRL-TR-02029	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle) (6) MEASUREMENTS OF ANGULAR MOMENTUM TRANSFER IN LIQUID-FILLED PROJECTILES.	5. TYPE OF REPORT & PERIOD COVERED (9) Final rept.,
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Andrew/Mark	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS USA Ballistic Research Laboratory Aberdeen Proving Ground, Maryland 21005	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS (16) RDT&E 1L161102AH54
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Materiel Development & Readiness Command 5001 Eisenhower Avenue Alexandria, VA 22333	12. REPORT DATE (11) NOV[redacted]77
	13. NUMBER OF PAGES (12) 55p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

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18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

In-flight measurements
liquid-filled projectile
spin-decay
yawsonde

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In-flight spin-decay measurements were performed on M687 liquid-filled projectiles by means of yawsondes. The shell spin-decay is used to infer the instantaneous liquid angular momentum and its rate of change by applying the moment of momentum equation. Through this inference one can deduce the time required for the liquid to acquire rigid-body-like rotation. This is called the spin-up time. This time becomes important when studying other aspects of the liquid-filled projectile problem; namely, that of projectile instability induced

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393 471

20. Abstract (Cont'd.)

by liquid eigenfrequency resonance with the projectile nutational frequency. The results indicate that for liquid Reynolds numbers below 10^6 the liquid spins up in less than 5 seconds. Above this Reynolds number the spin-up time is a strong function of Reynolds number reaching about 17 seconds at Re = 1.7 x ~~10^6~~. 1,000,000.

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I. INTRODUCTION

The stability of projectiles carrying liquid payloads has been a concern of the BRL since World War II. Early theoretical work by Stewartson[1] in 1959 addressed the problem of liquid resonance where the liquid was in rigid-body-like rotation. Measurements of liquid resonance effects were carried out by Karpov[2] and extensions to the Stewartson theory were made by Wedemeyer[3]. At present, a number of people in the Launch and Flight Division of BRL are actively working to develop theories to include the phenomenon of resonance during liquid spin-up. Existing theories account for resonant eigenfrequencies for the cases of fully filled or partially filled cylinders where the liquid is fully spun up. When a projectile exits the gun, however, the liquid is rarely fully spun up and the possibility of resonance during spin up is a condition which is currently being studied by Sedney and Kitchens[4].

A feature of the resonance during spin-up problem is the characteristic spin-up time. Recent developments in flight measurement technology at the BRL by Mermagen[5] have made it possible to obtain extremely accurate yaw and spin histories over entire trajectories through the use of yawsondes. Concurrent with the development of the yawsonde was the development of the binary projectile, 155mm M687. Yawsonde data from M687 firings showed some anomalous spin behavior and it was felt that methods could be developed to provide estimates of liquid spin-up times.

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1. *K. Stewartson, "On the Instability of a Spinning Top Containing Liquid," Journal of Fluid Mechanics, Vol. 5, Part 4, September 1959, pp. 577-592.*

2. *Karpov, B.G., "Liquid-Filled Gyroscope: The Effect of Reynolds Number on Resonance," Ballistic Research Laboratories Report 1302, Aberdeen Proving Ground, Maryland, October 1965. AD 479430.*

3. *Wedemeyer, E.G., "Viscous Corrections to Stewartson's Stability Criterion," Ballistic Research Laboratories Report 1325, June 1966, AD 489687.*

4. *W.C. Kitchens and N. Gerber, "Prediction of Spin-Decay of Liquid-Filled Projectiles," Ballistic Research Laboratory Report No. 1996, July 1977.* (AD #A043275)

5. *W.H. Mermagen and W.H. Clay, "Design of a Second Generation Yawsonde," Ballistic Research Laboratories Memorandum Report 2368, April 1974. AD 780064.*

During the initial development tests[6-16] of this liquid filled projectile, a tendency toward instability was discovered in the neighborhood of a charge zone 4 launch condition. The dispersions of a number of groups of M687 projectiles fired through May 1972 are shown in Figure 1. Impact range versus deflection is plotted with the line of fire indicated by an arrow. The open circles at the top of the Figure indicate

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6. *Anonymous, "Ballistic Firing Recovery, Split Round," Deseret Test Center, Interim Report No. 01-023-03, Deseret Test Center, Fort Douglas, Utah, July 1969.*

7. *Anonymous, "Ballistic Firings, Single Point Accuracy," Deseret Test Center Interim Report No. 01-046-01, Deseret Test Center Fort Douglas, Utah, 27 August 1969.*

8. *C. C. Sterns, "Feasibility Test for Projectile, 155mm, Binary XM687, EATP 69-2B, Ballistic Test," Deseret Test Center Data Report No. DR E949, Deseret Test Center, Fort Douglas, Utah, December 1969.*

9. *C. C. Sterns and B. Black, "Design Configurations Test for Projectile, 155mm, Binary, XM687," Deseret Test Center Data Report (DTCDR) E127, Deseret Test Center, Fort Douglas, Utah, 7 April 1971.*

10. *C. C. Sterns and K. Jones, "Phase 1 of Malfunction Investigation Test for Projectile, 155mm, GB2, XM687," Deseret Test Center Data Report (DTCDR) Phase 1-72-305, Deseret Test Center, Fort Douglas, Utah, October 1971.*

11. *C. C. Sterns and K. Jones, "First Article Test (Developmental Hardware) for Projectile, 155mm GS2, XM687," Deseret Test Center Data Report (DTCDR) 72-301, Deseret Test Center, Fort Douglas, Utah, 20 Jan 1972.*

12. *C. C. Sterns and K. Jones, "(Advanced Development) Ballistic (Match) Test for Projectile, 155mm, GB2, XM687," Deseret Test Center Data Report (DTCDR) 72-304, Deseret Test Center, Fort Douglas, Utah, 24 Feb 1972.*

13. *C. C. Sterns and K. Jones, "Phase 2 of Malfunction Investigation for Projectile, 155mm GB2, XM687," Deseret Test Center Data Report (DTCDR) 72-305, Deseret Test Center, Fort Douglas, Utah, 3 April 1972.*

14. *C. C. Sterns and K. Jones, "Phase 3 of Malfunction Investigation Test Projectile, 155mm, XM687," Deseret Test Center Data Report (DTCDR) 72-305, Deseret Test Center, Fort Douglas, Utah, 3 April 1972.*

15. *C. C. Sterns and K. Jones, "Services Developmental Test of Projectile, 155mm, XM687," Deseret Test Center Data Report (DTCDR) 72-311, Deseret Test Center, Fort Douglas, Utah, 15 May 1972.*

16. *A. Mark and W.H. Mermagen, "Measurement of Spin Decay and Instability of Liquid-Filled Projectiles via Telemetry," Ballistic Research Laboratories Memorandum Report No. 2333, October 1973. AD 771919.*

the dispersions of each volley and the percentage of liquid fill is printed nearby. Unstable rounds could not be included in the small dispersion circles and are plotted separately by numerals which indicate the percent of liquid fill. Solid filled M483 (the parent projectile) rounds are plotted separately for comparison, as are some M687 rounds which had a rigid filler.

The information shown in Figure 1 can be used to construct a plot of the probability of failure to achieve full range as a function of fill ratio. This is done in Figure 2. The Figure shows that the proposed service fill ratio of 85 to 95% would lead to a high probability of failure. Although a fill of 60% would probably be stable, the yield would be too small to be acceptable. Up until 1972, the length-to-diameter ratio of the liquid payload cavity was 4.9 and the projectile boattail was 0.5 caliber long. Experience with the M483 instability at transonic launch under high air density conditions (a Magnus instability) led to a reduction in boattail length to 0.25 caliber as a fix for the M483. This fix was also incorporated into the M687. Theoretical predictions based on Stewartson theory and gyroscope experiments showed that the 4.9 payload cavity ratio should not produce any liquid instabilities. Rounds with this payload geometry, however, did go unstable and personnel of the Chemical Research Laboratory developed a semi-empirical fix by reducing the cavity ratio to 4.43. In this configuration the M687 performed well under the most stringent conditions of critical Mach number and large induced yaw.

The semi-empirical fix to the M687 was theoretically unsatisfying and the problem of resonance during spin-up remained unaddressed until 1974 when the special capabilities of the BRL were concentrated in a team led by Sedney to address, in particular, the spin-up problem. Of importance to the development of theory was a measure of spin-up time. A great deal of data existed from yawsonde tests with the M687 and it was felt that the spin data could be treated from angular momentum considerations to provide estimates of spin-up times under a variety of firing conditions. The spin data from about five years of test firings were examined and spin-up time estimates derived. The results were correlated with Reynolds number where possible. The study shows that liquid spin-up times can vary from about 1 to 18 seconds for the M687 depending on the type of liquid payload (i.e. the Reynolds number). A method for dealing with the liquid-filled shell spin data has been developed and nominal spin-up times can be predicted.

II. EXPERIMENTAL PROCEDURES

A. The M687 Projectile

The external configuration of the 155mm, M687 binary projectile is almost identical to the M483. Internal modifications to the M483, in order to enable it to carry a chemical payload, result in the M687. Both projectiles are approximately 6 calibers long with a 3 caliber tangent-ogive and a quarter caliber boattail as depicted in Figure 3.

Internally, the M687 consists of two polyethylene-lined, hermetically sealed, tandemly oriented steel canisters. The forward canister contains the denser of two liquids. Two thin, adjacent diaphragms separate the chemicals and a toxic agent forms only after the diaphragms rupture as a result of launch acceleration. All the rounds documented in this report used either a nontoxic simulant or water. The simulant was chosen to duplicate some of the gross properties of the toxic agent such as density and viscosity at a single temperature and pressure. The water-filled rounds provided information on the liquid behavior or the liquid-container interaction. Furthermore, on some rounds, the canister was designed to be a smooth, unobstructed cavity in order to better delimit the flow properties. In these the diaphragms were omitted. A complete list of the internal configurations fired and all pertinent physical properties are listed in Table I.

B. Test Sites

1. The NASA Wallops Island Facility. The NASA Wallops Island launch facility was the site of a number of firings in May 1972 and May 1975. The launcher at Wallops consisted of an M126 cannon mounted on a sleigh. Propellant and rounds were conditioned to 70°F.

Tracking was accomplished with FPS-16 and MPS-19 radars providing both AGC records and x, y, z position versus time.

The NASA Wallops Telemetry Station consists of a multi-receiver network fed from a 28 db gain parabolic reflector antenna. This facility records received telemetry signals on tape and has the necessary equipment for immediate playback and analysis.

Velocimeters were not available for the May 1972 test series. The subsequent series in May 1975 did use a NERA doppler radar chronograph for muzzle velocity measurements. Since this instrument measures the velocity over a 2 metre interval at some distance in front of the muzzle, a slight correction had to be made to obtain the muzzle velocity. This amounted to adding 1.5 m/s to the indicated value for the 45° QE shots and 1.2 m/s for the 30° QE shots. None of the Wallops rounds were induced to yaw.

2. The Nicolet Facility. The 1975 and 1976 Winter Tests of the M687 and various other projectiles were accomplished at the Proof and Experimental Test Establishment (P.E.T.E.) at Nicolet, Quebec, Canada. This site was chosen because it was conveniently accessible and statistically favorable for cold weather high air density testing. The cold weather testing was necessary because of past misbehavior of the M483 family of shell at critical Mach number. This Mach number corresponds to the largest possible overturning moment for this shell and is most easily attained experimentally in a high density atmosphere.

TABLE I. PHYSICAL PROPERTIES OF XM687 AND FILLER

Round ID	I_a (Kg-m^2)	V_o (m/s)	QE (deg)	Liq Type	Fill %	Kin Visc. (m^2/s)	Canister Config.	Launch Yaw	α_{max} (deg)	Spin-Up Time (s)	$\sqrt{Re}$
E1-5977	.1601	310.0	28	water	90	1×10^{-6}	single	normal	3.0	17	1320
10G1	.1614	285.7	65	F113/ETH A	87	2.1×10^{-6}	dual	induced	6.8	3	948
10G2	.1614	281.3	65	F113/ETH A	87	2.1×10^{-6}	dual	induced	10.5	2.5	948
10G3	.1626	280.7	65	F113/ETH A	87	2.1×10^{-6}	dual	induced	10.5	2.5	948
10G4	.1626	313.2	30	F113/ETH A	87	2.1×10^{-6}	dual	induced	8.3	1.5	948
10G5	.1639	314.3	30	F113/ETH A	87	2.1×10^{-6}	dual	induced	8.8	2	948
E1-7670	.1621	284.7	30	Si oil	100	$.5\times10^{-3}$	single	normal	2.3	1.3	58
E1-7671	.1680	294.7	45		(SLUG)		single	normal	-	-	-
E1-7672	.1629	293.3	45	water	90	1×10^{-6}	single	normal	N.O.*	15	1320
E1-7673	.1630	292.9	45	Si oil	90	$.5\times10^{-3}$	single	normal	3.1	1.3	58
E1-7674	.1684	294.1	30		(SLUG)		single	normal	-	-	-
E1-7675	.1629	289.8	30	water	100	1×10^{-6}	single	normal	2.5	15	1320
E1-7676	.1631	291.8	30	water	90	1×10^{-6}	single	normal	4	15	1320
E1-7677	.1622	293.4	30	Si oil	90	$.5\times10^{-3}$	single	normal	1.0	1.3	58
E1-7450	.1621	292.8	30	Si oil	100	$.5\times10^{-3}$	single	normal	N.O.*	N.O.*	58

**N.O. - not observed, no data near muzzle or single pulse-train yawsonde only.*

NOTE: 1) E1 series were fired at Wallops Island, Virginia. All others at Nicolet, Quebec, Canada.

2) Canister aspect ratio (length to diameter) for all rounds was 4.43.

TABLE I. PHYSICAL PROPERTIES OF XM687 AND FILLER (Continued)

Round ID	I_a ($Kg\text{-}m^2$)	V_o (m/s)	QE (deg)	Liq Type	Fill %	Kin Visc. (m^2/s)	Canister Config.	Launch Yaw	α_{max} (deg)	Spin-Up Time (s)	$\sqrt{Re}$
3A1	.1605	295.4	30	F113/ETH A	87	2.1×10^{-6}	dual	normal	2	3.1	948
3B1	.1605	300.2	30	F113/ETH A	87	2.1×10^{-6}	dual	induced	10.5	2	948
3B2	.1605	294.3	30	F113/ETH A	87	2.1×10^{-6}	dual	induced	14.5	3	948
3B3	.1605	292.5	30	F113/ETH A	87	2.1×10^{-6}	dual	induced	12	3.4	948
3B4	.1605	292.5	30	F113/ETH A	87	2.1×10^{-6}	dual	induced	11	≈ 4	948
3B5	.1605	293.2	30	F113/ETH A	87	2.1×10^{-6}	dual	induced	11.5	≈10	948
3C1	.1605	300.6	30	F113/ETH A	87	2.1×10^{-6}	dual	induced	12	5	948
4A1	.1605	286.2	30	F113/ETH A	87	2.1×10^{-6}	dual	normal	5	2.7	948
4B1	.1605	287.3	30	F113/ETH A	87	2.1×10^{-6}	dual	normal	N.O.*	< 6	948

**N.O. - not observed, no data near muzzle or single pulse-train yawsonde only.*

NOTE: 1) E1 series were fired at Wallops Island, Virginia. All others at Nicolet, Quebec, Canada.

2) Canister aspect ratio (length to diameter) for all rounds was 4.43.

The launchers included two M109A1 self-propelled howitzers and an XM198 towed howitzer. Projectile tracking was accomplished with a Hawk doppler radar and an MPS-25 radar located a little more than 1 km behind the gun. The MPS-25 is the mobile version of the FPS-16 radar used at the Wallops Island facility. The purpose for using the Hawk system was to provide velocity histories. By on-site improvement of the system it was also possible to obtain projectile positional data. The MPS-25 provided positional data for almost all of the shells fired with a digital tape and plot board output.

BRL supplied its own telemetry van which consisted of a self-contained, portable receiving station complete from receiving antennas to tape recorder to playback systems. The telemetry van also had the capability of on-site data reduction.[17] Some of the rounds which flew poorly were analyzed immediately for yawing motion and spin history. Adjustments in yaw tip-off could thus be made.

Several radar chronograph velocimeters including the NERA DR890 and the GE MV201 were used during both Winter tests for redundancy in muzzle velocity measurement. Two types of yaw inducers were used in the Nicolet tests. They were designed to give a progressively larger kick to the projectile if rounds from a particular test series were found to exit the tube with consistently small yaw. The above devices are discussed more fully in References 18 and 19.

C. Flight Instrumentation

For a complete description and function of the BRL yawsonde, it is advisable to consult References 20 through 22. A brief account is given here for the sake of completeness. The BRL yawsonde consists of two

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17. *W.H. Clay, W.H. Mermagen, "The Portable Yaw Processor," Ballistic Research Laboratory Memorandum Report No. 2785, September 1977.* (AD #B022104L)
18. *V. Oskay and J.H. Whiteside, "Flight Behavior of 155mm (XM687 Mod I and XM687 Mod II) and 8-Inch (XM736 Mod I) Binary Shell at Nicolet, Canada, During the Winter of 1974-1975," Ballistic Research Laboratories Memorandum Report No. 2608, March 1976, AD B010566L.*
19. *W. Mermagen, W. Clay and V. Oskay, "Yawsonde Data From Firings in the Nicolet Winter Test Program 1974-1975," Ballistic Research Laboratories Memorandum Report No. 2612, April 1976. AD B011225L.*
20. *W.H. Mermagen, "Projectile High-g Telemetry for Long Range Dynamic Measurements," ISA, International Telemetering Conference Proceedings, Vol. III, 1971.*
21. *W.H. Mermagen, "Measurements of the Dynamical Behavior of Projectiles Over Long Flight Paths," Journal of Spacecraft and Rockets, Vol. 8, No. 4, April 1971, pp. 310-385.*
22. *W.H. Clay, "A Precision Yawsonde Calibration Technique," Ballistic Research Laboratories Memorandum Report No. 2263, January 1973, AD 758158.*

silicon solar cells mounted behind narrow slits such that their fields of view are essentially planar (see Figure 4). Since the slits are at an angle to each other, these planes intersect in space forming a "V". As the projectile rolls and yaws the sun's rays intercept different portions of the "V" with each rotation. The phase relationship between the pulses produced yields the solar aspect angle as a function of time through a simple geometrical relation. This data transmission link is depicted graphically in Figure 4. The solar aspect angle is defined as the angle between the solar vector and the projectile longitudinal axis. It is evident that pulses from one sensor only provide spin as a function of time. The reduction techniques become invalid when the roll period is of the same order of magnitude as the yawing period.

The output of the solar sensors is transmitted via FM/FM telemetry. The data pulses are fed through a voltage controlled oscillator, VCO, centered at a high enough frequency to insure adequate frequency response. The VCO in turn modulates a radio-frequency oscillator which develops approximately 100 mW at 250 MHz into a 50Ω load. The nose of the projectile serves as the transmitting antenna. The resulting nearly spherical radiation pattern insures signal reception over large changes in the curvature of the flight path.

III. TEST PROGRAMS

A. Wallops, May 1972

Five M687 projectiles were fired on this occasion at charge zone 4. The purpose of these tests was to make inflight measurements of projectile undamping and to verify a fix by changing the container dimensions and fill ratio. The cylindrical cavity for these projectiles consisted of the M687 dual canister configuration, 0.529 m long and 0.107 m in diameter (c/a=4.90). The rounds were conditioned to approximately 22°C. Three projectiles were filled to 80% of their volume with water to insure undamping as suggested by Figure 2. Although the launch was without yaw induction, the projectiles were unstable early in the flight. As a result of these confirming unstable flights, the cavity aspect ratio was changed to 4.43 in an effort to remove the primary eigenfrequency mode as far from the nutational frequency as practical payload capacity would permit. This was accomplished by shortening the canister. The remaining two projectiles were filled to 90% of their volume with water. Both exhibited stable flights under normal launch reaching a maximum yaw $\alpha_{max} = 2.5°$ within a few seconds out of the tube and maintaining that motion to impact. Since these flights were some of the earliest to use the yawsonde as a diagnostic tool, they do not contain all the refinements of present day yawsonde programs.

Telemetry data were acquired late by today's standard, two to four seconds after launch. This becomes critical when trying to infer angular momentum from the projectile spin history. The important portion of the projectile spin data, 0 to 2 seconds, where most of the momentum exchange takes place, was lost. Since this portion of the data is not predictable, the early portion of the flight cannot be reconstructed with any meaning.

Muzzle velocity was not specifically measured for these rounds. At that time, no doppler radar velocimeters were available. Muzzle velocity was therefore obtained by matching the FPS-16 radar data with a generated trajectory for most of the flight and extrapolating the trajectory to the muzzle. The muzzle velocity, then, was the velocity needed for a best fit to the trajectory. Unfortunately, the FPS-16 radar malfunctioned on all but the last round (E1-5977). Therefore, this is the only round for which any spin-up calculations can be made. The nominal Reynolds number for this liquid payload was 1.7×10^6.

B. Nicolet, February 1975

As a result of the anomalous transonic free flight behavior of the M483 family of shell at Nicolet, Quebec, Canada, during the winter of 1973-1974 a second test was scheduled for the winter of 1974-1975. Since the M687 belongs to the M483 shell family, it was scheduled as a matter of course. In an effort to maintain a ballistic match between the M483 and the M687, the M687 design was to include all the physical changes required by the effort to stabilize the M483 at critical Mach number. This involved shortening the boattail to 0.25 caliber. All the rounds launched with this shortened boattail flew well even though they were yaw induced with first maxima ranging from 6.8° to 10.5°. Three rounds designated 10G1 through 10G3 were fired at a quadrant elevation of 65° while 10G4 and 10G5 were fired at 30°.

The liquid contained in the canisters consisted of a 30% Freon 113/70% ethyl alcohol solution. In all cases the void was 10% at about 25°C and increased to 13% at 0°C because of payload shrinkage. Test conditions at Nicolet were nominally -16°C and no special precaution was taken to temperature condition the rounds. We may therefore safely assume the void to be slightly larger than 13% but no data are available below 0°C. At the request of the author the Binary Systems Office of Edgewood Arsenal performed kinematic viscosity measurements on the solution at low temperature. Results indicate that at -13.2°C the kinematic viscosity of the Freon 113/Ethyl alcohol solution is 2.1×10^{-6} m^2/s. With these liquid physicals and a nominal spin rate of 100 rps the liquid Reynolds numbers are approximately 0.9×10^6, which suggests a turbulent endwall boundary layer condition within the canister.

C. Wallops, May 1975

Eleven M687 yawsonde instrumented projectiles were fired on this occasion under the direction of the author and W. D'Amico. The overall purpose of this test was to establish the validity of Wedemeyer's[23] spin-up theory and to perform a carefully controlled experiment under both high and low Reynold number conditions. This was accomplished by using Dow Corning 200 silicon oil for the low Reynolds number case and water for the high Reynolds number.

Two of the rounds had slug payloads in an effort to establish two parameters, an aerodynamic spin damping coefficient and an effective tube twist (by projecting the spin back to the muzzle). Prior to this time, an average M483 spin damping coefficient based on a limited number of range firings was used for the M687 calculations and the tube twist was always taken to be the nominal value that applied to that particular tube. The results of these two dry rounds verified that C_{ℓ_p} exhibited a linear Mach number variation over the range $.50<M<.85$ and that the effective tube twist could have been 0.5% smaller than the nominal value (19.9 instead of 20.0). The latter plays a relatively minor role. The spin damping coefficient data are important as we shall see later. The slug tests supplemented our understanding and improved the angular momentum calculations.

The laminar end wall test condition was achieved with 500 cs oil and resulted in a Reynolds number of approximately 3400 at a nominal 100 rps spin rate. The turbulent endwall condition produced a Reynolds number of 1.7×10^6. These Wallops series were fired at both 30° and 45° quadrant elevation in order to evaluate an elevation effect on spin-up time. None was exhibited very clearly.

D. Nicolet, January 1976

Many different types of projectiles were fired during the winter test at Nicolet in 1976. Because of the previous configuration changes the M687 was redesignated as the XM687E1. It was tested in this series because of some anomalous spin behavior experienced in the previous Nicolet test[18]. Also, a test conducted with this round in 1975 at Yuma Proving Ground[24] suggested a possible misbehavior at Mach 2. A number of rounds were therefore, fired at this velocity. These rounds are not yet reducible for angular momentum because the spin damping coefficient

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23. *E.H. Wedemeyer, "The Unsteady Flow Within a Spinning Cylinder," Ballistic Research Laboratories Report 1225, October 1965, AD 431846.*

24. *Yuma Proving Ground Preliminary Firing Record data subject to confirmation. "Restructured Development Test II of the 155mm, HE, M483E1 Projectile in Howitzer, Self-Propelled, M109A1," TECOM Project No. 2-MU-003-483-030, Project Engineer: Dick Godly, 1976.*

has not been determined at this Mach number. A test will be forthcoming in the near future where C_{ℓ_p} will be determined as in the Wallops firings. The fill ratios, physicals, and weather conditions were almost identical to the previous Nicolet test.

IV. THEORETICAL CONSIDERATIONS

When launched from a cannon, projectiles which contain a liquid payload exhibit greatly different spin histories compared to those which carry a solid payload rigidly attached to the projectile. The most pronounced difference occurs near the muzzle where the liquid-casing interaction is the strongest. Here the liquid acts to supplement the aerodynamic torque to despin the projectile. Later in the flight, the liquid torque becomes negligible and aerodynamic torque predominates. Ideally, given a casing spin history, one would like to be able to calculate the liquid velocity profile at any instant in time. The velocity profiles are a necessary input for predicting unsteady liquid eigenfrequencies. Recall from the Introduction that the motivation for the study of liquid-filled projectiles was their erratic behavior caused by a liquid projectile interaction. This interaction resulted in projectile yaw instability as was evidenced in Figures 1 and 2.

Since the understanding of the stability problem involves several complicated steps, i.e., calculating or measuring the velocity profiles and then calculating the unsteady eigenfrequencies, it was thought that an appropriate measurement would be useful. Measuring the velocity profiles in flight or in the laboratory is impractical since it would require highly specialized techniques and instrumentation. We may, however, measure and infer something about the gross state of the liquid, such as its angular momentum. Then we can correlate the data, draw some conclusions and make predictions based on certain liquid flow parameters.

In free flight, the quantity that lends itself readily to measurement is projectile spin. By measuring the spin decrease of a projectile containing a liquid, one can computationally infer the liquid angular momentum history in the following way. Let us assume that the projectile yaw is small and that all the liquid rotation takes place about an axis which is coincident with the projectile longitudinal (spin) axis. If we then consider the moment of momentum equation about this axis we may write (refer to Figure 5):

$$M_{aero} + M_{liq} = I_a \dot{p} \tag{1}$$

where the left hand side represents the sum of the aerodynamic and liquid moments acting on the casing and the right hand side is the rate of change of angular momentum of the casing. I_a is the axial moment of inertia of the casing and p is the casing spin. One can then write the aerodynamic moment in its usual form

$$M_{aero} = 1/2\ \rho\ V^2\ S\ell\ (C_{\ell_p}\ \frac{p\ell}{V}) , \tag{2}$$

substitute Equation (2) into Equation (1) and integrate to obtain

$$L_{liq} = I_a(p_0 - p) + \frac{S\ell^2}{2} \int_o^t (C_{\ell_p}\ V\ \rho\ p)\ d\tau + L_{liq_0} . \tag{3}$$

In the above equation L_{liq} is the instantaneous angular momentum of the liquid, S and ℓ are projectile area and diameter respectively, C_{ℓ_p} is the aerodynamic spin damping coefficient, V is the projectile velocity, ρ is the air density, and the subscript 0 refers to muzzle conditions. L_{liq_0} is a constant of integration which corresponds to the angular momentum that the liquid has gained while being spun up in the tube. t=0 is taken to be the time when the base of the projectile clears the tube. Calculations by Kitchens and Gerber[4] have shown L_{liq_0} to be 19% of maximum rigid body liquid angular momentum ($L_{liq_{max}}$) for Reynolds numbers of approximately 3.4 X 10^3 whereas L_{liq_0} is only 2.5% for Reynolds numbers near 1.7 X 10^6. These Reynolds numbers correspond to laminar and turbulent endwall boundary layer conditions, respectively. $L_{liq_{max}}$ is the rigid body liquid angular momentum at maximum spin. Maximum spin occurs at muzzle exit for conventional projectiles. The above values are nominal for a charge zone 4 launch of a 155mm binary liquid-filled projectile. This means that to infer L_{liq} from any flight requires the knowledge of spin-up in the tube. This is a calculated result that needs to be introduced into an otherwise totally experimental program. It can be avoided by simply using the rate of change of liquid angular momentum, $\dot{L}_{liq}$, as opposed to the absolute value L_{liq}. To do this we write

$$\dot{L}_{liq} = I_a\ \dot{p} + 1/2\ \rho\ \ V\ p\ C_{\ell_p}\ S\ell^2 . \tag{4}$$

The physical interpretation of this equation is simple. Positive or negative values of $\dot{L}_{liq}$ means the liquid is gaining or losing angular momentum. This implies that the time until $\dot{L}_{liq} = 0$ is in a gross sense a measure of liquid spin-up time. To be consistent with Reference 4, however, we will define the spin-up time to be the interval when the liquid has reached 99% of its peak value. Whether the liquid takes a long or short time to spin-up is important because it is a measure of how long the transient eigenfrequencies can persist. In the limit, the problem becomes a Stewartson-type problem when the liquid is fully spun up. Instability in this case is predictable.
In order to be able to use equation (4), we must evaluate each term from the various data sources. The air density, ρ, is obtained from a meteorological sonde and is input in tabular form as a function of altitude. It is then interpolated where necessary by a divided difference scheme. The velocity function, V, is obtained from smoothed position-time data of a precision radar or from a trajectory computation with matched range, muzzle velocity and several positioned points along the trajectory. The aerodynamic damping coefficient, C_{ℓ_p}, is obtained by firing rounds of the same physical characteristics without the liquid. This coefficient is input as a function of Mach number.
By far the greatest difficulty is encountered with the spin reduction. The reciprocal of the time between successive discrete pulses from a single yawsonde sensor is the spin of the projectile. More correctly, the yawsonde pulses provide the roll rate, $\dot{\phi}$, of the projectile in a solar-fixed-plane coordinate system, as described by Murphy[25]. If the projectile is undergoing pitching and yawing motion, the effects of such motion are seen on the $\dot{\phi}$ data as small amplitude oscillations whose frequencies correspond to the fast and slow frequencies of motion of the shell. The average of these oscillations is a good representation of the spin of the projectile in an earth-fixed coordinate system[25]. The magnitude of the oscillations is a function of the magnitude of yaw of the shell. For all our cases, these oscillations in the $\dot{\phi}$ data amounted to a few percent of the nominal spin values. Despite these small amplitude fluctuations in $\dot{\phi}$, it was difficult to use the raw data in equation (3). The computed angular momentum, L_{liq}, showed excessively large amplitude variations when using the raw data. The results became meaningless when the derivative was substituted in equation (4). It became important, then to take an average of the $\dot{\phi}$ data using fitting procedures. The first and most straightforward fitting approach appears to work the best.

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25. *C.H. Murphy, "Effect of Large High-Frequency Angular Motion of a Shell on the Analysis of its Yawsonde Records," Ballistic Research Laboratories Memorandum Report 2581, February 1976. AD B009421L.*

The method consists of piecewise data fitting with a cubic polynomial in the least square sense. An arbitrary number of data points are fit but only the middle third of the fit is retained in the spin array. The number of points selected to be fit depends on the goodness of fit obtained after an initial trial. A new portion of data shifted by one-third the number of data points, is fit and again the middle third is retained. This procedure continues until all the spin data are fit. By retaining only the middle portion of the fitted spin data, a degree of smoothness is obtained such that subsequent differentiation does not produce large amplitude excursions in the derivative.

A second method, that of using cubic splines, was tried in order to be totally rid of any discontinuity in the spin derivative. The method consisted of fitting a section of data in the least square sense and forcing the derivatives at the end points from adjoining sections to match. This method works well in principle and indeed continuous derivatives are obtained at the junctions but in order to satisfy this condition each sectional fit oscillated wildly. It therefore, yielded results which were unacceptable when used in equations (3) or (4).

V. ANALYSIS OF RESULTS

A. Wallops, May 1972

It should be pointed out that with free flight projectiles the question of spin-up time is not answered in a straightforward manner. It happens that, since projectile casing spin is never a constant, the process of angular momentum interchange occurs continually. If casing spin were to reach a constant value, as is possible in the laboratory, spin-up time could be defined more rigorously since the limiting velocity profile would be a rigid body profile. This behavior can be simulated from free flight data by normalizing equation (3) by the rigid body angular momentum that the liquid would have at the instantaneous spin. In effect, it amounts to normalization by a variable which varies as the spin.

This type of analysis was used in calculating L_{liq} for Round E1-5977,[16,26] an M687 projectile. In the data shown in Figure 1 of Reference 26, which is a plot of L_{liq} as a function of time, the liquid does not appear to spin up over its entire flight time. The integration method in Reference 26 used a constant C_{ℓ_p} (= -.0135) to calculate the

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26. A. Mark, *"Transient Eigenfrequencies in Liquid-Filled Cylinders," AIAA Journal, Vol. 13, No. 2, 1975, pp 217-219.*

aerodynamic spin decay contribution. This value was the mean of a number of M483 aeroballistic range firings. The M483 has an external shape almost identical to the M687. In subsequent free flight yawsonde instrumented firings of the M687, several shell were set aside to be fired without the liquid filler. From these shell we were able to obtain a much better estimate of C_{ℓ_p}. The current relationship therefore, is the following:

$$C_{\ell_p} = -0.021 + 0.00972(M) \quad \text{for } .50<M<.85 \qquad (5)$$

In the above relationship M is the Mach number. As it turns out, $C_{\ell_p} = -.0135$ is almost the average value in the above linear relationship. The constant C_{ℓ_p} overestimates the aerodynamic moment by 10% at the beginning of the flight and underestimates it by 15% at the end. Since this moment is subtracted from the casing angular momentum to obtain L_{liq} (Eq. 3) the calculation becomes sensitive because it involves subtraction of two large numbers of similar value. When using Equation (5) to recalculate L_{liq}, or more appropriately $\dot{L}_{liq}$, a spin-up time of 17 seconds is obtained as compared to the failure to spin-up over the entire flight shown in Reference 26. This falls in line with other rounds of the same Reynolds number. Plots of the spin history and angular momentum exchange for E1-5977 are given in Figures 6 and 7. Because of late telemetry reception, the effect of the liquid is not evident in the spin record of Figure 6. The extreme smoothness of the curve also reflects the fact that the yaw amplitude was very small (≈2.5°).

B. Nicolet, February 1975

The same type of analysis (equation 4) was used on rounds 10G1 through 10G5. All five rounds were 87% filled M687 projectiles with dual canisters. The fill solution consisted of 30% Freon 113 and 70% Ethyl alcohol. Rounds 10G1 through 10G3 were fired at an elevation of 65 degrees whereas rounds 10G4 and 10G5 were fired at 30 degrees. Yaw was induced from 6.8° to 10.5° (α_{max}) on these rounds. At this point it is difficult to say what effect the yaw induction had on spin-up time. The angular momentum exchange is essentially completed between 1.5 and 3 seconds after emerging from the tube. Figures 8 and 9 depict a typical 65° (1156 mils) elevation result whereas Figures 10 and 11 depict a 30° QE (533 mils) result.

The line that is drawn through the oscillations found on the spin histories is a cubic, piecewise, least squares fit. The topmost curve in Figure 9 is the rate of change of angular momentum of the liquid and the lower curve is the rate of change of angular momentum of the casing.

There seems to be a slight discrepancy at the muzzle between the spin computed from radar velocimeter data and the initial yawsonde spin measurement. When computing the intial spin, p_o, from chronograph measured muzzle velocity and twist of the tube, we obtain a value slightly lower than the initial yawsonde record suggests. The values are lower by only about 1% but they strongly influence the calculations of rate of change of angular momentum. This is especially evident in Figure 11 where $\dot{L}_{liq}$ seems to increase for the first second and then reverses direction. The other rounds of this series are affected less strongly. The discrepancy may be due to velocimeter error or uncertainty as to the twist of the tube.

C. Wallops, May 1975

The May 1975 series of M687 rounds consisted of a set of projectiles where special care had been taken to obtain accurate measurements of the fill ratios, the masses, the axial moments of inertia, and the fluid mechanical properties of the liquid fill. The results of the spin-up measurements can, therefore, be more confidently correlated with such fluid parameters as Reynolds number. All the rounds were single canisters with an aspect ratio of 4.43. The canisters consisted of the usual polyethylene-lined-steel configuration. The additional precaution of fastening the polyethylene liner to the steel wall was accomplished by pinning the two with roll pins to preclude the possibility of independent spinning of the polyethylene liner. Pinning was not done in the service rounds nor in any of the other tests described in this report. The steel canisters were further keyed to the projectile body so that all metal and polyethylene parts would spin as a rigid body. The canisters were filled with either Dow Corning 200 silicon oil or water. The oil, with a kinematic viscosity of 500 cs, was designed to produce Reynolds numbers well in the laminar range whereas the water would produce a turbulent end wall boundary layer. The canisters were filled to 90% of their volume by removing 10% of the fully filled payload. The 100% canisters were filled by spinning them and filling through a centrally located port. In this way it was assured that no voids were left.

The spin-up times for the 100% and 90% oil-filled rounds were essentially the same, approximately 1.3 seconds. Sample spin and angular momenta exchange graphs of these fill ratios are presented in Figures 12 through 15. The spin-up time of 1.3 seconds is difficult to read from the graphs and comes rather from the computer tabulation for the graphs. This result agrees in general with spin-up calculations made by Kitchens and Gerber[4] for these particular rounds. Spin-up with water takes considerably longer. Here the times range about 15 seconds (to the nearest half second). Sample rounds E1-7675 and E1-7676 are given in Figures 16 through 19 respectively for the 100% and 90% filled cases. Rounds E1-7675 was the only 100% filled water round that was fired and therefore a cleaner spin history is not available. The straight line portion at the beginning of Figure 16 is a reconstruction with the initial point at t=0 computed from twist and muzzle velocity.

The NERA radar chronograph gave initial spins which consistently agreed with initial yawsonde spin. The two dry rounds, El-7671 and El-7674, were used to calculate the effective tube twist which turned out to be 19.89 and 19.91 respectively. An average of 19.90 was subsequently used for all calculations. Since the NERA measures the velocity at a fixed location in front of the muzzle, a drag correction of 1.5 m/s for the 45° shots and 1.2 m/s for the 30° shots was added to the NERA velocity and the result was termed V_o. The only slight complication was a late telemetry turn on for rounds El-7675 and El-7450. We also apparently lost information from one of the solar sensors on rounds El-7672 and El-7450 so that no yaw information was obtained.

D. Nicolet, January 1976

This series of Nicolet tests essentially repeated the previous year's firings. All the rounds contained the usual dual canister Freon/ethyl alcohol mixture filled to 87%. In general, spin-up times were comparable to the previous years Nicolet firings (2 to 3 sec). The discussion of section B applies here and need not be repeated. Only one slight variation appeared in the test. Three of the rounds, 3B4, 3B5, and 4B1 had canisters which were not keyed to the projectile body. The chronograph muzzle velocities for these rounds were about 2% lower than those computed from the first spin data points from the yawsondes. The spin records of Figures 20 and 22 for rounds 3A1 and 3B4 respectively are shown as contrasting illustrations of the early spin data. The spin histories of rounds 3B5 and 4B1 looked like 3B4. Because of the data anomaly near the muzzle, the rate of change of angular momentum for round 3B4 in Figure 23 contains the same type of reversal as discussed earlier. This is contrasted with the normal behavior for round 3A1 in Figure 21. Similar plots are obtained for round 3B5 and 4B1.

It is difficult to conceive of a physical mechanism which would produce an increase in the angular momentum of the projectile casing in flight. The transfer of angular momentum from the interior of the projectile outward is highly unlikely and the dissipative forces act to decrease the angular momentum of the system. The results shown in Figure 23 remain an anomaly and may be due to measurement error.

VI. DISCUSSION

This report has attempted to infer how long it takes a liquid contained in a projectile to spin up when the projectile is fired out of a cannon. The inference is made purely from experimental evidence by measuring the spin change of the projectile casing with yawsondes and applying the differentiated form of the moment of momentum equation. The results, which are depicted in the final graph, Figure 24, incorporate all M687 yawsonde instrumented rounds fired over the last five years for which there exist a suitable amount of yawsonde data. It turns out that all the rounds were launched near Mach 0.95 because that is where the problem was most critical. Some data at a Mach 2 launch condition were obtained but could not be reduced since no aerodynamic spin damping is presently available at this Mach number. It is understood that this situation will be remedied in the fall of 1977 and a report on the subject should follow.

The graph in Figure 24 depicts spin-up time, t_{su}, as a function of $\sqrt{Re}$. This functional dependence is chosen because the spin-up time is directly proportional to $\sqrt{Re}$.[27] The Reynolds number is based on the radius and is defined as a^2p/ν where a is the radius, p is the spin of the projectile, and ν is the liquid kinematic viscosity. Three distinct data sets are presented in the figure, at $\sqrt{Re}$ = 58, 948 and 1320.

Consistent spin-up times are achieved in the laminar regime with the silicon oil. Here all of the rounds spin up in approximately 1.3 seconds. This is true for the 100% filled rounds as well as the 90% filled rounds. This spin-up time agrees very well with computations of Kitchens and Gerber[4]. Their solution is based on a finite difference procedure of the viscous spin-up equation. This is important since it relieves the asymptotic Reynolds number restriction. Wedemeyer's[23] inviscid solution of the same problem is valid strictly for $Re \to \infty$. Numerical calculations for the turbulent spin-up case are at this point incomplete.

It is interesting to observe the qualitative spin behavior of some of the rounds in this report. If we focus on the spin history of 10G5, for example, Figure 10, we note a pronounced change in the curvature of this spin at approximately 2 seconds. Observing the yawsonde spin of round 10G2 of Figure 8, on the other hand, does not obviously lead to the same conclusions although it might become more evident on observing the fitted curve. The point is that some spin curves have an obvious "elbow" while others do not, although the launch conditions and Reynolds numbers were very similar. It should be pointed out that, for the rounds where the "elbow" is obvious to the eye, the decrease in projectile spin can be accounted for by an instantaneous spin-up of the liquid. Kitchens, Gerber and Sedney[4] conjecture that the sharp spin decay is the result of a Taylor-Görtler type of vortex instability initiated by the large pitching motion. Following this conjecture allows them to calculate the sudden spin decrease of the rounds in question. The dilemma is that there must be an a priori subjective decision as to whether the "elbow" exists or not. If, for example, an "elbow" is not assumed on round 10G2 and a spin calculation is made by the method of Reference 24,

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27. *H.P. Greenspan, The Theory of Rotating Fluids, Cambridge University Press, 1969.*

a spin-up time of approximately eighteen seconds is predicted whereas a 2.5 second spin-up time is predicted by the method of this report. Other rounds, most notably 3A1, fall into the same category. It is difficult to judge whether this round contains an "elbow". This large difference could possibly be accounted for if a Taylor-Görtler instability were incorporated. The straightforward calculations of this report may thus be used as an objective way to determine whether an elbow exists or not. The rounds which contain the obvious "elbow" were all fired at Nicolet with the Freon 113/ethyl alcohol mixture and are plotted in the middle portion in the graph of Figure 24. A series of tests are planned for the fall of 1977 which will duplicate some of the high Reynolds numbers at Wallops with yaw induction to try to produce an elbow.

From the data presented it appears that nominal spin-up times can be predicted for liquid-filled shells over a range of Reynolds numbers of interest to the Army. It is difficult to draw too many firm conclusions from three data regimes and intermediate data would be welcome. It is planned that in the fall of 1977 experiments would be conducted to ascertain the influence of angle of attack on spin-up with particular emphasis on spin anomalies near the muzzle. Concurrently, high Mach number spin-up experiments will also be conducted.

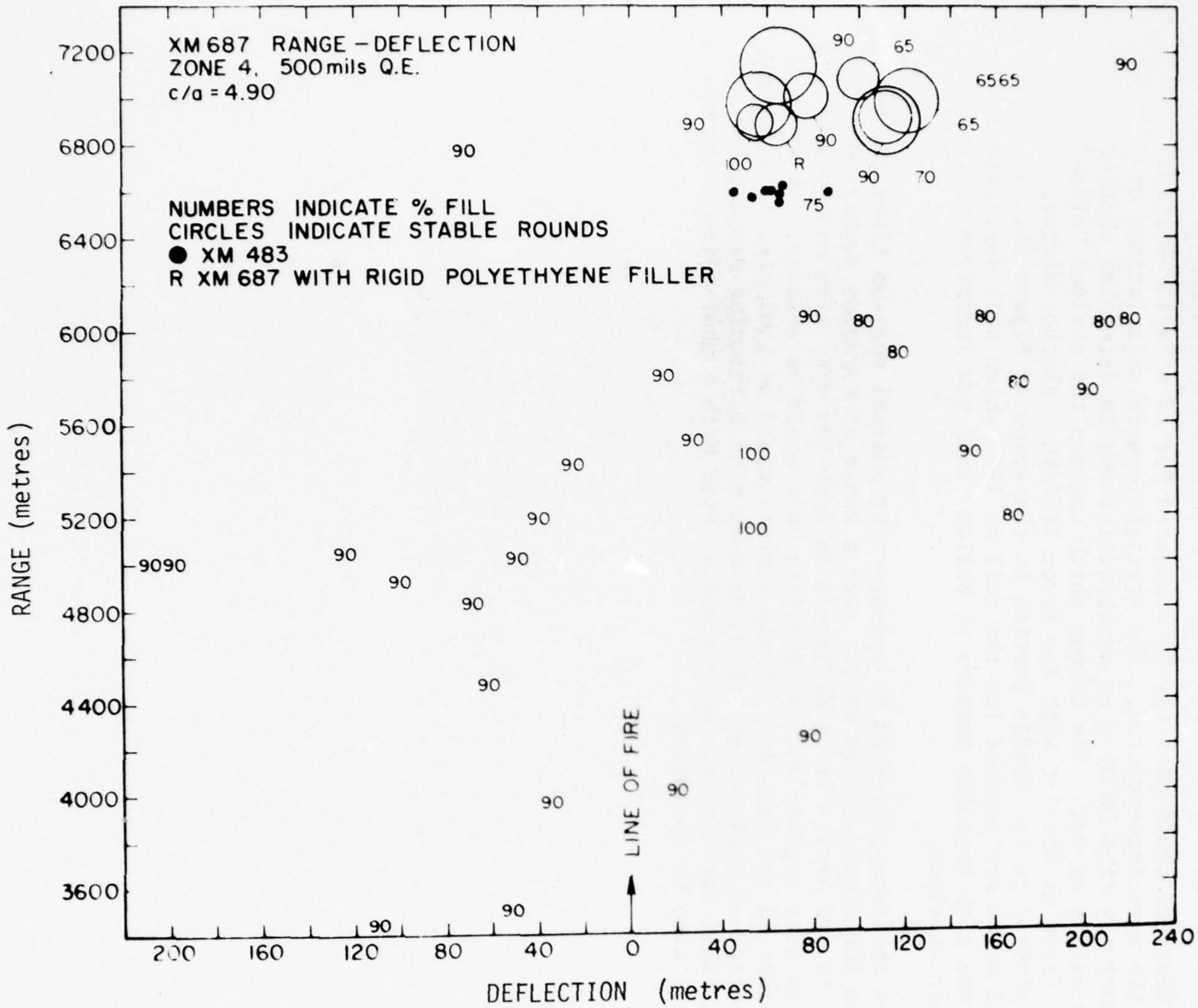

Figure 1. Range-Deflection for the XM687 at Zone 4, 500 mils Q.E.

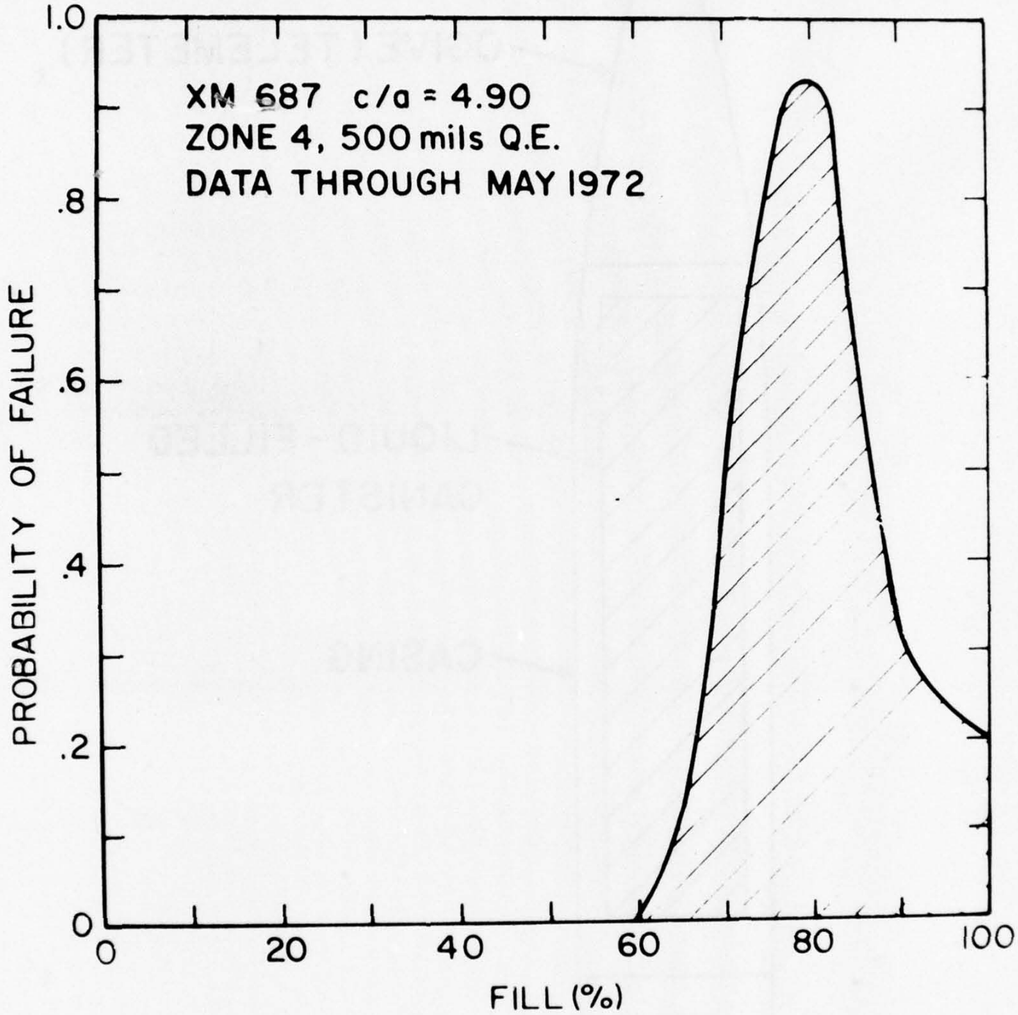

Figure 2. Probability of Instability of the XM687

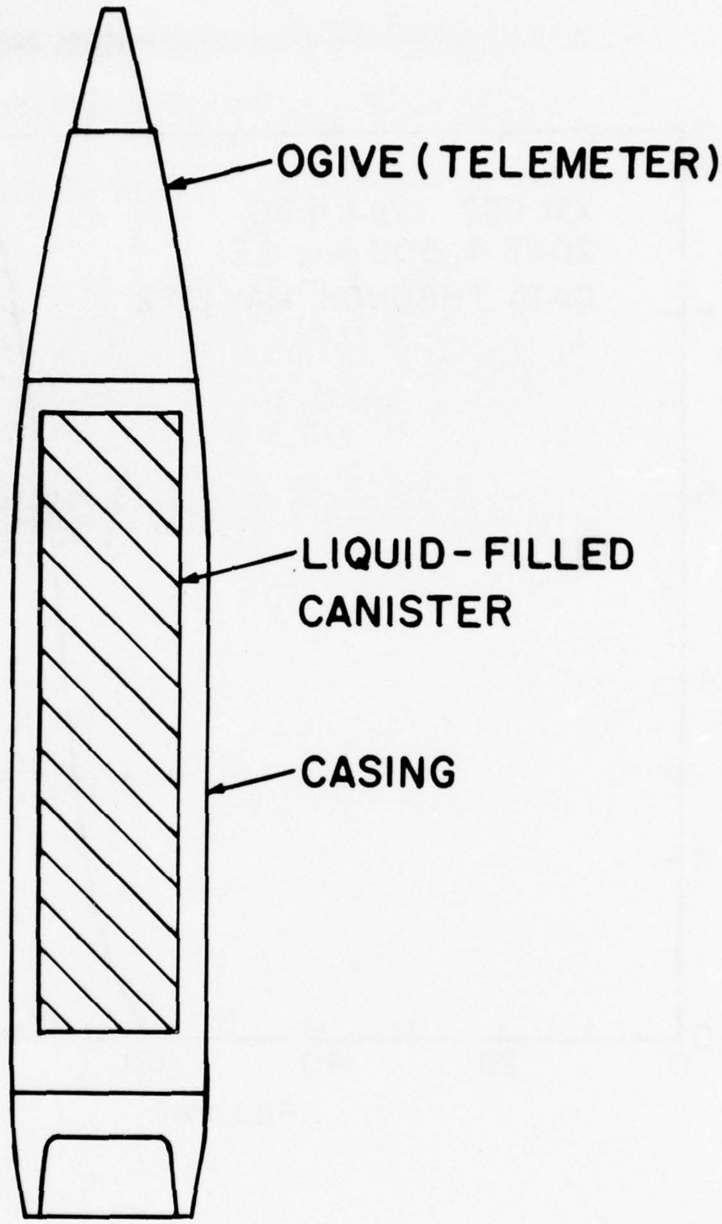

Figure 3. The XM687 Binary GB Projectile

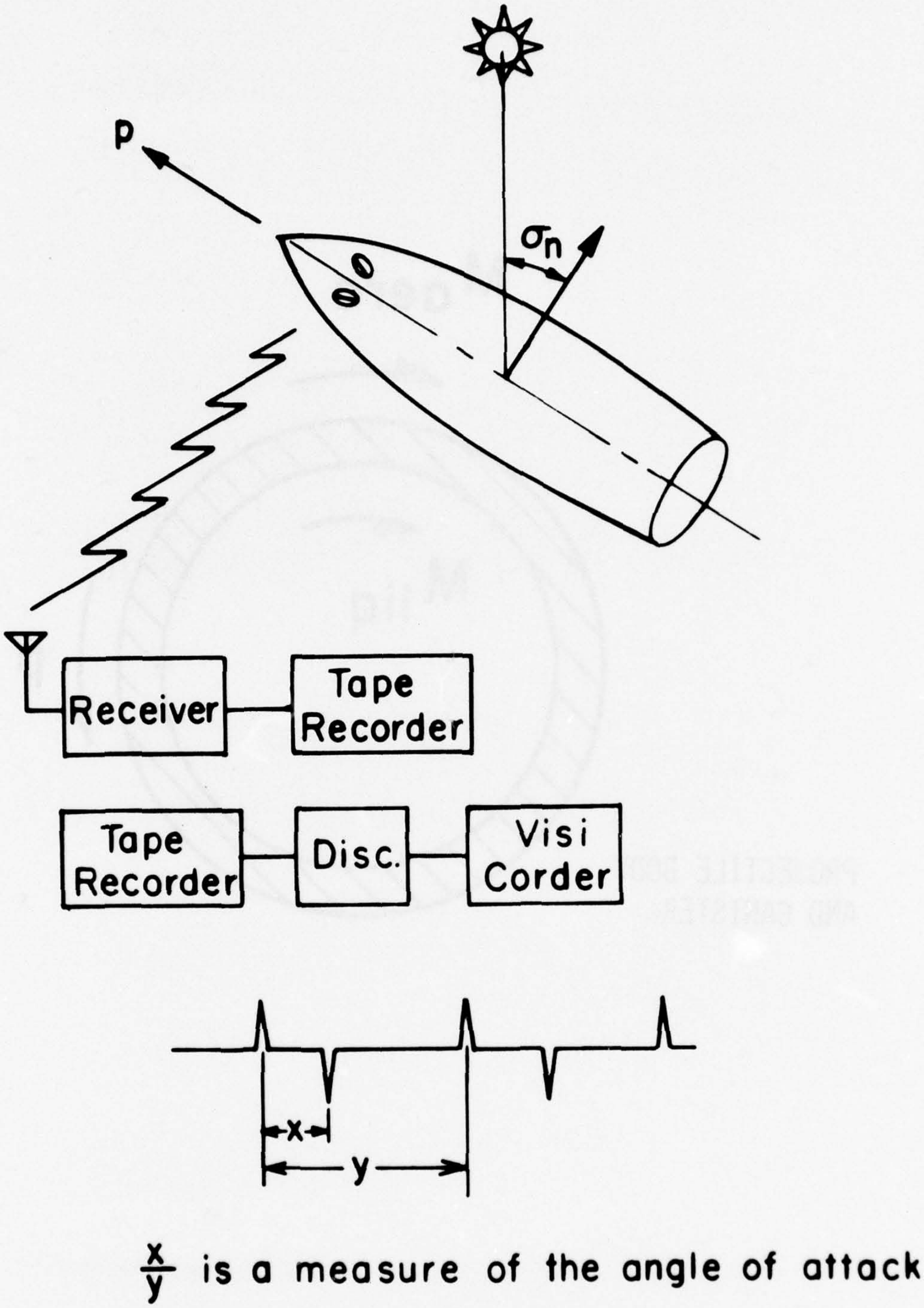

Figure 4. Yawsonde Data Transmission Link

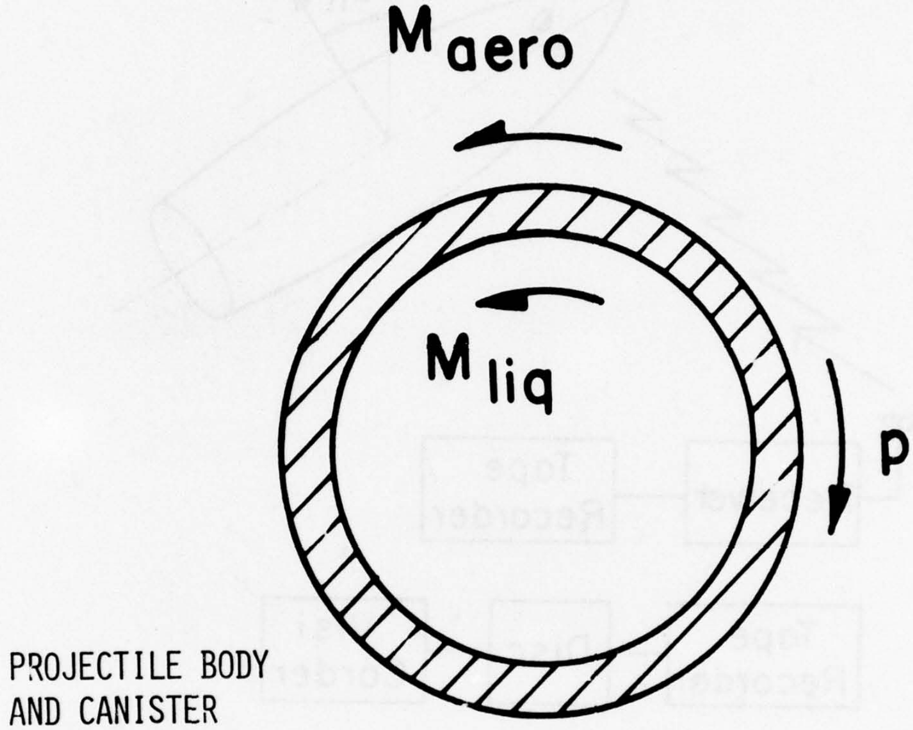

Figure 5. Torques Acting on a Liquid-Filled Projectile

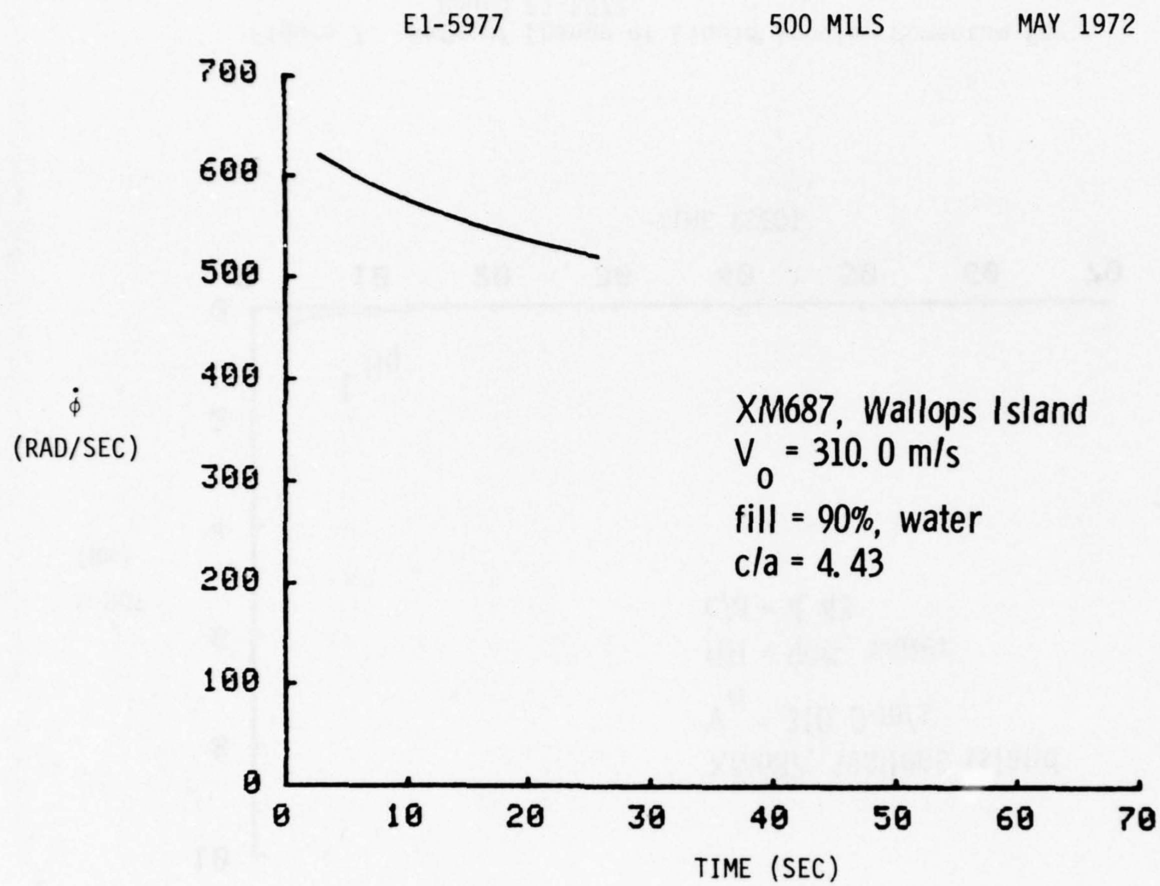

Figure 6. Spin History of Round E1-5977

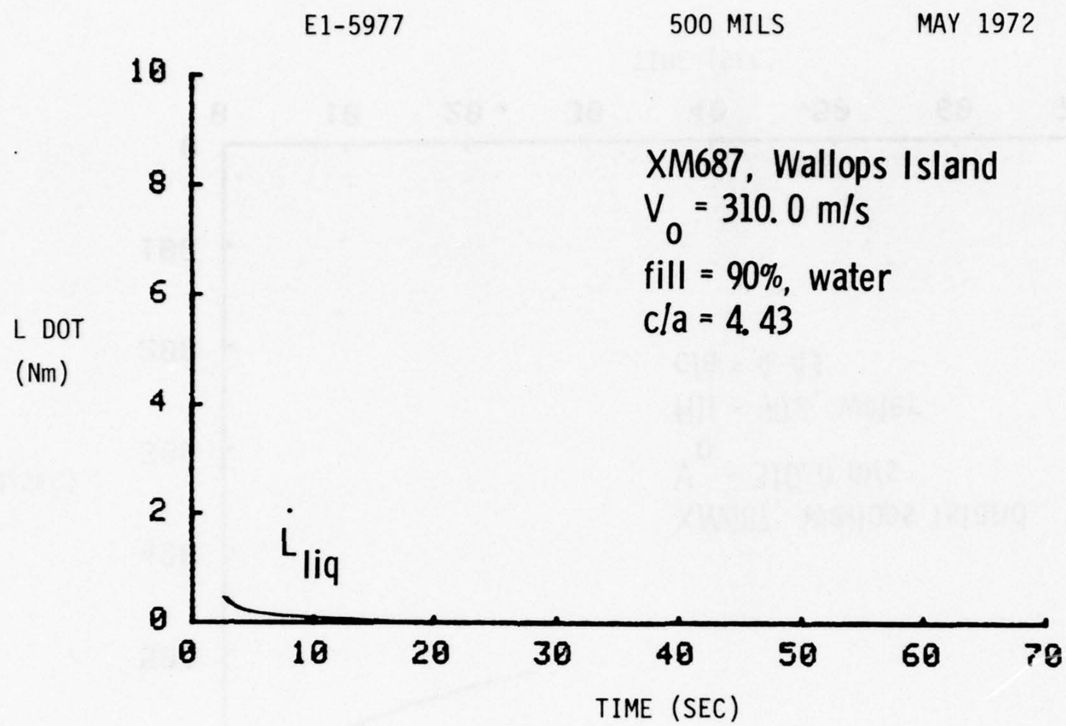

Figure 7. Rate of Change of Liquid Angular Momentum for Round E1-5977

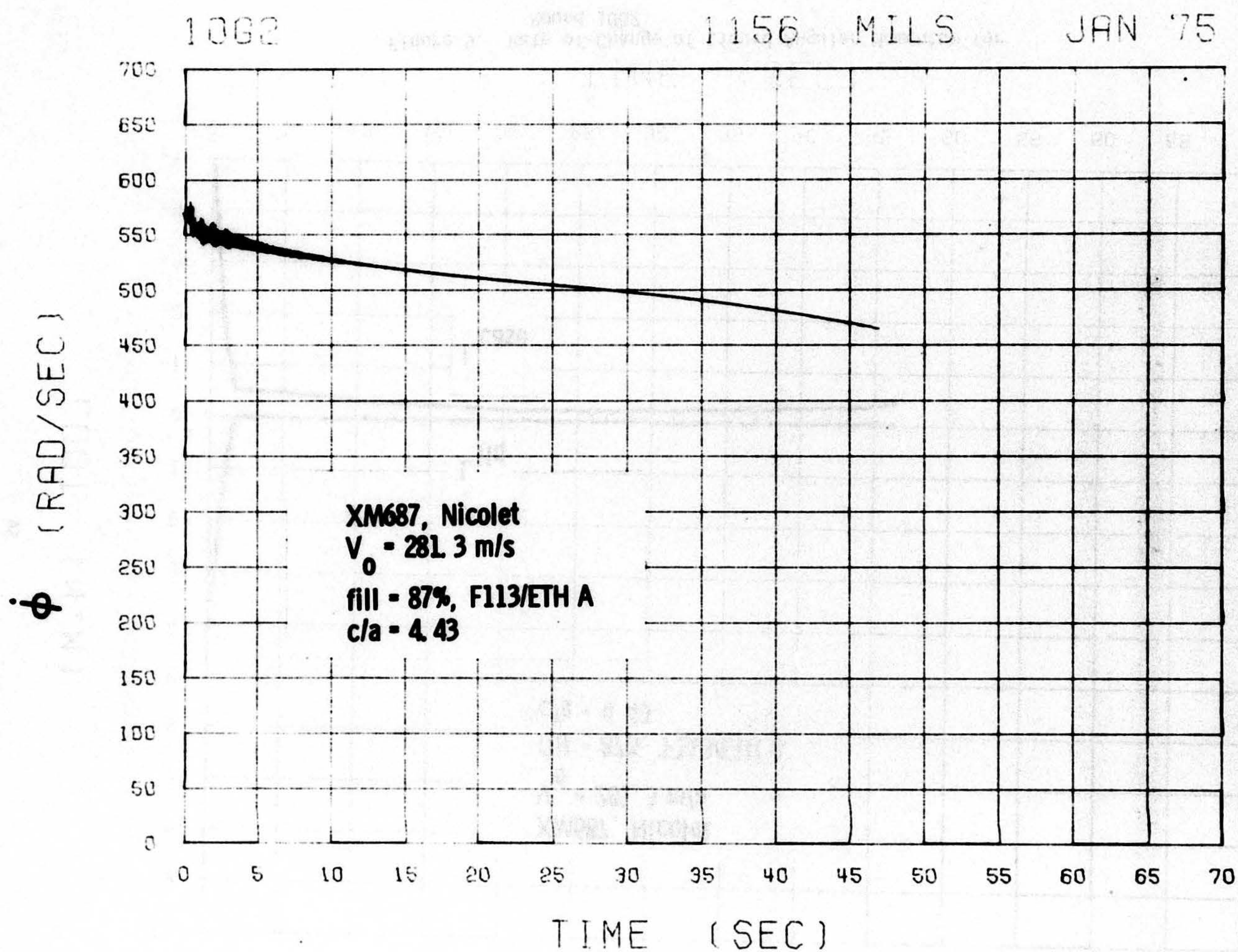

Figure 8. Spin History of Round 10G2

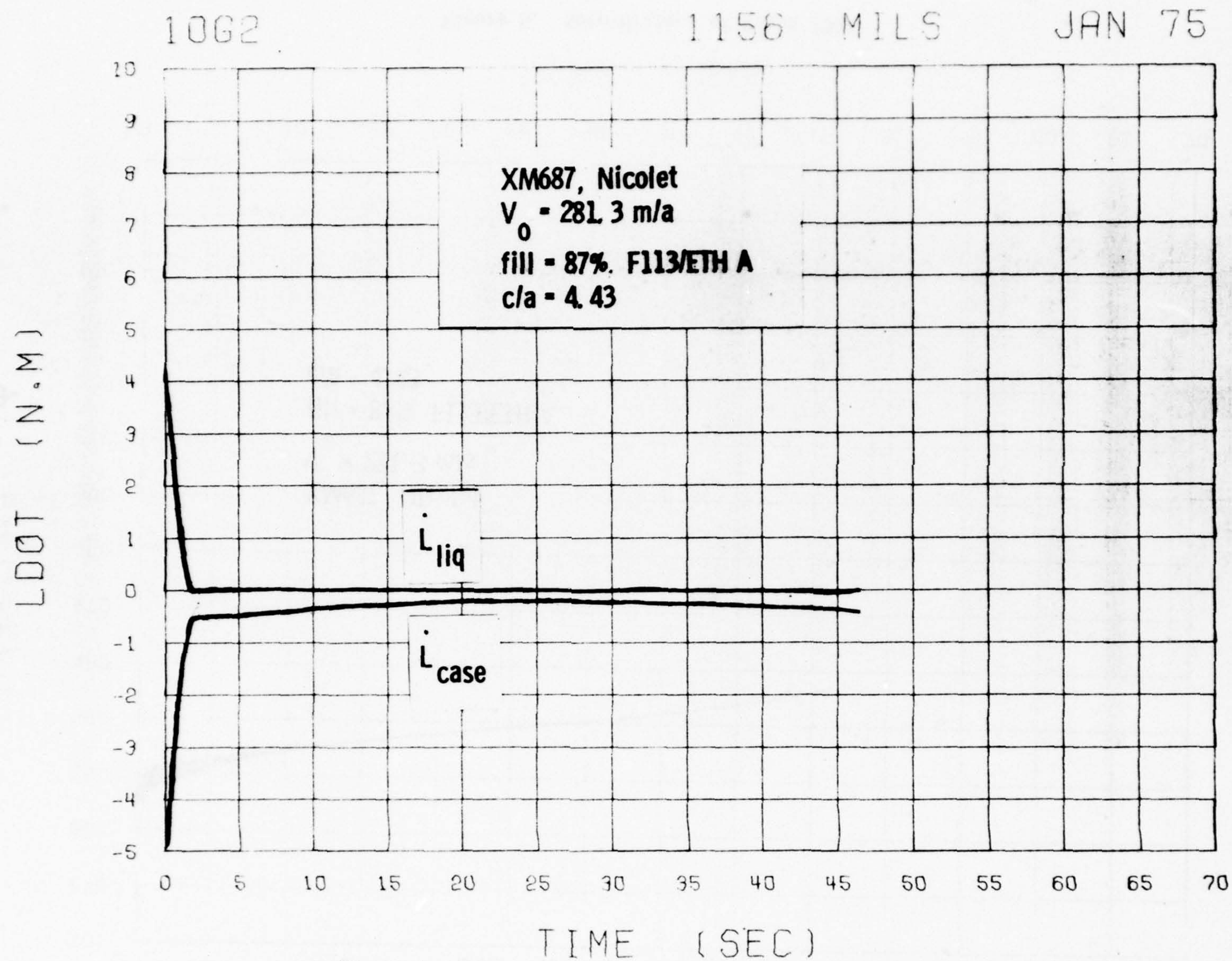

Figure 9. Rate of Change of Liquid Angular Momentum for Round 10G2

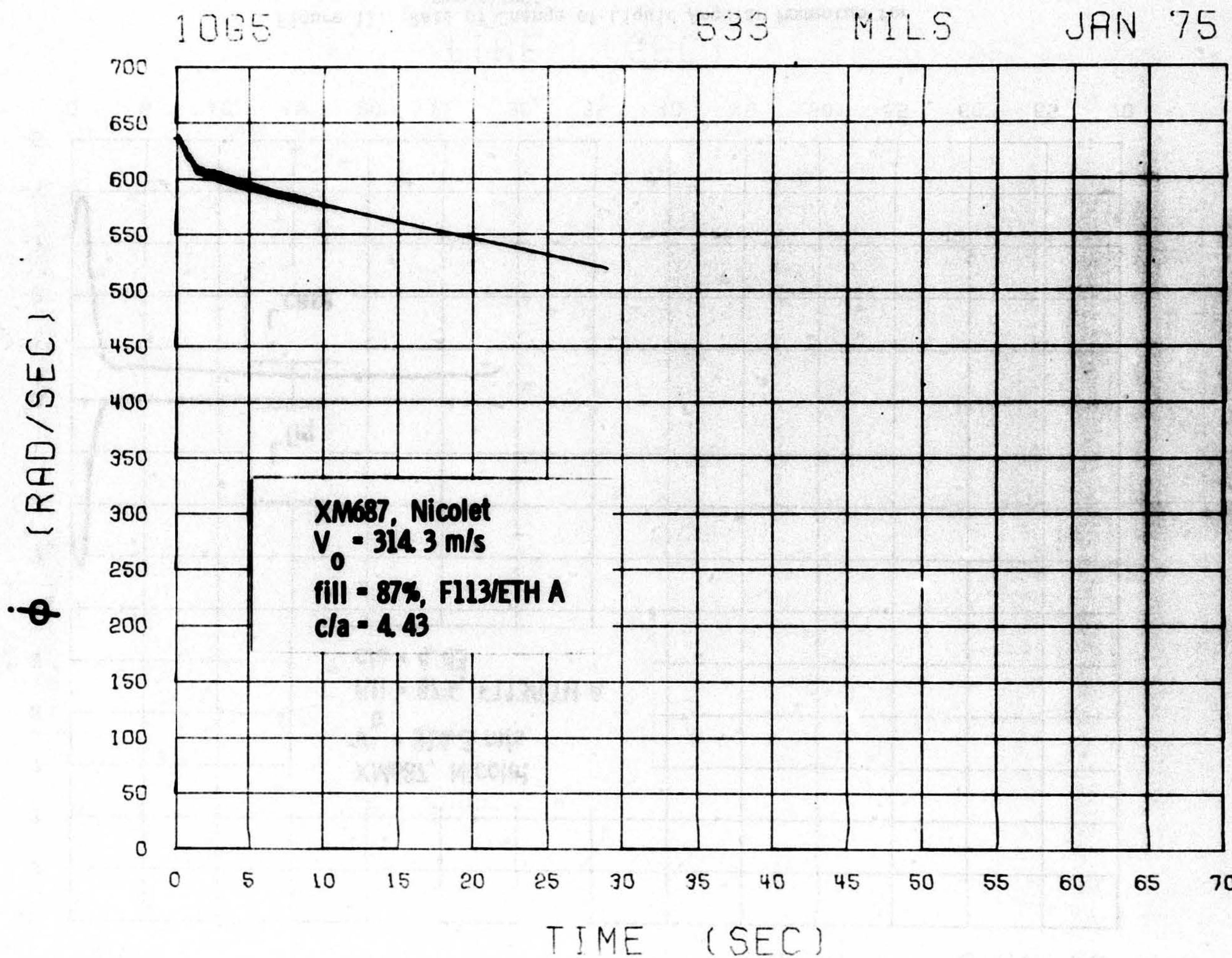

Figure 10. Spin History of Round 10G5

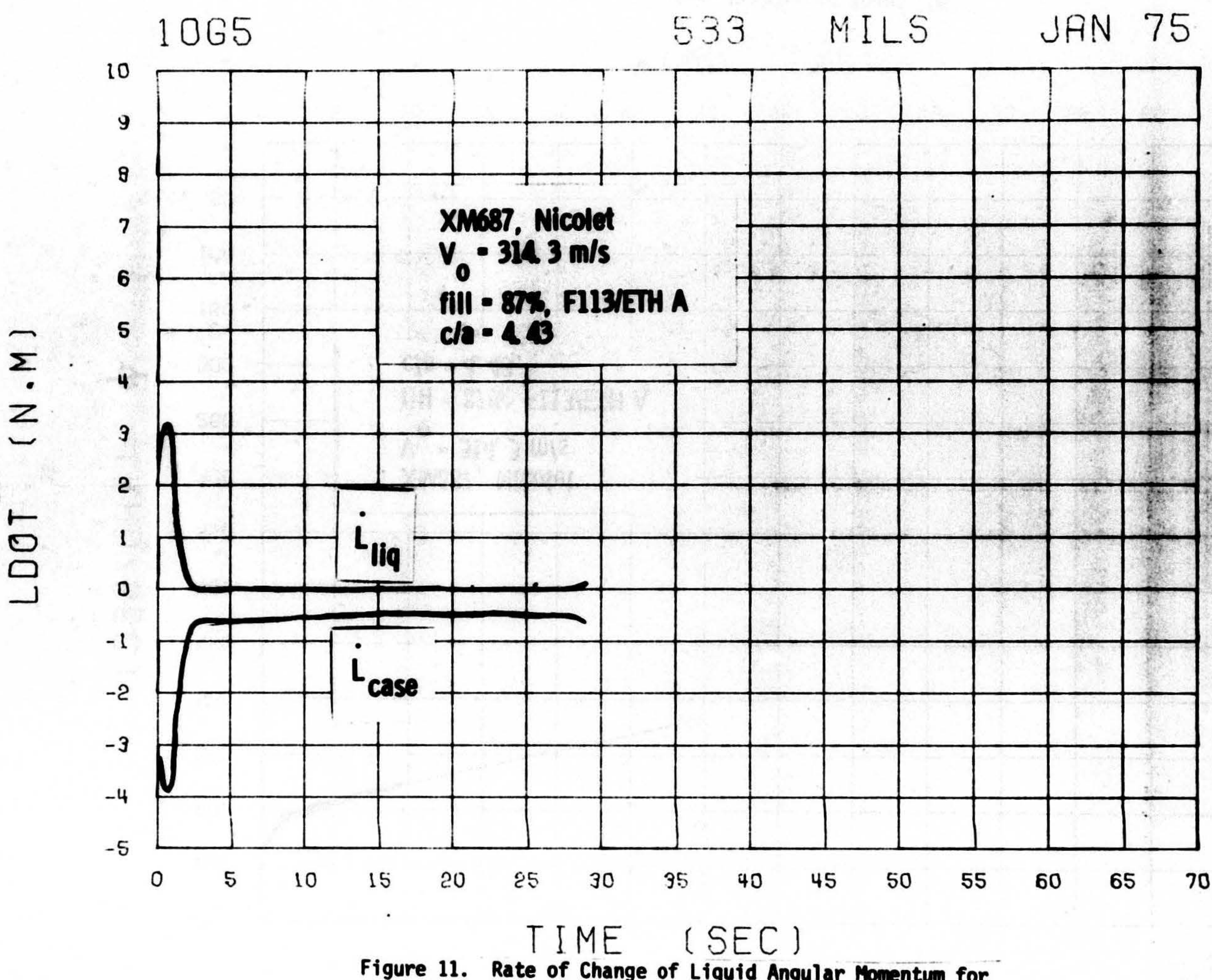

Figure 11. Rate of Change of Liquid Angular Momentum for Round 10G5

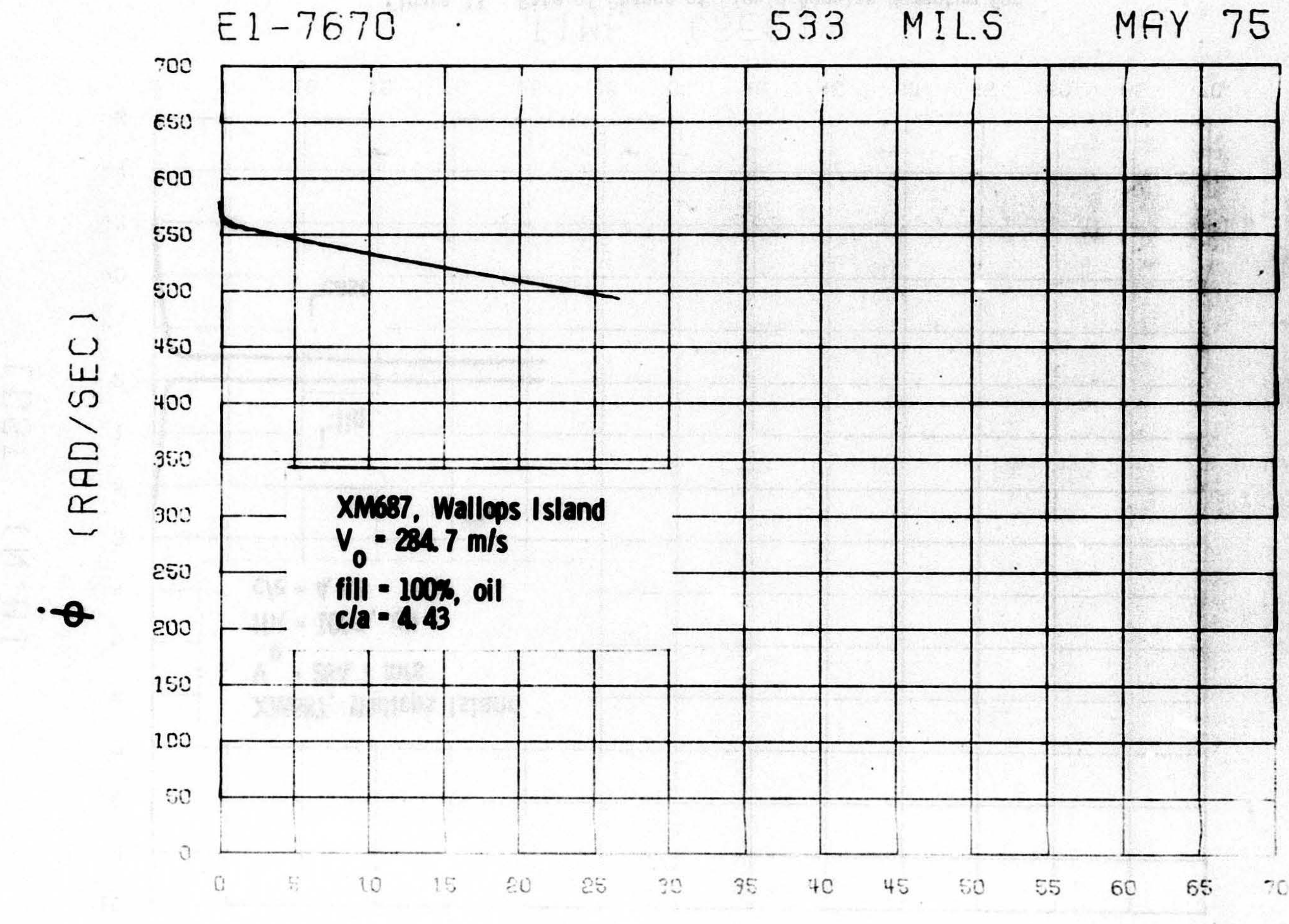

Figure 12. Spin History of Round E1-7670

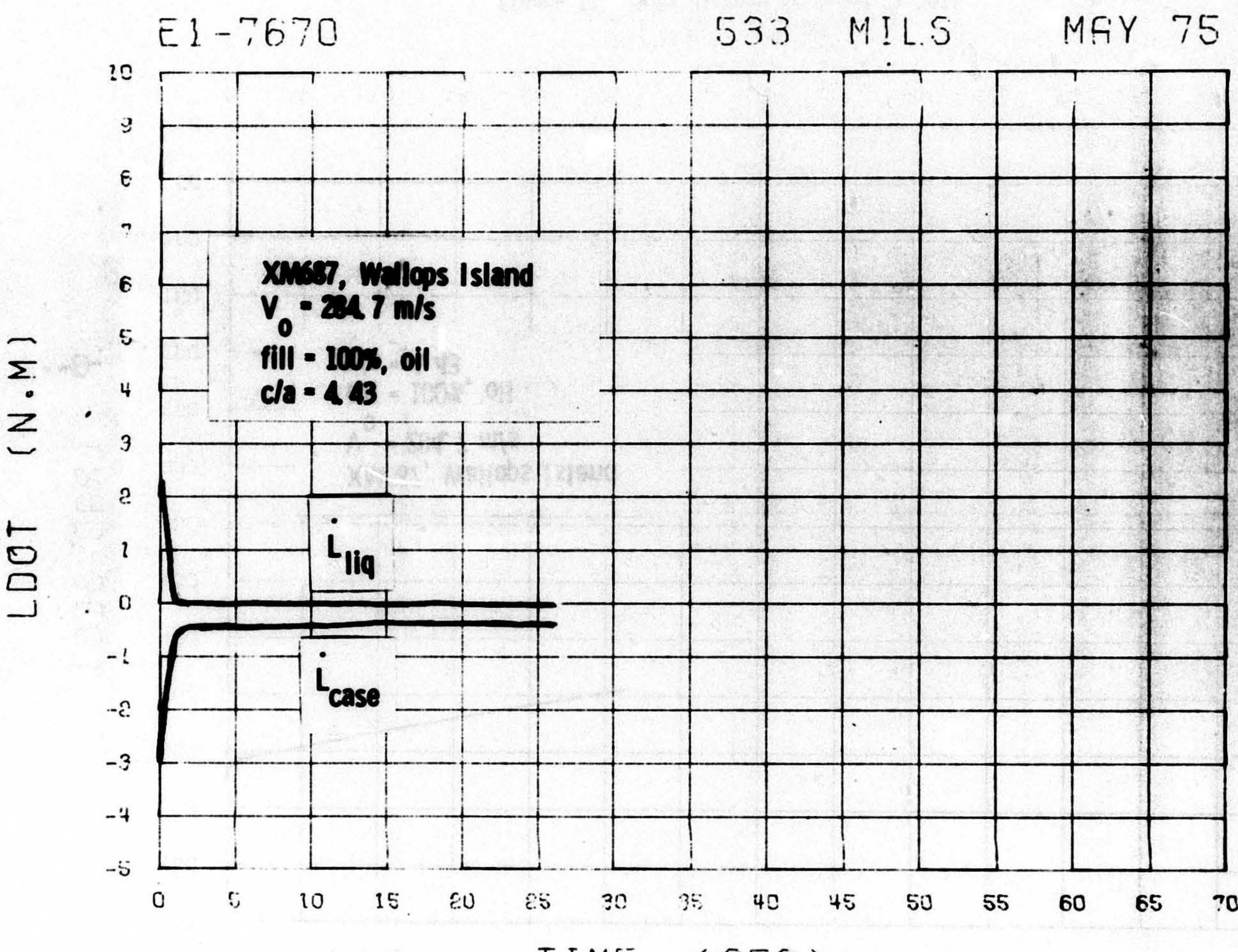

Figure 13. Rate of Change of Liquid Angular Momentum for Round E1-7670

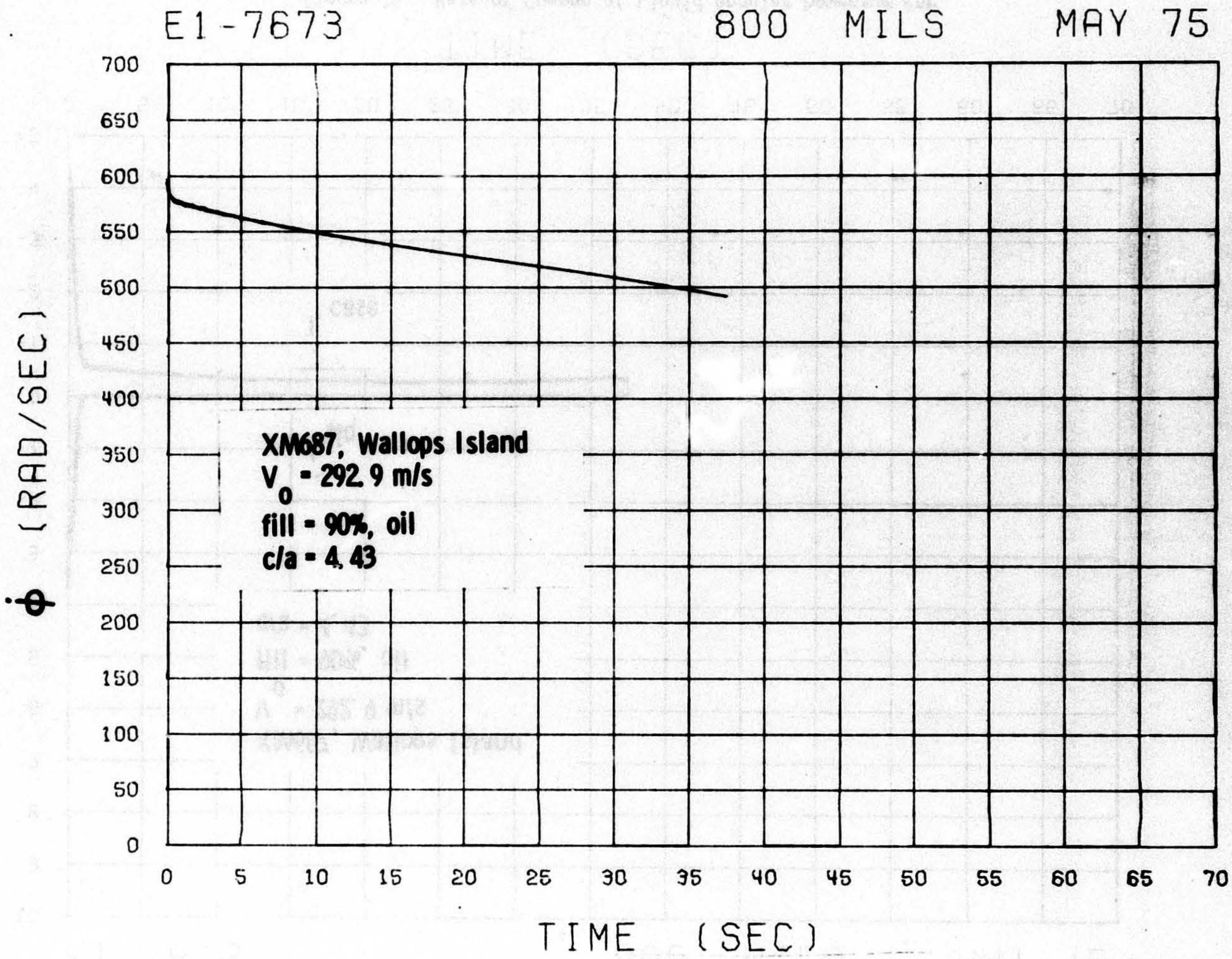

Figure 14. Spin History of Round E1-7673

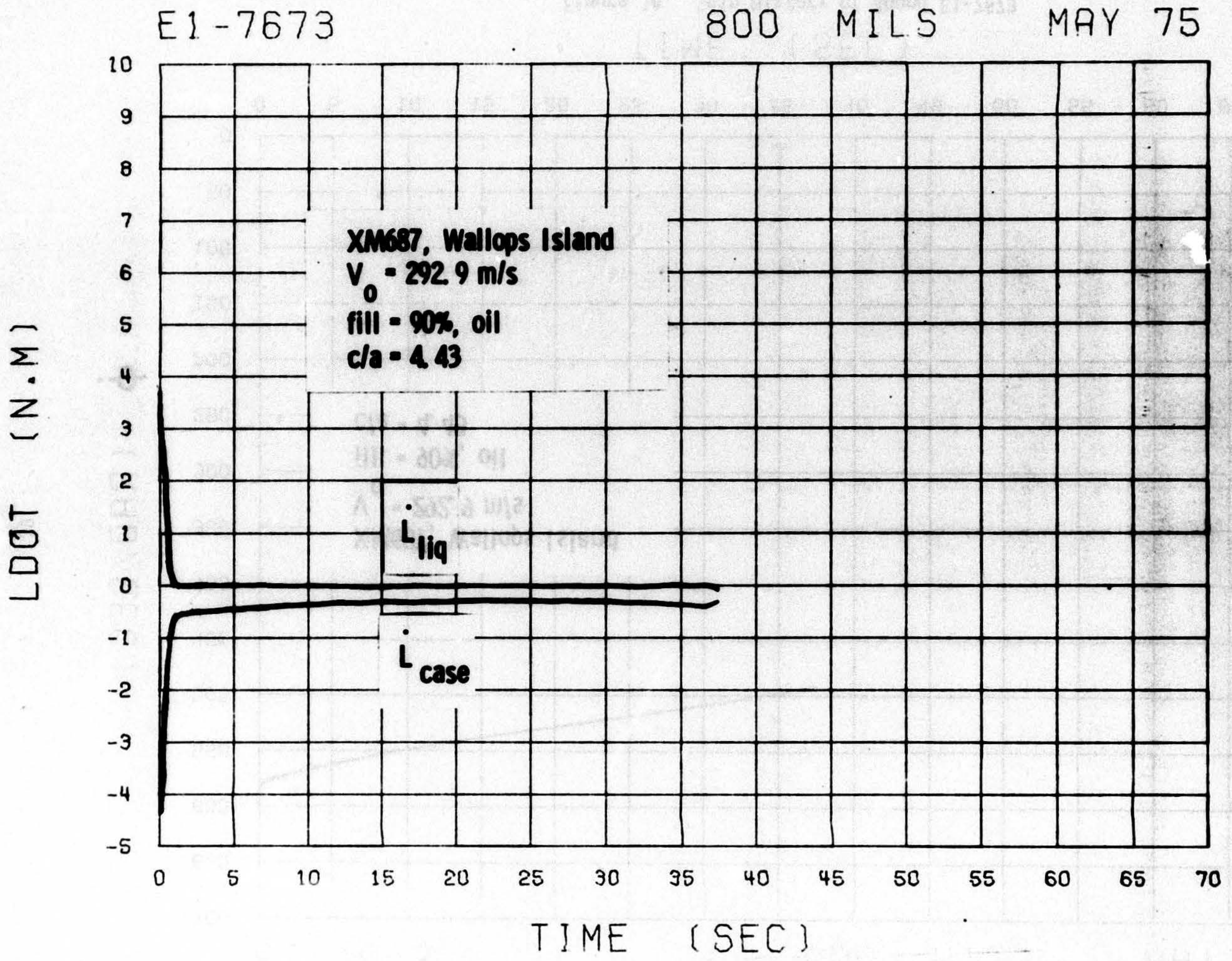

Figure 15. Rate of Change of Liquid Angular Momentum for Round E1-7673

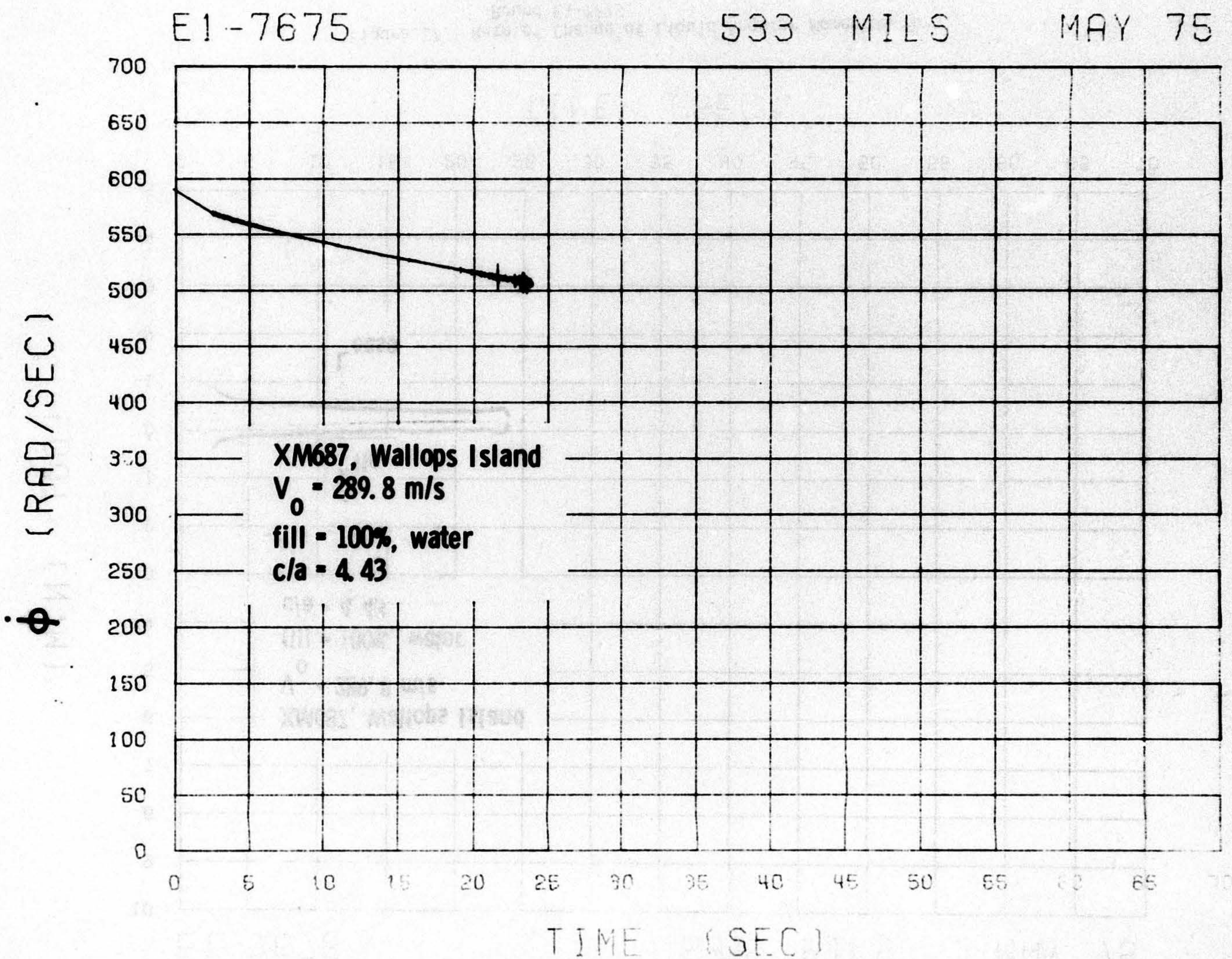

Figure 16. Spin History of Round E1-7675

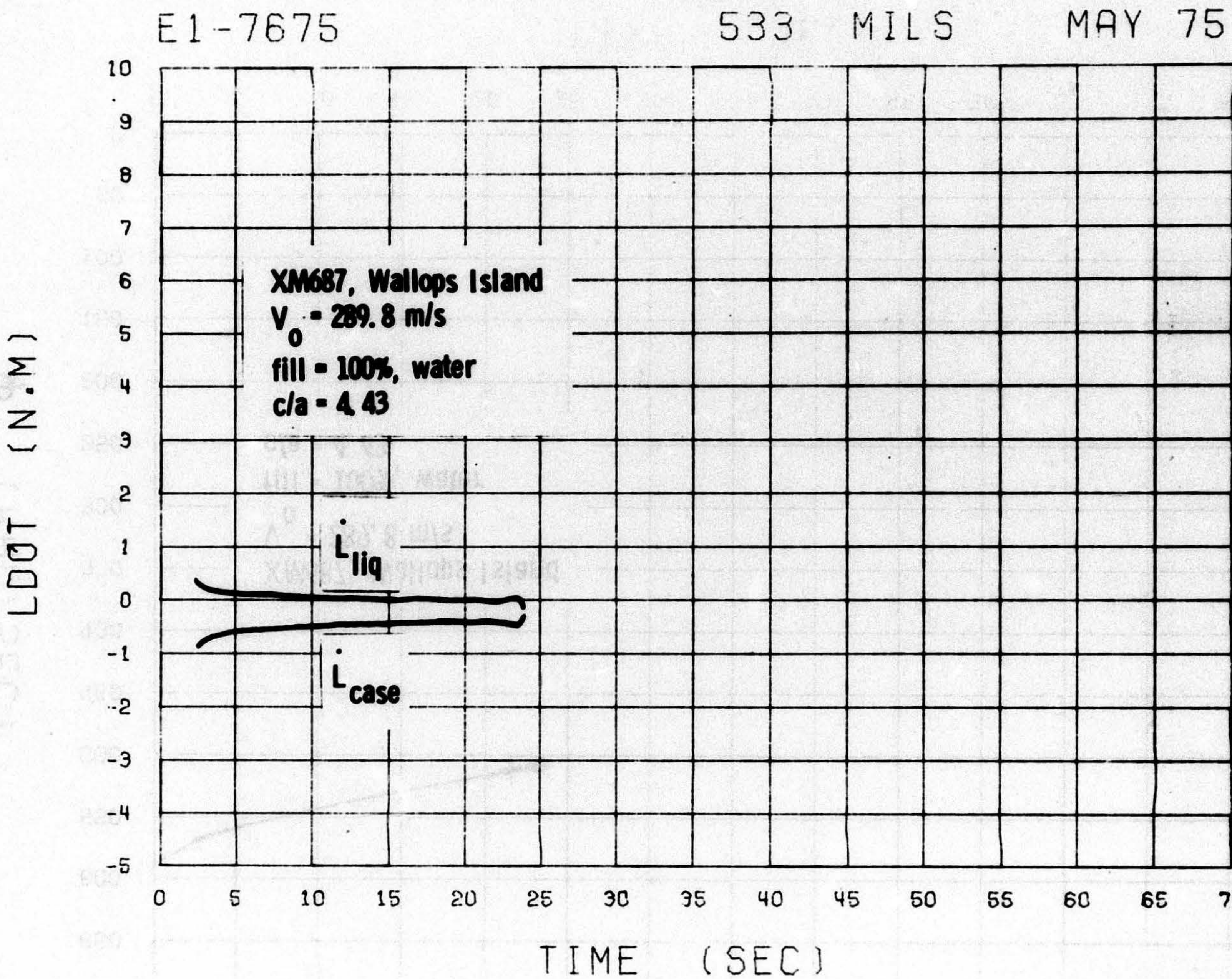

Figure 17. Rate of Change of Liquid Angular Momentum for Round E1-7675

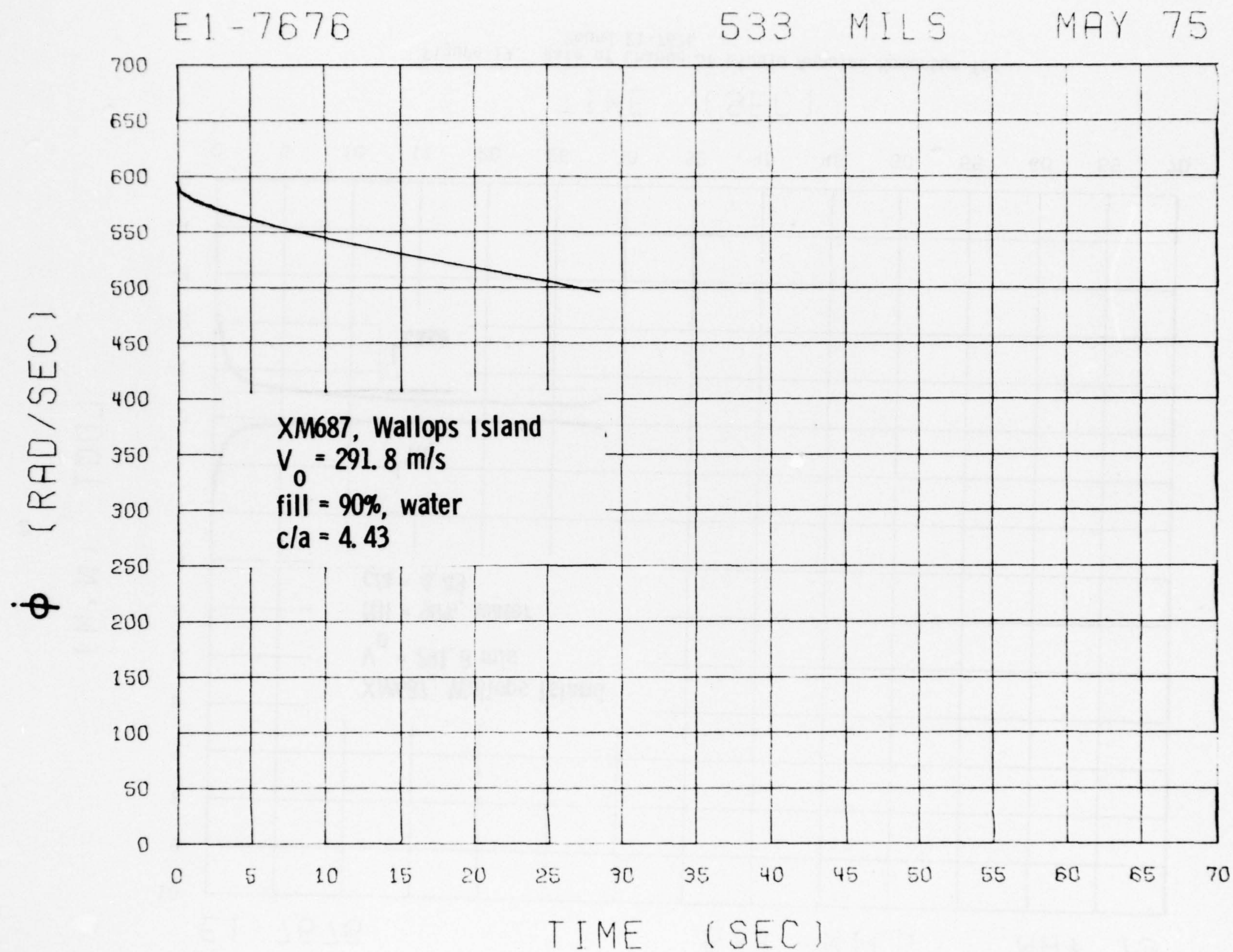

Figure 18. Spin History of Round E1-7676

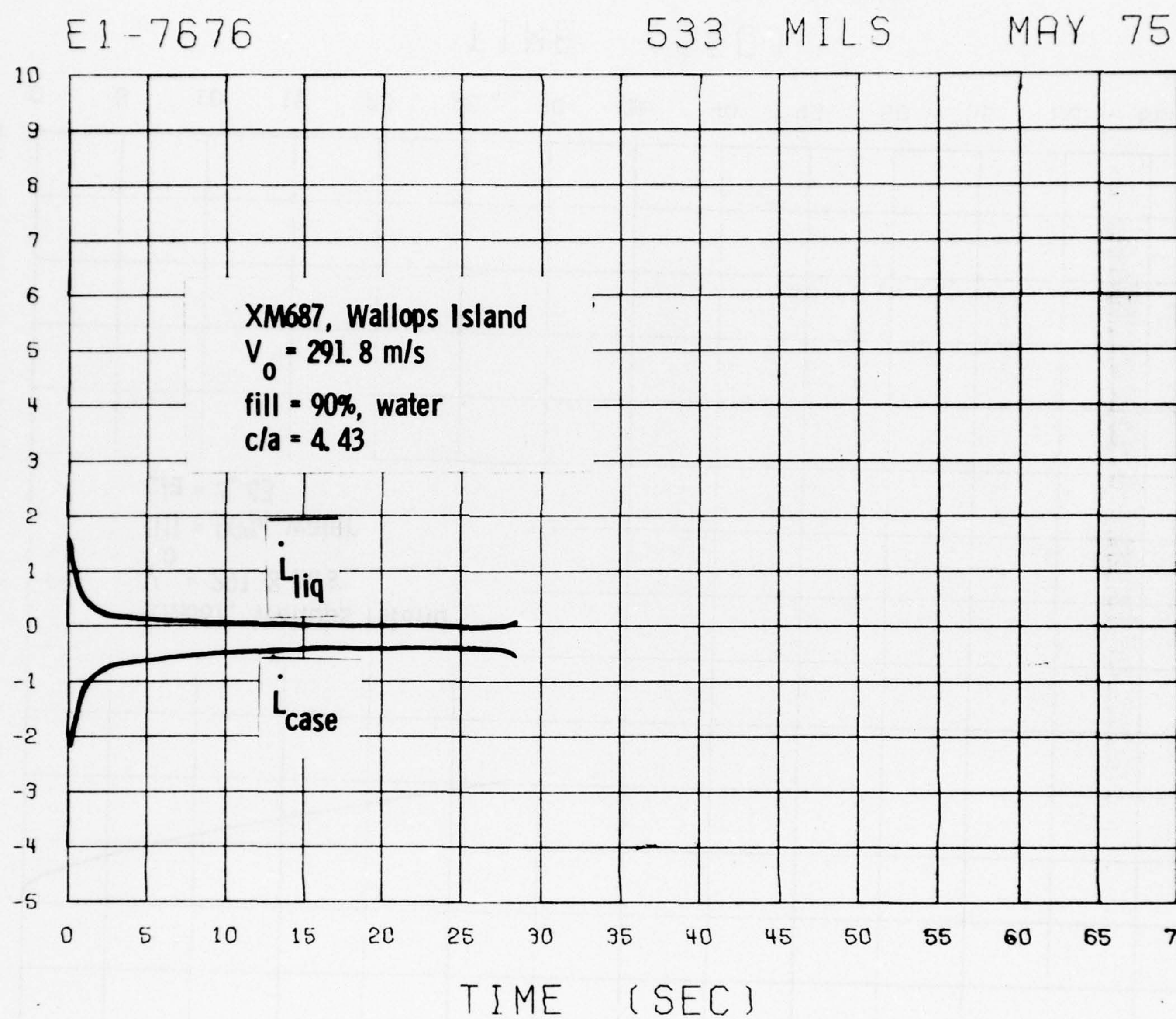

Figure 19. Rate of Change of Liquid Angular Momentum for Round E1-7676

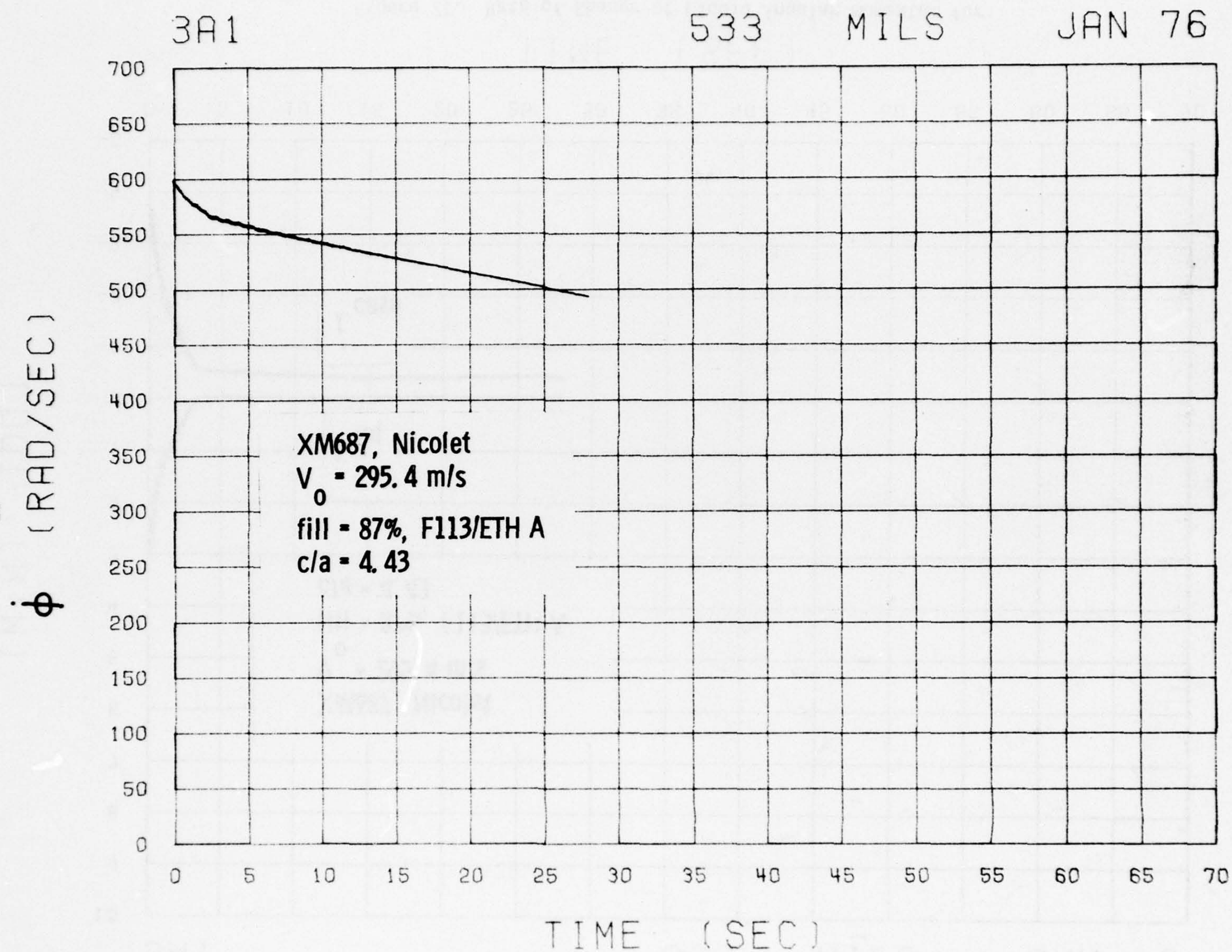

Figure 20. Spin History of Round 3A1

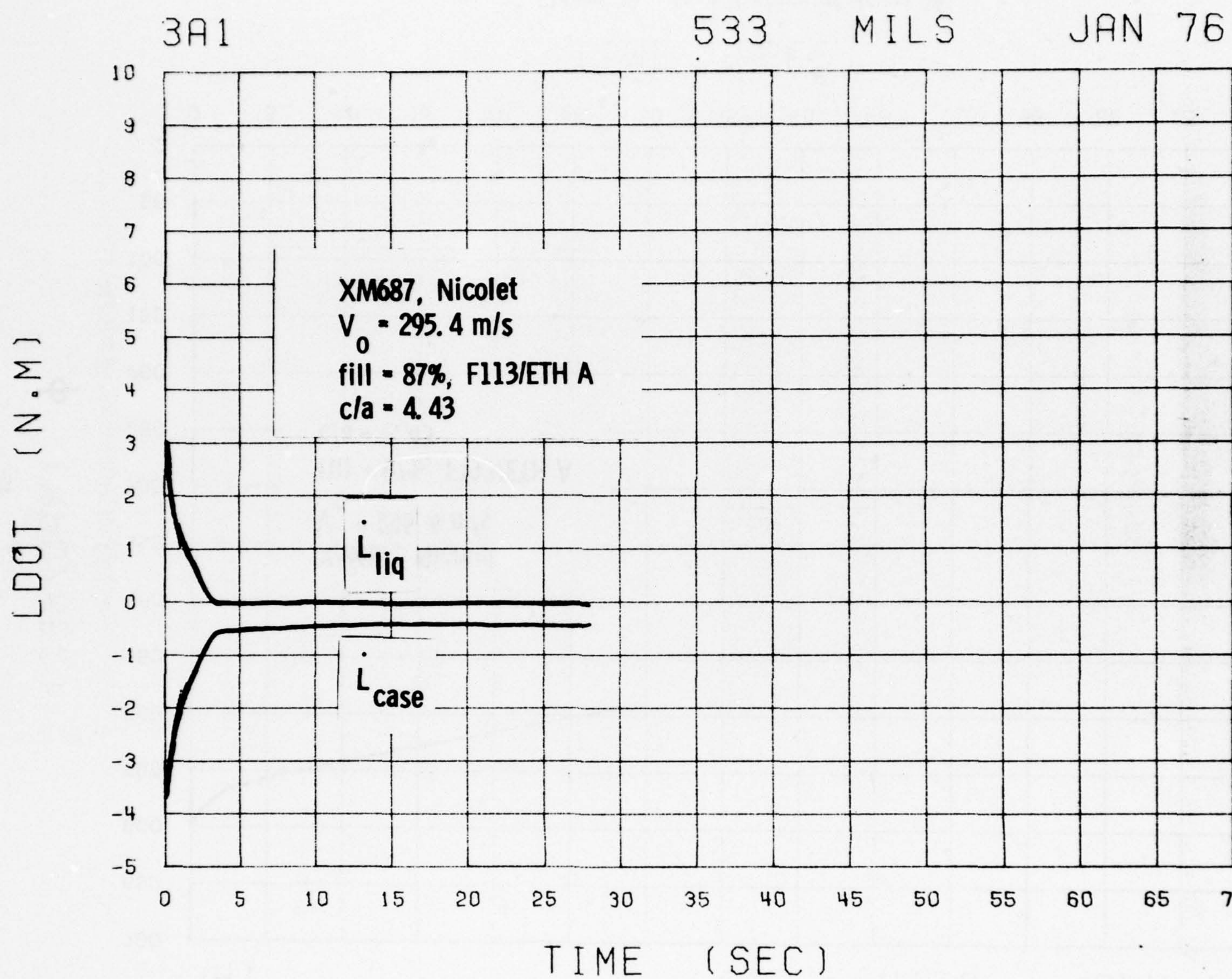

Figure 21. Rate of Change of Liquid Angular Momentum for Round 3A1

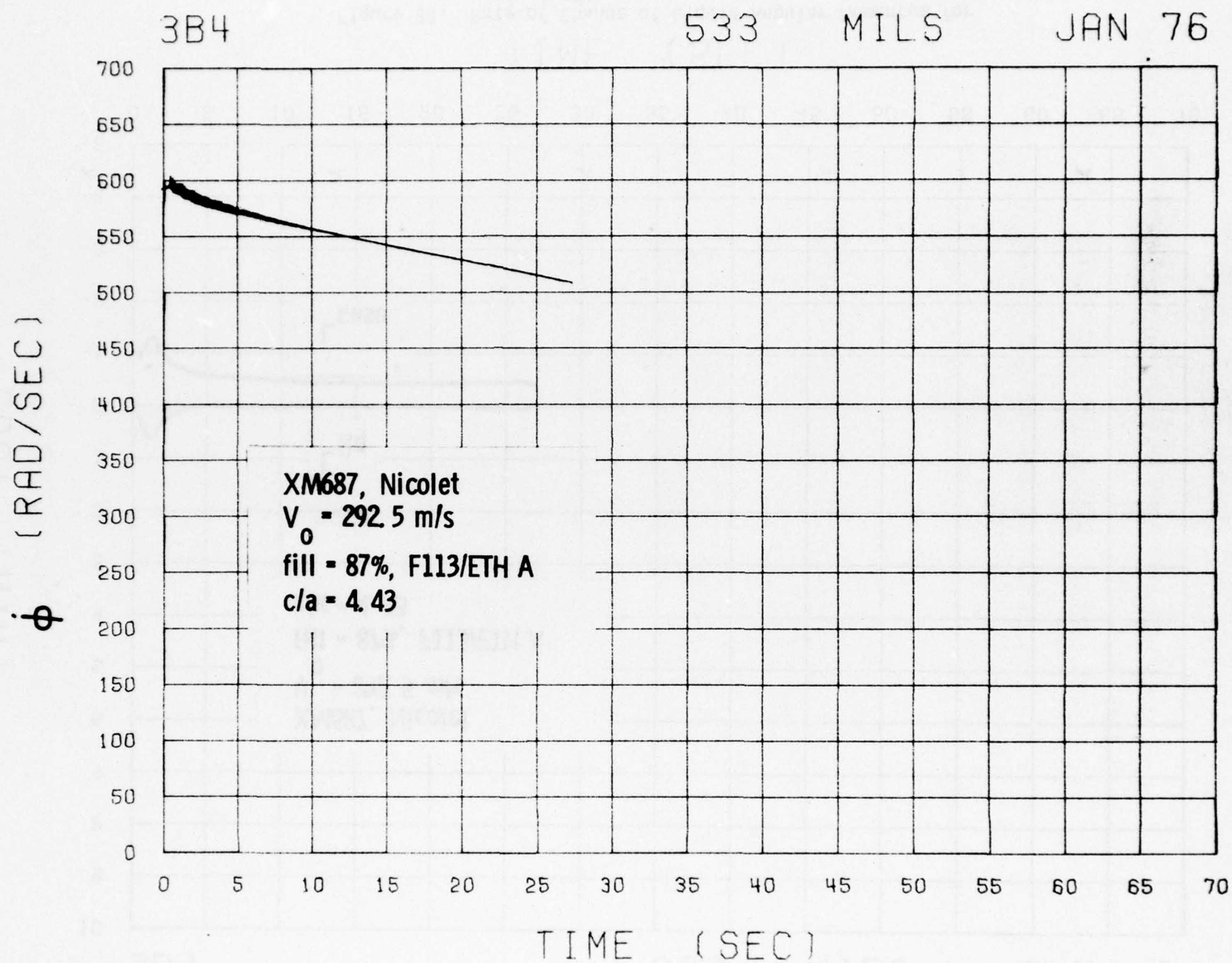

Figure 22. Spin History of Round 3B4

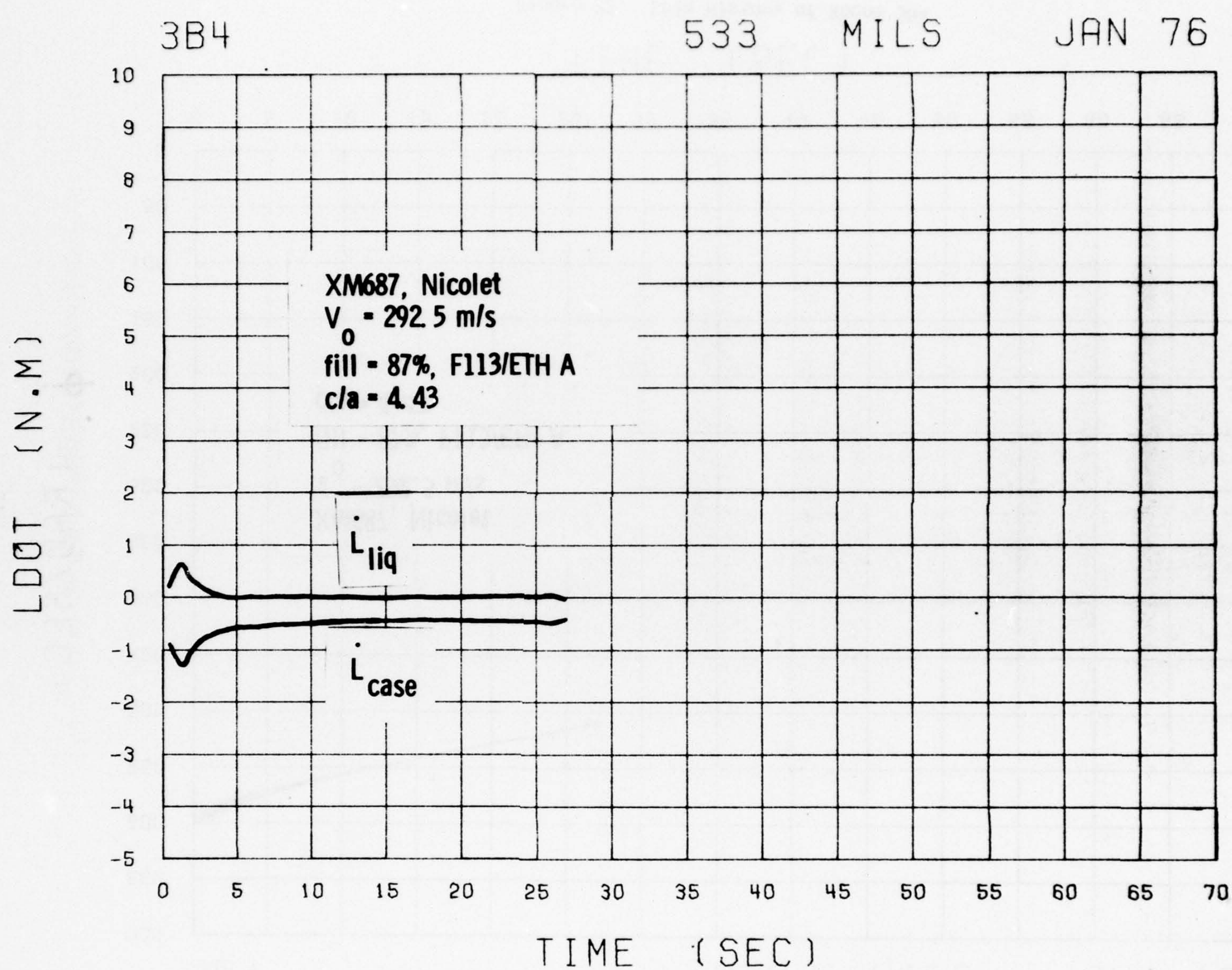

Figure 23. Rate of Change of Liquid Angular Momentum for Round 3B4

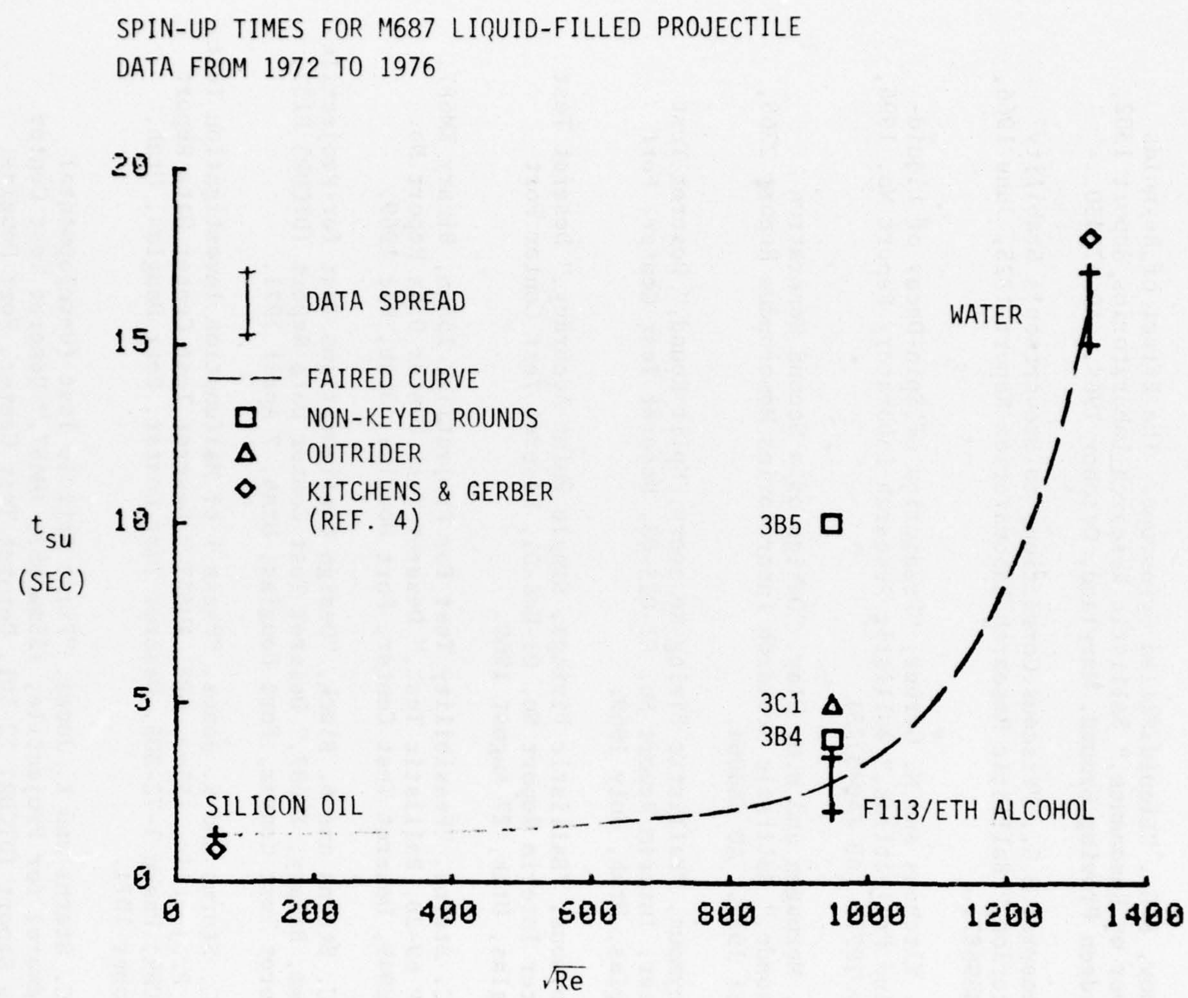

Figure 24. Spin-Up Times for XM687 at Various Reynolds Numbers

REFERENCES

1. K. Stewartson, "On the Instability of a Spinning Top Containing Liquid," Journal of Fluid Mechanics, Vol. 5, Part 4, September 1959, pp. 577-592.

2. Karpov, B.G., "Liquid-Filled Gyroscope: The Effect of Reynolds Number on Resonance," Ballistic Research Laboratories Report 1302, Aberdeen Proving Ground, Maryland, October 1965. AD 479430.

3. Wedemeyer, E.G., "Viscous Corrections to Stewartson's Stability Criterion," Ballistic Research Laboratories Report 1325, June 1966, AD 489687.

4. W.C. Kitchens and N. Gerber, "Prediction of Spin-Decay of Liquid-Filled Projectiles," Ballistic Research Laboratory Report No. 1996, July 1977. (AD #A043275)

5. W.H. Mermagen and W.H. Clay, "Design of a Second Generation Yawsonde," Ballistic Research Laboratories Memorandum Report 2368, April 1974. AD 780064.

6. Anonymous, "Ballistic Firing Recovery, Split Round," Deseret Test Center, Interim Report No. 01-023-03, Deseret Test Center, Fort Douglas, Utah, July 1969.

7. Anonymous, "Ballistic Firings, Single Point Accuracy," Deseret Test Center Interim Report No. 01-046-01, Deseret Test Center Fort Douglas, Utah, 27 August 1969.

8. C. C. Sterns, "Feasibility Test for Projectile, 155mm, Binary XM687, EATP 69-2B, Ballistic Test," Deseret Test Center Data Report No. DR E949, Deseret Test Center, Fort Douglas, Utah, Dec 1969.

9. C. C. Sterns and B. Black, "Design Configurations Test for Projectile, 155mm, Binary, XM687," Deseret Test Center Data Report (DTCDR) E127, Deseret Test Center, Fort Douglas, Utah, 7 April 1971.

10. C. C. Sterns and K. Jones, "Phase 1 of Malfunction Investigation Test for Projectile, 155mm, GB2, XM687," Deseret Test Center Data Report (DTCDR) Phase 1-72-305, Deseret Test Center, Fort Douglas, Utah, October 1971.

11. C. C. Sterns and K. Jones, "First Article Test (Developmental Hardware) for Projectile, 155mm GS2, XM687," Deseret Test Center Data Report (DTCDR) 72-301, Deseret Test Center, Fort Douglas, Utah, 20 Jan 1972.

REFERENCES (Continued)

12. C. C. Sterns and K. Jones, "(Advanced Development) Ballistic (Match) Test for Projectile, 155mm, GB2, XM687," Deseret Test Center Data Report (DTCDR) 72-304, Deseret Test Center, Fort Douglas, Utah, 24 Feb 1972.

13. C. C. Sterns and K. Jones, "Phase 2 of Malfunction Investigation for Projectile, 155mm, GB2, XM687," Deseret Test Center Data Report (DTCDR) 72-305, Deseret Test Center, Fort Douglas, Utah, 3 Apr 1972.

14. C. C. Sterns and K. Jones, "Phase 3 of Malfunction Investigation Test Projectile, 155mm, XM687," Deseret Test Center Data Report (DTCDR) 72-305, Deseret Test Center, Fort Douglas, Utah, 3 Apr 1972.

15. C. C. Sterns and K. Jones, "Services Developmental Test of Projectile, 155mm, XM687," Deseret Test Center Data Report (DTCDR) 72-311, Deseret Test Center, Fort Douglas, Utah, 15 May 1972.

16. A. Mark and W.H. Mermagen, "Measurement of Spin Decay and Instability of Liquid-Filled Projectiles via Telemetry," Ballistic Research Laboratories Memorandum Report No. 2333, October 1973, AD 771919.

17. W.H. Clay, W.H. Mermagen, "The Portable Yaw Processor," Ballistic Research Laboratory Memorandum Report 2785, September 1977. (AD #B022104L)

18. V. Oskay and J.H. Whiteside, "Flight Behavior of 155mm (XM687 Mod I and XM687 Mod II) and 8-Inch (XM736 Mod I) Binary Shell at Nicolet, Canada, During the Winter of 1974-1975," Ballistic Research Laboratories Memorandum Report No. 2608, March 1976, AD B010566L.

19. W. Mermagen, W. Clay and V. Oskay, "Yawsonde Data From Firings in the Nicolet Winter Test Program 1974-1975," Ballistic Research Laboratories Memorandum Report No. 2612, April 1976, AD B011225L.

20. W.H. Mermagen, "Projectile High-g Telemetry for Long Range Dynamic Measurements," ISA, International Telemetering Conference Proceedings, Vol. III, 1971.

21. W.H. Mermagen, "Measurements of the Dynamical Behavior of Projectiles Over Long Flight Paths," Journal of Spacecraft and Rockets, Vol. 8, No. 4, April 1971, pp. 310-385.

22. W.H. Clay, "A Precision Yawsonde Calibration Technique," Ballistic Research Laboratories Memorandum Report No. 2263, January 1973, AD 758158.

23. E. H. Wedemeyer, "The Unsteady Flow Within a Spinning Cylinder," Ballistic Research Laboratories Report 1225, October 1965, AD 431846.

REFERENCES (Continued)

24. Yuma Proving Ground Preliminary Firing Record data subject to confirmation. "Restructured Development Test II of the 155mm, HE, M483E1 Projectile in Howitzer, Self-Propelled, M109A1," TECOM Project No. 2-MU-003-483-030, Project Engineer: Dick Godly, 1976.

25. C.H. Murphy, "Effect of Large High-Frequency Angular Motion of a Shell on the Analysis of its Yawsonde Records," Ballistic Research Laboratories Memorandum Report 2581, February 1976. AD B009421L.

26. A. Mark, "Transient Eigenfrequencies in Liquid-Filled Cylinders," AIAA Journal, Vol. 13, No. 2, 1975, pp 217-219.

27. H.P. Greenspan, The Theory of Rotating Fluids, Cambridge University Press, 1969.

LIST OF SYMBOLS

a	cylinder cavity radius
c	1/2 cylinder cavity length
C_{ℓ_p}	$\dfrac{\text{Axial Moment}}{1/2\ \rho\ V^2\ S\ell\ (p\ell/V)}$
I_a	axial moment of inertia of projectile
ℓ	projectile diameter
L_{liq}	instantaneous axial angular momentum of liquid
L_{liq_o}	axial angular momentum of liquid at the muzzle
$L_{liq_{max}}$	axial angular momentum of liquid at maximum spin
M	Mach number
M_{aero}	aerodynamic moment
MHz	Megahertz
M_{liq}	liquid moment
m	metres
mW	milliwatts
p	projectile spin rate
p_o	projectile spin rate at the muzzle
Re	Reynolds number a^2p/ν
S	maximum cross-sectional area of projectile
s	second
t	time
V	projectile velocities

LIST OF SYMBOLS (Continued)

α_{max}	maximum projectile yaw
ν	kinematic viscosity
ρ	air density
ϕ	Eularian roll angle
σ	solar aspect angle
σ_n	complement of solar aspect angle
τ_n	projectile nutational frequency non-dimensionalized by spin
τ_o	eigenfrequency non-dimensionalized by spin
Ω	resistance (ohms)
$\cdot$	dot over symbol means time differentiation

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