COMPUTER PROGRAMS FOR H-FIELD, E-FIELD, AND COMBINED FIELD SOLUTIONS FOR BODIES OF REVOLUTION

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**Title:** Computer Programs for H-field, E-field, and Combined Field Solutions for Bodies of Revolution

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**Security Classification:** UNCLASSIFIED

**DISTRIBUTION STATEMENT:** Approved for public release; distribution unlimited

**Key Words:**
- Bodies of Revolution
- Combined field equation
- Computer programs
- Conducting bodies
- E-field equation
- Electromagnetic scattering
- H-field equation
- Method of Moments

**Abstract:**
A computer program is given to implement the H-field, E-field, and Combined-field solutions given in Interim Technical Report RADC-TR-77-109 for a perfectly conducting body of revolution excited by an oblique plane wave incident field. The program consists of several subroutines and a main program. The main program calculates the electric current on the body of revolution and the bistatic scattering cross section per square wavelength. Some examples of computations are given and discussed.
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I. INTRODUCTION

Computer programs for E-field, H-field and combined field solutions for a perfectly conducting body of revolution excited by an obliquely incident plane wave electromagnetic field are described and listed in this report. The H-field solution is obtained by applying the method of moments to the H-field integral equation. The E-field solution is obtained by applying the method of moments to the E-field integral equation. The matrix equation for the combined field solution is a linear combination of the matrix equations for the H-field and E-field solutions. The general theory and the method of computation are given in reference [1]. Equations numbers drawn from reference [1] are preceded by 1-. For instance, (1-3) denotes equation three of reference [1].

The computer program subroutine YMAT calculates the square moment matrix for the H-field solution and is described and listed in Section II. The subroutine ZMAT calculates the moment matrix for the E-field solution and is described and listed in Section III. The subroutine YZ calculates the moment matrices for both the H-field and E-field solutions simultaneously and is listed in Section IV. The subroutine PLANE calculates the plane wave measurement vectors and is described and listed in Section V. As shown in [1], the plane wave excitation column vectors for the H-field and E-field solutions and hence also for the combined field solution can be expressed in terms of certain plane wave measurement vectors. A main program which uses YMAT, ZMAT, YZ, and PLANE to obtain the H-field solution, the E-field solution, and the combined field solution is described and listed in Section VII. The final section shows some examples of computations made with the program and gives a discussion of them.

II. THE SUBROUTINE YMAT

A. Description:

The subroutine YMAT(NN,NP,NPHI,RH,ZH,X,A,Y) stores the matrix

\[
\begin{bmatrix}
Y^t
\n & Y^t\phi
\n & Y^\phi
\n & Y^\phi\phi
\end{bmatrix}
\]

appearing in the H-field matrix equation (1-17) by columns in Y. The elements of \([Y]\) are given by (1-31). The input variables are defined in terms of variables appearing in reference [1] by

\[
\begin{align*}
n & = \text{dependence, } (\rho_i, z_i), i = 1,2,\ldots P \\
N & = P \\
P & = N_P \\
N_{\phi} & = N_{\phi} \\
\rho_i & = k\rho_i \\
\rho_i & = k\rho_i \\
\rho_i & = k\rho_i \\
X(k) & = x_k \\
A(k) & = A_k
\end{align*}
\]

In summary, \(n\) denotes \(e^{in\phi}\) dependence, \((\rho_i, z_i), i = 1,2,\ldots P\) are coordinates on the generating curve, the \(k\) which multiplies \(\rho_i\) and \(z_i\) is the propagation constant, and \(x_k\) and \(A_k\) are the abscissas and weights for the \(N_{\phi}\) point Gaussian quadrature integration in \(\phi\).

Minimum allocations are given by

\[
\begin{align*}
\text{COMPLEX } & \ Y(4*N*N) \\
\text{DIMENSION } & \ RH(NP), ZH(NP), X(NPHI), A(NPHI), \\
& \ D(NG), PD(NG), CR(NPHI), C1(NPHI), \\
& \ C2(NPHI), C3(NPHI) \\
& \ COMMON \ RS(NG), ZS(NG), SV(NG), CV(NG), T(4*N)
\end{align*}
\]
where

\[ NG = NP - 1 \]
\[ N = \frac{(NG-2)}{2} \]

The variables in common make the results of some intermediate calculations done in YMAT available to the subroutine PLANE described in Section V.

DO loop 57 puts \( k_d \) of (1-28) in \( \mathcal{D} \), \( k_p \) of (1-27) in \( \mathcal{R} \), \( k_z \) of (1-27) in \( \mathcal{Z} \), \( \sin \nu_1 \) in \( \mathcal{S} \), \( \cos \nu_1 \) in \( \mathcal{C} \), and \( \frac{\pi}{k^2 d_1^p} \) in \( \mathcal{P} \). DO loop 68 puts (1-29) in \( \mathcal{T} \).

With regard to (1-11) and (1-24) - (1-26), DO loop 25 puts \( 4 \sin \left( \frac{\phi_k}{2} \right) \), \( \pi A_k \sin \left( \frac{\phi_k}{2} \right) \cos(nA_k) \), \( \pi A_k \cos \phi_k \cos(nA_k) \), and \( \pi A_k \sin \phi_k \sin(nA_k) \) in \( \mathcal{C} \), \( \mathcal{C}_1 \), \( \mathcal{C}_2 \), and \( \mathcal{C}_3 \) respectively where, as prescribed by (1-37),

\[ \phi_k = \frac{\pi}{2} (x_k + 1) \]

DO loop 62 initializes \( Y \).

The computation of (1-31) is sequential in \( s \) rather than \( i_j \). A computation sequential in \( s \) is preferable because it does not require storing \( (Y^t)_s \), \( (Y^2)_s \), \( (Y^3)_s \), or \( (Y^4)_s \) versus \( s \). Unfortunately, (1-31) as it stands tacitly implies computation sequential in \( i_j \). An alternative form of (1-31) somewhat more suggestive of computation sequential in \( s \) is given by

\[
\begin{pmatrix}
(Y_{tt}^n)_{pq} \\
(Y_{\phi t}^n)_{pq} \\
(Y_{\phi t}^t)_{pq} \\
(Y_{\phi t}^\phi)_{pq}
\end{pmatrix} = \begin{array}{c}
2p+2 \\
\sum_{i=2p-1}^{2p+2} \\
\sum_{j=2q-1}^{2q+2} \\
\end{array} \begin{pmatrix}
T_{2p-2} \\
T_{2p+1} \\
T_{2q-2} \\
T_{2q+1} \\
\end{pmatrix} \begin{pmatrix}
(Y_{1})_{ij} \\
(Y_{2})_{ij} \\
(Y_{3})_{ij} \\
(Y_{4})_{ij}
\end{pmatrix}
\]

If (1-38) is true, then

\[
\begin{align*}
((Y^t^n)_{pq})_{ij} &= -((Y^\phi^n)_{qp})_{ji} \\
((Y^\phi^n)_{pq})_{ij} &= -((Y^t^n)_{qp})_{ji} \\
((Y^t^n)_{pq})_{ij} &= -((Y^\phi^n)_{qp})_{ji}
\end{align*}
\]
where the additional subscripts $ij$ denote the contribution due to the $i,j$th term on the right-hand side of (2). From (3)

\[
\begin{align*}
\langle y^{tt}_n \rangle_{pq} &= \langle (\phi^t_n)_{pq} \rangle_{i \leq j} - \langle (\phi^t_n)_{qp} \rangle_{i \leq j} \\
\langle y^{tt}_n \rangle_{pq} &= \langle (\phi^t_n)_{pq} \rangle_{i \leq j} - \langle (\phi^t_n)_{qp} \rangle_{i \leq j} \\
\langle y^{tt}_n \rangle_{pq} &= \langle (\phi^t_n)_{pq} \rangle_{i \leq j} - \langle (\phi^t_n)_{qp} \rangle_{i \leq j} \\
\langle y^{tt}_n \rangle_{pq} &= \langle (\phi^t_n)_{pq} \rangle_{i \leq j} - \langle (\phi^t_n)_{qp} \rangle_{i \leq j}
\end{align*}
\]

(4)

where the subscript notation $i \leq j$ denotes the sum of the contributions due to all terms for which $i < j$ plus half of the $i = j$ term on the right-hand side of (2). According to (4), it is sufficient to calculate only the $i \leq j$ terms on the right-hand side of (2). Inspection of the limits of summation in (2) shows that the first term on the right-hand side of (4) is zero if $p > q + 1$ and that the second term is zero if $p < q - 1$.

The $ij$th term of the first of equations (2) contributes

\[
T_2(p + k - 1) + T_2(q + \ell - 1) + \langle Y_1 \rangle_{ij} \text{ to } \langle y^{tt}_n \rangle_{(p + k)(q + \ell)} \quad k = 1, 2, \ell = 1, 2
\]

(5)

where

\[
p = \lfloor (i + 1)/2 \rfloor - 2
\]

\[
q = \lfloor (j + 1)/2 \rfloor - 2
\]

where $[i]$ denotes the largest integer which does not exceed $i$. The $k=1$ term should be omitted from (5) for the first two values of $i$, and the $k=2$ term should be omitted for the last two values of $i$ because the triangle functions do not overlap at the ends of the generating curve. Similarly, the $\ell=1$ term should be omitted for the first two values of $j$, and the $\ell=2$ term should be omitted for the last two values of $j$. The last three of equations (2) imply relations similar to (5).
The indices I and J of nested DO loops 60 and 59 are respectively i and j of (5). DO loop 61 accumulates $G_1$, $G_2$, and $G_3$ of (1-24)-(1-26) for $I \neq J$ and $0.5G_1$, $0.5G_2$, and $0.5G_3$ for $I = J$ in $G_1$, $G_2$, and $G_3$. The $(Y_1)_{ij}$, $(Y_2)_{ij}$, $(Y_3)_{ij}$, and $(Y_4)_{ij}$ which appear in (5) and similar equations and which are given by (1-32)-(1-35) with the $\frac{\pi}{k_1d_{101}}$ term missing and the $i=j$ result halved are put in $Y_1$, $Y_2$, $Y_3$, and $Y_4$ just after DO loop 61. The indices K and L of nested DO loops 32 and 31 are respectively $k$ and $l$ of (5). The $Y(J1)$, $Y(J2)$, $Y(J3)$, and $Y(J4)$ in DO loop 32 are respectively $(Y_{tt})_{n}(p+k)(q+l)$, $(Y_{t\phi})_{n}(p+k)(q+l)$, $(Y_{t\phi})_{n}(p+k)(q+l)$, and $(Y_{\phi\phi})_{n}(p+k)(q+l)$.

DO loop 11 carries out (4) for

$$ (p,q) = \begin{cases} (J,J) & J = 1 \\ (J,J), (J-1,J), \text{ and } (J,J-1) & J > 1 \end{cases} $$

DO loop 11 also adds the contributions to (2) due to the $\frac{\pi}{k_2d_{101}}$ terms in (1-32) and (1-35). In DO loop 11, $Y(KD1)$, $Y(KD2)$, $Y(KD3)$, and $Y(KD4)$ are the diagonal $(J,J)$ elements of $Y_{tt}$, $Y_{t\phi}$, $Y_{t\phi}$, and $Y_{\phi\phi}$ respectively. Similarly, $Y(KU1)$, $Y(KU2)$, $Y(KU3)$, and $Y(KU4)$ are the $(J-1,J)$ elements while $Y(KL1)$, $Y(KL2)$, $Y(KL3)$, and $Y(KL4)$ are the $(J, J-1)$ elements. The first RI in DO loop 11 is the contribution to the diagonal elements of (2) due to the $\frac{\pi}{k_2d_{101}}$ terms. The second RI in DO loop 11 is the contribution to the off diagonal elements due to the $\frac{\pi}{k_2d_{101}}$ terms.

Nested DO loops 13 and 14 use (1-39) for $(i,j) = (I,J)$ to put $(Y_{tt})_{ij}$, $(Y_{t\phi})_{ij}$, $(Y_{t\phi})_{ij}$, and $(Y_{\phi\phi})_{ij}$ for $i \leq j$ in $Y(KL1)$, $Y(KL2)$, $Y(KL3)$, and $Y(KL4)$ respectively.
B. LISTING OF THE SUBROUTINE YMAT

SUBROUTINE YMAT(NN,NP,NPHI,RH,ZH,X,Y,A,Y)
COMPLEX U,Y(1600),G1,G2,G3,Y1,Y2,Y3,Y4
DIMENSION PH(43),ZH(43),X(20),A(20),D(42),CR(20),CI(20)
DIMENSION C2(20),C3(20)
DIMENSION RS(42),ZS(42),SV(42),CV(42),T(80)
P1=3.141593
DO 57 I=2,NP
   I2=I-1
   DR=RH(I)-RH(I2)
   DZ=ZH(I)-ZH(I2)
   D(I2) =SQRT(DR*DR+DZ*DZ)
   RS(I2)=.5*(RH(I)+RH(I2))
   ZS(I2)=.5*(ZH(I)+ZH(I2))
   SV(I2)=DR/D(I2)
   CV(I2)=CZ/C(I2)
   PC(I2)=PI/(C(I2)*RS(I2))
   CONTINUE
57 CONTINUE
NG=NF-1
N2=NC-2
N=N2/2
J1=1
J5=1
DO 68 J=1,N
   D1=C(J1)
   D2=D(J1+1)
   C3=C(J1+2)
   D4=D(J1+3)
   DEL1=D1+D2
   DEL2=D3+D4
   T(J5+1) =0.5*D1*D2/DEL1
   T(J5+2) =0.5*D3*D4/DEL2
   T(J5+3) =0.5*C1*C3/DEL2
   J1=J1-1
   J5=J5+4
68 CONTINUE
PI2=.5*PI
FN=NA
DO 25 K=1,NPHI
   PH=PI2*(X(K)+1.)
   PHN=PH*FN
   SN=SIN(.5*PH)
   CR(K)=4.*SN*SN
   R1=PI2*A(K)
   CI(K)=.5*R1*CR(K)*COS(PHN)
   C2(K)=R1*COS(PHN)*CCS(PHN)
   C3(K)=R1*SIN(PHN)*SIN(PHN)
   CONTINUE
25 CONTINUE
N2N=N2*N
N4N=N2N*2
DO 62 J=1,N4N
   Y(I,J)=0.
62 CONTINUE
U=(0.,1.)
DO 59 J=1,NC
   L1=1
   L2=2
   IF(J.LE.2) L1=2
59 CONTINUE
IF(J,GT,N2) L2=1
J8=J+1
J9=J8/2
JT=2*J9+J-6
J5=(J9-3)*N2-2
S1=1.
DO 60 I=1,J
I8=1+1
I9=I8/2
IT=2*I9+1-6
J6=I9+J5
RP=RS(J)-RS(I)
ZP=ZS(J)-ZS(I)
R2=RP*RP+ZP*ZP
IF(I.NE.J) GO TO 41
S1=5
R2=625*D(J)*D(J)
41 R3=4*S(*)*RS(J)
G1=0.
G2=0.
G3=0.
DO 61 K=1,NPHI
R4=R2+R3*CR(K)
R5=SQR(T(P4))
Y1=S1/(R4*R5)*(1.*U*R5)*(COS(R5)-U*SIN(R5))
G1=C1(K)*Y1+G1
G2=C2(K)*Y1+G2
G3=C3(K)*Y1+G3
61 CONTINUE
G3=U*G3
Y1=(RP*CV(J)-ZP*SV(J))*G2-RS(I)*CV(J)*G1
Y2=(RS(J)*SV(I)-RS(I)*SV(J))-ZP*SV(I)*SV(J))
Y3=ZP*G3
Y4=(RP*CV(I)-ZP*SV(I))*G2+RS(J)*CV(I)*G1
K1=1
K2=2
IF(I.LT.2) K1=2
IF(I.GT.N2) K2=1
DO 31 L=L1,L2
LT=J7+L+N2
DO 32 K=1,K2
KT=IT+K+K
TT=T*(LT)*T(KT)
J1=J7+K
J2=J1+N
J3=J1+N2N
J4=J3+N
Y(J1)=TT*Y1*Y(J1)
Y(J2)=TT*Y2*Y(J2)
Y(J3)=TT*Y3*Y(J3)
Y(J4)=TT*Y4*Y(J4)
32 CONTINUE
31 CONTINUE
60 CONTINUE
59 CONTINUE
A2P=A2+1
KD1=1
J1=1
J5=1
DO 11 J=1,N
KD2=KD1+N
KC3=KD1+2N
KC4=KD3+N
R2=T(J1+1)
R3=T(J1+2)
R4=T(J1+3)
J6=J5+1
R1=T(J1)*T(J1)*PD(J5)*R2*R2*PD(J6)+R3*R3*PD(J6+1)*R4*R4*PC(J6+2)
G1=Y(KD1)-Y(KD4)
Y(KD1)=R1+G1
Y(KD2)=0.
Y(KD3)=0.
Y(KD4)=R1-G1
IF(J.EQ.1) GO TO 22
KU1=KD1-1
KU2=KD2-1
KU3=KD3-1
KU4=KD4-1
KL1=KD1-N2
KL2=KD2-N2
KL3=KD3-N2
KL4=KD4-N2
R1=T(J1)*T(J1-2)*PD(J5)+R2*T(J1-1)*PD(J6)
G1=Y(KU1)-Y(KL4)
G2=Y(KU4)-Y(KL1)
Y(KU1)=R1+G1
Y(KU2)=Y(KU2)-Y(KL2)
Y(KU3)=Y(KU3)-Y(KL3)
Y(KU4)=R1+G2
Y(KL1)=R1-G2
Y(KL2)=Y(KL2)
Y(KL3)=Y(KU3)
Y(KL4)=R1-G1
22 KD1=KD1+N2P
J1=J1+4
J5=J5+2
11 CONTINUE
IF(N.LT.3) RETURN
J2=N2
DO 13 I=3,N
J2=J2+N2
J1=I-2
K1=1
DO 14 J=1,J1
KU1=J2+J
KU2=KU1+N
KU3=KU1+2N
KU4=KU3+N
KL2=KL1+N
KL3=KL1+2N
KL4=KL3+N
Y(KL1)=-Y(KL4)
Y(KL2)=-Y(KU2)
Y(KL3)=-Y(KU3)
Y(KL4)=-Y(KL1)
K1=K1+N2
14 CONTINUE
13 CONTINUE
RETURN
END
III. THE SUBROUTINE ZMAT

A. Description

The subroutine ZMAT (NN, NP, NPHI, RH, ZH, X, A, Z) stores the matrix

\[
[Z] = \begin{bmatrix}
Z_{nt} & Z_{n\phi} \\
Z_{nt} & Z_{n\phi}
\end{bmatrix}
\]

appearing in the E-field matrix equation (1-57) by columns in Z. The elements of [Z] are given by (1-69)-(1-72). The input variables are defined in terms of variables appearing in reference [1] by

\[
NN = n \\
NP = P \\
NPHI = N_\phi \\
RH(i) = k\rho_i \\
ZH(i) = k\zeta_i \\
X(k) = x_k \\
A(k) = A_k
\]

In summary, n denotes \(e^{in\phi}\) dependence, \((\rho_i, \zeta_i), i=1,2,\ldots P\) are coordinates on the generating curve, the k which multiplies \(\rho_i\) and \(\zeta_i\) is the propagation constant, and \(x_k\) and \(A_k\) are the abscissas and weights for the \(N_\phi\) point Gaussian quadrature integration in \(\phi\).

Minimum allocations are given by:

```
COMPLEX Z(4*N*N)
DIMENSION RH(NP), ZH(NP), X(NPHI), A(NPHI), D(NG),
TP(4*N), CR(NPHI), C2(NPHI), C3(NPHI), C4(NPHI)
COMMON RS(NG), ZS(NG), SV(NG), CV(NG), T(4*N)
```
where
\[ NG = NP - 1 \]
\[ N = (NG - 2)/2 \]
The variables in common make the results of some intermediate calculations
done in ZMAT available to the subroutine PLANE described in Section V.

DO loop 57 puts \( k_d \) of (1-28) in \( D \), \( k_p \) of (1-27) in \( R_S \), \( k_z \) of (1-27)
in \( Z_S \), \( \sin v_i \) in \( S_V \), and \( \cos v_i \) in \( C_V \). DO loop 68 puts (1-68) in \( T_P \) and
(1-29) in \( T \). With regard to (1-62)-(1-65), DO loop 25 puts
\[ 4 \sin^2 \left( \frac{\phi_k}{2} \right) \]
\[ \frac{\pi}{2} A_k \cos \phi_k \cos (n_k^p), \frac{\pi}{2} A_k \sin \phi_k \sin (n_k^p), \text{ and } \frac{\pi}{2} A_k \cos (n_k^p) \]
in \( C_R, C_2, C_3, \) and \( C_4 \) respectively where, as prescribed by (1-75),
\[ \phi_k = \frac{\pi}{2} (k + 1) \]

DO loop 62 initializes \( Z \).

Equation (1-69) can be rewritten as
\[
(Z_{tt}^n)_{pq} = \sum_{i=2p-1}^{2p+2} \sum_{j=2q-1}^{2q+2} \{ T_{i=2p-1}^{2p+2} T_{j=2q-1}^{2q+2} (G_5 \sin v_i \sin v_j + G_4 \cos v_i \cos v_j) \}
- T_{i=2p+1}^{2p+2} T_{j=2q+2}^{2q+2} G_4 \}
\]
where \( G_4 \) and \( G_5 \) are evaluated at \((\rho, z, \rho', z') = (\rho_i, z_i, \rho_j, z_j)\). From (7) and
similar equations for \( (Z_{t+}^t)_{pq} \), \( (Z_{t+}^c)_{pq} \), and \( (Z_{t+}^c)_{pq} \), we obtain
\[
((Z_{tt}^n)_{pq})_{ij} = ((Z_{tt}^n)_{qp})_{ji}
\]
\[
((Z_{t+}^c)_{pq})_{ij} = -((Z_{t+}^c)_{qp})_{ji}
\]
\[
((Z_{t+}^c)_{pq})_{ij} = ((Z_{t+}^c)_{qp})_{ji}
\]
where the additional subscripts \(i, j\) denote the contribution due to the \(ij^{th}\) term on the right-hand side of (7) and similar equations. Equation (8) implies that

\[
(Z_{ntpq}^t)_{pq} = ((Z_{ntpq}^t)_{pq})_{i=j} + ((Z_{ntpq}^t)_{pq})_{i\leq j}
\]

where the subscript notation \(i \leq j\) denotes the sum of the contributions due to all terms for which \(i < j\) plus half of the \(i = j\) term on the right-hand side of (7) and similar equations. According to (9), it is sufficient to calculate only the \(i < j\) terms on the right-hand side of (7) and similar equations. Inspection of the limits of summation in (7) and similar equations shows that the first term on the right-hand side of (9) is zero if \(p > q+1\) and that the second term is zero if \(p < q-1\).

The \(ij^{th}\) term of (7) and similar equations contributes

\[
\begin{align*}
T_2(p+k-1)+T_2(q+i-1)+j(Z1) + T_{2(p+k-1)}^t+T_{2(q+i-1)+j}(Z1A) & \to (Z_{ntpq}^t)_{(p+k)(q+i),} \\
T_2(p+k-1)+T_2(q+i-1)+j(Z2) + T_{2(q+i-1)+j}(Z2A) & \to (Z_{ntpq}^t)_{(p+k)(q+i),} \\
T_2(q+i-1)+T_2(p+k-1)+j(Z3) + T_{2(p+k-1)+j}(Z3A) & \to (Z_{ntpq}^t)_{(p+k)(q+i),} \\
\end{align*}
\]

and

\[
\begin{align*}
T_2(p+k-1)+T_2(q+i-1)+j(Z4) & \to (Z_{ntpq}^t)_{(p+k)(q+i),} \\
\end{align*}
\]

\(k=1,2\)

\(z=1,2\)
where

\[ Z_1 = \textstyle j(G_5 \sin v_i \sin v_j + G_4 \cos v_i \cos v_j) \]
\[ Z_{1A} = -jG_4 \]
\[ Z_2 = -G_6 \sin v_j \]
\[ Z_{2A} = -\frac{n}{k_p} G_4 \]
\[ Z_3 = G_6 \sin v_i \]
\[ Z_{3A} = \frac{n}{k_p} G_4 \]
\[ Z_4 = \textstyle j(G_5 - \frac{n^2}{k^2 p_4 p_j} G_4) \]

(11)

In (10),

\[ p = [(i+1)/2] - 2 \]
\[ q = [(j+1)/2] - 2 \]

where \([i]\) denotes the largest integer which does not exceed \(i\). The \(k=1\) term should be omitted from (10) for the first two values of \(i\), and the \(k=2\) term should be omitted for the last two values of \(i\) because the triangle functions do not overlap at the ends of the generating curve. Similarly, the \(l=1\) term should be omitted for the first two values of \(j\), and the \(l=2\) term should be omitted for the last two values of \(j\).

The indices \(i\) and \(j\) of DO loops 60 and 59 are respectively \(i\) and \(j\) of (10). DO loop 61 accumulates \(G_4, G_5, \) and \(G_6\) of (1-62)-(1-64) for \(I \neq J\) and \(.5G_4, .5G_5, \) and \(.5G_6\) for \(I = J\) in \(G_4, G_5, \) and \(G_6\). The \(Z's\) defined by (11) for \(I \neq J\) and these \(Z's\) divided by two for \(I = J\) are put in \(Z_1, Z_2, Z_2, Z_3, Z_3, \) and \(Z_4\) just after DO loop 61. The indices \(k\) and \(l\) of nested DO loops 32 and 31 are respectively \(k\) and \(l\) of (10). The \(Z(J1), Z(J2), Z(J3), \) and \(Z(J4)\) in DO loop 32 are
respectively $(Z_n^{\text{tt}})_{(p+k)(q+i)}$, $(Z_n^{\phi})_{(p+k)(q+i)}$, $(Z_n^{t\phi})_{(p+k)(q+i)}$, and

$(Z_n^{\phi \phi})_{(p+k)(q+i)}$.

DO loop 11 carries out (9) for

$$
(p, q) = \begin{cases} 
(J, J) & J = 1 \\
(J, J), (J-1, J), \text{ and } (J, J-1) & J > 1
\end{cases}
$$

In DO loop 11, $Z(KD1)$, $Z(KD2)$, $Z(KD3)$, and $Z(KD4)$ are the diagonal $(J, J)$ elements of $Z_n^{\text{tt}}$, $Z_n^{\phi t}$, $Z_n^{t\phi}$, and $Z_n^{\phi \phi}$ respectively. Similarly, $Z(KU1)$, $Z(KU2)$, $Z(KU3)$, and $Z(KU4)$ are the $(J-1, J)$ elements while $Z(KL1)$, $Z(KL2)$, $Z(KL3)$, and $Z(KL4)$ are the $(J, J-1)$ elements.

Nested DO loops 13 and 14 use (1-76) for $(i, j) = (I, J)$ to put

$(Z_n^{\text{tt}})_{1j}$, $(Z_n^{\phi t})_{1j}$, $(Z_n^{t\phi})_{1j}$, and $(Z_n^{\phi \phi})_{1j}$ for $i \leq i-2$ in $Z(KL1)$, $Z(KL2)$, $Z(KL3)$, and $Z(KL4)$ respectively.
B. LISTING OF THE SUBROUTINE ZMAT

SUBROUTINE ZMAT(NN, NP, NPHI, RH, ZH, X, A, Z)
COMPLEX U, Z(16CO), G1, G2, G3, G4, G5, G6, Z1, Z2, Z3, Z4
DIMENSION RH(43), ZH(43), X(20), A(20), D(42), TP(80), CR(20), C2(20)
DIMENSION C3(26), C4(20)
CCMMCN RS(42), ZS(42), SV(42), CV(42), T(80)
PI=3.141593
DO 57 I=2, NP
 IZ=I-1
 DR=RH(I)-RH(IZ)
 DZ=ZH(I)-ZH(IZ)
 D(I) = SQRT(DR*DR+DZ*DZ)
 RS(I)=5*(RH(I)+RH(IZ))/D(I)
 ZS(I)=5*(ZH(I)+ZH(IZ))/D(I)
 SV(I)=DR/D(I)
 CV(I)=DZ/D(I)
CONTINUE
NG=NP-1
N2=NG-2
N=N2/2
J1=1
J5=1
DO 68 J=1,N
 D1=0(J1)
 D2=0(J1+1)
 D3=0(J1+2)
 C4=0(J1+3)
 DEL1=D1*D2
 DEL2=D3*D4
 J6=J5+1
 J7=J6+1
 J8=J7+1
 TP(J5)=D1/DEL1
 TP(J6)=D2/DEL1
 TP(J7)=-C3/CNL2
 TP(J8)=-D4/CNL2
 T(J5)=.5*D1*TP(J5)
 T(J6)=(D1+.5*D2)*TP(J6)
 T(J7)=-(D4+.5*D3)*TP(J7)
 T(J8)=-.5*D4*TP(J8)
 J1=J1+2
 J5=J5+4
CONTINUE
P12=.5*PI
FN=NN
DO 25 K=1,NPHI
 PH=PI2*(X(K)+1.)
 PH=PH*FN
 SN=SN(0.5*PH)
 CR(K)=.5*SIN(PH/SN)
 R1=PI2*A(K)
 C2(K)=R1*COS(PH)*COS(PHN)
 C3(K)=R1*SIN(PH)*SIN(PHN)
 C4(K)=R1*CSS(PH+PN)
CONTINUE
A2N=N2*N
A4N=N2N*2
DO 62 J=1,N4N
Z(J)=0.
62 CONTINUE
U=(0,1)
DO 55 J=1,NG
FJ=FN/RS(J)
L1=1
L2=2
IF(J.LE.2) L1=2
IF(J.GT.N2) L2=1
J8=J+1
J9=J8/2
JT=2*J9+J-6
J5=(J9-3)*N2-2
S1=1.
DO 60 I=1,J
I8=I+1
I9=I8/2
IT=2*I9+I-6
J6=I9+J5
FI=FN/RS(I)
RP=RS(J)-RS(I)
ZP=ZS(J)-ZS(I)
R2=RP*RP+ZP*ZP
IF(I.NE.J) GC TO 41
S1=5
R2=.0625*C(J)*C(J)
41 R3=RS(I)*RS(J)
G4=0.*
G5=0.*
G6=0.*
DO 61 K=1,NPHI
R4=R2+R3*CR(K)
R5=SCRT(P4)
Z1=S1/R5*(CCS(R5)-U*SIN(R5))
G4=C4(K)*Z1+G4
G5=C2(K)*Z1+G5
G6=C3(K)*Z1+G6
61 CONTINUE
Z1=U*(SV(I)*SV(J)+G5+C(V(I)*C(V(J)*G4)
G1=-U*G4
Z2=-SV(J)*G6
G2=-F1*G4
Z3=SV(I)*G6
G3=FJ*G4
Z4=U*(G5-FI*G3)
K1=1
K2=2
IF(I.LE.2) K1=2
IF(I.GT.N2) K2=1
DO 31 L=L1,L2
LT=JT+L+L
J7=J6+L*N2
DO 32 K=K1,K7
KT=IT+K+K
TT=T(LT)*T(KT)
J1=J7+K
J2=J1+N
J3=J1+N2N
J4=J3+N
Z(J1)=TT*Z1+TP(LT)*TP(KT)*G1+Z(J1)
Z(J2)=TT*Z2+TP(LT)*T(KT)*G2+Z(J2)
\[
Z(j3) = TT * Z3 + TP(KT) * T(LT) * G3 + Z(j3)
\]
\[
Z(j4) = TT * Z4 + Z(j4)
\]

32 CONTINUE
31 CONTINUE
60 CONTINUE
59 CONTINUE
N2P = N2 + 1
KD1 = 1
DO 11 J = 1, N
KD2 = KD1 + N
KD3 = KD1 + N2
KD4 = KD3 + N
Z(KD1) = Z(KD1) + Z(KD1)
Z(KD2) = Z(KD2) - Z(KD3)
Z(KD3) = -Z(KD2)
Z(KD4) = Z(KD4) + Z(KD4)
IF(J.EQ.1) GO TO 22
KU1 = KD1 - 1
KU2 = KD2 - 1
KU3 = KD3 - 1
KU4 = KD4 - 1
KL1 = KD1 - N2
KL2 = KD2 - N2
KL3 = KD3 - N2
KL4 = KD4 - N2
Z(KU1) = Z(KU1) + Z(KL1)
Z(KU2) = Z(KU2) - Z(KL3)
Z(KU3) = Z(KU3) - Z(KL2)
Z(KU4) = Z(KU4) + Z(KL4)
Z(KL1) = Z(KU1)
Z(KL2) = -Z(KU3)
Z(KL3) = -Z(KL2)
Z(KL4) = Z(KU4)
22 KCL = KD1 + N2P
11 CONTINUE
IF(N.LT.3) RETURN
J2 = N2
DO 13 I = 3, N
J2 = J2 + N2
J1 = I - 2
KL1 = I
DO 14 J = 1, J1
KU1 = J2 + J
KU2 = KU1 + N
KL3 = KU1 + N2
KU4 = KU3 + N
KL2 = KL1 + N
KL3 = KL1 + N2
KL4 = KL3 + N
Z(KL1) = Z(KU1)
Z(KL2) = -Z(KL3)
Z(KL3) = -Z(KU2)
Z(KL4) = Z(KU4)
KL1 = KL1 + N2
14 CONTINUE
13 CONTINUE
RETURN
END
IV. THE SUBROUTINE YZ

A. Description:

The subroutine YZ(NN, NP, NPHI, RH, ZH, X, A, Y, Z) stores by columns [Y] of (1) appearing in the H-field matrix equation (1-17) and [Z] of (6) appearing in the E-field matrix equation (1-57) in Y and Z respectively. The subroutine YZ is the subroutines YMAT and ZMAT combined into one subroutine. If both [Y] and [Z] are required, some compile time and execution time can be saved by using YZ instead of YMAT and ZMAT. However, YZ requires that [Y] and [Z] be stored simultaneously whereas YMAT and ZMAT do not necessarily require separate storage locations for [Y] and [Z].

The input variables NN, NP, NPHI, RH, ZH, X, and A have the same meaning as in the subroutines YMAT and ZMAT.

Minimum allocations are given by

\[
\text{COMPLEX } Y(4*N*N), Z(4*N*N) \\
\text{DIMENSION RH(NP), ZH(NP), X(NPHI), A(NPHI), D(NG),} \\
P(D(NG), TP(4*N), CR(NPHI), C1(NPHI), C2(NPHI),} \\
C3(NPHI), C4(NPHI) \\
\text{COMMON RS(NG), ZS(NG), SV(NG), CV(NG), T(4*N)}
\]

where

\[
NG = NP-1 \\
N = (NG-2)/2
\]

The variables in common make the results of some intermediate calculations done in YZ available to the subroutine PLANE described in Section V.

Because all variables used in YZ can be traced back to YMAT and ZMAT, anyone who has gone through YMAT and ZMAT should have no trouble with YZ.
B. LISTING OF THE SUBROUTINE YZ

SLARCFUNTE YZ(N,P,PHI,RH,Z,PHI,A,Y,YPHIN)
COMPLEX Y(1600),Z(1600),G1,G2,G3,G4,G5,G6,Y1,Y2,Y3,Y4,Z1,Z2,Z3,Z4
DIMENSION R(43),VZH(43),X(20),A(20),D(42),TP(80),CR(20)
DIMENSION C1(20),C2(20),C3(20),C4(20)
COMMON RS(42),ZS(42),SV(42),CV(42),T(80)
PI=3.141593
DC 57 I=2,NP
I2=I-1
DR=RH(I)-RH(I2)
DZ=Z(I)-Z(I2)
DI2=SQRT(DR*CR+DZ*CZ)
RS(I2)=.5*(RH(I)+RH(I2))
ZS(I2)=.5*(ZH(I)+ZH(I2))
SV(I2)=CR/D(I2)
CV(I2)=DZ/D(I2)
PD(J2)=PI/(C(I2)*RS(I2))
CONTINUE

NG=NP-1
N2=NG-2
N=N2/2
J1=1
J5=1
DO 68 J=1,N
D1=D(J1)
D2=D(J1+1)
D3=D(J1+2)
D4=D(J1+3)
DEL1=D1*D2
DEL2=D3*D4
J6=J5+1
J7=J6+1
J8=J7+1
TP(J5)=D1/DEL1
TP(J6)=D2/DEL1
TP(J7)=-C3/DEL2
TP(J8)=-D4/DEL2
T(J5)=.5*D1*TP(J5)
T(J6)=(D1+.5*C2)*TP(J6)
T(J7)=-(D4+.5*C3)*TP(J7)
T(J8)=-.5*D4*TP(J8)
J1=J1+2
J5=J5+4
68 CONTINUE
PI2=.5*PI
FN=NN
DO 25 K=1,NPHI
PH=P12*(X(K)+1.)
PHN=PH*FN
SN=SIN (.5*PH)
CR(K)=4.*SN*SN
R1=PI2*A(K)
C1(K)=.5*R1*CR(K)*COS(PHN)
C2(K)=R1*COS(PH)*COS(PHN)
C3(K)=R1*SN(PH)*SIN(PHN)
C4(K)=R1*COS(PH)
25 CONTINUE
A2N=A2N*2
A4N=A2N*2
DO 62 J=1,N4
Y(J)=0.
Z(J)=0.
62 CONTINUE
U=0.*1.*
DO 59 J=1,NG
FJ=FN/R(S(J))
L1=1
L2=2
IF(J*LE.2) L1=2
IF(J*GT.N2) L2=1
J8=J+1
J9=J8/2
J1=2*J9+J-6
J5=(J9-3)*N2-2
S1=1.
DO 60 I=1,J
I8=I+1
I9=I8/2
J2=2*I9+I-6
J6=I9+J5
F1=FN/R(S(I))
RP=R(S(J)-R(S(I))
ZP=ZS(J)-ZS(I)
R2=RP*RP+ZP*ZP
IF(I.NE.J) GO TO 41
S1=.5
R2=.625*D(J)*C(J)
41 R3=R(S(I))*R(S(J))
G1=0.
G2=0.
G3=0.
G4=0.
G5=0.
G6=0.
C0 61 K=1,NPHI
R4=R2+R3*CR(K)
R5=SQRT(R4)
Z1=S1/R5*(CCS(R5)-U*SIN(R5))
Y1=Z1*(1.*U*R5)/R4
G1=C1(K)*Y1+G1
G2=C2(K)*Y1+G2
G3=C3(K)*Y1+G3
G4=C4(K)*Z1+G4
G5=C5(K)*Z1+G5
G6=C6(K)*Z1+G6
61 CONTINUE
G3=U*G3
Y1=(RP*CV(J)-ZP*SV(J))*G2-RS(I)*CV(J)*G1
Y2=(RS(I)+*SV(I)*CV(J)-RS(I)*SV(J))*CV(I)-ZP*SV(I)*SV(J))*G3
Y3=ZP*G3
Y4=(RP*CV(I)-ZP*SV(I))*G2+RS(J)*CV(I)*G1
Z1=U*(SV(I)*SV(J)*G5*CV(I)*CV(J)*G4)
G1=-U*G4
Z2=-SV(J)*G6
G2=-FI*G4
Z3=SV(I)*G6
G3=FJ*G4
Z4=U*(G5-FI*G3)
K1=1
K2 = 2
IF (I.AE.2) K1 = 2
IF (I.GT.N2) K2 = 1
DO 31 L = L1, L2
LT = J7 + J + L
J7 = J6 + L * K2
DO 32 K = K1, K2
KT = LT + K
TT = T(L) * T(KT)
J1 = J7 + K
J2 = J1 + N
J3 = J1 + N2
J4 = J3 + N
Y(J1) = TT * Y1 + Y(J1)
Y(J2) = TT * Y2 + Y(J2)
Y(J3) = TT * Y3 + Y(J3)
Y(J4) = TT * Y4 + Y(J4)
Z(J1) = TT * Z1 + TP(L) * TP(KT) * G1 + Z(J1)
Z(J2) = TT * Z2 + TP(L) * T(KT) * G2 + Z(J2)
Z(J3) = TT * Z3 + TP(KT) * T(L) * G3 + Z(J3)
Z(J4) = TT * Z4 + Z(J4)
32 CCNTINUE
31 CONTINUE
60 CONTINUE
59 CONTINUE
N2P = N2 + 1
KD1 = 1
J1 = 1
J5 = 1
DO 11 J = 1, N
KD2 = KD1 + K
KD3 = KD1 + N2
KD4 = KD3 + K
R2 = T(J1 + 1)
R3 = T(J1 + 2)
R4 = T(J1 + 3)
J6 = J5 + 1
R1 = T(J1) * T(J1) * PD(J5) + R2 * R2 * PD(J6) + R3 * R3 * PD(J6 + 1) + R4 * R4 * PD(J6 + 2)
G1 = Y(KD1) - Y(KD4)
Y(KD1) = R1 + G1
Y(KD2) = 0.
Y(KD3) = 0.
Y(KD4) = R1 - G1
Z(KD1) = Z(KD1) + Z(KD1)
Z(KD2) = Z(KD2) - Z(KD3)
Z(KD3) = -Z(KD2)
Z(KD4) = Z(KD4) + Z(KD4)
IF (J.EQ.1) GC TO 22
KU1 = KD1 - 1
KU2 = KD2 - 1
KU3 = KD3 - 1
KU4 = KD4 - 1
KL1 = KD1 - N2
KL2 = KD2 - N2
KL3 = KD3 - N2
KL4 = KD4 - N2
R1 = T(J1) * T(J1 - 2) * PD(J5) * R2 * T(J1 - 1) * PD(J6)
G1 = Y(KU1) - Y(KL4)
G2 = Y(KU4) - Y(KL1)
Y(KU1) = R1 + G1
\( Y(KU2) = Y(KL2) - Y(KL3) \)
\( Y(KU3) = Y(KU3) - Y(KL3) \)
\( Y(KU4) = R1 + G2 \)
\( Y(KL1) = R1 - G2 \)
\( Y(KL2) = -Y(KL2) \)
\( Y(KL3) = -Y(KL3) \)
\( Y(KL4) = R1 - G1 \)
\( Z(KU1) = Z(KU1) + Z(KL1) \)
\( Z(KU2) = Z(KU2) - Z(KL3) \)
\( Z(KU3) = Z(KU3) - Z(KL2) \)
\( Z(KU4) = Z(KU4) + Z(KL4) \)
\( Z(KL1) = Z(KU1) \)
\( Z(KL2) = -Z(KL3) \)
\( Z(KL3) = -Z(KL2) \)
\( Z(KL4) = Z(KU4) \)

22  KD1=KD1+N2P
    J1=J1+4
    JS=JS+2
11 CONTINUE
   IF(N.LT.3) RETURN
   J2=N2
   DO 14 J=3,N
      J2=J2+N2
      J1=J1-2
      KL1=I
      DO 14 J=J1+1,J
      KU1=J2+J
      KU2=KU1+N
      KU3=KU1+N2N
      KL4=KL4+A
      KL2=KL1+N
      KL3=KL1+N2N
      KL4=KL3+A
      Y(KL1)=-Y(KU4)
      Y(KL2)=-Y(KU2)
      Y(KL3)=-Y(KU3)
      Y(KL4)=-Y(KU1)
      Z(KL1)=Z(KU1)
      Z(KL2)=-Z(KU3)
      Z(KL3)=-Z(KU2)
      Z(KL4)=Z(KU4)
      KL1=KL1+N2
14 CONTINUE
13 CONTINUE
RETURN
END
V. THE SUBROUTINE PLANE

A. Description.

The subroutine PLANE(NN, N, NT, THR, R) stores by columns in R the matrix \( [R] \) given by

\[
[R] = \begin{bmatrix}
{R}_t^0 & {R}_t^\phi \\
{R}_n^0 & {R}_n^\phi
\end{bmatrix}
\]  

where the elements of the column vectors on the right-hand side of (12) are given by (1-95). The input variables NN and N are respectively \( n \) of (1-95) and the maximum value of \( i \) in (1-95). If \( NT = 1 \), then \( THR(i) \) is \( \theta_r \) of (1-95) in radians. If \( NT > 1 \), then the measurement matrices \( [R] \) of (12) for \( \theta_r = THR(i), i = 1,2,...NT \) are stored consecutively in R. The variables RS, ZS, SV, CV, and T appearing in the common statement early in the subroutine PLANE are input variables calculated by calling any one of the subroutines YMAT, ZMAT, or YZ beforehand. The calculated values of these variables depend only on the second, fourth and fifth arguments (NP, RH, and ZH) of either YMAT, ZMAT, or YZ.

Minimum allocations are given by

```plaintext
COMPLEX R(4*N*NT)
DIMENSION THR(NT), BJ(M)
COMMON RS(NG), ZS(NG), SV(NG), CV(NG), T(4*N)
```

where

\( NG = 2*N + 2 \)

and \( M \) is the largest of the values of \( M \) calculated by PLANE. The suggested allocation \( BJ(50) \) will work if the maximum circumference of the body of revolution is less than 26 wavelengths.

We rewrite (1-95) as
\[
\begin{bmatrix}
R^{t\theta}_{np} \\
R^{\phi\theta}_{np} \\
R^{t\phi}_{np} \\
R^{\phi\phi}_{np}
\end{bmatrix}
= \frac{1}{2} \sum_{i=2p-1}^{i+2p-2} T_{i+2p-2}
\begin{bmatrix}
R_{1} \\
R_{2} \\
R_{3} \\
R_{4}
\end{bmatrix}
\]

where
\[
R^{t\theta}_{1} = \pi j^{n}(-2J_{n} \sin \theta \cos v + J_{n+1} \cos \theta \sin v) e^{jkz \cos \theta}
\]
\[
R^{\phi\theta}_{2} = -\pi j^{n}(J_{n+1} + J_{n-1}) \cos \theta \sin v e^{jkz \cos \theta}
\]
\[
R^{t\phi}_{3} = \pi j^{n+1}(J_{n+1} + J_{n-1}) \sin v e^{jkz \cos \theta}
\]
\[
R^{\phi\phi}_{4} = \pi j^{n+1}(J_{n+1} - J_{n-1}) e^{jkz \cos \theta}
\]

In (14), \(\phi, z, \) and \(v\) are to be evaluated at \(t = t_{1}\). Equations (13) say that

\(T_{i+2p-2}R^{t\theta}_{1}\) contributes to \(R^{t\theta}_{np}\),

\(T_{i+2p-2}R^{\phi\theta}_{2}\) contributes to \(R^{\phi\theta}_{np}\),

\(T_{i+2p-2}R^{t\phi}_{3}\) contributes to \(R^{t\phi}_{np}\), and

\(T_{i+2p-2}R^{\phi\phi}_{4}\) contributes to \(R^{\phi\phi}_{np}\)

for
\[p = \left\lfloor \frac{i + 1}{2} \right\rfloor - 1\]

and
\[p = \left\lfloor \frac{i - 1}{2} \right\rfloor\]

Because the triangle functions do not overlap at the ends of the generating curve, the first value of \(p\) must be discarded if \(i\) is either 1 or 2 and the
second value of \( p \) must be discarded if \( i \) is either 2*N+1 or 2*N+2.

In DO loop 12, \( t_r \) of (14) is THR(L). DO loop 13 evaluates (14) at \( t = t_r \). The logic in DO loop 13 prior to statement 24 puts the Bessel functions \( J_{n-1}(x) \), \( J_n(x) \), and \( J_{n+1}(x) \) in BJ1, BJ2, and BJ3 respectively where

\[
x = k_0 I \sin \theta_r
\]

Choosing \( M \) so large that the magnitude of \( J_{M-1}(x) \) is around \( 10^{-8} \), we start with

\[
J_{M-1}(x) = 0
\]

\[
J_{M-2}(x) = 1
\]

and use the recurrence relation

\[
J_{n-1}(x) = \frac{2n}{x} J_n(x) - J_{n+1}(x)
\]

as given by (9.1.27) on page 361 of reference [2] to calculate \( J_n(x) \) for \( n = M-3, M-4, \ldots, 0 \) and then normalize \( J_n(x) \) according to

\[
1 = J_0(x) + 2J_2(x) + 2J_4(x) + 2J_6(x) + \ldots
\]

as given by (9.1.46) on page 361 of reference [2]. Statement 24 and the 6 statements following it put \( R_1, R_2, R_3, \) and \( R_4 \) of (14) in R1, R2, R3 and R4 respectively. DO loop 20 adds the contributions to (13) for \( i = I \) and

\[
p = \left[ \frac{I + 1}{2} \right] + k-2
\]

LISTING OF THE SUBROUTINE PLANE

SUBROUTINE PLANE(NA, A, NT, THR, R)
COMPLEX R(240), U, U1, U2, R1, R2, R3, R4
DIMENSION THR(50)
COMMON RS(42), ZS(42), SV(42), CV(42), T(80)
N2=2*N
NG=N2+2
U=(0.,1.)
U1=3.141593*U**NN
JR=4*N*NT
DO 22 J=1, JR
R(J)=0.
22 CONTINUE
J5=-2
DO 12 L=1, NT
CS=CCS(THR(L))
SN=2.*SIN(THR(L))
DO 13 I=1, NG
X=.25*RS(I)*SN
IF(X.LE.5E-7) GO TO 18
M=2.8*X+13.-2./X
IF(X.LT.5.) M=10.8*ALOG10(X)
IF(M.GE.(NN+2)) GO TO 19
18 BJ1=0.
BJ2=0.
BJ3=C.
IF(NN.EQ.1) BJ1=1.
IF(NN.EQ.0) BJ2=1.
GO TO 24
19 BJ(M)=0.
JM=M-1
BJ(JM)=1.
DO 14 J=3, M
JM=JM-1
BJ(JW)=JM/X*BJ(JM+1)-BJ(JM+2)
14 CONTINUE
S=0.
DO 15 J=3, M, 2
S=S+BJ(J)
15 CONTINUE
S=BJ(1)+2.*S
BJ2=BJ(NN+1)/S
BJ3=BJ(NN+2)/S
BJ1=-BJ3
IF(NN.GT.0) BJ1=BJ(NN)/S
24 ARG=ZS(I)*CS
U2=U1*(CCS(ARG)+U*SIN(ARG))
R4=(BJ3-BJ1)*U*U2
R2=(BJ3+BJ1)*U2
R1=-BJ2*CV(1)*SN*U2+CS*SV(1)*R4
R3=SV(1)*R2
R2=-CS*R2
I9=(I+1)/2
IT=2*I9+1-6
J7=I9+J5
K1=1
K2=2
IF(I.LE.2) K1=2
IF(I.GT.K2) K2=1
DO 20 K=K1,K2
I'=T(1T+K+K)
J1=J7+K
J2=J1+N
J3=J2+N
J4=J3+N
R(J1)=TT*R1+R(J1)
R(J2)=TT*R2+R(J2)
R(J3)=TT*R3+R(J3)
R(J4)=TT*R4+R(J4)
20 CONTINUE
13 CONTINUE
J5=J4-2
12 CONTINUE
RETURN
END
VI. THE SUBROUTINES DECOMP AND SOLVE

A. Description:

The subroutines DECOMP(N,IPS,UL) and SOLVE(N,IPS,UL,B,X) solve a system of N linear equations in N unknowns. These subroutines will be used in Section VII to solve the H-field matrix equations (1-17), the E-field matrix equations (1-57) and the combined field matrix equations (1-88). The input to DECOMP consists of N and the N by N matrix of coefficients on the left-hand side of the matrix equation stored by columns in UL. The output from DECOMP is IPS and UL. This output is fed into SOLVE. The rest of the input to SOLVE consists of N and the column of coefficients on the right-hand side of the matrix equation stored in B. SOLVE puts the solution to the matrix equation in X.

Minimum allocations are given by

```
COMPLEX UL(N*N)
DIMENSION SCL(N), IPS(N)
```

in DECOMP and by

```
COMPLEX UL(N*N), B(N), X(N)
DIMENSION IPS(N)
```

in SOLVE.

A description of DECOMP and SOLVE is on pages 46-49 of reference [3].

LISTING OF THE SUBROUTINE DECOMP

SUBROUTINE DECOMP(N, IPS, UL)
COMPLEX UL(N,160), PIVOT, EM
DIMENSION SCL(N), IPS(N)
DO 5 I=1,N
IPS(I)=I
RN=0.
J1=I
DO 2 J=1,N
UL=ABS(REAL(UL(J)))+ABS(AIMAG(UL(J)))
J1=J1+N
IF(RN=ULM) 1,2,2
1 RN=ULM
2 CONTINUE
SCL(I)=1/RN
5 CONTINUE
NM1=N-1
K2=0
DO 17 K=1,NM1
BIG=0.
DO 11 I=K,N
IP=IPS(I)
IPK=IP+K2
SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
IF(SIZE>BIG) 11,11,10
10 BIG=SIZE
IPV=I
11 CONTINUE
IF(IPV=K) 14,15,14
14 J=IPS(K)
IPS(K)=IPS(IPV)
IPS(IPV)=J
15 KPP=IPS(K)+K2
PIVOT=UL(KPP)
KP1=K+1
DO 16 I=KP1,N
KP=KP1
IP=IPS(I)+K2
EM=-UL(IP)/PIVOT
16 CONTINUE
UL(IP)=-EM
DO 18 J=KP1,N
IP=IP+N
KP=KP+N
UL(IP)=UL(IP)+EM*UL(KP)
18 CONTINUE
K2=K2+N
17 CONTINUE
RETURN
END

LISTING OF THE SUBROUTINE SCLVE

SUBROUTINE SCLVE(N, IPS, UL, B, X)
COMPLEX UL(N,160), R(N), X(N), SUM
DIMENSION IPS(N)
NP1=N+1
IP=IPS(1)
X(1) = R(IP)
DO 2 I=2,N
IP=IPS(I)
IPB=IP
IM1=I-1
SUM=C*
DO 1 J=1,IM1
SUM=SUM+LL(IP)*X(J)
1 IP=IP+N
2 X(I)=B(IPB)-SUM
K2=N*(N-1)
IP=IPS(N)+K2
X(N)=X(N)/UL(IP)
DO 4 IBACK=2,N
I=NP1-IBACK
K2=K2-N
IP1=IPS(I)+K2
IP1=I+1
SUM=C*
IP=IP1
DO 3 J=IP1,N
IP=IP+N
3 SUM=SUM+UL(IP)*X(J)
4 X(I)=(X(I)-SUM)/UL(IP)
RETURN
END
VII. THE MAIN PROGRAM

A. Description:

The main program uses all the subroutines listed in Sections II-VI to evaluate the surface currents (1-109) at one specified value of $\phi$ for both $\theta$ and $\phi$ transmitter polarizations and to evaluate the scattering cross section (1-111) at one specified $\tau$ of $\tau$ for the four different combinations of transmitter and receiver polarizations. The main program is not general enough to meet every user's needs. Its main purpose is to give an example of how these subroutines can be used and to provide the user with sample output from them so that he can be sure that they are working properly.

Input data is read from punched cards according to

```
READ(1,51) NM, NP, NPHI
51 FORMAT (3i3)
READ(1,50) BK, TT, P, TR, PR, ALP
50 FORMAT (5E14.7)
READ(1,46)(RH(I), I = 1, NP)
READ(1,46)(ZH(I), I = 1, NP)
46 FORMAT (10F8.4)
READ(1,50)(X(K), K = 1, NPHI)
READ(1,50)(A(K), K = 1, NPHI)
```

The above input variables are defined by

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>NM</td>
<td>one plus the maximum value of $n$ in (1-109) and (1-110)</td>
</tr>
<tr>
<td>NP</td>
<td>$P$</td>
</tr>
<tr>
<td>NPHI</td>
<td>$N_\phi$</td>
</tr>
<tr>
<td>BK</td>
<td>$k$</td>
</tr>
<tr>
<td>TT</td>
<td>$\theta_t$</td>
</tr>
<tr>
<td>P</td>
<td>$\phi$</td>
</tr>
<tr>
<td>TR</td>
<td>$\theta_r$</td>
</tr>
<tr>
<td>PR</td>
<td>$\phi_r$</td>
</tr>
<tr>
<td>ALP</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>RH(i)</td>
<td>$k \phi_i$</td>
</tr>
<tr>
<td>ZH(i)</td>
<td>$k z_i$</td>
</tr>
<tr>
<td>X(k)</td>
<td>$x_k$</td>
</tr>
<tr>
<td>A(k)</td>
<td>$A_k$</td>
</tr>
</tbody>
</table>

where all variables on the right-hand sides of the above equations appear in reference [1]. According to reference [1], $n$ denotes $e^{jn\phi}$ dependence, $(\phi_i, z_i), i=1,2,...P$ are coordinates on the generating curve, $k$ is the propagation constant, $\theta_t$ is the colatitude of the transmitter, and $\phi$ is
the longitude at which the surface current is evaluated. The angles \( \theta_r \) and \( \phi_r \) are the colatitude and longitude of the receiver. The real constant \( a \) is the relative weight of the E-field integral equation in the combined field formulation. Finally, \( x_k \) and \( A_k \) are the abscissas and weights for the \( N \phi \) point Gaussian quadrature integration in \( \phi \). All angles read into the main program are in degrees.

Minimum allocations are given by

\[
\text{COMPLEX TJ}(6\times N), \ PJ(6\times N), \ RT(4\times N), \ RR(4\times N), \\
Y(4\times N\times N), \ Z(4\times N\times N), \ R(2\times N), \ C(2\times N) \\
\text{DIMENSION RH}(NP), \ ZH(NP), \ X(NPHI), \ A(NPHI), \\
THT(1), \ THR(1), \ R2(N), \ IPS(2\times N)
\]

where

\[ N = (NP - 3)/2 \]

The surface current printed by the main program is not (1-109) per se, but (1-109) divided by the propagation constant \( k \). The heading above the surface current in the printed output denotes real and imaginary parts and \( t \) and \( \phi \) components. As further identification, \( \text{NHEC}=1 \) denotes the H-field solution, \( \text{NHEC}=2 \) denotes the E-field solution, \( \text{NHEC}=3 \) denotes the combined field solution, \( \text{KT}=1 \) denotes a \( \theta \) polarized transmitter, and \( \text{KT}=2 \) denotes a \( \phi \) polarized transmitter. The scattering cross section per square wavelength (1-111) is labeled \( \text{SIGMA}/(\text{LAMBDA})^2 \) and further identified by the parameters \( \text{NHEC} \), \( \text{KT} \), and \( \text{KR} \) where \( \text{KR}=1 \) denotes a \( \theta \) polarized receiver and \( \text{KR}=2 \) denotes a \( \phi \) polarized receiver.

The index \( K \) of DO loop 41 obtains the \( n=\ell+1 \) term in (1-109) and (1-110). In DO loop 58, \( \text{NHEC}=1 \) obtains the H-field solution, \( \text{NHEC}=2 \) obtains the E-field solution, and \( \text{NHEC}=3 \) obtains the combined field solution. In DO loop 27, \( \text{KT}=1 \) denotes a \( \theta \) polarized transmitter and \( \text{KT}=2 \) denotes a \( \phi \) polarized transmitter. In DO loop 16, \( \text{KR}=1 \) denotes a \( \theta \) polarized receiver and \( \text{KR}=2 \) denotes a \( \phi \) polarized receiver. The user can omit computations which are of no interest to him by changing the statements which introduce DO loops 41, 58, 27, and 16 so as to restrict the index of these DO loops. The logic in DO loop 58 before statement 59 puts the square matrix on the left-hand side of either the H-field matrix equation (1-17), the E-field matrix equation (1-57), or the combined
field matrix equation (1-88) in \( Y \). The logic in DO loop 27 before statement 53 uses (1-99), (1-100) and (1-104) to put the excitation column vector on the right-hand side of either the \( H \)-field, the \( E \)-field, or the combined field matrix equation in \( B \). Statement 53 puts the solution of the matrix equation in \( C \). If \( KT=1 \), the logic in DO loop 13 adds the \( n=K-1 \) term of the \( t \) and \( \phi \) components of the numerator of the first of equations (1-109) to \( TJ \) and \( PJ \) respectively. DO loop 15 adds the \( n=K-1 \) term of the \( t \) and \( \phi \) components of the numerator of the second of equations (1-109) to \( TJ \) and \( PJ \) respectively. DO loop 16 adds the \( n=K-1 \) term of

\[
\frac{jkr}{4\pi r} e^{-j\eta} E_{pq}^{s}(j)
\]

to \( E(KR) \) where \( E_{pq}^{s}(j) \) is given by (1-110). Here, \( p \) is either \( \theta \) or \( \phi \), and \( q \) is either \( \theta \) or \( \phi \).

Nested DO loops 28, 29, and 30 print out both (1-109) divided by the propagation constant \( k \) and (1-111). The indices \( NUEC, KT, \) and \( K R \) of these DO loops have the same meaning as in nested DO loops 58, 27, and 16.

The sample input and output data is for oblique incidence on a conducting sphere of radius 0.2 wavelengths. Only the \( n=0 \) and \( n=1 \) terms in (1-109) and (1-110) are considered. A few more terms may be needed to obtain reasonable accuracy.
B. LISTING OF THE MAIN PROGRAM

//PGM JOB (XXXX, XXX, 2), *MAUTZ, JOE*, REGION=200K
// EXEC WATFIV
//GO SYSTIN DD *

$JOB MAUTZ, TIME=2, PAGES=40

C THIS PROGRAM RAN FOR 45 SECONDS ON THE IBM 370/155.
C SUBROUTINES YMAT, ZMAT, YZ, PLANE, DECOMP, AND SCLVE ARE CALLED.
COMPLEX U, TJ(120), PJ(120), E(12), SN, SNR, RT(240), RR(240), Y(160)
COMPLEX Z(1600), B(40), C(40), U1, CCNJG
DIMENSION RH(43), ZH(43), X(20), A(20), TJ(3), THR(3), R2(20), IPS(40)
READ(1, 51) NM, NP, NPHI
51 FORMAT(3I3)
READ(1, 50) BK, TT, P, TR, PR, ALP
50 FORMAT(5E14.7)
READ(1, 150)(RH(I), I=1, NP)
READ(1, 150)(ZH(I), I=1, NP)
150 FORMAT(10F8.4)
READ(1, 46)(X(K), K=1, NPHI)
READ(1, 46)(A(K), K=1, NPHI)
46 FORMAT(10F8.4)
WRITE(3, 49) NM, NP, NPHI
49 FORMAT(10X, 2E14.1, 12X, PR, 12X, TR, 12X, 5E14.7, 7X, AL)
WRITE(3, 45) (RH(I), I=1, NP)
45 FORMAT(10X, 2E14.7, 7X, AL)
WRITE(3, 44) ZH(I), I=1, NP)
44 FORMAT(10X, 2E14.7, 7X, AL)
WRITE(3, 47) (X(K), K=1, NPHI)
47 FORMAT(10X, 2E14.7, 7X, AL)
WRITE(3, 43) (A(K), K=1, NPHI)
43 FORMAT(2E14.7, 7X, AL)
N2=NP-3
N3=N2+N
N4=N2*N2
N6=6*N
U=(0.9, 1.)
P1=3.141593
P4=0.0625/P1**3
P8=P1/180.
THT(1)=TT*P8
THR(1)=TR*P8
PR=PR*P8
DC 42 J=1, NP
RH(J)=BK*RH(J)
ZH(J)=BK*ZH(J)
24 CONTINUE
DO 42 J=1, N
R2(J)=1./RH(2*J+1)
42 CONTINUE
DO 54 J=1, N6
TJ(J)=0.
PJ(J)=0.
54 CONTINUE
DO 55 J=1, 12
F(J)=0.
55 CONTINUE
WRITE(3,9)
9 FORMAT(*OSAMPLE CUTPUT FROM SUBROUTINES*)
   DO 41 K=1,NM
      NN=K-1
      PN=NN*P
      CS=COS(PN)
      SN=2.*SIN(PN)*L
      PN=NN*PR
      CSR=COS(PN)
      SNR=2.*SIN(PN)*U
      IF(NK.EQ.0) GO TO 56
      CS=2.*CS
      CSR=2.*CSR
 56 LANE=0
   DO 58 NHEC=1,3
      GO TO (61,62,63,64),NHEC
   61 CALL YMAT(NA,NF,PHI,RH,ZH,X,A,Y)
      WRITE(3,8) Y(1),Y(2)
      GC TC 59
   62 CALL ZMAT(NA,NP,PHI,RH,ZH,X,A,Y)
      WRITE(3,8) Y(1),Y(2)
      GO TO 59
   63 CALL YZ(NA,NP,PHI,RH,ZH,X,A,Y,Z)
      WRITE(3,8) Y(1),Y(2),Z(1),Z(2)
   DO 66 J=1,N4
      Y(J)=Y(J)+ALP*2(J)
 66 CONTINUE
   CALL DECCMP(N2,IPS,Y)
      WRITE(3,8) Y(1),Y(2)
      IF(LANE.NE.1) GO TO 57
      LANE=1
      CALL PLANE(NN,N,1,THR,RT)
      CALL PLANE(NN,N,1,THT,RT)
      WRITE(3,8) RT(1),RT(2),RR(1),RR(2)
 57 DO 27 KT=1,2
      L=2*(NHEC-1)+KT
      GO TO (31,32,33,34,35,36),L
 31 DO 21 J=1,N
     B(J)=-RT(J+N3)
     B(J+N)=-RT(J+N2)
 21 CONTINUE
      GO TO 53
 32 DO 22 J=1,N
     JN=J+N
     B(J)=-RT(JN)
     B(JN)=-RT(J)
 22 CONTINUE
      GO TO 53
 33 DO 23 J=1,N
     B(J)=RT(J)
     JN=J+N
     B(JN)=-RT(JN)
 23 CONTINUE
      GO TO 53
 34 DO 24 J=1,N
     B(J)=-RT(J+N2)
     B(J+N)=RT(J+N3)
 24 CONTINUE
      GO TO 53
J = 1, N
B(J) = -RT(J+N3) + ALP*RT(J)
JN = J+N
B(JN) = -RT(J+N2) + ALP*RT(J+N3)

CONTINUE
GO TO 53

DO 26 J = 1, N
JN = J+N
B(J) = -RT(J+N) + ALP*RT(J+N2)
B(JN) = -RT(J) + ALP*RT(J+N3)

CONTINUE

CALL SCLVE(2, 1, IFSTY, BC)
WRITE(3, 8) C(1), C(2)
J1 = (L-1)*N
GO TO (11, 12), T

DO 13 J = 1, N
J2 = J+J1
T(J2) = TJ(J2) + CJ(J)*CS
PJ(J2) = PJ(J2) + C(J+N)*SN

CONTINUE
GO TO 14

DO 15 J = 1, N
J2 = J+J1
T(J2) = TJ(J2) + C(J)*CS
PJ(J2) = PJ(J2) + C(J+N)*SN

CONTINUE
GO TO 16

DO 16 J = 1, N2
U1 = U1 + RR(J)*C(J)

CONTINUE
E(K) = E(K) + U1*CSR
GO TO 16

DO 76 J = 1, N2
U1 = U1 + RR(J+N2)*C(J)

CONTINUE
E(K) = E(K) + U1*SNR
GO TO 16

DO 78 J = 1, N2
U1 = U1 + RR(J+N2)*C(J)

CONTINUE
E(K) = E(K) + U1*SNR
GO TO 16

DO 78 J = 1, N2
U1 = U1 + RR(J+N2)*C(J)

CONTINUE
E(K) = E(K) + U1*CSR

CONTINUE

DO 28 NHEC = 1, 3
DO 29 KT = 1, 2
WRITE(3, 18) NHEC, KT

FORMAT(* ONHEC=* , I3, * , KT=* , I3)
WRITE(3, 19)

FORMAT(* REAL JT IMAG JT REAL JP IMAG JP*)
J1=N*(2*NHEC+KT-3)
DO 37 J=1,N
J2=J+J1
TJ(J2)=TJ(J2)*R2(J)
P(J2)=P(J2)*R2(J)
WRITE(3,38) TJ(J2),P(J2)
38 FORMAT(1X,4E11.4)
37 CONTINUE
DO 3C
KR=I,2
J1=4*NHEC+2*KT+KR-6
SIG=P4*E(JI)*CCNJG(E(JI))
WRITE(3,10) NFECKTKRSIG
10 FORMAT( NHEC=*,13,4, KT=1,13,'0 KR=*,13,', SIGMA/(LAMBOA)**2=',E11.4)
30 CONTINUE
29 CONTINUE
28 CONTINUE
STOP
END
$CAT
A
2 21 20
C.1256637E+01 C.3CCCOOE+02 0.4500000E+02 0.6000000E+02 0.2000000E+02
C.2000000E+00
C.0300 C.1564 0.3090 0.4540 0.5878 0.7071 0.8090 0.8910 0.9511 0.9877
C.3000 C.9877 0.9511 0.8910 0.8090 0.7071 0.5878 0.4540 0.3090 0.1564
C.0000
-1.0000 -C.9877 -0.9511 -0.8910 -0.8090 -0.7071 -0.5878 -C.4540 -C.3090 -0.1564
C.0000
-C.9931286E+00-C.9635719E+00-C.9122344E+00-0.8391170E+00-0.7463319E+00
-0.6360537E+00-0.5168670E+00-0.3737061E+00-0.2277859E+00-0.1652652E+00
C.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6306537E+00
C.7463319E+00 0.8391170E+00 0.9122344E+00 0.9639170E+00 0.9931286E+00
C.1761401E-01 0.4060143E-01 0.6267205E-01 0.8327674E-01 0.1019301E+00
C.1181945E+00 0.1316866E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
C.1527534E+00 0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316866E+00
C.1181945E+00 0.1019301E+00 0.8327674E-01 0.6267205E-01 0.4060143E-01
C.1761401E-01
STOP
//
//
PRINTED OUTPUT
AN NP NPHI
2 21 20
C.1296637E+01 C.3CCCOOE+02 0.4500000E+02 0.6000000E+02 0.2000000E+02
C.2000000E+00
C.0300 C.1564 0.3090 0.4540 0.5878 0.7071 0.8090 0.8910 0.9511 0.9877
C.3000 C.9877 0.9511 0.8910 0.8090 0.7071 0.5878 0.4540 0.3090 0.1564
C.0000
-1.0000 -C.9877 -0.9511 -0.8910 -0.8090 -0.7071 -0.5878 -C.4540 -C.3090 -0.1564
C.0000
-C.9931286E+00-C.9635719E+00-C.9122344E+00-0.8391170E+00-0.7463319E+00
-0.6360537E+00-0.5168670E+00-0.3737061E+00-0.2277859E+00-0.1652652E+00
C.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6306537E+00
C.7463319E+00 0.8391170E+00 0.9122344E+00 0.9639170E+00 0.9931286E+00
C.1761401E-01 0.4060143E-01 0.6267205E-01 0.8327674E-01 0.1019301E+00
C.1181945E+00 0.1316866E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
C.1527534E+00 0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316866E+00
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//
//
SAMPLE OUTPUT FROM SLROUTINES

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SIGMA/(LAMBOA)**2= 0.3550E+00

N=EC = 1, KT= 1, KR= 1
SIGMA/(LAMBOA)**2= 0.3835E-01

N=EC = 1, KT= 2
SIGMA/(LAMBOA)**2= 0.3550E+00

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**Best Available Copy**

- The document contains Tables and numeric data related to various parameters such as HE, KT, KR, and SIGMA/(LAMBDA)**2**.
- The data is presented in a tabular format with columns for real and imaginary parts.
- The values are in scientific notation, indicating precision and magnitude.
\( N_{\text{EC}} = 3, \quad K_T = 2, \quad K_R = 1, \quad \sigma/(\Lambda^2) = 0.2903 \times 10^{-1} \)

\( N_{\text{EC}} = 3, \quad K_T = 2, \quad K_R = 2, \quad \sigma/(\Lambda^2) = 0.3845 \times 10^0 \)
VIII. EXAMPLES AND DISCUSSION

Some examples of computations obtained from the computer program are given in the previous report [1]. We here give some additional examples to illustrate the use of the program.

As discussed in [1], the parameter \( \alpha \) in the combined field solution should be taken \( 0 < \alpha \leq 1 \), with a value of the order of 0.2 normally giving good results. To give some additional information on this choice, Table 1 gives the RMS error in current (\( \Delta \)) and the backscattering \( (\sigma/\pi a^2) \) for a conducting sphere of radius \( a \). The three values \( k a = 1.50, 2.75, \) and 4.00 were chosen for \( k a \), and the parameter \( \alpha \) was varied from 0.2 to 1.0. Also shown are results for the H-field solution (\( \alpha = 0 \)) and for the E-field solution (\( \alpha = \infty \)). The exact values of \( \sigma/\pi a^2 \) are shown in the last row, computed from the spherical mode solution [4]. As is evident from the table, the value \( \alpha = 0.2 \) always gave good results, but the error was small for any value of \( \alpha \) from 0.2 to 1.0.

Table 1. RMS Error in current (\( \Delta \)) and Backscattering \( (\sigma/\pi a^2) \) for various solutions for a conducting sphere of radius \( a \). H-field solution corresponds to \( \alpha = 0 \). E-field solution corresponds to \( \alpha = \infty \).

<table>
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<tr>
<th>( k a )</th>
<th>( 1.50 )</th>
<th>( 2.75 )</th>
<th>( 4.00 )</th>
<th>( 1.50 )</th>
<th>( 2.75 )</th>
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<tr>
<td>( \alpha = 0.2 )</td>
<td>0.0132</td>
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<td>( \alpha = 0.6 )</td>
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<tr>
<td>( \alpha = 0.8 )</td>
<td>0.0127</td>
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<td>0.0447</td>
<td>1.065</td>
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<tr>
<td>( \alpha = 1.0 )</td>
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<td>0</td>
<td>1.076</td>
<td>0.8552</td>
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The problem of scattering by a conducting right circular cylinder of cross sectional radius \( a \) and height \( 2a \) was considered to illustrate further
the effect of internal resonance on the various solutions. The excitation is a plane wave axially incident on the flat end of the cylinder. The first internal resonance varying as $\cos \phi$ occurs at $ka = 2.42$ approximately. Figure 1 shows the normalized backscattering cross section ($\sigma/\lambda^2$) in the vicinity of the first internal resonance. The range of $ka$ is from 2.37 to 2.47 in increments of 0.005. Note that the internal resonance had no observable effect on the combined field solution with $\alpha = 0.3$ (triangles), a small effect on the E-field solution (circles), and a larger effect on the H-field solution (squares).

To estimate the RMS error in the current, the combined field solution was considered to be the exact solution. The RMS error is then defined as

$$\Delta = \left[ \frac{\iint |I - I^c|^2 \, ds}{\iint 1 \, ds} \right]^{1/2}$$

(17)

where $I^c$ is combined field solution and the integration is over the surface of the scatterer. Figure 2 shows $\Delta$ for the right circular cylinder vs. $ka$ in the vicinity of the first internal resonance. Note that the greatest RMS error occurs in the E-field solution (circles), with a somewhat smaller error in the H-field solution (squares). However, since the H-field eigencurrents radiate external to the cylinder, and the E-field eigencurrents do not (at least theoretically), the H-field backscattering cross section suffers a greater error at resonance than does the E-field backscattering cross section (Fig. 1).

In the E-field solution a resonance phenomena sometimes occurs that is unrelated to any internal resonance. For example, in the case of a sphere there was a small peak in $\Delta$ at about $ka = 1.3$ (Figure 5, reference [1]). In that case the resonance effect was too small to show up in the backscattering cross section (Figure 6, reference [1]). This E-field "false resonance" effect was also found to occur in the case of scattering by a flat-back cone. The cone geometry is shown in Fig. 3. The cone geometry is shown in Fig. 3. The excitation is a plane wave axially
Fig. 1. Normalized radar cross section ($\sigma/\lambda^2$) vs $ka$ in the vicinity of the first $\cos \phi$ internal resonance ($ka = 2.42$) for a closed conducting circular cylinder of radius $a$ and height $2a$. The $H$-field solution is shown by squares, the $E$-field solution by circles, and the combined field solution with $\alpha = 0.3$ by triangles. The excitation is an axially incident plane wave.
Fig. 2. RMS error ($\Delta$) in current vs. $ka$ in the vicinity of the first $\cos \phi$ internal resonance ($ka = 2.42$) for a closed conducting circular cylinder of radius $a$ and height $2a$. The combined field solution with $\alpha = 0$ was assumed to be the exact solution. The H-field solution is shown by squares and the E-field solution by circles. The excitation is an axially incident plane wave.
incident on the tip of the cone. Figure 4 shows the normalized cross section \( \sigma/\lambda^2 \) vs. frequency in the vicinity of an E-field false resonance, which occurred at 0.9 \( f_0 \), where \( f_0 \) is the frequency at which \( \alpha = 3\lambda/16 \). Note that the E-field solution (circles) suffers a considerable perturbation at this false resonance, while the H-field solution (squares) and the combined-field solution (triangles) are unaffected by it. Again \( \alpha \) is taken to be 0.3 in the combined field solution. It should also be noted that the H-field solution differs from the combined field solution by an appreciable amount in this range of frequencies.

Figure 5 shows the RMS error in current for the E-field and H-field solutions in the vicinity of the false resonance for the flat-back cone. Again the combined field solution with \( \alpha = 0.3 \) is taken to be the exact solution, with \( \Delta \) computed according to (17). Note that there is no resonance effect apparent in the H-field solution, even though there is a large effect in the E-field solution. It is felt that this false resonance is a consequence of energy storage due to the current being approximated by triangular expansion functions. If this is a correct conjecture, the resonance would probably not be present at the same frequency if the expansion functions were changed, but might occur at a different frequency.

In summary, we repeat that for the combined field solution \( \alpha \) should be chosen in the range \( 0.2 < \alpha < 1 \). At internal resonances, the RMS error in current is greater for the E-field solution than for the H-field solution. However, the scattering cross section error is usually smaller for the E-field solution than for the H-field solution. No effects due to internal resonances have ever been observed in the combined-field solution. Finally, the E-field solution sometimes exhibits a false resonance effect, giving incorrect solutions at some frequency which does not correspond to an internal resonance.
Fig. 3. A conducting flat-back cone of base radius $a$ and cone angle $90^\circ$. 
Fig. 4. Normalized radar cross section \((\sigma/\lambda^2)\) vs. frequency in the vicinity of an E-field solution false resonance \((a = 3\lambda/16\) at \(f_0\)) for a flat-back cone of radius \(a\) and 90° cone angle.

The H-field solution is shown by squares, the E-field solution by circles, and the combined field solution with \(a = 0.3\) by triangles. The excitation is a plane wave axially incident on the cone tip.
Fig. 5. RMS error (Δ) in current vs. frequency in the vicinity of an E-field solution false resonance (a = 3λ/16 at fo) for a flat-back cone of radius a and 90° cone angle. The combined field solution with a = 0.3 was assumed to be the exact solution. The H-field solution is shown by squares and the E-field solution by circles. The excitation is a plane wave axially incident on the cone tip.
REFERENCES


## Metric System

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* To be avoided where possible
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