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Geometric Theory of Moving Grid Wavefront Sensor

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A geometric optics analysis is made for a wavefront sensor that uses a moving Ronchi grid. It is shown that by simple data processing a signal proportional to the gradient of the distorted wavefront can be obtained for each element of an array of photodetectors that cover the entire image of a telescope aperture. The effect of diffraction is calculated for this system. This shows the limitations and provides a justification for the geometric analysis.

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I. INTRODUCTION

There are several types of adaptive optical systems being considered or being developed[1-3] for imaging an object through a turbulent atmosphere. Some of these use a wavefront sensor to measure the degree of wavefront deformation caused by atmospheric turbulence. This information is then used to deform the surface of a flexible mirror or control some other active element to compensate for the wavefront deformation and thereby improve the image quality. The wavefront sensor used in one of these adaptive optical imaging systems, which has been described by Feinleib and Hardy,[1] consists of a rotating Ronchi grid placed in the image plane of the telescope. This wavefront sensor is relatively simple and insensitive to external disturbances. The operation is explained by Feinleib and Hardy by means of a physical optics theory, in which the sensor is assumed to behave as a shearing interferometer. This report presents a geometric optics analysis of this wavefront sensor. The justification for the geometric optics approach is given in the Appendix. One result of this analysis is a simplification in the data processing required to translate sensor data into wavefront deformation information.

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[1] Julies Feinleib and J. W. Hardy, "Wideband Adaptive Optics for Imaging," Proc. SPIE 75, 103 (1976).

[2] A. Buffington, F. S. Crawford, R. A. Muller, A. J. Schwemin, and R. J. Smits, "Active Image Restoration with a Flexible Mirror," Proc. SPIE 75, 89 (1976).

[3] Virendra N. Mahajan, "Optical Wavefront Correction in Real Time," Proc. SPIE 75, 109 (1976).

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II. BASIC THEORY

Consider a simplified model of a telescope with a moving grid wavefront sensor, as shown schematically in Fig. 1. A flexible corrector mirror and an image sensor normally included in a compensated imaging system are not pertinent to this discussion and are not shown in Fig. 1. The telescope focuses a distant object, defined by the angular coordinates (α, β), onto an image plane (x, y). The aperture of the telescope lies in a (u, v) plane, and the deviation of the incoming wavefront from a plane wavefront at this aperture is described by $\Delta(u, v)$. It is assumed that the angular extent of the object is sufficiently small that all rays from the object to a given point on the aperture plane experience the same deviation in optical path length, i.e., the object is smaller than the aplanatic patch size. For a plane wavefront, the image formed by the telescope, $I(x, y)$, is a true mapping of the object, i.e.,

$$I_1(x, y) = I_0(x/F, y/F)$$

if the telescope is assumed to be perfect.

If the wavefront is distorted, the rays of light, which are perpendicular to the wavefront, are deviated by small angles of $[\partial\Delta(u, v)/\partial u]$ and $[\partial\Delta(u, v)/\partial v]$ in the x and y directions, respectively. Divide the aperture into a number of equal square segments of area ΔA_{ij} centered on the coordinates u_i and v_j. Also, make the elements sufficiently small that the gradient of the wavefront is essentially constant over each element. Each of these elements of the telescope aperture then forms a complete image that is shifted by $[\partial\Delta(u, v)/\partial u]_{ij}F$ in the x direction and by $[\partial\Delta(u, v)/\partial v]_{ij}F$ in the y direction. The elemental image formed by each ΔA_{ij} is

$$\Delta I_1^{ij}(x, y) = I_0\left[\frac{x}{F} - \left(\frac{\partial\Delta(u, v)}{\partial u}\right)_{ij}, \frac{y}{F} - \left(\frac{\partial\Delta(u, v)}{\partial v}\right)_{ij}\right]\frac{\Delta A_{ij}}{A} \qquad (1)$$

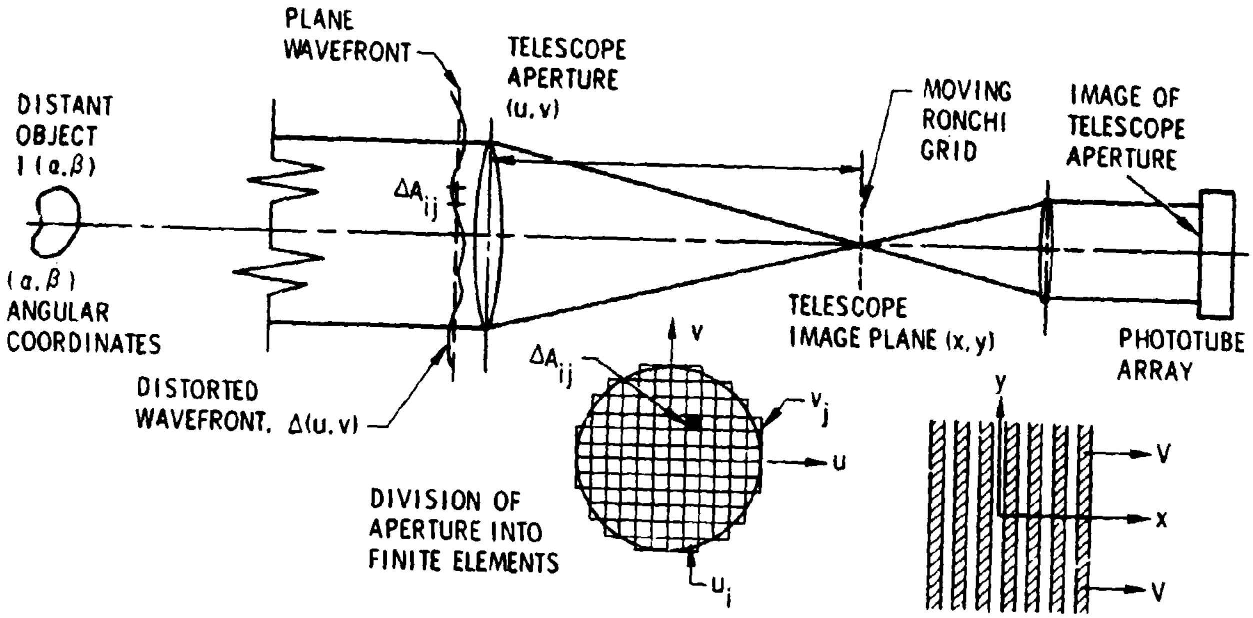

Figure 1. Telescope and Wavefront Sensor

and the complete image is a superposition of all ΔI_1^{ij}, i.e.,

$$I_1(x, y) = \sum_i \sum_j \Delta I_1^{ij}(x, y)$$

This is a noncoherent point of view for which it is assumed that the intensities of the elemental images add together without interference. This summation, however, could just as well be a sum of the complex amplitudes. The signals on the photodetectors or image sensors are then proportional to the absolute value of the square of the amplitudes. The gradients of the wavefront distortion in the x direction are determined by the use of a Ronchi grid, with bars oriented parallel to the y axis, moving in the x direction. The gradients of the distorted wavefront in the y direction are measured by a second moving Ronchi grid oriented at a right angle to the first. The second Ronchi grid is not shown in Fig. 1. The light transmitted through the grid is collected by a lens that projects an image of the telescope aperture onto a matrix of phototube detectors.

The operation of the wavefront sensor can be explained by considering only one grid, which can be defined by an aperture function h(x), such that

$$\begin{aligned} h(x) &= 1 \quad \text{for} \quad -d/4 \le x < d/4 \\ h(x) &= 0 \quad \text{for} \quad -d/2 \le x < -d/4 \quad \text{and} \quad d/4 \le x < d/2 \end{aligned} \qquad (2)$$

where h(x) is an even periodic function with a period d. The grid is translated with a velocity V in the x direction such that the transmission of the grid becomes h(x - Vt). Assume that d is made larger than the image size so the image is transmitted through only one slit at a time. The moving grid function is shown in Fig. 2.

The matrix of photodetectors corresponds to the square elements of the telescope aperture such that each photodetector measures the light

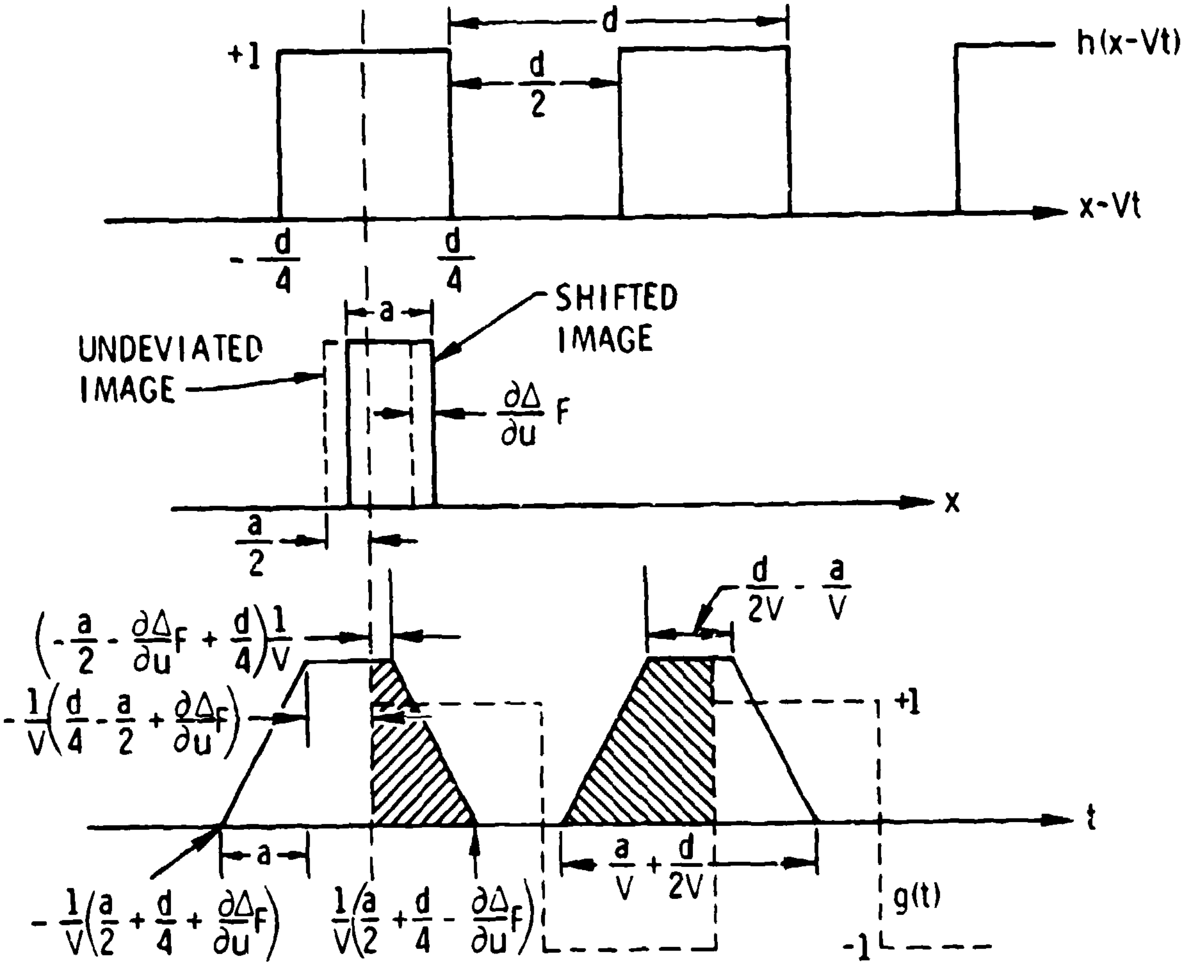

Figure 2. Transmitted Light for a Square Image Smaller than the Grid Opening

transmitted through a given element ΔA_{ij} of the aperture. The light incident on a given photodetector is given by

$$w_x^{ij} = \iint_I \Delta I_I^{ij}(x, y) h(x - Vt)\, dx\, dy \qquad (3)$$

where the integral is taken over the image area. With the use of Eq. (1), we obtain

$$w_x^{ij} = \frac{\Delta A_{ij}}{A} \iint_I I_0 \left[\frac{x}{F} - \left(\frac{\partial \Delta (u, v)}{\partial u} \right)_{ij} , \ \frac{y}{F} - \left(\frac{\partial \Delta (u, v)}{\partial u} \right)_{ij} \right] h(x - Vt)\, dx\, dy \qquad (4)$$

The problem is to extract the value of the wavefront gradient with a simple data processing scheme. Since h(x) was chosen to be an even function, we multiply $w_x^{ij}(t)$ by an odd function and integrate over one period. Define the function g(t) to be a periodic square wave, i.e., let

$$g(t) = +1 \quad \text{for} \quad 0 \le t < d/2v$$

$$g(t) = -1 \quad \text{for} \quad -d/2v \le t < 0$$

The integral over one period is

$$W_x^{ij} = \int_T w_x^{ij}(t)\, g(t)\, dt \qquad (5)$$

This integral is directly proportional to the gradient of the distorted wavefront with some restrictions.

A simple way to demonstrate the above assertion and to determine any restrictions is to consider the case of a square image of size a, such that $a < d/2$.

$$I_1(x) = I_0 \quad \text{for} \quad -a/2 \leq x \leq a/2$$

$$= 0 \quad \text{for} \quad x < -a/2 \quad \text{and} \quad x > a/2$$

The light transmitting through the grid is the convolution of the image distribution and the traveling grid function. From Fig. 2 it can be seen that the transmitted light is

$$w_x^{ij}(t) = I_0 \frac{\Delta A_{ij}}{A} \int_{-\frac{a}{2} + \frac{\partial \Delta}{\partial u} F}^{\frac{a}{2} + \frac{\partial \Delta}{\partial u} F} h(x - Vt)\, dx \tag{6}$$

The result of the integration for this case is a train of trapezoidal pulses as shown in Fig. 2. This temporal function is then multiplied by g(t) and integrated over one period

$$W_x^{ij} = \int_T w_x^{ij}(t)\, g(t)\, dt$$

This is done graphically in Fig. 2, and the result is

$$W_x^{ij} = -2I_0 \frac{\Delta A_{ij}}{A} \left(\frac{\partial \Delta(u, v)}{\partial u} \right)_{ij} \frac{F}{V} \tag{7}$$

Therefore, the integral over one period is indeed proportional to the gradient of the distorted wavefront. If the gradient is zero, the integral over one period is zero. In order to extract the wavefront gradient, it is necessary to divide by the integral of the transmitted light over one period, namely

$$S_x^{ij} = \int_T w_x^{ij}(t)\, dt$$

which, for this case, is

$$I_o \frac{\Delta A_{ij}}{A} \frac{d}{2V}$$

Then

$$\left(\frac{\partial \Delta(u,v)}{\partial u}\right)_{ij} = - \frac{d}{4F} \frac{W_x^{ij}}{S_x^{ij}} \tag{8}$$

The analysis becomes more complex if the flat top of the trapezoid does not extend beyond $t > 0$, ie, if

$$\frac{d}{4} - \frac{a}{2} - \left(\frac{\partial \Delta(u,v)}{\partial u}\right)_{ij} F \leq 0$$

In the latter case, quadratic terms are introduced, and the sensitivity is decreased.

Extension of the above analysis to an object of arbitrary shape can be made graphically with the information given in Fig. 3. The preceding analysis still holds if the transmitted pulse is approximately symmetric, and if the shifted image remains within the size of the grid opening at $t = 0$, viz, $-d/4 \leq x \leq d/4$.

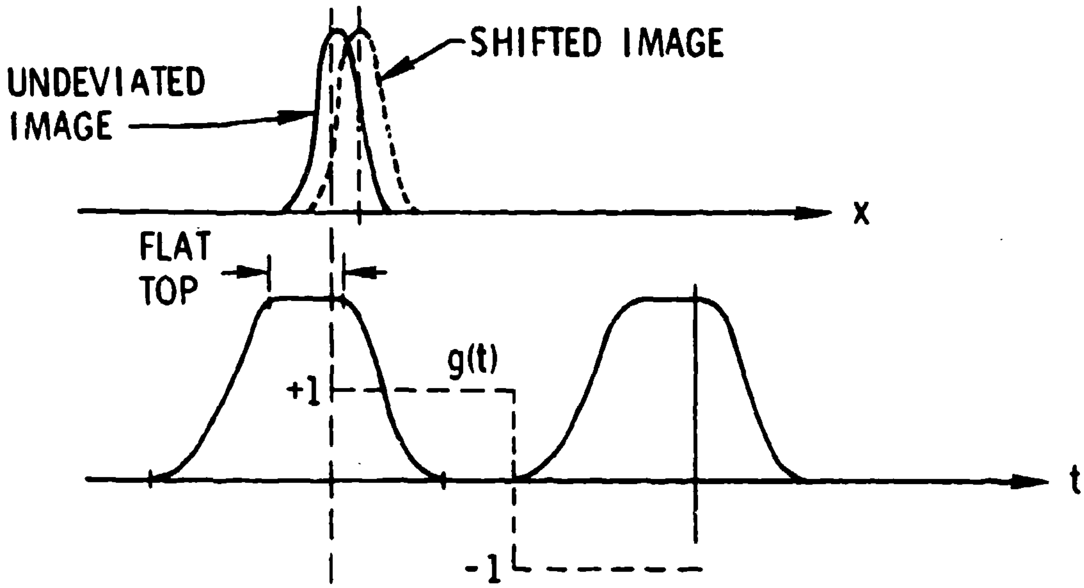

Figure 3. Arbitrary Shape Image

III. SUMMARY AND CONCLUSIONS

It has been shown that the operation of a moving grid wavefront sensor can be explained by a simple geometric optics analysis. The gradient of the distorted wavefront at any given aperture location can be extracted by multiplying the transmitted light by an odd square wave function and integrating over one period. A square object was used as an example to demonstrate the operation and to determine the approximate limits of operation.

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APPENDIX. EFFECTS OF DIFFRACTION IN THE WAVEFRONT SENSOR

The validity of the geometric analysis of the moving grid wavefront sensor for any specific case requires that the effects of diffraction on the operation of the sensor be small. The most important effect of diffraction is the spreading of light from one element of the aperture onto photodetectors associated with adjacent elements. This diffraction effect is caused by the presence of the Ronchi grid in the image plane. It is therefore necessary to calculate the fraction of light diffracted outside the geometric image of the aperture element and to determine the spatial period of the Ronchi grid for which this diffraction effect is acceptably small.

For a telescope of diameter D and focal length F the diameter of the point spread function, the Airy disc, at the focal plane is

$$\frac{2.44\lambda}{D} F = 1.342 \times 10^{-6} \frac{F}{D}$$

for a wavelength of 0.55 μm. Now, mask off the telescope aperture except for one square element of size a, where a is an integral fraction of the telescope diameter. The amplitude point spread function from this element is a sinc pattern in both the x and y directions

$$\Delta U(x, y) = a^2 \operatorname{sinc}\left(\frac{ak}{2F} x\right) \operatorname{sinc}\left(\frac{ak}{2F} y\right)$$

where $k = 2\pi/\lambda$.

The size of the central lobe of this diffraction pattern is $(2\lambda/a)F$. If N is the number of elements across the telescope diameter, then the size of this pattern is approximately N times the size of the Airy disc. If a screen or image sensor were placed at the focal plane of the telescope, all of the images from the individual elements would add together coherently to produce the

Airy disc; but in the absence of a recording medium, all of these images can be considered independently because of superposition.

Figure 1 shows how the aperture elements are imaged onto a phototube array by a lens placed behind the image plane of the telescope. In the absence of the Ronchi grid, the image of the telescope aperture formed by this lens would be quite sharp and would be limited only by the diffraction limit of this lens. The Ronchi grid, however, is a restrictive aperture that causes significant diffraction and spreading of the telescope aperture image. The geometric analysis determined that the aperture width of the Ronchi grid should be larger than the image size. A reasonable object size is 10 μrad, which is smaller than the usual size of the aplanatic patch. The period of the Ronchi grid should then be $d > 2 \times 10^{-5}F$. Let $d = 2.5 \times 10^{-5}F$. The image of the aperture element formed on the phototube array is approximately the fourier transform of the product of the Ronchi grid function and the amplitude point spread function of the square aperture element. Define the grid aperture by

$$g(x, y) = 1 \ \text{from} \ -d/4 \quad x \quad d/4 \ \text{and}$$

$$g(x, y) = 0 \ \text{for} \ x < -d/4 \ \text{and for} \ x < d/4$$

A single aperture has been assumed instead of a periodic function for $g(x, y)$; this can be accomplished by placing a slit aperture of width d in front of the Ronchi grid. We can therefore avoid the complex effects of partially coherent light transmitted through adjacent grid openings. The complex amplitude of the light just after transmission through the Ronchi grid is then $\Delta U(x, y)g(x, y) = \Delta U'(x, y)$. The image of the aperture on the plane of the phototype array, which is designated the (ξ, η) plane, is the fourier transform of $\Delta U'(x, y)$. This fourier transform is the convolution of the fourier transform of $\Delta U(x, y)$, which is just the geometric image of the square aperture element, and the fourier transform of $g(x, y)$. The latter is given by

$$G(\xi, \eta) = \int_{-d/4}^{d/4} e^{i\frac{k}{f}(x, \xi)} \, dx = \frac{d}{2} \operatorname{sinc}\left(\frac{kd}{4f}\right)\xi$$

The size of the central band (between the first zeroes) of this sinc function is $4\lambda f/d = 0.088\ f/F$ (meters), and the size of the geometric image is af/F (meters). A reasonable size for an aperture element is the coherence width of the light transmitted through the atmosphere, which varies from 5 to 15 cm. We therefore choose a value of 0.1 m for the size of an aperture element. In order to graphically represent the diffraction spreading, in Fig. 4, we plot the convolution of the sinc^2 function given above with a square element for two cases. The first case is for equal widths, which is approximately the preceding case, and the second is for an element image size twice that of the central lobe of the sinc^2 function, which is equivalent to doubling the period of the Ronchi grid. A graphical integration shows that approximately 20% of the light is diffracted to adjacent elements for the first case, and 12% of the light is diffracted out of the geometric image in the second case. An analysis of the effect of this diffracted light on system operation should be made to determine the acceptable level of light spilled into adjacent elements. It is clear, however, that a wider grid spacing is preferable. The diffraction in the first case may well be marginal or even unacceptable.

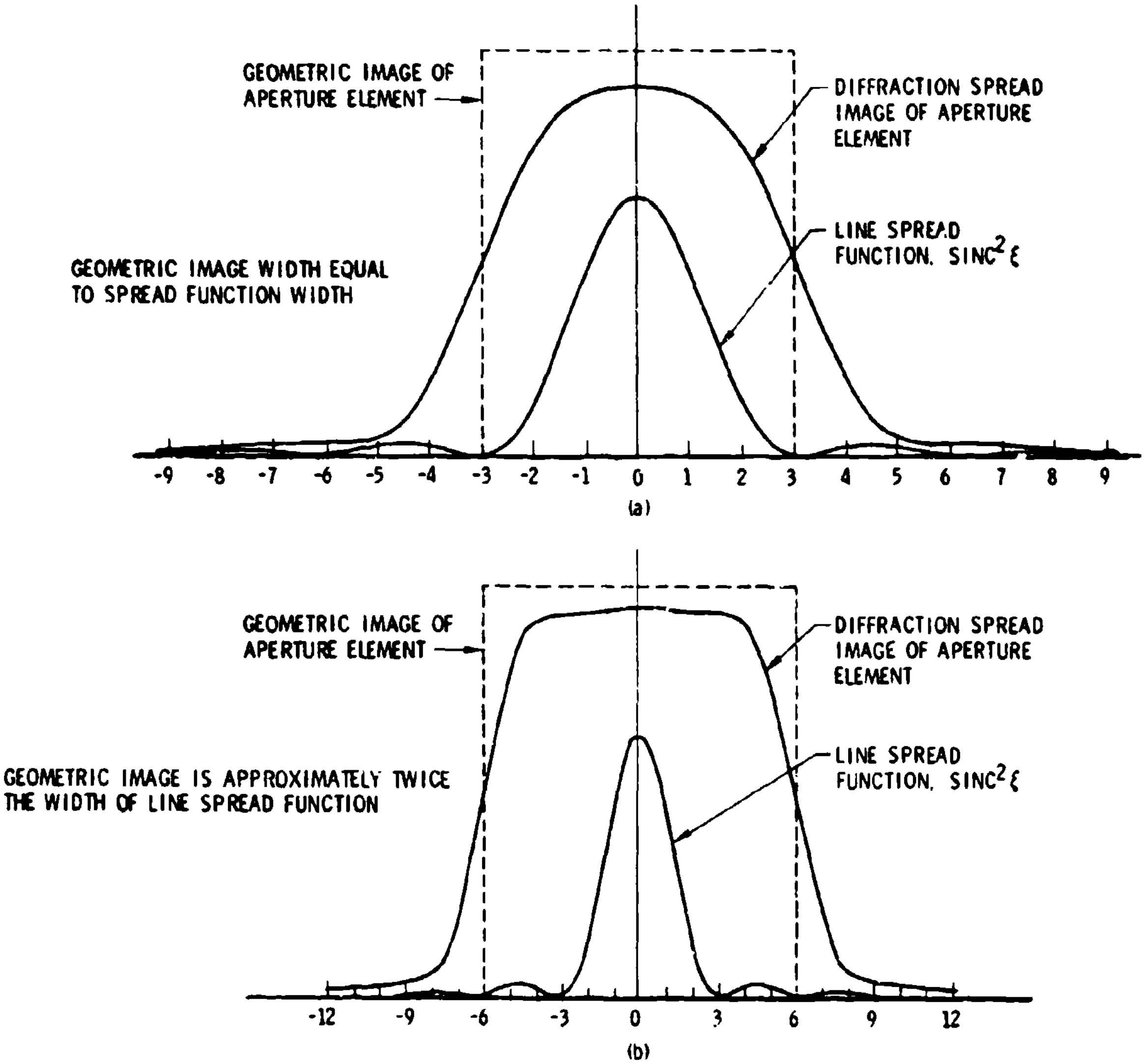

Figure 4. Diffraction Spreading of Image of Aperture Element