INTERFERENCE VERSUS FREQUENCY IN MEASUREMENTS IN A SHALLOW LAKE.

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22 May 61

USRL-RR-57

Reprinted from the Journal of the Acoustical Society of America
Vol. 33, No. 9, pages 1211-1213, September 1961
Interference Versus Frequency in Measurements in a Shallow Lake

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(Received March 22, 1961)

When an omnidirectional projector and hydrophone are closely spaced and in shallow water, and sound is transmitted from one to the other, a large interference signal is superimposed on the direct signal. If the received signal is plotted as a function of frequency, as in Fig. 2, the interference signal amplitude appears to be an incoherent series of irregular-shaped peaks and dips. Analytical analysis of the condition when the transducers are midway between a water-air surface and a bubble-covered bottom shows that the shape, amplitude, and frequency of the interference pattern are predictable as the result of a large number of multimode-path interference patterns.

INTRODUCTION

ACOUSTICAL measurements, particularly for the calibration of underwater sound transducers, are often made in shallow water. For economic or other practical reasons, the absence of good free field conditions must be tolerated.

The Underwater Sound Reference Laboratory calibrates transducers in a lake 25 to 30 ft deep. The depth is ample for measurements at ultrasonic frequencies because measurement transducers can be made directional to avoid difficulties with reflections from the top and bottom boundaries. Surface-reflected sound in the direction of minor lobes of the radiation pattern, and other types of interference, are easily recognized and accounted for. For example, Fig. 1 is a trace of a hydrophone output voltage versus frequency. The solid line is the measured voltage. The dashed line is the corrected voltage. The oscillating pattern can be identified as a surface reflection interference pattern

\[ \Delta \phi = \Delta \phi _s \]  \hspace{1cm} (1)

where \( \Delta \phi \) is the frequency interval between adjacent peaks or adjacent nulls, \( c \) is the speed of sound, and \( \Delta \phi _s \) is the path difference between the direct and the surface-reflected signal. Figure 1 is a decibel or logarithmic plot. On a linear plot the oscillating pattern would have an approximately sinusoidal shape. In either case, the correct level of the direct signal is easily ascertained from the maximum and minimum levels of the oscillations.

At audio frequencies the interference is not so simple. Figure 2 is an example of a hydrophone output voltage versus frequency. The sound projector was a USRL type H which has a smooth and almost constant transmitting response. The hydrophone was a Mace-type M115B, which has a constant receiving sensitivity. The 1.8 ratio of transducer separation to water depth was chosen to show the effect of interference in some detail. It is not the minimum ratio used at the USRL.

The interference oscillations appear to be monostatic, the frequencies are of irregular shape and the amplitude is much greater than can be accounted for by a single surface or bottom reflection. The oscillations do, however, repeat at the same frequency interval \( \Delta \phi _s \) as would a single surface reflection.

Because of the long wavelengths at audio frequencies, directional transducers are not feasible. Short projector-

Fig. 1 Measured hydrophone output voltage versus frequency, illustrating a typical oscillating interference pattern resulting from a surface reflection. Solid line is measured sum of direct and reflected signals. Dashed line is computed direct signal.

1 The formula is applicable to many types of interference. If standing waves are set up between two plane and parallel transducers, \( \Delta \phi _s \) is twice the distance between transducers. If "cross-talk," or the interference between the direct acoustic signal and a purely electromagnetic signal, is present, \( \Delta \phi _s \) is the distance between transducers.

Fig. 2 Measured hydrophone output voltage where projector and hydrophone are both at a depth midway between the water-air surface and bubble-covered bottom. The transducers are separated by a distance equal to 1 of the water depth. The projector transmitting response and hydrophone receiving sensitivity are both essentially constant with frequency.

Interference Versus Frequency in Measurements in a Shallow Lake

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(Received March 22, 1961)

When an omnidirectional projector and hydrophone are closely spaced and in shallow water, and sound is transmitted from one to the other, a large interference signal is superimposed on the direct signal. If the received signal is plotted as a function of frequency, as in a calibration measurement, the interference signal amplitude appears to be an inconsistent series of irregular step peaks and dips. A mathematical analysis of the condition where the transducers are midway between a water-air surface and a bubble-covered bottom shows that the shape, amplitude, and frequency of the interference pattern are predictable as the result of a large number of multireflection paths.

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ACOUSTICAL measurements, particularly for the calibration of underwater sound transducers, are often made in shallow water. For economic or other practical reasons the absence of good free-field conditions must be tolerated.

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Because of the long wavelengths at audio frequencies, directional transducers are not feasible. Short projector-

\[
\Delta f = c/\Delta d,
\]

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At audio frequencies the interference is not so simple. Figure 2 is an example of a hydrophone output voltage versus frequency. The sound projector was a USRL type J9 which has a smooth and almost constant transmitting response. The hydrophone was a Massa type M115B, which has a constant receiving sensitivity. The 1:8 ratio of transducer separation to water depth was chosen to show the effect of interference in some detail; it is not the minimum ratio used at the USRL.

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undergoes a phase reversal at each pressure-release boundary. Similarly, the signal from the two-reflection path is $A_2 \cos(\omega - kD - 2\pi)$, or from the $n$-reflection path is $A_n \cos(\omega - kd_n - n\pi)$. The reflection path distances are given by

\[
d_1 = 2\left(\frac{d_0}{2} + (D/2)^2\right),
\]

\[
d_2 = 4\left(\frac{d_0}{4} + (D/2)^2\right),
\]

\[
\vdots
\]

\[
d_n = 2n\left(\frac{d_0}{2n} + (D/2)^2\right).
\]

Since $D > d_n$, the $d_0/2n$ term can be neglected, and the distances reduce to

\[
d_1 = D,
\]

\[
d_2 = 2D,
\]

\[
\vdots
\]

\[
d_n = nD.
\]

The amplitudes of the reflected signals can all be given in terms of the path lengths and the amplitude $A_1$

\[
A_n = (d_1/d_n)A_1 = A_1/n.
\]

If $2n\left(\frac{d_0}{2n} + (D/2)^2\right) = nD < \lambda$, then the phase of the reflected signals can also be given in terms of the approximation $d_n = nD$ and the expressions for the reflected signals then become

\[
A_1 \cos(\omega - kD - \pi),
\]

\[
\left(A_1/2\right) \cos(\omega - 2kD - 2\pi),
\]

\[
\vdots
\]

\[
\left(A_1/n\right) \cos(\omega - nkD - n\pi).
\]

The total signal $H$ received by the hydrophone will be the sum of the direct and all reflected signals

\[
H = A_0 \cos(\omega - kd_0) + \sum_{n=1}^{\infty} \left(A_1/n\right) \cos(\omega - nkD - n\pi).
\]

The second term can be expanded as follows:

\[
\sum_{n=1}^{\infty} \left(A_1/n\right) \cos(\omega - nkD - n\pi)
\]

\[
= \sum_{n=1}^{\infty} \left(A_1/n\right) \left[\cos(\omega + n\pi) + \sin(\omega + n\pi)\right].
\]

Then

\[
H = A_0 \cos(\omega - kd_0)
\]

\[
+ \left[\sum_{n=1}^{\infty} n^{-1} \cos(nkD + n\pi)\right] \cos\omega
\]

\[
+ \left[\sum_{n=1}^{\infty} n^{-1} \sin(nkD + n\pi)\right] \sin\omega.
\]


If the substitutions
\[
\cos((nkD+n\pi) - (-1)^n\cos(nkD)
\]
\[
\sin((nkD+n\pi) - (-1)^n\sin(nkD)
\]
are made, Eq. (2) can be written
\[
H = A_0 \cos(\omega t - kd_0)
\]
\[
+ \left[ A_1 \sum_{n=1}^{\infty} (-1)^n \cos nkD \right] \cos \omega t
\]
\[
+ \left[ A_1 \sum_{n=1}^{\infty} (-1)^n \sin nkD \right] \sin \omega t
\]
(3)

The total signal then is the sum of the direct signal and two reflection or interference components. The two interference components, given by the second and third terms in Eq. (3), are 90° out of phase with each other, and the phase of both with respect to the direct signal depends on the angle \(kd_0\). Since the direct path \(d_0\) is much shorter than the one-reflection path \(D\), the amplitude \(A_0\) is much larger than the amplitude given by either bracketed term in Eq. (3). Therefore, an interference component which is in phase or 180° out of phase with the direct signal will add arithmetically to the direct signal, but an interference component which is 90° or 270° out of phase with the direct signal will have negligible effect on the amplitude of the total signal—that is,

\[
|H| = A_0 \left[ A_1 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nkD}{\sin \omega t} \right]
\]
(0° or 180° phase difference)

\[
|H| = A_0 \left| A_1 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nkD}{\sin \omega t} \right|^2 \approx A_0
\]
(90° or 270° phase difference).

The relative phase of the interference components and the direct signal will change continuously as a function of \(kd\) or frequency. Consider four conditions for \(d_0\) given in wavelengths \(\lambda\):

(a) \(kd_0 = \pi/2\) and \(\cos(\omega t - kd_0) = \sin \omega t\), then \(kd_0 = \pi/2\) and \(\cos(\omega t - kd_0) = -\omega t\).

(b) \(kd_0 = 3\pi/2\), \(5\pi/2\), \(7\pi/2\), etc. then \(kd_0 = \pi\) and \(\cos(\omega t - kd_0) = -\omega t\).

(c) \(kd_0 = 3\pi/4\), \(7\pi/4\), \(11\pi/4\), etc. then \(kd_0 = \pi/2\) and \(\cos(\omega t - kd_0) = -\omega t\).

(d) \(kd_0 = 0\), \(\lambda/2\), \(\lambda\), etc. then \(kd_0 = 0\) and \(\cos(\omega t - kd_0) = \omega t\).

For conditions (a) and (c), the sine component, or the third term in Eq. (3), will have a 0° or 180° phase relation with the direct signal. The cosine component, or the second term in Eq. (3), will have a 90° or 270° phase relation with the direct signal and can be neglected. Equation (3) then simplifies to

\[
H = \pm A_0 + A_1 \sum_{n=1}^{\infty} (-1)^n \cos nkD \sin \omega t.
\]
(4)

For conditions (b) and (d), the cosine component will have 0° or 180° phase relation with the direct signal. The sine component will have a 90° or 270° phase relation with the direct signal and can be neglected. Equation (3) then simplifies to

\[
H = \pm A_0 + A_1 \sum_{n=1}^{\infty} (-1)^n \cos nkD \cos \omega t.
\]
(5)

The summation term in Eq. (4) is the same as that for the Fourier series for an inverted* saw-toothed wave. If it is plotted as a function of \(kd\) or frequency, the saw-toothed interference wave in Fig. 4 is obtained. The term "wave" here applies to a periodic variable as a function of frequency rather than of time or distance as is the usual case, and will be referred to as an "interference wave" when necessary to distinguish it from the real acoustic wave. Where \(A_0\) and \(A_1\) have opposite signs, the inverted saw-toothed wave is inverted again, and a normal or positive* saw-toothed wave is obtained.

The summation term in Eq. (5) is, in spite of its simple form, not commonly used and not found in the usual references on Fourier series or wave forms. A plot of the sum of the first five terms is shown in Fig. 5. This interference wave shape will be referred to as a peak wave in the form shown, or when \(A_0\) and \(A_1\) have the same sign, and as an inverted peak wave in the negative sense, or when \(A_0\) and \(A_1\) have opposite signs. Of special significance is the amplitude of the sharp peak. When \(kd = \pi\), 3\(\pi\), 5\(\pi\), etc., the summation

*As used here, a positive or normal saw-toothed wave has a slant line with a positive slope. A negative or inverted saw-toothed wave has a slant line with a negative slope.

If amplitude is a harmonic series which is divergent. Thus, the amplitude of the sharp peak increases without limit as \( n \) increases. The negative amplitude is the sum of 
\(-1+1/2+1/3+1/4+\cdots+1/n\).

This is a harmonic series which is divergent. Thus, the amplitude of the sharp peak increases without limit as \( n \) increases. The negative amplitude is the sum of 
\(-1+1/2+1/3+1/4+\cdots+1/n\) which converges to \(-\ln 2\) or approximately \(-0.69\).

The terms in parentheses in Eqs. (4) and (5) describe the amplitude of the total acoustic signal under four different conditions. Since \( A_0 \) and \( A_1 \) are almost constants with frequency, the shapes of these amplitude functions, or interference waves, when \( kD \) or frequency is a variable, are described by the summation terms.

Table 1 summarizes the relations among the four conditions, the form of Eqs. (4) or (5) which applies, and the wave shape of the amplitude. Consequently, for the conditions assumed in this analysis, the amplitude wave shape should change from inverted saw-tooth to inverted peak to saw-tooth to peak, and then through the same sequence again, as the frequency is increased.

The frequencies at which each wave shape will appear are calculated from the \( d_n \) ratio. The inverted saw-toothed wave, for example, will appear when

\[ d_n = \lambda/4 = c/(4f) \]

or

\[ f = (1/4)(c/d_n) \]  

Similarly, the inverted peak wave will appear when \( f = (1/2)(c/d_n) \), the saw-toothed wave when \( f = (3/4)(c/d_n) \), and the peak wave when \( f = c/d_n \). At intermediate frequencies, the wave shape will, of course, be a complex combination of saw-toothed and peak waves.

The amplitude for the saw-toothed wave when \( n \to \infty \) is \( \pm 1.5714 \), and for the peak wave is \( +\infty \) and \(-0.6914\).

The ratio of \( A_1/A_n \) for a single boundary reflection would be \( d_n/D \); however, the amplitude \( A_n \) is the sum of one surface and one bottom reflection. Therefore, \( A_1 = 2.1d_n/D \), and the maximum amplitude of the interference waves becomes \( \pm 1.57(2.1d_n/D) \) for the saw-toothed wave, and \( +\infty \) and \(-0.69(2.1d_n/D) \) for the peak wave.

On a decibel scale, the interference wave amplitude is given by

\[ +20 \log(1+3.14d_n/D) \]

\[-20 \log(1-3.14d_n/D) \]

(\text{saw-tooth})

\[ +\infty \]

\[-20 \log(1-1.38d_n/D) \]

(peak).

For a typical ratio \( d_n/D = 0.12 \), these amplitudes become \(+2.8 \) and \(-4.1 \) db for the saw-toothed wave, and \(+\infty \) and \(-1.6 \) db for the peak wave. For similar amplitudes resulting from a single reflection from a single boundary the values are \( \pm 1.0 \) db.

**DATA AND DISCUSSION**

The interference waves in the curve in Fig. 2 clearly show the sequence of shapes predicted by the theory. At frequencies of 300 to 500 cps, the peak-to-peak amplitudes of the saw-toothed waves are approximately 7 db—again in good agreement with theory. At higher frequencies, the assumption of omnidirectional transducers loses validity. The effect of a directional transducer is to reduce the ratio of \( A_1/A_n \) and of the interference wave amplitude. It does not affect the wave shape.

The top and bottom boundaries in any practical situation are not perfect reflectors and some energy is lost through both boundaries. Because of this energy loss and the absence of other ideal conditions, the number of reflection paths effective in forming the interference waves is limited. Clues to how many reflection paths are effective are available in the number of secondary oscillations on the saw-toothed wave and the amplitude of the peak wave.

Analysis of data obtained when \( d_n/D \) is 0.10 to 0.12 indicates that 20 to 30 reflections are effective in forming the interference waves a surprisingly large number. It was assumed in the theory that \( d_n \) was approximately equal to \( nD \). The errors in this assumption are largest for \( n = 1 \) or the first reflection and at the highest frequencies. For \( d_n = 100 \) cm and \( D = 800 \) cm, the distance or magnitude error for \( n = 1 \) is 7 cm or less than 1 percent. The phase error is 33.6° at 2 kc and 8.4° at 500 cps. The phase error is obviously the more important of the 2, and along with the directivity, is responsible for degeneration of the interference pattern at high audio frequencies.
INTERFERENCE VS FREQUENCY IN SHALLOW WATER

Table 1. Summary of interference wave shape analysis.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Total signal amplitude</th>
<th>Amplitude wave shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d = \lambda /4, 5\lambda /4, \text{etc.})</td>
<td>(+A_r + A_1 \sum_{k=1}^{n} (\text{-}1)^k \sin k\theta)</td>
<td>Inverted saw-tooth</td>
</tr>
<tr>
<td>(d = \lambda /2, 3\lambda /2, \text{etc.})</td>
<td>(-A_r + A_1 \sum_{k=1}^{n} (\text{-}1)^k \cos k\theta)</td>
<td>Inverted peak</td>
</tr>
<tr>
<td>(d = 3\lambda /4, 7\lambda /4, \text{etc.})</td>
<td>(-A_r + A_1 \sum_{k=1}^{n} (\text{-}1)^k \sin k\theta)</td>
<td>Saw-tooth</td>
</tr>
<tr>
<td>(d = 0, \lambda, 2\lambda, \text{etc.})</td>
<td>(+A_r + A_1 \sum_{k=1}^{n} (\text{-}1)^k \cos k\theta)</td>
<td>Peak</td>
</tr>
</tbody>
</table>

The four types of interference waves appear at frequencies very close to those predicted by Eq. (6) and theory. A small, but consistent and unexplained, discrepancy can be noticed at the higher frequencies. The peak wave in Fig. 2, for example, appears at 1.5 to 1.9 kc, whereas the computed value \(f = c/d_o\) is 1.525 kc.

OTHER BOUNDARY CONDITIONS

With a single plane boundary such as the water-air surface, the total signal is the sum of the direct signal and one reflection

\[ H = B_0 \cos(\omega t) + B_1 \cos(\omega t + \theta), \]

where \(B_0\) and \(B_1\) are the amplitudes of the direct and reflected signal and \(\theta\) is the phase difference between them. This expression can be expanded and rearranged

\[ H = B_0 \cos(\omega t) + B_1 \cos(\omega t + \theta) = (B_0 + B_1 \cos \theta) \cos \omega t - (B_1 \sin \theta) \sin \omega t. \]

If \(B_0 \gg B_1\), the second term can be neglected and

\[ H = (B_0 + B_1 \cos \theta) \cos \omega t. \]

The amplitude \((B_0 + B_1 \cos \theta)\) will oscillate as a function of \(\theta\) and will repeat when

\[ \theta = 2n\pi \]

or

\[ 2\pi \Delta d / \lambda = 2n\pi, \]

where \(n\) is an integer and \(\Delta d\) is the difference in path length between the direct and reflected signal. Then

\[ \Delta d = n\lambda = c/\omega, \]

or

\[ f = \omega/c/\Delta d, \]

where \(f\) is the repetition frequency or frequency difference between adjacent interference peaks. Equation (7) is the same as Eq. (1).

Under some practical conditions, particularly in air acoustics, the top and bottom boundaries may both be rigid instead of pressure release—large, low-ceilinged rooms with hard floors and ceilings, for example. It can be shown that these boundary conditions merely eliminate the \((\text{-}1)^n\) terms in Eq. (3), and that this results in a shift of \(\pi\) in the interference waves—that is, in Figs. 4 and 5, the 0, 2\(\pi\), 4\(\pi\), etc., points on the abscissa would be shifted by \(\pi\) in either direction. Otherwise, the rigid boundary condition is the same as the pressure-release condition.

An implication of the theory is that when one boundary is rigid and the other is pressure release, the interference at points midway between will completely cancel. Attempts have been made to test this aspect of the theory. The results were inconclusive because of the practical difficulty of obtaining a large or wide-spread, rigid, bottom boundary in water.

Working at a depth shallower or deeper than the mid-depth does not eliminate the interference problem; it merely complicates the analysis of the situation. However, because of the symmetrical boundary conditions, the maximum coherence of the many interfering signals and therefore maximum interference would be expected at mid-depth.

CONCLUSION

Where an omnidirectional projector and hydrophone are closely spaced in shallow water, the acoustical interference measured as a function of frequency can be explained on the basis of a large number of multi-reflected sound rays. When the transducers are half-way between the water-air surface and a bubble pressure-release bottom, the true frequency response curve has superimposed upon it an interference pattern which is a sequence of saw-tooth and peak wave shapes. For a projector-hydrophone distance equal to one-tenth to one-eighth of the total water depth, the interference wave shapes can be expected to have amplitudes of 7 to 8 db.