

ADA 035879

VOLUME 9, NO. 2  
FEBRUARY 1977

2

11 Feb 77

12 98p.

6

# THE SHOCK AND VIBRATION DIGEST.

Volume 9  
Number 2, February 1977

A PUBLICATION OF  
THE SHOCK AND VIBRATION  
INFORMATION CENTER  
NAVAL RESEARCH LABORATORY  
WASHINGTON, D. C.

FOR RECORD AND ANNOUNCEMENT ONLY

NOT TO BE REPRODUCED FOR SALE  
OR DISTRIBUTION

may  
be purchased from

Shock and Vibration Information Center  
Naval Research Laboratory, Code 6020  
Washington, D. C. 20340

DDC  
RECEIVED  
FEB 23 1977  
RECEIVED

A



OFFICE  
OF THE  
DIRECTOR  
OF DEFENSE  
RESEARCH  
AND  
ENGINEERING



COPY AVAILABLE TO DDC DOES NOT  
PERMIT FULLY LEGIBLE PRODUCTION

Approved for public release; distribution unlimited

389004

YB

# THE SHOCK AND VIBRATION DIGEST

Volume 9 No. 2  
February 1977

## STAFF

**EDITORIAL ADVISORS:** Henry C. Pusey  
Robert Belshelm

**TECHNICAL EDITOR:** Ronald L. Eshleman

**EDITOR:** Judith Nagle-Eshleman

**RESEARCH EDITOR:** Milda Tamulionis

**BOARD OF EDITORS:**

R. Belshelm	W. D. Pilkey
R. L. Bort	A. Semmelink
J. D. C. Crisp	E. Sevin
C. L. Dym	J. G. Showalter
D. J. Johns	R. A. Skop
G. H. Klein	C. B. Smith
K. E. McKee	J. C. Snowdon
J. A. Macinante	R. H. Volln
C. T. Morrow	H. von Gierke
J. T. Oden	E. E. Ungar

A publication of  
**THE SHOCK AND VIBRATION  
INFORMATION CENTER**

Code 8404 Naval Research Laboratory  
Washington, D.C., 20375

Henry C. Pusey  
Director

Rudolph H. Volln

J. Gordan Showalter

Robert Belshelm  
Special Consultant

The Shock and Vibration Digest is a monthly publication of the Shock and Vibration Information Center. The goal of the Digest is to provide efficient transfer of sound, shock, and vibration technology among researchers and practicing engineers. Subjective and objective analyses of the literature are provided along with news and editorial material. News items and articles to be considered for publication should be submitted to:

Dr. R. L. Eshleman  
Vibration Institute  
Suite 206  
101 West 55th Street  
Clarendon Hills, Illinois 60514

Copies of articles abstracted are not available from the Shock and Vibration Information Center (except for those generated by SVIC). Inquiries should be directed to library resources, authors, or the original publishers.

This periodical is for sale on subscription at an annual rate of \$40.00. For foreign subscribers, there is an additional 25 percent charge for overseas delivery on both regular subscriptions and back issues. Subscriptions are only accepted for the calendar year, beginning with the January issue. Back issues are available by volume (12 issues) for \$15.00. Orders may be forwarded at any time, in any form, to SVIC, Code 8404, Naval Research Laboratory, Washington, D.C., 20375. Issuance of this periodical is approved in accordance with the Department of the Navy Publications and Printing Regulations, NAVEXOSP-35.

ACCESSION LIST	
NTIS	<input checked="" type="checkbox"/>
DDC	<input type="checkbox"/>
BRANKIN	<input type="checkbox"/>
JUSTICE	<input type="checkbox"/>
BY.....	
DISPATCHED	
DATE	.....
A 21	

# DIRECTOR NOTES

Looking back at previous Shock and Vibration symposia, a matter of perpetual concern has been the effectiveness of oral presentations and, in particular, the quality of visual aids used by the speakers. This problem is not unique to our symposia; it is recognized by every technical society concerned with the organization of technical meetings. The conference management in each case persistently prepares guidelines which are faithfully delivered to each accepted speaker, yet some poor presentations are still with us. The problem is recognized by managers of research institutions. Dr. Alan Berman, Director of Research at the Naval Research Laboratory recently insisted that NRL briefings be improved by eliminating illegible and cluttered slides. It is highly probable that most speakers are aware of the problem, yet many still violate the basic rules for planning their presentations and preparation of their visual aids. In doing this they are performing a serious disservice to themselves and to their prospective audience.

I do not propose to give detailed guidelines here, but I can recommend an excellent article by Hubbard\* for those who are interested in improving their presentations. In general, speakers should remember that their audience is not already familiar with their work. They should avoid complex mathematical derivations and should keep their slides simple and descriptive. It is distressing when researchers with significant accomplishments fall down in their efforts to communicate to others.

The 48th Shock and Vibration Symposium is now firmly scheduled for 18-20 October 1977 in Huntsville, Alabama. The U.S. Army Missile Command will be host. The first call for papers will be issued next month. It is hoped that all prospective speakers will do their homework on effective presentations. This effort will help to make the 48th Shock and Vibration Symposium a useful experience for all.

H.C.P.

\*H.H. Hubbard, "Guidelines for the Planning and Preparation of Illustrated Technical Talks," J. Acoust. Soc. Amer., 60 (5) (Nov 1976)

# EDITORS RATTLE SPACE

## MACHINE DIAGNOSIS AND PROGNOSIS: A NEW TECHNOLOGY

Machine diagnosis and prognosis is a new formal technology. The availability of complex signal processing equipment and sensors has enabled the engineer to use natural vibration and acoustic responses to carry out formal machine diagnosis. This new technology allows engineers to determine shut-down times for reasons of safety and to avoid loss of productivity. No longer does a machine have to run to destruction.

What is involved in machine diagnosis? First, the engineer must know the physics of the machine and its functions. Second, he must be able to measure and analyze machine vibration signals. Finally, he must be able to interpret the data properly.

Engineers tend to try to diagnose machine faults without knowing the physics of the machine, particularly its vibrational properties. Yet, how can the engineer associate vibration signals with machine faults or conclude that a problem is one of wear, misuse, corrosion, or process malfunction unless he knows about resonances, critical speeds, and conditions of stability? It is not sufficient to draw parallels between the frequencies of vibration signals and malfunction frequencies.

Many engineers try to match vibration signals with an excitation such as mass unbalance while watching for an increase in vibration level. Such a technique may occasionally give valid results, more often, however, the problem is too complicated to be so simply resolved. In order to solve machine fault problems routinely, the engineer must know his machine and why it vibrates at given levels.

In the near future machine prognosis will most likely become practical. Machine prognosis will allow the engineer to determine the most advantageous time for shut down. It will thus be possible to schedule repairs with no loss of production. The engineer will probably make a series of vibration measurements over a period of time. These measurements will be processed by a mathematical model of the machine and its process. In this way the state of the machine at some set date in the future can be determined. An alternative would be to determine when the machine will fail, so that maintenance can be scheduled. Prognosis is not yet practical because suitable wear models are not yet available and changes in machine excitation and conditions affecting stability as the machine is used cannot be predicted. Thus machine vibration is on the way to becoming an asset rather than a liability.

R.L.E.

## ABSORBERS AND ISOLATORS FOR TORSIONAL VIBRATION

John M. Vance\*

*This article describes devices used for reducing torsional vibration in rotating machinery. Consistent definitions are given for absorbers, isolators, and dampers.*

Devices for reducing torsional vibration in rotating machinery can be classified as absorbers, isolators, or dampers. These terms have been used inconsistently in the literature, and the definitions have been interchanged. To further confuse the issue, some devices are actually a combination of two, or even three, of these classes. The definitions used in this article are consistent with the classical literature on vibration theory. It is hoped that this will help the reader understand how a given device is meant to work.

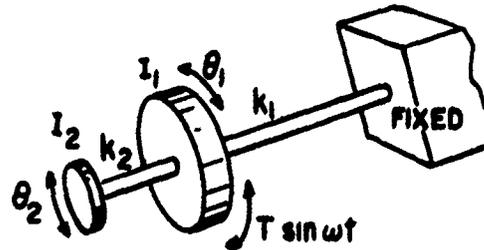
The number of devices for reducing torsional vibration that have been successfully applied is small compared to the number of ideas published and patents granted. The number of devices now being successfully marketed is also small, despite a demand that existing manufacturers cannot meet. Furthermore, most of the published literature on the reduction of torsional vibration is old; relatively little new information has appeared in the last decade.

Because this is the first of a series of reviews on the reduction of torsional vibration, older literature which the author believes is informative or potentially useful is briefly described.

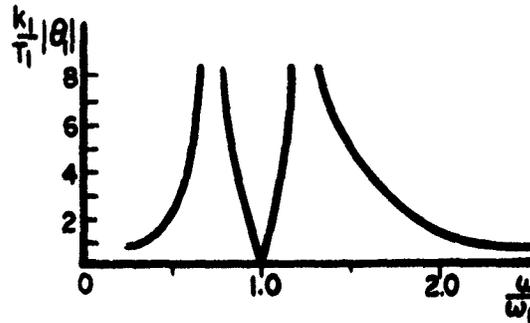
### Absorbers

Torsional vibration absorbers are based on Frahm's principle: an inertia  $I_2$  is attached to the vibrating member with a spring  $k_2$  that is tuned to be in resonance with the offending frequency. The added inertia appears infinite at the tuned frequency and converts the vibrating point to a node [1]. Figure 1 illustrates the basic idea for the case in which the exciting frequency  $\omega$  is near the natural frequency of the main system. This is the case for which an absorber will most likely be needed.

\*Department of Mechanical Engineering, University of Florida, Gainesville, Florida 32611



(a) Simple Torsional System  $k_1, I_1$  with Absorber  $k_2, I_2$



(b) Response of  $\theta_1$

**Figure 1: Torsional Absorber and Response of Main Inertia with Absorber Attached**

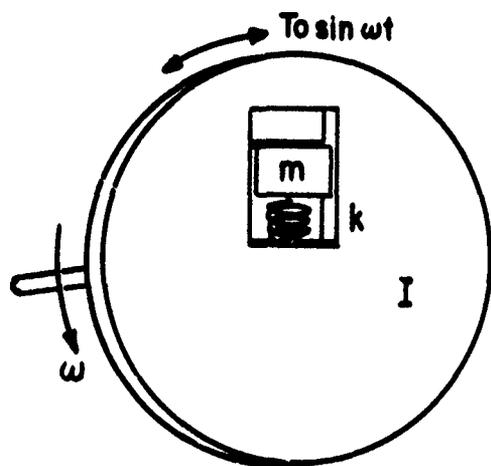
Practical problems associated with these absorbers are that they function only at one frequency, that the amplitude of  $\theta_2$  becomes so large as to break  $k_2$ , or that there is not enough space or support strength for the required size of  $I_2$ . Much of the literature on torsional absorbers has been concerned with novel designs aimed at overcoming these problems.

An early design was the pendulum damper\*. The inertia is a mass pinned and rotating so that the pendulum is aligned with a radius of the shaft [2]. The centrifugal force field produces a natural frequency that is proportional to shaft speed, thus

\*Commonly used terminology, but incorrect.

eliminating the single-frequency characteristic of the simple absorber. One of the few recent successful efforts to apply the pendulum damper involved a helicopter rotor [3]. Vibration of such rotors includes both translational and torsional modes.

An absorber has been described [4] in which the speed-proportional natural frequency is obtained by using an electromagnetic torsional spring; the stiffness varies with field current. Figure 2 illustrates the mechanism of another absorber [5]. The reaction of the inertia is through Coriolis forces, and the device departs somewhat from the basic Frahm principle. It is shown to be more effective than a pendulum damper when the order\*\* of torsional vibration is less than two.



**Figure 2: Centrifugal Governor Mechanism Used as a Torsional Absorber**

#### Isolators

Suppose that  $I_2$  in Figure 1(a) is not an inertia added to make an absorber, but is instead a rotating machine component that is susceptible to malfunction or damage by torsional vibration. In that case,  $I_2$  can be isolated from the torsional vibration of  $I_1$  by making the torsional spring  $k_2$  as soft as practical.

Successful use of an isolator depends on ensuring that all natural frequencies are well below the excitation frequency, as the basic principle is to operate the

system far to the right on the response curve for  $I_2$  (large  $\omega/\omega_a$ ,  $\omega_a = \sqrt{k_2/I_2}$ ).

Torsional isolators for rotating machinery take the form of shaft couplings with low torsional stiffness. A coupling with elastic elements and both low stiffness and high torque transmission capability presents a difficult design problem; therefore, most elastic isolator couplings are hybrids that incorporate damping with moderately low stiffness.

Very low torsional stiffness has been combined with high torque capability by designing couplings with nonlinear stiffness characteristics. These couplings use the centrifugal force on connecting links (between flanges) to produce torsional stiffness that is dependent in a nonlinear manner on speed and transmitted torque. Such a design [6] has been successfully tested in marine drives (to protect the gearbox in tugboats). A similar design has been developed in Russia [7].

It is possible to design centrifugal link couplings having zero torsional stiffness. This seeming anomaly has been explained [8], as has an application of the couplings to helicopter drive trains [8-10]. Although zero stiffness may be undesirable, (due to a 'hunting' characteristic), a small enough positive stiffness can practically eliminate the two-node resonance in a three-inertia drive train [6].

#### Dampers

A torsional damper is defined in this article as any device designed to dissipate the kinetic energy of torsional vibration into heat. Torsional dampers can be subdivided into two types: devices in which the damping element transmits none of the useful torque; and devices in which the element transmitting the torque also contains the damping. The second type always utilizes a specified and controlled stiffness in addition to the damping; it is discussed in the next section.

Dampers in which no useful torque is transmitted utilize an added inertia (flywheel). This inertia is driven by the rotating shaft only through the force of either coulomb (dry) or viscous friction. The added inertia can follow the speed oscillations only if the friction force is large enough to produce the required accelerations, hence, slipping occurs under some conditions, thereby dissipating energy as desired

\*\*Order is the number of cycles per shaft revolution.

The Lanchester damper was the original dry friction type. The theory and experimental results have been described [11]. The disadvantage of dry friction is that the friction coefficient is difficult to control; it varies with temperature and is sensitive to oils and other contaminants. Equations for the design of frictional dampers, based on experimental measurements have been derived [12].

The viscous damper does not have the disadvantage of temperature and contaminant sensitivity, if a fluid having a viscosity relatively insensitive to temperature is used. A viscous damper in which silicone is the working fluid [13] is available commercially. The theory and an analysis of a diesel drive application based on considerations of energy dissipated per cycle have been presented [14]. It appears to the author that the classical Holzer analysis [15] could be modified and used to calculate complex eigenvalues (including the real part) so as to allow computation of the damping effectiveness of dampers.

#### Hybrid Devices

Several devices available commercially employ a combination of the principles described above. All of these torsional couplings employ some type of elastic element -- i.e., radial leaf spring, rubber in compression, rubber in shear -- together with a mechanism for energy dissipation such as material hysteresis or fluid viscosity. Unless the excitation frequency is high, as would be the case for gear tooth impact, elastic stiffness elements are not soft enough to act as a true isolator. A change of stiffness is often sufficient to move the system off resonance and cure a problem, however. So far as the author can determine, technical information on these devices is obtainable only from commercial literature or proprietary reports

One complex hybrid device that has apparently not been marketed is the tuned and damped gyrostatic vibration absorber [16]. It is a combination of absorber and damper. A gyro wheel with the axis mounted normal to the machine shaft is the added inertia. The gyro speed provides a variable not possible with a pure absorber; the physical size of the inertia thus is not such a limiting factor as in the pure absorber. The gimbal is restrained with springs and dampers. "By a judicious choice of spring stiffness and viscous damping it is possible to ensure that the amplitude will not exceed some predetermined

limit, irrespective of the frequency of the disturbing torque." [15].

#### Suggested Future Work

There is a need for experiments and publication of results for various types of devices that are either in use or appear promising. The published test data available are old and practically obsolete for application to modern systems.

Testing should involve the devices themselves and their effects. Stiffness, damping, and inertia properties of the devices under operational dynamic conditions should be determined. The effects of devices on system response should also be studied. Such data could allow the engineer to better apply these devices where they are needed.

#### REFERENCES

1. Den Hartog, J.P., Mechanical Vibrations, McGraw-Hill, pp 87-93 (1956).
2. Ibid., pp 219-222.
3. Paul, W.F., "Main Rotor Absorber Reduces Helicopter Vibration to 0.1 G," Space/Aeronautics, 53, pp 46-48 (Mar 1970).
4. Ching-U, I. and Morse, I.E., "Variable Speed Torsional Vibration Absorber," Machine Design, pp 93-95 (Aug 7, 1958).
5. Pringle, O.A., "Use of the Centrifugal Governor Mechanism as a Torsional Vibration Absorber," ASME Trans., 75 (1), pp 59-62 (Jan 1953).
6. Chapman, C.W., "Zero (or Low) Torsional Stiffness Couplings," J Mech. Engr. Sci., 11 (1) (1969)
7. Balzhi, M.F. and Esin, G.D., "Flexible Metallic Couplings with Dynamic Links," Eng. Digest (Apr 1960).
8. Vance, J.M., Brown, R.A., and Darlow, M.S., "Feasibility Investigation of Zero Torsional Stiffness Couplings for Suppression of Resonance and Instability in Helicopter Drive Trains," USAAMRDL-TR-73-103, U.S. Army Air Mobility Res. Developm. Lab., Fort Eustis, VA.

9. Vance, J.M. and Brown, R.A., "Suppression of Torsional Vibration with Zero Torsional Stiffness Couplings," U.S. Naval Res. Lab., Shock Vib. Bull., 44, pp 43-54 (Aug 1974).
10. Darlow, M.S. and Vance, J.M., "Torsional Stability Analysis of a Gas Turbine Powered Helicopter Drive System," J. Engr. Power, Trans. ASME, pp 335-341 (Oct 1974).
11. Den Hartog, J.P. and Ormondroyd, P., "Torsional Vibration Dampers," ASME Trans., 52, APM-52-13-133 (1930).
12. Chichinadze, A.V. and Ternish, O.S., "Designing a Torsion-Vibration Damper," Russian Engr. J., (1), pp 13-16 (Jan 1971).
13. Geschelin, J., "Welding Facilitates Assembly of Silicone Engine Dampers," Automot. Indust., 97 (11), pp 24-25 (Dec 1, 1947).
14. Den Hartog, J.P., Mechanical Vibrations, McGraw-Hill, pp 210-216 (1956).
15. Ibid., pp 187-194.
16. Arnold, R.N., "Tuned and Damped Gyrostatic Vibration Absorber," Instn Mech. Engr. Proc., 157 (25), pp 1-15 (1947).

# LITERATURE REVIEW

survey and analysis  
of the Shock and  
Vibration literature

The monthly Literature Review, a subjective critique and summary of the literature, consists of two to four review articles each month, 3,000 to 4,000 words in length. The purpose of this section is to present a "digest" of literature over a period of three years. Planned by the Technical Editor, this section provides the DIGEST reader with up-to-date insights into current technology in more than 150 topic areas. Review articles include technical information from articles, reports, and unpublished proceedings. Each article also contains a minor tutorial of the technical area under discussion, a survey and evaluation of the new literature, and recommendations. Review articles are written by experts in the shock and vibration field.

This issue of the DIGEST contains review articles on Dynamic Stiffness and Damping of Fiber-Reinforced Composite Materials and Finite Element Modeling.

Professors Gibson and Plunkett of Iowa State University and University of Minnesota respectively, review the recent experimental and analytical efforts to characterize the dynamic mechanical properties of fiber-reinforced composite materials.

Professor Krishna Murty of the Indian Institute of Science, Bangalore, reviews finite element modeling techniques including lumped parameter, transfer matrix, finite element displacement and finite element force. Hybrid and quadratic eigenvalue methods are reviewed.

# DYNAMIC STIFFNESS AND DAMPING OF FIBER-REINFORCED COMPOSITE MATERIALS

R.F. Gibson\* and R. Plunkett\*\*

*Abstract - This paper reviews recent experimental and analytical efforts to characterize the dynamic mechanical properties of fiber-reinforced composite materials.*

Methods that utilize microscopic mechanics have been reported for finding the effective macroscopic moduli of fiber-reinforced composites, or orthotropic materials [1 - 6]. Determination of macroscopic dynamic orthotropic moduli using methods that combine microscopic analysis and geometric information with the isotropic moduli of the constituent materials are not as well verified. The purpose of this paper is to provide a critical review of the literature on the rapidly expanding subject of composite materials. The emphasis is on measurement; representative analytical approaches are mentioned, however, because they are referred to in discussions of experimental results. Most of the references are concerned with dynamic properties of materials during sinusoidal vibration. A few publications dealing with the propagation and attenuation of elastic waves are included because the authors' research interests have to do primarily with vibrations. With the exception of some analytical work on nonlinear effects and some experimental work on large amplitude vibration and the material damage it causes, the results described are for strains below the elastic limits of the constituent materials.

The material configurations of interest in this paper are based on the unidirectional ply. Such a ply is composed of a number of parallel, continuous, reinforcing fibers embedded in a thin layer of binder (matrix) material. Reinforcing fibers are generally composed of glass, boron, or carbon; the most commonly used matrix materials are cured epoxy resins except for high temperature applications. Unless otherwise stated, it is assumed that the unidirectional ply is orthotropic and statistically transversely isotropic, that is, the ply has two orthogonal planes of material property symmetry, and the material properties are statistically invariant to rotations

about the fiber axes. Cross-ply laminates of unidirectional plies are often used when strength and stiffness are needed in more than one direction.

## THEORETICAL CONSIDERATIONS

### Elastic Analysis

Much of the work in micromechanics has been based on static loading and deformation fields having characteristic lengths much greater than those of the material. The so-called effective modulus theories [13] used in this work are also applicable to dynamically loaded composites when the vibrational wavelength is much greater than the largest characteristic dimension of the representative volume element. That is, the averaging on which the effective modulus theories are based is valid so long as the deformation gradients are sufficiently small. Continuum theories have been developed in order to study the propagation and dispersion of high-frequency harmonic waves in composites [7 - 10]. References on wave propagation are also given in the articles by Ross and Sierakowski [11], Moon [12], and Achenbach [13].

The discussion that follows is primarily concerned with vibrations of sufficiently long wavelength that effective modulus theories are applicable. With these theories, dynamic properties differ from static properties only when the anelastic nature of the matrix (and to a much lesser extent, of the fibers) is taken into account.

Various approaches to elastic micromechanical analysis have been reported in the literature [14]. Heterogeneous materials in general have been reviewed [15]. The mechanics-of-materials approach [3, 4] involves certain simplifying assumptions about the stress or strain distribution in a composite material; for example, that plane sections of a composite beam remain plane in pure bending. The longitudinal strain is therefore linearly proportional to the distance from the centroidal axis. The exact stress or strain distribution in a composite material must be

\* Assistant Professor, Dept. of Engrg. Science and Mech., and Engrg. Research Institute, Iowa State University, Ames, Iowa 50011

\*\* Professor, Dept. of Aerospace Engrg. and Mech., University of Minnesota, Minneapolis, Minnesota 55455

known before its properties can be determined by the theory of elasticity. Alternatives to the solution of this formidable problem involve semi-inverse techniques: part of the solution is assumed and then checked to determine whether or not equilibrium and boundary conditions are satisfied. Variational principles such as minimum potential energy, complementary energy, or a mixed principle, can also be used to obtain alternatives to the exact solution. Methods based on variational principles provide only bounds on composite properties, but, in some cases, these bounds are very close to each other, and are sometimes coincidental.

The variational principles of minimum potential energy and complementary energy have been used to obtain coarse bounds on the elastic moduli of an isotropic, two-phase material of arbitrary phase geometry [16]. A unique mixed variational principle that reduces to the principles of minimum potential energy and complementary energy for two limiting cases has been developed [17]. The mixed principle involves comparison of two hypothetical materials and their respective states of stress and strain. One material is homogeneous, elastic, and isotropic, the other is nonhomogeneous, elastic, and anisotropic. The extremum condition for the variational functional is equivalent to the condition that the stresses and strains in the anisotropic material satisfy the proper constitutive equations.

This variational principle has been applied to orthotropic, transversely isotropic fiber-reinforced materials [18]. Bounds for three of the five independent moduli were obtained in terms of phase moduli and volume fractions. Hill [19] used a different method to derive bounds on one of these moduli and on the two moduli for which bounds had not been obtained. The combined work thus provides bounds on all five moduli. The results were identical for the one modulus bounded by both techniques. It was claimed that these are the best possible bounds obtainable without taking phase geometry into account. Closer bounds require additional constraints in the form of specified fiber arrays. Such bounds have been obtained for fiber-reinforced materials with hexagonal and random fiber arrays [20]. Both arrays are transversely isotropic.

It was concluded that the random array results are preferred because the random array bounds coincide, thus providing "exact" expressions for the moduli. Randomness was introduced in the fiber array with composite cylinders of various sizes containing fibers surrounded by outer cylinders of matrix material. The remaining volume within the composite cylinder assemblage was filled by composite cylinders of smaller and smaller size. The same fraction of the total volume was maintained for each cylinder. As a result of this randomness and filling, the bounds converged in the limit. From a practical standpoint, the introduction of randomness in the fiber array model is more appealing than the imposition of a regular pattern of fiber packing that never occurs in practice. Support for the random array hypothesis has been provided by Adams and Tsai [21], who developed finite element models representing random arrays. They found that the agreement between measured and predicted moduli improves as the degree of randomness in the fiber array increased.

The difficulty of finding exact solutions using the theory of elasticity has stimulated the development of various numerical techniques [22]. An interesting method is the interpolation technique [23]; it involves interpolation between previously derived bounds on elastic moduli. The interpolation factors are measures of the degree of reinforcement by the fibers and are chosen by comparison to other 'exact' numerical solutions.

The theories described thus far are based on linear elastic behavior of the composite material and its components. Nonlinear elastic behavior may be caused by large displacements (geometric nonlinearity) or by matrix-governed nonlinearity. Nonlinearity of fiber-reinforced composites is more apparent in their shear stress-shear strain response than in their corresponding uniaxial stress-strain response [3]. Constitutive equations that can describe this nonlinearity in shear have been formulated [24, 25]. According to Timoshenko's theory for vibrating beams, shear deformation becomes more important as the slenderness ratio decreases, as the mode number increases, and as the ratio of the extensional modulus to the shear modulus increases. It has been shown analytically [26] that the effects of this nonlinearity on free vibration frequencies of undamped, laminated beams become more important as the beam slenderness ratio decreases and as the mode number increases.

### Anelastic Analysis

Under certain conditions the mathematical models for the linear micromechanical theories discussed above can be modified to include the dynamic behavior of anelastic materials. Anelasticity is the term generally used to describe the stress and time dependence of elastic strain. That is, the strain lags the stress and the resulting hysteresis causes damping, or anelastic energy loss during vibration. A number of mathematical models for material damping have been discussed [27, 28, 29]. The complex modulus notation, which consists of a real part representing elastic stiffness and an imaginary part representing dissipation, is widely used to model the behavior of linear anelastic materials under sinusoidal vibration [27, 28, 30]. It has been shown that the complex modulus of a linear anelastic material may depend on vibration frequency, but not on amplitude [27]. With the elastic-viscoelastic correspondence principle [31], integral transforms in the time domain are used to obtain viscoelastic solutions from elastic ones.

Although complex modulus notation was developed for application to homogeneous, isotropic materials, it may also be applied to composite materials having one or more anelastic phases. A correspondence principle has been developed for isotropic viscoelastic composites and applied to a particulate composite [32]. The elastic moduli in an earlier elastic solution [33] were replaced with complex moduli; experimental and theoretical results were in agreement. The correspondence principle has been extended to anisotropic fiber-reinforced composites [34]; the elastic moduli from previous work [20] were replaced by complex moduli. The key simplifying assumptions were that the fibers are elastic and that the matrix is elastic in dilatation but viscoelastic in shear. An expression for the complex flexural modulus of a cross-ply laminated beam has been obtained [35]. The Hashin-Rosen random array model was used for the plies, and it was assumed that plane sections remain plane during flexure. It has been shown [36] that the effective moduli for viscoelastic composites can be related to phase properties and that all results from elasticity theory, including bounds, can be applied to viscoelastic materials with a correspondence principle. (It is appropriate to note again that the elastostatic theories of micromechanics and their viscoelastic counterparts may be used to determine dynamic properties only if the vibrational

wavelength is long in comparison with the characteristic dimension of the composite microstructure.)

The loss factor, or loss tangent, is the ratio of the imaginary part of the complex modulus to the real part [27]. The loss factor may also be defined as the ratio of the energy dissipated per cycle to the strain energy stored in the material at peak displacement. The effective loss factor for the composite may be found from the elastic stiffnesses and loss factors of the constituent materials and the corresponding volume fractions [35, 37].

## MEASUREMENT OF DYNAMIC PROPERTIES

### Experimental Techniques

Attempts to compare predicted results and actual measurements of composite materials invariably require the use of such mechanical properties of simple beams, rods, and plates as stiffness, frequency, dispersion, and damping ratio. Mechanical properties are used because it is difficult to determine the behavior of the individual components except by photoelastic methods and because many of the analytical methods neither use nor predict the details of the stress and strain fields.

Some of the most popular methods for measuring material damping have been reviewed and their limitations described [38]. Some currently used damping test techniques have also been discussed. Dynamic stiffness of a material is generally found by substituting measured data on specimen geometry, mass density, resonant frequency, and mode number in the eigenvalue equation for the specimen and solving for the effective elastic modulus. This straightforward procedure presents few difficulties. Measurement of damping of the material, however, is more difficult. The principal problem involves reducing the extraneous energy losses in the apparatus to an acceptable level. The experimentalist must consider support friction, air drag, acoustic radiation, and transducer mounting if the damping to be measured in the system is primarily material damping [35]. It is useful to calibrate the system with a material having known damping [35, 39, 40]. Aluminum is an excellent calibration material because its damping at low stress levels can be accurately predicted with the Zener thermoelastic theory [39].

Most damping measurements rely either on decay, resonant dwell, or band width. The decay test measures free vibration decay of a specimen after its release from an initial displacement [41, 42]. The resonant dwell test and the band width test are forced vibration techniques. The resonant dwell test involves measurements of the resonant mode shape of the specimen [35, 39, 40, 43]. In the band width test the predicted shape of the amplitude frequency function is determined; the resonant peak and the amplitudes at frequencies below and above resonance, usually at the two half-power points [43, 44], are measured. A fourth test that has been used successfully on viscoelastic materials is the impedance technique [53, 54]. Techniques in which the specimen is excited are generally applicable to a variety of configurations. Beam specimens excited in flexure are most commonly used, however, because flexure is probably the most important mode of structural vibration. Most structures have flexural members whose resonant frequencies generally lie within the frequency range of common sources of excitation. Bar specimens excited in torsional or extensional modes have also been used.

#### Experimental Results

Much of the experimental work on the dynamic properties of composite materials has been concerned with measuring frequency and temperature dependence. Typically, a long, slender beam specimen is tested at successively higher frequencies either by exciting higher modes or by reducing the length of the beam. Both aluminum and fiber-reinforced plastic beams have been tested by decay and resonant dwell techniques [40]. Tests were conducted in vacuo to minimize air damping; the results with aluminum were in good agreement with the Zener theory. The damping measured in fiberglass and boron-reinforced plastics was greater in the resonant dwell tests than in the decay tests; results with elastic moduli showed the reverse trend. This led to some doubts about the technique by which the resonant dwell specimens were mounted. Air damping was shown to be significant at low frequencies and corresponding large amplitudes. Paxson [43], in a continuation of earlier work [40], tested unidirectional and cross-ply laminated boron-reinforced plastics with resonant dwell, band width, and decay techniques in vacuo. Agreement among the three methods was poor.

Schultz and Tsai [44] used decay and band width techniques to test aluminum and glass fiber-reinforced plastics in flexural vibration. The damping in unidirectional beams increased with increasing frequency; dynamic elastic moduli did not correlate well with static elastic moduli. When the technique was extended to cross-ply laminates, however, damping decreased, leveled off, and increased with increasing frequency [45]. The tests were conducted in air, so that some of the inconsistency could have resulted from air damping. The elastic moduli of laminates predicted from measured ply properties agreed fairly well with measured values of the composite; predicted laminate damping values were significantly less than measured values.

An apparatus for measuring the complex flexural modulus of aluminum and carbon fiber composites at different temperatures and amplitudes has been developed [46, 47]. A resonant dwell technique was used to test a free-free beam in vacuo. Considerable effort was devoted to the reduction of extraneous energy losses in the apparatus. Data on damping at various stress levels were presented, but no conclusions were reported. This apparatus was also used to study the effects of fiber orientation and laminate geometry [48].

In tests of graphite reinforced epoxy beams through the eighth mode, it was found that resonant frequencies of the higher modes were markedly lower than those predicted if shear deformation is neglected [49]. Timoshenko's beam theory accounted reasonably well for the dependence of experimental resonant frequencies on mode number, slenderness ratio, and modulus ratio. The effects of resin type, degree of cure, and fiber type on damping in glass fiber-reinforced plastic beams at different temperatures and frequencies have been investigated [50]. Wright [51] conducted decay tests on epoxy beams reinforced with one type of glass and four types of carbon fibers. Work has been done on dynamic properties of fiber-reinforced metals [52]; decay tests were conducted on tungsten fiber-reinforced copper specimens in flexure. It was found that damping was independent of strain amplitude but was related to the residual strains induced by rolling.

A flexural resonant dwell technique has been used to test unidirectional and cross-ply fiberglass-reinforced epoxy beams in vacuo [35]. The effects of configuration and vibration frequency were studied both experimentally and analytically for small amplitudes, large amplitude effects were studied experimentally. Experimentally-determined complex moduli at small amplitudes were in good agreement with those predicted by the Hashin-Rosen random array model [20, 34] and were within the bounds predicted by the Hashin variational principle [18].

The complex moduli from beam vibration tests have been used to predict the velocities and attenuation coefficients of elastic waves traveling in fiber composites [55, 56]. Wave propagation was along the fiber axes of unidirectional material. The dynamic response of fiber-reinforced viscoelastic materials subjected to pulse loading along the fiber direction has been measured [57]; the measured response was compared with that predicted by effective modulus theories [20, 34] and by continuum theories [7, 10]. The accuracy of the models depended upon internal geometry and fiber packing geometry.

Modes of vibration other than flexural vibration have been investigated. Carbon and glass fiber-reinforced plastics have been tested in torsion and in flexure [47]. Lifshitz [58] used decay and pulse attenuation techniques to study the dynamic torsional and extensional moduli of pure epoxy and of unidirectional fiber composites. He showed that the properties of the epoxy matrix affect torsional moduli more than extensional moduli and that extensional moduli are essentially independent of frequency and temperature. A torsional pendulum apparatus has been used to study the damping of boron-epoxy composites at elevated temperatures [59]. Damping in glass-epoxy and boron-aluminum composites has been measured by observing the decay of extensional vibrations [60].

If vibration amplitudes in fiber-reinforced composites are sufficiently large, nonlinear inelastic behavior may be caused by microstructural damage [36, 61]. Because this damage is usually permanent, dynamic stiffness and damping properties will exhibit permanent changes. The behavior of cross-ply laminates at large strains is of particular interest. Static tensile

tests of cross-ply laminates showed that, as the ultimate fracture strain of the transverse plies (those plies perpendicular to the loading direction) was exceeded, the slope of the stress-strain curve was significantly reduced [62]. The reduction in stiffness was accompanied by matrix cracking along the direction of the transverse fibers. Gibson [35] has reported that the damping in cross-ply laminates in flexural vibration is independent of amplitude if the maximum strain amplitude does not exceed the fracture strain of the transverse plies. After the fracture strain has been exceeded, however, damping increases and remains near the higher level even after unloading and reloading. Corresponding reductions in stiffness do occur, but damping is far more sensitive to microstructural damage than is stiffness. Cracking of the fiber-matrix bond in transverse plies is apparently controlled by strain concentration in the matrix.

The attenuation of ultrasonic waves in unidirectional and cross-ply specimens under static uniaxial stress has been studied [63]. It was found that attenuation increases rapidly above a certain threshold stress and increases more rapidly for cross-ply laminates than for unidirectional material. It was noted that permanent damage apparently occurs because the attenuation does not return to its initial value after unloading. A sharp increase in acoustic emissions at the knee of the stress-strain curve has been reported for boron-epoxy-reinforced aluminum specimens [64]. Here again, the loading was static, but some failure mechanism was associated with the reduction in stiffness.

The shear modulus and damping of carbon fiber-reinforced epoxy rods in torsional vibration has been measured before and after cracks were induced by static twisting of the rods to shear failure [65]. It was found that damping increased and the shear modulus decreased after cracks were induced. Damping was much more sensitive to crack damage than was the shear modulus. Schultz and Warwick [66] used equipment similar to that used by Schultz and Tsai [44, 45] to measure the complex moduli of glass fiber-reinforced epoxy beams as a function of fatiguing time. They found that the amount of fatigue cracking correlated reasonably well with the amount of reduction in stiffness and the amount of increase in damping. Amplitude effects were not studied.

According to Broutman and Krock [1] transverse plies crack as a result of strain concentration in the matrix due to the fiber packing arrangement. A simple analysis for estimating the strain concentration for a regular fiber array has been developed [67]. Matrix cracking is only one of several mechanisms by which composite materials can fail. According to Salkind [68], composites may fail by matrix cracking, delamination, fiber failure, interface debonding, and other mechanisms, either separately or in combination. Salkind has proposed that, because the loss of stiffness in composite materials can result in structural failure long before complete fracture occurs, the fatigue failure criterion should be the number of cycles to a given change in stiffness, rather than the number of cycles to fractures. As has been shown by several of the references, the internal damping in fiber-reinforced composite materials is far more sensitive to these failure mechanisms than is stiffness so that damping measurements could be valuable tools for the early detection of structural damage.

### CONCLUSION

We have attempted to present a balanced picture of recent experimental and analytical efforts to characterize the dynamic mechanical properties of fiber-reinforced composite materials. This is a rapidly growing field. Most of the work cited has been done within the past decade. Although much experimental and analytical work has been done, more comparisons of measured and predicted properties are needed to evaluate the various micromechanical theories. For example, more definitive criteria could be established on the limits of validity of effective modulus theories for analyzing the dynamic behavior of composite structures. The actual mechanisms of damping in composite structures should be studied more extensively, both experimentally and analytically. Because damping is so sensitive to microstructural damage, damping measurements could provide valuable quality control and reliability information.

### ACKNOWLEDGMENT

Financial assistance for this work has been provided at different times by the U.S. Air Force Materials Laboratory, the University of Minnesota Graduate School, and the Engineering Research Institute at Iowa State University.

### REFERENCES

1. Broutman, L.J. and Krock, R.H., Modern Composite Materials, Addison-Wesley (1967).
2. Broutman, L.J. and Krock, R.H., Composite Materials, 1 - 8, Academic Press (1975).
3. Jones, R.M., Mechanics of Composite Materials, McGraw-Hill (1975).
4. Garg, S.K., Svalbonas, V., and Gurtman, G.A., Analysis of Structural Composite Materials, Marcel Dekker Co. (1973).
5. Bert, C.W. and Francis, P.H., "Composite Material Mechanics: Structural Mechanics," *AIAA J.*, 12 (9), pp 1173-1186 (Sept 1974).
6. Tsai, S.W., Mechanics of Composite Materials, AFML-TR-66-149, Parts I and II (1966).
7. Herrmann, G. and Achenbach, J.D., "Wave Propagation in Laminated and Fiber-Reinforced Composites," in *Mechanics of Composite Materials*, Proc. 5th Symp. Naval Struct. Mech., Pergamon Press (1970).
8. Achenbach, J.D. and Herrmann, G., "Dispersion of Free Harmonic Waves in Fiber-Reinforced Composites," *AIAA J.*, 6 (10), pp 1832-1836 (Oct 1968).
9. Sun, C.T., Achenbach, J.D., and Herrmann, G., "Continuum Theory for a Laminated Medium," *J. Appl. Mech.*, Trans. ASME, 35 (3), Ser. E, pp 467-475 (Sept 1968).
10. Bedford, A. and Stern, M., "On Wave Propagation in Fiber-Reinforced Viscoelastic Materials," *J. Appl. Mech.*, Trans. ASME, 37, pp 1190-1192 (1970).
11. Ross, C.A. and Sierakowski, R.L., "Elastic Waves in Fiber-Reinforced Composites," *Shock Vib Dig.*, 7 (1), pp 96-107 (Jan 1975).
12. Moon, F.C., "A Critical Survey of Wave Propagation and Impact in Composite Materials," NASA CR-121226 (1973).

13. Achenbach, J.D., "Waves and Vibrations in Directionally Reinforced Composites," in Composite Materials, (Broutman and Krock, Eds.) 2, Academic Press (1975).
14. Chamis, C C and Sendeckyj, G.P., "Critique on Theories Predicting Thermoelastic Properties of Fibrous Composites," J. Composite Matl., 2, pp 332-358 (July 1968).
15. Hashin, Z., "Theory of Mechanical Behavior of Heterogeneous Media," Applied Mechanics Surveys, pp 263-275, Spartan Books (1966).
16. Paul, B., "Prediction of Elastic Constants of Multiphase Materials," Trans. Metallurgical Soc. AIME, 218, pp 36-41 (Feb 1960).
17. Hashin, Z. and Shtrikman, S., "On Some Variational Principles in Anisotropic and Non-homogeneous Elasticity," J Mech. Phys. Solids, 10, pp 335-342 (1962).
18. Hashin, Z., "On Elastic Behavior of Fibre Reinforced Materials of Arbitrary Transverse Phase Geometry," J. Mech. Phys Solids, 13, pp 119-134 (1965).
19. Hill, R., "Theory of Mechanical Properties of Fibre-Strengthened Materials. I. Elastic Behavior," J Mech Phys. Solids, 12, pp 199-212 (1964).
20. Hashin, Z. and Rosen, B.W., "The Elastic Moduli of Fiber-Reinforced Materials," J Appl Mech., Trans ASME, pp 223-232 (June 1964) Errata, p 219 (March 1965).
21. Adams, D.F. and Tsai, S.W., "The Influence of Random Filament Packing on the Transverse Stiffness of Unidirectional Composites," J. Composite Matl , 3, p 368 (July 1969).
22. Adams, D.F. and Doner, D.R., "Transverse Normal Loading of a Unidirectional Composite," J Composite Matl., 1, pp 152-164 (Apr 1967).
23. Halpin, J C and Tsai, S.W., "Effects of Environmental Factors on Composite Materials," AFML-TR67-423 (June 1969)
24. Hahn, H.T. and Tsai, S.W., "Nonlinear Elastic Behavior of Unidirectional Composite Laminates," J. Composite Matl., 6, pp 102-118 (Jan 1973).
25. Hahn, H.T., "Nonlinear Behavior of Laminated Composites," J. Composite Matl., 6, pp 257-271 (Apr 1973).
26. Rehfield, L.W., "Nonlinear Considerations in the Vibration of Resin Matrix Composite Structures," Proc. 17th AIAA/ASME/SAE Struc., Struc. Dyn., Matl. Conf., King of Prussia, PA, pp 168-173 (May 1976).
27. Lazan, B.J., Damping of Materials and Members in Structural Mechanics, Pergamon Press (1968).
28. Lazan, D.J., "Damping Properties of Materials, Members, and Composites," Applied Mechanics Surveys, pp 703-715, Spartan Books (1966).
29. Bert, C.W., "Material Damping: An Introductory Review of Mathematical Models, Measures and Experimental Techniques," J. Sound Vib., 29 (2), pp 129-153 (July 1973).
30. Snowdon, J.C., Vibration and Shock in Damped Mechanical Systems, John Wiley & Sons (1968).
31. Lee, E.H., "Stress Analysis in Viscoelastic Bodies," Quart. Appl. Math., 13 (2), pp 183-190 (July 1955).
32. Hashin, Z., "Complex Moduli of Viscoelastic Composites I. General Theory and Application to Particulate Composites," Intl. J. Solids Struc , 6, pp 539-552 (1970)
33. Hashin, Z., "The Elastic Moduli of Heterogeneous Materials," J. Appl. Mech., Trans. ASME, 29, p 143 (1962).
34. Hashin, Z., "Complex Moduli of Viscoelastic Composites II Fiber Reinforced Materials," Intl J Solids Struc , 6, pp 797-807 (1970).

35. Gibson, R.F., "Elastic and Dissipative Properties of Fiber-Reinforced Composite Materials in Flexural Vibration," Ph.D. Dissertation, Univ. Minn., Minneapolis (Dec 1975).
36. Schapery, R.A., "Viscoelastic Behavior and Analysis of Composite Materials," Texas A&M Univ. Mech. Matl. Res. Ctr. Rep. No. MM 72-3 (Aug 1972).
37. Chang, S. and Bert, C.W., "Analysis of Damping for Filamentary Composite Materials," Composite Matl. Engr. Des., ASM, pp 51-62 (1973).
38. Plunkett, R., "Measurement of Damping," in Structural Damping (Rusicka, ed.), ASME, pp 117-131 (Dec 1959).
39. Granick, N. and Stern, J.E., "Material Damping of Aluminum by a Resonant Dwell Technique," NASA TN D-2893 (1965).
40. Mazza, L.T., Paxson, E.B., and Rodgers, R.L., "Measurement of Damping Coefficients and Dynamic Modulus of Fiber Composites," USAAVLABS Technical Note 2, U.S. Army Aviation Material Laboratories, Fort Eustis, VA (Feb 1970).
41. Baker, W.E., Woolam, W.E., and Young, D., "Air and Internal Damping of Thin Cantilever Beams," Intl J. Mech. Sci., 9, pp 743-766 (1967).
42. Hagel, W.C. and Clark, J.W., "The Specific Damping Energy of Fixed-Fixed Beam Specimens," J Appl. Mech., Trans. ASME, pp 426-430 (Sept 1957).
43. Paxson, E.B., Jr., "Real and Imaginary Parts of the Complex Viscoelastic Modulus for Boron Fiber Reinforced Plastics," J. Acoust. Soc Amer., 57 (4), pp 891-898 (Apr 1975).
44. Schultz, A.G. and Tsai, S.W., "Dynamic Moduli and Damping Ratios in Fiber-Reinforced Composites," J Composite Matl., 2 (3), pp 368-379 (July 1968).
45. Schultz, A.B. and Tsai, S.W., "Measurements of Complex Dynamic Moduli for Laminated Fiber-Reinforced Composites," J. Composite Matl., 3, pp 434-443 (July 1969).
46. Adams, R.D. and Bacon, D.G.C., "Measurement of the Flexural Damping Capacity and Dynamic Young's Modulus of Metals and Reinforced Plastics," J. Phys. D (Appl. Phys.), 6, pp 27-41 (1973).
47. Adams, R.D. and Bacon, D.G.C., "The Dynamic Properties of Unidirectional Fibre Reinforced Composites in Flexure and Torsion," J. Composite Matl., 7, pp 53-66 (Jan 1973).
48. Adams, R.D. and Bacon, D.G.C., "Effect of Fibre Orientation and Laminate Geometry on the Dynamic Properties of CFRP," J. Composite Matl., 7, pp 402-428 (Oct 1973).
49. Dudek, T.J., "Young's and Shear Moduli of Unidirectional Composites by a Resonant Beam Method," J. Composite Matl., 4, pp 232-241 (Apr 1970).
50. Ehrenstein, G.S. and Förstor, R., "Damping Properties of Glass Fiber-Reinforced Plastics," Transl. Proc. 7th Ann. Mtg. Working Group Reinforced Plastics, Freudensstadt, W. Ger., pp 33-1 to 33-7 (Oct 1968).
51. Wright, G.C., "The Dynamic Properties of Glass and Carbon Fibre Reinforced Plastic Beams," J. Sound Vib., 21 (2), p 205 (1972).
52. Yamane, T., Umakoshi, Y., Tomishima, M., and Kazuyoshi, N., "A Study of Internal Friction in Fiber-Reinforced Copper Composites (Tungsten Wire)," in Mech. Behavior Matl., Soc Matl. Sci., V, p 220, Kyoto, Japan (1972).
53. Edwards, J.L. and Hicks, D.R., "A Mechanical Impedance Technique for the Measurement of the Dynamic Properties of Materials," Proc. 1971 AFSC Sci. Engr. Symp., II, Dayton, OH (Oct 1971).

54. Laird, G.W. and Kingsbury, H.B., "A Method of Determining Complex Moduli of Viscoelastic Materials," *Exptl. Mech.*, 13 (3), pp 126-131 (Mar 1973).
55. Tauchert, T.R. and Moon, F.C., "Propagation of Stress Waves in Fiber-Reinforced Composite Rods," *Proc. 11th AIAA/ASME Struc., Struc. Dyn., Matl. Conf., Denver, CO*, p 176 (Apr 1970).
56. Jones, E.R., Sierakowski, R.L., and Ross, C.A., "Damping Measurements of a Controlled Composite Material," *J. Acoust. Soc. Amer.*, 57 (6), Part II, pp 1465-1472 (June 1975).
57. Sutherland, H.J. and Calvit, H.H., "A Dynamic Investigation of Fiber-Reinforced Viscoelastic Materials," *Exptl. Mech.*, 14 (8), pp 304-310 (1974).
58. Lifshitz, J.M., "Specimen Preparation and Preliminary Results in the Study of Mechanical Properties of Fiber Reinforced Material," *MML Rep. No. 15*, Technion, Israel Inst. Tech., Haifa (Jan 1969).
59. Schragar, M. and Carey, J., "Viscoelastic Behavior of Boron Fiber-Epoxy Resin Composites," *Polymer Engr. Sci.*, 10, p 369 (1970).
60. Pottinger, M.G., "Materials Damping of Glass Fiber Epoxy and Boron Fiber Aluminum Composites," *Aerospace Res. Labs, Wright-Patterson AFB, OH, NTIS-CSCL* (1970).
61. Schapery, R.A., "Deformation and Failure Analysis of Viscoelastic Composite Materials," in *Inelastic Behavior of Composite Materials*, ASME-AMD, 13, pp 127-155 (Nov 1975).
62. Lavengood, R.E. and Ishai, O., "The Mechanical Performance of Cross-Plied Composites," *Polymer Engr. Sci.*, 11 (3), pp 226-232 (May 1971).
63. Tauchert, T.R. and Hsu, N.N., "Influence of Stress upon Internal Damping in a Fiber Reinforced Composite Material," *J. Composite Matl.*, 7, pp 516-520 (Oct 1973).
64. Henneke, E.G., Herakovich, C.T., Jones, G.L., and Renieri, M.P., "Acoustic Emission from Composite Reinforced Metals," *Exptl. Mech.*, 15, p 10 (Jan 1975).
65. Adams, R.D., Flitcroft, J.E., Hancox, N.L., and Reynolds, W.N., "Effects of Shear Damage on the Torsional Behavior of Carbon Fibre Reinforced Plastics," *J. Composite Matl.*, 7, pp 68-75 (Jan 1973).
66. Schultz, A.B. and Warwick, D.N., "Vibration Response: A Nondestructive Test for Fatigue Crack Damage in Filament-Reinforced Composites," *J. Composite Matl.*, 5, pp 394-404 (July 1971).
67. Kies, J.A., "Maximum Strains in the Resin of Fiberglass Composites," *Naval Res. Lab. Rep. No. 5752* (1962).
68. Salkind, M.J., "Fatigue of Composites," in *Composite Materials: Testing and Design*, ASTM-STP-497, pp 143-169 (1972).

# FINITE ELEMENT MODELING OF NATURAL VIBRATION PROBLEMS

A. V. Krishna Murty\*

**Abstract - This is a review of finite element modeling techniques including lumped parameter, transfer matrix, finite element displacement, finite element force, hybrid and quadratic eigenvalue methods. Applications of the methods to natural vibration problems are given.**

Any structure will undergo distortion and oscillations if the proper force is applied. If the amplitude is sufficient, high stresses and damage to the structure can occur. The need to avoid structural damage resulting from vibration was the motivation for the study of structural vibrations. Natural frequencies and mode shapes have been used in the determination of the dynamic response of structures. The desire to achieve as simple an analytic procedure as possible and accurate results has motivated the development of many analytic models. The objective is to model the problem as close to reality as possible. In practice, for such structures as plates and shells, continuum treatment has been used. Such structures as frames and trusses can be handled with simpler discrete models. Discrete models often can be used to represent complex structural configurations. Among discrete models reported in the literature are those involving lumped mass, lumped inertia forces, finite differences, transfer matrix, and the finite element. This paper is intended as a review of the literature of modeling with finite elements.

In the formulation of a natural vibration problem using finite elements the structure is treated as an assembly of subdomains called elements. A typical finite element idealization is shown in the Figure. Each element is based on a physical or mathematical principle that allows calculation of quantities representing various properties of the structure: force displacement relationships, elastic and inertia forces, and strain and kinetic energies. The governing equations for the complete structure are derived using an appropriate assembly procedure. Boundary conditions are defined, and natural frequencies and mode shapes are obtained. Matrices are used in some methods, including the lumped mass method, transfer

matrix methods, and the matrix displacement method. There are variations of this concept which do not fit into this philosophy. These are not included in this review. For instance the finite difference method may be considered an exception to the finite element method. In the finite difference method, the equilibrium equation at a grid point involves a set of values at a specified set of neighboring grid points. Overlapping subdomains are used to form the governing equation at a point. The emphasis in this method is on forming equations of equilibrium directly at a point; thus, the finite difference method does not involve an assembly of subdomains. Instead, equilibrium and/or compatibility is satisfied in an average sense at a selected set of points.

Most finite element methods currently available can be placed under one of the following: lumped parameter methods, transfer matrix methods, finite element displacement methods, finite element force methods, hybrid methods, and quadratic eigenvalue methods.

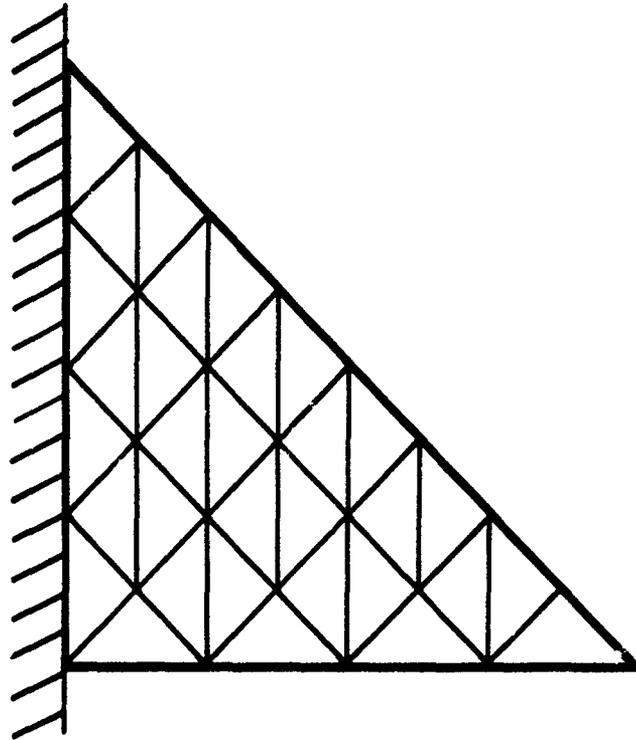
## Lumped Parameter Methods

The lumped mass method [121 - 123, 132] is perhaps the simplest approach to the analysis of natural vibrations. The mass of the structure is lumped at a set of selected grid points. Half the mass of the elements that meet at a grid point may be used as the criterion for lumping the masses. The stiffness influence coefficients with respect to the same grid points are obtained through either a separate analysis or an experiment. The governing equation is of the type shown in Equation (1).

$$[K] \{ \xi \} - \omega^2 [M] \{ \xi \} = 0 \quad (1)$$

The accuracy of the results depends upon the method of lumping the masses used. For example, the inertia force in each element may be lumped at the grid points. However, the representation of strain energy and kinetic energy is somewhat inconsistent. The representation is akin to using one displacement

\*Associate Professor, Dept. of Aeronautical Engineering, Indian Institute of Science, Bangalore, India



**FIG. 1. FINITE ELEMENT IDEALIZATION OF A TRIANGULAR PLATE .**

field in the element for the derivation of the stiffness matrix (strain energy) and a different displacement field in the same element for the derivation of the mass matrix (kinetic energy). It is because of this inconsistency that the nature of the bound of the eigenvalues cannot be predicted. The rate of convergence and the nature of the bound depend upon the method of lumping. It has been found [58] that, in the case of torsional oscillations of a shaft, the eigenvalue is estimated as a lower bound with the rate of convergence as  $1/m^2$ ,  $m$  being the number of elements used. The element mass matrix [121]

$$[m_1] = \frac{1l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (2)$$

gives upper bounds to eigenvalues, but the rate of convergence is the same as  $1/m^2$ . Another element mass matrix

$$[m_2] = \frac{1l}{12} \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix} \quad (3)$$

produces a lower bound to the eigenvalue with the rate of convergence being  $1/m^4$ . Similar trends have also been noted in the finite element analysis of plates [135]. The inconsistencies in the lumped parameter methods can thus become an advantage by the appropriate choice of mass matrices. In the literature the fact that the mass matrix governs the rate of convergence and the nature of bounds has been brought out by numerical correlative studies and by the use of simple classical examples for which error estimates are possible.

#### Transfer Matrix Methods

Transfer matrix methods [37, 50, 51, 93, 144, 150] have been used in vibration analysis since 1921 when Holzer developed his tabular method for torsional vibration problems. Later Mykelstad [247], and Prohl [134], adapted the Holzer Method to beam problems. This method will be described for a beam problem. The beam is divided into several elements. At each nodal point a state vector  $\{s\}$  is defined; it consists of the displacements and forces at that point. A transfer matrix  $[T]$  relating the state vector at two neighboring points is developed using the governing differential equation. Thus

$$\{s\}_{i+1} = [T_i] \{s\}_i \quad (4)$$

\*Most of the bibliography deals with this method

and

$$\{s\}_N = \prod_{i=1}^N [T_i] \{s\}_1 = [T] \{s\}_1 \quad (5)$$

$[T]$  is a transcendental matrix containing the eigenvalue  $\omega$ . A value of  $\omega$  that satisfies prescribed values in  $\{s\}_1$  and  $\{s\}_N$  is chosen by trial and error or a graphical procedure. The success of this method rests largely on the development of the transfer matrix  $[T]$ . This method is particularly suited to dimensional problems. The matrices are relatively small in comparison with those of the finite element displacement method. The finite element displacement method has replaced transfer matrix methods in many applications -- especially in natural vibration problems.

#### Finite Element Displacement Methods

A finite element displacement model\* was introduced into vibration analysis in the 1960s by Archer [15] and Argyris [17]. Because this model has become the most versatile tool for analyzing natural vibration problems, it is described in some detail. The method is derived from the Rayleigh-Ritz method, which may be stated as

$$\delta(U-T) = 0 \quad (6)$$

$U$  is the strain energy, and  $T$  is the kinetic energy. The structure is divided into a network of connected domains, or elements. In each element, the displacement field  $\{\bar{u}\}$  is chosen in terms of the element nodal displacement vector  $\{\delta\}$  as

$$\{\bar{u}\} = [N] \{\delta\} \quad (7)$$

$[N]$  is the matrix of shape functions. The vector of strains  $\{\epsilon\}$  in the element may be written as

$$\{\epsilon\} = [L] \{\bar{u}\} = [B] \{\delta\} \quad (8)$$

$[L]$  is an appropriate differential operation matrix. Equations (7) and (8) are used to determine the strain energy  $U^{(e)}$  and kinetic energy  $T^{(e)}$  in the element

$$U^{(e)} = \frac{1}{2} \{\delta\}^T [k] \{\delta\} \quad (9)$$

$$T(e) = \frac{1}{2} \{\delta\}^T [m] \{\delta\} \quad (10)$$

[k] and [m] are the element stiffness and mass matrices defined as

$$[k] = \int_V [B]^T [D] [B] dV \quad (11)$$

$$[m] = \int_V [N]^T \mu [N] dV \quad (12)$$

[D] is the elasticity matrix, and  $\mu$  is the mass density of the material. The total strain energy  $U$  and the kinetic energy  $T$  are obtained by adding the contributions of each element

$$U = \frac{1}{2} \{\xi\}^T [K] \{\xi\} \quad (13)$$

$$T = \frac{1}{2} \{\xi\}^T [M] \{\xi\} \quad (14)$$

where

$$[K, M] = [a]^T [k, m] [a] \quad (15)$$

[a] is the displacement transformation matrix and relates the elemental nodal displacements  $\{\delta\}$  and the global or structural displacements  $\{\xi\}$  as

$$\{\delta\} = [a] \{\xi\} \quad (16)$$

Equations (13) and (14) are used in equation (6) to obtain the governing equation

$$[K] \{\xi\} - \omega^2 [M] \{\xi\} = 0 \quad (17)$$

from which eigenvalues and eigenvectors are determined.

With finite element displacement models the displacements are treated as unknowns. Variations of the method described above are possible. Attempts have been made [185] to construct finite element models based on Temple's successive approximation scheme. Complete correspondence between finite element models and a modified Rayleigh-Ritz method has been established [177]. This method has proved useful for one-dimensional problems

### Finite Element Force Methods

Finite element force methods [60, 215 - 219] are based on complementary energy principles. Most of the work in this area has been done by Tabarrok [216 - 220] and his associates. A variational principle that is complementary to Hamilton's principle has been used to calculate plate frequencies. The method utilized the variational principle in the form

$$\delta \int_{t_0}^{t_1} (V - T) dt = 0 \quad (18)$$

$T$  is kinetic energy expressed in terms of the d'Alemberts impulses  $e$  as

$$T = \frac{1}{2\mu} \int e^2 dx dy \quad (19)$$

$V$  is the complementary strain energy. This formulation shows that the plate has an infinite number of zero frequency modes. It is essential to eliminate all of these modes in order to predict the nature of bound of the frequency. It has been demonstrated that the complementary energy formulation might result in natural frequencies lower than the true natural frequencies of the plates. In 1973, Tabarrok and Sodhi [220] generalized the stress function procedure and used Toupin's variational principle. Four conforming finite element models have now been developed and applied to linear bending vibration problems of plates. The convergence to the eigenvalues may proceed either from below or from above, depending upon the order of approximations employed for the stress functions. It has been suggested that the procedure could be extended to any problem for which stress functions have already been developed. It has also been noted that the stress function formulation is not always advantageous, especially with regard to suppressing any frequency modes, a concept introduced by Tabarrok and Sedhi

### Hybrid Methods

The combination of two or more methods to minimize disadvantages and maximize advantages is called a mixed, or hybrid, method [36, 59, 96, 97, 126, 149, 175, 206]. Finite element methods that compromise certain basic aspects of the parent continuum method are hybrid methods. Some hybrid methods concentrate on a specific advantage. The possibility that the force method might yield lower bounds to eigenvalues and the basic difficulties in

formulating the vibration problem in terms of forces suggests that hybrid methods that combine energy and complementary energy methods might be useful. Rao et al., [77, 78], attempted to formulate vibration problems using complementary energy. They treated the problem in terms of forces. Shape functions have been used to satisfy the equilibrium equations [36]. The shape functions contain the eigenvalue, an iterative method of solution is thus necessary. With another method [59], the stiffness matrix is derived by assuming suitable forms for stresses and boundary displacements. The mass matrix is based on an independently chosen displacement field in the element. Hybrid elements have been used to study cylindrical shell vibrations [97, 98]. In another case [149], masses have been represented by modified potential energy. The results have been more accurate than those obtained with conventional methods. The feasibility of a Pian-type hybrid model has been outlined [126]. These methods have not yet been fully explored; considerable further study is needed before viable hybrid models will be available for vibration problems.

#### Quadratic Eigenvalue Methods

Two types of formulation leading to a quadratic eigenvalue problem of the type

$$[A] \{q\} + \omega^2 [B] \{q\} + \omega^4 [C] \{q\} = 0 \quad (20)$$

have been reported [126, 179]. Prezemieniecki's work [179] is based on the premise that shape functions must contain the frequency. The problem is of the type

$$\left[ [A_1] - \omega^2 [A_2] - \omega^4 [A_3] \dots \right] \{q\} = 0 \quad (21)$$

Natural frequencies have been worked out retaining terms up to  $[A_3]$ . Results have confirmed that the accuracy of the natural frequencies and mode shapes might be improved by retaining the term with the frequency to the fourth power. This formulation contains negative eigenvalues, the reasons for their occurrence and their influence have not yet been established. Another quadratic eigenvalue problem is based on the least square method [126]. The results of this procedure are either a complex pair of eigenvalues or two real values corresponding to each natural frequency. When complex numbers are obtained, the modulus is an approximation of the frequency, and the imaginary value is an indication of

error. When two real values are obtained, they bound the natural frequency. In either case, therefore, an approximation of the eigenvalue and an indication of error are simultaneously provided. Quadratic eigenvalue methods are probably of greatest value in studying basic problems. Their application to complex problems has not been established. In particular, standard eigenvalue routines cannot be used to obtain numerical solutions; thus, special programming is necessary.

#### Other Methods

Any continuum method can be used to develop a finite element method. Most methods currently in use depend on a minimization principle. When this is not possible or convenient, it is useful to develop models based on methods for the solution of differential equations such as the Galerkin method [126, 175, 176], collocation [175, 176], or an orthogonality method. The differential equation governing the free vibration problem may be written as

$$L_1 (\bar{W}) - \omega^2 L_2 (\bar{W}) = 0 \quad (22)$$

$L_1$  and  $L_2$  are differential operators, and  $\omega$  is the eigenvalue. In a one-dimensional problem, the displacement distribution in the  $i$ -th element may be taken as

$$\bar{W}_i = [N] \{\delta\} \quad (23)$$

$[N]$  is the matrix of shape functions and  $\{\delta\}$  element nodal displacements.  $\bar{W}_i$  is substituted in the governing differential equation to obtain the expression for the error in the differential equation in the  $i$ -th element;  $\epsilon_i$  becomes

$$\epsilon_i = (L_{1i} - \omega^2 L_{2i}) [N] \{\delta\} \quad (24)$$

With the Galerkin procedure the integral of the product error in the differential equation and the components of the displacement functions are set equal to zero.

$$[E_k] \{\zeta\} - \omega^2 [E_m] \{\zeta\} = 0 \quad (25)$$

where

$$[\delta_e] = [a] \{\zeta\} \quad (26)$$

$$[E_k] = [a]^T \Gamma_{e_{k,j}} [a] \quad (27)$$

and

$$[e_m] = [a]^T e_{m_i} [a] \quad (28)$$

with

$$[e_{k_i}] = \int_e [N]^T [L_{1i} N] d\xi \quad (29)$$

and

$$[e_{m_i}] = \int_e [N]^T [L_{2i} N] d\xi \quad (30)$$

The eigenvalues and eigenvectors can be computed from equation (25). Small amplitude vibration problems have been studied using this method [126, 176]. Only linear eigenvalue problems have been studied thus far using these methods, but they are likely to prove useful for studying nonlinear oscillations.

### CONCLUSIONS

Some finite element models have been compared in the Table. The Rayleigh-Ritz finite element method is the most convenient and efficient one, at least, insofar as linear natural vibration problems are concerned. Other methods may be more efficient but have some disadvantages that must be eliminated.

### Acknowledgment

The author takes this opportunity to thank Mr. K. N. Shivakumar, Mr. T. S. R. Reddy, and Mr. S. S. Murthy for their assistance in compiling the references.

### REFERENCES

1. Abel, J.F. and Popov, E.P., "Static and Dynamic Finite Element Analysis of Sandwich Structures," P.S.C.M.M.S.M \*, Wright-Patterson AFB, Dayton, OH, AFFDL-TR-68-150, pp 213-246 (1968).
2. Ahmed, K.M., "Static and Dynamic Analysis of Sandwich Structures by the Method of Finite Elements," J. Sound Vib., 18 (1), pp 75-91 (1971)
3. Ahmed, K.M., "Free Vibration of Curved Sandwich Beams by the Method of Finite Elements," J. Sound Vib., 18 (1), pp 61-74 (1971)
4. Ahmed, K.M., "Dynamic Analysis of Sandwich Beams," J. Sound Vib., 21 (3), pp 263-276 (1972).
5. Ahmad, S., Anderson, R.G., and Zienkiewicz, O.C., "Vibration of Thick Curved Shells with Particular Reference to Turbine Blades," J. Strain Anal., 5, pp 200-206 (1970).
6. Jennings, Alan, "Mass Condensation and Simultaneous Iteration for Vibration Problems," Intl. J. Numer. Methods Engr., 6, pp 543-552 (1973).
7. Jennings, Alan and Orr, D.R.L., "Application of the Simultaneous Iteration Method to Undamped Vibration Problems," Intl. J. Numer. Methods Engr., 3, pp 13-24 (1971).
8. Cella, Aldo, Comments on "Rates of Change of Eigenvalues and Eigenvectors," AIAA J., 7, p 1655 (1969).
9. Berman, Alex and Flannely, William G., "Theory of Incomplete Models of Dynamic Structures," AIAA J., 9, p 1481 (1971).
10. Allik, H. and Hughes, T.J.R., "Finite Element Method for Piezoelectric Vibration," Intl. J. Numer. Methods Engr., 2, pp 151-159 (1970).
11. Allik, H., Webman, K.M., and Hunt, J.T., "Vibrational Response of Sonar Transducer Using Piezoelectric Finite Elements," J. Acoust. Soc. Amer., 56 (6), pp 1782-1791 (1974).
12. Al-Najafi, A.M.J. and Warburton, G B, "Free Vibration of Ring-stiffened Cylindrical Shells," J. Sound Vib., 13 (1), pp 9-25 (1970).
13. Anderson, D.T. and Milis, B., "Dynamic Analysis of Car Chassis by Finite Element Methods," Intl. J. Mech. Sci., 14 (12), pp 799-808 (1972)
14. Anderson, R.G., Irons, B.M., and Zienkiewicz, O.C., "Vibration and Stability of Plates Using Finite Elements," Intl. J. Solids Struct., 4, pp 1031-1051 (1968).

15. Archer, J.S., "Consistent Mass Matrix for Distributed Mass Systems," Proc. ASCE J. Struc. Div., 89, pp 161-178 (1963).
16. Archer, J.S., "Consistent Matrix Formulation for Structural Analysis Using Finite Element Techniques," AIAA J., 3, pp 1910-1918 (1965).
17. Argyris, J.H., "Continua and Discontinua," P.F.C.M.M.S.M.\*\* , Wright-Patterson AFB, Dayton, OH, pp 11-190 (1966)
18. Argyris, J.H., "On the Analysis of Complex Elastic Structures," Appl. Mech. Rev., 11, pp 331-338 (1958).
19. Argyris, J.H. and Patton, P.C., "Computer-oriented Research in a University Milieu," Appl. Mech. Rev., 19 (12), pp 1029-1039 (1966)
20. Barten, H.J., Scheurenbrand, J.A., and Scheer, D.D., "Stress and Vibration Analysis of Turbo-pump Inducer Blades by a Finite Element Method," J. Spacecraft Rockets, 8, pp 657-664 (1971)
21. Benfield, W.A. and Hruda, R.F., "Vibration Analysis of Structures by Component Mode Substitution," AIAA J., 9, pp 1255-1261 (1971).
22. Berman, J.H. and Sklerov, J., "Calculation of Natural Modes of Vibration for Free Structures in a Three-dimensional Space," AIAA J., 3, pp 158-160 (1965)
23. Best, G.C., "Vibration Analysis of a Cantilevered Square Plate by a Stiffness Matrix Method," P.F.C.M.M.S.M.\*\* , Wright-Patterson AFB, Dayton, OH, pp 849-862 (1966).
24. Bishop, R.E.D., Gladwell, G.M.L., and Michaelson, S., The Matrix Analysis of Vibration, Cambridge Univ Press (1965)
25. Bogner, F.K., Fox, F.L., and Schmit, Jr., L.A., "Addendum to the Generation of Inter-element Compatible Stiffness and Mass Matrices by the Use of Interpolation Formula," P.F.C.M.M.S.M.\*\* , Wright-Patterson AFB, Dayton, OH, pp 397-444 (1966).
26. Carnegie, W., Thomas, J., and Dokumaci, E., "An Improved Method of Matrix Displacement Method of Analysis in Vibration Problems," Aeron. Quart., 25, p 321 (1974).
27. Cheung, Y.K. and Chakrabarti, S., "Free Vibration of Thick, Layered Rectangular Plates by a Finite Layer Method," J. Sound Vib., 21 (3), pp 277-284 (1972).
28. Cheung, Y.K. and Cheung, M.S., "Vibration Analysis of Cylindrical Panels," J. Sound Vib., 22 (1), pp 59-73 (1972).
29. Cheung, M.S. and Cheung, Y.K., "Natural Vibrations of Thin, Flat-walled Structures with Different Boundary Conditions," J. Sound Vib., 18 (3), pp 325-337 (1971).
30. Mei, C., "Finite Element Analysis of Non-linear Vibrations of Beam Columns (Technical Note)," AIAA J., 11, p 115 (1973).
31. Mei, C., "Nonlinear Vibrations of Beams by Matrix Displacement Method (Technical Note)," AIAA J., 10, p 355 (1972).
32. Clough, R.W., "Use of Modern Computers in Structural Analysis," Proc. ASCE, 84 (ST3) p 1636 (1958).
33. Clough, R.W., "Analysis of Structural Vibrations and Dynamic Response," Recent Advances in Matrix Methods of Structural Analysis and Design, Univ. Alabama Press (1971).
34. Clough, R.W. and Bathe, K.J., "Finite Element Analysis of Dynamic Response," Second U.S.-Japan Seminar on Matrix Methods of Structural Analysis, pp 153-180 (1972).

35. Clough, R.W. and Penzien, J., Dynamics of Structures, McGraw-Hill (1975).
36. Cohen, E. and McCallion, H., "Improved Deformation Functions for the Finite Element Analysis of Beam Systems," *Intl. J. Numer. Methods Engr.*, 1, pp 163-167 (1969).
37. Cohen, E. and McCallion, H., "Economical Methods for Finding Eigenvalues and Eigenvectors," *J. Sound Vib.*, 5 (3), pp 397-406 (1967).
38. Cowdrey, D.R. and Willis, J.R., "Application of the Finite Element Method to the Vibrations of Quartz Plates," *J. Acoust. Soc. Amer.*, 56 (1), pp 94-98 (1974).
39. Craggs, A., "The Use of Simple 3-D Acoustic Finite Elements for Determining the Natural Modes and Frequencies of Complex Shaped Enclosures," *J. Sound Vib.*, 23 (3), pp 331-339 (1972).
40. Craig, R.R. and Bampton, M.C.C., "On the Iterative Solution of Semi-definite Eigenvalue Problems," *Aeron. J.*, 75, pp 287-290 (1971).
41. Damle, S.K. and Feeser, L.J., "Vibration of Fourpoint-supported Plates by a Finite Element Method," *Aeron. Soc. India J.*, 24 (3), p 375 (1972).
42. Margolias, David S. and Weingarter, Victor I., "Free Vibrations of Pressure-loaded Paraboloidal Shells," *AIAA J.*, 9, pp 2339-2345 (1971).
43. Davies, R., Henshell, R.D., and Warburton, G.B., "Constant Curvature Beam Finite Elements for In-plane Vibration," *J. Sound Vib.*, 25 (4), pp 561-576 (1972).
44. Davies, R., Henshell, R.D. and Warburton, G.B., "A Timoshenko Beam Element," *J. Sound Vib.*, 22 (4), pp 475-487 (1972).
45. Dawe, D.J., "The Application of the Finite Element Method for a Plate Vibration Problem," Ph.D. Thesis, Univ. Wales (Cardiff), (1966)
46. Dawe, D.J., "The Transverse Vibrations of Shallow Arches Using the Displacement Method," *Intl. J. Mech. Sci.*, 13 (8), pp 713-720 (1971).
47. Dawe, D.J., "A Finite Element Approach for Plate Vibration Problems," *J. Mech. Engrg. Sci.*, 7, pp 28-32 (1965).
48. Dawe, D.J., Comments on, "Free Vibrations of Finite Element Plates Subjected to Complex Middle-plane Force Systems," *J. Sound Vib.*, 28 (4), pp 759-763 (1973).
49. Dawe, D.J., "Vibrations of Rectangular Plates of Variable Thickness," *Intl. J. Mech. Engrg. Sci.*, 8, pp 42-51 (1966).
50. Dawson, B. and Davies, M., "An Improved Transfer Matrix Procedure," *Intl. J. Numer. Methods Engr.*, 8, pp 111-117 (1974).
51. Dawson, B. and Davies, M., "An Extended Matrix Transfer Method with an Automatic Root Search Capacity," *Intl. J. Numer. Methods Engr.*, 10, pp 67-76 (1976).
52. Deb Nath, J.M., "Use of Higher Order Displacement Functions in the Free Vibration Analysis of Shells of Revolution Having Meridional Singularities," *J. Sound Vib.*, 36 (2), pp 253-272 (1974).
53. Denke, P.H., Erde, G.R., and Picket, J., "Matrix Difference Equation Analysis of Vibrating Periodic Structures," *AIAA J.*, 13, pp 160-166 (1975).
54. Dickinson, S.M. and Henshell, R.D., "Clough-Tocher Triangular Plate Bending Element in Vibration," *AIAA J.*, 7, pp 560-561 (1969).
55. Dokainish, M.A. and Rawtani, S., "Vibration Analysis of Rotating Cantilever Plates," *Intl. J. Numer. Methods Engr.*, 3, pp 233-248 (1971).
56. Dong, S.B. and Joseph, Jr., A.W., "Effect of Transverse Shear Deformation on Vibrations of Planar Structures Composed of Beam-type Elements," *J. Acoust. Soc. Amer.*, 53, pp 120-127 (1973).

57. Norrie, Douglas H. and Devies, Gerard, The Finite Element Method (Fundamentals and Applications), Academic Press (1973).
58. Duncan, W.J., "A Critical Examination of the Representation of Massive Elastic Bodies by Systems of Rigid Masses Elastically Connected," Quart. J. Mech. Appl. Math., 5, Part 1, p 97 (1952).
59. Dungav, R., Severn, R.T., and Taylor, P.R., "Vibration of Plates and Shells Using Triangular Finite Elements," J. Strain Anal., 2, pp 73-83 (1967).
60. Elias, Z.M., "Dynamic Analysis of Frame Structures by the Force Method," Second U.S.-Japan Seminar on Matrix Methods of Structural Analysis and Design, pp 275-297, Univ. Alabama Press (Aug 1972).
61. Fox, R.L. and Kapoor, M.P., "A Minimization Method for the Solution of the Eigenproblem Arising in Structural Dynamics," P.S.C.M.M. S.M.\*, Wright-Patterson AFB, Dayton, OH, AFFDL-TR-68-150, pp 271-306 (1969).
62. Fox, R.L. and Kapoor, M.P., "Rates of Change of Eigenvalues and Eigenvectors (Technical Note)," AIAA J., 6, pp 2426-2428 (1968).
63. Frazer, R.A., Duncan, W.J., and Collor, A.R., Elementary Matrices, Cambridge Univ. Press (1946).
64. Fried, I., "Accuracy of Finite Element Eigenproblems," J. Sound Vib., 18 (2), pp 289-295 (1971).
65. Fried, I., "Optimal Gradient Minimization Scheme for Finite Element Eigenproblems," J. Sound Vib., 20 (3), pp 333-342 (1972).
66. Fried, I., "Bound on the Extremal Eigenvalues of the Finite Element Stiffness and Mass Matrices and Their Spectral Condition Number," J. Sound Vib., 22 (4), pp 407-418 (1972).
67. Fu, C.C., "Computer Analysis of Rotating Axial-Turbomechanic Blade in Coupled Bending-Bending Torsion Vibrations," Intl. J. Numer. Methods Engr., 8, pp 569-588 (1974).
68. Fujii, H., "Finite Element Schemes: Stability and Convergence," Second U.S.-Japan Seminar on Matrix Methods of Structural Analysis and Design, pp 201-218, Univ. Alabama Press (1972).
69. Gallagher, R.H. and Lee, C.H., "Matrix Dynamic and Instability Analysis with Non-uniform Elements," Intl. J. Numer. Methods Engr., 2, pp 265-275 (1970).
70. Galletly, G.D. and Mistry, J., "The Free Vibrations of Cylindrical Shells with Various End Closures," Nucl. Engr. Des., 30 (2), pp 249-268 (1974).
71. Geradin, M., "Error Bounds for Eigenvalue Analysis by Elimination of Variables," J. Sound Vib., 19 (2), pp 111-132 (1971).
72. Khabbaz, Ghassan R., "Dynamic Behaviour of Liquid in Elastic Tanks," AIAA J., 9, pp 1985-1990 (1971).
73. Gladwell, G.M.L. and Mason, V., "Variational Finite Element Calculation of the Acoustic Response of a Rectangular Panel," J. Sound Vib., 14 (1), pp 115-135 (1971).
74. Gladwell, G.M.L. and Tahbildar, U.C., "Finite Element Analysis of the Axisymmetric Vibrations of Cylinders," J. Sound Vib., 22 (2), pp 143-157 (1972).
75. Gladwell, G.M.L. and Zimmermann, G., "On Energy and Complementary Energy Formulations of Acoustic and Structural Vibrations Problems," J. Sound Vib., 3, pp 233-241 (1966).
76. Gladwell, G.M.L. and Vijay, D.K., "Errors in Shell Finite Element Models for Vibration of Circular Cylinders," J. Sound Vib., 42, pp 387-397 (1975).
77. Govinda Raju, S.P., "Some Applications of Matrix Force Methods," M Sc Thesis, Indian Institute of Science (1966).
78. Govindaraju, S.P. and Rao, A.K., "A Matrix Method for Vibration and Stability Problems," Aeron. Soc India J., 18, pp 90-99 (1966).

79. Greene, B.E., Jones, R.E., McLay, R.W., and Strome, D.R., "Dynamic Analysis of Shells Using Doubly Curved Finite Elements," P.S. C.M.M.S.M.\*, Wright-Patterson AFB, Dayton, OH, AFFDL-TR-68-150, pp 185-212 (1968).
80. Gupta, K.K., 'Eigenproblem Solution of Damped Structural Systems,' Intl. J. Numer. Methods Engr., 8, pp 877-911 (1974).
81. Gupta, K.K., 'Eigenproblem Solution by a Combined Strum Sequence and Inverse Iteration Technique' Intl. J. Numer. Methods Engr., pp 17-42 (1973).
82. Gupta, K.K., "On a Combined Strum Sequence and Inverse Iteration Technique for Eigenproblem Solution of Spinning Structures," Intl. J. Numer. Methods Engr., 7, pp 509-518 (1973).
83. Gupta, K.K., "Free Vibration Analysis of Spinning Structural Systems," Intl. J. Numer. Methods Engr., 5, pp 395-418 (1973).
84. Gupta, K.K., Solution of Eigenvalue Problems by the Strum Sequence Method," Intl. J. Numer. Methods Engr., 4, pp 379-404 (1972).
85. Gupta, K.K., "Vibration of Frames and Other Structures with Banded Stiffness Matrix," Intl. J. Numer. Methods Engr., 2, p 221-228 (1970).
86. Gupta, K.K., "Solution of Quadratic Matrix Equations for Free Vibration Analysis of Structures," Intl. J. Numer. Methods Engr., 6, pp 129-135 (1973)
87. Gupta, K.K., "Recent Advances in Numerical Analysis of Structural Eigenvalue Problems," Theory and Practice in Finite Element Analysis, Univ. Tokyo Press, pp 249-272 (1975).
88. Guyan, R.J., "Distributed Mass Matrix for Plate Elements in Bending," AIAA J., 3, p 567 (1965).
89. Guyan, R.J., "Reduction of Stiffness and Mass Matrices," AIAA J., 3, p 380 (1965).
90. Handa, K.N., "Analysis of In-plane Vibration of Box-type Structures by a Finite Element Method," J. Sound Vib., 21, pp 107-114 (1972).
91. Handa, K.N., "Analysis of In-plane Vibration of Shear Walls by a Finite Element Method," J. Sound Vib., 21, pp 169-180 (1972).
92. Handa, K.N. and Clarkson, B.L., "Application of a Finite Element Method to the Dynamic Analysis of Tall Structures," J. Sound Vib., 18, pp 391-403 (1971).
93. Henderson, J.P. and McDaniel, T.J., "The Analysis of Curved Multi-span Structures," J. Sound Vib., 18, pp 203-219 (1971).
94. Henshell, R.D. and Warburton, G.B., "Transmission of Vibration in Beam Systems," Intl. J. Numer. Methods Engr., 1, pp 47-66 (1969).
95. Henshell, R.D., Walters, D., and Warburton, G.B., "A New Family of Curvilinear Plate Bending Elements for Vibration and Stability," J. Sound Vib., 20 (3), pp 381-397 (1972).
96. Henshell, R.D., Neale, B.K., and Warburton, G.B., "A New Hybrid Cylindrical Shell Finite Element," J. Sound Vib., 16 (4), pp 519-531 (1971).
97. Henshell, R.D., Neale, B.K., and Warburton, G.B., "On Hybrid Cylindrical Shell Elements," J. Sound Vib., 21, pp 373-379 (1972).
98. Koeing, Herbert A., and Davids, Norman, "The Damped Transient Behaviour of Finite Beams and Plates," Intl. J. Numer. Methods Engr., 1, pp 151-162 (1969).
99. Hofmeister, L.D. and Evensen, D.A., "Vibration Problems Using Isoparametric Shell Elements," Intl. J. Numer. Methods Engr., 5, pp 142-145 (1972)
100. Hurty, W.C., "Dynamic Analysis of Structural Systems Using Component Modes," AIAA J., 3 (4), pp 678-685 (1965).

101. Hurty, W.C. and Rubinstein, M.F., Dynamics of Structures, Prentice-Hall (1964).
102. Hurty, W.C., "Truncation Errors in Natural Frequencies as Computed by the Method of Component Mode Synthesis," P.F.C.M.M.S. M.\*\*, Wright-Patterson AFB, Dayton, OH, pp 803-822 (1965).
103. Garnet, Hyman and Levy, Alvin, "Free Vibrations of Reinforced Elastic Shells," J. Appl. Mech., Trans. ASME, 36, pp 835-844 (1969).
104. Irons, B., "Eigenvalue Economisers in Vibration Problems," Aeron. Quart., 67, pp 526-528 (1963).
105. Irons, B., "Structural Eigenvalue Problems: Elimination of Unwanted Variables," AIAA J., 3, pp 961-962 (1965).
106. Irons, B.M. and Treharne, G., "A Bound Theorem in Eigenvalues and Its Practical Applications, P.T.C.M.M.S.M.\*\*\*, Wright-Patterson AFB, Dayton, OH, pp 245-254 (1971).
107. Fried, Issac and Schmitt, Karl-Hienz, "Numerical Results of the Application of Gradient Iterative Techniques to the Finite Element Vibration and Stability Analysis of Skew Plates," Aeron. Quart., 76, p 166 (1972).
108. Fried, Issac, "Basic Computational Problems in the Finite Element Analysis of Shells," Intl. J. Solids Struc., 7 (12), p 1705 (1971).
109. Fried, Issac, "Shape Functions and the Accuracy of Arch Finite Elements," AIAA J., 11, p 287 (1973).
110. Jennings, A., "Natural Vibration of a Free Structure," Aircraft Engr., 34, pp 81-83 (1962).
111. Robinson, John and Petyt, M., "Dynamic Analysis of Structures Using Rank Force Method," Intl. J. Numer. Methods Engr., 3, pp 103-117 (1971).
112. Jones, R.M. and Morgan, H.S., "Buckling and Vibration of Cross-ply Laminated Circular Cylindrical Shells," AIAA J., 13, p 664 (1975).
113. Kapur, K.K., "Prediction of Plate Vibrations Using a Consistent Mass Matrix," AIAA J., 4, p 565 (1966).
114. Kapur, K.K., "Stability of Plate Vibrations Using a Consistent Mass Matrix," AIAA J., 4, pp 565-566 (1966).
115. Kavana, K. and Taketo, K., "A Test on Convergence of Eigensolutions," Theory and Practice of Finite Element Analysis, Univ. Tokyo Press, pp 273-288 (1973).
116. Khatua, T.P. and Cheung, Y.K., "Bending and Vibration of Multilayer Sandwich Beams and Plates," Intl. J. Numer. Methods Engr., 6, pp 11-24 (1973).
117. Kirkhope, J. and Wilson, G.J., "Vibration of Circular and Annular Plates Using Finite Elements," Intl. J. Numer. Methods Engr., 4, pp 181-193 (1972).
118. Bathe, Klaus Jurgen and Wilson, E.L., "Solution Methods for Eigenvalue Problems in Structural Mechanics," Intl. J. Numer. Methods Engr., 6, pp 213-226 (1973).
119. Koopmann, G.H. and Petyt, M., "Building Vibrations and Acoustics," J. Sound Vib., 28, pp 471-485 (1973).
120. Krajcinovic, D., "A Consistent Discrete Elements Technique for Thin-walled Assemblages," Intl. J. Solids Struc., 5, pp 639-662 (1969).
121. Krishna Murthy, A.V., "A Lumped Inertia Force Method in Vibration Problems," Aeron. Quart., 17, pp 126-140 (1966).
122. Krishna Murthy, A.V. and Joga Rao, C.V., "Transverse Vibrations of Trusses," Aeron. Soc. India J., 18, pp 41-46 (1966).

123. Krishna Murty, A.B. and Prabhakaran, K.R., "Vibrations of Tapered Cantilever Beams and Shafts," *Aeron. Quart.*, 20, pp 171-177 (1969).
124. Krishnamurty, A.V. and Vankateswara Rao, G., "A Comparative Study of the Consistent and Simplified Finite Element Analysis of Eigenvalue Problems," *Aeron. Soc. India J.*, 22, p 183 (1970)
125. Krishnamurty, A.V. and Viswamurthy, G., "Design Modifications in Vibration Problems," *J. Spacecraft Rockets*, pp 870-872 (1968).
126. Krishnamurty, A. V., Rao, A. K., Prasad, K.S.R.K., and Rao, G.V., "Finite Element Modeling of Natural Vibration Problems," *Theory and Practice in Finite Element Analysis*, Univ. Tokyo Press, pp 323-332 (1973).
127. Krishnamurty, A.V., "Finite Element Analysis of Rotating Structural Elements," *Proc. First Intl. Conf. Finite Element Methods in Engineering*, Coimbatore, India, pp 159-178 (1973).
128. Lakis, A.A. and Paidoussis, M.P., "Free Vibration of Cylindrical Shells Partially Filled with Liquid," *J. Sound Vib.*, 19 (1), pp 1-15 (1971).
129. Laursen, H.L., Matrix Analysis of Structures, McGraw-Hill (1966).
130. Laursen, H.L., Subinski, R.P., and Clough, R.S., "Dynamic Matrix Analysis of Framed Structures," *Proc. Fourth U.S. Natl. Conf. Mech.*, 1, pp 99-105 (1962).
131. Leckie, F.A., "The Application of Transfer Matrices to Plate Vibrations," *Ing Arch.*, 32, p 100 (1963).
132. Leckie, F.A. and Lindberg, G.M., "The Effect of Lumped Parameters on Beam Frequencies," *Aeron. Quart.*, 14, pp 224-240 (1963).
133. Meirovitch, Leonard, "Comment on Non-zero Free-free Frequencies of Structures Idealized by Matrix Methods," *AIAA J.*, 8, p 1370 (1970).
134. Prohl, M.A., "A General Method for Calculating Critical Speeds of Flexible Rotors," *J. Appl. Mech., Trans. ASME*, 12 (3), pp A142-A148 (Sept 1945).
135. Lindberg, G.M. and Olson, M.D., "Convergence Studies of Eigenvalue Solutions Using Two Finite Plate Bending Elements," *Intl. J. Numer. Methods Engr.*, 2, pp 99-116 (1970).
136. Lindberg, G.M., Olson, M.D., and Tullock, H., "Closed Form Finite Element Solutions for Plate Vibrations," *NRC, Canada, Aeron. Rep.*, LR-518 (1969).
137. Livesley, R.K., Matrix Methods of Structural Analysis, Pergamon Press (1964).
138. Loewy, R.G., "A Matrix Holzer Analysis for Bending Vibrations of Clustered Launch Vehicles," *J. Spacecraft Rockets*, 3, pp 1625-1637 (1966).
139. Newman, Malcolm and Ojalvo, I.U., "Non-zero Free Frequencies of Structures Idealized by Matrix Methods," *AIAA J.*, 7, pp 2343-2441 (1969).
140. Paz, Mario and Dung, Lam, "Power Series Expansion of the General Stiffness Matrix for Beam Elements," *Intl. J. Numer. Methods Engr.*, 9, pp 449-459 (1975)
141. Martin, P.C., Introduction to Matrix Methods of Structural Analysis, McGraw-Hill (1966).
142. Mason, V., "Rectangular Finite Element for Analysis of Plate Vibrations," *J. Sound Vib.*, 7, pp 437-448 (1968).
143. Matsumoto, K., "Vibration of an Elastic Immersed in Fluid," *Second U.S.-Japan Seminar on Matrix Methods of Structural Analysis and Design*, Univ. Alabama Press, pp 259-274 (1972)
144. McDaniel, T.J., "Dynamics of Non-circular Stiffened Cylindrical Shells," *J. Sound Vib.*, 23 (2), pp 217-227 (1972)

145. McMinn, S.J., Matrices for Structural Analysis, John Wiley and Sons (1962).
146. Mei, C., "Coupled Vibrations of Thin-walled Beams of Open Section Using the Finite Element Method," Intl. J. Mech. Sci., 12 (10), pp 883-891 (1970).
147. Mei, C. and Yang, T Y., "Free Vibrations of Finite Element Plates Subjected to Complex Middle Plane Force Systems," J. Sound Vib., 23 (2), pp 145-156 (1972).
148. Mei, C., "Free Vibrations of Circular Membranes under Arbitrary Tension by Finite Element Method," J. Acoust. Soc Amer., 46, p 693 (1969)
149. Melosh, R.J. and Lang, T.E., "Modified Potential Energy Mass Representations of Frequency Prediction," P.F.C.M.M.S.M. \*\*, Wright-Patterson AFB, Dayton, OH, (1965).
150. Mercer, C.A. and Seavey, C., "Prediction of Natural Frequencies and Normal Modes of Skin Stringer Panel Rows," J. Sound Vib., 6, pp 149-162 (1967).
151. Monforton, G.R. and Schmit Jr., L.A., "Finite Element Analysis of Sandwich Plates and Cylindrical Shells with Laminated Faces," P.S.C.M.M.S.M. \*, Wright-Patterson AFB, Dayton, OH, AFFDL-TR-68-150, pp 573-616 (1968)
152. Murthy, V.R. and Nigam, N.C., "Dynamic Characteristics of Stiffened Rings by a Transfer Matrix Method," J. Sound Vib., 39, pp 237-245 (1975).
153. Nagaraja, V T, "Changes in Vibration Characteristics due to Change in Structure," J. Spacecraft Rockets, 7, pp 1499-1501 (1970).
154. Nagaraja, V.T, "The Influence of Concentrated Inertias on the Free Vibrations of Beams and Shafts," Aeron Soc. India J., 23, p 145 (1971).
155. Navaratna, D R, "Natural Vibrations of Deep Spherical Shells," AIAA J, 4, pp 2056-2058 (1966)
156. Nickel, R.E. and Secor, G.A., "Convergence of Consistently Derived Timoshenko Beam Finite Elements," Intl. J. Numer. Methods Engr., 5, pp 243-253 (1972).
157. Nosseir, T.A. and Dickinson, S.M., "The Free Vibration of a Model of a Car Body," J. Sound Vib., 15, pp 257-268 (1971).
158. Olson, M.D. and Lindberg, G.M., "Annular and Circular Finite Elements for Plate Bending," Intl. J. Mech. Sci., 12 (1), pp 17-33 (1970).
159. Olson, M.D. and Lindberg, G.M., "Dynamic Analysis of Shallow Shells with a Doubly Curved Triangular Finite Element," J. Sound Vib., 19, pp 299-318 (1971).
160. Olson, M.D. and Lindberg, G.M., "Vibration Analysis of Cantilever Curved Plates Using a New Cylindrical Shell Finite Element," P.S.C.M.M.S.M. \*, Wright-Patterson AFB, Dayton, Oh, AFFDL-TR-68-150, pp 247-270 (1968).
161. Paidoussis, M.P. and Lakis, A.A., "Vibration Characteristics of Cylindrical Shells with Several Axially Equi-spaced Constraints," J. Sound Vib., 24, pp 51-62 (1972).
162. Paul, P. Lynn and Balaur, S. Dhillon, "Convergence of Eigenvalue Solutions in Conforming Plate Bending Finite Elements," Intl. J. Numer. Methods Engr., 4, pp 217-234 (1972).
163. Pestel, E.C., "Dynamic Stiffness Matrix Formulation by Means of Hermitian Polynomials," P.F.C.M.M.S.M. \*\*, Wright-Patterson AFB, Dayton, OH (1966).
164. Pestel, E.C. and Leckis, F.A, Matrix Methods in Elastomechanics, McGraw-Hill (1963).
165. Petyt, M., "Vibration of Curved Plates," J. Sound Vib., 15, pp 381-395 (1971)
166. Petyt, M, "The Vibration Characteristics of a Tensioned Plate Containing a Fatigue Crack," J. Sound Vib., 8, pp 377-389 (1968).

167. Petyt, M. and Fleischer, C.C., "Free Vibration of a Curved Beam," *J. Sound Vib.*, 18, pp 17-30 (1971).
168. Petyt, M. and Fleischer, C.C., "Vibration of Multi-supported Curved Beams," *J. Sound Vib.*, 32, pp 359-365 (1974).
169. Petyt, M. and Mirza, W.H., "Vibration of Column-supported Floor Slabs," *J. Sound Vib.*, 21, pp 355-364 (1972).
170. Petyt, M. and Mirza, W.H., "Dynamic Behaviour of In-line Shear Walls Connected by Floor Slabs," *J. Sound Vib.*, 25, pp 349-357 (1972).
171. Petyt, M. and Mirza, W.H., "Vibration of Asymmetrical Coupled Shear Walls," *J. Sound Vib.*, 27, pp 573-581 (1973).
172. Pipes, L.A., Matrix Methods for Engineering, Prentice-Hall (1963).
173. Popplewell, N., "The Vibration of a Box-type Structure: I. Natural Frequencies and Normal Modes," *J. Sound Vib.*, 14, pp 357-365 (1971).
174. Popplewell, N., "The Vibration of a Box-type Structure: II. Response to a Travelling Pressure Wave," *J. Sound Vib.*, 18, pp 521-531 (1971).
175. Prasad, K.S.R.K., "Finite Element Adaptations of Continuum Methods for Vibration Problems," Ph.D. Thesis, Indian Institute of Science (1974).
176. Prasad, K.S.R.K. and Krishnamurty, A.V., "Galerkin Finite Element Method for Vibration Problems," *AIAA J.*, 11, p 544 (1973).
177. Prasad, K.S.R.K., Krishnamurty, A.V., and Rao, A.K., "A Finite Element Analogue of the Modified Rayleigh-Ritz Method for Vibration Problems," *Intl. J. Numer. Methods Engr.*, 5, pp 163-169 (1972).
178. Prentis, J.M. and Leckie, F.A., Mechanical Vibrations An Introduction to Matrix Methods, Longmans, Green, and Co., Ltd., London (1963).
179. Prezmieniecki, J.S., "Quadratic Matrix Equations for Determining Vibration Modes and Frequencies of Continuous Elastic Systems," P.F.C.M.M.S.M.\*\*., Wright-Patterson AFB, Dayton, OH, pp 777-802 (1966).
180. Prezmieniecki, J.S., Theory of Matrix Structural Analysis, McGraw-Hill (1968).
181. Raju, I.S., Prakasarao, B., and Venkateswara Rao, G., "Axisymmetric Vibrations of Linearly Tapered Annular Plates," *J. Sound Vib.*, 32, pp 507-512 (1974).
182. Ramamurthy, V., "Application of the Simultaneous Iteration Method to Torsional Vibration Problems," *J. Sound Vib.*, 29, pp 331-340 (1973).
183. Ramamurthy, V. and Ganesan, N., "Torsional Vibrations of Prismatic Shells," *J. Sound Vib.*, 38, pp 195-213 (1975).
184. Ramsden, J.N. and Stoker, J.R., "Mass Condensation: A Semi-automatic Method for Reducing the Size of Vibration Problems," *Intl. J. Numer. Methods Engr.*, 1, pp 333-349 (1969).
185. Rao, G.V., Rao, B.P., and Raju, I.S., "Vibrations of Inhomogeneous Thin Plates Using a High Precision Triangular Element," *J. Sound Vib.*, 34 (3), pp 444-445 (1974).
186. Rao, A.K., Ramamurthy, T.S., Murty, A.V.K., and Rao, G.V., "Bounds and Error Control of Eigenvalues," *Theory and Practice in Finite Element Structural Analysis*, Tokyo Univ. Press (1973).
187. Rao, S.S., "Finite Element Flutter Analysis of Multi-web Wing Structures," *J. Sound Vib.*, 38, pp 233-244 (1975).
188. Rao, S.S., "A Study of Some Finite Element Idealizations for the Dynamic Analysis of Aircraft Wing Structures," *Aeron. Soc. India J.*, 26 (3, 4) (1974).

189. Rao, C.K., Gupta, B.V.R., and Rao, D.L.N., "Torsional Vibrations of Thin-walled Beams on Continuous Elastic Foundation Using the Finite Element Method," Proc. First Intl. Conf. Finite Element Methods in Engineering, Coimbatore, India, pp 231-248 (1974).
190. Holzer, H., Torsional Vibration Calculations, Springer Verlag, Berlin (1921).
191. Rao, G.V. and Raju, K.K., "A Galerkin Finite Element Analysis of a Uniform Beam Carrying a Concentrated Mass and Rotary Inertia with a Spring Hinge," J. Sound Vib., 37, pp 567-569 (1974).
192. Rao, G.V., Sundararamaiah, V., and Raju, I.S., "Finite Element Analysis of Vibrations of Initially Stressed Thin Shells of Revolution," J. Sound Vib., 37 (1), pp 57-64 (1974).
193. Rao, Y.V.K.S. and Sinha, P.K., "Vibrations of Sandwich Plates under Uniaxial Compression," AIAA J., 12, p 1282 (1974).
194. Raymond, D. Krieg and Samuel, W. Key, "Transient Shell Response by Numerical Time Integration," Intl. J. Numer. Methods Engr., 7, pp 273-286 (1973).
195. Rehfield, L.W., "Higher Vibration Modes by Matrix Iteration," J. Aircraft, 9, p 505 (1972).
196. Rosen, Richard and Rubinstein, Moshe F., "Dynamic Analysis by Matrix Decomposition," Proc. ASCE, 94 (2), p 385 (1968).
197. Cook, Robert D., "Eigenvalue Problems with a 'Mixed' Plate Element," AIAA J., 7, p 982 (1969).
198. Robinson, J., Structural Matrix Analysis for the Engineer, John Wiley and Sons (1966).
199. Ross, C.T.S., "Free Vibration of Thin Shells," J. Sound Vib., 39, pp 337-344 (1975).
200. Ross, C.T.F., "Finite Elements for the Vibration of Cones and Cylinders," Intl. J. Numer. Methods Engr., 9, pp 833-845 (1975).
201. Craig, Jr., Roy R. and Bampton, Mervyn C.C., "Coupling of Structures for Dynamic Analysis," AIAA J., 6, pp 1313-1319 (1968).
202. Orris, Ruth M. and Petyt, M., "A Finite Element Study of the Vibration of Trapezoidal Plates," J. Sound Vib., 27, pp 325-344 (1973).
203. Orris, Ruth M. and Petyt, M., "A Finite Element Study of the Harmonic Wave Propagation in Periodic Structures," J. Sound Vib., 33, pp 223-236 (1973).
204. Sabir, A.B. and Ashwell, D.G., "A Comparison of Curved Beam Elements When Used in Vibration Problems," J. Sound Vib., 18, pp 555-563 (1971).
205. Seshadri, T.V., "Shock and Vibration Analysis Using Finite Element Techniques," Shock Vib. Dig., 7 (7), pp 75-95 (July 1975).
206. Sheng-Taur-Mau, Pain, T.H., and Pin Tong, "Vibration Analysis of Laminated Plates and Shells by Hybrid Stress Elements," AIAA J., 11, pp 1450-1452 (1973).
207. Shimizu, S., "Free Vibration Analysis of Stiffened Plates," Second U.S.-Japan Seminar on Matrix Methods of Structural Analysis and Design, Univ. Alabama Press, pp 219-236 (1972).
208. Shukut, T. and Ishihara, K., "The Analysis of the Acoustic Field in Irregularly-shaped Rooms by the Finite Element Method," J. Sound Vib., 29 (1), pp 67-76 (1973).
209. Singa Rao, K. and Amba Rao, C.L., "Vibrations of Beams with Overhangs," AIAA J., 11, pp 1445-1446 (1973).
210. Singarao, K., Venkateswara Rao, G., and Raju, I.S., "A Note on the Cylindrical Shell Finite Element," Intl. J. Numer. Methods Engr., 9, pp 245-250 (1975).
211. Slyper, H.A., "Development of Explicit Stiffness and Mass Matrices for a Triangular Plate Element," Intl. J. Solids Struct., 5, pp 241-249 (1969).

212. Klein, Stanley, "A Static and Dynamic Finite Element Shell Analysis with Experimental Verification," *Intl. J. Numer. Methods Engr.*, 3, pp 299-315 (1971).
213. Stroebel, G.J., "Dynamic Coupling in an Unsymmetrically Tapered Beam," *J. Sound Vib.*, 34, pp 275-283 (1974).
214. Subir, K. Sen and Phillip, L. Gould, "Free Vibrations of Shells of Revolution Using a Finite Element Method," *ASCE J. Engr. Mech. Div.*, 100 (2), pp 283-304 (1974).
215. Swanell, P., "The Automatic Computation of Natural Frequencies of Structural Frames Using Exact Matrix Techniques," *Theory and Practice in Finite Element Structural Analysis*, Tokyo Univ. Press, pp 289-304 (1973).
216. Tabarrok, B., "A Variational Principle for the Dynamic Analysis of Continua by a Hybrid Finite Element Method," *Intl. J. Solids Struct.*, 7 (3), pp 251-268 (1971).
217. Tabarrok, B. and Goss, N., "Vibrations of Cylindrical Shells by Hybrid Finite Element Methods (SYM)," *AIAA J.*, 10, p 1553 (1972).
218. Tabarrok, B. and Goss, N., Comment on "A New Hybrid Cylindrical Shell Finite Element," *J. Sound Vib.*, 21, pp 369-372 (1972).
219. Tabarrok, B., Discussion of a paper by John Robinson and M. Petyt, *Intl. J. Numer. Methods Engr.*, 4, p 143 (1972).
220. Tabarrok, B. and Sodhi, D.S., "On the Generalization of Stress Function Procedures for Dynamic Analysis of Plates," *Intl. J. Numer. Methods Engr.*, 5, pp 523-542 (1973).
221. Thangam Babu, P.V. and Reddy, D.V., "Frequency Analysis of Skew Orthotropic Plates by the Finite Strip Method," *J. Sound Vib.*, 18, pp 465-474 (1971).
222. Thomas, D.L. and Wilson, R.R., "The Use of Straight Beam Finite Element for Analysis of Vibrations of Curved Beams," *J. Sound Vib.*, 26, pp 155-158 (1973).
223. Thomas, D.L., Wilson, J.M., and Wilson, R.R., "Timoshenko Beam Finite Elements," *J. Sound Vib.*, 31, pp 315-330 (1973).
224. Thomas, J. and Dokumaci, E., "Simple Finite Elements for Pre-twisted Blading Vibration," *Aeron. Quart.*, 25, p 109 (1974).
225. Thomas, J. and Dokumaci, E., "Improved Finite Elements for Vibration Analysis of Tapered Beams," *Aeron. Quart.*, 24, p 39 (1973).
226. Turner, M.J., "Design of Minimum Mass Structures with Specified Natural Frequencies," *AIAA J.*, 5, pp 406-412 (1967).
227. Valathur, M., Dove, R.C., and Albrecht, B., "Wave Propagation in a Thin Hollow Cone by a Finite Element Method," *J. Sound Vib.*, 24, pp 211-218 (1972).
228. Venkateswara Rao, G., Venkataraman, J., and Prakasa Rao, B., "Vibrations of Thick Plates Using a High Precision Triangular Element," *Nucl. Engr. Des.*, 31 (1), pp 102-105 (1974).
229. Venkateswara Rao, G., Krishnamurty, A.V., and Kameswara Rao, A., "Bounds for Eigenvalues in Some Vibration and Stability Problems," *Intl. J. Numer. Methods Engr.*, 5, pp 237-242 (1972).
230. Venkateswara Rao, G., Raju, I.S., and Amba Rao, C.L., "Vibrations of Point Supported Plates," *J. Sound Vib.*, 29 (3), pp 387-391 (1973).
231. Venkateswara Rao, G., Prakasa Rao, B., and Raju, I.S., "Vibrations of Inhomogeneous Thin Plates Using a High Precision Triangular Element," *J. Sound Vib.*, 34 (3), pp 444-445 (1974).
232. Vysloukh, V.A., Kandidov, V.P., and Chesnokov, S.S., "Reduction of the Degrees of Freedom in Solving Dynamic Problems by the Finite Element Method," *Intl. J. Numer. Methods Engr.*, 7, pp 185-194 (1973).

233. Wang, C.K., Matrix Methods of Structural Analysis, International Textbook (1966).
234. Wang, P.C., Numerical and Matrix Methods in Structural Mechanics, John Wiley and Sons (1966).
235. Webster, J.J., "Free Vibration of Shells of Revolution Using Ring Finite Elements," *Intl. J. Mech. Sci.*, 9 (8), pp 559-570 (1967).
236. Wilson, R.R. and Brebbia, C.A., "Dynamic Behaviour of Steel Foundations for Turbo-alternators," *J. Sound Vib.*, 18, pp 405-416 (1971).
237. Wright, G.C. and Miles, G.A., "An Economical Method for Determining the Smallest Eigenvalues of Large Linear Systems," *Intl. J. Numer. Methods Engr.*, 3, pp 25-33 (1971).
238. Yamada, Y., "Dynamic Analysis of Structures," Second U.S.-Japan Seminar on Matrix Methods of Structural Analysis and Design, Univ. Alabama Press, pp 181-199 (1972).
239. Yang, T.Y. and Kim, H.W., "Asymmetrical Bending and Vibration of a Conical Shell Element (SNP)," *AIAA J.*, 12, p 251 (1974).
240. Yang, T.Y. and Kim, H., "Vibration and Buckling of Shells Under Initial Stresses," *AIAA J.*, 11, pp 1525-1531 (1973).
241. Yang, T.Y. and Sun, C.T., "Finite Elements for the Vibration of Framed Shear Walls," *J. Sound Vib.*, 27, pp 247-311 (1973).
242. Yang, T.Y. and Sun, C.T., "Axial Flexural Vibration of Frame Works Using a Finite Element Approach," *J. Acoust. Soc. Amer.* 53, pp 137-146 (1973).
243. Yoshiaki Muda, "A New Relaxation Method for Obtaining the Lowest Eigenvalue and Eigenvector of a Matrix Equation," *Intl. J. Numer. Methods Engr.*, 6, pp 511-529 (1973).
244. Zienkiewicz, O.C., "The Finite Element Method: From Intuition to Generality," *Appl. Mech. Rev.*, 23, p 249 (1970).
245. Zienkiewicz, O.C. and Cheung, Y.K., The Finite Element Method in Structural and Continuum Mechanics, McGraw-Hill 1967).
246. Zienkiewicz, O.C., The Finite Element Method in Engineering Science, McGraw-Hill (1971).
247. Myklestad, N.O., "A New Method for Calculating Natural Modes of Uncoupled Bending Vibrations of Airplane Wings and Other Types of Beams," *J. Aeron. Sci.*, 11, pp 153-162 (1944).
- \*P.S.C.M.M.S.M. Proceedings of the Second Conference on Matrix Methods in Structural Mechanics
- \*\*P.F.C.M.M.S.M. - Proceedings of the First Conference on Matrix Methods in Structural Mechanics
- \*\*\*P.T.C.M.M.S.M. - Proceedings of the Third Conference on Matrix Methods in Structural Mechanics

**TABLE - Comparison of Some**

Basis	Form of the Final Matrix Equation	Bound for Eigenvalue	Remarks
Rayleigh-Ritz (R-R) Method	$[K]\{\xi\} - \lambda [M]\{\xi\} = 0$	Upper Bound	<ol style="list-style-type: none"> <li>1. Easy formulation</li> <li>2. Convenient for computer programming</li> </ol>
Modified Rayleigh-Ritz Method [177]	$[F]\{\xi\} - \frac{1}{\lambda} [M]\{\xi\} = 0$	Upper bound	<ol style="list-style-type: none"> <li>1. Requires concepts of matrix force method</li> <li>2. Best suited for one-dimensional problems</li> </ol>
Galerkin Method [176]	$[E_k]\{\xi\} - \lambda [E_m]\{\xi\} = 0$	Cannot be predicted	<ol style="list-style-type: none"> <li>1. Element degrees of freedom are more</li> <li>2. Programming is similar to R-R method</li> <li>3. Can consider nonlinear problems for which a minimum energy principle may not exist</li> <li>4. Can be extended to initial value problems</li> </ol>
Least-Square Method [126]	$[P]\{\xi\} - \lambda [Q]\{\xi\} + \lambda^2 [R]\{\xi\} = 0$	Cannot be predicted	<ol style="list-style-type: none"> <li>1. Results in quadratic eigenvalue equations</li> <li>2. Eigenvalues are generally complex</li> <li>3. Programming complicated</li> </ol>

## Finite Element Models

Basis	Form of the Final Matrix Equation	Bound for Eigenvalue	Remarks
Pian's Type Hybrid Method [126]	$[F(\lambda)]\{\xi\} - \frac{1}{\lambda_0} [M(\lambda)]\{\xi\} = 0$	Cannot be predicted	<ol style="list-style-type: none"> <li>1. Results in transcendental matrix equation</li> <li>2. Conventional computer programs cannot be used directly</li> <li>3. Generally not a convenient formulation</li> </ol>
Collocation Method [175]	$[D]\{\xi\} - \frac{1}{\lambda}\{\xi\} = 0$	Cannot be predicted	<ol style="list-style-type: none"> <li>1. Requires concepts of the matrix force method</li> <li>2. Best suited for one-dimensional problems</li> </ol>
Prezemieniecki Method [179]	$[A_1 - \lambda A_2 - \lambda^2 A_3 \dots]\{\xi\} = 0$	Cannot be predicted	<ol style="list-style-type: none"> <li>1. Requires solution of a quadratic eigenvalue problem</li> <li>2. Programming complicated</li> <li>3. Negative eigenvalues are sometimes obtained</li> </ol>
Stress Function Method [219]	$[A]\{\xi\} - \lambda [B]\{\xi\} = 0$	Cannot be predicted	<ol style="list-style-type: none"> <li>1. Involves suppression of 'any frequency modes'</li> </ol>

# BOOK REVIEWS

## ELASTICITY IN ENGINEERING MECHANICS

Arthur P. Boresi and Paul Lynn  
Prentice-Hall, Inc. (1974)

This is an updated edition of an earlier book of the same title published by Boresi in 1965.

Chapter one is an introduction to elasticity and includes a description of boundary value problems. Chapter two deals with the theory of deformation and contains three appendices. Appendix 2A contains the derivation of strain-displacement relations in orthogonal curvilinear coordinates. Appendix 2B shows strain-displacement relations in general coordinates, and Appendix 2C is devoted to the derivation of displacement relations for special coordinates by cartesian methods.

The theory of stress is developed in Chapter 3. There is an appendix on the application of the principle of virtual work. The two-dimensional theory of elasticity in rectangular cartesian coordinates is the subject of the fifth chapter. Appendix 5A describes the effect of couple stresses on plane elasticity.

Two-dimensional elasticity in polar coordinates is developed in Chapter 6. Appendix 6A gives the results of stress concentration due to a circular hole in a plate under the assumption of couple stresses. Chapter seven describes the solution to the problem of a prismatic bar subjected to end load.

An elementary treatment of the thermal stress problem is found in Chapter 8. The final appendix recapitulates the mathematics necessary to make the book self-sufficient to the student.

The appendices to the individual chapters are the most refreshing part of the book. The derivations of the governing equations of elasticity in orthogonal curvilinear coordinates are especially valuable because they are seldom found in introductory books. The fact that the equations are derived vectorially makes them all the more attractive

It is unfortunate that the only method of solution emphasized is the semi-inverse method. Students are usually puzzled because direct methods are not presented in elasticity. Some introduction to the integral transform technique and complex variable methods should have been included. Nevertheless, this text can be of great value in a first course in elasticity, to both students and instructors, and it can be supplemented by handouts where desired.

Leon Y. Bahar  
Dept. of Mechanical Engineering and Mechanics  
Drexel University  
Philadelphia, Pennsylvania 19104

## THE FINITE ELEMENT METHOD FOR ENGINEERS

K. E. Huebner

John Wiley & Sons - New York (1975)

Finite element analysis now occupies a commanding position in the areas of analysis and design. Huebner's book is not elementary, as advertisements claim; rather, it belongs at the intermediate level. The text is divided into ten chapters and four appendices. The bibliography is excellent.

Fundamental theory is the subject of the first chapter. The second chapter contains discussions of simple linear spring systems, one-dimensional heat flow, electrical networks, and the two-dimensional structural constant strain triangular element. In Chapter III the author considers the composition of the finite element and the equivalence between the finite element method and the Rayleigh-Ritz variational method. His formulation of the finite element method is a good one.

In Chapter IV, which is the heart of the book, the author describes simple triangular and quadrilateral elements, as well as the more complicated tetrahedral and hexahedral elements used in three-dimensional studies. There is discussion of two- and three-dimensional isoparametric elements and interpolation schemes; consideration of hexahedral elements; and derivation of linear, quadratic, and cubic isoparametric elements for both two and three dimensions. The explanation is lucid and easy to follow.

The more complicated aspects of elasticity, including the role of variational methods, are found in the next chapter. The displacement method and its applications are considered. Fine mesh at a singularity, the plate bending problem, and a more detailed explanation of three-dimensional elements, including the hexahedral element, conclude the chapter.

Chapters VII and VIII consider such general problems as heat conduction and inviscid incompressible and compressible flow. Direct formulation of the finite elements using variational methods are used. Time-dependent fluid problems and transient motion utilizing recurrence relations are also described. The reviewer believes that the latter should have been expanded due to its importance in the dynamics of real structures.

The author applies various aspects of lubrication to gas bearings and journal bearings. He states that the isoparametric approach should have been considered in fluid flow.

In Chapter IX fluid mechanics problems are discussed in depth. A computer code with direct coding of a heat conduction problem is set up for the novice in Chapter X.

The reviewer thinks that the book should be part of the library of the practicing engineer and scientist involved in finite element applications. The book should be published as two volumes, however, one an elementary text, the other an advanced text. The section on structural dynamics should be expanded to include component mode methods, substructuring, and transient response. The section on gaussian quadrature applied to isoparametric elements and determination of transfer of stresses from the gaussian points to the nodes should also be expanded.

Herb Saunders  
General Electric Company  
LSTGD  
Schenectady, New York 12345

# NEWS BRIEFS

news on current  
and Future Shock and  
Vibration activities and events

## **ASTM PROPOSALS MEASURING IMPACT SOUND TRANSMISSION OF FLOOR-CEILING ASSEMBLIES NOW AVAILABLE**

Two proposals for methods to measure the impact sound transmission of floor-ceiling assemblies have been approved by the American Society for Testing and Materials (ASTM) Committee on Standards.

The two proposals are: "Proposed Alternative Method of Laboratory Measurement of Impact Sound Transmission through Floor-Ceiling Assemblies Using a Modified Tapping Machine" and "Proposed Alternative Method of Impact Sound Transmission through Floor-Ceiling Assemblies Using a Live Walker."

An ASTM Proposal is a specification, test method, classification, definition, or recommended practice which has been approved by the sponsoring committee for publication as an "information only" document. A proposal is not submitted to Society letter ballot and is not a standard. Copies of the two proposals can be obtained from ASTM, 1916 Race St., Philadelphia, PA 19103.

## **INSTITUTE OF ENVIRONMENTAL SCIENCES SCHEDULES SESSION ON NUCLEAR AND FOSSIL FUEL ELECTRIC POWER GENERATION STATION QUALIFICATION AT ITS 1977 ANNUAL MEETING**

A special all day technical session entitled "Total Qualification for Electric Power Generation Stations - Nuclear and Fossil Fueled" will be held on 25 April 1977, during the twenty-third Annual Meeting of the Institute of Environmental Sciences at the Marriott Hotel, Los Angeles, California. The speakers are prominent experts from government and industry, who will trace the requirements for qualifications of safety related equipment from the NRC Regulatory Guide through the QA plan to the completion of the program.

The session chairman is Paul M. Turkheimer, Director of Program Development, Wyle Laboratories, El Segundo, CA., Senior Member of the IES. The session co-chairman is Thomas R. Colandrea, Director of the Quality Assurance Div., General Atomic Co., San Diego, CA, and chairman of the Nuclear Div., ASQC. Frank Unmack, General Atomic Co., Staff Engineer responsible for equipment qualification, Fort St. Vrain Project, completes the planning staff.

The 9th Space Simulation Conference, being held at the same location from April 26 - 28, 1977, provides an international forum for the discussion of space simulation and related topics. The technical program is sponsored by the Institute of Environmental Sciences (IES), American Institute of Aeronautics and Astronautics (AIAA), the American Society for Testing and Materials (ASTM), and the National Aeronautics and Space Administration (NASA). The ninth in this series of space simulation conferences is being hosted by the IES and will be conducted in conjunction with the IES 23rd Annual Technical Meeting and Equipment Exposition.

The Exhibits in the IES Equipment Exposition of SPACE APPLICATIONS, ENERGY, TEST EQUIPMENT, and INSTRUMENTATION in addition to the Spacecraft and Test Facility Tours in the immediate vicinity are planned to enhance the technical program.

For further information contact:

B. L. Peterson, Executive Director  
Institute of Environmental Sciences  
940 East Northwest Highway  
Mount Prospect, Illinois 60056  
Tele. (312) 255-1561

# SHORT COURSES

## FEBRUARY

### MODELING IN ENGINEERING DYNAMICS

Dates: February 28 - March 4, 1977

Place: San Antonio, Texas

Objective: This course is recommended for prospective students who have a bachelor's degree in some field of engineering, physics, or mathematics. The intent of this course is to introduce and illustrate to engineers, physicists, and scientists investigating transient phenomena the powerful tool of model analysis, and although the course is directed toward experienced personnel, a newcomer to the fields of survivability analysis, terminal ballistics effects, safety engineering, engineering dynamics, etc., can keep pace by diligently applying himself.

Contact: Mr. Peter S. Westine, Southwest Research Institute, P.O. Box 28510, San Antonio, TX 78284

## MARCH

### CORRELATION AND COHERENCE ANALYSIS FOR ACOUSTICS AND VIBRATION PROBLEMS

Dates: March 7 - 11, 1977

Place: UCLA

Objective: This course covers the latest practical techniques of correlation and coherence analysis-- ordinary, multiple and partial -- for solving acoustics and vibration problems in physical systems

Contact: Continuing Education in Engineering and Mathematics, Short Courses, 6266 Boelter Hall, UCLA Extension, Los Angeles, CA 90024  
Tele (213) 825-1047

### DIGITAL SIGNAL PROCESSING

Dates: March 9 - 11, 1977

Place: Houston, Texas

Objective: This seminar covers theory, operation and applications -- plus additional capabilities such as transient capture, amplitude probability, cross spectrum, cross correlation, convolution coherence, coherent output power, signal averaging and plenty of demonstrations.

Contact: Mr. Bob Kiefer, Spectral Dynamics Corp. of San Diego, P.O. Box 671, San Diego, CA 92112  
Tele. (714) 565-8211

### DYNAMIC ANALYSIS OF STRUCTURES

Dates: March 14 - 17, 1977

Place: Detroit, Michigan

Objective: This seminar provides practical laboratory experience on getting good data and recognizing bad; diagnosing machinery with Vibration Spectrum Analyzers, solving structural problems with Transfer Function Analyzers; and demonstrations using state of the art FFT processors.

Contact: Mr. Bob Kiefer, Spectral Dynamics Corp. of San Diego, P.O. Box 671, San Diego, CA 92112  
Tele (714) 565-8211

### MEASUREMENT SYSTEMS ENGINEERING

Dates: March 14 - 19, 1977

Place: Phoenix, Arizona

Objective: Program emphasis is on how to obtain valid cost-effective data in the field and in the laboratory during the next decade through increased productivity of data acquisition systems and groups. The latest developments in the new Unified Approach to the Engineering of Measuring Systems to achieve these aims, will be presented.

Contact: Prof. P. Stein, Short Course Director, 5602 East Monte Rosa, Phoenix, AZ 85018  
Tele. (602) 945-4603/965-3124

## APRIL

### **INTRODUCTION TO VIBRATION AND SHOCK TESTING, MEASUREMENT, ANALYSIS AND CALIBRATION**

Dates: April 11 - 15, 1977

Place: Boston, Massachusetts

Objective: Firms manufacturing weapons, aircraft, missiles, naval or military vehicle systems or components should consider sending their environmental test personnel to this seminar. This course will concentrate upon equipments and techniques rather than upon theory.

Contact Mr. W. Tustin, Tustin Institute of Technology, Inc., 22 E. Los Olivos St., Santa Barbara, CA 93105 Tele. (805) 963-1124

### **RELIABILITY TESTING INSTITUTE**

Dates: April 25 - 29, 1977

Place: University of Arizona, Tucson

Objective To provide Reliability Engineers, Product Assurance Engineers and Managers and all other engineers and teachers with a working knowledge of analyzing component, equipment, and system performance and failure data to determine the distributions of their times to failure, failure rates and reliabilities; small sample size, short duration, Bayesian testing, suspended items testing; sequential testing; and others.

Contact Dr. D. Kececioglu, Institute Director, Aerospace and Mechanical Engrg. Dept., The Univ of Arizona, Bldg. 16, Tucson, AZ 85721  
Tele (602) 884-2495/884-3901/884-3054/884-1755

## MAY

### **TURBOMACHINERY BLADING SEMINAR**

Dates May 3 - 5, 1977

Place: Rochester Institute of Technology,  
Rochester, New York

Objective: To introduce the vibration technology involved in the design and operation of turbomachinery blades. Methods and instrumentation used to measure and analyze blade vibration will be described. Industrial experts and consultants will present theoretical background material and case histories. Panel sessions dealing with gas and steam turbine blading problems and their solutions will also be conducted.

Contact: Dr. R. L. Eshleman, Director, Vibration Institute, Suite 206, 101 W. 55th St., Clarendon Hills, IL 60514 Tele. (312) 654-2254/654-2053

### **FINITE ELEMENT METHOD AND NASTRAN USAGE**

Dates: May 16 - June 16, 1977

Place: Washington, D.C.

Objective A sequence of three professional development courses will be presented to provide an understanding of the technological content in general purpose finite element programs, and to provide training in the use of NASTRAN. The courses and dates are

Theory of Finite Elements, May 16 - 20, 1977

Static Structural Analysis Using NASTRAN,  
May 23 - 26, 1977

Dynamics and Nonlinear Structural Analysis  
Using NASTRAN, June 14 - 27, 1977

Contact Dr H Schaeffer, Schaeffer Analysis,  
P.O. Box 761, Berwyn Station, College Park, MD  
20740 Tele. (301) 721-3788

# ABSTRACTS FROM THE CURRENT LITERATURE

Copies of articles abstracted in the DIGEST are not available from the SVIC or the Vibration Institute (except those generated by either organization). Inquiries should be directed to library resources. Government reports can be obtained from the National Technical Information Service, Springfield, Va., 22151, by citing the AD-, PB-, or N- number. Doctoral dissertations are available from University Microfilms (UM), 313 N. Fir St., Ann Arbor, MI. Addresses following the authors' names in the citation refer only to the first author. The list of periodicals scanned by this journal is printed in issues 1, 6, and 12.

The proceedings of the "Conference on Vibrations in Rotating Machinery" held at the University of Cambridge, England, September 15-17, 1976, are available from the Institute of Mechanical Engineers, 1 Birdcage Walk, Westminster, London SW1 H9JJ England.

## ABSTRACT CONTENTS

### ANALYSIS AND DESIGN . . . .44

Analogs and Analog	
Computation . . . . .	44
Analytical Methods . . . . .	44
Numerical Analysis . . . . .	47
Perturbation Methods . . . . .	47
Stability Analysis . . . . .	47
Variational Methods . . . . .	47
Finite Element Modeling . . . . .	47
Modeling . . . . .	48
Parameter Identification . . . . .	48
Design Information . . . . .	48
Criteria, Standards, and	
Specifications . . . . .	48
Surveys . . . . .	48
Mode Synthesis . . . . .	48

### COMPUTER PROGRAMS . . . .49

General . . . . .	49
-------------------	----

### ENVIRONMENTS. . . . .49

Acoustic . . . . .	49
Random . . . . .	50
Seismic . . . . .	50
Shock . . . . .	50
General Weapon . . . . .	51

### PHENOMENOLOGY. . . . .52

Composite . . . . .	52
---------------------	----

Damping . . . . .	53
Elastic . . . . .	54
Fatigue . . . . .	54
Fluid . . . . .	54
Inelastic . . . . .	55
Viscoelastic . . . . .	55

### EXPERIMENTATION . . . . .55

Balancing . . . . .	55
Diagnostics . . . . .	56
Facilities . . . . .	56
Instrumentation . . . . .	56
Techniques . . . . .	57

### COMPONENTS. . . . .58

Absorbers . . . . .	58
Beams, Strings, Rods . . . . .	58
Bearings . . . . .	60
Blades . . . . .	61
Columns . . . . .	62
Cylinders . . . . .	63
Gears . . . . .	63
Linkages . . . . .	63
Mechanical . . . . .	63
Membranes, Films, and	
Webs . . . . .	64
Panels . . . . .	64
Pipes and Tubes . . . . .	64
Plates and Shells . . . . .	65
Rings . . . . .	68
Structural . . . . .	68

### SYSTEMS . . . . .69

Absorber . . . . .	69
Acoustic Isolation . . . . .	69
Noise Reduction . . . . .	69
Active Isolation . . . . .	70
Aircraft . . . . .	70
Bridges . . . . .	72
Building . . . . .	72
Construction . . . . .	73
Earth . . . . .	73
Helicopters . . . . .	73
Human . . . . .	74
Isolation . . . . .	74
Material Handling . . . . .	75
Mechanical . . . . .	75
Metal Working and	
Forming . . . . .	75
Off-Road Vehicles . . . . .	75
Pumps, Turbines, Fans,	
Compressors . . . . .	76
Rail . . . . .	76
Reactors . . . . .	77
Reciprocating Machines . . . . .	78
Road . . . . .	79
Rotors . . . . .	82
Satellite . . . . .	88
Ship . . . . .	88
Spacecraft . . . . .	89
Structural . . . . .	90
Turbomachinery . . . . .	90
Useful Application . . . . .	91

# ANALYSIS AND DESIGN

## ANALOGS AND ANALOG COMPUTATION

77-232

### The Dynamic Response of an Elastic Half Space with an Overlying Acoustic Fluid

B.E. Bennett and G. Herrmann

Dept. of Applied Mechanics, Stanford Univ., Palo Alto, CA., In: J. of Applied Mechanics, 98 (1), pp 39-42 (Mar 1976)

AD-A027 536/2GA

**Key Words:** Half-space, Elastic properties, Wave propagation, Elastic waves, Seismic waves

A class of dynamic problems involving a semi-infinite elastic solid with an overlying semi-infinite acoustic fluid, subjected at the plane interface to arbitrary normal loading is investigated. A method of solution is proposed which reduces the class of problems under study to that in which the fluid is absent. A specific example is presented for an expanding disk-shaped load including numerical results for the sub-seismic range.

## ANALYTICAL METHODS

77-233

### A Generalized Approach to the Solution of Variable Systems Subjected to Arbitrary Source Functions and Boundary Conditions

J.C. Hassab

Naval Underwater Systems Center, Newport, RI 02840, J. Sound Vib., 48 (2), pp 277-291 (Sept 1976) 15 refs

**Key Words:** Layered materials, Forcing function

Layered inhomogeneous systems with or without forcing functions as represented by sources and boundary conditions are discussed. A common formulation to the constraints and physical laws that apply is given by using the Fredholm integral equation. A unified and versatile solution is developed analytically to treat these problems under forced or unforced conditions. Then the approach is dissected to implement the results on the computer or to treat, at the outset,

the lumped approximation of the distributed system. This development modifies, extends, and generalizes previous studies that have been applied to a restricted class of systems. Throughout the article examples are given to introduce, motivate and illustrate the analysis.

77-234

### Mode Theory of Wave Propagation in a Bilinear Medium: The WKB Approximation

M. Hall

Navy Research Lab., Edgecliff 2027, Australia, J. Acoust. Soc. Amer., 60 (4), pp 810-814 (Oct 1976) 5 figs, 5 refs

**Key Words:** Wave propagation, Eigenvalue problems

Normal-mode calculations are made of wave propagation in a bilinear medium with a free surface (the surface duct). The WKB approximation is used. In the usual way, the integral expression for the field is transformed into an infinite series of the residues (normal modes) of the integrand at its poles (eigenvalues). Approximations for the eigenvalues of the trapped normal modes are obtained from WKB formulas. The eigenvalues of the untrapped modes are obtained by quadratic extrapolation of the eigenvalues of the trapped modes. The accuracy of the real part of the eigenvalues of all modes is improved by taking into account the imaginary part when solving the characteristic equation.

77-235

### Difference Methods for Boundary Value Problems with a Singularity of the First Kind

F.R. De Hoog and R. Weiss

Computer Centre, Australian National Univ., Canberra 2600, Australia, SIAM J. Numer. Anal. 13 (5), pp 753-760 (Oct 1976)

**Key Words:** Boundary value problems, Eigenvalue problems

The application of certain difference schemes (box, trapezoidal, Euler and backward Euler) to the numerical solution of boundary value problems for nonlinear first order systems of ordinary differential equations with a singularity of the first kind is examined. The solution of the linear eigenvalue problem is also considered.

77-236

### The Minimum Ratio of Two Eigenvalues

J.B. Keller

Courant Inst. of Mathematical Sciences, New York Univ., New York, NY 10012, SIAM J. Appl. Math., 31 (3), pp 485-491 (Nov 1976) 3 figs, 1 ref

Sponsored by ONR

**Key Words:** Eigenvalue problems

The first two eigenvalues,  $\lambda_1$  and  $\lambda_2$ , of the problem  $y'' + \lambda\phi(x)y = 0$ ,  $y(\pm\frac{1}{2}) = 0$  are considered. The method of analysis is applicable to other similar problems with inequality constraints.

**77-237**

**Analytical Theory of Nonlinear Oscillations. IV: The Periodic Oscillations of the Equation  $x - \epsilon(1-x^{2n+2})\dot{x} + x^{2n+1} = \epsilon a \cos \omega t$ ,  $a > 0$ ,  $\omega > 0$  Independent of  $\epsilon$**   
C. Obi

Dept. of Mathematics, Univ. of Lagos, Lagos, Nigeria,  
SIAM J. Appl. Math., 31 (2), pp 345-357 (Sept 1976)  
11 refs

**Key Words:** Nonlinear response, Periodic response

A form of the well-known equation of Van der Pol with a forcing term is  $\ddot{x} - \epsilon(1-x^2)\dot{x} + x = \epsilon a \cos \omega t$ . In this paper a generalized version of Van der Pol's equation is introduced  $\ddot{x} - \epsilon(1-x^{2n+2})\dot{x} + x^{2n+1} = \epsilon a \cos \omega t$ , where  $n \geq 1$  is an integer and  $a > 0$ ,  $\omega > 0$  are independent of  $\epsilon$ . A preliminary study of this equation for a general  $n$  and small  $\epsilon$  is made. In addition a more detailed study of its special case ( $n=1$  and  $\epsilon$  is small) is considered. Results on the exact number of its periodic oscillations, the relations between the stationary values (amplitudes), least periods of its periodic oscillations, and the ways some of the periodic oscillations appear or disappear as the parameters  $a$ ,  $\omega$  vary are given.

**77-238**

**Analytical Theory of Nonlinear Oscillations. II: Small Periodic Oscillations of Equations of the Second Order**

C. Obi

Dept. of Mathematics, Univ. of Lagos, Lagos, Nigeria,  
SIAM J. Appl. Math., 31 (2), pp 334-344 (Sept 1976)  
6 refs

**Key Words:** Nonlinear response, Periodic response

The equations considered are perturbations of the equation  $\ddot{x} + g(x) = 0$ . The results established are a number of theorems on the existence and number of those periodic oscillations (of the equations considered) which themselves are perturbations of the constant solutions of  $\ddot{x} + g(x) = 0$ . The theorems are stated in terms of small periodic oscillations.

**77-239**

**Frequency Modulation at a Moving Material Interface and a Conservation Law for Wave Number**

G.G. Kleinstein and M.D. Gunzburger

Joint Institute for Acoustics and Flight Sciences,  
The George Washington Univ., Hampton, VA 23665,  
J. Sound Vib., 48 (2), pp 169-178 (Sept 1976)  
3 figs, 4 refs

Sponsored by NASA

**Key Words:** Frequencies, Elastic waves, Wave number

An integral conservation law for wave numbers is considered. In order to test the validity of the proposed conservation law, a complete solution for the reflection and transmission of an acoustic wave impinging normally on a material interface moving at a constant speed is derived. Results are stated concerning frequency and wave number relations across a shock front as predicted by the proposed conservation law.

**77-240**

**Exact Solution for Guided Sound Transmission Through a Double Partition**

N. Romilly

Dept. of Applied Mathematical Studies, Univ. of  
Leeds, Leeds LS2 9JT, England, J. Sound Vib.,  
48 (2), pp 243-249 (Sept 1976) 5 refs

**Key Words:** Sound transmission, Walls, Membranes

The exact solution is obtained for the problem of the transmission of a symmetric sound wave through a double partition contained in a parallel-plane waveguide with rigid walls. A membrane model is used for the two leaves of the partition. The solution involves a number of infinite series which converge rapidly. No modifications are necessary for the region of the critical frequencies. Full details are given for the particular case of an incident plane wave. The result, which is suitable for computation, involves four converging infinite series.

**77-241**

**Spatial Cross-Correlation of Acoustic Pressures in Steady and Decaying Reverberant Sound Fields**

C.F. Chien and W.W. Soroka

Dept. of Mechanical Engrg., Univ. of California,  
Berkeley, CA 94720, J. Sound Vib., 48 (2), pp 235-  
242 (Sept 1976) 10 refs

**Key Words:** Cross-correlation technique, Sound pressures, Reverberation chambers

From the equation for the steady state sound pressure distribution produced in a rectangular reverberation chamber by a point source, and by using the usual high frequency approximations, the cross-correlation function for two points not too far apart for a random source position is discussed. When the same approach is used on the equation for sound pressure decay when the point source excitation is cut off, the cross-correlation function obtained for the initial portion of the decay corresponds with that determined experimentally by Balachandran and Robinson.

**77-242**

**A Comparison of Three Perturbation Methods for Non-Linear Hyperbolic Waves**

A.H. Nayfeh and A. Kluwick

Dept. of Engrg. Science and Mechanics, Virginia Polytechnic Inst. and State Univ., Blacksburg, VA, *J. Sound Vib.*, 48 (2), pp 293-299 (Sept 1976) 7 refs

**Key Words:** Perturbation theory, Wave propagation

Simple wave solutions of non-linear hyperbolic equations are studied by using the method of renormalization, the analytic method of characteristics, and the method of multiple scales. It is shown that the results of the method of renormalization depend on whether the potential function or the velocity is normalized.

**77-243**

**Perturbed Nonlinear Oscillations**

B. Kaper

Dept. of Mathematics, Univ. of Groningen, Groningen, The Netherlands, *SIAM J. Appl. Math.*, 31 (3), pp 519-546 (Nov 1976) 9 refs

**Key Words:** Oscillation, Nonlinear response, Perturbation theory

For a class of nonlinear oscillation problems described by a second order ordinary differential equation containing a small nonnegative perturbation parameter; asymptotic solutions which are uniformly valid as approaches zero are determined on intervals of the order of magnitude one over the perturbation parameter. These asymptotic solutions are obtained from formal asymptotic solutions subject to a condition in order that they approximate the exact solution asymptotically. These formal asymptotic solutions are constructed in the form of a finite generalized asymptotic power series involving functions of the periodic two variable type.

**77-244**

**Narrow-Band Excitation of a Nonlinear Oscillator**

W.C. Lennox and Y.C. Kuak

Dept. of Civil Engrg., Univ. of Waterloo, Waterloo, Ontario, Canada, *J. Appl. Mech.*, *Trans. ASME*, 43 (2), pp 340-344 (June 1976) 4 figs, 6 refs

**Key Words:** Oscillators, Narrow-band excitation

A nonlinear oscillator characterized by a hardening-type restoring force is excited by stationary narrow-band noise. The analysis is based on the concept of quasi-static amplitude and phase values which is exactly opposite to the stochastic or Markov approach with its associated Fokker-Planck equation. The approach, in effect, replaces a system with memory with one without memory. The existence of jumps is demonstrated and an expression for the probability of the occurrence of jumps is derived.

**77-245**

**The Forced Vibration of Singly Modified Damped Elastic Surface Systems**

R.G. Jacquot

Dept. of Electrical Engrg., Univ. of Wyoming, Laramie, WY 82070, *J. Sound Vib.*, 48 (2), pp 195-201 (Sept 1976) 2 figs, 15 refs

**Key Words:** Lumped parameter method, Natural frequencies, Mode shapes, Forced vibration, Plates, Shells, Membranes, Beams, Viscous damping

A technique is developed to predict the forced vibration of membranes, beams, plates or shells when they have attached to them at a single point a linear lumped parameter element or assembly of elements. The distributed parameter element is treated as viscously damped and the lumped parameter assembly may also contain viscous dampers. Solution is obtained in terms of generalized Fourier series in the unmodified eigenfunctions for the distributed portion of the system and the principle of superposition is used to handle the imposed forces and those generated at the attachment. The method is illustrated by investigating a uniformly forced simply supported rectangular plate with a lumped mass at its center and that of a point forced simple beam with a rigid pin support imposed at some arbitrary point.

**77-246**

**Periodic Solutions of Second Order Differential Equations with Nonlinear, Nondifferentiable Damping**

W.R. Utz

Dept. of Mathematics, Univ. of Missouri-Columbia,  
Columbia, MO 65201, SIAM J. Appl. Math., 31 (3),  
pp 504-510 (Nov 1976) 11 refs

**Key Words:** Equations of motion

For the equation  $x'' + Q(x') + f(x) = 0$ , periodic solutions are established without a requirement of differentiability of  $Q$ . By separate methods,  $Q$  is treated as even and then odd. In the odd case, a comparison theorem is provided when  $f(x) = w^2 x$ .

## NUMERICAL ANALYSIS

(See Nos. 319, 429)

## PERTURBATION METHODS

(See Nos. 282, 421)

## STABILITY ANALYSIS

(Also see Nos. 408, 427)

**77-247**

### Stability of Nonlinear Oscillatory Systems

J. Lin

Ph.D. Thesis, State Univ. of New York at Stony  
Brook, 203 pp (1976)  
UM 76-19, 691

**Key Words:** Stability methods, Oscillators

The averaging technique of Krylov-Bogoliubov-Mikopolskii (K-B-M) to solve several nonlinear models with oscillatory behavior used to study a class of one predator - one prey systems which incorporate a functional response of the predator. The K-B-M method allows us to calculate, under certain conditions, the radius of the limit cycle, and the renormalized frequency of the system's response.

**77-248**

### Parametric Instability Regions in Multi-Degree of Freedom Systems Under Quasi-Periodic Beating Input Excitation

D C McWhannell

Dept. of Mech Engrg., Univ. of Southampton, South-  
ampton SO9 5NH, England, J. Sound Vib, 48 (1),  
pp 73-81 (Sept 8, 1976) 2 figs, 4 refs

**Key Words:** Stability analysis, Multidegree of freedom systems, Parametric excitation

The existence of three different types of unstable region in multi-degree of freedom linear systems undergoing beating two-frequency parametric loading is demonstrated. Two and five degree of freedom digital system models exhibiting simple and combination resonant response to quasi-periodic parametric loading are discussed. Two degree of freedom mathematical models subjected to beating input situations are analyzed. Analytical, numerical, electronic analog and experimental studies are described and the practicalities and relevance of the methods used are indicated.

## VARIATIONAL METHODS

**77-249**

### A Reduction Scheme for Problems of Structural Dynamics

T.J.R. Hughes, H.M. Hilber and R.L. Taylor

Div. of Structural Engrg. and Structural Mech., Dept.  
of Civil Engrg., Univ. of California, Berkeley, CA.,  
Intl. J. Solids Struct., 12 (11), pp 749-767 (1976)  
6 figs, 14 refs

**Key Words:** Finite element technique, Variational methods, Dynamic structural analysis

A method for reducing the size of finite element systems in dynamics is presented. The technique is based upon a variational theorem in which it is admissible to describe the inertial properties of structures by way of independent displacement, velocity and momentum fields. This theorem allows us to construct reduced systems for problems in structural mechanics which retain the full rate of convergence of systems employing "consistent" mass matrices. An error analysis of the scheme and numerical examples are presented.

## FINITE ELEMENT MODELING

(Also see No. 249)

**77-250**

### Analysis of Elastic-Plastic Impact Involving Severe Distortions

G R. Johnson

Government and Aeronautical Products Div., Honey-  
well, Inc., Minneapolis, MN, J. Appl. Mech., Trans.  
ASME, 43 (3), pp 439-444 (Sept 1976) 5 figs, 12 refs

**Key Words:** Impact response, Finite element technique, Computer programs

A Lagrangian analysis technique is presented for two-dimensional axi-symmetric impact problems involving elastic-plastic flow. This technique is based on a triangular finite-element formulation rather than the quadrilateral formulation generally used in comparable finite-difference methods. The formulation of the technique and illustrative examples are included.

## MODELING

77-251

### Modal Simulation of the Intermittent Vibration of Turbo-rotors

B. Grabowski

VDI Fortschrittberichte (Progress Reports of the VDI), Series 11, (25), 122 pp (1976) 45 figs, Avail: VDI Verlag, GmbH, 4 Düsseldorf 1, Postfach 1139, Fed. Rep. of Germany. Summarized in VDI-Z, 118 (17/18) (Sept 1976)  
(In German)

**Key Words:** Rotors, Critical speeds, Modal models

Two different variations of the modal simulation of turbo-rotors are investigated. In one, the real eigenshapes of the mechanical system are used for the transformation; in the other the orthogonal vibration planes of an uncoupled mechanical system are used.

## PARAMETER IDENTIFICATION

(See Nos. 349, 373, 374)

## DESIGN INFORMATION

(See No. 428)

## CRITERIA, STANDARDS, AND SPECIFICATIONS

(See Nos. 347, 372)

## SURVEYS

(Also see No. 259)

77-252

### Noise and Blast (1956-1976): An Annotated Bibliography of Research Performed at the Human Engineering Laboratory

V.J. Confer

Human Engrg. Lab., Aberdeen Proving Ground, MD, 41 pp (July 1976) (Supersedes rept. dated Sept 1971, AD-731, 468)  
AD-A028 277/2GA

**Key Words:** Noise, Shock, Human response, Bibliographies

The bibliography is an annotated compilation of 95 formally published reports dealing with noise and blast. The research was performed by personnel of the U.S. Army Human Engineering Laboratory and covers the period from 1956 through July 1976.

## MODE SYNTHESIS

(Also see No. 261)

77-253

### Computed Restitution of Structural Natural Modes from Inappropriate Excitations

X.T. Nguyen

Office National d'Etudes et de Recherches Aero-spatiales, Paris, France, Rept. No. ONERA-NT-1975-9, 54 pp (1975)  
(In French; English summary)  
N76-22597

**Key Words:** Modal synthesis

After recalling the theoretical bases and experimental methods used for ground vibration tests, the fundamental problem of modal data determination is formulated. The proposed method makes it possible to determine the vibratory characteristics of a linear, weakly damped structure from scannings around the resonance frequency of each mode with different configurations of excitation forces.

77-254

### A Stationarity Principle for the Eigenvalue Problem for Rotating Structures

L. Meirovitch

Virginia Polytechnic Institute and State Univ., Blacksburg, VA, AIAA J., 14 (10), pp 1387-1394 (Oct 1976) 1 fig, 15 refs

**Key Words:** Eigenvalue problems, Rotating structures, Gyroscopes, Spacecraft

An important question associated with the eigenvalue problem for flexible gyroscopic systems is that of discretization of continuous elastic members. If discretization is performed by the assumed modes method, the question arises as to the type of functions to be used in series expansions. In particular, the question is whether one should use rotating-appendage eigenfunctions, fixed-based eigenfunctions, or any other set of admissible functions.

## COMPUTER PROGRAMS

### GENERAL

(Also see Nos. 281, 322, 337, 354, 440)

77-255

#### Dynamics of Inertia Variant Machinery

G.C. Hud

Ph.D. Thesis, Univ. of Pennsylvania, 247 pp (1976)  
UM 76-22, 706

**Key Words:** Computer programs, Equations of motion, Mechanical systems

It is shown in this dissertation how Lagrange's form of d'Alembert's Principle can be successfully adapted to a systematic modeling procedure for automatically generating the equations of motion, and associated equations of constraint, for multi-loop, multi-freedom mechanical systems. Using this principle, a general purpose, user-oriented computer program, DYMAC, for solving planar problems involving either lower or higher order pairings has been developed and tested.

## ENVIRONMENTS

### ACOUSTIC

(Also see Nos. 240, 241, 252, 291, 292, 351, 354, 371, 376, 387, 388)

77-256

#### Pulse Analysis of Acoustic Emission Signals

J.R. Houghton

Ph.D. Thesis, Vanderbilt Univ., 200 pp (1976)  
UM 76-22, 345

**Key Words:** Acoustic signatures, Spectrum analysis

A method for the signature analysis of pulses in the frequency domain and the time domain is presented. Fourier spectrum, Fourier transfer function, shock spectrum and shock spectrum ratio were examined in the frequency domain analysis and pulse shape deconvolution was developed for use in the time domain analysis. Comparisons of the relative performance of each analysis technique are made for the characterization of acoustic emission pulses recorded by a measuring system. To demonstrate the relative sensitivity of each of the methods to small changes in the pulse shape, signatures of computer modeled systems with analytical pulses are presented. Optimization techniques are developed and used to indicate the best design parameters values for deconvolution of the pulse shape. Several experiments are presented that test the pulse signature analysis methods on different acoustic emission sources.

77-257

#### Sound Production by Organ Flue Pipes

N.H. Fletcher

Dept. of Physics, Univ. of New England, Armidale, New South Wales 2351, Australia, J. Acoust. Soc. Amer., 60 (4), pp 926-936 (Oct 1976) 6 figs, 25 refs

**Key Words:** Sound generation, Musical instruments

A nonparametric treatment is devised to allow calculation of the transient and steady-state acoustic spectra of a flue organ pipe, given the geometry of its construction and the pressure variation in its foot. Explicit formulation is given for the fundamental and the first and second upper partials. A scaling law is set out, relating the behavior of pipes of different fundamental frequencies, and several illustrative calculations are performed.

77-258

#### Second Harmonic Generation in Elastic Surface Waves on an Isotropic Solid

G.V. Anand

Dept. of Electrical Communication Engrg., Indian Inst of Science, Bangalore-560012, India, Intl. J. Nonlinear Mech., 11 (4), pp 277-284 (1976) 15 refs

**Key Words:** Elastic waves, Wave propagation, Isotropy

A procedure for solving the problem of non-linear propagation of elastic surface waves is given. An expression for the particle displacement of the second harmonic is obtained.

77-259

**Urban Noise Pollution (A Bibliography with Abstracts)**

E.J. Lehmann

National Technical Information Service, Springfield, VA., 109 pp (July 1976) (Supersedes NTIS-PS-75/544 and NTIS/PS-74/106)  
NTIS/PS-76/0585/0GA

**Key Words:** Urban noise, Traffic noise, Noise reduction, Bibliographies

Aspects of noise in the urban environment are covered. The topics were chosen for their interest to urban planners and include citizen attitudes, transportation noise, and noise abatement techniques.

77-260

**Measurement and Prediction of Sound Attenuation by Buildings Using Acoustic Modeling Techniques**

E.S. Ivey

Ph.D. Thesis, Univ of Massachusetts, 117 pp (1976)  
UM 76-22, 266

**Key Words:** Urban noise, Noise reduction, Buildings, Mathematical models

The attenuation of sound as it propagates over building-size barriers is an important part of the general urban and suburban noise propagation problem. While the sound attenuation over finite thickness barriers has been studied analytically and experimentally, these studies have not produced a convenient methodology for predicting the noise attenuation characteristics of building-size barriers. These studies have, however, identified several factors which influence the noise attenuation of any particular barrier configuration.

77-261

**Mathematical Models of Acoustic and Acoustic-Gravity Wave Propagation in Fluids with Height-Dependent Sound Velocities**

W A. Kinney

Ph.D. Thesis, Georgia Inst of Technology, 172 pp (1976)  
UM 76-23, 735

**Key Words:** Elastic waves, Waves propagation, Modal synthesis, Mathematical models, Computer programs

Several problems which relate to the propagation of acoustic and acoustic-gravity waves in a medium whose properties vary with height only are considered with the intent of

refining existing schemes for the synthesis of waveforms. The contribution from very low frequencies to a modal synthesis of an acoustic-gravity waveform is clarified. A guide is provided for adapting a computer program to include such contributions in the synthesis of waveforms. A geometric acoustical scheme is outlined for the prediction of the amplitudes of waves that propagate over long distances. A number of FORTRAN subprograms are provided that exemplify the numerical implementation of this scheme. Recommendations are given for the refinement at low and high frequencies of schemes for the synthesis of waveforms.

**RANDOM**

(See No. 376)

**SEISMIC**

(Also see Nos. 331, 442)

77-262

**A Perturbation Scheme for Obtaining Partial Derivatives of Love-Wave Group-Velocity Dispersion**

D. Kosloff

Seismological Lab., California Inst. of Technology, Pasadena, CA, Rept. No. AFOSR-TR-76-0821, 10 pp (Feb 10, 1975) (Published in the Bull. of the Seismological Society of America, 65 (6), pp 1753-1760 (Dec 1975))

AD-A027 989/3GA

**Key Words:** Seismic waves, Perturbation theory, Computer programs

A method is derived for obtaining partial derivatives of Love-wave group-velocity spectra for a layered medium using a second-order perturbation theory. These partials are a prerequisite for systematic inversion of group-velocity spectra. Mathematically the equation of motion and boundary conditions for Love waves are a singular Sturm Liouville type eigenvalue problem.

**SHOCK**

(Also see Nos. 250, 252, 293, 360, 363, 365, 380, 381, 382, 383, 391, 392, 393, 394, 399, 435, 436)

77-263

**A Computer-Oriented Deterministic-Cum-Probabilistic Approach for the Extreme Load Design of Complex Structures**

H Kamil

Engineering Decision Analysis Co., Inc., 480 California Ave., Palo Alto, CA, Computers and Struct., 6 (4/5), pp 375-379 (Aug/Oct 1976) 16 refs

**Key Words:** Computer aided design, Shock-resistant design, Interaction: structure-foundation, Probability theory

To obtain a rational, economical, and reliable structural design under extreme loading conditions, a combination of deterministic and probabilistic approaches need to be used. Computers can be effectively used to combine these two approaches together and apply them to the design of structures. State-of-the-art techniques are reviewed for the deterministic and the probabilistic methods. Computers can be used for the application of a combined deterministic-cum-probabilistic approach in the following two areas: (1) determination of design loads for multiple load cases by combining the individual load cases probabilistically outside the structure-foundation system and then determining the element load cases deterministically inside the system; (2) determination of the reliability of the foundation-structure system by first modeling the member property variations at element level and then generating the system reliability using matrix algebra techniques, along with the system stiffness matrix. Methodologies are presented to describe how the above objectives can be achieved. Limitations of the proposed methodologies are described and recommendations for future studies are made.

**77-264**

**Correlation of Impact and Explosively Created Ground Shock Phenomena**

M B. Ford

Army Engineer Waterways Experiment Station, Vicksburg, MS, Rept. No. WES-MP-N-76-10, 74 pp (June 1976)

AD-A027 059/5GA

**Key Words:** Ground motion, Explosion effects

A technique for prediction of ground motions in the transition and far-out regions where outrunning surface waves are transporting most of the energy is described. A statistically significant number of surface ground motion velocity measurements were produced by detonating high-explosive (HE) spheres of differing yields and depths of burst. Similar measurements were obtained from energy sources realized by impacting free-falling weights. The tests in this study were conducted on a nearly homogeneous sandstone formation in October 1973 at a site near Grand Junction, Colorado, along with Project CENSE (Coupling Efficiency of Near Surface Explosions).

**77-265**

**One-Dimensional Shock and Acceleration Waves in Deformable Dielectric Materials with Memory**

P.J. Chen, M.F. McCarthy and T.R. O'Leary  
Sandia Labs., Albuquerque, NM, Archive Rational Mech. Anal., 62 (2), pp 189-207 (Aug 18, 1976)  
12 refs

Sponsored by ERDA

**Key Words:** Shock wave propagation

The behavior of one-dimensional shock and acceleration waves in deformable dielectric materials with memory are examined. The differential equation which the amplitude of the shock must obey was derived. The properties of the electric field during shock transition are examined and certain of its properties in terms of the material properties are derived. The differential equation which the amplitude of the acceleration wave must satisfy is derived.

**GENERAL WEAPON**

(Also see No. 285)

**77-266**

**Underground Ammunition Storage. Report I. Test Program, Instrumentation, and Data Reduction**

A. Skjeltorp, T. Hegdahl and A. Jenssen

Office of Test and Development, Norwegian Defence Construction Service, Oslo, Norway, Rept. No. Fortifikatorisk Notat-80/72, 68 pp (Sept 1975) (See also AD-A027 064)

AD-A027 063/7GA

**Key Words:** Underground structures, Tunnels, Ammunition, Storage, Explosion effects, Model testing

This report is the first in a series of five describing an extensive series of model tests on air blast propagation in underground ammunition storage sites. A general description is given of the scope and purpose of the tests as well as a presentation of the principles of scaling and various intrinsic uncertainties which may cause problems in the modeling technique. The report also surveys the test program, instrumentation, and data reduction for the whole test series.

**77-267**

**Underground Ammunition Storage. Blast Propagation in the Tunnel System. Report II. A. Chamber Pressure**

A. Skjeltorp, T. Hegdahl and A. Jenssen

Office of Test and Development, Norwegian Defence Construction Service, Oslo, Norway, Rept. No. Fortifikatorisk Notat-79/72, 42 pp (Sept 1975) (See also AD-A027 065)

AD-A027 064/5GA

**Key Words:** Underground structures, Tunnels, Ammunition, Storage, Explosion effects, Shock wave propagation

The pressure-time history for detonation and burning of various explosives in confined regions (chambers) is examined. In the detonation tests, a systematic study was performed for three types of explosives (TNT, PETN, and Dynamite) for different loading densities and chamber venting. Four additional explosives (AN/FO, RDX, ALUMIT, and COMP.B) were also tested over a limited range of loading densities in a closed chamber. To determine the maximum 'chamber pressure', carefully designed electrical filters were employed to remove the high frequency ringing in the pressure-time recordings. Some preliminary tests were performed to determine the ignitability for the explosives used in the detonation tests using the closed bomb method.

**77-268**

**Underground Ammunition Storage. Blast Propagation in the Tunnel System. Report III. A. Single Chamber Storage. Variable Tunnel Diameter and Variable Chamber Volume**

A. Skjeltop, T. Hegdahl and A. Jenssen

Office of Test and Development, Norwegian Defence Construction Service, Oslo, Norway, Rept. No. Fortifikatorisk Notat-81/72, 60 pp (June 1975) (See also AD-A027 066)

AD-A027 065/2GA

**Key Words:** Underground structures, Tunnels, Ammunition, Storage, Blast effects, Shock wave propagation, Model testing

Model tests have been performed to determine the blast wave propagation in the tunnel system of underground single chamber storage sites. A total of nine configurations were tested with variable tunnel diameters (6.7 - 13.5 cm), chamber volumes (7250 - 15200 cc), and TNT loading densities (1 - 70 kg/cu m). Comparisons are given with one large scale test and the importance of wall roughness attenuation is discussed.

**77-269**

**Underground Ammunition Storage. Blast Propagation in the Tunnel System. Report IV. A. Connected Chamber Storage. Variable Chamber Volume and Variable Angle between Branch and Main Passageway**

A. Skjeltop, T. Hegdahl and A. Jenssen

Office of Test and Development, Norwegian Defence Construction Service, Oslo, Norway, Rept. No. Fortifikatorisk Notat-82/72, 48 pp (Nov 1975) (See also AD-027 067)

AD-A027 066/0GA

**Key Words:** Underground structures, Tunnels, Ammunition, Storage, Blast effects, Shock wave propagation, Model testing

Model tests have been performed to determine the blast wave propagation in the tunnel system of underground ammunition storage sites with connected storage chambers. A total of 18 different configurations were tested with variable chamber volumes (300 - 15200 cc), angles between the branch and main passageway (35 - 90 deg.), and ratios between the cross sections of the branch and main passageway (0.125 - 0.5). The models were in linear scales 1:40 to 1:100 of typical full scale installations. By analyzing the data, it was possible to obtain relatively simple scaling relationships which contain all of the most important geometrical parameters. The model data are compared with one large scale test and fair agreement is found.

**77-270**

**Underground Ammunition Storage. Blast Propagation in the Tunnel System. Report V. A. Connected Chamber Storage Blast Load on Doors in Three Sites**

A. Skjeltop, T. Hegdahl and A. Jenssen

Office of Test and Development, Norwegian Defence Construction Service, Oslo, Norway, Rept. No. Fortifikatorisk Notat-83/72, 32 pp (Sept 1975) (See also AD-027 063)

AD-A027 067/8GA

**Key Words:** Underground structures, Tunnels, Ammunition, Storage, Blast effects, Shock wave propagation, Model testing

The result from airblast measurements in models of three particular types of underground ammunition storage sites is reviewed. The use of simple scaling relationships are shown to reproduce the blast data reasonably well for a wide range of geometrical parameters.

## PHENOMENOLOGY

### COMPOSITE

(Also see Nos. 283, 297, 340)

**77-271**

**Harmonic Waves in Three-Dimensional Elastic Composites**

S. Minagawa and S. Nemat-Nasser  
Dept. of Civil Engrg., Northwestern Univ., Evanston,  
IL 60201, Intl. J. Solids Struct., 12 (11), pp 769-  
777 (1976) 7 figs, 9 refs

**Key Words:** Composite materials, Wave propagation

For a periodic elastic composite which consists of a matrix and fibers with finite dimensions (i.e., a three-dimensional problem), estimates for eigenfrequencies and eigenfunctions are given. Calculations are based on a new quotient proposed by Nemat-Nasser. The periodic character of the eigenfrequencies is pointed out, and illustrative examples are given.

**77-272**

**Analysis of Wave Mode in Compound Elastic Circular Rods**

H. Toda and H. Fukuoka  
Dept. of Engrg. Science, Osaka Univ., Toyonaka,  
Japan, Bull. JSME, 19 (133), pp 755-760 (July 1976)  
5 figs, 7 refs

**Key Words:** Rods, Composite materials, Frequency equation

An analysis of the wave modes in compound elastic circular rods, which consists of a circular matrix rod and a circular sheath tube made of different materials, was executed.

**77-273**

**On Continuum Modeling of the Dynamic Behavior of Layered Composites**

G. Herrmann, R.K. Kaul and T.J. Delph  
Dept. of Mech. Engrg., Div. Appl. Mech., Stanford  
Univ., Stanford, CA 94305, Archives of Mechanics,  
28 (3), pp 405-421 (1976) 17 figs, 33 refs  
Sponsored by AFOSR

**Key Words:** Composite materials, Continuum mechanics, Mathematical models

A hierarchy of new approximate continuum theories to model the dynamic behavior of layered composites is advanced. In each approximation the approximate frequency spectra are matched as closely as possible with the exact spectra, and the range of validity of each approximation is ascertained. The procedure followed is like that employed in deriving approximate plate theories. Similarities to certain phenomena in solid state physics are pointed out, and an analogy to a system with one degree of freedom is drawn.

**77-274**

**Critical Factors for Frequency-Dependent Fatigue Processes in Composite Materials**

W.W. Stinchcomb, K.L. Reifsnider and R.S. Williams  
Engrg. Science and Mechanics Dept., Virginia Poly-  
technic Inst. and State Univ., Blacksburg, VA 24061,  
Exptl. Mech., 16 (9), pp 343-348 (Sept 1976) 6 figs,  
17 refs

Sponsored by AFOSR

**Key Words:** Composite materials, Fatigue (materials), Frequency response

Several fatigue-test parameters, including cyclic frequency, pre-fatigue material conditioning (preloading and step loading) and test-control modes (strain control and load control) are investigated and their effect on the fatigue response of composite materials is discussed. A conceptual model based on the test results is offered to aid in the understanding of fatigue processes in composite materials and the effect of frequency on fatigue response.

## DAMPING

**77-275**

**Damping Synthesis from Substructure Tests**

T.K. Hasselman  
J.H. Wiggins Company, Redondo Beach, CA, AIAA  
J., 14 (10), pp 1409-1418 (Oct 1976) 6 figs, 10 refs

**Key Words:** Modal damping

A new method is proposed for synthesizing structural damping from substructure test data. It utilizes the off-diagonal coupling terms in the substructure modal damping matrices as well as the diagonal terms which correspond to uncoupled modal damping. The coupling terms are evaluated from complex resonant response measured at each substructure mode. Tentative accuracy requirements on experimental data and data reduction are suggested.

**77-276**

**Steady Impact Vibration of a Body Having Hysteresis Collision Characteristics (2nd Report, System with Quadrilateral Hysteresis Loop Characteristics for Force of Restitution)**

S. Maezawa and T. Watanabe  
Dept. of Engrg., Yamanashi Univ., Kofu, Japan,  
Bull. JSME, 19 (134), pp 902-911 (Aug 1976)  
9 figs, 6 refs

**Key Words:** Hysteretic damping, Harmonic excitation

Steady impact vibrations in a mechanical system subjected to various harmonic exciting forces are analyzed by a perfect Fourier series method reinforced by convergent improvement by means of a series transformation. The hysteresis loop characteristics for force of restitution are assumed to be composed of four straight-line segments.

## ELASTIC

**77-277**

### **Vibrations of Solids in Terms of Quantum Equations of Elasticity**

S. Kaliski

Dept. of Mechanics of Continuous Media, Institute of Fundamental Technological Research, Polish Academy of Sciences, Swietokrzyska 21, 00-049 Warsaw, Poland, Bull. Acad. Polon. Sci., Ser. Sci. Tech., 24 (6), pp 293-297 (1976) 5 refs

**Key Words:** Plates, Elasticity theory, Eigenvalue problems, Natural frequencies

The problem of natural vibrations is solved for a rectangular parallelepiped and a plate, on the basis of equations of the theory of elasticity, taking into consideration the linearized quantum effects which are introduced by the methods of "classical" approach to quantum mechanics. The eigenvalues of the boundary value problems are calculated.

**77-278**

### **Wave Generation in an Elastic Half-Space by a Normal Point Load Moving Uniformly Over the Free Surface**

A. Ungar

Natl. Res. Inst. for Math. Sci., CSIR, Pretoria, South Africa, Intl. J. Engr. Sci., 14 (10), pp 935-945 (1976) 16 refs

**Key Words:** Half space, Elastic properties, Moving loads, Wave propagation

The motion is determined of an elastic, homogeneous and isotropic half-space, excited by a normal point load traveling uniformly over the free surface. Using the differential transform technique, exact, closed expressions in terms of algebraic functions are found for the displacements at any point of the half-space.

## FATIGUE

(See No. 274)

## FLUID

(Also see No. 331)

**77-279**

### **The Added Mass and Damping Coefficients of and the Excitation Forces on Four Axisymmetric Ocean Platforms**

K.J. Bai

Ship Performance Dept., David W. Taylor Naval Ship Res. and Dev. Ctr., Bethesda, MD, Rept. No. SPD-670-01, 33 pp (Apr 1976) AD-A027 377/1GA

**Key Words:** Floating bodies, Damping coefficients, Finite element technique, Oceans

Numerical results of the added mass and damping coefficients of vertical axisymmetric bodies on or under the free surface are presented. The excitation forces on these bodies due to an incident regular wave system are also computed. The numerical scheme employs a localized finite-element method, which is based on the theory of the calculus of variations. The excitation forces and moments on a submerged half-spheroid lying on the bottom are computed and compared with the results obtained by others.

**77-280**

### **Experiments on Transition to Turbulence in an Oscillatory Pipe Flow**

M. Hino, M. Sawamoto and S. Takasu

Dept. of Civil Engrg., Tokyo Inst. of Technology, O-okayama, Meguro-ku, Tokyo 152, Japan, J. Fluid Mech., 75 (2), pp 193-207 (May 27, 1976) 9 figs, 15 refs

**Key Words:** Fluid-filled containers, Pipes (tubes), Turbulence

Experiments on transition to turbulence in a purely oscillatory pipe flow were performed. Three types of turbulent flow regime were detected: weakly turbulent flow, conditionally turbulent flow and fully turbulent flow. In the conditionally turbulent flow, turbulence is generated suddenly in the decelerating phase and the profile of the velocity distribution changes drastically. In the accelerating phase, the flow recovers to laminar.

## INELASTIC

77-281

### Dynamic Plastic Analysis of Ductile Fracture -- the Charpy Specimen

D.J. Ayres

Combustion Engineering, Inc., Windsor, CT 06095  
Intl. J. Fract., 12 (4), pp 567-578 (Aug 1976) 15 figs, 12 refs

**Key Words:** Dynamic plasticity, Fracture properties, Finite element technique, Computer programs

A two-dimensional elastic-plastic dynamic finite element analysis of the precracked Charpy V-notch specimen is performed using the MARC finite element computer program. The results of the analysis are in good agreement with data produced from the instrumented precracked Charpy test. This agreement verifies the computational techniques for this type of analysis and provides a much more detailed view of the phenomena of ductile fracture. The analytical procedure developed will assist in the formulation and evaluation of ductile fracture criteria and the application of these criteria to pressure vessel and structural analysis.

77-282

### Application of an Energy Balance and an Energy Method to Dynamic Crack Propagation

W. Doll

Institut für Festkörpermechanik, 78 Freiburg i Br., Republic of Germany, Intl. J. Fract., 12 (4), pp 595-605 (Aug 1976) 8 figs, 27 refs

**Key Words:** Dynamic plasticity, Fracture properties

The speeds of fast running cracks in a range of quasi-brittle materials are measured as a function of the dynamic strain energy release rate. From the results, the heat output associated with the plastic work at the crack tip is calculated as a function of crack speed using an energy balance, and compared with the heat outputs determined experimentally using an energy method. A generally good agreement is found between the calculated and the experimentally measured variation of heat output with crack speed.

## VISCOELASTIC

(Also see No. 340)

77-283

### Vibration Analysis of Viscoelastic Bodies with Small Loss Tangents

Z. Hashin

School of Engineering, Tel-Aviv Univ., Israel, Rept. No. TAU-SOE/340-76, Scientific-4, AFOSR-TR-76-0830, 22 pp (Apr 1976)  
AD-A027 703/8GA

**Key Words:** Viscoelastic media, Forced vibration, Composite materials

The correspondence principle for vibrations of viscoelastic bodies is specialized to the case of small loss tangents, resulting in considerable simplification: analytical evaluation of oscillatory fields is greatly simplified: peak frequencies and peak amplitudes under forced vibrations can be simply and directly determined; numerical solution of viscoelastic vibration problems becomes no more complicated than that of elastic problems. Similar simplifications result for computation of real and imaginary parts of effective complex moduli of composite materials.

## EXPERIMENTATION

### BALANCING

(Also see Nos. 407, 408, 409, 410, 412, 413)

77-284

### The Dynamics of a Hooke's Joint Gyroscope with Mass Unbalance

J.S. Burdess and L. Maunder

Univ of Newcastle upon Tyne, England, "Conf. on Vibrations in Rotating Machinery," The Institution of Mechanical Engineers, Univ. of Cambridge, (Sept. 15-17, 1976) pp 195-199, 2 refs

**Key Words:** Gyroscopes, Unbalanced mass response

The dynamical characteristics of a Hooke's Joint Gyroscope with mass unbalance in both the rotor and the gimbal are described. For a constant rate input it is shown that the ideal gyroscope may operate as a device for measuring either angular rates of turn or angular displacements depending upon whether the gimbal suspension is tuned to the rotor speed.

## DIAGNOSTICS

(See Nos. 304, 369, 404, 405, 406, 445)

## FACILITIES

(Also see Nos. 400, 426)

**77-285**

### **A New Acoustics Centre for Noise Investigations by Simulating Real Life Conditions**

E. Schroder

Automobiltech. Z., 78 (7/8), pp 345-349 (July/Aug 1976) 6 figs, 15 refs

(In German)

**Key Words:** Test facilities, Motor vehicle noise

Detailed noise investigations on a vehicle are carried out in the new Acoustics Centre in Cologne, Merkenich. The anechoic chamber, equipped with built-in two-axle chassis dynamometer of latest design is described. The adjacent reverberation room serves to determine the sound power and to improve the noise characteristics of vehicle components such as engine, transmission, and fan. A special opening between the reverberation room and a transmitter room with reverberation room characteristics allows specific sound-transmission loss investigations to be made on damping materials fitted to body parts such as vehicle dash panel assemblies.

## INSTRUMENTATION

(Also see No. 292)

**77-286**

### **An Acoustic Monitoring System for the Vibroseis Low-Frequency Underwater Acoustic Source**

A.M. Young and G.D. Hugus

Naval Research Lab., Washington, D.C., Rept. No. NRL-MR-3316, 22 pp (June 1976)

AD-A028 239/2GA

**Key Words:** Underwater sound transmission, Acoustic measurement, Measuring instruments

One problem normally associated with the use of Low-frequency, high-power underwater acoustic sources is the ability to accurately monitor the generated sound pressure level. The design, fabrication and installation of an acoustic monitoring system for a large hydraulically actuated source,

the VIBROSEIS system, is described. The monitoring transducers and the associated calibration system were designed to operate in the 5- to 225-Hz frequency range and to minimize several problems associated with the particular source. The system was installed, calibrated at sea, and performed satisfactorily during the ocean exercise.

**77-287**

### **Resonant Pressure Instrumentation**

G.M. Schuster

Ph.D. Thesis, Polytechnic Inst. of New York, 225 pp (1976)

UM 76-23, 445

**Key Words:** Resonators, Measuring instruments, Design techniques, Calibrating

This dissertation is concerned with the development of analytical techniques which can be usefully applied to the design and calibration of resonant pressure transducers, with emphasis on those instruments which use an elastic resonator as the basic sensor. The dissertation shows how pressure-frequency relations may be developed for distributed resonators, both by direct solution of linear differential equations and by approximation methods for nonlinear cases. These relations are useful in establishing the operating frequency range and pressure sensitivity as functions of resonator dimensions and elastic properties, so that different resonator configurations may be compared.

**77-288**

### **Analysis of Induced EMF in Vibrating-Sample Magnetometers**

E. E. Bragg and M.S. Seehra

Physics Dept., West Virginia Univ., Morgantown, WV 26506, J. Phys. E. (Sci. Instr.), 9 (3), pp 216-222 (Mar 1976) 13 figs, 10 refs

**Key Words:** Vibration measurement, Measuring instruments

The theoretical induced EMF for vibrating-sample magnetometers is derived. Assuming a point magnetic dipole sample, located in a general position with respect to a single-turn circular pickup coil, the instantaneous flux cutting the coil and the induced EMF due to sample motion are found to be linear combinations of elliptic integrals of the first and second kind. An alternative representation of the flux by a power series is also given. These results can be applied to arbitrary versions of magnetometers by specifying coil size and sample position and orientation. The effect of sample misalignment on the induced EMF is computed. A comparison of the four systems is also made.

## TECHNIQUES

(Also see Nos. 308, 317, 318, 364,  
371, 387, 395, 422)

77-289

### Comparison of Methods for Measurement of Reverberation Time

T.E. Vigran and S. Sorsdal

Dept. of Acoustics, The Norwegian Inst. of Technology, Trondheim, Norway, *J. Sound Vib.*, **48** (1), pp 1-13 (Sept 8, 1976) 10 figs, 9 refs

**Key Words:** Acoustic measurement, Measurement techniques, Test facilities

Five methods for automatic measurement of reverberation time are studied with respect to the mean value and the variance of it. The measurements were performed in two reverberation chambers. For each 1/3-octave band and microphone position one hundred decay signals were evaluated. It is felt that an automatic method should be recommended to supplement the "straight line" method outlined in existing standards.

77-290

### Shock-Front Loading Method for Studies in Dynamic Photoelasticity

A.W. Miles

Dept. of Mech. Engrg., Univ. of Cape Town, Rondebosch, 7700, Cape Province, South Africa, *Exptl. Mech.*, **16** (9), pp 349-355 (Sept 1976) 11 figs, 17 refs

**Key Words:** Photoelastic analysis

A versatile technique for applying a well-defined dynamic load to models for studies in dynamic photoelasticity is described. The method utilizes the shock front produced in a gas-dynamic shock tube to apply a load to models by direct normal impact. The principles and scope of the method is suited to studies where simple variation and accurate determination of the load-cycle parameters, as well as precise reproducibility, are necessary. The method, in addition, permits close-field study of the initial response of materials to dynamic loading to be undertaken.

77-291

### On the Use of Acoustical Holography to Locate Sound Sources on Complex Structures

E.E. Watson and W.F. King, III

Environmental Acoustics Div., Universal Oil Products, Wolverine Tube Div., Decatur, AL, *J. Sound Vib.*, **48** (2), pp 157-168 (Sept 1976) 14 figs, 15 refs  
Sponsored by the Naval Sea Systems Command

**Key Words:** Noise source identification, Testing techniques, Fans, Acoustic holography

Until recently, it was extremely difficult to detect and locate far-field sound sources on complex structures. An experimental technique is presented in this paper for the identification of such sound sources on complex vibrating structures. Results are given for a preliminary investigation designed to determine the feasibility of applying a modified form of acoustical holography to the location of sound sources on ventilator fans. Such sources are identified on a three-bladed ventilator fan. Optical and computer sound source reconstructions are compared and analyzed.

77-292

### Measuring Noise -- The State-of-the-Art

A.J. Schneider

B & K Instruments, Inc., Cleveland, OH, 44142, SAE Paper No. 760672, 8 p, 8 figs, 3 refs

**Key Words:** Noise measurement, Measurement techniques, Measuring instruments

New instruments and new measurement concepts continually evolve in support of programs to reduce operator noise exposure and produce noise emission. The purpose of this paper is to review new equipment and new measurement practices which can assist in the direction of programs to develop quieter vehicles. Opportunity is taken to also discuss the closely associated field of whole-body vibration.

# COMPONENTS

## ABSORBERS

(Also see No. 399)

77-293

### Structural Analysis and Design for Energy Absorption in Impact

E.H. Lee and R.L. Mallett

Dept. of Applied Mechanics, Stanford Univ., CA, Rept. No. SUDAM-75-15, DOT/TST-76/44, 215 pp (Dec 1975)

PB-254 801/4GA

**Key Words:** Collision research (automotive), Beams, Shells, Energy absorption

A general assessment of the nature of the dynamic problem of analyzing the collision of a vehicle is first given. A vehicle commonly comprises an open structure of beams and thin shells of ductile metal and the kinetic energy to be absorbed as energy of deformation is usually sufficient to cause plastic flow. This is generally localized on hinges and is associated with appreciable elastic deformation. Thus elastic-plastic theory is needed including geometrical and material nonlinearities. Integration through the history of the motion is needed for solution. Theorems are given which provide approximations to the maximum plastic deformation and the duration of plastic flow without determining the whole solution. Application of these to simple models is presented. The theorems are also applied to determine the stiffness of a structural element so that it will transmit impulse to the rest of the structure without itself deforming appreciably, a so-called stiff interface. The ability of porous metal to absorb energy of deformation is also analyzed by solving the microscopic problem of collapse of the cavities. The overall macroscopic deformation laws can then be determined. Tests of the behavior of porous metal in impact are presented and related to quasistatic measurements of its deformation properties. Conclusions from these studies and recommendations for new work are given.

## BEAMS, STRINGS, RODS

(Also see Nos. 245, 272, 293, 313, 321, 341)

77-294

### Dynamic Torsion of a Two-Layer Viscoelastic Beam

A F. Johnson and A. Woolf

Natl. Phys. Lab., Teddington, Middlesex TW11 0LW, England, *J. Sound Vib.*, **48** (2), pp 251-263 (Sept 1976) 3 figs, 10 refs

**Key Words:** Beams, Viscoelastic properties, Torsional vibrations

The complex dynamic shear modulus of soft polymeric materials may be determined in principle at low and audio frequencies from torsion pendulum and torsional resonance experiments on metal strips coated with the polymer. In order to determine the polymer shear properties from such experiments, it is necessary to know the torsional rigidity of the two-layer compound beam. This is calculated in the paper by using classical elasticity theory, for the particular case when the metal shear modulus is much greater than that of the soft coating. The theory of dynamic torsion tests is then briefly reviewed and experiments are suggested for determining the polymer dynamic shear properties. A discussion is also given of the effectiveness of polymer coatings as a damping treatment for torsional vibrations.

77-295

### Dynamics of Timoshenko Beams Conveying Fluid

M.P. Paidoussis and B.E. Laithier

Dept. of Mech. Engrg., McGill Univ., Montreal, Canada, *J. Mech. Engr. Sci.*, **18** (4), pp 210-220 (Aug 1976) 8 figs, 13 refs

**Key Words:** Beams, Pipes (tubes), Timoshenko theory, Fluid-filled containers, Finite difference theory, Variational methods

The dynamics of pipes conveying fluid is described by means of the Timoshenko beam theory. The equations of motion are derived and solved (a) by a finite-difference technique, and (b) by a variational method. It is shown that the latter is the more efficient method. The eigenfrequencies of the system and its stability characteristics are compared with results obtained previously using the Euler-Bernoulli beam theory, and it is shown that in certain cases (e.g. short pipes) the two sets of results diverge. Experiments indicate that the present theory is more successful in predicting the observed behavior. Furthermore, the present theory shows that, in some cases, cantilevered pipes may lose stability by buckling, whereas previous theories indicate that the system always loses stability by flutter.

77-296

### Nonlinear Periodic Response of a Slender Beam-Column

B.P. Shafer

Ph D. Thesis, Univ of Denver, 183 pp (1976)

UM 76-27, 873

**Key Words:** Beams-columns, Periodic response, Nonlinear response

The nonlinear dynamic response of a perfect beam-column subjected to loading by axial stress waves was studied. Newton's method was used to solve the nonlinear equations iteratively and to determine the lateral displacement and axial strain as functions of time. An unstable linear approximation was used to start the iterative procedure, and to guide the search of the frequency domain for interesting nonlinear phenomena.

77-297

**Comparison of Solution Methods for Composite Material Beam Response Due to Impact**

P.C. Chou and W.J. Flis

Drexel Univ., Philadelphia, PA, Rept. No. NADC-76093-30, 59 pp (Dec 1975)  
AD-A028 317/6GA

**Key Words:** Beams, Composite materials, Impact response (mechanical), Computer programs

Several solution methods are compared as to their usefulness in treating impact problems. The problem used as a basis for comparison involves a simply supported beam impacted at mid-span by a mass. The early-time and late-time structural responses of the system have been computed by (1) an analytical solution, (2) the NASTRAN finite-element, structural-analysis computer program, using both the direct integration method and the modal analysis method, and (3) the method of characteristics. The results have been compared. The chief conclusions are that the method of characteristics is the most accurate method for early-time, and that the NASTRAN modal method is best suited for late-time structural response.

77-298

**Effects of Support Flexibility on the Stability of a Beam Under a Follower Thrust and Inertia**

J J Wu

Watervliet Arsenal, Watervliet, NY, Rept No WVT-TR-76024, 20 pp (June 1976)  
AD-A028 089/1GA

**Key Words:** Beams, Follower forces, Flexible foundations, Flutter, Stability analysis

The stability behavior of a slender structure, modeled by a beam, subjected to a follower force at one end and restrained at the other, is studied in detail in this paper. Eigenvalue curves which dictate the stability behavior are plotted for several combinations of parameters.

77-299

**Three-Dimensional Stochastic Response of Offshore Towers to Wave Action**

B. Berge and J. Penzien

Structural Engrg. Lab., California Univ., Berkeley, CA  
Rept. No. UCSESM-75-10, 158 pp (Oct 1975)  
PB-254 049/0GA

**Key Words:** Off-shore structures, Towers, Stochastic processes

A theory is developed to calculate the dynamic response of off-shore towers to random wave forces. Vibrations are considered simultaneously for translations in the orthogonal horizontal directions and for rotations about a vertical axis. The idealized structure for the dynamic analysis has its masses lumped at discrete horizontal levels. The ocean waves are considered to be a zero mean stationary ergodic Gaussian random process described by the directional wave spectrum, which specifies the distribution of wave energy with respect to frequency and direction.

77-300

**Vibrations of Suspended Cables**

J.G. Gale

Ph.D. Thesis, Oregon State Univ., 132 pp (1976)  
UM 76-23,515

**Key Words:** Cables, Equations of motion

The equations of motion of an elastic cable suspended in a viscous medium from two arbitrary points are derived. These are then linearized for the undamped, inextensible case, and it is found that the equations describing the in-plane motion of the cable are then independent of the equation describing the out-of-plane motion of the cable. Using the equations developed for the normal mode motion, a technique is presented for determination of the displacements and tensions throughout a cable when one end is subjected to a prescribed tangential displacement of known frequency. Maximum cable tensions are presented as a function of the forcing frequency for a variety of cable geometries.

77-301

**An Experimental Investigation of Wave Propagation in a Rubber String Impacted by a Projectile**

C. Riegel and J.L. Nowinski

Univ of Delaware, Newark, DE 19711, Intl. J. Nonlinear Mech., 11 (4), pp 229-237 (1976) 8 figs, 9 refs

**Key Words:** Strings, Elastomers, Dynamic tests

Transverse perpendicular impact on long rubber strings was studied using a special apparatus consisting of a system of oscilloscopes, stroboscopes, camera and air pressure gun. The rubbers used in tests came from two manufacturers and were classified as pure gum rubbers. Some of the results are compared with the theoretical data for the Mooney-Rivlin and the Isehara-Hashitsume-Tatibana-Zahorski materials, and others with the results of theoretical equations upon inserting experimental data.

**77-302**

**Dynamic Mechanical Properties of Cotton and Other Fibres**

J.L. Woo

Ph.D. Thesis, Univ. of New South Wales, Australia, (1976)

**Key Words:** Fibers

The dynamic elastic modulus of a polymer is a fundamental physical property which is specifically related to optical birefringence and molecular orientation. Such features of natural fibres as their extremely limited length, irregularity of shape and area of the fibre cross-section, relatively low yield point, spatially indeterminate geometry, fibre fineness and flexibility, and the time-dependence of the fibre mechanical properties, constitute problems which must be overcome before any meaningful and reproducible method can be developed for the determination of the dynamic mechanical properties of these fibres. Following a critical review of applicable physical principles and available experimental techniques, the acoustic pulse propagation method was chosen as the most suitable principle for the present application. In order to satisfy a set of imposed criteria, a kinematic solution was obtained on which the design and construction of a constant-fibre-strain scanner was based. Associated small tools, experimental techniques and procedures have been devised. The dynamic modulus-strain curves for cotton, wool and jute fibres have been obtained. By using numerical integration, the dynamic stress-strain curves were constructed from the dynamic modulus-strain curves, regarding the latter as the derivative stress-strain curves for the specimen.

**BEARINGS**

**77-303**

**Dynamics of Roller Bearings Considering Elasto-hydrodynamic Forces**

M A Molina, D M Sanborn and W.O Winer  
Univ of Carabobo, Valencia, Venezuela, Trans

Amer. Soc. Lubric. Engrs., 19 (4), pp 267-272 (Oct 1976) 8 figs, 13 refs

Sponsored by NASA

**Key Words:** Roller bearings, Elasto-hydrodynamic properties

The life and load carrying capacity of roller bearings are highly influenced by the sliding velocities at the contacts between rollers and races. In this investigation, a computational model was developed to predict traction forces, forces on the cage, sliding and spinning speeds of the roller when the bearing is lubricated with a fluid of known traction coefficient characteristics as functions of pressure and velocities.

**77-304**

**Information from Bearing Vibration**

R.C. Hemmings and J.D. Smith

R.H.P. Bearing Res. Centre, Chelmsford, England, "Conf. on Vibrations in Rotating Machinery," The Institution of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 117-121, 4 figs.

**Key Words:** Ball bearings, Diagnostic techniques, Vibration measurement, Testing techniques

The standard test methods for ball bearings are described and the relevance of testing to installation noise problems is discussed. It is shown that even the best current test specifications are inadequate for use in predicting vibration levels in situ. Variation of test speed allows extraction of further information about bearing errors and resonances to assist design. Further information about the bearing inner ring can be extracted from the measured vibration by making use of sampling techniques: Conventional "whitewashing" and wave analysis techniques are inadequate if used directly but if the signal is first sampled it may then be averaged to yield information about the inner and outer groove shapes separately. The technique is described and its use and limitations are discussed.

**77-305**

**A Theoretical and Experimental Investigation Illustrating the Influence of Non-Linearity and Misalignment of the Eight Oil Film Force Coefficients**

R.H Bannister

Cranfield Inst. of Technology, England, "Conf. on Vibrations in Rotating Machinery," The Institution of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 271-278, 9 figs, 5 refs

**Key Words:** Journal bearings, Hydrodynamic excitation, Lubrication

The results of a combined analytical and experimental investigation, into the hydrodynamic oil film coefficients operating in the laminar region are presented. An improved method for analyzing a dynamically loaded journal is developed, taking into account the second order coefficients characterizing non-linearity, whereby the theoretical elliptical whirl orbit can be deduced for any given periodic disturbing force. The investigation is also extended to take into account the effects of misaligning couples applied in the vertical plane, illustrating that for this kind of misalignment the oil film coefficients are increased, thus having a stabilizing effect on the bearing.

**77-306**

**Stabilization of Turbomachinery with Squeeze Film Dampers -- Theory and Applications**

E.J. Gunter, L.E. Barrett and P.E. Allaire

Univ. of Virginia, Charlottesville, VA, In: "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, 14 figs, 13 refs.

**Key Words:** Turbomachinery, Periodic response, Transient response, Squeeze film bearings, Rotor-bearing systems

This study investigates the steady-state and transient response of the squeeze film damper bearing. Both the steady-state and transient equations for the hydrodynamic bearing forces are derived. The steady-state equations are used to determine the damper equivalent stiffness and damping coefficients. The coefficients are used to find the damper configuration which will provide the optimum support characteristics based on a stability analysis of the rotor-bearing system. The effects of end seals and cavitated fluid film are included. The transient analysis of rotor-bearing systems is performed by coupling the damper and rotor equations and integrating forward in time. The effects of unbalance, bearing cavitation and retainer springs, aerodynamic forces, and internal friction are included in the analysis.

## BLADES

**77-307**

**Analytical and Experimental Study of Subsonic Stalled Flutter**

R.A. Arnoldi, F.O. Carta, R.H. Ni, and A.St. Hilaire Pratt and Whitney Aircraft, East Hartford, CT, Rept No PWA-5420, AFOSR-TR-76-0829, 73 pp (May 1976)

AD-A027 869/7GA

**Key Words:** Flutter, Turbine blades, Wind tunnel tests, Computer programs

A series of experimental measurements of airfoil pressure distributions were made in an eleven-airfoil oscillating cascade wind tunnel. The airfoils were oscillated in pitch about mid-cord pivots. A computer program was adapted to calculate oscillating lift and moment coefficients on the assumptions of incompressible flow, flat plate airfoils, fixed separation point, zero pressure amplitude in the separated region and in the wake, and all streamlines parallel to the airfoil surfaces.

**77-308**

**Experimental Methods for the Study of the Vibratory Behaviour of Steam Turbine Blades**

R. Bigret

GETT, St. Rateau, 93120 La Courneuve, France "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 159-164, 5 figs.

**Key Words:** Turbine blades, Testing techniques, Measurement techniques, Vibration response

The life of blading is greater, the smaller the tensile stresses which result from aerodynamic disturbances. During the running of turbines where the local strains in the blades are measured, power spectral densities can be determined and stresses can be calculated. Since at the design stage, the evaluation of the stresses is still marked by some uncertainty due to the difficulty of knowing precisely the exciting forces and the exact shape of the natural modes, this paper deals with frequency criteria. A theoretical experiment procedure which enables one to know the natural frequencies of the discs obtained is discussed.

**77-309**

**The Characteristics, Prediction and Test Analysis of Supersonic Flutter in Turbofan Engines**

D.G. Halliwell

Rolls-Royce (1971) Limited, Derby Engine Div., England, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 181-185, 6 figs, 3 refs

**Key Words:** Turbine blades, Flutter, Fans

The advent of high tip Mach Number, high work blading in the front fan stages of modern aero gas turbine engines has led to the recognition of a self excited flutter phenomenon in the high speed unstalled regime of the fan operating characteristics. This dangerous stress condition, unlike a resonance, cannot be avoided by passing through the affected speed range. This work, during design and development, has been devoted to obtaining a better understanding of the aero-elastic mechanisms involved. The subject is introduced by describing the vibration modes of unstalled, supersonic flutter in fan assemblies having part-span shrouds or clappers. The vibration pattern is seen to consist of discrete radial and circumferential nodes in the blade-disc assembly which can rotate relative to the rotor. A characteristic of this type of flutter is the selection of a unique, least-stable mode for any particular fan.

**77-310**

**Studies to Gain Insight into the Complexities of Blade Vibration Phenomena**

D.J. Ewins

Imperial College of Science and Technology, London, England, "Conf on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept 15-17, 1976, pp 165-172, 6 figs, 7 refs

**Key Words:** Turbine blades, Turbomachinery, Natural frequencies, Mode shapes, Test data

This paper summarizes a series of investigations which have been made in order to understand, explain, and predict the types of vibration response characteristics which are exhibited by turbomachine blades in their operating environment. The studies have been made using simplified but closely representative models of complete bladed disc assemblies, both with and without shrouding. Using these models, it is possible to demonstrate the vibration characteristics of this type of assembly, both in terms of the natural frequencies and mode shapes and also in terms of the vibration response under typical excitation conditions.

**77-311**

**The Unsteady Forces on Flat-Plate-Airfoils in Cascade Moving through Sinusoidal Gusts**

S Murata and Y. Tsujimoto

Dept of Mech. Engrg, Osaka Univ, Yamada-Kami, Suita, Osaka, Japan, Z Angew Math. Mech., 56 (5), pp 205-216 (May 1976) 16 figs, 12 refs

**Key Words:** Rotor blades (turbomachinery), Flutter

This paper presents an analysis for the determination of the lift fluctuation on flat-plate-airfoils in cascade due to transverse and chordwise gusts. The acceleration potential method is used in combination with conformal mapping method. This method gives an utterly simple analysis even for cascades, without the complicated integral equations inevitable for the determination of vortex distribution in velocity-field analysis. The computational results show that the pitch and the stagger angle of the cascade have much influence upon the magnitude of the fluctuating lift, but not so much upon phase angle.

**77-312**

**Effect of Phase Angle on Multibladed Rotor Flutter**

V.R. Murthy and G.A. Pierce

Dept. of Mech. Engrg. and Mechanics, Old Dominion Univ., Norfolk, VA 23508, J. Sound Vib., 48 (2), pp 221-234 (Sept 1976) 5 figs, 14 refs

**Key Words:** Rotary wings, Flutter, Helicopters

A theoretical technique for predicting the flutter characteristics of a helicopter rotor is presented. The effect of phase angle on flutter speed of a two-bladed rotor in hovering and axial flight is determined. For this purpose, a uniform and untwisted rotor blade with coupled flapwise bending and torsional degrees of freedom is considered. The transmission matrix method is used to obtain the natural vibration characteristics of the system. An unsteady aerodynamic theory is used to obtain the aerodynamic loading in compressible flow.

**COLUMNS**

(Also see No 296)

**77-313**

**Influence of an Elastic End Support on the Vibration and Stability of Beck's Column**

C Sundararajan

Foster Wheeler Energy Corp., Livingston, NY, Intl. J. Mech. Sci., 18 (5), pp 239-241 (May 1976) 4 figs, 8 refs

**Key Words:** Columns, Cantilevers, Elastic foundations, Vibration response

The influence of an elastic support on the vibration and stability of a non-conservatively loaded (follower force) undamped, linearly elastic system is studied. It is found that the support may destabilize the system in certain cases.

77-314

**Wind Induced Vibrations of Self-Supporting Conical Columns with End Mass**

L.J. Scerbo

Ph.D. Thesis, Polytechnic Institute of New York, 262 pp (1976)

UM 76-23,443

**Key Words:** Columns, Wind-induced excitation, Mathematical models

This study presents a mathematical model for predicting the wind induced vibrations of self-supporting, linearly tapered, truncated, conical columns with end mass. The columns considered can be made up of one or more linearly tapered uniform wall thickness sections, joined together to form a continuum. Their dynamic behavior is simulated by means of the Euler-Bernoulli bending differential equation, which is applicable for small deflections of variable cross section beams.

**CYLINDERS**

(See No. 329)

**GEARS**

77-315

**Statistical Analysis of Dynamic Loads on Spur Gear Teeth (Effect of Shaft Stiffness)**

T Tobe, K. Sato, and N Takatsu

Dept of Engrg, Tohoku Univ, Japan, Bull, JSME, 19 (133), pp 808-813 (July 1976) 13 figs, 4 refs

**Key Words:** Gears, Shafts, Statistical analysis, Dynamic response

In the previous paper, a statistical method of evaluation of the relation between transmission errors and dynamic loads on a pair of spur gears was established neglecting shaft stiffnesses. A more advanced study considering the shaft stiffness is developed in this paper. A Fokker-Planck equation is derived and the mean and the variance of dynamic loads are calculated numerically by applying statistical linearization to nonlinear terms. Monte-Carlo-Simulation is also performed to find maximum dynamic loads by means of an analogue computer.

**LINKAGES**

77-316

**Vibrational Loading of Mechanically Fastened Wood Joints**

T.L. Wilkinson

Forest Products Lab., Madison, WI, Rept. No. FSRP-FPL-274, 15 pp (1976)

AD-A026 642/9GA

**Key Words:** Joints (junctions), Vibration response

The paper presents the results of vibrational loading of nailed and bolted wood joints subjected to a range of frequencies and dead loads. Each joint was tested as a spring-mass system subjected to base excitation. Results indicate a considerable increase in joint stiffness under vibrational loading as compared to static loading with very small amounts of additional joint deformation.

**MECHANICAL**

77-317

**Technical Diagnostics of Driving Gear Elements**

J. Müller and D. Troppens

Wilhelm-Pieck-Universität Rostock, Dem. Rep. Germany, Maschinenbautechnik, 25 (8), pp 350-353 (Aug 1976) 4 figs, 16 refs

(In German)

**Key Words:** Diagnostic techniques, Crankshafts

After describing technical diagnostics, which has developed into a special branch of maintenance, the authors discuss its application in the analysis of driving gear elements and compile all known diagnostic techniques for the crank drive elements of high speed diesel motors. Only those parameters are investigated which are necessary for the determination of damage.

77-318

**Driveline Vibrations**

J.B. Large

General Motors School of Product Service, General Motors Corp., Warren, MI, Fleet Maintenance and Specifying, pp 46, 48-51 (Nov 1976)

**Key Words:** Motor vehicles, Ground vehicles, Trucks, Vibration control, Diagnostic techniques

Techniques for identifying and correcting truck driveline vibrations are described.

**77-319**

**The Dynamics of a Two Rotor System with Slipping Clutch**

J. Stewart

Twiflex Couplings, Ltd., England, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ of Cambridge, Sept. 15-17, 1976, pp 99-104, 7 figs, 3 refs

**Key Words:** Power transmission systems, Couplings, Clutches, Vibration control, Numerical analysis

The motion of a two mass linear or non-linear system with sinusoidal torsional excitation is studied with special reference to the presence of a slipping element, such as a centrifugal clutch coupling, during both transient and resonant conditions. Numerical solutions using parameters typical of a medium power diesel transmission show the strong detuning action and consequently high damping efficiency of the clutch in relation to a linear system. The model correctly predicts the anticipated characteristics of the non-linear system, and is shown to be accurate for known solutions.

**MEMBRANES, FILMS, AND WEBS**

(See Nos 240, 245)

**PANELS**

**77-320**

**Nonlinear Dynamic Buckling of Sandwich Panels**

N.K. Adi Murthy and R.S. Alwar

Dept of Appl. Mechanics, Indian Inst. of Technology, Madras, India, J. Appl. Mech., Trans ASME, 43 (3), pp 459-463 (Sept 1976) 4 figs, 13 refs

**Key Words:** Sandwich panels, Dynamic buckling, Snap through problems

Nonlinear dynamic buckling of simply supported rectangular sandwich panels with isotropic cores and with initial curvature under transverse loads is analyzed. The large deflection sandwich panel equations proposed by Reissner are solved using a trigonometric series spacewise and by Houbolt

scheme for timewise integration. The resulting nonlinear algebraic equations are solved by Newton's iterative method to yield the deflection coefficients at all time intervals. The present investigation essentially deals with the influence of dynamic loads on the snap-through behavior of sandwich panels. The effect of the initial curvature and core modulus is studied in detail. Static snap-through collapse loads are also calculated and compared with dynamic loads. As a check for the present analysis the results obtained for the static case for a homogeneous plate are compared with the available results.

**PIPES AND TUBES**

**77-321**

**Escape Piping Vibrations while Designing**

J.C. Wachel and C.L. Bates

Southwest Res. Inst., San Antonio, TX, Hydrocarbon Processing, 55 (10), pp 152-156 (Oct 1976) 5 figs, 8 refs

**Key Words:** Piping systems, Beams, Vibration response

In this paper the non-ideal conditions which exist in piping systems are compared to the ideal beam theory, in order to predict accurately the mechanical response of piping systems.

**77-322**

**A Composite Transfer Matrix-Synthesis Approach to Piping Dynamic Analysis**

P. Bezler

Ph.D. Thesis, Polytechnic Inst of New York, 246 pp (1976)

UM 76-23,394

**Key Words:** Piping systems, Natural frequencies, Mode shapes, Transfer matrix method, Computer programs

A composite transfer matrix-synthesis approach to piping system dynamic analysis is described. The transfer matrix method is used to determine the dynamic characteristics of simple configurations considered to represent typical piping components. The developed data, tabular listings of transfer matrix state vectors, include sufficient information to delineate the first five natural frequencies and their associated deflection and force mode shapes for five fixed-fixed component configurations over a range of their span length parameters. These data will define the dynamical characteristics of any three span configuration likely to occur in piping runs. Used with the fixed constraint modal synthesis method, they permit the ready analysis of the most generalized piping systems. Specific examples of applications to typical piping configurations, both simple and branched, with an assessment of technique accuracy are included.

77-323

**Optimizing Valve Jet Size and Spacing Reduces Valve Noise**

C. Reed

Masonelan International, Inc., Norwood, MA.,  
Control Engineering, 23 (9), pp 63-64 (Sept 1976)  
3 refs

**Key Words:** Valves, Noise reduction

The concept of using a spaced-array of small fluid jets to limit noise generation in process control valves has been applied by the author to several sizes of actual hardware. The resulting design was found to be very effective in limiting noise generation while using basic parts of the standard cage-guided valve.

77-324

**Pipewall Vibration Reveal ... Valve-Generated Noise?**

A.C. Fagerlund

Fisher Controls Co., Marshalltown, IA., Hydrocarbon Processing, 55 (10), pp 147-149 (Oct 1976) 7 figs

**Key Words:** Piping systems, Valves, Noise generation

Laminar fluid flow under steady pressure through a pipe of uniform cross-section can be almost noiseless. When flow is regulated, however, the valving may introduce varying amounts of turbulence or even cavitation. These forms of energy propagating through the fluid as a fluctuating pressure force the pipewalls to vibrate. The vibrations in turn cause pressure disturbances outside the pipe that radiate as sound. When measuring sound generated by a single control valve, extraneous noise sources and reflected sound make interpretation of the measurements difficult. This paper is concerned with the conversion of pipe vibrations into an equivalent sound pressure level in ambient air.

**PLATES AND SHELLS**

(Also see Nos 245, 277, 293, 341, 363)

77-325

**Static and Dynamic Analysis of Thermoviscoelastoplastic Fiber-Reinforced Composite Shells in Missile Structures**

T.J. Chung and R L Eidson

Dept of Mech. Engrg, Alabama Univ in Huntsville, AL, Rept. No. UAH-RR-182, 185 pp (May 1976)  
AD-A027 154/4GA

**Key Words:** Shells, Composite materials, Fiber composites, Missiles, Computer programs

It has been postulated that anisotropic structures such as the fiber composites should be governed by kinematic hardening. The feasibility of incorporating the changes of degrees of anisotropy during the post-yielding deformations of fiber composites is reported. The present report describes the development of kinematic plastic function theory. The method requires a considerable amount of computing time, but upon careful reprogramming, it is believed to be a powerful tool in the analysis of missile structures.

77-326

**Vibrations of Thin Elastic Shells - A New Approach**

M.J.A. O'Callaghan, W.A. Nash and P.M. Quinlan  
Dept. of Civil Engrg., Massachusetts Univ., Amherst, MA., Rept. No. AFOSR-TR-76-0731, 111 pp (Aug 1975) (AD-779 782)

AD-A027 706/1GA

**Key Words:** Spherical shells, Natural frequencies, Mode shapes, Computer programs

The present investigation extends the determination of natural frequencies and mode shapes of free vibration of thin elastic plates to the case of a shallow elastic spherical shell of n-sided polygonal plan-form. Natural frequencies and associated mode shapes together with boundary residuals (indicating how well the approximate solution has satisfied boundary conditions) are readily displayed by the computer program offered. Results obtained in the present work are shown to be in excellent agreement with existing values for specific boundary conditions treated. The present approach involves only modest computer efforts but offers the significant feature of rapid determination of how precisely the boundary conditions have been satisfied along each edge of the shell.

77-327

**Free Vibration of Fluid-Coupled Coaxial Cylindrical Shells of Different Lengths**

M.K. Au-Yang

Nuclear Power Generation Div., Babcock & Wilcox, Lynchburg, VA., J. Appl. Mech., Trans. ASME, 43 (3), pp 480-484 (Sept 1976) 3 figs, 9 refs

**Key Words:** Cylindrical shells, Submerged structures, Fluid-induced vibration

This paper considers the free vibration of two coaxial cylinders with finite lengths immersed in a restricted fluid medium. The natural frequencies in the fluid can be computed by including the virtual mass to its physical mass. Experimental results have been used to verify the analysis. Agreement between experimental and theoretical results is excellent.

**77-328**

**Free Vibrations of Circular Cylindrical Shells**

V. Ramamurti and J. Pattabiraman

Dept. of Appl. Mechanics, Indian Inst. of Tech.,  
Madras 600036, India, *J. Sound Vib.*, 48 (1), pp 137-155 (Sept 8, 1976) 14 figs, 27 refs

**Key Words:** Cylindrical shells, Free vibration, Finite element technique

The finite element method is used to predict the dynamic behavior of circular cylindrical shells in free vibrations. A suitable shape function for the circumferential displacement distribution has been proposed. This reduces the three-dimensional character of the problem to a two-dimensional one. The simultaneous iteration method to determine the eigenfrequencies and eigenvectors is utilized for solving the eigenvalue problem. The accuracy of the method has been checked by verifying the results of known cases. Finally an experimental shell structure containing elastic rings welded at the ends has also been analyzed and the experimental results compared with the theoretical ones.

**77-329**

**Vibration of Orthogonally Stiffened Waffle Cylinders Subjected to Initial Forces**

M.I. Baig and T.Y. Yang

Purdue Univ., West Lafayette, IN., *J. Spacecraft and Rockets*, 13 (10), pp 618-623 (Oct 1976) 9 figs, 6 refs

**Key Words:** Cylindrical shells, Equations of motion

The kinetic energy expression is formulated for the orthogonally stiffened waffle cylindrical shell. The equations of motion are formulated by using Hamilton's principle for simply supported edge conditions. The effect of initial middle-surface stresses is included. A variety of examples are performed, and results are compared, whenever possible, with alternative solutions.

**77-330**

**Steady State Response of Shell Structures**

R.K. Dubey

Ph.D. Thesis, Polytechnic Inst. of New York, 84 pp (1976)

UM 76-23, 402

**Key Words:** Shells, Periodic response

The steady state response of shell structures under axial and lateral excitation is reported. The two point boundary value problem is solved by reducing the equations of motion to a system of first order differential equations, and applying the method of stepwise integration. The axial vibration transmission characteristics for cylindrical shells as a function of circumferential mode number has been determined. The results for short and long cylindrical shells with various boundary conditions have been presented as elements of a transfer matrix. The response of laterally excited cylindrical shells and containment vessels has been obtained and the stresses, moments and displacements along the shell generator have been determined as a function of frequency.

**77-331**

**Dynamic Behavior of Fluid-Tank Systems**

J.Y. Yang

Ph.D. Thesis, Rice Univ., 214 pp (1976)

UM 76-21, 727

**Key Words:** Fluid-filled containers, Storage tanks, Earthquakes, Seismic excitation

Presented in this report are the results of a study of the hydrodynamic forces induced in fluid-filled storage tanks subjected to earthquake excitation and of the free vibration of the coupled fluid-tank systems. Throughout this study, the fluid is assumed to be incompressible and inviscid. The report can be divided into four parts: The first part deals with the hydrodynamic forces induced in a rigid tank subjected to a lateral earthquake excitation. In the second part of the report an approximate analysis is presented for the hydrodynamic forces induced in flexible fluid-filled tanks. The third part of the report deals with the free vibrational characteristics of flexible circular cylindrical tanks. The fourth and final part of the report deals with the axisymmetric free vibration of fluid-tank systems.

**77-332**

**Shells and Plates Loaded by Dynamic Moments with Special Attention to Rotating Point Moments**

W. Soedel

R.W. Herrick Laboratories, Purdue Univ., West Lafayette, IN 47907, *J. Sound Vib.*, 48 (2), pp 179-188 (Sept 1976) 3 figs, 6 refs

**Key Words:** Shells, Plates, Rotors, Dynamic response

The response of thin shells to line or point moment excitation is formulated by way of distributed moment fields. Twisting moments in the tangent plane are part of this formulation. The approach is illustrated by using Love's thin shell theory, but is valid for any other thin shell theory as well. Dirac delta functions are used to describe line and point moments. As a first example, the response of a plate to a rotating moment is evaluated and shown to be identical to the solution obtained by Bolleter and Soedel by a Green function approach. The three-directional response of a circular cylindrical shell to a rotating moment is given as the second example.

**77-333**

**Free Vibration of Two Span Rectangular Plates with Some Non-Classical Boundary Conditions**

S. Mirza and B. Setiawan

Dept. of Mech. Engrg., Univ. of Ottawa, Ontario, Canada, *Intl. J. Mech. Sci.*, 18 (4), pp 165-170 (Apr 1976) 7 figs, 10 refs

**Key Words:** Rectangular plates, Free vibration, Spring-mass systems

This paper deals with the vibration of two span rectangular plates resting on linear and torsional springs. The two opposite continuous edges of the plate are considered as simple supports. The resulting equations are general. Numerical results have been obtained by varying spring stiffness and the aspect ratio. Some limiting cases of interest have also been developed.

**77-334**

**Finite Element Methods for Nonlinear Eigenvalue Problems and the Postbuckling Behavior of Elastic Plates**

L. C. Wellford, Jr and G.M. Dib

Civ. Engrg Dept., Univ. of Southern California, Los Angeles, CA 90007, *Computers and Struc.*, 6 (4/5), pp 413-418 (Aug/Oct 1976) 6 figs, 16 refs

**Key Words:** Plates, Buckling, Eigenvalue problems, Finite element technique

In the following paper new variational methods for the solution of problems of postbuckling of elastic plates are formulated, and approximate solutions for various cases are obtained using finite element methods. Initially, the postbuckling problem is phrased as a nonlinear eigenvalue problem. Variational principles for the nonlinear eigenvalue problem of the postbuckling of structures are developed.

These variational principles have a form which can be characterized as a nonlinear Rayleigh principle, finite-element models are introduced and are used to implement the postbuckling variational principle. Incremental amplitude solution schemes are developed to solve the corresponding finite-element equations. Sample results are presented for the postbuckling behavior of circular plates.

**77-335**

**On the Non-Linear Dynamic Stability Problem for Thin Elastic Plates**

S.D. Kisiakov

Inst. of Civ. Engrg. and Arch., Sofia, Bulgaria, *Intl. J. Nonlinear Mech.*, 11 (4), pp 219-228 (1976) 7 figs, 7 refs

**Key Words:** Dynamic stability, Plates, Parametric excitation

In this paper the non-linear response of thin elastic plates under parametric excitation is investigated. A new analytical method is proposed. It gives the possibility to obtain all the characteristic features of the phenomenon considered, which are known from experiments - the existing of beats, their dependence on the excitation parameter, the influence of the initial conditions, the typical character of the vibrations in the different regions. Analog computer studies are carried out, and they show clearly the influence of different parameters on the output of the problem considered.

**77-336**

**Natural Frequencies of Orthotropic Rectangular Plates with Varying Thickness**

T. Sakata

Dept. of Mech. Engrg., Chubu Inst. of Technology, Kasugai, Nagoya-sub., Japan 487, *J. Acoust. Soc. Amer.*, 60 (4), pp 844-847 (Oct 1976) 2 figs, 13 refs

**Key Words:** Rectangular plates, Variable cross section, Natural frequencies

An approximate formula is derived for the estimation of the fundamental natural frequency of the simply supported orthotropic rectangular plate with thickness varying linearly in one direction. The accuracy of the formula and the influence of the flexural rigidity on the natural frequency are discussed.

**77-337**

**SPAN: A Computer Program for Static and Dynamic Analysis of Stiffened Plates and Grillages**

G.C. Mitchell, J.D. Clarke and C.S. Smith  
Naval Construction Research Establishment, Dunfermline, Scotland, Rept. No. NCRE/R630, DRIC-BR-50857, 67 pp (Feb 1976)  
AD-A026 865/6GA

**Key Words:** Computer programs, Plates, Grids (beam grids), Ship structural components, Finite element technique

A computer code for the static and dynamic analysis of flat plate grillages is described. The code was a finite element displacement method incorporating a geometric stiffness matrix to calculate the response to static, cyclic or transient loads. Natural frequencies and buckling loads can also be calculated. A full input description is given including a simplified data generation facility for regular grillages. Input and output for some representative sample problems are included.

**77-338**  
**Free Vibration Analysis of Stiffened Plates Using Finite Difference Method**

G. Aksu and R. Ali  
Dept. of Transport Technology, Univ. of Technology, Loughborough LE11 3TU, England, J. Sound Vib., 48 (1), pp 15-25 (Sept 8, 1976) 9 figs, 8 refs

**Key Words:** Plates, Free vibration, Finite difference method

Free vibration characteristics of rectangular stiffened plates having a single stiffener have been examined by using the finite difference method. A variational technique has been used to minimize the total energy of the stiffened plate and the derivative appearing in the energy functional are replaced by finite difference equations. The energy functional is minimized with respect to discretized displacement components and natural frequencies and mode shapes of the stiffened plate have been determined as the solutions of a linear algebraic eigenvalue problem. The analysis takes into consideration inplane deformation of the plate and the stiffener and the effect of inplane inertia on the natural frequencies and mode shapes. The effect of the ratio of stiffener depth to plate thickness on the natural frequencies of the stiffened plate has also been examined.

**77-339**  
**The Stability of a Spinning Elastic Disk with a Transverse Load System**

W.D. Iwan and T.L. Moeller  
Div of Engrg and Appl Science, California Inst. of Technology, Pasadena, CA, J. Appl. Mech., Trans. ASME, 43 (3), pp 485-490 (Sept 1976) 8 figs, 10 refs

**Key Words:** Disks, Rotating structures, Flexural vibration

This paper presents results of an investigation on the effect of a transverse load on the stability of a spinning elastic disk. The disk rotates at constant angular velocity and the load consists of a mass distributed over a small area of the disk, a spring, and a dashpot.

## RINGS

**77-340**  
**An Analytical Formulation of the Forced Responses of Damped Rings**

Y.P. Lu  
David W. Taylor Naval Ship Res. and Dev. Ctr., Annapolis, MD 21402, J. Sound Vib., 48 (1), pp 27-33 (Sept 8, 1976) 3 figs, 5 refs

**Key Words:** Rings, Composite structures, Viscoelastic damping, Forced vibration

An analysis in which an interaction formulation is used for the forced vibratory responses of a ring having a number of mass segments adhered to it by a viscoelastic material is presented. The mass segments are discretely distributed around the circumference of the ring, and the excitation is a concentrated vibratory radial force located on the surface of the ring. The mass segments may not have to be identical, nor do their distributions have to be uniform. The analysis can readily be extended to more complicated damped shell structural systems. The driving point mechanical impedances at a location midway between two mass segments for a given damped system are given as an example. These solutions compare very well with experimental data and theoretical results available. Also presented is a comparison of driving point mechanical impedances for two damped systems with different thicknesses of the viscoelastic layers.

## STRUCTURAL

**77-341**  
**Structural Vibrations and Fourier Series**

R. Greif and S.C. Mittendorf  
Dept. of Mech. Engrg., Tufts Univ, Medford, MA 02155, J. Sound Vib., 48 (1), pp 113-122 (Sept 8, 1976) 5 figs, 14 refs

**Key Words:** Structural elements, Beams, Plates, Shells, Variable material properties, Variable cross section, Component mode analysis

A method is introduced for vibration analysis of a wide class of beam, plate and shell problems including the effects of variable geometry and material properties. The method is based on the discrete technique of component mode analysis. For each of these components the mode shapes are written in terms of Rayleigh-Ritz expansions involving simple Fourier sine or cosine series. Due to the nature of these series, special attention must be given to end point behavior in the modal expansions and in the derivatives of these modal expansions. This is done via the mechanisms of Stokes' transformation. Continuity between components is enforced with Lagrange multipliers. The resulting frequency equation is exact and the associated eigenvector contains a combination of force and displacement types terms. Numerical solutions are found by truncating the series and monitoring the frequency determinant on a computer.

## SYSTEMS

### ABSORBER

(Also see Nos. 379, 380, 381)

77-342

**Dynamic Analysis of Impact Attenuation Systems Utilizing Plastic Deformations (Case in which Effect of Strain-Rate Sensitivity is Considered)**

K. Ohmata

Dept. of Engrg., Meiji Univ., Kawasaki, Japan, Bull. JSME, 19 (134), pp 884-901 (Aug 1976) 13 figs, 6 refs

**Key Words:** Shock absorption, Plastic deformation

An impact attenuation system whose idealized static load-displacement curve consists of an elastic range and unrestricted plastic flow is represented by a dynamically equivalent model which takes into account elasticity, viscosity, plasticity and strain-rate sensitivity of the structure, and its dynamic response to the impact by a moving mass is analyzed.

77-343

**Impact Absorber with Linear Spring and Quadratic Law Damper**

M.S. Hundal

Dept of Mech Engrg., Univ. of Vermont, Burlington, VT, J. Sound Vib., 48 (2), pp 189-193 (Sept 1976) 4 figs, 6 refs

**Key Words:** Shock absorbers

The performance of an impact absorber with a linear spring and a quadratic law damper is analyzed. The equation of motion is non-dimensionalized and a closed form solution is obtained. Chosen response variables are presented in terms of a dimensionless parameter, the quadratic damping ratio. The performance is compared with that of an absorber with viscous damper.

## ACOUSTIC ISOLATION

(See No. 260)

## NOISE REDUCTION

(Also see Nos. 259, 323, 353, 354, 367, 368, 372, 389, 402)

77-344

**Muffling Hydraulic Systems**

S.J. Skaistis and R.J. Becker

Systems Science Labs., Sperry Vickers Div., Sperry Rand Corp., Troy, MI, Mach. Des., 48 (24), pp 124-128 (Oct 21, 1976)

**Key Words:** Hydraulic equipment, Noise reduction

Techniques for selecting, mounting and shielding hydraulic components to avoid excess noise are described.

77-345

**Nonlinear Distortion in the Propagation of Intense Acoustic Noise**

F.M. Pestorius and D.T. Blackstock

Appl. Res. Lab., Texas Univ. at Austin, Austin, TX, Rept. No. AFOSR-TR-76-0925, 15 pp (Mar 1973) Sponsored by ONR AD-A027 650/1GA

**Key Words:** Sound transmission, Noise reduction

As sound pressure increases to 'finite' levels, nonlinear effects usually ignored in acoustics problems become increasingly important. In this paper, plane waves propagating through a pipe are considered. In the first section the distortion of a wave that is sinusoidal at the source is treated. In the second section the source signal is a pulse band-limited (500 to 3500 Hz) random noise. In both cases theoretical results are compared with experimental measurements. Extension of the analysis to the more practical case of outdoor propagation is indicated.

77-346

**Meeting European Noise Regulations for Rubber-Tired Front End Loaders**

D.F. Rudny

Pay line Div., International Harvester, SAE Paper No. 760599, 12 pp, 18 figs, 2 refs

**Key Words:** Construction equipment, Noise reduction

Major legislation has been established in Europe governing the noise levels of heavy construction equipment. Level limits, test procedures, data acquisition techniques, and future legislation of several European countries are presented specifically for rubber tired front end loaders. Techniques utilized in determining problem source components and methods employed in attenuation of the vehicle to comply with the existing legislation are also discussed. Special attention is given to the spectator level where the regulation limits are most stringent. However, many of the modifications shown are also effective in reducing operator station noise levels.

77-347

**Noise Reduction in the Generation of Energy from Brown Coal as an Environmental Production Problem**

R. Junghans and P. Koltzsch

KDT, Bergakademie Freiberg, Germany, Technik (Berlin), 31 (8), pp 532-536 (Aug 1976) 7 figs, 10 refs  
(In German)

**Key Words:** Noise reduction, Industrial facilities

After a short review of noise control regulations in the Democratic Republic of Germany, the application of modeling technology for noise control in the brown coal industry is described. Two examples are illustrated: one is a mathematical model for the investigation of noise propagation in power stations, the other a model for the investigation of the acoustic effect of noise isolators in ventilation cooling towers.

## ACTIVE ISOLATION

77-348

**Development of Computerized Active Vehicle Suspensions**

H K. Sachs and R.W. Siorek

Dept. of Mech. Engrg. Sciences, Wayne State Univ., Detroit, MI., Rept No. TACOM-TR-12126, 96 pp (Jan 1976)  
AD-A028 390/3GA

**Key Words:** Suspension systems (vehicles), Active isolation, Computer programs

An active, computerized, adaptive control system is discussed for the control of vehicle vibrations by means of adjustment of suspension parameters during the ride. The problem is subdivided into two basic parts. One, optimization of vehicle suspension parameters if forward speed, terrain statistics and vehicle descriptors are known a-priori. Two, methods of evaluating terrain data received from sensors on board the vehicle and subsequently processing it for comparison with the stored data contained in the memory.

## AIRCRAFT

(Also see Nos. 263, 436)

77-349

**Recursive Identification and Tracking of Parameters for Linear and Non-Linear Multivariable Systems**

M. Sidar

Ames Res. Ctr., NASA, Moffett Field, CA 94035, Intl. J. Control, 24 (3), pp 361-378 (Sept 1976) 6 figs, 26 refs

**Key Words:** Parameter identification, Aircraft, Stability analysis

The problem of identifying constant and variable parameters in multi-input, multi-output, linear and non-linear systems is considered, using the maximum likelihood approach. An iterative algorithm, leading to recursive identification and tracking of the unknown parameters and the noise covariance matrix, is developed. Agile tracking and accurate and unbiased identified parameters are obtained. Necessary conditions for a globally, asymptotically stable identification process are provided; the conditions proved to be useful and efficient. Among different cases studied, the stability derivatives of an aircraft were identified and some of the results are shown as examples.

77-350

**Dynamic Behavior of an Aircraft Encountering Aircraft Wake Turbulence**

R.C. Nelson

Univ. of Notre Dame, Notre Dame, IN, J. Aircraft, 13 (9), pp 704-708 (Sept 1976) 10 figs, 13 refs

**Key Words:** Aircraft, Aerodynamic response, Vortex induced vibration

This paper deals with the dynamic behavior of an airplane encountering aircraft wake turbulence. A digital computer simulation was developed to study the response of an aircraft flying into a trailing vortex wake. The simulation includes the complete six degree-of-freedom equations of motion, a description of the vortex velocity field, unsteady aerodynamics, and pilot control input. The parameters, varied in this simulation, include the penetration angle, separation distance, aircraft size (for both the penetrating and generating aircraft), and pilot control input (single- or multi-axes). Predicted vortex induced motions are presented for several probe aircraft.

77-351

**On Sound Transmission into a Thin Cylindrical Shell under "Flight Conditions"**

L.R. Koval

Dept. of Mech. and Aero. Engrg., Univ. of Missouri-Rolla. Rolla, MO 65401, J. Sound Vib., 48 (2), pp 265-275 (Sept 1976) 4 figs, 16 refs

**Key Words:** Cylindrical shells, Aircraft, Sound transmission, Mathematical models

In the context of the transmission of airborne noise into an aircraft fuselage, a mathematical model for sound transmission into a thin cylindrical shell is used to study sound transmission under "flight conditions": i.e., under conditions of external air flow past a pressurized cylinder at flight altitude. Numerical results for different incidence angles are presented for a typical narrow-bodied jet in cruising flight at 10 680m (35 000 ft) with interior pressure at 2440 m (8000 ft). A comparison is made between no-flow sound transmission at standard conditions on the ground to sound transmission under flight conditions.

77-352

**V/STOL Rotary Propulsion Systems Noise Prediction and Reduction. Volume I. Identification of Sources, Noise Generating Mechanisms, Noise Reduction Mechanisms, and Prediction Methodology**

B Magliozzi

Hamilton Standard Div., United Technologies Corp., Windsor Locks, CT., Rept. No. FAA-RD-76-49 1, 145 pp (May 1976) (AD-A027 390)  
AD-A027 389/6GA

**Key Words:** Vertical takeoff aircraft, Aircraft noise, Noise source identification, Noise prediction, Noise generation

The propulsion systems of current and future V/STOL vehicles can be defined as combinations of free-air propellers, shrouded propellers, variable pitch fans, fixed pitch fans, tilt

rotors, helicopter rotors, lift fans, gearboxes, and drive engines. In this report, noise sources for each of these propulsors, gearboxes, and drive engines are identified and rank ordered. The noise generating mechanisms for each of the propulsor noise sources identified are defined and systematically catalogued. Three approaches to reduction of propulsor noise are discussed: changes in physical geometry, changes in design operating conditions, and the use of acoustic treatments. Computerized and graphical procedures based on methodology from the open literature and at United Technologies Corp., are presented for predicting aerodynamic performance of and noise from the V/STOL propulsors identified in this study.

77-353

**V/STOL Rotary Propulsion Systems Noise Prediction and Reduction. Volume II. Graphical Prediction Methods**

B. Magliozzi

Hamilton Standard Div., United Technologies Corp., Windsor Locks, CT., Rept. No. FAA-RD-76-49.2, 299 pp (May 1976) (AD-A027 389)  
AD-A027 390/4GA

**Key Words:** Aircraft noise, Engine noise, Vertical takeoff aircraft, Noise prediction, Noise reduction, Graphic methods

Graphical procedures for estimating noise and performance of free-air propellers, variable pitch fans with inlet guide vanes, variable pitch fans with outlet guide vanes, fixed pitch fans, helicopter rotors, tilt rotors, and lift fans are presented. Noise prediction methods for drive engines, gearboxes, jets with and without bypass flow, as well as noise reduction and performance losses for partly sonic inlets and duct linings are also presented.

77-354

**V/STOL Rotary Propulsion Systems - Noise Prediction and Reduction. Volume III. Computer Program User's Manual**

B Magliozzi

Hamilton Standard Div., United Technologies Corp., Windsor Locks, CT., Rept. NO. FAA-RD-76-49-Vol-3, 300 pp (May 1976) (AD-A027 389)  
AD-A027 363/1GA

**Key Words:** Aircraft noise, Engine noise, Vertical takeoff aircraft, Noise prediction, Noise reduction, Ducts, Acoustic linings, Computer programs

A computer program is presented which allows a user to make performance and far-field acoustic noise predictions for free-air propellers, variable pitch fans with inlet guide vanes, variable pitch fans with outlet guide vanes, fixed pitch fans, helicopter rotors, tilt rotors, fixed pitch lift vanes with remote, integral, and tip-turbine drives, and variable pitch lift fans with remote and integral drives. Noise prediction methodology for drive engines, single stream and coaxial jets, and gearboxes are also included, as well as noise reduction and performance losses of partly sonic inlets and duct acoustic treatment. A description of the program, detailed instructions for its use, required inputs, and sample cases are presented.

**77-355**

**Pavement Response to Aircraft Dynamic Loads. Volume III. Compendium**

R.H. Ledbetter

Army Engineer Waterways Experiment Station, Vicksburg, MS., Rept. No. WES-TR-S-75-11-Vol-3, FAA-RD-74-39-3, 94 pp (June 1976) (See also Vol 1, AD-A016 450)

AD-A028 378/8GA

**Key Words:** Aircraft, Landing, Pavements, Runways

Instrumented aircraft were used to apply static and dynamic loads to instrumented pavement structures (both flexible and rigid) at the National Aviation Facilities Experimental Center (NAFEC), Atlantic City, NJ.

## BRIDGES

**77-356**

**Dynamic Analyses of Suspension Bridge Structures and Some Related Topics**

A M. Abdel-Ghaffar

Ph.D. Thesis, California Inst. of Technology, 460 pp (1976)

UM 76-23, 378

**Key Words:** Suspension bridges, Linear theories, Finite element technique

The thesis is divided into two parts. The first part develops a method of dynamic analysis for vertical, torsional and lateral free vibrations of suspension bridges, based on linearized theory and the finite-element approach. The method involves two distinct steps: (1) specification of the potential and kinetic energies of the vibrating members of the continuous structure, leading to derivation of the equations of motion by Hamilton's Principle, (2) use of the finite-element

technique to: (a) discretize the structure into equivalent systems of finite elements, (b) select the displacement model most closely approximating the real case, (c) derive element and assemblage stiffness and inertia properties, and finally (d) form the matrix equations of motion and the resulting eigenvalue problems. The stiffness and inertia properties are evaluated by expressing the potential and kinetic energies of the element (or the assemblage) in terms of nodal displacements. Detailed numerical examples are presented to illustrate the applicability and effectiveness of the analysis and to investigate the dynamic characteristics of suspension bridges with widely different properties.

**77-357**

**Structural Analysis and Retrofitting of Existing Highway Bridges Subjected to Strong Motion Seismic Loading**

R.R. Robinson, E. Privityzer, A. Longinow and K.H. Chu

IIT Research Inst., Chicago, IL., Rept. No. IITRI-J6320-FR, FHWA/RD-75-94, 330 pp (May 1975) PB-255 299/OGA

**Key Words:** Bridges, Earthquake resistant structures, Seismic response

The objective of this project was to determine cost effective means for modifying existing intermediate size bridges so as to better withstand the damaging effects of intense earthquake ground motions. Research studies were performed to identify and define, through structural analysis, practical techniques and criteria for retrofitting the bridges selected during the program. The need for retrofitting is based on the philosophy that damage should be limited so that collapse does not occur and traffic can be restored after minimum repairs. Seven different bridge structures were selected throughout the United States in high risk seismic regions. Seismic loads were determined for each bridge based on the soil conditions and seismicity at its site. A simplified analysis procedure was defined during the project and validated by selective comparisons with nonlinear response analyses.

## BUILDING

**77-358**

**Modal Control of Multistory Structures**

C.R. Martin and T.T. Soong

Dept. of Engrg. Sci., State Univ. of New York at Buffalo, Buffalo, NY, ASCE J. Engr. Mech. Div., 102 (EM4), pp 613-623 (Aug 1976) 4 figs, 7 refs

**Key Words:** Multistory buildings, Modal control technique

In this paper the design of active control systems for civil engineering structures is considered using modal control concepts. The theory of modal control is briefly presented with respect to the control of linear structures where the control objective is to direct changes of its specific dynamic modes and stiffness.

## CONSTRUCTION

(Also see No. 346)

77-359

### Vibratory Compaction of Bituminous Concrete Pavements

C.D. Burns

Army Engineer Waterways Experiment Station, Vicksburg, MS., Rept No WES-MP-S-76-10, 61 pp (June 1976)

AD-A026 843/3GA

**Key Words:** Vibrators (machinery), Compaction equipment, Compacting, Concretes, Pavements

This study was conducted to evaluate the effectiveness of vibratory rollers in the compaction of hot-mix asphaltic concrete and rubberized-tar concrete to satisfy the needs of the Air Force. The study consisted of over-laying an existing heavy gear load test section at the U.S. Army Engineer Waterways Experiment Station, which consisted of rigid and flexible pavements, with asphaltic concrete and rubberized-tar concrete pavements. The overlay pavements were compacted with two selected vibratory rollers, a Buffalo-Bomag BW210-A and a Dynapac CC-50A. A conventional steel-wheeled static roller and a pneumatic-tired static roller were also used for comparison. Variables included in the study were roller weight, frequency and amplitude of vibration, number of roller passes, type of roller (vibratory or static), type of foundation, and type and thickness of overlay pavements.

## EARTH

77-360

### Review and Analysis of Blasting and Vibrations at Bankhead Lock

R.J. Lutton

Army Engineer Waterways Experiment Station, Vicksburg, MS., Rept No WES-TR-S-76-6, 86 pp (June 1976)

AD-A026 735/1GA

**Key Words:** Locks (waterways), Excavations, Ground motion

The U.S. Army Corps of Engineers recently replaced the old Bankhead Lock on the Black Warrior River, Alabama, with a larger lock. The new lock required an approach canal excavated by the removal of more than 5 million cu yd of rock. Preliminary test shots and an exploratory excavation contract of 0.35 million cu yd established blasting and monitoring techniques that assured that vibration and settlement of the old lock would not be excessive. Shots were monitored with particle velocity pickups on and adjacent to the lock. A relationship between peak particle velocity and scaled distance was checked on the basis of these data after normalization for structure behavior, coupling, and transmissivity. Recommendations from the test shot program were used in specifications for the exploratory excavation contract. The exploratory excavation, with about 50 presplit and production shots, served as a second testing program for refining the previous observations and conclusions, according to various blasthold arrays and time delays.

## HELICOPTERS

(Also see Nos. 312, 352, 353, 354)

77-361

### In-Flight Far-Field Measurement of Helicopter Impulsive Noise

F.H. Schmitz and D.A. Boxwell

U.S. Army Air Mobility R&D Laboratory, Moffett Field, CA., J. Amer. Helicopter Soc., 21 (4), pp 2-16 (Oct 1976) 13 figs, 13 refs

**Key Words:** Helicopter noise, Measurement techniques

A new and highly successful method of collecting far-field acoustic data radiated by helicopters in forward flight has been developed, utilizing a quiet aircraft flying in formation ahead of the subject helicopter. The lead aircraft, flown as an acoustic probe, was equipped with tape-recording equipment and an external microphone. Spatial orientation of the helicopter with respect to the monitoring aircraft was achieved through visual flight reference. Far-field acoustic data defining the impulsive noise radiation characteristics of the UH-1H helicopter during high-speed flight and partial-power descents have been gathered with this technique. Three distinct types of impulsive waveforms have been identified and correlated with helicopter steady operating conditions.

77-362

### AH-1 Helicopter Vibration Levels for Stub Wing Mounted Equipment

G D. Welford and J.S. Boland, III  
Army Missile Research Development and Engrg. Lab.,  
Redstone Arsenal, AL., Rept. No. RG-76-30, 147 pp  
(Sept 1975)  
AD-A026 825/OGA

**Key Words:** Helicopters, Wing stores

This report presents measured translational and angular acceleration inputs to stub wing mounted equipment on the AH-1 helicopter. In this case the wing mounted electro-optical systems were the stabilized platform airborne laser system and the stabilized mirror airborne laser system. The data are presented in the form of power spectral density, amplitude versus frequency, and oscillograph charts.

## HUMAN

**77-363**

### **An Experimental Study of Package Cushioning for the Human Head**

Y.K. Liu and K.B. Chandran  
Biomechanics Lab., School of Medicine and Engrg.,  
Tulane Univ., New Orleans, LA., J. Appl. Mech.,  
Trans. ASME, 43 (3), pp 469-474 (Sept 1976)  
7 figs, 16 refs

Sponsored by the National Institute of Health and  
the National Science Foundation

**Key Words:** Head (anatomy), Impact shock, Isolation,  
Experimental data

An experiment was performed to determine the container acceleration and pressure distribution in a Plexiglass cylinder, filled either with water or 3 percent set-gelatin, and impacted against a wall. This experiment serves to quantitatively validate a theoretical model simulating an one-dimensional closed-head impact given earlier. The experiments showed important differences between the theoretical and experimental pressure measurements.

**77-364**

### **Dynamic Properties of the Human Head**

J.B. Smith and C.W. Suggs  
Dept. of Biological and Agricultural Engrg., North  
Carolina State Univ., Raleigh, NC., J. Sound Vib.,  
48 (1), pp 35-43 (Sept 8, 1976) 4 figs, 10 refs

**Key Words:** Head (anatomy), Mechanical impedance, Vibration response

Driving point mechanical impedance measurements were used to determine the dynamic response of the human head to sinusoidal vibration in the frequency range between 30 Hz and 5000 Hz at excitation levels of  $0.98 \text{ m/s}^2$  and  $3.4 \text{ m/s}^2$ . Because of the low excitation levels, the weight of the head was sufficient to couple the head to the vibration source. The response of the head to vibration can be simulated by a two degree-of-freedom, mass-excited system consisting of a series connection of a small driving mass, a damper, a spring and damper in parallel and a large final mass. Parameter values, derived by computer techniques, suggest that the large mass represents the total head, the small mass the tissue in contact with the vibration input and the spring the skull stiffness.

## ISOLATION

(Also see No. 401)

**77-365**

### **User Manual for Air Bag Restraint Program ABAG 19**

R.H. Dufort  
Vizex, Inc., Buffalo, NY., Rept. No. VCR-76-1,  
DOT-HS-801 929, 124 pp (July 1976)  
PB-256 115/7GA

**Key Words:** Air bags (safety restraint systems), Mathematical models, Computer programs

The major tasks covered by this report are the collection, interpretation, clarification and documentation of the accumulated but undescribed changes evolved by the various users of a computer simulation of air bag behavior.

**77-366**

### **Vibration Study of Radar Receiver/Transmitter M163 Vulcan Air Defense System**

J.H. Wiland  
Frankford Arsenal, Philadelphia, PA., Rept No.  
FA-TR-76017, 35 pp (Mar 1976)  
AD-A027 448/OGA

**Key Words:** Equipment mounts, Vibration control

This report describes a laboratory vibration study conducted on the Receiver/Transmitter unit of the M163 Vulcan Air Defense System. Field reports indicated an undesirable amount of vibration in the unit during vehicle operation on paved roads, resulting in excessive maintenance requirements. The laboratory study determined the equipment's critical frequencies likely to be excited by vehicle operation, and the associated mode shapes. The primary vibration problem is identified as insufficient support structure for the microwave chassis.

## MATERIAL HANDLING

(Also see Nos. 346, 347)

77-367

### A Systematic Approach to Noise Reduction of Army Fork Lift Trucks

E.F. Ellingson

Advanced Technology Center, Allis-Chalmers Corp.,  
SAE Paper No. 760600, 20 pp, 45 figs

**Key Words:** Hoists, Trucks, Noise reduction

This report discusses work performed under government contract to study and reduce noise emission of Army 6000 lb capacity fork lift trucks. Sound levels of stock fork lift trucks were studied in detail including operator exposure. Contributing noise sources were identified and studied, and several abatement techniques were established for each source.

77-368

### Noise Control on a Heavy-Duty Mobile Crane

J.R. Bernhagen

Harnischfeger Corp., SAE Paper No. 760601, 16 pp,  
7 figs, 26 refs

**Key Words:** Cranes (hoists), Noise reduction

The intent of this paper is to illustrate how simple testing techniques and application of elementary noise concepts helped attenuate the noise in the operator cabs of a heavy-duty mobile crane to within the 8 hour/90 dBA limit established by the Occupational Health and Safety Act of 1970.

## MECHANICAL

(Also see Nos 255, 406)

77-369

### Vibration Analysis as an Aid to the Detection and Diagnosis of Faults in Rotating Machinery

R M. Stewart

Inst. of Sound and Vibration Res, Univ. of Southampton, Southampton, England, "Conference on Vibrations in Rotating Machinery", The Inst. of Mech Engrs, Univ. of Cambridge, Sept 15-17, 1976, pp 223-229, 5 figs, 13 refs

**Key Words:** Rotating structures, Gears, Bearings, Machinery, Diagnostic techniques

The problems which confront the engineer wishing to monitor the condition of rotating machinery are first of all discussed in general terms. The subject of comparative diagnostics is then introduced and illustrated by reference to case histories of both bearing and gear failure.

## METAL WORKING AND FORMING

77-370

### Experimental and Analytical Investigation of Self-Excited Chatter in Metal Cutting

N. Saravanja-Fabris

Ph.D. Thesis, Illinois Inst. of Technology, 153 pp  
(1976)

UM 76-23, 498

**Key Words:** Metal working, Cutting, Chatter, Self-excited vibrations

Chatter in metal cutting is a nonlinear self excited vibration of the limit cycle type. The mechanism that causes it is not yet fully understood. The first part of this investigation is concerned with the analysis of chatter from the viewpoint of the describing function.

## OFF-ROAD VEHICLES

77-371

### Assessment of Occupational Noise Exposure and Associated Hearing Damage Risk for Agricultural Employees

J.D. Harris, B.J. Lindgren and R.L. Mann

J.I. Case Company, Milwaukee, WI., SAE Paper No. 760673, 12 pp, 9 figs, 8 refs

**Key Words:** Agricultural machinery, Noise measurement, Measurement techniques, Noise tolerance

This paper describes background information, measurement procedures, data analysis techniques, and results pertaining to assessment of agricultural employee occupational noise exposure. The criteria on which the proposed OSHA 85 db(A) 16 hour general industry noise regulation is based has been applied to predict hearing damage risk. Analysis of 237 man days of measured noise exposures shows that annual occupational noise exposure must be established to assess hearing damage risk. No evidence was found that agricultural employees are exposed to noise levels/durations which exceed the criteria for annual and lifetime noise exposure on which the proposed OSHA general industry regulation is based.

77-372

**Federal Regulation of Noise in Agricultural and Off-Highway Equipment**

F.A. Van Atta

Quinnipiac College, SAE Paper No. 760674, 3 pp, 4 refs

**Key Words:** Agricultural machinery, Noise reduction, Regulations

This paper seeks to answer questions pertinent to Federal regulations of noise in agricultural and off-highway equipment. It discusses the statutes under which authority lies, and whether or not the regulations actually accomplish the purposes for which they are intended.

**PUMPS, TURBINES, FANS,  
COMPRESSORS**

(Also see No. 307)

77-373

**Identification of an Automotive Gas Turbine. Part 1. Frequency Response Estimation**

P.E. Wellstead and D.J. Nuske

Control Systems Centre, Univ. of Manchester Inst. of Science and Technology, Sackville St., Manchester M60 1QD, England, Intl. J. Control, 24 (3), pp 297-309 (Sept 1976) 8 figs, 11 refs

**Key Words:** System identification, Motor vehicles, Gas turbines, Frequency response

A two-stage automotive gas turbine is subjected to an identification exercise. The system is studied using digital frequency response estimation procedures, and locally linearized transfer functions are obtained which span the normal working region of the engine.

77-374

**Identification of an Automotive Gas Turbine. Part 2. Parameter Fitting**

D.J. Nuske and P.E. Wellstead

Control Systems Centre, Univ. of Manchester Inst. of Science and Technology, Sackville St., Manchester M60 1QD, England, Intl. J. Control, 24 (3), pp 311-324 (Sept 1976) 9 figs, 10 refs

**Key Words:** System identification, Motor vehicles, Gas turbines

Using insight gained from a frequency-domain identification experiment a simple analytical model is formulated for an automotive gas turbine. With a structural model based upon

the analytical model a linear parametric representation is obtained by fitting to time-response data over the normal operating band of the engine.

**RAIL**

77-375

**DYNALIST II, A Computer Program for Stability and Dynamic Response Analysis of Rail Vehicle Systems. Volume 1. Technical Report**

T.K. Hasselman, A. Bronowicki and G.C. Hart

Transportation Systems Center, Cambridge, MA, Rept. No. DOT-TSC-FRA-74-14.1, FRA/ORD-75/22.1, 118 pp (Feb 1975)  
PB-256 046/4GA

**Key Words:** Interaction: rail-wheel, Railroad cars, Computer programs

A methodology and a computer program, DYNALIST II, have been developed for computing the response of rail vehicle systems to sinusoidal or stationary random rail irregularities. A modal synthesis procedure is used which permits the modeling of subsystems or components by partial modal representation using complex eigenvectors. Complex eigenvectors represent the amplitude and phase characteristics of rail vehicle systems which occur as a result of wheel-rail interaction, heavy damping in the suspension system and rotating machinery. Both vertical and lateral motion are handled by the program which allows up to twenty-five component and fifty system degrees of freedom.

77-376

**Acoustic Impacts of BART: Interim Service Findings**

Metropolitan Transportation Commission, Berkeley, CA., Bolt Beranek and Newman, Inc., Cambridge, MA., Rept. No. MTC-TM-16-4-76, 93 pp (Mar 1976)  
PB-254 966/5GA

**Key Words:** Rapid transit railways, Sound measurement, Vibration measurement

The report documents the findings and methodologies developed during a study of BART sound and vibration levels. The findings focus on: delineation of impacted regions, major factors affecting BART-generated sound, prototype vs. operational sound levels, BART vs. other transportation sound sources and BART-generated vibration levels. BART-generated sound levels were derived from direct wayside measurements and indirectly from on-board recording of sound levels throughout the BART system. Ambient community sound levels were based on predictive techniques verified by field measurements.

77-377

**Lateral Dynamics of a Tracked Air-Cushion Vehicle**

P.J. Davis and R.J. Hawks

General Electric Co., Philadelphia, PA., High-Speed Ground Transp. J., 10 (2), pp 135-146 (Summer 1976) 5 figs, 9 refs

**Key Words:** Tracked vehicles, Ground effect machines, Dynamic stability, Equations of motion

Linearized equations of motion for a TACV are developed in the three degrees-of-freedom of lateral displacement, yaw angle, and body roll angle. Air cushion forces and the aerodynamic loads on the body are included by means of stability derivatives. The lateral dynamic stability of the vehicle is analyzed by the use of root-locus diagrams.

## REACTORS

77-378

**Aeroelastic Modeling of Large Wind Turbines**

P.P. Friedmann

Mech. and Struct. Dept., University of California, Los Angeles, CA., J. Amer. Helicopter Soc., 21 (4), pp 17-27 (Oct 1976) 6 figs, 28 refs

**Key Words:** Turbine blades, Mathematical models, Equations of motion

A set of coupled flap-lag-torsional equations of motion for a single wind turbine blade are derived in a general, non-linear, partial differential form. These equations are suitable for determining the aeroelastic stability or response of large wind turbine blades. Methods for solving the equations together with some possible simplification of the equations are discussed. Finally, the formulation of the complete rotor-tower aeroelastic problem is considered in general terms.

77-379

**On Finite Element Analysis of Pipe Whip Problems**

S.M. Ma and K.J. Bathe

Bechtel Power Corp., San Francisco, CA 94119, Nucl. Engr. Des., 37 (3), pp 413-429 (June 1976) 14 figs, 10 refs

**Key Words:** Nuclear reactor containment, Pipes (tubes), Mathematical models, Finite element technique

The nonlinear dynamic finite element solution of pipe whip problems is presented. The finite element modeling used, the step-by-step incremental solution of the nonlinear equations

of motion and design considerations are discussed. The influence of various physical parameters on the response of the pipe and the restraint, and the effects of using different finite element models are considered. Specific emphasis is directed to the verification of the accuracy of the solutions obtained using energy balance checks.

77-380

**Energy Absorbers Used Against Impact Loading**

T. Kukkola

Nuclear Power Project Group, Imatran Voima Osa-  
keyhtiö, Helsinki, Finland, Nucl. Engr. Des., 37 (3),  
pp 407-412 (June 1976) 9 figs, 2 refs

**Key Words:** Nuclear reactor containment, Pipes (tubes), Energy absorption

In the WWER-440 reactor the primary piping consists of six horizontal loops going rapidly from the pressure vessel, each loop having a horizontal steam generator. In this reactor type the relatively long primary piping with many curved sections requires special attention in order to successfully eliminate the consequences of the design basis accident. Emergency supports are located in appropriate places to restrict the movements of the pipe. Under normal conditions there is a gap of some centimeters between the pipe and a support so that in the pipe can be deformed freely under changing loads. This paper deals with those energy-absorbing structures used at the Loviisa Nuclear Power Plant for protection against impact loading. Places and circumstances where energy-absorbing structures are employed are specified. Development and design of impact absorber elements are discussed and impact tests are described.

77-381

**The Use of Energy Absorbers to Protect Structures Against Impact Loading**

P. Hernalsteen and L.C. Leblois

TRACTIONEL, Societe de Traction et d'Electricite,  
B-1040 Brussels, Belgium, Nucl. Engr. Des., 37 (3),  
pp 373-406 (June 1976) 57 figs, 29 refs

**Key Words:** Nuclear reactor containment, Pipes (tubes), Energy absorption, Experimental data

When a flying missile impacts a fixed structure, the interface loading is dependent on the deformation characteristics of both impacting and impacted bodies. The following "energy absorption" materials and processes have been experimentally investigated, with an aim at pipe whipping restraint application for nuclear power plants; plastic extension of austenitic stainless steel rods, plastic compression of copper bumpers, and punching of lightweight concrete structures.

77-382

**Modelling Techniques for Pipe Whip Analysis**

D. Dini and L. Lazzeri

Comitato Nazionale Per l'Energia Nucleare, Divisione Sicurezza e Controlli, Rome, Italy, Nucl. Engr. Des., 37 (3), pp 361-371 (June 1976) 11 figs, 8 refs

**Key Words:** Nuclear reactor containment, Pipes (tubes), Mathematical models

The dynamic model used to describe the pipe and the restraint to predict their behavior during a shock phenomenon such as an earthquake is described. The pipe is simulated by means of a series of finite elements, having a complex elastoplastic behavior. Each element can be subdivided in to a number of subelements whose flexibilities are computed and internally condensed at the element level.

77-383

**Pipe Whip Analysis for Nuclear Reactor Applications**

N. Bisconti, L. Lazzeri and P.P. Strona

SAIGE, Societa de Architettura Industriale per gli Impianti di Generazione di Energia, I-16151 Genova-Sampierdarena, Italy, Nucl. Engr. Des., 37 (3), pp 347-360 (June 1976) 9 figs, 14 refs

**Key Words:** Nuclear reactor containment, Pipes (tubes), Computer programs

The main problems encountered in pipe whip analysis are discussed. Problems discussed are breakage locations, and force computations.

## RECIPROCATING MACHINE

77-384

**Dynamic Measurements of a Large Diesel Engine Using P.R.B.S. Techniques. Part 1. Development of Theory for Closed-Loop Sampled Systems**

J.O. Flower and G.P. Windett

School of Applied Sciences, Univ of Sussex, Falmer, Brighton, BN1 9QT, England, Intl. J. Control, 24 (3), pp 379-392 (Sept 1976) 8 figs, 7 refs

**Key Words:** System identification, Diesel engines

In attempting to identify the dynamic characteristics of a large medium-speed diesel engine using pseudo-random binary sequence (p.r.b.s.) and cross-correlation techniques inconsistent results were obtained. The cause of these is due to the sampled-data nature of the diesel engine operation and the difficulties were compounded by having to run so large an engine under closed-loop control conditions for safety reasons. Part 1 of this paper identifies the source of such problems arising from the use of such techniques for investigating sampled-data systems and develops a variation of the conventional theory which allows a wide class of both open and closed-loop sampled systems to be identified. Part 2 of the paper is concerned with the application of this theory to extensive measurements of a large engine.

77-385

**Dynamic Measurements of a Large Diesel Engine Using P.R.B.S. Techniques. Part 2. Instrumentation, Experimental Techniques and Results**

J.O. Flower and G.P. Windett

School of Applied Sciences, Univ. of Sussex, Falmer, Brighton, BN1 9QT, England, Intl. J. Control, 24 (3), pp 393-404 (Sept 1976) 11 figs, 6 refs

**Key Words:** Diesel engines, Dynamic properties, Measuring instruments, Measurement techniques

The work reported here in Part 2 concerns the application of the methods, discussed in Part 1, to obtain dynamic measurements from a large turbo-charged diesel engine. The small perturbations of this approach do not significantly affect normal engine running and can be well correlated with various linear models of the engine. The engine was a 750 b.h.p. rated unit designed specifically as the prime mover for electrical generation on a ship. In these experiments, however, the power produced was absorbed by a water-brake. For safety reasons it was not permitted to run the engine in other than a closed-loop configuration.

77-386

**On Piston Slap in Reciprocating Machinery**

Y. Fujimoto, T. Suzuki and Y. Ochiai

Dept. of Mech. Engrg, Tottori Univ., Japan, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept 15-17, 1976, pp 245-253, 13 figs, 14 refs

**Key Words:** Reciprocating engines, Pistons, Lubrication

Piston slap is a phenomenon when a piston impacts the cylinder wall, traveling across the small cylinder clearance space, during the course of its vertical movement along the cylinder axis in reciprocating machinery. Such lateral motion of the piston is caused by the change of the direction of the sidewise force acting on the piston. The dynamics of lateral and rotational motions of the piston were studied both theoretically and experimentally. Theoretical results obtained by numerical integration predict the continuous movements of the piston. The analysis includes the effect of the oil film formed on the piston skirt surface as well as the frictions at the rings. Experiments were carried out on a reciprocating compressor. Four miniature inductive displacement transducers on the piston skirt made it possible to determine both the transverse and rotational movements of the piston continuously. Comparison of calculated and measured movements showed the importance of oil cushion effect on piston slap.

77-387

**The Effect of Pistons in Diesel Engine Noise Generation**

M. Rohrlé

Ostfildern, MTZ Motortech. Z., 37 (7/8), pp 277-282 (July/Aug 1976) 9 figs, 16 refs (In German)

**Key Words:** Diesel engines, Engine noise, Noise measurement, Measurement techniques

A method for measuring diesel engine noise caused by piston hitting the cylinder wall is described.

77-388

**Interior Noise Problems of Small Diesel-Powered Buses**

G.L. Marlotte

H. L. Blachford, Inc., SAE Paper No. 760603, 12 pp, 15 figs, 12 refs

**Key Words:** Buses (vehicles), Diesel engines, Engine noise

This paper discusses interior noise problems characteristic of the combination of diesel engines with typical small bus geometries. One noise problem is the potential for low-frequency resonances. Natural air vibration modes for typical small bus interior dimensions are calculated and shown to give frequencies that can be stimulated by noise and vibration at engine firing frequencies. Measurements of noise and vibration at resonance are given. Other noise problems are associated with internal front engine covers, and leaks and low transmission loss paths in the console area. Some practical suggestions, with examples, are given to reduce interior noise.

77-389

**Mini RPV Engine Noise Reduction**

R.M. Shimovetz and D.L. Smith

Air Force Flight Dynamics Lab., Wright-Patterson AFB, OH, Rept. No. AFFDL-TR-76-28, 97 pp (Mar 1976)

AD-A027 638/6GA

**Key Words:** Engine noise, Reciprocating engines, Noise reduction

The purpose of this effort was to investigate the reduction in radiated acoustic noise associated with two types of engines considered for power plants in small (75-100 lb.) remotely piloted vehicles (Mini RPV) in the class of the Praelire and Calere Aircraft. The two engines considered are approximately 5 HP; the first a rotary combustion (RC), the second a two stroke cycle reciprocating (P). The sound pressure levels were recorded using a semicircle arrangement of microphones in a free field and with various engine noise reduction devices installed. The engines were rotated such that a spherical definition of the acoustic pressures were made.

77-390

**An Integrated Approach to Measurement of Exhaust and Intake Noise**

D.A. Olson, D.C. Flanders, and L.J. Eriksson

Corporate Research Dept., Nelson Industries, Inc., SAE Paper No 760602, 12 pp, 21 figs, 10 refs

**Key Words:** Engine noise, Noise measurement

An integrated approach for measurement of exhaust and intake noise during a single test is presented and typical results discussed. Internal pressure waveforms as well as external radiated noise are included for various intake and exhaust systems.

**ROAD**

(Also see Nos 285, 293, 348, 388)

77-391

**Biomechanics of Trauma in Children**

G. Ruter

Automobiltech Z., 78 (7/8), pp 339-342 (July/Aug 1976) 7 figs, 28 refs

(In German)

**Key Words:** Collision research (automotive), Human factors engineering

This paper reports about an international meeting of the International Research Committee on the Biokinetics of Impacts in Lyon, September 1974.

**77-392**

**Performance Evaluation of Child Test Dummies, Volume I. Technical Report**

M.P. Shah and V.G. Radovich

Transportation Research Center of Ohio, East Liberty, OH, Rept. No. DOT-HS-801-921, 261 pp (May 1976) (See also Vol 2, PB-255 119)

PB-255 118/2GA

**Key Words:** Collision research (automotive), Anthropomorphic dummies, Experimental data

Commerically available three- and six-year-old child dummies were evaluated for their anthropometric measurements and dynamic response characteristics in pendulum impact tests and simulated crashes in representative automobile-child seat restraint environments. Simulated crashes included 20- and 30-mile-per-hour frontal and 20-mph side impacts on automobile and specially designed bench seats.

**77-393**

**Performance Evaluation of Child Test Dummies, Volume II. Appendix**

M.P. Shah and V.G. Radovich

Transportation Research Center of Ohio, East Liberty, OH, Rept No. DOT-HS-801-922, 269 pp (May 1976) (See also Vol. 1, PB-255 118)

PB-255 119/0GA

**Key Words:** Collision research (automotive), Anthropomorphic dummies, Experimental data

This Appendix contains samples of the computer calculated data packages produced for each test. The digital data correlates with descriptions and data parameters discussed in the main body of the report.

**77-394**

**Development of Crash Energy Management Solutions**

H. Danckert

Volkswagenwerk AG, Germany, SAE Paper No. 760793, 20 pp, 19 figs 27 refs

**Key Words:** Collision research (automotive)

The mechanical properties of vehicles are investigated as a basis for favorable crash energy management solutions. Important parameters follow the similarity law of Cauchy. Different concepts of inter-vehicular compatibility are discussed. The tolerable closing speeds depending on the mass ratio and accelerations of colliding vehicles are calculated using realistic data of deformation forces, human load tolerances and different restraint system characteristics. Engine effects and measures to reduce engine aggressiveness are investigated. Possible solutions for accident types other than frontal collisions are briefly discussed.

**77-395**

**Radio Telemetry Applied to Tire Vibrations**

W.F. Reiter, Jr. and A.C. Eberhardt

Center for Acoustical Studies, N. Carolina State Univ., Raleigh, NC, SAE Paper No. 760745, 12 pp, 10 figs, 10 refs

**Key Words:** Automobile tires, Vibration measurement, Measurement techniques

This paper discusses the industrial radio telemetry system for use in typical automotive engineering measurements. Considerations of transducer type and measurement class (static or dynamic) are discussed in terms of selection of a telemetry system. Telemetry specifications and their interpretations are discussed along with characteristics and limitations of direct/FM and AM/FM systems. As an example application the measurement of in-service tire vibrations with radio telemetry is presented. The example includes selection and calibration of the telemetry system and presentation of typical telemetered tire vibration data.

**77-396**

**Proceedings of a Symposium on Commercial Vehicle Braking and Handling Held on May 5-7, 1975 at Ann Arbor, Michigan**

Highway Safety Research Inst., Michigan Univ., Ann Arbor, MI, Rept. No. UM-HSRI-PF-75-6, 599 pp (1975)

PB-255 985/4GA

**Key Words:** Motor vehicles, Tires, Brakes (motion arresters), Dynamic response, Proceedings

The symposium was to provide a review of the state of the art in commercial vehicle braking and handling. It was also designed to provide a forum in which members of various research organizations, industrial operations, and government agencies involved with the topic could communicate ideas and concerns over problems related to measuring and/or simulating the braking and directional response of commercial vehicles.

77-397

**Tentative Road Roughness Criteria Based Upon Vehicle Performance**

B.E. Quinn and S.R. Kelly

School of Mech. Engrg., Purdue Univ., Lafayette, IN,  
Rept. No. FHWA-RD-75-3, 107 pp (June 1975)  
PB-254 809/7GA

**Key Words:** Pavement roughness, Tire characteristics

Pavement roughness is defined in terms of two parameters of an equation used to represent the roughness spectrum. The relationship between these parameters and vertical tire force, passenger acceleration and lateral tire force is then developed. This is the third of three reports on pavement roughness and vehicle dynamics studies under this contract.

77-398

**Study of Aerodynamic Drag Reduction on a Full-Scale Tractor-Trailer**

L.L. Steers and L.C. Montoya

Dryden Flight Res. Ctr., NASA, Edwards AFB, CA,  
Rept. No. DOT-TSC-OST-76-13, 57 pp (Apr 1976)  
PB-254 571/3GA

**Key Words:** Articulated vehicles, Aerodynamic loads, Dynamic tests

Aerodynamic drag tests were performed on a tractor-trailer combination using the coast-down method on a smooth, nearly level runway. The tests included an investigation of drag reduction obtained with add-on devices that are commercially available or under development. The tests covered tractor-trailer speeds ranging from approximately 35 to 65 miles per hour and included fuel consumption measurements.

77-399

**An Assessment of the Crashworthiness of Existing Urban Rail Vehicles. Volume III. Train-Collision Model Users Manual**

D.J. Segal

Calspan Corp., Buffalo, NY, Rept. No. DOT-TSC-UMTA-75-21.III, UMTA-MA-06-0025-75-16.III, 66 pp (Nov 1975)  
PB-254 695/0GA

**Key Words:** Crashworthiness, Collision research (railroad), Energy absorption

The crashworthiness of existing urban rail vehicles (passenger cars) and the feasibility of improvements in this area were

investigated. Both rail-car structural configurations and impact absorption devices were studied. This final report issued under the crashworthiness effort covers the development of analytical tools to predict passenger threat - environment during collision; criteria for predicting passenger injury due to train collisions; an application of injury criteria and analytic models to predict passenger injuries resulting from collisions of trains that represent existing construction types; a preliminary investigation of applying impact absorption devices to transit vehicles; a design study of car structural configurations for improved impact energy management and a review of engineering standards for Urban Rail Car Crashworthiness.

77-400

**Vehicle Test Facilities at Aberdeen Proving Ground**

Army Test and Evaluation Command, Aberdeen Proving Ground, MD, Rept. No. TOP-1-1-011, 61 pp (Mar 17, 1976)

AD-A027 035/5GA

**Key Words:** Military vehicles, Tracked vehicles, Test facilities

The report describes Aberdeen Proving Ground facilities for testing wheeled and tracked vehicles including vehicular weapon systems. Includes photographs and drawings showing test course dimensions and characteristics. Does not cover equipment and instrumentation used on the courses nor laboratory facilities except for climatic test chambers.

77-401

**Control of Vehicle Dynamics Considering Guideway Design Criteria**

S.L. Graham and D.A. Hullender

Vought Corp., Dallas, TX, AIAA J., 14 (8), pp 1022-1025 (Aug 1976) 4 figs, 5 refs

**Key Words:** Interaction: vehicle-guideway, Suspension systems (vehicles)

The ride quality of a transportation vehicle is dependent on the effectiveness of the suspension system to filter out vibration due to guideway roughness and other sources such as wind gusts. To meet ride comfort specifications without unreasonable stroke requirements, it is sometimes necessary to compensate the suspension design with some form of either active or passive control. This paper addresses the subject of relating the tradeoff in ride comfort for suspension stroke to specific guideway profile design tolerances and constraints. Specifically, approximate lower bounds of RMS acceleration for any vehicle suspension system as a function of RMS suspension stroke are computed for specific guideway profile elevation variation constraints and construction surveying accuracies.

77-402

**Truck Noise VIII: The Determination of the Practical Noise Control Retrofitting of Pre-1970 Truck and Coach Models**

O.J. Bullard and G.M. Shaffer  
GMC Truck and Coach Div., General Motors Corp.,  
Pontiac, MI, Rept. No. DOT-TSC-OST-75-51, 120 pp  
(June 1976) (PB-241 114)  
PB-256 287/4GA

**Key Words:** Motor vehicle noise, Trucks, Noise reduction

A retrofit noise package was selected for four representative GMC vehicles to achieve optimum noise reduction. The selection of this material came from commercially available items submitted by various component suppliers. A new system of noise-source isolation was developed in order to evaluate the vendor-supplied material.

77-403

**Investigation on Vibration Characteristics of Unsprung Vehicles with Pneumatic Tyres**

A. Owzar  
Teheran, Iran, Automobil-tech. Z., 78 (9), pp 377-380 (Sept 1976) 8 figs, 10 refs  
(In German)

**Key Words:** Tractors, Seats, Vibration measurement

The purpose of this experiment was to examine the vibration characteristics of tractors. Practical measurements on the vibration characteristics of various vehicles were made to evaluate the effect of vibration on drivers.

## ROTORS

(Also see Nos 251, 332)

77-404

**Vibration Problems on Rotating Machinery in the Chemical Industry**

J.B. Erskine and C.W. Reeves  
Noise and Vibration Section, Agricultural Div.,  
ICI Ltd., Billingham Cleveland TS23 1LD, England,  
"Conf. on Vibrations in Rotating Machinery," The  
Inst of Mech. Engrs., Univ. of Cambridge, Sept.  
15-17, 1976, pp 209-214, 7 figs, 8 refs

**Key Words:** Rotors, Compressors, Diagnostic techniques

Vibration problems are encountered on different types of rotating equipment in Process Industries which can lead to very high economic losses due to the need to shut down for rectification. This paper outlines some of the more difficult problems met in recent years. Brief accounts are given of the symptoms of looseness and how they were identified on a centrifugal fan and a centrifugal compressor, the identification of vibration induced by rubbing of rotating elements on stationary parts, the cause of a modulating noise level from a gearbox driving an induced draught fan, problems arising from machinery being mounted on flexible bedplates and structures, and effects of bearing oil film on rotor vibration.

77-405

**Vibrations in Cracked Shafts**

T A. Henry and B.E. Okah-Avae  
Dept. of Engrg., Simon Engrg. Laboratories, Univ.  
of Manchester, England, "Conf. on Vibrations in  
Rotating Machinery," The Inst. of Mech. Engrs.,  
Univ. of Cambridge, Sept. 15-17, 1976, pp 15-19,  
6 figs, 3 refs

**Key Words:** Diagnostic techniques, Shafts, Rotors

The vibrations of a rotor are examined using analogue computer simulation. A crack is introduced as a reduction in shaft stiffness when the shaft deflects so as to open the crack. The interaction of gravity and the crack excites resonances at main and sub critical speeds. Out-of-balance combines with the crack to change the vibrations at the critical speed only in certain situations. The study, confirmed by experimental data, shows that the gravity effect dominates in horizontal shafts balanced to normal standards and that the growth of a crack would be detected from the vibrations at the main and sub critical speeds.

77-406

**Vibration in Rotating Machinery: Malfunction Diagnosis - Art & Science**

E. Downham  
The Univ. of Aston, Birmingham, England, "Conf.  
on Vibrations in Rotating Machinery," The Inst.  
of Mech. Engrs., Univ. of Cambridge, Sept. 15-17,  
1976, pp 1-6, 2 figs, 3 refs

**Key Words:** Diagnostic techniques, Rotating structures, Machinery

The generalized approach to vibration malfunction diagnosis of rotating machinery is shown to require a sound understanding of fundamental theoretical concepts associated with machine and machine element dynamics, an appreciation of the nature of the dynamic forces and instabilities which excite vibration in various types of machinery, together with the ability to plan concise experiments to obtain practical data regarding the cause or likelihood of failure. The wide variety of vibration phenomena which result in malfunction is illustrated by case studies on various types of industrial machinery.

**77-407**

**On the Determination of the Influence Coefficients in Rotor Balancing, Using Linear Regression Analysis**

L. Larsson

Balancing and Vibration Control Dept., ASEA, Sweden, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 93-98, 4 refs

**Key Words:** Rotors, Balancing techniques, Influence coefficient method

Balancing rotating bodies, using the conventional influence coefficient method does not always lead to a satisfactory result. The reason is often uncertainties in the response from a set of attached test weights which have been superimposed by internal, unknown alterations of the unbalance distribution caused by e.g. setting of the rotor windings or mechanical parts. In this paper, the emphasis is put on the determination of the influence coefficients, and a method for statistical evaluation is presented. The reliability of the so obtained influence coefficients is investigated on rigid and flexible rotors balanced in a test pit, and the application of one influence matrix for the balancing of other similar rotors is studied.

**77-408**

**Error Analysis in Vibration of Rotor/Bearing System**

T. Iwatsubo

Kobe Univ., Kobe, Japan, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 87-92, 6 figs, 11 refs

**Key Words:** Rotor-bearing systems, Error analysis, Balancing techniques, Stability analysis

This paper deals with the error analysis of the vibration of rotor bearing systems. If the statistical properties of errors (i.e., mean values and standard deviations) are known, the mean value and standard deviation of unbalance response and

critical speed may be obtained and the probability of instability may be calculated. The effects of influence coefficients measurement errors on the balancing weight determination are investigated for the balancing of flexible rotors.

**77-409**

**Modern Influence Coefficient Techniques for Multiplane Rotor Balancing in the Factory, Test Cell and Field**

R.H. Badgley

Mechanical Technology, Inc., Latham, NY, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 201-207, 2 figs, 8 refs

**Key Words:** Rotors, Balancing techniques, Influence coefficient matrix

Several modern techniques have been developed for performing multiplane-multispeed balancing by the influence coefficient procedure. These techniques include the use of digital minicomputers to control the performance of multiple low-cost spin-up stands in high-production factory environments, as well as the use of remotely accessed time-sharing computer systems for field balancing and occasional user applications. Significant improvements in balance quality are now achievable, together with manufacturing cost reductions. New machinery is now being designed to take advantage of this new capability.

**77-410**

**Systematic Combination of Experiments and Data Processing in Balancing of Flexible Rotors**

J. Drechsler

Technische Universität, Berlin, Germany, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 129-132, 7 refs

**Key Words:** Rotors, Balancing techniques, Influence coefficient matrix

Under the assumption that an approximate influence coefficient matrix is available, the balancing process of a flexible rotor can be considerably accelerated by a consequent combination of experiments and data processing, which yields balancing weights and an improved influence coefficient matrix after each trial run. Even if a trial run has to be interrupted because of excessive vibration amplitudes, the proposed method yields balancing weights, which will reduce the vibration amplitudes of the explored speed range and will not amplify the vibration level in the remaining unknown operation speed range.

77-411

**Modal Resolution and Balancing of Synchronous Vibrations in Flexible Rotors with Non-Conservative Cross Coupling**

H.F. Black and S.M. Nuttall

Heriot-Watt Univ., Edinburgh, Scotland, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 151-158, 4 figs, 14 refs

**Key Words:** Rotor-bearing systems, Balancing techniques, Mathematical models

The paper examines the response to mass unbalance of a generic rotor in hydrodynamic bearings. The non-conservative equations of motion are separated by bi-normal orthogonalization. Modal resolution of unbalance response is discussed in relation to balancing. Examples of modal resolution and balancing calculations are given.

77-412

**A Method for Using the Free Shaft Modes in Rotor Balancing**

J.W. Lund

Technical Univ. of Denmark, Lyngby, Denmark, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 65-71, 4 figs, 9 refs

**Key Words:** Shafts, Mathematical models, Modal analysis, Balancing techniques

From a correlation of measured and calculated natural frequencies and modes it is assumed that a representative analytical model can be established for the free shaft. With the shaft installed and running at some selected speed the unbalance response amplitudes are measured at several points along the rotor after which a resolution of the response into its free mode components is performed. The unknown bearing reactions, including damping forces, are evaluated either by trial weight runs, one run per bearing, or, in case of just two bearings, from a force balance. Elimination of the bearing reactions leads to the determination of the mass unbalance in terms of its modal expansion, and corresponding correction weights are then readily found.

77-413

**Practice of Flexible Rotor Balancing**

A. Giers

Carl Schenck AG, Darmstadt, W. Germany, "Conf on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 33-42, 16 figs, 15 refs

**Key Words:** Rotors, Balancing techniques

In the last few years the volume of literature concerning flexible rotor balancing has grown by leaps and bounds. Most authors concentrate on the theoretical aspects so that the introduction of new practical method is progressing slowly, and attempts at putting theory into practice are often quickly abandoned due to a lack of clear instructions from theoreticians. This paper is intended to provide practical guidelines for proper selection and application of flexible rotor balancing techniques.

77-414

**The Whirl Modes of Vibration of a Crankshaft**

D. Hodgetts

School of Automotive Studies, Cranfield Inst. of Technology, England, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 255-261, 4 figs, 3 refs

**Key Words:** Crankshaft, Whirling, Natural frequencies, Mode shapes

A theoretical three-dimensional model has been developed to provide the frequencies and modal shapes of the coupled whirling, axial and torsional modes of vibration of a crankshaft. A satisfactory correlation of the measured and theoretical frequencies of the vibrations was obtained for frequencies below 500Hz. The possibility of using the model to predict the amplitudes of vibration is discussed.

77-415

**Dynamic Stiffness Matrix Approach for Rotor-bearing System Analysis**

N.F. Rieger, C.B. Thomas and W.W. Walter

Rochester Inst. of Technology, Rochester, NY, "Conf on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 187-193, 8 figs 15 refs

**Key Words:** Rotor-bearing systems, Stiffness methods, Critical speeds, Unbalanced mass response, Stability analysis, Computer programs

The dynamic stiffness matrix concept is presented as a general technique for calculating the unbalance response, critical speeds (either damped or undamped) and stability threshold speed of a flexible rotor in damped flexible supports. A linear elastic rotor system in eight-speed dependent coefficient bearings is considered. A computer code based on this concept is described.

77-416

**A Method of Stabilizing Externally-Pressurized Gas Journal Bearings**

D.J. Woodford

Central Electricity Res. Labs., Leatherhead, England, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 111-115, 6 figs, 4 refs

**Key Words:** Rotors, Whirling, Stabilization

Rotors supported in externally-pressurized gas journal-bearings are subject to an instability known as self-excited translational whirl. The onset of whirl limits the maximum speed to which a rotor may be run. This paper shows how the upper bound to the mass parameter may be increased by means of a simple modification to the rotor's surface.

77-417

**Investigations into Load Dependent Vibrations of the High Pressure Rotor on Large Turbo-Generators**

S.H. Greathead and P. Bastow

CEGB NE Region Scientific Services Dept., Otley Road, Harrogate, England, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept 15-17, 1976, pp 279-285, 7 figs, 8 refs

**Key Words:** Turbomachinery, Rotors, Stability analysis

Vibrational instability of the high pressure rotor on certain types of large turbo-generator operated by the C.E.G.B. has sometimes developed above a particular load level. When this instability arises very high non-synchronous vibration of the rotor and bearings occurs necessitating load restrictions. In the past, this type of non-synchronous vibration was usually attributed to bearing instability but the present investigations show that the instability originates mainly in the steam glands at the high pressure inlet to the machine. The results show that the rotor instability is a self-excited vibration due to steam forces generated in these glands resulting from the steam leakage flow through them and that the rotor to gland alignment is an important parameter controlling the load at which instability occurs.

77-418

**Modal Analysis of Rotating Flexible Structures**

R.M. Laurenson

McDonnell Douglas Astronautics Co., East St. Louis, MO, AIAA J., 14 (10), pp 1444-1450 (Oct 1976) 9 figs, 14 refs

**Key Words:** Modal analysis, Rotating structures, Cantilevers

A discussion of the modal analysis of a rotating flexible structure is presented. Specifically this work is concerned with the free vibration analysis of a rotating cantilever structure. A brief discussion of the formulation of the equations of motion for the system is included to aid in understanding the origin of the various terms appearing in these equations.

77-419

**The Modal Interpretation of the Vibration of a Damped Rotating Shaft**

A.G. Parkinson

Mech. Engrg. Dept., University College, London, England, "Conf. on Vibrations in Rotating Machinery," The Inst. Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 261-269, 4 figs, 9 refs

**Key Words:** Rotors, Shafts, Modal analysis

Much of the theoretical progress in rotor dynamics has been made in terms of modal analysis. Practical applications of the theory, however, have resulted in many, often misleading, references to principal modes, damped modes and "twisted" modes. The relationships between these various concepts and the restrictions to their use are discussed. The intention is to remind the practical vibration engineer that no method of diagnosis and correction, such as balancing, is likely to be successful in an industrial context unless the actual shaft vibration is correctly interpreted.

77-420

**A Strategy for Investigating the Linear Dynamics of a Rotor in Bearings**

I. Fawzy and R.E.D. Bishop

Dept. of Mech. Des., Faculty of Engrg., Univ. of Cairo, Egypt, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 239-244, 1 fig, 7 refs

**Key Words:** Rotors, Rotor-bearing systems, Linear analysis, Modal analysis

Linear modal analysis of rotor dynamics necessitates the making of simplifying assumptions when the rotor is supported in bearings that are not 'ideal', i.e. when it is not housed in free, pinned, clamped or sliding bearings. A theoretical method is outlined by which this difficulty can be overcome. The rotor/bearing system is treated as the non-conservative system it is and a modified form of linear modal analysis is employed.

77-421

**Vibrations of a Rotating Asymmetric Shaft Carrying Two Disks, Supported in Asymmetric Bearings**

L. Forrai

Dept. of Mechanics, Technical Univ. of Heavy Industry, Miskolc, Hungary, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 43-48, 2 figs, 7 refs

**Key Words:** Shafts, Rotors, Perturbation theory

Parametrically excited lateral vibrations of a rotating asymmetric light shaft carrying two disks, supported in asymmetrically flexible bearings, are analyzed by the approximate perturbation-variation method of Hsu. The solutions of the equations of motion give simple explicit expressions for stability boundaries provided the shaft asymmetry is small. There is no limitation on the degree of bearing asymmetry. Three-dimensional diagram shows the unstable speed-regions for a rotor system with asymmetries of both the cross-section and the bearing.

77-422

**A Quick, Graphical Way to Analyze Rotor Whirl**

H.D. Nelson and D.A. Glasgow

Arizona State Univ., Tempe, AZ, Mach. Des., 48 (23), pp 124-130 (Oct 7, 1976) 6 figs

**Key Words:** Rotor-bearing systems, Rotors, Whirling, Mathematical models, Graphic methods

Complete design studies of rotor/bearing systems usually involve complex digital computer programs. But preliminary studies often can be based on greatly simplified math models. Here is a set of design curves that take some of the drudgery out of rotor analysis.

77-423

**A Practical Vibration Primer  
Part 6. Critical Speeds and Mode Shapes**

C. Jackson

Monsanto Co., Texas City, TX, Hydrocarbon Processing, 55 (10), pp 141-146 (Oct 1976) 12 figs

**Key Words:** Rotors, Critical speeds, Mode shapes

All turboequipment operates at certain speeds where the forcing or self-exciting frequencies are sympathetic (at synchronous or harmonics) to the resonances of a rotor, bearing, support system. When this resonance occurs at a finite operating speed, that speed is referred to as a critical speed. This basic article describes these phenomena.

77-424

**Rotor Vibration: The Choice of Optimal Journal Bearings**

D. Morrison

Y-ARD Ltd., Consulting Engineers, Glasgow, Scotland, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 7-14, 5 figs, 5 refs

**Key Words:** Rotors, Vibration response, Optimization, Journal bearings

A wide body of experience is now accumulating on the prediction of the vibratory behavior of rotor systems. Such predictions are in reasonable agreement with practical behavior and there is a close relationship with the general forms of behavior which are intrinsic in some early simple predictions. The relationship of these behavioral patterns to the absolute bearing clearance is noted as being particularly important and the numerical values involved are such that it appears possible to arrive at optimal bearing clearances for particular rotors, which are clearly not impractical. It is noted that wide ranges of bearing clearance ratio are in common use and hence that there is no real physical barrier to varying the clearances if this is vibrationally desirable. A simple (to carry out) method of determining an optimum clearance for a rotor is demonstrated by an example and the derivation is given.

77-425

**Characteristic Vibrations of Flexural Rotors in Journal Bearings**

E. Pollmann and H. Schwerdtfeger

Kraftwerk Union AG, Mulheim-Ruhr, W. Germany, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 21-26, 14 figs, 3 refs

**Key Words:** Journal bearings, Shafts, Rotors, Vibration response

The vibration behavior of an electrical generator was examined by considering similar geometric and dynamic conditions on a model with various types of bearings. When running through the first and second critical speeds, the unbalance vibrations show the large amplitudes typical for slender rotors. In order to examine stability behavior, the damping of a shock induced vibration was ascertained. In a wide range of rotation speeds low frequency dependent on the oil supply pressure occurred with a limited amplitude.

77-426

**Experimental Studies on the Shaft Dynamics of Large Turbogenerators for an Improved Surveillance**

A. Azzone, A. Clapis, R. Gariboldi, G. Lapini, G. Possa, and T. Rossini

Centro Informazioni Studi Esperienze, Segrate, Milano, Italy, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 173-179, 13 figs, 1 ref

**Key Words:** Shafts, Turbomachinery, Testing techniques, Test equipment

The paper presents the main results of some experimental studies on the shaft dynamics of two ENEL large turbogenerators, i.e. a 70 MWe unit at the Emilia power station (Piacenza) and a 320 MWe unit at the Turbigo Levante power station. The tests were performed within the frame of a research programme aimed at improving the continuous automatic surveillance techniques.

77-427

**Normal Mode Interaction in Self-Excited Rotor Vibrations**

E. Brommundt

Mechanikzentrum, Lehrstuhl A für Mechanik, Technische Universität Braunschweig, Germany, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 105-109, 5 figs, 5 refs

**Key Words:** Rotors, Stability analysis, Normal modes

When a rotor gets unstable, its oscillations are closely related to one of its normal modes. At different parameter sets, e.g., different speeds, different normal modes can act. When these simple motions, too, get unstable, combinatory motions with interaction of the normal modes can occur.

77-428

**The Impact of Rotor Dynamics Analysis on Advanced Turbo Compressor Design**

R G. Kirk

Ingersoll-Rand Co., Turbo Products Div., Phillipsburg, NJ, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 139-150, 7 figs, 16 refs

**Key Words:** Rotor-bearing systems, Compressors, Turbines, Stability, Design techniques

The advances that have been made in the past several years in the area of rotor-bearing analysis is changing the basic design procedure of the major industrial compressor manufacturers. The capability to calculate undamped critical speeds, forced response of damped rotor-bearing system, damped eigenvalues for linearized stability, and time transient response for large amplitude whirl and shock loading studies is now available to conduct design reviews of proposed rotor systems. Some of the problems that still exist in rotor dynamics analysis are discussed.

77-429

**Numerical Analysis of Free Vibrations of Damped Rotating Structures**

K.K. Gupta

Jet Propulsion Lab., Calif. Inst. of Technology, Pasadena, CA., "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 27-31, 1 fig, 7 refs

**Key Words:** Rotating structures, Damped structures, Numerical analysis, Free vibrations, Finite element technique, Component mode synthesis, Rotors

This paper is concerned with the efficient numerical solution of damped and undamped free vibration problems of rotating structures. While structural discretization is achieved by the finite element method, the associated eigenproblem solution is effected by a combined Sturm sequence and inverse iteration technique that enables the computation of a few required roots only without having to compute any other. For structures of complex configurations, a modal synthesis technique is also presented, which is based on appropriate combinations of eigenproblem solution of various structural components. Such numerical procedures are general in nature, which fully exploit matrix sparsity inherent in finite element discretizations, and prove to be most efficient for the vibration analysis of any damped rotating structure, such as rotating machineries, helicopter and turbine blades, spinning space stations, among others.

77-430

**The Solution of Vibration Problems with Simple Models**

E. Kramer, R. Nordman, and H.D. Klement

Technische Hochschule Darmstadt, German Federal Republic, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 79-86, 12 figs

**Key Words:** Shafts, Machinery, Elastic foundations, Natural frequencies

It is usual today to calculate the vibrations of turbo-sets in most cases with separate models for shaft and foundation, because the necessary combined calculation is expensive and time-wasting. In some cases more simple models are sufficient, whose behavior is also illustrative for the understanding of larger systems. This will be shown with the example of a machine shaft, mounted on a block foundation.

**77-431**

**Dynamic Behaviour of a Simple Rotor with a Cross-Sectional Crack**

R. Gasch

Technical University, Berlin, Germany, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 123-128, 9 figs, 2 refs

**Key Words:** Rotors, Cracked media, Stability, Whirling

In this paper the dynamic behavior of a rotor consisting of a disc on a flexible shaft with a cross-sectional crack is analyzed. The (non-linear) equations of motion are derived by replacing the crack by a simple mechanism. Experiments have shown, that the deflection-properties of this model come very close to that of the shaft with a real crack. Some results of an analog-computer-study are given for the influence of weight and unbalance on stability and whirl.

**77-432**

**Effect of Damping on the Flutter Boundary of Rotating Systems**

K. Huseyin

Univ. of Waterloo, Ontario, Canada, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 133-137, 4 figs, 9 refs

**Key Words:** Rotors, Flutter, Vibration damping

The effect of internal and external damping on the divergence and flutter boundaries of rotating systems under the influence of a conservative force (e.g. axial load) is studied in general terms on a comparative basis with the corresponding undamped system. Certain lower and upper bounds as well as convexity properties are established. As an example, the flexible rotating shaft subjected to axial load is analyzed.

**77-433**

**Large Amplitude Whirls of Rotors**

G. Capriz and A. Laratta

Istituto di Elaborazione della Informazione C.N.R., Pisa, Italy, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 49-52, 4 refs

**Key Words:** Shafts, Lubrication

Large amplitude vibrations are considered in shafts rotating on lubricated bearings, for a certain class of lubricant response; the behavior of the shaft is described in different ranges of speed of rotation. It is shown finally that the analysis applies partly also to more general cases of lubricant response (e.g., short bearings with cavitating oil film).

**SATELLITE**

(See No. 418)

**SHIP**

(Also see No. 337)

**77-434**

**Two Media Low Pressure Transient Measurements**

H.E. Krachman, L.G. Lindsey, and W.F. Caplan  
Developmental Sciences, Inc., City of Industry, CA,  
ISA Transactions, 15 (1), pp 17-21 (1976), 15 figs

**Key Words:** Hydrofoil craft, Ships, Slamming

Pressure transients were measured on the bow section of a model surface effect ship while investigating the bow slamming phenomenon.

**77-435**

**Nature of Ship Collisions within Ports**

J.E. Warwick and A.L. Anderson

Res. and Tech. Div., Todd Shipyards Corp., Galveston, TX, Rept. No. NMRC-336-40000-R1, MA-RD-920-76060, 48 pp (Apr 1976)  
PB-255 304/8GA

**Key Words:** Nuclear powered ships, Collision research (ships)

The report is a summary of the nature of ship collisions which have occurred in ports or while in the protected waters enroute to port and is based on information obtained for ship operations in seven representative U.S. ports over a five year period. In addition to the collisions which have occurred, port traffic information for the same time period is included.

77-436

**Structural Analysis of Aircraft Impact on a Nuclear Powered Ship**

R. Dietrich

Institut für Analagentchnik, Gesellschaft für Kernenergieverwertung in Schiffbau und Schifffahrt mbH., D-2054 Geesthacht-Tesperhude, Germany, Nucl. Engr. Des., 37 (3), pp 333-345 (June 1976) 19 figs, 16 refs

**Key Words:** Collision research (ships), Collision research (aircraft), Nuclear powered ships, Dynamic response, Finite element technique

As part of a general safety analysis, the reliability against structural damage due to an aircraft crash on a nuclear powered ship is evaluated. This structural analysis is an aid in safety design. It is assumed that a Phantom military jet-fighter hits a nuclear powered ship. The total reaction force due to such an aircraft impact on a rigid barrier is specified in the guidelines of the Reaktor-Sicherheitskommission (German Safety Advisory Committee) for pressurized water reactors. This paper investigates the aircraft impact on the collision barrier at the side of the ship.

**SPACECRAFT**

(Also see No. 325)

77-437

**Appendage Modal Coordinate Truncation Criteria in Hybrid Coordinate Dynamic Analysis**

P. Likins, Y. Ohkami, and C. Wong

Columbia Univ., New York, NY, J. Spacecraft and Rockets, 13 (10), pp 611-617 (Oct 1976) 1 fig, 9 refs

**Key Words:** Spacecraft, Dynamic analysis

Seven alternative candidates for hybrid and normal coordinate truncation criteria are offered for consideration, interpreted, and illustrated by example. Two of the criteria are based on system eigenvalues, two on system eigenvectors, and three on measures of controllability and observability. No definitive basis is provided for selection among the seven criteria, but comparisons are offered.

77-438

**Experimental Study on Oscillatory Combustion in Solid-Propellant Motors**

T. Koreki, I. Aoki, K. Shirota, Y. Toda, and K. Kuratani

Nissan Motor Co., Ltd., Tokyo, Japan, J. Spacecraft and Rockets, 13 (9), pp 534-539 (Sept 1976) 5 figs, 9 refs

**Key Words:** Solid rocket propellants, Combustion excitation

The relationship between the longitudinal mode oscillatory combustion phenomena in solid-propellant motors and the motor configuration is experimentally examined using small test motors with various grain configurations.

77-439

**Vibration of a Flexible Spacecraft with Momentum Exchange Controllers**

J.R. Canavin

Ph.D. Thesis, Univ. of California, Los Angeles, 172 pp (1976)  
UM 76-22,179

**Key Words:** Spacecraft, Vibration response, Rotating structures

Significant problems are presented in the vibration and rotation analysis of spacecraft with distributed structural flexibility and momentum exchange controllers. These systems exhibit gyroscopic coupling which depends on the rotor speed and orientation, which must remain explicit in the analysis as control variables. An investigation is made into floating reference frames in order to allow first order vibration analysis in the presence of large system rotations.

77-440

**Digital Program for Computing the Transfer Function and Frequency Response of the Fluidgenics Regulator (CCD-PROG-024)**

C.A. Roth

General Dynamics/Convair, San Diego, CA, Rept. No. GDC-DDE66-014, SAMSO-TR-76-173, 78 pp (Mar 1966)

AD-A027 769/9GA

**Key Words:** Pressure regulators, Frequency response, Computer programs, Missile components

The Fluidgenics Pressure Regulator Program was developed to evaluate the linearized, steady-state transfer function associated with the Fluidgenics pressure regulators used on the ATLAS LAUNCH VEHICLES. The regulator analysis was made in conjunction with the 5 cps longitudinal oscillation stability analysis. The important quantities generated by the program are the steady-state gains, damping ratios, and break frequencies for the transfer function; the transfer function in equation form, and the magnitudes and phase angles as a function of input frequency.

## STRUCTURAL

77-441

### Prediction of Noise Distribution in Various Enclosures from Free-Field Measurements

A.G. Galaitis and W.N. Patterson  
Bolt Beranek and Newman, Inc., Cambridge, MA  
02138, J. Acoust. Soc. Amer., 60 (4), pp 848-856  
(Oct 1976) 8 figs, 11 refs

Key Words: Rooms, Tunnels, Enclosures, Noise prediction

The image theory method is used to predict sound pressure levels generated by an acoustic source of known strength in regular rooms, tunnels, and flat rooms. For regular rooms, the results are identical to those of the well-established diffuse field theory. For tunnels and flat rooms, the predictions are verified by direct measurements taken in underground mines.

77-442

### Seismic Design Methods for Military Facilities -- Preliminary Recommendations

W.K. Stockdale  
Const. Engrg. Res. Lab., U.S. Army, Champaign, IL,  
Rept. No. CERL-IR-M-184, 80 pp (June 1976)  
AD-A027 384/7GA

Key Words: Seismic design, Military facilities

This report presents preliminary recommendations for the methods of structural analysis to be used in the design of critical facilities on military installations. The recommended dynamic analysis methods are described and discussed, and examples are presented which illustrate the elastic and inelastic response spectra methods. It is recommended that dynamic methods be used in all areas where the expected ground acceleration exceeds 0.10 g.

## TURBOMACHINERY

(Also see Nos. 306, 308, 310, 378, 417, 426)

77-443

### Dynamic Response of Cavitating Turbomachines

S.-L. Ng  
Ph.D. Thesis, California Inst. of Technology, 113 pp  
(1976)  
UM 76-23,385

Key Words: Turbomachinery, Cavitation

Stimulated by the pogo instability encountered in many liquid propellant rockets, the dynamic behavior of cavitating inducers is the subject of the present thesis. An experimental facility where the upstream and downstream flows of a cavitating inducer could be perturbed was constructed and tested. The upstream and downstream pressure and mass flow fluctuations were measured. Matrices representing the transfer functions across the inducer pump were calculated from these measurements and from the hydraulic system characteristics for two impellers in various states of cavitation. The transfer matrices when plotted against the perturbing frequency showed significant departure from steady state or quasi-steady predictions especially at higher frequencies.

77-444

### Acceptance Criteria for Compressors and Turbines in the Process Industry Based on the Response to Deliberate Unbalance

P.E. Simmons  
ICI Petrochemicals Div., Wilton, Middlesbrough, England, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 73-78, 4 figs

Key Words: Turbomachinery, Unbalanced mass response

A generation of large turbo-machines for the process industries is emerging in which it is difficult and may be undesirable to achieve the degree of separation between the operating speed range and the lateral critical speeds that is commonly specified. Alternative criteria are suggested which are based on the response of the rotor to deliberate unbalance.

77-445

**Vibrational Problems in Modern Power Station Plant**

W.G.R. Davies, A.W. Lees, I.W. Mayes, J.H. Worsfold, and F.J.P. Crampton

Mech. Sci. Section, CEGB Midlands Region, Scientific Services Dept., Radcliffe-on-Soar, Nottingham, England, "Conf. on Vibrations in Rotating Machinery," The Inst. of Mech. Engrs., Univ. of Cambridge, Sept. 15-17, 1976, pp 215-222, 7 figs, 3 refs

**Key Words:** Turbomachinery, Power plants (facilities), Diagnostic techniques

The paper describes the combined application of theoretical and standard experimental techniques to the diagnosis of operational problems on a modern power generation plant. In the first example, thermal changes in the relative heights of bearing pedestals were shown to cause one bearing of a large turbogenerator to become unloaded leading to instability and excessive vibration. At another location, a large fan was discovered to have critical speeds within the running speed range due to gyrodynamic forces on the overhung impeller.

77-446

**Vibration Excited by Lateral Forces Caused by the Leakage Flow in Thermal Turbomachinery**

K. Urlichs

Technische Universitat Munchen, Lehrstuhl f. Technische Kraftanlagen, Arcisstrasse 21, D-8000 Munchen 2, BRD, Germany, Ing. Arch., 45 (3), pp 193-208 (1976) 12 figs, 13 refs

(In German)

**Key Words:** Vibration excitation, Turbomachinery

Self excited vibrations of turbo rotors can be caused by lateral forces proportional to the displacement, which result at the turbine stages from the leakage flow between the rotor and the casing. For the calculation of these forces the variable clearances are represented by a set of stream tubes with various cross sections. So the local mass flux and the pressure distribution in the clearance follows by means of one-dimensional empirical relations, taking into account the circumferential velocity at the influx. Measurements on a test turbine gave good agreement with calculated results.

**USEFUL APPLICATION**

77-447

**Vibroflotation and Terra-Probe Comparison**

R.E. Brown and A.J. Glenn

Law Engrg. Testing Co., Washington, D.C., ASCE J. Geotech. Engr. Div., 102 (GT 10), pp 1059-1072 (Oct 1976)

**Key Words:** Vibratory techniques, Compacting, Soils

The most widely used techniques for deep vibratory compaction of cohesionless soils are Vibroflotation and Terra-Probe. The Vibroflotation process consists of penetrating the soil with a horizontally vibrating probe and simultaneously saturating and vibrating the soil as the probe is extracted. The Terra-Probe process consists of driving and retracting a 30-in. (760-mm) diam open-ended steel pipe with vertical vibrations induced by a vibratory pile driver. Three test patterns with different probe spacings were compacted with 100-hp Vibroflot probes and three test patterns were compacted by Terra-Probe. The test results are discussed in this paper.

# AUTHOR INDEX

Abdel-Ghaffar, A.M. . . . .	356	Chandran, K.B. . . . .	363	Giers, A. . . . .	413
Adi Murthy, N.D. . . . .	320	Chen, P.J. . . . .	265	Glasgow, D.A. . . . .	422
Aksu, G . . . . .	338	Chien, C.F. . . . .	241	Glenn, A.J. . . . .	447
Ali, R. . . . .	338	Chou, P.C. . . . .	297	Grabowski, B. . . . .	251
Allaire, P.E. . . . .	306	Chu, K.H. . . . .	357	Graham, S.L. . . . .	401
Alwar, R.S . . . . .	320	Chung, T.J. . . . .	325	Greathead, S.H. . . . .	417
Anand, G.V. . . . .	258	Clapis, A . . . . .	426	Greif, R. . . . .	341
Anderson, A.L . . . . .	435	Clarke, J.D. . . . .	337	Gunter, E.J. . . . .	306
Aoki, I . . . . .	438	Confer, V.J. . . . .	252	Gunzburger, M.D. . . . .	239
Arnoldi, R.A. . . . .	307	Crampton, F.J.P. . . . .	445	Gupta, K.K. . . . .	429
Au-Yang, M.K. . . . .	327	Danckert, H. . . . .	394	Hall, M. . . . .	234
Ayres, D.J. . . . .	281	Davies, W.G.R. . . . .	445	Halliwell, D.G. . . . .	309
Azzoni, A. . . . .	426	Davis, P.J. . . . .	377	Harris, J.D. . . . .	371
Badgley, R.H. . . . .	409	De Hoog, F.R. . . . .	235	Hart, G.C. . . . .	375
Bai, K.J. . . . .	279	Delph, T.J. . . . .	273	Hashin, Z . . . . .	283
Baig, M.I. . . . .	329	Dib, G.M. . . . .	334	Hassab, J.C. . . . .	233
Bannister, R.H . . . . .	305	Dietrich, R . . . . .	436	Hasselmann, T.K. . . . .	275, 375
Barrett, L.E. . . . .	306	Dini, D. . . . .	382	Hawks, R.J. . . . .	377
Bastow, P. . . . .	417	Doll, W. . . . .	282	Hegdahl, T. . . . .	266, 267
Bates, C.L. . . . .	321	Downham, E. . . . .	406		268, 269, 270
Bathe, K.J. . . . .	379	Drechler, J . . . . .	410	Hemmings, R.C. . . . .	304
Becker, R.J. . . . .	344	Dubey, R.K. . . . .	330	Henry, T.A. . . . .	405
Bennett, B.E . . . . .	232	Dufort, R.H. . . . .	365	Hernalsteen, P. . . . .	381
Berge, B . . . . .	299	Eberhardt, A.C. . . . .	395	Herrmann, G . . . . .	232, 273
Bernhagen, J.R. . . . .	368	Eidson, R.L. . . . .	325	Hilber, H.M . . . . .	249
Bezler, P . . . . .	322	Ellingson, E.F . . . . .	367	Hino, M. . . . .	280
Bigret, R. . . . .	308	Eriksson, L.J. . . . .	390	Hodgetts, D. . . . .	414
Bisconti, N . . . . .	383	Erskine, J.B. . . . .	404	Houghton, J.R. . . . .	256
Bishop, R.E.D. . . . .	420	Ewins, D.J . . . . .	310	Hud, G.C. . . . .	255
Black, H.F. . . . .	411	Fagerlund, A.C. . . . .	324	Hughes, T.J.R . . . . .	249
Blackstock, D.T . . . . .	345	Fawzy, I . . . . .	420	Hugus, G.D . . . . .	286
Boland, J.S., III . . . . .	362	Flanders, D.C. . . . .	390	Hullender, D.A. . . . .	401
Boxwell, D.A. . . . .	361	Fletcher, N.H. . . . .	257	Hundal, M.S. . . . .	343
Bragg, E.E. . . . .	288	Flis, W.J. . . . .	297	Huseyin, K . . . . .	432
Brommundt, E . . . . .	427	Flower, J.O. . . . .	384, 385	Ivey, E.S . . . . .	260
Bronowicki, A . . . . .	375	Ford, M.B. . . . .	264	Iwan, W.D . . . . .	339
Brown, R.E. . . . .	447	Forrai, L. . . . .	421	Iwatsubo, T. . . . .	408
Bullard, O.J. . . . .	402	Friedmann, P.P . . . . .	378	Jackson, C. . . . .	423
Burdess, J.S . . . . .	284	Fujimoto, Y. . . . .	386	Jacquot, R.G. . . . .	245
Burns, C.D . . . . .	359	Fukuoka, H . . . . .	272	Jenssen, A . . . . .	266, 267,
Canavin, J.R . . . . .	439	Galatsis, A.G . . . . .	441		268, 269, 270
Caplan, W.F . . . . .	434	Gale, J.G . . . . .	300	Johnson, A.F . . . . .	294
Capriz, G . . . . .	433	Gariboldi, R . . . . .	426	Johnson, G.R . . . . .	250
Carta, F.O . . . . .	307	Gasch, R . . . . .	431	Junghans, R . . . . .	347

Kaliski, S. . . . .	277	Mann, R.L. . . . .	371	Quinlan, P.M. . . . .	326
Kamil, H. . . . .	263	Marlotte, G.L. . . . .	388	Radovich, V.G. . . . .	392, 393
Kaper, B. . . . .	243	Martin, C.R. . . . .	358	Ramamurti, V. . . . .	328
Kaul, R.K. . . . .	273	Maunder, L. . . . .	284	Reed, C. . . . .	323
Keller, J.B. . . . .	236	Mayes, I.W. . . . .	445	Reeves, C.W. . . . .	404
Kelly, S.R. . . . .	397	Meirovitch, L. . . . .	254	Reifsnider, K.L. . . . .	274
King, W.F., III . . . . .	291	Miles, A.W. . . . .	290	Reiter, W.F., Jr. . . . .	395
Kinney, W.A. . . . .	261	Minagawa, S. . . . .	271	Riegel, C. . . . .	301
Kirk, R.G. . . . .	428	Mirza, S. . . . .	333	Rieger, N.F. . . . .	415
Kisliakov, S.D. . . . .	335	Mitchell, G.C. . . . .	337	Robinson, R.R. . . . .	357
Kleinstein, G.G. . . . .	239	Mittendorf, S.C. . . . .	341	Rohrle, M. . . . .	387
Klement, H.D. . . . .	430	Moeller, T.L. . . . .	339	Romilly, N. . . . .	240
Kluwick, A. . . . .	242	Molina, M.A. . . . .	303	Rossini, T. . . . .	426
Koltzsch, P. . . . .	347	Montoya, L.C. . . . .	398	Roth, C.A. . . . .	440
Koreki, T. . . . .	438	Morrison, D. . . . .	424	Rudny, D.F. . . . .	346
Kisloff, D. . . . .	262	Müller, J. . . . .	317	Ruter, G. . . . .	391
Koval, L.R. . . . .	351	Murata, S. . . . .	311	Sachs, H.K. . . . .	348
Krachman, H.E. . . . .	434	Murthy, V.R. . . . .	312	St. Hilaire, A. . . . .	307
Kramer, E. . . . .	430	Nash, W.A. . . . .	326	Sakata, T. . . . .	336
Kuak, Y.C. . . . .	244	Nayfeh, A.H. . . . .	242	Sanborn, D.M. . . . .	303
Kukkola, T. . . . .	380	Nelson, H.D. . . . .	422	Saravanja-Fabris, N. . . . .	370
Kuratani, K. . . . .	438	Nelson, R.C. . . . .	350	Sato, K. . . . .	315
Laithier, B.E. . . . .	295	Nemat-Nasser, S. . . . .	271	Sawamoto, M. . . . .	280
Lapini, G. . . . .	426	Ng, S.-L. . . . .	443	Scerbo, L.J. . . . .	314
Laratta, A. . . . .	433	Nguyen, X.T. . . . .	253	Schmitz, F.H. . . . .	361
Large, J.B. . . . .	318	Ni, R.H. . . . .	307	Schneider, A.J. . . . .	292
Larsson, L. . . . .	407	Nordman, R. . . . .	430	Schroder, E. . . . .	285
Laurenson, R.M. . . . .	418	Nowinski, J.L. . . . .	301	Schuster, G.M. . . . .	287
Lazzeri, L. . . . .	382, 383	Nuske, D.J. . . . .	373, 374	Schwerdtfeger, H. . . . .	425
Leblois, L.C. . . . .	381	Nuttall, S.M. . . . .	411	Seehra, M.S. . . . .	288
Ledbetter, R.H. . . . .	355	O'Callaghan, M.J.A. . . . .	326	Segal, D.J. . . . .	399
Lee, E.H. . . . .	293	O'Leary, T.R. . . . .	265	Setiawan, B. . . . .	333
Lees, A.W. . . . .	445	Obi, C. . . . .	237, 238	Shafer, B.P. . . . .	296
Lehmann, E.J. . . . .	259	Ochiai, Y. . . . .	386	Shaffer, G.M. . . . .	402
Lennox, W.C. . . . .	244	Ohkami, Y. . . . .	437	Shah, M.P. . . . .	392, 393
Likins, P. . . . .	437	Ohmata, K. . . . .	342	Shimovetz, R.M. . . . .	389
Lin, J. . . . .	247	Ikah-Avae, B.E. . . . .	405	Shirota, K. . . . .	438
Lindgren, B.J. . . . .	371	Olson, D.A. . . . .	390	Sidar, M. . . . .	349
Lindsey, L.G. . . . .	434	Owzar, A. . . . .	403	Siorek, R.W. . . . .	348
Liu, Y.K. . . . .	363	Paidoussis, M.P. . . . .	295	Simmons, P.E. . . . .	444
Longinow, A. . . . .	357	Parkinson, A.G. . . . .	419	Skaistis, S.J. . . . .	344
Lu, Y.P. . . . .	340	Pattabiraman, J. . . . .	328	Skjeltorp, A. . . . .	266, 267, 268, 269, 270
Lund, J.W. . . . .	412	Patterson, W.N. . . . .	441	Smith, C.S. . . . .	337
Lutton, R.J. . . . .	360	Penzien, J. . . . .	299	Smith, D.L. . . . .	389
McCarthy, M.F. . . . .	265	Pestorius, F.M. . . . .	345	Smith, J.B. . . . .	364
McWhannell, D.C. . . . .	248	Pierce, G.A. . . . .	312	Smith, J.D. . . . .	304
Ma, S.M. . . . .	379	Pollmann, E. . . . .	425	Soedel, W. . . . .	332
Maezawa, S. . . . .	276	Posa, G. . . . .	426	Soong, T.T. . . . .	358
Magliozzi, B. . . . .	352, 353, 354	Privitzer, E. . . . .	357	Soroka, W.W. . . . .	241
Mallett, R.L. . . . .	293	Quinn, B.E. . . . .	397		

Sorsdal, S. . . . .	289	Toda, Y. . . . .	438	Wellstead, P.E. . . . .	373, 374
Steers, L.L. . . . .	398	Troppens, D. . . . .	317	Wiland, J.H. . . . .	366
Stewart, J. . . . .	319	Tsujimoto, Y. . . . .	311	Wilkinson, T.L. . . . .	316
Stewart, R.M. . . . .	369	Ungar, A. . . . .	278	Williams, R.S. . . . .	274
Stinchcomb, W.W. . . . .	274	Urlich, K. . . . .	446	Windett, G.P. . . . .	384, 385
Stockdale, W.K. . . . .	442	Utz, W.R. . . . .	246	Winer, W.O. . . . .	303
Strona, P.P. . . . .	383	Van Atta, F.A. . . . .	372	Wong, C. . . . .	437
Suggs, C.W. . . . .	364	Vigran, T.E. . . . .	289	Woo, J.L. . . . .	302
Sundararajan, C. . . . .	313	Wachel, J.C. . . . .	321	Woodford, D.J. . . . .	416
Suzuki, T. . . . .	386	Walter, W.W. . . . .	415	Woolf, A. . . . .	294
Takasu, S. . . . .	280	Warwick, J.E. . . . .	435	Worsfold, J.H. . . . .	445
Takatsu, N. . . . .	315	Watanabe, T. . . . .	276	Wu, J.J. . . . .	298
Taylor, R.L. . . . .	249	Watson, E.E. . . . .	291	Yang, J.Y. . . . .	331
Thomas, C.B. . . . .	415	Weiss, R. . . . .	235	Yang, T.Y. . . . .	329
Tobe, T. . . . .	315	Welford, G.D. . . . .	362	Young, A.M. . . . .	286
Toda, H. . . . .	272	Wellford, L.C., Jr. . . . .	334		

CALENDAR

MEETING	DATE	LOCATION	CONTACT
	<u>1977</u> <u>MAR</u>		
NOISEXPO '77	14-17	Chicago, IL	NOISEXPO '77, 27101 E. Oviatt Rd, Bay Village, OH 44140 Tele. (216) 835-0101
Structures, Structural Dynamics and Materials Conference, AIAA	21-23	San Diego, CA	ASME or AIAA Hq.
15th Midwestern Mechanics Conference	23-25	Chicago, IL	Prof. T.C.T. Ting, Dept. of Materials Engrg., Univ. of Illinois at Chicago Circle, Box 4348, Chicago, IL 60680
Structural Dynamics Specialists Conference	24-25	San Diego, CA	AIAA Hq.
Gas Turbine Conference and Products Show, ASME	27-31	Philadelphia, PA	ASME Hq.
Joint Railroad Conference, IEEE/ASME	30-2	Washington, D.C.	IEEE Hq.
	<u>APR</u>		
American Power Conference, Ill. Inst. Tech.	18-20	Chicago, IL	R.A. Budenholzer, Dir. APC c/o IIT, 10 W. 35th St. Chicago, IL 60616
Design Engineering Conference and Show, ASME	18-21	New York, NY	ASME Hq.
Mini-Conference on Transportation	19-21	Ann Arbor, MI	Highway Safety Research Institute, The University of Michigan, Ann Arbor, MI 48109 Tele. (313) 764-2168
Diesel and Gas Engine Power Conference and Exhibit, ASME	24-28	Dallas, TX	ASME Hq.
IES Annual Meeting	24-27	Los Angeles, CA	IES Hq.
9TH Space Simulation Conference IES-AIAA-ASTM-NASA	26-28	Los Angeles, CA	IES Hq.
International Conference - Tribology	April	Cambridge, MA	Lt. R.S. Miller, Code 211 Office of Naval Research, Ballston Tower No. 1, Arlington, VA 22117 Tele. 692-4421
	<u>MAY</u>		
23rd International Instrumentation Symposium	1-5	Las Vegas, NV	ISA Hq.
31st Annual Technical Conference, ASQC	16-18	Philadelphia, PA	R.W. Shearman, ASQC Hq.
93rd Meeting of the Acoustical Society of America	17-20	State College, PA	J.C. Johnson, Chairman, ASA

<b>CALENDAR</b>			
<b>MEETING</b>	<b>DATE</b>	<b>LOCATION</b>	<b>CONTACT</b>
	<u><b>JUNE</b></u>		
<b>Fuels and Lubricants Meeting, SAE</b>	<b>7-9</b>	<b>Tulsa, OK</b>	<b>SAE Hq.</b>
<b>Applied Mechanics Conference, ASME</b>	<b>14-16</b>	<b>New Haven, CT</b>	<b>ASME Hq.</b>
<b>Lubrication Symposium, ASME</b>	<b>June</b>	<b>St. Louis, MO</b>	<b>ASME Hq.</b>
	<u><b>SEPT</b></u>		
<b>Vibrations Conference, ASME</b>	<b>26-28</b>	<b>Chicago, IL</b>	<b>ASME Hq.</b>

## CALENDAR ACRONYM DEFINITIONS AND ADDRESSES OF SOCIETY HEADQUARTERS

<p><b>AFIPS:</b> American Federation of Information Processing Societies 210 Summit Ave., Montvale, N.J. 07645</p>	<p><b>CCCCAM:</b> Chairman, c/o Dept. ME, Univ. Toronto, Toronto 5, Ontario, Canada</p>
<p><b>AGMA:</b> American Gear Manufacturers Association 1330 Mass. Ave., N.W. Washington, D.C.</p>	<p><b>IEEE:</b> Institute of Electrical and Electronics Engineers 345 E. 47th St. New York, N.Y. 10017</p>
<p><b>AIAA:</b> American Institute of Aeronautics and Astronautics, 1290 Sixth Ave. New York, N.Y. 10019</p>	<p><b>IES:</b> Institute Environmental Sciences 940 E. Northwest Highway Mt. Prospect, Ill. 60056</p>
<p><b>AIChE:</b> American Institute of Chemical Engineers 345 E. 47th St. New York, N.Y. 10017</p>	<p><b>IFT<sub>2</sub>MM:</b> International Federation for Theory of Machines and Mechanisms, US Council for TMM, c/o Univ. Mass., Dept. ME, Amherst, Mass. 01002</p>
<p><b>AREA:</b> American Railway Engineering Association 59 E. Van Buren St. Chicago, Ill. 60605</p>	<p><b>INCE:</b> Institute of Noise Control Engineering P.O. Box 3206, Arlington Branch, Poughkeepsie, N.Y. 12603</p>
<p><b>AHS:</b> American Helicopter Society 30 E. 42nd St. New York, N.Y. 10017</p>	<p><b>ISA:</b> Instrument Society of America 400 Stanwix St., Pittsburgh, Pa. 15222</p>
<p><b>ARPA:</b> Advanced Research Projects Agency</p>	<p><b>ONR:</b> Office of Naval Research Code 40084, Dept. Navy, Arlington, Va. 22217</p>
<p><b>ASA:</b> Acoustical Society of America 335 E. 45th St. New York, N.Y. 10017</p>	<p><b>SAE:</b> Society of Automotive Engineers 400 Commonwealth Drive Warrendale, Pa. 15096</p>
<p><b>ASCE:</b> American Society of Civil Engineers 345 E. 45th St. New York, N.Y. 10017</p>	<p><b>SEE:</b> Society of Environmental Engineers 6 Conduit St. London W1R 9TG, England</p>
<p><b>ASME:</b> American Society of Mechanical Engineers 345 E. 47th St. New York, N.Y. 10017</p>	<p><b>SESA:</b> Society for Experimental Stress Analysis 21 Bridge Sq. Westport, Conn. 06880</p>
<p><b>ASNT:</b> American Society for Nondestructive Testing 914 Chicago Ave. Evanston, Ill. 60202</p>	<p><b>SNAME:</b> Society of Naval Architects and Marine Engineers, 74 Trinity Pl. New York, N.Y. 10006</p>
<p><b>ASQC:</b> American Society for Quality Control 161 W. Wisconsin Ave Milwaukee, Wis 53203</p>	<p><b>SVIC:</b> Shock and Vibration Information Center Naval Research Lab., Code 8404 Washington, D.C. 20375</p>
<p><b>ASTM:</b> American Society for Testing and Materials 1916 Race St Philadelphia, Pa 19103</p>	<p><b>URSI-USNC:</b> International Union of Radio Science - US National Committee c/o MIT Lincoln Lab., Lexington, Mass. 02173</p>

DEPARTMENT OF THE NAVY  
NAVAL RESEARCH LABORATORY, CODE 8404  
SHOCK AND VIBRATION INFORMATION CENTER  
Washington, D.C. 20375  
OFFICIAL BUSINESS  
PENALTY FOR PRIVATE USE, \$300.

POSTAGE AND FEES PAID  
DEPARTMENT OF THE NAVY  
DoD-316



*A035-2366*  
*A035-310*

---

THE SHOCK AND VIBRATION DIGEST

---

VOLUME 9 No. 2

February 1977

---

EDITORIAL

- 1 Director Notes
- 2 Editors Rattle Space

ARTICLES AND REVIEWS

- 3 Feature Article *Partial contents;* ABSORBERS AND ISOLATORS FOR TORSIONAL VIBRATION;  
J.M. Vance
- 7 Literature Review
- 9 DYNAMIC STIFFNESS AND DAMPING OF FIBER-REINFORCED COMPOSITE MATERIALS;  
R.F. Gibson and R. Plunkett

19

*DEFINITE ELEMENT MODELING OF NATURAL VIBRATION PROBLEMS; and*  
A.V. Krishna Murty

Book Reviews

CURRENT NEWS

- 40 News Briefs
- 41 Short Courses

*ABSTRACTS FROM THE CURRENT LITERATURE.*

- 43 Abstract Contents
- 44 Abstracts: 77-232 to 77-447
- 92 Author Index

CALENDAR