U.S. COAST GUARD MOORED BUOY DATA HANDLING AND ANALYSIS

D. T. Milne
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SHIP PERFORMANCE DEPARTMENT

DECEMBER 1975

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DEPARTMENT 27

CENTRAL
INSTRUMENTATION
DEPARTMENT 29
**Title:** U. S. Coast Guard Moored Buoy Data Handling and Analysis  

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**Abstract:**  
The data handling and analysis of recorded data from a group of independent buoys deployed in the vicinity of the Chesapeake Light Tower is described. Included is a description of the digitization-quality checking progress, the analysis technique used and a discussion on the results.
TABLE OF CONTENTS

Page

ABSTRACT .................................................................................................................. 1
SUMMARY ................................................................................................................... 1
INTRODUCTION ......................................................................................................... 2
Section 1 - BACKGROUND ......................................................................................... 3
  1-1 Data Acquisition ................................................................................................ 3
  1-2 Analog Tapes ...................................................................................................... 6
  1-3 Data Logging and Quality Checking .................................................................. 7
  1-4 Digitization ....................................................................................................... 15
  1-5 Digitization Status .......................................................................................... 18
Section 2 - ANALYSIS TECHNIQUES ..................................................................... 21
  2-1 Statistical Analysis ............................................................................................ 21
  2-2 Correlations and Power Spectral Densities ....................................................... 22
Section 3 - ANALYSIS RESULTS ............................................................................ 26
Appendix A - COMPUTER GENERATED ANALYSIS RESULTS .......................... 110

LIST OF TABLES

Table 1 - Tape Recorder Data Channels ................................................................. 8
Table 2 - "A" Inputs for Analog Tape ...................................................................... 9
Table 3 - "B" Inputs for Analog Tape ...................................................................... 10
Table 4 - Breakdown of 109 Good Runs ............................................................... 20
Table 5 - Twenty-Five Runs Used in the Analysis ............................................... 27
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Two Types of Active Buoys</td>
<td>4</td>
</tr>
<tr>
<td>Figure 2</td>
<td>System Block Diagram</td>
<td>5</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Sample Coast Guard Recording Log Accompanying Analog Tapes</td>
<td>11</td>
</tr>
<tr>
<td>Figure 4</td>
<td>DTNSRDC Tape Log</td>
<td>12</td>
</tr>
<tr>
<td>Figure 5</td>
<td>USCG Analog Tape Status Summary</td>
<td>13</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Analog Tape Processing</td>
<td>14</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Block Diagram of the Processing Equipment for the Digitization of Analog Tapes</td>
<td>16</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Wave Height versus Wave Frequency in Sea State Two</td>
<td>30</td>
</tr>
<tr>
<td>Figure 9</td>
<td>RAO of Roll Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two</td>
<td>31</td>
</tr>
<tr>
<td>Figure 10</td>
<td>RAO of Roll Angle versus Wave Frequency for Buoy 4 (8 x 25 - 08) in Sea State Two</td>
<td>32</td>
</tr>
<tr>
<td>Figure 11</td>
<td>RAO of Roll Angle versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Two</td>
<td>33</td>
</tr>
<tr>
<td>Figure 12</td>
<td>RAO of Pitch Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two</td>
<td>34</td>
</tr>
<tr>
<td>Figure 13</td>
<td>RAO of Pitch Angle versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Two</td>
<td>35</td>
</tr>
<tr>
<td>Figure 14</td>
<td>RAO of Pitch Angle versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Two</td>
<td>36</td>
</tr>
<tr>
<td>Figure 15</td>
<td>RAO of Heave Displacement versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two</td>
<td>37</td>
</tr>
<tr>
<td>Figure 16</td>
<td>RAO of Heave Displacement versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Two</td>
<td>38</td>
</tr>
<tr>
<td>Figure 17</td>
<td>RAO of Heave Displacement versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Two</td>
<td>39</td>
</tr>
<tr>
<td>Figure 18</td>
<td>Phase Angle versus Wave Frequency of Cross Spectrum of Heave Displacement and Pitch Angle for Buoy 3 (OSPREY) in Sea State Two</td>
<td>40</td>
</tr>
</tbody>
</table>
Figure 19 - Phase Angle versus Wave Frequency of the Cross Spectrum of Heave Displacement and Pitch Angle for Buoy 4 (8 x 26 - 08) in Sea State Two ............. 41

Figure 20 - Wave Height versus Wave Frequency in Sea State Three. 42

Figure 21 - RAO of Roll Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Three ... 43

Figure 22 - RAO of Roll Angle versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Three .... 44

Figure 23 - RAO of Roll Angle versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Three ................. 45

Figure 24 - RAO of Pitch Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Three .......... 46

Figure 25 - RAO of Pitch Angle versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Three .... 47

Figure 26 - RAO of Pitch Angle versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Three ....... 48

Figure 27 - RAO of Heave Displacement versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Three .... 49

Figure 28 - RAO of Heave Displacement versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Three ....... 50

Figure 29 - RAO of Heave Displacement versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Three ....... 51

Figure 30 - Phase Angle versus Wave Frequency of the Cross Spectrum of Heave Displacement and Pitch Angle for Buoy 3 (OSPREY) in Sea State Three ..................... 52

Figure 31 - Phase Angle versus Wave Frequency of the Cross Spectrum of Heave Displacement and Pitch Angle for Buoy 4 (8 x 26 - 08) in Sea State Three .... 53

Figure 32 - Peak RAO of Roll Angle versus Wave Height Standard Deviation for Buoy 3 (OSPREY) ................. 54

Figure 33 - Peak RAO of Roll Angle versus Wave Height Standard Deviation for Buoy 4 (8 x 26 - 08) .......... 55

Figure 34 - Peak RAO of Roll Angle versus Wave Height Standard Deviation of Buoy 7 (8 x 26 - 05) .......... 56

Figure 35 - Peak RAO of Roll Angle versus Spectral Density at Wave Peak for Buoy 3 (OSPREY) .......... 57
Figure 36 - Peak RAO of Roll Angle versus Spectral Density at Wave Peak for Buoy 4 (8 x 26 - 08) ........................................... 58

Figure 37 - Peak RAO of Roll Angle versus Spectral Density at Wave Peak for Buoy 7 (8 x 26 - 05) ........................................... 59

Figure 38 - Peak RAO of Pitch Angle versus Wave Height Standard Deviation for Buoy 3 (OSPREY) ........................................... 60

Figure 39 - Peak RAO of Pitch Angle versus Wave Height Standard Deviation for Buoy 4 (8 x 26 - 08) ........................................... 61

Figure 40 - Peak RAO of Pitch Angle versus Wave Height Standard Deviation for Buoy 7 (8 x 26 - 05) ........................................... 62

Figure 41 - Peak RAO of Pitch Angle versus Spectral Density at Wave Peak for Buoy 3 (OSPREY) ........................................... 63

Figure 42 - Peak RAO of Pitch Angle versus Spectral Density at Wave Peak for Buoy 4 (8 x 26 - 08) ........................................... 64

Figure 43 - Peak RAO of Pitch Angle versus Spectral Density at Wave Peak for Buoy 7 (8 x 26 - 05) ........................................... 65

Figure 44 - Peak RAO of Heave Displacement versus Wave Height Standard Deviation for Buoy 3 (OSPREY) ........................................... 66

Figure 45 - Peak RAO of Heave Displacement versus Wave Height Standard Deviation for Buoy 4 (8 x 26 - 08) ........................................... 67

Figure 46 - Peak RAO of Heave Displacement versus Wave Height Standard Deviation for Buoy 7 (8 x 26 - 05) ........................................... 68

Figure 47 - Peak RAO of Heave Displacement versus Spectral Density at Wave Peak for Buoy 3 (OSPREY) ........................................... 69

Figure 48 - Peak RAO of Heave Displacement versus Spectral Density at Wave Peak for Buoy 4 (8 x 26 - 08) ........................................... 70

Figure 49 - Peak RAO of Heave Displacement versus Spectral Density at Wave Peak for Buoy 7 (8 x 26 - 05) ........................................... 71

Figure 50 - Mean Value of RAO of Roll Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two ........................................... 72

Figure 51 - Mean Value of RAO of Roll Angle versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Two ........................................... 73

Figure 52 - Mean Value of RAO of Roll Angle versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Two ........................................... 74
Figure 71 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 3 (OSPREY) ................................................................. 93
Figure 72 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 4 (8 x 26 - 08) ................................................................. 94
Figure 73 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 7 (8 x 26 - 05) ................................................................. 95
Figure 74 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 3 (OSPREY) ................................................................. 96
Figure 75 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 4 (8 x 26 - 08) ................................................................. 97
Figure 76 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 7 (8 x 26 - 05) ................................................................. 98
Figure 77 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 3 (OSPREY) ................................................................. 99
Figure 78 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 4 (8 x 26 - 08) ................................................................. 100
Figure 79 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 7 (8 x 26 - 05) ................................................................. 101
Figure 80 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 3 (OSPREY) ................................................................. 102
Figure 81 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 4 (8 x 26 - 08) ................................................................. 103
Figure 82 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 7 (8 x 26 - 05) ................................................................. 104
Figure 83 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 3 (OSPREY) ................................................................. 105
Figure 84 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 4 (8 x 26 - 08) ................................................................. 106
Figure 85 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 7 (8 x 26 - 05) ................................................................. 107
Figure 86 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 3 (OSPREY) ................................................................. 108
Figure 87 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 4 (8 x 26 - 08) ................................................................. 109
Figure 88 - Analog X-Y Plot of Roll and Pitch Angles for Buoy 7 (8 x 26 - 05 ) ................................................................. 110
ABSTRACT

The data handling and analysis of recorded data from a group of independent buoys deployed in the vicinity of the Chesapeake Light Tower is described. Included is a description of the digitization-quality checking process, the analysis technique used and a discussion on the results.

SUMMARY

The David W. Taylor Naval Ship Research and Development Center (DTNSRDC) has developed software to digitize and analyze U.S. Coast Guard Exposed Water Buoy data tapes. A total of 48 analog magnetic tapes from the initial buoy deployment (Dec 72 - Jul 73) have been logged, quality checked, digitized and analyzed. Each tape contains frequency multiplexed motion and tension signals from three moored buoys as well as wave height, time code, voice and calibration information. The analysis is based on data from 25 runs selected for quality and duration and is representative of sea states 2 and 3. The analysis results include both tabular and graphical output, such as power spectral densities, response amplitude operators, cross spectrums and statistical items, resulting directly from the data analysis computer programs and graphical displays derived from these, such as run to run comparisons, peak amplitude results and averaged results. Also included are X-Y plots of some of the raw analog data.

Section 1 of this report describes the data logging, quality checking and digitization procedures as well as some data acquisition background information. Section 2 includes a description of the analysis techniques used. Section 3 discusses the results of applying this analysis to the subject data and Appendix A is the foreword for the computer generated analysis results which, because of the large volume, are bound separately.
INTRODUCTION

Buoys have long been used as a marine navigational aid. The effectiveness of a given buoy design is a function of its cost, ease of handling, and its usefulness to the mariner, e.g., the detectability and recognition of its signal. Evaluation of alternative designs and comparison with the standard 8' x 26' buoy requires quantitative information on these factors. Since mooring costs and detectability are both greatly affected by the motion characteristics of the buoy, motion measurements are necessary to provide quantitative information for design evaluation.

Consequently, a buoy motion measurement system was designed and installed to provide buoy motion and driving function data for evaluating navigational buoy design concepts in the operational environment. A future goal of this program will be the implementation of a dynamic simulation of a moored buoy in a regular and irregular seaway. Tabulated and plotted results from the data analysis will be used for comparison with simulation generated data. The primary objective of this overall effort is to aid in selecting the most effective designs for new buoy-based navigation systems used by all vessels in coastal waterways.
SECTION 1 - BACKGROUND

1-1 Data Acquisition

The U. S. Coast Guard (USCG) Buoy Motion Measurement System was developed by the David W. Taylor Naval Ship Research and Development Center (DTNSRDC). Wave height at the light tower and roll angle, pitch angle, heave displacement and mooring line tension from up to seven buoys located near the tower are measured and recorded on command. Although the system can accommodate up to seven buoys, only buoys number 3, 4 and 7 were active during the collection of data for this report. Buoys numbers 4 and 7 are the standard 8' x 26' configuration and buoy number 3 is the OSPREY design (see Figure 1). Note that buoy number 3 is moored by a single chain from its central mooring eye, while the standard buoys are bridle moored. The weight in service of the OSPREY is approximately 7700 lbs and of the standard approximately 5000 lbs.

A block diagram of the entire USCG Buoy Motion Measurement System is shown in Figure 2. The buoy instrumentation is shown on the left side of the diagram. Buoy instrument packages are installed at the roll and pitch center of rotation and are powered by batteries located in the buoy. Buoy instrument packages are commanded to turn-on and calibrate by the command unit located at the ground station. Buoy roll, pitch, and heave are measured by a gyroscope and a vertically stabilized accelerometer located in the buoy package. Mooring cable tension is measured by means of an external tension dynamometer which is located in the mooring cable and electrically connected into the package. The conditioned data from the four transducers is multiplexed (mixed together) and transmitted to the data receiver located at the ground station.

The ground station equipment is shown in the center of the block diagram. The tape recorder is controlled by the command system. Multiplexed data from each of the seven buoys is recorded separately on one of the fourteen tape channels. In addition to buoy data, wave
Figure 1 - Two Types of Active Buoys
information from a Baylor Wave Profile System located on the outside of the tower, time code, a reference frequency, an "OK data" signal, and voice information are also recorded on the tape recorder.

The data tapes are transported to the data analysis center, represented on the right of Figure 2, where the information for each buoy is demultiplexed, quality checked, digitized and analyzed on a digital computer. The time code translator is utilized to locate times and dates on the data tapes.

1-2 Analog Tapes

DTNSRDC is furnished magnetic tapes containing data recorded in analog form on a Honeywell Model 5600 instrumentation tape recorder. The recorder uses a standard one inch magnetic tape, 1R1G standard record heads and the data was recorded at 1 7/8 inches per second. Data and information from the light tower (ground station) is recorded on Channels 1-7 ("A" inputs). The Channels 5 and 7 are reserved for water current not presently recorded. Channels 8 through 14 ("B" inputs) are reserved for the subcarrier multiplexed data from seven buoys. Each multiplexed channel contained 8 subcarrier measurements. The complete data set on the analog tape is described in Table 1. Table 2 describes the "A" inputs in detail and Table 3 describes the "B" inputs.

The analog tapes contain the wave height sensor data, data from the three buoys, and identification and control information. The time code channel is used to identify individual data runs. The OK data channel serves to indicate the beginning and ending of data runs and to identify calibration signals. Each of the experimental data channels, the wave height and the four buoy parameters are recorded at two different sensitivities. The appropriate sensitivity is selected during the data reduction process.

The analog data is reproduced at a tape speed of 30 inches per second, sixteen times real time. An eight channel demodulator and filter are used
to de-multiplex the recorded subcarrier buoy data into 8 separate analog data channels acceptable for input into the digital processing system.

1-3 Data Logging and Quality Checking

Analog tapes were received from the Coast Guard at a rate of about 5 tapes per week during the period between June and September 1973. Input logging of the analog tapes is a three step process. Figure 3 shows a typical Coast Guard Data Recording Log which is received with each analog tape. The analog tape is assigned a Tape Number and an DTNSRDC Tape Log is prepared (see Figure 4). The starting time codes listed on the Coast Guard Log are transposed to the DTNSRDC log and run numbers are assigned corresponding to tape numbers and time code. During the Quality Checking and Digitizing Process, the DTNSRDC log is completed to include the date digitized, the number of good records and quality remarks. Note that each analog tape will result in one DTNSRDC log per buoy (presently three). Figure 5 shows an analog Tape Status Summary which was established to provide the Coast Guard with continuing feedback relative to DTNSRDC progress during the digitization and analysis process.

Upon receipt of a group (usually 5 to 10) of analog tapes, a strip chart record of the most recent tape is generated and delivered, with comments, to DTNSRDC Code 296 for determination and analysis of problems in the data acquisition equipment. This early notification to the Coast Guard point of contact as to the condition of the analog tapes permits timely Coast Guard preventive action for future tapes.

Initial data quality checks on the analog tapes are performed by visual observation on a strip chart recorder in parallel with the digitization process (dotted line on Figure 6). The strip chart record of the next run is examined during the digitization process of the present run. Parallel digitization and analog quality checking with operator manual override is more efficient than performance of these operations.
### TABLE 1 - TAPE RECORDER DATA CHANNELS

<table>
<thead>
<tr>
<th>Head Bank</th>
<th>Tape Channel</th>
<th>Record Electronics</th>
<th>Data Recorded</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>VA</td>
<td>Voice</td>
<td>Remarks*</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>Direct</td>
<td>Time Code*</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>FM</td>
<td>OK DATA*</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>Direct</td>
<td>Reference</td>
</tr>
<tr>
<td>A</td>
<td>4</td>
<td>FM</td>
<td>25 FT Wave Height*</td>
</tr>
<tr>
<td>A</td>
<td>5</td>
<td>FM</td>
<td>SPARE</td>
</tr>
<tr>
<td>A</td>
<td>6</td>
<td>FM</td>
<td>5 FT Wave Height*</td>
</tr>
<tr>
<td>A</td>
<td>7</td>
<td>FM</td>
<td>SPARE</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>Direct</td>
<td>Buoy #1</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>Direct</td>
<td>Buoy #2</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>Direct</td>
<td>Buoy #3</td>
</tr>
<tr>
<td>B</td>
<td>11</td>
<td>Direct</td>
<td>Buoy #4</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>Direct</td>
<td>Buoy #5</td>
</tr>
<tr>
<td>B</td>
<td>13</td>
<td>Direct</td>
<td>Buoy #6</td>
</tr>
<tr>
<td>B</td>
<td>14</td>
<td>Direct</td>
<td>Buoy #7</td>
</tr>
</tbody>
</table>

* Wave Height Only (WHO) Recording System
TABLE 2 - "A" INPUTS FOR ANALOG TAPE

VA - (Voice) Time, date, run number, visual observations of wave height (P-P), wave period (sec), and general remarks.

CH1 - (Direct) IRIG B time code (days, hours, minutes, seconds). Log should indicate the approximate time code for the beginning of each run.

CH2 - (FM) "OK DATA" signal. Indicates when calibrating and when recording data.

CH3 - (Direct) 6.25 KHZ reference signal used for tape speed compensation in the discriminators on playback. Playback is at 30 IPS (16 x 1 7/8 IPS) which results in a compensation frequency of 100 KHZ.

CH4 - (FM) 50 ft full scale (+ 25 ft) wave height signal.

CH5 - (FM) spare channel reserved for ocean current.

CH6 - (FM) 10 ft full scale (+ 5 ft) wave height signal, clips signals greater than ± 5 ft.

CH7 - (FM) spare channel reserved for ocean current.
# TABLE 3 - "B" INPUTS FOR ANALOG TAPE

CH8-14- (Direct) multiplexed buoy data on IRIG telemetry channels 1-8.

<table>
<thead>
<tr>
<th>IRIG CH</th>
<th>MEASURAND</th>
<th>-CAL (OV)</th>
<th>0 CAL (2.5V)</th>
<th>+ CAL (5.0V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100° F.S. (±50°) ROLL</td>
<td>-50° STBD*</td>
<td>0°</td>
<td>+50° PORT*</td>
</tr>
<tr>
<td>2</td>
<td>20° F.S. (±10°) ROLL</td>
<td>-10° STBD*</td>
<td>0°</td>
<td>+10° PORT*</td>
</tr>
<tr>
<td>3</td>
<td>100° F.S. (±50°) PITCH</td>
<td>-50° AFT*</td>
<td>0°</td>
<td>+50° FWD*</td>
</tr>
<tr>
<td>4</td>
<td>20° F.S. (±10°) PITCH</td>
<td>-10° AFT*</td>
<td>0°</td>
<td>+10° FWD*</td>
</tr>
<tr>
<td>5</td>
<td>.1 G F.S. (±.5G) VERT. ACCEL.</td>
<td>-.5 G DWN</td>
<td>0 G</td>
<td>+.5 G UP</td>
</tr>
<tr>
<td>6</td>
<td>.2 G F.S. (± 1G) VERT. ACCEL.</td>
<td>-.1 G DWN</td>
<td>0 G</td>
<td>+.1 G UP</td>
</tr>
<tr>
<td>7</td>
<td>50 KIP F.S. TENSION</td>
<td>0 KIP</td>
<td>25 KIP</td>
<td>50 KIP</td>
</tr>
<tr>
<td>8</td>
<td>10 KIP F.S. TENSION</td>
<td>0 KIP</td>
<td>5 KIP</td>
<td>10 KIP</td>
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* Direction of rotation
<table>
<thead>
<tr>
<th>Time</th>
<th>Wave Height (ft)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>07/21</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>07/22</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>07/23</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>07/24</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>07/25</td>
<td>0.3</td>
<td></td>
</tr>
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<td>07/26</td>
<td>0.3</td>
<td></td>
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<tr>
<td>07/27</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>07/28</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>07/29</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>07/30</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>07/31</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

Figure 9: Sample Coast Guard Recording Log Accompanying Analog Tape.
<table>
<thead>
<tr>
<th>YMC Line No.</th>
<th>TARC ID</th>
<th>Date</th>
<th>NO. OF TIME/DATE OF DATA</th>
<th>Digitized No.</th>
<th>ANALYZED</th>
<th>PLotted</th>
<th>STATUS</th>
<th>V</th>
<th>V/C</th>
</tr>
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<tbody>
<tr>
<td>1973</td>
<td>#25</td>
<td>7-23-73</td>
<td>17</td>
<td>187-20</td>
<td>190-16</td>
<td>#3</td>
<td>P-6-73</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>#4</td>
<td>P-11-73</td>
<td>X</td>
<td>X</td>
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<td>190-20</td>
<td>192-12</td>
<td>#3</td>
<td>P-6-73</td>
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<td>#3</td>
<td>P-6-73</td>
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<td></td>
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<td>200-04</td>
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<td>X</td>
</tr>
<tr>
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<td>#41</td>
<td>8-28-73</td>
<td>17</td>
<td>205-12</td>
<td>208-09</td>
<td>#3</td>
<td>8/1/73</td>
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<td>X</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>#4</td>
<td>8/1/73</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>#7</td>
<td>8/1/73</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Figure 5 - USCG Analog Tape Status Summary
Figure 6 - Analog Tape Processing
sequentially. The following criteria is used for deciding which runs contain good information for a sufficient duration to warrant further analysis:

- calibration signals
- gyro fully erected
- buoys turned on
- steady zero lines
- excessive noise and spikes
- duration
- discontinuities in the data

If the calibrating signals are bad, the digitization process has the capability of providing artificial calibrations. When artificial calibrations are inserted it is noted under remarks on the DTNSRDC log sheet. Also listed on the log sheet is the operator’s estimate of the sea state in 1 FT peak-to-peak increments of wave height observed from the wave height signal on the strip chart recorder. If the data observed on the strip chart does not warrant digitization then a note is made in the DTNSRDC log. Normally, bad data is referred to in one of two categories; (1) NO GOOD - data did not meet examination criteria or (2) NO DATA - no data on the tape for the indicated time-code.

A final quality check is made at the end of the digitizing process when a read and print of the digital tape is made and the operator inspects the printout. Assuming the tape passes this final inspection it is transferred to the analysis computer for processing. The entire analog tape processing procedure just described is illustrated in Figure 6.

1-4 Digitization

A block diagram of the digitization system for the processing of the analog tapes is shown in Figure 7. The analog tape, which was recorded at 1 7/8 inches per second, is reproduced at 30 IPS (16 times real time) on an Ampex FR-1800 tape recorder. An eight channel demodulator is utilized to de-multiplex the individual data from each buoy to eight separate analog
Figure 7 - Block Diagram of the Processing Equipment for the Digitization of Analog Tapes
data signals (2 sensitivity levels for each of the four measurands). The
demodulator is equipped with a tape speed compensation unit. A 100 KHz
reference signal recorded at 6.25 KHz is utilized for this compensation. The
time code generator is used to properly locate and identify each run.

The de-multiplexed buoy signals along with the remainder of the tape
recorder signals are fed through 6 Pole Butterworth, Low-Pass filters to
remove high frequency fluctuations and noise. The filter half-power point
is set at 16 Hz which reflects a real time frequency of 1 Hz (6.28 rad/sec).
The filter outputs are simultaneously directed to the A/D converter and the
strip chart recorder. The A/D converter converts the analog signals to
digital format and feeds them into the computer. The computer is commanded
via the teletype. The digitized data is stored on magnetic tape and ID
arrays are outputted to the printer.

The first data recorded during a data run is a series of calibration
levels. These levels are identified by three negative excursions of the
OK data signal - the first indicating negative cal, the second zero cal
and the third positive cal. The experimental data, consisting of the two
wave height channels and the eight signals from the buoy of interest,
are digitized at discrete time intervals. The portion of the actual data to
be digitized is defined by a positive excursion of the OK DATA signal. The
digitized data and calibrations are recorded on an intermediate (scratch)
tape. The scratch tape is next re-read back into the computer. The program
selects the data range to be used for each variable based on the number of
times the lower range exceeds full scale. The three calibration levels
for the selected data channels are averaged to provide three single
calibration values for each variable. The plus and minus calibration levels
are summed to provide a single peak-to-peak calibration value for each
variable. The averaged zero calibration level is used to convert for zero
offset or drift in the data signals. The five data signals (variables)
selected and corrected along with the ID arrays (final peak-to-peak
calibration values, run numbers, no. of records, record no., buoy no., and Julian date) are outputted and stored on the final digitized tape.

The following procedure is followed in digitizing one run for one buoy from an analog tape:

1. Load analog tape to start of run (time code and voice is used to assist in this operation)
2. Based on prior inspection of the run, select or not select artificial calibration (switch on computer)
3. Load scratch and data tapes on digital tape recorders
4. Initialize computer, make proper logical unit assignments and input via teletype - run number, sampling rate, buoy no. and Julian date
5. Start tape recorder
6. Watch OK data line on strip chart recorder, when OK data goes negative for the first calibration - start the computer
7. After analog tape has completed run, let recorder continue while digitization process is being completed and quality check the data for the next run
8. Log in no. of records, no. of good records, remarks and sea state in ft peak-to-peak

1-5 Digitization Status

To date, a total of 48 analog tapes containing 1857 runs from the initial buoy deployment (Dec 72 - Jul 73) have been received, logged and quality checked. Of these 1857 runs, a total of 1126 were digitized after quality checking. The 731 runs not digitized were eliminated for the following reasons - no data on the tape, gyro not fully erected, buoys not turned on, unsteady zero lines, excessive noise and spikes, insufficient duration or discontinuities in the data. Further inspection of the data
revealed that the wave height signal on an additional 524 runs suffered from a periodic noise interference signal and thus these runs were of no further value for analysis. Therefore, 602 runs remained as candidates for analysis. Of these, 109 runs contained good data for all 3 buoys over a significant interval of time (i.e., greater than 10 minutes). Table 4 shows a breakdown of these runs with respect to visually observed wave height and number of good records. The 25 runs containing at least 100 records (greater than 14 minutes) were selected for further analysis.
## TABLE 4 - BREAKDOWN OF 109 GOOD RUNS

<table>
<thead>
<tr>
<th># GOOD RECORDS</th>
<th>1'</th>
<th>2'</th>
<th>3'</th>
<th>4'</th>
<th>5'</th>
<th>6'</th>
</tr>
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<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>131-135</td>
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<td>--</td>
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<td>1</td>
<td>--</td>
<td>--</td>
</tr>
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<td>121-125</td>
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<tr>
<td>116-120</td>
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<td>1</td>
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<td>--</td>
<td>--</td>
</tr>
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<td>111-115</td>
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<tr>
<td>106-110</td>
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<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>101-105</td>
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<td>2</td>
<td>2</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>&gt; 100 TOTALS</td>
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<td>8</td>
<td>6</td>
<td>1</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>96-100</td>
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<td>12</td>
<td>6</td>
<td>--</td>
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<td>4</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>81-85</td>
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<td>7</td>
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<td>--</td>
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</tr>
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<td>76-80</td>
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<td>8</td>
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<td>--</td>
<td>--</td>
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<td>&gt; 70 TOTALS</td>
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<td>45</td>
<td>19</td>
<td>2</td>
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20
SECTION 2 - ANALYSIS TECHNIQUES

Two analysis techniques are used in the data analysis computer module of the DTNSRDC Buoy Motion Measurement System. These are Statistical Analysis and Spectral Analysis the details of which are discussed in this Section.

2-1 Statistical Analysis

The wave height, buoy vertical acceleration, pitch and roll angles, and the mooring cable tension were analyzed as follows:

Let $X_i, i = 1, 2, \ldots, M$, be the time sequence digitized values of one of the 5 variables and $M$ is the total number of values. The following statistics can then be computed:

- **Average $\bar{x}$**
  
  \[
  \bar{x} = \frac{\sum_{i=1}^{M} x_i}{M}
  \]

- **Mean Square**
  
  \[
  M \frac{\sum_{i=1}^{M} x_i^2}{M}
  \]

- **Variance $C_o$**
  
  \[
  C_o = \frac{\sum_{i=1}^{M} x_i^2}{M} - \bar{x}^2
  \]

- **Standard deviation $\sigma$**
  
  \[
  \sigma = \sqrt{C_o}
  \]

- **Probability density**
  
  \[
  \text{frequency of occurrence}/M
  \]

- **Probability distribution at } b = \sum \text{ frequency of occurrence at } b_n
  
  for all $b_n \geq b$

In most instances, the full scale of a variable is divided into 50 sections. The frequency of occurrence of each section $b_n$ is tallied in order to compute the probability density.
For the sea wave the significant height is defined as

\[ H_{1/3} = 4 \sqrt{C_0} \]

where \( C_0 \) is the variance of the wave amplitude.

2-2 Correlations and Power Spectral Densities

The cross-covariance of two variables, \( x \) and \( y \), for \( k \) number of lags \( h \) (sample periods) is computed as

\[ C_{xyk} = \frac{1}{M-k} \sum_{i=1}^{M-k} x_i y_{i+k} - \bar{x} \bar{y} \]

The auto-covariance of \( x \) is the cross-covariance of \( x \) and \( x \) itself:

\[ C_{xxk} = \frac{1}{M-k} \sum_{i=1}^{M-k} x_i x_{i+k} - \bar{x}^2 \]

Auto or cross-covariance is the unbiased (or zero biased) auto or cross-correlation.

The power spectral density is computed in the following manner: Assume that the auto-covariance (or cross-covariance in the case of cross-spectral density) is becoming less and less correlated as \( k \) increases and is practically zero when \( k \) approaches \( N \). \( N \) is an integer less than \( M \). (As a rule of thumb, \( N \) should be less than \( M/10 \)). Consider then the auto-correlation is periodic with period \( T = (2N + 1) h \) from \(- (N + 1/2) h \). The Fourier series of this periodic function at \( kh, k = 0, 1, 2, \ldots, \) is
\[ C_{xx}(kh) = a_0/2 + \sum_{p=1}^{\infty} \left[ a_p \cos \frac{2\pi p k h}{(2N+1) h} + b_p \sin \frac{2\pi p k h}{(2N+1) h} \right] \]

with the coefficients

\[ a_p = 2 \int_{-T/2}^{T/2} C_{xx}(t) \cos \omega_0 t \, dt \quad p = 0, 1, 2, \ldots \]

\[ b_p = 2 \int_{-T/2}^{T/2} C_{xx}(t) \sin \omega_0 t \, dt \]

and the fundamental frequency is

\[ \omega_0 = \frac{2\pi}{(2N+1) h} \]

Both \( a_p \) and \( b_p \) are computed in the way described by Goertzel in Ralston and Wolf's Mathematical Methods for Digital Computers, pp. 258-262.

The spectral density of \( x \) is defined as

\[ S_{xx}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_{xx}(t) e^{-j\omega t} \, dt \]

Under the assumption that \( C_{xx} \) is practically zero for large \( k \) the spectral density at \( \omega = p\omega_0 \) is

\[ S_{xx}(p\omega_0) = \frac{1}{2} \int_{-T/2}^{T/2} C_{xx}(t) \left[ \cos p\omega_0 t - j \sin p\omega_0 t \right] dt \]

\[ = \frac{T}{4\pi} \left[ a_p - j b_p \right] \]

23
The auto-covariance is an even function and \( b_p = 0 \). For cross-spectrum \( \sigma_{xy}(p\omega_0) \), the expression is the same if it is computed from the cross-covariance \( C_{xy} \). However, \( C_{xy} \) is not an even function and \( b_p \) is not zero, although the assumption that \( C_{xy} \) is practically zero for \( k \) close to and beyond \( \pm N \) has to be applied in the same way.

Among the buoy variables (pitch and roll angles, vertical acceleration and cable tension on the buoy) the frequency response transfer function between two variables \( x \) and \( y \) is

\[
H(j\omega) = \frac{S_{yx}(\omega)}{S_x(\omega)}
\]

The cross-spectral density \( S_{yx} \) is a complex function of \( \omega \). It can be expressed in polar form with amplitude \( A_m \) and phase \( \alpha \).

\[
H(j\omega) = \frac{A_m}{S_x(\omega)} \ e^{j\alpha}
\]

The spectral density obtained in this way is discrete, only the values at \( p\omega_0 \), \( p = 0, 1, 2, ..., N \), are defined. The fundamental frequency \( \omega_0 \), therefore, determines the resolution of the spectrum. Suppose \( \omega_0 \) is taken to be \(.1 \) rad/sec or \(.1/2\pi = .0159 \) Hz. The fundamental period is then \( 1.0159 - 62.89 \) seconds. Since the sample period of the data points, or the lag, is \(.16 \) seconds, the fundamental period is then corresponding to \( 62.89/.16 = 382 \) lags. Therefore, to have a \(.1 \) rad/sec resolution of the spectrum, the auto or cross-covariance must be computed over 382 lags. Since the auto-covariance is an even function, it is computed for \( k = 0, 1, 2, ..., 196 \) only (\( N = 196 \)). For cross-covariance, we let \( k \) range from -196 to +196.

Finer resolution can be obtained with wave lags, but the computation of \( C_{xxk} \) or \( C_{xyk} \) is very time consuming. We consider the fundamental frequency \( \omega_0 = .1 \) rad/sec is adequate and use \( N = 196 \).
Due to the assumption of zero correlation for $k$ greater than $N$ and the finite period, the spectral values at 0 and .1 rad/sec are not accurate in actual practice. To smooth out the spectrum, a three point "hanning" process is also applied to the spectral density values. Because of this, the point at .2 rad/sec is not accurate either. Therefore, the first couple of points ($\omega < .2$ rad/sec) in the spectrum should not be used.

2-3 Response Amplitude Operators

For the sea wave with wave frequency $\omega$ (rad/sec) and length $\lambda$, the deep water relation is:

$$\lambda = \frac{2\pi g}{\omega^2}$$

The mean wave slope is defined as $2\zeta/\lambda$, where $\zeta$ is the wave amplitude. The wave slope is non-dimensional and its spectral density

$$\frac{4}{\lambda^2} S_\zeta(\omega) = \frac{\omega^2}{g^2} S_\zeta(\omega)$$

is used to compute the non-dimensional RAO (response amplitude operator) of the roll and pitch angles. The $S_\zeta(\omega)$ is the sea wave amplitude spectral density with amplitude in feet.

If $S_r(\omega)$ is the roll angle spectral density in degree$^2$-seconds, the RAO for roll is defined as

$$\frac{\pi g}{57.3 \omega^2} \frac{S_r(\omega)}{S_\zeta(\omega)} = \frac{1.754}{\omega^2} \frac{S_r(\omega)}{S_\zeta(\omega)}$$

The same applies to the pitch angle, replacing $S_r(\omega)$ by the pitch spectral density $S_p(\omega)$.

The RAO of buoy vertical acceleration is normalized to sea acceleration and is

$$\frac{1}{\omega^2} \frac{S_a(\omega)}{S_\zeta(\omega)}$$

where $S_a(\omega)$ is the buoy vertical acceleration spectral density with acceleration in ft/sec$^2$. 25
SECTION 3 - ANALYSIS RESULTS

This Section describes the results generated directly from the data analysis computer module of the DTNSRDC Buoy Motion Measurement System, as well as, those results derived indirectly from these. Interpretation of these results and conclusions based on them are also discussed.

Twenty-five runs were used in the analysis (see Table 5). Note that for each run, over 100 records were processed with respect to each of the three buoys. The output generated directly by the computer is bound separately, and is provided with Appendix A. It consists of a set of spectral analysis results and a set of statistical results. The 75 pairs (for each run, 3 pairs corresponding to the 3 buoys) of these sets are presented in the order in which they appear in Table 5.

At the beginning of each set of spectral analysis results, there are two tables of data upon which the plots that comprise the remainder of the set are based. The top of the first table contains two rows of summary data that are self-explanatory. The remainder of this table consists of the power spectral density (PSD) for the 5 parameters: wave height (indicated as sea), roll angle, pitch angle, heave displacement (indicated as accel), and tension and the response amplitude operator (RAO) for the latter four parameters. Both the PSD's and their RAO's correspond to frequencies between .1 and 5 radians/sec incremented by 1 radian/sec. The second table contains the following 5 items (tabulated for the frequencies just mentioned) representing the cross spectrum of heave displacement and pitch angle and the cross spectrum of heave displacement and tension: in phase term, in quadrature term, in magnitude, in phase angle, and transfer function. Reference is made to Section 2 for the mathematics related to the computation of the PSD's, RAO's and cross spectra.

Each set of statistical results is headed by run summary data almost identical to that heading the spectral analysis results. Plots of the probability density of wave height, heave acceleration, total angle,
TABLE 5 - TWENTY FIVE RUNS USED IN THE ANALYSIS

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Sea State</th>
<th>Number Rcds Processed</th>
<th>Standard Deviation of Wave Height</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>Buoy 3</td>
<td>Buoy 4</td>
</tr>
<tr>
<td>55.11</td>
<td>4</td>
<td>140</td>
<td>--</td>
</tr>
<tr>
<td>38.05</td>
<td>3</td>
<td>--</td>
<td>118</td>
</tr>
<tr>
<td>38.06</td>
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<td>110</td>
<td>123</td>
</tr>
<tr>
<td>38.07</td>
<td>3</td>
<td>105</td>
<td>120</td>
</tr>
<tr>
<td>36.10</td>
<td>3</td>
<td>140</td>
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</tr>
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<td>140</td>
</tr>
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<td>133</td>
<td>133</td>
</tr>
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<td>38.16</td>
<td>3</td>
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</table>
roll angle, pitch angle and tension are each preceded by related statistics such as maximum value, minimum value, average value, root mean square value, etc. Again, reference is made to Section 2 for the mathematics related to these statistical computations.

The indirectly derived results can be thought of as falling into three categories - run to run comparisons, peak amplitude results, and averaged results. The run to run comparisons (Figures 8-31) were generated by plotting results from 3 consecutive runs on the same set of axes. Two groups of such results were plotted, one corresponding to sea state 2 (runs 36.14, 37.05, 37.11) and one corresponding to sea state 3 (runs 36.12, 38.16, 36.13). In each group there are 12 plots: Wave Height vs Frequency, RAO of Roll Angle vs Frequency (one plot for each of the 3 buoys), RAO of Pitch Angle vs Frequency (one plot for each of the 3 buoys), RAO of Heave Displacement vs Frequency (one plot for each of the 3 buoys), and Phase Angle vs Frequency of the Cross Spectrum of Heave Displacement and Pitch Angle (for buoys 3 and 4). The Wave Height vs Frequency plot is not related to any one buoy since the wave height measurement is independent of the buoy. It is interesting to note that on each of the RAO plots, the values for the 3 runs are quite close. Further, the roll angle plots corresponding to buoys 4 and 7 (the Standard Buoys) are almost identical, whereas this similarity is not as strongly exhibited with respect to pitch angle and heave displacement. In these latter two cases, the RAO values corresponding to buoy 7 appear to be greater than those for buoy 4 over large segments of the frequency range (e.g., from 2.0 to 3.8 rad/sec for heave displacement, sea state 2).

The peak amplitude results (Figures 32-49) consist of the peak RAO of roll angle, pitch angle and heave displacement for each of the 3 buoys plotted as a function of wave height standard deviation and then density at wave peak. A single plot corresponds to a single buoy and one of the three buoy motion parameters versus either wave height standard deviation or density at wave peak. Each plot contains data from three sea states, the points corresponding to standard deviations greater than 1.30 (or wave peaks greater than 1.60) are from sea state 4 runs, the point corresponding to standard deviations less than .75 (or wave peaks less
than .25) are from sea state 2 runs and all the in between points are from sea state 3 runs.

The averaged results consist of plots of the means (indicated by \( \bar{x} \)) and 90% confidence limits for the RAO of roll angle, pitch angle and heave displacement. Two groups of results are plotted, one corresponding to sea state 2 (Figures 50-58) and one corresponding to sea state 3 (Figures 59-67). A single plot corresponds to one of the two sea states, one of the three buoys, and one of the three buoy motion parameters. For a given frequency, the mean is computed by averaging the RAO values corresponding to that frequency over the runs corresponding to the given sea state. For sea state 2 (see Table 5) the runs averaged over 36.14 to 37.15 inclusive and for sea state 3, 38.07 to 39.07 inclusive. The 90% confidence limits are given by

\[
\bar{x} \pm \frac{s}{\sqrt{n}} t(1-0.90) \sqrt{\frac{1}{n}}
\]

where \( \bar{x} \) = sample mean (i.e., the average over the appropriate runs)

\( s \) = sample of standard deviation

\( n \) = number of runs used to compute \( \bar{x} \) (or \( s \))

\( t \) = Student's t-distribution value corresponding to \( n-1 \) degrees of freedom and 90% confidence level

One other category of results is included here in an attempt to give the reader a better visual representation of the buoy motion - analog x-y plots of roll and pitch angles. The analog tape was played back at 1 7/8 ips through real time discriminators and then into an x-y recorder adjusted for appropriate scaling - 1° per minor division and 10° per major division. Pitch angle is plotted on the 90°/270° axis and roll angle on the 0°/180° axis. The graphs therefore display 50° full scale for roll angle and 35° full scale for pitch angle. Three runs were plotted - 36.10, 37.04 and 39.04. For run 36.10, time segments of 1.25 minutes (Figures 68-70), 2.50 minutes (Figures 71-73), 5 minutes (Figures 74-76), 10 minutes (Figures 77-79) and 20 minutes (Figures 80-82) were plotted. For the other two runs, only 10 minute time segments were plotted (Figures 83-88). In each case, all 3 buoys are shown.
Figure 9 - RAO of Roll Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two
Figure 10 - RAO of Roll Angle versus Wave Frequency for Buoy 4
(8 x 26 - 08) in Sea State Two
Figure 12 - RAO of Pitch Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two
Figure 13 - RAO of Pitch Angle versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Two
Figure 14 - RAO of Pitch Angle versus Wave Frequency for Buoy 7 (8 x 25 - 05) in Sea State Two
Figure 15 - RAO of Heave Displacement versus Wave Frequency
for Buoy 3 (OSPREY) in Sea State Two
Figure 17 - RAO of Heave Displacement versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Two
Figure 23 - RAO of Roll Angle versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Three
Figure 24 - RAO of Pitch Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Three
Figure 28 - RAO of Heave Displacement versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Three
Figure 30 - Phase Angle versus Wave Frequency of the Cross Spectrum of Heave Displacement and Pitch Angle for Buoy 3 (OSPREY) in Sea State Three
Figure 32 - Peak RAO of Roll Angle versus Wave Height Standard Deviation for Buoy 3 (OSPREY)
Figure 33 - Peak RAO of Roll Angle versus Wave Height Standard Deviation for Buoy 4 (8 x 26 - 08)
Figure 34 - Peak RAO of Roll Angle versus Wave Height
Standard Deviation for Buoy 7 (8 x 26 - 05)
Figure 35 - Peak RAO of Roll Angle versus Spectral Density at Wave Peak for Buoy 3 (OSPREY)
Figure 37 - Peak RAO of Roll Angle versus Spectral Density at Wave Peak for Buoy 7 (8 x 26 - 05)
Figure 38 - Peak RAO of Pitch Angle versus Wave Height Standard Deviation for Buoy 3 (0SP3cf)}
Figure 39 - Peak RAO of Pitch Angle versus Wave Height. Standard Deviation for Buoy 4 (8 x 26 - 08)
Figure 41 - Peak RAO of Pitch Angle versus Spectral Density at Wave Peak for Buoy 3 (OSPREY)
Figure 44 - Peak RAO of Heave Displacement versus Wave Height Standard Deviation for Buoy 3 (OSPREY)
Figure 45 - Peak RAO of Heave Displacement versus Wave Height Standard Deviation for Buoy 4 (8 x 26 - 08)
Figure 48 - Peak RAO of Heave Displacement versus Wave Height Standard Deviation for Buoy 7 (B x 26 - 05)
Figure 50 - Mean Value of RAO of Roll Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two
Figure 52 - Mean Value of RAO of Roll Angle versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Two
Figure 53 - Mean Value of RAO of Pitch Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two
Figure 54 - Mean Value of RAO of Pitch Angle versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Two
Figure 56 - Mean Value of RAO of Heave Displacement versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Two
Figure 57 - Mean Value of RAO of Heave Displacement versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Two
Figure 62 - Mean Value of RAO of Pitch Angle versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Three
Figure 63 - Mean Value of RAO of Pitch Angle versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Three
Figure 64 - Mean Value of RAO of Pitch Angle versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Three
Figure 65 - Mean Value of RAO of Heave Displacement versus Wave Frequency for Buoy 3 (OSPREY) in Sea State Three
Figure 66 - Mean Value of RAO of Heave Displacement versus Wave Frequency for Buoy 4 (8 x 26 - 08) in Sea State Three
Figure 67 - Mean Value of RAO of Heave Displacement versus Wave Frequency for Buoy 7 (8 x 26 - 05) in Sea State Three
Figure 6a - Analog X-Y Plot of Roll and Pitch Angles for Buoy 3 (OSPREY)
APPENDIX A - COMPUTER GENERATED ANALYSIS RESULTS

The David W. Taylor Naval Ship Research and Development Center (DTNSRDC) has developed software to digitize and analyze U. S. Coast Guard Exposed Water Buoy data tapes. The analysis includes data from a group of independent buoys deployed in the vicinity of the Chesapeake LT Tower during the initial buoy deployment (Dec 72-Jul 73). Each tape contains frequency multiplexed motion and tension signals from up to seven buoys as well as wave height, time code, voice and calibration information.

Although the system can accommodate up to seven buoys, only buoys number 3, 4 and 7 were active during the collection of data for this deployment. Buoy numbers 4 and 7 are the standard 8' x 26' configuration and buoy number 3 is the OSPREY design. Note that buoy number 3 is moored by a single chain from its central mooring eye, while the standard buoys are bridle moored. The weight in service of the OSPREY is approximately 7700 lbs and of the standard approximately 5000 lbs.

The analysis is based on data from 25 runs selected for quality and duration and is representative of sea states 2 and 3. These 25 runs contain at least 14 minutes of continuous data for buoys 3, 4 and 7. The data is divided into 75 separate files (3 buoys x 25 runs) and is tabulated in order of decreasing variance of wave height in Table A1. The analysis results contain both tabular and graphical output and include 75 pairs of statistical and spectral analysis results.

In addition to mooring cable tension, buoy motions include roll, pitch and heave acceleration all referenced to the vertical. Wave height was measured at the light tower and recorded along with the buoy motions and reference information.
In addition to basic statistics (max, min, average, RMS, MEANSQ, variance and sigma) the statistical segment also includes plots of probability amplitude densities and a probability density and distribution of total angle. Spectral Analysis results include tables and plots of power spectral density, Response Amplitude Operators, Cross Spectrums including magnitude and phase and Transfer functions. Some abbreviations used are as follows:

- SPS - samples per second
- FS - full scale sensitivity
- CAL - calibration
- PSD - power spectral density
- RAO - response amplitude operator
- SEA - wave height
- CSURFACE - wave height
- ACCEL - heave acceleration (statistical segment)
- ACCEL - heave displacement (spectral segment)

A detailed description of the digitization - quality checking process, the analysis technique used and a discussion on the results are included in DTNSRDC Report No SPD-657-01.
TABLE A1 - TWENTY-FIVE RUNS USED IN THE ANALYSIS - RUN #'s LISTED IN ORDER OF DECREASING VARIANCE OF WAVE HEIGHTS

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A3
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