MEASUREMENT OF
THE SPATIAL CORRELATION FUNCTION OF AN HF WAVE
AFTER PASSAGE THROUGH THE IONOSPHERE

Final Report Submitted to
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Arlington, Virginia
MEASUREMENT OF THE SPATIAL CORRELATION FUNCTION OF AN HF WAVE AFTER PASSAGE THROUGH THE IONOSPHERE.

This is a final report under Contract N00014-75-C-1178, which was a follow-on to N00014-67-A-0216-0033. This latter contract was initially awarded for the period 1 October 1973 to 30 September 1974, and a final report on the work through 30 September 1974 was submitted in December of 1974. Contract N00014-67-A-0216-0033 was extended from 1 October 1974 to 30 September 1975, and the N00014-75-C-1178 follow-on covered the period 1 August 1975 to 30 June 1976. This report covers the work done during the last year of N00014-67-A-0216-0033 and during the period of N00014-75-C-1178.

SUMMARY

The initial phase of an experimental data-taking program has been completed leading to the measurement of the spatial correlation function of the phase of an HF wave after passage through the ionosphere. The propagation path for the experiment was West-East, a distance of 2,480 kilometers. The transmitting site was in Boulder, Colorado. The receiving site, which was in the Delaware Valley, Pennsylvania, had a 40-kilometer baseline. The receiving system consisted of 8 phase-synchronized HF receivers. The raw data, which are in excess of $10^8$ bits, have been recorded digitally on magnetic tape. Data reduction and analysis will take about one year.

INTRODUCTION

Nonhomogeneities in the refractive index of the ionosphere induce time-varying spatial variations in the phase of a wave received from a distant high frequency (HF) radio source after passage of the wave through the ionosphere. The correlation distance of the phase approximately equals the nominal, maximum useful aperture length of an HF antenna whether it is designed for point-to-point communications or for over-the-horizon radar. In the former case large antenna size helps in providing a large signal-to-noise ratio (SNR) which in turn is required for the maintenance of a useful communications circuit. When the antenna size is less than the nominal correlation distance, the SNR grows linearly with antenna size; this is because all signal components received by the antenna are summed coherently. However when the antenna size exceeds the nominal correlation distance, signal decorrelation across the aperture precludes fully coherent summation and results in less than a linear buildup of SNR with antenna size.
In the second case, that of over-the-horizon radar, a second factor of importance arises. In addition to a linear improvement in SNR with antenna size, the angular directivity similarly improves (the beamwidth of a diffraction limited aperture is approximately the ratio of wavelength to aperture size). However, when the antenna length exceeds the spatial correlation distance, the aperture ceases to be diffraction limited and the angular directivity no longer grows with size. It is in the interest of overcoming some of the limitations in the HF radar problem that an experimental 40-kilometer HF data-taking array was constructed.

There is a third case in which knowledge of the spatial correlation function may prove useful in future system design: It is in the field of HF direction finding (HFDF). The apparent bearing of an HF source, as determined by the direction of arrival of an HF wave from that source, often appears to wander within a few degrees of the true bearing. The source of the apparent bearing wander is large-scale wave motion within the ionosphere, sometimes called travelling ionospheric disturbances. The long wavelengths (tens of kilometers) of these internal waves in the ionosphere and their speeds (the order of 100 meters per second) result in a time scale of bearing wander which is the order of minutes. Observation of the apparent bearing over several such cycles tends to disclose the true bearing, in essence by averaging the bearing data over time.

An average of bearing data also may be made over space, provided that the measurements extend over a suitably large distance; that is, over a measuring interval large compared to the spatial correlation distance. Hence a nondiffraction-limited aperture may prove useful in HFDF. Although an optimum signal processing theory for large aperture HFDF has yet to be developed, one of the dominant parameters of the theory is known to be the spatial correlation distance of the phase of the HF wave after it passes through the ionosphere. Thus the information to be extracted from the data may prove useful in HFDF.

The Valley Forge Research Center of the Moore School of Electrical Engineering, University of Pennsylvania, is engaged in an ongoing study of the limiting angular resolution properties of large apertures, and of self-correction techniques for overcoming the effects of nondiffraction-limited and time-varying...
apertures [1]. In a two-year study of the HF radar problem, a theory was developed of the maximum aperture which could be made to function as if it were diffraction limited through the use of adaptive, self-focusing signal processing techniques subject to the conditions experienced by HF signals in passage through the ionosphere [2,3]. The maximum size was found to be proportional to the phase correlation distance. When a search of the literature disclosed that this quantity had hitherto not been measured (perhaps because of the inherent difficulties in the measurement) an experimental system was developed to measure the necessary data, and five successful data recording experiments have been conducted. These data are stored digitally on magnetic tape. Later we expect to derive from these data the relevant spatial statistics of the phase of the HF signal received via the ionosphere during these five experiments.

THE EXPERIMENT*

An ionosonde of the Environmental Research Laboratory, National Oceanic and Atmospheric Administration, U. S. Department of Commerce, Boulder, Colorado, radiated an HF pulse train. The carrier frequency was 9.010 MHz, the pulse repetition frequency was 100 Hz, and the duration was 64 microseconds. Both the carrier frequency and the repetition frequency were derived from an atomic standard and hence were highly stable. The transmitting antenna was modified so as to depress the beam from the vertical in an easterly direction, thereby permitting a signal with adequate signal strength to be received in the Delaware Valley, Pennsylvania.

The receiving array consisted of eight receivers in the Delaware Valley.

[1] See articles 1 and 2 of VFRC Quarterly Progress Reports Nos. 8 through 14.

*Details on the experimental equipment are found in Valley Forge Research Center Quarterly Progress Reports Nos. 10 through 14, (August 1974 - August 1975).
The receivers were special units designed and built by VFRC staff for this experiment. The maximum separation was nearly 40 kilometers. The locations and spacings were so chosen that the set of inter-pair spacings would provide a useful distribution of abscissa values for sample points on the phase correlation function. The positions are given in the map of Figure 1. Table 1 lists the distances (km) and Table 2 lists the directions (degrees clockwise from North) between stations. The locations were at the Research Laboratory, at homes of faculty and staff members, at the University's Flower and Cook Observatory, and at a nearby brass foundry.

<table>
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<td>15.3</td>
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**Table 1. Interstation Distances (km).**

<table>
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<th>5</th>
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<td>156.0</td>
<td>85.1</td>
<td>85.5</td>
</tr>
</tbody>
</table>

**Table 3. Interstation Directions**

(Degrees clockwise from geographic North)
Figure 1. HF receiving array

Site 1 Location
(Valley Forge, Pa.)
40° 05', 14° N. Lat.
75° 29', 12° W. Long.

<table>
<thead>
<tr>
<th>Site Number</th>
<th>Name</th>
<th>Range (Km.)</th>
<th>Bearing (Degrees clockwise from North)</th>
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<tbody>
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<td>Upper Site</td>
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<td>Seidman</td>
<td>9.3</td>
<td>184.0</td>
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<td>Flower &amp; Cook</td>
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<td>29.5</td>
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<td>Garner</td>
<td>25.4</td>
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</tr>
<tr>
<td>8</td>
<td>Steinberg</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The function of the receiving system was to measure and to record phases of HF signals from Boulder received by the eight receivers. Accomplishment of this task required synchronizing the local oscillators in the receivers. These phase synchronized reference waves were obtained from the carrier of a local radio station, KYW, which broadcasts an acceptable signal strength at 1060 KHz throughout the Delaware Valley. Within each receiver the carrier was freed of its modulation by a crystal filter, resulting in a 1.06 MHz reference wave with approximately $1^\circ$ rms phase jitter. A frequency synthesizer in each receiver developed a large set of stable frequencies in the HF

![Diagram](image_url)

**FIGURE 14. OVERALL SYSTEM AT THE RECEIVING REGION**
region of the spectrum from this phase stable reference wave. Proper choice of transmitter frequency and local oscillator receiver frequency permitted phase detection of the received wave.

Figure 2 shows the organization of the receiving system. A phase comparison is made between the local reference and the succession of HF pulses from Colorado. The analog phase measurement in each receiver was sampled 100 times per second, digitized, and transmitted via data modem and telephone circuit to the Valley Forge Laboratory, where the signals were entered into a buffer digital store, adjusted in format, and read out onto a 7-track tape.

Figure 3 shows the receiving installation at Site 8. Figure 4 is a scope photo showing the envelope of the received pulse (upper trace) and a timing marker (just under the main pulse) showing the location of the pulse tracking gate.
Five successful tests were run during June, 1975. The dates, times and durations are given in Table 3. (Midpath time is approximately one hour earlier, and GMT is five hours later.) The recorded data are in excess of $10^8$ bits.

![Oscilloscope photo showing received signal (upper trace) and tracking gate](image)

**TABLE 3. SUCCESSFUL TESTS**

<table>
<thead>
<tr>
<th>DATE</th>
<th>TIME (EST)</th>
<th>DURATION (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 June</td>
<td>6:30 AM</td>
<td>60</td>
</tr>
<tr>
<td>21 June</td>
<td>5:30 AM</td>
<td>120</td>
</tr>
<tr>
<td>24 June</td>
<td>4:00 AM</td>
<td>60</td>
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<tr>
<td>25 June</td>
<td>Midnight</td>
<td>60</td>
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<tr>
<td>27 June</td>
<td>7:00 AM</td>
<td>60</td>
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The primary statistic sought in this work is the spatial correlation function of the phase of the received wave after its passage through the ionosphere. The abscissa of the correlation function is a distance on the ground. Samples of the correlation function will be measured at distances equal to the
spacings between receiving stations. Correlation information in the East-West direction (along the direction of propagation) and in the North-South direction (transverse to the direction of propagation) will be obtained. On theoretical grounds the correlation functions are expected to be similar. The resources of the experimental system were devoted roughly equally to measurements in these two directions and with some overlap to permit comparison. Table 4, constructed from the earlier tabular data, shows the interstation distances in the N-S and E-W directions. Fourteen other interstation separations permit measurement of correlation coefficients in a variety of directions; comparison will be made between these values and those measured along the direction of propagation (E-W) and transverse to it (N-S).

<table>
<thead>
<tr>
<th>N - S STATIONS</th>
<th>SEPARATIONS (km)</th>
<th>E - W STATIONS</th>
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The value of the correlation function at a distance \( d_{ij} \) corresponding to the spacing between the \( i \)th and \( j \)th stations may be calculated either by direct multiplication or from the variance of the phase difference function. The correlation coefficient

\[
\rho_{ij} = \frac{\phi_i \phi_j}{\sqrt{\phi_i^2 \phi_j^2}} = \frac{\phi_i \phi_j}{\phi^2}
\]

(1)

where \( \phi_i, \phi_j \) are the time samples of the received phase after removal of the artifacts and the slowly varying phase roll components ascribable to slight
frequency differences between the transmitter and receivers. The overbar represents a time average. Since the phase variations $\phi_i$ and $\phi_j$ are derived from a common source, it is assumed (in the expression above) that their mean square values are the same; this assumption will be tested by independent measurement of $\bar{\phi}_i^2$ and $\bar{\phi}_j^2$.

It is evident from (1) that $\rho_{ij}$ is available from the direct measurement of $\bar{\phi}_{ij}$ and $\bar{\phi}_i^2$. It also is available from the variance of the phase difference:

$$\sigma_{ij}^2 = (\bar{\phi}_i - \bar{\phi}_j)^2 = 2\bar{\phi}_i^2 - 2\bar{\phi}_i \bar{\phi}_j = 2\phi^2(1 - \rho_{ij}) \quad (2)$$

Although (1) is more direct, there may be instances in which (2) will prove useful, as when rapid movement in the height or the tilt of the ionosphere causes rapid but correlated variations in the phases of the received waves at two receivers. Such correlated variations are eliminated in calculations based on the phase differences.

The noise properties of the two measurement procedures are different, a factor which has to be reckoned with in the numerical calculations. If $n_i$ and $n_j$ are the time samples of adaptive phase noise in the $i$th and $j$th receivers, the average product of the phase measurements becomes

$$\bar{\phi}_{ij}^2 = (\bar{\phi}_i + n_i) (\bar{\phi}_j + n_j)$$

$$= \bar{\phi}_i \bar{\phi}_j + \bar{\phi}_i n_j + \bar{\phi}_j n_i + n_i n_j = \bar{\phi}_i \bar{\phi}_j \quad (3)$$

which is unbiased. The latter three terms reduce to zero under the assumptions (1) that a received wave and the receiver noise are independent, (2) that both are zero mean and (3) that the phase noises from receiver to receiver are statistically independent.

The phase difference variance, on the other hand, is not unbiased by the noise. Equation (2) becomes

$$\sigma_{ij}^2 = (\bar{\phi}_i + n_i - \bar{\phi}_j - n_j)^2 = 2\phi^2 - 2\bar{\phi}_i \bar{\phi}_j + \bar{n}_i^2 + \bar{n}_j^2 \quad (4)$$

under the same assumptions. Under a fourth assumption that the additive phase noises in the various receivers share common statistics, (4) becomes
The phase noise power \( \overline{n^2} \) can be measured using (5) provided that two of the stations are sufficiently close to each other such that their phases have nearly unity correlation, in which case \( \overline{\phi_i \phi_j} = \overline{\phi_j} \). \( \overline{n^2} \) also can be measured when two stations are sufficiently far apart that their phases have nearly zero correlation, in which case \( \overline{\phi_i \phi_j} = \overline{\phi_i} \overline{\phi_j} = 0 \) under assumption (2) above; comparison of (3) and (5) then yields \( \overline{n^2} \). If both situations can be found to exist in the data, then independent measurements of \( \overline{n^2} \) can be made and compared.

Another interesting statistic which may be available from the phase data is the first probability density function (pdf) of the signal phase. The measured phase in an arbitrary receiver is \( \hat{\phi} = \phi + n \) where \( \phi \) is the signal phase and \( n \) is the additive phase noise. Based on the first assumption made earlier (statistical independence of \( \phi \) and \( n \))

\[
\omega(\hat{\phi}) = \omega(\phi) \ast \omega(n)
\]

(6)

where \( \omega(\cdot) \) means pdf and \( \ast \) means convolution. Thus the shape of \( \omega(\hat{\phi}) \), which can be extracted from the data, differs from \( \omega(\phi) \), the pdf of the signal phase, because the latter is convolved with the pdf of the phase noise.

Although shape information may be obscured, the signal phase variance can be calculated from the variance relation in summing independent, random variables:

\[
\sigma^2_\phi = \sigma^2_\phi + \sigma^2_n
\]

(7)

Based on assumption (2) above (zero means of \( \phi \) and \( n \)), \( \sigma^2_\phi = \overline{n^2} \) and \( \sigma^2_n = \overline{n^2} \), the calculations of which were discussed earlier. Thus the signal phase variance is the difference between the mean square value of the total phase measurement and the value calculated for the noise.

Shape information also may be possible to extract. If \( \overline{n^2} \ll \overline{\phi^2} \), as is expected, \( \omega(\hat{\phi}) \) will be slightly broader than \( \omega(\phi) \) (according to (2)). Their shapes, however, will be very similar. In particular, if the phase noise power is small compared to the signal-phase power and if \( \omega(\hat{\phi}) \) appears Gaussian, it may be assumed that \( \omega(\phi) \) also is approximately Gaussian.
DISCUSSION

The spatial variation of the phase of a signal received along a line on the ground after the wave has passed through the ionosphere is believed to have at least two components. The first is due to scattering from random nonhomogeneities due to random ionization and recombination phenomena. The second is due to large-scale effects resulting from wave motion in the ionosphere. The former introduces a random phase variation across an aperture which sets an irreducible limit to the effective maximum diffraction-limited length which may be achieved. The latter is quasi-deterministic in a quasi-static sense; that is, time-varying curvatures and tilts in the ionosphere due to the passage of waves may induce relatively simple phase variations in the wave front. For example, at a given instant in time the phase variation of a wave across a large aperture might be well represented by a low order polynomial. The constant term represents the fixed phase. The linear term represents an angular tilt of the ionosphere. The quadratic term represents a curvature associated with the mean refractive index of a ray path through the ionosphere as a function of transverse position within it. This term may be identified with the properties of a spherical lens. High order terms would be the equivalent of distortion-inducing features of an optical system.

The spatial correlation function which we will attempt to extract from the recorded data will include both the effects of scattering and the large-scale phenomena. The result of the measurements will set bounds or lower limits to the maximum size that diffraction-limited apertures can be made under conditions in which the measurements were made. However, through the use of relatively sophisticated, adaptive signal processing circuits of the type being explored at VFRC some of the normally deleterious effects of the large scale phenomena may be removed. Specifically we believe that the linear term may be removed on a continuous, dynamic basis. We feel too that the slowly changing quadratic variation can be removed. Thus the irreducible distortion which would limit useful size of an aperture would be determined by the contributions to the spatial correlation of the phase of the energy scattered by the small-scale, random variations in refractive index within the ionosphere and by the higher order terms of a polynomial expansion of the phase variation induced by the large scale ionospheric phenomena.
It is not known at this writing the extent to which these phenomena will be separable in the data. The calculation of the spatial correlation function, as described earlier, ignores the distinction made in this section. During the course of the data reduction program, consideration will be given to the separation of the effects of these different phenomena. Attention will also be given to the development of any unexpected statistical or deterministic phenomena evident from the data.
MEASUREMENT OF THE SPATIAL CORRELATION FUNCTION OF AN HF WAVE AFTER PASSAGE THROUGH THE IONOSPHERE

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PHILADELPHIA, PA 19104

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IONOSPHERE REFRACTIVE VARIATIONS
LARGE-APERTURE ANTENNA ARRAYS
COHERENCE IN TIME-VARYING APERTURES

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Experimental equipment was designed, built, and used in an experiment for the purposes of acquiring data, performing analyses, testing algorithms, and evaluating system characteristics for measurement of the spatial correlation function of the phase of an HF wave after passage through the ionosphere. For the test, 8 phase-synchronized receivers were designed and built, distributed randomly over a 40-kilometer baseline, and used to receive HF signals from a transmitting site some 2500 kilometers distant. Raw data in excess of 10^{20} bits were collected.

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