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CONTINUOUS WHOLE-EARTH COVERAGE BY CIRCULAR-ORBIT SATELLITE PATTERNS

by

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Procurement Executive, Ministry of Defence
Farnborough, Hants

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17. Abstract

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SUMMARY

Earlier studies of continuous whole-Earth coverage by patterns of satellites in equal-period circular orbits have been extended by means of a computer program to patterns of up to 25 satellites and to the provision of up to seven-fold continuous coverage. The program is described and various factors which may influence the choice of pattern for a particular application are discussed, including the level of coverage provided, the minimum satellite separation, and the number and form of the Earth-tracks traced by a pattern at a particular orbital altitude. A method is given of selecting a short-list of potentially suitable patterns, prior to detailed study.

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1 INTRODUCTION

In a number of practical applications of satellites, including communications, navigation and various types of surveillance, it is desirable to use a system of several satellites in orbit simultaneously, so that the coverage provided by the system as a whole is substantially greater than that available from any single satellite. The requirement may be for single or multiple, intermittent or continuous coverage of some defined portion of the Earth's surface; a claim to provide 'world-wide coverage' has often been made for a system of quasi-geostationary satellites, in near-equatorial orbits, which can actually cover all longitudes at low and medium latitudes, but cannot reach the higher latitudes. However, in some cases the requirement may be for continuous coverage of the whole surface of the Earth by at least one satellite, or even by two, three or more satellites simultaneously; it is this that we refer to as 'continuous whole-Earth coverage', and which forms the subject of this Report.

It may be assumed that satellite station-keeping, to the degree of accuracy necessary to maintain a chosen orbital pattern, is (with some limitations) now within the state of the art. It is therefore appropriate to examine the question of how many satellites, and in what orbital pattern, will most efficiently and economically provide any required level of coverage. The study reported here has been pursued intermittently over several years. An initial approach was described, in 1970, in an earlier RAE Technical Report[1]; this identified two types of patterns of satellites in equal-period circular orbits, described as 'star patterns' and 'delta patterns' respectively. Star patterns had proved amenable to a relatively simple analysis, but the delta patterns, which appeared more promising, had not, so only systems incorporating limited numbers of satellites had been examined. It was recognised that the hand methods used up to that time would have to be replaced by a computerised approach in order to deal with larger numbers of satellites, and work on developing an appropriate computer program was just beginning. This program, when developed, was first used to check the numerical results obtained previously, and a few corrections found necessary were incorporated before publication of a shortened version[2] of the original report. It was subsequently used to extend the examination of delta patterns to cover larger numbers of satellites, and the principal results of a comprehensive examination of patterns containing up to 15 satellites were included in a short paper[3,4] prepared for the IEE International Conference on Satellites Systems for Mobile Communications and Surveillance, held in London in March 1973. This showed that patterns were available which could provide single, double, triple or quadruple coverage of the Earth's surface, using smaller numbers of satellites than had been suggested elsewhere.

Since then, the program has been used for an examination of all possible delta patterns containing 24 satellites, and of some other delta patterns consisting of from 16 to 25 satellites; these can provide up to seven-fold coverage. Multiple coverage has been stressed, bearing in mind that, if a requirement calls for continuous provision of n-fold coverage, the most economical way of meeting it is likely to be by choosing a pattern capable of providing continuous (n + 1)-fold coverage; then, even after failure of any one satellite, or of many of the possible combinations of two satellites, continuous n-fold coverage will still be maintained. Consideration of the Earth-tracks followed by delta patterns, with particular reference to the conditions under which they follow coincident repetitive tracks, has provided pointers to the pre-selection of some of the most suitable patterns. This work on delta patterns, including for completeness some of that reported previously[1,3,4], is covered in the main text of this Report; its relevance to a satellite navigation system is discussed in Appendix A, and Appendix B describes some limited use of the computer program to examine other types of pattern.

The provision of continuous whole-Earth coverage has been the subject of a number of studies elsewhere, particularly in the USA, so it is appropriate to examine the features in which this study differs from the others. They principally concern the type of orbital pattern selected for examination and the method of assessing coverage.

This study has been based, from the outset, on the expectation that continuous whole-Earth coverage would be provided most effectively by a system in which the distribution of satellites over the Earth's surface was maintained as uniform as possible, subject to the practical limitations imposed on a system necessarily involving multiple intersecting orbits. Thus circular orbits of equal period have been chosen as an essential feature of all the patterns considered; elliptical orbits are advantageous for coverage of limited areas, but the more uniform patterns provided by circular orbits appear preferable for whole-Earth coverage. With delta patterns (fully defined in section 2.1), identical satellite distributions recur frequently during a single orbital period.

For convenience of use with the computer program, circular orbit patterns having a uniform distribution of satellites within and between orbit planes have been identified[3,4] by a code reference T/P/F, where T is the total number of satellites in the pattern, P is the number of orbital planes between which they are evenly divided, and F is a non-dimensional measure of the relative phasing of satellites in different orbital planes; in general, F may have any value less than P , but for delta patterns F can take only integer values from 0 to (P - 1) . The choice of the value of F is thus an important aspect of the choice of pattern.

In this study it has been assumed that, in choosing pattern characteristics, the primary objective should be to minimise the total number of satellites needed to ensure that not less than a certain number of satellites are everywhere visible at all times above some minimum elevation angle; as a secondary objective, it is assumed that the minimum distance between adjacent satellites should be as large as possible. Coverage has been assessed by finding those points on the Earth's surface (namely the centres of the circumcircles of the relevant spherical triangles) which are furthest from appropriate sub-satellite points; each pattern has been optimised by varying the common inclination of the orbital planes, to reduce the worst-case value of the radius of the largest circumcircle until no further improvement is possible.

In contrast, several early US papers based their coverage assessments on finding the minimum strip-width continuously covered by a single ring of orbiting satellites, and hence the number of adjoining strips necessary to cover the Earth's surface. Vargo[5] and Lüders[6] made no allowance for any coverage advantage to be obtained from favourable phasing of satellites in adjacent co-rotating orbits, and so, though each considered two classes of pattern which broadly correspond to the star and delta patterns of Ref 1, they are not directly equivalent to those patterns. Gobetz[7], followed by Ullock and Schoen[8], recognised the advantage of synchronising satellites in co-rotating orbits, and reducing the spacing between contra-rotating orbits; they therefore anticipated the star patterns of Ref 1, but not the delta patterns. In particular, Gobetz purported to show that the minimum number of satellites which can provide continuous whole-Earth coverage is six, whereas Ref 1 showed that it is five, using a delta pattern.

Easton and Brescia[9] of NRL, in a study which appears to have been roughly contemporaneous with that reported in Ref 1, made use of the same concept of locating the point most distant from adjacent sub-satellite points; however, they considered only orthogonal two-plane patterns, which may be considered as either star or delta patterns, and again concluded that a minimum of six satellites is necessary to provide continuous whole-Earth coverage. Later NRL papers[10,11] consider three-plane patterns for use in a navigation system

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requiring multiple coverage, but give no details of the method of coverage assessment, nor any indication of the values of F for these patterns, so it is not clear whether they are actually delta patterns.

Morrison[12] examined multiple coverage by a few selected patterns, using both circular orbits in delta patterns and elliptical orbits. His approach to coverage analysis was to find the number of satellites visible from each of a number of points on the Earth's surface, making up a rectangular grid with 10° spacing in latitude and longitude; the disadvantages of this approach are discussed in section 4.2 of this Report. Bogen[13] used a similar approach, with 5° spacing in latitude and 10° in longitude.

Some work on the subject has also been published in the USSR. Mozhayev[14] presented tabulated results which clearly have some common ground with those in Refs 3 and 4 (and Table 2 of this Report), though unfortunately no English translation of the text of his paper is available.

Summarising, it appears that the approach used in Ref 1, and continued in this Report, of locating the point most distant from adjacent sub-satellite points in order to establish an accurate value of minimum elevation angle, has been used elsewhere only by Easton and Brescia[9], and then only for single coverage. Others[12,13] examining multiple coverage have used a less accurate grid approach. No comprehensive analysis of delta patterns has been found elsewhere, and in particular none of these authors (except perhaps Mozhayev[14]) appears to have recognised single-satellite-per-plane patterns as important members of the family of delta patterns. Moreover, the analysis in this Report of the conditions under which patterns produce coincident Earth-tracks does not appear to have any counterpart elsewhere.

Coverage will, of course, be only one of several, possibly conflicting, considerations to be taken into account in any complete study of a particular satellite system requirement; thus, for example, launching considerations might place constraints on the permissible range of orbital inclinations, or on the number of different orbital planes. However, even if such requirements should sometimes rule out the use of the particular patterns identified in this Report as providing optimum coverage, the methods of this Report may be used to identify which patterns, out of those which are compatible with the other system requirements, can most economically provide the required coverage.

2 CHARACTERISTICS OF DELTA PATTERNS

2.1 Definition

Following the publication of the first papers[1,2] on this continuing study of whole-Earth coverage by patterns of satellites in equal-period circular orbits, adaptation of the method of analysis to the development of a computer program made it desirable to adopt a revised nomenclature. This was introduced in subsequent papers[3,4] and is used, with some further development, in the present Report.

We denote by T the total number of satellites making up a complete pattern. In describing a pattern it is convenient to introduce a unit of $360^\circ/T$, which we describe as a 'pattern unit' (PU), for use in defining distances within the pattern (normally considered as projected on to the Earth's surface) in terms of the geocentric angle subtended. Thus, in a 9-satellite pattern a distance of 2 PUs subtends a geocentric angle of $2 \times 360^\circ/9 = 80^\circ$.

The term 'delta patterns' has been applied[1] to those patterns in which the T satellites are in equal-period circular orbits, evenly-spaced and all at the same inclination to a reference plane, with a uniform distribution of the satellites among and within the orbital planes. The characteristics of a delta pattern may be defined more fully as follows:

(1) The pattern, containing a total of T satellites, consists of S satellites evenly spaced in each of P orbital planes. Thus P and S may each equal any factor of T, including 1 and T, provided their relative values are such that $T = SP$.

(2) All orbital planes have the same inclination δ to a reference plane. This reference plane usually coincides with the equatorial plane, so that δ equals the orbital inclination i, but this need not necessarily be the case.

(3) The ascending nodes of the P distinct orbits are evenly spaced at intervals of S PUs (ie of $360^\circ/P$) in the reference plane.

(4) The relative positions of satellites in different orbital planes are such that there are equal intervals between passages of satellites in adjacent orbital planes through their respective ascending nodes (in the reference plane). When a satellite in one plane is at its ascending node, some satellite in the adjacent plane having a more easterly ascending node has covered F PUs, where F is an integer which may have any value from 0 to $(P - 1)$, since passing its ascending node.

The geocentric angle traversed by a satellite since passing its ascending node in the reference plane will be referred to in this Report as the 'phase angle' of the satellite. The phase angle of the whole pattern will be taken to be the phase angle of one particular satellite in the pattern which is treated as the reference satellite; this is considered further in section 2.2. The limitation on the values of F for delta patterns may be explained, in terms of phase angles, by the simple example of a two-plane pattern. In this case one uniform distribution has all the satellites in one plane at the same phase angles as the corresponding satellites in the other plane (ie $F = 0$), while the only other uniform distribution has satellites in one plane at phase angles midway between those of the satellites in the other plane (ie $F = 1$).

Any individual delta pattern may thus be identified by a three-integer code reference T/P/F, the values of these three integers being sufficient to determine the general shape of the pattern. To fix the precise positions of the orbital planes, it is necessary to specify their common inclination δ; this may be treated as a parameter, whose value may be chosen to optimise the pattern in accordance with any particular requirements. As an example, Fig 1 illustrates pattern 9/3/2 with an inclination δ of 60°; here the plane of the paper should be regarded as the reference plane, the continuous arcs representing the parts of the satellites' orbital paths which are in the hemisphere above the reference plane and the broken lines the parts that are below it, while the ✈ symbols represent instantaneous positions of satellites lying above the reference plane and the ↮ symbols represent those below it. Two separate positions are shown for each satellite; this aspect is discussed in section 2.2.

The individual satellites in the pattern are identified firstly by a letter of the alphabet (omitting I and O), always beginning with A for the reference satellite; and secondly by two suffices to this letter, which are integers representing the satellite's position in the pattern by two geocentric angles, both expressed in PUs and lying within the range of values from 0 to $(T - 1)$. The first of these is the east longitude, measured in the reference plane, of the satellite's ascending node; it is assumed that there is no relative motion between the orbital planes and the reference plane, and that the ascending node of the reference satellite is at zero longitude. The second integer represents the satellite's phase angle when the pattern phase angle is zero.

The reference satellite is therefore identified in all cases as A_{00}. With the nomenclature used previously[1,2], the next satellite to be identified would have been that ahead of A in the same orbit, which would therefore have the suffices 0, P ; however, with the revised nomenclature we identify one satellite in turn from each orbital plane, always taking as the next satellite the one which is in the next most easterly orbital plane and which has the same or next larger phase angle; thus the satellite after $A_{0,0}$ is identified as $B_{S,F}$ (unless all the satellites are in a single orbital plane, when it will appear as $B_{0,1}$). If there are more than two planes, the third satellite identified will be $C_{2S,2F}$ (or $C_{2S,(2F-T)}$ if $2F \geqslant T$). This continues until the plane containing the reference satellite is reached again, from which point each satellite identified has a phase angle which exceeds by P the phase angle of the previous satellite in the same plane; thus, in Fig 1, the plane of the reference satellite is reached again with the fourth satellite, which is taken to be $D_{0,P}$, followed in the next most easterly plane by $E_{S,(F+P)}$. Eventually all T satellites are identified in this fashion. If the process were continued to a (T + 1)th satellite, this would be found to coincide with the reference satellite.

It is evidently possible to program a digital computer to perform this identification of the characteristics of the individual satellites in a pattern, given as input data only the pattern reference code T/P/F. Moreover, given the values of δ and of the pattern phase angle, the instantaneous values of satellite latitude and longitude relative to the reference plane may be calculated for each satellite from the suffices to its identification letter.

When it is only necessary to distinguish one satellite in a pattern from the others, without full identification of the satellite characteristics, the alphabetical reference may be used without the suffices; however, their use may often help to clarify the elements of a complex pattern. In Fig 1, which shows the orbital paths of the satellites, it is not necessary to make use of the suffices to see which satellite is in which plane; but in Figs 4, 5 and 6 (discussed in section 2.3.2) it would be more difficult to relate the satellites to their orbital planes without the indication provided by the first suffix.

Since for each value of P there may be P different values of F , from 0 to (P − 1), and since P may be any factor of T , including 1 and T , it follows that for any value of T the number of possible delta patterns is equal to the sum of all the factors of T ; for example, for T = 5 the number of possible delta patterns is 1 + 5 = 6 , while for T = 6 it is 1 + 2 + 3 + 6 = 12.

2.2 Pattern repetition interval

Fig 1 shows two positions of the pattern 9/3/2: the first at a pattern phase angle of 0°, when satellite A is at its ascending node, and the second at a pattern phase angle of 10°, when satellite A is 10° past its ascending node and satellite C is 10° short of reaching its descending node. In this second condition the pattern is symmetrical about the plane which is perpendicular to the reference plane and which passes through the centre of the Earth and bisects the line joining the instantaneous positions of A and C. Throughout the next 10° of phase angle - as C moves on to reach its descending node - the pattern will appear as a mirror image of the corresponding configurations during the first 10°, with A interchanged with C, E with H, D with J and F with G. Over the following 20°, as C moves away from its descending node, the pattern will repeat in opposite hemispheres the configurations of the first 20° when A was moving away from its ascending node. Finally, at the end of this phase angle change of 40°, H will reach its ascending node and the pattern will start a repetition of the first 40°, offset by 120° in longitude.

Hence, though the pattern only truly repeats after 40° of phase angle, it passes through the full range of essentially dissimilar configurations every 10°; for instance, the first 10° includes the cases both of a satellite being at a node (satellite A at 0° phase angle) and of a satellite being at maximum latitude (satellite B at 10° phase angle). For this pattern it is therefore only necessary to study its characteristics over a phase angle range of 10°, starting or finishing with one or more satellites at an ascending or descending node; we call such a phase angle range the 'pattern repetition interval' (abbreviated to PRI).

In general, the PRI is half the phase angle range between successive nodal crossings (ascending or descending) by different satellites in the pattern. With pattern 9/3/2, shown in Fig 1, only one satellite at a time passes a node, and the PRI is $\frac{1}{4}$ PU. If n satellites are at ascending and/or descending nodes simultaneously, however, the PRI will be increased to $\frac{1}{4}n$ PUs. This increase may be regarded as compensated in one sense by the fact that the pattern then always consists of n identical sections; if y satellites reach their ascending nodes simultaneously then the pattern consists of y identical segments, while if satellites reach ascending and descending nodes simultaneously it consists of identical hemispheres either side of the reference plane.

Adopting the nomenclature $y = H[F,P]$ to indicate that y is the highest common factor (HCF) of F and P (so that when $F = 0$, $y = P$), and with

$z = H[2, T/y]$, the PRI is equal to $\frac{1}{4}yz$ PUs. Hence it is convenient to define a 'pattern repetition unit' (PRU), such that 1 PRU = $90^\circ \times yz/T$ and the PRI therefore always has a length of 1 PRU. It may then be specified, for instance, that a pattern shall be examined at intervals of 0.2 PRU from pattern phase angle $\phi = 0$ to $\phi = 1.0$ PRU.

Any arbitrary pattern of satellites in equal-period circular orbits could only be expected to repeat the previously-covered range of conditions after a phase angle of 180°; the fact that a delta pattern may have a PRI as short as $90^\circ/T$ (if there are no simultaneous nodal crossings) is a pointer to the high degree of uniformity of coverage which such patterns can achieve.

2.3 Earth-tracks

2.3.1 Tracks of single satellites

A diagram such as Fig 1, illustrating orbital patterns, is independent of the orbital period of the satellites, depending only on the particular delta pattern used and on its inclination to the reference plane. If the Earth were not rotating relative to the orbital planes, it could also be regarded as illustrating the Earth-tracks traced out by the satellites in the pattern; however, the shapes of the Earth-tracks followed by the satellites over the rotating Earth are in practice dependent on the orbital period of the satellites, as well as on their inclination to the reference plane and on the latter's inclination to the Earth's equatorial plane.

For each individual satellite in an inclined circular orbit, the Earth-track consists of a series of identical excursions alternately into the northern and southern hemispheres, each reaching a maximum latitude equal to the orbital inclination to the equator. Successive equatorial crossings (alternately at ascending and descending nodes) occur at eastward geographical longitude increments of $\frac{1}{2}(360^\circ - \Omega_R)$, where Ω_R is the rotation of the Earth relative to the orbital plane in one nodal period. In calculating the value of Ω_R account should be taken not only of the Earth's rotation round the Sun but also of the precession of the orbital plane due to the Earth's oblateness, which would amount (if uncorrected) to 0.0134° per orbit for synchronous equatorial orbits, and more for lower orbits. This precession is directly proportional to $\cos i$ and inversely proportional to the orbit radius to the power of $3\frac{1}{2}$, and its sense is such as to increase Ω_R if the orbit is direct and reduce it if retrograde; a fuller discussion is provided by Allan[15], among others. If $\Omega_R/360^\circ$ is equal to M/L , where L and M are coprime integers, than the Earth-track is repetitive after completion of L orbits in (approximately) M sidereal days, having covered 360(L - M) deg of geographical longitude; this condition may also be described as L:M resonance.

A selection of such repetitive Earth-tracks is illustrated, for an inclination i of 60°, in Fig 2. In each case the first ascending node is considered to occur at 0° longitude, and numbered arrows indicate the L successive ascending nodes which occur before the track starts to repeat with another ascending node at 0° longitude. Between ascending nodes, each track comprises excursions alternately into the northern and southern hemispheres, giving L of each.

Fig 2a shows the repetitive track for an 8-hour orbit ($L = 3$, $M = 1$). Since $L - M = 2$, this track repeats after covering 720° of longitude in one day, and the part of the track representing the second 360° inevitably crosses that covered during the first 360°, the cross-overs occurring at the nodes.

Fig 2b, c and d show the repetitive tracks for 12-hour ($L = 2$, $M = 1$), 16-hour ($L = 3$, $M = 2$) and 20-hour ($L = 6$, $M = 5$) orbits respectively. In each of these cases $L - M = 1$, so the track repeats after covering 360° of longitude (in 1, 2 and 5 days respectively) without having crossed itself at any point. This non-self-crossing characteristic of the Earth-tracks of orbits for which $L - M = 1$ is not, however, maintained for indefinitely increasing values of L; the loops formed by adjacent excursions into the same hemisphere may be seen to approach one another more closely as L is increased, and for $L \geq 8$ (when $i = 60^\circ$) adjacent loops overlap, so that the track crosses itself frequently.

Fig 2e shows the familiar figure-8 Earth-track of a 24-hour circular orbit ($L = 1$, $M = 1$), for which $L - M = 0$, so that successive nodes all occur at the same longitude, the track repeating after 1 day, and crossing itself at the node.

There may be less direct practical interest in the Earth-tracks for orbital periods exceeding 24 hours, but there are still reasons for examining them, as will be discussed in section 3.6. Fig 2f, g and h show the repetitive tracks for 30-hour ($L = 4$, $M = 5$), 36-hour ($L = 2$, $M = 3$) and 48-hour ($L = 1$, $M = 2$) orbits respectively. In each of these cases $L - M = -1$, so the track repeats after covering 360° of longitude (in 5, 3 and 2 days respectively) in a westerly, instead of an easterly, direction. In Fig 2h ($L = 1$) the track does not cross itself at any point, but in Fig 2g ($L = 2$) there are four small loops in the track with a cross-over point associated with each, and in Fig 2f there are eight cross-overs associated with eight loops which now constitute more than half the track.

The Earth-track of a 72-hour orbit (L = 1, M = 3) is more conveniently shown by a polar projection, as in Fig 3, in which the northern hemisphere portion of the track (which is duplicated in the southern hemisphere) is compared with the tracks of a 24-hour 60°-inclination orbit (already shown in Fig 2e) and a 24-hour 120°-inclination orbit. The 72-hour-orbit Earth-track repeats itself after covering 720° of longitude in a westerly direction (L - M = -2) in 3 days, while the 24-hour-orbit Earth-tracks do so after nominally covering 0° of longitude (L - M = 0) in 1 day, but the practical effect is the same for all three - the track is a figure-8 with a cross-over at the single nodal longitude. The 72-hour 60°-inclination track is very similar to the 24-hour 120°-inclination track, both being large figure-8s which encircle the poles, reaching a maximum latitude of 60° at 180° longitude; the 72-hour-orbit track shows a hump at 180° longitude, similar to those in the 48-hour-orbit track in Fig 2h, which represents a vestigial loop.

Retrograde orbits, such as the 120°-inclination 24-hour orbit of Fig 3, are considered further in section 2.4; meanwhile we may note from Fig 3 that, though the 60° and 120° 24-hour orbits would appear as mirror images of one another in inertial space, their tracks over the rotating Earth differ considerably.

2.3.2 Tracks of complete delta patterns

Having considered the forms which the Earth-track of an individual satellite in an inclined circular orbit may take, depending upon its period and upon whether the inclination is more or less than 90°, we may now consider the Earth-tracks of a complete delta pattern of such satellites, all having identical periods. Different considerations are involved, according to whether the reference plane is inclined to or coincident with the equatorial plane.

(a) Reference plane inclined to the equatorial plane

If the pattern reference plane is inclined to the Earth's equatorial plane, then different satellites in the pattern will have different inclinations to the equator, and so their Earth-tracks will take different forms; but if reference plane and equator coincide, all the satellites will have similar inclinations and their Earth-tracks will be similar. This is illustrated in Fig 4, for pattern 6/2/0 with L = M = 1 and $\delta = 60^{\circ}$. In Fig 4a the reference plane coincides with the equatorial plane, so that the Earth-tracks of all six satellites are similar, and correspond to the single-satellite track shown in Fig 2e. In Fig 4b the reference plane has been tilted through an angle of 60° about an axis through the common nodes of the orbital planes, so that one plane coincides with the equatorial plane while the inclination of the other is increased to 120^{o}; thus satellites in the former plane (symbol ⊕) are in geostationary orbit, while those in the latter plane are following retrograde orbits with Earth-tracks corresponding to that shown in Fig 3. In Fig 4c, on the other hand, the reference plane has been tilted through 90^{o} about an axis normal to the axis through the nodes, so that the orbital planes are both polar but unevenly spaced, their ascending nodes in the equatorial plane, at a_0 and a_3, being 120^{o} apart, and the satellites follow six separate, evenly-spaced figure-8s reaching their maximum latitude at the poles.

Although their Earth-track patterns are so different, the three different orientations of pattern 6/2/0 illustrated in Fig 4 retain all the common features associated with their identical orbital patterns; for instance, the overall standard of whole-Earth coverage provided, as indicated by the minimum elevation angle at which an observer anywhere on the Earth's surface could see the nearest satellite, is the same in each case. Moreover, all these three system configurations appear feasible from the point of view of satellite station-keeping requirements to overcome orbital perturbations. The configurations of Fig 4a and 4c each involve only a single orbital inclination for all six satellites, so that any perturbations due to the Earth's oblateness are similar for all six and could be offset, for the 60^{o} inclination satellites, by a slight altitude adjustment, though this would be unnecessary for the 90^{o} inclination satellites. The configuration of Fig 4b involves satellites in equatorial orbit as well as at one other inclination; and since equatorial satellites follow the same constant Earth-track regardless of the longitudes of their nodes, they can be synchronised with the satellites in inclined orbit by taking due account of the differential perturbations due to the Earth's oblateness when choosing orbital altitudes.

Any other orientation of the pattern would share the same basic characteristics, and would be equally valid in principle, but might be unacceptable in practice due to the long-term station-keeping requirements involved. For instance, midway between the conditions of Fig 4a and 4b would lie one with the reference plane of the pattern inclined at 30^{o} to the equatorial plane, so that the inclination of one plane was reduced to 30^{o} and that of the other increased to 90^{o}. Since, in synchronous orbit, the precession rate is $0.0134 \cos i$, this would involve a differential precession rate for the two planes of 0.0116^{o} per orbit, or about 4^{o} per year. Burt's analysis[16] of orbital manoeuvring thrust requirements shows that correcting this would involve a fuel expenditure several times as great as that involved in correcting the inclination perturbation at synchronous altitude, and so would probably be considered unacceptable; and the requirement would be still greater at lower altitude.

Thus a two-plane delta pattern can be re-oriented from the basic configuration (with the reference plane coinciding with the equator), either to a configuration with one of the two orbital planes coinciding with the equator, or to one in which both planes are polar. A three-plane delta pattern cannot be re-oriented to give three polar planes, but it can be re-oriented so that one orbital plane coincides with the equator and the other two have equal inclinations i such that $\sin \frac{1}{2}i = \frac{1}{2}\sqrt{3} \sin \delta$. Thus Fig 5a shows the Earth-tracks in synchronous orbit of the three-plane pattern 9/3/0, with $\delta = 60^{\circ}$ and the reference plane coinciding with the equator - in this case their spacing is such that three satellites, one from each plane, follow each of three similar figure-8s - while Fig 5 shows the effect on the Earth-tracks of tilting the same pattern (with $\delta = 60^{\circ}$) through 60°, so that one plane is equatorial and the other two have inclinations of 97.2°; three satellites are then geostationary (symbol ⊕), and the other six follow distinct figure-8 tracks encircling the poles. Once again, though the Earth-tracks shown in Fig 5a and 5b are very different, the orbital patterns are identical and the overall standard of whole-Earth coverage provided is the same; both these systems appear feasible.

With a delta pattern containing more than three orbital planes, it is not possible to re-orient the pattern to anything but the basic configuration (with the reference plane coinciding with the equator) without causing the different planes to have different precession rates.

With the arrangement of Fig 5b, changing the value of δ while keeping one orbital plane in the equatorial plane alters the spacing between the figure-8 tracks representing the other two orbital planes; Fig 5c shows that, if δ were increased from 60° to 70.5°, the six figure-8 tracks would merge into three, each followed by two satellites, one from each of the two orbital planes of inclination 109.5°. With the arrangement of Fig 5a, however, altering the value of δ merely alters the size of each figure-8 without altering either the spacing between their nodes or the number of satellites following each figure-8 track.

(b) Reference plane coincident with the equatorial plane

When the reference plane of a delta pattern coincides with the equator, as is likely to be the case apart from the few possible exceptions for two-plane or three-plane patterns just discussed, then the satellites follow similar and evenly-spaced, but usually distinct, Earth-tracks, as in Fig 4a. However, if certain conditions are met, the Earth-tracks of several of the satellites may coincide, as in Fig 5a; and it is even possible for the Earth-tracks of all the satellites in the pattern to coincide, as shown in Fig 6 for pattern 9/3/2 in an 18-hour orbit ($L = 4$, $M = 3$). In this case the Earth-track makes four excursions into each of the northern and southern hemispheres, each node occurring 45° to the east of the previous one, so that the Earth-track repeats itself after covering 360° of longitude (since $L - M = 1$) in 3 days, without having crossed itself at any point. Pattern 9/3/2 is the same pattern as was used for the illustration of orbital planes in Fig 1, and a comparison of Figs 1 and 6 confirms that, at the beginning of the pattern repetition interval, the positions of the satellites in the two illustrations are identical, while at the end of the 10° PRI they are identical apart from the clockwise displacement of those in Fig 6 due to the rotation of the Earth.

If we consider all the ways in which T satellites may be uniformly distributed between similar Earth-tracks corresponding to a particular pair of values of L and M, we find that the conditions which apply are similar to those already discussed in section 2.1 when defining the pattern reference code for a delta pattern in terms of the uniform distribution of satellites between orbital planes. We denote by $E_{L,M}$ the number of distinct Earth-tracks followed by a particular pattern for particular values of L and M; the condition that the satellite distribution is fully uniform implies (just as for P) that $E_{L,M}$ must be a factor of T. Hence the total number of distinct patterns, for given values of T, L and M, is equal to the sum of all the factors of T. This is the same as the total number of delta patterns for that value of T, and it will be evident that each such Earth-track pattern results from one of these delta patterns.

Fig 1, which represents the orbital paths of the satellites, might equally be considered as representing their Earth-tracks over a non-rotating Earth, ie for the condition $L = 1$ and $M = 0$. While we have chosen to quote reference codes for delta patterns in the form T/P/F, it would have been equally valid (though less generally convenient) to quote them in a comparable form $T/E_{L,M}/Q$, with Q suitably defined as the phase angle difference between two satellites in the pattern following adjacent Earth-tracks, so that Q could take any of the $E_{L,M}$ integer values from 0 to $(E_{L,M} - 1)$. Thus to the T/P/F code for the pattern 9/3/2 there might correspond $T/E_{L,M}/Q$ codes $9/3_{1,0}/2$ (from Fig 1),

$9/1_{4,3}/0$ (from Fig 6), and similarly for any other pair of values of L and M. However, use of a single code T/P/F for any one pattern appears preferable to introducing the multiplicity of possible codes $T/E_{L,M}/Q$, especially since Q would have to be measured in suitable time-related units along Earth-tracks which are not great circles, so this approach has not been pursued further.

It is nevertheless useful to have a simple method of determining the number of distinct repetitive Earth-tracks which a particular pattern will follow at an altitude corresponding to given integer values of L and M, and the relationship between $E_{L,M}$ and the pattern code reference T/P/F may be deduced as follows. For a single satellite, the ascending nodes of a repetitive Earth-track are spaced round the equator (not necessarily in chronological order) at intervals of T/L PUs. For successive satellites in the same orbital plane, as shown in Fig 7, the distance between corresponding ascending nodes is PM/L PUs, while for corresponding satellites in adjacent planes the distance between ascending nodes is $S + FM/L$ PUs. Defining $G = SL + FM$, the distance between ascending nodes of corresponding satellites in adjacent planes is G/L PUs.

Considering any arbitrary pair of satellites in the pattern, one may be displaced from the other by h orbital-plane steps and by k additional satellite steps within its plane, so that the distance between their ascending nodes is $(hG + kPM)/L$ PUs. The condition for two satellites to lie on the same Earth-track is that this should be equal to a multiple of T/L PUs, ie we must have

$$hG + kPM = qT$$

where h, k and q are all integers, and this may be re-written as

$$hG = P(qS - kM) \quad .$$

Now it can be shown that all possible positive values of an expression of the form $qS - kM$, where all four components are integers, are multiples of the HCF of S and M, which we shall denote by $J = H[S,M]$. Thus $hG = rPJ$, where r is an integer.

Putting $K = H[G,PJ]$, we have $G = gK$ and $PJ = fK$, where g and f are coprime integers, and since $hg = rf$, the minimum value of h is f, ie PJ/K. By definition of h it follows that the number of planes contributing a satellite to each Earth-track is P/h, ie K/J.

For satellites in the same orbital plane to follow the same Earth-track we simply have $kPM = qT$, ie $kM = qS$. As in the previous paragraph, putting $J = H[S,M]$, we have $M = mJ$ and $S = sJ$, and since $km = qs$, the minimum value of k is s , ie S/J . By definition of k it follows that there are s/k , ie J satellites from each plane following the same Earth-track.

Combining these results, we have J satellites from each of K/J planes, ie a total of K satellites, following each Earth-track. Hence the number of distinct repetitive Earth-tracks followed by a delta pattern is given by the formula

$$E_{L,M} = T/K$$

where $K = H[G,PJ]$, $G = SL + FM$, and $J = H[S,M]$.

Thus we may note that, for the case of Fig 4a, $G = 3 + 0 = 3$, $J = H[3,1] = 1$, $K = H[3,2] = 1$, so $E_{1,1} = T/1 = 6$; for the case of Fig 5a, $G = 3 + 0 = 3$, $J = H[3,1] = 1$, $K = H[3,3] = 3$, so $E_{1,1} = T/3 = 3$; and for the case of Fig 6, $G = 12 + 6 = 18$, $J = H[3,3] = 3$, $K = H[18,9] = 9$, so $E_{4,3} = T/9 = 1$.

The formula for $E_{L,M}$ may also be used to deduce, for a given value of T of which $E_{L,M}$ is a factor, which are the $E_{L,M}$ different patterns (ie pairs of values of P and F) which, at the orbital altitude indicated by the values of L and M , will produce $E_{L,M}$ distinct repetitive Earth-tracks - and, in particular, which is the single pattern which will produce a single repetitive Earth-track. For the general case we see that P, S and F must be such that both G and PJ are multiples of $T/E_{L,M}$. For $E_{L,M}$ to equal 1, G and PJ must be multiples of T . Hence J , ie $H[S,M]$, is a multiple of S , and so, since $H[S,M]$ cannot be greater than S, S is a common factor of M and T . Thus we can write $S = M/m$ and $hS = H[M,T]$, ie $h = H[m,P]$, where m and h are integers. Since T is a factor of G , ie of $SL + FM$, P is a factor of $L + Fm$, and we can write $pP = L + Fm$, ie $L = pP - Fm = h(pP/h - Fm/h)$, where p is an integer. Hence h is a factor of L . However, h is a factor of M , and L and M are coprime integers; therefore $h = 1$, and so

$$S = H[M,T] \quad .$$

Hence the value of P is determined, and the value of F is then given by

$$\begin{aligned} F &= kT/M - SL/M \\ &= (S/M)(kP - L) \quad , \end{aligned}$$

the value of k being uniquely determined such that F is an integer within the range from 0 to P − 1 . Thus, taking once again the example of Fig 6, and starting with the values T = 9, L = 4 and M = 3 , we find that S = H[3,9] = 3 , so that P = 3 , and F = 1(2 × 3 − 4) = 2 , ie the relevant single-track pattern is 9/3/2, as shown.

The formula for $E_{L,M}$ may be used in much the same way to deduce the altitudes or periods (ie pairs of values of L and M) at which a particular pattern will produce a particular number (a factor of T) of distinct repetitive Earth-tracks.

As a further illustration of the use of this formula, the twelve possible 6-satellite delta patterns are listed below, showing for each the number of distinct repetitive Earth-tracks which it would follow at the periods corresponding to seven different values of L/M , arbitrarily chosen to cover a range of values of L and M , and including several values for which tracks were illustrated in Fig 2. Also shown for each pattern is one example of the periods at which it would follow a single repetitive Earth-track, with the corresponding value of L/M ; the particular examples shown are those with M = S and with the minimum corresponding value of L , but other cases would exist with other values of L and with M equal to a multiple of S − as may be seen, for patterns 6/1/0, 6/3/2 and 6/6/0, from the previous columns. For each value of L/M it may be seen that one of the twelve patterns gives a single track, two give two tracks, three give three tracks and six give six tracks; for the case in which M = T , as would also occur with M equal to a multiple of T we see that $E_{L,M} = P$ in each instance.

Pattern	Period:	8h	16h	19.2h	20h	20.6h	24h	36h	For $E_{L,M}$ = 1:	
T/P/F	L/M:	3/1	3/2	5/4	6/5	7/6	1/1	2/3	Period (h)	L/M
	Number of separate Earth-tracks ($E_{L,M}$):									
6/1/0		6	3	3	6	1	6	2	144	1/6
6/2/0		6	6	6	3	2	6	1	36	2/3
6/2/1		3	6	6	6	2	3	2	72	1/3
6/3/0		2	1	3	2	3	6	6	16	3/2
6/3/1		6	3	3	6	3	2	6	9.6	5/2
6/3/2		6	3	1	6	3	6	6	48	1/2
6/6/0		2	2	6	1	6	6	3	4	6/1
6/6/1		3	6	2	6	6	3	6	4.8	5/1
6/6/2		6	6	6	3	6	2	3	6	4/1
6/6/3		1	2	6	2	6	3	6	8	3/1
6/6/4		6	6	2	3	6	6	3	12	2/1
6/6/5		3	6	6	6	6	1	6	24	1/1

Some early studies[17,18] of communications satellite systems considered the use of single repetitive Earth-track patterns at various altitudes. Applying to them the nomenclature of this Report, an 8-satellite 12-hour 30°-inclination system[18] would use pattern 8/8/6, a 14-satellite 8-hour 80°-inclination system[17] would use part of pattern 24/24/21, a 15-satellite 8-hour 30° inclination system[18] would use pattern 15/15/12, and a 17-satellite 6-hour 30°-inclination system[18] would use pattern 17/17/13.

2.4 Retrograde orbits

Retrograde orbits have already been mentioned briefly in section 2.3.1, in the discussion of the Earth-tracks traced by individual satellites; however, some points relating to complete delta patterns of satellites in retrograde orbits also deserve consideration.

Certain characteristics of a pattern are not dependent upon its inclination; for example, the number of separate repetitive Earth-tracks traced by a particular pattern at a particular altitude is independent of inclination (apart, of course, from the special cases $i = 0^\circ$ or 180°), even though the actual form of the Earth-tracks does depend upon the inclination. Such characteristics are, obviously, equally applicable to direct orbits (inclination less than 90°) and to retrograde orbits (inclination exceeding 90°).

On the other hand, there are various features which, applying to a particular pattern at a particular inclination and phase angle, are independent of orbital altitude, eg the angular separation between any pair of satellites, and the maximum angular distance of any point on the Earth's surface from the nearest sub-satellite point. It is worth remembering that patterns which are mirror images of one another will have identical characteristics in these respects; in particular, a pattern T/P/F at an inclination δ to the reference plane will have identical characteristics in these respects to the pattern T/P/(P-F) at an inclination of $180^\circ-\delta$ to the reference plane.

As an example, Fig 8a shows the pattern 5/5/1 with $i = \delta = 45^\circ$, and Fig 8b the pattern 5/5/4 with $i = \delta = 180^\circ - 45^\circ = 135^\circ$; their Earth-tracks for synchronous orbits ($L = M = 1$) are also shown. Though the Earth-tracks are very different (five small figure-8s for pattern 5/5/1, one large figure-8 for pattern 5/5/4), the satellite orbital patterns are mirror images of one another, and the overall standard of coverage they provide (eg in terms of minimum elevation angle and minimum satellite separation) is identical.

Hence, when calculating coverage standards for all the delta patterns having a particular pair of values of T and P , over a range of inclination angles, it is only necessary to perform the calculations for orbital inclinations up to 90°; the corresponding values in retrograde orbits may be deduced immediately as being identical to those of the complementary pattern (ie with F replaced by P-F) at the inclination which is the supplement of the retrograde inclination under consideration (though, as already noted, this reciprocity does not extend to matters - such as the number of distinct Earth-tracks - which are dependent on the Earth's rotation). However, it seems unlikely that there would be many cases in which a retrograde orbit would have advantages, due to its different Earth-track pattern, outweighing the direct orbit's benefits from an eastward launching direction.

3 SELECTION OF SUITABLE DELTA PATTERNS

3.1 General

This Report is concerned with the provision of continuous whole-Earth coverage, and in particular with its provision by means of satellites arranged in delta patterns. In the design of any particular satellite system it is necessary to work from the special requirements developed for that system, but for the purpose of this general study it has been assumed that certain requirements will normally be applicable.

The objective is taken to be the identification of those orbital patterns which will most economically provide a certain standard of continuous whole-Earth coverage. This standard is considered to be defined in terms of the level of coverage required, ie single coverage, double coverage, or, in general, n-fold coverage, and of the minimum acceptable elevation angle to the n^{th} nearest satellite from the least favoured point on the Earth's surface. When associated with a particular satellite altitude, the minimum acceptable elevation angle represents a maximum acceptable distance from any point on the Earth's surface to the n^{th} nearest sub-satellite point; and the problem is to find the pattern, containing the smallest possible number of satellites, which best ensures that this maximum acceptable distance is never exceeded. Section 4 discusses this aspect, and the relevant calculations, more fully.

It is further assumed that, as a secondary objective, the minimum separation between any two satellites in the pattern should be as large as possible. The direct importance of this objective may vary according to the system application; in a satellite navigation system, accuracy may well increase as the minimum distance between the satellites providing the fix increases, while in a satellite communications system it may only be necessary that the minimum distance should exceed some fixed value, to ensure that interference between transmissions in the same frequency band is acceptably small. However, it also has some indirect importance in relation to the main objective; the larger the minimum distance between satellites, the more uniform the distribution of the satellites over the Earth's surface, and hence the more likely that the pattern will provide relatively favourable values of the maximum distance to the n^{th} nearest sub-satellite point, for all relevant levels of coverage.

The calculation of the relevant parameters is described in section 4. However, as the calculations are somewhat time-consuming, even with the use of a computer, it is desirable to eliminate, in advance, any patterns which may clearly be recognized as unsuitable, and to select one or more patterns which may confidently be expected to be among the best, even if they are not the very best for a particular application. The remainder of section 3 shows how certain groups of delta patterns may be identified in this way.

3.2 Single-plane configurations

The family of delta patterns formally includes single-plane systems, with $P = 1$ and $F = 0$; however, these have been ignored in the computer study of coverage described later in this Report, since the coverage they provide can never reach as far as the poles of the orbital plane and they are therefore unable to meet a requirement for continuous whole-Earth coverage.

With some other patterns, the changing configuration may momentarily pass through a condition in which all the satellites lie in one plane; these patterns also may be rejected as unsatisfactory for providing continuous whole-Earth coverage. Two groups of patterns are identifiable in which all satellites pass through the reference plane simultaneously; these are

(i) those for which $P = T$ or $\frac{1}{2}T$ and $F = 0$, and

(ii) those for which $P = T$ and $F = \frac{1}{2}T$.

For example, among 6-satellite delta patterns, as listed in section 2.3, condition (i) applies to patterns 6/6/0 and 6/3/0, and condition (ii) applies to pattern 6/6/3.

It need hardly be said that, with any pattern, the value $\delta = 0^\circ$, or indeed any small value of δ, is unacceptable for the same reason. In particular, a system of three or more quasi-geostationary near-equatorial satellites, often described as providing 'global' coverage, cannot cover the polar regions and so does not provide whole-Earth coverage in the sense considered in this Report.

3.3 Two-parallel-plane configurations

As patterns which momentarily pass through a single-plane configuration were identified in section 3.2, so we may also identify patterns which momentarily pass through a configuration in which all the satellites lie in two parallel planes.

Consider first those patterns for which $P = T$ and $F = \frac{1}{3}T$ or $\frac{2}{3}T$; also those for which $P = \frac{1}{3}T$ and $F = 0$. With such patterns one-third of the total number of satellites arrive simultaneously at the maximum latitude δ relative to the reference plane, while the other two-thirds are in the opposite hemisphere and all at the same latitude β given by $\sin\beta = \sin\delta \sin 30^\circ$ – eg if $\delta = 60^\circ$ these two-thirds are all at a latitude of 25.7°, so that the pole of that hemisphere is 64.3° from the nearest sub-satellite point.

Whether or not such a situation is acceptable must depend on the total number of satellites in the pattern. For a large pattern, such as 24/24/8, it is clearly undesirable to have 16 of the 24 satellites distributed round a single parallel of latitude at more than 60° from the pole, but for a small pattern, such as 6/6/4 or 6/2/0, such an arrangement may be quite acceptable.

Rather similar considerations apply to those patterns for which $P = T$ and $F = \frac{1}{4}T$ or $\frac{3}{4}T$, and to those for which $P = \frac{1}{4}T$ and $F = 0$. With these, half the satellites are in one hemisphere passing simultaneously through a latitude β given by $\sin\beta = \sin\delta \sin 45^\circ$, while the other half are passing simultaneously through a similar latitude in the opposite hemisphere; if $\delta = 60^\circ$, the poles in both hemispheres are then 52.2° from the nearest sub-satellite point. This is quite acceptable for a small pattern such as 8/8/6 or 8/8/2, but is undesirable for a large pattern.

Thus, when the total number of satellites in the pattern is relatively large (say 12 or more), patterns of the types identified in this section are unlikely to be satisfactory choices.

3.4 Single-figure-8 patterns

If, in synchronous orbit, all the satellites in a pattern follow a single figure-8 Earth-track of the form shown in Fig 2e, with an inclination i substantially less than 90°, then clearly a large area around the antipodes of the node will not be covered. From the formula for $E_{L,M}$ given in section 2.3.2, it may be seen that for $E_{1,1}$ to equal 1 requires that $K = T$, and since $L = M = 1$ we have $J = S = 1$ and $P = T$; also $G = 1 + F = T$, ie $F = T - 1$. Thus a pattern whose code reference is of the form $T/T/(T - 1)$, and for which δ is substantially less than 90°, is unsuitable for providing whole-Earth coverage; and, since the relative positions of the sub-satellite points of the satellites in the pattern are independent of the inclination of the reference plane to the equatorial plane and of the altitude of the satellites, so is this conclusion. The relevant 6-satellite pattern is 6/6/5, and this is illustrated in Fig 9a, in which northern and southern hemispheres are shown separately, for greater clarity, rather than superimposed; although the Earth-tracks in 24-hour orbit (full lines) and in 12-hour orbit (broken lines) are very different, the instantaneous relative positions of the sub-satellite points are the same in both cases, all concentrated in one relatively small area of the Earth's surface.

If the value of δ were substantially greater than 90°, the position would be changed, on the lines indicated in section 2.4 (the coverage of a pattern T/P/F at an inclination δ being equivalent to that of pattern $T/P/(P - F)$ at inclination $(180^{\circ} - \delta)$). Thus in a retrograde orbit the single large figure-8 on which the satellites would be distributed in synchronous orbit would, as in Fig 8b, ensure satisfactory coverage by a pattern of the form $T/T/(T - 1)$, whereas a pattern of the form T/T/1 would have all the satellites concentrated near a single longitude, thus giving unsatisfactory coverage, even though in synchronous orbit they would be following T separate large figure-8s.

In contrast to pattern 6/6/5 in Fig 9a, Fig 9b shows that pattern 6/6/4 provides a very satisfactory distribution of the satellites; this pattern follows six separate figure-8s in synchronous orbit, but follows a single repetitive Earth-track in 12-hour orbit ($L = 2$, $M = 1$). The significance of this will be discussed further in section 3.6.

3.5 Patterns giving small inter-satellite separations

For most purposes it would be unsatisfactory to choose a satellite pattern in which pairs of satellites passed very close to one another; this might cause radio interference, reduce the number of independent observations available from the system, or cause other undesirable effects, even if the possibility of physical collision were discounted.

A number of delta patterns are such that, independent of the value of δ, the ascending node of one satellite coincides with the descending node of another as they pass simultaneously through the reference plane. The patterns involved are those for which P and (S - F) are both even numbers. The explanation for this is that, for an ascending and a descending node to coincide, two ascending nodes must be 180° apart, requiring P to be even; then, for a satellite in one plane to be at its descending node when one in the opposite plane is at its ascending node, its phase angle, which will be ($\frac{1}{2}$FP + nP) PUs, where n is some integer in the range from 0 to (S - 1), must equal an odd multiple (say q) of $\frac{1}{2}$T PUs. Dividing through by $\frac{1}{2}$P, we obtain qS - F = 2n , so that (S - F) must be even.

For most purposes, therefore, patterns in which both P and (S - F) are even numbers may be rejected as unsuitable without any further examination; with a few exceptions, they have not been studied in this survey. For example, among 6-satellite patterns this condition applies to 6/2/1, 6/6/1, 6/6/3 and 6/6/5.

With other patterns, the minimum angular separation between a pair of satellites will vary as δ varies, and may in some cases pass through zero for a particular pair of satellites at a particular inclination within the range of interest. Patterns for which this happens are unlikely to be a suitable choice; hence, if during this study it appeared that a pattern, which would have provided suitable coverage in other respects, led to a minimum satellite separation of less than 3° at the inclination which would otherwise have been chosen, then it was normally excluded from the short-list of patterns deserving a full examination. In a few such cases it appeared worthwhile to continue the calculations to resolve a point of interest, and these cases have been included in the tables of results discussed in section 5.

3.6 Selection of some suitable patterns

As noted in section 2.1, the number of different delta patterns to be considered, for any particular value of T , is equal to the sum of all the factors of T . The considerations listed in the preceding sections may make it possible to eliminate some of them from further consideration; for example, of the twelve 6-satellite patterns listed in section 2.4, it may be possible to eliminate the following seven, as noted in the relevant sections:

6/1/0 (section 3.2)
6/2/1 (section 3.5)
6/3/0 (section 3.2)
6/6/0 (section 3.2)
6/6/1 (section 3.5)
6/6/3 (sections 3.2 and 3.5) and
6/6/5 (sections 3.4 and 3.5).

However, about half the total number of patterns still remain to be considered, and would have to be subjected to a detailed comparative analysis unless some other, simpler, method could be found of choosing between them.

The basis for such a method actually exists in the repetitive-Earth-track analysis in section 2.3. However, while, for convenience of cross-reference to that section, we continue to refer in what follows to altitudes, periods and Earth-tracks, it should be realised that we are really only using the methods of Earth-track analysis previously developed so as to effect a transformation of the pattern into a rotating frame of reference, in order to throw light on certain characteristics of the pattern which are actually independent of altitude or period; it is not implied that the periods used in the analysis would be appropriate for use in a practical system.

In section 2.3.1, when discussing repetitive Earth-tracks followed by individual satellites, it was noted that the examples illustrated in Fig 2b, c and d represent periods such that $L - M = 1$, so that the Earth-tracks repeat themselves after covering exactly 360° of longitude, having made L excursions into each of the northern and southern hemispheres without having crossed themselves at any point. The 12-hour-period Earth-track of Fig 2b is also illustrated in Fig 9b, for pattern 6/6/4, whose characteristics are such that the complete pattern follows this single repetitive Earth-track; and, as noted in section 3.4, this appears to provide a satisfactorily uniform distribution of the satellites over the Earth's surface. Another example of a pattern following a single non-self-crossing repetitive Earth-track for which $L - M = 1$ (pattern 9/3/2 with $L = 4$, $M = 3$) is shown in Fig 6; here again a reasonably uniform satellite distribution, with relatively large minimum distances between the satellites in the pattern (for each of which two positions are shown, separated by 1 PRI), is seen to be achieved. Clearly, if T satellites are distributed evenly (in terms of time) along one of these single non-self-crossing repetitive Earth-tracks, such as are obtained with $L - M = 1$, the satellites can never approach one another closely and will always show a fairly even distribution over the Earth's surface; and a pattern which achieves this result will retain these characteristics, independent of orbital period. If, on the other hand, the satellites are distributed along multiple Earth-tracks, or along a single track which repeats itself after covering a longitude range other than exactly 360°, there will inevitably be points at which the tracks cross, so that two satellites following the relevant parts of the tracks may pass close to one another.

As noted in section 2.3.1, periods for which $L - M = -1$ may also produce non-self-crossing Earth-tracks, since in such cases the track repeats itself after covering exactly 360° of longitude, though in a westerly instead of an easterly direction. As seen in Fig 2h, for a 48-hour period ($L = 1$, $M = 2$) the Earth-track is non-self-crossing, while Fig 2g shows that, at 60° inclination, the track for a 36-hour period ($L = 2$, $M = 3$) is non-self-crossing apart from four small loops, which would not bring two satellites on the same track close to one another if the number distributed along the track was fairly small. For larger values of L, however, as in Fig 2f, the loops would occupy a larger part of the Earth-track and the likelihood of two satellites being close to a cross-over point would be increased.

As noted in section 2.3.1, the periods for which $L - M = 1$ do not produce non-self-crossing tracks, for $i = 60^{\circ}$, when $L > 8$. There are therefore only seven values of L/M for which the Earth-tracks are non-self-crossing when $i = 60^{\circ}$, these being 1/2, 2/1, 3/2, 4/3, 5/4, 6/5 and 7/6. Fig 2d shows that, for $L/M = 6/5$, adjacent loops in the track come to within 10° of one another; for $L/M = 7/6$ they close to within 3° of one another, so this configuration is unlikely to produce satisfactory single-track patterns with widely-spaced sub-satellite points. The value $L/M = 7/6$ has therefore been dropped from the list, and the value $L/M = 2/3$ added, in order to produce a list of seven values which, at first sight, appear likely to be such that, if the single repetitive Earth-track corresponding to each of those values had T satellites evenly distributed along it, the resulting delta pattern would have relatively suitable characteristics for providing whole-Earth coverage.

In section 2.3.2 it was found that, for given values of T, L and M, the values of P and F identifying the particular delta pattern which would follow a single repetitive Earth-track were given by

$$P = T/S = T/H[M,T] \quad \text{and} \quad F = (S/M)(kP - L) \quad .$$

These formulae have been used, for the seven selected values of L/M, and for values of T from 5 to 25, to draw up the list of potentially suitable delta patterns which appears in Table 1. To see whether the patterns identified in this table are indeed among those most suitable for providing continuous whole-Earth coverage, they have been compared with the short-listed patterns appearing in Tables 2 to 7 (which will be discussed in detail in section 5); this comparison is significant, since Tables 2 to 7 were originally prepared before Table 1. A ☆ against a pattern in Table 1 indicates, for values of T from 5 to 15, that this pattern has been identified in one of the Tables 2 to 5 as giving the best coverage for this value of T . A ◇ in Table 1 indicates, for values of T from 5 to 15, that this pattern has been short-listed in one of the Tables 2 to 5 without qualifying for a ☆ ; or, for values of T from 16 to 25, that it has been short-listed in Table 6 or Table 7. A x indicates that no comparison with Tables 6 and 7 was possible, since patterns of the form 16/16/F, 17/17/F and 25/25/F were not included in the computations from which those tables were compiled; a / indicates that this pattern can be identified as having a single-plane configuration, as in section 3.2; and a // that it has a two-parallel-plane configuration, as in section 3.3.

Examining the distribution of ☆ and ◇ symbols in Table 1, it may at first sight appear somewhat surprising that, while there are none in the 48-hour-period column, there are several in the 36-hour-period column, for values of T below 15, despite the loops which occur in the Earth-track at that period. The 12-hour-period column contains most ☆ and ◇ symbols for values of T up to 12, and the 16-hour-period column most for larger values of T . When we come to consider calculated values of the minimum inter-satellite distance, it appears that patterns in the 16-hour-period column give the largest minimum inter-satellite distance for values of T from about 10 upwards; below that, patterns in the 12-hour-period column give the largest value.

Many patterns short-listed in the later tables do not appear in Table 1, showing that the fact that a pattern would follow a single Earth-track at a period at which that track crosses itself several times does not necessarily imply that it has an unsatisfactory satellite distribution. In fact, the effect of cross-overs in the Earth-track depends on the spacing of the satellites along the track; if it is such that two satellites cannot be near a cross-over simultaneously, then the pattern may well provide adequate satellite separation and a satisfactory overall distribution over the Earth's surface. This is illustrated in Fig 10 for the case of the 6-hour-orbit (L = 4, M = 1) , where the single repetitive Earth-track has eight cross-over points, repeating itself only after covering 1080° of longitude; the effect is shown of distributing four different total numbers of satellites along this single track. Fig 10a shows pattern 5/5/1; when one satellite (eg D or E) is near a cross-over point, no other satellite is near it, as would be expected since this pattern appears twice in Table 1, with a ☆ , as some of the other periods at which it produces a single Earth-track correspond to non-self-crossing tracks. Fig 10b shows pattern 7/7/3; here two satellites (eg D and F) are fairly near one another at a cross-over point, and while this pattern is listed in Table 1 it does not qualify for a note. Pattern 10/10/6, shown in Fig 10c, has pairs of satellites (eg B and J, D and G) very close to one another at cross-over points, whereas pattern 12/12/8, shown in Fig 10d, is so spaced that, when one satellite is near a cross-over point, no other is near it. This is not, however, a pattern which has been short-listed in any of the tables; it is one of those in which two-thirds of the satellites are simultaneously at the same longitude, so that, while satellite separation is adequate, coverage is not of high standard.

Overall, it appears from consideration of Table 1 that, if it were desired to select for fuller examination, for a particular value or values of T , a short-list of patterns certain to include some from among those having the best coverage characteristics, then it would be appropriate to choose:

(a) for all values of T , the two patterns that give single Earth-tracks for L/M = 3/2 and 4/3 respectively;

(b) for T < 15 , the two patterns that give single Earth-tracks for L/M = 2/1 and 2/3 respectively;

(c) for T > 11 , the two patterns that give single Earth-tracks for L/M = 5/4 and 6/5 respectively.

The coverage characteristics of the patterns on this short-list (after eliminating any single-plane and, if appropriate, two-parallel-plane patterns) could then be calculated in detail by the methods to be described in section 4. However, if it were essential that the optimum pattern to meet a particular requirement should be found, or if there were system limitations (for example, on the number of orbital planes) which none of the short-listed patterns could meet, then it would be necessary to make the coverage calculations for a wider range of patterns, eliminating only those clearly shown to have unsatisfactory characteristics.

It must be emphasised that there is no suggestion that the particular orbital periods, used here as a basis for short-listing certain of the patterns, are especially desirable periods for use in a practical satellite system; this might apply only if a single repetitive Earth-track were a system requirement.

The reason for their use in the analysis is rather that they indicate that these patterns provide a favourable satellite distribution, and this favourable distribution will be applicable whatever the orbital period and whatever the number of separate Earth-tracks at that period.

4 METHODS OF ANALYSING COVERAGE

4.1 General approach

As already noted, the purpose of this study has been to develop methods of identifying those satellite orbital patterns which will most economically provide a required standard of continuous whole-Earth coverage, the level of coverage provided (single, double, ..., n-fold) being dependent on the elevation angle to the n^{th} nearest satellite always exceeding some specified minimum value, at every point on the Earth's surface. Section 3 considered the question of eliminating definitely unsuitable patterns, and selecting some of the more promising patterns, from the full list of delta patterns which might potentially be considered suitable; this section considers the method of evaluating the standard of coverage provided by any particular pattern under examination.

In several previous sections, attention has been concentrated on the Earth-tracks traced by a pattern at a particular orbital altitude. However, when we are considering whole-Earth coverage, what really matters is the uniformity of the distribution of the satellites in the pattern relative to a spherical surface; and this is probably more readily visualised in terms of the orbital paths of the satellites than in terms of the Earth-tracks they would follow at any particular orbital altitude or altitudes. For example, it appears better to think of pattern 9/3/2 on the basis of Fig 1 rather than Fig 6.

For the elevation angle ε to a satellite always to exceed some minimum value, the observer's geocentric angular distance d from the sub-satellite point must always be less than some maximum value which depends on the satellite altitude (or period). For a circular orbit, the relationship between ε and d is given by

$$\frac{\cos \varepsilon}{\cos (d + \varepsilon)} = \frac{r_s}{r_e} = 0.795 t_{sh}^{2/3}$$

where r_e is the radius of the Earth, r_s the radius of the satellite orbit, and t_{sh} the orbital period in sidereal hours; this is also plotted in Fig 11 (reproduced from Fig 1 of Ref 1).

The analysis of coverage has been based on the identification of those points at which the elevation angle to the nearest satellite (or the n^{th} nearest for n-fold coverage) is locally a minimum. These points are the centres of the circumcircles of the spherical triangles formed by the sub-satellite points of neighbouring satellites in the pattern. In Fig 12, which represents an undefined portion of the Earth's surface, the ♂ symbols labelled A to G represent the instantaneous positions of seven out of a total of, say, 10 sub-satellite points of some hypothetical 10-satellite pattern. 0_1, which is the instantaneous position of the centre of the circumcircle of the spherical triangle ABC, whose geocentric angular radius is R_1, is locally the point on the Earth's surface furthest from any sub-satellite point; at 0_1, an observer's distance d from each of A, B and C is equal to R_1, but another observer located a short distance from 0_1, in any direction, would be at a distance less than R_1 from at least one of those three sub-satellite points. Considering all the other spherical triangles, such as BCF and CDF, whose circumcircles do not enclose any other sub-satellite point, then the centre of the largest of those circumcircles is the point on the whole of the Earth's surface which is instantaneously furthest from any sub-satellite point. We will call the radius of that circumcircle $R_{max,1}$; here the suffix 1 indicates single coverage, and the suffix max (with a small 'm') that it is the *instantaneous* maximum value, over the whole of the Earth's surface, for that level of coverage.

If the requirement should be for double coverage (ie for not less than two satellites to be everywhere visible above the minimum elevation angle) then the problem may be tackled in similar manner, but considering circumcircles which enclose one other sub-satellite point - eg ABD in Fig 12, which encircles C. 0_2 is at a distance R_2 from A, B and D, and at a lesser distance from C; an observer at a little distance from 0_2 would be at a lesser distance than R_2 from at least one of A, B and D, as well as from C. This value of R_2 would have to be compared with the values of R_2 for those other circumcircles which each enclosed one other sub-satellite point, in order to find the instantaneous value of $R_{max,2}$.

Similar considerations apply if the requirement is for simultaneous visibility of any larger number of satellites. In Fig 12 the circumcircle of ABE encloses C and D, so its centre 0_3 is potentially - a qualification to be explained in the next-but-one paragraph - a locally least-favoured point for provision of triple coverage, being at a distance R_3 from A, B and E, and at lesser distances from C and D. Fig 12 shows 0_3 as also being at a distance

R_3 from a fourth sub-satellite point F; in this case O_3 is simultaneously the centre of the circumcircles of four spherical triangles - ABE, ABF, AEF and BEF. The next largest circumcircle shown as passing through A and B is that which also passes through G; it encloses C, D, E and F, so its centre O_5 is potentially a locally least-favoured point for provision of five-fold coverage. This clearly indicates that the centre and radius of ABEF should respectively be regarded not only as O_3 and R_3, relating to triple coverage, but also as O_4 and R_4, relating to quadruple coverage. As shown in Fig 12, F is just passing out of the circumcircle of ABE while E is passing into the circumcircle of ABF; previously ABF would have provided triple coverage and ABE quadruple, while subsequently ABE would provide triple coverage and ABF quadruple.

The circumcircle of ABG, as well as having a radius R_5 with centre at O_5 and enclosing C, D, E and F, also has a radius of $(180^\circ - R_5)$ with centre at the antipodes of O_5 and enclosing the three remaining sub-satellite points H, J and K - not shown in Fig 12 - of this 10-satellite pattern, so providing quadruple coverage. Likewise the circumcircle of ABD has a second radius of $(180^\circ - R_2)$ with centre at the antipodes of O_2 and enclosing the six sub-satellite points E to K, so providing seven-fold coverage; and the circumcircle of ABC has a second radius $(180^\circ - R_1)$ with centre at the antipodes of O_1 and enclosing the seven sub-satellite points D to K, so providing eight-fold coverage. To ensure that every locally least-favoured point has been examined as a possible source of the value of $R_{max,n}$, it is necessary to consider both larger and smaller circumcircle radii of all possible combinations of three sub-satellite points, to find the number of other sub-satellite points which are enclosed in each case; for though with many patterns, such as pattern 6/6/4 shown in Fig 9b, only the smaller radii are likely to be of practical interest, there are others, such as pattern 6/6/5 in Fig 9a, where the larger radii are significant.

It was noted earlier that O_3 in Fig 12, as centre of the circumcircle of ABE, is 'potentially' a locally least-favoured point for provision of triple coverage. The reason for this qualification may be explained, for a single-coverage example, by reference to Fig 13. Here A, B and C are three sub-satellite points whose circumcircle has a centre O_p which lies outside the spherical triangle ABC. Considering observers stationed at O_p, X and O_q, all on the perpendicular bisector of AB, at increasing distances from C, it is evident that the observer at X is further than the observer at O_p from all three of A, B and C, and the observer at O_q is still further from all three, so that O_p is not in fact a locally least-favoured point; the distance from A and

B increases from R_p at 0_p to d_X at X and to R_q at 0_q, while the increase in distance from C is even greater. However, 0_q is the centre of the circumcircle of ABD, where D is an adjacent sub-satellite point such that 0_q *does* lie inside ABD; hence 0_q *is* a locally least-favoured point, and R_q is a possible candidate for the value of $R_{max,1}$. Thus not all spherical triangles formed by combinations of three sub-satellite points necessarily provide valid candidates for the value of $R_{max,n}$; that value will be found among the radii of the circumcircles of those triangles which have centres falling within the triangles. However, such non-valid radii are effectively self-eliminating, since each has a larger, valid radius adjacent to it.

Returning to Fig 12, it is apparent that ABEF is a limiting case. R_3 does not qualify as a valid candidate for the value of $R_{max,3}$ by virtue of being the radius of the circumcircle of triangles ABE and ABF, since 0_3 lies outside those triangles; however, it does qualify in respect of triangles AEF and BEF, since 0_3 lies inside these triangles, and it must also be considered as a possible (though unlikely) candidate for the value of $R_{max,4}$.

We have so far considered only the determination of the value of $R_{max,n}$ for an instantaneous configuration of a satellite pattern, whereas we are really interested in the maximum value taken over the whole of a pattern repetition interval, which we shall call $R_{Max,n}$ (with a capital 'M'). The list of spherical triangles whose circumcircles enclose (n - 1) other sub-satellite points, and whose radii are therefore potential candidates for the value of $R_{Max,n}$, may well change as ϕ changes during the course of one PRI, as other sub-satellite points move into or out of the circumcircle of any one group of three; for example, in Fig 12, B is just passing from outside to inside the circumcircle of AEF, which will then be enclosing three other sub-satellite points instead of two, and so provide an example of quadruple, rather than just triple, coverage. In general, the actual value of $R_{Max,n}$ will be associated with one or more particular spherical triangles at one particular value of ϕ. It occasionally happens that the value of $R_{max,n}$, as given by a particular triangle, passes through a maximum at some value of ϕ, but it is more usual for $R_{Max,n}$ to occur when the relevant triangle is on the point of passing from the n-fold to the (n + 1)-fold coverage list; thus, in Fig 12, AEF is on the point of passing out of the triple coverage list and $R_{Max,3}$ for this pattern might well be found to equal this value of R_3, provided the circumradius of AEF is increasing with ϕ.

Values of $R_{Max,n}$, which represent maximum values of the sub-satellite distance d for a particular pattern inclination δ, mav be converted to values of the minimum elevation angle ε, for any satellite altitude, by use of Fig 11. Any value of $R_{Max,n}$ exceeding 81.3°, which corresponds to zero elevation angle for satellites in 24-hour orbits, is unlikely to be acceptable; for ε to be never less than 5°, $R_{Max,n}$ must not exceed 76.3° for 24-hour satellites, or 71.3° for 12-hour satellites. The value of $R_{Max,n}$ determined for a particular pattern is thus a very suitable criterion for assessing the merits of that pattern, at the particular inclination δ, for providing n-fold coverage; the smaller the value of $R_{Max,n}$, the larger will be the minimum elevation angle provided by a satellite system using that pattern at a given altitude, or the lower will be the altitude at which it can ensure that a requirement for a given minimum elevation angle is met.

If the pattern inclination δ is varied, the value of R_n for each spherical triangle, and the value of $R_{Max,n}$ for the whole pattern, will also vary; as δ is increased, R_n will be increased for some triangles and decreased for others. Fig 14 illustrates this for one of the simplest cases, $R_{Max,1}$ for pattern 5/5/1. When $\delta = 0°$, $R_{Max,1}$ must equal 90°, but as δ is increased the value of $R_{Max,1}$, which is associated with the group of satellites ACDE at $\phi = 1.0$ PRU, falls steadily until it reaches the value of 69.2° at $\delta = 43.7°$. In the meantime the value of R_1 associated with the triangle BDE at $\phi = 1.0$ will have been increasing as δ increased, until at $\delta = 43.7°$ it also reached the value of 69.2°; as δ is increased beyond 43.7°, so the value of R_1 for ACDE will continue to fall, but BDE now provides the value of $R_{Max,1}$, which increases steadily.

For other patterns the picture is similar in principle, though usually more complex in detail, with $R_{Max,n}$ switching more frequently from one triangle to another, and with the associated critical values of ϕ varying accordingly. Thus the plot of $R_{Max,n}$ against δ, which is the upper envelope of plots of R_n for all relevant triangles, usually shows more separate facets than the simple V-shaped plot of Fig 14, but always has a minimum value, which we shall call $R_{MAX,n}$ (with 'MAX' in capitals), at some value of δ which we shall call δ_{opt}. In a few cases $R_{MAX,n}$ and δ_{opt} are associated with a minimum in the plot of R_n against δ for one particular triangle, at one value of ϕ; but in most cases they are associated with a cross-over of the plots for two different triangles, each at a separate value of ϕ. In general, the present study has been directed towards finding, for each value of T (the total number of satellites in the pattern) and of n (the level of coverage), the pattern providing the minimum value of $R_{MAX,n}$. As noted in section 3.1, it has been taken as a secondary objective that the minimum separation between any two satellites in the pattern should be as large as possible. Using a parallel nomenclature to that for R, we use D_{min} to denote the minimum inter-satellite separation in a pattern for a particular value of ϕ , at a particular value of δ ; D_{Min} to denote the minimum separation measured over a whole PRI, for a particular value of δ ; and D_{MIN} to denote the maximum value of D_{Min} found for any value of δ . In general, this study has not set out to find values of D_{MIN} (though this would be easily done, and examples were given in Refs 1 and 2), but has rather found values of D_{Min} at the inclination δ_{opt} , as a guide to choosing between patterns having similar values of $R_{MAX,n}$.

In the discussion so far, it has been assumed that the requirement to be met is one for providing continuous coverage, through several different satellites arranged in a suitable pattern, of every point on the Earth's surface. Many practical applications will require intercommunication between two or more points via a satellite, so that these points must always lie, at any one time, within the coverage area of a single satellite, though this simultaneous coverage might be provided, over a period of time, by several satellites in turn. The approach via determination of values of $R_{MAX,n}$ is not appropriate for the analysis of systems required to provide the type of large-area coverage needed for intercontinental commercial communications[18], but it does have some application to possible requirements for limited-area multipoint coverage, as illustrated in Fig 15. Here A, B and C are the sub-satellite points of three satellites in a pattern· their circumcircle, of radius R_k , has its centre at O_1. O_1 is then also the point of intersection of circles of radius R_k centred on the three sub-satellite points. If coverage circles corresponding to the minimum acceptable elevation angle actually have a larger radius d_ℓ , then these intersect in pairs at three points X, Y and Z such that the whole of the triangle XYZ lies within the coverage circles of radius d_ℓ for all three satellites. The inscribed circle of XYZ has its centre at O_1 , and its radius d_m is equal to $d_\ell - R_k$; if R_k is equal to the value of $R_{Max,n}$ for this pattern at this inclination, then d_m is the radius of the largest circular area which can be guaranteed always to lie wholly within the coverage area of a single satellite. XX' is the shortest of the three distances between one of the vertices of XYZ, at the intersection of two of the coverage circles, and the nearest point on the third coverage circle; this is the greatest permissible distance between a pair of points such that it can be guaranteed that both points will always lie within the coverage area of a single satellite. This distance is approximately equal to $2d_m$ (though actually a little greater).

Hence, if a requirement should be expressed in the form that a satellite system must be able to provide intercommunication between any two points on the Earth's surface which are not more than $2d_m$ apart, or between all points within any circular area of radius not exceeding d_m, then an appropriate pattern would be one having a value of $R_{MAX,1}$ not exceeding $(d_\ell - d_m)$, where d_ℓ corresponds, in Fig 11, to the minimum acceptable elevation angle with satellites at the chosen orbital period. For example, continuous world-wide single-point coverage by 24-hour satellites above 5^o elevation requires a pattern having a value of $R_{MAX,1}$ not exceeding 76.3^o; pattern 5/5/1, at its δ_{opt} of 43.7^o, has $R_{MAX,1}$ equal to 69.2^o, so it can provide continuous simultaneous coverage, above 5^o elevation, of all points within any circular area of radius 7.1^o, ie 790 km, or of any two points not more than about 1580 km apart. Such 'points' need not be fixed points on the Earth's surface, but might, for instance, be ships; and the satellite period need not be limited to one of the values giving repetitive Earth-tracks. However, if the requirement included service to a few specified fixed points as well as to mobile stations, a rather better result could probably be obtained by choosing a repetitive-track period and nodal longitudes which ensured favourable elevation angles at those fixed points.

Hence, whether the system requirements allow freedom to choose the orbital inclination to give the required coverage most economically, or whether they constrain the inclination to a particular value (or range of values), the determination of values of $R_{MAX,n}$ and δ_{opt} in the former case, or of $R_{Max,n}$ at the required inclination in the latter, is a very powerful tool for assessing the relative merits of different satellite patterns and, with some consideration also for values of D_{Min}, for choosing the pattern most suited to the task of providing continuous world-wide coverage in any particular circumstances.

4.2 Computer analysis

The general approach described in section 4.1 has been used since the beginning of these studies, as recorded in Refs 1 and 2. In the early stages, as described in Ref 1, it was put into effect by:

(i) plotting the pattern on a globe, for phase angles corresponding to the beginning and end of a pattern repetition interval, and for large and small values of δ;

(ii) identifying for each case the triangle having the largest circumcircle, and finding its approximate radius, by visual inspection aided by a near-hemispherical scale, known as a 'geometer', used for measuring geocentric angles; and then

(iii) solving those triangles by simple formulae of spherical trigonometry in order to find δ_{opt} and obtain an accurate value for $R_{MAX,1}$ (which was at that time described as d_{max}).

The process described was practicable for patterns involving only small numbers of satellites, but the risk of error in identifying the triangle having the largest circumcircle increased rapidly as the number of satellites increased, and it was soon realised that a computerised approach would be necessary if the study was to be extended to larger numbers of satellites and higher levels of coverage. Ref 1 described the results obtained by the original hand methods; by the time it was published, work on the computer program had begun, and before publication of Ref 2 (a much shortened version of Ref 1) the original results had been checked on the computer and modifications had been found necessary to two out of the eight results quoted for delta patterns. Introduction of the computer program made it convenient to change the nomenclature used in Refs 1 and 2 in a number of respects; the revised system has been described in sections 2.1, 2.2 and 4.1 of this Report.

Some coverage studies elsewhere[12,13] have used computer programs which determined the number of satellites visible, above some minimum elevation angle, at each of a number of points forming a grid pattern on the Earth's surface. That approach was rejected at the outset, as it was thought that accuracy would be poor unless the grid points were very closely spaced, so that their number would be very large; instead, the computer program was based on the approach using circumcircle radii, described in the preceding section, which enables worst-case conditions to be determined precisely.

After passing through several stages of development this program, identified as COCO (for circular orbit coverage), and written in FORTRAN for use on the ICL computers installed at RAE, came to be used in three different versions, suitable for use at different stages in the process of pattern analysis: a 'full version' and a 'shortened version', both held in binary form on magnetic tape, and an 'express version', held in source form on punched cards for use in the EXPRESS RADS service at RAE.

Originally, only the full and express versions were developed, and these were initially dimensioned for a maximum pattern size of 12 satellites; this was later increased to 15 and eventually to 25. It is necessary to provide, as input data, a list of the specific values of δ at which the pattern is to be examined, and a similar list of values of ϕ, as well as information identifying the pattern itself; the latter could take the form, for each satellite, of the identification letter and the two integer suffices defining its position in the pattern, as described in section 2.1, but for a complete delta pattern it is only necessary to input the pattern reference code T/P/F, and the program can then derive the characteristics of individual satellites. The full program analyses the whole pattern, but the express version only examines specified pairs and trios of satellites, and so requires, as an additional input, the identification numbers (defined below) of the pairs and trios specified.

Either version of the program starts by taking the first values of δ and ϕ in the respective lists and using these to calculate the latitude and longitude of each satellite. Next, the satellites are taken in pairs, and given identification numbers ranging from 1 for AB and 2 for AC to $\frac{1}{2}T(T-1)$ for the last pair (eg 276 with 24 satellites). With the full version, the angular separation between each pair is calculated, these values are sorted into ascending order, and the sorted list of identification numbers and separations (if there are not more than 45 - otherwise only the smallest 30 and largest 14 are retained) later appears in the print-out. With the express version, the angular separation is only calculated between each specified pair, and for pairs making up specified trios, so no sorting is involved.

Next, the satellites are taken in groups of three, and each trio is allocated two identification numbers, ranging from 1 and 2 for ABC and 3 and 4 for ABD to $(\frac{1}{3}T(T-1)(T-2)-1)$ and $\frac{1}{3}T(T-1)(T-2)$ for the last group (eg 4047 and 4048 with 24 satellites). As noted in section 4.1, the circumcircle of each group of satellites has two radii; the smaller (odd) number is allotted to the smaller radius and the larger (even) number to the larger radius. For each group in turn (in the express version, specified trios only) the spherical triangle is solved by the standard formulae of spherical trigonometry; the sides are already known, as angular separations between pairs of satellites, and from these the angles of the triangles are found. If the three satellites are readily identifiable as lying on a great circle, the value of the circumradius R is set to 90°; if not, the smaller value of R is calculated from the standard formula

$$\tan R = \tan \tfrac{1}{2}a \sec \tfrac{1}{2}(B + C - A)$$

where a, b, c are the sides opposite the angles A, B, C respectively, as noted in Appendix D of Ref 1. From this, the coordinates of the centre of the circumcircle are calculated, the angular distance from the centre to each of the satellites in the pattern is found and compared with the circumcircle radius, and hence the number of satellites lying within the circle is found.

With the express version, this completes the calculations for the first of the listed values of δ and ϕ ; the print-out shows: (a) for each specified pair of satellites, the angular distance between them, and (b) for each specified trio: (i) the circumradius, (ii) the associated level of coverage (the number of satellites enclosed plus one), (iii) the coordinates of the three satellites and of the centre, and (iv) the angular distance from the centre to each satellite in the pattern. The process is then repeated for the other listed values of ϕ , for the same value of δ ; and subsequently, for all listed values of ϕ , for the other listed values of δ .

With the full version, each circumradius is compared with those previously calculated which correspond to the same level of coverage, for levels of coverage from single to seven-fold; values corresponding to higher levels of coverage are discarded. For these seven levels of coverage, lists are compiled, in descending order of radius, of trio reference and circumradius; when the number in any list would exceed 45, only those having the 30 largest radii and the 14 smallest radii are retained. After the smaller radius of each circumcircle has been dealt with, the larger radius is also examined. When all trios have been considered, the lists of trios and their radii for the seven levels of coverage are printed out, together with the list of pairs and separations calculated previously.

The process is then repeated, for the same value of δ , for the other listed values of ϕ , after which a summary is produced showing, for that value of δ , the apparent values (derived only from the listed values of ϕ) of $R_{Max,n}$, from $n = 1$ to $n = 7$, and of D_{Min} . The process is then repeated for the other specified values of δ ; and finally a summary is printed showing the apparent values (derived only from the specified values of δ and ϕ) of $R_{MAX,n}$, from $n = 1$ to $n = 7$, with the corresponding apparent values of δ_{opt} and D_{Min} , and the apparent value of D_{MIN} .

The overall procedure found appropriate, making use of the full and express versions of COCO in turn, is as follows. First, a list of potentially appropriate patterns is drawn up, based on the value of T expected to be suitable for the value of n required. Second, the considerations of sections 3.2 to 3.5 are used to eliminate clearly unsuitable patterns; alternatively, an even shorter list might be used, limited to the most promising patterns identified in accordance with section 3.6. Third, each of the remaining patterns is examined using the full version of COCO, probably considering three values of δ (say 45^o, 55^o and 65^o) and two or three values of ϕ (say 0 and 1.0, and perhaps also 0.5). From these results an initial estimate is made of the values of δ_{opt}, $R_{MAX,n}$ and D_{Min} for each pattern, and all except the two or three most promising patterns discarded. For each of the remaining patterns, the identification numbers of those triangles which appear liable to be involved in critical conditions determining $R_{MAX,n}$, with a range of suitable values of δ and ϕ, are then used as input data for a run of the express version of COCO. After interpolating between the printed results, and performing one or more further runs of the express version using improved estimates, it should be possible to make a precise estimate of the values of $R_{MAX,n}$ and δ_{opt}, with the associated triangle reference numbers and values of ϕ; a further run of the full version of COCO, using the selected value of δ, and about six or seven values of ϕ between 0 and 1.0, including the predicted critical values, should then confirm these values of $R_{MAX,n}$ and δ_{opt}, with the associated value of D_{Min}, or else provide further information from which the correct values may soon be determined.

This procedure, using full and express versions of COCO, was used successfully for studies of from single to quadruple coverage, using up to 15 satellites, results of which were given in Refs 3 and 4. However, the full version has the disadvantage that the number of triangles to be examined, and hence the computer running time, increases approximately as the cube of the value of T.

When it was desired to increase the maximum number of satellites in the patterns studied from 15 to 25, in the light of reported proposals[13] for a satellite navigation system using a delta-type pattern of 24 satellites, it was felt that this might lead to unacceptably long running times with the full version of COCO, so that a different approach should be explored. The running time of a program which determined the number of satellites visible from each of a number of grid points, though it would increase in inverse proportion to the square of the grid spacing, would only increase in direct proportion to the number of satellites in the pattern; a program, identified as GRID, was therefore prepared based on this approach.

GRID re-uses as many as possible of the elements of COCO, but differs in the basic calculations. Having determined the coordinates of the grid points (which are non-rotating relative to the orbital planes) and the coordinates of the satellites for the relevant values of δ and ϕ , it proceeds, for each grid point in turn, to find the angular distance to every sub-satellite point and to select from these the seven nearest, listed in increasing order of distance. As each successive grid point is examined, its distance to the nearest satellite is compared with the largest previous value of distance to the nearest satellite, second nearest to largest previous second nearest, and so on, only the larger value being retained in each case. When all grid points have been examined, these largest values of distances to the seven nearest satellites are printed and then, as with COCO, the process is repeated for other phase angles, and subsequently for other inclinations, the results being sorted and summaries printed at each stage.

The spacing of the grid points is treated as an input variable. The equator is divided into 2N equal segments, the zero meridian into N similar segments, and each of the (N - 2) parallels of latitude into M_ℓ equal segments, where M_ℓ is the nearest integer to 2N multiplied by the cosine of the latitude; a single grid point is also inserted at each pole. Other studies[12,13] elsewhere have not used this cosine factor, which maintains a relatively uniform distribution of grid points over the Earth's surface, avoiding the concentration of grid points which otherwise occurs at high latitudes as the meridians converge, and producing a valuable saving in computing time by reducing the total number of grid points; for example, with $N = 36$ (5° spacing) the number of grid points is reduced from 2522 to 1652.

It was found that, with $T = 15$ and 5° grid spacing, the running time of GRID was approximately equal to that of the full version of COCO; for larger values of T , GRID therefore has an increasing advantage. However, it was apparent that GRID also has substantial disadvantages relative to COCO. The results it produces inevitably underestimate the angular distance between the least-favoured point on the Earth's surface and a sub-satellite point. With 5° grid spacing, the error might be anything from zero to about 3.5°, with an average value of about 1.9°; with 10° spacing, as used elsewhere[12], these errors would be doubled. This uncertainty made it impossible to use the results to select a short-list of preferred patterns with any degree of confidence; it appeared desirable to reduce the grid spacing to about 3°, but this would so increase the running time that, even with $T = 24$, GRID would have a negligible advantage over the full version of COCO. Approximate relative running times of GRID and COCO are shown in Fig 16.

The other major disadvantage of GRID, relative to the full version of COCO, is that it produces much less information about the behaviour of the pattern, and provides no basis for a more detailed investigation of the critical conditions.

It therefore appeared that a downgraded version of COCO might be a useful compromise solution. The 'shortened version' which was then prepared differs from the full version only in that, as each triangle circumradius is calculated, it is compared with the largest circumradius previously found for the same level of coverage, and only the larger value is retained; the print-out, instead of listing up to 45 triangles for each level of coverage, quotes the largest only. This makes it rather less easy to pick out the critical triangles, for subsequent study using the express version of COCO, since their behaviour cannot be followed at values of ϕ at which their circumradius is not the largest for a particular value of n ; however, it approximately halves the running time, as shown in Fig 16. In consequence, no further use has been made of the GRID program, and the shortened version of COCO has generally been used instead of the full version for values of T exceeding 15.

5 NUMERICAL RESULTS FOR COVERAGE BY DELTA PATTERNS

Numerical results obtained through use of the program COCO to examine delta patterns are presented in Tables 2 to 7.

Tables 2 to 5 contain results for single, double, triple and quadruple coverage respectively, by delta patterns containing up to 15 satellites. These were obtained during 1971 and 1972, when the program was first used to extend the scope of the study[1,2] previously conducted using hand methods. Condensed versions of these tables have previously been presented in a paper[3,4] prepared for the IEE International Conference on Satellite Systems for Mobile Communications and Surveillance, held in London in March 1973.

Tables 6 and 7 contain results, for up to seven-fold coverage by delta patterns containing up to 25 satellites, which were obtained during late 1974 and early 1975 after the program had been modified, as described in section 4.2, to handle larger patterns. Whereas the earlier study had been quite general in nature, this effort was conducted primarily to clarify the background to US proposals[13] to use a 24-satellite, 12-hour, delta-type pattern for a satellite navigation system providing up to six-fold coverage, and for this reason it was concentrated on finding those patterns which appeared most suitable at a given inclination of 60^{o} or 63^{o}, rather than on finding the optimum inclination. In a few instances the optimum inclination was investigated, and these results appear in Table 6; results for 63^{o} inclination appear in Table 7. Results for 60^{o} inclination do not differ significantly from those at 63^{o}, and are not presented here. A discussion of the relevance of these results to a satellite navigation system appears in Appendix A, while the following discussion is more generally applicable.

The first column in each of Tables 2 to 7 lists the pattern reference codes of those delta patterns for which firm results were obtained. All possible delta patterns containing up to 25 satellites, with the exception of patterns of the forms 16/16/F, 17/17/F and 25/25/F, were examined during the course of these studies; however, as regards Tables 2 to 5, all except those listed were eliminated at an intermediate stage, when the examination had proceeded far enough to establish that, on the criteria being used (primarily a small value of $R_{MAX,n}$, but the values of D_{Min} and P were also considered), they were of less interest than other patterns retained in the list having the same value of T. It appears that, in most cases, the patterns providing the best coverage are found among those having only a single satellite in each plane; the instances in which the best results are obtained with several satellites in each of a small number of planes are comparatively rare. Some of the patterns listed have drawbacks (eg $D_{Min} = 0$) which would justify their elimination, and these were excluded from the tables in Refs 3 and 4, but they are included here for completeness, since they were originally considered to have sufficient interest to justify continuing till firm results were obtained. It is not suggested that the patterns omitted from these tables would be unsuitable for use under any circumstances; the criteria used here may not always be appropriate, and it may be found desirable to examine a wider range of patterns on the basis of criteria chosen to reflect particular system requirements.

In Tables 2 to 6, the second column shows, for the relevant value of n, the value of $R_{MAX,n}$ for each listed pattern; the third column shows the inclination δ_{opt} at which this value occurs; and the fourth column shows the value of D_{Min} at this inclination. Fig 17 shows, plotted against T, the lowest value of $R_{MAX,n}$ for each value of T, at each value of n, taken from these tables; no value is shown for T = 16 or 25, since not all the relevant patterns were examined. The general trends shown in this figure are those that would be expected, though the decrease in $R_{MAX,n}$ with increase in T is not entirely regular; it appears that, due to some quirk of geometry, the best 11-satellite delta pattern can provide a slightly smaller value of $R_{MAX,1}$ than can the best 12-satellite delta pattern, and the best 14-satellite delta pattern a slightly smaller value of $R_{MAX,1}$ than can the best 15-satellite delta pattern. Fig 17 is effectively an extended version of Fig 11 of Ref 1.

Of the 86 values of δ_{opt} shown in Tables 2 to 6, more than half lie between 50^o and 60^o; the smallest value is 43.7^o and the largest 71.8^o. For 3-plane patterns it might be expected that the optimum inclination would approximate to the value at which the three planes are mutually orthogonal, ie $\sin^{-1}(\cos 45^o/\cos 30^o)$, which equals 54.7^o; from these results it appears that 55^o would be a sound initial estimate for other patterns as well. There seems to be a tendency for the values of δ_{opt} to be slightly lower for single and double coverage than for higher levels of coverage, but this is not very significant relative to the overall scatter of values.

The values of D_{Min} listed in Tables 2 to 6 are those corresponding to the inclination δ_{opt}. If it were important, for a particular system, that the minimum satellite separation should be as large as possible, then it would be appropriate to optimise for D_{MIN} rather than for $R_{MAX,n}$, or at least to aim at a compromise between the two; the patterns short-listed in these tables would not necessarily be the best for that purpose, and section 3.6 (and Table 1) would provide a better basis for pattern selection. It may be seen, from Tables 2 and 6 respectively, that with a pattern containing only five satellites it is possible to obtain a value of D_{Min} greater than 80^o, while even with a pattern containing 24 satellites it is possible to obtain a value of D_{Min} greater than 30^o.

The fifth and sixth columns in Tables 2 to 6 show, respectively, the values of ϕ, and the latitudes (relative to the reference plane) of the circumcircle centres, corresponding to the critical conditions for $R_{MAX,n}$. These may be understood more readily by relating the entry for pattern 5/5/1 in Table 2 to Fig 14. For many patterns there are double entries in both the ϕ column and the latitude column, but with pattern 5/5/1, as with several other patterns, the value of ϕ at which R_1 is a maximum is 1.0 for both groups of satellites (ACDE and BDE) which contribute equal values of $R_{Max,1}$ to provide the value of $R_{MAX,1}$ at δ_{opt}. The centres of the circumcircles of these two groups, and other corresponding points at the latitudes listed in the sixth column, in both northern and southern hemispheres, and at several longitudes, are the points which experience the minimum elevation angle to the n^{th} nearest satellite which corresponds to the value of $R_{MAX,n}$. Even if the basic aim of a system were to provide uniformly satisfactory whole-Earth coverage, there might be reasons for providing above-average minimum elevation angles at a particular location - for example, where a telemetry, tracking and command station must be located - in which case it would probably be advisable to avoid any close correlation between the critical latitudes for the chosen pattern and the latitude of the station; single-station coverage was considered by Merson[19].

In most cases two groups of satellites are involved in the critical conditions, so two latitudes are quoted. With pattern 12/3/1 in Table 2 there are four groups having the same circumradius, centred at four different latitudes, while in a few other cases (such as pattern 5/5/3 in Table 2) $R_{MAX,n}$ occurs at a minimum in the plot of $R_{Max,n}$ against δ for the circumradius of a single group, so that there is only a single entry for latitude as well as for ϕ. For single coverage, critical conditions usually occur at one or both of the extremes of the PRI, when $\phi = 0$ or 1.0, but for higher levels of coverage they are more likely to occur at intermediate phase angles, and this was one of the difficulties which made it necessary to abandon the hand methods of analysis used for Ref 1 in favour of the computer program.

In Table 7, the second to eighth columns give values of $R_{Max,n}$ from $n = 1$ to $n = 7$, and the ninth column gives the value of D_{min}, for the single inclination of 63^o; this is the inclination which has been suggested[13] for use in the Navstar/GPS satellite navigation system, and it is not necessarily the optimum inclination for any particular level of coverage. Whereas all the values of $R_{MAX,n}$ quoted in Tables 2 to 6 have been refined by use of the express version of COCO, as described in section 4.2, in order to find the true maximum value at the most adverse phase angle, the values of $R_{Max,n}$ in Table 7 have not, and it is therefore possible that some of them are slight under-estimates of the true values. Most of the 24-satellite patterns, and all those containing 19 satellites or fewer, have been checked for six or more values of ϕ, and any errors are likely to be very small; but patterns in the sidelined block in the centre of the table have been checked only for three values of ϕ (0, 0.5 and 1.0), and the possibility that the results are under-estimates is therefore somewhat greater, though significant errors are unlikely.

Three of the patterns listed in Table 7 have the characteristic described in section 3.3, having $P = T$ and $F = \frac{1}{3}T$ or $\frac{2}{3}T$, so that in the worst case two-thirds of the total number of satellites lie on a single circle centred on one pole, with a radius, for 63^o inclination, of 63.5^o. Thus, for patterns 21/21/7, 21/21/14 and 24/24/8, 63.5^o is the value not only of $R_{Max,1}$ but also of $R_{Max,2}$ and $R_{Max,3}$, and for pattern 24/24/8 it is also the value of $R_{Max,4}$; but these patterns nevertheless have relatively favourable values of $R_{Max,5}$ and $R_{Max,6}$, which led to their inclusion in the table.

The ninth column in Table 7 gives values of D_{Min} for 63° inclination, and the tenth column shows the number of separate Earth-tracks which each of these patterns would follow in 24-hour and in 12-hour orbits, the latter being the period suggested[13] for the Navstar/GPS system.

The last column in each of Tables 2 to 7 contains a note (☆ or ◇) indicating the correlation with Table 1, in which patterns are listed on the basis of their compatibility with non-self-crossing single Earth-tracks, as discussed in section 3.6. The correlation is generally good, particularly considering that some patterns were included in Tables 2 to 7 because of the flexibility available from their use of only two or three orbital planes, rather than the provision of a favourable value of $R_{MAX,n}$, and others have been included despite a zero value of D_{Min}. However, it is apparent that some potentially useful patterns are omitted from Table 1.

It may be noted that, among the patterns short-listed in the various tables, a high proportion have a relatively large number of orbital planes, with relatively few satellites (often only one) in each plane.

An elevation angle of 5° is usually regarded as the minimum value for operational planning purposes; this corresponds to a value of $R_{MAX,n}$ not exceeding 76.3° for 24-hour orbits, or 71.2° for 12-hour orbits. From the values shown in Tables 2 to 7 and Fig 17, it appears that this requirement can be met by selected delta patterns which contain at least the numbers of satellites listed below. The minimum values for five-fold and six-fold coverage are not definitely established, so are shown in brackets; in particular, it seems likely that the number required for five-fold coverage using 12-hour orbits might be reduced after investigation of inclinations other than 63°.

Level of coverage	24-hour orbits	12-hour orbits
Single	5	5
Double	7	8
Triple	11	12
Quadruple	14	15
Five-fold	(18)	(21)
Six-fold	(21)	(23)
Seven-fold	24	

In general it appears that at either of these orbital periods, a unit increase in the level of coverage requires an increase of about 3 or 4 in the number of satellites in the optimum pattern.

6 CONCLUSIONS

(1) Considering only systems of satellites in equal-period circular orbits, the merits of a particular orbital pattern of satellites, as regards the provision of a particular (n-fold) level of continuous whole-Earth coverage, are largely determined by the value (preferably small) of $R_{MAX,n}$, the maximum angular distance of an observer from the n^{th} nearest sub-satellite point, at the optimum orbital inclination (or else by $R_{Max,n}$, if an inclination other than the optimum has to be used); and also by the value (preferably large) of D_{Min}, the minimum angular separation between any two satellites in the pattern, at the inclination in question. Values of $R_{MAX,n}$ and D_{Min} for a particular pattern are independent of orbital altitude; however, because of the dependence on orbital altitude of the relationship between the observer's distance R from a sub-satellite point and the elevation angle ε at which the satellite is visible, the maximum acceptable value of $R_{MAX,n}$ increases as altitude increases.

(2) Selected members of the family of equal-period circular-orbit patterns, described previously[1,2] as delta patterns, provide a particularly uniform satellite distribution, leading to relatively large values of D_{Min} and relatively small values of $R_{MAX,n}$; these patterns are likely to be a particularly suitable choice when multiple coverage is required.

(3) A delta pattern may be fully identified and described by (i) a three-integer code reference T/P/F, where T is the total number of satellites in the pattern, P is the number of planes between which they are equally divided, and F is a measure of the relative phasing of satellites in different planes; and (ii) the inclination δ of all the orbital planes to a reference plane, which would usually (but need not necessarily) be the equator. The instantaneous positions of the satellites in their orbits are then given by the pattern phase angle ϕ. These characteristics are independent of orbital altitude. For any value of T, the number of possible delta patterns is equal to the sum of all the factors of T, including 1 and T.

(4) Unlike the orbital patterns, Earth-track patterns are dependent upon orbital altitude (or period) and on the inclination of the reference plane to the equator. For long-term station-keeping to be practicable, configurations are likely to be limited to (i) two-plane and three-plane delta patterns in which one plane is equatorial and (ii) patterns containing any number of planes of equal inclination to the equator.

(5) For delta patterns whose reference plane coincides with the equator, the number of separate Earth-tracks followed by the complete pattern may correspond to any factor of T, including 1 and T. For an altitude corresponding to L:M resonance, at which the Earth-tracks are repetitive after L orbits in M days, it is possible to deduce from the pattern code reference (T/P/F) either (i) how many separate tracks a particular pattern will follow, or (ii) which of the patterns having a particular value of T will produce a number of Earth-tracks equal to a particular factor of T (the number of such patterns being equal to this factor, eg only one such pattern will produce a single repetitive track). It is also possible to deduce, for a given pattern, at what orbital altitudes it will produce the number of Earth-tracks corresponding to a particular factor of T.

(6) From the pattern code reference T/P/F, it is possible to identify certain delta patterns as having unsatisfactory values of $R_{Max,n}$ or of D_{Min}, and to identify others as having favourable values of D_{Min}, and hence also of $R_{MAX,n}$, likely to place them among the best available. However, positive identification of the optimum pattern for any particular requirement would involve a computer study of the coverage provided by a number of potentially suitable patterns.

(7) For study of the coverage provided by satellites in equal-period circular orbits, a computer program based on determination of the radii of circumcircles of sub-satellite points, as in the program COCO, has advantages over one based on determination of distances of sub-satellite points from a network of points on the Earth's surface, as in the program GRID, in respect both of accuracy and, except for the largest patterns, of computing time needed.

(8) The results presented, obtained with the program COCO, show minimum numbers of satellites necessary to achieve various levels of continuous whole-Earth coverage which range (for 24-hour orbits and 5° minimum elevation angle) from 5 satellites for single coverage to 24 satellites for seven-fold coverage. In most cases the patterns providing the best coverage are found among those having a relatively large number of separate orbital planes with relatively few satellites (often only one) in each plane; and in most cases the optimum inclination is between 50° and 60° (or between 120° and 130° for retrograde orbits).

Appendix A

EXAMPLE OF APPLICATION OF RESULTS TO A SATELLITE NAVIGATION SYSTEM

Earlier papers[1,3,4] have discussed the relevance of some of the results of this study of whole-Earth coverage to mobile communications systems. Previously unpublished results included in this Report deal with the extension of the study to patterns of up to 25 satellites, which followed press reports of proposals for a satellite navigation system (Navstar/GPS) using an orbital pattern of 24 satellites.

When developing a system design making use of a multiple-satellite pattern to provide whole-Earth coverage, it is desirable to develop coverage criteria based on known system requirements and constraints in order to guide the study of coverage patterns and the eventual choice of a preferred pattern. During the present study the particular system requirements and constraints for the satellite navigation system were not known, and no attempt has been made to synthesize such requirements in any extended form; these comments are based only on a fairly elementary consideration of a few relevant points.

GPS has been described[13] as using 8 satellites in each of three 12-hour orbits; at that altitude $R_{Max,n}$ must not exceed 71.2° if the minimum elevation angle is not to be less than 5°. Earlier work[3,4] in this study (see also Table 5) has shown that 15 satellites are enough to provide quadruple coverage with $R_{MAX,4}$ not exceeding 71.2°; Table 7 indicates that, at 63° inclination, 21 or 22 satellites would provide five-fold coverage and 23 or 24 satellites would provide six-fold coverage. A GPS system user would calculate his position in three dimensions by radio determination of his range from four satellites at known orbital positions; the fact that no less than six satellites should always be visible provides redundancy which allows both for failure of some satellites and for choice of those satellites which provide the best geometry for a fix.

Without entering into a detailed analysis of requirements, elementary considerations suggest the following three criteria for selecting promising patterns; these involve no more information than is available in Table 7.

(i) If two satellites have only a small separation, one of them is effectively useless; the minimum satellite separation D_{Min} should therefore be as large as possible.

(ii) If the value of $R_{Max,7}$ for seven-fold coverage is only a little greater than 76.1°, the value corresponding to zero elevation angle with satellites in 12-hour orbits, then experience of the manner in which R usually varies with ϕ suggests that a seventh satellite is likely to be below the horizon for only a small proportion of the time; whereas if $R_{Max,7}$ is much greater than 76.1°, then the seventh satellite is probably below the horizon for long periods. Hence, while it is essential that $R_{Max,6}$ should be no greater than 71.2°, it is also desirable that $R_{Max,7}$ should be as small as possible, in order to minimise the effects of any satellite failures.

(iii) If the observer and all four satellites being used lie in or nearly in the same plane, then the position-fixing accuracy in the direction perpendicular to the plane will be poor. Apart from satellites which are permanently in the same orbital plane, other satellites will pass through that plane as the orbits intersect. With eight satellites in each of three orbital planes, and to a lesser extent with six satellites in each of four orbital planes, it will regularly happen that an observer can see four satellites in the same plane; there is therefore a risk that failures of other satellites might leave these as the only four usable satellites. With larger numbers of orbital planes, containing fewer satellites each, this is unlikely; patterns containing less than six satellites in each plane therefore appear preferable.

Considering first the three 3-plane, 24-satellite patterns in Table 7, it appears that pattern 24/3/1 is least desirable because of its very small value of D_{Min} ; this would fall to zero if the inclination increased to 63.7°. There is not a great deal to choose between patterns 24/3/0 and 24/3/2 in terms of values of D_{Min} and of $R_{Max,7}$; one difference is that 24/3/2 traces only eight separate Earth-tracks in 12-hour orbits, whereas 24/3/0 follows 24 tracks.

If a limitation to three orbital planes is not an overriding requirement, some patterns involving larger numbers of planes appear worth consideration. Pattern 24/4/3 gives a slightly better value of D_{Min} , but no significant improvement on criterion (ii) or (iii). However, the 24-satellite patterns in Table 7 which have six or more planes show a definite improvement in terms of criterion (ii), with $R_{Max,7}$ reduced by about 10°, as well as criterion (iii). Pattern 24/8/4 (12 tracks) also has D_{Min} increased to about 20°, and pattern 24/12/9 (24 tracks) has D_{Min} increased to about 30°. In addition, pattern 23/23/14 (23 tracks) probably has comparable values of $R_{Max,6}$ and $R_{Max,7}$, with D_{Min} exceeding 20°. The possible suitability of these patterns appears to deserve further consideration.

From considerations solely of coverage, 63° is not necessarily the optimum inclination. Table 5 shows that, for the minimum value of $R_{MAX,6}$, the optimum inclination for both patterns 24/3/1 and 24/3/2 is 55.1°, very close to the inclination of 54.7° at which the three planes are orthogonal; however, the reduction in $R_{Max,6}$ between 63° and 55.1° inclination is less than 2°.

Table 7 provides a basis for discussion of the effects of even larger changes in the system parameters. Thus (if the not-fully-checked figures are assumed to be reliable) the 21-satellite patterns 21/7/1 (3 tracks) and 21/7/3 (21 tracks) have values of D_{Min} approaching 20°, and 21/21/9 a value of D_{Min} well over 30°; a sixth satellite will sometimes be below 5° elevation, but never below the horizon. It might be considered whether, as compared with the 3-plane 24-satellite patterns, this low minimum elevation of the sixth satellite would be balanced by the reduced risk of having only four coplanar satellites available, the larger value of D_{Min}, and the smaller total number of satellites required. Alternatively, consideration might be given to the balance of advantage if the orbital period of the system were increased (eg to 24 hours) so that the sixth satellite visible in these 21-satellite patterns would always be above 5° elevation.

Table 7 also provides some information relevant to a gradual build-up to (or run-down from) a full 24-satellite system; for example, pattern 18/3/0 (six instead of eight satellites in each of three planes) and pattern 18/6/2 (three instead of four satellites in each of six planes) both have values of $R_{Max,4}$ less than 71.2°, but the six-plane pattern has the more favourable values of $R_{Max,5}$ and D_{Min}. Pattern 16/8/5 (two instead of three satellites in each of eight planes) has a value of $R_{Max,4}$ which only slightly exceeds 71.2°.

The foregoing discussion is intended to illustrate the amount of relevant information which a simple coverage study of this sort can contribute to the early stages of examination of even a highly complex system, such as would eventually require much deeper analysis. However, it must be noted that Bogen[13] implies that the preferred choice for Navstar/GPS is pattern 24/3/1, which we would have rejected on the basis of our simple analysis; this may perhaps show the simple approach to be inadequate.

Appendix B

USE OF THE PROGRAM COCO WITH OTHER TYPES OF PATTERN

While the program COCO was developed primarily with the objective of studying larger delta patterns than had been possible by hand methods, it was designed to be capable of handling other types of pattern also. This is largely a matter of the manner in which data regarding satellite orbital characteristics are input to the program.

As noted in section 4.2, there are two possibilities: to supply data on each individual satellite, or on the pattern as a whole. Where feasible, the latter approach has been adopted. In the case of the star patterns, discussed in Ref 1, there are two significant differences from delta patterns: F need not have an integer value, and the ascending nodes are not evenly distributed over the full 360^o of the equator, but all occur within one 180^o arc. The latter point is dealt with by introducing an input parameter W, which modifies the spacing of the nodes; thus the ascending nodes are separated by $W \times 360^o/P$, where $W = 1$ for a delta pattern and has a value a little greater than 0.5 (usually in the range 0.525 to 0.625) for a star pattern. In Ref 1 the separation between ascending nodes of a star pattern is identified as 2α; hence $W = P \times \alpha/180^o$. All satellites in a star pattern are treated as having a common inclination of 90^o, and the pattern is optimised by varying W (for a given value of F) to find the value W_{opt} at which the minimum value $R_{MAX,n}$ occurs.

In Ref 1 it was suggested, when discussing the results obtained, that delta patterns were most appropriate for use when medium inclination orbits were required, and star patterns when polar orbits were required, but that if near-polar orbits were required it seemed likely that a modified star pattern would prove most appropriate. To test this suggestion, the program COCO was used to make a brief comparison of such 'open-centred-star patterns', having orbital inclinations of 85^o and 80^o, with star patterns of 90^o inclination. This was done, for double coverage, with 9-satellite, three-plane patterns having values of F of 2.0 and 2.25; the results obtained were as follows:

Pattern	Inclination	$R_{MAX,2}$	W_{opt}	D_{Min}
9/3/2.0	90^o	74.2^o	0.542	16.6^o
	85^o	74.0^o	0.548	23.4^o
	80^o	73.9^o	0.552	15.8^o
9/3/2.25	90^o	74.0^o	0.544	24.2^o
	85^o	73.9^o	0.541	16.5^o
	80^o	73.9^o	0.535	9.5^o

These results confirm a small reduction in the value of $R_{MAX,2}$ for these open-centred-star patterns as the inclination is reduced below 90°; however, it remains substantially larger than the value of $R_{MAX,2}$ for comparable delta patterns of moderate inclination, as shown in Table 3.

No further general investigation of open-centred -star patterns was undertaken as, in the absence of any specific requirement, it appeared that the results were unlikely to be of sufficient interest to justify the time and effort involved.

Another pattern, which formed the subject of Fig 3 of Ref 1, was a five-satellite pattern consisting of two satellites in geostationary orbit, at longitudes ±s relative to a reference longitude, and three satellites in polar or near-polar synchronous orbits following a single figure-8 Earth-track with its node 180° from the reference longitude. Estimates of optimum conditions for this pattern, as quoted in Ref 1, were based on measurements on a globe, and their accuracy was uncertain; COCO was therefore used to perform accurate calculations. For this purpose data were supplied for each satellite separately, the basic program being slightly modified so that two satellites were given zero inclination instead of the common inclination i used for the other three. It was found that, in general, values of the radius of the circumcircle identified in Ref 1 as $BC_2E_2D_2$ are about 1.5° smaller than originally estimated; in consequence, optimum conditions occur at $i = 82.4°$ (instead of 78°) and $s = 41.1°$ (instead of 42°), giving a value of $R_{MAX,1}$ of 76.6° (instead of 77°) and hence a minimum elevation angle of 4.7° (instead of 4.3°), with D_{Min} equal to 44.1°. This is clearly inferior to both the delta patterns 5/5/1 and 5/5/3, as shown in Table 2, since in synchronous orbit these provide minimum elevation angles of 12.3° and 5.8° respectively, with values of D_{Min} of 60.9° and 82.2°.

Table 1

PATTERNS GIVING SINGLE REPETITIVE EARTH-TRACKS

L/M	1/2	2/3	2/1	3/2	4/3	5/4	6/5
Period / T	48 h	36 h	12 h	16 h	18 h	19.2 h	20 h
5	5/5/2	5/5/1 ★	5/5/3 ◇	5/5/1 ★	5/5/2	5/5/0 /	5/1/0 /
6	6/3/2	6/2/0 ◇**/**	6/6/4 ★**/**	6/3/0 /	6/2/0 ◇**/**	6/3/2	6/6/0 /
7	7/7/3	7/7/4	7/7/5 ★	7/7/2 ★	7/7/1 ◇	7/7/4	7/7/3
8	8/4/3	8/8/2 ★**/**	8/8/6 ★**/**	8/4/1	8/8/4 /	8/2/1 ◇	8/8/2 ★**/**
9	9/9/4	9/3/1	9/9/7 ★	9/9/3 **/**	9/3/2 ★	9/9/1	9/9/6 **/**
10	10/5/4	10/10/6	10/10/8 ★	10/5/2 ★	10/10/2 ★	10/5/0 /	10/2/0 ◇
11	11/11/5	11/11/3 ★	11/11/9 ★	11/11/4 ★	11/11/6	11/11/7	11/11/1
12	12/6/5	12/4/2 ★	12/12/10 ◇	12/6/3	12/4/0 **/**	12/3/1 ★	12/12/6 /
13	13/13/6	13/13/8	13/13/11	13/13/5 ★	13/13/3 ★	13/13/2 ★	13/13/4 ★
14	14/7/6	14/14/4 ★	14/14/12 ◇	14/7/4 ★	14/14/8	14/7/1	14/14/10 ★
15	15/15/7	15/5/3	15/15/13	15/15/6 ★	15/5/1 ◇	15/15/10 **/**	15/3/0
16	16/8/7	16/16/10 x	16/16/14 x	16/8/5 ◇	16/16/4 x**/**	16/4/3 ◇	16/16/2 x
17	17/17/8 x	17/17/5 x	17/17/15 x	17/17/7 x	17/17/10 x	17/17/3 x	17/17/9 x
18	18/9/8	18/6/4	18/18/16	18/9/6 ◇	18/6/2 ◇	18/9/2	18/18/6 **/**
19	19/19/9	19/19/12	19/19/17	19/19/8 ◇	19/19/5	19/19/13	19/19/14
20	20/10/9	20/20/6	20/20/18	20/10/7 ◇	20/20/12	20/5/0 **/**	20/4/2
21	21/21/10	21/7/5	21/21/19	21/21/9 ◇	21/7/3 ◇	21/21/4	21/21/3
22	22/11/10	22/22/14 ◇	22/22/20	22/11/8 ◇	22/22/6	22/11/3	22/22/12
23	23/23/11	23/23/7	23/23/21	23/23/10	23/23/14 ◇	23/23/16	23/23/8
24	24/12/11	24/8/6	24/24/22	24/12/9 ◇	24/8/4 ◇	24/6/1 ◇	24/24/18 **/**
25	25/25/12 x	25/25/16 x	25/25/23 x	25/25/11 x	25/25/7 x	25/25/5 x	25/5/4 ◇

Notes on Tables 1 to 5

★ This pattern appears both in Table 1 and in one or more of Tables 2 to 5, giving the minimum value of $R_{MAX,n}$ for this value of T.

◇ This pattern appears both in Table 1 and in one or more of Tables 2 to 7.

Notes on Table 1 only

x No comparison with Tables 2 to 7 can be made for this pattern, since patterns of the forms 16/16/F, 17/17/F and 25/25/F were not included in the calculations from which those tables were compiled.

/ This pattern is of one of the forms which indicate a single-plane configuration (see section 3.2).

/ This pattern is of one of the forms which indicate a two-parallel-plane configuration (see section 3.3).

Table 2

SINGLE COVERAGE

Pattern T/P/F	$R_{MAX,1}$ (degrees)	δ_{opt} (degrees)	D_{Min} (degrees)	Critical phase angles (PRUs)	Latitudes (degrees)	Notes*
5/5/1	69.2	43.7	60.9	1.0	25.5, 51.3	☆
5/5/3	75.5	51.8	82.2	1.0	51.8	◇
6/6/4	66.4	53.1	73.7	1.0	30.0, 90.0	☆
6/2/0	66.7	52.2	35.7	1.0	14.5, 90.0	◇
7/7/5	60.3	55.7	57.0	1.0	15.4, 60.6	☆
7/7/1	60.5	48.0	42.5	1.0	12.5, 68.4	◇
8/8/6	56.5	61.9	56.3	0, 1.0	2.4, 38.7	☆
8/2/1	56.9	48.2	29.5	0, 1.0	8.7, 72.8	◇
9/9/7	54.8	70.5	43.1	1.0	9.8, 20.6	☆
9/9/2	57.9	61.3	26.0	1.0	40.5	
9/3/0	61.9	70.5	33.6	1.0	19.5, 90.0	
10/5/2	52.2	57.1	46.6	0, 1.0	12.2, 63.5	☆
10/5/1	52.3	47.4	14.8	0	26.0, 77.8	
10/10/2	52.5	48.8	37.9	1.0	33.8, 90.0	◇
10/2/0	53.2	47.7	24.0	1.0	5.5, 90.0	◇
11/11/4	47.6	53.8	49.0	1.0	6.1, 47.0	☆
12/3/1	47.9	50.7	26.0	0.20, 1.0	2.8, 32.2, 56.8, 69.2	☆
12/3/2	48.3	58.8	24.1	0.05, 0.54	15.7, 72.8	
12/12/2	49.6	48.5	16.8	0	23.5, 90.0	
12/12/10	50.2	57.5	42.5	0, 1.0	3.4, 25.3	◇
12/2/1	50.4	46.5	20.5	0, 1.0	3.9, 79.0	
13/13/5	43.8	58.4	45.9	1.0	6.4, 70.0	☆
14/7/4	42.0	54.0	42.5	0, 1.0	18.4, 79.8	☆
14/2/0	49.3	46.4	17.6	1.0	2.8, 90.0	
15/3/1	42.1	53.5	20.2	0.09, 1.0	14.1, 77.1	
15/5/1	42.7	53.5	32.1	1.0	2.5, 82.7	◇
15/15/6	42.7	65.3	32.0	1.0	9.7, 90.0	◇

* See footnote to Table 1.

Table 3

DOUBLE COVERAGE

Pattern T/P/F	$R_{MAX,2}$ (degrees)	δ_{opt} (degrees)	D_{Min} (degrees)	Critical phase angles (PRUs)	Latitudes (degrees)	Notes*
7/7/2	76.0	61.8	37.1	0.38, 1.0	18.3, 21.0	☆
8/8/2	71.0	57.1	9.6	0.42, 0.80	18.1, 41.6	☆
8/8/6	74.0	58.0	61.3	0.39, 0.53	20.0, 54.0	◇
8/8/5	74.2	56.5	0	0	30.5	
9/3/2	66.2	62.1	24.2	0.53, 0.77	13.3, 53.5	☆
9/3/0	66.8	65.5	29.4	0.91, 1.0	7.7, 37.9	
10/10/2	64.1	61.6	21.2	0.67, 0.82	11.5, 41.9	☆
10/5/2	65.2	52.6	52.8	0.97, 1.0	25.4, 45.4	◇
10/10/8	65.5	49.4	46.7	0, 0.13	34.3, 65.3	
10/2/0	73.1	44.4	25.5	0.46, 1.0	24.5, 63.1	◇
11/11/9	62.0	52.7	44.1	0.68, 0.82	24.2, 51.3	☆
12/3/1	56.6	57.0	18.2	0, 0.13	7.8, 76.5	☆
12/6/2	56.6	54.0	0	0.39, 0.46	16.7, 65.5	
12/3/2	56.7	58.5	23.9	0, 0.33	5.6, 30.0	
12/12/10	59.3	50.8	42.2	0.40, 0.58	14.8, 38.7	◇
12/2/0	63.7	45.7	0	0.64, 1.0	18.0, 71.1	
12/2/1	64.3	45.0	21.1	0	15.0, 75.0	
13/13/3	54.7	52.8	38.0	0.54, 0.68	19.3, 69.2	☆
14/14/10	52.4	53.8	16.7	0.41, 0.70	2.1, 38.4	☆
14/14/3	52.8	53.5	0	0.52, 1.0	16.2, 80.6	
14/2/0	59.0	44.6	18.2	0.19, 1.0	10.4, 76.3	
15/3/1	51.3	55.3	21.5	0.51, 1.0	2.2, 79.3	
15/15/11	51.5	58.6	24.0	0.30, 0.45	1.5, 62.1	

* See footnote to Table 1.

Table 4

TRIPLE COVERAGE

Pattern T/P/F	$R_{MAX,3}$ (degrees)	δ_{opt} (degrees)	D_{Min} (degrees)	Critical phase angles (PRUs)	Latitudes (degrees)	Notes*
10/10/8	80.3	60.0	51.5	0.57, 0.86	12.7, 33.7	☆
11/11/3	74.6	59.8	3.9	0.80, 1.0	14.8, 39.5	☆
11/11/9	75.8	52.0	43.8	0.63, 0.92	32.5, 54.7	◇
12/4/2	70.9	60.0	5.4	0, 1.0	10.9, 22.2	☆
12/4/3	71.1	49.5	0	0, 1.0	0, 53.1	
12/12/10	71.9	53.8	41.0	0.15, 0.94	25.2, 46.7	◇
12/12/2	73.7	53.2	20.0	0.87, 1.0	20.5, 53.2	
12/3/1	79.0	61.7	12.1	0.42, 1.0	12.6, 39.9	◇
13/13/4	68.0	50.0	27.1	1.0	34.1, 61.5	☆
14/14/4	66.1	47.6	4.6	1.0	24.4, 72.2	☆
14/14/12	66.4	49.4	34.0	0.80, 0.91	35.0, 55.5	◇
14/2/0	77.5	45.3	18.0	1.0	32.1, 58.6	
15/15/6	63.2	57.0	41.9	0.87, 1.0	7.9, 40.8	☆
15/15/2	65.5	71.8	6.8	1.0	11.2, 15.1	

Table 5

QUADRUPLE COVERAGE

Pattern T/P/F	$R_{MAX,4}$ (degrees)	δ_{opt} (degrees)	D_{Min} (degrees)	Critical phase angles (PRUs)	Latitudes (degrees)	Notes*
13/13/2	77.1	45.7	29.3	0.05, 0.25	33.5, 41.2	☆
14/14/4	75.8	69.6	4.3	0.79, 0.87	14.7, 30.7	☆
14/14/2	75.8	62.4	9.0	0.17, 1.0	16.3, 17.9	
14/7/4	75.9	62.0	36.8	0.72, 1.0	18.1, 24.0	◇
14/14/11	76.0	70.1	0	0.79, 0.89	14.5, 30.9	
14/14/9	76.0	61.8	0	0.38, 1.0	18.3, 21.0	
15/15/2	70.9	55.7	21.8	0.13, 0.52	23.4, 44.2	
15/5/4	71.7	67.3	6.1	0, 0.65	8.9, 39.8	
15/3/1	73.1	54.4	20.8	0.21, 1.0	22.2, 63.8	

* See footnote to Table 1.

Table 6

HIGHER LEVEL COVERAGE: SELECTED PATTERNS

Pattern T/P/F	$R_{MAX,n}$ (degrees)	δ_{opt} (degrees)	D_{Min} (degrees)	Critical phase angles (PRUs)	Latitudes (degrees)	Notes
	$R_{MAX,4}$		QUADRUPLE COVERAGE			
16/4/3	74.3	58.0	23.4	0, 0.80	24.6, 54.6	◇
18/3/0	65.1	57.5	17.6	0.75, 1.0	8.2, 90.0	
18/6/2	66.6	62.1	24.2	0.60, 0.77	13.7, 53.5	◇
21/3/[illegible]	58.8	54.4	15.5	0.64, 0.70	6.2, 63.1	
	$R_{MAX,5}$		FIVE-FOLD COVERAGE			
18/6/2	73.6	64.6	20.5	0.22, 0.97	9.0, 30.7	◇
	$R_{MAX,6}$		SIX-FOLD COVERAGE			
24/4/3	68.3	56.7	14.1	0, 0.78	8.0, 38.5	
24/3/2	69.2	55.1	10.9	0.43, 0.62	17.5, 52.1	
24/3/1	69.2	55.1	10.2	0.40, 0.64	20.6, 45.0	
24/12/9	69.6	62.8	30.2	0, 0.40	20.7, 25.8	◇
24/6/1	69.9	57.0	18.2	0, 0.47	22.3, 47.0	◇
24/8/4	69.9	65.0	16.9	0, 0.90	13.4, 36.3	
25/5/4	71.0	51.9	15.6	0.75, 1.0	38.9, 73.1	◇
	$R_{MAX,7}$		SEVEN-FOLD COVERAGE			
24/8/4	75.8	59.9	24.2	0.32, 0.77	21.5, 43.7	◇

◇ This pattern is also listed in Table 1.

Table 7

COVERAGE WITH LARGER PATTERNS AT 63° INCLINATION

Pattern T/P/F	$R_{Max,1}$	$R_{Max,2}$	$R_{Max,3}$	$R_{Max,4}$	$R_{Max,5}$	$R_{Max,6}$	$R_{Max,7}$	D_{Min}	Number of Earth-tracks Period: 24 h, 12 h	Notes
24/3/0	39.7	47.3	52.6	58.6	69.2	70.8	86.2	8.7	24, 24	
24/3/1	42.0	48.7	53.6	58.6	68.5	70.9	85.6	0.9	8, 24	
24/3/2	41.1	48.2	52.5	59.1	68.0	70.7	89.6	10.4	24, 8	
24/4/3	42.0	44.2	55.9	60.8	65.1	70.8	85.1	13.5	24, 24	
24/6/1	37.8	45.5	56.3	60.1	66.2	72.1	76.1	10.4	24, 8	◇
24/8/4	39.5	47.2	54.3	60.3	68.3	70.1	76.3	19.8	24, 12	◇
24/12/9	36.2	43.4	55.5	60.2	65.0	69.7	77.5	30.3	24, 24	◇
24/24/8	63.5	63.5	63.5	63.5	65.3	69.9	77.9	12.9	8, 12	
24/12/7	37.8	42.8	60.6	60.6	65.2	69.2	76.1	3.0	8, 24	
24/24/2	46.2	5[illegible].5	60.2	61.2	65.2	71.1	77.3	3.0	8, 6	
23/23/14	36.0	48.7	56.3	60.4	69.1	70.0	77.2	21.5	23, 23	◇
22/11/8	36.9	45.3	59.1	61.6	67.1	75.0	80.1	32.8	22, 22	◇
22/22/14	56.7	57.5	63.2	65.2	68.7	72.4	86.9	7.5	22, 11	◇
21/7/1	46.6	47.7	61.3	64.3	69.1	75.4	85.7	18.9	21, 3	
21/7/3	37.2	48.6	60.4	63.5	69.8	74.8	84.7	19.9	21, 21	◇
21/7/6	59.5	60.2	61.2	62.7	67.5	75.6	88.3	6.9	21, 21	
21/21/7	63.5	63.5	63.5	64.3	70.1	74.6	86.1	12.1	21, 7	
21/21/9	37.6	46.4	60.3	61.8	69.0	75.6	84.0	34.2	21, 21	◇
21/21/14	63.5	63.5	63.5	63.9	68.2	76.7	84.0	18.6	7, 21	
21/21/18	59.3	60.1	61.4	63.3	67.9	74.1	86.4	3.2	21, 21	
20/10/7	38.3	48.5	60.7	63.6	72.0	77.5	84.4	35.7	20, 20	◇
20/20/2	48.8	56.9	61.6	63.9	71.5	78.7	89.9	14.9	20, 5	
19/19/3	45.4	52.5	67.8	69.6	72.0	79.1	88.3	17.5	19, 19	
19/19/6	59.5	60.7	61.4	65.0	73.1	83.4	97.5	7.8	19, 19	
19/19/8	39.0	50.2	61.7	66.1	73.2	82.4	86.2	35.7	19, 19	◇
18/3/0	43.4	54.1	61.4	66.2	85.8	89.0	90.0	10.4	6, 6	
18/6/2	41.2	53.2	65.4	66.6	73.9	87.9	89.3	22.9	18, 9	◇
18/9/6	39.8	51.6	60.7	68.4	75.8	82.5	90.0	35.0	18, 18	◇
18/18/14	43.5	51.9	69.7	69.8	74.7	80.0	90.4	12.3	6, 9	
16/8/5	41.4	55.5	64.2	71.9	81.1	90.0	90.0	37.4	16, 16	◇
15/3/1	46.1	55.6	72.8	74.5	90.0	90.0	103.8	17.9	5, 15	

* For the sidelined patterns, values of $R_{Max,n}$ have been checked for ϕ = 0, 0.5 and 1.0 only.

◇ This pattern is also listed in Table 1.

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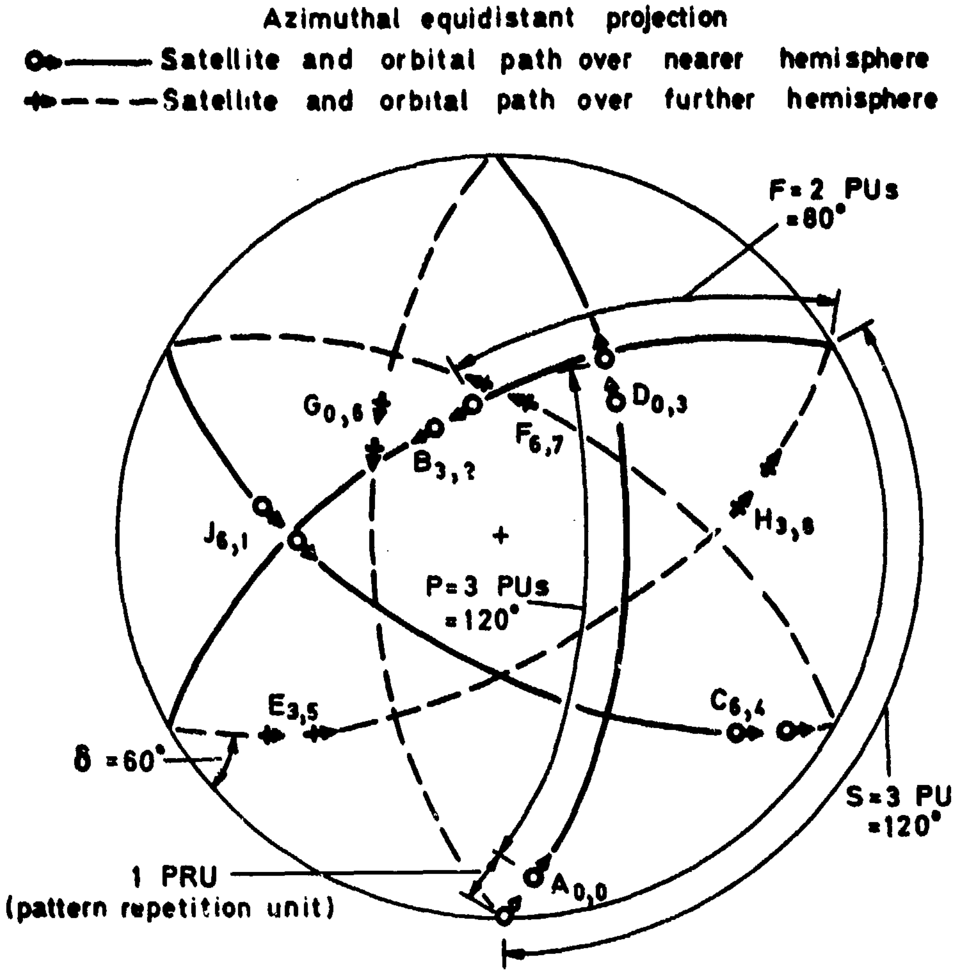


Pattern 9/3/2

The pattern code reference has the form T/P/F, where
Total number of satellites in the pattern = T=9
Number of separate orbital planes = P=3
Phase difference (in PUs) between planes = F=2

Also:
1 pattern unit (PU) = 360°/T = 40°
Number of satellites per plane = longitudinal spacing (in PUs) of ascending nodes = S = T/P = 3
Phase difference (in PUs) between satellites in same plane = P = 3
Pattern repetition interval (PRI) = $\frac{1}{4}$ PU = 10° = 1 PRU

Fig 1 Example of pattern identification (pattern 9/3/2)

TR 77044

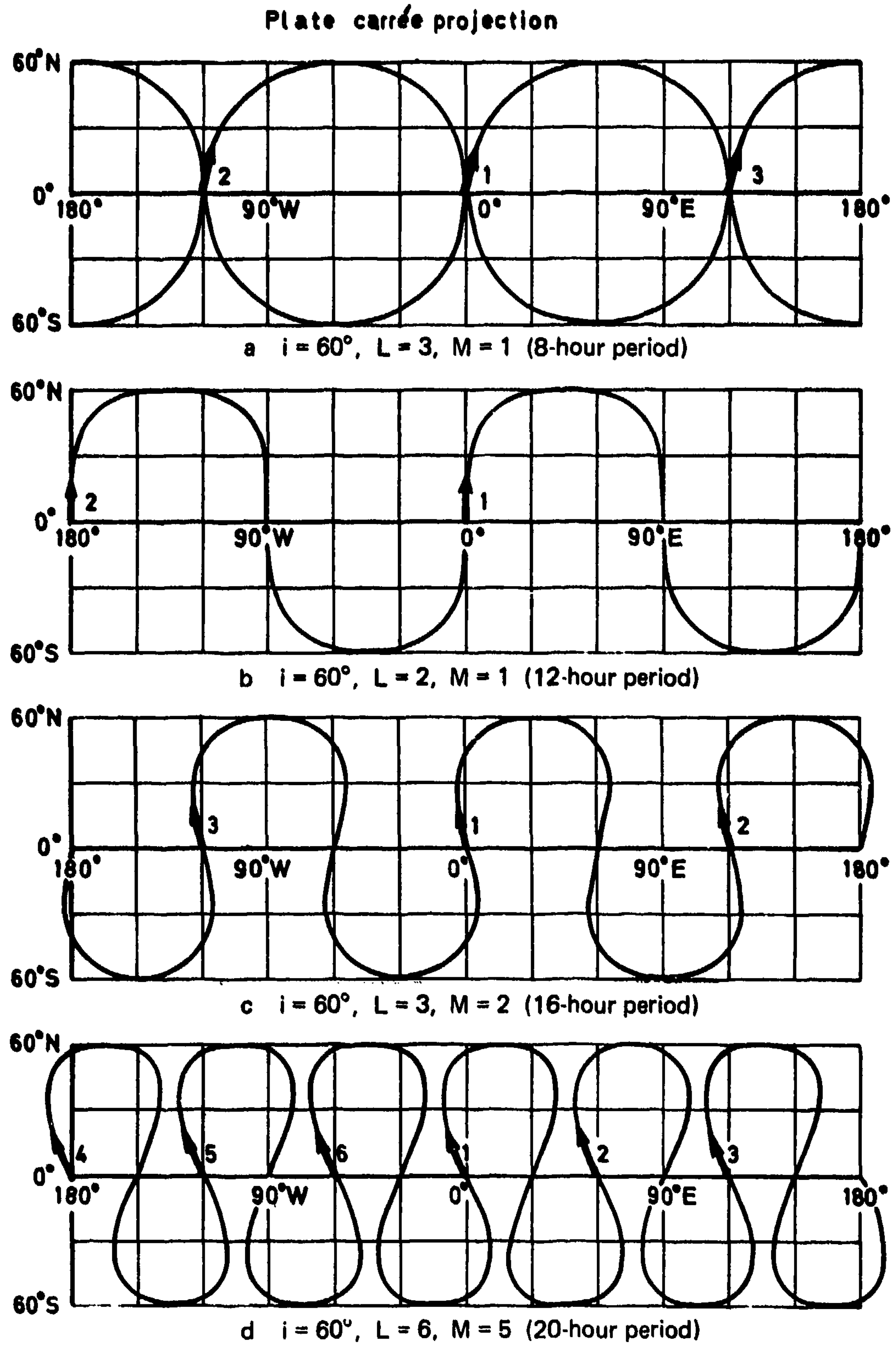


Fig 2a-d Single-satellite Earth-tracks

TR 77044

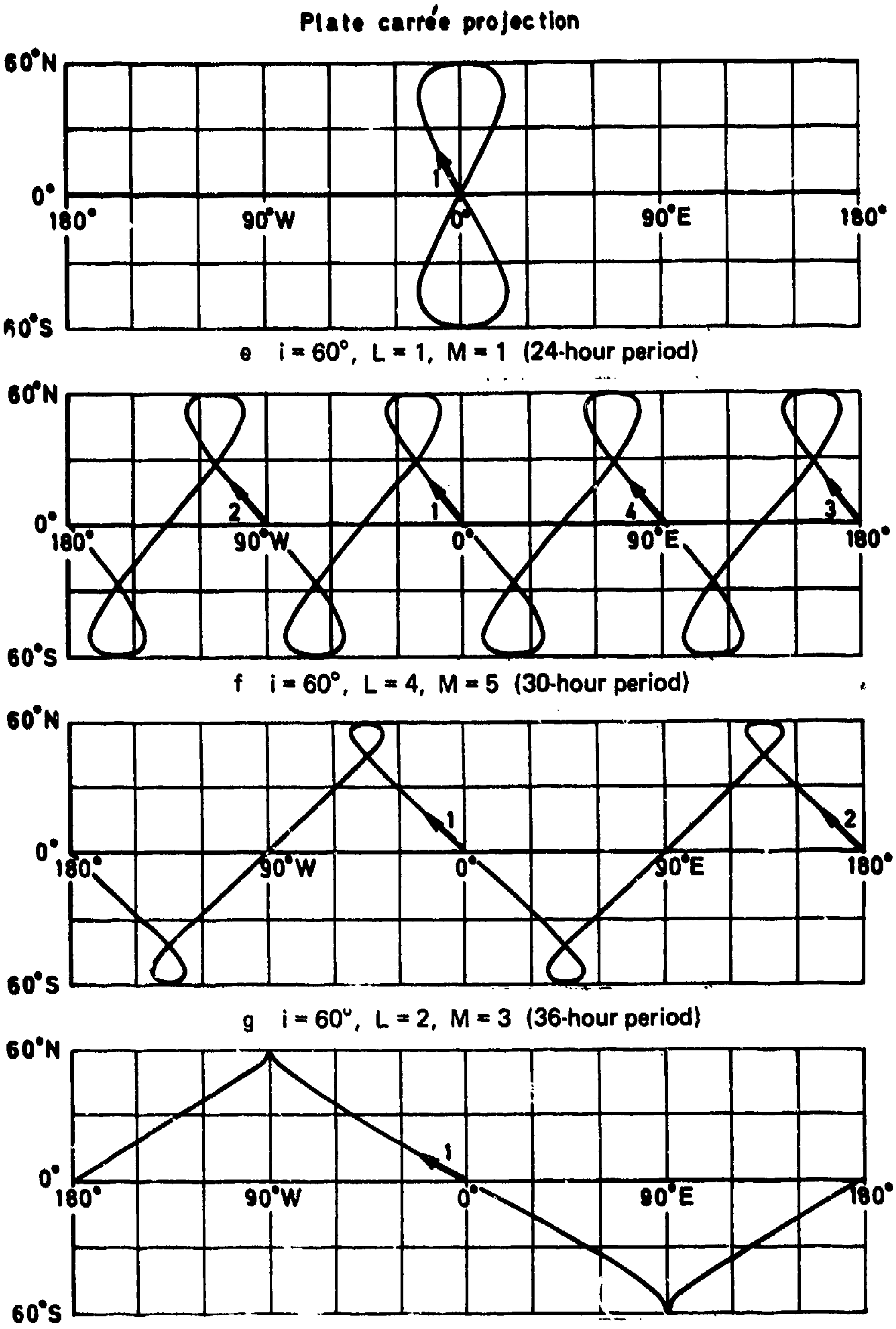


Fig 2e-h Single-satellite Earth-tracks

TR 77044

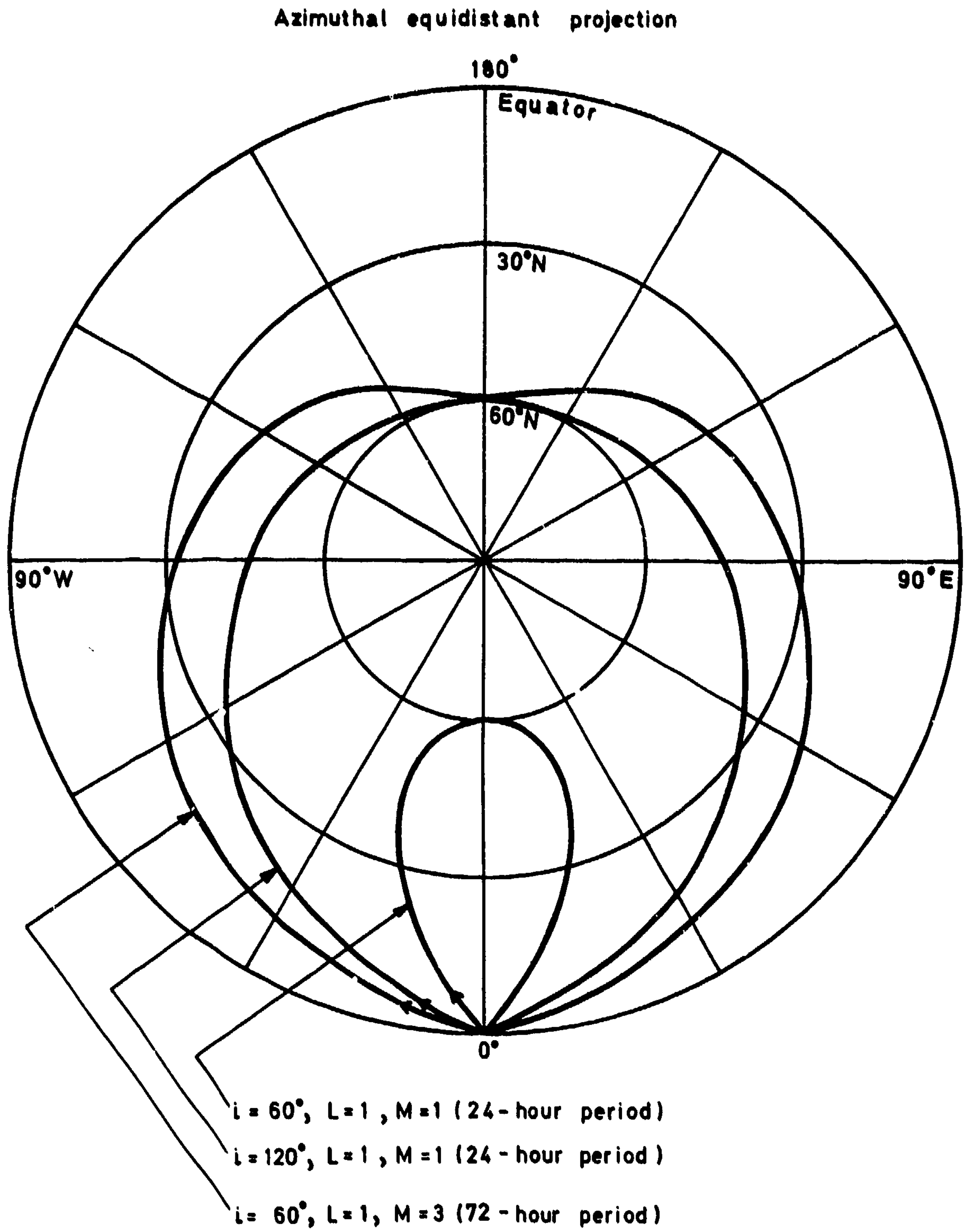
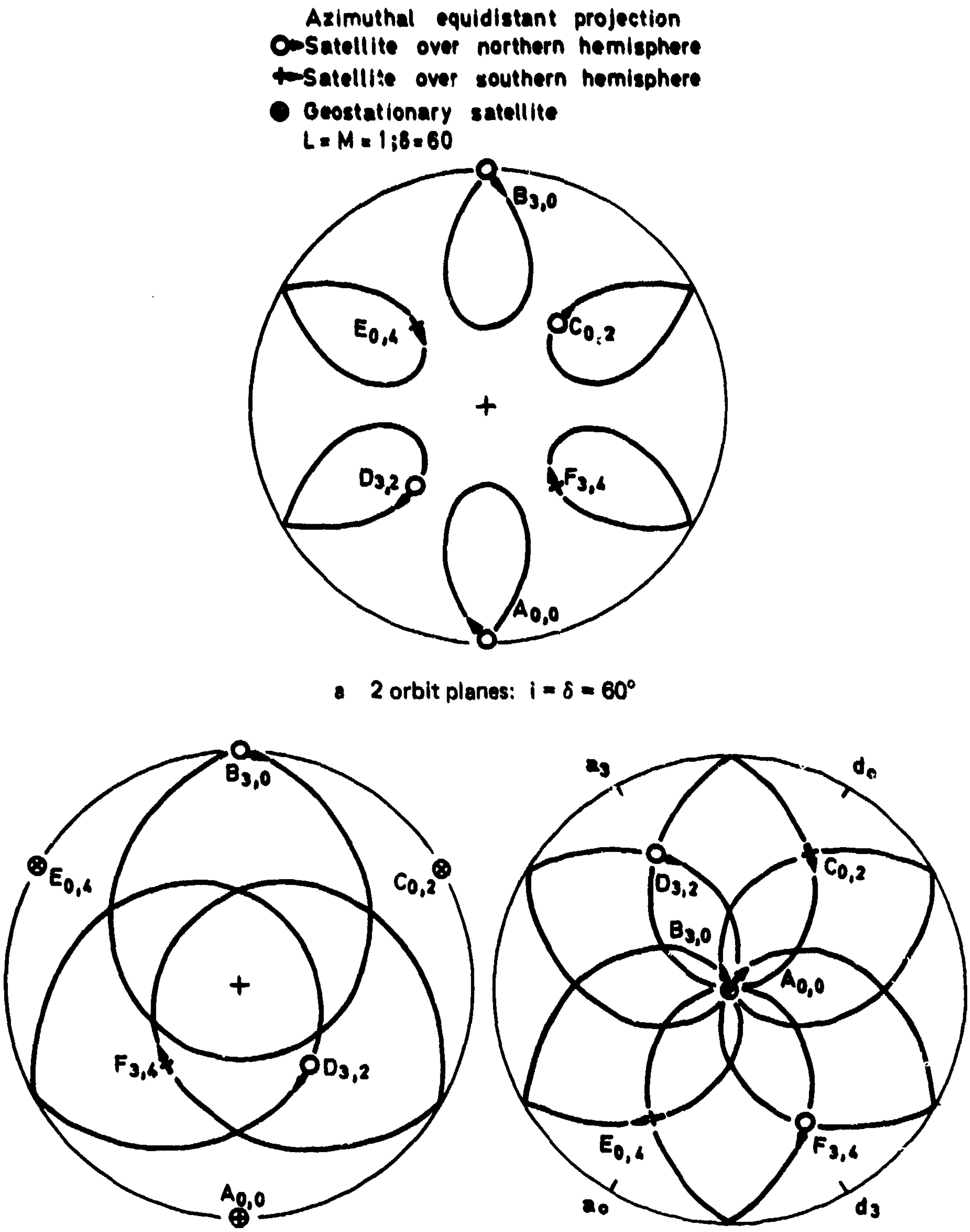


Fig 3 Single-satellite Earth-tracks



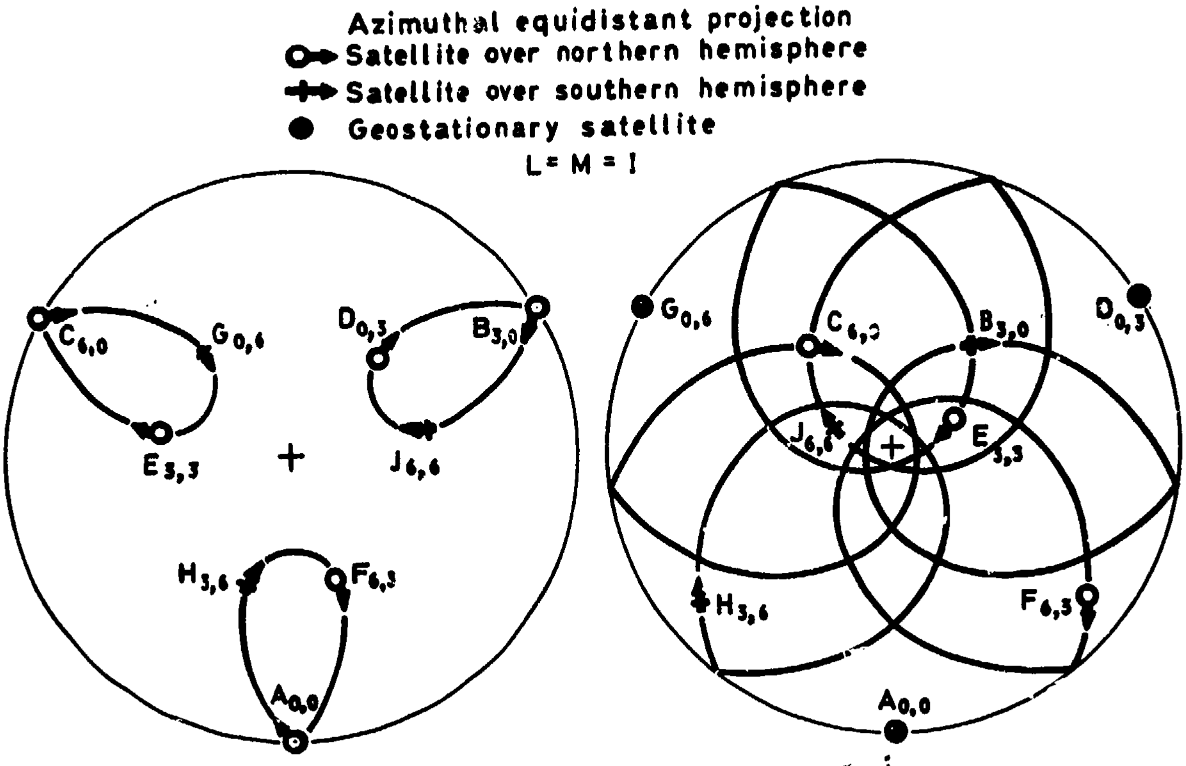
a 2 orbit planes: $i = \delta = 60°$

b 1 orbit plane: $i = 0°$
1 orbit plane: $i = 120°$

c 2 orbit planes: $i = 90°$

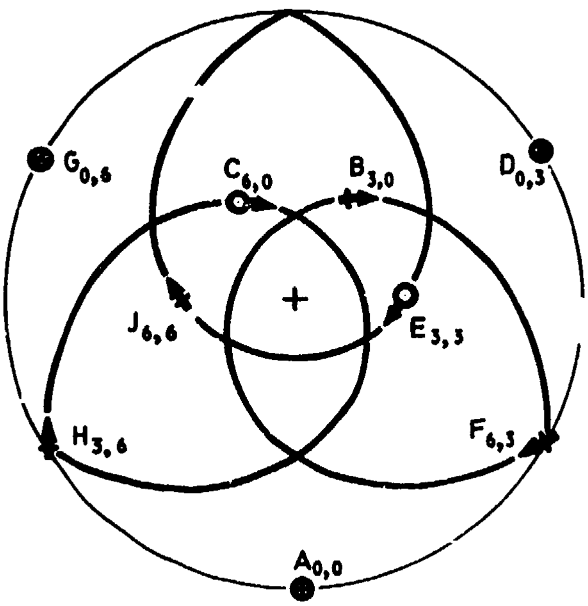
Fig 4a-c Earth-tracks: pattern 6/2/0: 24-hour orbit

TR 77044



a 3 orbit planes: i = δ = 60°

b δ = 60°
1 orbit plane: i = 0°
2 orbit planes: i = 97.2°



c δ = 70.5°
1 orbit plane: i = 0°
2 orbit planes: 109.5°

Fig 5a-c Earth-tracks: pattern 9/3/0: 24-hour orbit

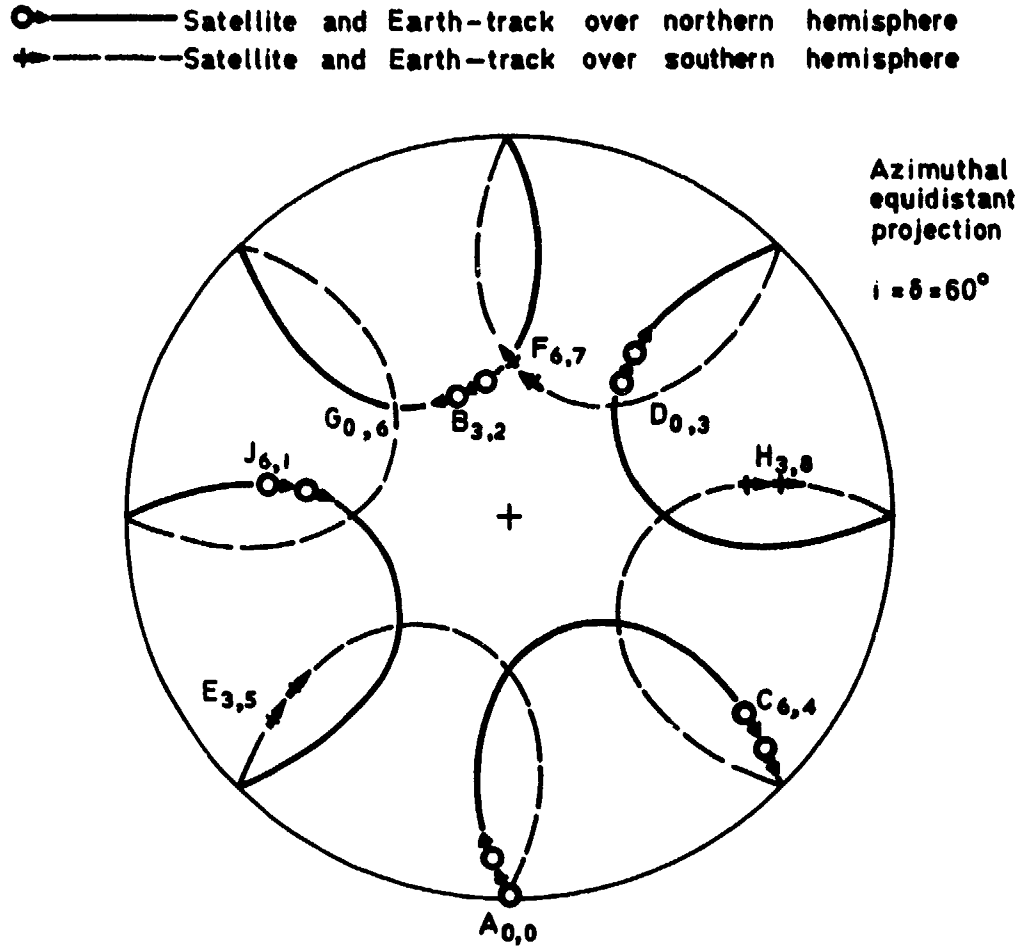


Fig 6 Single Earth-track: pattern 9/3/2: L = 4, M = 3 (18-hour period)

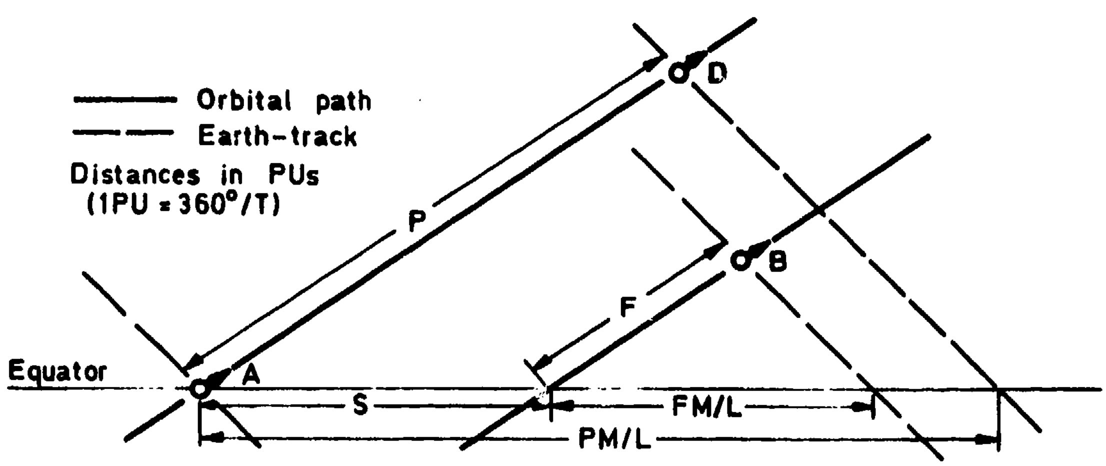


Fig 7 Derivation of condition for Earth-tracks to coincide

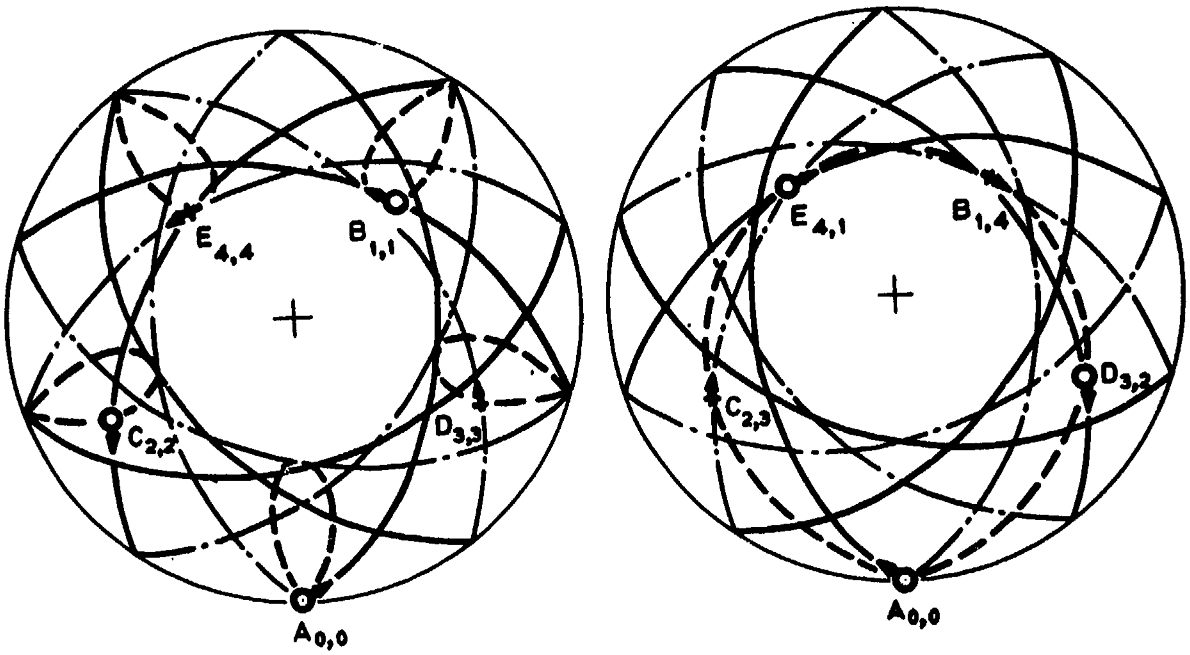
Fig 8a&b

Azimuthal equidistant projection

Orbital path over northern hemisphere

Orbital path over southern hemisphere

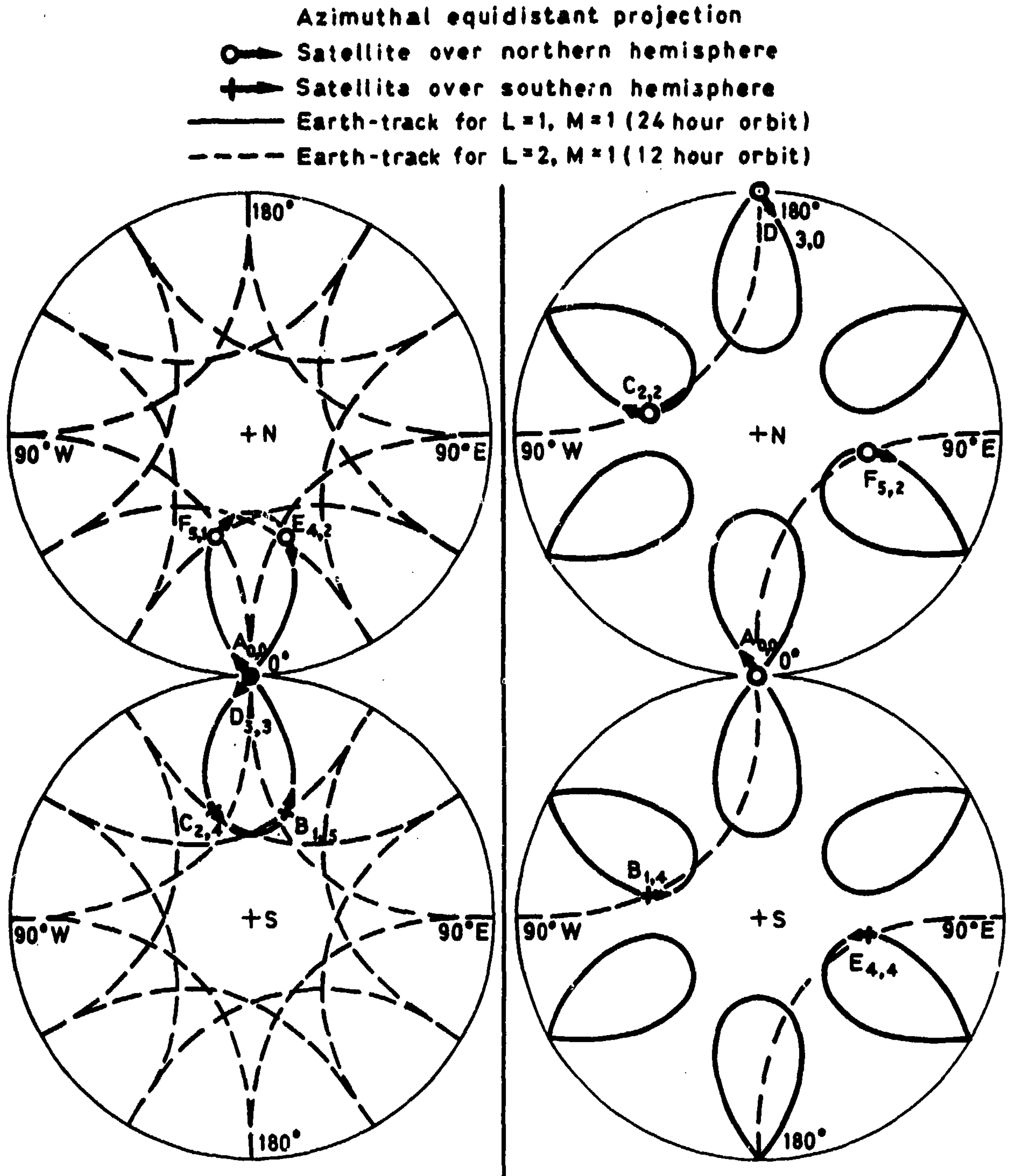
Instantaneous position of Earth-track for L = M = 1 (24 hour orbit)



a Pattern 5/5/1: $i = \delta = 45°$

b Pattern 5/5/4: $i = \delta = 135°$

Fig 8a&b Orbital paths and Earth-tracks: patterns 5/5/1 and 5/5/4



a Pattern 6/6/5: i = δ = 60°

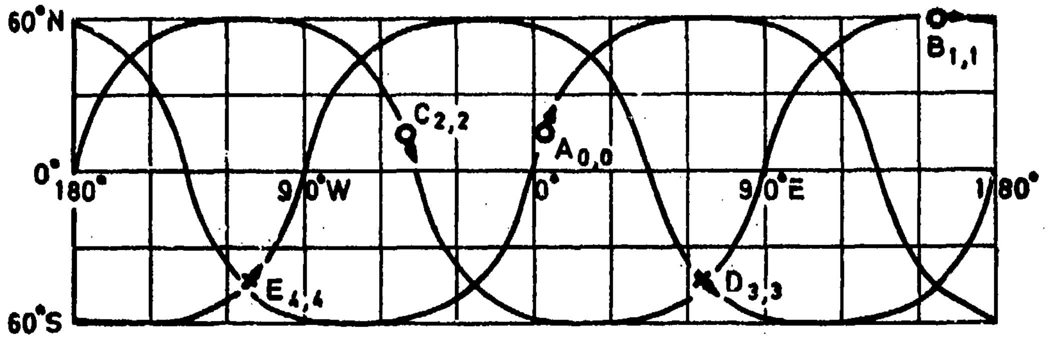
b Pattern 6/6/4: i = δ = 60°

Fig 9a&b Earth-tracks: patterns 6/6/5 and 6/6/4

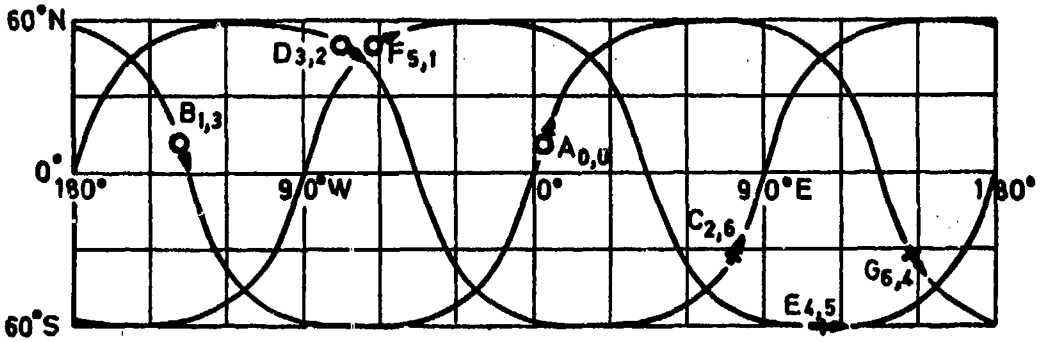
TR 77044

Plate carrée projection

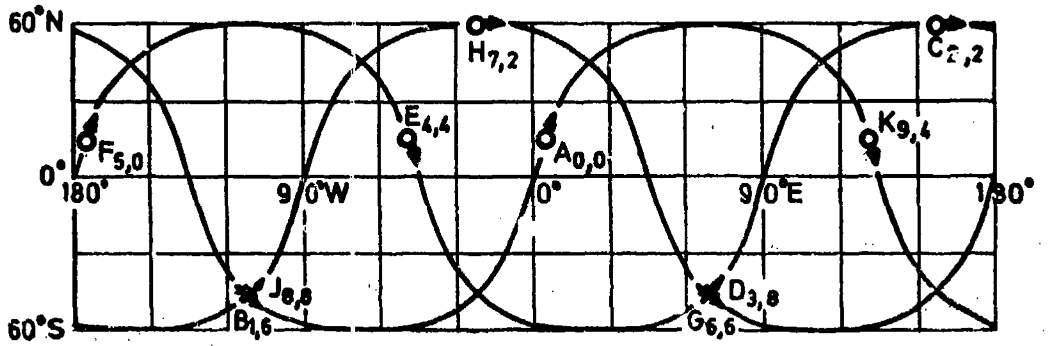
L = 4 M = 1 i = δ = 60°



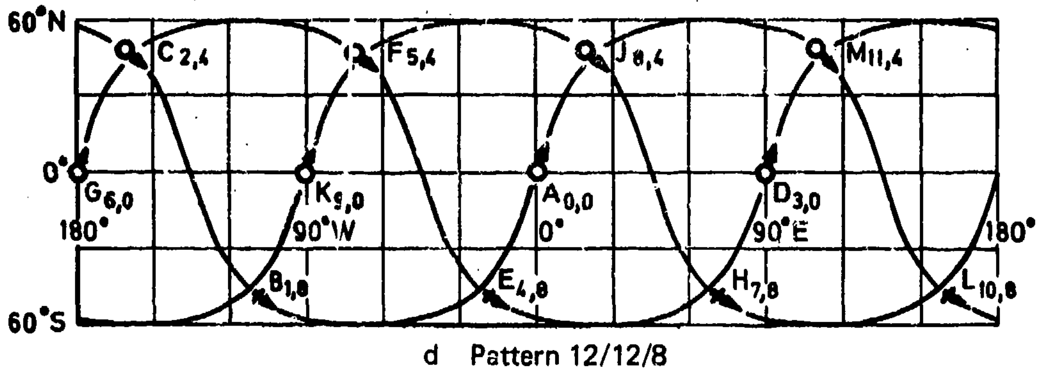
a Pattern 5/5/1



b Pattern 7/7/3



c Pattern 10/10/6



d Pattern 12/12/8

Fig 10a-d Earth-tracks: 6-hour orbit single-track patterns

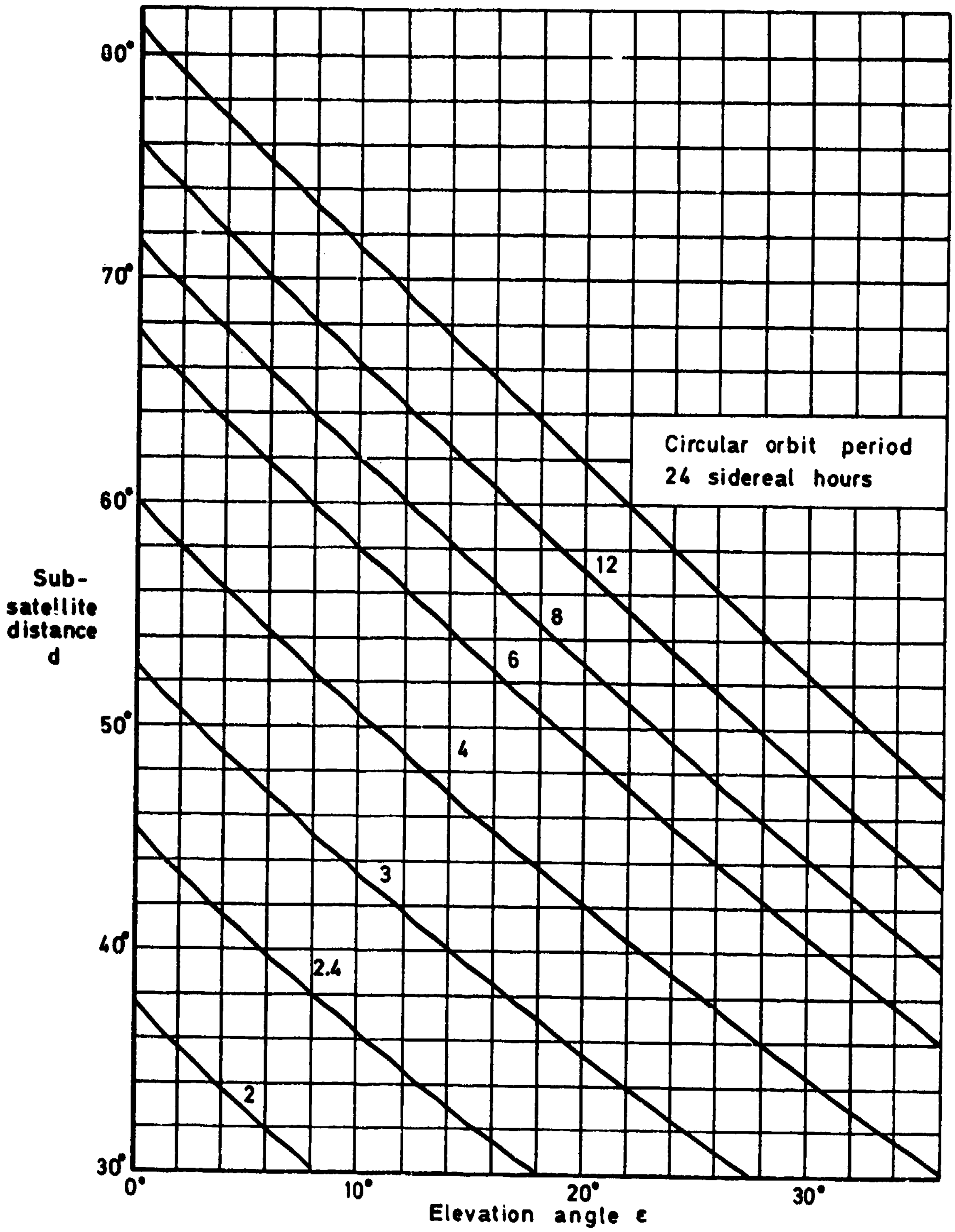


Fig 11 Dependence of elevation angle on sub-satellite distance and orbit period

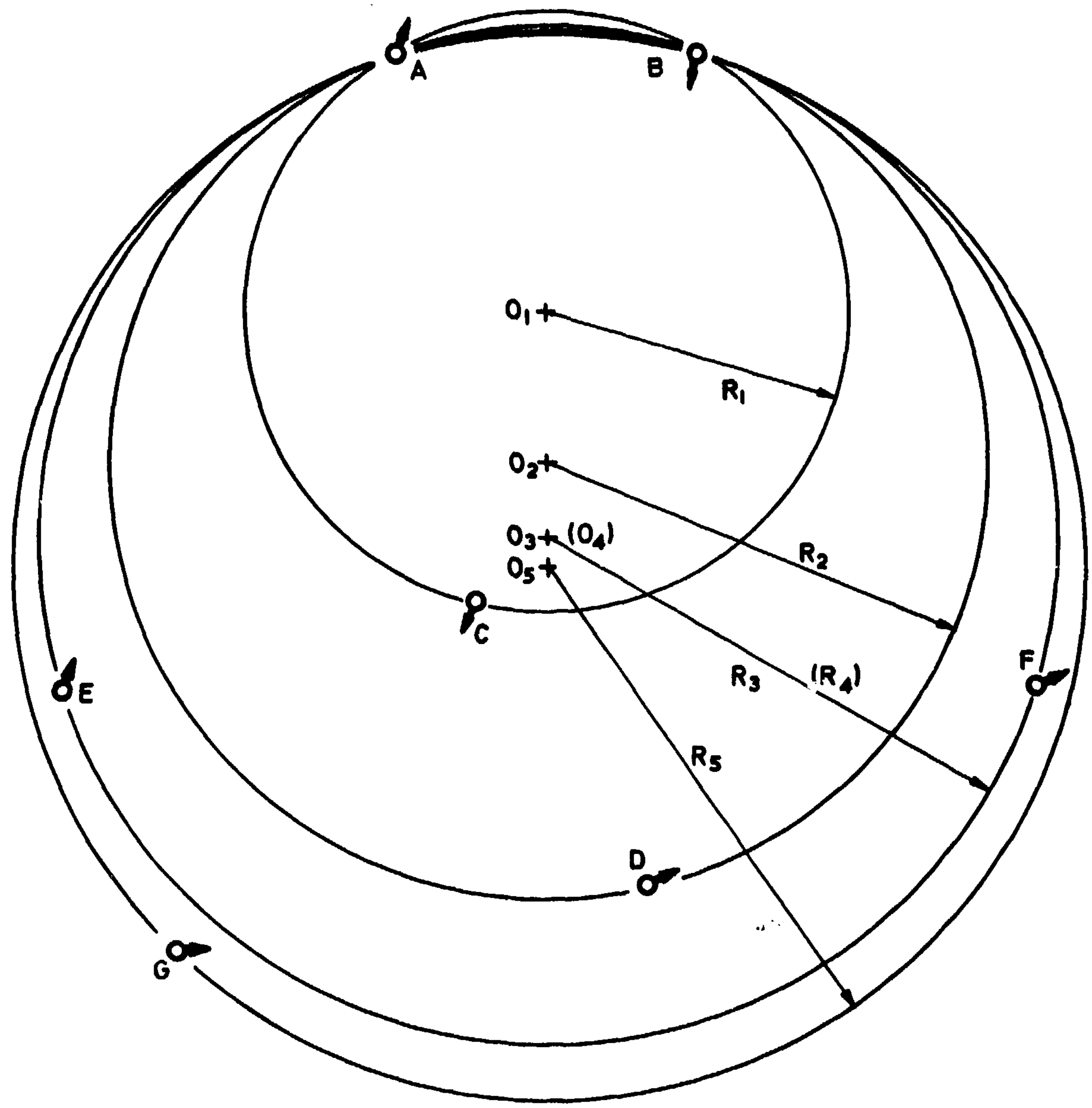


Fig 12 Example of determination of R_n, for from single to five-fold coverage

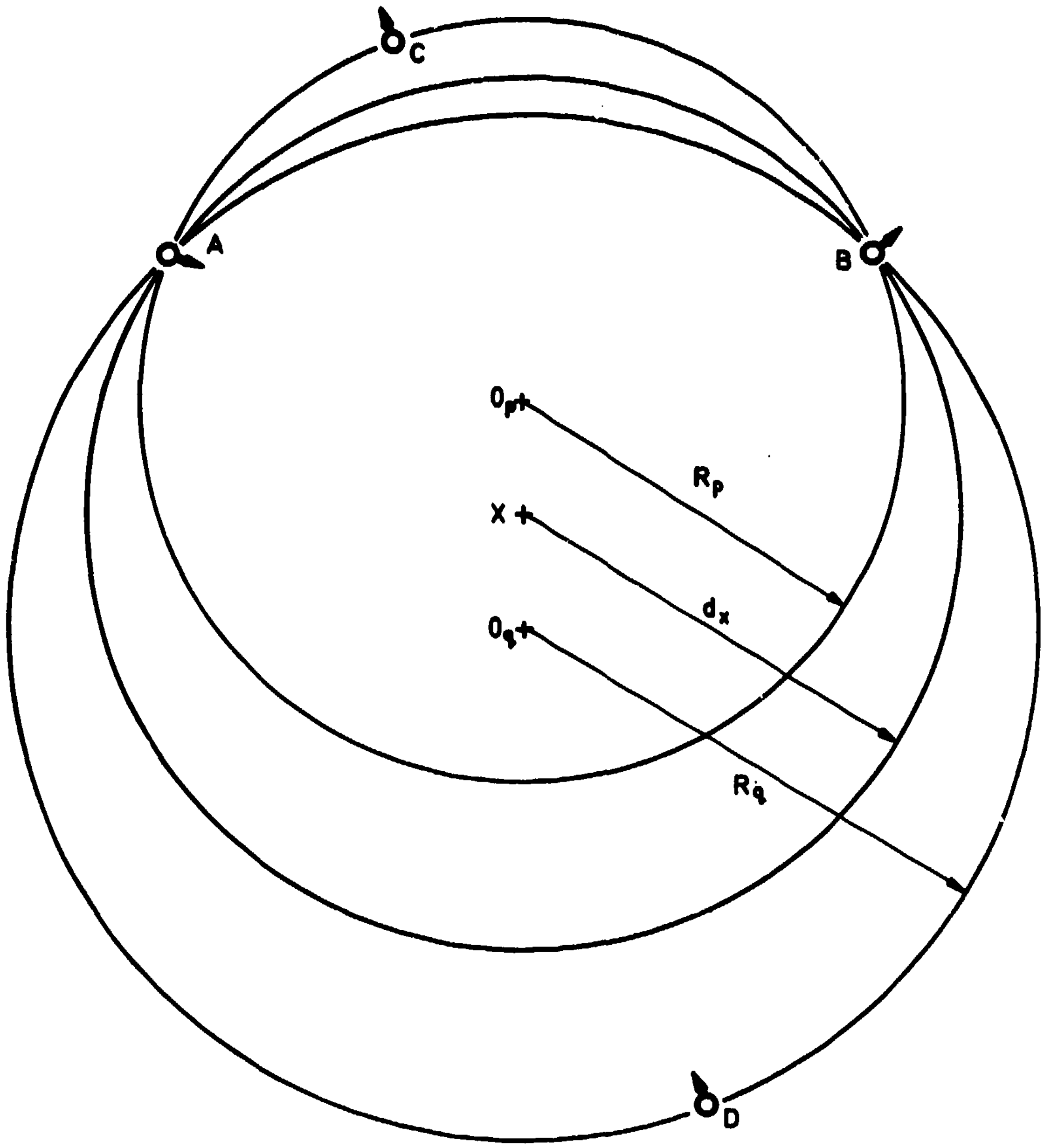


Fig 13 Example of determination of R_n : centre of circumcircle outside triangle

Fig 14

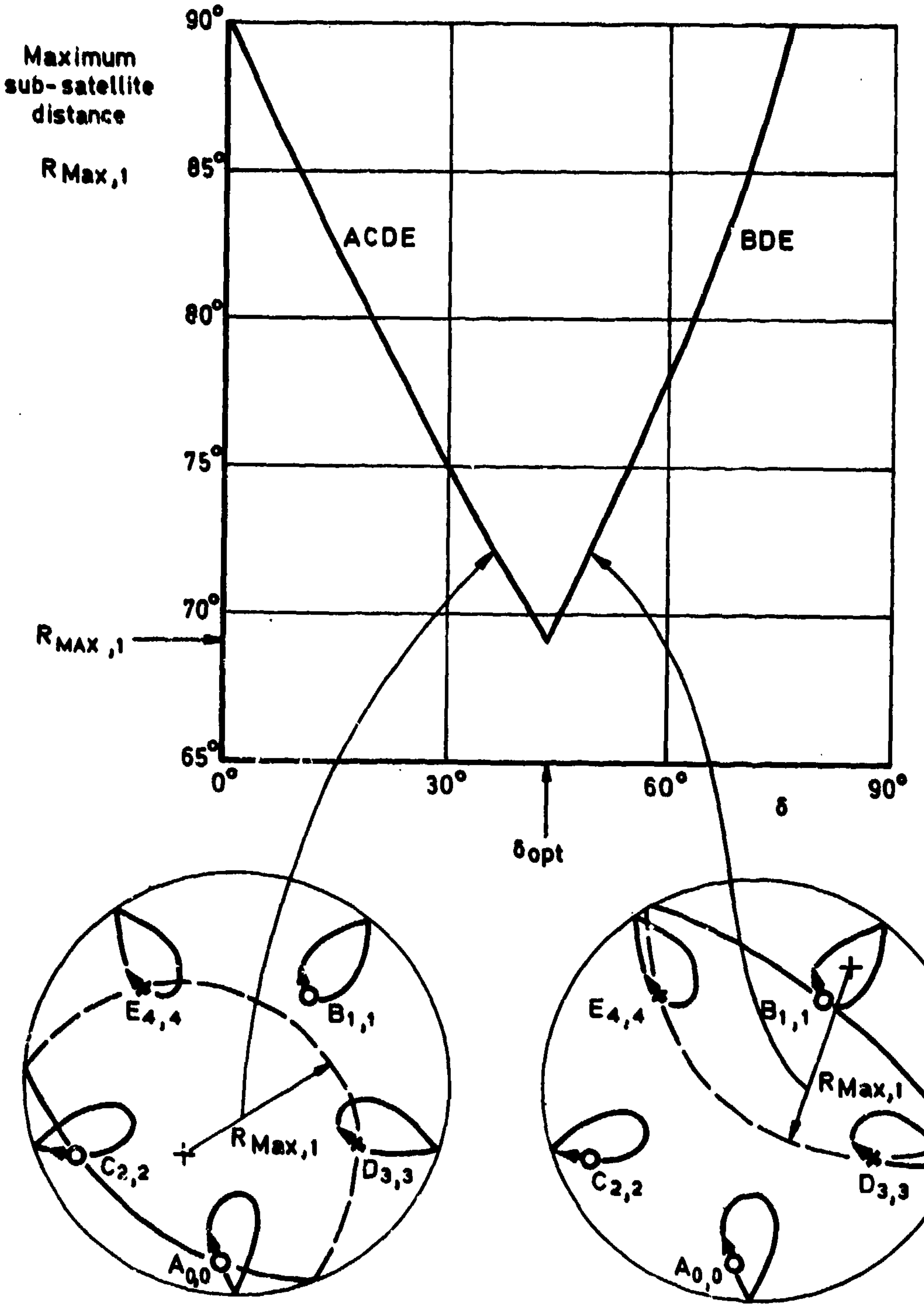


Fig 14 Pattern 5/5/1: variation of $R_{Max,\,1}$ with inclination to reference plane (δ)

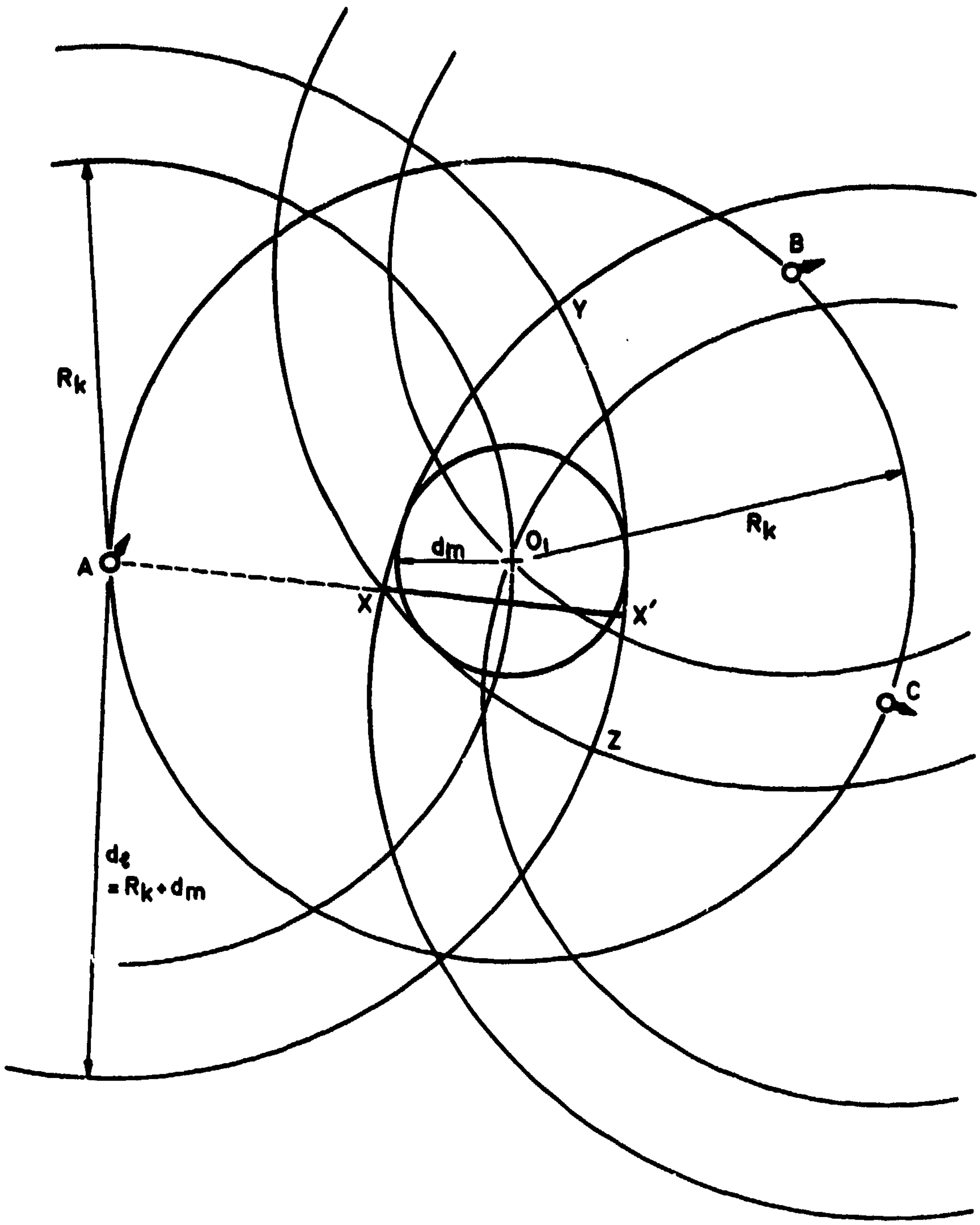


Fig 15 Simultaneous coverage of two points, or of a circular area

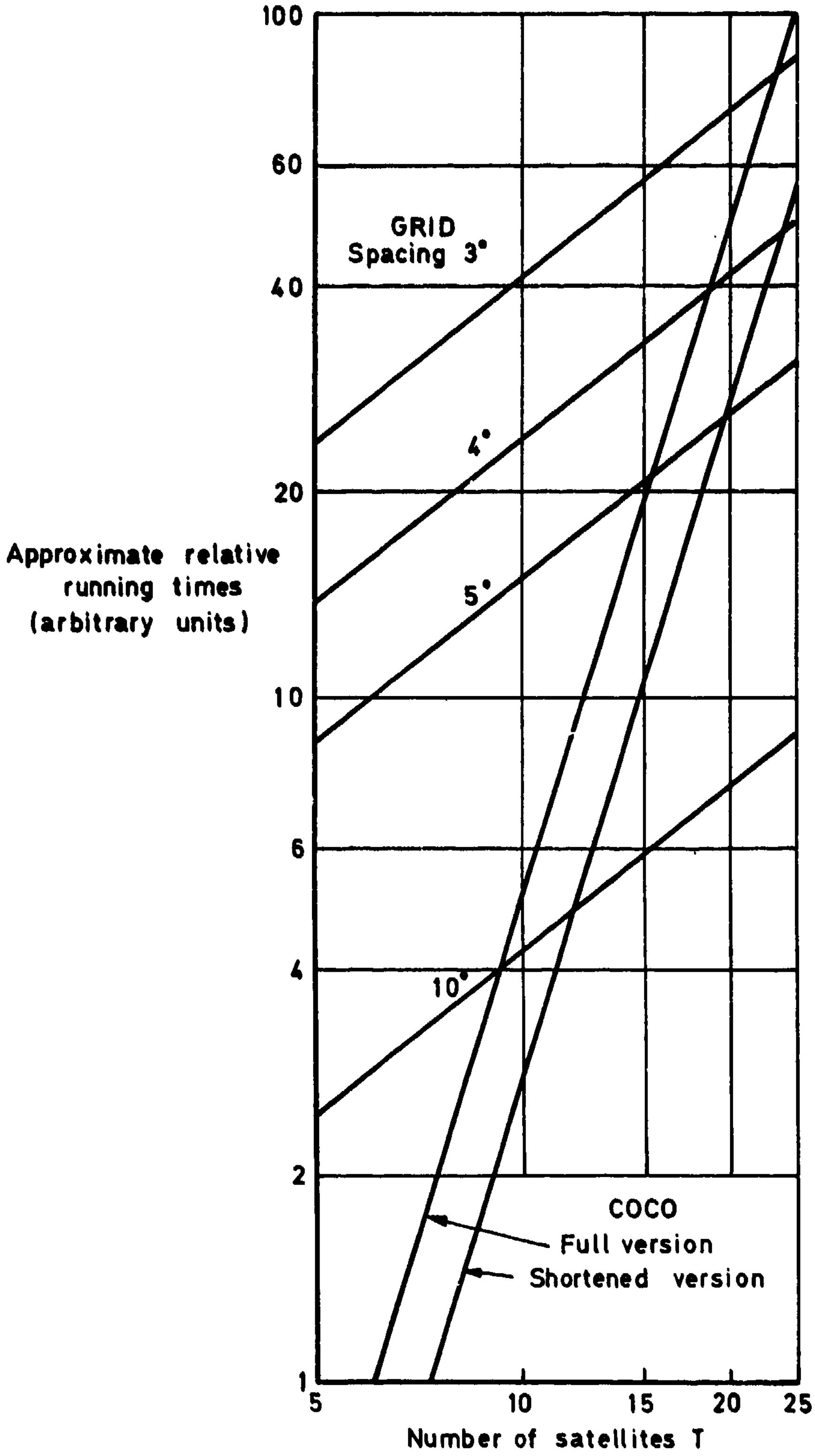


Fig 16 Relative running times of COCO and GRID programs

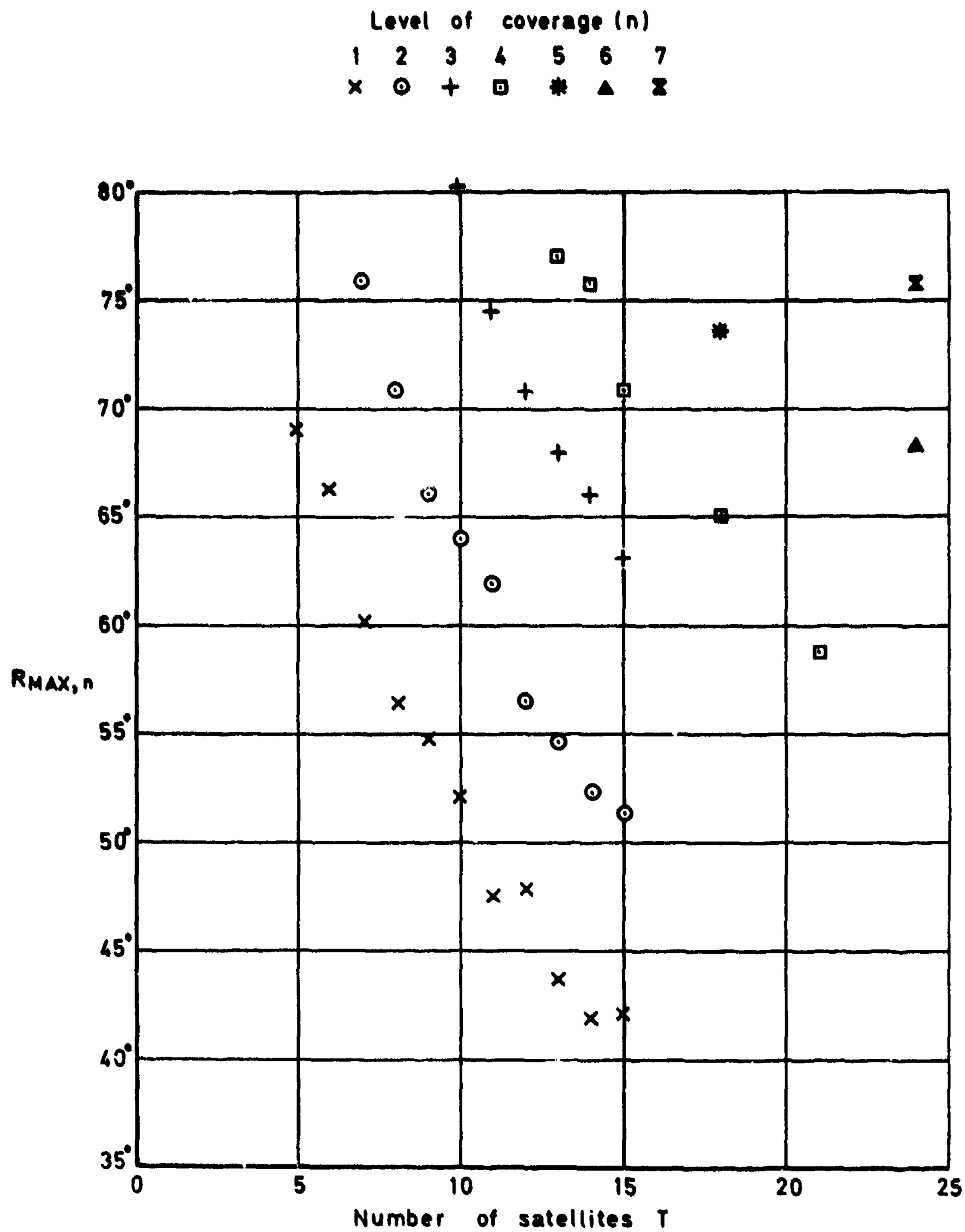


Fig 17 Minimum values of $R_{MAX,\,n}$ found for delta patterns containing different numbers of satellites