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GENERAL SLENDER CONE SIMULATION AT REENTRY FLOW CONDITIONS

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PREFACE

The research reported herein was conducted at the Department of Physics, Rutgers University, New Brunswick, New Jersey, under U. S. Air Force Contract No. F40600-75-C-0001, subcontract No. 76-20-VKF at the request of the Directorate of Technology, Research and Development Division, Arnold Engineering Development Center, Air Force Systems Command, Arnold Air Force Station, Tennessee, under Program Element 65807F. The work was done under ARO Project No. V45G. The author of this report was Joseph A. Johnson, III, Department of Physics, Rutgers University, New Brunswick, N. J. 08903. The manuscript (ARO Control No. ARO-VKF-TR-77-38) was submitted for publication on May 13, 1977.

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1.0 INTRODUCTION

In the past decade, experimental and theoretical investigations at a large variety of industrial, governmental, and university research centers have revealed an unexpected complexity in the physics of hypersonic phenomena. The quest for new simple fundamentally consistent and universal scaling laws, even in nonreacting flows, has been repeatedly frustrated. Furthermore, the increasingly mysterious coupling of turbulence and molecular interactions at high Reynolds numbers and high Mach numbers has introduced new dimensions to the uncertainty ordinarily associated with the predictability of high-speed flow.

By contrast (as a result of the same investigations), the capability of the systems designer for detailing the requirements of specific systems has increased. This, in turn, enhances the likely impact on the current understanding of high-speed phenomena of a carefully planned and focused attempt at simulation. Thus, both to serve the interests of current system problem-solvers and to attack anew the uncertain physics as well, the elements of model similitude deserve a re-examination.

With this motivation, the problem of slender cone simulation will be reviewed. There have been more than 100 years of attention given to the problem of gas dynamic similitude (Ref. 1). In this report, the approaches developed by Zierep (Ref. 2) and Thompson (Ref. 3) are followed, and the theoretical analyses of Eschenroeder (Ref. 4) have been used extensively. After a brief summary of theoretical similitude, the various flow regimes will be cataloged. Finally, the overall approach in this report will be applied to a specific example of current interest.

2.0 SIMILITUDE: A REVIEW

The variety of ways in which a physical law can be expressed is limited by the requirement, among others, that its form be independent of a change in dimensions. Thus, even when the law is not known, certain restrictions can be defined. Consider, therefore, that a list of important dimensional parameters for a given observable (X_1) can be defined. When this list is ordered, one has (including X_1):

$$X_1, X_2, \ldots X_i, \ldots X_n$$

The law characterizing these parameters can always be put in the form

$$G\ (X_1, X_2, \ldots X_i, \ldots X_n) = 0$$

Usually, there are only p of these parameters ($p \leq n$), which are dimensionally independent; that is, only ($n - p$) parameters have dimensions which can be found as products of the dimensions of one or more of the remaining p parameters. Further, if the number of fundamental dimensions (such as mass, length, time, charge, and temperature) relevant to the problem is q, then p must be equal to or less than q. Let's rearrange our list of X's into two groups so that the dimensionally independent parameters are listed first:

$$X_1, X_2, \ldots X_p, \ \vdots\ X_{p+1}, \ldots X_n$$

The dimensions of the ($n - p$) parameters in the second group ($j \geq p + 1$),

$$\left[X_j\right] = \left[X_1\right]^{\beta_1} \left[X_2\right]^{\beta_2} \ldots \left[X_p\right]^{\beta_p}$$

can now be expressed in terms of the first group and a dimensionless parameter in each case results when the following prescription is used:

$$Y_j \equiv \frac{X_j}{\left[X_1\right]^{\beta_1} \left[X_2\right]^{\beta_2} \ldots \left[X_p\right]^{\beta_p}}$$

From this, an alternative and equivalent form of the physical law is

$$b\ (Y_1,\ Y_2,\ \ldots\ Y_{n-p}) = 0$$

as the law now allows only (n − p) independent variables.

If the form of the law is assumed to be known, then there are at least two resulting implications. First, it is implied that a decision has already been made concerning the important physical parameters. Secondly, an adequate set of boundary conditions on any relevant differential equations must exist and be expressable in terms of dimensionless dependent and independent variables and using nondimensional constant coefficients.

Thus, the similarity analysis allows a reduction in and an interdependence among the significant physical parameters. In addition, for gas dynamic problems, the similarity solution is a prerequisite for the development of model rules since it is desired to transform the data of flow determined from one body to another body. Sometimes, it is even possible through the modeling transformation to obtain insight not otherwise available by treating an ostensibly unrelated problem whose equations can be placed in a form similar to the gas dynamic equations.

3.0 THE SHARP CONE IN AN IDEAL GAS

Now, one can apply these considerations to the general problem of high-speed gas flow around sharp cones. That is, for cone angle (θ), it is assumed that $\sin^2 \theta << 1$. After first discussing flow without chemistry, the treatment will be generalized to include reacting species.

3.1 EQUILIBRIUM FLOW

Let's assume that the overall field is describable (for an ideal gas) by eighteen parameters (n = 18), allowing for all fluid dynamical possibilities, as shown below:

1. x; ⎫
2. y; ⎬ spatial coordinates
3. z; ⎭
4. w; characteristic velocity
5. p; pressure
6. ρ; density
7. ℓ; characteristic length
8. g; acceleration due to gravity
9. β; coefficient of expansion
10. ΔT; temperature difference
11. σ; surface tension
12. λ; conductivity
13. C_p; specific heat, constant pressure
14. C_v; specific heat, constant volume
15. t; time
16. Q; energy source or sink
17. ν; viscosity
18. α; heat transfer rate

Take the value p = 4 (as this is the maximum number corresponding to mass length, time, and temperature as basic units) and use $X_1 = \rho$, $X_2 = C_p$, $X_3 = \nu$, and $X_4 = \ell$. This leaves (n − p = 14) fourteen independent variables:

1. x; x/ℓ
2. y; y/ℓ
3. C_v; C_p/C_v or γ
4. w; $w\ell/\nu$ or Re
5. z; z/ℓ
6. λ; $\nu\rho C_p/\lambda$ or Pr
7. p; $w^2\rho/\gamma p$ or M^2
8. g; $w^2/g\ell$ or Fr
9. σ; $\ell\rho w^2/\sigma$ or We
10. β; $g\ell^3\beta\Delta T/\nu^2$ or Gr
11. t; $\ell^2 C_p\rho/\lambda t$ or Fo
12. α; $\alpha/w\rho C_p$ or St
13. ΔT; $w^2/C_p\Delta T$ or Ec
14. Q; $Q/C_p\Delta T$ or $\overline{D}$

where

Re = Reynolds number
Pr = Prandtl number
M = Mach number
Fr = Froude number
We = Weber number
Gr = Grashof number
Fo = Fourier number
St = Stanton number
Ec = Eckert number
$\overline{D}$ = Damköhler number

Thus, a general solution for this problem can be expressed in terms of the well-known similarity parameters:

$$b\ (\vec{r}/\ell;\ \gamma,\ Re,\ Pr,\ M,\ Fr,\ We,\ Gr,\ Fo,\ St,\ Ec,\ \overline{D}) = 0$$

When this result is applied to the general problem of modeling hypersonic flow, it is clear that complete simulation of the flow is extremely complicated. Lrt's look at three limiting cases. For frictionless flow and no important unsteady, gravitational or thermodynamic effects,

$$b_1(\vec{r}/\ell;\ \gamma,\ M) = 0$$

and simulation is trivial. If viscous effects are important as well,

$$b_2(\vec{r}/\ell;\ \gamma,\ M,\ Re) = 0$$

This implies that, ignoring thermodynamics, the modeling can be accomplished for viscous flows by

$$b_{2a}(\vec{r}/\ell,\ M^n,\ Re^m) = 0$$

Let

$$Z = M^n/Re^m$$

where n and m are determined by the parameter of interest. For example, $n = m = 1$ corresponds to Z = Kundsen number; here, Z is the well-known characterization of the transition from continuum to free molecular flow. For $n = 3$ and $M = 1/2$, Z characterizes the influence of shock wave-boundary layer interference (since $M\frac{\delta^*}{\ell} \sim \frac{M^3}{Re^{1/2}}$). Figure 1 shows these possible regimes. Adding unsteadiness, the equation for $b(Y_i)$ becomes

$$b_3(\overline{r}/\ell,\ \gamma,\ M,\ Re,\ Fo) = 0$$

In this last case, the modeling and approximating problem is obviously more complicated.

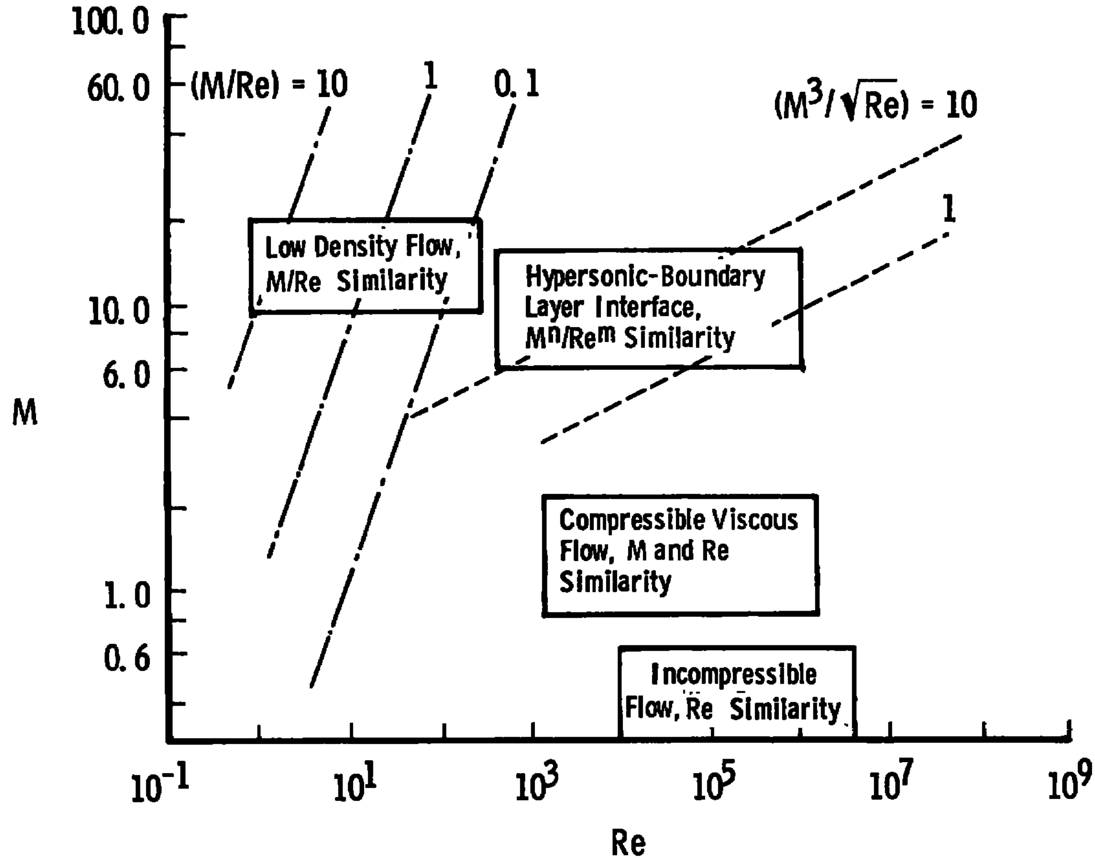


Figure 1. Boundaries of flow regimes in the M-Re plane (from Ref. 2).

3.2 NON-EQUILIBRIUM EFFECTS

When molecular and/or atomic interactions require a period of time in order to reach equilibrium (t_r) which is comparable to local flow field time (t_f ($\sim \ell/w$)), then it becomes necessary to introduce a second time variable. This results in a new time similarity parameter:

$$D = \frac{t_f}{t_r} = \frac{\ell}{wt_r}$$

Either $t_f >> t_r$ or $t_f << t_r$ will make D irrelevant. However, if D is important, then, in order to scale both D and Re in modeling, since

$$D = \frac{\ell}{wt_r} \text{ and } Re = \frac{w\ell}{\nu}$$

the different dependence on w requires that one be able to vary t_r and/or ν. Thus, an important limitation is imposed on these parameters.

If there are several processes with relaxation times (t_r), then each one has a parameter (D_i) given by

$$D_i = \frac{t_f}{t_{r_i}}$$

When the slender cone in air is considered (Ref. 4), for example, the i^{th} two-body forward reaction for the j^{th} species has the form

$$D_{fij}^{(2)} = \frac{\rho_\infty \ell \nu_{ij} k_{fi}^{(2)}(T_o)}{w\mathfrak{M}_\infty}$$

where $\mathfrak{M}_\infty$ is the mean molecular weight of the cold, undisturbed, flight medium, T_o is a typical flow field temperature, ν_{ij} are the stoichiometric coefficients, k_{ij} are the rate constants, and the other symbols have their usual meaning. Similarly, for three-body reactions:

$$D_{fij}^{(3)} = \frac{\rho_\infty^2 l \nu_{ij} k_{fi}^{(3)}(T_o)}{w\mathfrak{M}_\infty^2}$$

In the above, the gas dynamic equations have been used, the diffusion coefficient is assumed to be proportional to T^α/ρ, and viscosity is proportional to $(T)^\beta$ where α and β are unknown powers. From this, the simultaneous preservation of both D's would not generally be possible since the requirements of $\rho_\infty \ell$ and $\rho_\infty^2 \ell$ are contradictory.

Obviously, the long list of similarity parameters must be reduced if modeling is to be possible. This might be accomplished by a space-time transformation so that the general three-dimensional steady flow problem becomes a problem in one-dimensional unsteady flow. This might be accomplished, alternatively, by determining and exploiting the flexibility of neglecting certain effects in the various flight regimes. Furthermore, the similarity problem might be simplified by exactly reproducing certain aspects of the prototype problem so that the range of modeling to be attempted is reduced.

4.0 APPLICATION TO THE REENTRY CONE

The techniques just outlined can be used in the problem of model simulation for the reentry cone. Here, one can take advantage of the possibility of neglecting various terms in appropriate regimes. For our purposes, we shall restrict our application, implicitly, to simulation in a ballistic range. In addition, we assume (incorrectly, but with calculable error) that there is exact duplication of temperature; i.e., $T_\infty \approx T_{Test}$.

4.1 GENERAL CONSIDERATIONS

If flight velocity is duplicated, then collision efficiencies and temperature are also essentially duplicated. Furthermore, at high altitudes, where viscous effects are minor and three-body effects are negligible compared with two-body effects, simulation is accomplished by Mach number M and by $D^{(2)}$ or $\rho_\infty \ell$ scaling. Conversely, at low altitudes where three-body effects might dominate, $D^{(3)}$ or $\rho_\infty{}^2 \ell$ scaling would be more appropriate; however, the additional importance of viscous and unsteady influence on the flow field serves (as has been shown) to complicate considerably the modeling. Figure 2 qualitatively suggests (Ref. 5) these regimes for slender cone flow.

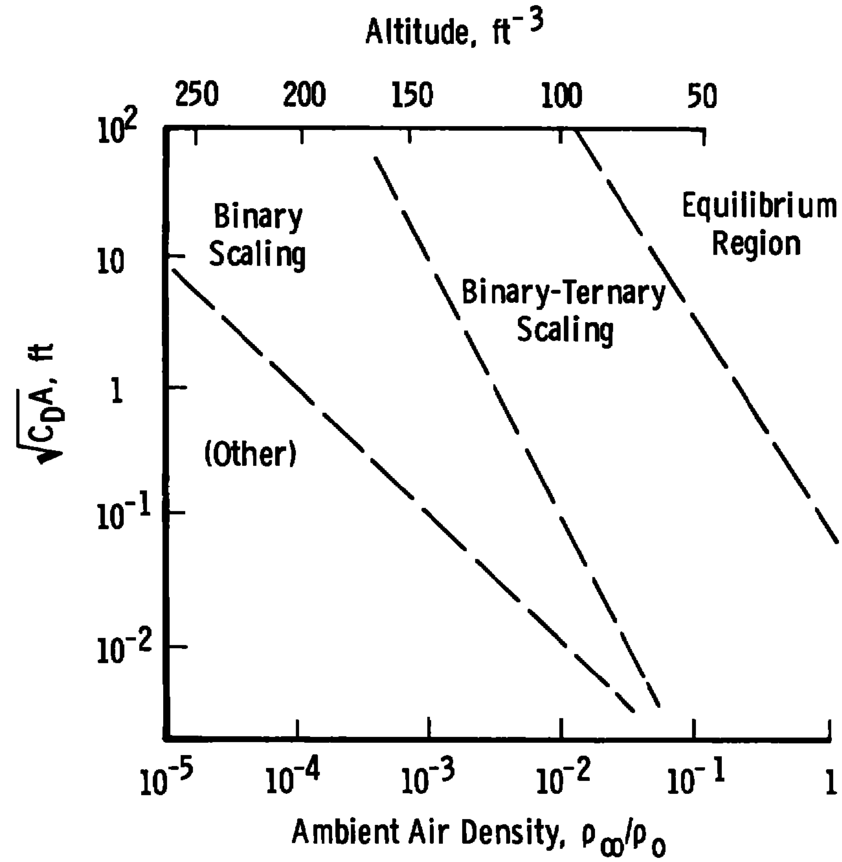


Figure 2. Approximate scaling regimes (from Ref. 5).

Other altitude related complications exist with regard to flow observables which add to the modeling difficulties. Figure 3 shows, for comparison, a cone and a sphere of equal drag area (hence producing approximately equal numbers of electrons). The variety of behaviors for the slender cone is much greater than that for the sphere. For example, there are four regimes in so far as wake effects are concerned: (1) high altitude - laminar boundary layer, modest electron density; (2) medium altitude - laminar boundary layer, and moderate electron density; (3) medium altitude - laminar boundary layer, and high electron density; (4) medium to low altitude - turbulent boundary layer, and high electron density. Figure 4 qualitatively illustrates these regimes in terms of wake velocities from backscattering radar signals and shows a comparison with an alternative history for another reentry vehicle shape.

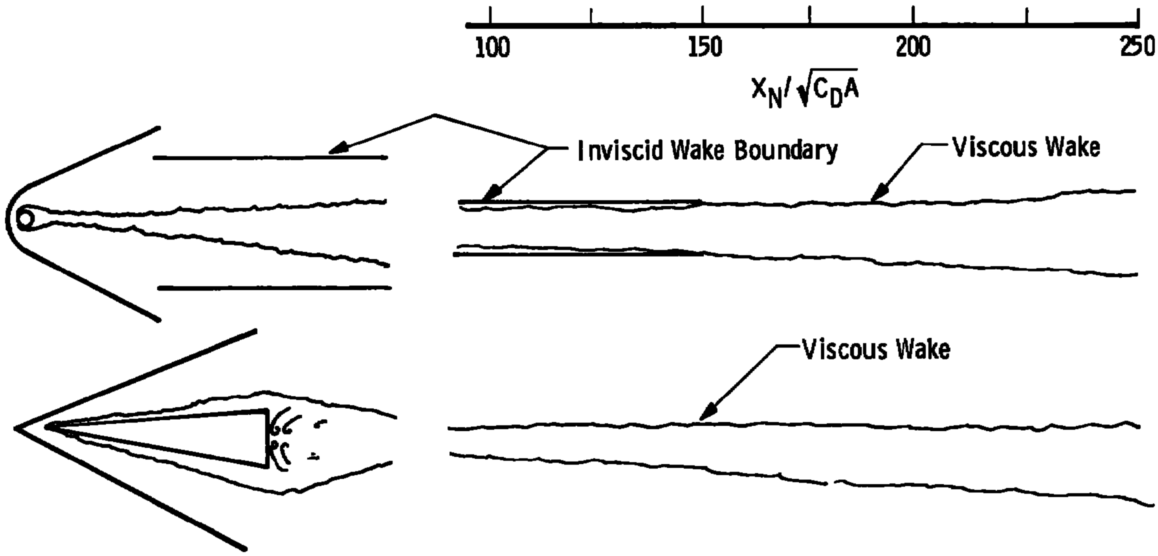
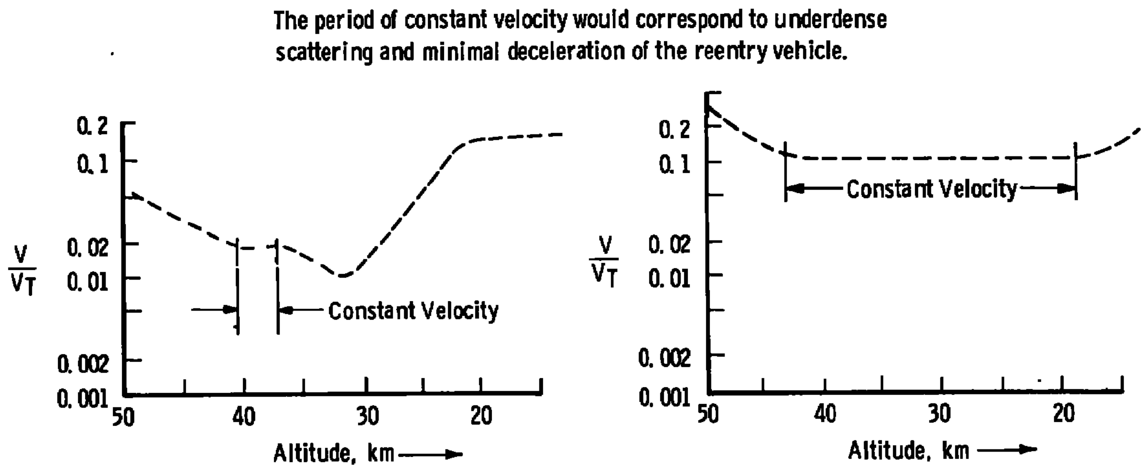


Figure 3. A schematic for shape influences on wake effects, $(C_D A)_{cone} = (C_D A)_{sphere} \times M_\infty >> 1$.

Using Fig. 3, it is possible to explore more explicitly the application of binary scaling to hypersonic conical flow. For example, in the inviscid shock layer, if local flow-field temperatures can be held constant by preserving M (sin θ), then not only is $\rho_\infty \ell$ useful but $\rho_\infty \ell / w$ is a good scaling parameter as well (Ref. 6). This allows a direct scaling of shock layer radiation. Notice, however, that modeling the radar backscattering still presents problems since the plasma frequencies depend on the square root of electron density, and the appearance of critical density effects would not be modeled.



a. A sharp-nosed body

b. A blunt-nosed body

Figure 4. A schematic for shape influences on altitude histories; wake velocity measurements at a fixed distance from the nose.

In the boundary layer, binary scaling should still dominate at high altitudes if there are no unique surface effects. This requires, specifically, a non-turbulent, non-ablative boundary layer so that Re and St can be ignored. However, a contaminating boundary layer (in order to be scaled) would require the scaling of viscous and heat transfer phenomena. Thus, the modeling of flow with binary scaling at high altitudes where ablation and/or seeding are present requires $\rho_\infty \ell$, Re, and St similarity.

With regard to the reentry wake, there seem to be three distinct regions; (1) base flow, between the cone and the neck; (2) far-wake flow, where asymtoptic laws are obeyed; and (3) the region intermediate between (1) and (2) above. Even at high altitudes, it is likely that three-body and two-body processes are at least competitive in regions (1) and (2). However, the proper scaling for the electron density decay in region (3) is probably characterized by Fig. 2. Thus, at high altitudes, the rate of dissipation in radiation and (for underdense scattering) in radar backscattering should follow binary scaling even though the absolute levels of electron densities may not scale.

There are several additional comments which are appropriate. First, the usual choice for a length scale would be $\ell \sim \sqrt{C_D A}$. This allows for the possibility of investigating modeling effects (assuming $C_D \sim \sin^2\theta$) in the scaling of wake phenomena which could be attributable to changes in the cone's angle. However, the use of this parameter implies a truly sharp cone ($R_n/R_B << 1$). Secondly, it is generally conceded that diffusion effects, molecular and turbulent, are not properly treated in the similarity analyses (Ref. 7). In particular, for example, intense small-scale turbulence could produce relaxation effects which would not appear in this approach. Furthermore, the flow sequence at the base of the cone from stagnant flow to neck flow to laminar transition to turbulence is probably inadequately treated in this approach; this is an important limitation since the position of the neck plays a crucial role in determining the absolute level of electron density in the wake and, therefore, the quality of the radar backscattering.

4.2 AN ILLUSTRATION

Now let's apply these results to the problem of a prototype high-altitude cone (50 km < h < 90 km) with $\theta = 10$ deg and $w \simeq 6.1$ km/sec. It is to be modeled by a ballistic range conical projectile with $\theta = 10$ deg but length scale off by a factor of approximately 7.5. Further, allow for a generator inside the projectile injecting hot gas into the boundary-layer flow at the base. The test conditions should be (with $T_\infty = T_{Test}$):

(1) Mach number duplication, or $w_{Test} \sim 6$ km/sec

(2) Binary scaling; for $\rho_\infty \ell \simeq \rho_{Test} \frac{\ell}{7.5}$, then $\rho_{Test} \sim 7.5\ \rho_\infty$

This latter condition gives

$$8.1 \times 10^{-6} \lesssim \rho_{Test} \lesssim 8.1 \times 10^{-8}\ \text{g/cm}^3 \text{ or}$$

$$0.04 \lesssim p_{Test} \lesssim 4 \text{ mm Hg}$$

for the altitude range specified.

The ballistic range observables will then have the following relationship to full scale:

1. Radiation from the inviscid shock layer will scale with $\sqrt{C_D A}$ since there will be Mach number duplication.
2. In the absence of a firm prediction on critical electron density, the radar backscattering from the inviscid shock layer will probably not scale.
3. In the absence of Re and St similarity, the injected hot gas will not produce a scalable effect on absolute electron density or radiation since binary scaling is presumed applicable to the prototype problem.
4. Radiation and radar backscattering in the base region and in the far wake will not scale.
5. Relative radiation decay will scale in the wake for intermediate regions.
6. Relative decay in radar backscattering will scale, assuming that the scattering is underdense.
7. Ionization trail lengths will not scale because of effect (3) above, viz, the nonscaling of absolute electron densities and radiation.

5.0 CONCLUSIONS AND RECOMMENDATIONS

When the most general approach is taken, the cone similarity problem can be defined in a straightforward fashion. Some limiting cases are especially amenable to analysis even though these cases might not be of present day relevance. The possibility of usefully applying the techniques of similitude to carefully selected specific problems has been shown and should be evaluated since, for example, modeling and simulation are important issues in threat-decoy discrimination. It is recommended, therefore, that experimental tests of predictions of the sort which have been illustrated be conducted. In addition, these procedures should be extended to some of the several blunt body types.

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