

APPROXIMATE SOLUTIONS FOR CERTAIN OPTIMAL STOPPING PROBLEMS

BY

ALBERT JOHN PETKAU

TECHNICAL REPORT NO. 13

JANUARY 5, 1978

PREPARED UNDER GRANT

DAAG29-77-G-0031

FOR THE U.S. ARMY RESEARCH OFFICE

Reproduction in Whole or in Part is Permitted  
for any purpose of the United States Government

Approved for public release; distribution unlimited.

DEPARTMENT OF STATISTICS  
STANFORD UNIVERSITY  
STANFORD, CALIFORNIA



APPROXIMATE SOLUTIONS FOR CERTAIN OPTIMAL STOPPING PROBLEMS

BY

ALBERT JOHN PETKAU

TECHNICAL REPORT NO. 13

JANUARY 5, 1978

Prepared under Grant DAAG29-77-G-0031  
For the U.S. Army Research Office  
Herbert Solomon, Project Director

Approved for public release; distribution unlimited.

DEPARTMENT OF STATISTICS  
STANFORD UNIVERSITY  
STANFORD, CALIFORNIA

Partially supported under Office of Naval Research Contract N00014-75-C-0555  
(NR-042-331) - Dept. of Mathematics, Massachusetts Institute of Technology  
and issued as Technical Report No. 9.

THE FINDINGS IN THIS REPORT ARE NOT TO BE  
CONSTRUED AS AN OFFICIAL DEPARTMENT OF  
THE ARMY POSITION, UNLESS SO DESIGNATED  
BY OTHER AUTHORIZED DOCUMENTS.

# Approximate Solutions for Certain Optimal Stopping Problems

Albert John Petkau

## 1. Introduction

The following optimal stopping problem (which is one of several different problems which have come to be known as the one-armed bandit problem) has arisen in a number of statistical applications (Chernoff and Ray (1965), Chernoff (1967), Mallik (1971)): Let  $X(t)$  be a Wiener process described by  $E[dX(t)] = \mu \cdot dt$  and  $\text{Var}[dX(t)] = \sigma^2 \cdot dt$  where  $\sigma^2$  is presumed known. One is permitted to stop observing the process  $X(t)$  at any time  $t$ ,  $0 \leq t \leq N$ , and receive a payoff  $X(t)$ . The unknown parameter  $\mu$  is assumed to have a  $N(\mu_0, \sigma_0^2)$  prior. What is the optimal stopping procedure?

It is easy to verify that the posterior distribution of  $\mu$  given  $X(t')$ ,  $0 \leq t' \leq t$ , is  $N(Y^*(s^*), s^*)$  where

$$(1.1) \quad Y^*(s^*) = \frac{\mu_0 \sigma_0^{-2} + X(t) \sigma^{-2}}{\sigma_0^{-2} + t \sigma^{-2}}$$

and

$$(1.2) \quad s^* = (\sigma_0^{-2} + t \sigma^{-2})^{-1}.$$

Here  $s^*$  varies from  $s_0^* = \sigma_0^2$  to  $s_1^* = (\sigma_0^{-2} + N \sigma^{-2})^{-1}$ . Furthermore, the process  $Y^*(s^*)$  is a Wiener process (in the  $-s^*$  scale) described by

$E[dY^*(s^*)] = 0$  and  $\text{Var}[dY^*(s^*)] = -ds^*$ , starting from  $Y^*(s_0^*) = \mu_0$ .  
 The loss upon stopping at  $(Y^*(s^*), s^*)$  is  $-X(t)$  which from (1.1) is a  
 linear function of  $-Y^*(s^*)/s^*$ . Applying the transformation  $s = s^*/s_1^*$ ,  
 $Y(s) = Y^*(s^*)/s_1^{1/2}$  leads to a normalized version of this stopping problem  
 where  $s$  varies from  $s_0 = \sigma_0^2(\sigma_0^{-2} + N\sigma^{-2})$  to  $s_1 = 1$  and in which the  
 stopping cost is given by  $d(y, s)$  defined by

$$(1.3) \quad d(y, s) = -y/s$$

for  $s \geq 1$  with stopping enforced at  $s = 1$ .

This normalized problem is a special case of the following optimal  
 stopping problem: Given a Wiener process  $\{Y(s), s \geq s_1\}$  in the  $-s$   
 scale described by  $E[dY(s)] = 0$  and  $\text{Var}[dY(s)] = -ds$  and starting at  
 $Y(s_0) = y_0$ , find the stopping time  $S$  to minimize  $E[d(Y(S), S)]$ . If we  
 define  $\rho(y_0, s_0) = \inf b(y_0, s_0)$  where  $b(y_0, s_0)$  is the risk associated  
 with a particular stopping time and the infimum is taken over all such  
 stopping times,  $\rho(y, s)$  represents the best that can be achieved once  
 $(y, s)$  has been reached, irrespective of how it was reached. An optimal  
 procedure is then described by the continuation set  $\mathcal{C} = \{(y, s): \rho(y, s) < d(y, s)\}$ .  
 Chernoff (1968) has demonstrated that one should expect the solution  $(\rho, \mathcal{C})$   
 of the stopping problem to be a solution of the following free boundary problem:

$$(1.4) \quad \begin{aligned} \frac{1}{2} \rho_{yy}(y, s) &= \rho_s(y, s) & \text{for } (y, s) \in \mathcal{C} , \\ \rho(y, s) &= d(y, s) & \text{for } (y, s) \in \mathcal{C}^e , \\ \rho_y(y, s) &= d_y(y, s) & \text{for } (y, s) \in \partial \mathcal{C} . \end{aligned}$$

Furthermore, for any such stopping problem, Van Moerbeke (1974) has shown that one should never stop at points  $(y,s)$  at which  $\frac{1}{2} d_{yy}(y,s) - d_s(y,s) < 0$ .

Applying this criterion to the normalized version of the stopping problem described above, hereinafter referred to as the one-armed bandit problem, one finds that  $\{(y,s): y > 0, s > 1\}$  is a subset of the optimal continuation region  $\mathcal{C}$ . Chernoff and Ray (1965) have shown that for this problem  $\mathcal{C}$  can be described as  $\mathcal{C} = \{(y,s): y > \tilde{y}(s), s > 1\}$  and have determined asymptotic expansions for the boundary curve  $\tilde{y}(s)$  in the regions of large  $s$  and  $s$  close to 1. The leading terms in these expansions are given by

$$\begin{aligned} \tilde{y}(s) &\sim -(2s \ln s)^{1/2} && \text{as } s \rightarrow \infty \\ \tilde{y}(s) &\sim -0.64(s-1)^{1/2} && \text{as } s \rightarrow 1. \end{aligned}$$

The scale  $z = -y/s$  and  $t = 1/s$  is more appropriate for applications and these expansions are illustrated in this scale in Chernoff and Ray (1965). It is evident from this illustration that these asymptotic expansions are inadequate as a complete description of the optimal continuation region. An approximation to the optimal continuation region is required as a description of the optimal procedure in the region where the asymptotic expansions are clearly inadequate.

Although it is possible that refined methods of asymptotic analysis could lead to expansions which would provide an adequate description of the optimal procedure, the purpose of the present paper is to describe simple methods which lead to arbitrarily accurate numerical approximations to the optimal continuation region for the one-armed bandit problem. Although

most of the discussion in the present paper will concentrate on the one-armed bandit problem, these same methods could be applied with equal facility to any optimal stopping problem of the general form described above.

## 2. An Analogous Discrete Problem

Consider the process  $Y'(s')$  which starts at  $Y'(1+n\cdot\Delta) = y'$  and is defined by  $Y'(s' - \Delta) = Y'(s') \pm \Delta^{1/2}$  each with probability  $\frac{1}{2}$ . This process is observed for at most  $n$  successive times and the cost associated with stopping the process at any point  $(y', s')$  is given by  $d(y', s')$  defined by (1.3). The problem is to find a stopping time to minimize the expected loss. We shall denote the optimal expected loss by  $\rho'(y', s')$ . For this problem, a backward induction algorithm becomes

$$(2.1) \quad \rho'(y', 1+n\cdot\Delta) = \min\{d(y', 1+n\cdot\Delta), \frac{1}{2}[\rho'(y' + \Delta^{1/2}, 1+(n-1)\cdot\Delta) + \rho'(y' - \Delta^{1/2}, 1+(n-1)\cdot\Delta)]\}$$

for  $n > 1$  with  $\rho'(y', 1) = d(y', 1)$ . It is easy to verify using the methods of Chernoff and Petkau (1976) that the optimal stopping set can be described as  $\{(y', 1+n\cdot\Delta): y' \leq \tilde{y}_n(\Delta), n \geq 1\}$  where for each fixed value of  $\Delta$ ,  $\{\tilde{y}_n(\Delta)\}$  is a non-increasing non-positive sequence. Note that this set does not depend upon the initial point. Further note that direct application of (2.1) yields  $\tilde{y}_1(\Delta) \equiv 0$ .

Since  $Y'(s')$  is a process of independent increments with mean zero and variance one per unit change in  $-s'$ , any stopping problem for the

Wiener process  $Y(s)$  of the previous section can be imitated by the use of a small value of  $\Delta$  in the  $Y'(s')$  process. As  $\Delta$  approaches zero, the solution of the analogous discrete problem would be expected to converge to the solution of the Wiener process problem. In particular, for the one-armed bandit problem this leads to the initial approximation

$$(2.2) \quad \tilde{y}(1+n\cdot\Delta) \approx \tilde{y}_n(\Delta)$$

where  $\tilde{y}(1+n\cdot\Delta)$  denotes the optimal boundary for the one-armed bandit problem evaluated at  $s = 1+n\cdot\Delta$ .

It remains to evaluate the sequence  $\{\tilde{y}_n(\Delta)\}$ . Consider the  $Y'(s')$  process defined as above on the grid of points  $\{(y', s') : s' = 1+n\cdot\Delta, y' = c + k\cdot\Delta^{1/2}; n = 0, 1, 2, \dots, k = 0, \pm 1, \pm 2, \dots\}$ . Note that the grid is completely specified by the parameter  $c$  (for convenience, assume  $0 \leq c < \Delta^{1/2}$ ). Examination of (2.1) with the particular form of  $d(y, s)$  given in (1.3) reveals that for any given choice of  $\Delta$ , if the points  $\{(y', 1+n\cdot\Delta) : y' \leq y^*\}$  are stopping points then so are the points  $\{(y', 1+(n+1)\cdot\Delta) : y' \leq y^* - \Delta^{1/2}\}$ . This observation, together with the fact that the sequence  $\{\tilde{y}_n(\Delta)\}$  is non-increasing, implies that when using the backward induction algorithm (2.1) to classify the grid points as either stopping or continuation points, the comparisons implied by (2.1) need be carried out at only a single value of  $y'$  for each fixed value of  $s'$ . The algorithm (2.1) can now be easily implemented in a direct fashion.

Due to the special nature of the one-armed bandit problem, namely the fact that all points  $(y, s)$  with  $y > 0$  and  $s > 1$  are continuation points, one might expect to be able to improve somewhat upon the naive approach outlined above. Consider a particular path of the  $Y'(s')$  process

originating at the point  $(y', s') = (c+2\Delta^{1/2}, 1+n\Delta)$ . The path of the  $Y'(s')$  process could hit the line  $y' = c+\Delta^{1/2}$  for the first time at  $s' = 1+(n-1)\Delta, 1+(n-3)\Delta, \dots$ . Alternately, the path could remain above the line  $y' = c+\Delta^{1/2}$  all the way to  $s' = 1$ . Noting that the points  $(c+\Delta^{1/2}, s')$  are continuation points for all  $s' > 1$  (since  $\tilde{y}_1(\Delta) = 0$  and the sequence  $\tilde{y}_n(\Delta)$  is non-increasing) leads to the relation

$$(2.3) \quad \rho'(c+2\Delta^{1/2}, 1+n\Delta) = \sum_{m=1}^n p_m \rho'(c+\Delta^{1/2}, 1+(n-m)\Delta) + \sum_{k=1}^{n+1} q_{n,k} d(c+(k+1)\Delta^{1/2}, 1)$$

where  $p_m$  is the probability that an ordinary random walk starting at 0 first passes through 1 at time  $m$  and  $q_{n,k}$  is the probability that an ordinary random walk starting at 0 stays above -1 until time  $n$  and achieves level  $k-1$  at time  $n$ . From Feller (1968, p. 89, Theorem 2) one finds

$$(2.4) \quad p_m = \begin{cases} 0 & \text{for } m \text{ even} \\ \frac{1}{m} \cdot \binom{m}{\frac{m+1}{2}} \cdot 2^{-m} & \text{for } m \text{ odd.} \end{cases}$$

In addition we have the recursive relation  $p_{m+2} = \frac{m}{m+3} p_m$  with  $p_1 = \frac{1}{2}$  and  $p_2 = 0$ . From Feller (1968, p. 73, Ballot Theorem) one also finds that

$$(2.5) \quad q_{n,k} = \begin{cases} 0 & \text{for } n \text{ even and } k \text{ odd} \\ 0 & \text{for } n \text{ odd and } k \text{ even} \\ \frac{k}{n+1} \cdot \left( \frac{n+1}{\frac{n+k+1}{2}} \right) \cdot 2^{-n} & \text{otherwise.} \end{cases}$$

The relation (2.3) provides a modified method of carrying out the backward induction which we shall call the truncation method: At  $s' = 1$ , the risks are specified by  $d(y,s)$ . At any stage  $s' = 1+n\cdot\Delta$ , compute the risk at  $y' = c+2\cdot\Delta^{1/2}$  by means of (2.3). The risks at the levels  $y' = c+k\cdot\Delta^{1/2}$  for  $k = 1,0,-1,-2,\dots$  are computed using the algorithm (2.1) in the fashion described above.

Returning for a moment to the one-armed bandit problem, it is well-known (and obvious from (1.4)) that changing the stopping cost function  $d(y,s)$  by adding to it any solution of the heat equation leaves the optimal continuation region unchanged. For the present purposes, it is convenient to consider the new stopping cost function  $d_0(y,s)$  defined by  $d_0(y,s) = d(y,s) + y$ , that is,

$$(2.6) \quad d_0(y,s) = y - y/s$$

for  $s \geq 1$  with stopping enforced at  $s = 1$ . Note that  $d_0(y,1) \equiv 0$ . Denoting the corresponding optimal risk by  $\rho_0(y,s)$ , the algorithm (2.1) becomes

$$(2.7) \quad \rho'_0(y', 1+n\Delta) = \min\{d'_0(y', 1+n\Delta), \frac{1}{2}[\rho'_0(y'+\Delta^{1/2}, 1+(n-1)\Delta) + \rho'_0(y'-\Delta^{1/2}, 1+(n-1)\Delta)]\}$$

for  $n > 1$  with  $\rho'_0(y', 1) \equiv 0$ . Further, the relation (2.3) becomes

$$(2.8) \quad \rho'(c+2\Delta^{1/2}, 1+n\Delta) = \sum_{m=1}^n p_m \cdot \rho'_0(c+\Delta^{1/2}, 1+(n-m)\Delta).$$

This reduces the computation involved in carrying out the truncation method.

To this point we have simply described the direct and truncation methods of carrying out the backward induction algorithm for the  $Y'(s')$  process when the motion of the process is restricted to a grid specified by a parameter  $c$ . Implementing the algorithm for a sequence of  $c$  values (with the same fixed value of  $\Delta$ ) allows the sequence  $\{\tilde{y}_n(\Delta)\}$  to be determined to within any desired degree of accuracy.

### 3. A Refined Approximation

In the previous section we have presented simple methods of obtaining initial approximations to the solutions of optimal stopping problems for a zero drift standard Wiener process. These methods involve replacing the Wiener process problem by an analogous discrete problem involving dichotomous random variables. The relation of the solution of any such Wiener process problem to the solutions of an entire class of analogous discrete problems is considered in Chernoff and Petkau (1976). A particular result of that paper is the following simple approximate relation between the optimal boundary of the Wiener process and the optimal boundary for the particular

analogous discrete problem described in the previous section

$$(3.1) \quad \tilde{y}(1+n\cdot\Delta) = \tilde{y}_n(\Delta) \pm 0.5 \Delta^{1/2}$$

(the sign being determined so as to make the continuation region for the Wiener process problem larger). For the one-armed bandit problem, this leads to the following refinement of (2.2)

$$(3.2) \quad \tilde{y}(1+n\cdot\Delta) \approx \tilde{y}_n(\Delta) - 0.5 \Delta^{1/2} .$$

To illustrate the accuracy of these approximations, it would be desirable to evaluate these approximations in a Wiener process problem for which the exact solution is known. A normalized version of a problem discussed in Van Moerbeke (1974) is the following:  $\{X(s), 0 < s \leq 1\}$  is a zero drift standard Wiener process. One is permitted to stop the process at any time  $s$ ,  $0 < s \leq 1$ . The reward for stopping the process at the point  $(x,s)$  is given by  $g(x,s)$  defined by

$$(3.3) \quad g(x,s) = \begin{cases} 1-s+2x^2 & \text{for } x \geq 0 \\ 1-s & \text{for } x \leq 0 . \end{cases}$$

The problem is to find a stopping time that maximizes the expected reward. Van Moerbeke (1974) proves that the optimal continuation region for this problem can be described as  $\{(x,s): x > \tilde{x}(s), 0 < s \leq 1\}$  where  $\tilde{x}(s) = -\alpha (1-s)^{1/2}$  and  $\alpha$  is the solution of the simple equation

$\alpha \cdot \int_0^{\infty} \exp[\lambda\alpha - \lambda^2/2] d\lambda = 1$  which can easily be determined to be

$\alpha \doteq 0.5061$ . Modifying the reward function to be  $\hat{g}(x,s) = g(x,s) - 2[x^2 + 1 - s]$  does not change the solution of this problem and it is easily seen that the methods of the previous section are directly applicable (in particular,  $\hat{g}(x,1) \equiv 0$  for  $x \geq 0$ ).

This Wiener process problem has been approximated by three different analogous discrete problems, those corresponding to  $\Delta = 0.01, 0.0025$  and  $0.000625$ . In addition, for each fixed value of  $\Delta$  the computations were carried out for values of the parameter  $c$  varying from 0 to  $\Delta^{1/2}$  in steps of 0.001. Thus each individual member of each of the three sequences  $\{\tilde{x}_n(\Delta)\}$  is located to within an error of 0.001. In addition the corrected sequences  $\{\tilde{x}_n^*(\Delta)\}$  defined by  $\tilde{x}_n^*(\Delta) = \tilde{x}_n(\Delta) - 0.5\Delta^{1/2}$  were evaluated. These six approximating sequences and the exact solution  $\tilde{x}$  are illustrated in Figure 1 in the  $(x,s)$  scale. Here  $\tilde{x}_1 = \{\tilde{x}_n(0.01)\}$ ,  $\tilde{x}_2 = \{\tilde{x}_n(0.0025)\}$ ,  $\tilde{x}_3 = \{\tilde{x}_n(0.000625)\}$  and similarly  $\tilde{x}_1^* = \{\tilde{x}_n^*(0.01)\}$ ,  $\tilde{x}_2^* = \{\tilde{x}_n^*(0.0025)\}$ ,  $\tilde{x}_3^* = \{\tilde{x}_n^*(0.000625)\}$ . This figure clearly illustrates that for this particular problem whereas the approximations provided by  $\tilde{x}_1, \tilde{x}_2$  and  $\tilde{x}_3$  are quite crude, the approximations provided by  $\tilde{x}_1^*, \tilde{x}_2^*$  and  $\tilde{x}_3^*$  (particularly both  $\tilde{x}_2^*$  and  $\tilde{x}_3^*$ ) are exceptionally accurate, being virtually indistinguishable from each other and from the exact solution. The fact that  $\tilde{x}_1^*$  is not too accurate reflects the fact that when using these approximations, one must begin with a reasonably small value of  $\Delta$ .

The exceptional performance of the refined approximation (3.1) in Van Moerbeke's problem leads us to hope that the same type of behavior will occur in the one-armed bandit problem. In order to examine this possibility, the one-armed bandit problem was approximated by three different analogous discrete problems, those corresponding to  $\Delta = 1.0, 0.25$  and  $0.0625$ . For each fixed value of  $\Delta$  the computations were carried out for the region  $1 \leq s \leq 100$  and for values of the parameter  $c$  varying from 0 to  $\Delta^{1/2}$  in steps of 0.01. Thus each individual member of each of the three sequences  $\{\tilde{y}_n(\Delta)\}$  is located to within an error of 0.01. In addition the corrected sequences  $\{\tilde{y}_n^*(\Delta)\}$  defined by  $\tilde{y}_n^*(\Delta) = \tilde{y}_n(\Delta) - 0.5\Delta^{1/2}$  were evaluated. These six approximating sequences are illustrated in Figure 2. Here  $\tilde{y}_1 = \{\tilde{y}_n(1.0)\}$ ,  $\tilde{y}_2 = \{\tilde{y}_n(0.25)\}$ ,  $\tilde{y}_3 = \{\tilde{y}_n(0.0625)\}$  and similarly  $\tilde{y}_1^* = \{\tilde{y}_n^*(1.0)\}$ ,  $\tilde{y}_2^* = \{\tilde{y}_n^*(0.25)\}$ ,  $\tilde{y}_3^* = \{\tilde{y}_n^*(0.0625)\}$ . This figure clearly indicates that for the one-armed bandit problem the approximations provided by  $\tilde{y}_1^*$ ,  $\tilde{y}_2^*$  and  $\tilde{y}_3^*$  are exceptionally accurate, these curves being indistinguishable from one another.

As pointed out in the introduction, for applications of the solution of the one-armed bandit problem, the  $(z,t)$  scale where  $z = -y/s$  and  $t = 1/s$  is more appropriate. In order to obtain an accurate approximation to the optimal stopping boundary in the  $(z,t)$  scale in as efficient a manner as possible, the computations were carried out as follows: Beginning with a very small value of  $\Delta$ , the boundary was approximated in a small interval of  $s$  in the manner described above. Successively larger values of  $\Delta$  were then employed to approximate the boundary in successively larger intervals of  $s$ . These approximations to the optimal boundary,

determined in overlapping intervals of  $s$ , were then superimposed to obtain the final approximation to the optimal boundary. Since the values of  $\Delta$  used were chosen in such a way as to yield the desired accuracy, only the value  $c = 0$  was used in these computations. The computations were carried out using both the direct and the truncation method. The truncation method reduced the computation time required by a factor of two. The resulting approximation to the optimal stopping boundary and the asymptotic expansions of Chernoff and Ray (1965) are illustrated in Figure 3. Here  $\tilde{z}_0$  and  $\tilde{z}_1$  denote the boundaries obtained using the asymptotic expansions for  $t$  close to 0 ( $s$  large) and  $t$  close to 1 ( $s$  close to 1) respectively and  $\tilde{z}$  denotes the boundary obtained by means of the computations described above.

#### 4. Discussion

The optimal stopping boundary for the one-armed bandit problem has been approximated before and appears in the literature in both Mallik (1971) and Chernoff (1972). These previous approximations were determined without the benefit of the "correction for continuity" given in (3.1). The purpose of the present paper was to describe explicitly how these boundaries could be determined, to demonstrate that exceptional accuracy is possible and to indicate that these computations can be quite efficient. Obtaining the present approximation to the boundary of the one-armed bandit problem involved ten separate runs, the  $i$ -th run approximating the boundary in the region  $s = 1$  to  $s = 1 + 1000 \cdot \Delta$  using a grid determined by  $\Delta = 10^{-4} \cdot 4^{i-1}$ . The entire computation, approximating the boundary in the

region  $1 \leq s \leq 25,000$ , required just 14 seconds of computation time on the IBM 370/168 at U.B.C. The objective in the present computation was to obtain an accurate approximation in the  $(z,t)$  scale. Detailed examination of the output leads to the empirical estimate that in the  $(z,t)$  scale the boundary has been located to within an error of about 0.002.

As indicated in the introduction, these same methods could be applied with equal facility to any optimal stopping problem of the general form described there. (In Petkau (1977) these methods have been employed to obtain the optimal continuation region for a stopping problem in which the optimal continuation region can be described as the set  $\{(y,s): y_1(s) < y < y_2(s), s > 1\}$  where  $y_1(s) \neq y_2(s)$ .) The connection between such optimal stopping problems and free boundary problems involving the heat equation of the form (1.4) makes it clear that these same methods could be used to determine numerical solutions of such free boundary problems. The problem of obtaining numerical solutions to free boundary problems has received considerable attention in the literature (see for example Sackett (1971) and Meyer (1977)). Whether the methods proposed here provide a reasonable alternative to these more general methods is a question that remains to be answered.

#### Acknowledgment

The author would like to acknowledge several helpful conversations with Herman Chernoff.

## References

1. Chernoff, H. (1967), "Sequential models for clinical trials", Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability, IV, 805-812.
2. \_\_\_\_\_ (1968), "Optimal stochastic control", Sankhyā, Series A, 30, 221-252.
3. \_\_\_\_\_ (1972), Sequential Analysis and Optimal Design, SIAM monograph, Philadelphia.
4. \_\_\_\_\_ and Petkau, A.J. (1976), "An optimal stopping problem for sums of dichotomous random variables", Annals of Probability, 4, 875-889.
5. \_\_\_\_\_ and Ray, S.N. (1965), "A Bayes sequential sampling inspection plan", Annals of Mathematical Statistics, 36, 1387-1407.
6. Feller, W. (1968), An Introduction to Probability Theory and its Applications, Vol. 1, 3rd edition, John Wiley and Sons, Inc., New York.
7. Mallik, A.K. (1971), "Sequential estimation of the common mean of two normal populations", Technical Report No. 42, Department of Statistics, Stanford University.
8. Meyer, G.H. (1977), "One-dimensional parabolic free boundary problems", SIAM Review, 19, 17-34.
9. Petkau, A.J. (1977), "Sequential medical trials for comparing an experimental with a standard treatment", Journal of the American Statistical Association (to appear).

10. Sackett, G.G. (1971), "Numerical solution of a parabolic free boundary problem arising in statistical decision theory", Mathematics of Computation, 25, 425-434.
11. Van Moerbeke, P.L.J. (1974), "Optimal stopping and free boundary problems," Rocky Mountain Journal of Mathematics, 4, 539-578.

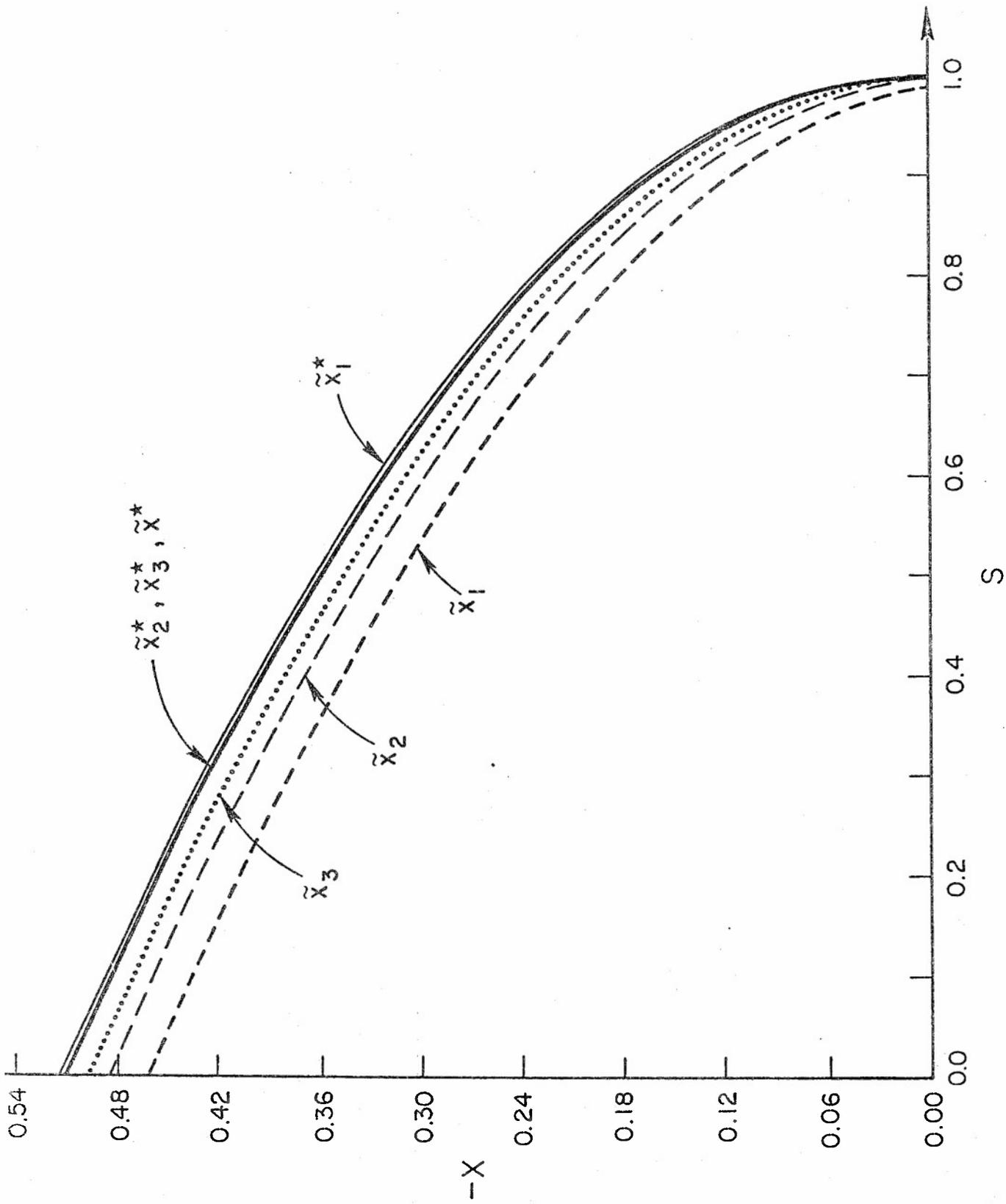


Figure 1. Van Moerbeke's Problem

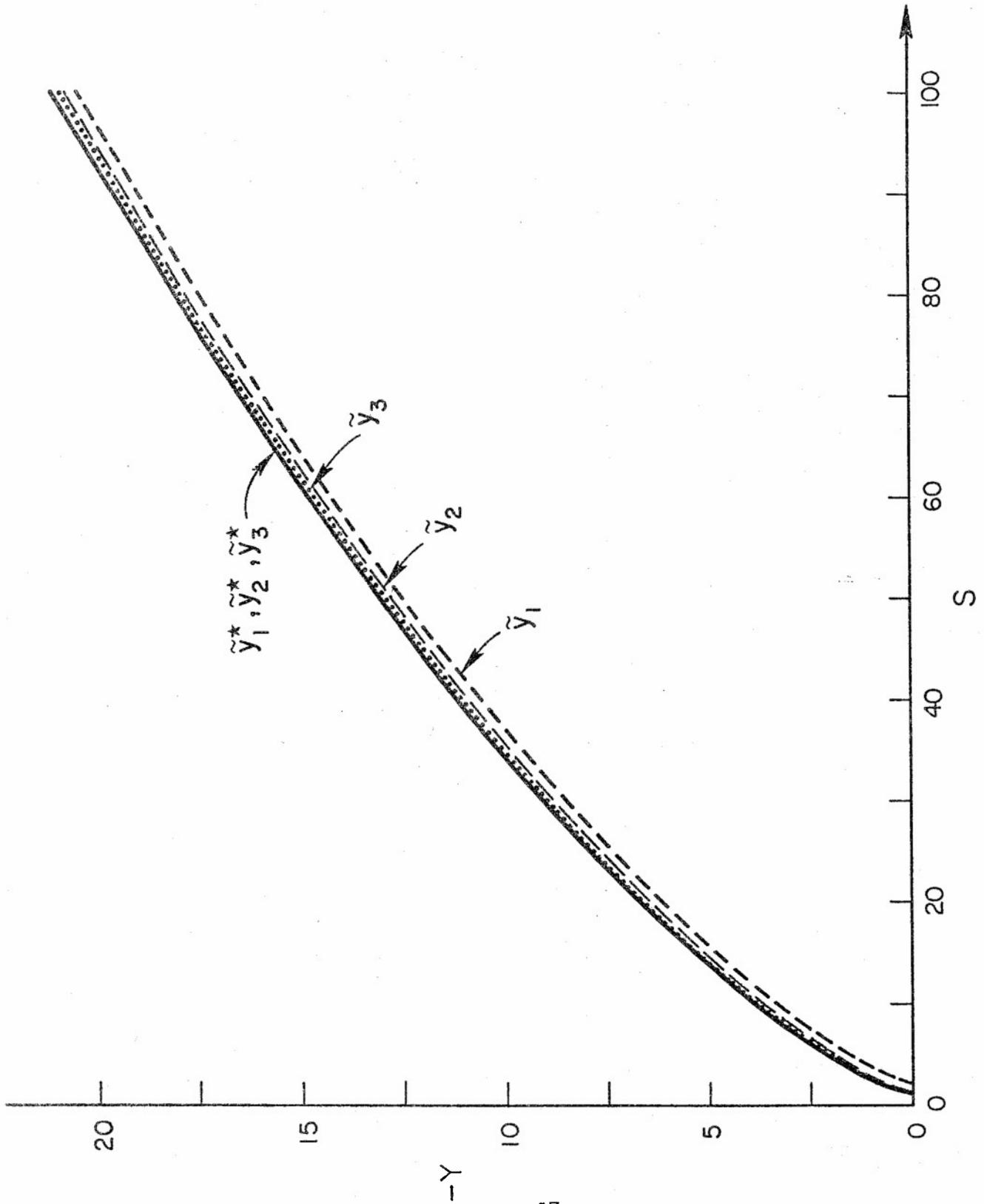


Figure 2 One-Armed Bandit Problem

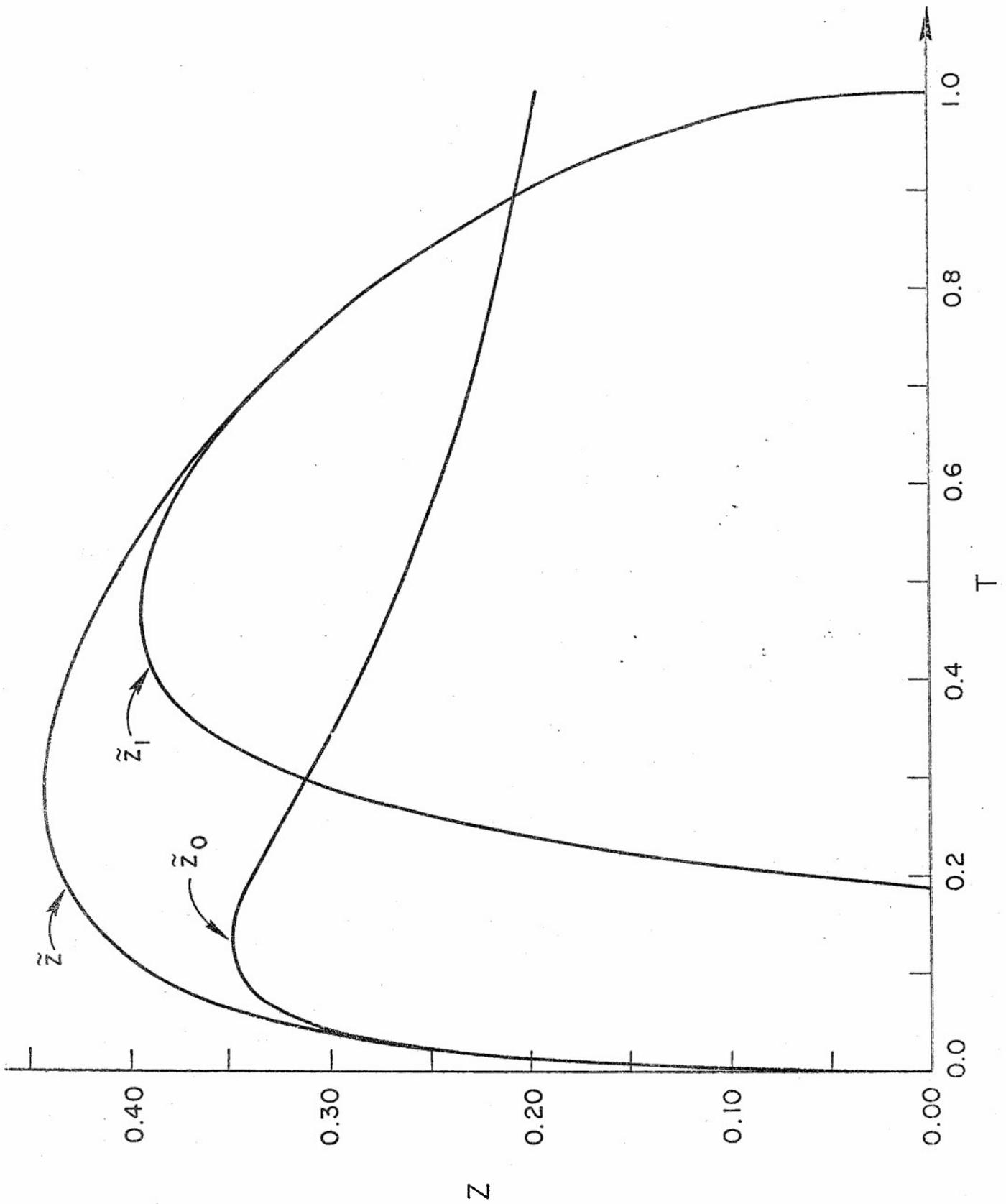


Figure 3 One-Armed Bandit Problem

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 13	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Approximate Solutions for Certain Optimal Stopping Problems		5. TYPE OF REPORT & PERIOD COVERED TECHNICAL REPORT
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Albert John Petkau		8. CONTRACT OR GRANT NUMBER(s) DAAG29-77-G-0031
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Statistics Stanford University Stanford, CA 94305		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS P-14435-M
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE January 5, 1978
		13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution Unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents. This report partially supported under Office of Naval Research Contract N00014-75-C-0555 (NR-042-331), Dept. of Math. Massachusetts Inst. of Tech.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Optimal stopping, Wiener process, backward induction, one-armed bandit		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) See reverse side		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 68 IS OBSOLETE  
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Simple methods of determining numerical approximations to the optimal continuation regions of certain optimal stopping problems for a zero drift standard Wiener process are described. The methods involve the use of the solutions of analogous discrete problems together with a "correction for continuity" due to Chernoff and Petkau (Annals of Probability (1976), 4, 875-889). A problem due to Van Moerbeke, the exact solution of which is known, and the one-armed bandit problem are considered as illustrations of the methods.