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Sidelobe Reduction in Polyphase Codes

BEN H. CANTRELL
Radar Analysis Staff
Radar Division

April 13, 1977
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Ben H. Cantrell

Naval Research Laboratory
Washington, D.C. 20375

October 1977

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A means of reducing the sidelobes in Frank polyphase codes was obtained. This was achieved by applying a stepped amplitude weighting function across the length of the code. An example which reduces the sidelobes by about 10 dB for a code of length 256 is demonstrated.
SIDELOBE REDUCTION IN POLYPHASE CODES

INTRODUCTION

This report describes a means of reducing the sidelobes of a Frank polyphase code [1-3]. This is basically achieved by applying a stepped weighting across the code. The polyphase code is first reviewed, and then the means of weighting is described. Finally an example is given.

FRANK POLYPHASE CODE

The Frank polyphase code is probably most easily described by noting the elements of the code are the same as the elements in the argument portion of the discrete Fourier transform (DFT). The DFT is described by

\[ S(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}, \text{ for } k = 0, \ldots, N-1. \]  

(1)

This can be written in matrix form

\[ S = W X, \]  

(2)

where

\[ S = \begin{bmatrix} S(0) \\ S(1) \\ \vdots \\ S(N-1) \end{bmatrix}, \quad W = \begin{bmatrix} W_{0,0} & W_{0,1} & \cdots & W_{0,N-1} \\ W_{1,0} & W_{1,1} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ W_{N-1,0} & \cdots & \cdots & W_{N-1,N-1} \end{bmatrix}, \quad \text{and} \quad X = \begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix}. \]

The elements \( W_{k,n} \) are

\[ W_{k,n} = e^{-j2\pi nk/N}. \]  

(3)

The Frank polyphase code is obtained by sequentially reading the elements of \( W \) one row at a time from left to right starting from element \( W_{0,0} \). For example for \( N = 2 \) the resulting code would be

\[ W_{0,0} \quad W_{0,1} \quad W_{1,0} \quad W_{1,1}. \]

The code is compressed by crosscorrelating the returned signal with the inverted-order complex conjugate of the code. The reverse-order complex conjugate would be obtained by reading the elements of \( W \) one row at a time from right to left starting from element \( W_{N-1,N-1} \). For example for \( N = 2 \) the resulting code to crosscorrelate with is

\[ W_{1,1}^* \quad W_{1,0}^* \quad W_{0,1}^* \quad W_{0,0}^*. \]

A circuit for performing the crosscorrelation is given in Fig. 1, which is a common correlator.

Fig. 1 — Circuit for compressing an $N = 2$ Frank polyphase code

The coherent summer adds the real and imaginary parts separately. The compressed pulse for $N = 16$, or a compression ratio of 256, is shown in Fig. 2. The results show peak sidelobes in the order of 33 dB.

**SIDELOBE REDUCTION**

The sidelobes of the code can be reduced by applying an additional weighting to the sequence used in the crosscorrelation process. The weights used in the crosscorrelation multiplier is specified by

$$Z_{k,N} = C_k \cdot W_{k,N}.$$  \hspace{1cm} (4)
Fig. 3 — Circuit for compressing an $N = 3$ Frank polyphase code with weighting for sidelobe reduction

![Circuit diagram]

The weight $C_k$ remains constant for each row of the $W$ matrix. The weight $C_k$ used is a Hamming weight [3]. The equation is

$$C_k = a + (1 - a) \cos \frac{\pi (k)}{2} \left[ \frac{N}{N - 1} \right] - 1,$$

where $N$ is even and $k = 0, 1, 2, ..., N - 1$. The crosscorrelator structure for $N = 3$ is shown in Fig. 3. The compressed pulse for $N = 16$ using the weighting specified by (4) and (5) for $a = 0.15$ is shown in Fig. 4. Comparing Figs. 2 and 4, we find the sidelobes have been reduced and the peak sidelobe has been reduced in the order of 10 dB.

**SUMMARY**

A means of weighting for reducing the sidelobe levels in the compressed pulse of a Frank polyphase code was obtained. This was achieved by using a stepped weighting such as a Hamming weighting across the code. The results showed improvements of the order of 10 dB in peak sidelobe levels.
ACKNOWLEDGMENTS

I thank Frank Kretschmer and Bernard Lewis for discussing the problem with me.

REFERENCES

