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DESIGN AND ANALYSIS OF BIFURCATED TWIN DIELECTRIC SLAB LOADED RECTANGULAR WAVEGUIDE DUAL FREQUENCY ARRAY ELEMENTS

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**Title:** Design and Analysis of Bifurcated Twin Dielectric Slab Loaded Rectangular Waveguide Dual Frequency Array Elements

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**Abstract:**
The design and analysis of a unique dual frequency array element and three probe element exciter for aperture sharing at two widely separated frequency bands is presented. The bifurcated twin dielectric slab loaded rectangular waveguide element simultaneously supports a single low frequency band phase center and four independently controllable high frequency phase centers, resulting in the formation of an independent radiating beam in each band. The principle element design objective is to minimize the...
20. Abstract (Cont.)

...element count while maximizing the rejection of high frequency grating lobes. It is shown that the slab loaded element results in 21 percent fewer phase and frequency control per unit array area than an alternate diplexed wide band element dual frequency concept for a scanning array operated over 15 percent bands centered at 4 and 8 GHz.

The radiation and coupling properties of the array element are developed from a scattering formulation of the feedguide - free space discontinuity for the fully excited infinite array. Comparison of theoretical performance for triangular and rectangular grid configurations shows that considerable improvement in high frequency grating lobe rejection is obtained from the triangular lattice.

To characterize the transverse aperture fields, a complete modal description of propagation in the inhomogeneously loaded guide is obtained through a component-by-component comparison of the degenerate eigenfunctions of the structure.

Simulator measurement of an element designed to provide 60° scan coverage over 15 percent bands centered at 4 and 8 GHz shows excellent agreement with theoretical predictions for main beam gain loss in the measurement bands 4. - 4.32 GHz and 7.36 - 8.08 GHz. Over the remainder of the operating bands, the agreement is assumed to be equally close.

Initial experimental design data for unidirectional stripline fed notch is presented. The notch probe is shown to result in better than 2:1 mismatch over greater than 10 percent measurement bands when looking into load terminated twin slab feedguides, and greater than 50 dB probe isolation over the 4 GHz band. Over the 8 GHz, probe isolation is approximately 12 dB and remains a problem for future design efforts.
1. This report is the Final Report of Contract F19628-75-C-0197. It covers the analytical and experimental investigations of the bifurcated twin dielectric slab loaded rectangular waveguide dual frequency array element. The report describes analytical studies of the infinite array and documents operation at two 15% frequency bands centered at 4GHz and 8GHz. In addition, the report presents the experimental results and examination of the strip-line fed notch exciter which is used at both frequency ranges.

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# TABLE OF CONTENTS

1. INTRODUCTION AND SUMMARY ........................................... 1

2. ANALYSIS OF INFINITE PHASED ARRAYS OF BIFURCATED TWIN DIELECTRIC SLAB LOADED RECTANGULAR WAVEGUIDE DUAL FREQUENCY ELEMENTS
   2.1 Active Array Element Pattern .................................. 13
   2.2 Numerical Results .............................................. 29
   2.3 Comparison with Limiting Cases ................................ 42
   2.4 Convergence of Numerical Results ............................. 56

3. PROPAGATION CHARACTERISTICS OF TWIN DIELECTRIC SLAB LOADED RECTANGULAR WAVEGUIDE
   3.1 Mode Functions ............................................... 60
   3.2 Mode Spectrum ................................................ 81

4. ARRAY APERTURE DESIGN .............................................. 98
   4.1 Aperture Design Trade-Offs .................................... 100
   4.2 Experimental Evaluation of the Bifurcated Twin Dielectric Slab Loaded Rectangular Waveguide Dual Frequency Element ........ 122

5. ELEMENT EXCITER DESIGN ............................................ 134
   5.1 Exciter Concept ............................................... 134
   5.2 Experimental Investigation of the Stripline Fed Flared Notch Exciter ........................................... 138

6. CONCLUSIONS .......................................................... 148
# List of Illustrations

1. Bifurcated Twin Dielectric Slab Loaded Rectangular Waveguide Element in Triangular Grid ... 3
2. Symmetric Inhomogeneously Loaded Rectangular Waveguide ... 5
3. Dual Frequency Element ... 9
4. Lattice Definitions for Rectangular Grids ... 11
5. Lattice Definitions for Triangular Grid - Either Frequency Band ... 12
6. Network Representation of Unit Cell Discontinuity ... 14
7. Reflected Power in Upper and Lower Element Halves ... 27
8. Grating Lobe Diagrams for WR187 and WR137 Elements in Rectangular and Triangular Configurations ... 33
9. WR187 Element Power Transmission Coefficient - Rectangular Grid, Low Frequency (=2.5GHz) ... 34
10. WR187 Element Power Transmission Coefficient - Triangular Grid, Low Frequency (=2.5GHz) ... 35
11. WR137 Element Power Transmission Coefficient - Rectangular Grid, Low Frequency (=4 GHz) ... 36
12. WR137 Element Power Transmission Coefficient - Triangular Grid, Low Frequency (=4 GHz) ... 37
13. Effects of Grating Lobe Correction ... 39
14. Grating Lobe Levels Obtained from Rectangular and Triangular Grid Configurations of WR137 Elements ... 41
15. Comparison of Main Beam Power Levels for Rectangular and Triangular Grid Configurations of WR137 Elements - H Plane ... 43
16. Comparison of Main Beam Power Levels for Rectangular and Triangular Grid Configurations of WR137 Elements - E Plane ... 44
17. Rectangular Grid of Rectangular Waveguides ... 46
18. Comparison of Exact (8) and Approximate Modal Solutions for Active TE_{10} Reflection Coefficients - Thin Walled Square Elements H-Plane \( D_x/\lambda = .5714 \).
List of Illustrations (Continued)

19. Comparison of Exact (8) and Approximate Modal Solutions for Active $TE_{10}$ Reflection Coefficient - Thin Walled Square Elements H-Plane $D_x/\lambda = 0.6205$.

20. Comparison of Exact (8) and Approximate Modal Solutions for Active $TE_{10}$ Reflection Coefficient - Thin Walled Square Elements H-Plane $D_x/\lambda = 0.6724$.

21. Comparison of Modal Solutions for Active $TE_{10}$ Reflection Coefficient - Thick Walled Square Elements - H-Plane $D_x/\lambda = 0.5714$, $t = 0.1D_x$.

22. Comparison of Modal Solutions for Active $TE_{10}$ Reflection Coefficient - Thick Walled Square Elements - H-Plane $D_x/\lambda = 0.6205$, $t = 0.1D_x$.

23. Comparison of Modal Solutions for Active $TE_{10}$ Reflection Coefficient - Thick Walled Square Elements - H-Plane $D_x/\lambda = 0.6724$, $t = 0.1D_x$.

24. Comparison of Modal Solutions for H-Plane Element Pattern When Aperture Field is Approximated by $TE_{10}$ Mode.

25. Comparison of Modal Solutions for H-Plane Element Pattern When Aperture Field is Approximated by $TE_{10} + TE_{20}$ Modes.


27. Equivalent Transmission Line Representation of Wave Propagation in x.

28. $LSM_{nm}$ Mode Functions.

29. $LSE_{nm}$ Mode Functions.

30. $e_{y10}(x,y)$ Distribution for WR187 Bifurcated Guide With .250" Thick $\varepsilon_r = 9$ Loading. 2.5 GHz.

31. $e_{y10}(x,y)$ Distribution for WR187 Bifurcated Guide With .250" Thick $\varepsilon_r = 9$ Loading. 6 GHz.

32. $e_{y10}(x,y)$ and $e_{y20}(x,y)$ Distributions for WR187 Bifurcated Guide With .250" Thick $\varepsilon_r = 9$ Loading. 6 GHz.

33. $e_{y10}(x,y)$ and $e_{y20}(x,y)$ Distributions for WR187 Bifurcated Guide With .250" Thick $\varepsilon_r = 5$ Loading. 6 GHz.
List of Illustrations (continued)

34. Disposition of Roots of $D(K,K_e)$ Along Real $K_e$ Axis • • • 84

35. LSE Dispersion Diagram for Bifurcated WR187 Guide With • 85
   250" Thick $\epsilon_r = 9$ Loading

36. LSE Dispersion Diagram for Bifurcated WR137 Guide With • 89
   250" Thick $\epsilon_r = 3$ Loading

37. LSE Dispersion Diagram for Bifurcated WR137 Guide With, • 90
   .250" Thick $\epsilon_r = 5$ Loading

38. LSE Dispersion Diagram for Bifurcated WR137 Guide With • 91
   .250" Thick $\epsilon_r = 7$ Loading

39. LSE Dispersion Diagram for Bifurcated WR137 Guide With • 92
   .250" Thick $\epsilon_r = 9$ Loading

40. LSE Dispersion Diagram for Bifurcated WR137 Guide With • 93
   .063" Thick $\epsilon_r = 5$ Loading

41. LSE Dispersion Diagram for Bifurcated WR137 Guide With • 94
   .125" Thick $\epsilon_r = 5$ Loading

42. LSE Dispersion Diagram for Bifurcated WR137 Guide With • 95
   .188" Thick $\epsilon_r = 5$ Loading

43. LSE Dispersion Diagram for Bifurcated WR137 Guide With • 96
   .375" Thick $\epsilon_r = 5$ Loading.

44. Equilateral Triangle Lattice Configuration Which • • • 101
    Minimizes Element Count @ 4 GHz.

45. Triangular Grid Configuration for Dual Frequency Operation • 108
    Operation over 16% Bands Centered at 4 GHz and 8 GHz

46. E- and H- Plane Performance of Designed Element, • • • 110
    $f = 3.68$ GHz

47. E- and H- Plane Performance of Designed Element, • • • 111
    $f = 4.0$ GHz

48. E- and H- Plane Performance of Designed Element, • • • 112
    $f = 4.32$ GHz

49. Propagating Beam Power Levels - H-Plane Scan, • • • 113
    $f = 7.36$ GHz

50. Propagating Beam Power Levels - H-Plane Scan, • • • 114
    $f = 8.0$ GHz

51. Propagating Beam Power Levels - H-Plane Scan, • • • 115
    $f = 8.64$ GHz
List of Illustrations (continued)

52. Propagating Beam Power Levels - E-Plane Scan, . . . . . . . 116
    $f = 7.36 \text{ GHz}$

53. Propagating Beam Power Levels - E-Plane Scan, . . . . . . . 117
    $f = 8.0 \text{ GHz}$

54. Propagating Beam Power Levels - E-Plane Scan, . . . . . . . 118
    $f = 8.64 \text{ GHz}$

55. Waveguide Simulator for Designed Element . . . . . . . . . . 119

56. Simulator Imaging of Element Section . . . . . . . . . . . . 123

57. Measured Simulator Port, Impedance 4.0 - 4.32 GHz . . . . . 124
    Sampled in 80 MHz Increments

58. Comparison of Measured and Predicted Reflections . . . . . . 127
    Coefficient Magnitude at the Simulator Port, 4.0 - 4.32 GHz

59. Port Definitions for 7.32 - 8.08 GHz Simulator . . . . . . . 129

60. Measured Simulator Port Impedance 7.32 - 8.64 GHz . . . . . 131
    Sampled in 160 MHz Increments

61. Comparison of Measured and Predicted Reflection . . . . . . 133
    Coefficient Magnitude at the Simulator Port, 7.32 - 8.00 GHz

62. Stripline Fed Notch Exciter for Twin Dielectric Slab . . . . 135
    Loaded Rectangular Waveguide Dual Frequency Array Element

63. Basic Notch Exciter . . . . . . . . . . . . . . . . . . . . . . . . . 137

64. Baseline Notch Exciter Design . . . . . . . . . . . . . . . . . . 140

65. Measured VSWR for the P5 Exciter, 4 to 6 GHz . . . . . . . . 142

66. Measured VSWR for the P1 Exciter, 7.5 to 8.5 GHz . . . . . . 144

67. Measured Probe Isolation, 3.5 to 8.5 GHz . . . . . . . . . . . 146
List of Tables

1 Dimensions (in.) of WR187 and WR137 Elements ........... 31
2 Dispersion Relations for Inhomogeneously Loaded Rectangular Waveguide ........... 82
3 Prescribed Array Performance ........... 99
4 Performance of Dual Frequency Element ........... 105
5 Open Circuit Stub Lengths for Experimental Exciters ........... 141
1. INTRODUCTION AND SUMMARY

This report summarizes the analytical and experimental investigations of the infinite array radiation and coupling properties of bifurcated twin dielectric slab loaded rectangular waveguide dual frequency array elements conducted under Contract F19628-75-C-0197. Specifically, the report presents:

. The complete analysis of the element in infinite array configurations.

Theoretically determined element/grid design trade-off conclusions, leading to a proposed configuration for operation over 15% bands centered at 4GHz and 8GHz.

. Experimental verification of the analytical results and the examination of a unique strip-line fed notch antenna mode exciter.

. The computational details and computer programs developed during the study.
The bifurcated twin dielectric slab loaded rectangular waveguide dual frequency array element shown in Figure 1 is a unique concept for providing simultaneous aperture usage at two widely separate frequency bands. At low frequency both upper and lower halves of the waveguide are excited in-phase with equi-amplitude signals. For moderate slab loading (assuming relatively thin slabs) this array will behave similarly to a rectangular waveguide array excited in the \( \text{TE}_{10} \) mode with scan behavior associated with these elements in the basic lattice (either rectangular or triangular). At the high frequency the first odd and even half-waveguide modes can be independently specified such that four phase centers are defined; within a single low frequency cell the fields are confined predominately to the slab regions.

For practical array designs, the low and high frequency lattice cells are identical, and the principal element/grid design trade-off is to minimize the number of phase centers (or phase shifters) while maintaining main beam purity and gain over a specified scan volume, particularly in the high frequency band. Consequently the lattice is selected such that element spacing is not optimum for either band, but presents the best compromise of element count versus grating
Figure 1. Bifurcated Twin Dielectric Slab Loaded Rectangular Waveguide Element in Triangular Grid
lobe free scan volume in the high frequency band. At scan conditions for which high frequency grating lobes are entering or present, the multimode aperture excitation is adjusted (slightly) to cancel the grating lobe. These considerations are treated fully in Section 2 which gives details of the analysis of array performance, and section 4 which summarizes design tradeoffs and experimental results.

The application of the twin dielectric slab loaded element to dual frequency aperture sharing follows from the unique propagation properties of the inhomogeneously loaded structure. The symmetric loaded guide, shown in Figure 2, is inherently wideband. At sufficiently low frequency, a single guide mode (the LSE\textsubscript{10} mode) propagates and has an electric field distribution somewhat broader than the homogeneously loaded guide TE\textsubscript{10} distribution. As frequency is increased, the LSE\textsubscript{20} mode enters, having electric field distribution similar to the TE\textsubscript{20} distribution of the homogeneously loaded guide. Concurrently, the LSE\textsubscript{10} distribution begins to develop a minimum along the guide mid-plane. At sufficiently high frequency, the distributions become essentially identical, except for symmetry about the mid-plane, and the ratio of longitudinal wavenumbers approaches unity.
Figure 2. Symmetric Inhomogeneously Loaded Rectangular Waveguide
By appropriate selection of guide, slab permittivity, and operating points such that only the two modes propagate, the guide will simultaneously support a single propagating low frequency mode with phase center at the midplane, and a conglomerate high frequency distribution with two independent phase centers at (roughly) the slab centers. The analysis of propagation in the inhomogenously loaded guide is given in Section 3.

In general, the dispersion in the inhomogeneously loaded guide is not linear in frequency. Consequently, the use of a bidirectional exciter requires load terminations at the back of the guide to ensure the proper aperture field phase at both frequencies, and results in a 3 dB power loss. This difficulty is alleviated by a unique unidirectional exciter consisting of three stripline fed flared notch antennas \((2, 3, 4)\) inserted into the back of the feed-guide in such a manner as to provide a minimum of 25 dB frequency band isolation. Preliminary experimental investigation of this exciter design concept was begun during this study, and is discussed in Section 5.

Four appendices are included. Appendices A, B, and C give the details of the analysis. Explicit expansions of modal coupling coefficient integrals are given in Appendix A. In Appendix B, the derivation of the differential equations
relating feedguide modal fields is given. And in Appendix C, explicit expressions for feedguide mode orthonormalization integrals are given. The remaining appendix gives complete listings of all programs required to reproduce the numerical results given in this report.

A time dependence $e^{j\omega t}$ is assumed throughout.
2.0 ANALYSIS OF INFINITE PHASED ARRAYS
OF BIFURCATED TWIN DIELECTRIC SLAB LOADED RECTANGULAR
WAVEGUIDE DUAL FREQUENCY ELEMENTS

In this section, the formal solution for the radiation
properties of the element in infinite array configuration
is presented. The unique property of the bifurcated twin
dielectric slab loaded rectangular waveguide dual freq-
quency array element is that it possesses five phase centers:
one, associated with the low frequency band operation; and
the remaining four, with a high frequency band. By appropri-
ate exciter design, the element can simultaneously operate
over both bands.

The basic element is shown in Figure 3. Arrays are
formed by stacking these elements in rectangular or tri-
angular grid configuration. The element consists of a
rectangular waveguide bifurcated in the E-plane by a
septum of thickness $\delta$. Outer dimensions are $D_x$ and $D_y$, and
inner dimensions, $A$ and $B'$, where $D_x$ and $A$ are associated
with the $x$ coordinate. Four half height lossless dielectric
slabs of thickness $\delta$ and relative dielectric constant $\varepsilon_r$
are located at distance $\alpha + \delta/2$ (on center) from the narrow
Figure 3. Dual Frequency Element
walls. At low frequency, a phase center is maintained at the element center (i.e., over the septum) by exciting the \( \text{LSE}_{10} \) mode equally in the two half guides. At high frequency, four independent phase centers, at roughly the four slab centers, are formed by appropriately exciting the \( \text{LSE}_{10} \) and \( \text{LSE}_{20} \) modes in each half guide.

When the elements are arrayed in a rectangular grid the element lattice vectors, \( \mathbf{s}_1 \) and \( \mathbf{s}_2 \) are as shown in Figure 4a, provided the septum thickness, \( \delta \), is not equal to \( \delta_y - B'k \). When \( \delta = \delta_y - B'k \), the low frequency lattice is as shown in Figure 4a, and the high frequency lattice is as shown in Figure 4b. When arrayed in a triangular grid, the lattice vectors are defined as in Figure 5, regardless of operating frequency or septum thickness.

In section 2.1, the formal solution for active array element pattern is given. The method of solution is similar to that developed by Lewis, et al \((5)\) for the analysis of a parallel plate array with protruding dielectric. In the present work, the formalism is extended to two dimensional array cells, and the unique dual frequency unit cell geometry is accounted for. In section 2.2 numerical results are presented and particular attention is given to the disposition of high frequency band grating lobes. Discussion of grating lobe suppression is deferred
Figure 4. Lattice Definitions for Rectangular Grids

(a) Septum Thickness, $s = D_{y-B}$; High and Low Frequencies

(b) Septum Thickness $g_s = D_{y-B}$; High Frequency
Figure 5. Lattice Definitions for Triangular Grid - Either Frequency Band
to section 4.1. In section 2.3, numerical results are checked against published data for several limiting geometries.

2.1 Active Array Element Pattern

The active array element pattern is determined from a unit cell formulation of scattering at the feedguide free space interface. The interface is taken as coincident with the z = 0 plane, with the array elements occupying the z < 0 half space. The scattering matrix, $S$, which relates feedguide modal voltages to the modal voltages of the space harmonics, in the manner indicated by the network in Figure 6, is obtained by matching the transverse-to-z fields in the cell across the interface. The field matching is accomplished via Galerkin's method, from which the scattering formalism follows directly. Active array transmission coefficient is then obtained from the network. In the following discussion, the assumed cell configuration is that shown in Figure 4a. The extension of these results to either of the other two cases is straightforward.

For the configuration of Figure 4a, the unit cell perimeter may be taken as coincident with the element outside perimeter. Thus, the unit cell overlays two
Figure 6. Network Representation of Unit Cell Discontinuity
independent aperture regions and the modal representation of total transverse-to-z electric and magnetic fields at 
\( z = 0^- \) is given as

\[
\begin{align*}
E^t(s) &= U(y) \sum_{i>0}^1 V_{i>0} e_i^>(s) + U(-y) \sum_{i>j}^j V_{<j} e_i^<(s) \\
H^t(s) &= U(y) \sum_{i>0}^1 I_{i>0} h_i^>(s) + U(-y) \sum_{i>j}^j I_{<j} h_i^<(s)
\end{align*}
\]

where

\[
U(\xi) = \begin{cases} 
1, & \xi > 0 \\
0, & \xi > 0 
\end{cases}
\]

The subscripts \( \rangle \) and \( \langle \) are used to distinguish the two aperture regions. \( V_{\xi i} \) and \( I_{\xi i} \) are modal voltage and current coefficients, and are related by

\[
I_{\xi i} = Y_{\xi i} V_{\xi i}
\]

where \( Y_{\xi i} \) is the modal admittance of the \( i^{th} \) aperture mode, and the single ordering index \( i \) is used to simplify notation. The mode functions \( e \) and \( h \) are given in section 3.1.
At \( z = 0^+ \), the unit cell guide representation of transverse-to-z fields, over the full cell, is

\begin{equation}
E^+_{t}(s) = \sum_{pqr} V_{apqr} e_{apqr}(s) \tag{5}
\end{equation}

\begin{equation}
H^+_{t}(s) = \sum_{pqr} I_{apqr} h_{apqr}(s) \tag{6}
\end{equation}

where (6)

\begin{equation}
h_{apqr}(s) = z_o x e_{apqr}(s) \tag{7}
\end{equation}

\begin{equation}
e_{apqr}(s) = \frac{e^{-j\Delta t^* s}}{\sqrt{|k_{tpq}|}} \left[ (2-r)k_{tpq} + (r-1)k_{tpq} x z_o \right] \tag{8}
\end{equation}

\begin{equation}
r = \begin{cases} 
1, & \text{for E modes with respect to } z \\
2, & \text{for H modes with respect to } z 
\end{cases} \tag{9}
\end{equation}

\begin{equation}
k_{tpq} = k_{xpq} x + k_{ypq} y = k \sin \theta_o \left( \cos \phi_o x + \sin \phi_o y \right) + pt_1 + qt_2 \tag{10}
\end{equation}

\begin{equation}
k_{xpq} = k \sin \theta_o \cos \phi_o + pt_1 x + qt_2 x \tag{11}
\end{equation}
\[(12) \quad k_{ypq} = k \sin \theta_0 \sin \phi_0 + p t_1 y + q t_2 y\]

\[(13) \quad t_i \cdot s_j = 2 \pi \delta_{ij}, \quad i, j = 1, 2\]

\[(14) \quad c = |s_1 \times s_2|\]

\(\delta_{ij}\) is Kronecker's delta, \(s_j\) are the lattice vectors, as shown, for example, in Figures 4 a, b, and 5, and \(\theta_0\) and \(\phi_0\) are the spatial look angles. The mode function normalization is taken such that \(V_{apqr} I^{*}_{apqr}\) is power, and such that the modal voltage and current are related by

\[(15) \quad I_{apqr} = Y_{apqr} V_{apqr}\]

where \(Y_{apqr}\) is a modal admittance, given as

\[(16) \quad Y_{apqr} = \begin{cases} \frac{k_o}{k_{zpq} \eta_o}, & r = 1 \\ \frac{k_{zpq}}{k_o \eta_o}, & r = 2 \end{cases}\]

\(k_o = 2\pi/\lambda\) is the free space wavenumber and \(\eta_o\) is the free space impedance 377 ohms. The indices \(pqr\) and \(pq\), which appear explicitly in equations (5) through (12) will henceforth be replaced by the single index \(\alpha\).

Matching transverse fields at the aperture plane of the unit cell gives
\[ \Sigma V_{a_0-e_{a_0}}(s) = \begin{cases} U(y) \Sigma V_{i \geq i} (s) + U(-y) \Sigma V_{j \leq j} (s), & \text{in the aperture} \\ 0, \text{elsewhere} \end{cases} \]

and

\[ \Sigma I_{a_0-h_{a_0}}(s) = U(y) \Sigma I_{i \geq i} (s) + U(-y) \Sigma I_{j \leq j} (s), \]

in the aperture.

Approximate solutions for the parameters of the network in Figure 6 are obtained when the modal series are truncated. As a result of the truncation, the continuity equations (i.e., equations (17) and (18)) can no longer be exactly satisfied, and vector error terms \( \Delta_1 \) and \( \Delta_2 \) must be inserted to restore the equality. These error terms are given as

\[ \Delta_1 = \begin{cases} \Sigma V_{a_0-e_{a_0}}(s) - U(y) \Sigma I_{i \geq i} (s) + U(-y) \Sigma V_{j \leq j} (s), & \text{in the apertures} \\ \Sigma V_{a_0-e_{a_0}}(s), \text{elsewhere} \end{cases} \]

and

\[ \Delta_2 = \Sigma I_{a_0-h_{a_0}}(s) - U(y) \Sigma I_{i \geq i} (s) - U(-y) \Sigma I_{j \leq j} (s), \]

in the apertures.
It is now required that the projections of $A_1$ and $A_2$ onto the appropriate modal spaces be zero.

The domain of definition of $E_t^+(s)$ is over the entire unit cell, and the domain of $E_t^-(s)$ may be artificially extended over the metallic portions of the cell. Thus, the domain of $A_1$ is the unit cell, and the modal subset spanning the space are the $h_a(s)$. Requiring orthogonality of $A_1$ to the $h_a(s)$ and performing the inner products over the cell results, after manipulation, in

\[(21) \quad V_a = E^> V > + E^< V <\]

where $E^>$ is a matrix of coupling coefficients, the elements of which are given as

\[(22*) \quad \hat{E}^>_{\sigma, i} = \int_{1/2 \text{ cell } \xi} dA_{\sigma i}(s) \cdot (h^*(s)xz_0)\]

The elements of the column vectors $V >$ are the feedguide modal voltages.

The domain of definition of $A_2$ is over the aperture only. Therefore, the appropriate basis spanning this space is formed by the concatenation of the truncated modal

---

*Complete expressions for $\hat{E}^>_{\sigma, i}$ are given in Appendix A*
sets which individually span only one or the other of the aperture region spaces. Such a basis may be represented by the partitioned vector $\mathbf{B}(s)$, given as

$$
\mathbf{B}(s) = \begin{pmatrix}
U(y)e_i^>(s) \\
\vdots \\
U(-y)e_j^<(s) \\
\vdots
\end{pmatrix}
$$

(23)

Requiring orthogonality of $A_2$ on the space spanned by $\mathbf{B}(s)$ results in

$$
I = \begin{pmatrix}
I_{>i} \\
\vdots \\
I_{<j}
\end{pmatrix} = \begin{pmatrix}
\lambda_{>i}^> \\
\vdots \\
\lambda_{<j}^<
\end{pmatrix} I_a
$$

(24)

where $\lambda_{>i}$ are submatrices of coupling coefficients, the elements of which are given as

$$
\lambda_{>i}^> = \int dA[h_{ao}(s)xz_o]e_i^>(s) e_i^>(s) \quad \text{1/2 aperture } \gamma
$$

(25)

Since the $e_i^>(s)$ and $e_j^<(s)$ may be artificially extended to individually span the appropriate entire half cell,
It is convenient, then, to define the partitioned vector \( \mathbf{V} \) such that

\[
\mathbf{V} = \begin{bmatrix} \mathbf{V}^\succ \\ \mathbf{V}^\prec \end{bmatrix}
\]

and the partitioned matrix \( \mathbf{E} \) as

\[
\mathbf{E} = (\mathbf{E}^\succ | \mathbf{E}^\prec)
\]

Then, the voltage and current equations, (3-21) and (3-24), respectively, take the form

\[
\mathbf{V}_a = \mathbf{E} \mathbf{V}
\]

\[
\mathbf{I} = \mathbf{E}^* \mathbf{I}_a
\]

where the asterisk denotes conjugation and the t denotes the transpose operation.

The vectors \( \mathbf{V}, \mathbf{V}_a, \mathbf{I}, \) and \( \mathbf{I}_a \) in equations (29) and (30) are total modal voltages and currents. Using the conventions established for the network in Figure 6,
assuming that all external sources are zero (i.e. \( V_a^- = 0 \)), and manipulating equations (29) and (30) results in an expression for feedguide reflected field voltage coefficients, \( V^- \), in terms of the active modal excitations, \( V^+ \), given as

\[
V^- = \{2[Y + \frac{E^*}{\omega} + \frac{E}{\omega}]^{-1}Y-1\}V^+
\]

where \( \frac{1}{\omega} \) is the identity matrix.

Let the scattering matrix of the network be defined by

\[
\begin{pmatrix}
\begin{array}{c}
V_a^-
\end{array}
\end{pmatrix}
= \begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
V^+
\end{pmatrix}
\]

where the scattering blocks have the usual meaning. Then, from equation (31)

\[
S_{11} = 2[Y + \frac{E^*}{\omega} + \frac{E}{\omega}]^{-1}Y-1
\]

and from equation (29)

\[
S_{21} = \frac{E}{\omega}(1 + S_{11})
\]
with \( \mathbf{V}^{-} \equiv 0 \).

In more standard array configurations, the feedguide and aperture are designed for single feedguide mode propagation in a single frequency band. In these configurations, the active array reflection coefficient is simply the one element of \( S_{11} \) corresponding to reflections in the dominant mode. Defining that matrix element as \( T(\theta_0, \phi_0) \), normalized active array element gain pattern is

\[
\tilde{g}_e(\theta_0, \phi_0) = (1 - |\Gamma(\theta_0, \phi_0)| \cos \theta_0
\]

provided no grating lobes have entered real space. Following the entrance of the first grating lobe, it is necessary to track the propagating beams individually, and the relative power in the \( \sigma \)th beam due to the single excited mode (\( i=1 \)) is

\[
P_{\sigma}(\theta_0, \phi_0) = |T_{\sigma}(\theta_0, \phi_0)|^2 \frac{Y_{a0}}{Y_1}
\]

where \( T_{\sigma}(\theta, \phi) \) is the \((\sigma, i)\)th element of the partitioned block \( S_{21} \), and \( \theta_\sigma \) and \( \phi_\sigma \) are the actual location angles of the \( \sigma \)th beam.
Active array element pattern is defined in a different manner for the array of bifurcated twin dielectric slab loaded rectangular waveguides. In this instance, the aperture is always considered to be multimode. In the low frequency range, the LSE$_{10}$ mode (ordered as the $i^{th}$) is excited equally, in amplitude and phase, in both regions of the aperture. Consequently, for an I-mode feedguide aperture field approximation in both upper and lower regions, the power in the main beam, $\sigma=m$, is given, for principle plane scan*, as

\[ P_m(\theta_0, \phi_0) = \frac{1}{2} \left| S^{m,i}_{21} (\theta_0, \phi_0) + S^{m,i+1}_{21} (\theta_0, \phi_0) \right|^2 Y_{am}/Y_i \]

where $S^{m,i}_{21} (\theta, \phi)$ is the $(m,i)^{th}$ element of $S_{21}$, and the $i^{th}$ feedguide mode is the propagating LSE$_{10}$ mode. By the definitions in equations (27), (32), and (34), $S^{m,i}_{21} (\theta_0, \phi_0)$ is the voltage transmission coefficient for coupling from the $i^{th}$ mode in the upper aperture region to the $m^{th}$

Equation (3.37) is strictly valid only for $\theta_0 \neq 0$ and in the principle planes ($\phi=0, \Pi$, or $\phi=\Pi/2, 3\Pi/2$). For all other scan planes (and at broadside), the total power in the main beam is the sum of the powers in the dominant ($p=q=0$) E and H modes.
beam in free space; and $S_{11}^{M,i+I}$ is the voltage transmission from the $i^{th}$ mode of the lower aperture region to the $m^{th}$ beam*.

Active array reflection coefficient is also obtained via superposition. For the multimode aperture configuration, it is necessary to independently track all propagating waves in the feedguide. Hence, for low frequency excitation, the total reflected power in the $\text{LSE}_{10}$ mode of the upper aperture region is given, for any scan angle, as

\begin{equation}
R^>(\theta_0, \phi_0) = \frac{1}{2} | S_{i,i}^i(\theta_0, \phi_0) + S_{i+I,i}^{i+I}(\theta_0, \phi_0) |^2
\end{equation}

For the lower aperture region, the reflected power is

\begin{equation}
R^<(\theta_0, \phi_0) = \frac{1}{2} | S_{i+I,i}^{i+I}(\theta_0, \phi_0) + S_{i+I,i}^{i+I}(\theta_0, \phi_0) |^2
\end{equation}

where $S_{i+I,i}^{i+I}(\theta_0, \phi_0)$ is the voltage scattering coefficient from the $r^{th}$ aperture region mode to the $t^{th}$ aperture region mode.

It is evident from these two equations that the reflected powers in the two regions are not necessarily equal. Indeed, it is found that for low frequency excitation, $R^>(\theta_0, \phi_0)$ equals $R^<(\theta_0, \phi_0)$ only in the $H$ plane of scan.

*It is assumed that the mode ordering in the two aperture regions is the same. This assumption will carry through the remainder of the report.
As an example, reflected power is shown versus $E$ plane scan angle in Figure 7. The element is a WR187 guide with .250" thick slabs of $\varepsilon_r=9$ dielectric located .450", on center, from either narrow wall and with a .032" septum. The operating frequency is 2.5 GHz. There is considerable difference in reflected power between upper and lower regions throughout the scan range .17 < sin $\theta$ < .95. Consequently, low frequency excitation of the upper and lower regions of the element from a common post phase shifter feed point, as is desirable for several low frequency feed concepts, will produce an imbalance at the outputs of the power divider network. Since the impact of this effect on feed and exciter design is beyond the scope of this study, it will be given no further consideration in this report.

At the high frequency band, the element is excited such that four independently controllable phase centers are distributed in the aperture. To maintain this phase center distribution, four propagating modes, two in each region, are excited. The modes are $LSE_{10}$ and $LSE_{20}$. For sufficiently high frequency and dielectric constants, these two modes have nearly equal dispersion. In addition, to a crude approximation, the modal field distributions, $e^{"}_{y_10}$ and $e^{"}_{y_{20}}$ differ
Figure 7. Reflected Power in Upper and Lower Element Halves - E Plane Scan
only in symmetry and behave like $|\sin(2\pi x/A)|$ and $\sin(2\pi x/A)$, respectively. By exciting the $\text{LSE}_{10}$ and $\text{LSE}_{20}$ modes of the upper aperture region with voltages

\begin{equation}
V_1 = V_1^\dagger = \cos(0.25k_o D_x \sin \theta_o \cos \phi_o)
\end{equation}

and

\begin{equation}
V_2 = V_2^\dagger = j \sin(0.25k_o D_x \sin \theta_o \cos \phi_o)
\end{equation}

respectively, and the modes of the lower aperture region with voltages

\begin{equation}
V_3 = V_3^\dagger = V_1^\dagger \exp\{j 0.5k_o B \sin \theta_o \sin \phi_o\}
\end{equation}

and

\begin{equation}
V_4 = V_4^\dagger = V_2^\dagger \exp\{j 0.5k_o B \sin \theta_o \sin \phi_o\}
\end{equation}

the four phase centers are established at roughly, $x = \pm A/4$ in each region. The beam is scanned to $(\theta_o, \phi_o)$. 

28
As for low frequency, the normalized power in the high frequency propagating beams is determined via superposition (i.e., using equation (29)). The total power delivered is

\[ P = 2 \{ Y_1 |V_1^\ast| ^2 + Y_2 |V_2^\ast| ^2 \} = 2(Y_1 + Y_2) \]

The power in the \( \sigma^{\text{th}} \) beam is therefore, given as

\[ P_{\sigma}(\theta_\sigma, \phi_\sigma) = \sum_{i=1}^{4} S_{21}^{\sigma,i}(\theta_\sigma, \phi_\sigma) V_i |^2 Y_{\sigma_0} / P \]

where the ordering of the elements of \( V^+ \) has been altered to simplify the equation. Taking the same liberty with mode ordering, the power reflected in the jth mode \( (j = 1, 2, 3, 4) \) is given as

\[ R_j(\theta_o, \phi_o) = \sum_{i=1}^{4} S_{1j}^{i}(\theta_o, \phi_o) V_i |^2 Y_j / P \]

2.2 Numerical Results

In this section, numerical results are presented for several element and grid geometries to illustrate the principle performance characteristics of the bifurcated

*See footnote to equation (37).*
twin dielectric slab loaded rectangular waveguide dual frequency array element. Element/grid design is discussed more fully in section 4.

For purpose of discussion, it is convenient to present performance data in a somewhat unusual format. Rather than present realized gain pattern, (i.e., normalized directive gain) power transmission coefficient is given for each radiating beam. The advantage gained by this form of presentation is that it allows a direct comparison of the power in the radiated beams. If \( P_0(\theta_0, \phi_0) \) is the power associated with the \( n \)th beam when the main beam is scanned to \( (\theta_0, \phi_0) \), then the directive gain of this beam is proportional to \( P_0(\theta_0, \phi_0)\cos\theta_0 \). Consequently, for a given scan angle, comparison of beam directive gains includes the comparison of projected aperture at the various beam locations.

Data are presented for rectangular and triangular grid configurations. Two elements are discussed: a WR187 guide with .250", \( \varepsilon_r=9 \) slab loading; and a WR137 guide with .125", \( \varepsilon_r=4.75 \) slab loading. The former is operated at 2.5 GHz (\( kA/2=1.246 \)) and 6.0 GHz (\( kA/2=2.990 \)). The later is operated at 4 GHz (\( kA/2=1.463 \)) and 8 GHz (\( kA/2=2.926 \)). The dimensions of the elements are given in Table 1. In the following, the elements will be distinguished by the WR number of the rectangular guides.
<table>
<thead>
<tr>
<th>Dimension</th>
<th>WR187</th>
<th>WR137</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x$</td>
<td>2.000</td>
<td>1.500</td>
</tr>
<tr>
<td>$D_y$</td>
<td>1.000</td>
<td>.750 or .960</td>
</tr>
<tr>
<td>$A$</td>
<td>1.872</td>
<td>1.374</td>
</tr>
<tr>
<td>$B'$</td>
<td>.872</td>
<td>.622 or .832</td>
</tr>
<tr>
<td>$\delta$</td>
<td>.032</td>
<td>.032</td>
</tr>
<tr>
<td>$B$</td>
<td>.420</td>
<td>.295 or .400</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>.325</td>
<td>.281</td>
</tr>
<tr>
<td>$\beta$</td>
<td>.361</td>
<td>.281</td>
</tr>
<tr>
<td>$\delta$</td>
<td>.250</td>
<td>.125</td>
</tr>
</tbody>
</table>

**Table 1**  
Dimensions (in.) of WR187 and WR137 Elements
The grating lobe diagrams for the four grids are shown in Figures 8a,b,c,d. Each diagram shows the near in grating lobe locations for the two operating points. Low frequency grating lobes are indicated by solid boxes, and high frequency lobes, by solid dots. What is immediately obvious from the figures is that the triangular grid provides a grating lobe free scan region for all directions in the plane at high frequency. One consequence of this is improved high frequency broadside match, as is shown below.

Figures 9 through 12 show power transmission coefficient in the principle planes at low frequency for each of the four grids. In the scan range \( \sin \theta < 0.95 \), no grating lobes enter, as can be seen from the grating lobe diagrams in Figure 8. In general, the performance of the four configurations is the same. There is some improvement in scan coverage of the WR137 element over the WR187 element, but the difference is not large enough to show up on the scale of the figures. In the E-plane \((\sin \phi = 1.0)\), the fall off is nearly \( \cos^{1/2} \theta \), out to \( 60^\circ \) - the WR187 element shows slightly greater scan loss. In the H-plane, the scan loss exceeds \( \cos^{-1/2} \theta \) by approximately .5 db at \( \theta = 60^\circ \). As might be expected, the performance of the rectangular and triangular grid configurations, as measured by power trans-
Figure 8. Grating Lobe Diagrams for WR187 and WR137 Elements in Rectangular and Triangular Configurations
Figure 9. WR187 Element Power Transmission Coefficient - Rectangular Grid, Low Frequency (=2.5GHz)
Figure 10. WT187 Element Power Transmission Coefficient - Triangular Grid, Low Frequency (=2.5GHz)
Figure 11. WR137 Element Power Transmission Coefficient - Rectangular Grid, Low Frequency (=4 GHz)
Figure 12. WR137 Element Power Transmission Coefficient - Triangular Grid, Low Frequency (=4 GHz)
mission coefficient, is not distinguishable, one grid from the other.

In Figures 8a and 8c, the high frequency grating lobes of the rectangular grid configurations are shown residing in real space for the no scan condition. In this situation, a broadside power loss occurs, and a high farout sidelobe condition is created. To a certain extent, the power delivered to these lobes can be reduced by introducing a complex, multiplicative correction for the voltage excitation coefficients, $V_2$ and $V_4$, given by equations (41) and (43). Ideally, this correction is independent of scan and frequency (for small bandwidths). Typical grating lobe levels, with and without the correction, are shown in Figure 13 for a thin walled element in a rectangular grid. Without correction, grating lobe levels reach -16.7 db. With the multiplier $1.164 - j.291^*$, the maximum grating lobe level is -20.4 db, and beyond $\sin \theta_o = .3$, the level is below -25 db. Differences in the main beam due to the two excitations are too small to be represented in the figure.

*The case shown here is supplied by R. Mailloux, the multiplier $1.164 - j.291^*$ was also found suitable for the WR187 element in a rectangular grid.
Figure 13. Effects of Grating Lobe Correction
By configuring the WR187 and WR137 elements in triangular lattice, the difficulties encountered due to the location of the high frequency grating lobes are largely eliminated. Figures 8b and 8d, show the disposition of high frequency grating lobes for triangular grid. No grating lobes enter real space along any cut plane for $\sin \theta_0 < 0.391$.

In the principle cut planes, only the main beam is propagating out to $\sin \theta_0 = 0.820$, or nearly $60^\circ$ scan.

For wide angle scan applications, the triangular grid configuration does not fully eliminate the necessity for the type of excitation correction described above. Consider Figures 8c and 8d. The $(1,0)$ harmonics of the structures follow the same track for E plane scan, and while they are in real space, one is the image of the other, reflected about the $k_x/k_0$ axis. Hence, for the same voltage correction term, roughly the same amount of power will be dumped in the triangular and rectangular grid lobes when the E plane scan sines, $\sin \theta_t$ and $\sin \theta_r'$, respectively are related by

\begin{equation}
\sin \theta_r = \frac{\lambda}{D_y} - \sin \theta_t
\end{equation}

This is illustrated in Figure 14 for the WR137 element operating at 8.64 GHz. Both E and H mode space harmonics are shown. It is clear from the high power levels shown in the figure that grating lobe control is required for either
Figure 14. Grating Lobe Levels Obtained from Rectangular and Triangular Grid Configurations of WR137 Elements
grid type, even though the $\cos \theta$ beam broadening factor is less than .4 throughout the scan range.

Because of the high grating lobe levels obtained for the rectangular grids, there is significant (approximately .1 db) main beam loss at broadside relative to main beam levels obtained for the triangular grid. For the WR137 element, the difference is .13 db. A comparison of principle plane main beam levels for the two grids is shown in Figures 15 and 16. The operating point is 8 GHz. In the H plane, Figure 15, the beams show the .13 db difference at broadside, and smoothly coalesce. In the E plane, Figure 16, there are sharp jogs in the curves. For the rectangular grid, the jog occurs as the grating lobe pair exits from real space. For the triangular grid, the sudden power loss at $\sin \theta = .82$ is due to the grating lobe pair entering real space. The main beam falloff is slightly greater than $\cos^{1/2} \theta$ for both grids in either plane.

2.3 Comparison with Limiting Cases

To check the analysis, and in particular the details of the computational procedure, several limiting cases have been examined. For these cases, the slab dielectric constant is set to $\varepsilon_r=1$ and the LSE$_{10}$ mode is independently
Figure 15. Comparison of Main Beam Power Levels for Rectangular and Triangular Grid Configurations of WR137 Elements - H Plane
Figure 16. Comparison of Main Beam Power Levels for Rectangular and Triangular Grid Configurations of WR137 Elements - E Plane
excited in the upper and lower aperture regions.

For $\epsilon = 1$, the LSE_modes degenerate to the TE_modes of empty rectangular waveguide. Therefore, an appropriate set of check cases include $\Pi$ plane scanned thin wall rectangular grid arrays of rectangular elements and special triangular grid examples which have appeared in the literature. Thick wall rectangular grid cases can also be used as checks provided the wall thickness is not too great.

The geometry for the rectangular grid examples is shown in Figure 17. The aperture dimensions are $A$ and $B$, and the lattice vectors are

\begin{equation}
S_1 = d_x X_0
\end{equation}

and

\begin{equation}
S_2 = d_y Y_0
\end{equation}

For $A = d_x$ and $B = d_y$, exact solutions for $E$ and $\Pi$ plane element patterns have been obtained using function theoretic techniques to construct the reflection coefficient of the driven $TE_{10}$ mode. Figures 18 through 20 show magnitude and phase of active reflection coefficient, $\Gamma$, of the $TE_{10}$ mode of square waveguide in thin-wall square lattice configuration for $\Pi$
Figure 17. Rectangular Grid of Rectangular Waveguides
Figure 18. Comparison of Exact (8) and Approximate Modal Solutions for Active TE$_{10}$ Reflection Coefficients - Thin Walled Square Elements H-Plane $D_x/\lambda = .5714$

$|\Gamma|$ vs $\sin \theta$

$\angle \Gamma$ vs $\sin \theta$
Figure 19. Comparison of Exact and Approximate Modal Solutions for Active TE_{10} Reflection Coefficient - Thin Walled Square Elements H-Plane $D_x/\lambda = 0.6205$
Figure 20. Comparison of Exact(8) and Approximate Modal Solutions For Active $TE_{10}$ Reflection Coefficient - Thin Walled Square Elements H-Plane $D_x/\lambda = .6724$
plane scan. The solid curves are the exact solution, as obtained by Wu and Galindo. The dashed curves are obtained from the current formalism using the first five $TE_{no}$ modes to approximate the aperture field distribution. The minor discrepancies between the results are removable by including additional higher order modes in the modal computations.

Figures 21 through 23 show magnitude of $r$ as a function of $H$ plane scan angle when the $H$ plane metallic walls have finite thickness, $t = 0.1d_x$. The lattice remains square. The solid curves were obtained by Galindo and Wu\(^{(9)}\) using 30 feedguide modes to represent the aperture field. The dashed curves are from the present analysis using the first nine $TE_{no}$ modes. The agreement is excellent.

Aperture field approximations consisting of the first few $TE_{no}$ mode functions may be used to compute $H$ plane element patterns of triangular grid arrays of rectangular apertures.

Figures 24 and 25 show $H$ plane element pattern, $[(1 - |r|^2)\cos\theta]$, as computed by Amitay, Galindo, and Wu and using the approximate limiting case of the present analysis.
Figure 21. Comparison of Modal Solutions for Active $\text{TE}_{10}$ Reflection Coefficient - Thick Walled Square Elements - $H$-Plane $D_x/\lambda = .5714$, $t = .1D_x$
Figure 22. Comparison of Modal Solutions for Active TE$_{10}$ Reflection Coefficient - Thick Walled Square Elements - H-Plane $Dx/\lambda = 0.6205$, $t = 1D_x$
Figure 23. Comparison of Modal Solutions for Active $TE_{10}$ Reflection Coefficient - Thick Walled Square Elements -
H-Plane $Dx/\lambda = 0.6724$, $t = 0.1D_x$
Figure 24. Comparison of Modal Solutions for H-Plane Element Pattern When Aperture Field is Approximated by $TE_{10}$ Mode
Figure 25. Comparison of Modal Solutions for H-Plane Element Pattern When Aperture Field is Approximated by $TE_{10} + TE_{20}$ Modes
The lattice is 45° triangular, with lattice vectors

\[(50) \quad S_1 = 1.008\lambda x_o \]

and

\[(51) \quad S_2 = 0.504\lambda x_o + 0.504\lambda y_o \]

The rectangular apertures are .905λ by .4λ. In figure 24, the aperture field is approximated using only the TE\textsubscript{10} feedguide mode. For Figure 25, the TE\textsubscript{20} mode is added. Again, the comparison is quite good. The resonance, near \(\sin \theta = .6\), was found experimentally by Diamond\textsuperscript{11} in an investigation of the central element pattern of a 95 element array.

The deviation of computed results in Figures 24 and 25 arise from two sources. The first is an inability to read the published curves to sufficient accuracy. The second source of error is number of space harmonics used to obtain the published curves and the present results.

2.4 Convergence of Numerical Results

From the results of the previous section, it is event that the numerical solution implemented here converges uniformly as the numer of aperture modes, space modes or both is increased. However, to ensure that the convergence is indeed uniform, the array properties of the WR187 element were examined as the mode
count was varied in the aperture and free space regions. One to eight aperture modes were considered and the circle of convergence was varied from $k_0 (= 2\pi/\lambda)$ to $13.9k_0$ at 2.5GHz and $33.4k_0$ at 6 GHz.

For maximum circle of convergence, the solution converged rapidly as the number of aperture modes was increased. At both frequencies, four or five aperture modes were found to be sufficient.

With the number of aperture modes held fixed, the circle of convergence was uniformly increased. Beyond roughly $5k_0$ at 2.5GHz and $10k_0$ at 6GHz, the solution became stable.

Except for the usual effect of truncating the modal series at an inordinately premature point, no convergence anomalies were evident in the computations.
3.0 PROPAGATION CHARACTERISTICS OF TWIN DIELECTRIC SLAB LOADED RECTANGULAR WAVEGUIDE

The analysis of the radiation properties of an infinite array of dual band elements requires a complete description of wave propagation in the inhomogeneously loaded feedguide. The geometry of the dual band element is shown in Figure 26. The element consists of a rectangular waveguide bifurcated in the E-plane by a septum of thickness, \( \delta \), and symmetrically loaded with full height slabs of lossless dielectric parallel to the guide narrow wall. Both slab thickness and spacing are arbitrary in the ranges \( 0 < 2\delta/A < 1 \), \( 0 < 2\beta/A < 1 \), and \( \varepsilon_r \), the relative dielectric constant, may take on any real value, \( \varepsilon_r > 1 \).

Several investigators have studied wave propagation in similar guiding structures, primarily to provide bases for perturbation calculations of ferrite phase shifter properties. Collin \(^{(15)}\) has obtained general expressions for mode functions and modal propagation constants for the asymmetric single slab case. These results are equivalent to the symmetric twin slab results for short-circuit symmetry in \( x \). Seckelmann \(^{(12)}\) has obtained general expressions for \( LSE_{no} \) (i.e., \( TE_{no} \)) mode functions and propagation
Figure 26. Dual Band Element
The modal fields of the inhomogeneously loaded rectangular waveguide shown in Figure 26 are obtained in a straightforward manner. Recognizing that E and H modes with respect to \( x_o \) remain decoupled at the dielectric interfaces, arbitrary waveguide fields can be decomposed into E-type (LSM) and H-type (LSE) modes with respect to \( z_o \). Thus, an arbitrary field may be expressed in terms of either complete mode set. A one-to-one correspondence exists between the modes of each set (i.e., eigenvalues of the E and H mode set are eigenvalues of the type mode set for the given boundary value problem). The LSE and LSM modal fields are then obtained via a component-by-component comparison of the modal fields in either set corresponding to a particular eigenvalue. The components of the type mode functions are then proportional to either the voltage or current distributions of the equivalent circuit.

The modal spectrum for the structure is obtained via a transverse resonance technique.

3.1 Mode Functions

It is well known that E and H modes with respect to surface normals remain decoupled at planar interfaces.
between dielectrics. Thus, in the inhomogeneously (in x) loaded guide shown in Figure 26, decoupled modes of the structure will be E and H with respect to x, or, equivalently, E-type (LSM) and H-type (LSE) with respect to z.

For the infinite phased array of dual frequency elements, the aperture fields are expressed as the superposition of transverse-to-z mode functions which satisfy the vector equations (see Appendix B).

\begin{align}
(52) & \quad \gamma_i \gamma'_i h'_i(x,y) = \omega c [I + \frac{\nabla \cdot \nabla}{k^2}] \cdot (z_0 \times e'_i(x,y)) \\
(53) & \quad h'' \gamma_i z'' e''_i(x,y) = \omega \mu [\bar{I} + \frac{\nabla \cdot \nabla}{k^2}] \cdot (h''_i(x,y) \times z_0)
\end{align}

In equations (52) and (53), \(\gamma_i\) is the longitudinal (z directed) wavenumbers; \(\gamma'_i\) and \(z''\) are modal immitances; \(I\) is the unit transverse-to-z diadic,

\begin{equation}
(54) \quad I = x_0 x_0 + \nu_0 \nu_0;
\end{equation}

and \(\nabla_t\) is the transverse-to-z gradient operator,
The prime (') is used to denote LSM modes [for which $h_x(x,y) \equiv 0$]; the double prime, to denote LSE modes [for which $e_x(x,y) \equiv 0$]; and single index is used rather than a double index.

The desired modal representation of transverse fields is

\begin{align}
(56) \quad & E_t(x,y,z) = \sum_i V^i(z)e^i(x,y) + \sum_j V^n(z)e^n(x,y) \\
(57) \quad & H_t(x,y,z) = \sum_i I^i(z)h^i(x,y) + \sum_j I^n(z)h^n(x,y)
\end{align}

where $V^\alpha_n(z)$ and $I^\alpha_n(z)$ ($\alpha = ', "$) are $z$ dependent modal voltages and currents satisfying the transmission line equations

\begin{align}
(58) \quad & \frac{d}{dz} V_i(z) = -j\gamma_i z_i I_i(z) \\
(59) \quad & \frac{d}{dz} I_i(z) = -j\gamma_i y_i V_i(z)
\end{align}

Since the guide is uniform (and assumed infinite) in $z$, the $z$ dependence of the modal voltages and currents is
\[
\exp[-j\gamma z], \text{ hence}
\]

(60) \( V_i(z) = V_ie^{-j\gamma z} \)

(61) \( I_i(z) = I_ie^{-j\gamma z} \)

Equations (52) and (53) may be solved to obtain relationships between the mode components. For LSM modes, the \( x \) component of \( h'(x,y) \) is taken as zero \( (h_x' \equiv 0) \), and equation (52) results in:

(62) \( h_i'(x,y) = \varepsilon_r(x) \varepsilon_{xi}'(x,y) \gamma_o \)

(63) \( e_i'(x,y) = e_{xi}'(x,y) \gamma_o \)

\[
+ \frac{1}{k^2 \varepsilon_r(x) - \kappa_i^2(x)} \frac{\partial^2}{\partial x \partial y} e_{xi}'(x,y) \gamma_o
\]

(64) \( Z_i' = \frac{k \varepsilon_r(x) - \kappa_i^2(x)}{\gamma_i \omega \varepsilon_o} \)

where \( e_{xi}'(x,y) \) is a solution of the scalar wave equation.
\( (\nabla^2 + k_{ti}^2(x)) e'(x,y) = 0 \)

with \( k_{ti}^2 = \kappa_i^2(x) + k_{yi}^2 = k_0^2 \varepsilon_r(x) - \gamma_i^2 \), subject to the boundary conditions of the guide cross-section. \( \varepsilon_r(x) \) is the \( x \)-dependent dielectric constant of the cross-section. For LSE modes, the \( x \) component of \( e''(x,y) \) is zero \( (e''_x \equiv 0) \), and solution of (53) gives:

(66) \[ e''_i(x,y) = -h''_i(x,y) \gamma_0 \]

(67) \[ h''_i(x,y) = h''_i(x,y) \gamma_0 \]

\[ + \frac{1}{k^2 \varepsilon_r(x) - \kappa_i^2(x)} \frac{\partial^2}{\partial x \partial y} h''_i(x,y) \gamma_0 \]

(68) \[ \gamma''_i = \frac{2 \kappa \varepsilon_r(x) - \kappa_i^2(x)}{\omega \mu \gamma''_i} \]

with \( h''_i(x,y) \) satisfying

(69) \[ (\nabla^2_t + k_{ti}^2(x)) h''_i(x,y) = 0 \]
over the cross-section. The wave immittances given by equations (64) and (68) are defined such that the direct proportionalities of equations (62) and (66) are obtained. It is shown in Appendix C that the type modes possess the following orthonormality property for the bounded cross-section (CS) of Figure 26.

\[
(70) \quad \int_{CS} \int \text{d}x \text{d}y \varepsilon_{n}^{\alpha} \cdot (\alpha_{m}^{\beta} \times \text{z}_{o}) = \delta_{\alpha}^{\beta} \delta_{nm}
\]

where \( \alpha, \beta = (',") \).

Equations (62) through (69), with appropriate boundary conditions, are the complete formal solution for the mode functions of the symmetric twin dielectric slab loaded rectangular guide shown in Figure 26. However, due to the complexity of boundary conditions along \( x \), the scalar wave equations (65) and (69) are difficult to solve. If the transmission line direction is temporarily taken along \( x \), the fields in the guide may be put in a representation of \( E \) and \( H \) modes with respect to \( x \). Due to the degeneracy of the rectangular cross-section, eigenvalues of the \( E(H) \) mode set are also eigenvalues of the LSM (LSE) mode set. Thus, corresponding to each eigenvalue of the
structure, there are two expressions for total field. These expressions are compared component-by-component, resulting in particular expressions for LSE and LSM mode functions in the symmetric twin dielectric slab loaded rectangular waveguide.

The transverse-to-x modes of the twin slab loaded guide are obtained in standard fashion and are given as:

**E modes** ($h'_x \equiv 0$)

\[
\hat{\mathbf{e}}_i(y,z) = \frac{\nabla^2 \psi_E}{k_{ti}}
\]

\[
= -\frac{Ae}{k_{ti}} e^{-jY_i z} \frac{\cos m\pi y}{b} \frac{\sin m\pi y}{b} \frac{\sin m\pi y}{b} \frac{\sin m\pi y}{b} \frac{\sin m\pi y}{b}
\]

\[
\hat{h}'_i(y,z) = k_{ti} \hat{\mathbf{e}}_i(y,z)
\]

\[
\lambda'_i = \frac{\kappa_i(x)}{\omega_{\sigma} \epsilon_r(x)}
\]

\[
\psi_E = Ae^{-jY_i z} \sin \frac{m\pi y}{b}
\]
H Modes \( \left( e''_x = 0 \right) \)

\[
(75) \quad \hat{h}''_i (y,z) = -\frac{\nabla_t \psi_H}{k_{ti} z} \\
= \frac{Be^{jy_i z}}{k_{ti}} \left[ \frac{m_i y}{b} \text{ sin } \frac{m_i y}{b} \left( \gamma \frac{m_i y}{b} \gamma \right) + jy_i \text{ cos } \frac{m_i y}{b} \gamma \right]
\]

\[
(76) \quad \hat{e}''_i (y,z) = \hat{h}''_i (y,z) \times \mathbf{x}_0
\]

\[
(77) \quad \hat{\gamma}_i = \frac{k_t (x)}{\omega}
\]

\[
(78) \quad \psi_H = \frac{-jy_i z}{b} \text{ cos } \frac{m_i y}{b}
\]

In the above equations, \( \hat{\cdot} \) is used to indicate results in the transverse-to-\( x \) representation, and \( k_{ti} = \sqrt{\left( \frac{m_i y}{b} \right)^2 + y_i^2} \).

The modal representation of transverse-to-\( x \) fields is

\[
(79) \quad \hat{E}_t (x,y,z) = \sum_i \hat{e}_i^i (y,z) \hat{n}_i (y,z) + \sum_j \hat{e}_j^j (y,z) \hat{n}_j (y,z)
\]

\[
(80) \quad \hat{H}_t (x,y,z) = \sum_i \hat{n}_i^i (y,z) \hat{n}_i (y,z) + \sum_j \hat{n}_j^j (y,z) \hat{n}_j (y,z)
\]
where $\hat{V}_i(x)$ and $\hat{I}_i(x)$ are modal voltages and currents which satisfy the transmission line equations:

\begin{align}
\frac{d}{dx} \hat{V}_i(x) &= -j\kappa_i(x) Z_i \hat{I}_i(x) \\
\frac{d}{dx} \hat{I}_i(x) &= -j\kappa_i(x) Y_i \hat{V}_i(x)
\end{align}

The single mode transverse-to-$z$ magnetic field corresponding to eigenvalue $\gamma_i$ of the LSM and E modal subsets must be equal. Thus, since $h_x'(x,y) = h_x'(y,z) = 0$,

\begin{equation}
\hat{I}_i(x) h_{yi}(y,z) \gamma_i = I \ e^{-j\gamma z} e_{xi}(x) e_{yi}(x,y) \gamma_i
\end{equation}

Using (71) in (72), and letting $A = I \ k_i / N_i \gamma_i$ results in an expression for $e_{xnm}'(x,y)$ in terms of the $x$ dependent modal current distribution $\hat{I}_n(x)$:

\begin{equation}
e_{xnm}'(x,y) = -j \frac{1}{N_i \epsilon_r(x)} \hat{I}_n(x) \sin \frac{ny}{b}
\end{equation}

*in equations (84) and (36), the double subscript is used to explicitly indicate $x$ and $y$ dependence.
Note that for \( m = 0 \), the LSM mode does not exist.

Similarly, the single mode tranverse-to-\( z \) electric field corresponding to eigenvalue \( \gamma \) of the LSE and H modal sub-sets are equal giving

\[
\hat{V}''_{\ell}(x)\hat{e}''_{y,z}(y,z)\gamma_o = -\hat{V}''_{\ell}e^{-j\gamma z}h''_{x,y}(x,y)\gamma_o
\]

Letting \( B_k = V''_{\ell}k_{t\ell}/N''_{\ell}\gamma_k \) and using (75) in (76) results in

\[
\hat{h}''_{x,\text{nm}}(x,y) = -j\frac{1}{N''_{\text{nm}}} \hat{V}''_{n}(x)\cos\frac{m\pi y}{b}
\]

The coefficients \( N''_{\text{nm}} \) and \( N''_{\text{nm}} \) appearing in equations (84) and (86) are normalization constants determined by application of equation (70). Complete expressions for the normalizations are given in Appendix C.

Since the inhomogeneously (in \( x \)) loaded guide is symmetric about the midplane (\( x = 0 \)), the modes will be either: symmetric, corresponding to an open circuit plane at \( x = 0 \) for LSE modes, or short circuit plane for LSM modes; or anti- symmetric, for the converse. Consider the
equivalent transmission line representation of wave propagation in x shown in Figure 27. The complete current and voltage distributions may be written down by inspection for either of the indicated terminations. The resultant distributions are then appropriately inserted in (84) or (86) to obtain the cross-sectional dependence of the x components of LSM electric and LSE magnetic mode functions. The vector mode functions are summarized in Figures 28 and 29.

In normal operation as a dual frequency phased array element, the excited (propagating) modes of the symmetric twin dielectric slab loaded rectangular waveguide will be the \(LSE_{10}\) mode in the low frequency band, and \(LSE_{10}\) and \(LSE_{20}\) modes in the high frequency band. At either band, the tendency will be for the field strength interior to the dielectric slabs to exceed the field strength elsewhere. This characteristic is clearly evident in the equations of Figure 29 for slow wave propagation. Examination of the modal voltage expressions shows that for \(\kappa_n = -j|\kappa_n|\), \(e_{y10}(x,y)\) is proportional to \(\cosh(|\kappa_n|)\) in \(-\beta<x<0\), and to \(\sinh(|\kappa_n|)(x+a/2)\) in \(-a/2<x<-\beta\). Thus, for greatly slowed waves, i.e., \(\gamma_{nm} = k\sqrt{\varepsilon_r}\), the \(e_{y10}(x,y)\) is nearly exponential in the air regions, and nearly constant in the dielectric. Similar characteristics
Figure 27. Equivalent Transmission Line Representation of Wave Propagation in $x$. 
I. Transverse-to-z Mode Function

\[ h'_{nm}(x,y) = \varepsilon_r(x) e^{i2\pi x_{nm}(x,y)} V_0 \]

\[ e^{i2\pi x_{nm}(x,y)} = e^{i2\pi x_{nm}(x,y) \gamma_0} + \frac{1}{k^2 \varepsilon_r(x) - \varepsilon_n^2} \frac{\partial^2}{\partial x \partial y} e^{i2\pi x_{nm}(x,y)} V_0 \]

\[ z'_{nm} = \frac{k^2 \varepsilon_r(x) - \varepsilon_n^2}{\gamma_{nm} \varepsilon_0} \]

where

\[ s_n^2(x) = \frac{k^2 \varepsilon_r(x)}{b} - \gamma^2 = \begin{cases} \frac{\varepsilon_n \varepsilon_r(x)}{\varepsilon_r} & \varepsilon_r(x) = \varepsilon_r \\ \frac{2}{\varepsilon_n \varepsilon_r(x)} & \varepsilon_r(x) = 1 \end{cases} \]

\[ e^{i2\pi x_{nm}(x,y)} = \frac{1}{\gamma_{nm} \varepsilon_r(x)} I_n(x) \sin \frac{\mu y}{b} \]

\[ \int_{-a/2}^{a/2} \frac{1}{\varepsilon_r(x)} |I_n'(x)|^2 dx \]

II. Longitudinal Modal Currents, \( \hat{I}_n'(x) \)*

symmetric: \( \hat{I}_{ymn}(|x|,y) = \hat{I}_{ymn}(-|x|,y) \)

antisymmetric: \( \hat{I}_{ymn}(|x|,y) = -\hat{I}_{ymn}(-|x|,y) \)

\[ \hat{I}_n'(x) = \begin{cases} \cos \varepsilon_{\pm n}(x+\delta), & -a/2 < x < a/2 \\ \varepsilon_{\pm n}(x+a/2), & -a/2 < x < a/2 \end{cases} \]

\[ \hat{I}_n'(x) = \begin{cases} \sin \varepsilon_{\pm n}(x+\delta), & -a/2 < x < a/2 \\ \varepsilon_{\pm n}(x+a/2), & -a/2 < x < a/2 \end{cases} \]

*The coefficients \( G, B, C; E, F, E' \) and \( F' \) are given in Appendix C.*

Figure 28

LSM\(_{nm}\) Mode Functions
I. Transverse-to-z Mode Functions

\[ e_{nm}''(x,y) = -h_{xnm}(x,y)Y_0 \]
\[ h_{nm}''(x,y) = h_{xnm}(x,y)X_0 + \frac{1}{k_{\varepsilon r}(x) - k_n^2(x)} \frac{\partial^2}{\partial x \partial y} h_{xnm}(x,y)Y_0 \]
\[ \gamma_{nm}^2 = \frac{k_{\varepsilon r}^2(x) - k_n^2(x)}{\omega \gamma_{nm}} \]

where

\[ k_n^2(x) = \frac{|\varepsilon_n^2|}{b} + \gamma_{nm}^2 \]

II. Longitudinal (in x) Modal Voltages, \( \hat{V}_n''(x) \star \)

Symmetric: \( e_{ynm}(|x|,y) = e_{ynm}(-|x|,y) \)

Antisymmetric: \( e_{ynm}(|x|,y) = -e_{ynm}(-|x|,y) \)

\[ \hat{V}_n''(x) = \begin{cases} \cos \kappa_n x, & -\beta < x < 0 \\ \sin \kappa_n x, & 0 < x < \beta \end{cases} \]

(The coefficients \( B_1'', B_2'', C'', E_1'', E_2'' \), and \( F'' \) are given in Appendix C)

Figure 29

ISE_{nm} Mode Function
a.e evident for the $LSE_{20}$ mode (the first anti-symmetric in $x$ mode).

Typical $e''_{y10}(x,y)$ distributions are shown in Figures 30 and 31 for an element operating at 2.5 GHz and 6.0 GHz, respectively. The element is a WR187 guide with .250" (=\delta) slabs of $\epsilon_r = 9$ dielectric located .325" (=\alpha) from either narrow wall. At low frequency, the distribution is roughly uniform between slabs, with some field concentration in the vicinity of the interior air-dielectric interfaces. In the region $|x|>\beta+\delta$, the field behaves very nearly like $\cos(\pi x/A)$. At 6 GHz, the $e'_{y10}(x,y)$ distribution is entirely different, showing well defined field concentration about the dielectric, with very low field strength in the air-filled regions. The distributions are roughly symmetric about the slabs, with non-zero field at the guide center. As the dielectric constant is increased the fields become more heavily concentrated in the dielectric, and, consequently, the field strength at the guide center approaches zero.

At 6 GHz, the antisymmetric $LSE_{20}$ mode is also propagating, and all other modes are well beyond cut-off. The $e''_{y20}(x,y)$ distribution is shown overlayed on the $e''_{y10}(x,y)$ distribution in Figure 32. This comparison shows that the
Figure 30. $e_{y10}(x,y)$ Distribution for WR187 Bifurcated Guide With .250" Thick $\varepsilon_r = 9$ Loading. 2.5 GHz
Figure 31. $e_{y10}(x,y)$ Distribution for WR187 Bifurcated Guide With .250"Thick $\varepsilon_r = 9$ Loading. 6 GHz
Figure 32. $e_{y10}$ and $e_{y20}$ $(x,y)$ Distributions for WR187 Bifurcated Guide With .250" Thick $\varepsilon_r = 9$ Loading. 6 GHz
LSE$_{10}$ and LSE$_{20}$ modal field distributions are quite similar, differing by no more than a few percent in relative magnitude for $|x/A|>0.15$, but having opposite symmetries. It is this high frequency propagation characteristic and the fact that $\gamma_{20}/\gamma_{10} \approx 1.0$ for appropriately chosen dielectric constants and guide geometries which provide the unique dual frequency array element potential of the symmetric twin dielectric slab loaded rectangular waveguide. For, assuming the functions are exactly identical in magnitude and that $\gamma_{20}/\gamma_{10} = 1$, the LSE$_{10}$ and LSE$_{20}$ modes, by magnitude control only, may be excited such that two independent phase centers are located at roughly the positions of the slabs.

The similarity of the $e''_{y10}(x,y)$ and $e''_{y20}(x,y)$ distributions, and hence, the achievable high frequency phase center independence, is directly related to dielectric constant for fixed geometry. As the dielectric constant is decreased, holding cross-section fixed, the LSE$_{10}$ distribution approaches the TE$_{10}$ distribution of empty guide; and the LSE$_{20}$ approaches the empty guide TE$_{20}$ distribution. These trends are evident in Figure 33, where the $e''_{y10}(x,y)$ and $e''_{y20}(x,y)$ distributions are shown overlayed for the WR187 guide with $\varepsilon_r = 5$ loading. The departure in magnitude
Figure 33. $e_{y10}$ and $e_{y20}$ $(x,y)$ Distributions for WR187 Bifurcated Guide With .250" Thick $\varepsilon_r = 5$ Loading, 6 GHz
between the distributions is significantly greater for the lower dielectric constant than for the higher.

While the above discussion shows that there is a strong influence of dielectric constant on achievable high frequency phase center independence, it should not be construed that large dielectric constant is generally preferable to low dielectric constant. In particular, it will be shown in section 4-1 that for certain geometries, dielectric constants on the order of \( \varepsilon_r = 9 \) may lead to large aperture reflections which are difficult, if not impossible, to match out.
3.2 Mode Spectrum

The modal spectrum of the symmetric twin dielectric slab loaded rectangular waveguide is obtained via a transverse resonance procedure. Representing the loaded guide as an E or H mode transmission line in x, as in Figure 27, and requiring that x = 0 be either an open or short circuit plane results in four dispersion relations, in x, of the form,

\[ D(\kappa_n, \kappa_{en}) = 0. \]

where

\[ \kappa_n^2 = k^2 - \left( \frac{m\pi}{B} \right)^2 - \gamma_{nm}^2 \]

\[ \kappa_{en}^2 = \kappa_n^2 + k_n^2(\varepsilon_r - 1) \]

k = 2\pi/\lambda is the free space wave number, m\pi/B is the y directed wave number, and \gamma_{nm} is the z directed wave-number. One dispersion relation is obtained for each symmetry condition, in x, of each modal subset. The four dispersion relations are given in Table 2. It should be noted that the forms given are computationally unstable due to both the various tangent evaluations,
<table>
<thead>
<tr>
<th>Mode</th>
<th>Symmetry</th>
<th>Condition at x = 0</th>
<th>$D (\kappa, \kappa_{\varepsilon})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSM</td>
<td>Symmetric</td>
<td>S.C. or PEC</td>
<td>$\varepsilon_x \kappa \tan \alpha + \frac{\kappa_{\varepsilon}}{\varepsilon_x} \kappa \tan \beta \tan_{\varepsilon} \delta$</td>
</tr>
<tr>
<td>LSM</td>
<td>Anti-Symmetric</td>
<td>O.C. or PMC</td>
<td>$\varepsilon_x \kappa \tan \alpha + \frac{\kappa_{\varepsilon}}{\varepsilon_x} \cot \kappa \beta \tan_{\varepsilon} \delta$</td>
</tr>
<tr>
<td>LSE</td>
<td>Symmetric</td>
<td>O.C. or PMC</td>
<td>$\kappa_{\varepsilon} \tan \beta \tan_{\varepsilon} \delta - \kappa$</td>
</tr>
<tr>
<td>LSE</td>
<td>Anti-Symmetric</td>
<td>S.C. or PEC</td>
<td>$\kappa_{\varepsilon} \tan \beta + \kappa \tan_{\varepsilon} \delta$</td>
</tr>
</tbody>
</table>

Table 2, Dispersion Relations for Inhomogeneously Loaded Rectangular Waveguide
and the indicated divisions. In practice, the expressions are converted to combinations of sines and cosines, and all divisions are removed.

In the LSE and LSM modal subsets, the mode indexing is \((n,m)\). The first index, \(n\), is associated with the infinite sequence of zeros of \(D(\kappa_\epsilon, \kappa_\epsilon)\) which are arrayed along the real \(\kappa_\epsilon\) axis, as illustrated in Figure 34. The first zero is always associated with the even symmetry solution of the modal subset (i.e. open circuit symmetry for LSE modes; short circuit symmetry for LSM modes). Thereafter, the roots of the even and odd symmetry dispersion relations are interleaved. Hence, for even symmetry, \(n\) is always odd, and conversely. The second index, \(m\), appears explicitly in the auxiliary dispersion equation, (88), and is directly interpreted as the order of the \(\gamma\) variation of the modal field.

The potential of the symmetric twin dielectric slab loaded rectangular waveguide as a dual frequency array element arises, in part, due to the unique migration of the roots \(\gamma_{10}\) and \(\gamma_{20}\) with frequency. Figure 35 shows an LSE mode dispersion diagram for half height WR187 guide with \(\epsilon = 9\) loading. Only the \((n,0)\), \(n = 1,2,3\) and \((n,1)\), \(n = 1,2\) roots are shown, other LSE roots being considerably
Figure 34. Disposition of Roots of $D(K, K_\varepsilon)$ Along Real $K_\varepsilon$ Axis.

\[ K = \sqrt{K_\varepsilon^2 - k_0^2 (\varepsilon - 1)} \]
Figure 35. LSE Dispersion Diagram for Bifurcated WR187 Guide With 250" Thick \( \varepsilon_r = 9 \) Loading
more evanescent. For $kA/2 < 1.40$, only the $LSE_{10}$ mode is propagating. As frequency is increased, the $LSE_{20}$ mode begins to propagate, and $\gamma_{20}$ rapidly approaches $\gamma_{10}$, the ratio $\gamma_{20}/\gamma_{10}$ being nearly unity for $kA/2 > 2.5$. As frequency is further increased, the $LSE_{11}$ and $LSE_{21}$ modes begin to propagate, though they may, of course, be pushed further out by decreasing the guide height. The result is that over the range $2.5 < kA/2 < 3.0$, the propagating modes of the structure have virtually identical dispersion. Hence, if in this range, the $LSE_{10}$ and $LSE_{20}$ mode functions differs only in symmetry, as in Figure 32, the half height WR187 guide with $\varepsilon_r=9$ slab loading will support two independent phase centers over an 18% frequency band centered at $kA/2 = 2.75$; and over multiple guide wavelengths, $2\pi/\gamma_{10}$, in $z$.

The dashed line, $\gamma = k$, in Figure 35 may be used to estimate the mismatch at the feedguide-free space interface for broadside excitation (only the $LSE_{10}$ is excited). The modal admittance of the $LSE_{10}$ mode is $\gamma_{10}/\omega\mu$, and the modal admittance of the dominant space harmonic is $k/\omega\mu$ at broadside. As an estimate, then, the reflection coefficient at the aperture will be in the
neighborhood of

\[ |\Gamma| = \frac{Y_{10}-k}{Y_{10}+k} \]  

(90)

for broadside excitation. For the WR187 guide with \( \varepsilon_r = 9 \) loading, operating at \( kA/2 = 2.99 \), equation (4-39) gives \( |\Gamma| = 0.383 \), whereas the exact value for a rectangular thin wall array of these guides is \( |\Gamma| = 0.424 \). For the same guide operating at \( kA/2 = 1.25 \), equation (90) gives \( |\Gamma| = 0.110 \), and the exact value is \( |\Gamma| = 0.285 \). In both cases, the implication is that the aperture susceptance, which is ignored in (90), is significant. This is not entirely surprising, since it is well known* that equation (90) is exact for thin walled rectangular grid arrays of empty rectangular grid guide, for which the set of transverse wavenumbers of the feedguide is the set of transverse wavenumbers for the unit cell guide.

For fixed guide wall dimensions, the parameters which most strongly influence the dispersion curves are dielectric constant and slab thickness. The location of the slabs, denoted by the ratio \( \alpha/\beta \), has second order effects in the regions \( 0.5 < \alpha/\beta < 2.0 \). \( \alpha/\beta \) ratios outside this region are not of interest due to the irregularly spaced high freq-

*See, for example, Amitay, Galindo, and Wu, (10) pg 132ff.
uency phase centers which would result.

Figures 36 through 39 are LSE dispersion diagrams for half height WR137 guide with $\delta = .250''$ and $a/b = 1.00$. The curves progress from loading of $\varepsilon_r=3$ to $\varepsilon_r=9$. As the loading increases, $\gamma_{20}$ approaches $\gamma_{10}$, in general, and the trend is toward steeper slopes.

The migration of the $LSE_{30}$ cutoff frequency toward lower frequency produces one of the critical trade-offs inherent to the design of the symmetric twin dielectric slab rectangular waveguide dual frequency array element. In general, the overriding array design criterion will be to minimize the number of radiators (or, equivalently maximize array cell size). Thus, for dual frequency operation, the high frequency band center operating point will be in the vicinity of $kA/2 = \pi$. In this region, the higher dielectric constants ($\varepsilon_r = 7, 9$) provide $\gamma_{20}/\gamma_{10}$ ratios very close to unity, but the $LSE_{30}$ mode is propagating. Since it is necessary to push this mode out, lower dielectric constant is required.

Similar results are obtained by holding $\varepsilon_r$ fixed and varying only slab thickness, $\delta$. Figures 40 through 43 are LSE dispersion diagrams for four slab thicknesses in half height WR137 guide. The dielectric constant is 5, and $a/b = 1.00$. In general, the behavior with thickness
Figure 36. LSE Dispersion Diagram for Bifurcated WR137 Guide With 25\(\text{\textmu}\) Thick \(\varepsilon_r = 3\) Loading

\[ \alpha/\beta = 1.000 \]
\[ \delta/\lambda = 0.1522 \]
\[ \varepsilon = 3.00 \]

LEGEND
- LSE0100
- LSE0200
- LSE0300
- LSE0101
- LSE0201
- LSE0301
Figure 37. LSE Dispersion Diagram for Bifurcated WR137 Guide With .250" Thick $\epsilon_z = 5$ Loading
Figure 38. LSE Dispersion Diagram for Bifurcated WR137 Guide With 0.250" Thick $\epsilon_r = 7$ Loading
Figure 39. LSE Dispersion Diagram for Bifurcated WR137 Guide With .250" Thick $\varepsilon_r = 9$ Loading
Figure 40. LSE Dispersion Diagram for Bifurcated WR137 Guide With .063" Thick $\epsilon_r = 5$ Loading
Figure 41. LSE Dispersion Diagram for Bifurcated WR137 Guide With .125" Thick $\varepsilon_r = 5$ Loading
Figure 42. LSE Dispersion Diagram for Bifurcated WR137 Guide With .188" Thick $\varepsilon_r = 5$ Loading
Figure 43. LSE Dispersion Diagram for Bifurcated WR137 Guide With .375" Thick $\varepsilon_I = 5$ Loading
is very like the behavior with $\varepsilon_r$. However, comparison of Figures 39 and 43 shows that high dielectric constant is preferable to extreme thickness. In the two figures, the LSE$_{30}$ mode enters at roughly the same frequency, but the ratio of $z$ directed wave numbers, $\gamma_{20}/\gamma_{10}$, is significantly nearer to unity for $\varepsilon_r = 9$ than for $\varepsilon_r = 5$. 
4.0 ARRAY APERTURE DESIGN

In this section, the trade-offs leading to a practical array aperture design are presented for the hypothetical array performance given in Table 3. The operating bands are 15% centered at 4 and 8 GHz. Sixty degree (60°) principle plane scan coverage is required. The array is to provide 30 db gain for 60° principle plane scan at 4 GHz. The first sidelobe level is to be below 20 db and RMS sidelobe levels are to be below 35 db. Nominal feed losses of 1.5 db at 4 GHz and 2.8 db at 8 GHz are assumed. To reduce the inherent difficulties in matching out the aperture, an aperture mismatch loss at broadside of no greater than .6 db is required.

To provide the required performance, a Taylor, \( \bar{n}=3 \), 25 db SLL distribution is selected, resulting in an aperture gain of 37 db at 4 GHz.

The principle result of this section is the determination of an element/grid configuration which minimizes element count while holding high frequency grating lobe contributions to levels consistent with the sidelobe requirements.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>4.8 GHz ±7.5%</td>
</tr>
<tr>
<td>Scan Coverage</td>
<td>±60° in either principle plane</td>
</tr>
<tr>
<td>Aperture Gain @ 60°</td>
<td>30 db @ 4 GHz</td>
</tr>
<tr>
<td>Principle Plane Scan</td>
<td></td>
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<tr>
<td>1st Sidelobe Level</td>
<td>20 db</td>
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<tr>
<td>RMS Sidelobe Level</td>
<td>35 db</td>
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<tr>
<td>Feed Losses</td>
<td>1.5 db @ 4 GHz</td>
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<tr>
<td></td>
<td>2.8 db @ 8 GHz</td>
</tr>
<tr>
<td>Aperture Mismatch</td>
<td></td>
</tr>
<tr>
<td>Loss @ 0° Scan</td>
<td>&lt;.6 db in both bands</td>
</tr>
</tbody>
</table>

Table 3
Prescribed Array Performance
4.1 Aperture Design Trade-Offs

It is well-known\(^{(16)}\) that an equilateral triangle lattice configuration minimizes element count for a given scan requirement. As a basis for comparison, it is therefore, convenient to first consider a lattice specifically tailored to minimize element count at low frequency. This lattice and the near-in grating lobe diagrams for 4 and 8 GHz are shown in Figure 44. The lattice base is 1.780" (.602\(\lambda\) @ 4 GHz).

It is immediately apparent from Figure 44c that the six near-in high frequency grating lobes will migrate well inside the coverage sector for all scan directions and pose a potential limitation on achievable peak sidelobe level and main beam gain. This difficulty may be alleviated somewhat by appropriate selection of the element configuration and high frequency excitation modifier, \(R\), for the LSE\(_{2\theta}\) mode as discussed in section 2.2. However, it should be noted that the six grating lobe locations actually represent twelve independent beams (six each E and H with respect
Figure 44. Equilateral Triangle Lattice Configuration Which Minimizes Element Count @ 4 GHz
to the array normal). Since there are only nine design parameters (half aperture dimensions A and B; septum thickness; \( \delta \); relative permittivity, \( \varepsilon \); slab thickness, \( \delta \); ratio \( \alpha/\beta \); lattice spacings, \( d_x \) and \( d_y \); and R), it is clear that there is insufficient control for the cancellation of all beams.

The only available means of cancelling the beams residing along the v axis, the \( (0,1) \) and \( (0,-1) \) lobes is to separate the half apertures by one half the y lattice spacing \( d_y \), resulting in a scanning sub-array pattern with nulls coincident with the grating lobe. This coincidence is maintained for H-plane scanning, however degrades in all other scan planes due to the imbalance in half-aperture reflections induced by the phase taper. In the circumstance that the half apertures are spaced at less than 0.5 \( d_y \), a significant fraction of the radiated power is delivered to the E-mode \( (0,1) \) and \( (0,-1) \) beams for all scan directions, including broadside. In calculations with the half apertures separated by 0.26 \( d_y \), it was found that as much as 20% of the power was delivered to each of the two E-mode beams at broadside scan.
Cancellation of the remaining eight beams is a considerably more difficult problem, and turns out to be virtually impossible for this large lattice spacing. Again, effective reduction of radiated power into the unwanted lobes must be obtained by locating the null of a scanning subarray pattern at or near the grating lobe location. One subarray spacing has already been used to cancel the on-axis lobes; the remaining degree of freedom is therefore, the \( x \) separation of the apparent high frequency phase centers. In the event that these separations can be extended to 0.5 \( d_x \), good grating lobe rejection can be achieved. However, as a practical matter, such wide \( x \) displacements of apparent high frequency phase centers are not possible.

As was shown in section 3.2, a reasonable upper limit on aperture \( x \) dimension is in the vicinity of \( A=\lambda \) at the center of the high frequency band for moderate relative permittivity loading (\( \varepsilon_r=5 \)). Larger aperture dimensions can be achieved by reducing the permittivity, or using very thin slabs; however, such an approach is generally counter-productive in that bandwidth is reduced and H-plane scan loss is increased at high frequency due to the widening dissimilarity.
between $LSE_{10}$ and $LSE_{20}$ modes which occurs for parameter variations in these directions. Consequently, in order to achieve the wide apparent phase center displacement for this grid, it is necessary to place the slabs very close to the narrow feedguide walls, giving $\alpha/\beta$ ratios of, typically, 0.56 to 0.62. This is not a very attractive solution since it produces considerable aperture field asymmetry about the apparent phase centers, thereby invalidating the primary assumption that the high frequency aperture distributions are roughly symmetric about these phase centers. As a practical matter, then, it is impossible to kill off the eight remaining beams.

The best result obtained for the 1.78" base equilateral grid are summarized in Table 4. The element dimensions are given in the table caption. Clearly, the 1.05 db power loss to the grating lobes is intolerable, and it is necessary to reduce the grid size significantly.

As mentioned above, cancellation of off-axis high frequency grating lobes requires that the apparent phase centers be separated by roughly $0.5 \ d_x$. In addition, to maintain reasonable aperture field symmetry about these phase centers, ratios $\alpha/\beta$ should lie in the approximate range $0.8 < \alpha/\beta < 1.25$. Allowing for
<table>
<thead>
<tr>
<th></th>
<th>4 GHz</th>
<th>8 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broadside Reflection Loss (db)</td>
<td>1.18</td>
<td>1.05</td>
</tr>
<tr>
<td>*H-plane Scan Loss at 60° (db)</td>
<td>6.44</td>
<td>6.80</td>
</tr>
<tr>
<td>Power Loss to Grating Lobes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(broadside scan)</td>
<td></td>
<td>1.05</td>
</tr>
<tr>
<td>Broadside Gain Loss (db)</td>
<td>1.18</td>
<td>2.43</td>
</tr>
</tbody>
</table>

**Table 4**

Performance of Dual Frequency Element in 1.78" Base Equilateral Triangular Grid at 4 and 8 GHz. \( A = 1.480" , \) \( \beta = .412" , \) \( \delta = .358" , \) \( d_x = 1.780" , \) \( d_y = 1.540" , \) \( \alpha = .241" , \) \( \beta = .301" , \) \( \delta = .198" , \) \( \varepsilon_{r} = 5.0 \)

\*Includes cos \( \theta \) beam broadening factor.
a wall thickness of 0.062", a practical set of element and grid x-dimensions is then $A = 1.376"$, $d_x = 1.500"$, and $\beta + \delta/2 = 0.375"$, resulting in the specification of slab thickness, $\delta$, as no greater than 0.130", or $\delta/A < 0.094$ and $\alpha/\beta < 0.835$. From Figures 36 through 43, it is seen that a relative permittivity of 5 will provide the desired feedguide characteristics in both frequency bands for this aperture size with $\alpha/\beta = 1$ and $\delta/A = 0.091$. Bringing the $\alpha/\beta$ ratio into the required range produces only a small perturbation on the dispersion curves shown in Figure 41. Consequently, the selection of $\alpha = 0.247"$, $\beta = 0.309"$, and $\delta = 0.132"$ will provide the necessary control of off-axis grating lobes while keeping higher order feedguide modes well below cutoff. By requiring that the LSE$_{11}$ mode be attenuated by 8 db per wavelength at the high end of the 8 GHz band, the y aperture dimensioned is obtained as $B = 0.400"$.

The aperture/grid parameters determined so far guarantee some cancellation of the off-axis grating lobes, and it might be assumed that by reducing $d_Y$ for the original grid by the ratio 1.5/1.78 would result in an acceptable configuration, giving a 29%
increase in element count over the "optimum" number. Unfortunately this spacing is still on the large size and results in considerable power loss to the grating lobes at relatively small scan angles. It is therefore necessary to further reduce the cell along the E-plane to $d_y = 0.960"$.

The final configuration, shown in Figure 45, with its low and high frequency grating lobe diagrams has a cell area roughly half that for the grid optimized for element count at low frequency, and results in an array of 2444 elements. With five phase shifters per element, this increase seems (at first glance) rather unattractive. However, this comparison is quite misleading. The twin dielectric slab dual frequency array element concept provides for simultaneous excitation by two entirely independent feed systems, and consequently the simultaneous radiation of two independent beams at two widely separated frequency bands. If the alternatives for multifrequency operation are considered, the dual frequency element is suddenly very attractive.

One such alternative is the use of wideband elements in the grid depicted in Figure 44 to obtain simultaneous aperture usage. The obvious disadvantage
Figure 45. Triangular Grid Configuration for Dual Frequency Operation over 16% Bands Centered at 4 GHz and 8 GHz
of this scheme is that it requires eight phase shifters and four diplexers per element quartet or unit cell. In comparison with the dual frequency element, this configuration requires twice the number of low frequency controls, and half the number of high frequency controls per unit area of the array, plus diplexers.

A second alternative, in a broad sense, is to design two entirely independent single frequency systems. However, it is clear that only under very special circumstances could this system be considered a viable multifrequency concept.

Predicted principle scan plane performance for the configuration in Figure 45, is shown in Figures 46 through 54, at the end and midpoints of the 4 and 8 GHz bands. In these figures, the performance measure is taken as the power transmission coefficient into the radiating beam(s). Mainbeam scan loss is determined by adding $10 \log_{10} (\cos \theta)$ to the curve for the $(0,0)$ beam.

Figures 46, 47, and 48 show E and H plane performance at the low end, middle, and high end of the 4 GHz band, respectively, for scan out to $\sin \theta = 0.975$. 

109
Figure 46. E- and H-Plane Performance of Designed Element, 
\( f = 3.68 \) GHz
Figure 47. E- and H-Plane Performance of Designed Element, 
f = 4.0 GHz

Legend

\[ \Delta \text{ H-PLANE} \quad \text{-- -- E-PLANE} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>4.00 GHz</td>
</tr>
<tr>
<td>( a )</td>
<td>0.247 in.</td>
</tr>
<tr>
<td>( b )</td>
<td>0.309 in.</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.132 in.</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>0.000</td>
</tr>
<tr>
<td>( A )</td>
<td>1.376 in.</td>
</tr>
<tr>
<td>( B )</td>
<td>0.400 in.</td>
</tr>
<tr>
<td>( D_x )</td>
<td>1.500 in.</td>
</tr>
<tr>
<td>( D_y )</td>
<td>0.960 in.</td>
</tr>
<tr>
<td>( \text{SEP} )</td>
<td>0.032 in.</td>
</tr>
</tbody>
</table>

CUT PLANE \( \sin \theta = 0.000 \)
Figure 48. E- and H- Plane Performance of Designed Element, 
f = 4.32 GHz
Figure 49. Propagating Beam Power Levels - H-Plane Scan, $f = 7.36$ GHz
Figure 50. Propagating Beam Power Levels - H-Plane Scan
\( f = 8.0 \, \text{GHz} \)
Figure 51. Propagating Beam Power Levels - H-Plane Scan, $f = 8.64$ GHz

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle$</td>
<td>$(0, 0, 2)$</td>
</tr>
<tr>
<td>$\square$</td>
<td>$(-1, -1, 2)$</td>
</tr>
<tr>
<td>$\circ$</td>
<td>$(-1, -1, 1)$</td>
</tr>
<tr>
<td>$\diamondsuit$</td>
<td>$(-2, -1, 2)$</td>
</tr>
</tbody>
</table>

Legend:
- $F = 8.64$ GHz
- $\alpha = 0.247$ in.
- $\beta = 0.309$ in.
- $\delta = 0.132$ in.
- $\epsilon = 5.00$

Cut Plane $\sin \theta = 0.000$

A = 1.376 in.
B = 0.400 in.
$D_x = 1.500$ in.
$D_y = 0.960$ in.
SEP = 0.032 in.
**Figure 52. Propagating Beam Power Levels - E-Plane Scan,**

\[ f = 7.36 \text{ GHz} \]
Figure 53. Propagating Beam Power Levels - E-Plane Scan, \( f = 8.0 \text{ GHz} \)
Figure 54. Propagating Beam Power Levels - E-Plane Scan

f = 8.64 GHz

LEGEND

\(\Delta\) (0, 0, 1)  \(\bigcirc\) (0, -1, 1)
\(\square\) (-1, -1, 1)  \(\bigtriangleup\) (-1, -1, 2)

\(F = 8.64\, \text{GHz}\)
\(A = 1.376\, \text{in.}\)
\(\alpha = 0.247\, \text{in.}\)
\(B = 0.400\, \text{in.}\)
\(\beta = 0.309\, \text{in.}\)
\(D_x = 1.500\, \text{in.}\)
\(\delta = 0.132\, \text{in.}\)
\(D_y = 0.960\, \text{in.}\)
\(\epsilon = 5.00\)
\(\text{SEP} = 0.032\, \text{in.}\)

CUT PLANE \(\sin\phi = 1.000\)
Figure 55. Waveguide Simulator for Designed Element
In this range, no grating lobes enter real space. Maximum broadside mismatch loss in this band is less than .3 db and is readily matched out by any number of means. The E-plane fall-off out to approximately 60° is somewhat better than might be expected for the subarray array factor at these spacings, and decreases at the upper end of the band. The H-plane fall-off is typical of a planar array of rectangular apertures.

Figures 49, 50 and 51 show the H-plane power levels in the propagating beams in the 8 GHz band. To evaluate the array characteristics over this band, the LSE₁₀ and LSE₂₀ feedguide modes are assumed to be generated 2λ₉₁₀ behind the aperture plane, where λ₉₁₀ is the LSE₁₀ guide wavelength evaluated at 8 GHz. This long feedguide phase length results in considerable excitation of the (-l,-l) E-mode at the band extremes. Evidently, this path must be shortened for such wide band operation.

At midband, Figure 50, the (-l,-l) and (-l,0) grating lobes are well within the desired range of rms sidelobe level and will not result in any significant perturbation of the far-out sidelobe region. This
level of cancellation is achieved using the modifier 
\[ R = 1.2 \exp(-j14^\circ) \], which seems to also result in 
significant reduction of the H-mode \((-1,-1)\) and 
\((-1,0)\) beams at the band edges as seen in Figures 49
and 51. However, the modifier clearly has little 
effect on the E-mode beams, and these levels must be 
controlled by a proper choice of a feedguide phase 
length.

In Figure 51, it is seen that the \((-2,-1)\) H-
mode beam is heavily excited at the upper end of the 
band. However, this occurs only at the H-plane 
extreme of the scan volume, and will be of only minor 
consequence.

The 8 GHz band E-plane performance is shown in 
Figures 52 through 54. In general, the best perform-
ance occurs at the lower end of the band. Due to the 
choice of \(y\) lattice spacing, the grating lobes remain 
outside real space throughout most of the scan volume 
at this end of the band, and are only moderately 
excited upon entering. From the results at 8 and 8.64 
GHz, it is clear that this is the most effective means 
of controlling spurious beam levels in this plane.

At the high end of the band, the \((-1,-1)\) E 
and H mode beams are very heavily excited and will
have considerable impact on the general sidelobe level for scanning beyond 25°. However, except for the small dips occurring near (-1,-1) lobe incipience, there is little effect on main beam gain. Again, as for the H-plane results, a general improvement in grating lobe levels may be expected for shorter feedguide phase lengths.

4.2 Experimental Evaluation of the Bifurcated Twin Dielectric Slab Loaded Rectangular Waveguide Dual Frequency Element

The dual frequency element design shown in Figure 45 was built and tested in H-plane waveguide simulators over an 8% frequency band centered at 4.6 GHz and a 16% band centered at 8 GHz. In general, the experimental results were in excellent agreement with predictions.

The simulators are shown in Figure 55. They are constructed of brass and are soft soldered. A single element section and parallel wall simulator section is used for both bands, resulting in near broadside simulation in the upper band, and wide angle scan simulation in the lower band. The parallel wall section has 2.250" x .960" cross-section and provides imaging as shown in Figure 56. In the low frequency band, only the TE₁₀ mode propagates in the simulator. In the high frequency band, up to eight modes can propagate.
Figure 56. Simulator Imaging of Element Section
Figure 57. Measured Simulator Port, Impedance 4.0 - 4.32 GHz
Sampled in 80 MHz Increments
The element section, shown in Figure 56 with the simulator imaging, consists of two half elements and two quarter elements, resulting in $19.1^\circ$ and $41^\circ$ H-plane scan simulations at the centers of the high and low bands, respectively. The dielectric slabs are styacast HiK, $\varepsilon_r = 5$. The element section is 7.2" long ($2\lambda_g$ at 4 GHz), with tapered 3.6" 300Ω card loads inserted at the rear of the section on either side of the dielectric slabs and the feedguide mid-plane. The long tapers are necessary to eliminate rearward radiation and reduce reflections at the load discontinuities.

Figure 57, shows measured results in the upper half of the low frequency band. Results in the lower half of the band were not obtained due to the frequency limitation of the network analyzer. To ensure measurement accuracy, the experimental band was sampled discretely in 80 MHz increments, and a short circuit reference was established at each frequency step. Measured reflection coefficient magnitude is in the range .34 to .41* with the peak and minimum at 4.24 and 4.08 GHz, respectively, and is typical of the variation in H-plane gain loss with scan observed in many broadside matched phased arrays. The phase of the reflection coefficient is nearly constant.

*For H-plane scan, it is permissible to represent feedguide port pairs (upper and lower element halves) by a single "effective port". Consequently, the reflection coefficient magnitude at the simulator port is a factor $\varepsilon_r^{\frac{1}{2}}$ greater than that in either half element.
Measured and predicted simulator results are shown in Figure 58. The calculated results lie well within the range of the measurements. Also shown in the figure is a least mean square straight line fit to the experimental data and \( \sqrt{1 - \cos^2 \theta} \), where \( \theta \) is the simulator angle. These two curves, in comparison with the predictions, show quite clearly that the analytical model provides an excellent description of the array. The scatter of experimental data about the theoretical results is due primarily to errors in element section fabrication which resulted in small air gaps between the feedguide broadwalls and the dielectric primarily in the interior of the section. By assuming a maximum reflection coefficient magnitude of 0.035 at the internal discontinuities, the discrepancies in the results are accounted for throughout the band.

In the 8 GHz band, up to eight waveguide modes will propagate in the simulator. Below 8.08 GHz, the first five simulator modes will propagate. However, for a properly fabricated element section, only the TE\(_{10}\) mode is excited. Above 8.08 GHz, three of the eight modes will be excited by the array interface. These
Figure 58. Comparison of Measured and Predicted Reflections Coefficient Magnitude at the Simulator Port, 4.0 - 4.32 GHz
are the TE\textsubscript{10}, TE\textsubscript{21}, and TM\textsubscript{21} modes, corresponding to
the (0,0); H mode (1,0) and (-1,-1); and E mode (1,0)
and (-1,-1) beams, respectively.

The equivalent network representing the simulator
discontinuity is a 3-port below 8.08 GHz, and a 5-
port above 8.08 GHz. Consequently, below the (2,1)
waveguide mode cut-off, the measured reflection coe-
cfficient at the simulator port is given as

\[(91) \quad |S_{11}|^2 = -1 + |S_{22}|^2 + |S_{33}|^2 + |S_{23}|^2 \frac{Y_2}{Y_3} + |S_{32}|^2 \frac{Y_3}{Y_2}\]

where the ports are defined in Figure 59. Above the
cut-off frequency, the simulator dominant mode self
reflection term is complicated function of the self
and cross coupling scattering parameters of the re-
main ing ports in the network. Since, in the analysis
presented here, the interface scattering blocks
\(S_{12}\) and \(S_{22}\) are unnecessary for the determination of
element performance, they are not calculated*; and it

*The calculation of \(S_{12}\) and \(S_{22}\) requires prohibitively
large amounts of computer core. Roughly 120K, decimal,
works are required for \(S_{22}\) for the convergence radii
considered here.
Figure 59. Port Definitions for 7.32 - 8.08 GHz Simulator
and it is therefore, not possible to compare theoretical and measured results above 8.08 GHz.

Measured results in the 7.32 to 8.64 GHz band are shown in Figure 60. As for the 4 GHz band, the experimental band was sampled discretely to ensure measurement accuracy. The sampling rate is roughly every 160 MHz, with an exact short circuit reference established for each sample point. The reflection coefficient magnitude is in the range .35 to .46, and the phase is nearly constant.

In the measurements, no attempt was made to load terminate the higher order modes of the simulator. This leads to an inherent error in the results which is associated with the reactive termination of the higher order modes by the simulator flare transition. This error should be insignificant for a reasonably well fabricated element section since the higher order beams are only weakly excited above 8.08 GHz. However, as discussed above, some irregularities in element section fabrication did occur, resulting in weak excitation of the TE_{20} simulator mode.
Figure 60. Measured Simulator Port Impedance 7.32 - 8.64 GHz
Sampled in 160 MHz Increments
A comparison of measured and predicted results in the 7.32 to 8.00 GHz region of the experimental band is shown in Figure 61. Also shown in the figure is a straight line least square fit to the experimental data. As in the 4 GHz band, the theoretical data falls within the range of the experimental results, and the least square fit has approximately the same slope and magnitude as the calculated curve. The maximum deviation of measured reflection coefficient from the theoretical value is .051 and occurs at 8 GHz.
Figure 61. Comparison of Measured and Predicted Reflection Coefficient Magnitude at the Simulator Port, 7.32 - 8.00 GHz
5.0 ELEMENT EXCITER DESIGN

The design of an exciter for the slab loaded dual frequency element presents a particularly intriguing problem. As demonstrated in section 3, the dominant mode dispersion in the inhomogeneously loaded guide is roughly linear with frequency for practical element configurations. However, the slope of $\gamma/k$ is, in general, considerably greater than unity. Consequently, the use of a bidirectional exciter, such as a stub or slot, requires load termination at the back of the feedguide to ensure proper aperture excitation, and results in a 3dB power loss.

5.1 Exciter Concept

A unique uni-directional exciter concept, shown in Figure 62, has been developed which alleviates this difficulty. The exciter consists of three stripline fed flared notch antennas\(^{(2,3,4)}\) configured to provide maximum coupling to the driven feedguide modes in either band. The center probe is the low frequency exciter, and is placed well in advance of the high frequency exciters (outer two probes) to maximize the low frequency isolation. The high frequency exciters butt directly into the dielectric slabs.
Figure 62. Stripline Fed Notch Exciter for Twin Dielectric Slab Loaded Rectangular Waveguide Dual Frequency Array Element
The basic exciter is shown in Figure 63. It is formed by symmetrically etching the outer conductor of symmetric stripline board to form flared notches which terminate with maximum aperture at the board edge and short circuit at the notch bottom. The stripline center conductor is configured to cross the notch region at a right angle to the notch center line, and terminates in an open circuit. By appropriately selecting the distance $x_2$, from the center conductor center line to the notch short circuit, and the distance $y_2$, from the notch edge to the center conductor open circuit, the exciter is matched in the feedguide environment over the operating band.

A particularly attractive feature of the flared notch exciter is that the stripline board is plugged into the back of the element. This considerably simplifies feed design. In addition, the phase shifter and exciter may be integrated into a single unit.

The exciter geometry shown in Figure 62 results in natural isolation between the low and high frequency probes in the low frequency band. Since the stripline outer conductor is etched only near the board edge, the center probe provides a short circuit bifurcation of the guide in the
Figure 63. Basic Notch Exciter
vicinity of the high frequency probes. Consequently, the outer probes are effectively below cut-off guide in the low frequency band, and their excitation is governed by the longitudinal separation between center probe notch bottom and leading edges of the outer probes.

In the high frequency band, the outer probes couple into the \( LSE_{10} \) mode of the loaded half width guide and some natural probe isolation is achieved due to the low field strength along the outer conductors of the center probe. However, the discontinuity at the center probe termination results in scattering back into the low frequency port. This difficulty may be removed by introducing an appropriately placed shorting stub along the low frequency probe center conductor, but the impact of this technique on high frequency aperture field distributions has not been determined and remains a design problem for future consideration.

5.2 Experimental Investigation of the Stripline Fed Flared Notch Exciter

Experimental investigation of the stripline fed flared notch exciter and exciter design were initiated during the contract period. In general, the anticipated results were obtained, and demonstrate the viability of the concept.
Notch exciter designs were developed to provide better than 2:1 VSWR looking into the load terminated feedguide over greater than 10% band widths, and greater than 50dB probe isolation in the low frequency band.

The baseline exciter design, a scaled version of a previously designed antenna element, is shown in Figure 64. The dielectric is Rexolite. The etched notch region has uniform width of .058" for a length of .154" from the notch short circuit, and then flares smoothly to a width of .400" at the board edge. The centerline of the stripline center conductor crosses the notch .135" from the short circuit. In the vicinity of the notch, the stripline impedance is 80 ohms, and transitions smoothly to 50 ohms at the connector junction. For the experimental investigation, only the center conductor open circuit location, \( y_2 \), was varied.

Five exciter elements, differing only in open circuit location, were fabricated and tested using an HP Automatic Network Analyzer (ANA). The values of \( y_2 \) for these elements, designated P1 through P5, are given in Table 5. In the 4GHZ band, exciter P5 produced the best overall match. In the 8 GHZ band, P1 gave best results.

Measured VSWR for the P5 exciter is shown in Figure 65.
Figure 64. Baseline Notch Exciter Design
<table>
<thead>
<tr>
<th>Exciter</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.070&quot;</td>
</tr>
<tr>
<td>P2</td>
<td>0.090&quot;</td>
</tr>
<tr>
<td>P3</td>
<td>0.110&quot;</td>
</tr>
<tr>
<td>P4</td>
<td>0.130&quot;</td>
</tr>
<tr>
<td>P5</td>
<td>0.150&quot;</td>
</tr>
</tbody>
</table>

Table 5
Open Circuit Stub Lengths for Experimental Exciters
Figure 65. Measured VSWR for the P5 Exciter, 4 to 6 GHz
The measurement band is 4 to 6 GHz. From 4 to 4.7 GHz, the exciter is poorly matched, and is operating in the vicinity of its low frequency cutoff. From 4.7 GHz to 5.75 GHz, the mismatch is below 2:1. The low frequency cut-off phenomenon is known to occur when the open and short circuit stub lengths (x and y in Figure 64) become electrically short. Consequently, by increasing the notch depth and center conductor stub length, the exciter operating band may be readily reduced to the band of interest.

The ripple in the well-matched region of the experimental band may also be largely eliminated by judicious selection of the stub lengths. In the study of isolated notch antennas, it was found that similar ripple in reflection coefficient occurred when the electrical lengths of the stubs were significantly different. Since the spectrum of guided waves in the region of the notch short circuit may be determined in a straight forward manner, the proper stub length ratio is obtainable by either analytical or experimental means.

Measured VSWR for the P1 exciter looking into load terminated half width loaded guide is shown in Figure 66. The measurement band is 7.5 to 8.5 GHz. The match is below 2:1 throughout the band. It is evident from the figure that
Figure 66. Measured VSWR for the Pl Exciter, 7.5 to 8.5 GHz
the low end of the experimental band is quite close to the low frequency cutoff for the exciter and that some stub lengthening is required to extend the bandwidth to cover the full 16% operating band. The discontinuity at 8 GHz is due to the automatic head change on the ANA. While the ripple shown in the Figure is not large, some improvement can be achieved by adjusting the stub length ratio.

Probe isolation is determined by exciting a single high frequency probe in the three probe load terminated configuration and measuring return power in the center probe. In the high frequency band, the measured result corresponds, roughly, to equal excitation of $LSE_{10}$ and $LSE_{20}$ modes in the probe free region of the test device, and consequently is an improper excitation for high frequency operation. Below the loaded halfwidth guide cut-off frequency, the measurement approximates the band isolation at broadside.

Measured isolation from 3.5 to 8.5 GHz is shown in Figure 67. The center probe is the P5 exciter and is inserted into the guide 1.80" beyond the P1 exciters. The high frequency exciters are butted against the dielectric slabs and arranged such that the slab and exciter midplanes are coplanar. Below 5.8 GHz (the halfwidth loaded guide cutoff frequency) the measured isolation is greater than 40dB, and
Figure 67. Measured Probe Isolation, 3.5 to 8.5 GHz
exceeds 50 dB from 3.5 to 5.0 GHz. As anticipated, the isolation in dB shows a roughly linear decrease from 5.5 to 5.8 GHz. Above 5.8 GHz, the isolation rapidly degenerates to 12 dB at 8.5 GHz.

The weak isolation in the 7.5 to 8.5 GHz band is due to the LSE₁₀ mode constituent of the waveguide field in the plane of the center probe board edge. As was shown in section 3.2, the LSE₁₀ relative modal electric field strength along the loaded guide centerline will be on the order of .3 to .4 for a slab relative permittivity of 5. Consequently, improved high frequency isolation can be obtained by either tapering the center probe board thickness near the edge, or by introducing an appropriately located shorting stub along the low frequency probe center conductor. In general, the first alternative seems best since it minimizes aperture perturbations, but may be impractical. The second approach has several difficulties associated with perturbations of the field at the radiating aperture, but is readily implemented. Clearly, the high frequency isolation presents a problem which will only be resolved through further study.
6.0 CONCLUSIONS

The bifurcated twin dielectric slab loaded rectangular waveguide has been shown to be a viable candidate as a dual frequency array element which results in a considerable reduction in electronic components relative to other multi-frequency aperture techniques. In comparison with wide band elements, the bifurcated twin dielectric slab loaded rectangular waveguide element requires 21\% fewer controls per unit aperture area for equivalent scan and gain specifications.

To provide the multi-mode aperture control required for dual frequency operation, a unidirectional stripline fed notch exciter concept has been investigated which results in greater than 50dB isolation between high and low frequency probes in the low frequency operating band. Preliminary exciter designs have been shown to remain well matched over greater than 10\% bandwidths.

An element design for operation over 16\% frequency bands centered at 4 and 8 GHz has been fabricated and tested in waveguide simulators. Measured results are in excellent agreement with theoretical predictions in the 4.0 to 4.32 GHz and 7.32 to 8.08 GHz bands. Measured results outside these bands were not obtained due to limitations of measurement.
equipment (below 4 GHZ) and computational practicallity (above 8.08 GHZ), but are expected to show similar close agreement with theory in small array testing.
APPENDIX A

EVALUATION OF THE $E_{pqr,nm}$ FOR RECTANGULAR LATTICE

The coefficients $E_{pqr,nm}$ are defined by the integral

$$E_{pqr,nm} = \int_{b_1}^{B+L_1} \int_{-A/2}^{A/2} dy \int_{-A/2}^{A/2} dx \, e_{nm}^p(x,y) \cdot e_{a_{pqr}}^*(x,y)$$

where $A$ is the x dimension of the guide, $B$ is the y dimension of the guide, $b_1$ is the half-thickness of the septum, $e_{nm}^p(x,y)$ is an LSM ($p = \prime$) or LSE ($p = \"$) feedguide electric mode function, and $e_{a_{pqr}}(x,y)$ is a cell guide electric mode function which is an E mode with respect to the array normal ($r = 1$) or an H mode ($r = 2$). By appropriate choice of ordering in both feedguide and cell guide regimes, single subscripts may be used to identify the modes. These indices are taken as $i$ for the feedguide mode modes, and $\sigma$, for the cell modes, giving

$$E_{\sigma,i} = \int_{b_1}^{B+L_1} \int_{-A/2}^{A/2} dy \int_{-A/2}^{A/2} dx \, e_{i}(x,y) \cdot e_{a_{\sigma}}^*(x,y)$$

Expressions for $e_{i}(x,y)$ are given in chapter 3, and expressions for $e_{a_{\sigma}}(x,y)$ are given in equation 8.
In all, there are eight forms of the integral $E_{\beta\sigma}$, corresponding to the possible inner products of feedguide and cell guide modes. These are:

1. LSM-Symmetric Modes with E-Modes
2. LSM-Symmetric Modes with H-Modes
3. LSM-Antisymmetric Modes with E-Modes
4. LSM-Antisymmetric Modes with H-Modes
5. LSE-Symmetric Modes with E-Modes
6. LSE-Symmetric Modes with H-Modes
7. LSE-Antisymmetric Modes with E-Modes
8. LSE-Antisymmetric Modes with H-Modes

Since the integrals possess many terms in common, these terms are defined first and will be used as simple variables to shorten the expressions. These terms are:

\begin{align*}
R_1 &= \beta \frac{\sin(k_x \kappa + \kappa_i^\beta)}{(k_{\sigma}x + \kappa_i^\beta)} \\
R_2 &= \beta \frac{\sin(k_{\sigma}x - \kappa_i^\beta)}{(k_{\sigma}x - \kappa_i^\beta)} \\
R_3 &= \frac{\cos(k_{\sigma}x + \kappa \epsilon_i^\delta)}{k_{\sigma}x + \kappa \epsilon_i^\delta} \\
R_4 &= \frac{\cos(k_{\sigma}x - \kappa \epsilon_i^\delta)}{k_{\sigma}x - \kappa \epsilon_i^\delta}
\end{align*}
\begin{align*}
R_5 &= \frac{\sin(k \cdot x_0 + \kappa \cdot i) \delta}{(k \cdot x_0 - \kappa \cdot i) \delta} \\
R_6 &= \frac{\sin(k \cdot x_0 - \kappa \cdot i) \delta}{(k \cdot x_0 - \kappa \cdot i) \delta} \\
R_7 &= \frac{\cos(k \cdot x_0 - \kappa \cdot i) \delta}{k \cdot x + \kappa \cdot i} \\
R_8 &= \frac{\cos(k \cdot x_0 - \kappa \cdot i) \delta}{k \cdot x + \kappa \cdot i} \\
R_9 &= \alpha \frac{\sin(k \cdot x_0 - \kappa \cdot i) \alpha}{(k \cdot x_0 - \kappa \cdot i) \alpha} \\
R_{10} &= \alpha \frac{\sin(k \cdot x_0 - \kappa \cdot i) \alpha}{(k \cdot x_0 - \kappa \cdot i) \alpha} \\
S &= e^{\frac{j k_y b_1}{m \parallel} (1 - e^{-k_x b_1})} \cos m \parallel \\
&= e^{\frac{j k_y b_1}{m \parallel} (1 - e^{-k_x b_1})} \cos m \parallel \\
&= \frac{m \parallel}{B} - k_y b_1 \\
\end{align*}

The coefficients $B'_1$, $B'_2$, $C'$, $E'_1$, $E'_2$, $F'$, $B''_1$, $B''_2$, $C''$, $E''_1$, $E''_2$, $F''$, $N'_1^S$, $N'_i^A$, $N''_1^S$, and $N''_i^A$ which appear in the following expressions are defined in Appendix D.

1. LSM-Symmetric Modes with E-Modes
\[ (A-3) \quad E_{\alpha,i} = \frac{S}{N_1^s \sqrt{\epsilon_1 k_\epsilon_0}} \left\{ k_{x,0} I_1 + k_{y,0} I_2 \right\} \]

where

\[ (A-4) \quad I_1 = \frac{\pi l} B \left[ R_1 + R_2 + \frac{1}{\epsilon_r} B_1 \right] \left[ R_3 + R_4 - \frac{2k_{x_0}}{k^2 - k_{\epsilon i}^2} \right] \sin k_{x_0} \beta \]

\[ + \frac{1}{\epsilon_r} B_1 \left[ R_5 + R_6 \right] \cos k_{x_0} \beta \]

\[ + \frac{1}{\epsilon_r} B_2 \left[ \frac{2k_{x_0}}{k^2 - k^2_{\epsilon i}} + R_3 - R_4 \right] \]

\[ + \frac{1}{\epsilon_r} B_2 \left[ R_5 - R_6 \sin k_{x_0} \beta \right] \]

\[ + C' \left[ \frac{2k_{x_0}}{k^2 - k_{\epsilon i}^2} - R_7 - R_8 \right] \sin k_{x_0} \beta \]

\[ + C' \left[ R_3 + R_1 \right] \cos k_{x_0} \beta \]

and

\[ (A-5) \quad I_2 = -\frac{\pi l}{B} k_{y,0} \left\{ \frac{k_{i}}{k^2 - k_{\epsilon i}^2} \right\} \left[ R_2 - R_1 \right] \]

\[ + \frac{k_{\epsilon i} B_1}{k^2_{\epsilon x} - k^2_{\epsilon i}} \left[ R_4 - R_3 - \frac{2k_{\epsilon i}}{k^2_{x_0} - k^2_{\epsilon i}} \right] \sin k_{x_0} \beta \]
\[
\begin{align*}
&+ \frac{\kappa \epsilon_i B_i}{k^2 - \kappa^2 \epsilon_i} [R_6 - R_5] \cos k x_0 \beta \\
&+ \frac{\kappa \epsilon_i B_i}{k^2 - \kappa^2 \epsilon_i} \left[ \frac{2k x_0}{k^2 - \kappa^2 \epsilon_i} - R_3 - R_4 \right] \cos k x_0 \beta \\
&+ \frac{\kappa \epsilon_i B_i}{k^2 - \kappa^2 \epsilon_i} \left[ R_5 + R_6 \right] \sin k x_0 \beta \\
&+ \frac{\kappa \epsilon_i B_i}{k^2 - \kappa^2 \epsilon_i} \left[ \frac{2\kappa_i}{k^2 - \kappa^2 \epsilon_i} + R_7 - R_8 \right] \sin k x_0 \beta \\
&+ \frac{\kappa \epsilon_i B_i}{k^2 - \kappa^2 \epsilon_i} \left[ R_10 - R_9 \right] \cos k x_0 \beta \\
&+ \frac{\kappa \epsilon_i B_i}{k^2 - \kappa^2 \epsilon_i} \left[ R_10 - R_9 \right] \cos k x_0 \beta \\
\end{align*}
\]

2. LSM Symmetric Modes with H-Modes.

\begin{equation}
(A-6) \quad E_{\sigma, i} = \frac{S}{N_1 s \sqrt{c} |k_{\tau 0}|} \{k_{y \sigma} I_i - k_{x \sigma} I_i\}
\end{equation}

where $I_i$ and $I_i^\dagger$ are defined in equations (A-4) and (A-5), respectively.

3. LSM-Antisymmetric Modes with E-Modes
\begin{equation}
E_{\sigma,i} = \frac{S}{N_i A \sqrt{G} |k_{t\sigma}|} \left\{ k_{x\sigma} I_1 + k_{y\sigma} I_2 \right\}
\end{equation}

where

\begin{align*}
I_1 &= -j \frac{m_{\Pi}}{B} \left\{ R_2 - R_1 + \frac{1}{\varepsilon_r} E_1 \left[ \frac{2k_{x\sigma}}{k_{x\sigma} - k_{i\sigma}} - R_3 - R_4 \right] \cos k_{x\sigma} \theta \right. \\
&\quad + \frac{1}{\varepsilon_r} E_1' \left[ R_5 - R_6 \right] \sin k_{x\sigma} \theta \\
&\quad + \frac{1}{\varepsilon_r} E_2' \left[ \frac{2k_{i\sigma}}{k_{x\sigma} - k_{i\sigma}} + R_3 - R_4 \right] \sin k_{x\sigma} \theta \\
&\quad + \frac{1}{\varepsilon_r} E_2' \left[ R_5 - R_6 \right] \cos k_{x\sigma} \theta \\
&\quad + F' \left[ R_7 + R_8 \right] - \frac{2k_{x\sigma}}{k_{x\sigma} - k_{i\sigma}} \cos k_{x\sigma} \theta \\
&\quad + F' \left[ R_9 + R_{10} \right] \sin k_{x\sigma} \theta
\end{align*}

and

\begin{equation}
I_2 = j k_{x\sigma} \frac{m_{\Pi}}{B} \left\{ \frac{\kappa_{i\sigma}}{k_{x\sigma} - k_{i\sigma}} \left[ R_1 + R_2 \right] \\
+ \frac{\kappa_{i\sigma} E_1}{k_{x\sigma} - k_{i\sigma}} \left[ \frac{2k_{i\sigma}}{k_{x\sigma} - k_{i\sigma}} + R_3 - R_4 \right] \cos k_{x\sigma} \theta \right\}
\end{equation}
\[ + \frac{k^2 \epsilon_i E_i}{k^2 - k^2} \left[ R_6 - R_5 \right] \text{sink}_x \sigma \beta \]

\[ + \frac{k^2 \epsilon_i E_i}{k^2 - k^2} \left[ R_3 + \frac{2k^2}{k^2 - k^2} \right] \text{sink}_x \sigma \beta \]

\[ + \frac{k^2 \epsilon_i E_i}{k^2 - k^2} \left[ R_5 + R_6 \right] \text{cosk}_x \sigma \beta \]

\[ + \frac{k^2 F'}{k^2 - k^2} \left[ R_8 - R_7 - \frac{2k^2}{k^2 - k^2} \right] \text{cosk}_x \sigma \frac{A}{2} \]

\[ + \frac{k^2 F'}{k^2 - k^2} \left[ R_{10} - R_9 \right] \text{sink}_x \sigma \frac{A}{2} \]

4. LSM-Antisymmetric Modes with H-Modes.

\[ (A-10) \quad E_{\sigma, i} = \frac{S}{N_1 \sqrt{C}} \left\{ k_y \sigma I_1^i - k_x \sigma I_1^i \right\} \]

where \( I_1^i \) and \( I_1^i \) are defined in equations \((A-8)\) and \((A-9)\), respectively.
5. LSI: Symmetric Modes with E-Modes

\[(A-11) \quad E'_i = \frac{S}{N'_i} \frac{k}{\sqrt{c}} k_{y'} I'_y\]

where

\[(A-12) \quad I'_y = j y \bigg\{ R_1 + R_2 + \frac{2k_i}{k_{x_0} - \kappa^2} \bigg( R_1 - R_4 \bigg) \cos k x_0\beta \bigg\} \]

\[+ B' \bigg[ R_6 - R_5 \bigg] \sin k x_0\beta \bigg]\]

\[+ B' \bigg[ R_4 + R_5 \bigg] \sin k x_0\beta \bigg]\]

\[+ B' \bigg[ R_5 + R_6 \bigg] \cos k x_0\beta \bigg\} \]

\[+ C' \bigg[ R_6 - R_5 \bigg] \cos k x_{z_0} A \bigg]\]

\[+ C' \bigg[ R_4 + R_5 \bigg] \sin k x_{z_0} A \bigg]\]
6. LSE-Symmetric Modes with H-Modes

\[ E_{\sigma,i} = \frac{S}{N_1 N_i A \sqrt{\sigma} |k_{t0}|} (-k_{\sigma} I^i_1) \]

where \( I^i_1 \) is defined in equation (A -12).

7. LSE-Antisymmetric Modes with E-Modes

\[ E_{\sigma,i} = \frac{S}{N_1 A \sqrt{\sigma} |k_{t0}|} (j k_{\sigma} I^i_1) \]

where

\[ I^i_1 = j k_{\sigma} \{ R_2 - R_1 + E^i_2 [R_4 - R_3 - \frac{2k_{\sigma} i \epsilon_i}{k^2 - \kappa^2}] \} \sin k_{x0} \beta \]

\[ + E^i_2 [R_6 - R_5] \cos k_{x0} \beta \]

\[ + E^i_1 [R_3 + R_4 - \frac{2k_{\sigma} \epsilon_i}{\kappa_{x0}^2 - \kappa^2}] \cos k_{x0} \beta \]

\[ + E^i_1 [R_3 + R_4 - \frac{2k_{\sigma} \epsilon_i}{\kappa_{x0}^2 - \kappa^2}] \cos k_{x0} \beta \]
+ $E_\theta [-(R_5 - R_6) \text{sink}_x \theta$}

+ $F'' \left[ \frac{2\kappa_i}{\kappa_x \sigma - \kappa_i^2} + (R_7 - R_8) \text{sink}_x \sigma \right]$

+ $F'' [R_{10} - R_9] \cos k \sigma \frac{A}{x \sigma}$

8. LSE-Antisymmetric Modes with H-Modes

(A-16) $E_{\phi, i} = \frac{S}{N_1 A \sqrt{\sigma} |k_\phi \sigma|} (-jk_x \sigma I''_\phi)$

where $I''_\phi$ is defined in equation (A-15)
APPENDIX B
DERIVATION OF DIFFERENTIAL EQUATION RELATING \( \epsilon \) AND \( \mu \)

The homogeneous Maxwell field equations are

\[(B-1) \quad \nabla \times \mathbf{E}(x) = -j\omega \mu \mathbf{H}(x)\]

\[(B-2) \quad \nabla \times \mathbf{H}(x) = j\omega \epsilon \mathbf{E}(x)\]

To separate out longitudinal components (\( z_0 \) directed) take vector and scalar products of \((B-1)\) and \((B-2)\) with \( z_0 \).

Thus

\[(B-3) \quad j\omega \mu \mathbf{H}(x) \times z_0 = z_0 \times (\nabla \times \mathbf{E}(x)) = -\frac{\partial}{\partial z} \mathbf{E}(x) + \nabla E_z(x)\]

\[= \frac{\partial}{\partial z} \left( z_0 E_z(x) + \hat{e}(x) \right) + \left( \nabla \times z_0 \frac{\partial}{\partial z} \right) E_z(x)\]

\[= \nabla E_z(x) - \frac{\partial}{\partial z} \hat{e}(x)\]
where \( \hat{e}(x) \) is the transverse to z electric field.

Similarly,

\[
(B-5) \quad j\omega \varepsilon z \times E(x) = \nabla_t H_z(x) - \frac{\partial}{\partial z} \hat{h}(x)
\]

\[
(B-6) \quad j\omega \varepsilon z(x) = \nabla_t \cdot (H \times Z_0)
\]

Substituting for \( E_z(x) \) in (B-3) from (B-6) gives:

\[
(B-7) \quad -\frac{\partial}{\partial z} \hat{e}(x) = j\omega \mu [I + \frac{\nabla_t \nabla_t}{k^2}] \cdot (\hat{h}(x) \times Z_0)
\]

Recognizing that

\[
\hat{e}(x) = e^{-jyz} \hat{e}(x,y)
\]

\[
\hat{h}(x) = e^{-jyz} \hat{h}(x,y)
\]

for uniform (in \( z_0 \)) media, (B-7) reduces to
(B-8) \[ \frac{Y}{\omega} e(x,y) = [\mathbf{I} + \frac{\nabla^2}{k^2}] \cdot (h(x,y) \times Z_o) \]

And from (B-4) and (B-5)

(B-9) \[ \gamma h(x,y) = \omega e \left[ \mathbf{I} + \frac{\nabla^2}{k^2} \right] \cdot (Z_o x e(x,y)) \]

For the inhomogeneously filled, uniform in \( Z_o \) waveguide, uncoupled modes will be either LSE \( (e'' = 0) \) or LSM \( (h_x' \equiv 0) \). For LSE modes, solution of (B-8) gives

\[ e'' (x,y) = - \frac{\omega Y_m}{k^2 - k_x^2} h'' (x,y) \]

\( e'' \) is related to \( h'' \) by an impedance. It is convenient, therefore, to define a modal admittance \( Y'' \), such that

\[ e'' (x,y) = - h'' (x,y) \]

and

\[ \int \int dA (h \times Z_o) \cdot e_j = \delta_{ij} \]
In this manner, (B-8) may be rewritten as

\begin{equation}
(B-10) \quad \gamma Z'' e''(x,y) = \omega \mu [I + \frac{\nabla t \cdot \nabla t}{k^2}] \cdot (h''(x,y) x z_o)
\end{equation}

Similarly

\begin{equation}
(B-11) \quad \gamma Y' h'(x,y) = \nu \epsilon [I + \frac{\nabla t \cdot \nabla t}{k^2}] \cdot (z_o \times e'(x,y))
\end{equation}
APPENDIX C

ORTHONORMALIZATION OF FEEDGUIDE MODE FUNCTIONS

Let $E_n, H_n$ and $E_m, H_m$ be linearly independent characteristic solutions of Maxwell's source free equations in the cylindrical guide shown in Figure D-1. The guide is uniform in $z$, resulting in $z$ dependencies given by

$$e^{-j\gamma_n z}, e^{-j\gamma_m z}$$

Assuming a lossless region, then the conjugates of the characteristic solutions are also solutions of Maxwell's Equations. Consider the curl equations

\begin{align*}
(c-1a, b) \quad & \nabla \times E_n = -j\omega H_n \\
& \nabla \times E_n^* = j\omega H_n^*
\end{align*}

\begin{align*}
(c-1c, d) \quad & \nabla \times H_n = j\omega E_n \\
& \nabla \times H_n^* = -j\omega E_n^*
\end{align*}

where $*$ denotes conjugation. Taking scalar products of (C-1a,b) with $H_m^*$ and $H_m^*$, respectively, and adding, gives,

\begin{align*}
(c-2) \quad & H_m^* \cdot \nabla \times E_n + H_n \cdot \nabla \times E_n^* = 0
\end{align*}

Similarly, from (c-1c,d)
Adding (C-2) and (C-3) and manipulating results in

\[(C-4) \quad 0 = \nabla_t \cdot (E_m^{*} \times H_n - E_n^{*} \times H_m) + \frac{\partial}{\partial z} (E_m^{*} \times H_n - E_n^{*} \times H_m)\]

where \(\nabla_t\) is the transverse gradient operator, and the longitudinal gradient has been specifically shown. Since the entire \(z\) dependence is embodied in the exponentials, (C-4) becomes

\[(C-5) \quad \nabla_t \cdot (E_m^{*} \times H_n - E_n^{*} \times H_m) = j(\gamma_n - \gamma_m) z_0 \cdot (E_t^* x H_t + E_t x H_t^*)\]

where the transverse field is indicated by the subscript \(t\). In equation (C-5) the longitudinal components of the fields have been ignored on the right hand side due to the \(z_0\) operator. Let

\[(C-6) \quad E_{t_n} = e_n(x,y) e^{-j\gamma_n z}\]

and similarly for the other explicitly transverse field quantities (subscript \(t\)). Then, application of the divergence theorem (\(z\)-dependent integrals cancel out) results in
where $S$ is the cylindrical cross-section and $d\sigma$ is the differential unit of transverse area.

The fields $E_n$, $H_n$, $E_m$, and $H_m$ are assumed to be linearly independent. Hence $\gamma_n \neq \gamma_m$, and (C-7) may be rewritten as

\[
\int \int_{S} \mathbf{z} \cdot (\mathbf{e}_m \times \mathbf{h}_n + \mathbf{e}_n \times \mathbf{h}_m^*) d\sigma = 0
\]  

Since the direction of propagation ($\pm z$) should not affect the result (C-8), consider the $z$ dependencies

\[
e^{-j\gamma_n z}, e^{+j\gamma_m z}
\]

Then, by entirely equivalent steps,

\[
(C-9) \quad (\gamma_n + \gamma_m) \int \int_{S} \mathbf{z} \cdot (\mathbf{e}_m \times \mathbf{h}_n - \mathbf{e}_n \times \mathbf{h}_m^*) d\sigma = 0
\]

or, for $\gamma_n \neq -\gamma_m'$,

\[
(C-10) \quad \int \int_{S} \mathbf{z} \cdot (\mathbf{e}_m \times \mathbf{h}_n - \mathbf{e}_n \times \mathbf{h}_m^*) d\sigma = 0
\]

Adding and subtracting (C-8) and (C-9) results in
Equations (C-11) and (C-12) are the desired mode orthogonality relations.

In the instance that \( \gamma_n = \gamma_{m'} \) then, assuming the eigenvalues are not degenerate, equation (C-7) is satisfied independent of the value of the integral. From equation (C-10), with \( m + n \)

\[
(C-13) \quad e_n^* \times h_n = e_n \times h_n^*
\]

Substituting (C-13) into

\[
(C-14) \quad \int_S \int s \cdot (e_n^* \times h_n + e_n \times h_n^*) d\sigma = \text{constant}
\]

yields the desired mode function normalization integral

\[
(C-15) \quad \int_S \int e_n \cdot (h_n^* \times z_0) d\sigma = \text{constant}
\]

If the constant in equation (C-15) is taken as 1, then
\[ E = \sum_{i} V_i e_i \]
\[ H = \sum_{i} I_i h_i \]

\( V_i I_i^* \) has units of power.

Using the expressions for \( e^r_i \) and \( h^r_i \) \( (r', r) \) from section 4 in equation (C-15) with unit constant, and rearranging results in the following integrals for the normalization constant \( N^r_i \):

1. LSM symmetric modes \( (r'=r) \)

\[
\left( N_{\text{sym}}^{s_i} \right)^2 = \int_{-\lambda/2}^{\lambda/2} dx \int_0^{\pi} dy \frac{1}{r(x) \sin^2 \phi} |l_i^{s_i}(x)|^2
\]

\[
= b \left\{ \frac{s_1}{2} (2 \pi)^2 + \frac{1}{s_{r1}} \frac{s_2}{2} (2 \pi)^2 \right\}
\]

\[
+ \frac{1}{s_{r2}} (2 \pi)^2 (2 \pi)^2 \right\}
\]

\[
+ t (\delta_1^r) \delta_{r1} \delta_1 (2 \pi)^2 \right\}
\]

where

\[ \delta_1^r = \cos \psi \]
\[ \delta_2^r = \frac{r}{s} \sin \psi \]
\[ C' = \frac{s_{r1}}{\sin \psi} \left[ \cos \delta + \frac{\delta'}{s_{r2}} \cot \psi \sin \psi \right] \]
and

\[ t(x) = \begin{cases} 
1, & x = |x| \\
-1, & x = -|x| 
\end{cases} \]

\[ s(x) = \begin{cases} 
1, & x = |x| \\
-1, & x = -|x| 
\end{cases} \]

\[ S_1(x) = \frac{\sin x}{x} + 1 \]

\[ S_2(x) = 1 - \frac{\sin x}{x} \]

\[ S_3(x) = t(x) \frac{\sin^2 x}{x} \]

2. LSM antisymmetric modes \((r'=1)\)

\[
\left( N_{nm}^{a} \right) = \int_{-A/2}^{A/2} \int_{b}^{\infty} \frac{1}{e(x)} \sin^{2m} \frac{\pi y}{b} |r_{n}^{a}|^{2} \sin x dy \cos \theta x dx \\
= b \left\{ \frac{\delta S_{2}(2\kappa' \beta)}{2} + \frac{1}{\varepsilon_{\kappa}} |E_{1}^{1}|^{2} \delta S_{1}(2\kappa' \delta) \right\} \\
+ \frac{1}{\varepsilon_{\kappa}} |E_{1}^{1}|^{2} \frac{\delta S_{2}(2\kappa' \delta)}{2} \sigma(\kappa') \cos \frac{\delta}{r} S_{1}(\kappa' \delta) \\
- \frac{\sigma(\kappa')}{\varepsilon_{\kappa}} F S_{1}(\kappa' \delta) \\
+ |F'|^{2} \frac{\delta S_{1}(2\kappa' \alpha)}{2} \right\}
\]

where

\[ E_{1}^{1} = \sin \kappa' \beta \]

\[ E_{2}^{1} = \frac{\varepsilon_{0} \kappa'}{\varepsilon_{\kappa}} \cos \kappa' \beta \]
\[
F' = \frac{\cos \kappa' \beta}{\sin \kappa' \alpha} \left[ \cos \kappa' \delta - \frac{\kappa' \delta \tan \kappa' \beta \sin \kappa' \delta}{\epsilon} \right]
\]

\[
F = \begin{cases} 
\text{Re}(E_1 E_2^*), & \kappa'_\epsilon = |\kappa'_\epsilon| \\
\text{Im}(E_1 E_2^*), & \kappa'_\epsilon = -j|\kappa'_\epsilon| 
\end{cases}
\]

3. LSE symmetric modes (r=)

\[
\frac{(N''^S)}{n_m} = \int_{-A/2}^{A/2} dx \int_{0}^{b} dy \cos \frac{2m\pi y}{b} |v''^S(x)|^2
\]

\[
= r_m b \left\{ 8'' S_1 (2 \kappa'' \beta) + 8'' S_2 (2 \kappa'' \delta) \right\}
\]

where

\[
r_m = \text{Neumann factor} = \begin{cases} 
1, & m \neq 0 \\
2, & m = 0 
\end{cases}
\]

\[
8''_1 = \cos \kappa'' \beta
\]

\[
8''_2 = \frac{\kappa''}{\kappa'_\epsilon} \sin \kappa'' \beta
\]

\[
C'' = \frac{\cos \kappa'' \beta}{\sin \kappa'' \alpha} \left[ \cos \kappa'' \delta - \frac{\kappa'' \delta \tan \kappa'' \beta \sin \kappa'' \delta}{\epsilon} \right]
\]
4. LSE antisymmetric modes ($r''$)

\[
\begin{align*}
(N^a_{nm}) &= \int_{-A/2}^{A/2} \int_{0}^{b} dy \cos^2 \frac{2 \pi y}{b} |v_n^a(x)|^2 \\
&= r_m b \frac{\delta}{2} S_2 (2 \kappa'' \beta) \sigma (\kappa'') + |E_1''|^2 \frac{\delta}{2} S_1 (2 \kappa'' \delta) \\
&\quad + |E_2''|^2 \frac{\delta}{2} S_2 (2 \kappa'' \delta) \sigma (\kappa'') \\
&\quad - F \delta S_3 (\kappa'' \delta) \\
&\quad + |F''|^2 \frac{\delta}{2} S_2 (2 \kappa'' \alpha) \}
\end{align*}
\]

where

\[
\begin{align*}
E_1'' &= - \sin \kappa'' \beta \\
E_2'' &= \frac{\kappa''}{\kappa''} \cos \kappa'' \beta \\
F'' &= - \frac{\sin \kappa'' \beta}{\sin \kappa'' \alpha} \left[ \cos \kappa'' \delta + \frac{\kappa''}{\kappa''} \cot \kappa'' \beta \sin \kappa'' \delta \right] \\
F &= \begin{cases} 
E_1'' E_2'', & \kappa'' = |\kappa''| \\
\text{Re} (E_1'' E_2''), & \kappa'' = - j |\kappa''|, \kappa'' = |\kappa''| \\
j 0.5 \{ E_1'' - E_1'' \} E_2'', & \kappa'' = - j |\kappa''|, \kappa'' = - j |\kappa''|
\end{cases}
\end{align*}
\]
APPENDIX D

PROGRAM LISTINGS

This Appendix gives listings of all programs, and subprograms required to reproduce the numerical results presented in this report. In general, the listings are self-explanatory. The language is FORTRAN (extended) and the programs are designed to run on CDC 6600, 6700 and CYBER 73 series computers.

The Appendix has two subsections. The first gives listings of main programs. The second section gives listing of subroutines and function subprograms required for execution of the main programs.
D.1 Program Listings
PROGRAM PLIPAT (INPUT,OUTPUT,TAPES=INPUT,TAPES=OUTPUT)  

COMPUTATION OF BIFURCATED TWIN DIELECTRIC SLAB LOADED RECTANGULAR 
WAVEGUIDE ARRAY ELEMENT RADIATION CHARACTERISTICS.  

REQUIRES CALCOMP LIBRARY AND SUBROUTINES GIVEN IN APPENDIX D.  

DIMENSION POW(10),ST(51),S1(I2),S2(I2),MORDU(20),IP(10),IG(10), 
1IR(10),JJ(10)  
DIMENSION IBUF(1000)  

REAL *K,KL  
INTEGER P1,P11,TU(10),W0(10),RU(10),SIG0(10),SIG(20),SIG10(10)  
COMPLEX Y,B,LU,C20,C1,C2,AC3,R1,R2,C4  
COMPLEX S1(20),S2(250),S2(2)  
COMMON /ARRAY/ AL,LU,R1,SEPTL,TP1,EPS,S1(2),S2(2)  
COMMON /CNKM/ Y(20),Y(250)  
COMMON /CMOS/ K(20),KE(20),GAMMA(20),MODE(20),ISY(20),NN(20), 
1MM(20),MODM(20)  
COMMON /PO/ P1,P1  
DATA TP1,C/0.2831853071796,11,8,028852/  
DATA AJ/(0.1)/  
CALL PLOTS (IBUF,1000,4)  
KNT=0  

C INPUT> FREQUENCY IN GHz, FO# RELATIVE SLAB PERMITTIVITY, EPS#  
C FEEDGUIDE HEIGHT, RD, IN INCHES# A(ALPHA), B(BETA), AND D(DELTA),  
C IN INCHES# AND APERTURE PLANE TO REFERENCE PLANE SEPARATION IN  
C FEEDGUIDE AVELENGTHS, ALEN.  

100 READ (5,800) FO,EPS,AD,A,B,D,SEP,ALEN  
IF (EOF(5),NE,0) GO TO 160  
A1=2*(A+B*D)  
WRITE (6,490) FO,EPS,A,B,D,SEP,ALEN  

C INPUT> LIMITS OF SAMPLED GRATING LORE SPACE, P1 AND Q1  

READ (5,820) P1,Q1  

C INPUT> LATTICE VECTORS S1 AND S2, IN INCHES.  

READ (5,800) S1,S2  
DO 105 I=1,2  
S1(I)=S1(I)  
S2(I)=S2(I)  
105 CONTINUE  
IGR=1  
S=S1(1)*S2(1)+S1(2)*S2(2)  
IF (ABS(S).LT.1.E-10) IGR=2  
Dx=S1(1)  
DY=S2(2)  

C INPUT> NUMBER OF MODES TO ESTABLISH ORDERING, NMODE, MAX OF  
C 20# AND FREQUENCY HAND DESIGNATION, LOH=LU OR HI#.  

READ (5,810) NMODE,LUHI  
LUHI=LUHI  
IF (SEP.61,1.E-10) IMLD=2ML0  

C INPUT> SINE SPACE SCAN RANGE AND INCREMENTS.  

174
INPUT NUMBER OF FEEDGUIDE MODES TO BE USED FOR APERTURE FIELD APPROXIMATION, NMODES# AND NUMBER OF BEAMS TO BE PLOTTED, LUBES.

READ (5,820) NMODES,LUBES
NM=NMODES+1
DO 106 I=1,NM

INPUT# NMODES FEEDGUIDE MODE DESIGNATIONS #E.G., LSE0100#
LSM0101, LSE0201#.

READ (5,830) MORD(I)
CONTINUE
IF (LUBES.EQ.0) GO TO 106
IF (LUBES.GT.10) STOP 'LOBES>10'

INPUT# LUBES# BEAM DESIGNATIONS IN FORM P*U,R

DO 107 I=1,LOBES
READ (5,820) TO(I),Q0(I),RO(I)
SIG0(I)=Q0(I)+Q1+1*(TO(I)+P1)*(2*Q1+1)+(RO(I)-1)*(2*P1+1)+
1(2*Q1+1)

CONTINUE

INPUT# COMPLEX LSE020U MODE VOLTAGE MODIFIERS, R1# AND COMPLEX UPPER ELEMENT HALF VOLTAGE MODIFIER, R2#.

READ (5,800) R1,R2
WRITE (6,904) R1,R2
WRITE (6,910) S1,S2
WRITE (6,915) P1,Q1,NMODES
L=F0/C
ALE=AL*L
BL=B*L
DL=D*L
BI=BD*L
SEPT=SEP*L

CALL L5MLSE (NMODE1)
ALEN=ALEN/(L*GAMMA(1))
IF (ALEN,GT.1,E=05) ALEN=ALEN
ALEN=ALEN1
C10=C*EXP(AJ*TP1*GAMMA(1)*ALEN*L)
C20=C*EXP(AJ*TP1*GAMMA(2)*ALEN*L)

INPUT# HALF BANDWIDTH (INTEGER) IN PERCENT# IBW# AND PERCENT BANDWIDTH STEP# ISTEP#.

READ (5,820) IBW,STEP
NBW=2*IBW+1

COMPUTE PERFORMANCE OVER BAND.

DO 155 JBW=1,NBW,STEP
BW=JNW+0.01*(JNW-1*IBW)

175
FBSW*FO
IF (IBW.EQ.0) GO TO 109
WRITE (6,905) B*F*EPS*A1*B*D*A*B*D*SEP*ALEN
109 L*F/C
AL#A*L
BL#B*L
DL#D*L
BI#D*L
SEPIL#SEP*L
B1=#0.5*(B1+SEPIL)
IF (IBW.EQ.0) GO TO 1091
CALL LMSLE (NMODE1)
1091 C1=C10*CEXP(-AJ*TPI*GAMMA(1)*ALEN*L)
C2=C20*CEXP(-AJ*TPI*GAMMA(2)*ALEN*L)
DO 110 I=1,2
S1(I)=S11(I)*L
S2(I)=S21(I)*L
110 CONTINUE
J=0
DO 116 I=1,NMDES
J=j+1
IF (MODORD(J).NE.MORD(I)) GO TO 115
K(I)=K(J)
KE(I)=KE(J)
GAMMA(I)=GAMMA(J)
NN(I)=NN(J)
MM(I)=MM(J)
MODE1(I)=MODE1(J)
ISYM(I)=ISYM(J)
MODORD(I)=MORD(I)
115 CONTINUE
IWRITE=1
CALL NORM (NMODES,IWRITE)
SPH#SPHS=SPHI
DO 150 IPH=1,NPH
KNT=KNT+1
DO 1164 I=1,10
DO 1163 ITH=1,51
POW(I,ITH)**=100.00
1163 CONTINUE
1164 CONTINUE
SPH#SPHS=SPHI
CPH#SQRT(1.-SPH**2)
STH#SQRT(I.2THH)
IF (IHLO.EQ.2*HLO) WRITE (6,920) SPH
IF (IHLO.EQ.2*HMI) WRITE (6,930) SPH
J1=0
KJ1=0
C
C TAKE THETA CUTS.
C
DO 140 ITH=1,NTH
STH=STH+STH
ST(ITH)=STH
BTH=SQRT(I.2THH)
U=SI#CPH
V=STH*SPH
C3=CEXP(AJ*TPI*V*H11)
C4=92/C3
C3=R2/C3
EVALUATE SCATTERING BLOCKS S11 AND S21

CALL SCMT (IHLO,NMODES,U,V,S11,S21,NMUDSIG)
J=J+1
KJ=KJ+1
DO 117 1=1,ISIG
IF (REAL(YA(I)) .LE. 0.) GO TO 117
IF (J1.EQ.0) GO TO 1162
DO 1161 T=1,1
IF (J.EQ.ISIG(I1)) GO TO 117
1161 CONTINUE
1162 J=J+1
IF (J.EQ.10) GO TO 1165
KJ=KJ+1
SIG10(KJ)=I
1165 SIG(J)=I
117 CONTINUE
J1=J
J2=J
IF (J.EQ.10) J2=10
KJ1=KJ
IF (LOMI.EQ.2MMI) GO TO 120
P=2.*CABS(Y(1))
P0=0.

COMPUTE POWER TRANSMISSION COEFFICIENTS FOR FIRST 10 PROPAGATING BEAMS IN GRATING Lobe SEQUENCE, AT LOW FREQUENCY AND POWER REFLECTION COEFFICIENTS, P=INPUT POWER, P0=SUM OF REAL POWER IN ALL BEAMS AND FEEDGUIDE MODES.

DO 118 I=1,J1
ISG=SIG(I)
PT=(CABS(S21(ISG+1)+S21(ISG,NM))**2)*REAL(YA(ISG))/P
S=REAL(YA(ISG))=1.0
IPT=1
IF (ABS(S)*LT.1.E-10.AND.ABS(SPH*CPh)*LT.1.E-10) IPT=IPT+1
IF (ABS(SPH)*LT.1.E-10.AND.ISG.GT.ISIG/2) IPT=IPT+1
IF (ABS(CPH)*LT.1.E-10.AND.ISG.LE.ISIG/2) IPT=IPT+1
PT=IABS(IPT)
PT=PT+PT
IF (PT.EQ.0) PT=1.
118 CONTINUE

CONTINUE
PRU=(CABS(S11(1+1)+S11(1+NM))**2)*CABS(Y(1))/P
P0=P0+PRU
IF (PRU.EQ.0) P0=1.
PRU=10.*ALOG10(PRUE)
PRU=(CABS(S11(NM+1)+S11(NM+NM))**2)*CABS(Y(1))/P
P0=P0+PRU

CHECK CONSERVATION OF ENERGY, IPO AND IM ARE MINUS THE NUMBER OF DIGITS TO WHICH CONSERVATION OF ENERGY IS APPROXIMATED BY SOLUTION.

PO=ALOG10(ABS(P0=1.0)+1.E-80)
IPO=PI
IF (IPO.LT.-99) IPO=-99
IF (PRL.LT.1.E-10) PRL=1.E-10
PRL=10.*ALOG10(PRL)
CALL CONSRV (S11*S21*NMOD*ISIG*IM)
WRITE (6,E400) STH,PR1,PR2,IP0,IM,(PW(I,ITH)+I=1,J2)
GO TO 130

C compute power transmission and reflection coefficients at hig frequency.

120
V2=AN(S11*25*TPI*IS1(1 JTextField)
V0=R1*V2
V2=ABS(V0)
P=ABS(Y(1))+ABS(Y(2))*V2*V2
IF (IMOD,NE.ISI1) GO TO 124

C C C INFINITELY THIN SEPTUM AND RECTANGULAR GRID.

P0=0.0
DO 121 ISG=1,J1
PT=REAL(YA(ISG))*(ABS(S21(ISG+1)+S21(ISG+2)*V0)**2)/P
S=REAL(YA(ISG))=1.0
IPT=1
IF (ABS(S11*LT.1.E-10.AND.EXTRA(SPH*CPH),LT.1.E-10) IPT=IPT+1
IF (ABS(S21*LT.1.E-10.AND.SPH.ISIG/2) IPT=IPT+2
IF (ABS(CPH*LT.1.E-10.AND.SPH.LE.ISIG/2) IPT=IPT+2
IPT=IABS(IP1)
PT=IPT+IP
PO=PO+PT
IF (PT.LT.1.E-10) PT=1.E-10
IF (I,GT.10) GO TO 121
PON(I,ITH)=10.*ALOG10(PT)
CONTINUE
PR1=ABS(Y(1))*(S11(1)+S11(2)*V0)**2)/P
PO=PO+PR1
IF (PR1.LT.1.E-10) PR1=1.E-10
PR1=10.*ALOG10(PR1)
PR2=ABS(Y(2))*(S11(2)+S11(2)*V0)**2)/P
PO=PO+PR2

C CHECK CONSERVATION OF ENERGY.

P0=ALOG10(ABS(P0=1.0)+1.E-80)
IP0=P0
IF (IP0.LT.99) IP0=99
IF (PR2.LT.1.E-10) PR2=1.E-10
PR2=10.*ALOG10(PR2)
CALL CONSRV (S11*S21*NMOD*ISIG*IM)
WRITE (6,E400) STH,PR1,PR2,IP0,IM,(PW(I,ITH)+I=1,J2)
GO TO 130

C CASE 2> THICK SEPTUM OR TRIANGULAR GRID.

124
P=2.0+P
PO=0.0
DO 125 ISG=1,J1
PT=REAL(YA(ISG))*CABS((S21(ISG+1)+S21(ISG+2)*NM)*C3)*C1+
S=REAL(YA(ISG))=1.0
178
IP = 1
IF (ABS(S) .LT. 1.E-10 .AND. ABS(SPH*CPH) .LT. 1.E-10) IPT = IPT + 1
IF (ABS(SPH) .LT. 1.E-10 .AND. ISG.GT.1SIG/2) IPT = IPT + 1
IF (ABS(CPH) .LT. 1.E-10 .AND. ISG.LE.1SIG/2) IPT = IPT + 1
IPT = IPT + PT
P0 = PO + PT
IF (PT .LT. 1.E-10) PT = 1.E-10
IF (SIG .GT. 10) GO TO 125
POW(I,ITH) = 10 .* ALOG10(PT)
CONTINUE
PRU = CABS(Y(1) * ((S11(1,1) * C4 + S11(1,NM) * C3) * C1 + (S11(1,2) * C4 + S11(1,NM+1) * C3) * C2 * V0) * *2) / P
PRU = PRU + CABS(Y(2) * ((S11(2,1) * C4 + S11(2,NM) * C3) * C1 + (S11(2,2) * C4 + S11(2,NM+1) * C3) * C2 * V0) * *2) / P
P0 = PO + PRU
IF (PRU .LT. 1.E-10) PRU = 1.E-10
PRU = 10 .* ALOG10(PRU)
PRL = CABS(Y(1) * ((S11(NM,1) * C4 + S11(NM,NM) * C3) * C1 + (S11(NM,2) * C4 + S11(NM,NM+1) * C3) * C2 * V0) * *2) / P
PRL = PRL + CABS(Y(2) * ((S11(NM+1,1) * C4 + S11(NM+1,NM) * C3) * C1 + (S11(NM+1,2) * C4 + S11(NM+1,NM+1) * C3) * C2 * V0) * *2) / P
P0 = P0 + PRL

CHE.CK CONSERVATION OF ENERGY.

PO = ALOG10(ABS(P0-1.0)+1.E-10)
IP0 = PO
IF (IP0 .LT. 99) IP0 = 99
IF (PRL .LT. 1.E-10) PRL = 1.E-10
PRL = 10 .* ALOG10(PRL)
CALL CONS8V (S11,S21,NM,0D,ISIG,IM)
WRITE (6,940) STH + PRU + PRL, IP0, IM, (POW(I,ITH) * I = 1,J2)
CONTINUE
CONTINUE

PRINT BEAM DESIGNATIONS.

IQ = 2 * Q1 + 1
ISG = ISIG / 2
DO : 41 I = 1,J2
DO 42 I = 1,1000
IF (SIG(I) .LE. SIG0(I1)) GO TO 1410
J(I) = I
GO TO 1411
CONTINUE
CONTINUE

IR(I) = 1
IF (SIG(I) .GT. ISG) IR(I) = 2
IP(I) = (SIG(I)-1*(IR(I)-1)*ISG)/IQ1 = P1
IQ(I) = SIG(I) - 1 = Q1 = (IR(I) - 1) * ISG = (IP(I) + P1) * IQ1
CONTINUE
WRITE (6,950) (IP(I) + IQ(I), IR(I), I = 1,J2)
IF (KJ1.EQ.0) GO TO 143
DO 42 I = 1,KJ1
IR(I) = 1
IF (SIG(I) .GT. ISG) IR(I) = 2
IP(I) = (SIG(I) - 1*(IR(I) - 1)*ISG)/IQ1 = P1
IQ(I) = SIG(I) - 1 = Q1 = (IR(I) - 1) * ISG = (IP(I) + P1) * IQ1
CONTINUE
WRITE (6,960) (IP(I) + IQ(I), IR(I), I = 1,KJ1)
143 IF (LOBES.LE.4) GO TO 150
IF (KNTS.EQ.1) CALL PLOT (0.,.5+3)
C
C      PLOT @LOBES@ BEAMS
C
CALL PLOT (0.,.2+2)
ISTRT=1
LOBE=MINO(4,LOBES)
CALL PLOTCAL (A,B,D+F,EPS*A1,BD+DX,DY,SEP,POW*SIG0,LOBE,ISTRT,JJ,
IST+SPH,IGRD,NTH)
IF (LOBES.LE.4) GO TO 150
CALL PLOT (0.,.2+2)
LOBE=MINO(4,LOBES=4)
ISTRT=5
CALL PLOTCAL (A,B,D+F,EPS*A1,BD+DX,DY,SEP,POW*SIG0,LOBE,ISTRT,JJ,
IST+SPH,IGRD,NTH)
IF (LOBES.LE.8) GO TO 150
CALL PLOT (0.,.2+2)
LOBE=MINO(2,LOBES=8)
ISTRT=9
CALL PLOTCAL (A,B,D+F,EPS*A1,BD+DX,DY,SEP,POW*SIG0,LOBE,ISTRT,JJ,
IST+SPH,IGRD,NTH)

150 CONTINUE
155 CONTINUE
GO TO 100
160 CALL PLOT (0.,.2+2)
CALL PLOT (10.0,.999)
ENDFILE
CALL EXIT
C
800 FORMAT (8F10.0)
810 FORMAT (15,A2)
820 FORMAT (16I5)
830 FORMAT (A7)
900 FORMAT (1H1,46X,37HDUAL FREQUENCY ARRAY ELEMENT PATTERNS,///
15X,13HELEMENT DATA,///,10X,1SHFU = ,F5.2,5X,6HEPS = ,F5.2,5X,4HA = ,
2F5.3,5X,6HBL = ,F5.3,5X,10X,8HALPHA = ,F5.3,5X,7HEF = ,F5.3,5X
3HSDELTA = ,F5.3,5X,9HSEP = ,F5.3,5X,14HLENE/LAMDA = ,F5.3,///)
904 FORMAT (5X,18HVOLTAGE MONITORS,///,10X,1SHR1 = ,2F7.4,1HJ+5X,
1HR2 = ,2F7.4,1HJ+///)
905 FORMAT (1H1,10X,7HCASL& ,F4.2,2HF0 = ,10X,4HF = ,F5.2,5X,
16HEPS = ,F5.2,5X,4HA = ,F5.2,5X,4HB = ,F5.3,///,10X,8HALPHA = ,
2F5.3,5X,7HEF = ,F5.3,5X,8HSEP = ,F5.3,5X,9HSEP = ,F5.3,///)
910 FORMAT (5X,11HARRAY DATA,///,10X,4HB1 = ,F6.3,1H+,F5.3,5X,4HB2 =
1F6.3,1H+,F5.3)
915 FORMAT (10X,4HPO = ,13+5X,8MNODES = ,13+/10X,
12HALL DIMENSIONS IN INCHES,///)
920 FORMAT (1H1,5X,11HIN(PHI) = ,F5.2,///,1X,7HIN(TH) = ,X,3HPRU=5X,
13HPR1,7X,3HPPO=2X,2HIM=3X,3IPPOWER IN EXCITED BEAMS (DB) =///)
930 FORMAT (1H1,5X,11HIN(PHI) = ,F5.2,///,1X,7HIN(TH) = ,X,3HPR1=5X,
13HPR2,7X,3HPPO=2X,2HIM=3X,3IPPOWER IN EXCITED BEAMS (DB) =///)
940 FORMAT (2X,F5.2,4X,2(1X,F6.2,1X),3X,2(1X,13+1X),10(1X,F6.2,1X))
950 FORMAT (/5X,8HP+Q* = ,126X,10(1X,12+,1H+,12+,1H+,113),///)
960 FORMAT (/5X,2HSPACE MODES NOT PRINTED ARE & /5X,8HP+Q*K = ,
110(2(12+,1H*)+11,3X))

END
PROGRAM DE.SMOD (INPUT=OUTPUT,TAPF5=INPUT,TAPE6=OUTPUT,TAPE7)

COMPUTATION OF FIRST FOUR ROUTS OF SYMMETRIC AND ANTI-SYMMETRIC LSE AND LSM MODE DISPERSION RELATIONS WITH M=0 FOR TWIN DIELECTRIC SLAB LOADED RECTANGULAR WAVEGUIDE IN RANGE 1.0E2K*A/2. AND
K*2.0E-4.

CREATES DATA FILE INPUT FOR PROGRAM DESGN.

DIMENSION BB(16),G(16)
REAL K(16)*KA2
COMMON /WAVGD/ A+B*D,B1,TPI,EPS
DATA CPI,TPI/3.7569646676283185308/

INPUT: A(ALPHA), B(BETA), AND D(DELTA), IN INCHES# GUIDE HEIGHT, B1, IN INCHES# AND RELATIVE SLAB PERMITTIVITY E PS.

100 READ (5,800) A,B,D,B1,EPS
IF (EDF(5)).NE.0 CALL EXIT
AA=A**(A+B+B)
WRITE (6,910)
WRITE (6,900) A,B,D,B1,EPS
WRITE (7,900) A,B,D,B1,EPS
EPSI=SQRT(EPS/1.0)
DO 110 I=1,16
BB(I)=EPSQ+1.E-10
110 CONTINUE
KA2=97
DO 130 I=1,101
KA2=KA2+0.03
F=CPI*KA2/AA
CALL FOURMD8 (F,BB)
DO 120 J=1,16
K(J)=KA2*BB(J)
T=KA2+2*K(J)*ABS(K(J))
G(J)=SORT(ABS(T))
IF (T,LT.0) G(J)=G(J)
BB(J)=BB(J)=1.0
IF (BB(J),LT.,EPSQ) BB(J)=EPSQ+1.E-10
120 CONTINUE
WRITE (7,900) KA2
WRITE (6,900) KA2
WRITE (7,900) (K(J),J=1,16)
WRITE (6,900) (G(J),J=1,16)
130 CONTINUE
GO TO 100

800 FORMAT (8F10.0)
900 FORMAT (8F10.6)
910 FORMAT (1H1)
END
PROGRAM DESGN (INPUT,OUTPUT,TAPES=INPUT,TAPE6=OUTPUT,TAPE10)

COMPUTATION OF BROADSIDE SCAN ELEMENT MISMATCH FOR INFINITE ARRAY OF BIFURCATED TWIN DIELECTRIC SLAB LOADED RECTANGULAR WAVEGUIDES VS. K*A/2. USE IN CONJUNCTION WITH PROGRAM DESMOU WHICH CREATES TAPE10.

DIMENSION NM(10)*NE(10),T1(16),T(64),IMODE(2),PT(2),PTNB(2*101),
PR1(2),PR1DB(2),SS1(2),SS2(2)
REAL KEKA(101),M2B
INTEGER P1,U1,SYM(2)
COMPLEX Y,YA,S11(20,20),S12(250,20)
COMMON /ARRAY/ AL,BL,UL,B1L,SEPTL,PTL,EPSS1(2),S2(2)
COMMON /CNRSV/ Y(20),YA(250)
COMMON /MODES/ KE(20),GAMMA(20),MOD1(20),ISYM(20)*NM(20),
IMM(20)*MODORD(20)
COMMON /PUA/ P1,U1
DATA TPI/6.2831853071796/
DATA IMODESYM/3HLS,9J1'HSE1HA,1HS/
WRITE=0

INPUT> SEPTUM THICKNESS* SEPT* IN INCHES* AND LATTICE VECTORS SS1 AND SS2 IN INCHES.

READ (5*800) SEPT,SS1,SS2

INPUT> NUMBER OF MOVES FOR APERTURE FIELD APPROXIMATION* AND LIMITS OF SAMPLED LEBE SPACE* P1 AND Q1.

READ (5*810) NMODES,P1,U1
MLO=1
IF (SEPTLT1,E=10) MLO=2
NM0=NMODES+1
NM0=NM0

READ (10*800) A,B,D,B1,EPS
IF (EPS10,N,0) GO TO 190
AA=2.*(A*B*D)
WRITE (6*900) AA,B1,A,B,D,SEPT,EPS,NM0,SS1,SS2,P1,U1
AP=2./(AA*TPI)
WRITE (6*940)
DO 180 IF=1,101
READ (10*800) KA(IF)
AP=KA(IF)*AP
AL=AA*AP1
BL=B*AP1
DL=D*AP1
B1L=B1*AP1
M2B=(0.5/B1L)**2
SEPTL=SEPT*AP1
S1(1)=AP1*SS1(1)
S1(2)=AP1*SS1(2)
S2(1)=AP1*SS2(1)
S2(2)=AP1*SS2(2)
S1=S1(2)
S2=S2(2)
READ (10*800) (T1(I)*I=1,16)
I0=0
DO 120 I=1,16
T1(I)=T1(I)/KA(IF)
M0=(I-1)/4+1
120 CONTINUE
M = n0/3
DO 110 J = 1, 4
I0 = I0 + 1
M*M + 1
S1,J = M2B*M**2 = T1(I) * AHS(T1(I))
GS = SQRT(ABS(S))
IF (S, LT, 0.) GS = G
T(10) = G
110 CONTINUE
120 CONTINUE
J = 1
130 X = 1.E + 25
DO 140 I = 1, 64
IF (X < GT, T(I)) GO TO 140
IA = 1 + (I - 1)/16
IR = (1 + (IA - 1) * 16)/4 + 1
IC = (1 + (IA - 1) * 16 - (IB - 1) * 4
ID = 1
X = T(I)
140 CONTINUE
GAMMA(J) = X
T(ID) = 1.E + 30
IE = 4 * (IA = 1) + IB
K(J) = T(I)
G*EPS = 1. * K(J) * AHS(K(J))
S*SQRT(ABS(G))
IF (G, LT, 0.) S = S
KE(J) = S
MODE = IA/3 + 1
MODE1(J) = MODE(MODE)
MODE = MOD(IA + 2) + 1
SYM(J) = SYM(MODE)
NM(IC) = NM(IC) + IA/3
NE(IC) = NE(IC) + IA/3
N2 = (1 - IA/3)*NM(IC) + (IA/3)*NE(IC)
IC1 = IC = IA/3
NM(J) = N2
MM(J) = IC1
IF (IA, NE, 3, OR, IC1, NE, 0) GO TO 130
ENCODE (10 + 910, MODORD(J)) MODE1(J) = N2 + IC1
J = J + 1
IF (J, LE, NMODE) GO TO 130
CALL NORM (NMODE, IWRI TE)
U = U,
V = 0.
ISG*(3)*P1 + 1)*(2*Q1 + 1) + 01 + 1
DO 170 ILO=MLO*MLU
L0H1 = 2*HLO
IF (ILO + LE2) LUM1 = 2*HMI
IF (ILO + LE2) S1(2) = 0.5 * S12
IF (ILO + LE2) S2(2) = 0.5 * S22
CALL SCIMAT (L0H1 = NMODE, I + V, S11, S21, MLO, MLU, 1SIG)
IF (ILO + LE2) GO TO 150
PT(1L0, EQ2 = 0) = SCBS(S21(1SIG) + S21(1SIG) + 01)
PT(1L0, EQ2 = 0) = REAL(yA(ISG)/Y(1)) * PT(1L0)
IF (PT(1L0, LT, 1, E = 10) PT(1L0) = 1, E = 10
PTDB(1L0, IF) = 10. * ALUG10(PT(1L0))
PRI(1L0) = 0.5 * SCBS(S11(1SIG) + S11(1SIG) + 01)
IF (PRI(1L0, LT, 1, E = 10) PRI(1L0) = 1, E = 10
PR1DB(1L0) = 10. * ALUG10(PRI(1L0))
PR2=0.5*CABS(S11(NO,1)+S11(NO+NO)**2
IF (PH2<LT.1.E-10) PR2=1.E-10
PR2DE=1.0*ALOG10(PR2)
GO TO 160
150 PT(ILO)*REAL(YA(ISG)/Y(1))*CABS(S21(ISG,1)**2
PT(ILO)*2.*ARS(PT(ILO))
IF (PT(ILO)<LT.1.E-10) PT(ILO)=1.0
PTDB(ILO,IF)=10.*ALOG10(PT(ILO))
PRI(ILO)=CABS(S11(1,1)**2
IF (PH1(ILO)<LT.1.E-10) PRI(ILO)=1.0
PR1DB(ILO)=10.*ALOG10(PRI(ILO))
160 CALL CONSV(S11,S21,WKMOD,ISIG,IM)
IF (ILU.EQ.1) WRITE (6,950) KA(IF),LOHI,P1(ILU),PTDB(ILU,IF),
1PRIDB(ILO),PR2DE*IM
IF (ILU.EQ.2) WRITE (6,960) KA(IF),LOHI,P1(ILU),PTDB(ILU,IF),
1PRIUB(ILO),IM
170 CONTINUE
180 CONTINUE
GO TO 100
190 CALL EXIT
C
800 FORMAT (8F10.0)
810 FORMAT (16I5)
900 FORMAT (1H1,44X,42MHUAL FREQUENCY ARRAY ELEMENT DESIGN CURVES,
1///,10X,13HELEMENT DATA///,15X,3HA =F0,3*5X,3HB =F0,3*5X,
27HALPHA =F0,3*5X,6HBLTA =F6,3*5X,7HDELTA =F6,3*5X,
38HEPTUM =F6,3*5X,
415X*PHEPS =F5,2*5X,12*6H MODES///,10X,11HARRAY DATA///,
515X*4HSL =F6,3*1H,1F5,3*5X,4HSE =F6,3*1H,1F5,3*5X,4HP1 =13,
65X*4HUL =13*/15X*24HALL DIMENSIONS IN INCHES)
910 FORMAT (A3,21Z,13)
920 FORMAT (2X,A7,3X,AD,2X,3F10.5,14)
930 FORMAT (1H1,46X,3HPOWER TRANSMISSION FACTOR AT BROADSIDE///,
15X*12HVERSUS K&A/2///,10H K&A/2 +10MEXCITATION+10H PT
2*10H PT(IUB) +10H PH1(DB) +10H PH2(DB) +10H I&M ///,
310X*10H TYPE ///)
950 FORMAT (3XF5.2,6X*A2,4X*4(2X+F7,3*1X)+17)
960 FORMAT (3XF5.2,6X*A2,4X*3(2X+F7,3*1X)+10X+17)
END
D.2 Subroutine and Function Subprogram Listings
SUBROUTINE LSMLE (NMODES)

COMPUTE CHARACTERISTIC ROOTS OF DISPERSTIVE RELATION FOR SYMMETRIC
INHOMOGENEOUSLY LOADED RECTANGULAR GUIDE.

DEFINITIONS

B = GUIDE NARROW DIMENSION DIVIDED BY
FREE SPACE WAVELENGTH
EPS = RELATIVE DIELECTRIC CONSTANT OF
LOADING
AL = DISTANCE FROM GUIDE WALL TO NEAREST
EDGE OF LOADING SLAB *WAVELENGTHS*
BL = DISTANCE FROM SLAB EDGE TO SYMMETRY
PLANE *WAVELENGTHS*
DL = SLAB THICKNESS *WAVELENGTHS*
SEPTL = SEPTUM THICKNESS WITH RESPECT TO FREE
SAPLE WAVELENGTH

TPI = 6.28318531...
S1 = FIRST LATTICE VECTOR
S2 = SECOND LATTICE VECTOR
K = X-DIRECTED VAVEVECTOR IN AIR REGION OF
GUIDE WITH RESPECT TO FREE SPACE
WAVELENGTH
KE = X- DIRECTED VAVEVECTOR IN DIELECTRIC
REGION OF GUIDE WITH RESPECT TO FREE
SPACE WAVELENGTH

GAMMA = LONGITUDINAL WAVEVECTOR NORMALIZED TO
FREE SPACE WAVEVECTOR == ORDERED BY
INCREASING CUT-OFF FREQUENCY

MODE1 = MODE TYPE (1.E+ LSE OR IS")
ISYM = SYMMETRY (1.E+ S OR A)
NN = X DIRECTING PARAMETER
MM = Y DIRECTING PARAMETER

MODOH = VECTOR CONTAINING MORE DESIGNATIONS ==
ORDERED BY INCREASING CUT-OFF
FREQUENCY

DIMENSION GAM(4*10+11),MODE(4*71),SU(4*10+11),SE(4*10+11),NM(11)*
1NE(11)
REAL KAP*KAPE+M2B+K*KE+XAC
COMMON /ARRAY/ AL,BL,DL,R,SEPTL,TPI,EP1,EP1,SI(2),S2(2)
COMMON /MODES/ K(20),KE(20),GAMM(20),MODE1(20),ISYM(20),NN(20)*
1MM(20),MODOH(20)
A1=TPI*AL
B1=TPI*BL
D1=TPI*DL
EPS1=EPS1.1.
EPS0=SORT(EPS1)
M2B=(0.5/B)**2

COMPUTE K FOR M=0

DO 180 MODE=1,4
M4=1HS
IF ((MODE/2)*2.5.LE.MODE) M4=1HA
M1=3HLSH
IF (MODE.GT.2) M1=3HLSLE
M2=10
IF (MODE.GT.2) M2=11 186
M3=MODE/3
KAP=EPSQ+1.0=10
M0=1
M=M0=M3
DO 160 N=1+1
J=1/N
S=1.01
IF (KAP LT 0.0) S=S0.99
KAP=KAP+(1-J)*S*KAP
KINC=U.314
KAP=KAP=KINC
I=0
100 KAP=KAP+KINC
DIFF=DIFF1
I=I+1
KAPEEPS1+KAP*ABS(KAP)
S=SURT(ABS(KAPE))
IF (KAPE LT 0.0) S=S0
KAPE=S
DIFF1=DIFF1*(MODE*KAP*KAPE*AI.R1+L1,EPS)
IF (I=EQ,1) GO TO 120
IF (KINC LT 1.0) GO TO 130
IF (ABS(DIFF+DIFF1)*LE.E=10) GO TO 130
IF (DIFF=DIFF1) 110,130,100
110 KAP=KAP=KINC
KINC=U.0=S=KINC
DIFF1=DIFF
GO 10 100
120 DIFF=DIFF1
GO 10 100
130 IF (ABS(DIFF)+1.0.1ABS(DIFF)) GO TO 140
GO 10 150
140 KAP=KAP=KINC
KAPEEPS1+KAP*ABS(KAP)
S=SURT(ABS(KAPE))
IF (KAPE LT 0.0) S=S0
KAPE=S
150 S0(MODE*N+MU)=KAP
SE(MODE*N+MU)=KAP
G=1.*M2*M=ABS(KAPE)*KAP
S=SURT(ABS(G))
IF (G LT 0.0) S=S0
GAM(MODE*N+MU)=S
160 CONTINUE
180 CONTINUE
C INCLUDE Y (1.0. M) DEPENDENCE
C
DO 183 MODE=1+4
M3=MODE/3
M2=10+M3
DO 182 N=1+10
KAP=S0(MODE*N+1)
KAPE=SE(MODE*N+1)
DO 181 M0=2+M2
S0(MODE*N+M0)=KAP
SE(MODE*N+M0)=KAP
M=M(M0=M3)**2
BML=M2*M
G2=KAP*ABS(KAP)+1.0=BML
GAM(MODE=N*4) = SWEAT(A-I*R2)
IF (G2L2*N.0) GAM(MODE=N*4) = GAM(MODE=N*4)

181 CONTINUE
182 CONTINUE
183 WRITE (6*400)
DO 185 I=1+11
N(I)=0
NE(I)=0
185 CONTINUE
186 I=1

GAMM1(I) = 1.0*36
DO 220 NDF=1.4
M=1+16
IF (NDF/2 steril 0) M=1+16
M=1+3
IF (MODE*G2*2.0) M=1+3
M=1+10
IF (MODE*G2*2.0) M=1+11
M=1+3
DI 210 NO=1.4
M=1+3
DO 220 NO=1.4
IF (GAMMA(I)*G2*GAM(MODE-N)*4) G(I) IN 200
GAMMA(I) = GAM(MODE-N)*4
ENCOD1 (10+4101*MODE(I)) =1.9+1.0
IA=MODE
THEN
ICM=0
200 CONTINUE
210 CONTINUE
220 CONTINUE

ORDER MODES

GAM1A+1B*IC) = 1.0*36
K(1) = S0A(Ia+1R+1C)
KE(1) = SLA(Ia+1R+1C)
M=1+3
IF (IA+G2*2) M=1+3
IF (IA+G2*3) M=1+3
IF (IA+G2*2) M=1+3
M=1+10
M=1+3
IF (IA+G2*2) M=1+3
ICM=ICM*I/3
MODE1(T) = M
NN(I) = N
MM(I) = IC
SYM(1) = 1+8
IF (IA/2*2, 0) I=RIA I=1+3
ENCOD1 (10+9001*MODE(I)) M=1+2*IC
WRITE (6*420) INODE(I) = ENCOD1(I)*GAMMA(I)*1
I=1+1
IF (M,I*LE.NM) GOS TO 190
SEC=SECUL1(I)
WRITE (6*930) SEC
RETURN

900 FORMAT (I1,1)
910 FORMAT (A3+1A+212.2)
920 FORMAT (5X*8.5*X*7.5*E10.5,5*X*15)
930 FORMAT (5*F20.3,8X 5FCORDR)
940 FORMAT (A3*212.2)
   END
FUNCTION DISP(M*K*KE*A*B*D*E)  
COMPUTE DISPERSION RELATION D(K*KE) FOR ARBITRARY K AND KE  

DEFINITIONS  
M = MODE FUNCTION DESIGNATION = SEE CODE  
K = X-DIRECTED WAVENUMBER IN AIR REGION WRT KO  
KE = X-DIRECTED WAVENUMBER IN DIELECTRIC REGION WRT KO  
A = ALPHA/LAMBDAO  
B = BETA/LAMBDAO  
D = DELTA/LAMBDAO  
ER = RELATIVE PERMITTIVITY OF SLABS  

REAL K*KE  
SA=SINC(K*A)  
CA=COSC(K*A)  
SB=SINC(K*B)  
CB=COSC(K*B)  
SD=SINC(KE*D)  
CD=COSC(KE*D)  
SKA=SXOX(K*A)*A  
SKB=SXOX(K*B)*B  
SKE=SXOX(KE*D)*D  
S1=SIGN(1.,K)  
S2=SIGN(1.,KE)  
GO TO (100,110,120,130), M  

LSM SYMMETRIC MODES  
100 DISP=ER*K*SA*S1*(CB*CD=ER*K*SB*S1*SKE)  
DISPDISP*CA*(ER*K*SB*S1*CD+KE*SD*S2*CB)  
RETURN  

LSM ANTI-SYMMETRIC MODES  
110 DISP=ER*SA*S1*(SB*CD+ER*K*SKE*CH)+CA*(KE*SD*S2*SKE=ER*CD*CB)  
RETURN  

LSL SYMMETRIC MODES  
120 DISP=SKA*(KE*SD*S2*CB*K*SB*S1*CD)+CA*(K*SB*S1*SKE=CB*CD)  
RETURN  

LSL ANTI-SYMMETRIC MODES  
130 DISP=SKA*(CB*CD=KE*SKBE*SD=S?)+CA*(SKR*CD+SKE*CR)  
RETURN  
END
SUBROUTINE NORM (NMODES, IWRITE)

SUBROUTINE NORM computes the fieldguide mode normalizations, and
values for the coefficients appearing in the expressions for what
and what of which see documentation.

real or imaginary. Not complex. All other coefficients are
always real. Also, normalizations carry an additional 1/lambda
dependence which cancels in scattering matrix computations, but
not in mode function computations.

Definitions: see subroutine LSLMSE

Suffixes:

P = prime (LSM modes)
DP = double prime (LSE modes)
S = symmetric
A = antisymmetric

******************************************************************************

REAL KAP, KAPE, KE, NPS, NPA, NDD, NPA, NDD
COMMON /ARRAY/ AL, RL, DL, RIL, SEPIL, TPI, EPS, S1(2), S2(2)
COMMON /COEFS/ B1P(10), B2P(10), B1DP(10), B2DP(10), CP(10), CP(10),
1E1P(10), E2P(10), E1DP(10), E2DP(10), FP(10), FP(10), NPS(10), NPS(10),
2NDD(10), NDD(10), NDD(10), NDD(10)
COMMON /MODES/ KAP(20), KAPE(20), Gamma(20), Mod.el(20), Sym.1(20),
1NN(20), 1MM(20), Mod.Ord(20)
A = TPI*AL
B = TPI*BL
D = TPI*DL
IF (IWRITE, EQ, 1) WRITE (6, 900)
ILSM = 0
ILMA = 0
ILSE = 0
ILSE = 0
DO 140 I = 1, NMODES
   IK = KAP(I)
   KE = KAPE(I)
   AK = a
   BK = b
   DK = D*KE
   MODE = MODE(I)
   ISYM = ISYM(I)
   MM = MM(I)
   IF (MODE.EQ.3 .AND. ISYM.EQ.1) GO TO 101
   LSM MODES
   IF (ISYM.EQ.1 .AND. ISYM.EQ.1) GO TO 100
   SYMMETRIC
   ILSM = ILSM + 1
   KOUNT(I) = ILSM
   B1P(1LSM) = COSC(BK)
   B2P(1LSM) = EPS*N*SINC(BK)/KE
   B2P(1LSM) = B2P(1LSM)*SIGN(1, K)*SIGN(1, KE)
   CP(1LSM) = SINC(BK)*COSC(DKE)*COSC(KE)*SINC(DKE)/B2P(1LSM)
   CP(1LSM) = CP(1LSM)*SINC(AK)
140 CONTINUE

100 CONTINUE

101 CONTINUE

900 FORMAT (6, 900)

ANTI-SYMMETRIC

ILSMA=ILSMA+1
KOUNT(I)=ILSMA
BL=BL*SIGN(1.*K)
E1P(ILSMA)*=SINC(BK)
E2P(ILSMA)=EPS*K*CSC(BK)/KE
FP(ILSMA)*=CSC(BK)*(CSC(DKE)=KE*TANT(BK)*SINC(DKE)*SIGN(1.*KE)/
1(EPS*K))+SIGN(1.*K)/SINC(AK)
F=E1P(ILSMA)*E2P(ILSMA)
ANORM=0.5*BL*(BL*S1(2.*BK)+DL*(S1(2.*DKE)*E1P(ILSMA)**2+
1(S2(2.*DKE)*E2P(ILSMA)**2=2.*F*S3(DKE))*SIGN(1.*KE))/EPS+
2AL*S1(2.*AK)*FP(ILSMA)**2)
NDPA(ILSMA)=SQR(ABS(ANORM))
BL=BL*SIGN(1.*K)
GO TO 130

LSE MODES

RMM=1.0
IF (M.EQ.0) RMM=2.0
IF (ISYM.EQ.1) GO TO 120

ILSES=ILSES+1
KOUNT(I)=ILSES
B1P(ILSES)=CSC(BK)
B2P(ILSES)=SINC(BK)*SIGN(1.*K)*SIGN(1.*KE)/KE
CDP(ILSES)=CSC(BK)*(CSC(DKE)=KE*TANT(BK)*SINC(DKE)*SIGN(1.*KE)/
1(SIGN(1.*K))/SINC(AK)
ANORM=0.5*RM*AL*(BL*S1(2.*BK)+DL*(S1(2.*DKE)*E1P(ILSES)**2+
1SIGN(1.*KE)*E2P(ILSES)**2)=2.*F*S3(DKE))*SIGN(1.*KE))/EPS+
2AL*S2(2.*AK)*FP(ILSES)**2)
NDPA(ILSES)=SQR(ABS(ANORM))
GO TO 130

ANTI-SYMMETRIC

ILSEA=ILSEA+1
AL=AL*SIGN(1.*K)
BL=BL*SIGN(1.*K)
KOUNT(I)=ILSEA
E1DP(ILSEA)=SINC(BK)
E2DP(ILSEA)=K*CSC(BK)/KE
FPD(ILSEA)=SINC(BK)*(CSC(DKE)+K*SINC(DKE)/(KE*TANT(BK)))/SINC(AK)
1)
F=E1DP(ILSEA)*E2DP(ILSEA)
ANORM=0.5*RM*AL*(BL*S2(2.*BK)+DL*(S2(2.*DKE)*E1DP(ILSEA)**2+
1SIGN(1.*KE)*E2DP(ILSEA)**2=2.*F*S3(DKE)))
+2AL*S2(2.*AK)*FPD(ILSEA)**2)
NDPA(ILSEA)=SQR(ABS(ANORM))
AL=AL*SIGN(1.*K)
BL*HL*SIGN(1.+K)
130 IF (IWRITE.EQ.0) GO TO 140
WRITE (6,910) I*K*KE*GAM(1),MU0(1),ANORM
140 CONTINUE
RETURN
C
900 FORMAT (1H1,5H 1 ,1H K ,10H KE ,10H GAM
110H MUDE ,1H NORM:*2 ,/)
910 FORMAT (1X,13X,1X,13F10.5,2X,7X,1X,F10.5)
   **
END
SUBROUTINE SCTMAT (LOHI, NMODES, U0, V0, S11, S21, NMUD, SIG1)

SCIMAT COMPUTES THE FEEDGUIDE - FREE SPACE SCATTERING MATRIX
BLOCKS S11 AND S21 FOR AN INFINITE RECTANGULAR GRID ARRAY OF
THIN DIELECTRIC SLAB LOADED RECTANGULAR WAVEGUIDES.
LATTICE VECTORS S1 AND S2

LOHI SPECIFIES THE FREQUENCY BAND: LOHI=2MLN FOR LOW FREQUENCY
BAND, LOHI=2HMI FOR HIGH FREQUENCY BANDs.

NOTE: IF THE SEPTUM THICKNESS IS NOT EQUAL TO THE WALL THICKNESS
LOHI=2MLN AND THE HIGH AND LOW FREQUENCY UNIT CELLS ARE
IDENTICAL.

DEFINITIONS:
LOHI = 2MLN, FOR TRIANGULAR GRID OR THICK
SEPTUM, 2HMI, FOR THIN SEPTUM, HIGH
FREQUENCY BAND, AND RECTANGULAR GRID

NMODES = NUMBER OF FEEDGUIDE MODES USED TO
APPROXIMATE APERTURE FIELD

U0 = SIN(THETA)*COS(PHI)
V0 = SIN(THETA)*SIN(PHI)

S11 = FEEDGUIDE SELF REFLECTION SCATTERING
BLOCK

S21 = FEEDGUIDE TO SPACE MODE VOLTAGE
TRANSMISSION COEFFICIENT

NMUD = NUMBER OF FEEDGUIDE MODES IN UNIT CELL

SIG1 = NUMBER OF SPACE MODES = 2*PI*Q1

Y = FEEDGUIDE MODE ADMITTANCE

YA = SPACE MODE ADMITTANCE

*** FOR OTHER DEFINITIONS SEE SUB LSMLSE ***

SUFFIXES: SEE SUBROUTINE NAME

DIMENSION AM(2U),
REAL KAP*KAPE,K*KLKLMPS*PA*NSP*NDPA*KZ1*KT(250)*KX(250)*KY(250),
COMPLEX ZAJ*ZC*ZEM(250)*ZDN(250)*S11(20+20),Y*YAE*EXPB(250),
IS21(250+20)*EXPB(250)*TRF(20+20),INTEGR
INTEGER P*Q*PI*SIG1*SIG2
COMMON /ARRAY/ AL, BL, UL, BL, SEP11, TP11, EPS, SIG1, SIG2
COMMON /CNSHV/ Y(20)*YA(250)
COMMON /CUEFS/ B1P(10)+B2P(10)+B2D(10)+B2D(10)+CP(10)+CPD(10),
1E1P(10)+E2P(10)+E1DP(10)+E2DP(10)+FP(10)+FDP(10)+NPS(10)+NPA(10),
1NDPS(10)+NDPA(10)*KOUNT(20)
COMMON /COSIN/ SKPAP(20)+SKPAP(20)+SKPAP(20)+
1SKAPD(20)+SKAPD(20)+SKAPC(20)+SKAPC(20)+
2SKAPD(20)+SKAPD(20)+SKAPC(20)+SKAPC(20)+
SKWX(250)+SKXL(250)+SKXL(250)+SKXL(250)+
SKX(250)+SKX(250)+SKX(250)+SKX(250)+
COMMON /MULES/ KAP(20)+KAP(20)+KAP(20)+
NAM(20)+NAM(20)+NAM(20)+
1MM(20)+MODD(20)+
COMMON /PU/ P*Q
DATA AJ(0,1,1)/
100 AMPI+AL
B*TP1*BL
D*TP1*DL
B*TP1*BL
SEP=PI*SEP11
FPSU=1./TP1**2
P1=2*P+1
Q1=2*Q+1

194
SIG1=2*PI*Q1
SIG2=SIG1/2
NMOD=NMODES
IF (LOHI.EQ.2.HLO) NMODE=2*NMODES
DO 110 N=1,NMODES
KE=KAP(N)*U
WaKAP(N)*A
XsKAP(N)*B
AM(N)=0.5*MM(N)/Q1L
T=GAM(N)/(1.0*KAP(N)*ABS(KAP(N)))
Y(N)=CMPLX(T,0.0)
IF (GAM(N).LT.0.) Y(N)=CMPLX(0.,T)
IF (MODE(N).EQ.3ILSE) Y(N)=I.*Y(N)
SKAPAL(N)=SINC(N)
CKAPAL(N)=COSC(N)
SKAPB(N)=SINC(X)
CKAPB(N)=COSC(X)
SKAPD(N)=SINC(KE)
CKAPD(N)=COSC(KE)
IF (LOHI.EQ.2.HLO) GO TO 110
Y(N+NMODES)=Y(N)
110 CONTINUE.
C
C COMPUTE INVERSE LATTICE
C
T=S1(1)*S2(2)-S2(1)*S1(2)
CELLA=1./SQRT(ABS(T))
T=1./T
T1X=T*S2(2)
T2X=T*S1(2)
T1Y=T*S2(1)
T2Y=T*S1(1)
SIG=0
C
C COMPUTE FREE SPACE WAVE NUMBERS AND WAVE ADMITTANCES
C
DO 130 L=1,2
L1=L=L
RL1=L1
L1=L1=L
RL2=L1
DO 130 J=1,P1
J=J+1
U=VU+J*T1X
V=VO+J*T1Y
DO 130 K=1,Q1
SIG=SIG+1
KX(SIG)=U+K*T2X
KY(SIG)=V+K*T2Y
KZ1=1.*KX(SIG)**2+KY(SIG)**2
AA=SQRT(ABS(KZ1))
KZ=CMPLX(AA,0.)
IF (KZ1.LT.0.) KZ=CMPLX(0.,1.*AA)
KT(SIG)=SQRT(ABS(1.+KZ1))
IF (AA.LT.1.E-10) GO TO 120
YA(SIG)=PL1/KZ+HL2*KZ
GO TO 130
120 YA(SIG)=RL2*KZ
130 CONTINUE
**COMPUTE SINES AND COSINES OF KX*(ELEMENT DIMENSIONS)**

```
DO 140 SIG=1SIG1
   T=KX(SIG)*A
   SKXAL(SIG)=SIN(T)
   CKXAL(SIG)=COS(T)
   T=KX(SIG)*B
   SKXU(SIG)=SIN(T)
   CKXU(SIG)=COS(T)
   T=KX(SIG)*(A+B+U)
   SKXAS(SIG)=SIN(T)
   CKXAS(SIG)=COS(T)
   T=KY(SIG)*BB
   EXPB1(SIG)=EXP(AJ*1)
   T=KY(SIG)*BB+SEP)
   EXPB(SIG)=EXP(AJ*T)
   CONTINUE
```

**COMPUTE COUPLING COEFFICIENTS: ESN(SIG,N)**

```
DO 180 N=1,NMODES
   T2=AM(N)*2
   IF (MM(N).EQ.0) T2=1,0
   DO 160 SIG=1SIG1
      T1=ABS(KY(SIG))
      IF (ABS(T1=112)*LT.1.E-10) GO TO 150
      COEFM(SIG)=(1,E-EXPB1(SIG)*(-1)**MM(N))*T2/(T1=112)
      IF (MIDE(N).EQ.3HLE) COEFM(SIG)=COEFM(SIG)*KY(SIG)/T2
      GO TO 155
   150 COEFM(SIG)=0.5*AJ*BB
   IF (MM(N).EQ.0) CUEFM(SIG)=2.*CUEFM(SIG)
   IF (MIDE(N).EQ.3HLSM) CUEFM(SIG)=CUEFM(SIG)*SIG1(1.,KY(SIG))
   155 COEFM(SIG)=CUEFM(SIG)*CEL
   160 CONTINUE
   DO 170 SIG=1SIG1
      LR=1
      IF (SIG.GT.SIG2) LR=2
      ESN(SIG+N)=INTGHL(KX(SIG)*KY(SIG),KT(SIG)+LR.SIG+N)
      ESN(SIG+N)=FPSO*COEFM(SIG)*ESN(SIG+N)
      IF (LOHL.EQ.2HII) GO TO 170
      ESN(SIG+N+NMODES)=CONJG(EXP(SIG))*ESN(SIG+N)
      CONTINUE
   170 CONTINUE
```

**FORM SCATTERING MATRIX BLOCKS S11 AND S21**

```
FORM MATRIX TRIPLE PRODUCT
   MAT(TRIP)=MAT(CONJG(ESN))*MAT(YA)*MAT(ESN)
   AND MATRIX FOR INVERSION:
      DIAG(Y)*MAT(TRIP)
```

```
DO 210 IA=1,NMOD
DO 200 IB=1,NMOD
   TRIP(1A+IB)=(0.,0.,)
```
S11(IA+IB)=(0,0),
IF (IA .EQ. IB) S11(IA+IB)=2.*Y(IA)
DO 190 SIG=1,SG1
TRIP(IA+IB)=TRIP(IA+IB)+CONJG(ESN(SIG*IA))*YA(SIG)*ESN(SIG+IB)
190 CONTINUE
200 CONTINUE
210 CONTINUE
DO 220 IA=1,NMOD
TRIP(IA+IA)=Y(IA)+TRIP(IA+IA)
220 CONTINUE
C
CSIMEQ RETURNS MAT(S11+DEL(I,J)) WHERE DEL(I,J) IS THE KRONOECKER DELTA FUNCTION
CALL CSIMEQ (TRIP,NMOD,S11,NMOD,KS)
C
SOLVE
MAT(S21)=MAT(ESN)*MAT(S11+DEL(I,J))
C
DO 250 SIG=1,SG1
DO 240 IB=1,NMOD
S21(SIG+IB)=(0,0)
DO 230 IA=1,NMOD
S21(SIG+IB)=S21(SIG+IB)+ESN(SIG+IA)*S11(IA+IB)
230 CONTINUE
240 CONTINUE
250 CONTINUE
C
SOLVE
MAT(S11)=MAT(S11+DEL(I,J))
C
DO 260 IA=1,NMOD
S11(IA+IA)=S11(IA+IA)=1.0
260 CONTINUE
RETURN
END
COMPLEX FUNCTION INTGR1 (KK, KY, KT, LH, SIG, N)

FUNCTION INTGR1 COMPUTES THE INTEGRAL (IN X) PORTION OF THE COUPLING COEFFICIENTS, ESN(SIG, N)

NOTE: MEMORANDUMS ARE CHOSEN TO COINCIDE WITH NOTATION IN REPORT

DEFINITIONS:

KK = X-DIRECTED WAVELENGTH OF LOBE WRT KO
KY = Y-DIRECTED WAVELENGTH OF LOBE WRT KO
KT = TRANSVERSE WAVELENGTH OF LOBE WRT KO
LR = =1 FOR E-MODES, =2 FOR H-MODES
SIG = NUMBER OF GATING LOBE IN INTERNAL ORDERING
N = NUMBER OF APERTURE MODE IN INTERNAL ORDERING

*** FOR OTHER DEFINITIONS SEE SUB LS=ELSE ***

SUFFIXES: SEE SUBROUTINE NOMM

COMPLEX R1, R2, R3, R4, R5, R6, R7, R8, R9, R10, Z1, Z2, Z3, Z4, Z5, Z6, T1, T2
REAL KK, KY, KT, LH, EPS, EPS1, S1(2), S2(2)

DATA SGH2/(1.41421356237309)/

INITIALIZE TEMPORARY STORAGE:

R1 = (0, 0)
R2 = (0, 0)
R3 = (0, 0)
R4 = (0, 0)
R5 = (0, 0)
R6 = (0, 0)
R7 = (0, 0)
R8 = (0, 0)
R9 = (0, 0)
R10 = (0, 0)
Z1 = (0, 0)
Z2 = (0, 0)
Z3 = (0, 0)
Z4 = (0, 0)
Z5 = (0, 0)
Z6 = (0, 0)

MODE=NODE1(N)
ISYM=ISYM1(N)
ALR=LH
ALR=1.5=LH
IKT=0
IF (KT.LT.1.E-10) IKT=1

COMPUTE TERMS COMMON TO ALL INTEGRALS

KAP IMAGINARY

IF (K.GT.0.) GO TO 100
TU=AJ*K/(1.-K*ABS(K))
TU=AJ
R1=(SKXB(SIG)*CKAPB(N)+AJ*SKAPB(N)*CKXB(SIG))/(KX+AJ*K)
R2=(SKXB(SIG)*CKAPB(N)+AJ*SKAPB(N)*CKXB(SIG))/(KX+AJ*K)
R7=(CKXAL(SIG)*CKAPAL(N)+AJ*SKXAL(SIG)*SKAPAL(N))/(KX+AJ*K)
R8=(CKXAL(SIG)*CKAPAL(N)+AJ*SKXAL(SIG)*SKAPAL(N))/(KX+AJ*K)
R9=(SKXAL(SIG)*CKAPAL(N)+AJ*SKXAL(SIG)*CKXAL(SIG))/(KX+AJ*K)
R10=(SKXAL(SIG)*CKAPAL(N)+AJ*SKXAL(SIG)*CKXAL(SIG))/(KX+AJ*K)

KAP REAL

100 T=ABS(KK)
TU=KE/(EPS=KE**2)
T1=(1.0+U.)
IF (ABS(T=K).GT.1.E-15) GO TO 110
R3=(1.0+270.)
IF (KX.LT.0.) A3=(CKXD(SIG)*CKAPD(N)+SKXD(SIG)*SKAPD(N))/(KX+KE)
R4=(1.0+270.)
IF (KX.GT.0.) A4=(CKXD(SIG)*CKAPD(N)+SKXD(SIG)*SKAPD(N))/(KX+KE)
A5=TRIBL
IF (KX.LT.0.) A5=(SKXD(SIG)*CKAPD(N)+CKXD(SIG)*CKAPD(N))/(KX+KE)
A6=TRIBL
IF (KX.GT.0.) A6=(SKXD(SIG)*CKAPD(N)+CKXD(SIG)*CKAPD(N))/(KX+KE)
GO TO 120

110 A3=(CKXD(SIG)*CKAPD(N)+SKXD(SIG)*SKAPD(N))/(KX+KE)
A4=(CKXD(SIG)*CKAPD(N)+SKXD(SIG)*SKAPD(N))/(KX+KE)
A5=(SKXD(SIG)*CKAPD(N)+CKXD(SIG)*CKAPD(N))/(KX+KE)
A6=(SKXD(SIG)*CKAPD(N)+CKXD(SIG)*CKAPD(N))/(KX+KE)

KAP REAL

120 IF (K.LT.0.) GO TO 140
TU=K/(1.0+118)
T1=(1.0+U.)
IF (ABS(T=K).GT.1.E-15) GO TO 130
A1=TRIBL
IF (KX.GT.0.) A1=(SKXB(SIG)*CKAPR(N)+SKAPB(N)*CKXB(SIG))/(KX+K)
A2=TRIBL
IF (KX.GT.0.) A2=(SKXB(SIG)*CKAPR(N)+SKAPB(N)*CKXB(SIG))/(KX+K)
R7=(1.0+270.)
IF (KX.LT.0.) A7=(CKXAL(SIG)*CKAPAL(N)+SKXAL(SIG)*SKAPAL(N))/(KX=K)
PB=(1.0+270.)
IF (KX.GT.0.) A8=(CKXAL(SIG)*CKAPAL(N)+SKXAL(SIG)*SKAPAL(N))/(KX+K)
A9=TRIBL
IF (KX.GT.0.) A9=(CKXAL(SIG)*CKAPAL(N)+SKXAL(SIG)*SKAPAL(N))/(KX+K)
A10 = TPI * AL
IF (KX*.GT.0.) A10 = (SXXAL(SIG) * CKAPAL(N) + CKXAL(SIG) * SKAPAL(N)) / (KX + 1K)

GO TO 140

130 A1 = (SXXB(SIG) * CKAPB(N) + SXXB(SIG) * CKXB(SIG) + SKAPB(N) * CKX(N)) / (KX + K)
A2 = (SXXB(SIG) * CKAPB(N) + SXXB(SIG) * CKXB(SIG) + SKAPB(N) * CKX(N)) / (KX = K)
A3 = (CKXAL(SIG) * CKAPAL(N) + SKXAL(SIG) * SKAPAL(N)) / (KX = K)
A4 = (CKXAL(SIG) * CKAPAL(N) + SKXAL(SIG) * SKAPAL(N)) / (KX = K)
A5 = (SXXAL(SIG) * CKAPAL(N) + CKXAL(SIG) * SKAPAL(N)) / (KX = K)

A6 = SXXAL(SIG) * CKAPAL(N) + CKXAL(SIG) * SKAPAL(N) / (KX - K)

GO TO 140

140 A11 = 2. * KX / (KX + 2 * KX * ABS(KE))
A12 = 2. * KX / (KX + 2 * KX * ABS(KE))
A13 = 2. * KX / (KX + 2 * KX * ABS(KE))
A14 = 2. * KX / (KX + 2 * KX * ABS(KE))

IF (MODE, EQ, 3HLSE) GO TO 320
IF (SYM, EQ, 1HA) GO TO 230

C

LSM = SYMMETRIC AND LSE = ANTI-SYMMETRIC

C

145 IF (A3*GT.1.E+25 .OR. A4*GT.1.E+25) GO TO 150
Z1 = (R3+R4=A11) * SXXB(SIG)
Z2 = (TV1*A12=R3+R4) * CKX(SIG)
Z3 = (R3=R4+TV1*A12) * SKX(SIG)
Z4 = (A11=R3+R4) * CKX(SIG)

GO TO 180

150 IF (A3*GT.1.E+25) GO TO 170
IF (A4*GT.1.E+25) GO TO 160
B1 = SKAPB(N) ** 2 / KE
B2 = B1
B3 = B1 * SKX(SIG)
B4 = B2 * CKX(SIG)
B5 = B1
B6 = B2

GO TO 180

160 B1 = SKAPB(N) ** 2 / KE
B2 = B1
B3 = B1 * SKX(SIG)
B4 = B2 * CKX(SIG)
B5 = B1
B6 = B2

GO TO 180

170 B1 = 0.
B2 = 0.
B3 = 0.
B4 = 0.

180 IF (A7, GT, 1.E+25 .OR. A8, GT, 1.E+25) GO TO 190
Z5 = (A13=R7=R8) * SXXA(SIG)
Z6 = (TV1=A14=R7+R8) * SKX(SIG)

GO TO 220

190 IF (A7*GT.1.E+25) GO TO 210
IF (A8*GT.1.E+25) GO TO 200
B5 = SXXAL(SIG) * SKAPAL(N) ** 2 / K
B6 = B5

GO TO 220

200 B5 = SXXAL(SIG) * SKAPAL(N) ** 2 / K
B6 = B5

GO TO 220

210 B5 = 0.
B6 = 0.

220 IF (MODE, EQ, 3HLSE) GO TO 340
TT = (1.0)  
1P = R1 + R2 + (H1P(M) *(Z1 + (R5 + R6) * CKXH(SIG)))  
1+TT + B2P(M) *(Z2 + (R5 + R6) * CKXH(SIG))) / EPS  
+CP(M) *(Z5 + (R5 + R10) * CKXH(SIG))  
12P = TU*(R2 = R1) + V*(B1P(M) *(Z3 + (R5 + R6) * CKXB(SIG)))  
1 + B2P(M) *(Z4 + (R5 + R6) * CKXB(SIG))) + TU*CP(M) *(Z6 + (R9 + R10) * CKXH(SIG))  

C

LSM = SYMMETRIC

WITH E=MODES

INTGRL = XX * I1P = I2P * KY**2

WITH H=MODES

IF (LH.EQ.2) INTGRL = KY * I1P + XX * KY * I2P  
IF (IKT.EQ.1) INTGRL = I1P / SQR2  
IF (IKT.EQ.0) INTGRL = INTGRL / KI  
INTGRL = INTGRL / NPS(M)  
RETURN

C

LSM = ANTISYMMETRIC AND LSE = SYMMETRIC

Z1 = (A11 = R3 = H4) * CKXH(SIG)  
Z2 = (TV1 = A12 = R3 + H4) * CKXH(SIG)  
Z3 = (TV1 = A12 = R3 + H4) * CKXH(SIG)  
Z4 = (R3 + R4 = A11) * CKXH(SIG)  
GO TO 270

240 IF (A5.GT.1.E+50) GO TO 260  
IF (A4.GT.1.E+25) GO TO 250  
B1 = SKAPU(N)**2 / KE  
B2 = B1  
B1 = B1 * CKXH(SIG)  
B2 = B2 * SKXH(SIG)  
B3 = B1  
B4 = B2  
GO TO 270

250 B1 = SKAPD(N)**2 / KE  
B2 = B1  
B1 = B1 * CKXH(SIG)  
B2 = B2 * SKXH(SIG)  
B3 = B1  
B4 = B2  
GO TO 270

260 B1 = 0.  
B2 = 0.  
B3 = 0.  
B4 = 0.  

Z5 = (R7 + R8 = A13) * CKXH(SIG)  
Z6 = (R7 + R8 = A14) * CKXH(SIG)  
GO TO 310

280 IF (A7.GT.1.E+50) GO TO 300  
IF (A8.GT.1.E+25) GO TO 290  
B5 = CKXH(SIG) * SKAPAL(N)**2 / K  
B6 = B5  
GO TO 310

290 B5 = CKXH(SIG) * SKAPAL(N)**2 / K
SUBROUTINE CONSRV (S11, S21, NMOD, SIG, IM)

CONSERVATION OF ENERGY CHECK

NOTE: DOES NOT GUARANTEE THAT ANSWER IS CORRECT. ONLY GUARANTEES SELF-CONSISTENCY.

DEFINITIONS:

S11 - FEEDGUIDE SELF-REFLECTION SCATTERING BLOCK
S21 - FEEDGUIDE TO SPACE MODE VOLTAGE TRANSmission COEFFICIENT
NMOD - NUMBER OF FEEDGUIDE MODES IN UNIT CELL
IM - MINUS THE NUMBER OF DIGITS TO WHICH CONSERVATION OF ENERGY IS APPROXIMATED BY SOLUTION

INTEGER SIG
COMPLEX S11(20,20), S21(250,20), S(20,20), Y, YA
COMMON /CNSRV/ Y(20), YA(250)
IM=-10000

DO 150 IA=1,NMOD
DO 140 IB=1,NMOD
S(IA,IB)=(0.,0.)
DO 100 SIG=1,ISIG
S(IA,IB)=S(IA,IB)+S21(SIG,IA)*CONJG(YA(SIG)*S21(SIG,IB))
100 CONTINUE

DO 110 NN=1,NMOD
DELR=0.
DELS=0.

IF (NN.EQ.IA) DELR=1.
IF (NN.EQ.IB) DELS=1.
S(IA,IB)=S(IA,IB)+(DELR+S11(NN,IA))*CONJG(Y(NN)*(DELS-S11(NN,IB)))

A=CABS(S(IA,IB))
IF (A.GT.1.E-20) GO TO 120
IM=1000
GO TO 130

120 I=ALOG10(A)
130 IM=MAX0(I+IM)
140 CONTINUE
150 CONTINUE
RETURN
END
SUBROUTINE PLTCAL (AqboFEPSAI,81,DXDYSEP9PT,50.LOBE,ISTNT,
1JJ*ST,SPH*IGRD*NTH)

CALCOMP PATTERN PLOTTING ROUTINE
MAX OF 10 CURVES; 51 POINTS EACH.

DEFINITIONS:
A = ALPHA
B = BETA
D = DELTA
F = FREQUENCY (GHz)
EPS = RELATIVE PERMITTIVITY OF SLABS
A1 = X-DIMENSION OF GUIDE
B1 = Y-DIMENSION OF GUIDE
DX = X GRID SPACING
DY = Y GRID SPACING
SEP = SLOT THICKNESS
PT = POWER TRANSMISSION COEFFICIENT
S0 = LOBE NUMBER IN INTERNAL ORDERING
LOBE = NUMBER OF BEAMS TO BE PLOTTED
ISTRT = ARRAY PICKUP VALUE FOR CURRENT PAGE
JJ = LURE SELECTION VECTOR
ST = SIN(THETI) ARRAY
SPH = SIN(Phi) OF PLOT
IGRD = GRID TYPE
NTH = NUMBER OF POINTS TO BE PLOTTED

DIMENSION PT(10-51),JJ(10),ST(51),ST1(51),P(51)
INTEGER 50(10)

I=1
IF (NTH.LE.21) L=2
CALL AXIS (2.,3.,1H=15.,0.,0.,0.,2)
CALL SYMBOL (4.,6.,2.,5.,10.,3H=90.,0.,3)
CALL GREEK (4.,6.,2.,5.,15.,0.,8)
CALL SYMBOL (4.,7.,5.,10.,11.,0.,4.,1)
CALL IAX13 (2.,3.,0.,3H=POWER TRANSMISSION FACTOR (DB),30.,.,90.,
1-30.,5.)
IF (IGRD.EQ.1) CALL SYMBOL (3.,7.,5.,9.,6.,1.,15H=TRIANGULAR GRID,0.,6.
IF (IGRD.EQ.2) CALL SYMBOL (3.,7.,5.,9.,6.,1.,16H=RECTANGULAR GRID,0.,16)
CALL PLOT (2.,3.,=3)
I3=ISTRT-1
DO 100 I=1,LOBE
I3=I3+1
I4=JJ(I3)
N1=NTH
N=0
I=1
90 DO 91 I=I1*NTH
IF (PT(I4+1),GT,=32.4999) Go to 92
N=1
91 CONTINUE
92 N=N+1
IF (N,GT,NTH) Go to 96
DO 93 I=N,NTH
IF (PT(I4+1),LE,=32.49999) Go to 94
N=1
93 CONTINUE
94 N2=N1+N1
DO 95 I=N,N1
P(I)=PT(I4+1)
ST1(I) = ST(I)
CONTINUE
P(N+1) = 30.0
P(N+2) = 5.0
ST1(N+1) = 0.0
ST1(N+2) = 0.2
CALL LINE (ST1(N) * P(N) * 2 + 1, L + 1 + 1)
IF (N1 >= N1TH) GO TO 96
I1 = N1TH
GO TO 90
96 CONTINUE
100 CONTINUE
CALL PLUT (2.0, 0.0, -3)
CALL SYMBOL (2.5, 0.75, 1.0 + 1 + 1 = 0.0 + 1)
CALL NUMBER (499, 0.0, 0.0)
CALL SYMBOL (999, 0.999, 1.4 + 1 + 0.4)
CALL SYMBOL (5.5, 0.75, 1.0 + 1 + 1 = 0.0 + 1)
CALL NUMBER (999, 0.999, 1.0 + 0.0 + 1)
CALL SYMBOL (999, 0.999, 1.4 + 1 + 0.4)
CALL GREEK (2.5, 1.0, 1 + 0.0 + 1)
CALL SYMBOL (2.75, 1.0, 2 = 0.0 + 1)
CALL NUMBER (999, 0.999, 1 + 0.3)
CALL SYMBOL (999, 0.999, 1.0 + 1 + 0.4)
CALL SYMBOL (5.5, 1.25, 1.0 + 0.0 + 1)
CALL SYMBOL (999, 0.999, 1 + 0.3)
CALL SYMBOL (999, 0.999, 1.0 + 1 + 0.4)
CALL SYMBOL (999, 0.999, 1 + 0.3)
CALL GREEK (2.5, 1.55, 1 + 0.0 + 1)
CALL SYMBOL (2.75, 1.5, 1 + 0.3)
CALL NUMBER (999, 0.999, 1 + 0.3)
CALL SYMBOL (999, 0.999, 1 + 0.4)
CALL SYMBOL (5.5, 1.5, 1 + 0.3)
CALL SYMBOL (999, 0.999, 1 + 0.3)
CALL SYMBOL (999, 0.999, 1 + 0.4)
CALL NUMBER (999, 0.999, 1 + 0.3)
CALL SYMBOL (999, 0.999, 1 + 0.4)
CALL GREEK (2.5, 1.8, 1 + 0.0 + 1)
CALL SYMBOL (2.75, 1.75, 1 + 0.0 + 1)
CALL NUMBER (999, 0.999, 1 + 0.3)
CALL SYMBOL (5.5, 1.75, 1 + 0.3)
CALL NUMBER (999, 0.999, 1 + 0.3)
CALL SYMBOL (999, 0.999, 1 + 0.4)
CALL SYMBOL (3.5, 0.2, 1.0 + 0.0 + 1)
CALL GREEK (4.0, 0.205, 1.0, 0.0)
CALL SYMBOL (4.95, 0.0, 1.0 + 1 + 1 = 0.0 + 1)
CALL NUMBER (999, 0.999, 1 + 0.3)
CALL SYMBOL (1.5, 0.25, 1.0 + 0.0 + 1)
CALL SYMBOL (1.75, 0.25, 1.0 + 1 + 1 = 0.0 + 1)
CALL GREEK (2.25, 0.25, 1 + 0.0 + 1)
CALL SYMBOL (2.35, 0.25, 1 + 0.0 + 1)
AS = S0 (ISTRT)
CALL NUMBER (999, 0.999, 1 + 0.0 + 1)
IF (LUGE LT 2) GO TO 110
CALL SYMBOL (5.0 = 2.45 + 13,0.0 = 1)
CALL GREEK (5.5 = 2.5 + 15,0.0 = 1)
CALL SYMBOL (5.0 = 2.5 + 13,0.0 = 1)
AS = SU (ISTR + 1)
CALL NUMBER (994.0, 999.0, 1.0, 3.0)
IF (LUGE LT 3) GO TO 110
CALL SYMBOL (1.75 = 2.70 + 14,0.0 = 1)
CALL GREEK (2.25 = 2.75 + 15,0.0 = 1)
CALL SYMBOL (2.35 = 2.75 + 13,0.0 = 1)
AS = SU (ISTR + 2)
CALL NUMBER (994.0, 999.0, 1.0, 3.0)
IF (LUGE LT 4) GO TO 110
CALL SYMBOL (5.0 = 2.7 + 15,0.0 = 1)
CALL GREEK (5.5 = 2.75 + 15,0.0 = 1)
CALL SYMBOL (5.0 = 2.75 + 13,0.0 = 1)
AS = SU (ISTR + 3)
CALL NUMBER (994.0, 999.0, 1.0, 3.0)
110 CALL PLUT (6.5, 3.0, 0.0)
RETURN
END
SUBROUTINE FORMDS (F, R, B)

SPECIAL DISPERSION RELATION SOLVER FOR RAPID COMPUTATION OF ELEMENT MISMATCH AT BRESIDE.

DEFINITIONS:

F  =  FREQUENCY IN GHZ
R  =  X-DIRECTED WAVENUMBER VECTOR
A  =  ALPHA
H  =  BETA
D  =  DELTA
B  =  GUIDED HEIGHT
TPI  =  6.283  ...
EPS  =  RELATIVE PERMITTIVITY OF SLARS

******************************************************************************************

DIMENSION RH(1)
REAL K, KE, KINC
COMMUN /AVGO/ ANDU+RH+TPI+EPS
DATA L/11.+H/2850/ TPI=1.P+1/F/C
EPS=EPS=1.+0
A=1.P+0*A
B=1.P+0*B
D=1.P+0*D
L=0
DO 170 M=1,4
DO 160 F=1,4
L=L+1
J=1/N
S=1.01
IF (K.LT.0.) S=0.99
K=K+T(L)+1-J)S*K
KINC=0.314
K=K+KINC
J=0
100 K=K+KINC
DIFF=DIFF1
I=I+1
KE=EPS1*K*ABS(K)
S=SURT(ABS(KE))
IF (KE.LT.0.) S=-S
KE=S
DIFF1=DISP(MUE*K*KE*A1*B1+11*EPS)
IF (J.EQ.1) GO TO 120
IF (KINC.LE.10.E10) GO TO 130
IF (ABS(DIFF+DIFF1).LE.1.E10) GO TL 130
IF (DIFF*DIFF1).LT.1.E10
110 K=K+KINC
KINC=0.5*KINC
DIFF1=DIFF
GO TO 100
120 DIFF=DIFF1
GO TO 100
130 IF (ABS(DIFF+DIFF1).LT.0ABS(DIFF1)) GO TO 140
RH(L)=K
GO TO 160
140 K=K+KINC
RH(L)=K
160 CONTINUE
170 CONTINUE
    RETURN
END
SUBROUTINE KRET (NUMODES)

SPECIAL SUBROUTINE FOR CREATING LSE(N*U) AND LSE(N+1) K=KETA
DIAGRAMS

DEFINITIONS

NUMODES = DUMMY ARGUMENT
HG = NORMA LIZED LONGITUDINAL WAVELENGTH
KA = K*A/PI
IMODE = MODE DESIGNATION
A = ALPHA/LAMBDA
B = BETA/LAMBDA
D = DELTA/LAMBDA
BB = (LAMINATE HEIGHT)/LAMBDA
TPF = 6.283185307
EPS = RELATIVE PERMITTIVITY OF SLABS

******************************************************************************

DIMENSION GAM(2*2*2),SU(2*2,2),SF(2*2*2)
REAL K0,K*K,KE,M2R,KINC
COMMON /KHETA/ HG(6,1U1),KA(1U1),IMODE(6)
COMMON /NAVGD/ A,B,D,FP,TPF,EPS
DATA C/11.8028528/ AA=A+B+0
FO=C/(TPF*AA)
DF=0.03*FO
NF=1U1
EPSU=EPS=1.0
FPSU=SQR(T(PF1)) F=FO=DF
WHITE (6900) (IMODE(1),I=1,6)
DN 200 IF=1,NF
F=FO+DF
K0=TPF*AA
KA(IF)*=0*AA
A1=K0*AA
F1=K0*0
D1=K0*D
M2A(1,5)*C/(HH*F))**2
DN 180 M=MODE+2
K=S=EPSU+1,E=1U1
M0=1
DN 180 N=1+3
J=1/N
S1=1,U1
IF (K*L1,U1) S=0.99
KJ=K*(1=J)*S*K
KINC=0.014
KE=K*KINC
I=0

100 KE=KINC
DIFF=0IF1
I=I+1
KE=EPS1+K*ABS(K)
S=SUR1(ABS(KE))
IF (KE,LT,U1) S=5
KE=S
DIFF1=1ISU(M1,K,KE,A1+R1*U1,EPS)
IF (I,EW1) GO TO 120
IF (KINC.LT.1,E=10) GO IN 130

210
IF (ABS(DIFF*DIFF1) .LE. 1.0) GO TO 130
IF (DIFF*DIFF1) 110, 130, 100

110
K*K**KINC
KINC=0.5*KINC
DIFF1=DIFF
GO TO 100

120
DIFF=DIFF1
GO TO 100

130
IF (ABS(DIFF) .LT. ABS(DIFF1)) GO TO 140
GO TO 150

140
K*K**KINC
G=EPS1+K*ABS(K)
S=SQRT(ABS(G))
IF (G .LT. 0.) S=-S
KE=S

150
SO(MODE+N*MU)=K
SE(MODE+N*MU)=KE
G=1.0*K*ABS(K)
S=SQRT(ABS(G))
IF (G .LT. 0.) S=-S
GAM(MODE+N*M0)=S*KA(IF)

160
CONTINUE

180
CONTINUE
II=0
DO 190 I=1,2
DO 190 I=1,2
II=II+1
IF (II .EQ. 4) GO TO 190
BG(II,IF)=GAM(I*II+1)
K=SO(I*II+1)
GZ=K*ABS(K)+1.0=M2B
G=SQRT(ABS(G2))*KA(IF)
IF (G .LT. 0.) G=-G
BG(I+3*IF)=G

190
CONTINUE
WRITE (6,910) KA(IF)*(BG(1*IF)*I1+6)

200
CONTINUE
C
900 FORMAT (1H1,5X,10H/ KA/2,10H*GAMMA*A/2,15X*6(2X,A7,1X))/
910 FORMAT (8X,F5.2,2X,16(2X,F6.3,2X))
END

**
SUBROUTINE LSEMOD (N)

COMPUTE LSE MODE FUNCTION FY(X)

DEFINITIONS> SEE SUBROUTINE LSMLSE

C************************************************************************************
C C*****************************************************************************
C C************************************************************************************
C C*****************************************************************************

DIMENSION VX(101)
REAL KAP, KAPE, VPS, VPA, VP1, VPS, VPA
COMMON /AKHPY/ AL+PL+U*H1, SEPI, TPI, EP, S(4)
COMMON /MPKU/ KAP(20)+KAP(20)+KAP(20)+KAP(20)+KAP(20)+KAP(20)
COMMON /MMPK/ M1K(20)+M1K(20)

M=MIN(N)
A=1.0*(AL+BL+DL)
XDEL=VPS*U*PS*U*PS
R=1.0*P1
D=1.0*(AL+DL)
NI=K1(1-M-1)
N1=K1(1-M-1)
R=5.0/U/A
D=5.0/U/A
IF (1SY+1H+1A) G0 110
X=1.02*A
DO 130 I=1,51
X=X+XDEL
Y=ABS(X)
IF (X.GT.U) G0 110
VX(1)=CDP(N)+VPS*C(CAP(N)+(X+A))+VPS*CAP(N)/MPK(N)
VX(102)=VX(1)
G0 130
110 IF (X.GT.B) G0 120
T=KAP(N)*(B-X)
VX(1)=E1P(N)+VPS*C(CAP(N)*(X+A))/MPK(N)
VX(102)=VX(1)
G0 130
120 VX(1)=VPS*(KAP(N)+Y)/MPK(N)
VX(102)=VX(1)
CONTINUE
G0 130
130 VX=1.02*A
DO 170 I=1,51
X=X+XDEL
Y=ABS(X)
IF (X.GT.U) G0 150
VX(1)=VPS*CAP(N)+SINC(KAP(N)+(X+A))/MDP(K1)
VX(102)=VX(1)
G0 170
150 IF (X.GT.B) G0 160
T=KAP(N)*(B-X)
VX(1)=E1P(N)+VPS*C(CAP(N)*(X+A))/MDP(K1)
VX(102)=VX(1)
G0 170
160 VX(1)=SINC(KAP(N)+Y)/MDP(K1)
VX(102)=VX(1)
CONTINUE
C C CALL PRINT/PLOT ROUTINE
C 180 CALL PRINT (VX,3MLS8F,R1D1)
SUBROUTINE LSMMODE (N)

COMPUTE LSM MODE FUNCTION H Y(X)

DEFINITIONS > SEE SUBROUTINE LSMU1SE

REAL KAP, KAPE, NPS, NPA, NDPS, NDPA, IX(101)
COMMON /ARRAY/ AL, BL, DL, B1L, SEPLT, TPI, EPS, S(4)
COMMON /CUEFS/ BIP(10), B2P(10), B1DP(10), B2DP(10), C1P(10), C2P(10),
               1EP1(10), E2P(10), E1DP(10), E2DP(10), F1P(10), F2P(10)
COMMON /NPS/ NPS(10), NPA(10), NDPS(I0), NDPA(I0), KOUNT(20)
COMMON /MODES/ KAP(20), KAPE(20), GAM(20), MULE(20), ISYM(20), NN(20),
               MM(20), MOMODD(20)

M=MM(N)
A=TPI*(AL+BL+DL)
XDE[L]=0.02*A
B=TPI*BL
D=TPI*(BL+DL)
N1=KOUNT(n)
B1S=50.*R/A
D1S=50.*D/A

IF (ISYM(N)) EQ. 1MA) GO 10 100
X=1.02A
D0 130 I=1+51
X=X+XDE[L]
Y=ABS(X)

IF (X GT A) GO TO 110
IX(1)=CP(N1)*COSC(KAP(I)*(X+A))/NPS(N1)
IX(102-I)=IX(1)
GO TO 130

110 IF (X GT B) GO TO 120
T=KAPE(N)*B=X
IX(1)=BIP(N1)*COSC(T)+B2P(N1)*SINC(T))/NPS(N1)
IX(102-I)=IX(1)
GO TO 130

120 IX(1)=COSC(KAP(N)*Y)/NPS(N1)
IX(102-I)=IX(1)
130 CONTINUE
GO TO 180

140 X=1.02*A
D0 170 I=1+51
X=X+XDE[L]
Y=ABS(X)

IF (X GT A) GO TO 150
IX(1)=FP(1.1)*COSC(KAP(N)*(X+A))/NPA(N1)
IX(102-I)=IX(1)
GO TO 170

150 IF (X GT B) GO TO 160
T=KAPE(N)*B=X
IX(1)=EP1(N1)*COSC(T)+E2P(N1)*SINC(T))/NPA(N1)
IX(102-I)=IX(1)
GO TO 170

160 IX(1)=SINC(KAP(N)*Y)/NPA(N1)
IX(102-I)=IX(1)
170 CONTINUE

CALL PRINT/PLOT ROUTINE

CALL PRNT (IX, 3*LSM, H1, H1)
RETURN

214
SUBROUTINE PRNT (VX, IM, B1, D1)

PRINT/ PLOT ROUTINE FOR MODE FUNCTIONS

DEFINITIONS:

VX = MODE AMPLITUDE
IM = MODE TYPE
B1 = NORMALIZED DIELECTRIC BOUNDARY POINT
D1 = NORMALIZED DIELECTRIC BOUNDARY POINT

(* To be continued *)

DIMENSION VX(1), IX(101), IP(102), IC(50)
INTEGER DOT, BLANK, PLUS, ZERO, SLASH
DATA DOT, BLANK, PLUS, MINUS, ZFRC, SLASH / 1, 1H, 1H + 1H = 1H0, 1H /
IF (IM E Q 3HLSM) IF = 3H = Y
IBL = B1
IDL = D1
IBR = 52 + IBL
IBL = 52 - IBL
IDR = 52 + IDL
IDL = 52 - IDL
XMAX = 0,
DO 100 I = 1, 101
T = ABS (VX(I))
XMAX = AMAX1 (XMAX, T)
100 CONTINUE
WRITE (6, 900) IM, IF
DO 110 I = 1, 21
WRITE (6, 910) (JJ = 51, VX(JJ), JJ = I, 101 + 21)
110 CONTINUE
DO 120 I = 1, 101
T = 50 * VX(I) / XMAX + 0.25 * SIGN (1., VX(I))
IX(I) = T
120 CONTINUE
WRITE (6, 920) XMAX
IP(1) = DOT
DO 130 I = 2, 102
IP(I) = BLANK
130 CONTINUE
IP(IDC) = SLASH
IP(IDL) = SLASH
IP(IBL) = SLASH
IP(IDR) = SLASH
DO 140 I = 1, 51
I1 = I - 1
IRA = 100 - 211
E = 0.01 * IB
N = 0
DO 140 J = 1, 101
IT = 51 = IABS (IX(JJ))
IF (IT NE I) GO TO 140
N = N + 1
IC(N) = JJ + 1
IP(JJ) = PLUS
IF (IX(JJ) .EQ. 0) IP(JJ) = MINUS
IF (IX(JJ) .EQ. 0) IP(JJ) = ZERO
140 CONTINUE
IF ((I1/10) .EQ. IJ) GO TO 150
WRITE (6, 930) (IP(JJ) * JJ = 1, 102)
GO TO 160

(* To be continued *)
150 WRITE (6,940) X*(IP(JJ)+.1J#1+102)
160 IF (N.EQ.0) GO TO 180
    DO 170 JJ=1,J
170 CONTINUE
    IP(1B)=BLANK
    IP(1D)=SLASH
    IP(1H)=SLASH
    CONTINUE
180 WRITE (6,950) (I=51,I=1,101,20)
    RETURN
900 FORMAT (1H1,5X,A3,8H CUE*, *A3,18H COMPONENT)*LAMBDA,//
15(10H 10U*X/A,10H V*LAMBDA )//)
910 FORMAT (5(3X,14,3X=F8.2x))
920 FORMAT (1H1+1UX,7HVMAX = *F10.6//
930 FORMAT (1UX,102A1)
940 FORMAT (5X,F5.2+102A1)
950 FORMAT (11X,1H+1U(10H*....,....,....,....,1U,13+5(17X+13),*58X*
17M100*X/A))
END
SUBROUTINE GREEK (X, Y, H, T, M)

SUBROUTINE TO PLOT GREEK CHARACTERS

DEFINITIONS:

X = X LOCATION OF CHARACTER
Y = Y LOCATION OF CHARACTER
H = CHARACTER HEIGHT
T = PLOTTING ANGLE
M = CHARACTER NUMBER (SEQUENCE IS GREEK ALPHABET)

DIMENSION K(120), L(25)

DATA K/7741, 4225, 1504, 211, 2144, 4577, 1526, 3645, 3477, 1434, 4342, 3121, 1127, 7704, 1522, 7710, 2245, 7724, 1312, 2132, 3315, 1626, 3577, 7703, 3377, 23515, 4021, 1131, 7711, 2131, 1336, 3525, 1677, 7701, 1314, 3771, 1335, 4420, 3771, 1425, 3544, 4313, 1221, 3142, 4377, 7725, 1121, 2277, 7711, 1577, 4515, 4417, 7701, 2577, 1625, 3141, 2477, 4432, 3177, 1221, 3277, 7705, 1511, 3435, 5771, 2031, 1102, 1304, 1525, 3477, 1333, 7713, 2434, 4342, 3171, 1213, 7703, 6145, 7744, 3077, 1024, 1324, 3443, 3212, 7754, 2443, 1221, 3142, 4334, 7703, 7143, 3377, 2410, 7703, 1424, 1120, 3041, 5477, 7720, 3577, 3424, 1312, 2131, 8424, 3477, 4477, 414, 3040, 7710, 2477, 515, 312, 2233, 4577, 7714, 302, 1131, 9424, 3477, 2123/

DATA L/1, 7, 14, 16, 24, 29, 34, 39, 46, 49, 53, 57, 67, 65, 72, 77, 82, 85, 90, 94, 199, 106, 109, 115, 121/

CALL CALCM (X + Y + 00, 1)

H*4*Y/60

M*4*Y/60

T*4*0, 174533 + 1

S*1*N (T) + C*C*S (T)

C*H*4*H

S*H*S*H

C*H*4*H

S*H*S*H

IZ = 99

I = I + L (M)

I = (I + 1) * 1

CALL CALCM (X + Y + 00, 1)

DO 3 I = I, 1

J*K (I) / I 10

DO 3 D = 1, 2

I * 10/15

IF (I = 7) 2, 1, 2

IZ = 0

GOTO 10

3

I = I + 10 * IX

CALL CALCM (X + H*4*I - S*H*4*Y + 4 + C*H*4*Y + 4 + S*H*4*Y + 4 + IZ, 1)

IZ = 99

J*K (I) = 1000

CALL CALCM (X + C*H*4*Y + S*H*4*Y, 1)

RETURN

END
SUBROUTINE AXIS(XPAGE,YPAGE,IRCNOCHAXLEN,ANGLE,FIRSTV,DELTA)

CONTAINS CALLS TO CALCMPE INSTEAD OF PLUT

CONTAINS AXIS ENTRY POINT - WRITES VALUES IN INTEGER FORMAT

XPAGE YPAGE COORDINATES OF STARTING POINT OF AXIS, IN INCHES

IBCNOCH AXIS TITLE.
NCHAR NUMBER OF CHARACTERS IN TITLE, + FOR C.C=+ SIDE.
AXLEN FLOATING POINT AXIS LENGTH IN INCHES.
ANGLE ANGLE OF AXIS FROM THE X-DIRECTION IN DEGREES.
FIRSTV SCALE VALUE AT THE FIRST TIC MARK.
DELTA CHANGE IN SCALE BETWEEN TIC MARKS ONE INCH APART

DIMENSION IBCNOCH(1)

15 NDEC=2
15 INT=2
15 GO TO 8

ENTRY IAXIS

IF (ABS(DELTAV).LT.1.) GO TO 15

NDEC=1
15 INT=1
8

KN=NC
VIEH=1.
A=1.
1

IF(KN)12,2
A=A

2

EX=0.0

ADX=ABS(DELTAV)

IF(AEX).LT.13 GO TO 3
13

IF(AEX).LT.14 GO TO 4

ADX=ADX/10.0
EX=EX+1.0
GO TO 3
4

ADX=ADX*10.0
EX=EX-1.0

6

IF(AEX).LT.7.57 GO TO 7
7

XVAL=FIRSTV*10.0**(-EX)
ADX=DELTA*10.0**(-EX)
STH=ANGLE*0.0174533
CTH=COS(STH)
STH=TH(SIH)
CTHTIC=CTH*VIEW
STHTIC=STH*VIEW

DXB=0.1
POSN=.5

IF(VIEW).LT.9)POSN=.25
DYN=POSN/.A=0.05

XN=XPAGE+UXB*CTH+UYB*STH
YN=YPAGE+UYB*CTH+UXB*STH
NTIC=AXLEN+1.0
NTIC=NTIC/2

DO 20 I=1,NTIC
20

GO TO 10 (9,10),INT

NEG=0

IF(XVAL.LT.0) NEG=1
AXVAL=ABS(XVAL)
NDIG=2-NEG

IF(XVAL.GE.9.5)NDIG=1-NEG
SUBROUTINE CSIMEQ(A,N,B,M,KS)
SCALL CSIMEQ
CDC6700***CSIMEQ

PURPOSE
OBTAIN SOLUTION OF A SET OF SIMULTANEOUS LINEAR EQUATIONS, A*X=B

USAGE
CALL CSIMEQ(A,N,B,M,KS)

DESCRIPTION OF PARAMETERS
A = MATRIX OF COEFFICIENTS STORED COLUMNWISE. THESE ARE
DESTROYED IN THE COMPUTATION. THE SIZE OF MATRIX A IS
N BY N.
B = MATRIX OF ORIGINAL CONSTANTS (LENGTH N BY M). THESE ARE
REPLACED BY FINAL SOLUTION VALUES. MATRIX X.
N = NUMBER OF EQUATIONS
M = NUMBER OF SETS OF SOLUTIONS
KS = OUTPUT DIGIT
0 FOR A NORMAL SOLUTION
1 FOR A SINGULAR SET OF EQUATIONS

REMARKS
MATRIX A MUST BE GENERAL.
IF MATRIX IS SINGULAR, SOLUTION VALUES ARE MEANINGLESS.

METHOD
METHOD OF SOLUTION IS HY ELIMINATION USING LARGEST PIVOTAL
DIVISOR. EACH STAGE OF ELIMINATION CONSISTS OF INTERCHANGING
ROWS WHEN NECESSARY TO AVOID DIVISION BY ZERO OR SMALL
ELEMENTS.
THE FORWARD SOLUTION TO OBTAIN VARIABLE A IS DONE IN
N STAGES. THE BACK SOLUTION FOR THE OTHER VARIABLES IS
CALculated BY SUCCESSIVE SUBSTITUTIONS. FINAL SOLUTION
VALUES ARE DEVELOPED IN VECTOR B, WITH VARIABLE 1 IN R(1),
VARIABLE 2 IN H(2), . . . . . . VARIABLE N IN H(N),
IF NO PIVOT CAN BE FOUND EXCEEDING A TOLERANCE OF 0.05,
THE MATRIX IS CONSIDERED SINGULAR AND KS IS SET TO 1. THIS
TOLERANCE CAN BE MODIFIED BY REPLACING THE FIRST STATEMENT.

FORWARD SOLUTION

COMPLEX A,B,BIGA
DIMENSION A(20,20),B(20,20)
TOL=0.0
KS=0
DO 200 J=1,N
BIGA=(0.,0.)
C Search FOR MAXIMUM COEFFICIENT IN COLUMN
DO 120 I=1,N
IF(CABS(BIGA)=CABS(A(I,J)))110,120,120
110 BIGA=A(I,J)
IMAX=I
120 CONTINUE
TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)

IF(ABS(REAL(BIGA)) + ABS(IMAG(BIGA)) = TOL) 130, 130, 140

CONTINUE

INTERCHANGE ROWS IF NECESSARY

DIVIDE EQUATION BY LEADING COEFFICIENT

ELIMINATE NEXT VARIABLE

BACK SOLUTION

END
FUNCTION SINC(X)
IF (X.LT.0.) GO TO 100
SINC=SIN(X)
RETURN
100 SINC=SINH(X)
RETURN
END
FUNCTION COSC(X)
IF (X.LT.0.) GO TO 100
COSC = COS(X)
RETURN
100 COSC = COSH(X)
RETURN
END
FUNCTION TAN\(X\)\)
IF (X LT 0) GO TO 100
TAN\(X\)
RETURN
100 TANH\(X\)
RETURN
END
FUNCTION S1(X)
IF (ABS(X).GT.1.E-15) GO TO 100
S1=2.0
RETURN
100 S1=SINC(X)/X+1.0
RETURN
END
FUNCTION S2(X)
IF (ABS(X).GT.1.E-15) GO TO 100
S2=0.
RETURN
100 S2=1.0-SINC(X)/X
RETURN
END
FUNCTION S3(X)
IF (ABS(X).GT.1.*F=15) GO TO 100
S3=0.*U
RETURN
100 S3=(SINC(X)**2)/ABS(X)
RETURN
END
FUNCTION SXUX(x)
SXUX=1.0
IF (ABS(x).LT.1.f=10) RETURN
SXUX=SINC(x)/x
RETURN
END
FUNCTION TXUX (X)
IF (ABS(X),GT,1,E-15) GO TO 100
TXOX = 1.0
RETURN
100 TXOX = TAN(T)/(X)
RETURN
END
REFERENCES


7. Private communication from R.J. Mailloux


REFERENCES (cont.)


## Metric System

### Base Units:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>SI Symbol</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>metre</td>
<td>m</td>
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</tr>
<tr>
<td>mass</td>
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<td>time</td>
<td>second</td>
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<td>electric current</td>
<td>ampere</td>
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<td>thermodynamic temp.</td>
<td>kelvin</td>
<td>K</td>
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<tr>
<td>amount of substance</td>
<td>mole</td>
<td>mol</td>
<td></td>
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<tr>
<td>luminous intensity</td>
<td>candela</td>
<td>cd</td>
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</tbody>
</table>

### Supplementary Units:

| Plane angle         | radian        | rad       |         |
| solid angle         | steradian     | sr        |         |

### Derived Units:

| Acceleration        | metre per second squared | m/s       |         |
| activity (radioactive source) | disintegration per second | (disintegration)/s |         |
| angular acceleration | radian per second squared | rad/s     |         |
| angular velocity    | radian per second     | rad/s     |         |
| area                | square metre         | m         |         |
| density             | kilogram per cubic metre | kg/m     |         |
| electric capacitance | farad             | F         | A·V     |
| electrical conductance | siemens         | S         | A/V     |
| electric field strength | volt per metre     | V         | V/m     |
| electric inductance | henry               | H         | V·s/A   |
| electric potential difference | volt           | V         | V/A     |
| electric resistance | ohm                 | V         | W/A     |
| electromotive force | volt               | V         | W/A     |
| energy              | joule               | J         | N·m     |
| entropy             | joule per kelvin    | J         | J/K     |
| force               | newton              | N         | kg/m/s  |
| frequency           | hertz               | Hz        | cycle/s |
| illuminance         | lumen               | lx        | lm/m    |
| luminance           | candelas per square metre | cd/m   |         |
| luminous flux       | lumen               | lm        | cd·s    |
| magnetic field strength | ampere per metre    | A·m       |         |
| magnetic flux       | weber               | Wb        | V·s     |
| magnetic flux density | tesla                | T         | Wb/m    |
| magnetomotive force | ampere              | A         |         |
| power               | watt                | W         | J/s     |
| pressure            | pascal              | Pa        | N/m     |
| quantity of electricity | coulomb            | C         | A·s     |
| quantity of heat    | joule                | J         | N·m     |
| radiant intensity   | watt per steradian  | W         | sr      |
| specific heat       | joule per kilogram-kelvin | J/kg·K     |         |
| stress              | pascal              | Pa        | N/m     |
| thermal conductivity | watt per metre-kilogram-kelvin | W/m·K |         |
| velocity            | metre per second    | m         | s       |
| viscosity, dynamic  | pascal-second       | Pa·s      |         |
| viscosity, kinematic | square metre per second | m/s     |         |
| voltage             | volt                | V         | W/A     |
| volume              | cubic metre         | m         |         |
| wavenumber          | reciprocal metre    | J         | (wavelength/m) |
| work                | joule                | J         | N·m     |

### SI Prefixes:

<table>
<thead>
<tr>
<th>Multiplication Factors</th>
<th>Prefix</th>
<th>SI Symbol</th>
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* To be avoided where possible