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**NSWC/WOL/TR 76-145**

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# **NSWC** **TECHNICAL** **REPORT** **WOL** **C**

**WHITE OAK LABORATORY**

**WUNDY CALCULATIONS OF THE COLLAPSE VELOCITIES OF A STAGED IMPLODING CYLINDER**

**BY**  
**James P. Coughlin**

**18 APRIL 1977**

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC/WOL/TR 76-145	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) WUNDY CALCULATIONS OF THE COLLAPSE VELOCITIES OF A STAGED IMPLoding CYLINDER		5. TYPE OF REPORT & PERIOD COVERED Final
7. AUTHOR(s) James P. Coughlin		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center White Oak Laboratory White Oak, Silver Spring, Maryland 20910		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 0; 0; 0; 0;
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE 18 April 1977
		13. NUMBER OF PAGES 55
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Hydrocalculations                      Cylinders Implosion WUNDY Staging		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report describes numerical computations for imploding staged cylinders. The calculations were made with the WOL's one-dimensional code, Wundy. Six different geometries were examined and the positions and velocities of the inner and outer surfaces of both the core and the staging were calculated. In the course of these calculations, a spurious oscillation arose and the steps taken to deal with it are described.		

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S/N 0102-014-6601

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WUNDY CALCULATIONS OF THE COLLAPSE VELOCITIES OF A STAGED IMPLoding  
CYLINDER

The work reported here was done at the request of Dr. F. I. Grace of the Ballistics Research Laboratory (BRL), Aberdeen, Maryland, on funding supplied by BRL. It will be of interest to anyone calculating cylindrical implosion problems and it points out the need for a linear artificial viscosity term in performing numerical calculations of this type.

*J. Enig, for*

JULIUS W. ENIG  
By direction

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## WUNDY CALCULATIONS OF THE COLLAPSE VELOCITIES OF A STAGED IMPLoding CYLINDER

### INTRODUCTION

Computational work was undertaken to determine the effect of staging on the collapse velocity of a copper cylinder under an imploding cylindrical detonation wave. A pie-shaped section of the cylinder is shown in Figure 1. The inner copper cylinder is called the core and the outer one is called the staging. There were six problems in all, although the first two had the same geometry, differing only in the modes of initiation. By varying the dimensions four additional problems were obtained. This report will describe the calculations, the difficulties encountered, and the steps taken to overcome these difficulties.

### THE FIRST TWO PROBLEMS

For the first problem, the dimensions are given in Table I. (All dimensions for all problems were supplied by Dr. F. I. Grace of the Ballistics Research Laboratory.)

TABLE I

<u>Radius</u>	<u>Value</u>
r <sub>1</sub>	2.00 cm
r <sub>2</sub>	2.18 cm
r <sub>3</sub>	2.68 cm
r <sub>4</sub>	3.25 cm
r <sub>5</sub>	3.33 cm
r <sub>6</sub>	5.00 cm

The explosive was initiated circumferentially at r<sub>3</sub> and r<sub>6</sub> producing a detonation wave travelling inward. The equation of state for the detonation products was that of a gamma law gas:

$$p = (\gamma - 1)E/V$$

where  $V = \rho_0/\rho$  and  $\rho_0$  is taken to be 1.6 gm/cc,  $\gamma = 2.742$  and the initial energy was .0078624 megabar-cc/cc

The equation of state used for the copper was a modified Mie-Grüneisen equation with empirical constants determined by Rice, McQueen and Walsh (Compression of Solids by Strong Shock Waves in Solid State Physics, ed. by Seitz and Turnbull, (Academic Press Inc., New York City) Vol. 6, pp. 1-63). The second problem was identical to the first, except that the explosive energy was released immediately (constant volume detonation - which is equivalent to an infinite detonation velocity). For each copper interface (viz:  $r_1$  and  $r_2$ ,  $r_4$  and  $r_5$  of Figure 1) plots were made of the position and velocity as functions of time. The results for the first problem are shown in Figures 2,3,4,5, and 6 and those for the second problem in Figures 7,8,9,10, and 11.

Turning first to Figure 2, the positions of the copper staging (above) and of the copper core can be seen. Examining the staging first, a slight "bump" in the curve is observed at A. In Figures 5 and 6, the velocities are seen to be positive at this point and this result is somewhat surprising, although easily explained. The detonation products from the inner explosive cross the gap and collide with the staging, accelerating it radially outwards, before the detonation wave from the outside explosive (which starts at  $r_6$  in Figure 1) has arrived. This detonation wave reverses the process and drives the staging inward toward the axis ( $r=0$ ). The process is finally reversed (point B) when the relief wave from the outer surface comes back through the detonation products and impinges on the staging, allowing it to expand.

The most striking feature of Figure 2 ( and it occurs in the other plots of position vs. distance) is the lower curves which show the swelling of the copper core as it collapses on the origin. (Observe how the vertical distance between the core surfaces increases between points C and D in the figure.) At first this seems anomalous, but careful consideration reveals the underlying causes. The pressure forces arise to resist changes in volume. If the core of Figure 1 is driven towards the origin, then (using the notation of Figure 1) the volume will be given by:

$$V = \pi (r_2^2 - r_1^2)h$$

where  $h$  is a unit height of cylinder and  $r_2$  and  $r_1$  are functions of time. Let  $\Delta r = r_2 - r_1$ . Then:

$$V = \pi h \Delta r (2r_1 + \Delta r)$$

As  $r_1$  becomes smaller,  $\Delta r$  must increase or the volume will be reduced. If the volume is reduced, a pressure arises to resist the reduction. This pressure accelerates  $r_1$  inward (toward the axis) and  $r_2$  outward, causing the velocity of the inner surface to increase

in magnitude and that of the outer surface to decrease in magnitude. This causes the observed swelling of the core. This is confirmed by the velocity plots of Figures 3 and 4. In Figure 3, there is a sharp decrease in velocity (the velocity is negative and decreases serve to increase the magnitude) as the shock arrives (about 2 microseconds), followed by a levelling off (from 5 to 10 microseconds) and a new deceleration caused by the shock from the outer explosive (from 14 to 24 microseconds) followed by a very rapid decrease in velocity (about 25 microseconds) produced by the pressure forces resisting the convergence. At a time greater than 29.8 microseconds, the inner surface of the core reaches the symmetry axis and its velocity is reduced to zero.

In Figure 4, the velocity of the outer surface of the copper core is plotted against time and the graph is similar to Figure 3 (the inner surface) until about 20 microseconds where this velocity begins to level off and finally increase due to the pressure forces created by the convergence. The velocity becomes zero at approximately the time the inner surface arrives at the axis. Its velocity becomes positive thereafter as the copper expands, but is reduced somewhat by the inertia of the reacted gases and seems to be declining near 40 microseconds, which was the end of the computation.

In Figures 5 and 6 the velocity of the inner and outer surfaces (respectively) of the copper staging is plotted. At about 4 microseconds, both surfaces are accelerated outwards by the arrival of the detonation products from the inner explosive. When the outer detonation wave arrives, this motion is quickly stopped and the staging is accelerated inward. The inward motion is arrested by the inertia of the reacted gases from the inner explosion and later by the arrival of the relief wave from the outside. Thereafter the velocities begin increasing and finally become positive as the staging is forced outward.

The corresponding plots for the "constant volume detonation" are found in Figures 7,8,9,10, and 11. Here there is no detonation wave, but the chemical energy of the explosive is released instantaneously and the detonation pressure appears everywhere in the explosive at the same time. Figure 7 shows the plots of position of the liner and the staging against time and resembles the corresponding plots of Figure 2. In Figures 8 and 9, plots of the velocity of the inner and outer surfaces (respectively) of the copper core are shown. They resemble the corresponding plots for problem one except for an oscillation superimposed on the general motion.

#### THE OSCILLATION

The first question to arise, in connection with this oscillation is whether or not it is genuine or merely a numerical instability. On the one hand, it does not seem to grow in time, and the period

corresponds roughly to the shock transit time across the copper core. Nevertheless it is not a genuine ringing of the metal. To begin with, no such oscillation appears in problem 1 and, further, consideration of the shock wave system shows that there is no force available to slow down the inward motion of the inner surface of the core. The velocity curves of Figure 8 must slope downward as time increases. In order to discover why the velocity increases towards zero, curves of pressure versus distance at 0.2 microseconds (see Figure 12) were examined. There, sharp "spikes" (at A and E in Figure 12) were found. These pressures exceed the detonation pressure (G or H) by 10% or more. This is physically unrealistic. Even if such a pressure rise could occur, the pressures at B and F would lie between the spike pressure and the detonation pressure. As is seen from the graph, these pressures actually lie below both. In a plane wave case, for which solutions are available, the pressure profile would be as shown in Figure 13. The cylindrical wave should show some increase due to convergence, but at 2 cm, this effect should be much smaller than in Figure 12.

An explanation was sought for the anomalous pressure build up in the copper and, when none was found, a cure was sought instead. It should be observed, before passing on, that the pressure spike, on reflection from the free surface becomes a rarefaction and the interaction of this rarefaction with the reduced pressure behind causes a tension to appear in the copper. This tension produces the velocity upswing which is then propagated back and forth across the case causing the oscillation. Similar remarks apply to the copper staging. If the pressure buildup can be prevented, then the oscillation would be prevented also.

The first and most obvious answer is that the problem is a starting problem, caused by the discontinuous nature of the pressure function at time zero. Several schemes were used to get the problem started correctly:

1. Fine zoning and small initial time steps. This was not very successful.
2. Adjusting the artificial viscosity. It is known that this term smooths out shocks and discontinuities and enables an otherwise unsuccessful finite difference scheme to proceed. In the case at hand, there is an initially discontinuous pressure function. Consequently, if a much higher value of the coefficient for the artificial viscosity term is used, the smoothing out of the discontinuity should be much more rapid. The coefficient can then be gradually reduced to its normal value. This approach also failed.
3. Velocity cut-offs. The shock transmission problem can be solved for two dimensional flow (see Appendix A) and the pressure and particle velocity transmitted to the copper can be calculated. Now the pressure discontinuity at the copper-explosive interfaces causes these interfaces to have an infinite acceleration. The calculated acceleration will, accordingly, be very large - so large that the product of acceleration and time step will result in an interface velocity greater than the particle velocity calculated

from the shock transmission problem. Then this interface will move too rapidly towards its neighbor causing an overcompression which in turn leads to the overpressure. To prevent the overpressure, a velocity cut-off is introduced: no interface is allowed to move with a velocity greater than the particle velocity computed in Appendix A. This method also failed.

4. The linear viscosity. When von Neumann and Richtmyer first introduced the concept of artificial viscosity ("A Method for the Numerical Calculation of Hydrodynamic Shocks," J. Appl. Phys. Vol 21, p. 232 (1950)), they considered the various functional forms. The one considered best was to make the viscosity term dependent on the square of the velocity gradient. Later mathematicians introduced a dependence on the first power of the velocity gradient as well. A combination of these two (linear and quadratic viscosity) was used for the problem under consideration. This is the method that worked. Accordingly, problem 2 was rerun using this function for viscosity. The low energy explosive used in problems 1 and 2 was discarded in favor of Comp B which has a density of 1.68 gm/cc and an energy per unit volume of  $7.6824 \times 10^{-2}$  megabars (NAVORD Report 4510), and a gamma of 2.742. The results are plotted in Figures 14,15,16,17, and 18. Figure 15 shows the velocity of the inner surface of the case and it is quite satisfactory. The velocity proceeds downwards, more or less in stages as a qualitative consideration of the shock fronts would predict. The annoying oscillation has been removed (see Appendix B).

#### DISCUSSION OF THE CURVES

The dimensions for problem 2 are given in Table I and the dimensions for problems 3 through 6 are given in Table II.

TABLE II

<u>Problem Number</u>	$r_1$	$r_2$	$r_3$	$r_4$	$r_5$	$r_6$
3	1.50 cm	1.65 cm	1.95 cm	2.30 cm	2.555 cm	4.05 cm
4	1.50 cm	1.65 cm	1.95 cm	2.65 cm	2.90 cm	4.4 cm
5	1.50 cm	1.65 cm	1.95 cm	3.00 cm	3.25 cm	4.75 cm
6	1.50 cm	1.65 cm	1.95 cm	3.35 cm	3.60 cm	5.1 cm

In all four problems, the core is thinner and closer to the origin than in the first problem. The distance to the staging is systematically varied as well as the mass of copper in the staging and the mass of explosive behind the staging. Constant volume detonation is used throughout.

The results of the calculation for problem 3 are shown in Figures 19,20,21,22, and 23 while those for problem 4 appear in Figures 25,26,27,28, and 29. Similarly the results of problem 5

are shown in Figures 30 through 34 inclusive and those of problem 6 in Figures 35 through 39. In Figures 14,19,25,30, and 35, the positions of the four copper interfaces are plotted against time. The staging (the upper curves) is initially pushed inward by the outer explosive, but is finally driven outwards and compressed. The core (lower curves) collapses on the axis and begins to swell as it approaches the axis, as observed earlier. After the collapse, the core begins to swell to an enormous size (especially in Figure 14, although Figures 19,25,30, and 35 show the outer radius levelling off for large times). The core fills almost the entire volume originally occupied by the core and inner void although the plots of the outer surface velocity vs. time (Figures 16,21,27,32, and 37) show the velocity going towards zero. The expansion is over, leaving the copper at an extremely high temperature and in a state of tension.

In reality, this expansion would not occur. It is found in the calculations only because the model used in the WUNDY code does not permit either fracture or rebound - the copper cannot tear apart nor can it ever come away from the axis. Further the tension in the copper is limited to 3kb which prevents the restoring forces in the copper from building up. The combination of these factors lead to the excessive expansion of the core, as shown in the figures.

The velocity of the inner surface of the core is plotted in Figures 15,20,26,31, and 36. As was noted earlier, the velocities proceed downwards in stages in the early part of the motion, corresponding to the arrival of the shock from the inner explosive, followed by the arrival of the reflection of the reflected relief wave. The arrival of the shock produced by the collision of the staging with the inner explosive produces a new downturn in the velocity curve and the curve turns downwards. At first the rate at which the curve slopes downward seems to be decreasing (the curvature of the curve is positive) but then there is a transition to an increasingly rapid descent (at about 7 microseconds in Figure 20). This is the convergence effect referred to earlier. Thereafter the velocity becomes very negative very fast until the axis is reached, at which time the velocity is reduced to zero.

The plots for the velocity of the outer surface of the copper core are found in Figures 16,21,27,32, and 37. The motion of the outer surface parallels that of the inner surface until the convergence effect makes itself felt. When the inner surface velocity "takes off", the outer surface velocity levels off and starts back towards zero. It swings positive after the collision, reaching a high peak from which it gradually decays towards zero. In Figure 16, the decay is very gradual compared with the others. In Figures 21, 27,32, and 37, the velocity curve passes through zero and oscillates about that value. In these four problems (3,4,5, and 6) the staging was much more massive than in problem 2 and this may account for the more rapid decline in velocity.

In Figures 17,22,28,33 and 38, the velocity of the inner surface of the copper staging is plotted against time. In Figure 17, there is a slight dip at about 1 microsecond, but it quickly disappears. The same cannot be said for the remaining figures and a new oscillation seems to be rearing its ugly head. Accordingly, the shock wave system giving rise to the plots of Figure 22, was examined in detail. In Figure 24, the shock wave system is represented. The detonation shock (A in Figure 24) starts the inner surface moving. These shocks are reflected as relief waves (B's in Figure 24) which will in turn be reflected as a compression wave (not shown for the staging in Figure 24, but analogous to C in the core) which will give the front surface another downward velocity increment. This motion is arrested by the collision of the staging with the expanding detonation products from the inner explosive (slightly less than 0.8 $\mu$ sec). The resulting collision shock (H in Figure 24) causes the velocity to increase sharply towards zero, followed by a gradual reexpansion of the copper which causes a slight downturn in the velocity which is accentuated sharply by the arrival of the relief wave (F in Figure 24) from the liner-explosive interface (slightly greater than 0.8 $\mu$ sec). There results a rapid downturn in velocity which is arrested by the arrival of a relief wave from the outer surface of the staging (L in Figure 24). This drives the velocity back upwards and the interplay of these two relief waves moving back and forth across the staging produces an oscillation of diminishing amplitude until the arrival of the reflection (from the copper liner) of the collision shock at about 4.7 $\mu$ sec (I in Figure 24) which starts the staging moving out. The velocity ultimately swings positive and levels off after 18 $\mu$ sec. Similar conclusions hold for Figures 28,33, and 38.

The plots of the velocity of the outer surface of the staging are given in Figures 18,23,29,34, and 39. They parallel the plots of the velocity of the inner surfaces and require little comment.

### CONCLUSIONS

The initial objective of this study was to determine the effects of staging on the implosion of a copper core. In carrying out the calculations, a difficulty arose with a spurious oscillation and this difficulty was overcome only through the use of a linear artificial viscosity term. With this improvement, the calculation proceeded as would be expected from a qualitative analysis of the system.

The quantity of greatest interest in the calculation was a collapse velocity for the core. At first glance, the velocity of the inner surface of the core at the time of collapse would seem to be the most relevant number. But the fact that the velocity decreases so rapidly near the collapse time makes this number a poor choice. If the time steps near the collapse time were cut in half, say, then the calculated velocity would be much higher near

the collapse time. Consider the curves of Figures 15,20,26,31, and 36. The slopes are almost vertical at the time step before collapse. If a velocity for a time intermediate between  $t_n$ , the last time for which  $X$  is positive, and  $t_{n+1}$  were extrapolated from these curves, the result would be significantly less due to the rapid fall. Hence, these numbers are not very reliable as they are sensitive to the time difference between  $t_n$  (referred to above) and the actual collapse time.

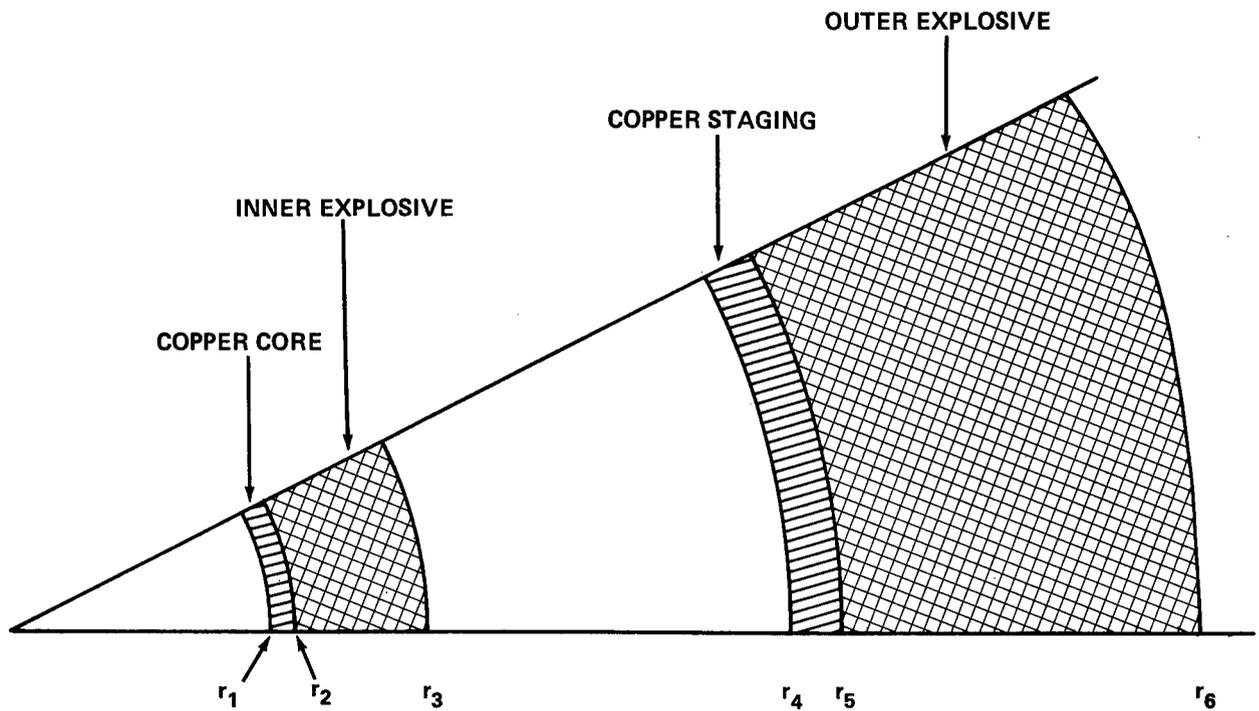
The only number that does seem to be varying slowly enough for use in comparing the problems is the minimum value of velocity for the outer surface of the core. These values are given in Table III.

TABLE III

Problem Number	Velocity	Time	CMO
2	-.21283	7.8 $\mu$ sec	4.99
3	-.17268	6.5 $\mu$ sec	1.51
4	-.17052	7.3 $\mu$ sec	1.49
5	-.16600	7.4 $\mu$ sec	1.45
6	-.15980	8.4 $\mu$ sec	1.42

The last column of the table is the ratio of mass of the outer explosive to mass of copper staging, which varies slowly. It should be observed that, in problem 2, the liner starts out 2 cm from the axis, but in the other four, 1.5 cm. This accounts for the large velocity coupled with the longer collapse time. The collapse velocity varies monotonically with this number and the results are graphed in Figure 40. So the collapse velocity can be improved by increasing the mass of the outer explosive, decreasing the mass of the copper staging, or both. But if the mass of the copper staging is too far decreased (as in problem 2) the core will "swell" after collision (see Figure 14) whereas a more massive copper staging will prevent this (see Figures 19,25,30, and 35). Of course, the realism of Figure 14 is open to question since the copper core occupies a greater volume than both the liner and inner void did originally.

It is doubtful that this could occur experimentally, but the alternatives (which would be either a fracture appearing in the copper or a rebound of the copper from the axis) would undoubtedly occur. The more massive staging should be useful in preventing either of these occurrences.



A PIE-SHAPED SECTOR OF THE CYLINDER UNDER STUDY. THE  $r$ 'S REPRESENT THE DISTANCE FROM THE ORIGIN TO THE INTERFACE AND THEIR DIFFERENT VALUES ARE GIVEN IN THE TEXT. THE CROSSHATCHED REGIONS ARE COMP B AND THE LINEARLY SHADED REGIONS ARE COPPER. THE NON-SHADED REGIONS ARE VOIDS.

FIG. 1 GEOMETRY OF THE PROBLEMS

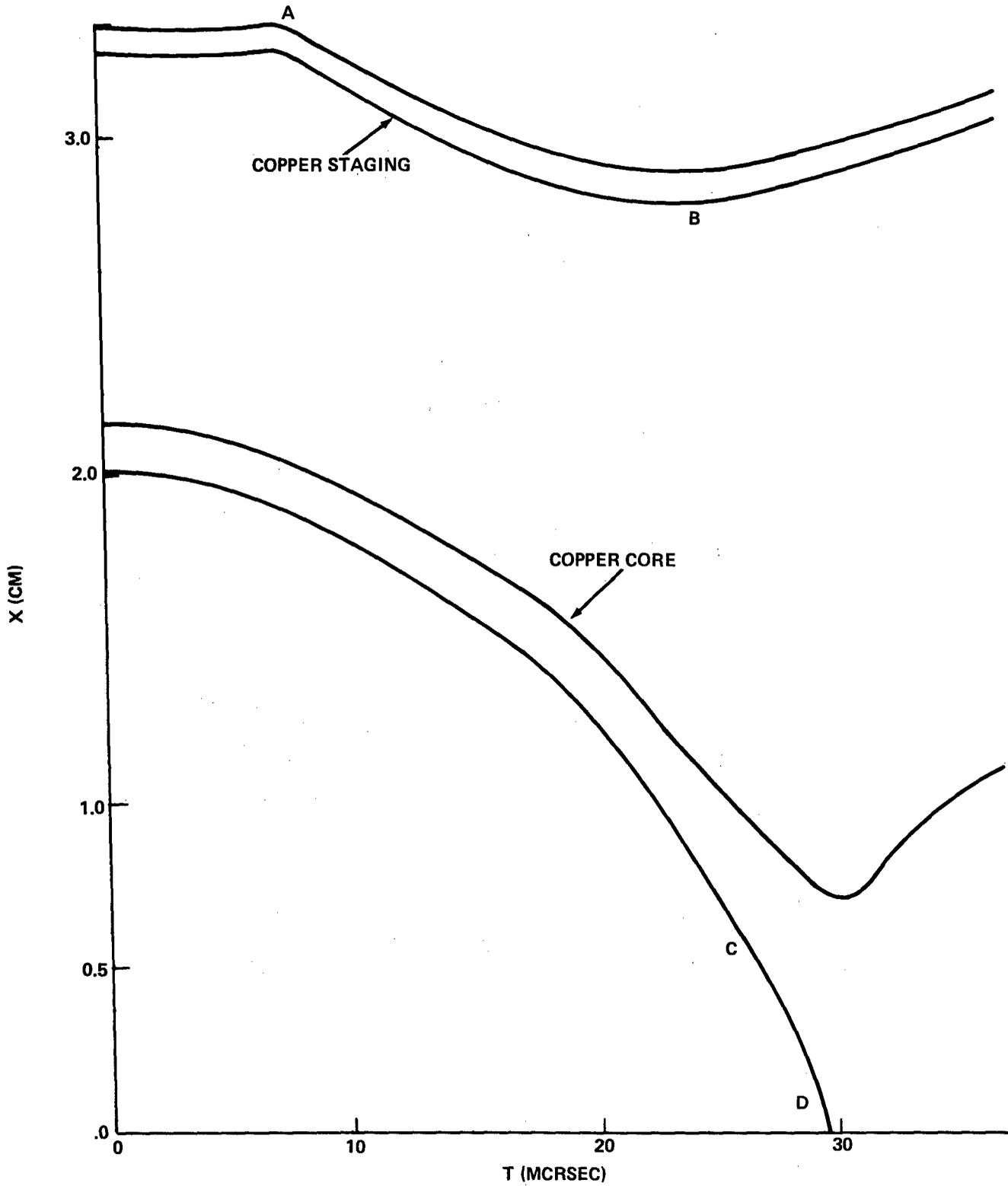


FIG. 2 PLOT OF CASE POSITION VS. TIME FOR PROBLEM 1

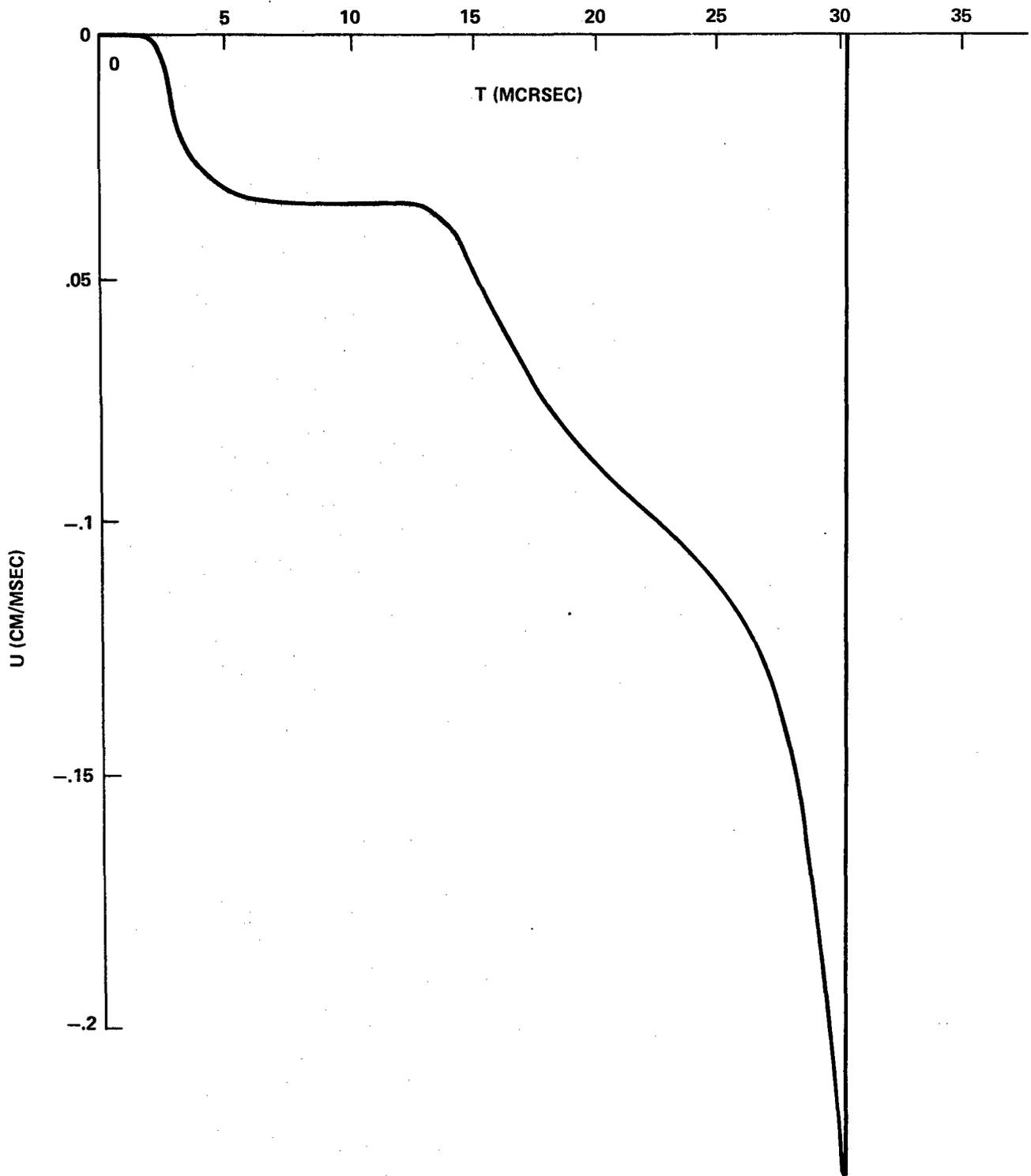


FIG. 3 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE CORE VS. TIME (PROBLEM 1)

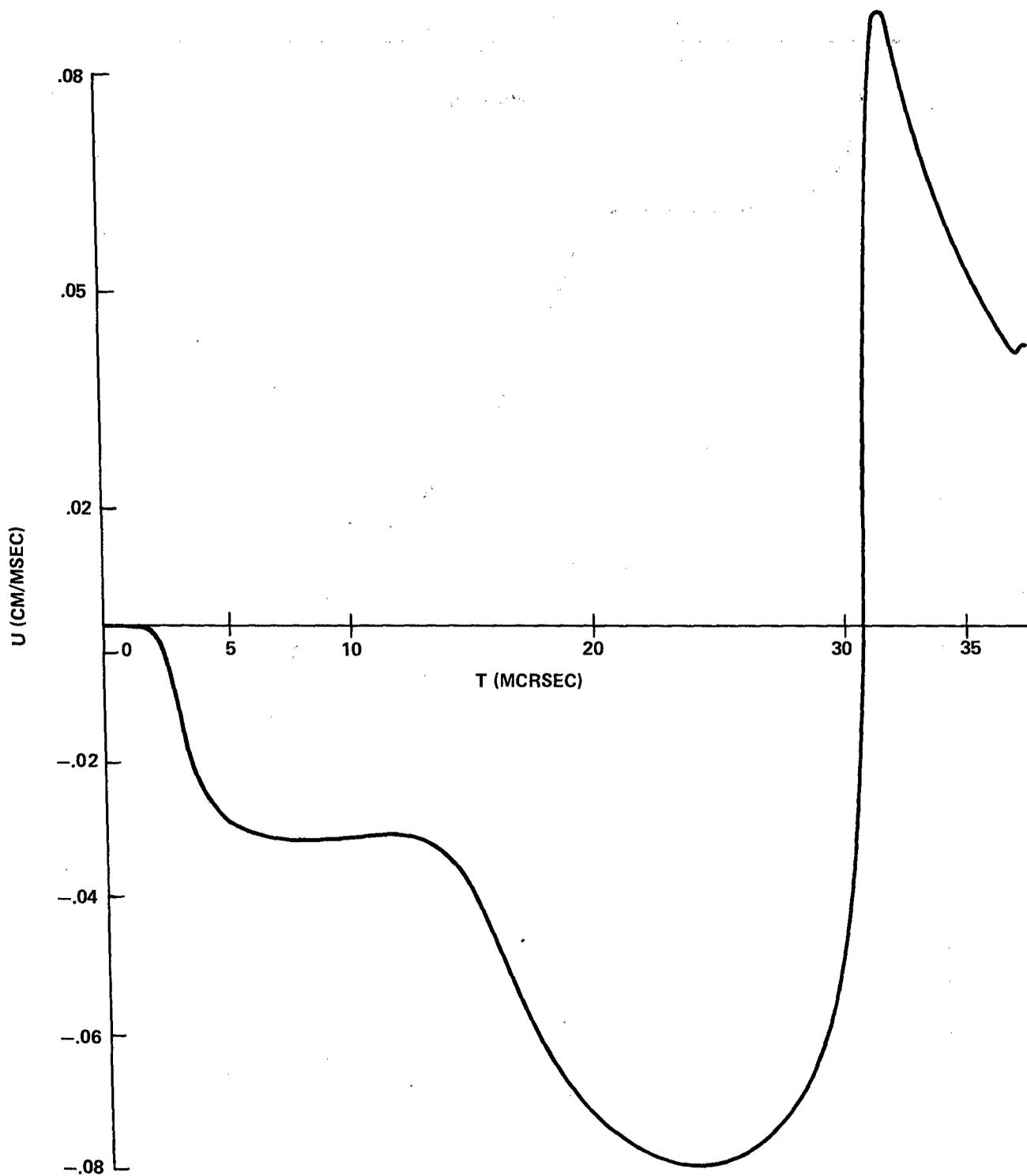


FIG. 4 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE CORE VS. TIME (PROBLEM 1)

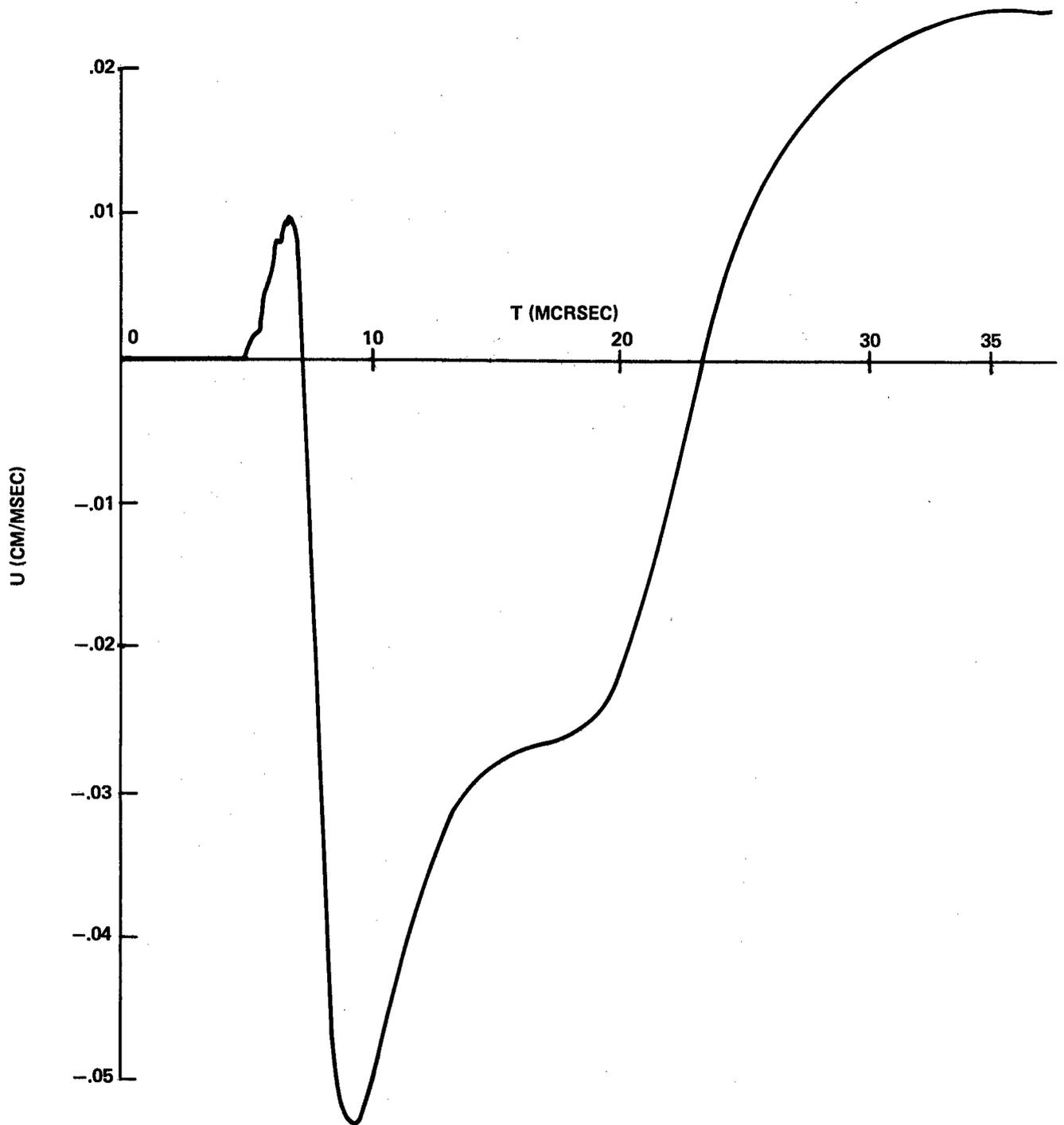


FIG. 5 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE STAGING VS. TIME (PROBLEM 1)

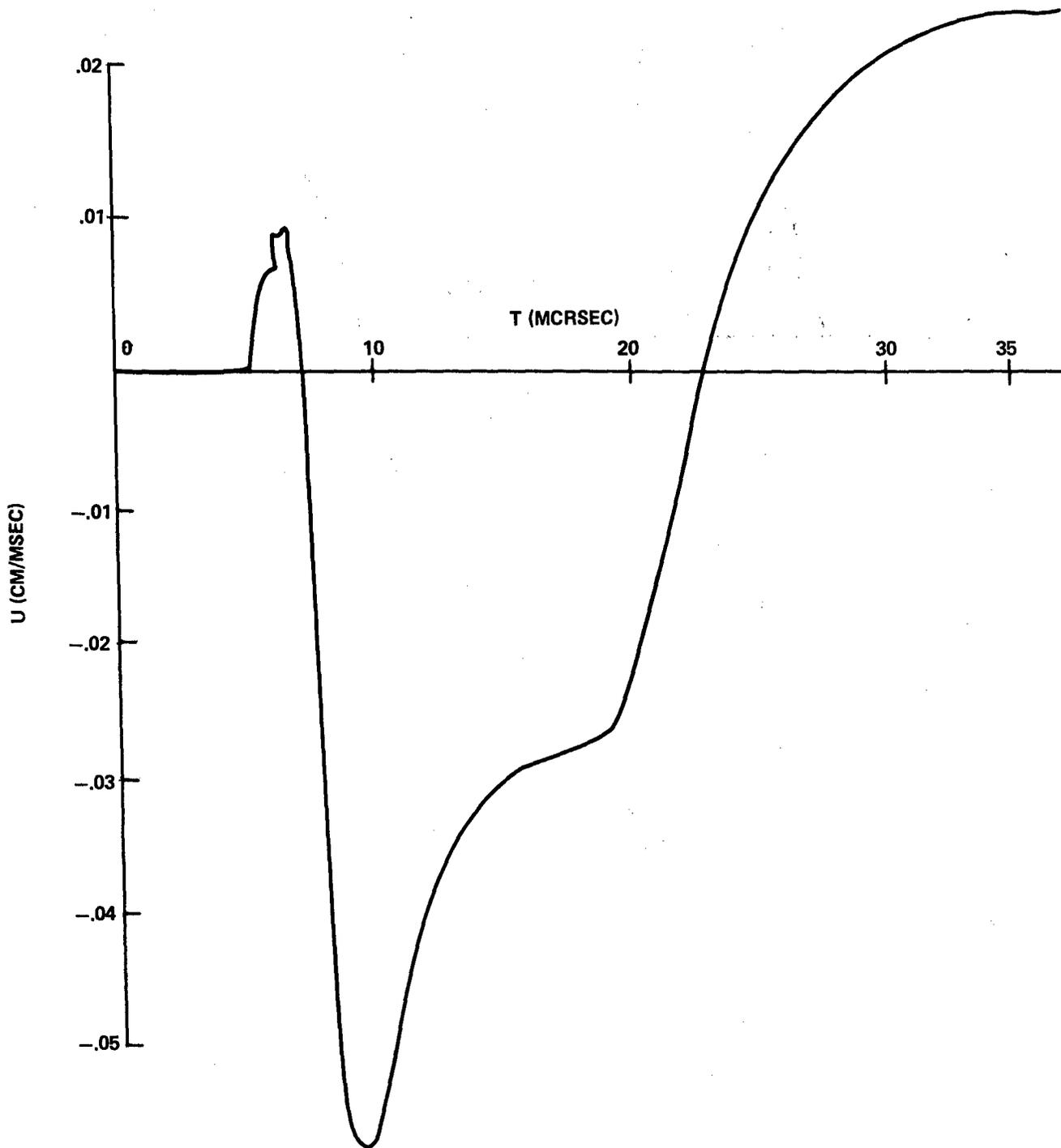


FIG. 6 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE STAGING VS. TIME (PROBLEM 1)

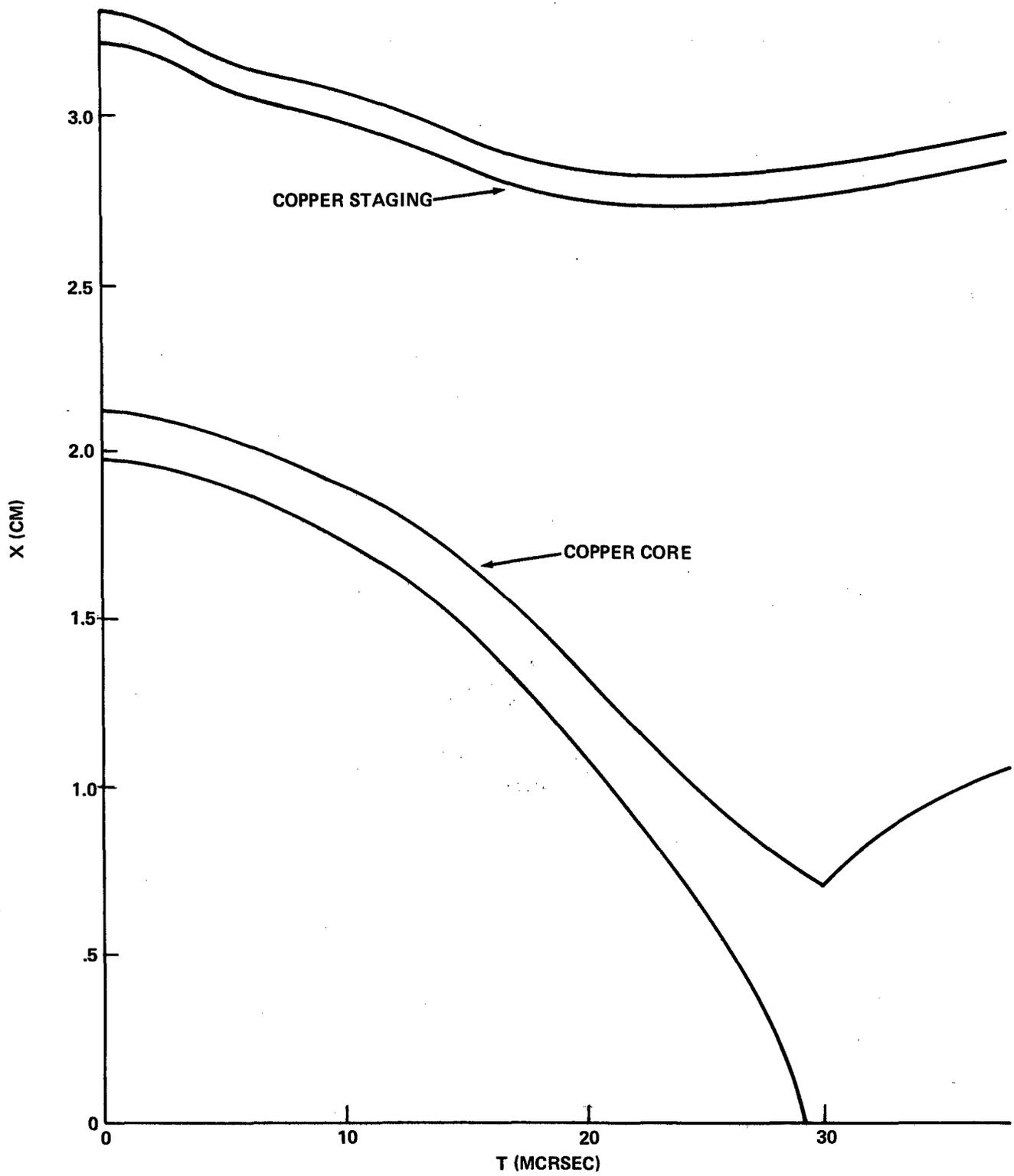


FIG. 7 PLOT OF CASE POSITION VS. TIME FOR PROBLEM 2

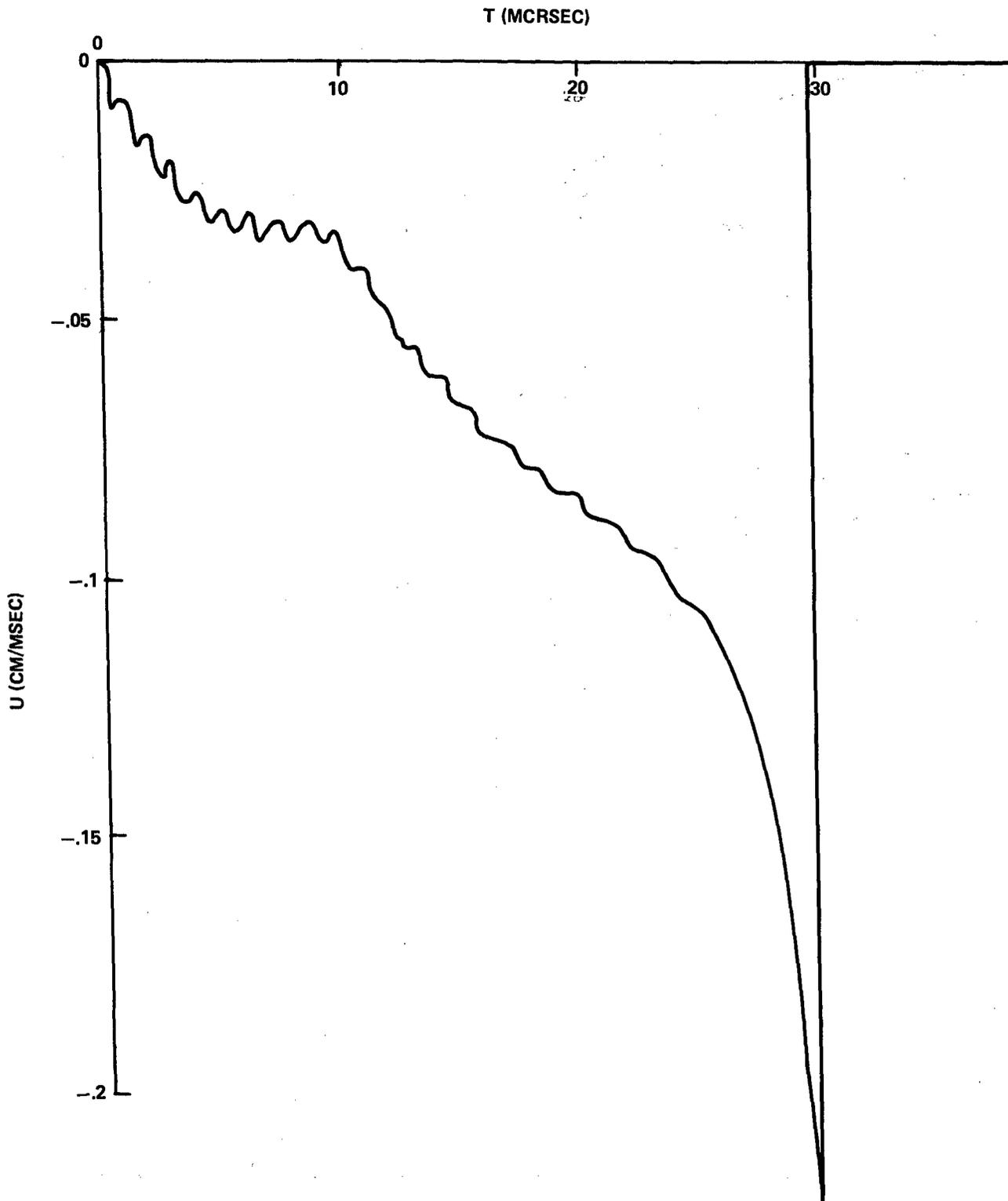


FIG. 8 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE CORE VS. TIME (PROBLEM 2)

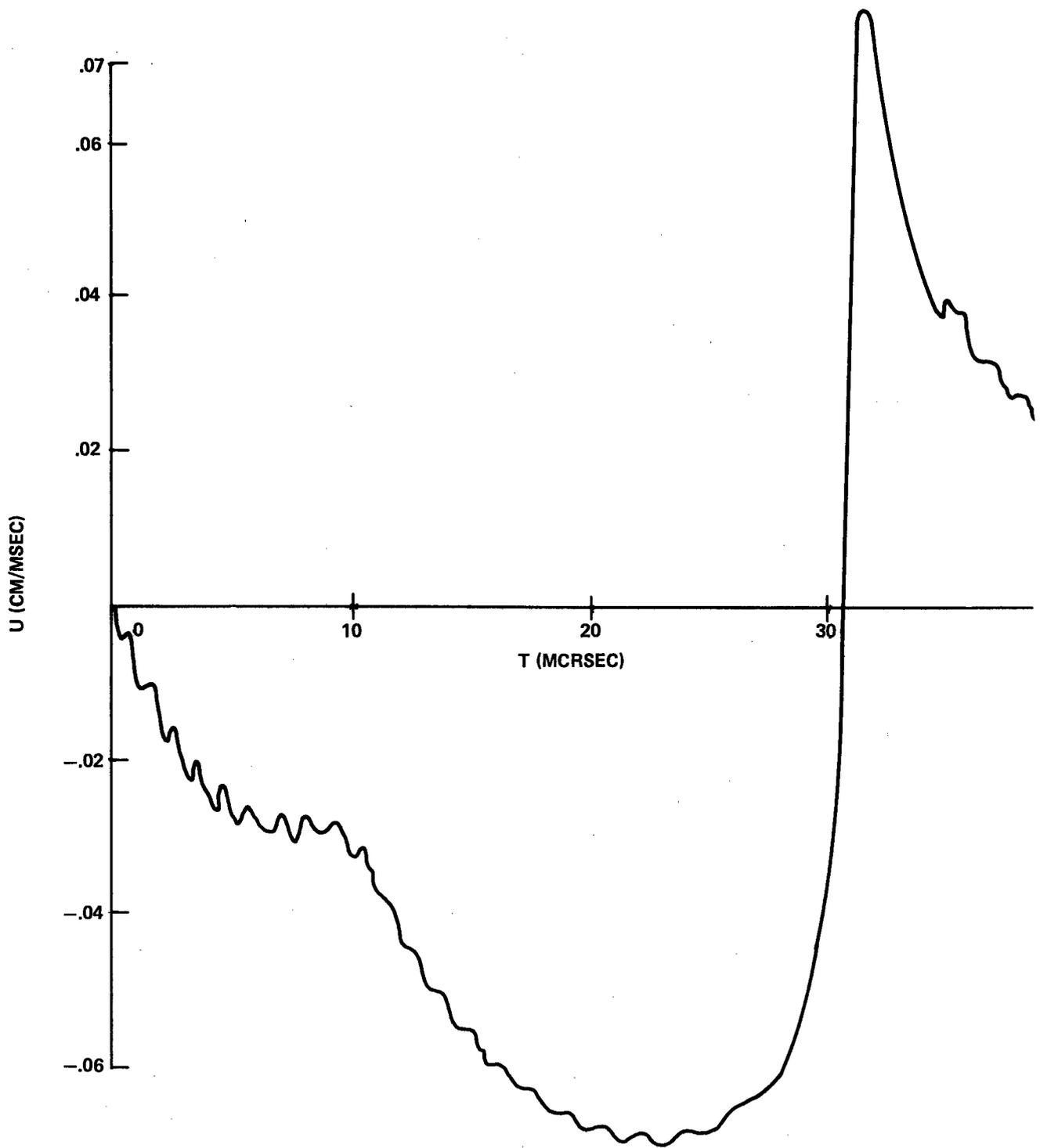


FIG. 9 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE CORE VS. TIME (PROBLEM 2)

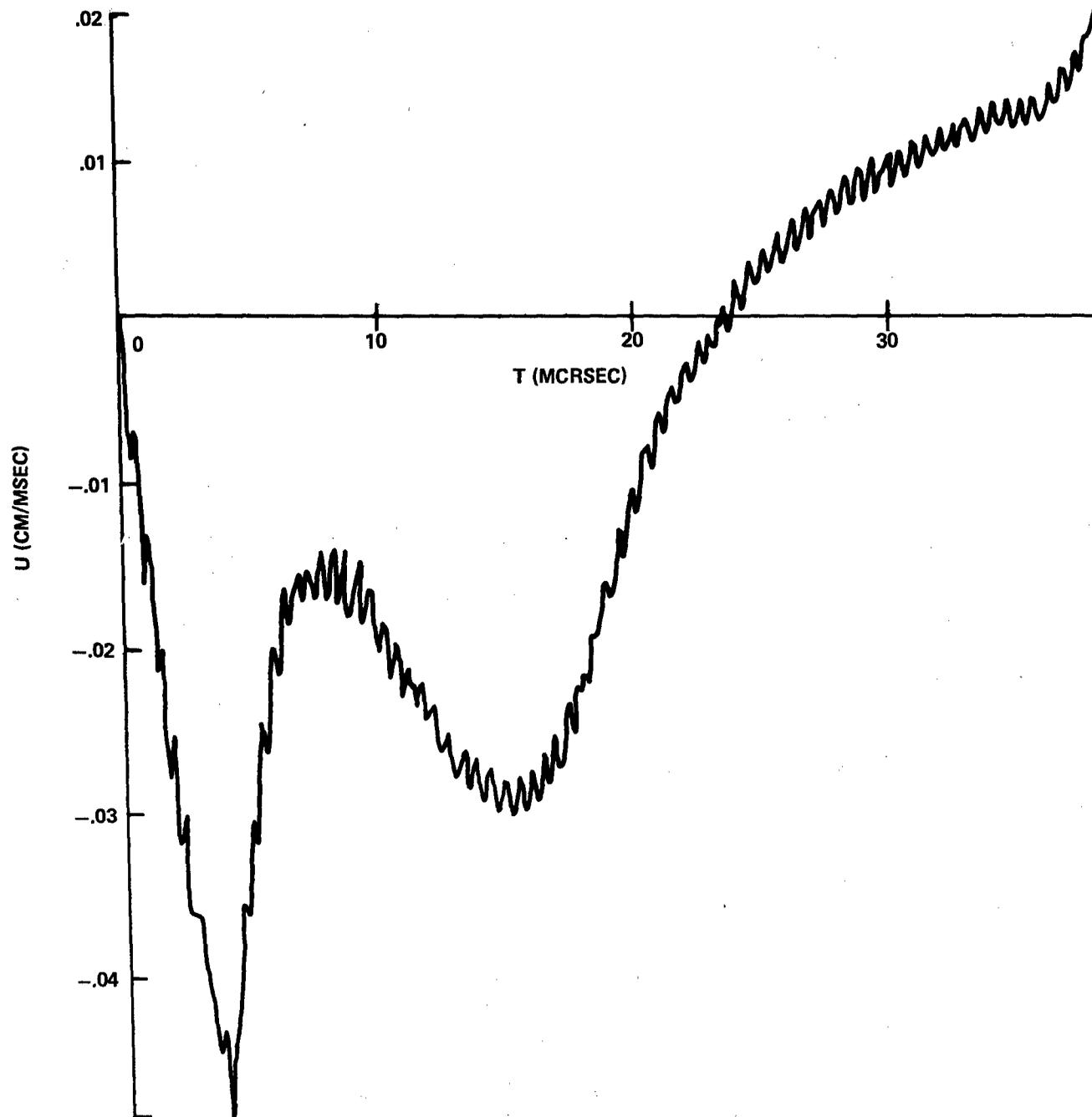


FIG. 10 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE STAGING VS. TIME (PROBLEM 2)

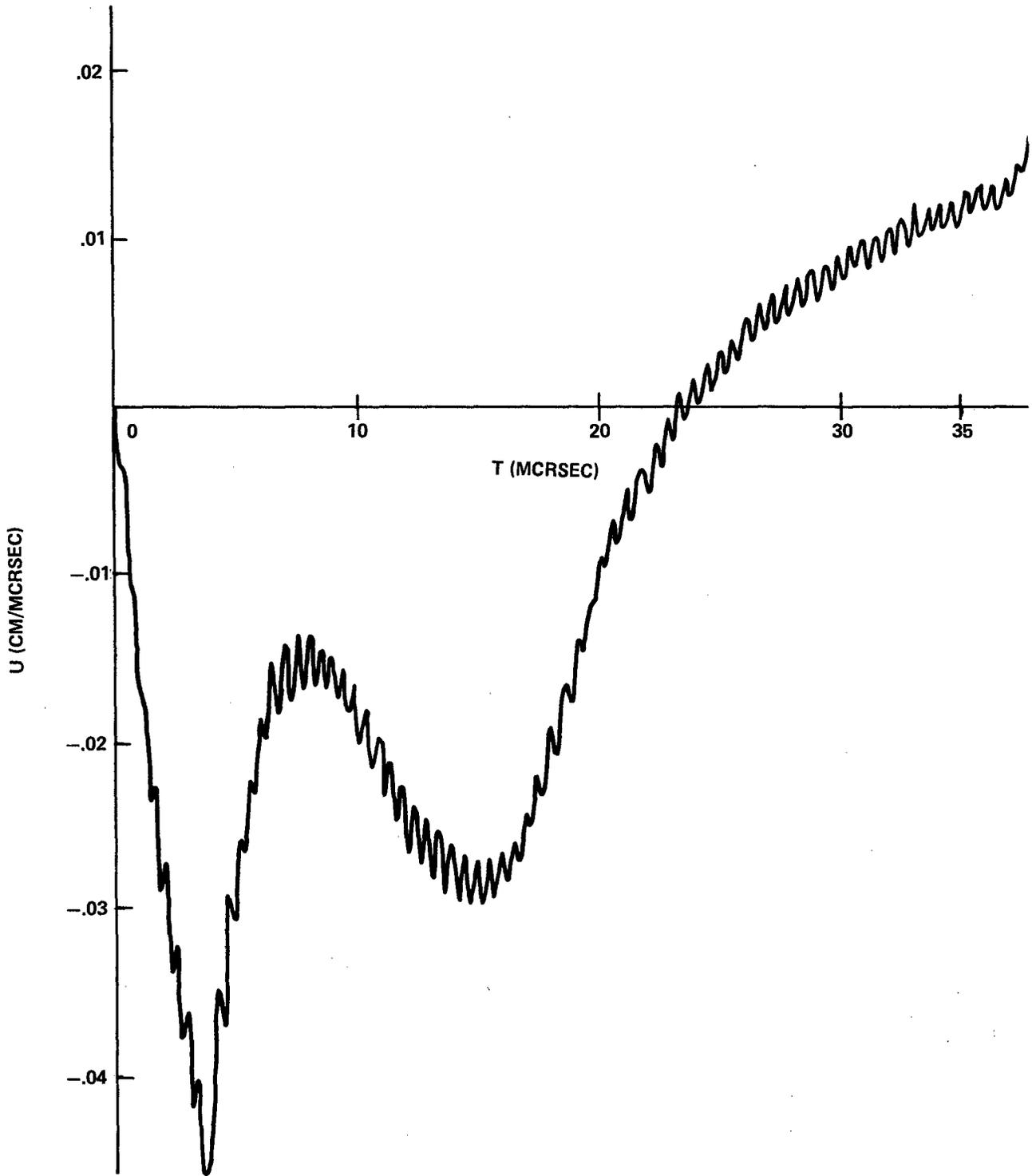


FIG. 11 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE STAGING VS. TIME (PROBLEM 2)

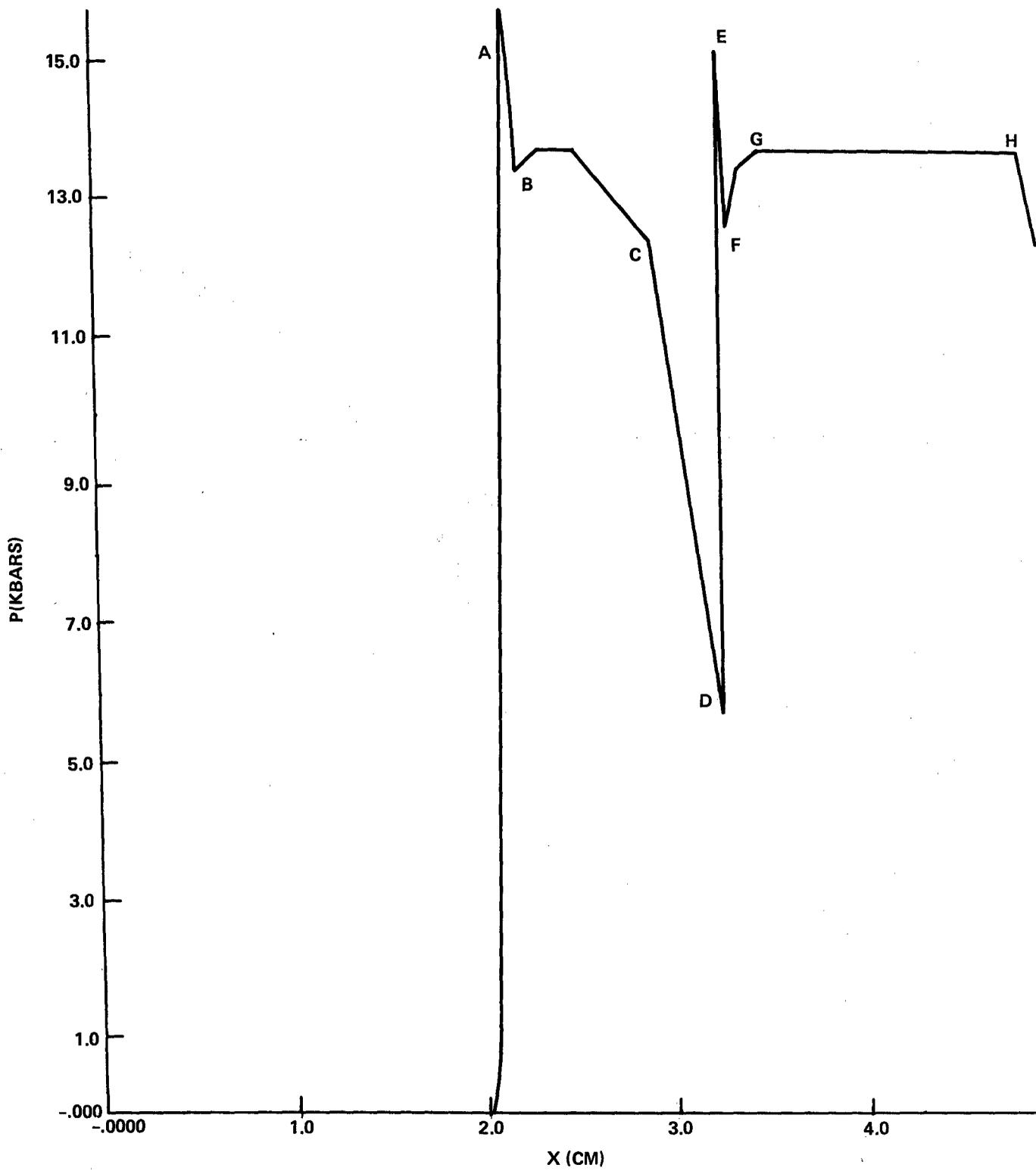


FIG. 12 PLOT OF PRESSURE VS. DISTANCE AT .2 $\mu$  SEC (PROBLEM 2)

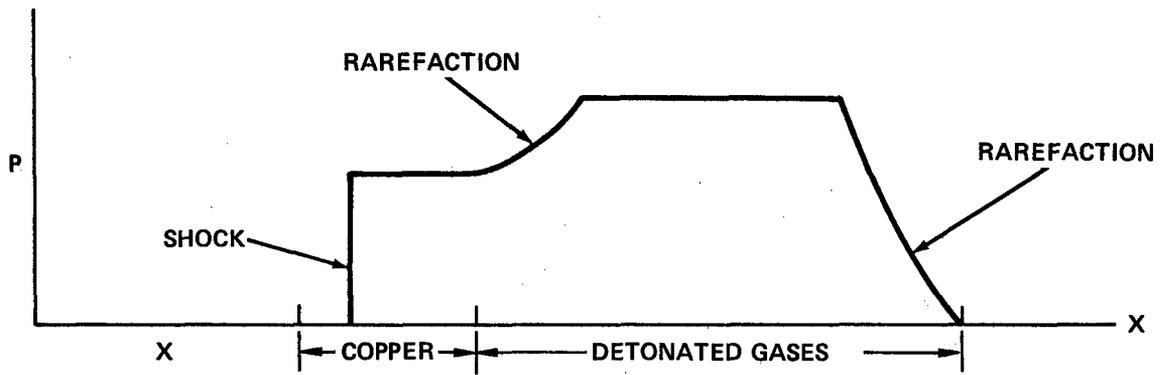


FIG. 13 PLOT OF PRESSURE VS. DISTANCE (THEORETICAL)

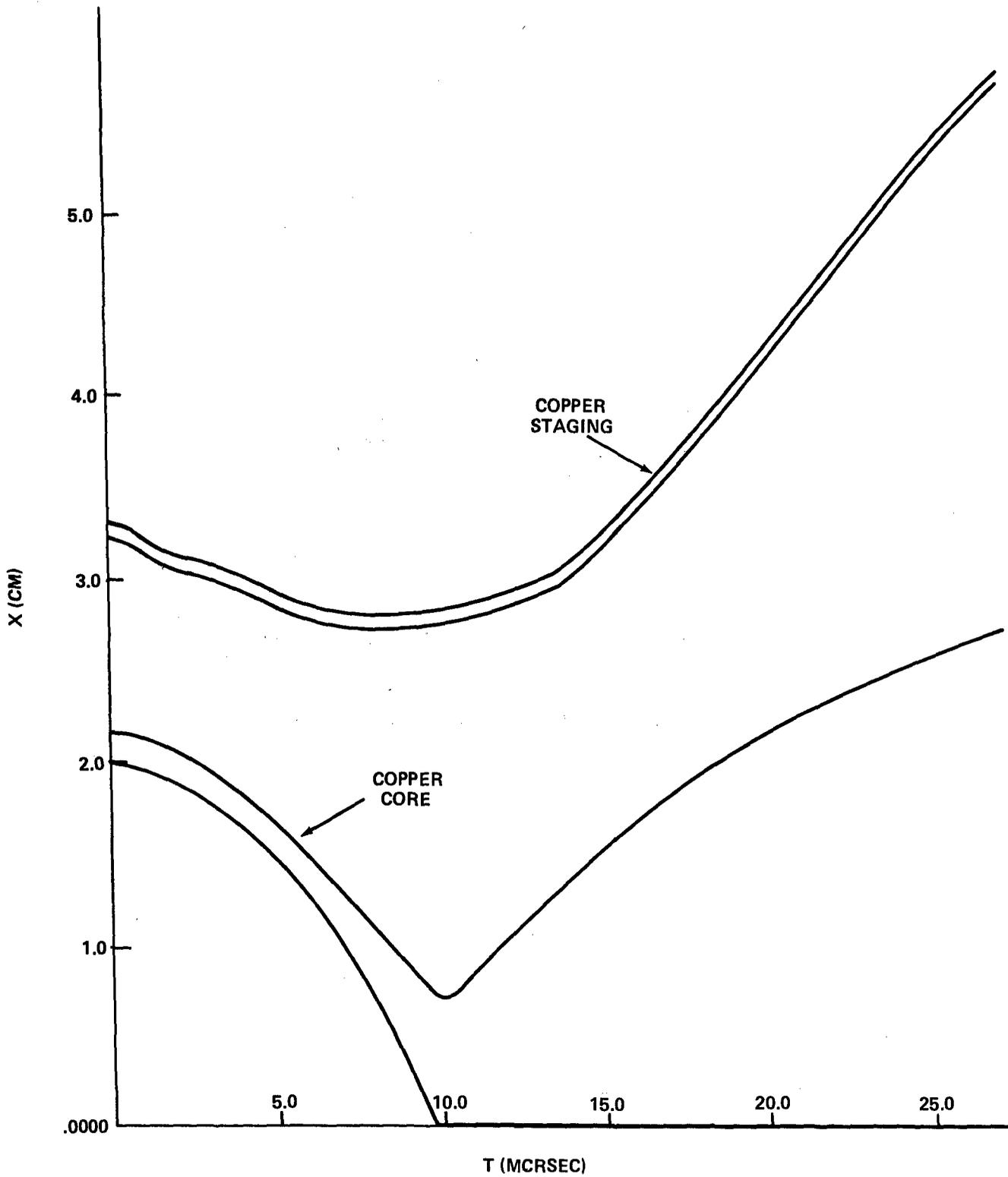


FIG. 14 PLOT OF CASE POSITION VS. TIME (PROBLEM 2, REVISED)

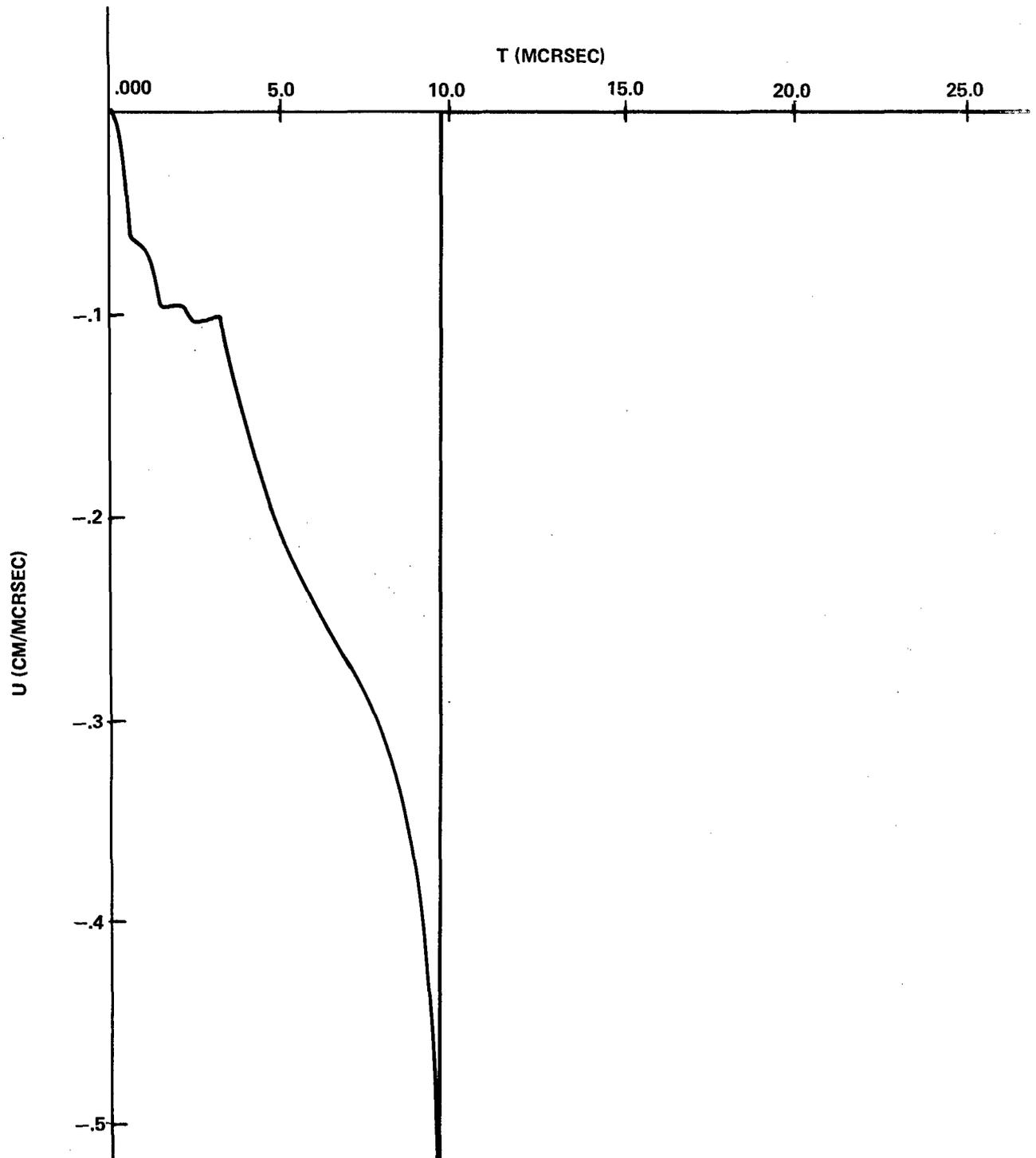


FIG. 15 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE CORE VS. TIME (PROBLEM 2, REVISED)

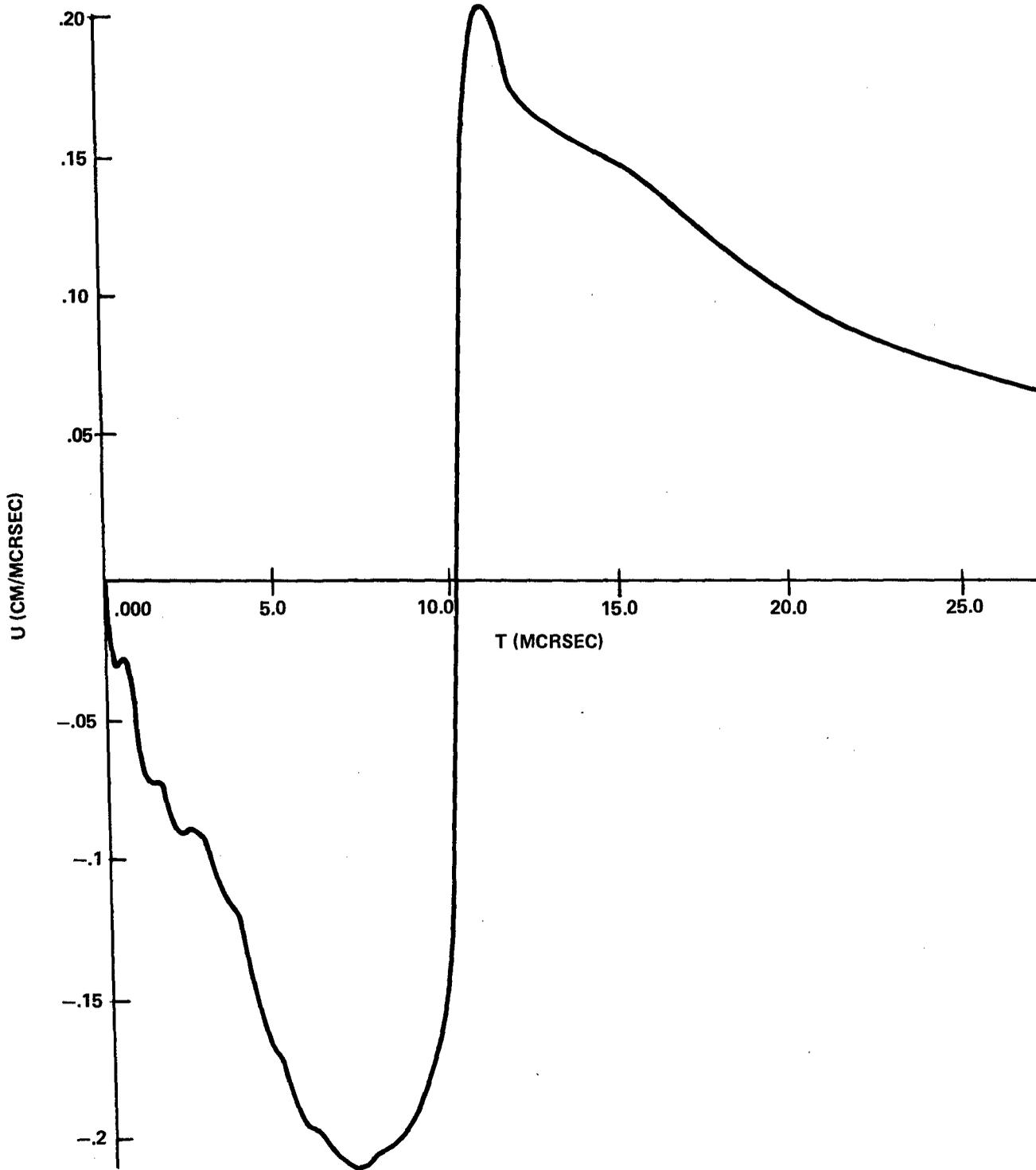


FIG. 16 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE CORE VS. TIME (PROBLEM 2, REVISED)

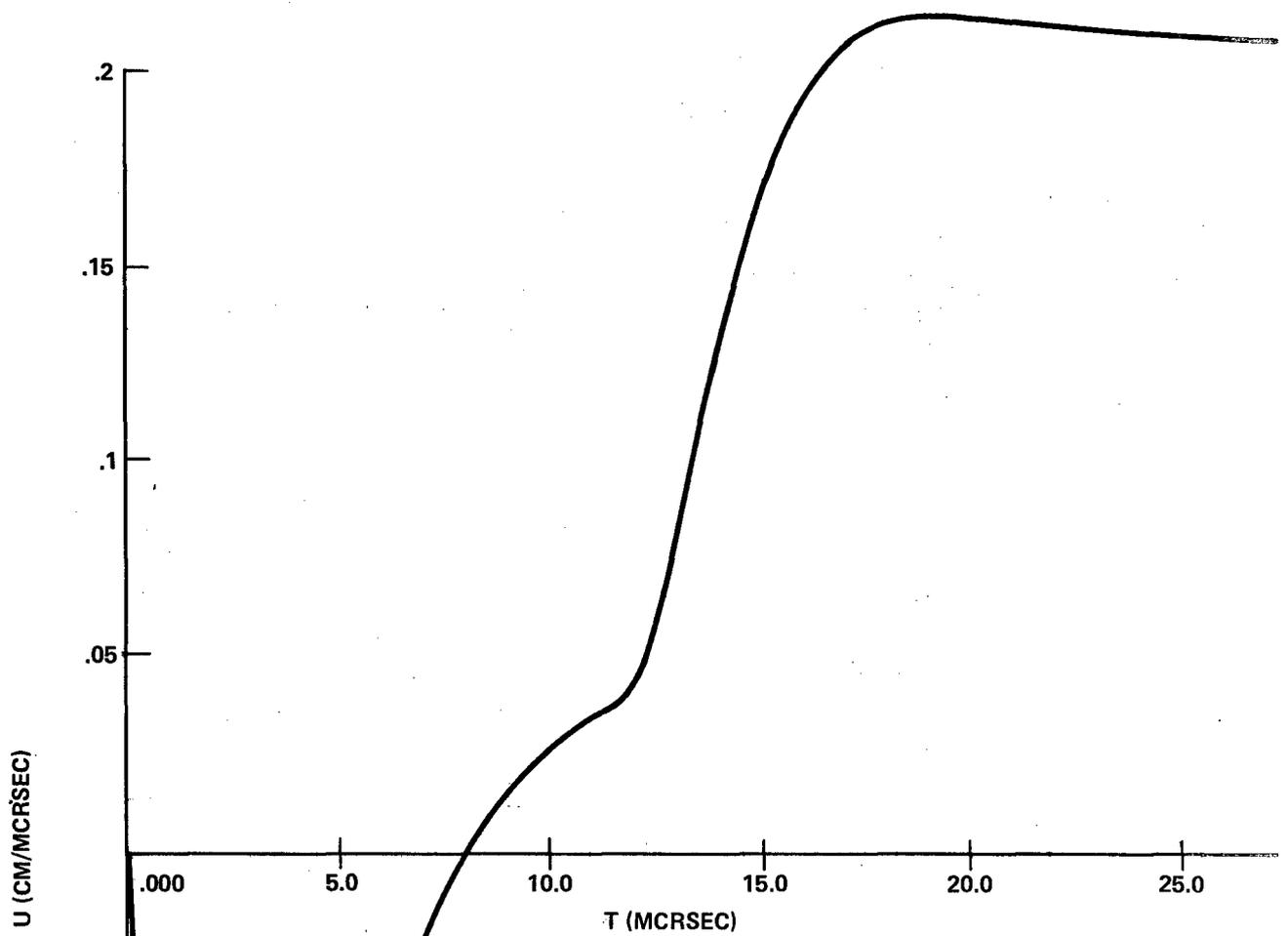


FIG. 17 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE STAGING VS. TIME (PROBLEM 2, REVISED)

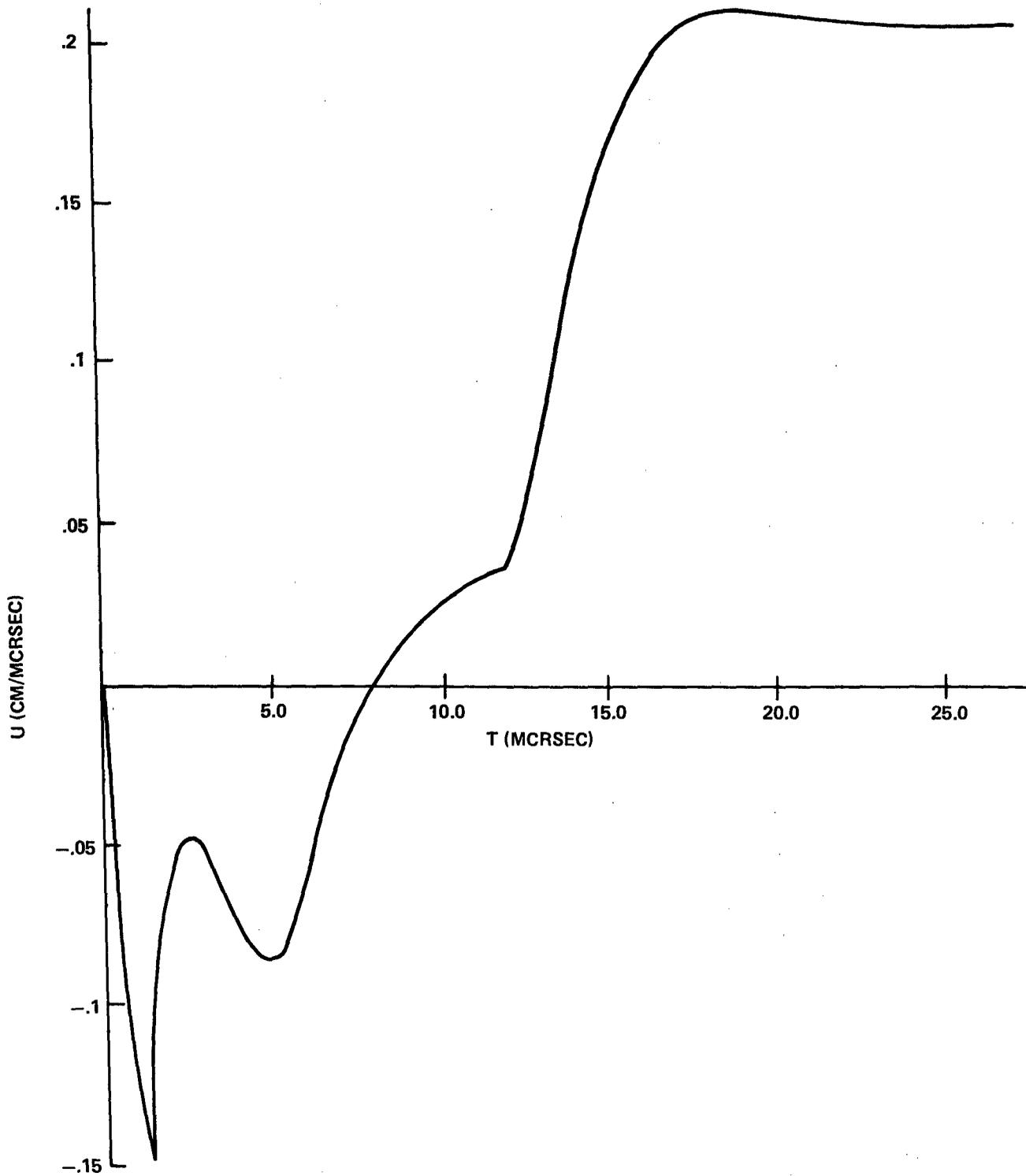


FIG. 18 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE STAGING VS. TIME (PROBLEM 2, REVISED)

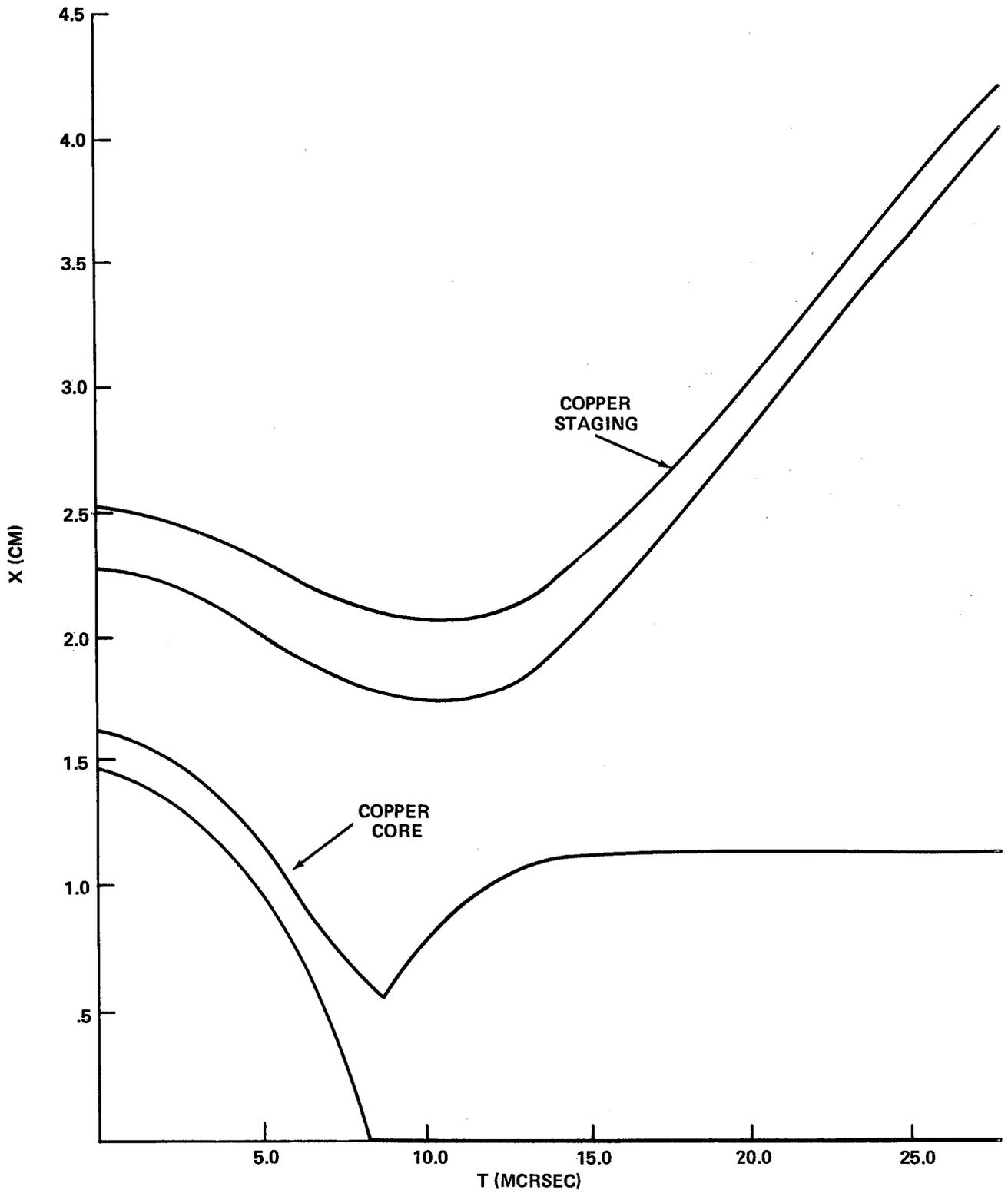


FIG. 19 PLOT OF CASE POSITION VS. TIME (PROBLEM 3)

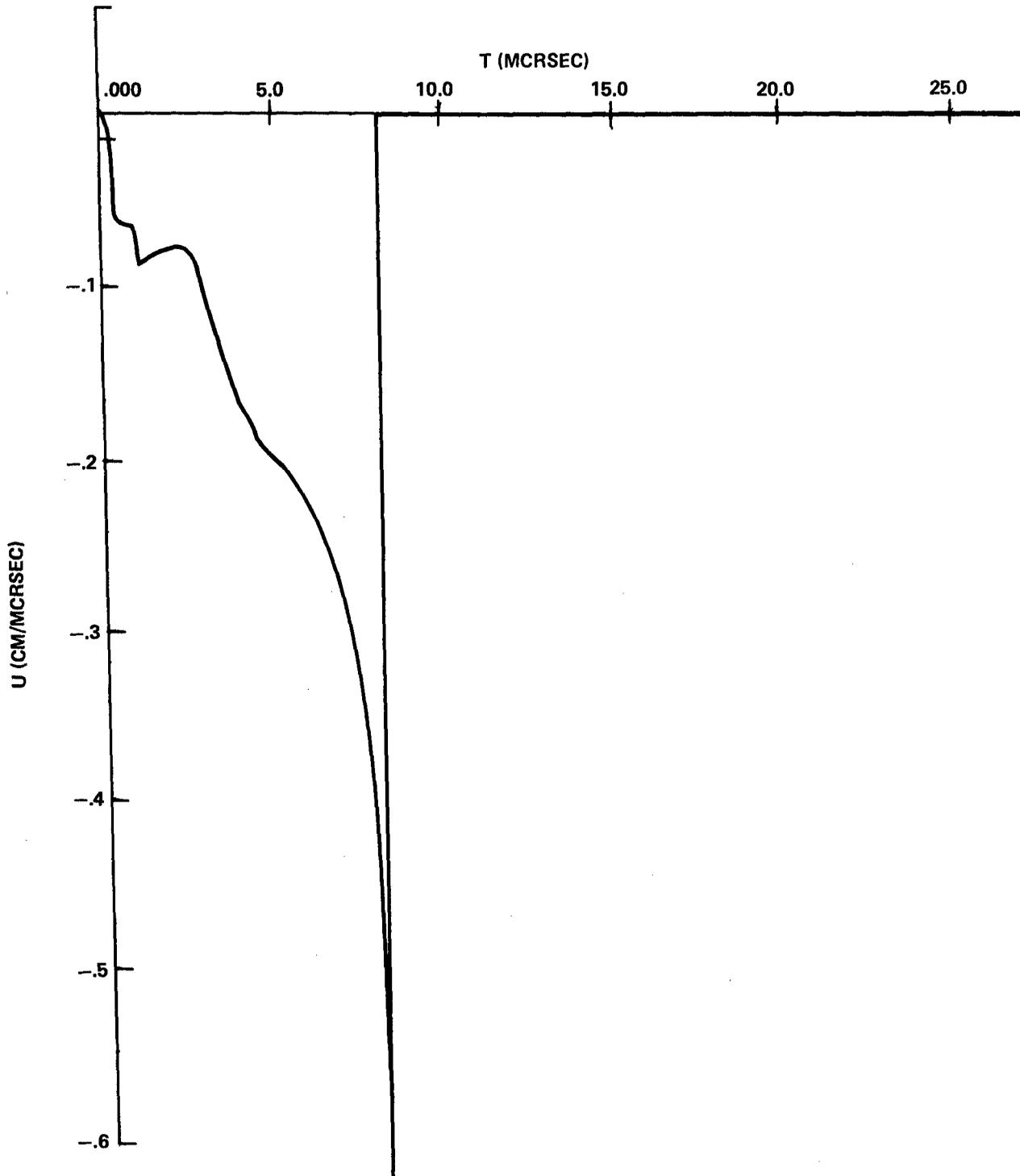


FIG. 20 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE CORE VS. TIME (PROBLEM 3)

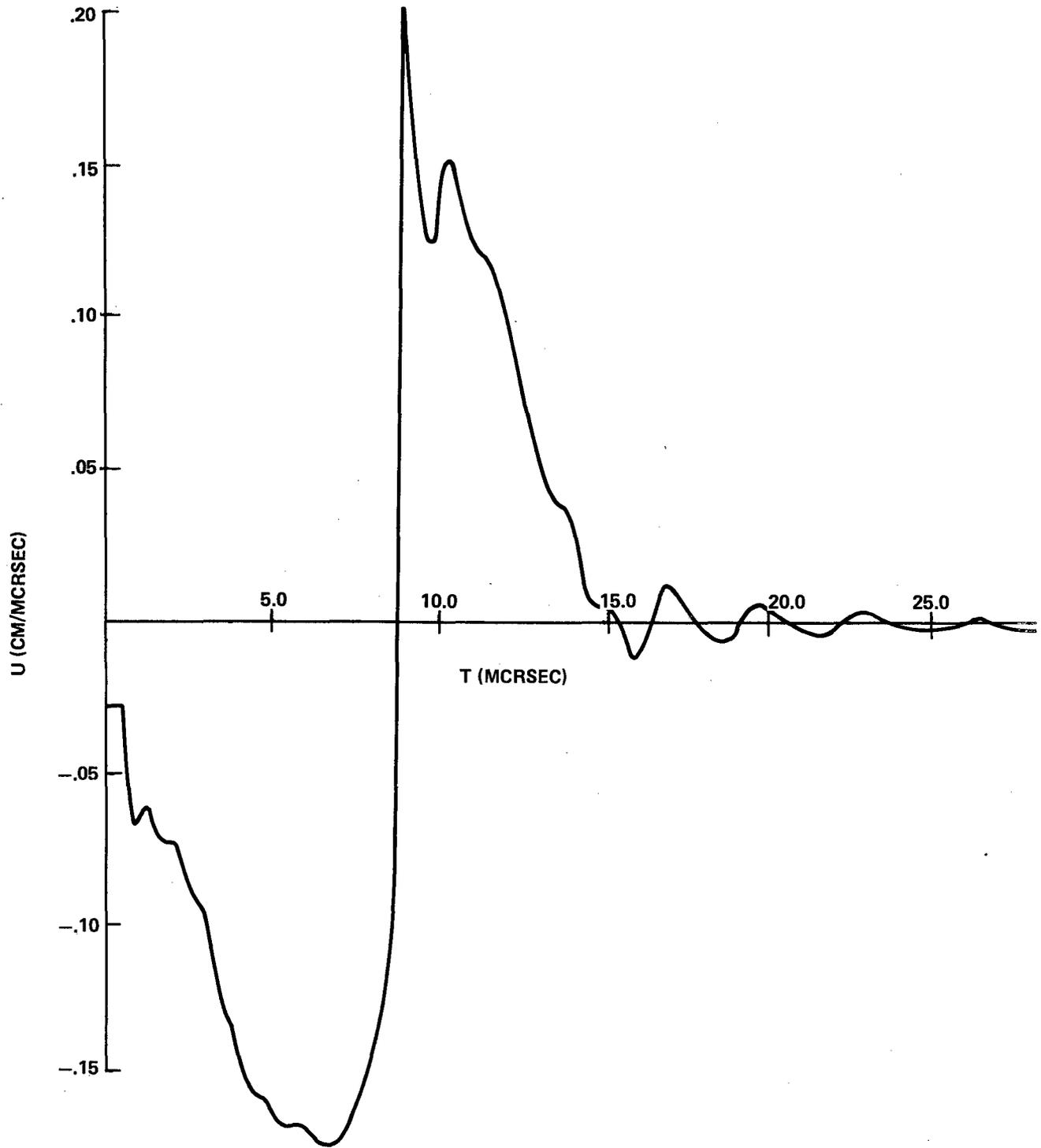


FIG. 21 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE CORE VS. TIME (PROBLEM 3)

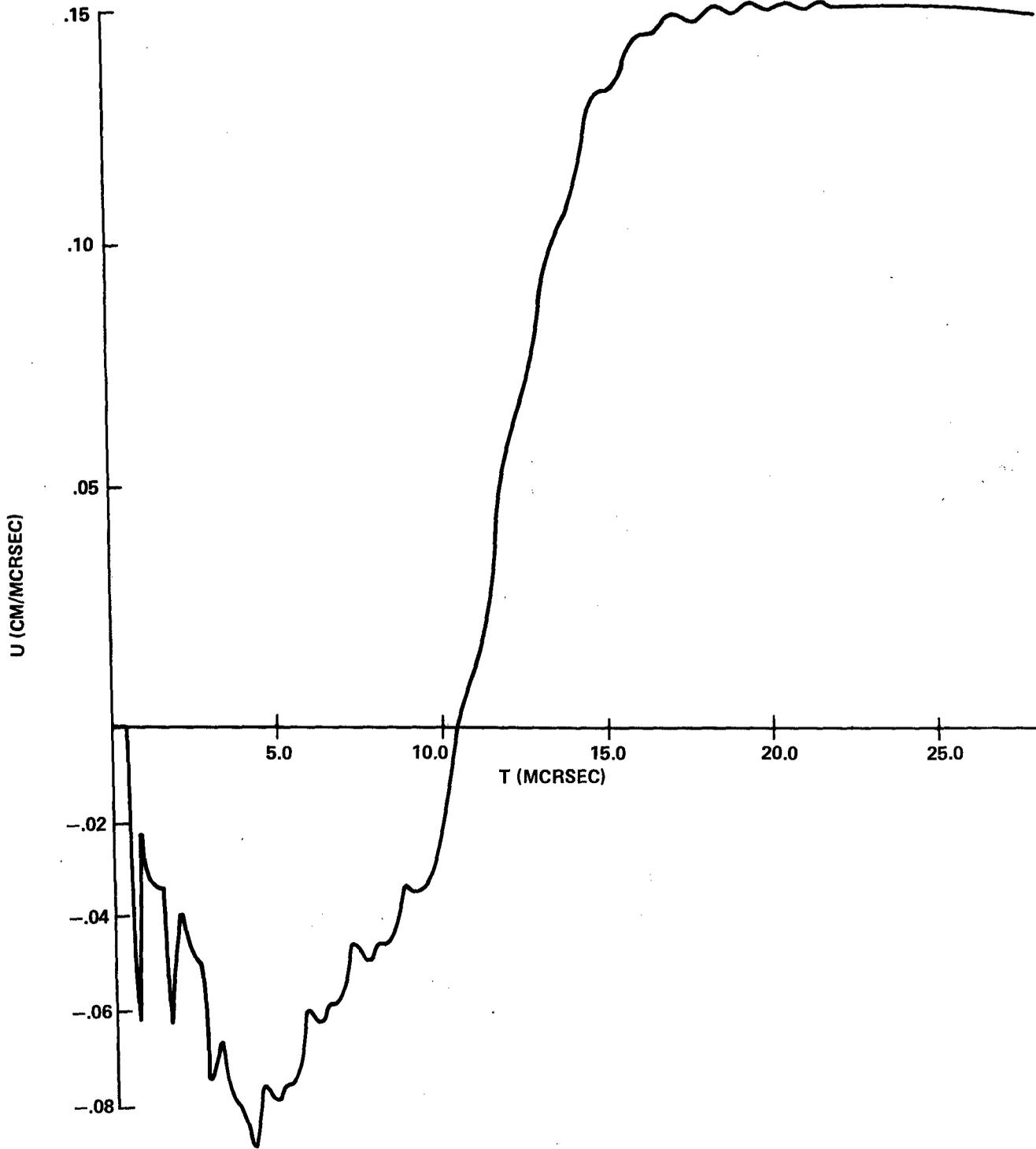


FIG. 22 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE STAGING VS. TIME (PROBLEM 3)

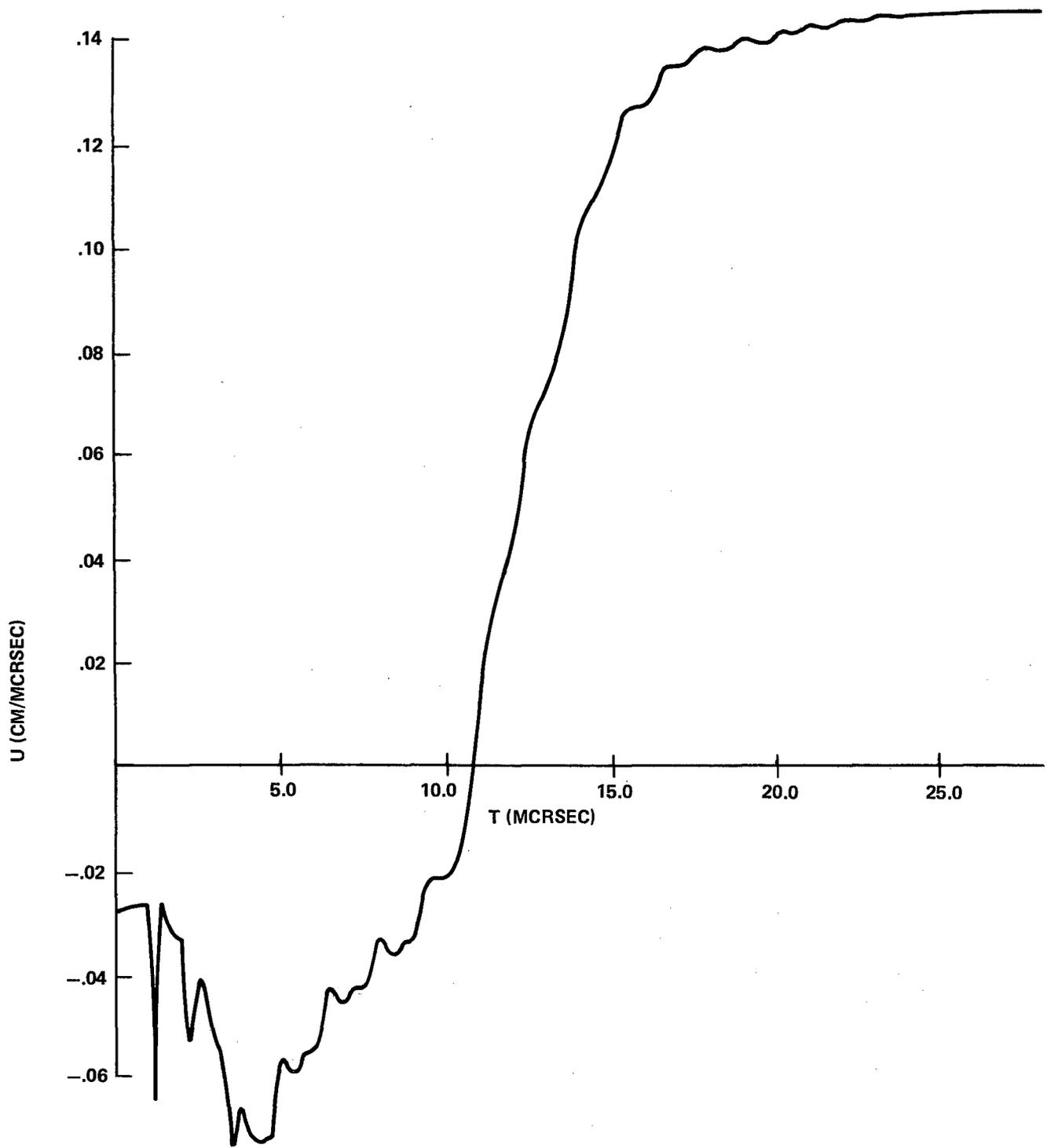
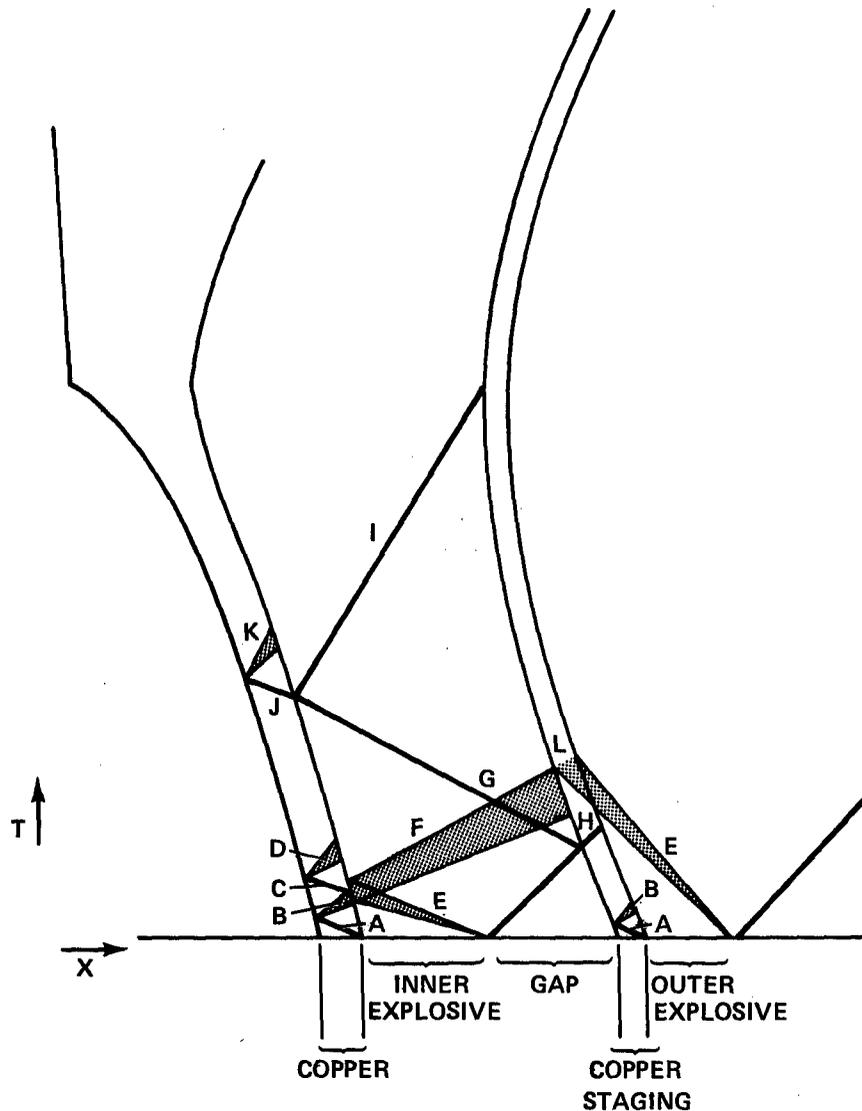


FIG. 23 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE STAGING VS. TIME (PROBLEM 3)



- A DETONATION SHOCKS FROM THE EXPLOSIVE INTO THE COPPER
- B THE DETONATION SHOCK IS REFLECTED AS A RELIEF WAVE FROM THE FREE SURFACE
- C THE RELIEF WAVE IS REFLECTED FROM THE GASES AS A SERIES OF COMPRESSION WAVES WHICH COALESCE INTO A NEW SHOCK
- D THE NEW SHOCK IS REFLECTED FROM THE FREE SURFACE AS A RELIEF WAVE
- E A RELIEF WAVE FROM THE FREE SURFACE PROCEEDS INTO THE EXPLOSIVE GASES
- F AND IS REFLECTED FROM THE COPPER CORE AS A RELIEF WAVE
- G THE COLLISION OF THE INNER EXPLOSIVE'S OUTER SURFACE WITH THE COPPER STAGING PRODUCES A COLLISION SHOCK IN THE GASES
- H AND IN THE STAGING
- I THE COLLISION SHOCK IS REFLECTED FROM THE CORE AND
- J TRANSMITTED TO THE CORE
- K THE SHOCK IN THE CORE IS REFLECTED AS A RELIEF WAVE
- L THE COLLISION SHOCK IN THE STAGING IS REFLECTED AS A RELIEF WAVE

FIG. 24 SHOCK WAVE SYSTEM IN THE COPPER

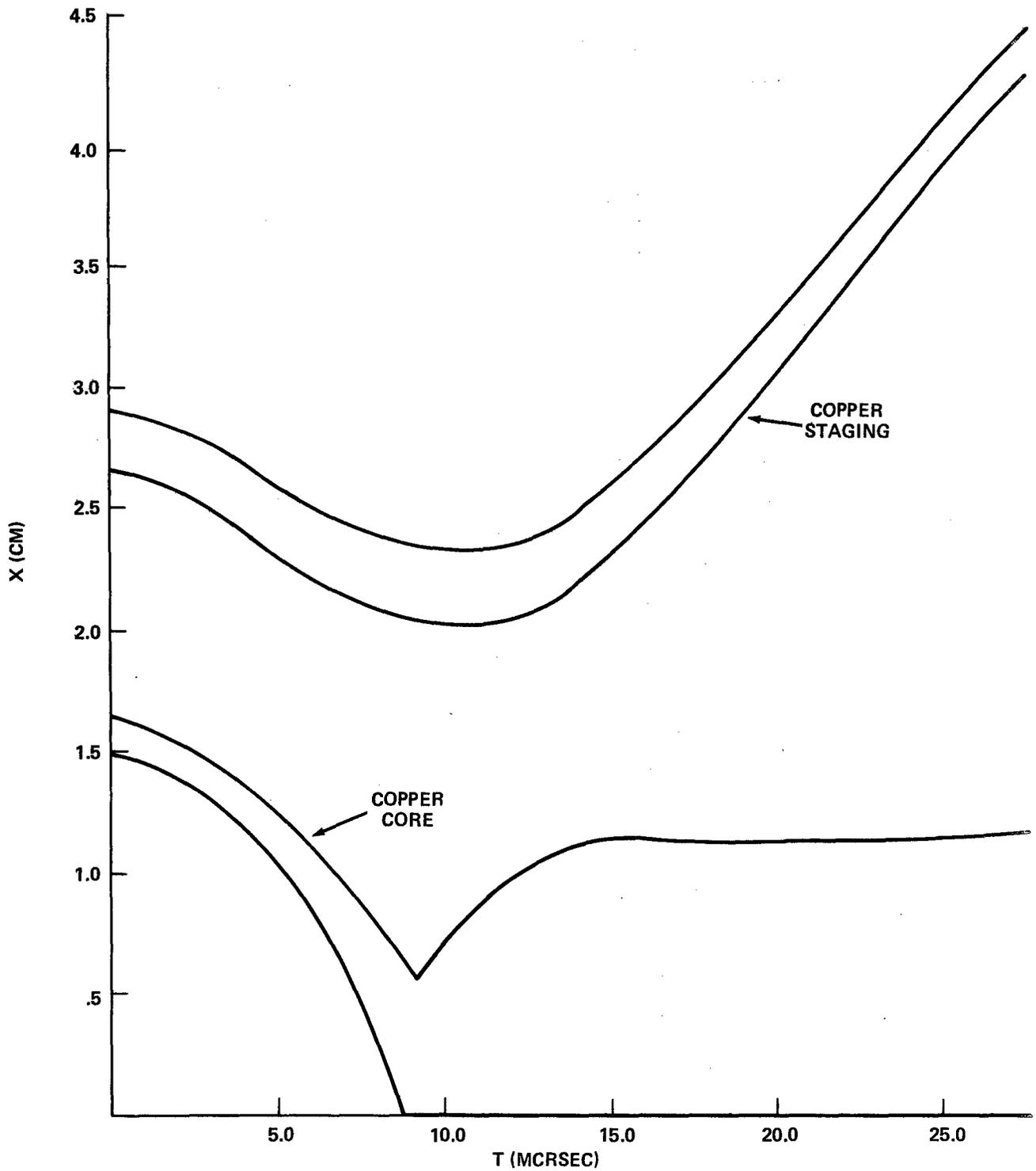


FIG. 25 PLOT OF THE CASE POSITIONS VS. TIME (PROBLEM 4)

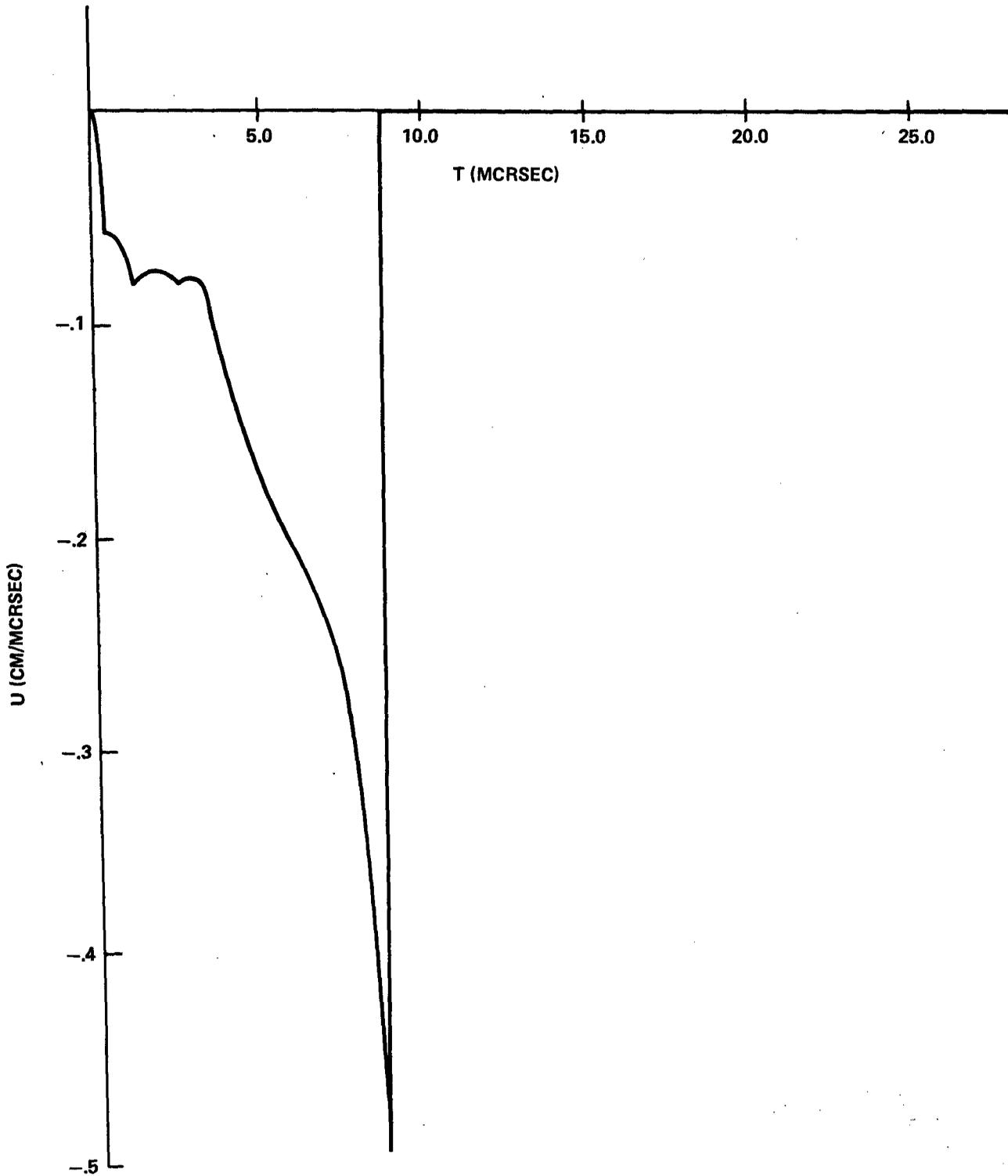


FIG. 26 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE CORE VS. TIME (PROBLEM 4)

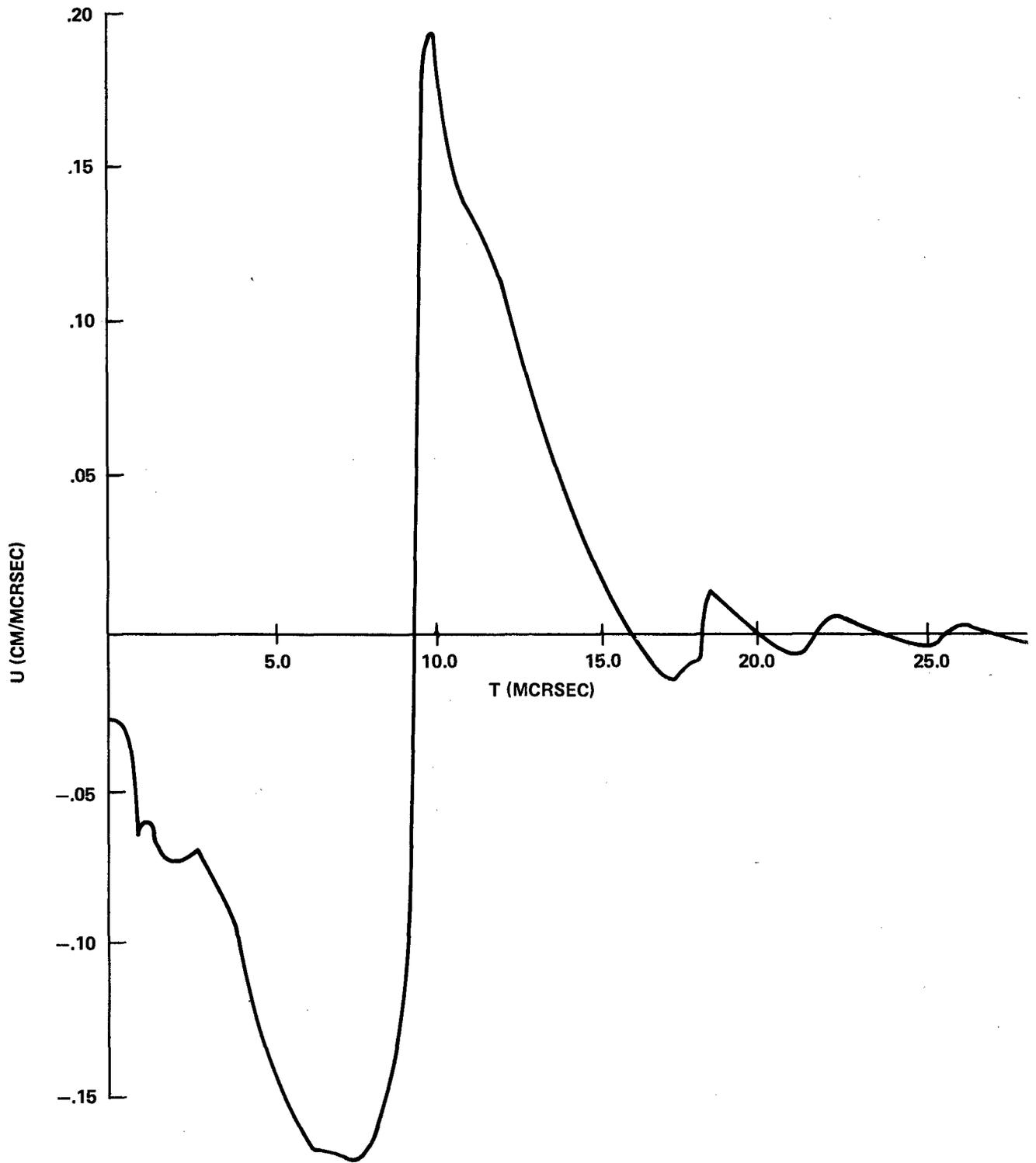


FIG. 27 PLOT OF VELOCITY OF THE OUTER SURFACE OF THE CORE VS. TIME (PROBLEM 4)

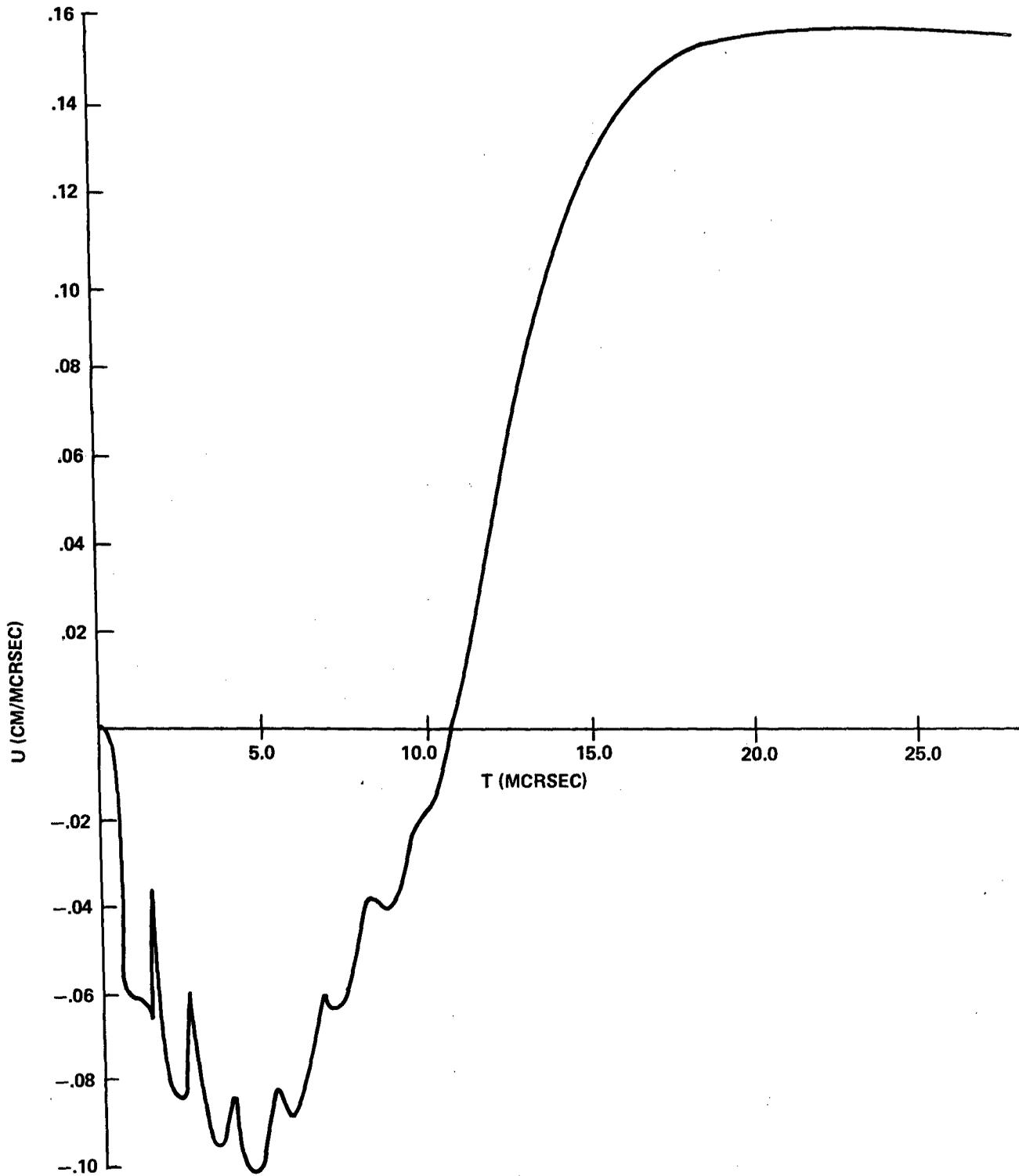


FIG. 28 PLOT OF VELOCITY OF THE INNER SURFACE OF THE STAGING VS. TIME (PROBLEM 4)

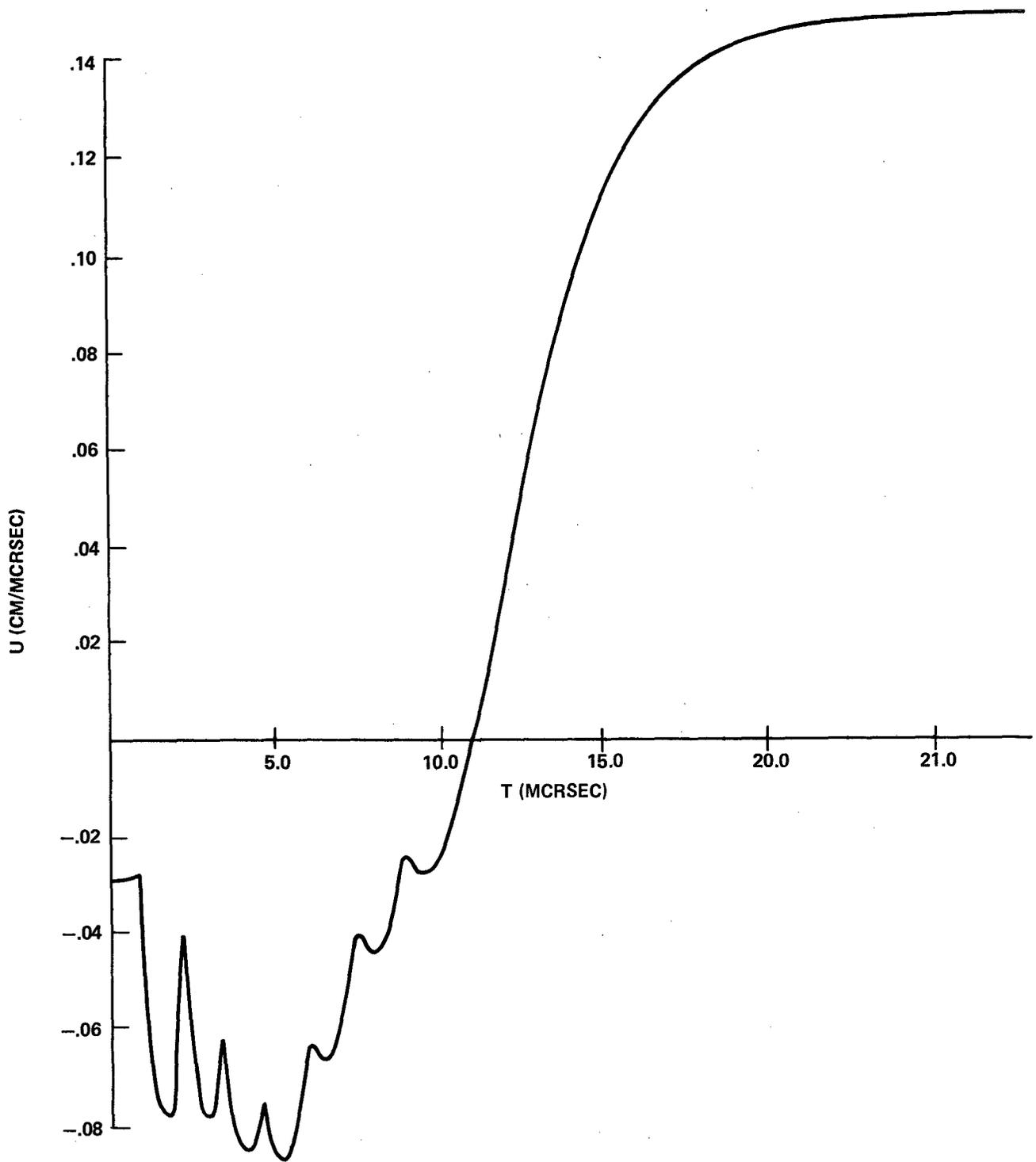


FIG. 29 PLOT OF VELOCITY OF THE OUTER SURFACE OF THE STAGING VS. TIME (PROBLEM 4)

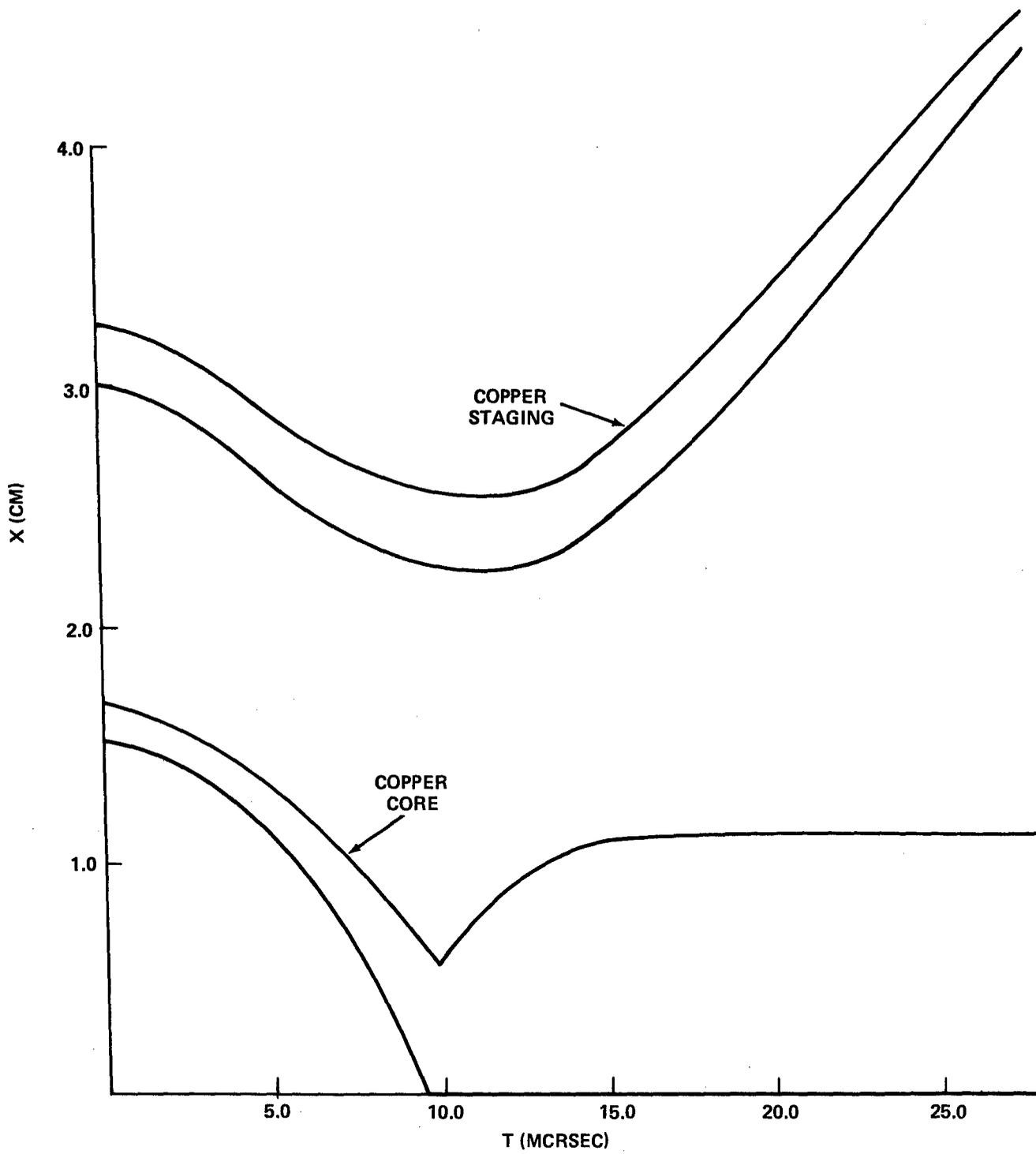


FIG. 30 PLOT OF CASE POSITION VS. TIME (PROBLEM 5)

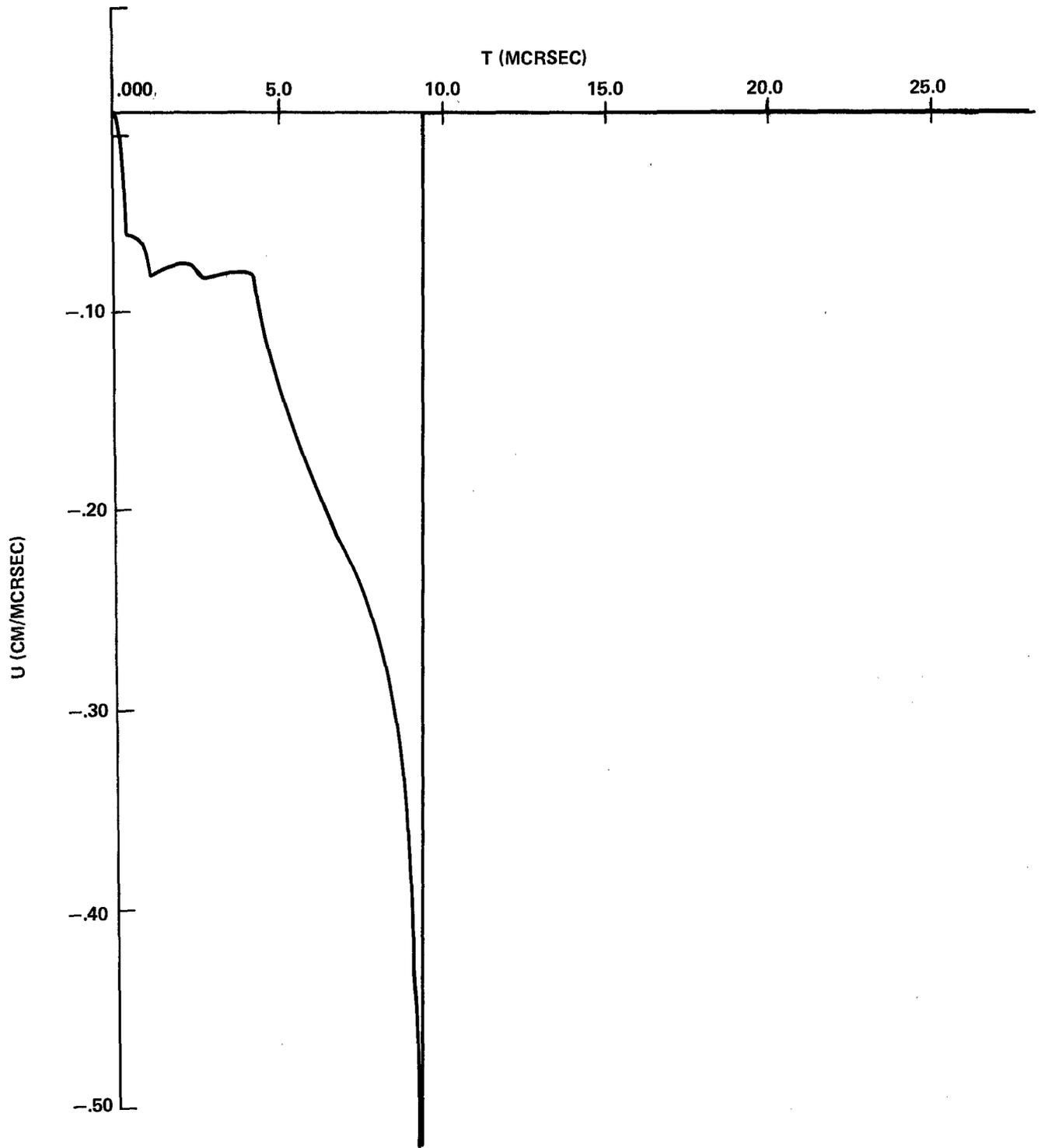


FIG. 31 PLOT OF VELOCITY OF THE INNER SURFACE OF THE CORE VS. TIME (PROBLEM 5)

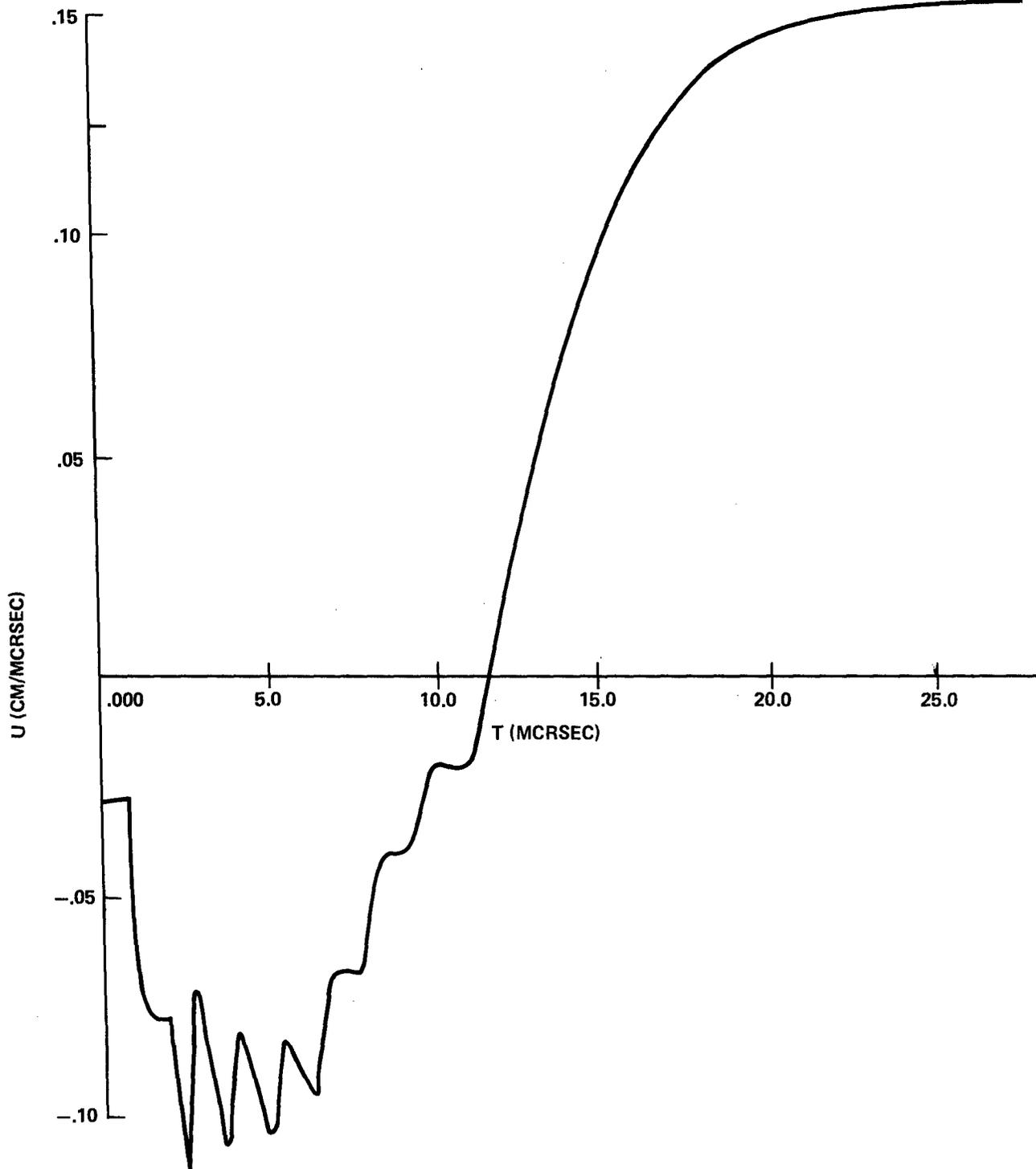


FIG. 32 PLOT OF VELOCITY OF THE OUTER SURFACE OF THE CORE VS. TIME (PROBLEM 5)

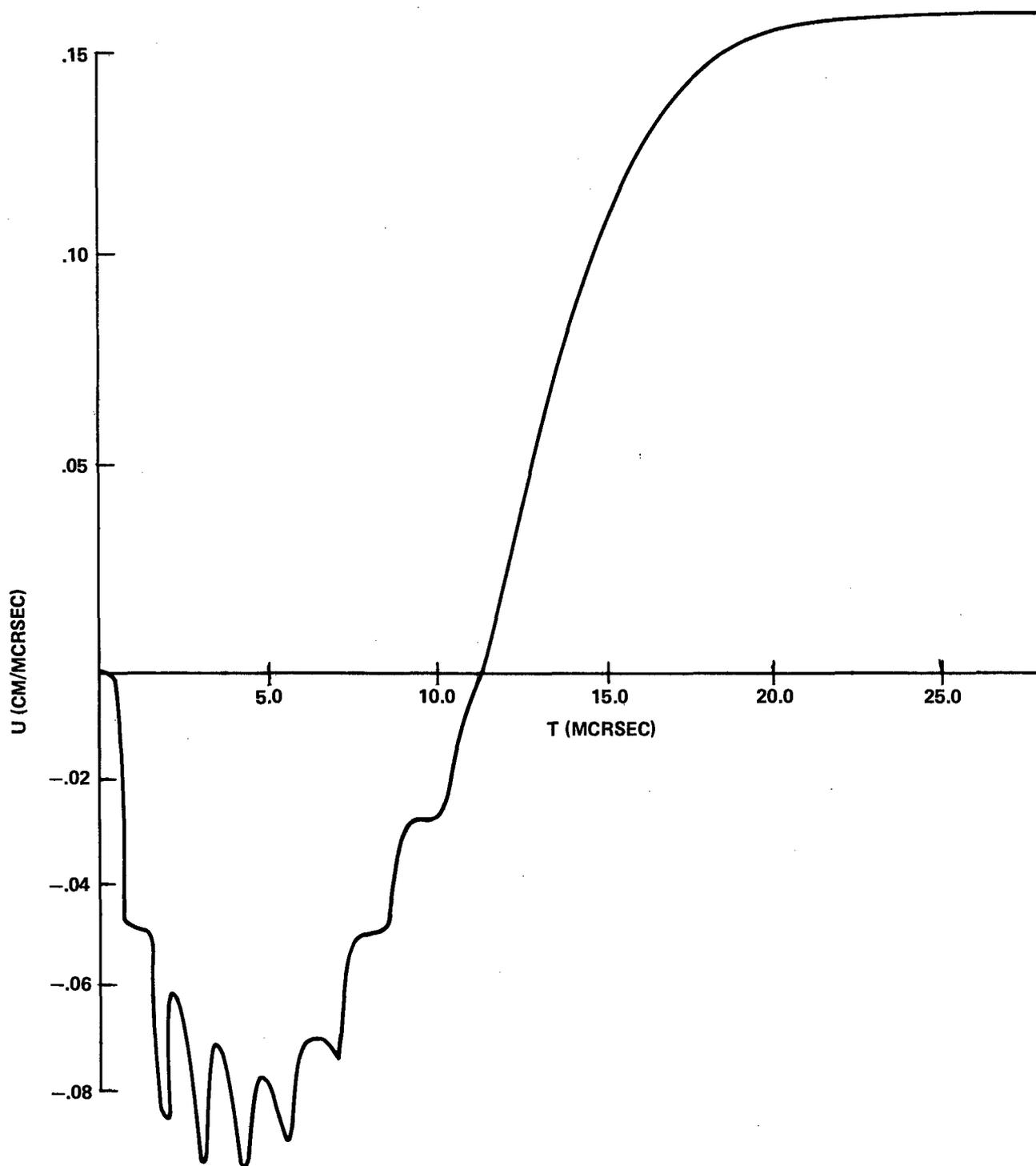


FIG. 33 PLOT OF VELOCITY OF THE INNER SURFACE OF THE STAGING VS. TIME (PROBLEM 5)

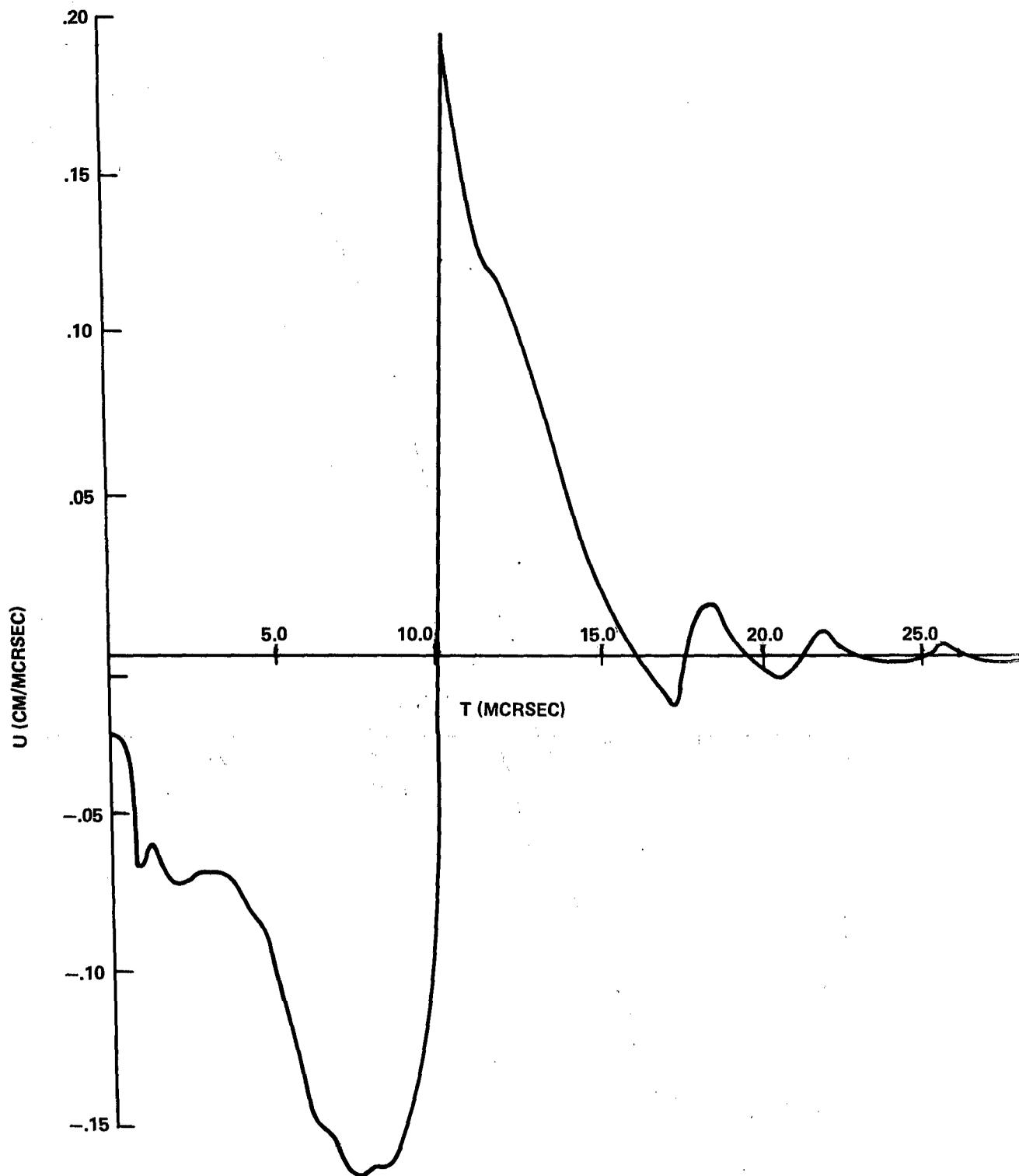


FIG. 34 PLOT OF VELOCITY OF THE OUTER SURFACE OF THE STAGING VS. TIME (PROBLEM 5)

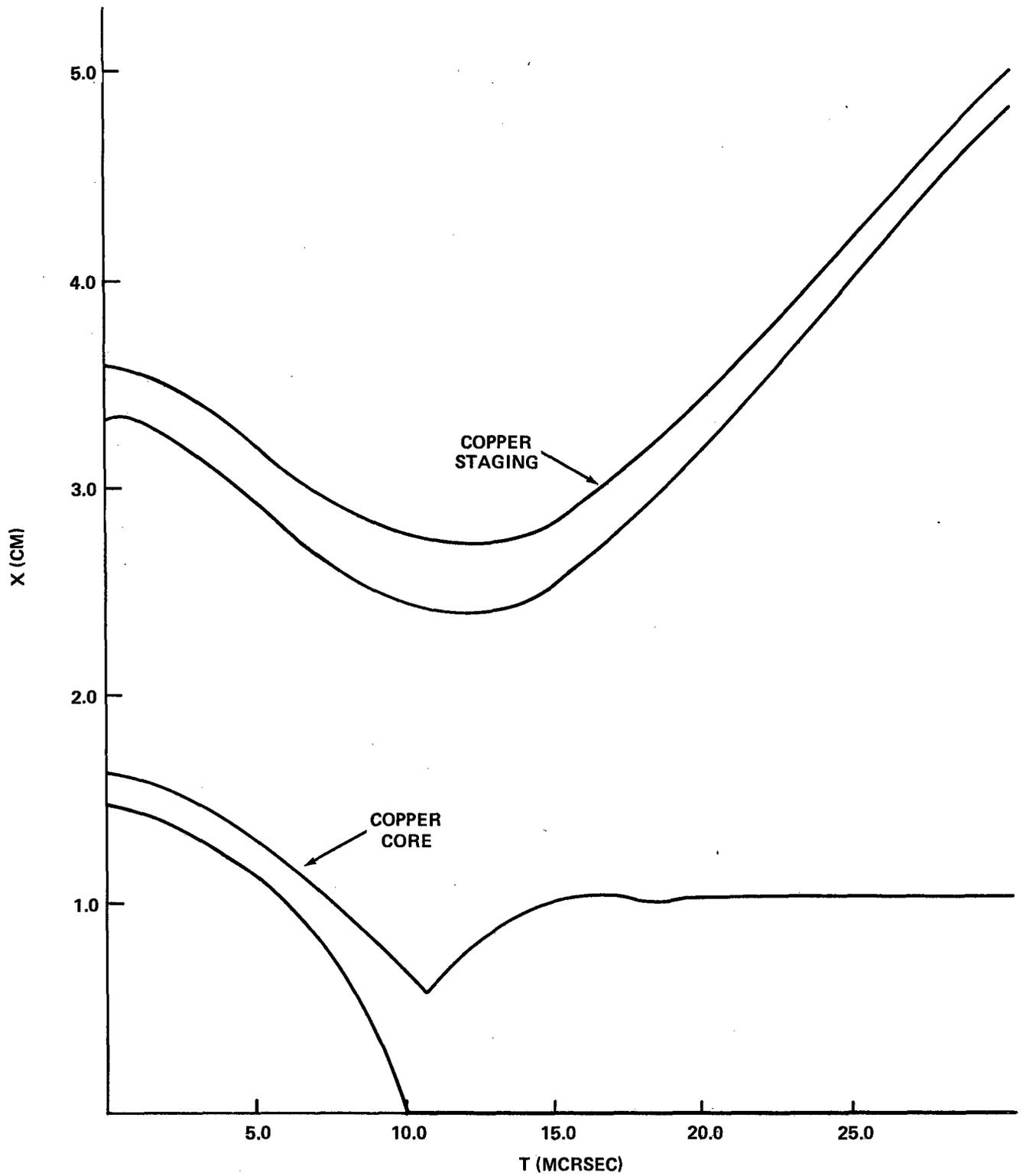


FIG. 35 PLOT OF CASE POSITION VS. TIME (PROBLEM 6)

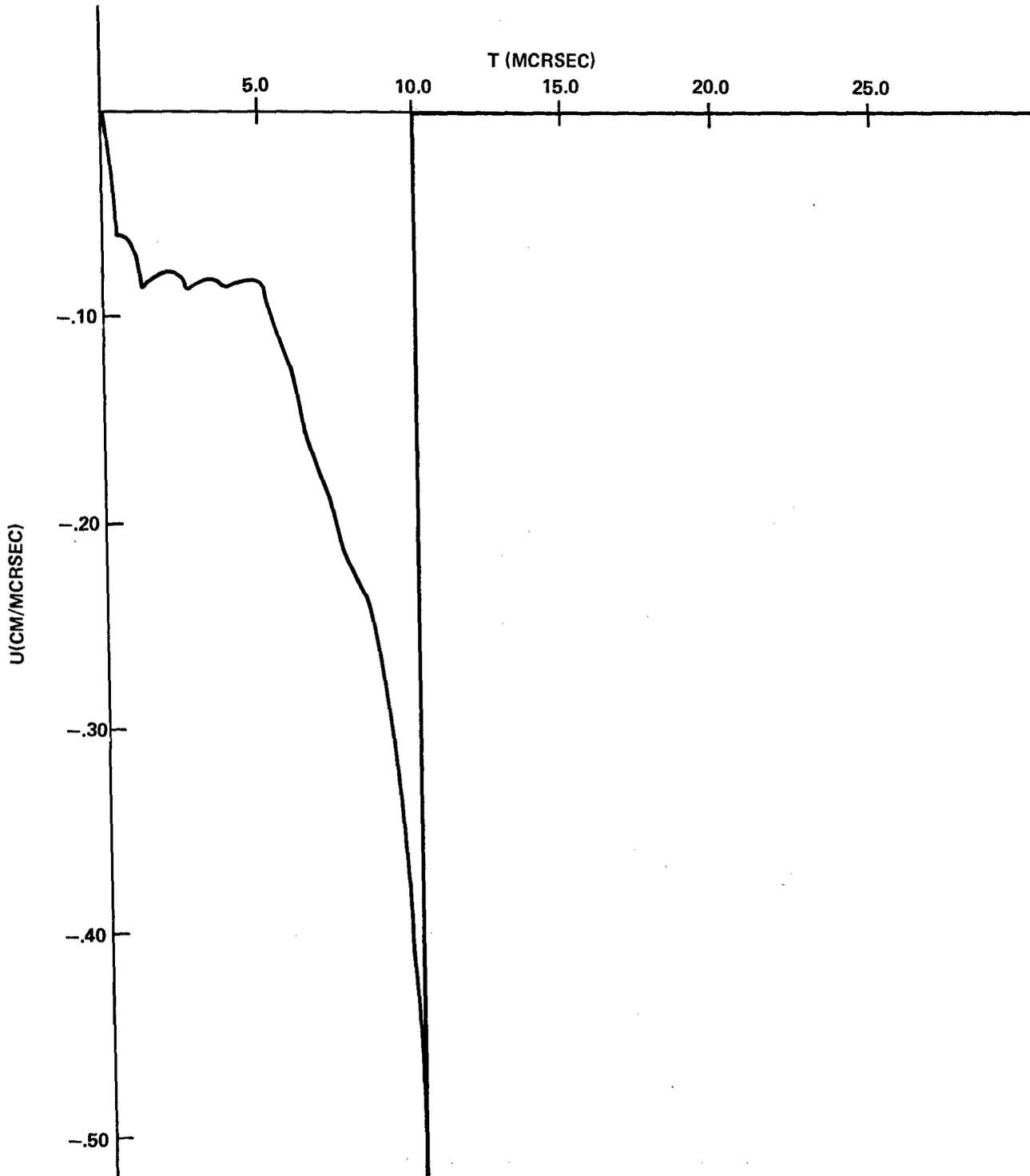


FIG. 36 PLOT OF VELOCITY OF THE INNER SURFACE OF THE CORE VS. TIME (PROBLEM 6)

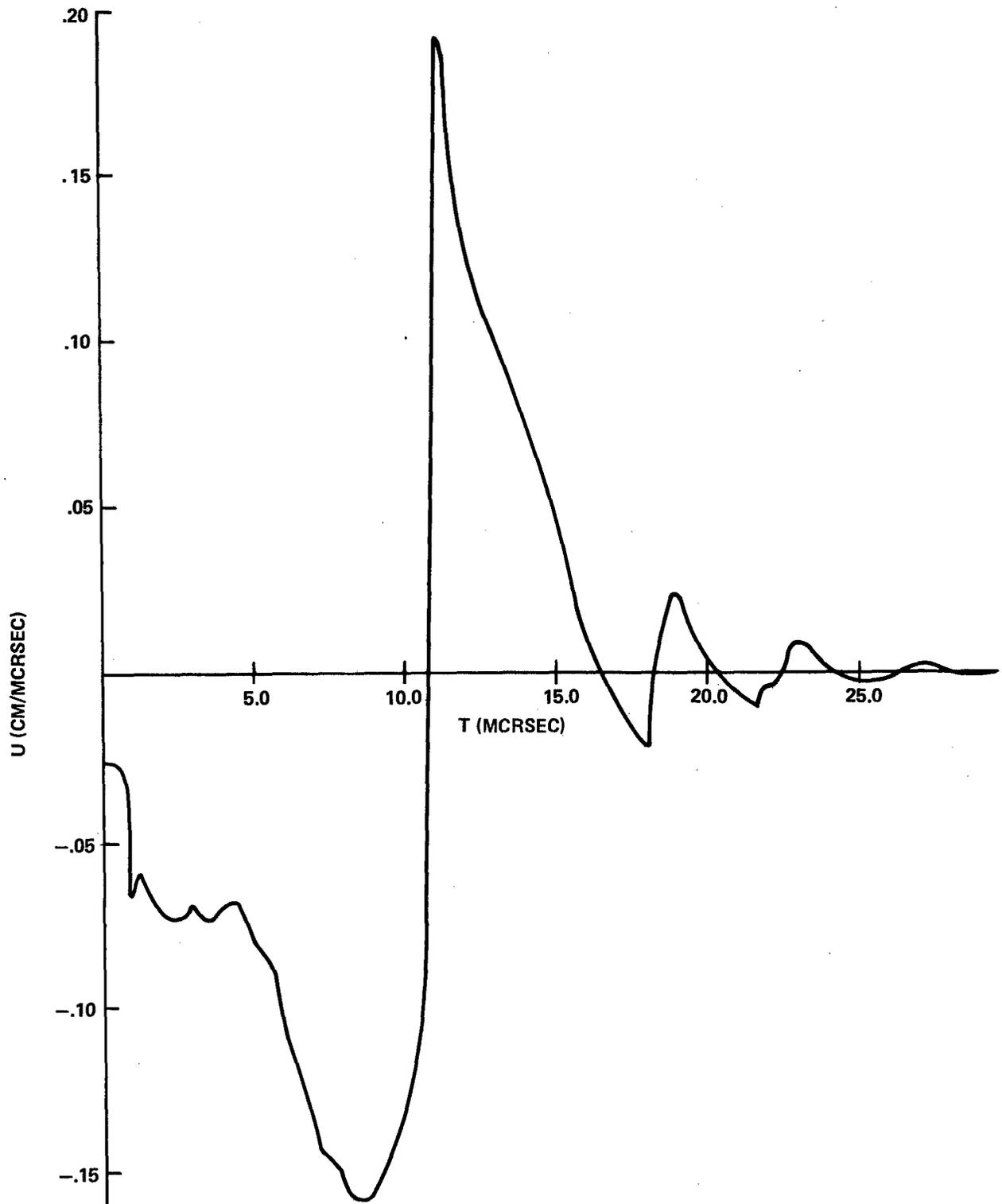


FIG. 37 PLOT OF VELOCITY OF THE OUTER SURFACE OF THE CORE VS. TIME (PROBLEM 6)

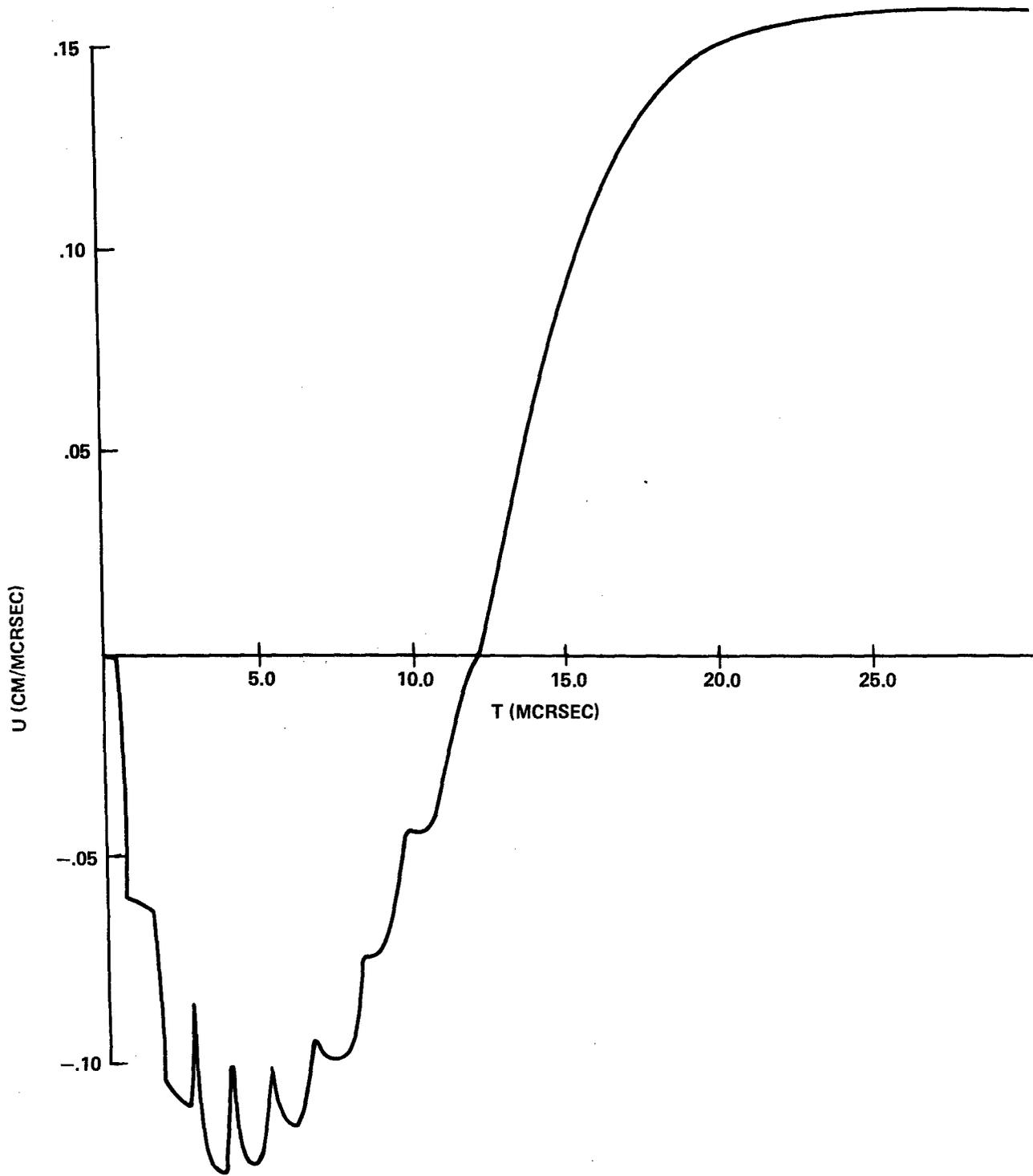


FIG. 38 PLOT OF THE VELOCITY OF THE INNER SURFACE OF THE STAGING VS. TIME (PROBLEM 6)

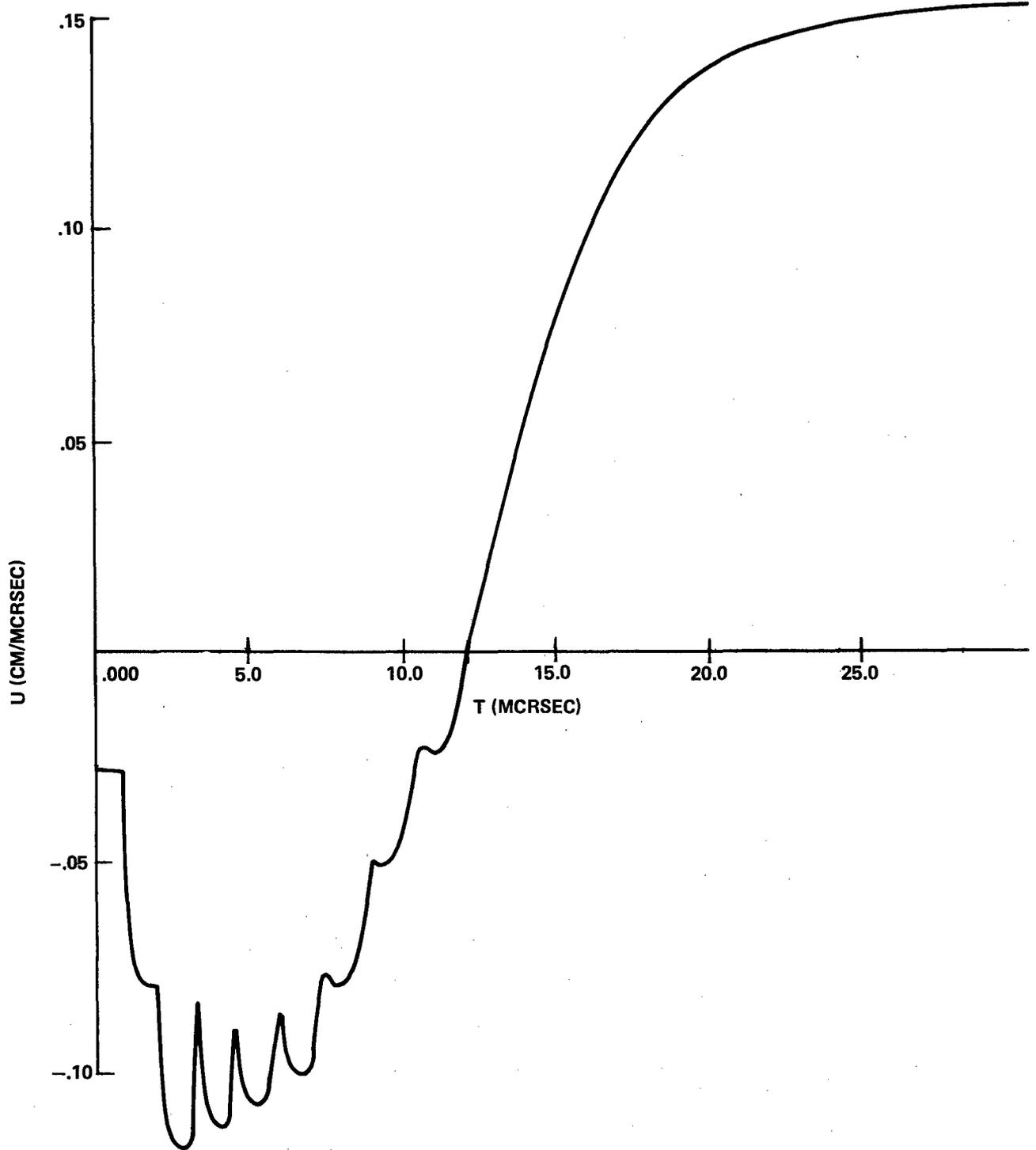


FIG. 39 PLOT OF THE VELOCITY OF THE OUTER SURFACE OF THE STAGING VS. TIME (PROBLEM 6)

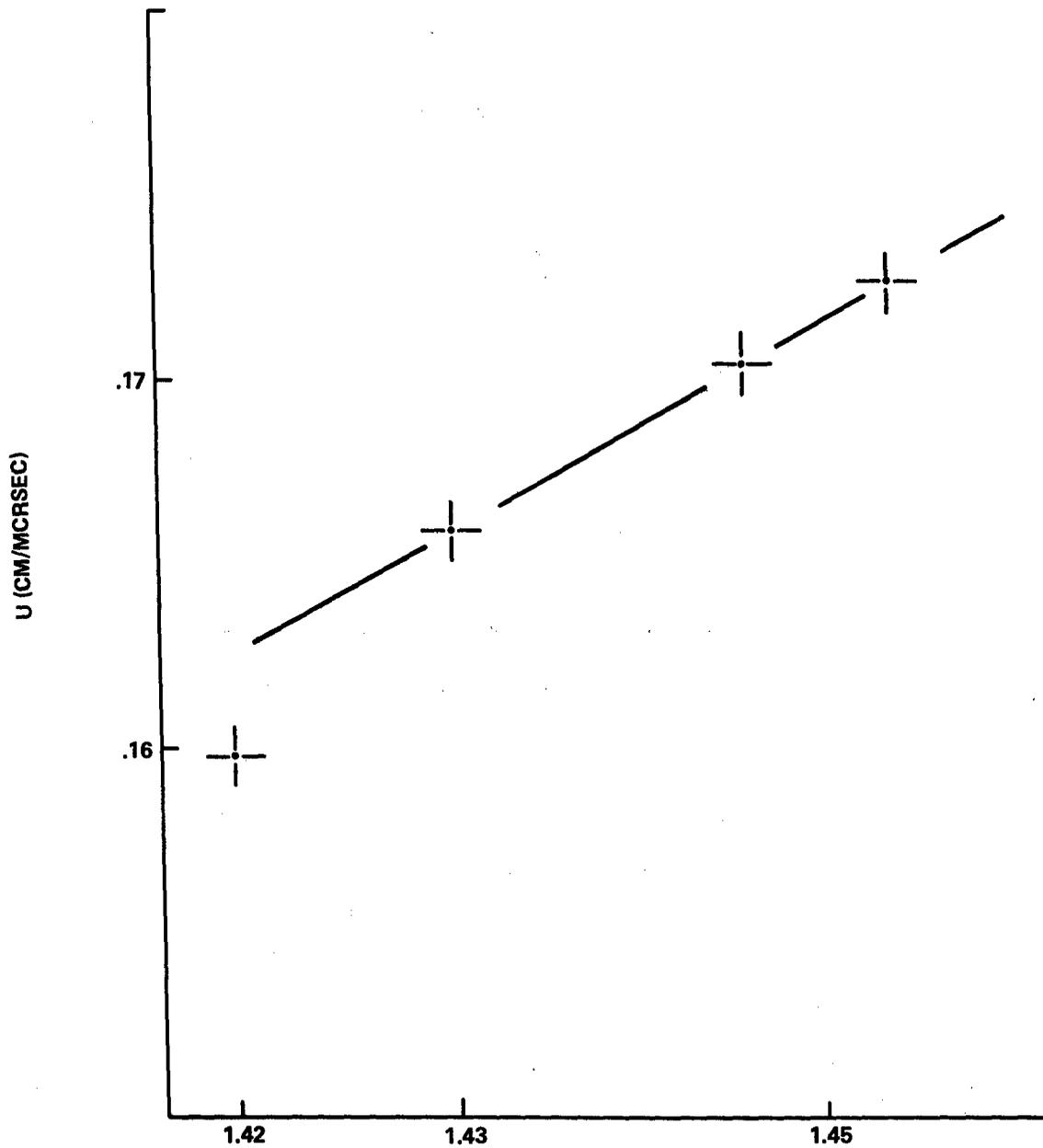


FIG. 40 PLOT OF MINIMUM VELOCITY OF THE OUTER SURFACE OF THE CORE VS. C/M RATIO OF THE STAGING

## APPENDIX A

Solution to the Shock Transmission Problem (Ideal Case)

A sheet of copper .18 cm thick is adjacent to a high pressure (.1369 megabars) gas at time zero. A shock enters the copper and a relief wave enters the gas. At the interface between the copper and the gas, the pressure and particle velocity are continuous.

In the copper, the pressure and density are related by the Hugoniot and the particle velocity can be obtained from the equations for conservation of mass and momentum. The pressure and particle velocity in the copper must be the same as that obtained from the relief wave equations for the gas.

Let  $\rho/\rho_0 - 1$  be  $\mu$  and consider the usual equations for conservation of mass and momentum

$$\rho_0 U = \rho(U-u)$$

$$\rho_0 U^2 = p + (\rho_0 U)(U-u)$$

Solve the first of these for U, substitute in the second and obtain:

$$u = -\sqrt{\frac{\mu}{1+\mu} \frac{p}{\rho_0}} \quad (1)$$

Now p can be expressed as a polynomial in  $\mu$ , using the data of Rice, McQueen, and Walsh (op. cit.).

Now the pressure and particle velocity are continuous across the interface and can be computed from the relationship for a gamma law gas

$$p = p_0 \left\{ 1 + \frac{1}{2} (\gamma-1) u/c_0 \right\} \frac{2\gamma}{\gamma-1} \quad (2)$$

To find the actual values of p and u, assume a value of  $\mu$  and calculate p from the polynomial equation. Use this value of p to calculate u from (1) and use u to calculate p from (2). The two values of p will not agree, but by varying  $\mu$  it can be found that the difference between them changes sign between  $\mu = .071$  and  $.072$ . A Newton Raphson iteration scheme then shows that  $\mu = .0711$ . The other values of the variables are:

$$\begin{aligned} p &= .1154 \text{ mbars} \\ u &= -.0293 \text{ cm}/\mu\text{sec} \\ V &= .9336 \\ E &= .0038 \text{ mbars} \\ U &= .4416 \text{ cm}/\mu\text{sec} \end{aligned}$$

The transit time of the shock across the copper will be the thickness (.18 cm) divided by the shock speed (.4416) and is equal to .4076 $\mu$ sec.

## APPENDIX B

The Oscillation in Problem 1

After the oscillation has been eliminated in problem 2, the question arises as to why there was no similar oscillation in problem 1 which was geometrically identical to problem 2. In problem 1, the shock has to traverse 10 zones (5 in the explosive and 5 in the copper) before arriving at the inner free surface of the copper. This allows a certain time for the shock to smear out. In problem 2 (the constant volume detonation case) there are only 5 zones (all in the copper) between the shock and the free surface, and the shock which arrives at the free surface is sharper because it does not have time to smear out. In order to make the shock of problem 1 comparable, more zones must be added. The shock will still be smeared over three zones (approximately) but the zones will be smaller, thus making the shock sharper. When the number of zones was doubled, the same oscillation appeared in problem 1 as in problem 2.

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