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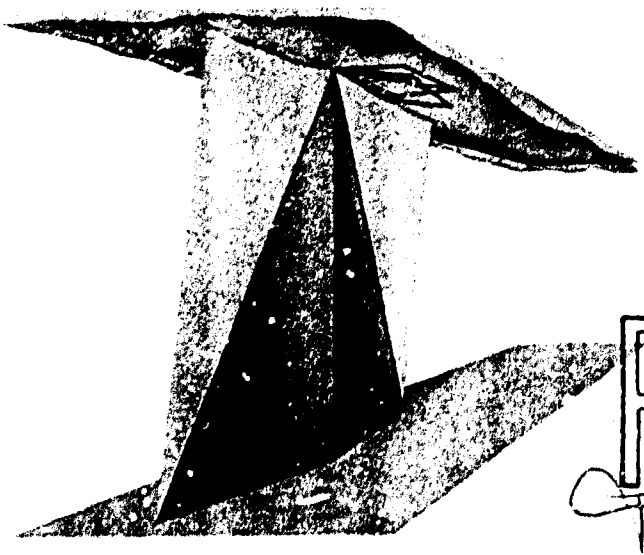
"MODESRCH", AN IMPROVED COMPUTER PROGRAM FOR OBTAINING ELF/VLF/LF MODE CONSTANTS IN AN EARTH-IONOSPHERE WAVEGUIDE

Interim Report No. 77T

D. G. Morfitt and C. H. Shellman



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'MODESRCH', An Improved Computer Program for Obtaining ELF/VLF/LF Mode Constants in an Earth-Ionosphere Waveguide.

by

D. G. MORFITT

and

C. H. SHELLMAN

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ABSTRACT:

MODESRCH is a FORTRAN computer program designed to obtain rapidly, economically, and accurately, the mode constants needed to describe propagation in the earth-ionosphere waveguide at ELF, VLF and LF (10 Hz to 60 kHz). It is suggested that MODESRCH, along with required mode summing procedures, be used to replace "waveguide" and "wave-hop" codes currently used in network analysis programs which cover the ELF to LF frequency bands. This report contains a discussion of the analytical approach taken in MODESRCH, a FORTRAN listing of the program, instructions for using the program and some sample calculations.

INTRODUCTION

Radio field strengths at ELF, VLF and LF (10 Hz to about 60 kHz) are usually calculated using procedures based on either normal mode theory (reference 1) or on a semi-geometrical optics approach (reference 2).

Normal mode theory treats the field as being composed of one or more discrete families (modes) of plane waves confined to the natural waveguide bounded by the earth and the ionosphere. The number of waveguide modes needed to adequately represent the field is proportional to the earth-ionosphere separation measured in radio wavelengths. Thus, only a single mode is required below 3 kHz, about ten to fifteen modes at 30 kHz, and as many as twenty-five modes may be important at 60 kHz. The propagation parameters for each mode depend on the characteristics of the earth and the ionosphere, and are independent of the relative positions of the transmitter and receiver. For a homogeneous waveguide, once all of the important modes have been found, the radio fields can be determined as a function of distance by a simple mode summing calculation.

Computer programs based on waveguide mode theory have been brought to a high degree of sophistication and versatility in recent years, making it possible to calculate fields for any antenna configuration at any altitude, for both vertically and horizontally polarized waves, and for inhomogeneous waveguides. Unfortunately, solutions to the modal equation cannot be determined explicitly. Instead, the complex angles of incidence at the waveguide boundaries, which uniquely identify each mode, must be determined by iterative techniques. At frequencies above about 30 kHz, where there is a large number of modes to be found, the mode finding procedure of reference 1 may miss some solutions and it has taken human judgment to determine if the set of modes

found was complete. If the set of modes was judged incomplete, new initial conditions had to be resubmitted to the computer in an attempt to compute the missing modes, resulting in additional computer and manpower costs. Systems analysis codes do not allow human intervention, and so they have not been able to rely on mode theory when a sizeable number of modes is needed to represent the radio fields.

Previous practice has been to compute radio fields at frequencies below 30 kHz using waveguide mode theory and to use a wave-hop theory, such as described in reference 2, for computing fields at and above 30 kHz. The wave-hop procedure considers the total field at the receiver to be composed of a groundwave component and a number of skywaves following unique geometric ray paths connecting the transmitter and receiver with intermediate reflections off the earth and the ionosphere. Calculations of radio field strength, using the wave-hop procedure of reference 2, agree well with waveguide mode calculations for horizontally homogeneous normal ionospheres. They do not agree for severely depressed ionospheres over low conductivity ground.

The wave-hop approach has features which make it somewhat inconvenient for system analyses. For each separation distance between transmitter and receiver there is a unique ray geometry. Thus, for calculations of fields as a function of distance; e.g., for aircraft operations, certain inherent calculations must be repeated many times. Also, for inhomogeneous waveguides such as across day-night ionospheric gradients or across nuclear-induced ionospheric depressions, the geometry is ill-defined and the calculations tend toward approximations.

Aside from the limitations of either the waveguide approach or the wave-hop approach to long-wave propagation prediction, it would be advantageous to

use a single method across the entire ELF, VLF, LF frequency bands. Systems codes used for network analyses would be considerably reduced in size, saving computer costs. Comparisons of systems performance as a function of frequency would be more meaningful if a single propagation theory were invoked.

The MODESRCH computer program described in this report has the required ability to compute waveguide mode constants rapidly, economically and accurately over the frequency band of interest. MODESRCH contains all of the basic features of the previously reported waveguide mode computational procedures of reference 1 and several important additions. The new additions virtually guarantee that all waveguide modes of importance will be found at any frequency up to at least 60 kHz. MODESRCH, used with presently available mode summing techniques, will provide the radio fields required for systems analysis.

This report describes the MODESRCH theory. It also contains a listing of the FORTRAN program, instructions for the user, and examples of MODESRCH calculations.

II. DISCUSSION OF MODESRCH PROCEDURE

A. General Background

In the propagation of ELF, VLF and LF terrestrial radio waves to great distances, the waves are confined within the space between the earth and the ionosphere. This space acts as a waveguide, and the "waveguide concept" is applicable for characterizing the propagated fields as a function of distance.

The energy within the waveguide is considered to be partitioned among a series of modes. Each mode represents a resonant condition; i.e., for a discrete set of angles of incidence of the waves on the ionosphere, resonance occurs and energy will propagate away from the source. The complex angles (θ) for which this occurs are called the eigenangles. They may be obtained using the 'full-wave' procedures described in reference 1 by solving the determinantal equation

$$F_1(\theta) = |\ \underset{\sim}{\bar{R}}_d\ (\theta)\ \underset{\sim}{R}_d\ (\theta) - \underset{\sim}{1}\ | = 0 \tag{1}$$

$$\text{where } \underset{\sim}{R}_d(\theta) = \begin{bmatrix} {}_{\parallel}R_{\parallel d}(\theta) & {}_{\perp}R_{\parallel d}(\theta) \\ {}_{\parallel}R_{\perp d}(\theta) & {}_{\perp}R_{\perp d}(\theta) \end{bmatrix} \tag{2}$$

is the complex ionospheric reflection coefficient matrix looking up into the ionosphere from height "d" and

$$\underset{\sim}{\bar{R}}_d(\theta) = \begin{bmatrix} {}_{\parallel}\bar{R}_{\parallel d}(\theta) & 0 \\ 0 & {}_{\perp}\bar{R}_{\perp d}(\theta) \end{bmatrix} \tag{3}$$

is the complex reflection matrix looking down from height "d" towards the ground.

Note: Any variable "A" denoted by $\underset{\sim}{A}$ in this report implies a matrix.

The complex angle, θ, is the angle of incidence at height $z = H$ where the modified index of refraction given by

$$n^2 = 1 - \alpha(H - Z) = 1 - \frac{2}{r_e}(H - Z) \tag{4}$$

is unity. Here r_e is the radius of the earth.

The notation $\parallel$ for the R's and $\bar{R}$'s denotes vertical polarization while the notation, $\perp$, denotes horizontal polarization. The first subscript on the R's refers to the polarization of the incident wave while the second applies to the polarization of the reflected wave.

To find solutions of (1) for the eigenangles, θ, the well known Newton-Raphson method may be used. The procedure is to guess a solution θ_0 to the equation $F_1(\theta) = 0$. The function $F_1(\theta)$ is then reevaluated for $\theta_0 + \delta\theta$ and the correction to θ_0 found from the equation

$$\Delta\theta = - \frac{F_1(\theta_0)\delta\theta}{F_1(\theta_0 + \delta\theta) - F_1(\theta_0)}. \tag{5}$$

The correction determined by (5) is then evaluated and the process repeated until the quantities $|\Delta\theta_r|$ and $|\Delta\theta_i|$ are reduced to within a preassigned tolerance. The subscripts r and i denote the real and imaginary parts, respectively. The final solution for mode "n" is then given by

$$[\theta_n]_j = [\theta_n]_{j-1} + \Delta\theta_n \quad j = 1, 2, 3, \ldots \tag{6}$$

B. Introduction to MODESRCH Theory

The "MODESRCH" procedure also obtains the full-wave solution of equation 1 in terms of the eigenangles. In this case; however, the $\underset{\sim}{R}$ and $\underset{\sim}{\bar{R}}$ terms are modified in order to make simpler equations.

The basic premise of the MODESRCH theory is to modify the function $F_1(\theta)$ so that it contains no poles and then in this form, apply a newly-developed root-finding algorithm to obtain the complete set of eigenangles.

In particular, the $F_1(\theta)$ equation of reference 1 becomes upon modification (see Appendix A):

$$F_3(\theta)=({}_{\parallel}n_{\parallel}-{}_{\parallel}\underline{\bar{X}}_{\parallel}{}_{\parallel}d_{\parallel})\ ({}_{\perp}n_{\perp}\ -{}_{\perp}\underline{\bar{X}}_{\perp}{}_{\perp}d_{\perp})\ -\ {}_{\parallel}\underline{\bar{X}}_{\perp}{}_{\perp}\underline{\bar{X}}_{\parallel}{}_{\parallel}d_{\perp}{}_{\perp}d_{\parallel} \tag{7}$$

where $\underline{\bar{X}} = (\underset{\sim}{R} + 1)/c$ (8)

and $c = \cos\theta$ (θ = complex angle of incidence)

or ${}_{\parallel}\underline{\bar{X}}_{\parallel} = ({}_{\parallel}R_{\parallel} + 1)/c$

$$\begin{aligned} {}_{\parallel}\underline{\bar{X}}_{\perp} &= {}_{\parallel}R_{\perp}/c \\ {}_{\perp}\underline{\bar{X}}_{\parallel} &= {}_{\perp}R_{\parallel}/c \\ {}_{\perp}\underline{\bar{X}}_{\perp} &= ({}_{\perp}R_{\perp} + 1)/c \end{aligned} \tag{9}$$

and $\underset{\sim}{n}\ \underset{\sim}{d}^{-1} = (\underset{\sim}{\bar{R}}^{-1} + 1)/c$

$$\text{or}\quad \frac{{}_{\parallel}n_{\parallel}}{{}_{\parallel}d_{\parallel}} = \left(\frac{{}_{\parallel}\bar{R}_{\parallel} + 1}{{}_{\parallel}\bar{R}_{\parallel}}\right)/c \tag{10}$$

$$\frac{{}_{\perp}n_{\perp}}{{}_{\perp}d_{\perp}} = \left(\frac{{}_{\perp}\bar{R}_{\perp} + 1}{{}_{\perp}\bar{R}_{\perp}}\right)/c$$

The modified variables:

$$\left(\frac{\underset{\sim}{R}+1}{c}\right) \quad \left(\frac{\underset{\sim}{\bar{R}}^{-1}+1}{c}\right)$$

are related to making simpler equations in the 'MODESRCH' procedure and in removing the zeros in $F_3(\theta)$ at $\theta = 90°$. The functions $F_1(\theta)$ and $F_3(\theta)$ are not the same, but except for the point where $\theta=90°$, they have the same zeros.

The solutions to the modal equation; (i.e., equation 7) for $F_3(\theta)=0$ are the complex eigenangles (θ) needed to obtain the mode constants (e.g., phase velocity, attenuation rate, and excitation factor) which

---

Note: From this point on, let $F_3(\theta)$ be denoted by $F(\theta)$ for convenience.

are the parameters of interest in computing field strengths. The total field at any point along the guide may be computed from the vector sum of the contributions from each individual mode.

As in reference 1, the MODESRCH procedure requires that the upper boundary of the waveguide be assigned arbitrary electron and ion density distributions with height as well as variable collision frequency profiles with height. Allowance is also made for the anisotropy of the ionosphere. The lower boundary of the guide is taken to be a smooth homogeneous earth characterized by an adjustable conductivity and permittivity. Earth curvature is also included via the artifice of the modified refractive index (e.g., Equation (4)).

The (X,Y,Z) coordinate system used is such that Z is directed into the ionosphere, the X-Z plane is the plane of propagation, and invariance in the Y-axis is assumed.

C. General Steps in the Solution of the Modal Equation

(1) For VLF/LF cases the input electron and ion density profiles vs height are modified by eliminating those heights above which and below which the corresponding particle densities will have negligible effects on the modal solutions. For ELF cases, profiles are not cutoff at the top or the bottom heights but instead the complete input profile is used in subsequent calculations.

(2) The modal solutions of Equation (7), the complex eigenangles ($\theta = \theta_{real} + i\theta_{imaginary}$), are determined within a bounded region of the complex "THETA" plane defined by a set of limiting THETA values. This bounded region will be called the "contour rectangle" and the limiting THETA values are denoted by the corners of the rectangle (e.g., A,B,C, and D). These parameters are portrayed in Figure 1.

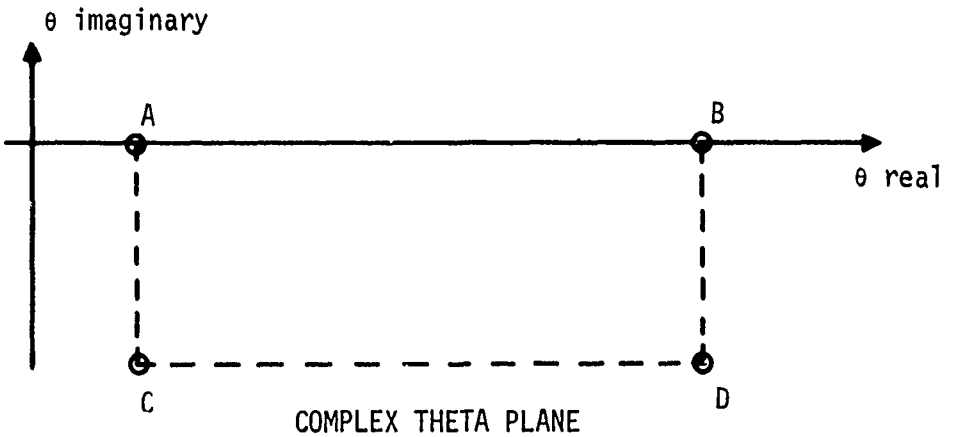


Figure 1. The Contour Rectangle ABCD

(3) Four fixed angles (θT_i, i=1,2,3,4) are chosen in the complex THETA plane. These angles are somewhat outside the corners A,B,C, and D of the contour rectangle. For VLF and LF calculations these angles are the given values of θ at which full-wave solutions are to be carried out for use in a Lagrange interpolation procedure. At ELF (e.g., < 1000 Hz) the purpose of these angles is to identify the location of the contour rectangle in the THETA plane.

(4) At VLF and at LF, the differential equations for the ionospheric terms $\underset{\sim}{\overline{X}}(\theta)$ are integrated at the angles θT_i, to give full-wave values of $\underset{\sim}{\overline{X}}(\theta T_i)$ at the base of the modified electron-ion density profiles. The results obtained by integrating to the bottom of the profile (e.g., $Z=Z_b$) will be designated as $\underset{\sim}{\overline{X}}_b(\theta T_i)$.

(5) For VLF/LF cases a further upward integration of the $\underset{\sim}{X}(\theta T_i)$ equations from the height Z_b to a reference height 'd' is then performed assuming a free space environment over a curved earth. This reference height may be taken to be an effective reflection height. At this height there will be

no poles in the function $\underset{\sim}{\overline{X}}(\theta)$ for the region of interest in the complex THETA plane. The determination of this height is important because the general root-finding algorithm used to locate the zeros of $F(\theta)$ requires that there be no poles in the vicinity of the zeros to be found. This reference height is also appropriate for performing Lagrange interpolation on the function $\underset{\sim}{\overline{X}}(\theta)$.

(6) At the reference height 'd', for VLF and LF, the contour rectangle is subdivided into small mesh squares. The value of $F(\theta)$ at each corner of each mesh square is determined by Lagrange interpolation of $\underset{\sim}{\overline{X}}(\theta)$ based on the given values of $\underset{\sim}{\overline{X}}(\theta T_i)$.

Recall from Equation (7) that the function $F(\theta)$ is computed from $\underset{\sim}{\overline{X}}(\theta)$, $\underset{\sim}{n}(\theta)$ and $\underset{\sim}{d}(\theta)$ so that for any angle, θ_j the value of $\underset{\sim}{\overline{X}}(\theta_j)$ is found from the Lagrangian interpolation of $\underset{\sim}{\overline{X}}(\theta T_i,\ i=1,2,3,4)$ whereas $\underset{\sim}{n}(\theta_j)$ and $\underset{\sim}{d}(\theta_j)$ are calculated directly.

(7) For ELF (e.g., <1000 Hz), the angles used in the full-wave integration of the $\underset{\sim}{\overline{X}}(\theta)$ equation consists of the usual corners of the contour rectangle plus all additional angles formed from the four corners of each mesh square into which the contour rectangle is also divided. In this case the integration is extended downward to the earth's surface (e.g., Z=0) and no further integration back up to the height 'd' is done. No Lagrangian interpolation is involved since the full-wave solution is computed for all angles of interest to the problem. It is important to realize at this point that the MODESRCH procedure has not been formulated so as to eliminate the poles for ELF cases. This may result in no solution being found to the modal equation due to the presence of poles. Because of this result, it may be advisable to compute modes for the ELF cases by the methods of reference 1. This problem has not; however, occurred in practice.

(8) The zeros of $F(\theta)$ at 'd' for VLF/LF (or at '0' for ELF are then found by applying the root-finder algorithm to the mesh squares.

(9) Since $F(\theta)$, as determined from step (8) is approximate, the exact solution of $F(\theta)$ is then obtained by applying the usual Newton-Raphson procedures where the full-wave solution is used to obtain $\underset{\approx}{\overline{X}}(\theta)$

(10) With the determination of the exact solutions of the θ's (eigenangles), the required mode constants and excitation factors, may be calculated.

D. Optimization of Input Ionospheric Density Profiles

Computing efficiency for VLF and LF may be improved by cutting off the top and bottom heights of the input profile.

(1) Exponential Profiles (electrons only)

An exponential, electrons only, ionospheric profile may be specified as input to the MODESRCH procedure. The profile is computed as described in Wait's NBS TN 300, reference 3, where

$$\omega_p(z) = 2.5\text{X}10^5 \exp[\beta(z-h'] = 3.18254\text{X}10^9 \, N(z)/\nu(z) \qquad (11)$$

and β is in inverse kilometres, z and h' are in kilometres, N(z) is the electron density in electrons per cubic centimetre and $\nu(z)$ is the collision frequency in collisions per second. The exponential collision frequency is given in reference 3 as

$$\nu(z) = 1.86 \cdot 10^{11} \cdot \exp(-0.15z) \qquad (12)$$

The top of the profile may be modified by choosing a value for the variable SCLHTS. The relationship is that:

$$\text{Top Height} = \text{Integer}\left(H' + \frac{\text{SCHLTS}}{\beta} + 0.5\right) \qquad (13)$$

The nearest integral value of altitude as given by 'Top Height' is used to set the top of the profile. All heights above this value are neglected in the MODESRCH computations.

The bottom of the exponential profile is set by assigning a minimum value (i.e., ENMIN) of electron density in electrons/cm^3. All altitudes, at which the corresponding density is less than the value of ENMIN, will be neglected.

(2) VLF/LF - Tabular Electron-Ion Density Profiles

When the particle density profiles are in tabular form, the top and bottom of the profile may be cut off using the following criteria, as presented in reference 4. Here the particle profiles may consist of ions as well as of electrons.

The (isotropic) squared index of refraction is written as:

$$n^2_{(z)} = 1 - A - jB \tag{14}$$

where

$$A = A_e + A_+ + A_-$$

$$B = B_e + B_+ + B_-$$

or

$$A = \underbrace{\frac{N_e e^2}{M_e \varepsilon_0 [\omega^2 + \nu_e^2]}}_{A_e} + \underbrace{\frac{N_+ e^2}{M_+ \varepsilon_0 [\omega^2 + \nu_+^2]}}_{A_+} + \underbrace{\frac{N_- e^2}{M_- \varepsilon_0 (\omega^2 + \nu_-^2)}}_{A_-} \tag{15}$$

and

$$B = \frac{\nu_e}{\omega} A_e + \frac{\nu_+}{\omega} A_+ + \frac{\nu_-}{\omega} A_- \tag{16}$$

with the identification that

ν_e = electron-neutral particle collision frequency (s^{-1})

ν_+ = positive ion-neutral particle collision frequency (s^{-1})

ν_- = negative ion-neutral particle collision frequency (s^{-1})

N_e = electron density (cm^{-3})

N_+ = positive ion density (cm^{-3})

N_- = negative ion density (cm^{-3})

ω = frequency (radians s^{-1})

m_e = mass of the electron (kg)

m_+ = mass of the positive ions (kg)

m_- = mass of the negative ions (kg)

ε_0 = permittivity of free space

The top part of the ionospheric profile which may be neglected in the calculations is defined as that altitude(z) above which:

$$B(z) > \text{Maximum}\ (\text{TEMPO-B},\ 20\ \frac{\omega}{\nu_e}) \tag{17}$$

where 'TEMPO-B' is assigned a value (e.g., 2.0).

An additional requirement for cutting off the top of the profile is that the electron density must be as great as 'TOPEN'. The variable 'TOPEN' is assigned a value (e.g., 1000 electrons/cm^2). The top of the profile is taken to be the higher of the two heights corresponding to 'B' (equation (17)) and to 'TOPEN'.

The bottom of the input ionospheric profile is neglected (for VLF/LF cases) at those altitudes for which the computed value of B in equation (16) is less than 'CUTOFF'. The variable 'CUTOFF' is assigned a value (e.g., 0.0001). The relationship for cutoff is:

$$B(z) < \text{CUTOFF} \tag{18}$$

(3) ELF Tabular Electron-Ion Density Profiles

In the case of ELF the profile top and bottom are not cut off and the complete profile is used in the calculations.

E. Integration of the Differential Equations for $\underset{\approx}{X} = (\underset{\sim}{R} + \underset{\sim}{1})/C$ Through the Ionosphere

The differential equations for the ionosphere reflection matrix, $\underset{\sim}{R}$, are given in reference 5.

The differential equations are of the form

$$d\underset{\sim}{R}/dz = -\, ik/2\left\{\underset{\sim}{S}^{(21)} + \underset{\sim}{S}^{(22)}\underset{\sim}{R} - \underset{\sim}{R}\underset{\sim}{S}^{(11)} - \underset{\sim}{R}\underset{\sim}{S}^{(12)}\underset{\sim}{R}\right\} \tag{19}$$

where z is the height variable and $S^{(kl)}$ are 2 X 2 partitions of a 4 X 4 matrix given by:

$$\underset{\sim}{S} = \left[\begin{array}{cc|cc} T_{11}+T_{44}+T_{14}/C+CT_{41} & -T_{12}/C-T_{42} & -T_{11}+T_{44}+T_{14}/C-CT_{41} & -T_{12}/C-T_{42} \\ -T_{31}-T_{34}/C & C+T_{32}/C & T_{31}-T_{34}/C & -C+T_{32}/C \\ \hline -T_{11}+T_{44}-T_{14}/C+CT_{41} & T_{12}/C-T_{42} & T_{11}+T_{44}-T_{14}/C-CT_{41} & T_{12}/C-T_{42} \\ T_{21}+T_{34}/C & C-T_{32}/C & -T_{31}+T_{34}/C & -C-T_{32}/C \end{array}\right] \tag{20}$$

where

$$\begin{aligned} C &= \cos\theta \\ \theta &= \text{complex angle of incidence (deg).} \end{aligned} \tag{21}$$

The intermediate matrix T relates the electromagnetic field components to their derivatives. In turn, T is defined in terms of the susceptibility matrix M:

where T is defined as

$$\underset{\sim}{T} = \begin{bmatrix} \left(-\dfrac{SM_{31}}{1+M_{33}}\right) & \left(\dfrac{SM_{32}}{1+M_{33}}\right) & (0) & \left(\dfrac{C^2+M_{33}}{1+M_{33}}\right) \\ (0) & (0) & (1) & (0) \\ \left(\dfrac{M_{23}M_{31}}{1+M_{33}} - M_{21}\right) & \left(C^2+M_{22} - \dfrac{M_{23}M_{32}}{1+M_{33}}\right) & (0) & \left(\dfrac{SM_{23}}{1+M_{33}}\right) \\ \left(1+M_{11} - \dfrac{M_{13}M_{31}}{1+M_{33}}\right) & \left(\dfrac{M_{32}M_{13}}{1+M_{33}} - M_{12}\right) & (0) & \left(\dfrac{SM_{13}}{1+M_{33}}\right) \end{bmatrix} \tag{22}$$

where S = sin (θ).

Finally, the susceptibility $\underset{\sim}{M}$ matrix is defined as

$$\underset{\sim}{M} = \frac{-P}{U(U^2-Y^2)} \begin{bmatrix} (U^2-\ell^2Y^2-2(H-z)/r_e) & (-jUnY-\ell mY^2) & (jUmY-\ell nY^2) \\ (jUnY-\ell mY^2) & (U^2-m^2Y^2-2(H-z)/r_e) & (-jU\ell Y-mnY^2) \\ (-jUmY-\ell nY^2) & (jU\ell Y-mnY^2) & (U^2-n^2Y^2-2(H-z)/r_e) \end{bmatrix} \quad (23)$$

where

$$\begin{aligned} j &= -1 \\ P &= (\omega_p/\omega)^2 \\ U &= 1 - j(\nu/\omega) \\ Y &= \mu_o\omega_m/\omega \text{ henrys} \\ \omega_m &= \text{magnetic gyrofrequency } (s^{-1}) \\ \omega_p &= \text{plasma frequency } (s^{-1}) \\ \ell &= \cos\theta_D \cos\varphi_a \\ m &= \cos\theta_D \sin \\ n &= -\sin\theta_D \\ \nu &= \text{collision frequency } (s^{-1}) \\ \omega &= \text{wave frequency } (s^{-1}) \\ \mu_o &= \text{free space permeability (henrys/m)} \\ \theta_D &= \text{magnetic dip angle (deg)} \\ r_e &= \text{earth's radius (km)} \\ H &= \text{altitude where the index of refraction is unity (km)} \\ \varphi_a &= \text{direction of propagation east of magnetic north (deg)} \\ z &= \text{altitude (km)} \end{aligned} \quad (24)$$

The matrix $\underset{\sim}{M}$ described above is general in that it represents a single ionospheric constituent. Three species are considered by this model: electrons and positive and negative ions. Thus for each species there will

be a matrix $\underset{\sim}{M}_i$, and the matrix elements needed for computing the $\underset{\sim}{T}$ matrix elements are found from:

$$\underset{\sim}{M} = \sum_{i=1}^{3} \underset{\sim}{M}_i \tag{25}$$

Earth curvature terms have been added to the diagonal elements of $\underset{\sim}{M}$, by the terms $-2(h-z)/r_e$.

Equation (19) may be rearranged to form the following differential equations for $\underset{\approx}{\overline{X}} = (\underset{\sim}{R} + 1)/C$, (see Equation (8)), giving:

$$\frac{d\underset{\approx}{\overline{X}}}{dZ} = \frac{-ik}{2} \underset{\sim}{A} + \underset{\sim}{B}\,\underset{\approx}{\overline{X}} + \underset{\approx}{\overline{X}}\,\underset{\sim}{C} + \underset{\approx}{\overline{X}}\,\underset{\sim}{D}\,\underset{\approx}{\overline{X}} \tag{26}$$

where k is the wave number, z is height, and

$$\underset{\sim}{A} = (\underset{\sim}{S}^{21} - \underset{\sim}{S}^{22} + \underset{\sim}{S}^{11} - \underset{\sim}{S}^{12})/C = \begin{pmatrix} 4T_{41} & 0 \\ 0 & 4 \end{pmatrix} \tag{27}$$

$$\underset{\sim}{B} = \underset{\sim}{S}^{22} + \underset{\sim}{S}^{12} = 2 \begin{pmatrix} T_{44} = CT_{41} & -T_{42} \\ 0 & -C \end{pmatrix} \tag{28}$$

$$\underset{\sim}{C} = -\underset{\sim}{S}^{11} + \underset{\sim}{S}^{12} = 2 \begin{pmatrix} T_{11} - CT_{41} & 0 \\ T_{31} & -C \end{pmatrix} \tag{29}$$

$$\underset{\sim}{D} = -C\,\underset{\sim}{S}^{12} = \left(\begin{array}{c|c} C(T_{11}-T_{14}) - T_{14} + C^2T_{41} & T_{12} + CT_{42} \\ \hline -CT_{31} + T_{34} & C^2 - T_{32} \end{array}\right) \tag{30}$$

Equation (26) may be integrated numerically by the Runge-Kutta method. The integration is carried out in the direction of decreasing ionospheric height. Error testing is based on comparing the results of integrating through a height interval with one Runge-Kutta step with the result of integrating through the same height interval using two steps. If the difference is greater than a specified tolerance, the step size in that interval is cut in half. The initial set of integration steps is taken to be the input profile segments. However, the integration steps used in one integration are taken to be the initial set of steps for the next integration.

Note that since the elements of $\underset{\sim}{\overline{X}}(\theta)$ are complex, the integration is that of eight real coupled differential equations for the real and imaginary parts of the elements of $\underset{\sim}{\overline{X}}(\theta)$.

The classical procedure of assuming a semi-infinite homogeneous ionosphere at the top of the profile is followed. The solution for reflection from this semi-infinite medium is taken from reference 6 and is used as the starting solution for the integration.

This method of integration is identified as the "full-wave" procedure for obtaining ionospheric reflection coefficients. The values obtained from the integration are denoted as $\underset{\sim}{\overline{X}}(\theta T_i)_b$, where z=b is the lower cutoff height of the integration.

F. Integration of $\underset{\sim}{\overline{X}} = (\underset{\sim}{R} + \underset{\sim}{1})/C$ Through Free Space Over a Curved Earth

As was stated in Section II, C-5, the general root-finding algorithm used to locate the zeros of $F(\theta)$ requires that there be no poles in the vicinity of the zeros to be found. This requires that there be no poles in $\underset{\sim}{\overline{X}} = (\underset{\sim}{R} + \underset{\sim}{1})/C$ in this region of the θ-plane. This can be assured by integrating $\underset{\sim}{\overline{X}}$ from the bottom of the profile upward through free-space over a curved earth to an effective "reflection height", defined by the condition

$$\sum_{\theta\text{-points}}^{4}{}_{1} \quad \sum_{\text{elements}}^{4}{}_{1} \left| \frac{d\underset{\sim}{\overline{X}}(\theta T_i)}{dC} \right|^2 \rightarrow \text{Minimum} \tag{31}$$

where the summation is over the four elements of $\underset{\sim}{\overline{X}}$ and over a set of points (e.g., θT_i $i = 1,2,3,4$) in the θ-plane distributed over the region in which zeros of $F(\theta)$ are to be located. This condition is also appropriate for defining a level at which Lagrange interpolation can best be performed.

In practice the set of points (θT_i) are used in defining the "effective reflection height" and these are taken to be the "given" points in the θ-plane used for Lagrange interpolation.

As in previous waveguide formulations, e.g., reference 1, the earth curvature effect is approximated by assuming a dielectric medium which varies linearly in the square of the refractive index over a flat earth.

Considering the usual right handed cartesian coordinate system, where +Z is taken vertically upward, formulation for such an integration from the bottom of the profile at "Z_b" to another height "Z_t", can be obtained following reference 7. The derivation of the equations is given in Appendix B. The final equations are listed and discussed as follows.

Just below the free space-ionosphere boundary (e.g., Z_b), let the total electric wave field be composed of two upward (e.g., +Z direction) traveling incident components and two downward traveling reflected components. They are defined by $E^i_{11,y}$ and $E^r_{11,y}$ respectively. E_{11} is in the X-Z plane and E_y is parallel to the y-axis. The total-field components just below the boundary are given in reference 7, page 118 as

$$\left.\begin{aligned} E_x &= (E^i_{11} - E^r_{11}) \cos\theta_I \\ E_y &= E^i_y + E^r_y \\ H_x &= (E^r_y - E^i_y) \cos\theta_I \\ H_y &= E^i_{11} + E^r_{11} \end{aligned}\right\} \qquad (32)$$

where θ_I is the angle in the x-y plane which the incident wave normal makes with the vertical z axis. The following identifications are made:

$$
\left.
\begin{aligned}
{}_{\perp}R_{\perp} &= E_y^r/E_y^i \\
{}_{\parallel}R_{\perp} &= E_y^r/E_{11}^i \\
{}_{\parallel}R_{\parallel} &= E_{11}^r/E_{11}^i \\
{}_{\perp}R_{\parallel} &= E_{11}^r/E_y^i
\end{aligned}
\right\} \tag{33}
$$

Equations (32) and (33) are combined with the differential equations to be satisfied by the wave fields at oblique incidence (reference 7, pp. 140), to give two independent sets of relationships. Either of these sets of field variables may be set equal to zero without affecting the other, so that the corresponding waves are propagated independently.

For the first set of equations the electric field is everywhere parallel to the y-axis and the waves are said to be "horizontally polarized". For the second set of equations the electric field is parallel to the X-Z plane and the waves are said to be vertically polarized, although the electric vector is not in general vertical.

The equations become (as derived in Appendix B):

For <u>Horizontal Polarization</u>:

$$(-A/C)h_1(\zeta) + (-B/C)h_2(\zeta) + E_y^i\left(\frac{{}_{\perp}R_{\perp} + 1}{C}\right) + E_{11}^i\left(\frac{{}_{\parallel}R_{\perp}}{C}\right) = 0 \tag{34}$$

$$(-A/C)\left\{ch_1(\zeta) + Kh_1'(\zeta)\right\} + (-B/C)\left\{ch_2(\zeta) + Kh_2'(\zeta)\right\} + 2E_y = 0 \tag{35}$$

For <u>Vertical Polarization</u>:

$$(-Q/C)h_1(\zeta) + (-G/C)h_2(\zeta) + E_{11}^i\left(\frac{{}_{\parallel}R_{\parallel} + 1}{C}\right) + E_y^i\left(\frac{{}_{\perp}R_{\parallel}}{C}\right) = 0 \tag{36}$$

$$(-Q/C)\left\{ch_1(\zeta) + [Kh_1'(\zeta) + Lh_1(\zeta)]/n^2\right\} +$$

$$(-G/C)\left\{ch_2(\zeta) + [Kh_2'(\zeta) + Lh_2(\zeta)]/n^2\right\} + 2E_{11}^i = 0 \tag{37}$$

The coefficients A, B, Q, and G are arbitrary constants to be determined. The functions $h_1(\zeta)$ and $h_2(\zeta)$ satisfy the differential equation

$$\frac{d^2w}{d^2\zeta} + \zeta w = 0 \tag{38}$$

and are the modified Hankel functions or order 1/3 (e.g., $h_1(\zeta)$ and $h_2(\zeta)$) as defined by the Computation Laboratory at Cambridge, Massachusetts (reference 8). The primes on these quantities denote derivatives with respect to the argument. Also

$$\zeta = (k/\alpha)^{2/3}\,[C^2 + \alpha(Z-H)] \tag{39}$$

$$\begin{aligned} C &= \cos(\theta) \\ \alpha &= 2/r_e \\ k &= 2\pi/\lambda \\ K &= i(\alpha/k)^{1/3} \\ L &- i(\alpha/2K) \\ i &= \sqrt{-1} \end{aligned} \tag{40}$$

where

H: is the altitude where the index of refraction is unity.

λ: is the free space wave length.

n: is the index of refraction of the medium.

r_e: is the radius of the earth.

Note that the $\underset{\sim}{R}$ terms may be written as:

$$\begin{aligned} {}_{\parallel}\underline{\overline{X}}_{\parallel} &= \frac{{}_{\parallel}R_{\parallel} + 1}{C} \\ {}_{\perp}\underline{\overline{X}}_{\perp} &= \frac{{}_{\perp}R_{\perp} + 1}{C} \\ {}_{\perp}\underline{\overline{X}}_{\parallel} &= \frac{{}_{\perp}R_{\parallel}}{C} \\ {}_{\parallel}\underline{\overline{X}}_{\perp} &= \frac{{}_{\parallel}R_{\perp}}{C} \end{aligned} \tag{41}$$

The "effective reflection height" is found by integrating $\underline{\overline{X}}(\theta)$ upward to heights at, say one kilometer intervals above the bottom of the profile. That height at which the minimizing condition, Equation (31), is satisfied is the "effective reflection height". This height is chosen as the reference height, "d", used in classical waveguide theory as the height at which R and $\overline{R}$ (i.e., $\underline{\overline{X}}$, n, and d) are computed and as the height at which Lagrange interpolation is performed. For evaluations of $F(\theta)$, Z_t is then always taken to be at z=d.

The upward free space integration of the ionospheric $\underline{\overline{X}}(\theta)$ parameters is carried out in the following steps.

STEP I:

Two sets of boundary conditions (B.C.) are applied to equations (34), (35), (36) and (37) at the Z_b level. These sets are

SET I: $E^i_{\parallel} = 1,\ E^i_y = 0$

SET II: $E^i_{\parallel} = 0,\ E^i_y = 1$

Here $\underline{\overline{X}}_b(\theta)$ is the result of the Runge-Kutta integration of Equation (26) at what will be denoted as the "from level", ("f"). Where

$$\zeta_f = (k/\alpha)^{2/3}\,[C^2 + \alpha(Z_b - H)] \tag{42}$$

For <u>Horizontal Polarization</u> at the "from level" (Z_b), the B.C. give:

For B.C. SET I:

$$\left(\frac{A}{C}\right)_I = \|\underline{\overline{X}}^f_{\perp} \cdot \left(\frac{a_{22}}{\Delta_1}\right) \tag{43}$$

$$\left(\frac{B}{C}\right)_I = -\|\underline{\overline{X}}^f_{\perp} \cdot \left(\frac{a_{21}}{\Delta_1}\right) \tag{44}$$

For B.C. SET II:

$$\left(\frac{A}{C}\right)_{II} = ({}_{\perp}\underline{\overline{X}}^{f}_{\perp} \cdot a_{22} - 2A_{12})/\Delta_1 \tag{45}$$

$$\left(\frac{B}{C}\right)_{II} = (2a_{11} - {}_{\perp}\underline{\overline{X}}^{f}_{\perp} \cdot a_{21})/\Delta_1 \tag{46}$$

where

$$\left.\begin{aligned} a_{11} &= h_1(\zeta_f),\ a_{12} = h_2(\zeta_f) \\ a_{21} &= Ch_1(\zeta_f) + Kh_1'(\zeta_f) \\ a_{22} &= Ch_2(\zeta_f) + Kh_2'(\zeta_f) \\ \Delta_1 &= a_{11}\, a_{22} - a_{21}\, a_{12} \end{aligned}\right\} \tag{47}$$

For <u>Vertical</u> <u>Polarization</u> at the "from level" (Z_b), the B.C. give:

For B.C. SET I:

$$(Q/C)_I = ({}_{\parallel}\underline{\overline{X}}^{f}_{\parallel} \cdot a_{22} - 2a_{12})/\Delta_2 \tag{48}$$

$$(G/C)_I = (2a_{11} - {}_{\parallel}\underline{\overline{X}}^{f}_{\parallel} \cdot a_{21})/\Delta_2 \tag{49}$$

For B.C. SET II:

$$(Q/C)_{II} = {}_{\perp}\underline{\overline{X}}^{f}_{\parallel} \cdot \frac{a_{22}}{\Delta_2} \tag{50}$$

$$(G/C)_{II} = -{}_{\perp}\underline{\overline{X}}^{f}_{\parallel} \cdot \frac{a_{21}}{\Delta_2} \tag{51}$$

where:

$$\left.\begin{aligned} a_{11} &= h_1(\zeta_f),\ a_{12} = h_2(\zeta_f) \\ a_{21} &= Ch_1(\zeta_f) + [Kh_1'(\zeta_f) + Lh_1(\zeta_f)]/n_f^2 \\ a_{22} &= Ch_2(\zeta_f) + [Kh_2'(\zeta_f) + Lh_2(\zeta_f)]/n_f^2 \\ \Delta_2 &= a_{11} \cdot a_{22} - a_{21}\, a_{12},\ n_f^2 = 1 + \alpha(Z_b - H) \end{aligned}\right\} \quad (52)$$

STEP II:

The values of E_{11}^i and E_y^i at the "to" level (e.g., at the height Z_t) are determined from the values of the coefficients; A/C, B/C, Q/C, and G/C, for the SET I and SET II boundary conditions.

Here the ζ parameter at the "to" level will be denoted by:

$$\zeta_t = (k/\alpha)^{2/3}\,[C^2 + \alpha\,(Z_t - H)] \quad (53)$$

For <u>Horizontal Polarization</u> at the Z_t "to" level the results are

$$((E_y^i)_t)_I = ((\tfrac{A}{C})_I \cdot a_{21} + (\tfrac{B}{C})_I \cdot a_{22})/2 \quad (54)$$

$$((E_y^i)_t)_{II} = ((\tfrac{A}{C})_{II} \cdot a_{21} + (\tfrac{B}{C})_{II} \cdot a_{22})/2 \quad (55)$$

where

$$\left.\begin{aligned} a_{21} &= Ch_1(\zeta_t) + Kh_1'(\zeta_t) \\ a_{22} &= Ch_2(\zeta_t) + Kh_2'(\zeta_t) \\ K &= i(\alpha/k)^{1/3} \end{aligned}\right\} \quad (56)$$

For <u>Vertical</u> <u>Polarization</u> at the Z_t "to" level the results are:

$$((E^i_{11})_t)_I = ((Q/C)_I \cdot a_{21} + (G/C)_I \cdot a_{22})/2 \tag{57}$$

$$((E^i_{11})_t)_{II} = (Q/C)_{II} \cdot a_{21} + (G/C)_{II} \cdot a_{22})/2 \tag{58}$$

where

$$\left.\begin{aligned} a_{21} &= Ch_1(\zeta_t) + [Kh_1'(\zeta_t) + Lh_1(\zeta_t)]/n_t^2 \\ a_{22} &= Ch_2(\zeta_t) + [Kh_2'(\zeta_t) + Lh_2(\zeta_t)]/n_t^2 \\ L &= i(\tfrac{\alpha}{2k}),\ n_t^2 = 1 + \alpha(Z_t - H) \end{aligned}\right\} \tag{59}$$

STEP III:

Knowing the values of

$$((E^i_{11})_t)_{I,II} \text{ and } ((E^i_y)_t)_{I,II}$$

at the "to" level from sets of the coefficients A/C, B/C, Q/C and G/C the elements of the ionospheric $\underset{\sim}{\overline{X}}$ (i.e., $\underset{\sim}{R}$) can be found at the "to" level.

For <u>Horizontal</u> <u>Polarization</u> at the "to" level the results are:

$$({}_{\parallel}\overline{X}_{\perp})_t = [V_1((E^i_y)_t)_{II} - V_2((E^i_y)_t)_I]/W_1 \tag{60}$$

$$({}_{\perp}\overline{X}_{\perp})_t = [V_2((E^i_{11})_t)_I - V_1((E^i_{11})_t)_{II}]/W_1 \tag{61}$$

where:

$$\left.\begin{aligned} W_1 &= ((E^i_{11})_t)_I \cdot ((E^i_y)_t)_{II} - ((E^i_{11})_t)_{II} \cdot ((E^i_y)_t)_I \\ V_1 &= (\tfrac{A}{C})_I\, h_1(\zeta_t) + (\tfrac{B}{C})_I\, h_2(\zeta_t) \\ V_2 &= (\tfrac{A}{C})_{II}\, h_1(\zeta_t) + (\tfrac{B}{C})_{II}\, h_2(\zeta_t) \end{aligned}\right\} \tag{62}$$

For Vertical Polarization at the "to" level the results are:

$$({}_{\parallel}\underline{\overline{X}}_{\parallel})_t = [V_1((E_y^i)_t)_{II} - V_2((E_y^i)_t)_I]/W_2 \tag{63}$$

$$({}_{\perp}\underline{\overline{X}}_{\parallel})_t = [V_2((E_{\parallel}^i)_t)_I - V_1((E_{\parallel}^i)_t)_{II}]/W_2 \tag{64}$$

where

$$\left.\begin{aligned} W_2 &= ((E_{\parallel}^i)_t)_I \cdot ((E_y^i)_t)_{II} - ((E_{\parallel}^i)_t)_{II} \cdot ((E_y^i)_t)_I \\ V_1 &= (Q/C)_I\, h_1(\zeta_t) + (G/C)_I\, h_2(\zeta_t) \\ V_2 &= (Q/C)_{II}\, h_1(\zeta_t) + (G/C)_{II}\, h_2(\zeta_t) \end{aligned}\right\} \tag{65}$$

Thus having the values of the ionospheric reflection coefficients $\underline{\overline{X}}(\theta)$ at Z_b the resulting values of these terms at the height Z_t=d are obtained by applying the minimizing condition, e.g., so that the function $F(\theta)$ may be computed in a form where poles are absent.

The values of the terms ${}_{\parallel}R_{\parallel}$, ${}_{\perp}R_{\perp}$, ${}_{\parallel}R_{\perp}$ and ${}_{\perp}R_{\parallel}$ at height Z may be obtained from the results of the above equation in terms of $\underline{\overline{X}}(\theta)$ by the relations of Equation (41).

G. <u>The Ground Reflection Coefficient Matrices</u>

The ground reflection coefficients (e.g., ${}_{\parallel}\overline{R}_{\parallel}$ and ${}_{\perp}\overline{R}_{\perp}$) are needed in the computation of the $F(\theta)$ equation. The particular form of these coefficients, as used in the MODESRCH procedure, is identified by the ratios given in Appendix A by the equations A-9 and A-10:

$$\frac{{}_{\parallel}n_{\parallel}}{{}_{\parallel}d_{\parallel}} = \frac{{}_{\parallel}\bar{R}_{\parallel} + 1}{{}_{\parallel}\bar{R}_{\parallel}C} \tag{66}$$

$$\frac{{}_{\perp}n_{\perp}}{{}_{\perp}d_{\perp}} = \frac{{}_{\perp}\bar{R}_{\perp} + 1}{{}_{\perp}\bar{R}_{\perp}C} \tag{67}$$

The equations needed to determine the values of ${}_{\parallel}\bar{R}_{\parallel}$, ${}_{\perp}\bar{R}_{\perp}$ (in terms of ${}_{\parallel}n_{\parallel}$, ${}_{\perp}n_{\perp}$, ${}_{\parallel}d_{\parallel}$, and ${}_{\perp}d_{\perp}$) at a reference height "d" may be obtained from equations of Section F.

Needed substitutions to those equations are:

$$Z = 0 \text{ for } Z = Z_b$$

$$({}_{\perp}\underline{\bar{X}}_{\parallel})^f = ({}_{\parallel}\underline{\bar{X}}_{\perp})^f = 0 \tag{68}$$

$$\frac{{}_{\parallel}\overline{R^o}_{\parallel} + 1}{C\,{}_{\parallel}\overline{R^o}_{\parallel}} = \left(\frac{{}_{\parallel}n_{\parallel}}{{}_{\parallel}d_{\parallel}}\right)\Bigg|_{Z=0} \quad \text{for } ({}_{\parallel}\underline{\bar{X}}_{\parallel})_b \tag{69}$$

$$\frac{{}_{\perp}\overline{R^o}_{\perp} + 1}{C\,{}_{\perp}\overline{R^o}_{\perp}} = \left(\frac{{}_{\perp}n_{\perp}}{{}_{\perp}d_{\perp}}\right)\Bigg|_{Z=0} \quad \text{for } ({}_{\perp}\underline{\bar{X}}_{\perp})_b \tag{70}$$

Where the Fresnel reflection coefficients for the ground-free space interface are

$${}_{\parallel}\overline{R^o}_{\parallel} = \frac{C\,n_g^2 - \sqrt{n_g^2 - S^2}}{C\,n_g^2 + \sqrt{n_g^2 - S^2}} \tag{71}$$

and

$${}_{\perp}\overline{R^o}_{\perp} = \frac{C - \sqrt{n_g^2 - S^2}}{C + \sqrt{n_g^2 - S^2}} \tag{72}$$

with

$$n_g^2 = (\varepsilon - j\,\sigma/\omega)/\varepsilon_o \tag{73}$$

and

ε = dielectric constant of the ground.

σ = conductivity of the ground.

ω = angular wave frequency.

C = $\cos\theta$, $S = \sin\theta$.

Well behaved forms of $\left(\frac{{}_{\parallel}n_{\parallel}}{{}_{\parallel}d_{\parallel}}\right)\Big|_{z=0}$ and $\left(\frac{{}_{\perp}n_{\perp}}{{}_{\perp}d_{\perp}}\right)\Big|_{z=0}$ at the "from" level (e.g. Z = 0), are derived in appendix C, and are given as:

$$\left.\begin{aligned} {}_{\parallel}n_{\parallel} &= 1 \quad , \quad {}_{\parallel}d_{\parallel} = \frac{1}{2}\left[C - \left(\frac{\sqrt{n_g^2 - S^2}}{n_g^2}\right)\right], \\ {}_{\perp}n_{\perp} &= \frac{1}{\sqrt{n_g^2 - S^2}} \quad , \quad {}_{\perp}d_{\perp} = \frac{1}{2}\left[\left(C/\sqrt{n_g^2 - S^2}\right) - 1\right] \end{aligned}\right\} \quad (74)$$

At the "from" level (e.g., z=0), Equations (45) and (46) become:

<u>For Horizontal Polarization</u>:

$$(A/C)_{II} = ({}_{\perp}n_{\perp} \cdot a_{22} - 2\,{}_{\perp}d_{\perp}\, a_{12})/(\Delta_1 \cdot {}_{\perp}d_{\perp}) \quad (75)$$

$$(B/C)_{II} = (2 \cdot {}_{\perp}d_{\perp} \cdot a_{11} - {}_{\perp}n_{\perp} \cdot a_{21})/(\Delta_1 \cdot {}_{\perp}d_{\perp}) \quad (76)$$

where:

$$\left.\begin{aligned} a_{11} &= h_1(\zeta_0),\ a_{12} = h_2(\zeta_0) \\ a_{21} &= Ch_1(\zeta_0) + Kh_1'(\zeta_0) \ ;\ \zeta_0 = (k/\alpha)^{2/3}\,[C^2 - \alpha H] \\ a_{22} &= Ch_2(\zeta_0) + Kh_2'(\zeta_0) \ ;\ \alpha = 2/r_e \\ \Delta_1 &= a_{11} \cdot a_{22} - a_{21} \cdot a_{12} \end{aligned}\right\} \quad (77)$$

For Vertical Polarization:

Equations (48) and (49) become:

$$(Q/C)_I = ({}_{\|}n_{\|} \cdot a_{22} - 2 \cdot {}_{\|}d_{\|} \cdot a_{12})/(\Delta 2 \cdot {}_{\|}d_{\|}) \tag{78}$$

$$(G/C)_I = (2 \cdot {}_{\|}d_{\|} \cdot a_{11} - {}_{\|}n_{\|} \cdot a_{21})/(\Delta 2 \cdot {}_{\|}d_{\|}) \tag{79}$$

where:

$$\left.\begin{aligned} & a_{11} = h_1(\zeta_o),\ a_{12} = h_2(\zeta_o) \\ & a_{21} = Ch_1(\zeta_o) + ((Kh_1'(\zeta_o) + Lh_1(\zeta_o))/n_o^2 \\ & a_{22} = Ch_2(\zeta_o) + ((Kh_2'(\zeta_o) + Lh_2(\zeta_o))/n_0^2 \\ & \Delta 2 = a_{11}\ a_{22} - a_{21}\ a_{12},\ n_0^2 = 1 - \alpha H \end{aligned}\right\} \tag{80}$$

At the "to" level (e.g., z=d), Equation (55) becomes, for Horizontal Polarization:

$$((E_y^i)_d)_{II} = ((A/C)_{II} \cdot a_{21} + (B/C)_{II} \cdot a_{22})/2 \tag{81}$$

where

$$\left.\begin{aligned} & a_{21} = Ch_1(\zeta_d) + Kh_1'(\zeta_d) \\ & \qquad\qquad \zeta_d = (k/\alpha)^{2/3}\left[C^2 + \alpha(d - H)\right] \\ & a_{22} = Ch_2(\zeta_d) + Kh_2'(\zeta_d) \end{aligned}\right\} \tag{82}$$

and Equation (57) becomes, for Vertical Polarization:

$$((E_{11}^i)_d)_I = (Q/C)_I\ a_{21} + (G/C)_I\ a_{22})/2 \tag{83}$$

where

$$\left.\begin{aligned} & a_{21} = Ch_1(\zeta_d) + \left[Kh_1'(\zeta_d) + Lh_1(\zeta_d)\right]/n_d^2 \\ & a_{22} = Ch_2(\zeta_d) + \left[Kh_2'(\zeta_d) + Lh_2(\zeta_d)\right]/n_d^2 \\ & n_d^2 = 1 + \alpha\ (d - H) \end{aligned}\right\} \tag{84}$$

At the "to" level (e.g. Z = d) define:

$$\left.\frac{{}_{\parallel}n_{\parallel}}{{}_{\parallel}d_{\parallel}}\right|_{z=d} = \frac{{}_{\parallel}N_{\parallel}}{{}_{\parallel}D_{\parallel}} \tag{85}$$

and

$$\left.\frac{{}_{\perp}n_{\perp}}{{}_{\perp}d_{\perp}}\right|_{z=d} = \frac{{}_{\perp}N_{\perp}}{{}_{\perp}D_{\perp}} \tag{86}$$

Equations (61) and (62) for <u>Horizontal Polarization</u> give:

$${}_{\perp}N_{\perp} = (A/C)_{II}\ h_1(\zeta_d) + (B/C)_{II}\ h_2(\zeta_d) \tag{87}$$

$${}_{\perp}D_{\perp} = ((E_y^i)_d)_{II} \tag{88}$$

and Equations (64) and (65) for <u>Vertical Polarization</u> give:

$${}_{\parallel}N_{\parallel} = (Q/C)_I\ h_1(\zeta_d) + (G/C)_I\ h_2(\zeta_d) \tag{89}$$

$${}_{\parallel}D_{\parallel} = ((E_{11}^i)_d)_I \tag{90}$$

The results of Equations (87) - (90) can be rearranged by reference to Equation (10)

where

$$\frac{{}_{\parallel}N_{\parallel}}{{}_{\parallel}D_{\parallel}} = \frac{\left(\frac{1}{{}_{\parallel}\bar{R}_{\parallel}} + 1\right)}{C}$$

and

$$\frac{{}_{\perp}N_{\perp}}{{}_{\perp}D_{\perp}} = \frac{\left(\frac{1}{{}_{\perp}\bar{R}_{\perp}} + 1\right)}{C}$$

to give explicit expressions for $({}_{\parallel}\bar{R}_{\parallel})_d$ and $({}_{\perp}\bar{R}_{\perp})_d$ as

$$({}_{\parallel}\bar{R}_{\parallel})_d = \frac{1}{C(\frac{{}_{\parallel}N_{\parallel}}{{}_{\parallel}D_{\parallel}}) - 1} = \frac{{}_{\parallel}D_{\parallel}}{C\,{}_{\parallel}N_{\parallel} - {}_{\parallel}D_{\parallel}} \tag{91}$$

and

$$({}_{\perp}\bar{R}_{\perp})_d = \frac{1}{C(\frac{{}_{\perp}N_{\perp}}{{}_{\perp}D_{\perp}}) - 1} = \frac{{}_{\perp}D_{\perp}}{C\,{}_{\perp}N_{\perp} - {}_{\perp}D_{\perp}} \tag{92}$$

Inspection of the equations will reveal that ${}_{\parallel}N_{\parallel}$, ${}_{\parallel}D_{\parallel}$, ${}_{\perp}N_{\perp}$, and ${}_{\perp}D_{\perp}$ contain no poles for any finite value of θ.

The resulting equations of this section are equivalent to those given in reference 9.

H. Excitation Factors, Height Gains and Auxiliary Functions

(1) Excitation Factors

The modal excitation factor and the modal height gain functions are two parameters needed in computing electric field strengths. The excitation factor formulas, as presented in reference 10, are summarized in the table 1. The column headings only apply to excitation of the electric field components E_z, E_y and E_x and the row headings apply to excitation by a vertical dipole (λ_V), horizontal dipole end on (λ_E) and a horizontal dipole broadside (λ_B). The direction of z is taken positive into the ionosphere. Positive x is the direction of propagation and y is normal to the plane of propagation.

Table 1 - EXCITATION FACTORS

Field Component → / Exciter ↓	E_z	E_y	E_x
λ_V	$B_1 \dfrac{(1+{}_{\parallel}\bar{R}_{\parallel})^2(1-{}_{\perp}\bar{R}_{\perp}{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\, D_{11}}$	$\dfrac{-B_1}{S}\, \dfrac{{}_{\parallel}R_{\perp}(1+{}_{\parallel}\bar{R}_{\parallel})(1+{}_{\perp}\bar{R}_{\perp})}{D_{12}}$	$\dfrac{B_1}{S}\, \dfrac{(1+{}_{\parallel}\bar{R}_{\parallel})^2(1-{}_{\perp}\bar{R}_{\perp}{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\, D_{11}}$
λ_E	$B_2 \dfrac{(1+{}_{\parallel}\bar{R}_{\parallel})^2(1-{}_{\perp}\bar{R}_{\perp}{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\, D_{11}}$	$-\dfrac{B_2}{S}\, \dfrac{{}_{\parallel}R_{\perp}(1+{}_{\parallel}\bar{R}_{\parallel})(1+{}_{\perp}\bar{R}_{\perp})}{D_{12}}$	$\dfrac{B_2}{S}\, \dfrac{(1+{}_{\parallel}\bar{R}_{\parallel})^2(1-{}_{\perp}\bar{R}_{\perp}{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\, D_{11}}$
λ_B	$B_2\, \dfrac{{}_{\perp}R_{\parallel}(1+{}_{\perp}\bar{R}_{\perp})(1+{}_{\parallel}\bar{R}_{\parallel})}{D_{12}}$	$-\dfrac{B_2}{S}\, \dfrac{(1+{}_{\perp}\bar{R}_{\perp})^2(1-{}_{\parallel}\bar{R}_{\parallel}{}_{\parallel}R_{\parallel})}{{}_{\perp}\bar{R}_{\perp}\, D_{22}}$	$\dfrac{B_2}{S}\, \dfrac{{}_{\perp}R_{\parallel}(1+{}_{\perp}\bar{R}_{\perp})(1+{}_{\parallel}\bar{R}_{\parallel})}{D_{12}}$

The R and $\bar{R}$'s are the elements of the reflection matrix looking into the ionosphere and towards the ground from the reference level "d" within the guide.

The reference height "d" is that value where solutions to $F(\theta) = 0$ are determined.

B_1 and B_2 are given by:

$$B_1 = S^{5/2} \Big/ \left.\frac{\partial F}{\partial \theta}\right|_{\theta=\theta_n} \tag{93}$$

$$B_2 = -B_1/S \tag{94}$$

where S is the sine of the eigenangle and the denominator is the derivative of the modal equation (i.e. eq. 7) at an eigenangle, θ.

The definitions of the D_{ij}'s are given in the next section.

To compute electric field strengths the excitation factors must be supplemented with definitions of the height gains.

(2) Height Gain Terms

The height gain relationships at height Z are described below.

From Equation (39):

$$\zeta = (k/\alpha)^{2/3} [c^2 + \alpha(Z - H)] \tag{95}$$

At any height Z.

The height gain for the vertical electric field (E_z) is $f_{\parallel}$ (z) and is given by:

$$f_{\parallel} (Z) = [(Q/C)_I \, h_1(\zeta) + (G/C) \, h_2(\zeta)] \exp[(Z - d)1/r_e] \tag{96}$$

where d is the reference height for solving $F(\theta) = 0$ for the eigenangle and where $(Q/C)_I$ and $(G/C)_I$ are given by Equations (48) and (49) respectively. The term "r_e" is the radius of the earth.

Also, the height gain for the horizontal electric field component (E_y) normal to the plane of propagation at height Z is given by:

$$f_{\perp} (Z) = [(A/C)_{II} \, h_1(\zeta) + (B/C)_{II} \, h_2(\zeta)] \tag{97}$$

where $(A/C)_{II}$ and $(B/C)_{II}$ are given by Equations (45) and (46) respectively.

The height gain for the horizontal electric field components (E_x) in the plane of propagation at height Z is given by:

$$g(Z) = \frac{1}{ik} \frac{d}{dZ} (f_{\parallel}(Z)) \text{ or}$$

$$g(Z) = -i \exp\left(\frac{Z - d}{a}\right) \left\{ \left(\frac{2}{ak}\right)^{1/3} [(Q/C)_I \, h_1'(\zeta) + (G/C)_I \, h_2'(\zeta)] + \left(\frac{2}{ak}\right) [(Q/C)_I \, h_1(\zeta) + (G/C)_I \, h_2(\zeta)] \right\} \tag{98}$$

At height Z = d (i.e. the reference height) the argument is:

$$\zeta_d = (k/\alpha)^{2/3} \, [c^2 + \alpha(d - H)] \tag{99}$$

and

$$D_{11} = f_{\parallel}^2(\zeta_d), \; D_{12} = f_{\parallel}(\zeta_d) \, f_{\perp}(\zeta_d) \tag{100}$$

$$D_{22} = f_{\perp}^2(\zeta_d)$$

(3) Modified Excitation Terms

Various combinations of the modal excitation factor and the height gain function are required for computing field strengths by different mode-sum programs. Reference 11 is an example.

Table 2 illustrates the case where the excitation factors as described in Table 1 and the height gains from Equations (96), (97) and (98) are lumped together and defined as modified excitation factors. Table 2 illustrates combinations obtained for computing the vertical electric field (E_z). These particular combinations will be discussed further in Section V-C-2 for the option NPUNCH = 7.

Table 2

Excitation Factors for Computing the

Vertical Electrical Field (E_z)

Vertical source:

$$X_{VZ} = B_{\perp} \frac{(1 + {}_{\parallel}\bar{R}_{\parallel})^2 \, (1 - {}_{\perp}\bar{R}_{\perp} \, {}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel} \cdot D_{11}} \cdot f_{\parallel} \, (Z_t) \cdot f_{\parallel} \, (Z_r)$$

Horizontal broadside source:

$$X_{BZ} = \frac{{}_{\perp}R_{\parallel}(1 + {}_{\perp}\bar{R}_{\perp})(1 + {}_{\parallel}\bar{R}_{\parallel}) \cdot f_{\perp}(Z_t) \cdot f_{\parallel}(Z_r)}{D_{12}}$$

Horizontal end-on source:

$$X_{EZ} = \frac{(1 + {}_{\parallel}\bar{R}_{\parallel})^2}{{}_{\parallel}\bar{R}_{\parallel} \cdot D_{11}} (1 - {}_{\perp}\bar{R}_{\perp} \, {}_{\perp}R_{\perp}) \cdot g(Z_t) \cdot f_{\parallel} \, (Z_r)$$

Z_t is height of the transmitter and

Z_r is the height of the receiver

The excitation factor usually considered is the vertical component of the electric field vector for vertical receiving and transmitting point dipoles, where both receiver and transmitter are at the ground (e.g., z = 0). The excitation factor X_{vz} is denoted in this case by X_0. Wait's excitation factor is computed as $\Lambda_0 = -.5ikh_rX_v$ where k is the wave number in km^{-1}, h_r is REFLHT and $i = \sqrt{-1}$.

(4) Fast Mode Conversion

Reference 12 presents equations and a computer program for calculating the vertical electric field strength at a receiver by the procedure of mode conversion. An input parameter required for these calculations

is denoted as 'FOVR'. The mode conversion program requires that the reference height used for computing ionospheric and ground reflection coefficients be at the ground (i.e., z=0).

The reference height used in the MODESRCH computer program is at some height "d" ($d \neq 0$). Therefore the equation needed to obtain the required 'FOVR' term is given by"

$$\text{FOVR} = \left\{ \frac{(1 + {}_{\perp}\bar{R}_{\perp})(1 - {}_{\parallel}R_{\parallel}\ {}_{\parallel}\bar{R}_{\parallel})}{(1 + {}_{\parallel}\bar{R}_{\parallel})({}_{\perp}R_{\parallel}\ {}_{\perp}\bar{R}_{\perp})} \right\}\Bigg|_{z=d} \cdot \frac{f_{\parallel}(d)}{f_{\perp}(d)}$$

$$\text{or} \quad \left\{ \frac{(1 + {}_{\perp}\bar{R}_{\perp})({}_{\parallel}R_{\perp}\ {}_{\parallel}\bar{R}_{\parallel})}{(1 + {}_{\parallel}\bar{R}_{\parallel})(1 - {}_{\perp}R_{\perp}\ {}_{\perp}\bar{R}_{\perp})} \right\}\Bigg|_{z=d} \frac{f_{\parallel}(d)}{f_{\perp}(d)} \tag{101}$$

I. Use of Lagrange Interpolation

The general root-finding algorithm, (section III), used to locate the zeros of a function, $f(\theta)$, requires that values of $f(\theta)$ and its derivatives wrt θ be available upon call for any value of θ in the region of interest. In Section F a criterion was given for choosing a reference height, 'd', so as to minimize the variation of $\bar{\underset{\sim}{X}} = (\underset{\sim}{R} + 1)/C$ wrt θ thereby placing the dominant waveguide effects into $\underset{\sim}{n}$ and $\underset{\sim}{d}$ (i.e., into $\bar{\underset{\sim}{R}}$). (See Equation 7.) On the other hand, far more computation is required to evaluate $\bar{\underset{\sim}{X}}$ than to evaluate $\underset{\sim}{n}$ and $\underset{\sim}{d}$. This suggests the utilization of an approximate representation of $\bar{\underset{\sim}{X}}$ to be used within the general root-finder, and then following the zeros of $f(\theta)$ (see Equation 7) to their locations which will be consistent with the exact formulation for $\bar{\underset{\sim}{X}}$.

The general root-finder requires that the function $f(\theta)$ be analytic with no poles and therefore an approximate representation of $\bar{\underset{\sim}{X}}$ must be analytic with no poles. A polynomial representation has been chosen, which can be represented by the Lagrange interpolation formula

$$\underset{\approx}{\overline{X}}(\theta)_{\text{approx.}} = \sum_{i=1}^{n} \left\{ \prod_{\substack{k=1 \\ k\neq i}}^{n} (\theta-\theta_k) \right\} P_i \quad \text{and where } P_i = \frac{\underset{\approx}{\overline{X}}(\theta_i)}{\prod_{\substack{k=1 \\ k\neq i}}^{n} (X_i - X_k)} \tag{102}$$

where $\underset{\approx}{\overline{X}}(\theta_i)$ are evaluations of $\underset{\approx}{\overline{X}}$ using the exact formulation for $\underset{\approx}{\overline{X}}$ at n selected angles, θ_i, distributed over the θ-plane in the region of interest.

Since $F(\theta)$ (see Equation 7) in both approximate and exact (complete) forms is analytic with no poles, each zero of $F(\theta)_{\text{approx.}}$ may be followed uniquely to a corresponding zero of $F(\theta)_{\text{exact}}$. The continuous transition from one to the other may be represented, for example, by

$$F(\theta)_{\text{comb.}} = F(\theta)_{\text{approx.}} \cdot \cos^2\beta + F(\theta)_{\text{exact}} \cdot \sin^2\beta \tag{103}$$

with β varying continuously from 0 to 90°. In practice β is incremented in finite steps with a Newton-Raphson iteration at each step. How far the zero of $F(\theta)$ "travels" in the θ-plane as β is varied from 0 to 90° depends; however, on how well $F(\theta)$ (i.e., $\underset{\approx}{\overline{X}}(\theta)$) is represented by the Lagrange interpolation. It is possible, in principle, that a zero may travel from the region of interest to infinity, or vice-versa. There are no guarantees and although, in most cases in the VLF and LF range, the use of Lagrange interpolation may save on computation by an order of magnitude, it nevertheless represents the "weakest link" in formulating a self-contained mode search. In some cases, especially in the ELF range, it is necessary to use the (exact) formulation for F.

J. Comments of the Differentiation of $F(\theta)$ wrt θ

The excitation factor components given in Section H require the computation of the derivative $F(\theta)$ wrt θ for each eigen-value of θ. In

addition, the general root-finder, (section III), requires, in its later stages, some evaluations of $dF/d\theta$, and the following of zeros by the use of Newton-Raphson iteration described in Section I requires computation of $dF/d\theta$. Traditionally this derivative has been evaluated by computing F at two closely-spaced values of θ, using a large number of significant digits in the computation. An alternative is to analytically differentiate F, which implies analytic differentiation of $\underset{\sim}{X}$, $\underset{\sim}{n}$ and $\underset{\sim}{d}$. This alternative was chosen for the mode search program.

Note that differentiation of the numerical (Runge-Kutta) integration described in Section E is performed by the same type numerical integration, with the integrand analytically differentiated wrt θ.

In certain sections of MODESRCH where $F(\theta)$ is computed in terms of Lagrange interpolation, the formulation of the derivative wrt θ is given by

$$\frac{dF(\theta)}{d\theta} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left\{ \prod_{\substack{k=1 \\ k\neq i \\ k\neq j}}^{n} (F(\theta)-F_k(\theta)) \right\} \cdot \left\{ P_i + P_j \right\} \tag{104}$$

where

$$P_\ell = \frac{F(\theta_\ell)}{\prod_{\substack{k=1 \\ k\neq \ell}}^{n} (X_\ell - X_k)} .$$

and n is the number of points.

III. THE GENERAL ROOT-FINDER

There is a need to formulate a procedure to determine the complex zeros of a complex function. For the method presented here, the search for such zeros is based on the fact that the lines of constant phase of any complex function, $F(\theta)$, may be discontinuous only at points where $F(\theta) = 0$ or $F(\theta) \to \infty$. The function $F(\theta)$, to be discussed, is such that it contains no poles and only $F(\theta) = 0$ is considered. Also, the lines of constant phase around $F(\theta) = 0$ progress only in a counter-clockwise direction.

Let:

$$F(\theta) = F_R(\theta_r,\theta_i) + j\, F_I(\theta_r,\theta_i) \tag{106}$$

where

$$\theta = \theta_r + j\theta_i$$

Also

$$F(\theta) = [F_R(\theta_r,\theta_i)^2 + F_F(\theta_r,\theta_i)^2]^{1/2} e^{j\phi} \tag{107}$$

where

$$\phi = \tan^{-1}\left(\frac{F_I(\theta_r,\theta_i)}{F_R(\theta_r,\theta_i)}\right) \tag{108}$$

and

$F_R(\theta)$ is the real part of the complex function $F(\theta)$

$F_I(\theta)$ is the imaginary part of the complex function $F(\theta)$

θ_r is the real part of the complex number (θ)

θ_i is the imaginary part of the complex number (θ).

If $F(\theta)$ has no poles, this implies that the line of any particular constant phase value $\phi = \phi_c$, radiating from a zero of $F(\theta)$, must cross a closed contour containing that zero at least once. Furthermore, no other zero of $F(\theta)$ may be on this phase line. These relations are illustrated in Figure 2.

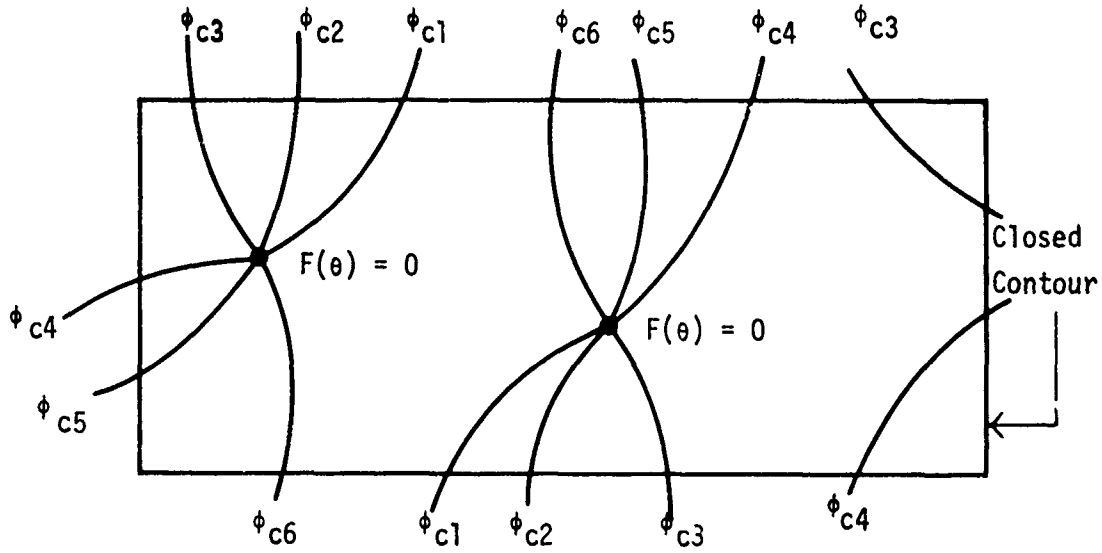


Figure 2. Constant Phase Lines of $F(\theta)$.

A line of constant phase (e.g., $\phi = \phi_c$) which crosses the contour may be followed inward until it leads to a zero of $F(\theta)$ or until the line again reaches the contour.

The procedure is as follows.

(A) Let $\phi_c = 0°$ (or 180°), this implies that

$$F_I(\theta_r, \theta_i) = 0 \tag{109}$$

Also let $\phi_c = 90°$ (or 270°), this implies that

$$F_R(\theta_r, \theta_i) = 0 \tag{110}$$

(B) Construct a "contour rectangle" in the complex θ plane. Denote the corners of this rectangle by:

TLEFT - value of real part of Theta at left edge of rectangle.

TRIGHT - value of real part of Theta at right edge of rectangle.

TBOT - value of imag. part of Theta at bottom edge of rectangl

TTOP - value of imag. part of Theta at top edge of rectangle.

Define the following parameters:

TMESH - equal to about half the average spacing between zeros within the rectangle.

TOL - tolerance to which zeros are to be found.

If two zeros are closer than 'TOL', the zeros will not be found by this method.

(C) Divide the 'contour rectangle' up into small mesh squares where the dimensions of the squares is such that the length of a side is smaller than the average separation of the zeros of $F(\theta)$ (e.g., size of a mesh side = 'TMESH').

(D) Normalize the value of the corners of the "contour rectangle" into mesh units. The corners are then denoted as JLT, JRT, JTOP, and JBOT. That is θ's for the "contour rectangle" are:

θ_1 at left edge: JLT = TLEFT/TMESH

θ_2 at right edge: JRT = TRIGHT/TMESH

θ_3 at top edge: JTOP = TTOP/TMESH

θ_4 at bottom edge: JBOT = TBOT/TMESH.

(E) Figure 3 illustrates the rectangular region of the complex θ-plane within which the roots of $F(\theta) = 0$ are to be found.

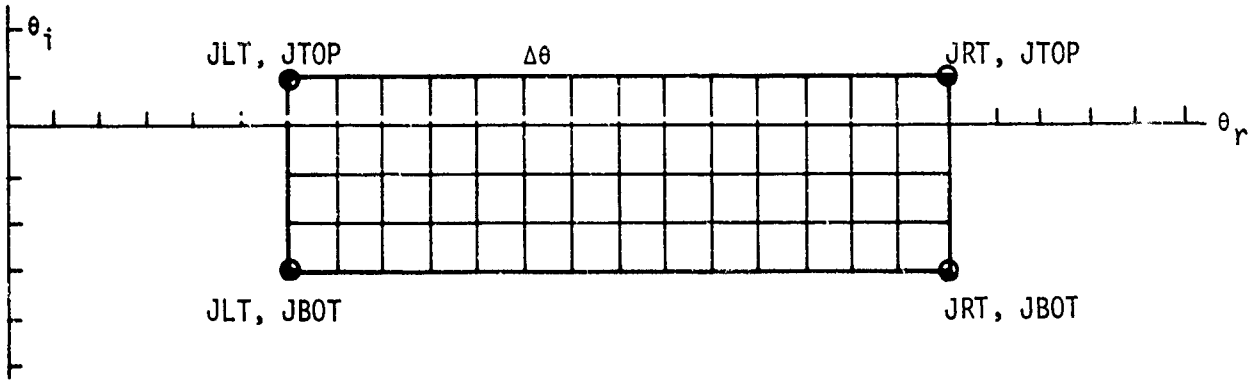


Figure 3. "Contour Rectangle" with "MESH" Squares.

In the above example all the axes are marked with tic marks, with spacing $\Delta\theta$ = TMESH. The mesh size within the rectangle is equal to TMESH.

In the above figure, JLT = 6, JRT = 20, JTOP = 1, JBOT =-3.

Since some "slop" occurs at the rectangle boundary, the limits of the rectangle are made one TMESH larger on all sides than actually desired.

TMESH should be no larger than about one-half the average spacing between roots.

(F) The procedure then is to examine the character of $F(\theta)$ in a counter clockwise direction around the contour rectangle (see Figure 4). Intermediate steps of investigating mesh squares are also carried out in a counterclockwise direction. Again see Figure 4.

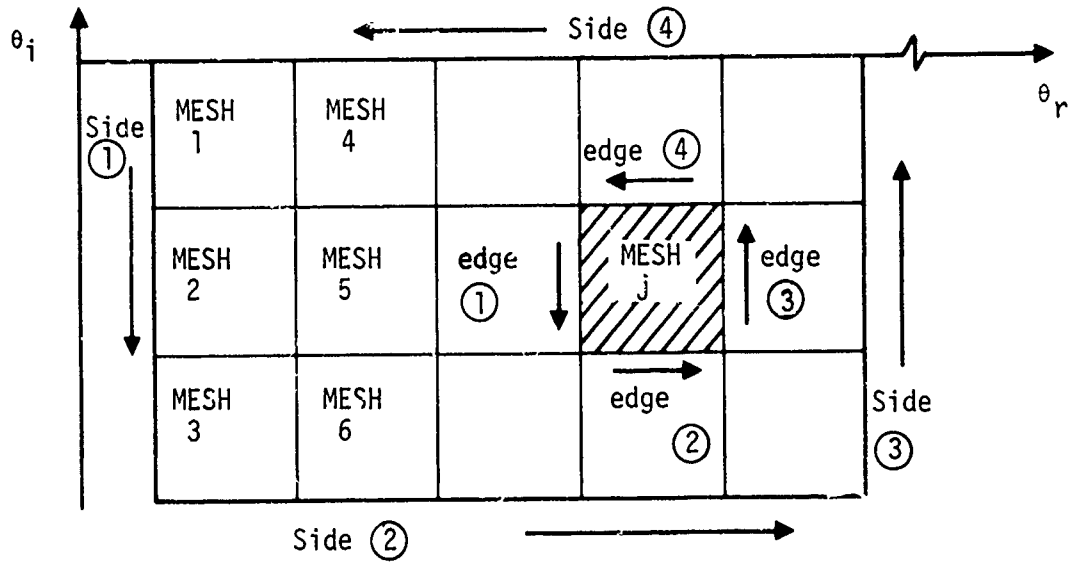


Figure 4. Directions for Determining Zeros of $F(\theta)$.

(G) Consider an individual mesh square (e.g., mesh no. j). The corners of the square may be labeled in terms of $F(\theta_r,\theta_i)$ where each edge is one-mesh unit in length. Figure 5 illustrates normalized coordinates of the mesh.

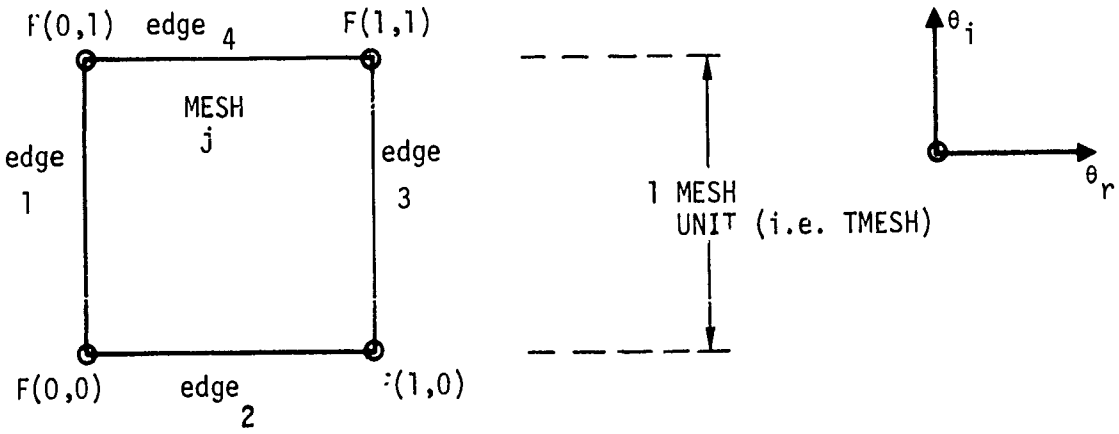


Figure 5. Normalized Coordinates of Each Mesh Square for $F(\theta_r,\theta_i)$.

Note that for each unit mesh square the lower left hand corner is taken as the reference position.

A basic assumption put on $F(\theta)$ is that $F_I(\theta)$ and $F_R(\theta)$ are to be linear along each edge of a mesh square.

The coordinates of the corners of mesh square No. 1 (see Figure 4) are given by:

$$\begin{aligned} F(0,1) &= F(JLT,JTOP) \\ F(0,0) &= F(JLT,JTOP-1) \\ F(1,0) &= F(JLT+1,JTOP-1) \\ F(1,1) &= F(JLT+1,JTOP \end{aligned} \tag{111}$$

All other squares are identified by increasing or decreasing the θ_r and/or θ_i coordinates by increments of 1-mesh unit.

(H) The root-finder procedure continues as follows. First, mesh square No. 1 is examined. The values of $F_I(0,1)$ and $F_I(0,0)$ for mesh edge No. 1 are computed. If the signs of these two values are identical, there can be no line with constant value of $F_I(\theta) = 0$ passing through this edge. Next, investigate mesh square No. 2 (see Figure 4). Compute the values of $F_I(o,1)$ and $F_I(0,0)$ for this mesh square. The value of $F_I(0,1)$ for mesh square No. 2 is identical to that of $F_I(0,0)$ for mesh square No. 1, however, the value of $F_I(0,0)$ for mesh square No. 2 will be a new value. The coordinates of mesh square No. 2 are identified by:

$$\begin{aligned} F(0,1) &= F(JLT,JTOP-1) \\ F(0,0) &= F(JLT,JTOP-2) \\ F(1,0) &= F(JLT+1,JTOP-2) \\ F(1,1) &= F(JLT+1,JTOP-1) \end{aligned} \tag{112}$$

If the signs of $F_I(0,1)$ and $F_I(0,0)$ for mesh square No. 2 are opposite, this implies that the line $F_I(\theta) = 0$ enters mesh square No. 2 somewhere along edge No. 1. Because of the linearity assumption of $F_I(\theta)$ along

the edge of a mesh square, the intersection between the edge and the constant phase line $F_I(\theta)=0$ is (as shown in Appendix D, eq. 13) to be at:

$$\theta_r = 0, \theta_i = \left(\frac{-F_I(0,0)}{F_I(0,1)-F_I(0,0)}\right) \tag{113}$$

θ_r is identified as ENTER-R

θ_i is identified as ENTER-I

(I) It has now been determined that the constant line $F_I(\theta) = 0$ has entered mesh square No. 2. The problem is to find where this line exits from the square. Also it must be determined whether a zero to $F(\theta)$ lies within the mesh square. Several tests must be made:

1) Computations are made to obtain the values of $F_I(1,0)$ and $F_I(1,1)$ for mesh square No. 2. At this point values are known for $F_I(\theta)$ at all four corners of mesh square No. 2.

2) Test for there being two (hyperbolic) lines entering and leaving the mesh square along each line of which $F_I(\theta) = 0$.

   (a) If only one line, the parameter LTWO is set to zero.

   (b) If the values of $F_I(0,0)$ and $F_I(1,1)$ are each greater than zero; and the values of $F_I(1,0)$ and $F_I(0,1)$ are each less than zero then there are two hyperbolic lines of $F_I(\theta) = 0$ entering and leaving the mesh square. This condition is illustrated in Figure 6.

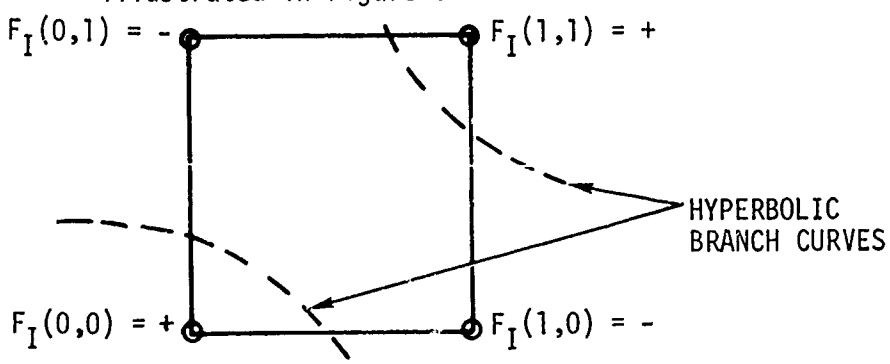


Figure 6. Hyperbolic Branch Curves of $F_I(\theta)$ Within a Mesh Square.

For this case, the parameter "LTWO" is set equal to 1.

(c) Also, if the values of $F_I(0,0)$ and $F_I(1,1)$ are each less than zero, and the values of $F_I(1,0)$ and $F_I(0,1)$ are each greater than zero, the result is identical to (b) above.

3) Test for there being at least one hyperbolic branch line entering and leaving the mesh square along which $F_R(\theta) = 0$.

(a) If the values of $F_R(0,0)$, $F_R(0,1)$, $F_R(1,0)$ and $F_R(1,1)$ are all of the same sign, then there is no line of $F_R(\theta) = 0$ passing through the mesh square and the variable "I90" is set to zero.

(b) If the condition of (a) above, is not met, then there is at least one branch line entering and leaving the mesh and the variable 'I90' is set equal to 1.

4) Test to see if there exists the conditions for the presence of hyperbolic lines at both $F_I(\theta) = 0$ and $F_R(\theta) = 0$ within the mesh square.

(a) If the condition is not present (e.g., "LTWO" is equal to 0 and "I90" is equal to 0) then determine which mesh edge (e.g., 2,3, or 4) where the original entry line exits and proceed with an analysis of the appropriate mesh square. For example, as can be observed from Figure 4, edge No. 2 leads to mesh square No. 3, edge No. 3 leads to mesh square No. 5 and edge No. 4 leads to mesh square No. 1.

The line $F_I(\theta) = 0$ is followed continuously from mesh to mesh until the line exits from the "contour rectangle". When this occurs, the value of the leading edge (counter clockwise direction) of the mesh square,

through which the $F_I(\theta) = 0$ line exits, is noted. By identifying the mesh square, in which this line exit occurs, it is possible to later avoid re-entering this mesh square through this exit edge while following along the outer edge of the "contour rectangle". Figure 7 illustrates this situation. Note that it is possible that a solution to $F(\theta) = 0$ will be found as the line $F_I(\theta) = 0$ progresses through the various mesh squares. This case will be discussed in the following Section (b).

The next step after exit from mesh square No. k is to examine mesh square No. 3 for intersections of the line $F_I(\theta) = 0$ on edge No. 1 of mesh square No. 3.

(b) If hyperbolic lines of both $F_I(\theta) = 0$ and $F_R(\theta) = 0$ do exist within mesh square No. 2, then the following hyperbolic relations are examined.

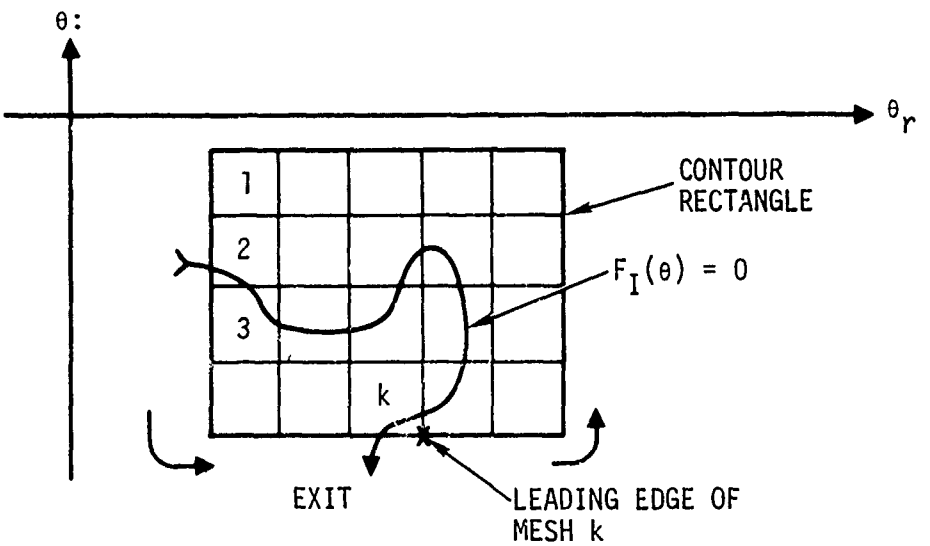


Figure 7. Contour Rectangle, Mesh Squares and the line $F_I(\theta) = 0$.

It is shown in Appendix D that the relationship for $F(\theta)$ based on a knowledge of $F(\theta)$ at each of the corners of a mesh square is given by

$$F(\theta) = a + b\theta_i + c\theta_r + d\theta_r\theta_i \tag{114}$$

where

$$\begin{aligned} a &= F(0,0) \\ b &= F(0,1) - F(0,0) \\ c &= F(1,0) - F(0,0) \\ d &= F(0,0) + F(1,1) - F(0,1) - F(1,0) \end{aligned} \tag{115}$$

These coefficients describe the variation of $F(\theta)$ within the mesh square given the values of $F(\theta)$ at the corners of the square and that a linear variation exists between values of $F(\theta)$ along the edges of the square.

When $F_R(\theta) = 0$ (or $F_I(\theta) = 0$), Equation 114 is the equation of an equilateral hyperbola with vertical and horizontal asymptotes.

Now consider the constant phase line $F_I(\theta) = 0$. From Equation 114 this is

$$(a_i+b_i\theta_i) + (c_i+d_i\theta_i)\theta_r = 0$$

or (116)

$$\theta_r = -\frac{a_i+b_i\theta_i}{c_i+d_i\theta_i}$$

The following equation is also obtained from Equation 114.

$$(a_i+c_i\theta_r) + (b_i+d_I\theta_r)\theta_i = 0$$

or (117)

$$\theta_i = -\frac{a_i+c_i\theta_r}{b_i+d_i\theta_r}$$

The position of the crossings of the hyperbolic asymptotes (which are parallel to the sides of the square) for the lines $F_I(\theta) = 0$ are computed since both lines (hyperbolic branches) may enter and leave the square.

For the function $F_I(\theta) = 0$ the line of the vertical asymptote is given in Appendix D as

$$\theta_r = - b_i/d_i \tag{118}$$

where θ_r is identified as CENTER-R, while the line of the horizontal asymptote is given by

$$\theta_i = - c_i/d_i \tag{119}$$

where θ_i is identified as CENTER-I.

There are two possible points at which there may be crossings of the (hyperbolic) lines $F_R(\theta) = 0$ and $F_I(\theta) = 0$. A crossing point is chosen to be a zero of the function if it lies within the current mesh square and if it lies on the hyperbolic branch of $F_I(\theta) = 0$ currently being followed. Note that in the example being examined the hyperbolic branch line being examined enters mesh square No. 2 along edge No. 1 and exits along edge No. 2.

Both Equation (116) and Equation (117) must be examined since in any given case either form (but not both forms) may be indeterminate.

Equation (116) may be considered as two equations, that is for real and imaginary parts of the constants a, b, c, and d:

$$\theta_r = - \frac{(a_r+b_r\theta_i)}{(c_r+d_r\theta_i)} = - \frac{a_i+b_i\theta_i}{c_i+d_i\theta_i} \tag{120}$$

and, equating the two right hand equations the result is

$$P_1\theta_i^2 + Q_1\theta_i + W_1 = 0 \tag{121}$$

where

$$\begin{aligned} P_1 &= b_r d_i - b_i d_r \\ Q_1 &= a_r d_i + b_r c_i - a_i d_r - b_i c_r \\ W_1 &= a_r c_i - a_i c_r \end{aligned} \tag{122}$$

This equation is solved (see Appendix E) for θ_i. Then, knowing θ_i, θ_r can be determined from Equation (116). In the same manner Equation (117) gives

$$\theta_i = -\frac{a_r+c_r\theta_r}{b_r+d_r\theta_r} = -\frac{a_i+c_i\theta_r}{b_i+d_i\theta_r} \tag{123}$$

and equating the two right hand equations gives

$$P_2\theta_r^2 + Q_2\theta_r + W_2 = 0 \tag{124}$$

where

$$\begin{aligned} P_2 &= c_r d_i - c_i d_r \\ Q_2 &= a_r d_i + c_r b_i - a_i d_r - c_i b_r \\ W_2 &= a_r c_i - a_i c_r \end{aligned} \tag{125}$$

this equation may be solved for θ_r (see Appendix E). Then, knowing θ_r, θ_i can be determined from Equation (117).

If

$$|b_r d_i - b_i d_r| < |c_r d_i - c_i d_r| \tag{126}$$

Equations (116) and (121) are solved for θ_r and θ_i. If the inequality of Equation (126) is not true then Equations (117) and (124) are solved for θ_r and θ_i.

A solution of Equations (116) and (121) (or Equations (117) and (124)) is accepted if it lies within the current mesh square. That is, if the solution is denoted θ_s or $(\theta_{rs} + j\theta_{is})$, then θ_s must satisfy the relationships

$$0 \geq \theta_{rs} \leq 1 \text{ and } 0 \geq \theta_{is} \leq 1 \tag{127}$$

to be a solution.

Also the solution must lie on the hyperbolic branch of $F_I(\theta) = 0$ which is currently being followed. The relationship is that (using Equation (113) and Equations (118) and (119):

$$\text{if } (\text{ENTER R} - \text{CENTER R}) \cdot (\theta_{rs} - \text{CENTER R}) > 0$$

and

$$\text{if } (\text{ENTER I} - \text{CENTER I}) \cdot (\theta_{is} - \text{CENTER I}) > 0 \tag{128}$$

then the solution of $F(\theta)$ is on the current branch line. If Equation (128) is not true then the solution θ_s is not on the current branch; however, the improper solution will be selected later when following another constant phase line of $F_I(\theta) = 0$.

Figure 8 illustrates the case where θ_s is a proper solution to $F(\theta) = 0$. Figure 9 illustrates the case where θ_s is not a proper solution to $F(\theta) = 0$.

If the value θ_s is a solution to $F(\theta) = 0$ the line $F_I(\theta) = 0$ is followed out of mesh square No. 2 through edge No. 2 into mesh square No. 3 through its edge No. 4 (see Figure 4). The line $F_I(\theta) = 0$ is then followed from mesh to mesh until the line $F_I(\theta) = 0$ exits from the contour rectangle (see Figure 10).

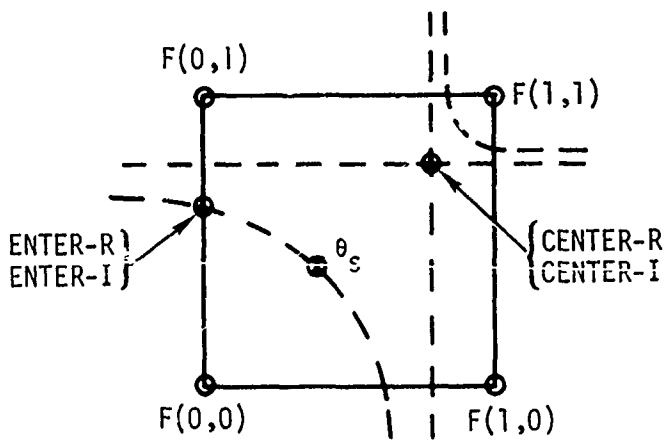


Figure 8. θ_s is a Proper Solution to $F(\theta) = 0$.

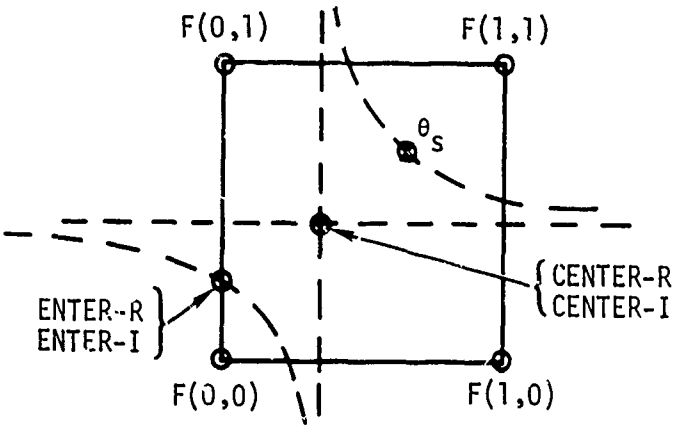


Figure 9. θ_s is not the Proper Solution to $F(\theta) = 0$.

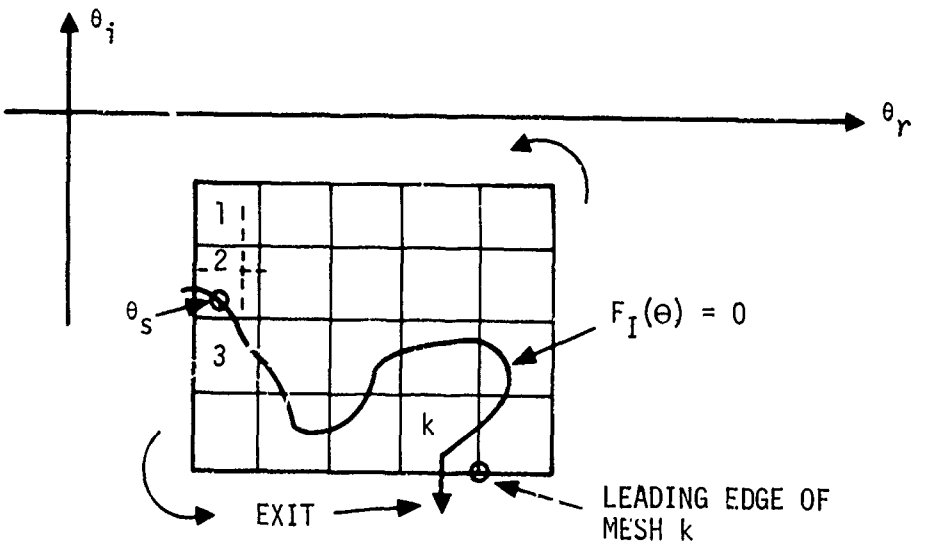


Figure 10. Contour Rectangle, Mesh Squares and Hyperbolic Lines of $F_I(\theta) = 0$. θ_s is a Solution of $F(\theta) = 0$.

At this point the value of the leading edge (counter clockwise direction) of the mesh square, through which the $F_I(\theta) = 0$ line exits, is noted. This mesh edge will also be along one of the contour rectangle sides. By identifying the mesh square in which this line e it occurs, it is then possible to later avoid re-entering this mesh through this exit edge while following the sides of the contour rectangle. This is important because if the search were allowed to re-enter at this location, the same zero of $F(\theta)$ would be found twice.

The next step is to examine mesh square No. 3 for intersections of the line $F_I(\theta)$ and edge No. 1 of that mesh.

(J) The entire contour rectangle is now searched, by the methods described in Section I, so that finally all of the zeros of the function $F(\theta) = 0$, which lie within the contour rectangle, are known. These solutions must be considered approximate; however, in that the hyperbolic approximation has been used in solving for $F(\theta) = 0$.

(K) To alleviate this problem, use is made of the Newton-Raphson iteration method to remove the mesh approximation where:

$$\Delta\theta = - \left. \frac{F(\theta)}{\frac{dF(\theta)}{d\theta}} \right|_{\theta = \text{mesh}} \tag{129}$$

If two approximate solutions are nearly equal, they may coverge to a single value after application of the Newton-Raphson procedure.

In order to resolve the two solutions further analysis is needed. Let the two approximate solutions be $\theta_{s,1}$ and $\theta_{s,2}$. Consider a set of 9 mesh squares arranged in a 3x3 formation with the one containing the solution $\theta_{s,1}$ at the center. There are 16 corner points associated with the corners of these squares (e.g. $(\theta_x)_i$, i=1,2,3,...16). The functions $F((\theta_x)_i)$ are then computed

for each of the 16 corners. These values will be denoted as $F_{mesh}((\theta_x)_i)$. The set of functions given by the following equation are also computed.

$$F_{\substack{no\\mesh}}((\theta_x)_i) = F(\theta_{s,1}) + \left.\frac{dF}{d\theta}\right|_{\theta_{s,1}} (\theta_{xi}-\theta_{s,1}) \qquad (i=1,2,3,\ldots,16) \tag{130}$$

The next step is to compute the following function at each of the 16 corners:

$$F_{total}((\theta_x)_i) = F_{mesh}((\theta_x)_i)\cos^2\beta + F_{\substack{no\\mesh}}((\theta_x)_i)\sin^2\beta \tag{131}$$

where β is incremented from 0° to 90° in whatever increments are required to resolve the zeros of $F(\theta)$. Considering the 9 squares one at a time the hyperbolic equation (Eq. 114) is solved to get 0,1, or 2 solutions in each of the squares. Only those solutions $(\theta_{q,j})$ which exist inside of a given square are kept. The final solution $(\theta_{p,1})$ is chosen to be that one solution from the set $(\theta_{q,j})$ which is closest to the original $\theta_{s,1}$.

The same type of analysis is then carried out for the solution $\theta_{s,2}$. The final solutions $\theta_{p,1}$ and $\theta_{p,2}$ are iterated by the Newton-Raphson procedure to give the exact solutions. If these two solutions again converge to a single number, the value of β in Equation 131 is incremented and the entire procedure is again carried through with the values $\theta_{p,1}$ and $\theta_{p,2}$ replacing the values of $\theta_{s,1}$ and $\theta_{s,2}$. Note that completeness (in that the full set of zeros has been determined) is assured in Steps I and J whereas exact values must await Step K.

IV STRUCTURE AND DESCRIPTION OF THE MODESRCH COMPUTER PROGRAM

(A) Program Structure

The MODESRCH computer program finds the waveguide mode constants of the earth-ionosphere waveguide for radio propagation at ELF-VLF-LF. These mode constants are determined in the form of modal: eigenangles, attenuation rates, phase velocities and excitation factors.

The individual subroutines of the MODESRCH program are listed, in Table 3. In this table, those subroutines are divided into special sub-groups which are in accordance with their function in the total program structure.

Figures 11 through 14 illustrate the structure of the MODESRCH computer program. In particular, Figure 11 presents the general flow of the program showing the order in which the various subroutines and sub-groups are executed. This sequence is indicated, in the figure, by the numerals 1 through 6. Figure 12 shows the structure of the Ionospheric Full-Wave Integration procedure. Figure 13 gives the Free Space Integration Structure along with the structure of the routines for computing the values of the function $F(\theta)$ and the derivative of $F(\theta)$ with respect to θ. Figure 14 illustrates the structure of the Root-Finder procedure.

TABLE 3

LIST OF PROGRAM

ROUTINES FOR MODESRCH

MAIN FLOW

MAIN

WVGUID

PRFLIN

SETRH

NEOUT

NPOUT

ROOT-FINDER

FZEROS

NOMESH

FINDF

QUAD

F-FUNCTION

FCTVAL

FDFDT

FINAL

LAGRNG

RBARS

MDHNKL

FULL WAVE INTEGRATION

INTEG

INITLR

QUARTC

TMTRX

SMTRX

ENNU

DIFFEQ

ROUT

FREE SPACE INTEGRATION

FSINTG

MDHNKL

AUXILIARY ROUTINES

CASIN

CANG

XFER

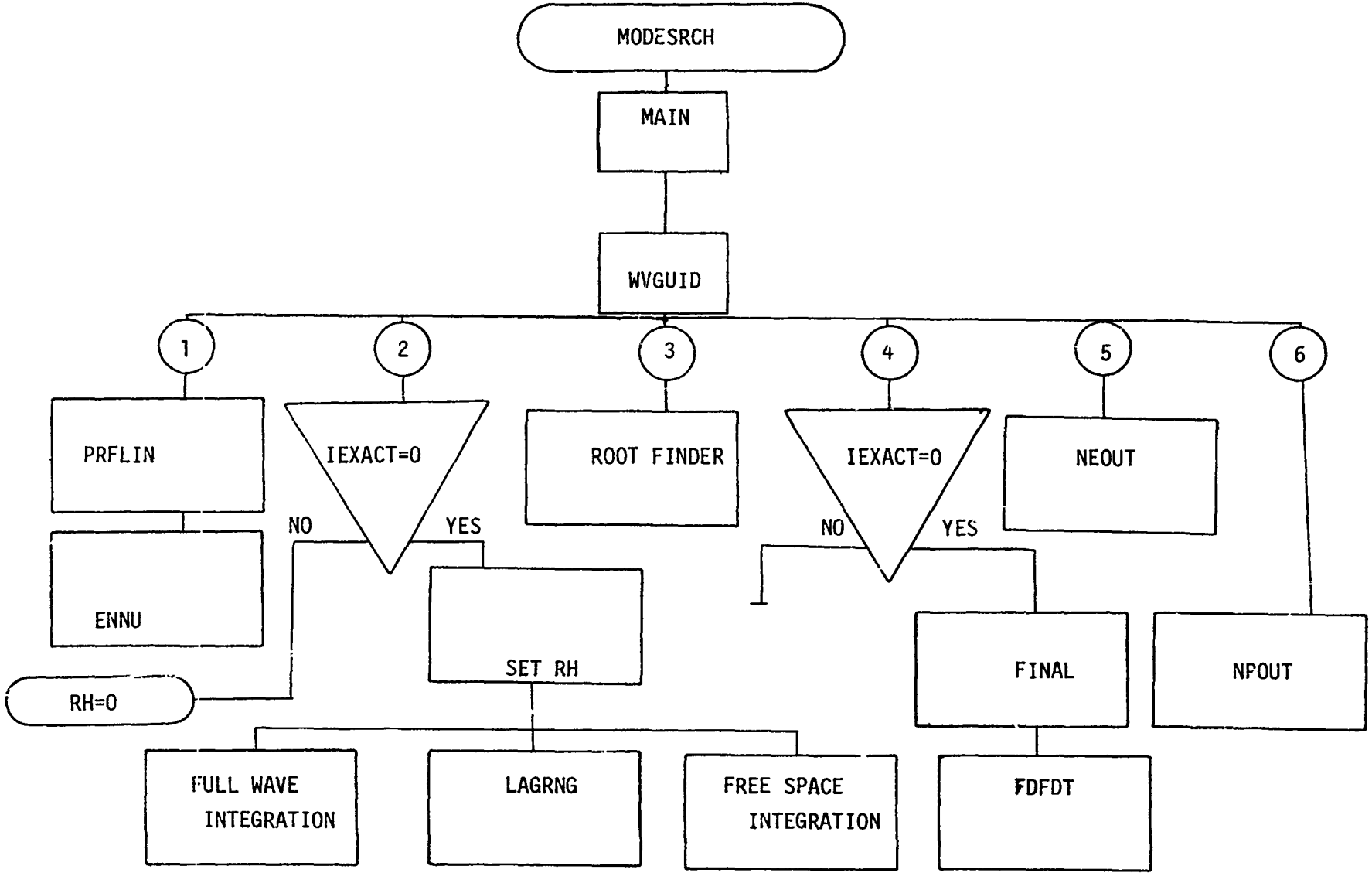


Figure 11. Structure of MODESRCH Computer Program.

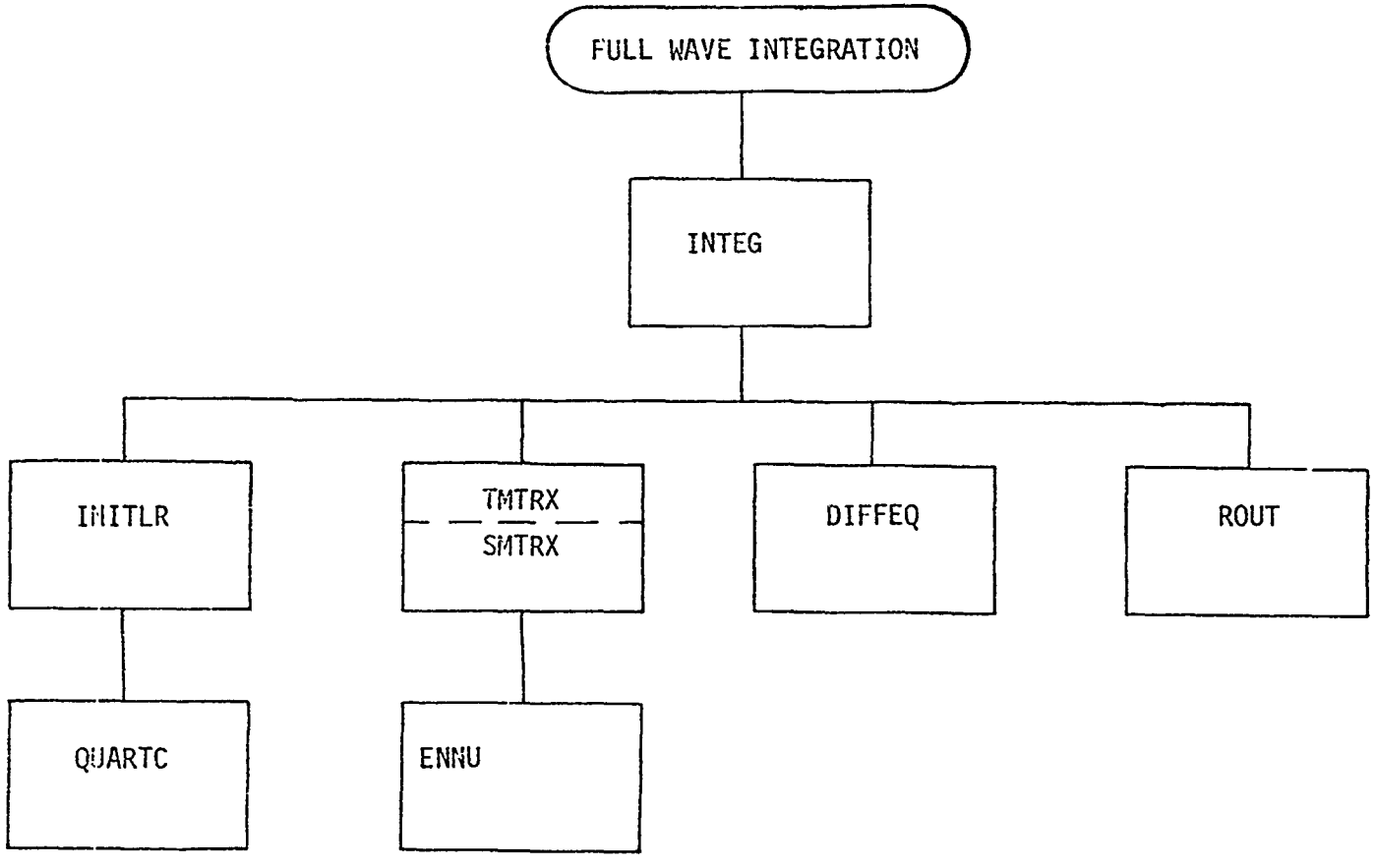


Figure 12. Structure of the Ionospheric Full-Wave Integration Procedure for the MODESRCH Computer Program.

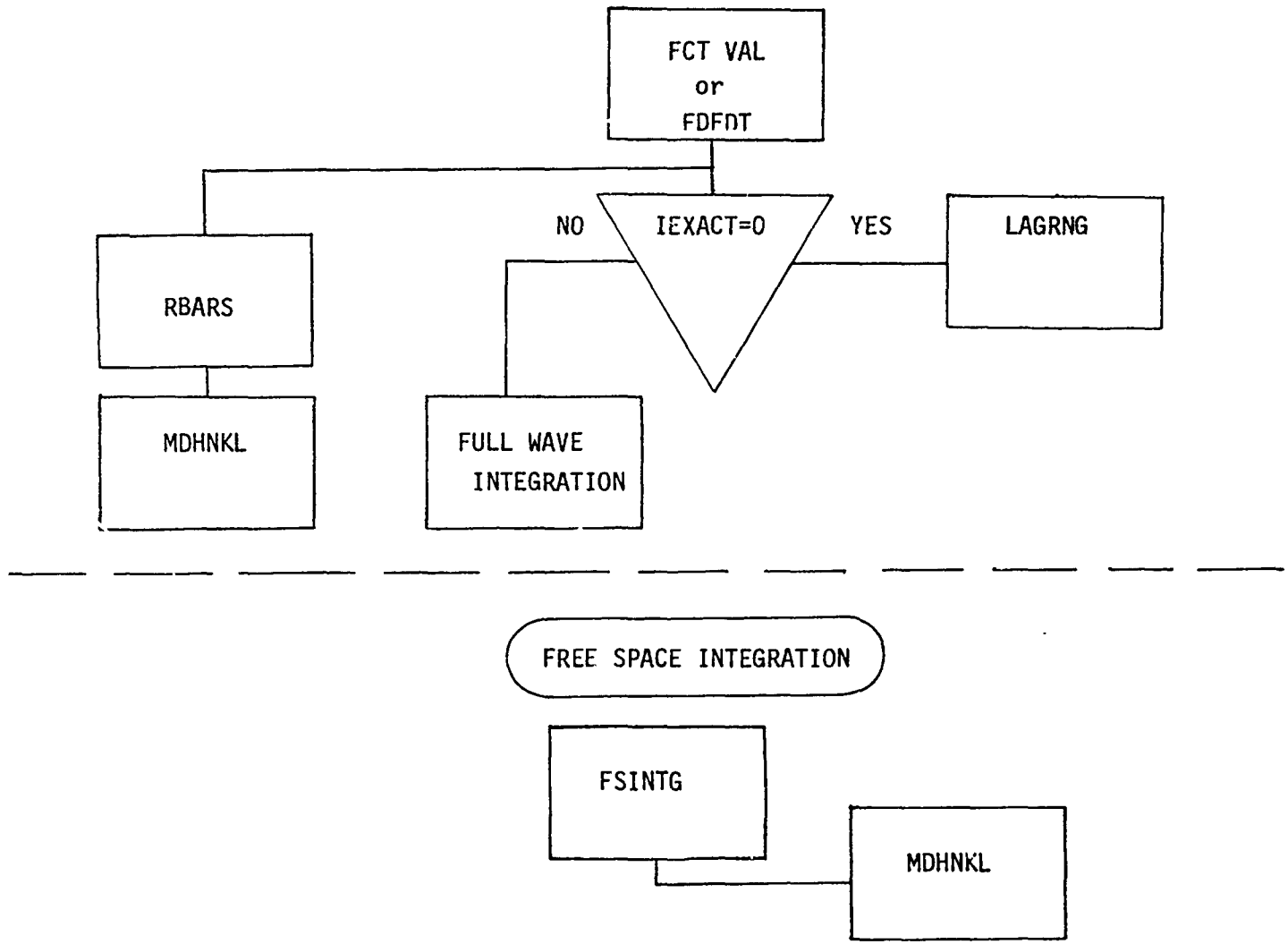


Figure 13. Structure of the Free Space Integration Procedure and Structure of the Function Value Routine and Derivative wrt θ of the Function Value Routine.

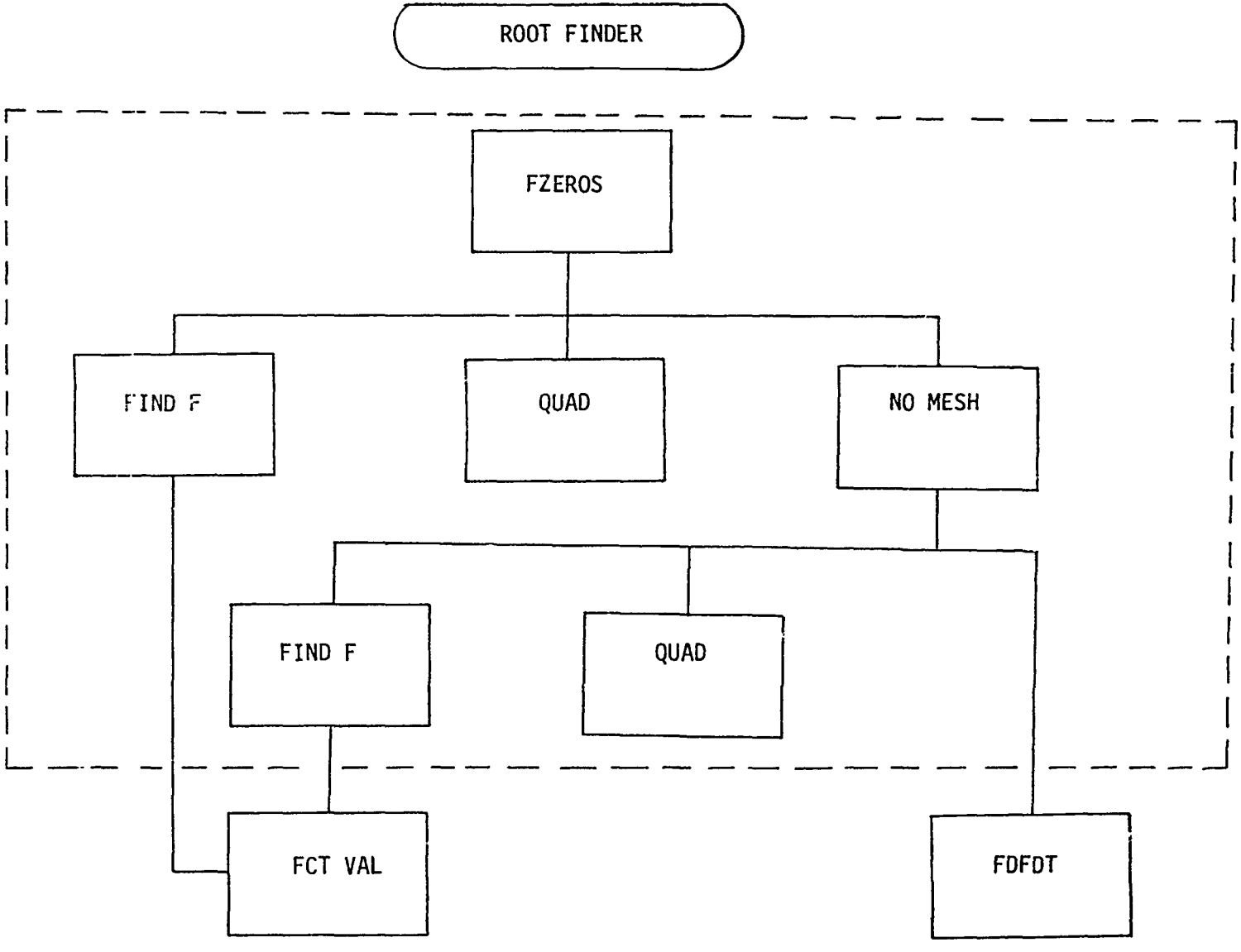


Figure 14. Structure of the Root Finder Procedure for the MODESRCH Computer Program.

(B) Computer Program Description

The function of each program subroutine is listed below:

(1) MAIN ROUTINE:

This routine controls the sequence of input and execution. It provides for NAMELIST input.

(2) SUBROUTINE PRFLIN:

This routine provides for input of electron density and collision frequency profile cards and for limiting the extent of the top and bottom of the electron density profile. Input of ion profiles is also provided for.

(3) SUBROUTINE WVGUID:

This routine controls the running sequence of the total computer program. Specific operations are to:

(a) Initialize the full-wave solution of the ionospheric integration routines.

(b) If the propagation frequency is greater than 1000 Hz and the variable 'IEXACT' is set equal to zero, then a reference height, (RK), is set through the routine SET RK. If either of the above is not true, the reference height is taken as zero.

(c) Initialize the ground reflection coefficient equations.

(d) Initialize the free-space integration calculations.

(e) Find the zeros (i.e., eigenangles) of the function $F(\theta)$ (see Equation 7), through the routine FZEROS. Lagrange interpolation is used if the frequency is greater than 1000 Hz or if the variable IEXACT is greater than zero. Otherwise, full wave solutions of the ionospheric reflection coefficients are computed.

(f) If the variable IEXACT is equal to zero or the frequency is greater than 1000 Hz, then the use of the routine FINAL is used to iterate on the eigenangle solutions of $F(\theta) = 0$ as the function is changed from that using Lagrange Interpolation to that using exact full-wave solutions of the ionospheric reflection coefficients.

(g) Order the eigenangles according to the value at the real part of the eigenangles.

(h) Compute and punch the eigenangles if the variable 'NEIGEN' is set equal to one. This is done through the routine NEOUT.

(i) Compute and punch the mode constant parameters if the variable 'NPUNCH' is set greater than zero. This is done through the routine NPOUT.

(4) <u>SUBROUTINE SETRH</u>:

(a) This routine selects the values at THETA, (θ) at which full-wave solutions of the ionospheric reflection coefficients are to be carried out for use in Lagrange interpolation.

(b) This routine also selects the height, to the nearest whole kilometer, at which the function:

$$\sum_{\theta\text{-points}} \left(\sum_{R\text{-elements}} \left\{ \left| \frac{d(\frac{\underset{\sim}{R}+1}{c})}{d\,C} \right|^2 \right\} \right) \qquad (31$$

is minimized. In the above expression $C = \cos(\theta)$, $\underset{\sim}{R}$ is the reflection coefficient matrix and the summations are over all θ-points at which full-wave solutions are

to be carried out for use in Lagrange interpolation. The summation is also over all elements of the $\underset{\sim}{R}$ reflection matrix. Derivatives of the full-wave solutions are not computed; rather, derivatives of the Lagrange interpolation formula are used. Note that the full-wave solutions are carried out to the bottom of the profile, using the ionospheric reflection coefficient integration procedures (i.e., subroutine INTEG) after which, the full wave solution is then carried out in an upward direction through free space over a curved earth (by use of the free space integration routine FSINTG) to various heights which are increased successively by intervals of one kilometer. That height at which the minimum occurs is printed out as the 'REFLECTION HEIGHT' and is used; thereafter, as the Reference Height at which the function, $F(\theta)$ is to be computed.

(5) SUBROUTINE NEOUT:

This routine provides for punched card output of eigenangles.

(6) SUBROUTINE NPOUT:

This routine provides for computing and printing tables and punched card output associated with the 'NPUNCH' option.

(7) SUBROUTINE INTEG:

This subroutine performs an integration of the differential equations for the ionosphere reflection matrix using Runge-Kutta integration formulas. The integration variables are

the elements of the matrix $(\underset{\sim}{R}+1)/C$ where $\underset{\sim}{R}$ is the reflection matrix described in Equation 2. If 'IDERIV' is set non-zero, the derivatives of the $(\underset{\sim}{R}+1)/C$ elements wrt C = COS (THETA) are also integration variables. The set of heights at which pairs of integration steps begin and end is stored in the array 'RKHTS'. Sizes of the steps are determined by comparing the values of the elements of $(\underset{\sim}{R}+1)/C$ after a pair of steps and values of the same variables after a single double-size comparison step. The $(\underset{\sim}{R}+1)/C$ variables used in the comparison step are denoted as 'X'. If the difference is too large, the step size pairs are cut in half by adding a new height to the list in 'RKHTS'. In order to enhance stability in other parts of the program, no heights are deleted from the list unless the list is reset by a call to the SET RK subroutine.

(8) SUBROUTINE ENNU:

Computation of electron density and collision frequency (and ion density if appropriate) as a function of height. If profile was input from formatted cards, logarithmic interpolation is used. See also notes in subroutine PRFLIN.

(9) SUBROUTINE INITLR:

This routine computes the coefficients of the Booker Quartic equation and solves for its roots. The routine then computes the value of $(\underset{\sim}{R}+1)/C$. For reflection from a sharply bounded anisotropic ionosphere of semi-infinite extent where $\underset{\sim}{R}$ is the reflection coefficient matrix. The solution is used as the initial condition for Runge-Kutta integration.

The solution is based on the material of reference 6. Derivatives wrt C=COS (THETA) are also computed if the variable 'IDERIV' is set non-zero.

(10) SUBROUTINE QUARTIC:

This routine obtains the solution for the roots of a fourth-order polynomial (QUARTIC Equation). A summary of pertinent equations is given in reference 6.

(11) SUBROUTINE TMTRX:

This routine computes the elements of the 'M' matrix of Equation 23. Those combinations of 'M' matrix elements used in the 'T' matrix which do not include use of THETA are the final output of this routine. Computations of quantities which are not functions of height or of THETA are carried out in the routine INIT T.

(12) SUBROUTINE SMTRX:

This routine computes the coefficients used in the differential equations for $(\underset{\sim}{R}+1)/C$. These are stored in common area 'S MTX' and are analogous to 'S' matrix elements given by Equation 20. Derivatives of these coefficients wrt C=COS (THETA) are also computed if 'IDERIV' is set non-zero.

(13) SUBROUTINE DIFFEQ:

This routine computes the derivative of the differential equations for $(\underset{\sim}{R}+1)/C$, where $\underset{\sim}{R}$ is the ionospheric reflection matrix. The equations are derived from the differential equations for 'R'as given in reference 5. Storage into the array 'R MTRX' is made so that the equations may be used with either the main set of integration variables,

or with the comparison set, X. Differential equations for the derivatives of (R+1)/C wrt C=COS (THETA) are also used if 'IDERIV' is set non-zero. Note that the variable called 'R' in this routine is actually (R+1)/C.

(14) SUBROUTINE RBARS:

This subroutine computes values of variables which may be used to form the elements of the RBAR matrix, where RBAR represents reflection of an ELM wave from the earth's surface. Namely, RBAR11=DEN11/(C*NUM11-DEN11), or (1.0/RBAR11+1.0)/C=NUM11/DEN11, and RBAR22=DEN22/(C*NUM22-DEN22), or (1.0/RBAR22+1.0)/C=NUM22/DEN22. Derivatives wrt C=COS (THETA) are also computed if 'IDERIV' is set non-zero. Note that the equations are formulated in such a way that a smooth transition is made from the 'CURVED EARTH' form to the 'FLAT EARTH' form. See also notes in subroutine MDHNKL regarding definitions of hankel function parameters. The value of earth's radius is such as to make 2.0/RE=3.14E-4. Computation at THETA=90° is not excluded.

(15) SUBROUTINE MDHNKL:

This subroutine computes values of variables which may be combined to form modified hankel functions as described in 'TABLES OF THE MODIFIED HANKEL FUNCTIONS OF ORDER ONE THIRD AND OF THEIR DERIVATIVES', by the staff of the computation laboratory, Harvard University Press, 1945. The notation is such that the actual H1=H1*EXP(-E). The actual H2=H2*EXP(E). Actual derivatives wrt argument, Z, are H1P*EXP(-E) and H2P*EXP(E).

(16) SUBROUTINE FSINTG:

This routine performs an integration of the differential equations for the ionosphere reflection matrix through a free space region over a curved earth. The integration may be performed in either a positive or negative height direction, but in this program the integration is always upward. The integration variables are the elements of the matrix $(\underset{\sim}{R}+1)/C$ where $\underset{\sim}{R}$ is the reflection matrix. The solution is based on Budden, reference 7, in particular the material on pp.118, 327-329, 336-338, and 343-345. If 'IDERIV' is set non-zero, the derivatives of $(\underset{\sim}{R}+1)/C$ elements wrt C=COS (THETA) are also integration variables. Computation at THETA=90 is not excluded. Also, the equations are formulated in such a way that a smooth transition is made from the 'CURVED EARTH' form to the 'FLAT EARTH' form for appropriate values of THETA. The initial and final values of the integration variables are stored in the common area 'INTEGR'. This routine carries out the computation of height-gain coefficients Q,G,A,B for two conditions on the upgoing wave at each height=Z_f, namely E11=1,EY=0 and E11=0,EY=1. Each of the coefficients Q,G,A,B is divided by C=COS (THETA). Computation of the upgoing fields E11 and EY at height=Z_t for the two conditions described above are also done. The integration proceeds from height Z_b to height Z_t when 'FSINTG' is called.

(17) SUBROUTINE LAGRNG:

This routine performs Lagrange interpolation of reflection coefficients in the COS (THETA) plane. Also to obtain derivatives of interpolated values WRT COS (THETA) the Lagrange interpolation formula itself is differentiated WRT COS (THETA).

(18) SUBROUTINE FINAL:

The routine for following the eigen-angles as the F function is changed from that using Lagrange interpolation to that using exact full-wave solution values of reflection coefficients.

(19) SUBROUTINE FZEROS: (TLEFT,TRIGHT,TBOT,TTOP,TMESH,TOL, MPRINT,ZEROS,NR Z)

For Root-Finder FZEROS is a routine for finding the zeros of a complex function, F, which lie within a specified rectangular region of the complex (THETA) plane, provided the function has no poles in the vicinity of the rectangle.

EXPLANATION OF PARAMETERS--

TLEFT - value of real part of THETA at left edge of rectangle.

TRIGHT - value of real part of THETA at right edge of rectangle.

TBOT - value of imag part of THETA at bottom edge of rectangle.

TTOP - value of imag part of THETA at top edge of rectangle.

TMESH - set equal to about half the average spacing between zeros within the rectangle. A smaller value may be used as a safety measure, but too small a value will result in excessive run time.

TOL - tolerance to which zeros are to be found. If two zeros are closer than 'TOL', the root-finder will stop with an error message.

MPRINT - normally set to zero. A non-zero value leads to print-out for debugging.

ZEROS - output list of (complex) values of THETA at which zeros are found.

NR Z - the number of zeros found.

SUBROUTINES TO BE PROVIDED--

FCT VAL - (THETA,F) - to return the value of the function, F, at the point in the complex plane specified by 'THETA'.

F DFDT - (THETA,F,DFDT) > same as 'FCT VAL' except that the derivative, DFDT, of the function WRT THETA must also be returned.

(20) <u>SUBROUTINE QUAD</u>:

For Root-Finder finds the solution for the real roots of a Quadratic Equation of the form A*X**2+2.0*B*X+C=0.0, where X is called 'SOL' in this routine. The number of real roots found is given by 'NR SOL'. A value of 1 for 'NR SOL' results from the Quadratic Equation approaching linearity. Used by subroutines FZEROS and NO MESH.

(21) SUBROUTINE NO MESH:

For Root-Finder the routine for finding exact (in the sense of NO MESH approximation) locations of zeros of the function, F, forwhich a complete, but approximate, set was found in subroutine FZEROS.

(22) SUBROUTINE FINDF:

This routine calls upon the routine FCT VAL to compute $F(\theta)$.

(23) SUBROUTINE FCT VAL:

This routine is for computing F function values at mesh points, called on principally by subroutines FZEROS and NO MESH. This is the modified F function which has no poles and which has no zeros at THETA=90.

(24) SUBROUTINE F DFDT:

This routine for computing function $F(\theta)$ values and their derivatives WRT THETA at arbitrary values of THETA. Called on principally by subroutines NO MESH and FINAL. This is the modified F function which has no poles and which has no zeros at THETA=90.

(25) SUBROUTINE XFER:

Routine for transferring one array into another.

(26) SUBROUTINE R OUT

Output of Runge-Kutta integration variables, used only for debugging.

(27) FUNCTION CASIN

Routine for computing the complex sine function of a complex argument.

(28) FUNCTION CANG:

Routine for computing the arc tangent of a complex angle.

V. RUNNING THE PROGRAM

A. Control Cards

These cards contain mnemonic words denoting the program operations to be executed. These words will be assumed to begin in column 1 of the card. In the text of this outline, these words will be enclosed in apostrophes. Blanks will be denoted by a space indicated by '_' to clarify positioning of blank columns. These cards are summarized in Table 4.

(1) 'PROFILE-i' initiates reading of the ionospheric profile. An alternative method for specifying an ionospheric profile is presented in Section B.

The value of i indicates the number of species to be used. It may have one of two possible values: 1 or 3. If the position for i is left blank, i = 1 is assumed.

The 'PROFILE_i' card must be followed by an alphanumeric card which may be used to identify the profile.

The profile cards contain the following parameters punched according to the format (F7.2,4X,5(1X,E9.2): height in kilometers and the specie densities in number per cubic centimeter. The first specie is electrons. The cards are input in order of descending altitude and the profile is terminated with a dummy height with value less than zero. A maximum of 101 profile cards may be used.

In the case of i = 3, the program assumes that the first two species read are electrons, N_e and singly charged positive ions, N_i^+. The required third specie is thus singly charged negative ions, N_i^-, and their density is computed and printed by the program as $N_i^- = N^+_i - N_e$.

In the integration of the ionospheric reflection elements through the ionosphere, the program interpolates exponentially between input values. Thus, for example, an exponential profile can be input with only the top and bottom densities specified.

(2) 'COLFREQ' initiates reading of an ionospheric collision frequency profile. An alternative form for specifying exponential collision frequencies is discussed in Section B. This profile, if used, must follow the 'PROFILE_i' input because i specifies the number of species.

The 'COLFREQ' card is immediately followed by the profile. The input format and order is the same as used for 'PROFILE_i', except that collision frequencies in collisions per second for electrons, and when i is greater than 1 for positive ions and negative ions, must be input. The profile is input in order of descending height and is terminated by a dummy height of value less than zero. A maximum of 50 collision frequency profile cards may be used. As with the ionospheric specie profiles, the program interpolates exponentially between input values and there need be no correspondence between the altitudes used to define the two profiles.

When 'COLFREQ' is used, the collision frequency profile can be over-ridden later in the job by the exponential collision frequency specification described in Section B-1. Note that in this case the exponential parameters, 'COEFNU' and 'EXPNU', must be input through NAMELIST.

(3) 'ID' indicates that the next card is identification for the data being run. This identifying card is merely read and printed. It may contain any alphanumeric information desired.

(4) '_&DATUM' and '_&END' initiate and terminate reading of Namelist. The form of this input is described below.

All cards appearing between '_&DATUM' and '_&END' must have a blank in column 1. In the sample data decks this will be indicated by '_'.

Variable names are punched with no imbedded blanks. The name is followed by an equal sign (=) and the value of the variable punched in any valid format. Integer variables may only be input with integer numbers and real variables may be input with either integer numbers or floating point numbers. The value of the variable must be followed by a comma. All blanks in the values punched are treated as zeros. A comma is the only valid delimiter. For example, 'I=1_,' would result in I=10. If a variable name is repeated, the second value will be used.

If the variable defines an array, the array may be filled by a string of values separated by commas. There need not be as many values specified in the string as there are in the dimension of the array. The string of variables may be continued on additional cards by not repeating the variable name. Alternatively, the value of single elements may be specified by subscripting the variable name before the equal sign such as 'ARRAY(3) = -6.E3,'.

Complex variables must be input in order of real and imaginary parts. Arrays of complex variables are input the same as described above except that twice as many values are required. For example, 'CARRAY = 1., 3., -1., -3.,' is used to input CARRAY(1) = (1., 3.) and CARRAY(2) = (-1., -3.).

(5) 'QUIT' signals normal termination of a job. This is not required provided a /* card is used at the end of the input deck.

(6) A reasonable order of input is shown in Table 4.

TABLE 4

SUMMARY OF CONTROL CARDS

PROFILE_i	read ionospheric profiles (i=1 or 3)*
COLFREQ_i	read ionospheric collision frequency profiles
ID	read and print identification card(80 card columns)
_&DATUM	begin Namelist read
_&END	end Namelist read and begin Waveguide calculations
QUIT	end of job (optional)
/*	

*Note: If i not specified, 1 is assumed.

B. Input Variables

(1) General Input variables needed for computations of the modal solutions.

'FREQ' is the frequency in kilohertz.

'MRATIØ is the ratio of the mass of the specie to the mass of an electron. Default values are for an electron, 'MRATIØ = 1'. For positive or negative ions 'MRATIØ = 58000'.

An exponential collision frequency may be specified for each specie. 'CØEFNU' is the collision frequency, in collisions per second, at the ground. 'EXPNU' is the height variation in inverse kilometers. The collision frequency at an altitude of z kilometers in this case is defined by:

$$\nu(z)j = \text{CØEFNU}(j)*\exp(\text{ENPNU}(j)*z)$$

where j=1 is for electron-neutral particle collisions, j=2 is for positive ion-neutral particle collisions and j=3 is for negative ion-neutral particle collisions.

The parameter 'NUFLAG' allows for a choice of exponential collision frequency profiles to be used in the calculations. In this instance values are not assigned to COEFNU(j) and EXPNU(j); instead default values, which are built into the MODESRCH program are used in the calculations. With 'NUFLAG = 0', the values of the exponential parameters 'COEFNU' and 'EXPNU' (as described in Wait's TN 300, Reference 3) are used. If 'NUFLAG = 1', the values of these exponential parameters (as computed by Moler, Reference 13) are used. The special values are:

'NUFLAG = 0' (Wait):

COEFNU(1) = 1.816E+11, coll/sec

COEFNU(2) = 4.540E+9, coll/sec

COEFNU(3) = 4.540E+9, coll/sec

EXPNU(1) = -0.15 km^{-1}

EXPNU(2) = -0.15 km^{-1}

EXPNU(3) = -0.15 km^{-1}

'NUFLAG = 1' (Moler):

COEFNU(1) = 4.303E+11 coll/sec

COEFNU(2) = 1.076E+10 coll/sec

COEFNU(3) = 1.076E+10 coll/sec

EXPNU(1) = -0.1622 km^{-1}

EXPNU(2) = -0.1622 km^{-1}

EXPNU(3) = -0.1622 km^{-1}

If an exponential electron collision frequency profile is to be used other than those described by Wait or Moler, 'NUFLAG' must be set = 0 and the proper values assigned to 'COEFNU(j)' and 'EXPNU(j)'.

It is important to note that if the collision frequency profile is entered as tabulated values via the control card 'COLFREQ' and then in the same job step, additional calculations are wanted for new collision

frequency values, as input through the Namelist variables 'COEFNU(j)' and EXPNU(j)'; then, 'NUFLAG' must be set equal to zero and new values assigned to 'COEFNU(j)' and 'EXPNU(j)'.

An exception to this procedure occurs when the new values of 'COEFNU(j)' and 'EXPNU(j)' are those denoted as "Moler's values". In this case the parameter 'NUFLAG' is set equal to 1 and no values need be assigned to 'COEFNU(j)' or 'EXPNU(j)' in that this is done automatically within the program.

'RHO' is the distance in Mm from the transmitter. This variable is used only in punched output. It is useful to further identify punched data that is being generated along a propagation path.

The geomagnetic field is specified by three variables. 'AZIM' is the clockwise angle between magnetic north and the horizontal propagation direction in degrees east of north. 'CODIP' is the magnetic co-dip angle in degrees from the vertical down. The magnetic equator is specified by 'CODIP = 90'. 'MAGFLD' is the magnetic field intensity in webers per square meter.

The ground conditions are specified by two variables. 'SIGMA' is the conductivity in mhos per meter and 'EPSR' is the relative dielectric constant.'

'H' is the height in kilometers at which the modified refractive index is unity. This is the height to which the eigenangles are referred.

'REFLHT' is the height in kilometers which is used to normalize the excitation factor defined by Wait. 'REFLHT' must not be zero.

'GMAX' is the maximum separation (in degrees) of the theta angles at which full wave solutions, to be used in Lagrange interpolation, are carried out.

'SEP' is a real variable which defines the minimum separation between eigen solutions.

'RTOL' is the variable which determines the precision of the Runge-Kutta integration for the ionospheric reflection elements.

'NPUNCH' determines the specific parameters to be punched on output cards.

'CARDPN' is an option for punching or not punching on cards the outputs from the 'NPUNCH' option.

'CARDPT' is an option for printing, or not printing, the values computed using the 'NPUNCH' option.

'NEIGEN' is the option for punching eigenangles on the cards.

'TALT' is the height of the transmitting antenna in km.

'RALT' is the height of the receiving antenna in km.

Note: Input of TALT and RALT is only applicable to the calculation of modal excitation factors as described in Section V-C-2. Computed output values, as a function of 'RALT' and 'TALT', are obtained only through the option NPUNCH = 7.

'TMESH' is the angular dimension of the mesh squares contained in the contour rectangle. This value is in degrees. 'TMESH' should always be smaller than the average separation of the eigenangles. Also 'TMESH' should be chosen smaller than the separation between the values of 'RANGER' (maximum) and 'RANGER' (minimum) or the values of 'RANGEI' (maximum) and 'RANGEI' (minimum). Unless 'TMESH' is assigned a value via NAMELIST, it is computed within the program as:

$$TMESH = \sqrt{3.75/freq_{kHz}} \tag{132}$$

'LUB' is a real variable used to terminate the iterative process. 'LUB' is used to test the iterative change in the magnitude of the complex

eigenangles. The iteration is stopped when the change in the magnitude of the complex eigenangle is less than or equal to 'LUB'. Unless 'LUB' is assigned a value via NAMELIST, it is computed within the program as:

$$LUB = \sqrt{15.0/freq_{kHz}} \cdot (0.01) \tag{133}$$

'IEXACT' is an option that determines whether Lagrangian interpolation or the "full wave" solution is used in initial stages of the program. IEXACT = 0 is for Lagrangian. IEXACT = 1 is for "full wave".

(2) Exponential profiles (electrons only)

An exponential, electrons only, ionospheric profile may be specified. The profile is computed as described in Wait's NBS TN 300, Reference 3, where

$$\omega_r(z) = 2.5X10^5 \exp[\beta(z-h')] = 3.18254X10^9 N(z)/\nu(z) \tag{11}$$

and β is in inverse kilometers, z and h' are in kilometers, N(z) is the electron density in electrons per cubic centimeter and $\nu(z)$ is the collision frequency in collisions per second. The values used to compute $\nu(z)$ are 'COEFNU(1)' and 'EXPNU(1)' and are given in Section V-B-1. Note that the term 'BETA' is β and 'HPRIME' is h'.

The equation for computing the electron density (N(z), electrons/cm^3) at height z is given by:

$$N(z) = 7.8554X10^{-5} \cdot COEFNU(1) \cdot [\exp(((BETA + EXPNU(1))z) - BETA \cdot HPRIME)] \tag{134}$$

'SCLHTS' is a real variable which indicates the number of scale heights above h' which will be used to denote the top of the profile when the β,h' exponential electron density profile is used.

The top of the profile may be modified by choosing a value for the variable SCLHTS. The relationship is that:

$$\text{Top Height} = \text{Integer}\left(\text{HPRIME} + \frac{\text{SCLHTS}}{\text{BETA}} + 0.5\right) \tag{135}$$

The nearest integral value of altitude as given by 'Top Height' is used to set the top of the profile. All heights above this value are neglected in the MODESRCH computations.

The bottom of the exponential profile is set by assigning a minimum value (i.e. ENMIN) of electron density in electrons/cm^3. All altitudes (z), at which the corresponding density is less than 'ENMIN' will be neglected.

(3) VLF/LF - Tabular Electron-Ion Density Profiles.

Computing efficiency for VLF and LF may be improved by cutting off the top and bottom heights of the input ionospheric profile.

When the particle density profiles are input to the program via 'PROFILE_i' (i = 1 implies electrons only) (i = 3 implies electrons and ions), the top and bottom of the profile may be cut off using the criteria, as presented in Section II, D-2. In that section it is stated that the top part of the input ionospheric profile which may be neglected in the calculations is defined as that altitude(z) above which:

$$B(z) > \text{Maximum}\left(\text{TEMPO-B},\ 20\,\frac{\omega}{\nu_e}\right) \tag{17}$$

where 'TEMPO-B' is assigned a value (e.g. 2.0) and B(z) is given by eq. 16. Also an additional requirement for cutting off the top of the profile is that the electron density must be as great as 'TOPEN'. The variable 'TOPEN' is assigned a value (e.g. 1000 electrons/cm^2). The top of the profile is taken to be the higher of the two heights corresponding to B(z), (equation 17), and to 'TOPEN'.

The bottom of the input ionospheric profile is neglected (for VLF/LF cases) at those altitudes for which the computed value of B in equation 16 is less than 'CUTOFF'. The variable 'CUTOFF' is assigned a value (e.g. 0.0001). The relationship for Cutoff is:

$$B(z) < CUTOFF \tag{18}$$

If it is not desired to cut off the profile bottom for certain calculations then the Namelist parameter 'ICUT' must be set equal to zero.

(4) ELF Tabular Electron-Ion Density Profiles

The same criteria for the variable 'PROFILE_i' as was presented for the VLF/LF case is applicable to ELF, (i.e. i = 1 or blank implies electrons only and i = 3 corresponds to electrons and ions).

In the case of ELF the profile top and bottom are not cut off and the Namelist variable 'ICUT' is not applicable.

(5) Namelist Variables with Initial Values.

Initial values of the Namelist variables are presented in Table 5.

TABLE 5

NAMELIST VARIABLES AND INITIAL VALUES

NAME	VALUE	UNITS	NAME	VALUE	UNITS
NPROF	None	-	ICUT	1	
FREQ	0.0	kHz	NUFLAG	0	
H	50.0	km	COEFNU(3)	1.816+11, 4.540E9, 4.540E9	coll/sec " "
REFLHT	50.0	km	EXPNU(3)	-0.15, -0.15, -0.15	km^{-1} "
RHO	0.0	Mm			
AZIM	None	degrees	SEP	0.1	degrees
CODIP	None	degrees	RTOL	0.001	degrees
MAGFLD	None	$WEBERS/m^2$	GMAX	5.0	degrees
SIGMA	4.64	mho/m			
EPSR	81.0	-			
BETA	None	km^{-1}	TOPEN	1000.	$electrons/cm^3$
HPRIME	-99.0	km	ENMIN	0.1	$electrons/cm^3$
SCLHTS	3		RANGER	None	degrees
MRATIO(3)	1,58000.0 58000.0	-	RANGEI	None	degrees
			TEMPOB	2.0	-
CUTOFF	0.0001	-	TALT	0.0	km
NEIGEN	0	-	RALT	0.0	km
NPUNCH	1	-	IEXACT	0	-
CARDPT	0	-	TMESH	-1.0	degrees
CARDPN	1	-	LUB	-1.0	-

Notes: Numbers in parenthesis after Name indicate dimensions.
Lack of decimal point indicates integer variables.

(6) The Eigenangle (θ)

The modal solutions of equation 1, which are denoted as complex eigenangles ($\theta = \theta_{real} + i\theta_{imaginary}$), are determined within a bounded region of the complex "THETA" plane defined by a set of limiting THETA values. This bounded region will be called the "contour rectangle" and the limiting THETA values are denoted by "RANGER" (minimum and maximum) and 'RANGEI' (maximum and minimum). These parameters are portrayed in Figure 15.

Where Point:

A: RANGER (minimum), RANGEI (maximum)

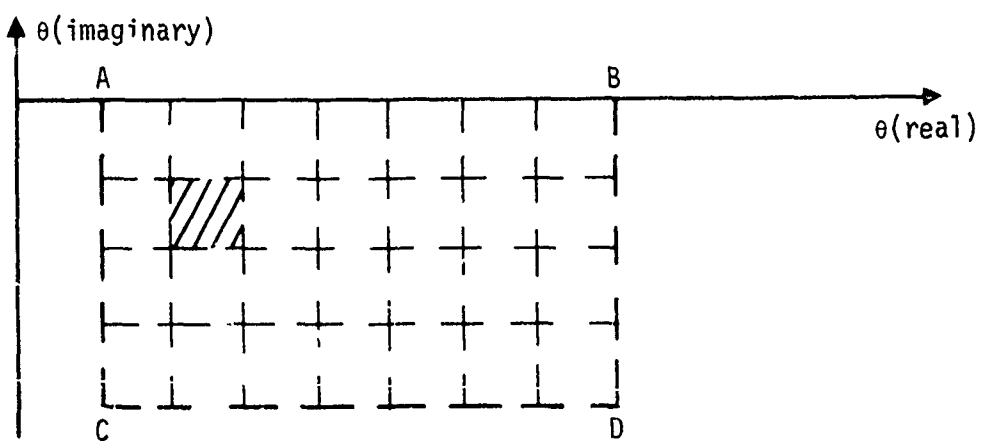
B: RANGER (maximum), RANGEI (maximum)

C. RANGER (minimum), RANGEI (minimum)

D: RANGER (maximum), RANGEI (minimum)

The small squares inside the "contour rectangle" are called the "Mesh Squares". 'RANGER' is the real part in degrees of the minimum and maximum values assigned to the contour rectangle.

'RANGEI' is the imaginary part in degrees of the minimum and maximum values assigned to the contour rectangle.



COMPLEX THETA PLANE

Figure 15. Showing the Contour Rectangle ABCD and Mesh Squares

i.e. RANGER = Minimum, maximum,

RANGEI = Maximum, minimum

Note: The minimum value must be given before the maximum value for RANGER while the maximum value must be given first for RANGEI.

During program execution, the contour rectangle, as described by RANGER and RANGEI, is automatically divided into small sub-rectangles. This is done to increase computation efficiency. The division process is applied only to the RANGER values while the RANGEI values remain the same as those input through NAMELIST.

The algorithm is that the number of segments 'NS' into which the original contour rectangle is divided is given by:

$$NS = \text{TRUNCATION}\ \{[\frac{RANGER(max)-RANGER(min)-LUB}{GMAX}] + 1\} \qquad (136)$$

The number of degrees in each segment is given by:

$$\Delta^\circ = \frac{RANGER\ (max) - RANGER(min)}{NS}$$

C. Program Outputs

For purposes of clarity in describing program outputs, the following symbols will be used:

k = wave number in km^{-1}

h_r = reference height for Wait's excitation factor (REFLHT)

i = $\sqrt{-1}$

θ = (θ_r, θ_i) = final mode solution referenced to H

θ' = (θ'_r, θ'_i) = θ referenced to the ground

α = $-8686\ k\ Im(\sin\theta')$ = attenuation rate in dB/Mm

v/c = $1/Re(\sin\theta')$ = phase velocity/speed of light

Note: The above procedure for sub-dividing the RANGER value is done only for VLF/LF calculations and is not applied for calculations at ELF.

λ_{mn} = excitation factor

X_{mn} = modified excitation factor

Λ_{mn} = normalized excitation factor = $-.5ikh_r X_{mn}$

The terms, X_{mn}, Λ_{mn} and λ_{mn}, are elements of 3 by 3 complex matrices. The individual elements are subscripted first by m = V, E or B to specify a vertical, end fire or broadside dipole and second by n = Z, Y or X to specify the field component excited. Thus, X_{vz} is the z component of the electric field excited by a vertical dipole source. The terms X_{mn} and Λ_{mn} are also functions of the transmitter and receiver altitudes (TALT and RALT). These are the excitation factors mentioned in Section II, H. The terms X_o and Λ_o specify X_{vz} and Λ_{vz} for both transmitter and receiver at the ground. In both printed and punched output, each X_o, X_{mn} and λ_{mn} is given as magnitude and phase in radians and each Λ_o and Λ_{mn} is given as 20 log (magnitude) and phase in radians. Note that in the MODESRCH computer program all the λ_{mn} terms are used whereas only X_{vz}, X_{ez}, X_{BZ}, X_o and Λ_o of the other excitation terms are utilized.

The output from the computer program consists of the following:

(1) Computer Listing Output"

(a) ID Information

(b) PROFILE information

(c) NAMELIST information

(d) Values of 'TMESH' and 'LUB'

Also, heights and electron densities of top and bottom of profile.

(e) Value of TEMPO-B at top of the profile, value of Wait's OMEGA-R at top of the profile or height of the profile where Wait's OMEGA-R = $2.5*10^5$

(f) Reflection height -- height at which ionospheric reflection coefficients are interpolated and the height at which the ground reflection coefficients are computed (e.g., the height where $F(\theta)$ is computed).

(g) Theta range for each 'contour rectangle'.

(h) H: the height at which the eigenangle solutions are computed

(i) Mode No. n

(j) THETA (θ_r, θ_i) The final mode solution referenced to H (θ_r and θ_i are in degrees)

(k) $\alpha(\theta_n')$: Attenuation rate dB/Mm where α = -8686*K*Imag (SIN θ') and where K is the wave number in km^{-1}

(l) $V/C(\theta_n')$: The phase velocity/speed of light

$$V/C = \frac{1}{\text{Real}(\text{SIN}\theta')}$$

(m) Magnitude and Phase of the modal excitation factor as defined by Wait. $(\Lambda_0(\text{dB}), \phi_{\Lambda 0}$ (radians))

(n) THETAP (θ_r', θ_i') The θ eigenangle referenced to the ground (θ_r', θ_i' are in degrees)

(o) Polarization mixing ratio:

$$\rho = \left| \frac{(1 - {}_{\perp}\bar{R}_{\perp}\ {}_{\perp}R_{\perp})}{({}_{\parallel}R_{\perp}\ {}_{\perp}\bar{R}_{\perp})} \right| = \left| \frac{({}_{\perp}R_{\parallel}\ {}_{\parallel}\bar{R}_{\parallel})}{(1 - {}_{\parallel}\bar{R}_{\parallel}\ {}_{\parallel}R_{\parallel})} \right| \qquad (138)$$

When NEIGEN = 1 that ionospheric height where the electron density is equal either to the value of the input variable ENMIN (for β, h' electron-density profiles) or to the lowest height of the input profile (for tabular electron density profiles) is printed. This parameter is

denoted as "D" and corresponds to the variable "D" in the reference 1 WAVEGUIDE program where D is defined as the height in kilometers below which ionospheric integration is terminated.

Also when NEIGEN = 1, individual eigenangles (referenced to the height H) are printed.

If NEIGEN = 0, the above variables are not printed.

(2) Punched Output

When NEIGEN = 1 the value of "D" as described above is punched and also the individual eigenangle solutions are punched. These parameters are punched in the format of usual NAMELIST variables. When NEIGEN = 0, these variables are not punched.

Additional punched output from the program is obtained with the variable NPUNCH.

NPUNCH options for obtaining output cards:

NPUNCH=1***	gives output cards for horizontally homogeneous mode-sum or WKB-sum in terms of (T'S).
NPUNCH=2***	gives output cards in terms of SNVLF output.
NPUNCH=7***	gives output cards for input into ELF-WKB mode-sums.
NPUNCH=8***	gives output cards for input to mode conversion in terms of (FOVR).
NPUNCH=9***	gives output cards for input into mode conversion in terms of both (T'S) and (FOVR).
NPUNCH=0***	the above variables are not computed.
CARDPN=0***	gives usual mode summary print out but no cards.
CARDPN=1***	gives usual mode summary print and cards.
CARDPT=0***	causes the output cards to not be listed.
CARDPT=1***	causes the output cards to be listed.

When NPUNCH equals 1, 2, 7, 8, or 9, each entry into '&DATUM' results in a card containing RHO, FREQ, AZIM, CODIP, MAGFLD, SIGMA and EPSR in the format:

('R',F7.3,' F',F8.4', A',F8.3,' C',F8.3,' M',E10.3,' S',E10.3,' E',F5.1)

and each exit via '&END' results in a blank card. In other program descriptions, the first card is referred to as the RFACMSE header card.

NPUNCH = 1 generates two cards per mode in the order

$(1,\theta',T_1,T_2)$ and $(2,\theta',T_3,T_4)$

with format (I1,2F9.5,4E15.8).

Where:

$$T_1 = \frac{S^{\frac{1}{2}} \; (1 + {}_{\parallel}\bar{R}_{\parallel})^2 (1 - {}_{\perp}R_{\perp} \; {}_{\perp}\bar{R}_{\perp})}{\left.\frac{\partial F}{\partial \theta}\right|_{\theta=\theta_n} \; {}_{\parallel}\bar{R}_{\parallel} D_{11}}$$

$$T_2 = \frac{S^{\frac{1}{2}} \; (1 + {}_{\perp}\bar{R}_{\perp})^2 (1 - {}_{\parallel}R_{\parallel} \; {}_{\parallel}\bar{R}_{\parallel})}{\left.\frac{\partial F}{\partial \theta}\right|_{\theta=\theta_n} \; {}_{\perp}\bar{R}_{\perp} \; D_{22}}$$

$$T_3 = \frac{S^{\frac{1}{2}} \; (1 + {}_{\parallel}\bar{R}_{\parallel}) \; (1 + {}_{\perp}\bar{R}_{\perp}) \; {}_{\parallel}R_{\perp}}{\left.\frac{\partial F}{\partial \theta}\right|_{\theta=\theta_n} \; D_{12}}$$

$$T_4 = \frac{{}_{\perp}R_{\parallel}}{{}_{\parallel}R_{\perp}} \tag{139}$$

θ' = complex eigenangle referenced to the ground and is punched as θ_r' (degrees) and θ_i' (degrees). Examination of Table 1 and equation 139 shows that the excitation "λ" values can be obtained from the "T" values. The "T" values are punched as complex numbers.

NPUNCH = 2 generates a single card per mode containing (α, V/C, $|\Lambda_o|$,$\phi_{\Lambda o}$) with format (4F10.5). Here Λ_o is in dB above one μV/meter and $\phi_{\Lambda o}$ is in radians. This format is denoted as SNVLF.

NPUNCH = 7 generates two cards per mode. The first card contains θ' with a comma after the real and imaginary parts. Here θ_r' and θ_i' are in degrees. The second card contains X_{VZ},X_{BZ} and X_{EZ} for transmitter and receiver heights of RALT and TALT. There is a comma after each term. This form of output is suitable for a Namelist type input.

These values are punched as magnitude and phase of the complex excitation number. Two comments which are pertinent to the NPUNCH = 7 output are:

(a) The values of the X_{mz}'s are computed as functions of the transmitter and receiver heights (TALT and RALT).

(b) If the punched output is to be used as input in the WDB-ELF Field Strength computer program (reference 14), then the NAMELIST terms TALT and RALT, input to the MODESRCH program must be zero for compatibility.

NPUNCH = 8 generates one card per mode. The card contains the variables (θ',$|X_o|$,ϕ_{xo}, FOVR, HOFWR) using the format (0P2F9.5,1PE14.5, 0PF9.5,1P2E16.8,0PF7.2) Here θ' is punched as θ_r' and θ_i' each in degrees. ϕ_{xo} is in radians and 'FOVR' is a complex number. The variable 'HOFWR' is the profile height (km) which corresponds to a value of Wait's OMEGA-R = $2.5*10^{+5}$.

D. Examples of Program Inputs and Outputs

Example I illustrates some of the options for data input. This example is for an exponential electron density profile defined by 'BETA' and 'HPRIME' parameters. In this case the exponential collision frequency profile, corresponding to that described by Wait, (Reference 3), is included by setting the 'NUFLAG' variable to zero. Also, putting NPUNCH = 1 or greater causes the mode constants to be printed in the output listing and if 'CARDPN=1', (which is the default value) functions of the excitation factors are punched. The value NEIGEN = 1 gives both a listing and punched cards for the eigenangles at a height of 50 km. The printed output is also shown in example I.

Example II lists the output cards as computed from 'NPUNCH = 1' as presented in example I. Note that the first card gives the values for RHO, FREQ, AZIM, CODIP, MAGFLD, SIGMA and EPSR as used in the modal calculations. The card will be denoted as the RFACMSE card. Also the complex eigenangles at the ground are also punched on the cards as θ_r' and θ_i'. There are two cards per mode corresponding to T_1(Real), T_1(Imaginary) and T_2(Real) and T_2(Imaginary) on card No. 1. Card No. 2 contains T_3(Real) and T_3(Imaginary), T_4(Real), and T_4(Imaginary).

Where 'NEIGEN = 1' as in example I, the values of the eigenangles at a height of H = 50 km are also punched one per card as θ(real) and θ(imaginary). The value of D as punched corresponds to the bottom of the electron density profile.

Example III lists the punched card output obtained from the input parameters of example I when 'NPUNCH = 2'. Here the first card is the usual RFACMSE card. The second card gives the propagation frequency. The following cards give, for each mode, the values of the attenuation

rate (dB/Mm); the ratio V/C; the magnitude of Wait's excitation factor (dB); and the phase of Wait's excitation factor (radians).

Example IV lists the punched card output obtained from the input parameters of example I where 'NPUNCH = 8'. This output is suitable for input into the mode conversion computer program described in reference 12. The first output card is the RFACMSE card. The following cards give for each mode the values of θ'(real) (degrees), θ'(imaginary) (degrees); the excitation component from a vertical exciter for the vertical electric field at z=0, (magnitude (not dB) and phase (radians)); the real and imaginary parts of the complex number 'FOVR'; and the height, in km, used for mode conversion, 'HOFWR'.

Example V presents input data where the electron density and positive ion density are entered in tabular form. Note that in this instance the collision frequencies for electrons, positive and negative ions are all entered automatically within the program through the variable 'NUFLAG = 1'. This corresponds to the use of these terms as computed by Moler (reference 13).

Example VI shows the printed output obtained from the input of example V. Note that for this case the profile top is not cutoff whereas the bottom is cutoff through the variable 'CUTOFF'.

Example VII lists the punched card output obtained from example V for NPUNCH = 9. Note that this punch option gives a combination of NPUNCH = 1 and NPUNCH = 8. There are three cards per mode.

Example VIII presents input data similar to example V. In this case the variable 'TOPEN' has been set to a small number so as to illustrate the output messages obtained when the top of the profile is cutoff.

Example IX shows the input when both particle density (electrons and positive ions) and collision frequency profiles (electron, positive ions and negative ions) are input in tabular form. This procedure will generally be the case for computations at ELF.

Example X illustrates the usual printed output obtained for NPUNCH greater than 0. Note that since this is an ELF case only one mode is found.

Example XI lists the output obtained for NPUNCH = 7. This output is compatible with a program which computes WKB-Fields at ELF (reference 14). Note however in this regard the NAMELIST variables 'TALT' and 'RALT' must both be zero. The punched card output for NPUNCH = 7 contains the following: the RFACMSE card; a card with θ_r' (degrees) and θ_i' (degrees); and a single card with the magnitude (not dB) and phase (radians) of the vertical excitation factors for vertical, end-on and broadside exciters.

Example XII shows the output obtained at 10 kHz for propagation over sea water at night. Whereas Examples XIII and XIV illustrate the results for propagation for the same conditions at 60 kHz. Note that considerably more modes are required to compute the total field at 60 kHz than at 10 kHz.

```
&DATUM
BETA=0.5,HPRIME=87.0,SCLHTS=6.0,ENMIN=0.1,FREQ=27.0,
NUFLAG=0,NPUNCH=1,NEIGEN=1,RANGEK=70.0,87.0,RANGEI=0.0,-1.0,
AZIM=283.0,CODIP=21.0,MAGFLD=5.44E-5,EPSK=15.0,SIGMA=1.0E-2,
&END
```

INPUT:

TMESH = 0.373 LOB = 0.0075

PRINTED OUTPUT:

COMPUTED PROFILE PARAMETERS:

H	NE	NU	X	Y	Z	WR
99.0	2.045E 03	6.454E 04	2.262E 02	-5.640E 01	3.865E-01	1.009E 08
70.6	1.000E-01	4.543E 02	1.106E-02	-5.640E 01	2.678E 01	7.005E 01

OMEGA-R = 2.5E05 AT TOP HEIGHT = 87.00

REFLECTION HT SET TO 90.64 KM THETA RANGE = 82.750 TO 87.000

REFLECTION HT SET TO 90.64 KM THETA RANGE = 78.500 TO 82.750

REFLECTION HT SET TO 90.64 KM THETA RANGE = 74.250 TO 78.500

REFLECTION HT SET TO 91.64 KM THETA RANGE = 70.000 TO 74.250

H = 50.00

MODE	THETA		ATTEN	VOVERC	WAIT MAG	WAIT ANG	THETAP		POL
1	85.889	-0.342	2.117	0.99469	-25.537	0.188	89.762	-5.921	2.29915
2	83.364	-0.167	1.664	0.99884	-9.550	-0.448	89.002	-2.792	0.83315
3	82.750	-0.214	2.335	1.00014	-6.858	0.256	88.555	-1.079	0.76029
4	82.750	-0.214	2.335	1.00014	-6.858	0.256	88.555	-1.079	0.76029
5	80.304	-0.293	4.265	1.00651	-6.406	-0.396	83.463	-0.437	1.15665
6	79.520	-0.224	3.528	1.00897	-9.021	0.508	82.346	-0.309	0.78515
7	76.820	-0.269	5.300	1.01898	-11.287	-0.721	78.920	-0.321	0.76962
8	76.005	-0.349	7.298	1.02248	-6.501	0.407	77.956	-0.408	1.28896
9	73.277	-0.224	5.580	1.03595	-17.624	-1.090	74.858	-0.249	0.50270
10	72.312	-0.507	13.330	1.04134	-5.372	0.236	73.792	-0.557	2.03806

D = 70.64,

EIGEN =		ATTEN	VOVERC
85.889,	-0.342,	2.117	0.99469
83.364,	-0.167,	1.664	0.99884
82.750,	-0.214,	2.335	1.00014
82.750,	-0.214,	2.335	1.00014
80.304,	-0.293,	4.265	1.00651
79.520,	-0.224,	3.528	1.00897
76.820,	-0.269,	5.300	1.01898
76.005,	-0.349,	7.298	1.02248
73.277,	-0.224,	5.580	1.03595
72.312,	-0.507,	13.330	1.04134
0.0 ,	0.0 ,		

EXAMPLE I

TIME REQUIRED FOR THIS PROFILE = 83.20 SECONDS

PUNCHED OUTPUT (NPUNCH=1):
(NEIGEN=1):

R 0.0 F 27.0000 A 283.000 C 21.000 M 5.440E-05 S 1.000E-02 E 15.0

1	89.76163	-5.921301	3.31041636E-04	-1.73154031E-03	-2.94311020E-10	-3.63712532E-11
2	89.76163	-5.921301	4.33465349E-07	5.66232529E-07	1.01706014E 00	6.01177737E-02
1	89.60211	-2.792252	-4.84679267E-03	-1.00973534E-02	-5.70600633E-09	-2.80088286E-09
2	89.60211	-2.792252	-6.05480818E-06	-5.76844832E-06	1.01618481E 00	5.83054982E-02
1	88.55502	-1.079442	3.89140495E-03	-1.48055814E-02	-8.39861054E-09	2.21654717E-09
2	88.55502	-1.079442	8.31343914E-06	7.84578697E-06	1.01579657E 00	5.76923676E-02
1	88.55502	-1.079452	3.89146036E-03	-1.48056000E-02	-8.39877146E-09	2.21652718E-09
2	88.55502	-1.079452	8.31343186E-06	7.84573876E-06	1.01579762E 00	5.76919802E-02
1	83.46289	-0.436671	-6.26920909E-03	-1.50769539E-02	-1.92630680E-08	-1.35022731E-08
2	83.46289	-0.436671	-1.25950228E-05	-1.48307736E-05	1.01577678E 00	5.48705570E-02
1	82.34634	-0.308832	5.92788309E-03	-1.06003359E-02	-3.67607476E-08	1.27407240E-08
2	82.34634	-0.308832	1.42949139E-05	1.61712815E-05	1.01286888E 00	5.39279357E-02
1	78.91954	-0.321452	-6.28706813E-03	-7.17989355E-03	-7.72604634E-08	-3.67887552E-08
2	78.91954	-0.321452	-2.30863225E-05	-1.65923120E-05	1.00946999E 00	4.99686189E-02
1	77.95581	-0.407701	6.64458051E-03	-1.52903795E-02	-4.21412700E-08	3.61722612E-08
2	77.95581	-0.407701	2.47947028E-05	1.73886656E-05	1.00832462E 00	4.85481583E-02
1	74.85843	-0.249042	-4.21220809E-03	-2.20749388E-03	-1.60071254E-07	-4.58113050E-08
2	74.85843	-0.249042	-2.79344094E-05	-1.00054694E-05	1.00267124E 00	4.41214181E-02
1	73.79199	-0.556671	4.70567495E-03	-1.91233456E-02	-2.16391030E-08	4.50759252E-08
2	73.79199	-0.556671	2.97281222E-05	9.96970630E-06	1.00072384E 00	4.15326096E-02
	θ_r'	θ_i'	T_r	T_i	T_r	T_i

D = 70.64,
FIGEN =
85.889, -0.342,
83.364, -0.167,
82.750, -0.214,
82.750, -0.214,
80.304, -0.293,
79.520, -0.224,
76.820, -0.269,
76.005, -0.349,
73.277, -0.224,
72.312, -0.507,
0.0 , 0.0 ,

EXAMPLE II

PUNCHED OUTPUT (NPUNCH=2):

R 0.0 F 27.0000 A 283.000 C 21.000 M 5.440E-05 S 1.000E-02 E 15.0

27.000			
2.11708	0.99469	-25.53743	0.18805
1.66405	0.99834	-9.54964	-0.44820
2.33522	1.00014	-6.85628	0.25607
2.33524	1.00014	-6.85827	0.25607
4.20478	1.00051	-6.40642	-0.39581
3.52849	1.00897	-9.02146	0.50844
5.29968	1.01898	-11.28702	-0.72139
7.29815	1.02248	-6.50111	0.40691
5.58048	1.03595	-17.62401	-1.09043
13.32971	1.04154	-5.37156	0.23563
α	V/c	$\lvert\Lambda_o\rvert$	Φ_{Λ_o}

EXAMPLE III

PUNCHED OUTPUT (NPUNCH=8):

R 0.0 F 27.0000 A 283.000 C 21.000 M 5.440E-05 S 1.000E-02 E 15.0

89.78163	-5.92130	3.73651E-05	1.75885	-2.68774806E-04	3.04608140E-04	87.00
89.60211	-2.79225	2.35427E-02	1.12259	6.98302174E-04	-2.64585018E-04	87.00
88.55502	-1.07944	3.20940E-02	1.82687	-3.57784096E-04	6.55593583E-04	87.00
88.55502	-1.07945	3.20940E-02	1.82687	-3.57785728E-04	6.55590557E-04	87.00
83.40289	-0.43667	3.38000E-02	1.17499	1.13481912E-03	-3.63505445E-04	87.00
82.34634	-0.30883	2.50187E-02	2.07924	-5.87876653E-04	1.67732174E-03	87.00
78.91954	-0.32145	1.92747E-02	0.84940	2.90093734E-03	-6.73920149E-04	87.00
77.95581	-0.40770	3.34412E-02	1.97770	-3.63842817E-04	1.77970389E-03	87.00
74.85843	-0.24934	9.29259E-03	0.48037	6.17924705E-03	-8.63190508E-04	87.00
73.79199	-0.55867	3.80355E-02	1.80643	-1.30885470E-04	1.56675434E-03	87.00
Θ_r'	Θ_i'	$\lvert X_o\rvert$	Φ_{X_o}	$(FOVR)_r$	$(FOVR)_i$	HOFWR

EXAMPLE IV

```
PROFILE 3
NFLCNUCL NITE    40000 SEC     6.00 MM      DNA NIGHTTIME
 100.00        9.74E 02   1.04E 03
  95.00        8.14E 02   6.05E 02
  90.00        3.69E 02   4.39E 02
  85.00        1.50E 02   3.63E 02
  80.00        5.10E 01   4.15E 02
  75.00        3.80E 00   4.47E 02
  70.00        3.40E-02   4.50E 02
  65.00        1.14E-02   5.60E 02
  60.00        5.77E-03   7.65E 02
  55.00        3.40E-03   1.09E 03
  50.00        2.06E-03   1.54E 03
  45.00        1.21E-03   2.11E 03
  40.00        6.73E-04   2.73E 03
  35.00        3.46E-04   3.25E 03
  30.00        1.67E-04   3.55E 03
  25.00        7.70E-05   3.68E 03
  20.00        3.43E-05   3.65E 03
  15.00        1.50E-05   3.63E 03
  10.00        6.58E-06   3.72E 03
   5.00        3.43E-06   4.48E 03
   0.0         2.17E-06   5.50E 03
  -9.00
 &DATUM
 AZIM=283.0,CODIP=21.0,MAGFLD=5.44E-5,
 FREQ=27.0,EPSK=15.0,SIGMA=1.0E-2,NUFLAG=1,NPUNCH=9,
 RANGER=70.0,87.0,RANGEI=0.0,-1.0,
 &END
```

INPUT:

EXAMPLE V

PRINTED OUTPUT:

TMESH = 0.373 LUB = 0.0075

WARNING--THE TOP OF THE PROFILE CAN NOT BE CUT OFF. THE TOP OF THE PROFILE MAY BE TO LOW.

TEMPO-B AT TOP = 2.344E 01 WAITS OMEGA-R AT TOP = 7.962E 07

THE BOTTOM PROFILE HEIGHT IS = 39.414 AT B(CUTOFF) = 0.100E-03

OMEGA-R EQUALS 2.5E05 AT TOP HEIGHT = 80.98

REFLECTION HT SET TO 82.41 KM THETA RANGE = 82.750 TO 87.000

REFLECTION HT SET TO 83.41 KM THETA RANGE = 78.500 TO 82.750

REFLECTION HT SET TO 83.41 KM THETA RANGE = 74.250 TO 78.500

REFLECTION HT SET TO 84.41 KM THETA RANGE = 70.000 TO 74.250

H = 50.00

MODE	THETA		ATTEN	VOVERC	WAIT MAG	WAIT ANG	THETAP		POL
1	86.468	-0.590	3.143	0.99399	-45.101	0.036	89.068	-6.308	0.44411
2	85.326	-0.305	2.148	0.99545	-20.244	0.169	89.739	-5.484	2.03634
3	82.849	-0.352	3.784	0.99991	-4.393	-0.065	88.501	-1.666	1.66767
4	82.118	-0.225	2.666	1.00161	-12.869	-0.046	86.710	-0.542	0.45272
5	79.574	-0.439	6.863	1.00876	-5.471	-0.072	82.412	-0.606	1.55826
6	78.624	-0.240	4.098	1.01202	-12.795	0.120	81.153	-0.311	0.61951
7	75.819	-0.401	9.774	1.02330	-9.170	-0.240	77.738	-0.536	1.03780
8	74.693	-0.292	6.564	1.02765	-9.468	0.267	76.074	-0.333	0.97222
9	71.953	-0.440	11.798	1.04546	-13.349	-0.611	73.399	-0.481	0.67283
10	70.957	-0.405	11.417	1.04936	-6.698	0.275	72.346	-0.439	1.53786

TIME REQUIRED FOR THIS PROFILE = 169.08 SECONDS

EXAMPLE VI

PUNCHED OUTPUT (NPUNCH=9):

3	1.0	E 67.000 A	683.000 C	61.000 M	5.443E-05 S 1.000E-06 E 15.0	
1	89.66703	-6.308402	6.861234406E-06	-1.645662685E-04	-9.886125886E-10	-4.583979861E-10
2	89.66708	-6.308404	-2.342781681E-07	-3.781033039E-07	9.985203374E-01	1.855798881E-02
	89.66708	-6.30840	3.92915E-04	1.00668	1.998655271E-03	-1.343547835E-03	80.98
1	89.73874	-5.483711	5.463352866E-04	-3.193761691E-03	-5.352078876E-10	3.635960411E-11
2	89.73874	-5.483711	8.931040959E-07	9.714358838E-07	9.576269931E-01	1.744917776E-02
	89.73874	-5.48371	6.87267E-02	1.75938	-2.490375195E-03	3.222601825E-04	80.98
1	88.57098	-1.686141	-1.287414468E-02	-2.023633205E-02	-3.791996055E-09	-1.022329136E-09
2	88.50398	-1.686141	-5.710983045E-06	-8.881022227E-06	9.904036027E-01	1.300726598E-02
	88.20098	-1.68614	4.262741E-02	1.50572	3.565303305E-03	-2.593539985E-04	80.98
1	86.71039	-0.541542	-3.419513861E-04	-7.660325625E-03	-1.879427105E-08	-1.879386576E-09
2	86.71039	-0.541542	8.344357491E-06	8.708765556E-06	9.956481408E-01	1.233266667E-02
	86.71039	-0.54154	1.65852E-02	1.52510	-1.182936835E-03	1.055434590E-03	80.98
1	82.41203	-0.605841	-1.255320991E-03	-1.818728807E-02	-1.886546058E-08	-4.643599771E-09
2	82.41203	-0.605841	-1.220423725E-05	-1.442165374E-05	9.936822265E-01	5.840588636E-03
	82.41203	-0.60584	3.76538E-02	1.49907	8.352886549E-04	-6.133785947E-04	80.98
1	81.15297	-0.310832	9.578533859E-04	-7.835750415E-03	-5.729397407E-08	7.194278651E-11
2	81.15297	-0.310832	1.419460116E-05	1.595392318E-05	9.919152886E-01	3.164670658E-03
	81.15297	-0.31083	1.82534E-02	1.09972	-1.789052904E-03	2.023771713E-03	80.98
1	77.73752	-0.536411	-2.358738416E-03	-1.191437626E-02	-7.017314376E-08	-2.386723539E-08
2	77.73752	-0.536411	-2.033294005E-05	-2.247521485E-05	9.885921438E-01	-7.271253256E-03
	77.73752	-0.53641	4.42933E-02	1.33124	2.170889876E-03	-1.165708612E-03	80.98
1	76.07305	-0.332982	3.177975546E-03	-1.150974286E-02	-8.458954435E-08	1.875223896E-08
2	76.07305	-0.332982	2.216316356E-05	2.302013304E-05	9.801420225E-01	-1.209545146E-02
	76.07305	-0.33298	2.37642E-02	1.83744	-1.412335584E-03	2.315625321E-03	80.98
1	73.39934	-0.481372	-4.236709335E-03	-6.109301005E-03	-1.645238867E-07	-5.855763965E-08
2	73.39934	-0.481372	-3.342777183E-05	-2.314687305E-05	9.809329325E-01	-3.023517885E-02
	73.39934	-0.48137	1.43519E-02	0.95945	4.890728745E-03	-1.569050285E-03	80.98
1	72.34846	-0.438911	4.733480351E-03	-1.646003135E-02	-7.285643735E-08	5.267299845E-08
2	72.34846	-0.438911	3.203276115E-05	2.341601107E-05	9.784926790E-01	-3.764095478E-02
	72.34846	-0.43891	3.26906E-02	1.84594	-7.975203925E-04	2.173622845E-03	80.98

EXAMPLE VII

```
PROFILE 3
NELCNUC1 NITE    40000 SEC     6.00 MM      DNA NIGHTTIME
 100.00        9.74E 02   1.04E 03
  95.00        6.24E 02   6.65E 02
  90.00        3.69E 02   4.39E 02
  85.00        1.90E 02   3.63E 02
  80.00        5.10E 01   4.15E 02
  75.00        3.80E 00   4.47E 02
  70.00        3.40E-02   4.50E 02
  65.00        1.14E-02   5.60E 02
  60.00        5.77E-03   7.65E 02          INPUT:
  55.00        3.40E-03   1.09E 03
  50.00        2.06E-03   1.54E 03
  45.00        1.21E-03   2.11E 03
  40.00        6.73E-04   2.73E 03
  35.00        3.46E-04   3.25E 03
  30.00        1.67E-04   3.55E 03
  25.00        7.70E-05   3.68E 03
  20.00        3.43E-05   3.65E 03
  15.00        1.50E-05   3.63E 03
  10.00        6.58E-06   3.72E 03
   5.00        3.43E-06   4.48E 03
   0.0         2.17E-06   5.50E 03
  -9.00
 &DATUM
 TOPEN=100.0,
 AZIM=283.0,CODIP=21.0,MAGFLD=5.44E-5,
 FREQ=27.0,EPSR=15.0,SIGMA=1.0E-2,NUFLAG=1,NPUNCH=9,
 RANGER=70.0,87.0,RANGEI=0.0,-1.0,
 &END
```

```
TMESH =   0.373     LUB = 0.0075
                                          PRINTED OUTPUT (PARTIAL):
THE TOP OF THE PROFILE IS SET TO BE--
  89.10        3.28E 02 4.24E 02 8.24E 01

 AT THE TOP OF THE PROFILE B =    1.724E 01 OMEGA-R =    4.584E 06

THE BOTTOM PROFILE HEIGHT IS =      39.414 AT B(CUTOFF) =      0.100E-03
```

EXAMPLE VIII

```
PROFILE 3
SATELLITE NIGHT AMBIENT PROFILE
 250.00        1.00E 05   1.00E 05
 225.00        5.00E 03   5.00E 03
 220.00        3.20E 03   3.20E 03
 210.00        1.68E 03   1.68E 03
 200.00        1.00E 03   1.00E 03
 190.00        6.50E 02   6.50E 02
 180.00        4.60E 02   4.60E 02
 170.00        3.45E 02   3.45E 02
 160.00        2.82E 02   2.82E 02
 155.00        2.60E 02   2.60E 02
 150.00        2.50E 02   2.50E 02
 145.00        2.55E 02   2.55E 02
 140.00        2.80E 02   2.80E 02
 130.00        3.70E 02   3.70E 02
 120.00        5.80E 02   5.80E 02
 112.00        1.10E 03   1.10E 03
 110.00        1.30E 03   1.30E 03
 106.00        1.70E 03   1.70E 03
 104.00        1.90E 03   1.90E 03
 102.00        1.98E 03   1.98E 03
 100.00        2.00E 03   2.00E 03
  99.00        1.95E 03   1.95E 03
  95.00        5.62E 02   6.50E 02
  90.00        3.15E 02   4.11E 02
  85.00        1.97E 02   4.11E 02
  80.00        5.09E 01   4.11E 02
  75.00        3.14E 00   4.11E 02
  70.00        3.27E-02   4.11E 02
  65.00        1.00E-02   5.33E 02
  60.00        5.73E-03   7.06E 02
  55.00        3.39E-03   1.09E 03
  50.00        2.06E-03   1.54E 03
  45.00        1.22E-03   2.11E 03
  40.00        6.74E-04   2.73E 03
  35.00        3.47E-04   3.25E 03
  30.00        1.67E-04   3.55E 03
  25.00        7.70E-05   3.69E 03
  20.00        3.43E-05   3.65E 03
  15.00        1.50E-05   3.63E 03
  10.00        6.58E-06   3.72E 03
   5.00        3.43E-06   4.40E 03
   0.0         0.0        0.0
 -99.99
CULFREQ 3
 250.00        1.05E 02   4.50E-01   4.50E-01
 225.00        3.50E 01   9.00E-01   9.00E-01
 220.00        3.00E 01   1.00E 00   1.00E 00
 210.00        3.30E 01   1.30E 00   1.30E 00
 200.00        4.50E 01   2.00E 00   2.00E 00
 150.00        1.60E 03   4.50E 01   4.50E 01
 120.00        1.00E 04   3.00E 02   3.00E 02
 100.00        3.90E 04   8.00E 03   8.00E 03
   0.00        4.30E 11   1.07E 10   1.07E 10
 -99.99
 &DATUM
 RANGER=78.0,85.0,RANGEI=-30.0,-40.0,
 NUFLAG=0,
  AZIM = 90.0,  COLIP = 16.0,  MAGFLD = 5.50E-05,
```

INPUT:

```
NUFLAG=0,AZIM=90.0,COLIP=16.0,MAGFLD=5.5E-05,EPSR=10.0,SIGMA=3.0E-04,
REFLHT=60.,FREQ=0.075,NFUNCH=7,
&END
```

EXAMPLE IX

PRINTED OUTPUT:

```
TMESH =   7.071     LUB = 0.1414

          H = 50.00
 MODE       THETA        ATTEN    VOVERC    WAIT MAG   WAIT ANG        THETAP              PUL
   1   83.117 -33.751    1.029   0.84794     -5.630     -0.150     83.246 -34.567      4.92227

TIME REQUIRED FOR THIS PROFILE =       60.56 SECONDS
```

EXAMPLE X

PUNCHED OUTPUT (NPUNCH=7):

```
R  0.0   F  0.0750 A  90.000 C  16.000 M 5.500E-05 S 3.000E-04 E 10.0
      83.24591, -34.56673,
```

1.1091E 01,	1.420,	3.4725E-02,	5.411,	4.5238E-03,	4.200,
X_{VZ}	$\Phi_{X_{VZ}}$	X_{EZ}	$\Phi_{X_{EZ}}$	X_{BZ}	$\Phi_{X_{BZ}}$

EXAMPLE XI

INPUT:

```
ID
 HAWAII TO CALIFORNIA (NIGHTTIME)
 &DATUM
 FREQ=10.0,
 AZIM=58.5,COSTH=59.0,MAGFLD=4.51E-5,
 DELTA=0.4,HPRIME=87.0,SCLHTS=6.0,
 RANGER=50.0,90.0,RANGEI=0.0,-5.0,
 &END
```

PRINTED OUTPUT:

```
TMESH =   0.612     LOB = 0.0122

COMPUTED PROFILE PARAMETERS:
   H          NE          NU          X           Y           Z           WF
 102.0    1.004E 03   4.115E 04   2.051E 03  -1.200E 02   6.550E-01   1.009E 08
  84.1    1.000E-01   1.212E 07   8.061E-02  -1.200E 02   1.929E 02   2.625E 01

      MEGA-K = 2.5E05 AT TOP HEIGHT =    87.00

REFLECTION HT SET TO   86.10 KM      THETA RANGE =    85.000 TO    90.000

REFLECTION HT SET TO   86.10 KM      THETA RANGE =    80.000 TO    85.000

REFLECTION HT SET TO   86.10 KM      THETA RANGE =    75.000 TO    80.000

REFLECTION HT SET TO   86.10 KM      THETA RANGE =    70.000 TO    75.000

REFLECTION HT SET TO   87.10 KM      THETA RANGE =    65.000 TO    70.000

REFLECTION HT SET TO   87.10 KM      THETA RANGE =    60.000 TO    65.000

REFLECTION HT SET TO   88.10 KM      THETA RANGE =    55.000 TO    60.000

REFLECTION HT SET TO   88.10 KM      THETA RANGE =    50.000 TO    55.000

REFLECTION HT SET TO   89.10 KM      THETA RANGE =    45.000 TO    50.000

REFLECTION HT SET TO   90.10 KM      THETA RANGE =    40.000 TO    45.000

REFLECTION HT SET TO   90.10 KM      THETA RANGE =    35.000 TO    40.000

REFLECTION HT SET TO   91.10 KM      THETA RANGE =    30.000 TO    35.000

          H = 50.00
 NO.      THETA        ATTEN    VOVERC    WAIT MAG   WAIT ANG        THETAP          FOL
  1   82.961  -0.129    0.505   0.99960   -3.835     0.114     89.415  -1.556    1.10605
  2   79.915  -0.496    2.704   1.00780   -27.154    1.851     82.035  -0.797    0.22010
  3   74.282  -0.367    3.182   1.03067   -6.742     0.052     75.981  -0.413    3.89191
  4   69.915  -0.947   10.418   1.05625   -25.802    1.453     71.194  -1.017    0.35914
  5   63.733  -0.500    7.165   1.10555   -7.430    -0.027     64.723  -0.530    2.56060
  6   59.103  -1.732   26.406   1.15564   -19.901    1.100     59.571  -1.766    0.54162
  7   52.235  -0.555   10.340   1.25248   -9.950    -0.232     52.977  -0.567    1.79337
  8   45.998  -3.217   70.824   1.36546   -17.417    0.702     47.074  -3.271    0.72755
  9   39.140  -0.551   13.676   1.57172   -14.287   -0.300     39.510  -0.559    1.43496

TIME REQUIRED FOR THIS PROFILE =      132.91 SECONDS
```

EXAMPLE XII

INPUT:

```
ID
 HAWAII TO CALIFORNIA (NIGHTTIME)
 &DATUM
 AZIM=58.5,CODIP=39.0,MAGFLD=4.31E-5,
 BETA=0.0,HPRIME=80.0,SCLHTS=6.0,
 FREQ=60.0,
 RANGER=60.0,90.0,RANGEI=0.0,-2.0,
 &END
```

PRINTED OUTPUT (PARTIAL):

```
TMESH =   0.250     LUB = 0.0050

COMPUTED PROFILE PARAMETERS:
   H          NE          NU          X           Y           Z           WR
  90.0    4.765E 03   1.012E 05   1.072E 02  -2.011E 01   2.685E-01   1.505E 08
  79.4    1.000E-01   1.217E 06   2.239E-03  -2.011E 01   3.228E 00   2.615E 02

     OMEGA-R = 2.5E05 AT TOP HEIGHT =    88.00

REFLECTION HT SET TO  89.42 KM     THETA RANGE =   85.000 TO   90.000

REFLECTION HT SET TO  89.42 KM     THETA RANGE =   80.000 TO   85.000

REFLECTION HT SET TO  89.42 KM     THETA RANGE =   75.000 TO   80.000

REFLECTION HT SET TO  90.42 KM     THETA RANGE =   70.000 TO   75.000

REFLECTION HT SET TO  93.42 KM     THETA RANGE =   65.000 TO   70.000

REFLECTION HT SET TO  93.42 KM     THETA RANGE =   60.000 TO   65.000
```

EXAMPLE XIII

PRINTED OUTPUT CONTINUED:

H = 50.00

MODE	THETA		ATTEN	VOVERC	WAIT MAG	WAIT ANG	THETAF		POL
1	86.739	-0.030	0.325	0.99376	-79.481	0.107	89.985	-6.417	0.64230
2	86.048	-0.166	2.196	0.99451	-66.344	0.317	88.891	-6.018	1.46497
3	84.583	-0.017	0.306	0.99660	-57.818	0.064	89.981	-4.730	0.66656
4	84.159	-0.110	2.146	0.99733	-22.706	0.239	89.346	-4.197	1.34101
5	83.234	-0.024	0.543	0.99911	-7.619	-0.117	89.933	-2.420	0.97130
6	82.886	-0.063	2.011	0.99985	-8.946	0.286	89.469	-1.136	0.71320
7	82.024	-0.064	1.699	1.00184	-5.170	-0.166	88.521	-0.147	1.35340
8	81.513	-0.060	2.281	1.00313	-11.498	0.427	85.467	-0.151	0.63947
9	80.590	-0.083	2.601	1.00568	-5.233	-0.206	83.904	-0.128	1.28110
10	80.025	-0.091	3.032	1.00738	-11.742	0.439	83.060	-0.132	0.73214
11	79.062	-0.093	3.384	1.01051	-7.010	-0.259	81.730	-0.123	1.15225
12	78.482	-0.105	4.021	1.01254	-11.043	0.426	80.972	-0.134	0.85126
13	77.494	-0.100	4.163	1.01626	-8.506	-0.327	79.756	-0.123	1.02989
14	76.904	-0.120	5.215	1.01864	-10.152	0.408	79.021	-0.144	0.98782
15	75.902	-0.106	4.954	1.02296	-10.060	-0.409	77.337	-0.123	0.92130
16	75.302	-0.136	6.609	1.02571	-9.244	0.382	77.143	-0.156	1.14250
17	74.292	-0.111	5.764	1.03064	-11.629	-0.503	75.993	-0.125	0.82648
18	73.679	-0.152	8.195	1.03381	-8.401	0.352	75.306	-0.169	1.31816
19	72.666	-0.115	6.507	1.03935	-13.243	-0.610	74.185	-0.127	0.74454
20	72.026	-0.168	9.966	1.04239	-7.812	0.319	73.491	-0.184	1.51967
21	71.026	-0.120	7.506	1.04915	-14.922	-0.731	72.592	-0.130	0.67454
22	70.374	-0.184	11.889	1.05334	-7.297	0.286	71.667	-0.198	1.75372
23	69.370	-0.125	8.492	1.06013	-16.056	-0.867	70.610	-0.134	0.61567
24	68.692	-0.200	13.964	1.06495	-8.907	0.254	69.886	-0.213	2.02821
25	67.697	-0.132	9.602	1.07237	-18.411	-1.023	68.629	-0.139	0.56825
26	66.988	-0.215	16.170	1.07792	-8.650	0.222	68.080	-0.227	2.35314
27	66.005	-0.139	10.880	1.08600	-20.157	-1.203	67.044	-0.146	0.53159
28	65.263	-0.230	18.458	1.09236	-8.451	0.191	66.266	-0.241	2.73990
29	64.291	-0.148	12.371	1.10115	-21.847	-1.409	65.249	-0.155	0.50595
30	63.513	-0.244	20.896	1.10849	-8.358	0.161	64.458	-0.254	3.19900
31	62.553	-0.160	14.129	1.11799	-23.434	4.638	63.439	-0.166	0.49134
32	61.737	-0.257	23.361	1.12643	-8.342	0.130	62.592	-0.206	3.73157
33	60.788	-0.173	16.208	1.13671	-24.053	4.369	61.609	-0.179	0.48751

TIME REQUIRED FOR THIS PROFILE = 335.80 SECONDS

EXAMPLE XIV

E. Error Messages

(1) EIGENANGLES AT θ_1 AND θ_2 ARE TO BE RESOLVED (program execution stops).

Solution: Assign to RANGER and RANGEI values which constitute a small "contour rectangle" which incloses the above two eigenangles and re-execute the program. An alternative is to decrease 'GMAX'.

(2) ERROR IN CONTROL CARD (Program execution stops).

Solution: Check control cards (i.e., PROFILE, COLFREQ, &DATUM, or &END)

(3) TOP OF PROFILE CANNOT BE SET -- At the bottom of the PROFILE, B is greater than the value of TEMPO-B (program execution stops).

Solution: Problem is due to incompatibilities between values of TEMPO-B and the actual electron and ion density values.

(4) WARNING -- THE TOP OF THE PROFILE CANNOT BE CUTOFF. THE TOP OF THE PROFILE MAY BE TOO LOW (program execution continues).

Reason: In top card of electron density profile the computed B value is less than the input value of TEMPO-B.

(5) THE PROFILE BOTTOM CANNOT BE CUTOFF. ALL B-VALUES (AS A FUNCTION OF HEIGHT) ARE LESS THAN CUTOFF. THIS IMPLIES THAT THE PROFILE IS VERY CLOSE TO FREE SPACE (Program execution stops).

Solution: Check values of 'CUTOFF', and individual values of electron and ion densities. If o.k., put 'ICUT' = 0 and re-execute.

(6) THE BOTTOM OF THE PROFILE CANNOT BE CUTOFF USING 'CUTOFF' = XXX AT HEIGHT = YYY BECAUSE THE VALUE OF B AT THE BOTTOM OF THE PROFILE IS LARGER THAN 'CUTOFF' (Program execution continues).

(7) NO MODES (program execution continues).

There are no eigenangle solutions located inside the contour rectangle as chosen. Solutions: Change RANGER and RANGEI values and re-execute the program.

(8) INVALID MODE AT θ (program execution continues).

Solution: Cut down the size of the contour rectange in the region of θ.

(9) STEP TOO SMALL IN INTEG OR TOO MANY STEPS IN INTEG (program execution stops).

Solution: Examine PROFILE (e.g. too dense at top or too sparse at bottom) and other ionospheric parameters such as magnetic field, collision frequency, etc. First decrease 'GMAX' and re-execute the program. If problem persists, increase 'ENMIN' and again re-execute.

(10) PROBLEM IN SORTING Q VALUES OR Q FAILS TO CONVERGE IN QUARTIC (program execution stops).

Solution: Again look at ionospheric parameter values. RANGER (minimum) is probably too small. Increase the value of RANGER (minimum) and re-execute program.

(11) X MODES FOUND ON SAME PHASE LINE (program execution continues).

Solution: Information only.

(12) WARNING X MODES FOUND ON Y PHASE LINES (program execution continues).

Solution: Information only.

(13) If run time is very long it is probably related to #8 above and thus solution is the same as for #8.

(14) NO EXIT FROM MESH SQUARE (program execution stops)

Solution: The value of TMESH should be changed slightly (1%) and the program rerun. If message continues - STOP.

(15) ONLY X MODES FOUND ON Y PHASE LINES (program execution stops).

Solution: Reduce TMESH by 1/2 and re-execute program.

(16) PROBLEMS IN SUBROUTINE NO MESH (program execution stops).

Solutions: Reduce TMESH by 1/2 and re-execute program. If message continues - STOP.

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APPENDIX A

Modification of the Modal Equation

Given:

$$F_1(\theta) = (1 - {}_{\parallel}\bar{R}_{\parallel}\,{}_{\parallel}R_{\parallel})\,(1 - {}_{\perp}\bar{R}_{\perp}\,{}_{\perp}R_{\perp}) - {}_{\parallel}R_{\perp}\,{}_{\perp}R_{\parallel}\,{}_{\parallel}\bar{R}_{\parallel}\,{}_{\perp}\bar{R}_{\perp} \tag{A-1}$$

where $R = R(\theta)$ and $\bar{R} = \bar{R}(\theta)$.

Let

$$ {}_{\parallel}\bar{R}_{\parallel} = \frac{{}_{\parallel}N_{\parallel}}{{}_{\parallel}D_{\parallel}}\ ,\ {}_{\perp}\bar{R}_{\perp} = \frac{{}_{\perp}N_{\perp}}{{}_{\perp}D_{\perp}} \tag{A-2}$$

where ${}_{\parallel}N_{\parallel}$, ${}_{\parallel}D_{\parallel}$, ${}_{\perp}N_{\perp}$ and ${}_{\perp}D_{\perp}$ are in such form that they contain no poles in the region of interest in the θ-plane.

Then

$$F_1(\theta) = (1 - \frac{{}_{\parallel}N_{\parallel}}{{}_{\parallel}D_{\parallel}}\,{}_{\parallel}R_{\parallel})\,(1 - \frac{{}_{\perp}N_{\perp}}{{}_{\perp}D_{\perp}}\,{}_{\perp}R_{\perp}) - {}_{\parallel}R_{\perp}\cdot{}_{\perp}R_{\parallel}\cdot\frac{{}_{\parallel}N_{\parallel}}{{}_{\parallel}D_{\parallel}}\cdot\frac{{}_{\perp}N_{\perp}}{{}_{\perp}D_{\perp}} \tag{A-3}$$

and

$$\begin{aligned} F_2(\theta) = {}_{\parallel}D_{\parallel}\cdot{}_{\perp}D_{\perp}F_1(\theta) &= ({}_{\parallel}D_{\parallel} - {}_{\parallel}N_{\parallel}\,{}_{\parallel}R_{\parallel})({}_{\perp}D_{\perp} - {}_{\perp}N_{\perp}\,{}_{\perp}R_{\perp}) \\ &\quad - {}_{\parallel}R_{\perp}\,{}_{\perp}R_{\parallel}\,{}_{\parallel}N_{\parallel}\,{}_{\perp}N_{\perp} \end{aligned} \tag{A-4}$$

Also

$$\left.\begin{aligned} F_2(\theta) = {} & [{}_{\parallel}D_{\parallel} - {}_{\parallel}N_{\parallel}\,C\,(\frac{{}_{\parallel}R_{\parallel}+1}{C}) + {}_{\parallel}N_{\parallel}] \\ & [{}_{\perp}D_{\perp} - {}_{\perp}N_{\perp}\,C\,(\frac{{}_{\perp}R_{\perp}+1}{C}) + {}_{\perp}N_{\perp}] \\ & - [C^2\,\frac{{}_{\parallel}R_{\perp}}{C}\cdot\frac{{}_{\perp}R_{\parallel}}{C}\cdot{}_{\parallel}N_{\parallel}\cdot{}_{\perp}N_{\perp}] \end{aligned}\right\} \tag{A-5}$$

Then

$$\left.\begin{aligned} F_3(\theta) &= \frac{F_2(\theta)}{C^2} = \frac{{}_{\parallel}D_{\parallel}\ {}_{\perp}D_{\perp}\ F_1(\theta)}{C^2} = \\ &\left[\frac{{}_{\parallel}N_{\parallel} + {}_{\parallel}D_{\parallel}}{C} - {}_{\parallel}N_{\parallel}\ \frac{({}_{\parallel}R_{\parallel} + 1)}{C}\right] \cdot \left[\frac{{}_{\perp}N_{\perp} + {}_{\perp}D_{\perp}}{C} - {}_{\perp}N_{\perp}\ \frac{({}_{\perp}R_{\perp} + 1)}{C}\right] \\ &- \left[{}_{\parallel}N_{\parallel} \cdot {}_{\perp}N_{\perp} \cdot \frac{{}_{\parallel}R_{\perp}}{C}\ \frac{{}_{\perp}R_{\parallel}}{C}\right] \end{aligned}\right\} \tag{A-6}$$

Let $\underset{\sim}{\overline{X}} = \dfrac{\underset{\sim}{R} + 1}{C}$

$$\frac{({}_{\parallel}N_{\parallel} + {}_{\parallel}D_{\parallel})}{C} = {}_{\parallel}n_{\parallel} \qquad \frac{({}_{\perp}N_{\perp} + {}_{\perp}D_{\perp})}{C} = {}_{\perp}n_{\perp} \tag{A-7}$$

$${}_{\parallel}N_{\parallel} = {}_{\parallel}d_{\parallel} \qquad {}_{\perp}N_{\perp} = {}_{\perp}d_{\perp}$$

Then

$$\begin{aligned} F_3(\theta) &= \frac{{}_{\parallel}D_{\parallel}\ {}_{\perp}D_{\perp}\ F_1(\theta)}{C^2} \\ &= [{}_{\parallel}n_{\parallel} - {}_{\parallel}d_{\parallel}\ {}_{\parallel}\overline{X}_{\parallel}]\ [{}_{\perp}n_{\perp} - {}_{\perp}d_{\perp}\ {}_{\perp}\overline{X}_{\perp}] - {}_{\parallel}d_{\parallel}\ {}_{\perp}d_{\perp}\ {}_{\parallel}\overline{X}_{\perp}\ {}_{\perp}\overline{X}_{\parallel} \end{aligned} \tag{A-8}$$

Note that:

$$\frac{{}_{\parallel}n_{\parallel}}{{}_{\parallel}d_{\parallel}} = \frac{{}_{\parallel}N_{\parallel} + {}_{\parallel}D_{\parallel}}{C\ {}_{\parallel}N_{\parallel}} = \frac{{}_{\parallel}D_{\parallel}/{}_{\parallel}N_{\parallel}+1}{C} = \left(\frac{1}{{}_{\parallel}\overline{R}_{\parallel}} + 1\right) 1/C = \frac{{}_{\parallel}\overline{R}_{\parallel} + 1}{C\ {}_{\parallel}\overline{R}_{\parallel}} \tag{A-9}$$

$$\frac{{}_{\perp}n_{\perp}}{{}_{\perp}d_{\perp}} = \frac{{}_{\perp}N_{\perp} + {}_{\perp}D_{\perp}}{C\ {}_{\perp}N_{\perp}} = \frac{{}_{\perp}D_{\perp}/{}_{\perp}N_{\perp}+1}{C} = \left(\frac{1}{{}_{\perp}\overline{R}_{\perp}} + 1\right) 1/C = \frac{{}_{\perp}\overline{R}_{\perp} + 1}{C\ {}_{\perp}\overline{R}_{\perp}} \tag{A-10}$$

Also note that no poles in N or D implies that there are no poles in n or d.

APPENDIX B

Derivation of the Equations for Free Space Integration

In the MODESRCH routine the differential equations for the ionospheric and ground reflection coefficient matrices have to be integrated through a free space region over a curved earth. In the following derivation of the necessary differential equations for the ionospheric reflection matrices a number of references are made to Budden's work, reference 7.

A right handed Cartesian system is assumed with the X and Y axes lying in a horizontal plane boundary between the bottom of the ionosphere and the free space below. The positive Z direction is upward. A plane wave is incident upon the ionosphere from below with the wave normal in the X-Z plane at an angle θ_I to the Z axis.

The differential equations describing wave fields at oblique incidence (Op. cit., P. 140, eqns. 9.49-9.54) may be separated into two independent sets. Either of these sets of field variables may be set equal to zero without affecting the other, so that the corresponding waves are propagated independently.

For the first set the electric field is everywhere parallel to the Y-axis, and the waves are said to be '<u>horizontally polarized</u>'. The equations are:

$$\frac{\partial E_y}{\partial Z} = i\ k\ H_x \tag{B-1}$$

$$i\ k\ S\ Ey = -i\ k\ H_z \tag{B-2}$$

$$\frac{\partial H_x}{\partial z} + i\ k\ S\ H_z = i\ k\ n^2\ Ey \tag{B-3}$$

where $i = \sqrt{-1}$

$S = \sin \theta_I$

$k = 2\pi/\lambda$

λ = free space wavelength

n = index of refraction of the medium

and $H = Z_0 H$

where $Z_0 = E_x/Hy$

H_z may be eliminated from eqns. 2 and 3 to give

$$\frac{\partial H_x}{\partial Z} = i\ k\ (n^2 - S^2)\ Ey \tag{B-4}$$

Elimination of H_x between eqns. 1 and 4 gives

$$\frac{d^2\ Ey}{dZ^2} + k^2\ q^2\ Ey = 0 \tag{B-5}$$

where $q^2 = (n^2 - S^2)$

For the second set of equations the electric field is parallel to the X-Z plane and the waves are said to be '<u>vertically polarized</u>' even though the electric vector is not, in general, vertical.

The equations are:

$$\frac{\partial\ E_x}{\partial Z} + i\ k\ S\ E_z = -i\ k\ Hy \tag{B-6}$$

$$\frac{\partial\ Hy}{\partial Z} = -i\ k\ n^2\ E_x \tag{B-7}$$

$$-i\ k\ S\ Hy = i\ k\ n^2\ E_z \tag{B-8}$$

The field component E_z may be eliminated from eqns. (6) and (8) to give

$$\frac{\partial\ E_x}{dZ} = -i\ k\ \frac{q^2}{n^2}\ Hy \tag{B-9}$$

Elimination of E_x between eqns. 7 and 9 gives

$$\frac{d^2 Hy}{dZ^2} - \frac{1}{n^2}\frac{d(n^2)}{dZ}\frac{d\,Hy}{dZ} + k^2 q^2 Hy \qquad \text{(B-10)}$$

Just below the free space-ionosphere boundary the total electric wave field is composed of two upward traveling incident components and two downward traveling reflected components. They are $E^i_{\parallel,y}$ and $E^r_{\parallel,y}$ respectively, where $E_\parallel$ is in the X-Y plane. Following Budden (OP Cit. Pg. 118 eq. 8.72) the equations for free space integration of reflection at oblique incidence may be derived.

The total field components just below the boundary are

$$E_x = (E_\parallel{}^i - E_\parallel{}^r)\cos\theta_I \qquad \text{(B-11,a)}$$

$$E_y = E_y{}^i + E_Y{}^r \qquad \text{(B-11,b)}$$

$$H_x = (E_y{}^r - E_y{}^i)\cos\theta_I \qquad \text{(B-11,c)}$$

$$H_y = E_\parallel{}^i + E_\parallel{}^r \qquad \text{(B-11,d)}$$

Define:

$$_\perp R_\perp = E_y{}^r/E_y{}^i \qquad \text{(B-12,a)}$$

$$_\parallel R_\perp = E_y{}^r/E_\parallel{}^i \qquad \text{(B-12,b)}$$

$$_\parallel R_\parallel = E_\parallel{}^r/E_\parallel{}^i \qquad \text{(B-12,c)}$$

$$_\perp R_\parallel = E_\parallel{}^r/E_y{}^i \qquad \text{(B-12,d)}$$

where the R's are functions of θ_I.

For <u>Horizontal Polarization</u>:

Combine eqns. (11,b), (12,a) and (12,b) to get

$$E_y = E_y{}^i + E_y{}^i\,{}_\perp R_\perp + E_\parallel{}^i\,{}_\parallel R_\perp$$

or

$$E_y = E_y^{\,i}\,({}_{\perp}R_{\perp} + 1) + E_{\parallel}^{\,i}\;{}_{\parallel}R_{\perp} \tag{B-13}$$

The wave equation for <u>Horizontal Polarization</u> (e.g., eq 5) is:

$$\frac{d^2 E_y}{dZ^2} + k^2 q^2 E_y = 0 \tag{B-5}$$

where $q^2 = n^2 - S^2$

and $S = \sin \theta_I$

To include a curved earth let

$$n^2 = 1 + \alpha\,(Z-H)$$

where:

n is the index of refraction of the medium,

$\alpha = 2/r_e$, where r_e is the radius of the earth.

H is that height where n = 1.

then $q^2 = 1 + \alpha\,(Z-H) - S^2 = C^2 + \alpha\,(Z-H)$

where $C = \cos \theta_I$

so that

$$\frac{d^2 E_y}{dZ^2} + k^2\,[C^2 + \alpha\,(Z-H)]\,E_y = 0 \tag{B-14}$$

to get standard form:

Let $\rho = (\frac{k}{\alpha})^{2/3}\,[C^2 + \alpha\,(Z-H)]$ (B-15)

then $\frac{d\rho}{dZ} = (\frac{k}{\alpha})^{2/3}\,\alpha = (k^2\,\alpha)^{1/3}$

$$\frac{d^2\rho}{dZ^2} = 0$$

and then expanding gives

$$\frac{d^2 E_y}{dZ^2} = \frac{d}{dZ}\left\{\frac{d E_y}{dZ}\right\} = \frac{d}{dZ}\left\{\frac{d E_y}{dZ\rho}\frac{d\rho}{dZ}\right\}$$

$$= \frac{d^2 E_y}{d\rho^2}\left(\frac{d\rho}{dZ}\right)^2 + \frac{d E_y}{d\rho}\frac{d^2\rho}{dZ^2}$$

now, substituting the above results into eq. B-14 gives

$$\frac{d^2 E_y}{dZ^2} = (k^2 \alpha)^{2/3} \frac{d^2 E_y}{d\rho^2} \tag{B-16}$$

Also, since

$$(k^2 \alpha)^{2/3} \left(\frac{k}{\alpha}\right)^{2/3} = k^2$$

then

$$k^2 q^2 = (k^2 \alpha)^{2/3} \left(\frac{k}{\alpha}\right)^{2/3} [C^2 + \alpha (z-h)]$$

or

$$k^2 q^2 = (k^2 \alpha)^{2/3} \rho$$

so that the wave equation (eq. 5) becomes upon substitution of the above:

$$(k^2 \alpha)^{2/3} \left\{\frac{d^2 E_y}{d\rho^2} + \rho E_y\right\} = 0$$

or

$$\frac{d^2 E_y}{d\rho^2} + \rho E_y = 0 \tag{B-17}$$

The general solution to this equation is

$$E_y(\rho) = A h_1 (\rho) + B h_2 (\rho) \tag{B-18}$$

where A and B are arbitrary constants and $h_1(\rho)$ and $h_2(\rho)$ are modified Hankel functions of order 1/3. (See reference 8).

To relate E_y and H_x:

equation (1) is:

$$\frac{d\,E_y}{dZ} = i\,k\,H_x$$

or

$$H_x = -\frac{i}{k}\,\frac{d\,E_y}{dZ}$$

$$= -\frac{i}{k}\,\frac{d\,E_y}{d\rho}\,\frac{d\rho}{dZ} = -\frac{i}{k}\,(k^2\,\alpha)^{1/3}\,\frac{d\,E_y}{d\rho}$$

$$H_x = -k\,\left(\frac{\alpha}{k}\right)^{1/3}\,\frac{d\,E_y}{d\rho}$$

let $\quad K = i\,\left(\frac{\alpha}{k}\right)^{1/3}$ (B-19)

then $\quad H_x = -K\,\frac{d\,E_y}{d\rho}$

or

$$H_x = -K\{A\,h_1'\,(\rho) + B\,h_2'\,(\rho)\} \qquad \text{(B-20)}$$

The primes on the above quantities denote derivatives with respect to the argument.

From Eq. 13,

$$E_y = C\,E_y{}^i\left(\frac{{}_\perp R_\perp + 1}{C}\right) + C\,E_\parallel{}^i\left(\frac{{}_\parallel R_\perp}{C}\right) \qquad \text{(B-21)}$$

and from eq. 11c

$$H_x = CE_y^r - CE_y{}^i$$

also from eq. 11,b

$$E_y{}^r = E_y - E_y{}^i$$

Substitution for E_y^r into eq. (11,c) gives

$$C\,E_y - C\,E_y^i = H_x + C\,E_y^i$$

or

$$C\,E_y - H_x = 2C\,E_y^i \qquad \text{(B-22)}$$

Substitutions into eq. 21 for E_y from eq. 18 gives

$$A\,h_1(\rho) + B\,h_2(\rho) = C\,E_y^i\left(\frac{{}_\perp R_\perp + 1}{C}\right) + C\,E_\parallel^i\left(\frac{{}_\parallel R_\perp}{C}\right)$$

and substitution into eq. 22 for E_y and H_x from equations 18 and 20 gives

$$C\,\{A\,h_1(\rho) + B\,h_2(\rho)\} + K\,\{A\,h_1'(\rho) + B\,h_2'(\rho)\} = 2\,C\,E_y^I$$

or

$$\boxed{(-\tfrac{A}{C})\,h_1(\rho) + (-\tfrac{B}{C})\,h_2(\rho) + E_y^i\left(\frac{{}_\perp R_\perp + 1}{C}\right) + E_\parallel^i\left(\frac{{}_\parallel R_\perp}{C}\right) = 0} \qquad \text{(B-23)}$$

$$\boxed{(-\tfrac{A}{C})\{Ch_1(\rho) + Kh_1'(\rho)\} + (-\tfrac{B}{C})\{Ch_2(\rho) + Kh_2'(\rho)\} + 2E_y^i = 0} \qquad \text{(B-24)}$$

Equations 23 and 24 are the equations to be used for <u>'horizontal polarization'</u> in the free space integration procedure.

For <u>Vertical Polarization</u>:

Applying previous equations (e.g., equations 11 and 12) gives:

$$H_y = E_\parallel^i + E_\parallel^r \qquad \text{(B-11,d)}$$

$$E_x = (E_\parallel^i - E_\parallel^r)\cos\theta_1 \qquad \text{(B-11,a)}$$

$${}_\parallel R_\parallel = E_\parallel^r / E_\parallel^i \qquad \text{(B-12,c)}$$

$${}_\perp R_\parallel = E_\parallel^r / E_y^i \qquad \text{(B-12,d)}$$

Combine eqns. (11,d), (12,c) and (12,d) to get

$$H_y = E_{\parallel}^{\,i} + E_{\parallel}^{\,i}\ {}_{\parallel}R_{\parallel} + E_y^{\,i}\ {}_{\perp}R_{\parallel}$$

also combining eqns. (11,a), (12,c) and (12,d)

$$E_x = (E_{\parallel}^{\,i} - E_{\parallel}^{\,i}\ {}_{\parallel}R_{\parallel} - E_y^{\,i}\ {}_{\perp}R_{\parallel})\ C$$

In different form:

$$H_y = C\ E_{\parallel}^{\,i}\ \frac{({}_{\parallel}R_{\parallel}+1)}{C} + C\ E_y^{\,i}\left(\frac{{}_{\perp}R_{\parallel}}{C}\right) \qquad \text{(B-25)}$$

where $C = \cos\theta_I$

and from eqns. (11,a) and (11,d)

$$E_x = E_{\parallel}^{\,i}\ C - H_y\ C + E_{\parallel}^{\,i}\ C$$

or

$$C\ H_y + E_x = 2E_{\parallel}^{\,i}\ C \qquad \text{(B-26)}$$

To obtain the wave equation for vertical polarization, follow Budden (op. ct. pg. 343). Note that in this case the electric vector has both vertical (E_z) and horizontal (E_y) components.

The wave equation for <u>vertical polarization</u> is given by equation 10 as:

$$\frac{d^2 H_y}{dZ^2} - \frac{1}{n^2}\frac{d(n^2)}{dZ}\frac{d\ H_y}{dZ} + k^2\ q^2\ H_y = 0 \qquad \text{(B-10)}$$

Any linear second-order differential equation with a term containing the first derivative (e.g., $d\ H_y/dZ$) can be reduced to its 'normal form', that is a form without a first derivative term, by a change in the dependent variable.

Let $\quad V = H_y/n \qquad$ (B-27)

or $\quad H_y = nV \qquad$ (B-28)

then equation 10 becomes:

$$\frac{d^2V}{dZ^2} + k^2 \left[q^2 + \frac{1}{2k^2\ n^2} \frac{d^2(n^2)}{dZ^2} - \frac{3}{4k^2\ n^4} \left\{ \frac{d(n^2)}{dZ} \right\}^2 \right] V = 0 \qquad \text{(B-29)}$$

where for a curved earth:

$$q^2 = C^2 + \alpha(Z-H)$$

Note that:

$$\frac{d(n^2)}{dZ} = \frac{d}{dZ}\ (1 + \alpha(Z-H)) = \alpha \qquad \text{(B-30)}$$

and

$$\frac{d^2(n^2)}{dZ} = 0 \qquad \text{(B-31)}$$

The wave equation, Eq. 29, then becomes, after substitution of eqns. 30, and 31:

$$\frac{d^2V}{dZ^2} + k^2 \left[C^2 + \alpha(Z-H) - \underbrace{\frac{3\ \alpha^2}{4k^2\ n^4}}_{\text{neglect this term}} \right] V = 0 \qquad \text{(B-32)}$$

let

$$\rho = \left(\frac{k}{\alpha}\right)^{2/3} [C^2 + \alpha(Z-H)] \qquad \text{(B-33)}$$

Compare the resulting form of eq. 32 with eq. 17

Therefore the solution is

$$V = Q\ h_1(\rho) + G\ h_2(\rho) \qquad \text{(B-34)}$$

and since

$$H_y = nV \qquad \text{(B-28)}$$

$$H_y = n\ \{Q\ h_1(\rho) + G\ h_2(\rho)\} \qquad \text{(B-35)}$$

Note:

$$\frac{d(n^2)}{dZ} = 2\,n\,\frac{dn}{dZ} \qquad \text{or} \quad \frac{dn}{dZ} = \frac{1}{2n}\,\frac{d(n^2)}{dZ}$$

but from eq. 30 $\qquad \dfrac{d(n^2)}{dZ} = \alpha$

$$\therefore \frac{dn}{dz} = \frac{\alpha}{2n} \tag{B-36}$$

Also from eq. 33,

$$\frac{d\rho}{dZ} = \left(\frac{k}{\alpha}\right)^{2/3} \alpha = \left(\frac{k^2\,\alpha^3}{\alpha^2}\right)^{1/3} = (k^2\,\alpha)^{1/3} \tag{B-37}$$

To relate H_y and E_x:

From eq. (7):

$$\frac{d\,H_y}{dZ} = -i\,k\,n^2\,E_x \tag{B-7}$$

or

$$E_x = \frac{i}{k\,n^2}\,\frac{d\,H_y}{dZ} = \frac{i}{k\,n^2}\,\frac{d}{dZ}\,(nV)$$

or

$$E_x = \frac{i}{k\,n^2}\left\{n\,\frac{dV}{dZ} + \frac{dn}{dZ}\,V\right\}$$

and

$$E_x = \frac{i}{k\,n^2}\left\{n\,\frac{dV}{d\rho}\,\frac{d\rho}{dZ} + \frac{dn}{dZ}\,V\right\} \tag{B-38}$$

Substituting eqns. 36 and 37 into eq. 38 gives

$$E_x = \frac{i}{k\,n^2}\left\{n\,(k^2\alpha)^{1/3}\,\frac{dV}{d\rho} + \frac{\alpha}{2n}\,V\right\}$$

or

$$E_x = \frac{1}{n}\left\{i\,\left(\frac{\alpha}{k}\right)^{1/3}\,\frac{dV}{d\rho} + i\,\frac{\alpha}{2k}\,\frac{V}{n^2}\right\} \tag{B-39}$$

Now define

$$K = i \left(\frac{\alpha}{k}\right)^{1/3} \tag{B-40}$$

and

$$L = i \left(\frac{\alpha}{2k}\right) \tag{B-41}$$

Then

$$E_x = \frac{1}{n} \left\{ K \frac{dV}{dZ} + L \frac{V}{n^2} \right\} \tag{B-42}$$

Since L is small let $n^2 \approx 1$

Then

$$\frac{1}{n} E_x = \frac{1}{n^2} \left\{ K \frac{dV}{d\rho} + L V \right\} \tag{B-43}$$

Substituting for V from Eq. (34) gives:

$$\frac{1}{n} E_x = \frac{1}{n^2} \{ K [Q h_1' (\rho) + G h_2' (\rho)]$$

$$+ L [Q h_1(\rho) + G h_2(\rho)]\} \tag{B-44}$$

Substituting eq. 35 into eq. 25 for H_y gives

$$n \left\{ Q h_1(\rho) + G h_2(\rho) \right\} = C E_{\parallel}{}^{i} \left(\frac{{}_{\parallel}R_{\parallel}+1}{C} \right) + C E_y{}^{i} \left(\frac{{}_{\perp}R_{\parallel}}{C} \right) \tag{B-45}$$

Substituting eqns. 35 and 44 into eq. (26) for H_y and E_x gives:

$$n \Big[C \{ Q h_1(\rho) + G h_2(\rho) + \frac{1}{n^2} \{ K [Qh_1' (\rho) + Gh_2' (\rho)]\} +$$

$$\frac{1}{n^2} \{ L [Qh_1(\rho) + Gh_2 (\rho)]\} \Big] = 2E_{\parallel}{}^{i} C \tag{B-46}$$

Now neglect the multiplier n so that $\perp$ and $\parallel$ will be alike for coupling to get from equation 45:

$$\boxed{(-\frac{Q}{C}) h_1(\rho) + (-\frac{G}{C}) h_2(\rho) + E_{\parallel}{}^{i} \left(\frac{{}_{\parallel}R_{\parallel} + 1}{C} \right) + E_y{}^{i} \left(\frac{{}_{\perp}R_{\parallel}}{C} \right) = 0} \tag{B-47}$$

and from equation 46:

$$\boxed{\begin{aligned}&\left(-\frac{Q}{C}\right)\{C\,h_1(\rho) + [K\,h_1'(\rho) + L\,h_1(\rho)]/n^2\}\\ &+\left(-\frac{G}{C}\right)\{C\,h_2(\rho) + [K\,h_2'(\rho) + L\,h_2(\rho)]/n^2\} + 2\,E_{\parallel}{}^{i} = 0\end{aligned}} \qquad \text{B-48}$$

Equations 47 and 48 are the equations to be used for <u>'vertical polarization'</u> in the free space integration procedure.

The four equations for ionospheric reflection at oblique incidence (e.g., eqns. 23, 24, 47 and 48) may be used to accomplish the full-wave integration of the four ionospheric reflection coefficients ($\tilde{R}$) through a free space medium over a curved earth. The integration is to be carried out from some starting height "Z_f", where the values of the R's are known, to some other level "Z_t" where the values of the R's are to be determined.

The free space integration procedure consists of three steps (I, II and III) which are as follows:

I. At the "from" level (i.e., $Z = Z_f$) the known values of the elements of $\underset{\sim}{R}$ may be substituted into equations (23, 24, 47, and 48) and then two sets of boundary conditions, B. C., applied successfully. That is for Set I assign ($E_{\parallel}{}^{i} = 1$, $E_y{}^{i} = 0$) and solve for the coefficients A_I and B_I from equations 23 and 24; and for the coefficients Q_I and G_I from equations 47 and 48. Next, for Set II assign ($E_y{}^{i} = 1$, $E_{\parallel}{}^{i} = 0$) and solve for the coefficients A_{II} and B_{II} from equations 23 and 24; and for the coefficients Q_{II} and G_{II} from equations 47 and 48.

The procedure includes a need for solving two simultaneous equations. That is if:

$$a_{11}\overline{A} + a_{12}\overline{B} = V_1 \qquad \text{(B-49,a)}$$

$$a_{21}\overline{A} + a_{22}\overline{B} = V_2 \qquad \text{(B-49,b)}$$

then

$$\bar{A} = \frac{(V_1\ a_{22} - V_2\ a_{12})}{(a_{11}\ a_{22} - a_{21}\ a_{12})} \qquad \text{(B-50,a)}$$

$$\bar{B} = \frac{(V_2\ a_{11} - V_1\ a_{21})}{(a_{11} a_{22} - a_{21} a_{12})} \qquad \text{(B-50,b)}$$

For <u>Horizontal Polarization</u> at the Z_f (from) level, the following identifications are made:

Identify terms of eq. 23 with eq. (49,a) and terms of eq. 24 with (49,b). That is let:

$$\left.\begin{aligned}
&\bar{A} = -\left(\frac{A}{C}\right)\ ,\quad \bar{B} = -\left(\frac{B}{C}\right)\ ; \\
&\text{where} \\
&C = \cos\theta_I\ , \qquad \rho_f : \text{ implies a function of } \rho \text{ at the "from" level.} \\
&\rho_f = \left(\frac{k}{\alpha}\right)^{2/3} [C + \alpha(Z_f - H)] \\
&a_{11} = h_1(\rho_f)\ , \qquad a_{12} = h_2(\rho_f) \\
&a_{21} = C\ h_1(\rho_f) + K\ h_1'(\rho_f) \\
&a_{22} = C\ h_2(\rho_f) + K\ h_2'(\rho_f) \\
&V_1 = -E_y^{\,i}\left(\frac{{}_{\perp}R_{\perp}^{f} + 1}{C}\right) - E_{\parallel}^{\,i}\left(\frac{{}_{\parallel}R_{\perp}^{f}}{C}\right) \\
&V_2 = -2\ E_y^{\,i} \\
&\text{where from (19)} \\
&K = i\ \left(\frac{\alpha}{k}\right)^{1/3}
\end{aligned}\right\} \text{(B-51)}$$

For Set I B.C.: $(E_{11}^{i} = 1,\ E_{y}^{i} = 0)$ Applying these boundary conditions and the relations of equation 51 to equation 23 and to equation 24 gives:

$$\left\{\begin{array}{l} \left(-\frac{A}{C}\right)_{I} a_{11} + \left(\frac{-B}{C}\right)_{I} a_{12} = \left(\frac{{}_{\parallel}R_{\perp}^{f}}{C}\right) \\ \\ \left(-\frac{A}{C}\right)_{I} a_{21} + \left(\frac{-B}{C}\right)_{I} a_{22} = 0 \end{array}\right\}$$

Solving these equations simultaneously by the application of equation 50(A,B) gives:

$$\boxed{\left(\frac{A}{C}\right)_{I} = \frac{{}_{\parallel}R_{\perp}^{f}}{C} \cdot \frac{a_{22}}{\Delta_1}} \qquad \text{(B-52,a)}$$

$$\boxed{\left(\frac{B}{C}\right)_{I} = -\frac{{}_{\parallel}R_{\perp}^{f}}{C} \cdot \frac{a_{21}}{\Delta_1}} \qquad \text{(B-52,b)}$$

where

$$\boxed{\Delta_1 = (a_{11}a_{22} - a_{21}a_{12})} \qquad \text{(B-53)}$$

with a_{ij} as defined in equation 51

For Set II B.C.: $(E_{\parallel}{}^{i} = 0,\ E_{y}{}^{i} = 1)$ Applying these B.C. conditions and the relations of equation 51 to equation 23 and equation 24 gives:

$$\left(-\frac{A}{C}\right)_{II} a_{11} + \left(-\frac{B}{C}\right)_{II} a_{12} = -\frac{\left({}_{\perp}R_{\perp}^{f} + 1\right)}{C}$$

$$\left(-\frac{A}{C}\right)_{II} a_{21} + \left(-\frac{B}{C}\right)_{II} a_{22} = -2$$

Solving these equations simultaneously by the application of equations 50(A,B) gives:

$$\boxed{\left(\frac{A}{C}\right)_{II} = \left(\frac{\left({}_{\perp}R^{f}_{\perp} + 1\right)}{C} a_{22} - 2a_{12}\right)\frac{1}{\Delta_1}} \qquad \text{(B-54,a)}$$

$$\boxed{\left(\frac{B}{C}\right)_{II} = \left(2a_{11} - \left(\frac{{}_{\perp}R^{f}_{\perp} + 1}{C}\right)a_{21}\right)\frac{1}{\Delta_1}} \qquad \text{(B-54,b)}$$

Where Δ_1 is again defined in equation 53.

For <u>Vertical Polarization</u> at the Z_f, "from", level the following identifications are made.

Identify terms of equation 47 with equation (49,a) and terms of equation 48 with (49,b). That is let:

$$\left.\begin{aligned}
&\overline{A} = \left(-\frac{Q}{C}\right),\ \overline{B} = \left(-\frac{G}{C}\right)\\
&a_{11} = h_1(\rho_f),\ a_{12} = h_2(\rho_f)\\
&a_{21} = C\,h_1(\rho_f) + [K\,h_1'(\rho_f) + L\,h_1(\rho_f)]/n_f^{\,2}\\
&a_{22} = C\,h_2(\rho_f) + [K\,h_2'(\rho_f) + L\,h_2(\rho_f)]/n_f^{\,2}\\
&\text{Where from equation 40 and 41}\\
&K = i\left(\frac{\alpha}{k}\right)^{1/3},\ L = i\left(\frac{\alpha}{2k}\right)\\
&\text{and}\\
&V_1 = -E^i_{\parallel}\left(\frac{{}_{\parallel}R^{f}_{\parallel} + 1}{C}\right) - E^i_y\left(\frac{{}_{\perp}R^{f}_{\parallel}}{C}\right)\\
&V_2 = -2E^i_{\parallel}
\end{aligned}\right\} \qquad \text{(B-55)}$$

For Set I B.C.: $(E_{\parallel}^{i} = 1,\ E_{y}^{i} = 0)$ Applying these B.C. conditions and the relations of equation 55 to equations 47 and 48 gives:

$$\left(\frac{Q}{C}\right)_{I} a_{11} + \left(\frac{G}{C}\right)_{I} a_{12} = \left(\frac{{}_{\parallel}R_{\parallel}^{f} + 1}{C}\right)$$

$$\left(\frac{Q}{C}\right)_{I} a_{21} + \left(\frac{G}{C}\right)_{I} a_{22} = 2$$

Solving these equations simultaneously by the application of equations 50(A,B) gives:

$$\boxed{\left(\frac{Q}{C}\right)_{I} = \left[\left(\frac{{}_{\parallel}R_{\parallel}^{f} + 1}{C}\right) a_{22} - 2a_{12}\right] \frac{1}{\Delta_2}} \qquad \text{(B-56,a)}$$

$$\boxed{\left(\frac{G}{C}\right)_{I} = \left[2a_{11} - \left(\frac{{}_{\parallel}R_{\parallel}^{f} + 1}{C}\right) a_{21}\right] \frac{1}{\Delta_2}} \qquad \text{(B-56,b)}$$

Where

$$\Delta_2 = (a_{11}a_{22} - a_{21}a_{12})$$

with a_{ij} as defined in equation 55

For Set II B.C.: $(E_{\parallel}^{i} = 0,\ E_{y}^{i} = 1)$ Applying these boundary conditions and the relations of equation 55 to equations 47 and 48 gives:

$$\left(\frac{Q}{C}\right)_{II} a_{11} + \left(\frac{G}{C}\right)_{II} a_{12} = \frac{{}_{\perp}R_{\parallel}^{f}}{C}$$

$$\left(\frac{Q}{C}\right)_{II} a_{21} + \left(\frac{G}{C}\right)_{II} a_{22} = 0$$

Solving this equation simultaneously by the application of equations 50(A,B) gives:

$$\boxed{\left(\frac{Q}{C}\right)_{II} = \frac{{}_{\perp}R_{\parallel}^{f}}{C} \frac{a_{22}}{\Delta_2}} \qquad \text{(B-57,a)}$$

$$\boxed{\left(\frac{G}{C}\right)_{II} = -\frac{{}_{\perp}R_{\parallel}^{f}}{C} \frac{a_{21}}{\Delta_2}} \qquad \text{(B-57,b)}$$

Where Δ_2 is again defined in equation 55

II. The values of the electric field terms $E_{\parallel}^{i}$ and E_{y}^{i} at the "to" level (e.g., at height = Z_t) may be determined from the values of the coefficients (A, B, F and G) as determined in Step I for the Set I and Set II boundary conditions.

For Horizontal Polarization at the Z_t "to" level, the following identifications are made.

Identify terms of equation 24 with equation 49,b. That is:

$$\left.\begin{array}{l}
\bar{A} = \left(-\frac{A}{C}\right),\ \bar{B} = \left(-\frac{B}{C}\right) \\
a_{21} = C\, h_1(\rho_t) + K\, h_1'(\rho_t);\ V_2 = -2\left(E_y^{\,i}\right)_t \\
a_{22} = C\, h_2(\rho_t) + K\, h_2'(\rho_t) \\
\text{Where} \\
\rho_t = \left(\frac{k}{\alpha}\right)^{2/3} \left[C^2 + \alpha(Z_t - H)\right] \\
\rho_t\text{: implies a function of } \rho \text{ at "to" level} \\
K = i\left(\frac{\alpha}{k}\right)^{1/3}
\end{array}\right\} \qquad \text{(B-58)}$$

For Set I B.C.: (e.g. $(E_{\parallel}^{i})_f = 1$; $(E_y^{\,i})_f = 0$)

$$\left(-\frac{A}{C}\right) a_{21} + \left(-\frac{B}{C}\right) a_{22} = -2\left(\left(E_y^{\,i}\right)_t\right)_I$$

or

$$\left(\left(E_y^{i}\right)_t\right)_I = \left(\frac{\left(\frac{A}{C}\right)_I a_{21} + \left(\frac{B}{C}\right)_I a_{22}}{2}\right) \qquad \text{(B-59)}$$

Where the values of $(A/C)_I$ and $(B/C)_I$ were determined by equation 52 and the a_{ij} are given in equation 58.

For Set II B.C.: $\left(\text{e.g. } \left(E_{\parallel}{}^{i}\right)_f = 0,\ \left(E_y{}^{i}\right)_f = 1\right)$

$$\left(-\frac{A}{C}\right)_{II} a_{21} + \left(-\frac{B}{C}\right)_{II} a_{22} = -2\left(\left(E_y{}^{i}\right)_t\right)_{II}$$

or

$$\boxed{\left(\left(E_y{}^{i}\right)_t\right)_{II} = \left(\frac{\left(\frac{A}{C}\right)_{II} a_{21} + \left(\frac{B}{C}\right)_{II} a_{22}}{2}\right)} \tag{B-60}$$

Where the values of $(A/C)_{II}$ and $(B/C)_{II}$ were determined by equation 54 and the a_{ij} are given by equation 58.

For <u>Vertical Polarization</u> at the Z_t "to" level, the following identifications are made.

Identify terms of equation 48 with equation (49,b). That is:

$$\left.\begin{aligned}
\bar{A} &= \left(-\frac{Q}{C}\right);\ \bar{B} = \left(-\frac{G}{C}\right) \\
a_{21} &= \left\{C\, h_1(\rho_t) + \left[K\, h_1'(\rho_t) + L\, h_1(\rho_t)\right]/h^2\right\} \\
a_{22} &= \left\{C\, h_2(\rho_t) + \left[K\, h_2'(\rho_t) + L\, h_2(\rho_t)\right]/h^2\right\} \\
V_2 &= -2\left(E_{\parallel}{}^{i}\right)_t \\
L &= i\left(\frac{\alpha}{2k}\right) \text{ from equation 41}
\end{aligned}\right\} \tag{B-61}$$

For Set I B.C.: $\left(\text{e.g. } \left(E_{\parallel}{}^{i}\right)_f = 1;\ \left(E_y{}^{i}\right)_f = 0\right)$

$$\left(-\frac{Q}{C}\right)_I a_{21} + \left(-\frac{G}{C}\right)_I a_{22} = -2\left(\left(E_{\parallel}{}^{i}\right)_t\right)_I$$

$$\boxed{\text{or } \left(\left(E_{\parallel}{}^{i}\right)_t\right)_I = \frac{\left[\left(\frac{Q}{C}\right)_I a_{21} + \left(\frac{G}{C}\right)_I a_{22}\right]}{2}} \tag{B-62}$$

Where $(A/C)_I$ and $(G/C)_I$ are given by equation 56 and the a_{ij} are given by equation 61.

For Set II B.C.: $\left(\text{e.g. } \left(E_{\parallel}{}^{i}\right)_f = 0;\ \left(E_y{}^{i}\right)_f = 1\right)$

$$\left(-\frac{Q}{C}\right)_{II} a_{21} + \left(-\frac{G}{C}\right)_{II} a_{22} = -2\left(\left(E_{\parallel}{}^{i}\right)_t\right)_{II}$$

$$\boxed{\text{or}\quad \left(\left(E_{\parallel}{}^{i}\right)_t\right)_{II} = \frac{\left[\left(\frac{Q}{C}\right)_{II} a_{21} + \left(\frac{G}{C}\right)_{II} a_{22}\right]}{2}} \qquad \text{(B-63)}$$

Where $(Q/C)_{II}$ and $(G/C)_{II}$ are given by equation 57 and the a_{ij} are given by equation 61.

III. Knowing the values of

$$\left(\left(E_{\parallel}{}^{i}\right)_t\right)_I \quad \left(\left(E_{\parallel}{}^{i}\right)_t\right)_{II},\ \left(\left(E_y{}^{i}\right)_t\right)_I \text{ and } \left(\left(E_y{}^{i}\right)_t\right)_{II}$$

at the "to" level, from both sets of coefficients A, B, Q and G, as determined in Step II, the elements of the ionospheric R's can be found at the "to" level.

For <u>Horizontal Polarization</u> equation 23 at the "to" level is:

$$\boxed{\begin{aligned}&\left(-\frac{A}{C}\right)h_1(\rho_t) + \left(-\frac{B}{C}\right)h_2(\rho_t) + \left[E_y{}^{i}\right]_t\left(\frac{{}_{\perp}R_{\perp} + 1}{C}\right)_t \\ &+ \left[E_{\parallel}{}^{i}\right]_t\left(\frac{{}_{\parallel}R_{\perp}}{C}\right)_t = 0\end{aligned}} \qquad \text{(B-64)}$$

Applying the B.C. of Set I for $(A/C)_I$ and $(B/C)_I$ to equation (64) and applying the B.C. of Set II for $(A/C)_{II}$ and $(B/C)_{II}$ to equation (64) gives two equations.

$$\left(\left(E_y^{\,i}\right)_t\right)_I \left(\frac{{}_{\perp}R_{\perp} + 1}{C}\right)_t + \left(\left(E_{\parallel}^{\,i}\right)_t\right)_I \left(\frac{{}_{\parallel}R_{\perp}}{C}\right)_t$$

$$+ \left(-\frac{A}{C}\right)_I h_1(\rho_t) + \left(-\frac{B}{C}\right)_I h_2(\rho_t) = 0 \qquad \text{(B-65)}$$

and

$$\left(\left(E_y^{\,i}\right)_t\right)_{II} \left(\frac{{}_{\perp}R_{\perp} + 1}{C}\right)_t + \left(\left(E_{\parallel}^{\,i}\right)_t\right)_I \left(\frac{{}_{\parallel}R_{\perp}}{C}\right)$$

$$+ \left(-\frac{A}{C}\right)_{II} h_1(\rho_t) + \left(-\frac{B}{C}\right)_{II} h_2(\rho_t) = 0 \qquad \text{(B-66)}$$

Identifying terms of equations 65 and 66 with equations 49,a and 49,b respectively gives:

$$\left.\begin{aligned}
&\overline{B} = \left(\frac{{}_{\perp}R_{\perp} + 1}{C}\right)_t \qquad \overline{A} = \left(\frac{{}_{\parallel}R_{\perp}}{C}\right)_t \\
&a_{11} = \left(\left(E_{\parallel}^{\,i}\right)_t\right)_I \qquad a_{12} = \left(\left(E_y^{\,i}\right)_t\right)_I \\
&a_{21} = \left(\left(E_{\parallel}^{\,i}\right)_t\right)_{II} \qquad a_{22} = \left(\left(E_y^{\,i}\right)_t\right)_{II} \\
&V_1 = \left(\frac{A}{C}\right)_I h_1(\rho_t) + \left(\frac{B}{C}\right)_I h_2(\rho_t) \\
&V_2 = \left(\frac{A}{C}\right)_{II} h_1(\rho_t) + \left(\frac{B}{C}\right)_{II} h_2(\rho_t)
\end{aligned}\right\} \qquad \text{(B-67)}$$

Then from equation (50,a):

$$\boxed{\left(\frac{{}_{\parallel}R_{\perp}}{C}\right)_t = \frac{V_1\left(\left(E_y^{\,i}\right)_t\right)_{II} - V_2\left(\left(E_y^{\,i}\right)_t\right)_I}{\left(\left(E_{\parallel}^{\,i}\right)_t\right)_I \left(\left(E_y^{\,i}\right)_t\right)_{II} - \left(\left(E_{\parallel}^{\,i}\right)_t\right)_{II} \left(\left(E_y^{\,i}\right)_t\right)_I}} \qquad \text{(B-68)}$$

And from equation (50,b):

$$\boxed{\left(\frac{{}_{\perp}R_{\perp} + 1}{C}\right)_t = \frac{V_2\left(\left(E_{\parallel}^{\,i}\right)_t\right)_I - V_1\left(\left(E_{\parallel}^{\,i}\right)_t\right)_{II}}{\left(\left(E_{\parallel}^{\,i}\right)_t\right)_I \left(\left(E_y^{\,i}\right)_t\right)_{II} - \left(\left(E_{\parallel}^{\,i}\right)_t\right)_{II} \left(\left(E_y^{\,i}\right)_t\right)_I}} \qquad \text{(B-69)}$$

For Vertical Polarization equation 47 at the "to" level is:

$$\boxed{\left(-\frac{Q}{C}\right)h_1\rho_t + \left(-\frac{G}{C}\right)h_2(\rho_t) + \left(E_{\parallel}{}^{i}\right)_t \left(\frac{{}_{\parallel}R_{\parallel} + 1}{C}\right)_t + \left(E_y{}^{i}\right)_t \left(\frac{{}_{\perp}R_{\parallel}}{C}\right)_t = 0} \qquad \text{(B-70)}$$

Applying the conditions of Set I for $(Q/C)_I$ and $(G/C)_I$ to equation 70 and applying the conditions of Set II for $(Q/C)_{II}$ and $(G/C)_{II}$ to equation (70) gives the two equations:

$$\left(\left(E_{\parallel}{}^{i}\right)_t\right)_I \left(\frac{{}_{\parallel}R_{\parallel} + 1}{C}\right)_t + \left(\left(E_y{}^{i}\right)_t\right)_I \left(\frac{{}_{\perp}R_{\parallel}}{C}\right)_t + \left[\left(-\frac{Q}{C}\right)_I h_1(\rho_t) + \left(-\frac{G}{C}\right)_I h_2(\rho_t)\right] = 0 \qquad \text{(B-71)}$$

and

$$\left(\left(E_{\parallel}{}^{i}\right)_t\right)_{II} \left(\frac{{}_{\parallel}R_{\parallel} + 1}{C}\right)_t + \left(\left(E_y{}^{i}\right)_t\right)_I \left(\frac{{}_{\perp}R_{\parallel}}{C}\right)_t + \left[\left(-\frac{Q}{C}\right)_{II} h_1(\rho_t) + \left(-\frac{G}{C}\right)_{II} h_2(\rho_t)\right] = 0 \qquad \text{(B-72)}$$

Identifying terms of equation 65 with equation 49,a

$$\left.\begin{aligned}
\bar{A} &= \left(\frac{{}_{\parallel}R_{\parallel} + 1}{C}\right)_t & \bar{B} &= \left(\frac{{}_{\perp}R_{\parallel}}{C}\right)_t \\
a_{11} &= \left(\left(E_{\parallel}{}^{i}\right)_t\right)_I & a_{12} &= \left(\left(E_y{}^{i}\right)_t\right)_I \\
a_{21} &= \left(\left(E_{\parallel}{}^{i}\right)_t\right)_{II} & a_{22} &= \left(\left(E_y{}^{i}\right)_t\right)_{II} \\
V_1 &= \left(\frac{Q}{C}\right)_I h_1(\rho_t) + \left(\frac{G}{C}\right)_I h_2(\rho_t) \\
V_2 &= \left(\frac{Q}{C}\right)_{II} h_1(\rho_t) + \left(\frac{G}{C}\right)_{II} h_2(\rho_t)
\end{aligned}\right\} \qquad \text{(B-67)}$$

Then from equation (50,a)

$$\boxed{\left(\frac{{}_{\parallel}R_{\parallel} + 1}{C}\right)_t = \frac{V_1 \left(\left(E_y{}^i\right)_t\right)_{II} - V_2 \left(\left(E_y{}^i\right)_t\right)_I}{\left(\left(E_{\parallel}{}^i\right)_t\right)_I \left(\left(E_y{}^i\right)_t\right)_{II} \left(\left(E_{\parallel}{}^i\right)_t\right)_{II} \left(\left(E_y{}^i\right)_t\right)_I}} \quad \text{(B-73)}$$

and from equation (50,b)

$$\boxed{\left(\frac{{}_{\perp}R_{\parallel}}{C}\right)_t = \frac{V_2 \left(\left(E_{\parallel}{}^i\right)_t\right)_I - V_1 \left(\left(E_{\parallel}{}^i\right)_t\right)_{II}}{\left(\left(E_{\parallel}{}^i\right)_t\right)_I \left(\left(E_y{}^i\right)_t\right)_{II} - \left(\left(E_{\parallel}{}^i\right)_t\right)_{II} \left(\left(E_y{}^i\right)_t\right)_I}} \quad \text{(B-74)}$$

APPENDIX C

MODIFIED GROUND REFLECTION COEFFICIENTS

Given: The Fresnel Reflection Coefficients at the ground (i.e., z = 0)

$$ {}_{\parallel}\bar{R}^{\circ}_{\parallel} = \frac{CN_g^2 - W}{CN_g^2 + W} \tag{C-1} $$

$$ {}_{\perp}\bar{R}^{\circ}_{\perp} = \frac{C - W}{C + W} \tag{C-2} $$

where

$$ W = \sqrt{N_g{}^2 - S^2} \tag{C-3} $$

with

$S = \sin\theta$

N_g = index at refraction of ground

Define modified reflection coefficients as:

$$ A = \left(\frac{1}{{}_{\parallel}\bar{R}^{\circ}_{\parallel}} + 1\right)/C \tag{C-4} $$

$$ B = \left(\frac{1}{{}_{\perp}\bar{R}^{\circ}_{\perp}} + 1\right)/C \tag{C-5} $$

where $C = \cos\theta$

substituting equations 1 and 2 into equations 4 and 5 gives:

$$ A = \left[\frac{1}{\left(\dfrac{CN_g^2 - W}{CN_g^2 + W}\right)} + 1\right] 1/C \tag{C-6} $$

$$ B = \left[\frac{1}{\dfrac{C - W}{C + W}} + 1\right] 1/C \tag{C-7} $$

then with further manipulation:

$$A = \left[\frac{CN_g^2 + W}{CN_g^2 - W} + 1\right] /C \qquad = \left[\frac{CN_g^2 + W + CN_g^2 - W}{CN_g^2 - W}\right] /C$$

$$= \left[\frac{2\ CN_g^2}{(CN_g^2 - W}\right] /C \qquad = \frac{1}{1/2[C - W/N_g^2]} \qquad \text{(C-8)}$$

and

$$B = \left[\frac{C + W}{C - W} + 1\right] /C \qquad = \left[\frac{C + W + C - W}{C - W}\right] /C$$

$$= \left[\frac{2\ C}{C - W}\right] 1/C \quad = \frac{2 \cdot 1/W}{(C - W)} \cdot 1/W \quad = \frac{\frac{1}{W}}{\frac{1}{2}\left[\frac{C}{W} - 1\right]} \qquad \text{(C-9)}$$

then define:

$$A = \frac{{}_{\parallel}n_{\parallel}}{{}_{\parallel}d_{\parallel}} \quad \text{so that:} \quad \left.\begin{aligned} {}_{\parallel}n_{\parallel} &= 1 \\ {}_{\parallel}d_{\parallel} &= \tfrac{1}{2}[C - W/N_g^2] \end{aligned}\right\} \qquad \text{(C-10)}$$

and

$$B = \frac{{}_{\perp}n_{\perp}}{{}_{\perp}d_{\perp}} \quad \text{So that:} \quad \left.\begin{aligned} {}_{\perp}n_{\perp} &= 1/W \\ {}_{\perp}d_{\perp} &= \tfrac{1}{2}[\tfrac{C}{W} - 1] \end{aligned}\right\} \qquad \text{(C-11)}$$

The final forms of A and B as presented above are well behaved functions as $N_g \to \infty$ and $\theta \to 90°$.

APPENDIX D

MESH SQUARE ANALYSIS

Consider an individual mesh square. See figure D-I. Any point in the complex θ plane which lies within (or on an edge of) the square is described by the complex function $F(\theta_r,\theta_i)$. That is:

$$F(\theta) = F_R(\theta_r,\theta_i) + j\ F_I(\theta_r,\theta_i) \tag{D-1}$$

and

$$\theta = \theta_r + j\theta_i \tag{D-2}$$

where

$F_R(\theta)$ is the real part of the complex function $F(\theta)$

$F_I(\theta)$ is the imaginary part of the complex function $F(\theta)$

θ_r is the real part of the complex number (θ)

θ_i is the imaginary part of the complex number (θ).

The lower left hand corner of the square is taken as the reference position and each side of the square is one-mesh unit in length. The geometry of the mesh square is shown below where the $F(\theta_r,\theta_i)$ values of the corners of the square are identified.

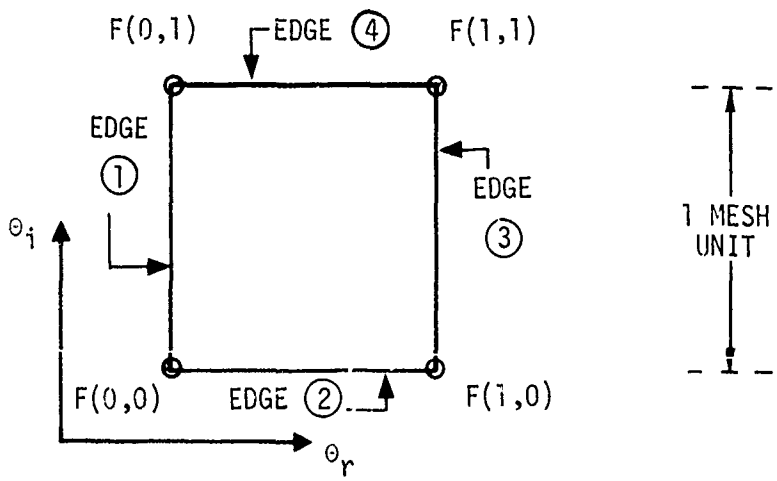

Figure D-I. Mesh Square Geometry

A basic assumption put on $F(\theta_r,\theta_i)$ is that $F_R(\theta_r,\theta_i)$ and $F_I(\theta_r,\theta_i)$ are to be linear along each edge of the mesh square.

The function of $F(\theta_r,\theta_i)$ at any point within the square is given by:

$$\begin{aligned} F(\theta_r,\theta_i) = {} & F(0,0)(1 - \theta_r - \theta_i + \theta_r\theta_i) \\ & + F(0,1)(\qquad\quad \theta_i - \theta_r\theta_i) \\ & + F(1,0)(\quad \theta_r \qquad - \theta_r\theta_i) \\ & + F(1,1)(\qquad\qquad\quad \theta_r\theta_i) \end{aligned} \tag{D-3}$$

or expanding:

$$\begin{aligned} F(\theta_r,\theta_i) = {} & F(0,0) + \theta_i(F(0,1) - F(0,0)) + \theta_r(F(1,0) - F(0,0)) \\ & + \theta_r\theta_i(F(0,0) + F(1,1) - F(0,1) - F(1,0)) \end{aligned} \tag{D-4}$$

which can be written as

$$F(\theta_r,\theta_i) = a + b\theta_i + c\theta_r + d\theta_r\theta_i \tag{D-5}$$

(a) <u>Hyperbolic and Asymptotic Lines</u>:

Consider the imaginary part of $F(\theta_r,\theta_i)$ and $F_I(\theta_r,\theta_i) = 0$. This gives (from equation D-5):

$$a_i + b_i\theta_i + c_i\theta_r + d_i\theta_r\theta_i = 0 \tag{D-6}$$

Also, in the same way the real part of $F(\theta_r,\theta_i)$ gives:

$$a_r + b_r\theta_i + c_r\theta_i + d_r\theta_r\theta_i = 0 \tag{D-7}$$

Equation D-6 (or equation D-7) is the equation of an "equilateral" or "rectangular" hyperbola with vertical and horizontal asymptotes.

To determine the center of the asymptotes for the hyperbolic curve of equation D-6, that equation must be solved for θ_r and also for θ_i.

That is, (from equation D-6):

$$(a_i + b_i\theta_i) + (c_i + d_i\theta_i)\ \theta_r = 0$$

or

$$\theta_r = -\left(\frac{a_i + b_i\theta_i}{c_i + d_i\theta_i}\right) \tag{D-8}$$

Also from equation D-6:

$$(a_i + c_i\theta_r) + (b_i + d_i\theta_r)\theta_i = 0$$

or

$$\theta_i = -\left(\frac{a_i + c_i\theta_r}{b_i + d_i\theta_r}\right) \tag{D-9}$$

Now let $\theta_r \to \infty$ in equation (D-8) so that:

$$c_i + d_i\theta_i = 0$$

or

$$\theta_i = -\frac{c_i}{d_i} \tag{D-10}$$

The line described by equation (D-10) is the horizontal asymptote to the equilateral hyperbola.

From equation D-9 let $\theta_i \to \infty$ so that:

$$b_i + d_i\theta_r = 0$$

or

$$\theta_r = -\frac{b_i}{d_i} \tag{D-11}$$

The line described by equation (D-11) is the vertical asymptote to the equilateral hyperbola. These asymptotic lines in relation to the mesh square are illustrated in the following figure:

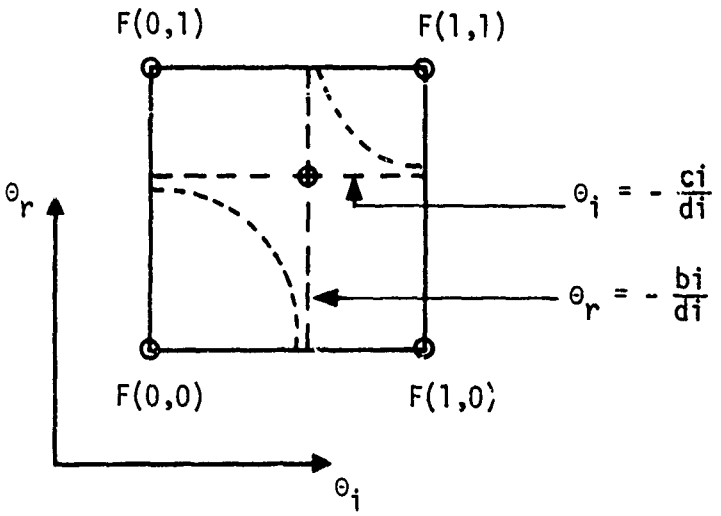

Figure D-II. Vertical and Horizontal Asymptotes for an equilateral hyperbola.

(b) Intersection of Hyperbolic Lines with the Edges of the Mesh Square:

Identify the edges of the mesh square, as shown in figure D-I, as edges 1, 2, 3 and 4. Also, consider only the imaginary part of $F(\theta_r,\theta_i)$ of equation D-4 and equate $F_I(\theta_r,\theta_i)$ to 0. Equation D-4 becomes:

$$F_I(0,0) + \theta_i(F_I(0,1) - F_I(0,0)) + \theta_r(F_I(1,0) - F_I(0,0)) + \theta_r\theta_i(F_I(0,0) + F_I(1,1) - F_I(0,1) - F_I(1,0)) = 0 \qquad \text{(D-12)}$$

To find the intersection of a branch of the equilateral hyperbola, described by equation D-12, with edge No. 1 of the square, set $\theta_r = 0$ and

note that θ_i varies between 0 and 1 along edge No. 1. Equation D-12 becomes:

$$F_I(0,0) + \theta_i(F_I(0,1) - F_I(0,0)) = 0$$

or

$$\theta_i = \frac{-F_I(0,0)}{F_I(0,1) - F_I(0,0)} \qquad \text{(D-13)}$$

Therefore the point of intersection for edge No. 1 is:

$\theta_r = 0$, θ_i is as given by equation D-13.

The point of intersection between the hyperbola and the remaining three sides may be computed in a similar manner. The results are as follows:

The point of intersection for edge No. 2 is:

$$\theta_r = -\frac{F_I(0,0)}{F_I(1,0) - F_I(0,0)}, \quad \theta_i = 0 \qquad \text{(D-14)}$$

The point of intersection edge No. 3 is:

$$\theta_r = 1, \quad \theta_i = -\frac{F_I(1,0)}{F_I(1,1) - F_I(1,0)} \qquad \text{(D-15)}$$

The point of intersection edge No. 4 is:

$$\theta_r = -\frac{F(0,1)}{F_I(1,1) - F_I(0,1)}, \quad \theta_i = 1 \qquad \text{(D-16)}$$

APPENDIX E

SOLUTION OF THE QUADRATIC EQUATION FOR THE GENERAL ROOT-FINDER.

Given the Quadratic equation as used in the Root-Finder procedure:

$$\alpha X^2 + 2bX + C = 0 \qquad \text{(E-1)}$$

$$X = -\frac{2b \pm \sqrt{4b^2 - 4ac}}{2a} = \frac{b}{a}\left[-1 \pm \sqrt{1 - \frac{ac}{b^2}}\right]$$

let $\varepsilon = -ac/b^2$

and then $(1+\varepsilon)^{1/2} = 1 + \frac{1}{2}\varepsilon + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2}\varepsilon^2$

$$+ \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3 \cdot 2}\varepsilon^3 + - - -$$

Now $X = \frac{b}{a}\left[-1 \pm \sqrt{1 + \varepsilon}\right]$

so that

$$X_1 = \frac{b}{a}\left\{\frac{1}{2}\left(\frac{-ac}{b^2}\right) + \frac{1}{2}\frac{\left(\frac{1}{2} - 1\right)}{2}\left(\frac{-ac}{b^2}\right)^2 + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3 \cdot 2}\left(\frac{-ac}{b^2}\right)^3 + - - -\right\}$$

$$X_1 = -\frac{1}{2}\frac{c}{b}\left\{1 + \frac{\left(\frac{1}{2} - 1\right)}{2}\left[-\frac{ac}{b^2}\right] + \frac{\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3 \cdot 2}\left[\frac{-ac}{b^2}\right]^2 + - - -\right\} \qquad \text{(E-2)}$$

let $t_0 = 1$

$$t_1 = \frac{\left(\frac{1}{2} - 1\right)}{2}\left[-\frac{ac}{b^2}\right]$$

$$t_2 = \frac{\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3 \cdot 2}\left[\frac{ac}{b^2}\right]^2$$

or $$t_i = t_{i-1} \left(\frac{\frac{1}{2} - 1}{i + 1}\right) \left[- \frac{ac}{b^2}\right]$$

or $$t_i = t_{i-1} \left(\frac{i - 1/2}{i + 1}\right) \left[\frac{ac}{b^2}\right]$$

and $$X_1 = - \frac{1}{2} \frac{c}{b} \left\{t_0 + t_1 + t_2 + - - - + t_N\right\} \qquad \text{(E-3)}$$

Now $$X_1 = \frac{b}{a} \left[-1 + \sqrt{1 + \varepsilon}\right] \qquad \text{(E-4)}$$

also $$X_2 = b/a \left[-1 - \sqrt{1 + \varepsilon}\right]$$

Then: $$-X_1 = \frac{b}{a} \left[1 - \sqrt{1 + \varepsilon}\right]$$

and $$\frac{-2b}{a} - X_1 = \frac{b}{a} - \frac{b}{a} \sqrt{1 + \varepsilon} \quad \frac{-2b}{a}$$

or $$\frac{-2b}{a} - X_1 = - \frac{b}{a} - \frac{b}{a} \sqrt{1 + \varepsilon}$$

or $$\frac{-2b}{a} - X_1 = \frac{b}{a} \left[-1 - \sqrt{1 + \varepsilon}\right]$$

such that:

$$\frac{-2b}{a} - X_1 = X_2 \qquad \text{(E-5)}$$

Equations E-3 and E-5 give the solutions of the Quadratic equation (i.e., Equation E-1) as solved in the Root-Finder routine.

APPENDIX F

FORTRAN LISTING OF THE MODESRCH COMPUTOR PROGRAM

```
C   PROGRAM '''MODESRCH'''
C   THIS PROGRAM FINDS THE WAVEGUIDE MODES FOR THE EARTH-IONOSPHERE
C   WAVEGUIDE FOR ELF-VLF-LF
C   MAIN
C   FOR MS
C   PROGRAM FOR COMPUTING EARTH-IONOSPHERE WAVEGUIDE EIGEN-ANGLES AND
C                 EXCITATION FACTORS WHICH ARE OUTPUT ON PUNCHED CARDS
C                 FOR USE IN A PROGRAM WHICH COMPUTES FIELD STRENGTHS.
C
C   THIS ROUTINE CONTROLS THE SEQUENCE OF INPUT AND EXECUTION AND
C                 PROVIDES FOR NAMELIST INPUT.  A DESCRIPTION OF THE
C                 INPUT PARAMETERS IS GIVEN IN A DNA REPORT.
C
C
      COMMON/IC COM/IDENT(20)
      COMMON/RTALT/ TALT,RALT
      COMMON/BPRT/ BPRINT
      COMMON /FP/HUFWK
      COMMON /RTOL C/RTOL
      COMMON/IXACT C/IEXACT
      COMMON/FLD COM/AZIM,CODIP,MAG FLD
      COMMON/FRQ COM/FREQ
      COMMON/GND COM/EPSR,SIGMA
      COMMON/BFP COM/BETA,HPRIME,SCLHTS,ENMIN
      COMMON/WR COM/OMEGAR,TEMPOB
      COMMON/TOPEN C/TOPEN
      COMMON/EXPNU C/COEFNU(3),EXPNU(3)
      COMMON/ION COM/IONS,MRATIO(3)
      COMMON/LIMS C/RANGER(2),RANGEI(2)
      COMMON/GMAX C/GMAX
      COMMON/PARAM C/TMESH
      COMMON/SEP COM/SEP
      COMMON/TCL COM/LUB
      COMMON/NEIGN C/NEIGEN
      COMMON/NPNCH C/NPUNCH
      COMMON/CRD PRT/CARDPT
      COMMON/CRD PNC/CARDPN
      COMMON/CUT OF/ ICUT,CUTOFF
      COMMON/RHO COM/RHO,REFLHT
      COMMON/H COM/H
      COMMON/MPRNT/MPRINT
      COMMON/XPRNT/XPRINT
      COMMON/EXTRA9/IBUFFR(500)
C
C
      NAMELIST/DATUM/AZIM,CODIP,MAG FLD,
     $          FREQ,TALT,RALT,
     $          EPSR,SIGMA,
     $          BETA,HPRIME,SCLHTS,ENMIN,
     $          TOPEN,TEMPOB,
     $          COEFNU,EXPNU,NUFLAG,
     $          MRATIO,TMESH,LUB,
     $          RANGER,RANGEI,
```

```
     $            GMAX,SEP,RTOL,IEXACT,
     $            NEIGEN,
     $            NPUNCH,RHO,REFLHT,
     $            H,
     $            ICUT,CUTOFF,
     $            BPRINT,NPROF,
     $            MPRINT,XPRINT,CARDPT,CARDPN
C
C
      INTEGER PROF,COLF,ADAT,QUIT
      INTEGER CARDPT,CARDPN
      INTEGER XPRINT
      INTEGER BPRINT
      REAL MAGFLD,MRATIO,LUB
      DIMENSION IBCD(20)
      DATA PROF/'PROF'/,COLF/'COLF'/,IDEN/'ID  '/,
     $            WAVE/'WAVE'/,ADAT/' &DA'/,IEND/' &EN'/,
     $            QUIT/'QUIT'/
      DATA IBLANK/'    '/
      DATA ITHREE/'3   '/
C
C
      HOFWR=0.0
      OMEGAR=2.5E05
C
C  INITIAL VALUES OF NAMELIST PARAMETERS.
C
      IEXACT=0
      ICUT = 1
      CUTOFF=0.0001
      TGPEN = 1.0E3
      SCLHTS = 3.0
      ENMIN = 1.0E-1
      TEMPOR=2.0
      HPRIME=-99.0
      FREQ = 0.0
      RHO = 0.0
      REFLHT = 50.0
      H = 50.0
      TALT=0.0
      RALT=0.0
C
      GMAX = 5.0
      RTOL=0.001
      SEP = 0.1
      TMESH=-1.0
      LUB=-1.0
C
      NUFLAG = 0
      COEFNU(1) = 1.816E11
      COEFNU(2) = 4.540E9
      COEFNU(3) = 4.540E9
      EXPNU(1) = -0.15
```

```
      EXPNU(2) = -0.15
      EXPNU(3) = -0.15
C
      MRATIO(1) = 1.0
      MRATIO(2) = 58000.0
      MRATIO(3) = 58000.0
C
      IONS = 0
      NOIGEN = 0
      NPUNCH = 1
C
      BPRINT=0
      MPRINT = 0
      XPRINT = 0
      CARDPT=0
      CARDPN=1
C
      DO 7 I = 1,20
    7 IDENT(I) = IBLANK
C
C
C     DEFAULT VALUES OF GROUND PARAMETERS ARE TAKEN AS SEA WATER
      SIGMA=4.64
      EPSR=81.
C
C  CONTROL OF SEQUENCE OF INPUT AND EXECUTION.
   10 READ(5,901,END=9999)IBCD
      PRINT 902,IBCD
      IF (IBCD(1) .EQ. PROF) GO TO 11
      IF (IBCD(1) .EQ. COLF) GO TO 12
      IF (IBCD(1) .EQ. IDEN) GO TO 13
      IF (IBCD(1) .EQ. ADAT) GO TO 20
      IF(IBCD(1) .EQ. QUIT) GO TO 9999
      PRINT 900
  900 FORMAT (' ','ERROR IN CONTROL CARD')
      STOP
C
 9999 PRINT 9998
 9998 FORMAT('0','***THE CALCULATIONS ARE COMPLETE***')
      STOP
C
   11 IONS=0
      IF (IBCD(3) .EQ. ITHREE) IONS = 1
      CALL PREL IN
      HPRIME=-99.0
      GO TO 10
   12 CALL COLF IN
      NUFLAG=0
      COEFNU(1) = -99.0
      GO TO 10
   13 READ 901,IDENT
      PRINT 902,IDENT
      GO TO 10
```

```
C
C
C   NAMELIST INPUT.
C
   20 IBUFFR(1) = IBCD(1)
      IBUFFR(2) = IBCD(2)
      J = 3
      DO 21 L=1,25
      READ 901,IBCD
      PRINT 902,IBCD
      IF(IBCD(1) .EQ. IEND) GO TO 22
      DO 21 I=1,20
      IBUFFR(J) = IBCD(I)
      J = J+1
   21 CONTINUE
   22 IBUFFR(J) = IBCD(1)
      J = J+1
      IBUFFR(J) = IBCD(2)
      CALL CODE (IBUFFR,4*J)
      READ(1,CATUM)
      HCFWK=0.0
C
      IF(TMESH .LT. 0) TMESH=SQRT(3.75/FREQ)
      IF(LUB .LT. 0) LUB=SQRT(15.0/FREQ)*0.01
      PRINT 205,TMESH,LUB
  205 FORMAT('0','TMESH = ',F6.3,4X,'LUB = ',F6.4)
      IF(FREQ .LE. 1.0) IFXACT=1
C
      IF (NUFLAG .EQ. 0) GO TO 40
      COEFNU(1) = 4.303E11
      COEFNU(2) = 1.076E10
      COEFNU(3) = 1.076E10
      EXPNU(1) = -0.1622
      EXPNU(2) = -0.1622
      EXPNU(3) = -0.1622
C
C   MISCELLANEOUS INPUT OPERATIONS AND CALL TO CONTROL ROUTINE.
   40 IF(HPRIME .GT. 0) CALL SET EN
      IF(HPRIME .LT. 0  .AND.  FREQ .GT. 1.0) CALL CUT TOP
      IF (RANGER(1) .LT. RANGER(2)) GO TO 41
      TEMP = RANGER(1)
      RANGER(1) = RANGER(2)
      RANGER(2) = TEMP
   41 IF (RANGEI(1) .GT. RANGEI(2)) GO TO 42
      TEMP = RANGEI(1)
      RANGEI(1) = RANGEI(2)
      RANGEI(2) = TEMP
   42 CONTINUE
      INCR=ITIME(ICK)
      CALL WVGUID
      INCR=ITIME(ICK)-INCR
      TIME=FLOAT(INCR)/100.
      PRINT  615,TIME
```

```
615   FORMAT('0',/' TIME REQUIRED FOR THIS PROFILE = ',F10.2,' SECONDS')
      PRINT 980
  980 FORMAT('1')
      IF(HPRIVE .LT. 0 ) CALL REPEAT
      GO TO 10
C
  901 FORMAT(20A4)
  902 FORMAT(' ',20A4)
C
      END
```

```
      SUBROUTINE WVGUID
C  FOR MS
C  CONTROL ROUTINE.
C
      COMMON/TOL COM/TOL
      COMMON/IXACT C/IEXACT
      COMMON/MPRNT/MPRINT
      COMMON/NPNCH C/NPUNCH
      COMMON/NEIGN C/NEIGEN
      COMMON/LIMS C/RANGEP(2),RANGEI(2)
      COMMON/PARAM C/TMESH
      COMMON/GMAX C/GMAX
      COMMON/NR OF AS/NR A
      COMMON/HTS COM/HTS(101)
      COMMON/SIDES C/TLEFT,TRIGHT,TBOT,TTOP
      COMMON/THETA C/THETA
      COMMON/FIG COM/EIGEN(100),NR E
      COMMON/KXACT C/ KEXACT
      COMMON/D COM/D
      COMPLEX THETA,EIGEN,
     $        ELIST(100),
     $        TEMP
C
C
      IF (NPUNCH .EQ. 0 .AND. NEIGEN .EQ. 0) GO TO 90
C
C  INITIALIZE FULL-WAVE SOLUTION.
      CALL SET RK
      CALL INIT T
C
C  SET UP SMALL RECTANGLES.
      NR EL = 0
      NR BOXS = (RANGER(2)-RANGER(1)-TOL)/GMAX+1.0
      IF(IEXACT .NE. 0) NR BOXS = 1
      RLSPAN = (RANGER(2)-RANGER(1))/NR BOXS
      IBOX = 1
      TTOP = RANGEI(1)
      TBOT = RANGEI(2)
      TLEFT = RANGER(2)
C
   20 TRIGHT = TLEFT
      TLEFT = TRIGHT-RLSPAN
C
C  SET REFERENCE HEIGHT AND INITIALIZE LAGRANGE INTERPOLATION.
      D = 0.0
      IF(IEXACT .EQ. 0) CALL SET RH
C
C  INITIALIZE RBAR SOLUTION AND FREE-SPACE INTEGRATION.
      CALL INIT RB
      CALL INIT FS(HTS(NRA),D)
C
C  FIND ZEROS OF F FUNCTION - LAGRANGE INTERPOLATION USED IF NOT ELF.
      KEXACT=0
```

```
      CALL FZEROS (TLEFT,TRIGHT,TBOT,TTOP,TMESH,TOL,MPRINT,
     $             EIGEN,NR E)
      IF(NR E .EQ. 0) GO TO 77
C
C  ORDER EIGEN ANGLES ACCORDING TO VALUE OF REAL PART.
      IF (NR E .EQ. 1) GO TO 64
      NREM1 = NR E-1
      DO 63 JE=1,NREM1
      JEP1 = JE+1
      DO 62 J=JEP1,NR E
      IF (REAL(EIGEN(J)) .LT. REAL(EIGEN(JE))) GO TO 62
      TEMP = EIGEN(J)
      EIGEN(J) = EIGEN(JE)
      EIGEN(JE) = TEMP
   62 CONTINUE
   63 CONTINUE
   64 CONTINUE
C
C  FOLLOW EIGEN ANGLES AS F FUNCTION IS CHANGED FROM THAT USING
C                LAGRANGE INTERPOLATION TO THAT USING EXACT
C                FULL-WAVE SOLUTION VALUES OF REFLECTION COEFFICIENTS.
      IF(IEXACT .EQ. 0) CALL FINAL
      IF(NR E .EQ. 0) GO TO 77
C
C  DISCARD EIGEN-ANGLES WHICH ARE OUTSIDE CURRENT SMALL RECTANGLE.
      J=0
      DO 71 JE=1,NR E
      IF (REAL(EIGEN(JE)) .LT. TLEFT-2.0*TOL .OR.
     $             REAL(EIGEN(JE)) .GT. TRIGHT+2.0*TOL) GO TO 71
      J = J+1
      EIGEN(J) = EIGEN(JE)
   71 CONTINUE
      NR E = J
C
C  STORAGE OF EIGEN ANGLES.
      IF(NR E .EQ. 0) GO TO 77
      DO 76 JE=1,NR E
      NR EL = NR EL+1
   76 ELIST(NR EL) = EIGEN(JE)
   77 IBOX = IBOX+1
      IF (IBOX .LE. NR BOXS) GO TO 20
      NR E = NR EL
      IF (NR E .EQ. 0) GO TO 85
      DO 78 JE=1,NR E
   78 EIGEN(JE) = ELIST(JE)
C
C  COMPUTE AND PUNCH NPUNCH OUTPUT AND PRINT TABLE.
      IF (NPUNCH .NE. 0) CALL NP OUT
C
C  PRINT AND PUNCH EIGEN ANGLES IF NEIGEN SET TO 1.
      IF (NEIGEN .NE. 0) CALL NE OUT
      RETURN
C
```

```
   85 PRINT 985
  985 FORMAT ('0','N) MODES')
      RETURN
C
  990 PRINT 990
  990 FORMAT ('0','SET NPUNCH OR NEIGEN')
      STOP
C
      END
```

```
      SUBROUTINE SET RH
C   FOR NS
C   ROUTINE FOR SELECTING GIVEN POINTS IN THE LAGRANGE INTERPOLATION
C                 AND FOR SELECTING THE REFERENCE HEIGHT.
C
      COMMON/MPRNT/MPRINT
      COMMON/PARAM C/TMESH
      COMMON/HTS COM/HTS(101)
      COMMON/NR OF AS/NR A
      COMMON/SIDES C/TLEFT,TRIGHT,TBOT,TTOP
      COMMON/THETA C/THETA
      COMMON/IDERV C/IDERIV
      COMMON/INTEGR/R(4),DRDC(4)
      COMMON/D COM/D
      DIMENSION PART(2)
      COMPLEX THETA,R,DRDC,
     $          PARTS,
     $          THETAG(20),
     $          FBOT(4,20),R GIVEN(4,20),R TEMP(4,20),R SAVE(4,20)
      EQUIVALENCE (PART(1),PARTS)
      DATA HT INC/1.0/
      DATA HMARGN/20.0/
C
C
C   SELECT VALUES OF THETA AT WHICH FULL-WAVE SOLUTIONS ARE TO BE
C                 CARRIED OUT FOR USE IN LAGRANGE INTERPOLATION.
      JLT = TLEFT/TMESH-1.0
      JRT = TRIGHT/TMESH+2.0
      JBOT = TBOT/TMESH-2.0
      JTOP = TTOP/TMESH
      JTOP = JTOP+1
C
      PART(1) = (JRT+1)*TMESH
      PART(2) = (JTOP+1)*TMESH
      THETAG(1) = PARTS
      PART(1) = (JLT-1)*TMESH
      PART(2) = (JTOP+1)*TMESH
      THETAG(2) = PARTS
      PART(1) = (JRT+1)*TMESH
      PART(2) = (JBOT-1)*TMESH
      THETAG(3) = PARTS
      PART(1) = (JLT-1)*TMESH
      PART(2) = (JBOT-1)*TMESH
      THETAG(4) = PARTS
      NG = 4
      CALL INIT LG (THETAG,NG)
      NG8 = NG*8
C
C   SELECT THE HEIGHT (TO NEAREST WHOLE KILOMETER) AT WHICH
C                 THE MINIMUM OCCURS IN
C                 THE SQUARE OF THE MAGNITUDE OF THE DERIVATIVE OF
C                 (R+1.0)/C WRT C=COS(THETA), SUMMED OVER ALL FOUR
C                 ELEMENTS OF R=REFLECTION MATRIX AND SUMMED OVER ALL
```

```
C                 POINTS AT WHICH FULL-WAVE SOLUTIONS ARE CARRIED OUT
C                 FOR USE IN LAGRANGE INTERPOLATION.  DERIVATIVES OF THE
C                 FULL-WAVE SOLUTIONS ARE NOT COMPUTED.  RATHER,
C                 DERIVATIVES OF THE LAGRANGE INTERPOLATION FORMULA
C                 ARE USED.  NOTE THAT THE FULL-WAVE SOLUTIONS ARE
C                 CARRIED OUT TO THE BOTTOM OF THE PROFILE AFTER WHICH
C                 THE FULL-WAVE SOLUTION IS CARRIED OUT IN AN UPWARD
C                 DIRECTION THROUTH FREE SPACE OVER A CURVED EARTH TO
C                 HEIGHTS AT ONE KILOMETER INTERVALS.  THE HEIGHT AT
C                 WHICH THE MINIMUM OCCURS IS PRINTED OUT AS THE
C                 'REFLECTION HT' AND IS THEREAFTER USED AS THE
C                 REFERENCE HEIGHT AT WHICH THE F FUNCTION IS COMPUTED.
      IF (MPRINT .NE. 0) PRINT 911
  911 FORMAT ('1','INTEGRATION TO DETERMINE REFLECTION HT--')
      IDERIV = 0
      RHTEMP = 9.9E9
      RFSAVE = 9.9E9
      LIMIT = NG
      GB = 0.0
      DO 12 JG=1,NG
      THETA = THETAG(JG)
      CALL INTEG
   12 CALL XFER (R,R GIVEN(1,JG),8)
C
      CALL XFER (R GIVEN,RBOT,NG8)
      CALL XFER (R GIVEN,R TEMP,NG8)
      CALL XFER (R GIVEN,R SAVE,NG8)
      CALL SET LAG (NG,R GIVEN)
      DO 13 JG=1,LIMIT
      THETA = THETAG(JG)
      CALL LAG DER
      DO 13 I=1,4
   13 GB = GB+CABS(DRDC(I))**2
      GB MIN = GB
      IF (MPRINT .NE. 0) PRINT 912,HTS(NR A),GB
      HTFRST = HTS(NR A)
   14 HTLAST = HTFRST+HT INC
      CALL INIT FS (HTS(NR A),HTLAST)
      GB = 0.0
      DO 15 JG=1,NG
      THETA = THETAG(JG)
      CALL XFER (RBOT(1,JG),R,8)
      CALL FS INTG
   15 CALL XFER (R,R GIVEN(1,JG),8)
      CALL SET LAG (NG,R GIVEN)
      DO 16 JG=1,LIMIT
      THETA = THETAG(JG)
      CALL LAG DER
      DO 16 I=1,4
   16 GB = GB+CABS(DRDC(I))**2
      IF (MPRINT .NE. 0) PRINT 912,HTLAST,GB
  912 FORMAT (' ','AT HT = ',F7.2,1X,'GB = ',1PE9.2)
      IF (GB .GT. GB MIN) GO TO 17
```

```
      RHTEMP = HTLAST
      GB MIN = GB
      CALL XFER (R GIVEN,R TEMP,NGB)
      GO TO 16
   17 RHSAVE = RHTEMP
      CALL XFER (R TEMP,R SAVE,NGB)
   16 HTFRST = HTLAST
      IF (HTFRST .LT. (HTS(1)+ HMARGN) ) GO TO 14
      IF(RHSAVE .GT. 9.0E9) GO TO 90
      D = RHSAVE
      CALL XFER (R SAVE,R GIVEN,NGB)
      CALL SET LAG (NG,R GIVEN)
C
      PRINT 913,D,TLEFT,TRIGHT
  913 FORMAT ('0','REFLECTION HT SET TO',F7.2,1X,'KM',
     $ 5X,'THETA RANGE = ',F8.3,' TO ',F8.3)
C
      IF (MPRINT .EQ. 0) GO TO 39
      PRINT 931
  931 FORMAT ('0','REFLECTION COEFFICIENTS AT THE GIVEN POINTS--')
      DO 32 JG=1,NG
   32 PRINT 932,THETAG(JG),(R GIVEN(I,JG),I=1,4)
  932 FORMAT ('0','AT THETA = ',2F9.3,5X,'R = ',4(2X,2F9.5))
   39 CONTINUE
      RETURN
C
   90 PRINT 900,TLEFT,TRIGHT
  900 FORMAT('0','REFLECTION HT CANNOT BE SET FOR THETA RANGE = ',
     $F8.3,' TO ',F8.3)
      STOP
C
      END
```

```
      SUBROUTINE PRFL IN
C FOR MS
C  THIS ROUTINE PROVIDES FOR INPUT OF ELECTRON DENSITY AND COLLISION
C                FREQUENCY PROFILE CARDS AND FOR LIMITING THE EXTENT
C                OF THE TOP AND BOTTOM OF THE ELECTRON DENSITY PROFILE.
C                INPUT OF ION PROFILES IS ALSO PROVIDED FOR.  FURTHER
C                DESCRIPTION OF THE INPUT IS GIVEN IN A DNA REPORT.
C
      COMMON/HTLCGN/NR NU,HTS NU(50),ALCNU(50,3)
      COMMON/BPRT/ BPRINT
      COMMON/CUT OF/ ICUT,CUTOFF
      COMMON/FLD COM/AZIM,CODIP,MAG FLD
      COMMON/FRQ COM/ FREQ
      COMMON/NR OF AS/NR A
      COMMON/HTS COM/HTS(101)
      COMMON/ALOGN C/ALOGEN(101,3)
      COMMON/BHP COM/BETA,HPRIME,SCLHTS,ENMIN
      COMMON/EXPNU C/COEFNU(3),EXPNU(3)
      COMMON/ICN COM/IONS,MRATIO(3)
      COMMON/EN COLL/HT,EN(3),CNU(3)
      COMMON/WR COM/OMEGAR,TEMPOB
      COMMON/HF/HOFWR
      COMMON/TOPEN C/TOPEN
      COMMON/JAY COM/JAY
      REAL MAGFLD,MRATIO
      DIMENSION IBCD(20),ENS(3),
     *ITITLE(20),SAVHT(101),SAVEN(101,3)
C
      INTEGER BPRINT
C
C*****************************************************************
C
C  INPUT OF AN ELECTRON DENSITY PROFILE, IONS OPTIONAL.
C
      READ 901,IBCD
      PRINT 902,IBCD
      DO 113 K =1,20
  113 ITITLE(K)=IBCD(K)
      J = 1
   11 READ 904,HT,ENS(1),ENS(2)
  904 FORMAT (F7.2,5X,E9.2,E10.2)
      IF (HT .LT. 0.0) GO TO 15
      HTS(J) = HT
      IF(J .NE. 1 .AND. HTS(J) .GE. HTS(J-1)) STOP
      ENS(3) = ENS(2)-ENS(1)
      NTOT=J
      SAVHT(J)=HT
      SAVEN(J,1)=ENS(1)
      SAVEN(J,2)=ENS(2)
      SAVEN(J,3)=ENS(3)
      PRINT 916,HT,ENS(1),ENS(2)
  916 FORMAT(' ',F7.2,5X,1PE9.2,E10.2,E10.2)
      DO 13 K=1,3
```

```
      IF (ENS(K) .LT. 1.0E-10) ENS(K) = 1.0E-10
   13 ALOGEN(J,K) = ALOG(ENS(K))
      NR A = J
      J = J+1
      GO TO 11
   15 PRINT 906,HT
      RETURN
C
C*****************************************************************
C
      ENTRY REPEAT
C
C     THIS SECTION IS USED WHEN THE ORIGINAL UNMODIFIED PROFILE
C     IS TO BE USED IN THE CALCULATIONS
C
      NR A=NTOT
      DO 134 K = 1,3
      DO 134 KJ=1,NTOT
      HTS(KJ) = SAVHT(KJ)
      IF(SAVEN(KJ,K) .LT. 1.0E-10) SAVEN(KJ,K) = 1.0E-10
  134 ALOGEN(KJ,K)=ALOG(SAVEN(KJ,K))
      RETURN
C
C*****************************************************************
C
      ENTRY COLF IN
C
C  INPUT OF COLLISION FREQUENCY PROFILE CARDS
C
      J = 1
   21 READ 901,(IBCD(L),L=1,20)
      PRINT 902,(IBCD(L),L=1,20)
      CALL CORE (IBCD,80)
      READ (69,905) HT,CNU
  905 FORMAT (F7.2,4X,3(1X,E9.2) )
      IF (HT .LT. 0.0) GO TO 23
      HTS NU(J) = HT
      DO 22 K=1,3
      IF(CNU(K) .LT. 1.0E-10) CNU(K) = 1.0E-10
   22 ALCNU(J,K) = ALOG(CNU(K))
      J = J+1
      GO TO 21
   23 NR NU=J-1
      RETURN
C
C
C*****************************************************************
C
      ENTRY SET EN
C
C  USE OF VARIOUS NAMELIST PARAMETERS TO SET UP A DESCRIPTION OF AN
C                EXPONENTIAL ELECTRON DENSITY PROFILE IN TERMS OF
C                VALUES AT THE TOP AND BOTTOM OF THE PROFILE.
```

```
C
      HTS(1) = AINT(HPRIME+SCLHTS/BETA+0.5)
      FNA= 7.85535E-5*COEFNU(1)*EXP((EXPNU(1)+BETA)*HTS(1)-BETA*HPRIME)
      ALOGEN(1,1) = ALOG(FNA)
      HTS(2) = (ALOG(ENMIN/(7.85535E-5*COEFNU(1)))+BETA*HPRIME)/
     $          (EXPNU(1)+BETA)
      IF(HTS(2) .GT. 0.0) GO TO 501
      HTS(2) = 0.0
      ENO= 7.85535E-5*COEFNU(1)*EXP(-BETA*HPRIME)
      ALOGEN(2,1) = ALOG(ENO)
      GO TO 502
  501 ALOGEN(2,1) = ALOG(ENMIN)
  502 NKA = 2
  951 FORMAT (' .',F7.2,5X,1PE9.2)
C
C     -----COMPUTED PROFILE PARAMETERS-----
C
      POW=6.283185307D03*FREQ
      POY=-1.758796D11/POW*MAGFLD
      PRINT 213
213   FORMAT(/' COMPUTED PROFILE PARAMETERS:'/
     $       T5,'H',T15,'NE',T27,'NU',T39,'X',T51,'Y',T63,'Z',T75,'WR')
      DO 25 L=1,2
      PONU=COEFNU(1)* EXP(EXPNU(1)*HTS(L))
      PONE=7.85535D-05*PONU* EXP(BETA*(HTS(L)-HPRIME))
      POWR=3.182357D09*PONE/PONU
      POX =3.182357D09*PONE/(POW*POW)
      POZ=PONU/POW
25    PRINT 214,    HTS(L), PONE, PONU, POX, POY, POZ, POWR
214   FORMAT(F7.1,1P6E12.3)
C
C     DETERMINATION OF HEIGHT WHERE OMEGA-R EQUALS 2.5E05
      HOFWR=HPRIME
      PRINT 952,HOFWR
  952 FORMAT('C',5X,'OMEGA-R = 2.5E05 AT TOP HEIGHT = ',F7.2)
C
      RETURN
C
C*****************************************************************
C
      ENTRY CUT TOP
C
C  USE OF TEMPC-B AND TOPEN TO LIMIT THE EXTENT OF THE TOP OF AN
C                ELECTRON DENSITY PROFILE WHICH HAS BEEN INPUT FROM
C                PROFILE CARDS.  APPLICABLE ALSO TO THE ION PROFILES
C                IF IONS ARE AN INPUT.
C
C     IN THIS SECTION OF THE PROGRAM THE TOP OF THE PROFILE IS DETERMINE
C     THE NOTATION USED IS THAT WAITS ('OMEGA-R') IS DENOTED BY 'WR'.
C     THE VALUE OF ''B'' AS DESCRIBED IN G.E. TEMPO REPORT 75-TMP-9,V.7,
C     PAGES 11 AND 12 IS DENOTED AS 'BCUT'
C     AND IS INPUT IN TO THIS PROGRAM AS THE NAMELIST VARIABLE 'TEMPOB'
C
```

```
C
C       TO DETERMINE THE VALUE OF'BCUT' TO BE USED
C          AND TO CUT OFF THE TOP OF THE PROFILE
        OMEGA=6.28319E3*FREQ
        PLASCN=3.182357E9
        DO 560 JREV=1,NR A
        J=NR A-JREV+1
        HT=HTS(J)
        JAY=J
        IF(J .EQ. NR A) JAY = NR A-1
        CALL EN NU
        OMOVNU=10.0*OMEGA/CNU(1)
        BCUT=2.0*OMOVNU
        IF(BCUT .LT. TEMPOB) BCUT = TEMPOB
        B=(PLASCN*EN(1)*CNU(1))/(OMEGA*(OMEGA**2+CNU(1)**2))
        IF(IONS .NE. 0) B=B+
     $   (PLASCN*EN(2)*CNU(2))/(OMEGA*(OMEGA**2+CNU(2)**2)*MRATIO(2))
     $  +(PLASCN*EN(3)*CNU(3))/(OMEGA*(OMEGA**2+CNU(3)**2)*MRATIO(3))
        IF(B .GT. BCUT .AND. EN(1) .GT. TOPEN) GO TO 581
  580   BBOT=B
        WR=PLASCN*EN(1)/CNU(1)
        IF(IONS .NE. 0) WR=WR+
     $   PLASCN*EN(2)/(CNU(2)*MRATIO(2))
     $  +PLASCN*EN(3)/(CNU(3)*MRATIO(3))
        PRINT 608
  608   FORMAT('0','WARNING--THE TOP OF THE PROFILE CAN NOT BE CUT OFF.',
     $  '  THE TOP OF THE PROFILE MAY BE TO LOW.' )
        PRINT 610,B,WR
  610   FORMAT('0','  TEMPO-B AT TOP = ',1PE11.3,5X,'WAITS OMEGA-R AT ',
     $'TOP = ',1PE11.3)
        GO TO 900
C
C
C          HERE THE VALUE OF ---B--- AT THIS HEIGHT IS GREATER THAN TEMPO-
  581   IF(J .NE. NR A) GO TO 583
C
C          IF THE HEIGHT IS THE BOTTOM OF THE PROFILE
        PRINT 603,B,TEMPOB
  603   FORMAT('0','THE TOP OF THE PROFILE CAN NOT BE SET. ',/2X,
     $'AT THE BOTTOM OF THE PROFILE, B = ',1PE13.3,', WHICH',
     $ ' IS GTEATER THAN THE VALUE OF TEMPO-B( = ',1PE13.3,').',/
     $5X,'CHECK THE INPUT VALUES OF --TEMPOB--AND--TOPEN--.' , /
     $ 5X,'ALSO CHECK THE VALUES OF THE ELECTRON AND ION DENSITIES ',
     $ 'AT THE BOTTOM OF THE PROFILE.',//
     $    30X,     '***PROGRAN EXECUTION STOPS***')
C
        STOP 1111
C
C
C          ***INTERPOLATION***
C
  583   BTOP = B
        HTTOP=HTS(J)
```

```
      HTBOT=HTS(J+1)
   64 HT=(HTTOP+HTBOT)/2.0
      CALL EN NU
      B=(PLASCN*EN(1)*CNU(1))/(OMEGA*(OMEGA**2+CNU(1)**2))
      IF(IONS .EQ. 0) B2=0.0
      IF(IONS .NE. 0) B2=
     $  (PLASCN*EN(2)*CNU(2))/(OMEGA*(OMEGA**2+CNU(2)**2)*MRATIO(2))
     $ +(PLASCN*EN(3)*CNU(3))/(OMEGA*(OMEGA**2+CNU(3)**2)*MRATIO(3))
      B=B+B2
      IF(HTTOP-HTBOT .LT. 0.10) GO TO 566
      IF(B .GT. BCUT .AND. EN(1) .GT. TOPEN) GO TO 565
      BBOT =B
      HTBOT = HT
      GO TO 64
  565 BTOP=B
      HTTOP=HT
      GO TO 64
  566 WR=PLASCN*EN(1)/CNU(1)
      IF(IONS .NE. 0) WR=WR+
     $  PLASCN*EN(2)/(CNU(2)*MRATIO(2))
     $ +PLASCN*EN(3)/(CNU(3)*MRATIO(3))
      PRINT 600
  600 FORMAT('C','THE TOP OF THE PROFILE IS SET TO BE--')
      IF(IONS .EQ. 0) PRINT 906,HT,EN(1)
  906 FORMAT (' ',F7.2,5X,1PE9.2,1PE9.2,1PE9.2)
      IF(IONS .NE. 0) PRINT 906,HT,EN(1),EN(2),EN(3)
      PRINT 614,BCUT,WR
  614 FORMAT('C',' AT THE TOP OF THE PROFILE B = ',1PE11.3,
     $ ' OMEGA-R = ',1PE11.3)
      HTS(1) = HT
      ALOGEN(1,1) = ALOG(EN(1))
      IF(IONS .EQ. 0) GO TO 67
      ALOGEN(1,2) = ALOG(EN(2))
      ALOGEN(1,3) = ALOG(EN(3))
   67 CONTINUE
      IF(J .EQ. 1) GO TO 900
      JN=1
      JP1 = J+1
      DO 68 JJ = JP1,NR A
      JN=JN+1
      HTS(JN) = HTS(JJ)
      DO 68 K = 1,3
   68 ALOGEN(JN,K) = ALOGEN(JJ,K)
      NR A = JN
  900 CONTINUE
C
C
C*************************************************************************
C
C         **BOTTOM CUT OFF**
C
C
C     THIS SECTION CUTS OFF THE BOTTOM OF THE PROFILE.
```

```
C       IF ( N**2 = 1-A-JB), THAN THOSE PROFILE DENSITIES WHICH COMPUTE
C       TO "B" LESS THEN "CUTOFF" ARE NEGLECTED
C
C       IF THE PROFILE BOTTOM IS TO BE CUTOFF THEN "ICUT= ",
C       IF NOT THEN ICUT=0
        IF(ICUT .LT. 1) GO TO 702
C
        DO 590 JREV = 1,NR A
        J=NR A-JREV+1
        HT=HTS(J)
        JAY=J
        IF(JAY .EQ. NR A) JAY =NR A-1
        CALL EN NU
        B1=(PLASCN*EN(1)*CNU(1))/(OMEGA*(OMEGA**2+CNU(1)**2))
        IF(IONS .EQ. 0) B2=0.0
        IF(IONS .NE. 0)  B2 =
     $   (PLASCN*EN(2)*CNU(2))/(OMEGA*(OMEGA**2+CNU(2)**2)*MRATIO(2))
     $ +(PLASCN*EN(3)*CNU(3))/(OMEGA*(OMEGA**2+CNU(3)**2)*MRATIO(3))
        B=B1+B2
        IF(B .GT. CUTOFF) GO TO 6300
        BBOT=B
  590   CONTINUE
        PRINT 6030
 6030   FORMAT('0',' THE PROFILE BOTTOM CAN NOT BE CUT OFF.',/
     $ ' ALL B-VALUES (AS A FUNCTION OF HEIGHT) ARE LESS THAN CUTOFF.',/
     $ ' THIS IMPLIES THAT THE PROFILE IS VERY CLOSE TO FREE SPACE.'//
     $    30X,     '***PROGRAN EXECUTION STOPS***')
C
        STOP 2222
C
 6300   IF(J .NE. NR A) GO TO 6301
        PRINT 704,CUTOFF,HTS(NR A),B
704     FORMAT('0',' THE BOTTOM OF THE PROFILE CAN NOT BE CUT OFF',
     *' USING CUTOFF = ',E13.3,' AT HEIGHT = ',F10.3,/
     $ ' BECAUSE THE VALUE OF  B  AT THE BOTTOM OF THE PROFILE',
     $', WHERE B = ',1PE13.3,', IS LARGER THAN CUTOFF.'/)
        IF(PRINT .EQ. 0) GO TO 4111
        PRINT 4112,B1,B2,B
 4112   FORMAT(' THE VALUES ARE B-ELECTRONS = ',1PE11.4,'  B-IONS = ',
     * 1PE11.4,'  B-TOTAL = ',1PE11.4/)
 4111   CONTINUE
        GO TO 702
C
C          ***INTERPOLATION***
C
 6301   BTOP=B
        HTTOP=HTS(J)
        HTBOT=HTS(J+1)
  364   HT=(HTTOP+HTBOT)/2.0
        CALL EN NU
        B1=(PLASCN*EN(1)*CNU(1))/(OMEGA*(OMEGA**2+CNU(1)**2))
        IF(IONS .EQ. 0) B2=0.0
        IF(IONS .NE. 0)  B2 =
```

```
     $  (PLASCN*EN(2)*CNU(2))/(OMEGA*(OMEGA**2+CNU(2)**2)*MRATIO(2))
     $ +(PLASCN*EN(3)*CNU(3))/(OMEGA*(OMEGA**2+CNU(3)**2)*MRATIO(3))
      B=B1+B2
      IF(HTTOP-HTBOT .LT. 0.10) GO TO 366
      IF(B .GT. CUTOFF) GO TO 365
      BBOT =B
      HTBOT = HT
      GO TO 364
  365 BTOP=B
      HTTOP=HT
      GO TO 364
  366 CONTINUE
      A1=EN(1)
      IF(IONS .EQ. 0) GO TO 6661
      A2= EN(2)
      A3= EN(3)
      GO TO 6662
 6661 A2 = 0.0
      A3 = 0.0
 6662 CONTINUE
      PRINT 703,HT,CUTOFF
  703 FORMAT('0','THE BOTTOM PROFILE HEIGHT IS = ',F10.3,
     $' AT B(CUTOFF) = ',E13.3,//)
      IF(BPRINT .EQ. 0) GO TO 4113
      PRINT 4114,A1,A2,A3,B1,B2,B
 4114 FORMAT(' ',
     $          ' THE ELECTRON,POSITIVE ION AND NEGATIVE ION DENSITIES',
     *' ARE EQUAL TO ',// 1X,1PE10.3,3X,1PE10.3,3X,1PE10.3,//
     *1X,'B-ELECTRONS = ',1PE11.4,'  B-IONS = ',1PE11.4,'  B-TOTAL = ',
     *1PE11.4/)
 4113 CONTINUE
      NR A = J+1
      HTS(NR A) = HT
      ALOGEN(NR A,1) = ALOG(EN(1))
      IF(IONS .EQ. 0) GO TO 667
      ALOGEN(NR A,2) = ALOG(EN(2))
      ALOGEN(NR A,3) = ALOG(EN(3))
  667 CONTINUE
C
  702 CONTINUE
C
C
      PRINT 913
913   FORMAT('1')
C
C*****************************************************************
C
C     THIS SECTION IS USED TO DETERMINE THE PROFILE HEIGHT WHERE
C     THE VALUE OF OMEGA-R EQUALS 2.5E05
C     THE VARABLE 'WR' IS USED AS WAIT'S OMEGA-R
C
      DO 750 JREV=1,NR A
      J=NR A-JREV+1
```

```
      HT=HTS(J)
      JAY=J
      IF(J .EQ. NR A) JAY = NR A-1
      CALL EN NU
      WR=PLASCN*EN(1)/CNU(1)
      IF(IUNS .NE. 0) WR=WR+
     $  PLASCN*EN(2)/(CNU(2)*MRATIO(2))
     $ +PLASCN*EN(3)/(CNU(3)*MRATIO(3))
      IF(WR .GT. OMEGAR) GO TO 761
      BBOT=B
  790 CONTINUE
      HOFWR=HTS(1)
      PRINT 3608,HOFWR
 3608 FORMAT('C',5X,'WARNING THE TOP OF THE PROFILE IS TO LOW',
     *' FOR OMEGA-R TO BE EQUAL TO 2.5E05.',/5X,'THE VALUE OF',
     *' TOP-HEIGHT IS THEREFORE TAKEN AS THE HEIGHT AT THE TOP OF ',
     * 'THE PROFILE',//10X,'TOP HEIGHT = ',F7.2)
      GO TO 9000
C
  761 IF(J .NE. NR A) GO TO 763
      HOFWR=HTS(NR A)
      PRINT 3603,HOFWR
 3603 FORMAT('C',5X,'THE VALUE OF THE COMPUTED --OMEGA-R-- -- AT THE ',
     $ 'BOTTOM OF THE PROFILE  --HOFWR = ',F7.2,' -- ',/5X,'IS ',
     *'GREATER THAN THE INPUT LIMIT OF --OMEGA-R--')
      GO TO 9000
C
C
C         ***INTERPOLATION***
C
  763 BTOP=B
      HTTOP=HTS(J)
      HTBOT=HTS(J+1)
  764 HT=(HTTOP+HTBOT)/2.0
      CALL EN NU
      WR=PLASCN*EN(1)/CNU(1)
      IF(IUNS .NE. 0) WR=WR+
     $  PLASCN*EN(2)/(CNU(2)*MRATIO(2))
     $ +PLASCN*EN(3)/(CNU(3)*MRATIO(3))
      IF(HTTOP-HTBOT .LT. 0.10) GO TO 766
      IF(WR .GT. OMEGAR) GO TO 765
      BBOT=B
      HTBOT=HT
      GO TO 764
  765 BTOP=B
      HTTOP=HT
      GO TO 764
  766 HOFWR=HT
      PRINT 3600,HOFWR
 3600 FORMAT('C',5X,'OMEGA-R EQUALS 2.5E05 AT TOP HEIGHT = ',F7.2)
 9000 CONTINUE
      RETURN
C
```

```
901 FORMAT (20A4)
902 FORMAT (' ',20A4)
C
      END
```

```
      SUBROUTINE FINAL
C  FOR MS
C
C  ROUTINE FOR FOLLOWING THE EIGEN-ANGLES AS THE F FUNCTION IS CHANGED
C                FROM THAT USING LAGRANGE INTERPOLATION TO THAT USING
C                EXACT FULL-WAVE SOLUTION VALUES OF REFLECTION
C                COEFFICIENTS.
C
      COMMON/KXACT C/ KEXACT
      COMMON/THETA C/THETA
      COMMON/SEP COM/SEP
      COMMON/TOL COM/TOL
      COMMON/EIG COM/EIGEN(100),NR E
      COMMON/MPRNT/MPRINT
      COMMON/IDERV C/IDERIV
      DIMENSION FRACTS(32)
      COMPLEX
     $            THETA,
     $            SAVE T,
     $            EIGEN,
     $            F,DFDT,DELT,
     $            FI,DFIDT
      DATA MAX NR/6/
C
C
      IF (MPRINT .NE. 0) PRINT 900
  900 FORMAT ('1','ITERATIONS FOR EXACT SOLUTION')
      IDERIV = 1
C
      DO 19 JE=1,NR E
   10 FRACTS(1) = 0.0
      FRACTS(2) = 1.0
      NR STPS = 2
      NS = 2
      IF (MPRINT .NE. 0) PRINT 901,JE
  901 FORMAT (' ',I8)
      THETA = EIGEN(JE)
   11 SAVE T = THETA
      NCOUNT = 0
      FE FRCT = FRACTS(NS)
      FI FRCT = 1.0-FE FRCT
   12 IF (MPRINT .NE. 0) PRINT 902,THETA
  902 FORMAT (' ',10X,'THETA =',2F9.4)
      KEXACT=1
      CALL F DFDT (THETA,F,DFDT)
      IF (MPRINT .NE. 0) PRINT 104,F,DFDT
  104 FORMAT (' ',60X,'EXACT F =',2E11.3,3X,2E11.3)
      IF (NS .EQ. NR STPS) GO TO 14
C
   13 KEXACT=0
      CALL F DFDT (THETA,FI,DFIDT)
      IF (MPRINT .NE. 0) PRINT 105,FI,DFIDT
  105 FORMAT (' ',60X,'APPRX F =',2E11.3,3X,2E11.3)
```

```
      F = FI FRCT*FI+FE FRCT*F
      DFDT = FI FRCT*DFIDT+FE FRCT*DFDT
      IF (MPRINT .NE. 0) PRINT 106,F,DFDT
  106 FORMAT (' ',60X,'COMBN F =',2E11.3,3X,2E11.3)
   14 DELT = -F/DFDT
      THETA = THETA+DELT
      NCOUNT = NCOUNT+1
      IF (CABS(DELT) .GT. SEP .AND. NCOUNT .GE. 2) GO TO 20
      IF (CABS(DELT) .LT. 0.3*TOL) GO TO 16
      IF (NS .NE. NR STPS .AND. CABS(DELT) .LT. SEP) GO TO 16
      GO TO 12
C
   20 IF (MPRINT .NE. 0) PRINT 906
  906 FORMAT ('0','RESET')
      IF (NR STPS .GE. MAX NR) GO TO 17
      THETA = SAVE T
      DO 22 NREV=NS,NR STPS
      N = NR STPS-NREV+NS
   22 FRACTS(N+1) = FRACTS(N)
      FRACTS(NS) = (FRACTS(NS-1)+FRACTS(NS+1))/2.0
      NR STPS = NR STPS+1
      GO TO 11
C
   16 IF (MPRINT .NE. 0) PRINT 907
  907 FORMAT ('0')
      NS = NS+1
      IF (NS .LE. NR STPS) GO TO 11
      EIGEN(JE) = THETA
      IF (MPRINT .NE. 0) PRINT 902,THETA
      GO TO 19
   17 PRINT 109,THETA
  109 FORMAT ('0','INVALID MODE AT',2F9.4, 'TRY SMALLER RECTANGLE AROUND
     $ THIS AREA WITH APPROPRIATE RANGER AND RANGEI VALUES AND ',
     $ 'RE-EXECUTE')
      NR E = NR E-1
      IF(NR E .EQ. 0) GO TO 33
      DO 18 JJ=JE,NR E
   18 EIGEN(JJ) = EIGEN(JJ+1)
      GO TO 10
   19 CONTINUE
C
      IF (NR E .LE. 1) GO TO 33
      ISTOP = 0
      NREM1 = NR E-1
      DO 32 J=1,NREM1
      JP1 = J+1
      DO 32 JJ=JP1,NR E
      IF (CABS(EIGEN(J)-EIGEN(JJ)) .GT. TOL) GO TO 32
      PRINT 905,J,JJ
  905 FORMAT ('0','EIGENS NR',I3,1X,'AND',I3,1X,'ARE TO BE RESOLVED',/
     $                                    ' TRY SMALLER RECTANGLE AROUND
     $ THIS AREA WITH APPROPRIATE RANGER AND RANGE VALUES AND RE-EXECUTE
     $')
```

```
      PRINT 302,EIGEN(J),EIGEN(JJ)
  302 FORMAT ('0','EIGEN ANGLES AT',2F9.4,1X,'AND',2F9.4,1X,'ARE TO BE R
     $ESOLVED')
      ISTOP = 1
   32 CONTINUE
      IF (ISTOP .NE. 0) STOP
   33 RETURN
C
   90 PRINT 990
  990 FORMAT ('0','PROBLEMS IN SUBROUTINE FINAL')
      STOP
C
      END
```

```
      SUBROUTINE NE OUT
C  FOR MS
C  ROUTINE PROVIDING FOR PUNCHED OUTPUT OF EIGEN ANGLES
C
      COMMON/EIG COM/EIGEN(100),NR E
      COMMON/HTS COM/HTS(101)
      COMMON/NR OF AS/NR A
      COMMON/H COM/H
      COMMON/FRQ COM/FREQ
      COMPLEX EIGEN,
     $          THETA,
     $          S,SO
      DATA PI/3.141593/
      DATA RTD/57.29578/
      DATA VEL LT/2.997928E5/
      DATA RE/6369.427/
C
C
      WAVE NR = 2.0*PI*FREQ*1000.0/VEL LT
      ACOEF = -8.6859*WAVE NR*1000.0
      CAP K = 1.0/(1.0-H/RE)
C
      PUNCH 971,HTS(NR A)
  971 FORMAT (1X,'D = ',F7.2,',')
      PRINT 972,HTS(NR A)
  972 FORMAT ('0',1X,'D = ',F7.2,',')
      PUNCH 975
  975 FORMAT (1X,'EIGEN = ')
      PRINT 976
  976 FORMAT (' ',1X,'EIGEN = ',20X,'ATTEN   VOVERC')
      DO 77 JE=1,NR E
      THETA = EIGEN(JE)
      PUNCH 977,THETA
  977 FORMAT (1X,2(F8.3,','))
      S = CSIN(THETA/RTD)
      SO = S*CAP K
      ATTEN = ACOEF*AIMAG(SO)
      VOVERC = 1.0/REAL(SO)
      PRINT 978,THETA,ATTEN,VOVERC
   77 CONTINUE
  978 FORMAT (' ',1X,2(F8.3,','),F15.3,F9.5)
      THETA = 0.0
      PUNCH 977,THETA
      PRINT 978,THETA
      RETURN
C
      END
```

```
      SUBROUTINE NP OUT
C FOR MS
C  ROUTINE FOR COMPUTING AND PRINTING TABLE AND PUNCHED CARD OUTPUT
C                ASSOCIATED WITH NPUNCH OPTION.  FURTHER DESCRIPTION
C                OF OUTPUT IS GIVEN IN A DNA REPORT.
C
      COMMON/ID COM/IDENT(20)
      COMMON/MPRNT/MPRINT
      COMMON/NPNCH C/NPUNCH
      COMMON/CRD PRT/CARDPT
      COMMON/CRD PNC/CARDPN
      COMMON/RHO COM/RHO,WAIT RH
      COMMON/FLD COM/AZIM,CODIP,MAGFLD
      COMMON/GND COM/EPSR,SIGMA
      COMMON/FRQ COM/FREQ
      COMMON/H COM/H
      COMMON/WN/WAVE NR
      COMMON/HF/HOFWR
      COMMON/THETA C/THETA
      COMMON/EIG COM/EIGEN(100),NR E
      COMMON/INTEGR/X11,X21,X12,X22
      COMMON/RB COM/NUM11,NUM22,DEN11,DEN22
      COMMON/F1F2 C/F1,F2,HG,F1ZT,F1ZR,F2ZT,F2ZR,GF1ZT
      COMPLEX
     $          THETA,EIGEN,
     $          X11,X21,X12,X22,
     $          NUM11,NUM22,DEN11,DEN22,
     $          F1,F2,HG,F1ZT,F1ZR,F2ZT,F2ZR,GF1ZT,
     $          C,
     $          R11,R21,R12,R22,
     $          RBAR11,RBAR22,D11,D22,
     $          THETAP,TERM1,TERM2,TERM3,TERM4,
     $          RB11P1,RB22P1,
     $          FM,DFMDT,
     $          DFDT,FACTOR,
     $          S,CP,CASIN,
     $          Q1,Q2,Q3,FOVRD,FOVRO,
     $          XVZ,XEZ,XBZ,
     $          SP,STOREI,ECOMP,EXLOG
      REAL MAGFLD
      INTEGER CARDPT,CARDPN
      DATA RE/6369.427/
      DATA RTD/57.29578/
      DATA ALN TEN/2.302585/
      DATA PI/3.141593/
      DATA RTD/57.29578/
C
C
C     PRINT/PUNCH 'ID' INFORMATION
      IF(MPRINT .NE. 0) PRINT 901,IDENT
  901 FORMAT('C',20A4)
      IF(CARDPN .GT. 0) PUNCH 902,IDENT
  902 FORMAT(20A4)
```

```
C
C  PUNCH RHO CARD, PRINT H VALUE AND TABLE HEADINGS.
      IF (MPRINT .NE. 0) PRINT 971,RHO,FREQ,AZIM,CODIP,MAGFLD,SIGMA,EPSR
  971 FORMAT ('0','R',F7.3,' F',F8.4,' A',F8.3,' C',F8.3,' M',1PE10.3,
     $            ' S',E10.3,' E',0PF5.1)
      IF(CARDPN .GT. 0)
     $PUNCH 972,RHO,FREQ,AZIM,CODIP,MAGFLD,SIGMA,EPSR
  972 FORMAT ('R',F7.3,' F',F8.4,' A',F8.3,' C',F8.3,' M',1PE10.3,
     $            ' S',E10.3,' E',0PF5.1)
      PRINT 974,H
  974 FORMAT ('0',9X,'H =',F6.2)
      PRINT 975
  975 FORMAT (' ',1X,'MODE',6X,'THETA',6X,'ATTEN',3X,'VUVERC',3X,
     $            'WAIT MAG',2X,'WAIT ANG',8X,'THETAP',13X,'POL')
      IF(NPUNCH .EQ. 2 .AND. CARDPN .EQ. 1) PUNCH 939,FREQ
  939 FORMAT(F8.3)
C
      ACUEF = -8.6858896*WAVE NR*1000.0
      MODE = 0
      IDERIV = 1
C
      DO 79 JE=1,NR E
      THETA = EIGEN(JE)
C
C  FORMATION OF THE USUAL REFLECTION MATRIX, R, FROM (R+1.0)/C AND
C                FORMATION OF THE USUAL RBAR MATRIX VALUES FROM
C                FORM USED TO FIND EIGEN ANGLES.  ALSO, COMPUTATION OF
C                DERIVATIVES FOR USE IN COMPUTING DFDTHETA.
      CALL F DFDT (THETA,FM,DFMDT)
      C = CCOS(THETA/RTD)
      R11 = C*X11-1.0
      R21 = C*X21
      R12 = C*X12
      R22 = C*X22-1.0
      D11 = C*NUM11-DEN11
      D22 = C*NUM22-DEN22
      RBAR11 = DEN11/D11
      RBAR22 = DEN22/D22
      RB11P1 = C*NUM11/D11
      RB22P1 = C*NUM22/D22
C
      DFDT = DFMDT*C**2/(D11*D22)*RTD
      S = CSIN(THETA/RTD)
      FACTOR = CSQRT(S)/DFDT
C
      ITERM=1
      IF(CABS(1.0-R11*RBAR11) .GE. CABS(1.0-R22*RBAR22)  ) ITERM=2
C
C  VLF-LF EXCITATION TERMS (E.G. THE ''T'' TERMS TO BE USED FOR
C  NPUNCH=1 OR NPUNCH = 9 ).
      TERM1 = RB11P1**2*(1.0-RBAR22*R22)*FACTOR/(RBAR11*F1*F1)
      TERM2 = RB22P1**2*(1.0-RBAR11*R11)*FACTOR/(RBAR22*F2*F2)
      TERM3 = RB11P1*RB22P1*FACTOR*R21/(F1*F2)
```

```
      TERM4 = F12/R21
C
C  ELF EXCITATION TERMS (E.G. THE TERMS TO BE USED FOR NPUNCH = 7 ).
      XVZ=TERM1*S*S*F1ZT*F1ZR
      XEZ=-TERM1*S*GF1ZT*F1ZR
      XBZ=-TERM3*TERM4*S*F2ZT*F1ZR
C        IN TERMS OF MAGNITUDE AND PHASE THESE TERMS ARE GIVEN BY
      XVZM=CABS(XVZ)
      XVZA=CANG(XVZ)
      XEZM=CABS(XEZ)
      XEZA=CANG(XEZ)
      XBZM=CABS(XBZ)
      XBZA=CANG(XBZ)
C
C  EIGEN ANGLE REFERRED TO GROUND LEVEL.
      RATIO = H/RE
      CP = CSQRT(C**2+(C**2-1.0)*(2.0*RATIO+3.0*RATIO**2))
      THETAP = 90.0-CASIN(CP)*RTD
C
C  ATTENUATION AND PHASE VELOCITY REFERRED TO GROUND LEVEL.
      SP = S/(1.0-H/RE)
      ATTEN = ACOEF*AIMAG(SP)
      VOVERC = 1.0/REAL(SP)
C
C  WAIT'S EXCITATION.
      STORE1 = RB11P1**2*(1.0-RBAR22*R22)
     $          *FACTOR/RBAR11
      ECOMP = FG**2*STORE1*S**2
      EXLOG = CLOG(ECOMP*0.5*WAVE NK*WAIT RH)
      WM = 20.0*REAL(EXLOG)/ALN TEN
      WA = AIMAG(EXLOG)-PI/2.0
      IF (WA .LT. -PI/2.0) WA = WA+2.0*PI
C
C  MODE CONVERSION TERMS TO BE USED FOR NPUNCH=8 AND NPUNCH=9.
      XM= CABS(ECOMP)
      XA= CANG(ECOMP)
      Q1=1.0-R11*RBAR11
      Q2=1.0-R22*RBAR22
      Q3= RB22P1/RB11P1
      IF( CABS(Q1) .GE.  CABS(Q2) ) FOVRD=Q3*Q1/(R12*RBAR22)
      IF( CABS(Q2) .GE.  CABS(Q1) ) FOVRD = Q3*(R21*RBAR11)/Q2
      FOVRC=(FOVRD*F1)/F2
C
C  POLARIZATION.
      D1 = CABS(R21*RBAR22)
      D2 = CABS(1.0-RBAR11*R11)
      IF (D1 .GE. D2) POL = CABS(1.0-RBAR22*R22)/D1
      IF (D2 .GE. D1) POL = CABS(R12*RBAR11)/D2
C
C  PRINT VALUES FOR TABLE.
C
      MODE = MODE+1
      PRINT 932,MODE,THETA,ATTEN,VOVERC,WM,WA,THETAP,POL
```

```
  932 FORMAT (' ',I4,2F8.3,F6.3,F9.5,F10.3,F9.3,3X,2F8.3,F12.5)
C
C
C  NPUNCH OPTION FOR OBTAINING OUTPUT CARDS
C
C        NPUNCH=1*** GIVES OUTPUT CARDS FOR HORIZONTALY HOMOGENOUS
C                     MODE-SUM OR WKB-SUM IN TERMS OF (T'S)
C        NPUNCH=2*** GIVES OUTPUT CARDS IN TERMS OF SNVLF OUTPUT
C        NPUNCH=7*** GIVES OUTPUT CARDS FOR INPUT INTO ELF-WKB MODE-SUMS
C        NPUNCH=8*** GIVES OUTPUT CARDS FOR INPUT TO MODE CONVERSION
C                     IN TERMS OF (FOVR)
C        NPUNCH=9*** GIVES OUTPUT CARDS FOR INPUT INTO MODE CONVERSION
C                     IN TERMS OF BOTH (T'S) AND (FOVR)
C
C        CARD PN=0*** GIVES USUAL MODE SUMMARY PRINT OUT BUT NO CARDS
C        CARD PN=1*** GIVES USUAL MODE SUMMARY PRINT AND CARDS
C        CARD PT=0*** CAUSES THE OUTPUT CARDS TO NOT BE LISTED
C        CARD PT=1*** CAUSES THE OUTPUT CARDS TO BE LISTED
C
C  PUNCH ''NPUNCH ''' OPTION CARDS----'MPRINT' USED ONLY FOR DEBUGGING
C
      IF (MPRINT .NE. 0) PRINT 933,THETAP,TERM1,TERM2,
     $           THETAP,TERM3,TERM4
  933 FORMAT ('0','1',0P2F9.5,1X,1P4E15.8/' ','2',0P2F9.5,1X,1P4E15.8)
      IF(MPRINT .NE. 0) PRINT 930
  930 FORMAT(' ')
C
      IF(NPUNCH .EQ. 1 .AND. CARDPT .EQ. 1 ) PRINT 930
      IF(NPUNCH .EQ. 1 .AND. CARDPT .EQ. 1 )
     * PRINT 931,THETAP,ITERM,TERM1,TERM2,THETAP,ITERM,TERM3,TERM4
  931 FORMAT (' ','1',0P2F9.5,I1,1P4E15.8/' ','2',0P2F9.5,I1,1P4E15.8/)
      IF(NPUNCH .EQ. 1 .AND. CARDPN .EQ. 1 )
     * PUNCH 934,THETAP,ITERM,TERM1,TERM2,THETAP,ITERM,TERM3,TERM4
  934 FORMAT ('1',0P2F9.5,I1,1P4E15.8,/,'2',0P2F9.5,I1,1P4E15.8)
      IF(NPUNCH .EQ. 1) GO TO 600
      IF(NPUNCH .EQ. 2 .AND. CARDPT .EQ. 1 )
     *PRINT 937,ATTEN,VOVERC,WM,WA
  937 FORMAT('0',4F10.5)
      IF(NPUNCH .EQ. 2 .AND. CARDPN .EQ. 1 )
     *PUNCH 938,ATTEN,VOVERC,WM,WA
  938 FORMAT(4F10.5)
      IF(NPUNCH .EQ. 2) GO TO 600
      IF(NPUNCH .EQ. 7 .AND. CARDPT .EQ. 1 ) PRINT 930
      IF(NPUNCH .EQ. 7 .AND. CARDPT .EQ. 1 )
     * PRINT 626,THETAP,XVZM,XVZA,XEZM,XEZA,XBZM,XBZA
  626 FORMAT(   2(F10.5,','),/5X,3(1PE13.4,',',0PF8.3,','),/)
      IF(NPUNCH .EQ. 7 .AND. CARDPN .EQ. 1 )
     * PUNCH 627,THETAP,XVZM,XVZA,XEZM,XEZA,XBZM,XBZA
  627 FORMAT(5X,2(F10.5,','),/5X,3(1PE13.4,',',0PF8.3,',') )
      IF(NPUNCH .EQ. 7) GO TO 600
      IF(NPUNCH .EQ. 8 .AND. CARDPT .EQ. 1 ) PRINT 930
      IF(NPUNCH .EQ. 8 .AND. CARDPT .EQ. 1 )
     *PRINT 935, THETAP,XM,XA,FOVRO,HOFWR
```

```
  935 FORMAT(     0P2F9.5,1PE14.5,0PF9.5,1P2E16.8,0PF7.2/)
      IF(NPUNCH .EQ. 8 .AND. CARDPN .EQ. 1 )
     *PUNCH 936, THETAP,XM,XA,FOVRO,HOFWR
  930 FORMAT(0P2F9.5,1PE14.5,0PF9.5,1P2E16.8,0PF7.2)
      IF(NPUNCH .EQ. 8) GO TO 600
      IF(NPUNCH .EQ. 9 .AND. CARDPT .EQ. 1 ) PRINT 930
      IF(NPUNCH .EQ. 9 .AND. CARDPT .EQ. 1 )
     * PRINT 931,THETAP,ITERM,TERM1,TERM2,THETAP,ITERM,TERM3,TERM4
      IF(NPUNCH .EQ. 9 .AND. CARDPT .EQ. 1 )
     *PRINT 935, THETAP,XM,XA,FOVRO,HOFWR
      IF(NPUNCH .EQ. 9 .AND. CARDPN .EQ. 1 )
     * PUNCH 934,THETAP,ITERM,TERM1,TERM2,THETAP,ITERM,TERM3,TERM4
      IF(NPUNCH .EQ. 9 .AND. CARDPN .EQ. 1 )
     *PUNCH 936, THETAP,XM,XA,FOVRO,HOFWR
  600 CONTINUE
   79 CONTINUE
C
      IF(CARD PN .GT. 0) PUNCH 700
  700 FORMAT (' ')
      RETURN
C
      END
```

```
      FUNCTION CASIN(S)
C  ROUTINE FOR COMPUTING THE COMPLEX SINE FUNCTION OF A COMPLEX
C                ARGUMENT.
C  FOR MS
C
      COMPLEX CASIN,S,SSQ,SPOWER,SUM,TERM,PARTS,C,CPLX I
      DATA CPLX I/(0.0,1.0)/
      DIMENSION PART(2)
      EQUIVALENCE (PART(1),PARTS)
C
C
      IF (CABS(S) .GT. 0.7) GO TO 20
      FACTOR = 1.0
      SPOWER = S
      SSQ = S**2
      SUM = S
      IF (CABS(SUM) .LT. 1.0E-7) GO TO 12
      DO 11 K=3,99,2
      FACTOR = FACTOR*(K-2.0)/(K-1.0)
      SPOWER = SPOWER*SSQ
      TERM = FACTOR*SPOWER/K
      SUM = SUM+TERM
      PARTS = TERM/SUM
      IF (ABS(PART(1))+ABS(PART(2)) .LT. 1.0E-10) GO TO 12
   11 CONTINUE
   12 CASIN = SUM
      RETURN
C
   20 C = CSQRT(1.0-S**2)
      CASIN = -CPLX I*CLOG(C+CPLX I*S)
      RETURN
C
      END
```

```
FUNCTION CANG(ARG)
IMPLICIT REAL    (A-H,O-Z)
COMPLEX     ARG,MINUSI/(0.,-1.)/
ARGR=ARG
ARGI=MINUSI*ARG
CANG=ATAN2(ARGI,ARGR)
IF(ARGI .LT. 0. )  CANG= CANG+6.283185
RETURN
END
```

```
      SUBROUTINE INTEG
C FOR MS
C  THIS SUBROUTINE PERFORMS AN INTEGRATION OF THE DIFFERENTIAL
C                 EQUATIONS FOR THE IONOSPHERE REFLECTION MATRIX
C                 USING RUNGE-KUTTA INTEGRATION FORMULAS.  THE
C                 INTEGRATION VARIABLES ARE THE ELEMENTS OF THE MATRIX
C                 (R+1.0)/C WHERE R IS THE REFLECTION MATRIX DESCRIBED
C                 BY BUDDEN.  IF IDERIV IS SET NON-ZERO, THE DERIVATIVES
C                 OF (R+1.0)/C ELEMENTS WRT C=COS(THETA) ARE ALSO
C                 INTEGRATION VARIABLES.  THE SET OF HEIGHTS AT WHICH
C                 PAIRS OF INTEGRATION STEPS BEGIN AND END IS STORED
C                 IN THE ARRAY 'RKHTS'.  SIZES OF THE STEPS ARE
C                 DETERMINED BY COMPARING THE VALUES OF THE ELEMENTS
C                 OF (R+1.0)/C AFTER A PAIR OF STEPS AND VALUES OF THE
C                 SAME VARIABLES AFTER A SINGLE DOUBLE-SIZE COMPARISON
C                 STEP.  THE (R+1.0)/C VARIABLES USED IN THE COMPARISON
C                 STEP ARE DENOTED AS 'X'.  IF THE DIFFERENCE IS TOO
C                 LARGE, THE STEP SIZE PAIRS ARE CUT IN HALF BY ADDING
C                 A NEW HT TO THE LIST IN 'RKHTS'.  IN ORDER TO
C                 ENHANCE STABILITY IN OTHER PARTS OF THE PROGRAM, NO
C                 HEIGHTS ARE DELETED FROM THE LIST UNLESS THE LIST IS
C                 RESET BY A CALL TO 'SET RK'.
C
      COMMON/NR OF AS/NR A
      COMMON /RTOL C/RTOL
      COMMON/HTS COM/HTS(101)
      COMMON/XPRNT/XPRINT
      COMMON/INTEGR/R(16),DRDH(16)
      COMMON/X INTGR/X(8),DXDH(8)
      COMMON/EN COLL/HT
      COMMON/WN/WAVE NR
      COMMON/IDERV C/IDERIV
      COMMON/N COM/N
      COMMON/OVRFLO/IOVFLO
      COMMON/JAY COM/JAY
      DIMENSION
     $          R0(16),HDELR0(16),DELR1(16),DELR2(16),
     $          X0(8),HDELX0(8),DELX1(8),DELX2(8),
     $          R SAVE(16),
     $          RKHTS(201)
      INTEGER XPRINT
      DATA MAX NR/201/
      DATA DHMIN/0.01/
C
C
      N = 8
      IF (IDERIV .NE. 0) N = 16
      IOVFLO = 0
      IF(XPRINT .NE. 0) CALL R COLS
      THIRD = 1.0/3.0
      CALL INIT S
      JAY = 1
      JRK = 1
```

```
      HT = HTS(1)
      CALL S MTRX
      CALL INITL R
C
      CALL XFER (R,X,8)
      IF(XPRINT .NE. 0) PRINT 901
  901 FORMAT ('0')
      IF(XPRINT .NE. 0) CALL PRINT R
C
      NRAM1 = NR A-1
      DO 79 J=1,NRAM1
      JAY = J
   30 CALL XFER(R,R SAVE,N)
      HT = RKHTS(JRK)
      CALL XFER (R,X,8)
   31 DELH = (RKHTS(JRK+1)-RKHTS(JRK))*0.5
      IF (ABS(DELH) .LT. DHMIN) GO TO 90
      DH = DELH*WAVE NR/2.0
      HDH = 0.5*DH
      TDH = 2.0*DH
C
C  BEGIN FIRST RUNGE-KUTTA STEP OF THE PAIR AND THE COMPARISON STEP.
      CALL R DERIV
      IF (IOVFLC .NE. 0) GO TO 60
C
      DO 32 I=1,N
      RO(I) = R(I)
      HDELRO(I) = DRDH(I)*HDH
   32 R(I) = RO(I)+HDELRO(I)
C
      DO 33 I=1,8
      XO(I) = X(I)
      HDELXO(I) = HDELRO(I)*2.0
   33 X(I) = XO(I)+HDELXO(I)
C
      HT = HT+0.5*DELH
      CALL S MTRX
      CALL R DERIV
      IF(IOVFLC .NE. 0) GO TO 60
C
      DO 34 I=1,N
      DELR1(I) = DRDH(I)*DH
   34 R(I) = RO(I)+0.5*DELR1(I)
C
      CALL R DERIV
      IF(IOVFLC .NE. 0) GO TO 60
C
      DO 35 I=1,N
      DELR2(I) = DRDH(I)*DH
   35 R(I) = RO(I)+DELR2(I)
C
      HT = HT+0.5*DELH
      CALL S MTRX
```

```
      CALL R DERIV
      CALL X DERIV
      IF(IOVFLC .NE. 0) GO TO 60
C
      DO 36 I=1,N
      HDELR3 = DRDH(I)*HDH
      DELR4 = (HDELR0(I)+DELR1(I)+DELR2(I)+HDELR3)*THIRD
   36 R(I) = RC(I)+DELR4
C
C  BEGIN SECOND RUNGE-KUTTA STEP OF THE PAIR.
      DO 37 I=1,8
      DELX1(I) = DXDH(I)*TDH
   37 X(I) = X0(I)+0.5*DELX1(I)
C
      CALL R DERIV
      CALL X DERIV
      IF(IOVFLC .NE. 0) GO TO 60
C
      DO 42 I=1,N
      R0(I) = R(I)
      HDELR0(I) = DRDH(I)*HDH
   42 R(I) = R0(I)+HDELR0(I)
C
      DO 43 I=1,8
      DELX2(I) = DXDH(I)*TDH
   43 X(I) = XC(I)+DELX2(I)
C
      HT = HT+C.5*DELH
      CALL S MTRX
      CALL R DERIV
      IF(IOVFLC .NE. 0) GO TO 60
C
      DO 44 I=1,N
      DELR1(I) = DRDH(I)*DH
   44 R(I) = R0(I)+0.5*DELR1(I)
C
      CALL R DERIV
      IF(IOVFLC .NE. 0) GO TO 60
C
      DO 45 I=1,N
      DELR2(I) = DRDH(I)*DH
   45 R(I) = RC(I)+DELR2(I)
C
      HT = HT+C.5*DELH
      CALL S MTRX
      CALL R DERIV
      CALL X DERIV
      IF(IOVFLC .NE. 0) GO TO 60
C
      DO 46 I=1,N
      HDELR3 = DRDH(I)*HDH
      DELR4 = (HDELR0(I)+DELR1(I)+DELR2(I)+HDELR3)*THIRD
   46 R(I) = R0(I)+DELR4
```

```
C
      DO 47 I=1,8
      HDELX3 = DXDH(I)*DH
      DELX4 = (HDELX0(I)+DELX1(I)+DELX2(I)+HDELX3)*THIRD
   47 X(I) = X0(I)+DELX4
C
C  ESTIMATE TRUNCATION ERROR AND CUT STEP SIZE IF APPROPRIATE.
      IF(XPRINT .NE. 0) CALL PRINT R
      DO 51 I=1,7,2
      ERRSQ = (R(I)-X(I))**2+(R(I+1)-X(I+1))**2
      RSQ = R(I)**2+R(I+1)**2
      IF (RSQ .GT. 1.0) ERRSQ = ERRSQ/RSQ
      IF (ERRSQ .GT. RTOL**2) GO TO 61
   51 CONTINUE
      GO TO 70
C
   60 IF (XPRINT .NE. 0) CALL PRINT R
   61 IF (XPRINT .NE. 0) PRINT 960
  960 FORMAT (' ','OVRFLO')
      IF (NR RK .GE. MAX NR) GO TO 95
      IOVFLO = 0
      CALL XFER(R SAVE,R,N)
      CALL XFER (R SAVE,X,8)
      HT = RKHTS(JRK)
      CALL S MTRX
C
      NRRKM1 = NR RK-1
      DO 62 JCOUNT=JRK,NRRKM1
      JJ = NRRKM1-JCOUNT+JRK
   62 RKHTS(JJ+2) = RKHTS(JJ+1)
      RKHTS(JRK+1) = (RKHTS(JRK)+RKHTS(JRK+2))*0.5
      NR RK = NR RK+1
      GO TO 31
C
   70 JRK = JRK+1
      IF (ABS(RKHTS(JRK)-HTS(J+1)) .GT. 0.001) GO TO 30
   79 CONTINUE
      RETURN
C
C
C  RESET THE INTEGRATION STEP PAIRS TO BE THE INPUT PROFILE SEGMENTS.
      ENTRY SET RK
      DO 81 J=1,NR A
   81 RKHTS(J) = HTS(J)
      NR RK = NR A
      RETURN
C
   90 PRINT 990
  990 FORMAT ('0','STEP TOO SMALL IN INTEG')
      STOP
C
   95 PRINT 995
  995 FORMAT ('0','TOO MANY STEPS IN INTEG')
```

```
      STOP
C
      END
```

```
      SUBROUTINE EN NU
C FOR MS
C  COMPUTATION OF ELECTRON DENSITY AND COLLISION FREQUENCY (AND ION
C                 DENSITY IF APPROPRIATE) AS A FUNCTION OF HEIGHT.
C                 IF PROFILE WAS INPUT FROM FORMATTED CARDS, LOGARITHMIC
C                 INTERPOLATION IS USED.  SEE ALSO NOTES IN SUBROUTINE
C                 PRELIN.
C
      COMMON/NR OF AS/NR A
      COMMON/ICN COM/IONS
      COMMON/JAY COM/J
      COMMON/HTS COM/HTS(101)
      COMMON/HTLOGN/NR NU,HTS NU(50),ALCNU(50,3)
      COMMON/ALOGN C/ALOGEN(101,3)
      COMMON/EXPNU C/COEFNU(3),EXPNU(3)
      COMMON/EN COLL/HT,EN(3),CNU(3)
C
      DATA JNU/1/
C
      FACTR = 1.0/(HTS(J)-HTS(J+1))
      NR SPEC = 1
      IF (IONS .NE. 0) NR SPEC = 3
      F1 = (HT-HTS(J+1))*FACTR
      F2 = (HTS(J)-HT)*FACTR
      DO 11 K=1,NR SPEC
   11 EN(K) =  EXP(ALOGEN(J,K)*F1+ALOGEN(J+1,K)*F2)
      IF(COEFNU(1) .LT. 0.0) GO TO 15
      DO 12 K=1,NR SPEC
   12 CNU(K) = COEFNU(K)*EXP(EXPNU(K)*HT)
      RETURN
C
   15 IF(JNU .GT. NR NU-1) JNU=1
   16 IF (HT .LE. HTS NU(JNU) .OR. JNU .EQ. 1) GO TO 17
      JNU = JNU-1
      GO TO 16
   17 IF (HT .GE. HTS NU(JNU+1) .OR. JNU .EQ. NR NU-1) GO TO 18
      JNU = JNU+1
      GO TO 17
   18 DO 19 K = 1, NR SPEC
      ALN = ALCNU(JNU,K)+(ALCNU(JNU+1,K)-ALCNU(JNU,K))
     $          *(HT-HTS NU(JNU))/(HTS NU(JNU+1)-HTS NU(JNU))
   19 CNU(K) = EXP(ALN)
      RETURN
C
      END
```

```
      SUBROUTINE R OUT
C FOR MS
C  OUTPUT OF RUNGE-KUTTA INTEGRATION VARIABLES, USED ONLY FOR DEBUGGING.
C
      COMMON/INTEGR/R(8)
      COMMON/X INTGR/X(8)
      COMMON/EN COLL/HT,EC OMIT(2)
C
C
      ENTRY R COLS
      PRINT 900
  900 FORMAT('0',8X,'HT',10X,'11R11',16X,'11R1',16X,'1R11',17X,'1R1')
      RETURN
C
C
      ENTRY PRINT R
      PRINT 901,HT,(X(I),I=1,8)
      PRINT 901,HT,(R(I),I=1,8)
  901 FORMAT(' ',F10.2,4(2X,F9.5,F9.5))
      PRINT 902
  902 FORMAT ('0')
      RETURN
C
      END
```

```
      SUBROUTINE XFER (A,B,N)
C FOR MS
C  ROUTINE FOR TRANSFERRING ONE ARRAY INTO ANOTHER.
C
      DIMENSION A(1),B(1)
C
C
      DO 11 J=1,N
   11 B(J) = A(J)
      RETURN
C
      END
```

```
      SUBROUTINE INITL R
C FOR MS
C  COMPUTATION OF (R+1.0)/C FOR REFLECTION FROM A SHARPLY BOUNDED
C                ANISOTROPIC IONOSPHERE OF SEMI-INFINITE EXTENT
C                WHERE R IS THE REFLECTION COEFFICIENT MATRIX AS DEFINED
C                BY BUDDEN.  SOLUTION IS USED AS INITIAL CONDITION
C                FOR RUNGE-KUTTA INTEGRATION.  THE SOLUTION IS BASED
C                ON RADIO SCIENCE PAPER VOL. 3, AUG. 1968, P.792.
C                DERIVATIVES WRT C=COS(THETA) ARE ALSO COMPUTED IF
C                IDERIV IS SET NON-ZERO.
C
      COMMON/THETA C/THETA(2)
      COMMON/XPRNT/XPRINT
      COMMON/M MTX/M11,M21,M31,M12,M22,M32,M13,M23,M33
      COMMON/INTEGR/R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC
      COMMON/C COM/C
      COMMON/S COM/S
      COMMON/IDERV C/IDERIV
      COMPLEX
     $          C,CSQ,S,
     $          M11,M21,M31,M12,M22,M32,M13,M23,M33,
     $          R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC,
     $          QTEMP(4),EIPI04,Q(2),
     $          B4,B3,B2,B1,B0,
     $          D11,D12,D13,D31,D32,D33,
     $          P(2),T(2),
     $          DEN,FACTOR,
     $          DSDC,DQDC(2),DDENDC,
     $          DB3DC,DB2DC,DB1DC,DB0DC,
     $          DD11DC,DD13DC,DD31DC,DD33DC,
     $          DPDC(2),DTDC(2)
      INTEGER XPRINT
      DATA EIPI04/(0.707107,0.707107)/
C
C
      CSQ = C**2
C
C  BOOKER QUARTIC COEFFICIENTS AND ITS FOUR ROOTS.
      B4 = 1.0+M33
      B3 = S*(M13+M31)
      B2 = -(CSQ+M33)*(1.0+M11)+M13*M31-(1.0+M33)*(CSQ+M22)+M23*M32
      B1 = S*(M12*M23+M21*M32-(CSQ+M22)*(M13+M31))
      B0 = (1.0+M11)*(CSQ+M22)*(CSQ+M33)+M12*M23*M31+M13*M21*M32
     $          -M13*(CSQ+M22)*M31-(1.0+M11)*M23*M32-M12*M21*(CSQ+M33)
C
      CALL QUARTC(B4,B3,B2,B1,B0,QTEMP)
C
C  SELECTION OF THE TWO ROOTS CORRESPONDING TO UPWARD TRAVELLING
C                WAVES AS BEING THOSE OF ANGLE LESS THAN 45 BUT
C                GREATER THAN -135 DEGREES IN THE COMPLEX PLANE.
      K = 0
      DO 21 KT=1,4
      IF (REAL(EIPI04*QTEMP(KT)) .LT. 0.0) GO TO 21
```

```
      K = K+1
      IF (K .GE. 3) GO TO 90
      Q(K) = QTEMP(KT)
   21 CONTINUE
      IF (K .NE. 2) GO TO 90
C
      IF(XPRINT .NE. 0) PRINT 901,THETA,(Q(J),J=1,2)
  901 FORMAT('C','THETA = ',2F6.3,5X,'Q = ',2(E15.4,E13.4),/)
C
C  REMAINDER OF THE SOLUTION AND ITS DERIVATIVES.
      DO 31 J=1,2
      D11 = 1.0+M11-Q(J)**2
      D12 = M12
      D13 = M13+Q(J)*S
      D31 = M31+Q(J)*S
      D32 = M32
      D33 = CSQ+M33
      DEN = D11*D33-D13*D31
      P(J) = (-D12*D33+D13*D32)/DEN
   31 T(J) = Q(J)*P(J)-S*(-D11*D32+D12*D31)/DEN
C
      IF (IDERIV .EQ. 0) GO TO 50
      DSDC = -C/S
      DB3DC = DSDC*(M13+M31)
      DB2DC = -2.0*C*(1.0+M11+1.0+M33)
      DB1DC = (DSDC/S)*B1-S*2.0*C*(M13+M31)
      DB0DC = 2.0*C*((1.0+M11)*(CSQ+M22+CSQ+M33)-M13*M31-M12*M21)
C
      DO 41 K=1,2
      DQDC(K) = -(((DB3DC*Q(K)+DB2DC)*Q(K)+DB1DC)*Q(K)+DB0DC)
     $          /(((4.0*B4*Q(K)+3.0*B3)*Q(K)+2.0*B2)*Q(K)+B1)
      DD11DC = -2.0*Q(K)*DQDC(K)
      DD13DC = Q(K)*DSDC
      DD31DC = Q(K)*DSDC
      DD33DC = 2.0*C
      DDENDC = D11*DD33DC+DD11DC*D33-D13*DD31DC-DD13DC*D31
      DPDC(K) = -DDENDC*P(K)/DEN+(-D12*DD33DC+DD13DC*D32)/DEN
   41 DTDC(K) = Q(K)*DPDC(K)+DQDC(K)*P(K)
     $          -(-D11*D32+D12*D31)*(DSDC-S*DDENDC/DEN)/DEN
     $          -S*(-DD11DC*D32+D12*DD31DC)/DEN
C
   50 DEN = (T(1)*C+P(1))*(C+Q(2))-(T(2)*C+P(2))*(C+Q(1))
      FACTOR = 2.0/DEN
      R11 = (T(1)*(C+Q(2))-T(2)*(C+Q(1)))*FACTOR
      R22 = ((T(1)*C+P(1))-(T(2)*C+P(2)))*FACTOR
      R12 = -(T(1)*P(2)-T(2)*P(1))*FACTOR
      R21 = -(Q(1)-Q(2))*FACTOR
C
      IF (IDERIV .EQ. 0) RETURN
      DDENDC = (T(1)*C+P(1))*(1.0+DQDC(2))
     $          +(T(1)+DTDC(1)*C+DPDC(1))*(C+Q(2))
     $          -(T(2)*C+P(2))*(1.0+DQDC(1))
     $          -(T(2)+DTDC(2)*C+DPDC(2))*(C+Q(1))
```

```
      DR11DC  = -R11*DDENDC/DEN
     $           +(T(1)*(1.0+DQDC(2))+DTDC(1)*(C+Q(2))
     $           -T(2)*(1.0+DQDC(1))-DTDC(2)*(C+Q(1)))*FACTOR
      DR22DC  = -R22*DDENDC/DEN
     $           +((T(1)+DTDC(1)*C+DPDC(1))
     $           -(T(2)+DTDC(2)*C+DPDC(2)))*FACTOR
      DR12DC  = -R12*DDENDC/DEN
     $           -(T(1)*DPDC(2)+DTDC(1)*P(2)
     $           -T(2)*DPDC(1)-DTDC(2)*P(1))*FACTOR
      DR21DC  = -R21*DDENDC/DEN
     $           -(DQDC(1)-DQDC(2))*FACTOR
      RETURN
C
   90 PRINT 900
  900 FORMAT('0','PROBLEM IN SORTING Q VALUES')
      PRINT 995,THETA,QTEMP
  995 FORMAT ('0','THETA = ',2F8.3,5X,'Q = ',4(1PE13.3,1PE11.3))
      STOP
C
      END
```

```
      SUBROUTINE QUARTC(B4,FOUR B3,SIX B2,FOUR B1,ONE B0,Q)
C FOR MS
C  SOLUTION FOR THE ROOTS OF A FOURTH-ORDER POLYNOMIAL (QUARTIC
C                EQUATION) TAKEN FROM BURNSIDE AND PANTON (1904) THE
C                THEORY OF EQUATIONS.  A SUMMARY OF PERTINENT EQUATIONS
C                IS GIVEN IN RADIO SCIENCE VOL. 3, AUG. 1968,PP.792-795.
C
      COMPLEX FOUR B3,SIX B2,FOUR B1,ONE B0,Q,B4,
     $          H,I,G,H PRIME,G PRIME,
     $          SQ ROOT,P POS,P,LOG P,
     $          CUB RT0,CUB RT1,CUB RT2,OMEGA1,OMEGA2,
     $          ROOT P,ROOT Q,ROOT R,
     $          TWLV B3,TWLV B2,F,DFDQ,DEL Q,FB4
      COMPLEX*16 B3,B2,B1,B0,B3 SQ
      REAL MAG POS,MAG NEG
      DIMENSION Q(4),P RI(2)
      DATA OMEGA1/(-0.5,0.8660254038)/,OMEGA2/(-0.5,-0.8660254038)/
      EQUIVALENCE(P,P RI(1) )
C
C
      B3 = FOUR B3/(4.0*B4)
      B2 = SIX B2/(6.0*B4)
      B1 = FOUR B1/(4.0*B4)
      B0 = ONE B0/B4
C
      B3 SQ = B3**2
      H = B2-B3 SQ
      I = B0-4.0*B3*B1+3.0*B2**2
      G = B1+B3*(-3.0*B2+2.0*B3 SQ)
      H PRIME = -I/12.0
      G PRIME = -G**2/4.0-H*(H**2+3.0*H PRIME)
C
      SQ ROOT = CSQRT(G PRIME**2+4.0*H PRIME**3)
      P = (-G PRIME+SQ ROOT)*0.5
      MAG POS = ABS(P RI(1))+ABS(P RI(2))
      P POS = P
      P = (-G PRIME-SQ ROOT)*0.5
      MAG NEG = ABS(P RI(1))+ABS(P RI(2))
      IF(MAG POS .GT. MAG NEG) P = P POS
      LOG P = CLOG(P)
      CUB RT0 = CEXP(LOG P/3.0)
      CUB RT1 = OMEGA1*CUB RT0
      CUB RT2 = OMEGA2*CUB RT0
C
      ROOT P = CSQRT(CUB RT0-H PRIME/CUB RT0-H)
      ROOT Q = CSQRT(CUB RT1-H PRIME/CUB RT1-H)
      ROOT R = CSQRT(CUB RT2-H PRIME/CUB RT2-H)
      IF(CABS(G) .LT. 1.0E-50) GO TO 21
      SIGN = -ROOT P*ROOT Q*ROOT R*2.0/G
      IF(SIGN .LT. 0.0) ROOT R = -ROOT R
   21 Q(1) = +ROOT P+ROOT Q+ROOT R-B3
      Q(2) = +ROOT P-ROOT Q-ROOT R-B3
      Q(3) = -ROOT P+ROOT Q-ROOT R-B3
```

```
      SUBROUTINE QUARTC(B4,FOUR B3,SIX B2,FOUR B1,ONE B0,Q)
C FOR MS
C  SOLUTION FOR THE ROOTS OF A FOURTH-ORDER POLYNOMIAL (QUARTIC
C                EQUATION) TAKEN FROM BURNSIDE AND PANTON (1904) THE
C                THEORY OF EQUATIONS.  A SUMMARY OF PERTINENT EQUATIONS
C                IS GIVEN IN RADIO SCIENCE VOL. 3, AUG. 1968,PP.792-795.
C
      COMPLEX FOUR B3,SIX B2,FOUR B1,ONE B0,Q,B4,
     $          H,I,G,H PRIME,G PRIME,
     $          SQ ROOT,P POS,P,LOG P,
     $          CUB RT0,CUB RT1,CUB RT2,OMEGA1,OMEGA2,
     $          ROOT P,ROOT Q,ROOT R,
     $          TWLV B3,TWLV B2,F,DFDQ,DEL Q,FB4
      COMPLEX*16 B3,B2,B1,B0,B3 SQ
      REAL MAG POS,MAG NEG
      DIMENSION Q(4),P RI(2)
      DATA OMEGA1/(-0.5,0.8660254038)/,OMEGA2/(-0.5,-0.8660254038)/
      EQUIVALENCE(P,P RI(1) )
C
C
      B3 = FOUR B3/(4.0*B4)
      B2 = SIX B2/(6.0*B4)
      B1 = FOUR B1/(4.0*B4)
      B0 = ONE B0/B4
C
      B3 SQ = B3**2
      H = B2-B3 SQ
      I = B0-4.0*B3*B1+3.0*B2**2
      G = B1+B3*(-3.0*B2+2.0*B3 SQ)
      H PRIME = -I/12.0
      G PRIME = -G**2/4.0-H*(H**2+3.0*H PRIME)
C
      SQ ROOT = CSQRT(G PRIME**2+4.0*H PRIME**3)
      P = (-G PRIME+SQ ROOT)*0.5
      MAG POS = ABS(P RI(1))+ABS(P RI(2))
      P POS = P
      P = (-G PRIME-SQ ROOT)*0.5
      MAG NEG = ABS(P RI(1))+ABS(P RI(2))
      IF(MAG POS .GT. MAG NEG) P = P POS
      LOG P = CLOG(P)
      CUB RT0 = CEXP(LOG P/3.0)
      CUB RT1 = OMEGA1*CUB RT0
      CUB RT2 = OMEGA2*CUB RT0
C
      ROOT P = CSQRT(CUB RT0-H PRIME/CUB RT0-H)
      ROOT Q = CSQRT(CUB RT1-H PRIME/CUB RT1-H)
      ROOT R = CSQRT(CUB RT2-H PRIME/CUB RT2-H)
      IF(CABS(G) .LT. 1.0E-50) GO TO 21
      SIGN = -ROOT P*ROOT Q*ROOT R*2.0/G
      IF(SIGN .LT. 0.0) ROOT R = -ROOT R
   21 Q(1) = +ROOT P+ROOT Q+ROOT R-B3
      Q(2) = +ROOT P-ROOT Q-ROOT R-B3
      Q(3) = -ROOT P+ROOT Q-ROOT R-B3
```

```
      SUBROUTINE T MTRX
C FOR MS
C  COMPUTATION OF M MATRIX ELEMENTS AS DEFINED BY BUDDEN IN PROC.
C                 ROY. SOC. (LONDON), A227, PP.516-537 (1955).  THOSE
C                 COMBINATIONS OF M MATRIX ELEMENTS USED IN THE
C                 T MATRIX WHICH DO NOT INCLUDE USE OF THETA ARE THE
C                 FINAL OUTPUT OF THIS ROUTINE.  THE DUMMY INDEXING
C                 ON L IS USED IN VIEW OF POSSIBLE STORAGE (AS FUNCTIONS
C                 OF HEIGHT) AND RE-USE OF THESE COMBINATIONS FOR
C                 DIFFERENT VALUES OF THETA.
C
      COMMON/FRQ COM/FREQ
      COMMON/FLD COM/AZIM,CODIP,MAG FLD
      COMMON/H COM/H
      COMMON/ION COM/IONS,MRATIO(3)
      COMMON/WN/WAVE NR
      COMMON/EN COLL/HT,EN(3),CNU(3)
      COMMON/M MTX/M11,M21,M31,M12,M22,M32,M13,M23,M33
      COMMON/STORE X/X11(001),X44(001),X12(001),X34(001),X14(001),
     $            X31(001),X42(001),X32(001),X41(001)
      COMPLEX
     $            U,USQ,D,IUD,
     $            USQD,TA,TB,
     $            D TEMP,CPLX I,UDTEMP,
     $            M13D,M31D,M23D,M32D,
     $            M2332D,M1331D,M2331D,M3213D,
     $            M11,M21,M31,M12,M22,M32,M13,M23,M33,
     $            X11,X44,X12,X34,X14,X31,X42,X32,X41
      REAL MAG FLD,LY,MY,NY,IUD PRT,MRATIO,
     $            LSQYSQ,MSQYSQ,NSQYSQ,LMYSQ,LNYSQ,MNYSQ
      DATA CPLX I/(0.0,1.0)/
      DATA PI/3.141592653/
      DATA DTR/0.01745329252/
      DATA COEFF X/3.182357E03/,COEFF Y/1.758796E11/
      DATA VEL LT/2.997928E05/
      DATA RE/6369.427/
      DIMENSION U PARTS(2),USQ PRT(2),D PARTS(2),IUD PRT(2)
      EQUIVALENCE (U,U PARTS(1) ),(USQ,USQ PRT(1) ),(D,D PARTS(1)),
     $          (IUD,IUD PRT(1) )
C
C
      CALL EN NU
C
      IF(IONS .NE. 0) GO TO 20
      X = COEF EN*EN(1)
      Z = CNU(1)*OV OMGA
      U PARTS(1) = 1.0
      U PARTS(2) = -Z
      USQ PRT(1) = 1.0-Z**2
      USQ PRT(2) = -Z-Z
      D = -X''U*(USQ-YSQ))
      IUD PRT(1) = Z*D PARTS(1)-D PARTS(2)
      IUD PRT(2) = D PARTS(1)+Z*D PARTS(2)
```

```
      USQD = USQ*D
      GO TO 30
C
   20 D=0.0
      USQD=0.0
      IUD=0.0
      DO 22 K = 1,3
      X=COEF EN*EN(K)/MRATIO(K)
      Z=CNU(K)/OMEGA
      U=1.0-CPLX I*Z
      USQ=U**2
      DTEMP = -X/(U*(USQ-YSQ/MRATIO(K)**2))
      D=D+DTEMP/MRATIO(K)**2
      UDTEMP=U*DTEMP/MRATIO(K)
      IF(K .EQ. 2) UDTEMP=-UDTEMP
      IUD = IUD +CPLX I*UDTEMP
   22 USQD = USQD+USQ*DTEMP
C
   30 CRVTRM = 2.0*(HT-H)/RE
C
      M11 = USQD-LSQYSQ*D+CRVTRM
      M22 = USQD-MSQYSQ*D+CRVTRM
      M33 = USQD-NSQYSQ*D+CRVTRM
      TA = NY*IUD
      TB = LMYSQ*D
      M21 = +TA-TB
      M12 = -TA-TB
      TA = MY*IUD
      TB = LNYSQ*D
      M13 = +TA-TB
      M31 = -TA-TB
      TA = LY*IUD
      TB = MNYSQ*D
      M32 = +TA-TB
      M23 = -TA-TB
C
      D = 1.0/(1.0+M33)
      M13D = M13*D
      M31D = M31*D
      M23D = M23*D
      M32D = M32*D
      M2332D = M32*M23D
      M1331D = M31*M13D
      M2331D = M31*M23D
      M3213D = M32*M13D
C
      L = 1
      X11(L) = -M31D
      X44(L) = -M13D
      X12(L) = M32D
      X34(L) = M23D
      X14(L) = D
      X31(L) = M2331D-M21
```

```
      X42(L) = M3213D-M12
      X32(L) = M22-M2332D
      X41(L) = 1.0+M11-M1331D
      RETURN
C
C
C  COMPUTATION OF QUANTITIES WHICH ARE NOT FUNCTIONS OF HEIGHT
C                 OR OF THETA.
      ENTRY INIT T
      OMEGA = 2.0*PI*FREQ*1000.0
      WAVE NR = OMEGA/VEL LT
      COEF EN = COEFF X*1.0E06/OMEGA**2
      OV OMGA = 1.0/OMEGA
      SIN DIP = SIN(CODIP*DTR)
      DIR CS L = SIN DIP*COS(AZIM*DTR)
      DIR CS M = SIN DIP*SIN(AZIM*DTR)
      DIR CS N = -COS(CO DIP*DTR)
      Y = -COEFF Y*MAG FLD/OMEGA
      YSQ = Y**2
      LY = DIR CS L*Y
      MY = DIR CS M*Y
      NY = DIR CS N*Y
      LSQYSQ = DIR CS L**2*YSQ
      MSQYSQ = DIR CS M**2*YSQ
      NSQYSQ = DIR CS N**2*YSQ
      LMYSQ = DIR CS L*DIR CS M*YSQ
      LNYSQ = DIR CS L*DIR CS N*YSQ
      MNYSQ = DIR CS M*DIR CS N*YSQ
      RETURN
C
      END
```

```
      SUBROUTINE S MTRX
C  FOR MS
C  COMPUTATION OF COEFFICIENTS USED IN THE DIFFERENTIAL EQUATIONS
C                 FOR (R+1.0)/C.  THESE ARE STORED IN COMMON AREA 'S MTX'
C                 AND ARE ANALOGOUS TO S MATRIX ELEMENTS GIVEN BY
C                 BUDDEN (1955).  DERIVATIVES OF THESE COEFFICIENTS
C                 WRT  C=COS(THETA) ARE ALSO COMPUTED IF IDERIV
C                 IS SET NON-ZERO.
C
      COMMON/THETA C/THETA
      COMMON/C COM/C
      COMMON/S COM/S
      COMMON/STORE X/X11(001),X44(001),X12(001),X34(001),X14(001),
     $            X31(001),X42(001),X32(001),X41(001)
      COMMON/S MTX/A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
     $            DB11DC,DB22DC,DC11DC,DC22DC,DD11DC,DD21DC,DD12DC
      COMMON/IDERV C/IDERIV
      COMPLEX
     $            THETA,
     $            C,S,CSQ,CS,
     $            DSDC,DCSDC,
     $            X11,X44,X12,X34,X14,X31,X42,X32,X41,
     $            TEMP,TWOC,
     $            A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
     $            DB11DC,DB22DC,DC11DC,DC22DC,DD11DC,DD21DC,DD12DC
      DATA RTD/57.29578/
C
C
      CALL T MTRX
C
      L = 1
      TEMP = X41(L)+X41(L)
      A11 = TEMP+TEMP
      A22 = 4.0
      TEMP = S*X44(L)-C*X41(L)
      B11 = TEMP+TEMP
      B12 = -X42(L)-X42(L)
      TWOC = C+C
      B22 = -TWOC
      TEMP = -S*X11(L)-C*X41(L)
      C11 = TEMP+TEMP
      C21 = X31(L)+X31(L)
      C22 = -TWOC
      D11 = CS*(X11(L)-X44(L))-(1.0-X14(L))+CSQ*(X41(L)-X14(L))
      D21 = -C*X31(L)+S*X34(L)
      D12 = S*X12(L)+C*X42(L)
      D22 = -X32(L)
C
      IF (IDERIV .EQ. 0) RETURN
      TEMP = DSDC*X44(L)-X41(L)
      DB11DC = TEMP+TEMP
      DB22DC = -2.0
      TEMP = -DSDC*X11(L)-X41(L)
```

```
      DC11DC = TEMP+TEMP
      DC22DC = -2.0
      DD11DC = DCSDC*(X11(L)-X44(L))
     $          +2.0*C*(X41(L)-X14(L))
      DD21DC = -X31(L)+DSDC*X34(L)
      DD12DC = DSDC*X12(L)+X42(L)
      RETURN
C
C
C  INITIALIZATION FOR GIVEN VALUE OF THETA.
      ENTRY INIT S
      C = CCOS(THETA/RTD)
      CSQ = C**2
      S = CSIN(THETA/RTD)
      CS = C*S
C
      IF (IDERIV .EQ. 0) RETURN
      DSDC = -C/S
      DCSDC = S-CSQ/S
      RETURN
C
      END
```

```
      SUBROUTINE DIFF EQ
C FOR MS
C  DIFFERENTIAL EQUATIONS FOR (R+1.0)/C WHERE R IS THE REFLECTION
C                MATRIX AS DEFINED BY BUDDEN.  THE EQUATIONS ARE
C                DERIVED FROM THE DIFFERENTIAL EQUATIONS FOR R
C                GIVEN BY BUDDEN IN   QC. RCY. SOC. (LONDON), A227,
C                PP.516-537 (1955).  STORAGE INTO THE ARRAY 'R MTRX'
C                IS MADE SO THAT THE EQUATIONS MAY BE USED WITH
C                EITHER THE MAIN SET OF INTEGRATION VARIABLES, OR
C                WITH THE COMPARISON SET, X.  DIFFERENTIAL EQUATIONS
C                FOR THE DERIVATIVES OF (R+1.0)/C WRT C=COS(THETA) ARE
C                ALSO USED IF IDERIV IS SET NON-ZERO.  NOTE THAT THE
C                VARIABLE CALLED 'R' IN THIS ROUTINE IS ACTUALLY
C                (R+1.0)/C.  MULTIPLICATION BY -I IS EFFECTED BY
C                APPROPRIATE TRANSFER OF VALUES FROM 'DERIV' TO
C                COMMON AREA VARIABLES.
C
      COMMON/INTEGR/R(16),DRDH(16)
      COMMON/X INTGR/X(8),DXDH(8)
      COMMON/S MTX/A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
     $             DB11DC,DB22DC,DC11DC,DC22DC,DD11DC,DD21DC,DD12DC
      COMMON/EXTRA//R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC,
     $             DR11DH,DR21DH,DR12DH,DR22DH,
     $             DR11CH,DR21CH,DR12CH,DR22CH
      COMMON/IDERV C/IDERIV
      COMMON/N COM/N
      COMMON/CVREFLD/ICVFLD
      COMPLEX
     $             R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC,
     $             DR11DH,DR21DH,DR12DH,DR22DH,
     $             DR11CH,DR21CH,DR12CH,DR22CH,
     $             A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
     $             DB11DC,DB22DC,DC11DC,DC22DC,DD11DC,DD21DC,DD12DC,
     $             D11R11,D12R21,D21R12,D22R22,
     $             R11R22,R12R21,
     $             G11R11,G12R21,G21R12,
     $             D21R11,D12R11,D11R21,D22R21,D11R12,D22R12,D21R22,D12R22
      DIMENSION R MTRX(16),DERIV(16)
      EQUIVALENCE(R11,R MTRX(1) ),(DR11DH,DERIV(1) )
C
C
      ENTRY P DERIV
      CALL XFER (R,R MTRX,N)
      IXFLAG = 0
   20 DO 21 I=1,8
      IF(ABS(R MTRX(I)) .GT. 1.0E4) GO TO 90
   21 CONTINUE
C
      D11R11 = D11*R11
      D12R21 = D12*R21
      D21R12 = D21*R12
      D22R22 = D22*R22
      R11R22 = R11*R22
```

```
      R12R21 = R12*R21
C
      DR11DH = A11+(B11+C11+D11R11+D12R21+D21R12)*R11
     $          +B12*R21+C21*R12+D22*R12R21
      DR21DH = (B22+C11+D11R11+D12R21+D22R22)*R21
     $          +C21*R22+D21*R11R22
      DR12DH = (B11+C22+D11R11+D21R12+D22R22)*R12
     $          +B12*R22+D12*R11R22
      DR22DH = A22+(B22+C22+D12R21+D21R12+D22R22)*R22
     $          +D11*R12R21
      IF (IXFLAG .NE. 0) GO TO 40
C
      IF (IDERIV .EQ. 0) GO TO 22
      G11R11 = DD11DC*R11
      G12R21 = DD12DC*R21
      G21R12 = DD21DC*R12
C
      D21R11 = D21*R11
      D12R11 = D12*R11
      D11R21 = D11*R21
      D22R21 = D22*R21
      D11R12 = D11*R12
      D22R12 = D22*R12
      D21R22 = D21*R22
      D12R22 = D12*R22
C
      DR11CH = (DB11DC+DC11DC+G11R11+G12R21+G21R12)*R11
     $          +(B11+C11+D11R11+D11R11+D12R21+D21R12)*DR11DC
     $          +(D12R11+B12+D22R12)*DR21DC
     $          +(D21R11+C21+D22R21)*DR12DC
      DR21CH = (DB22DC+DC11DC+G11R11+G12R21)*R21+DD21DC*R11R22
     $          +(D11R21+D21R22)*DR11DC
     $          +(B22+C11+D11R11+D12R21+D12R21+D22R22)*DR21DC
     $          +(D22R21+C21+D21R11)*DR22DC
      DR12CH = (DB11DC+DC22DC+G11R11+G21R12)*R12+DD12DC*R11R22
     $          +(D11R12+D12R22)*DR11DC
     $          +(B11+C22+D11R11+D21R12+D21R12+D22R22)*DR12DC
     $          +(D22R12+B12+D12R11)*DR22DC
      DR22CH = (DB22DC+DC22DC+G12R21+G21R12)*R22+DD11DC*R12R21
     $          +(D12R22+D11R12)*DR21DC
     $          +(D21R22+D11R21)*DR12DC
     $          +(B22+C22+D12R21+D21R12+D22R22+D22R22)*DR22DC
C
   22 DO 24 I=1,N,2
      DRDH(I) = DERIV(I+1)
   24 DRDH(I+1) = -DERIV(I)
      RETURN
C
C
      ENTRY X DERIV
      CALL XFEF (X,R MTRX,8)
      IXFLAG = 1
      GO TO 20
```

```
C
   40 DO 42 I=1,7,2
      DXCH(I) = DERIV(I+1)
   42 DXCH(I+1) = -DERIV(I)
      RETURN
C
   90 IOVFLO = 1
      RETURN
C
      END
```

```
      SUBROUTINE RBARS
C   FOR MS
C   THIS SUBROUTINE COMPUTES VALUES OF VARIABLES WHICH MAY BE USED TO
C                 FORM THE ELEMENTS OF THE RBAR MATRIX, WHERE RBAR
C                 REPRESENTS REFLECTION OF AN ELM WAVE FROM THE EARTH'S
C                 SURFACE.  NAMELY, RBAR11=DEN11/(C*NUM11-DEN11),
C                 OR (1.0/RBAR11+1.0)/C=NUM11/DEN11,
C                 AND RBAR22=DEN22/(C*NUM22-DEN22),
C                 OR (1.0/RBAR22+1.0)/C=NUM22/DEN22.  DERIVATIVES
C                 WRT C=COS(THETA) ARE ALSO COMPUTED IF IDERIV
C                 IS SET NON-ZERO.  NOTE THAT THE
C                 EQUATIONS ARE FORMULATED IN SUCH A WAY THAT A SMOOTH
C                 TRANSITION IS MADE FROM THE 'CURVED EARTH' FORM TO
C                 THE 'FLAT EARTH' FORM.  SEE ALSO NOTES IN SUBROUTINE
C                 MOHNKL REGARDING DEFINITIONS OF HANKEL FUNCTION
C                 PARAMETERS.  THE VALUE OF EARTH'S RADIUS IS SUCH AS
C                 TO MAKE 2.0/RE=3.14E-4.  COMPUTATION AT THETA=90
C                 IS NOT EXCLUDED.
C
      COMMON/RTALT/ TALT,RALT
      COMMON/THETA C/THETA
      COMMON/C COM/C
      COMMON/RB COM/NUM11,NUM22,DEN11,DEN22,DNM1DC,DNM2DC,DDN1DC,DDN2DC
      COMMON/F1F2 C/F1,F2,HG,F1ZT,F1ZR,F2ZT,F2ZR,GF1ZT
      COMMON/FRQ COM/FREQ
      COMMON/H COM/H
      COMMON/D COM/D
      COMMON/GND COM/EPSREL,SIGMA
      COMMON/IDERV C/IDERIV
      COMPLEX
     $ EZ,FZ,
     $          THETA,C,
     $          NUM11,NUM22,DEN11,DEN22,
     $          DNM1DC,DNM2DC,DDN1DC,DDN2DC,
     $          F1,F2,HG,HG FCTR,F1ZT,F1ZR,F2ZT,F2ZR,DF1ZT,GF1ZT,
     $          CSQ,SSQ,
     $          NGSQ,I,
     $          SQR,
     $          Z,DZDC,
     $          H1,H1P,H1PP,
     $          H2,H2P,H2PP,
     $          E,E0,EXPON,
     $          KK,KX,
     $          A1,A2,A3,A4,B1,B2,B3,B4,
     $          DA1DC,DA2DC,DA3DC,DA4DC,DB1DC,DB2DC,DB3DC,DB4DC
      REAL KVRADT,KVRATT,NDSQ,NDSQ
      DATA RTD/57.29578/
      DATA I/(0.0,1.0)/
      DATA EPSLN0/8.85434E-12/
      DATA VEL LT/2.997928E5/
      DATA PI/3.141593/
      DATA RE/6369.427/
C
```

```
C
      C = COS(THETA/RTD)
      CSQ = C**2
      SSQ = 1.0-CSQ
C
C     HEIGHT GAIN FOR Z=0
      SQR = CSQRT(NGSQ-SSQ)
      NUM11 = 1.0
      DEN11 = (C-SQR/NGSQ)/2.0
      NUM22 = 1.0/SQR
      DEN22 = (C/SQR-1.0)/2.0
      IF (IDERIV .EQ. 0) GO TO 15
      DNM1DC = 0.0
      DEN1DC = (1.0-C/(SQR*NGSQ))/2.0
      DNM2DC = -C/SQR**3
      DDN2DC = -(C**2/SQR**3+1.0/SQR)/2.0
C
   15 Z = KVRATT*(CSQ+EMO)
      CALL MDHNKL (Z,H1,H2,H1P,H2P,EO)
      B1 = C*H1+(KK*H1P+KX*H1)/NOSQ
      B2 = C*H2+(KK*H2P+KX*H2)/NOSQ
      B3 = C*H1+KK*H1P
      B4 = C*H2+KK*H2P
      A1 = NUM11*B1-2.0*DEN11*H1
      A2 = NUM11*B2-2.0*DEN11*H2
      A3 = NUM22*B3-2.0*DEN22*H1
      A4 = NUM22*B4-2.0*DEN22*H2
      IF (IDERIV .EQ. 0) GO TO 20
      DZDC = KVRATT*2.0*C
      H1PP = -Z*H1
      H2PP = -Z*H2
      DB1DC = H1+(C*H1P+(KK*H1PP+KX*H1P)/NOSQ)*DZDC
      DB2DC = H2+(C*H2P+(KK*H2PP+KX*H2P)/NOSQ)*DZDC
      DB3DC = H1+(C*H1P+KK*H1PP)*DZDC
      DB4DC = H2+(C*H2P+KK*H2PP)*DZDC
      DA1DC = NUM11*DB1DC+DNM1DC*B1-2.0*(DEN11*H1P*DZDC+DDN1DC*H1)
      DA2DC = NUM11*DB2DC+DNM1DC*B2-2.0*(DEN11*H2P*DZDC+DDN1DC*H2)
      DA3DC = NUM22*DB3DC+DNM2DC*B3-2.0*(DEN22*H1P*DZDC+DDN2DC*H1)
      DA4DC = NUM22*DB4DC+DNM2DC*B4-2.0*(DEN22*H2P*DZDC+DDN2DC*H2)
      HG FCTR = (A2*H1-A1*H2)/(-KK)
C
C     HEIGHT GAINS FOR ELEVATED TRANSMITTER AND ELEVATED RECEIVER
      Z = KVRATT*(CSQ+EMZT)
      CALL MDHNKL(Z,H1,H2,H1P,H2P,EZ)
      EXPON = CEXP(EZ-EO)
      H1 = H1/EXPON
      H2 = H2*EXPON
      H1P=H1P/EXPON
      H2P=H2P*EXPON
      F1ZT = (A2*H1-A1*H2)/(-KK)*EXP((TALT-D)/RE)
      F2ZT = (A4*H1-A3*H2)*(-SQR/KK)
C     PART OF THE EQUATION OF THE DERIVATIVE OF HEIGHT GAIN FOR
C     FPERAL AT HEIGHT Z
```

```
      DF1ZT = (A2*H1P-A1*H2P)/(-KK)*EXP((TALT-D)/RE)
C     THE DERIVATIVE OF THE HEIGHT GAIN FOR FPERL AT HEIGHT Z
C     EXCLUSIVE OF THE D11 TERM
      DF1ZT= -I*(AVRKCT*DF1ZT+ARG*F1ZT/2.0)
C
      Z = KVRATT*(CSQ+EMZR)
      CALL MDHNKL(Z,H1,H2,H1P,H2P,EZ)
      EXPON = CEXP(EZ-EO)
      H1 = H1/EXPON
      H2 = H2*EXPON
      F1ZR = (A2*H1-A1*H2)/(-KK)*EXP((RALT-D)/RE)
      F2ZR = (A4*H1-A3*H2)*(-SQR/KK)
C
C     HEIGHT GAINS AT Z=D
   20 Z = KVRATT*(CSQ+EMD)
      CALL MDHNKL (Z,H1,H2,H1P,H2P,E)
      EXPON = CEXP(E-EO)
      H1 = H1/EXPON
      H2 = H2*EXPON
      H1P = H1P/EXPON
      H2P = H2P*EXPON
C
      B1 = C*H1+(KK*H1P+KX*H1)/NDSQ
      B2 = C*H2+(KK*H2P+KX*H2)/NDSQ
      B3 = C*H1+KK*H1P
      B4 = C*H2+KK*H2P
      DEN11 = A2*B1-A1*B2
      DEN22 = A4*B3-A3*B4
      NUM11 = 2.0*(A2*H1-A1*H2)
      NUM22 = 2.0*(A4*H1-A3*H2)
      IF (IDERIV .EQ. 0) RETURN
      H1PP = -Z*H1
      H2PP = -Z*H2
      DB1DC = H1+(C*H1P+(KK*H1PP+KX*H1P)/NDSQ)*DZDC
      DB2DC = H2+(C*H2P+(KK*H2PP+KX*H2P)/NDSQ)*DZDC
      DB3DC = H1+(C*H1P+KK*H1PP)*DZDC
      DB4DC = H2+(C*H2P+KK*H2PP)*DZDC
      DDN1DC = A2*DB1DC+DA2DC*B1-A1*DB2DC-DA1DC*B2
      DDN2DC = A4*DB3DC+DA4DC*B3-A3*DB4DC-DA3DC*B4
      DNM1DC = 2.0*(A2*H1P*DZDC+DA2DC*H1-A1*H2P*DZDC-DA1DC*H2)
      DNM2DC = 2.0*(A4*H1P*DZDC+DA4DC*H1-A3*H2P*DZDC-DA3DC*H2)
C
C  FOLLOWING FOUR STATEMENTS NEEDED ONLY FOR COMPUTING EXCITATION
C                 FACTORS.
C     HEIGHT GAIN FOR FPERL AT Z=D
      F1 = (A2*H1-A1*H2)/(-KK)
C     HEIGHT GAIN FOR FPERP AT Z=D
      F2 = (A4*H1-A3*H2)*(-SQR/KK)
C     HEIGHT GAIN FOR FPERL AT Z=0 DIVIDED BY
C     HEIGHT GAIN FOR FPERL AT Z=D
      HG = EXP(-D/RE)*HG FCTR/F1
      RETURN
C
```

```
C
C  COMPUTATION WHICH IS INDEPENDENT OF THETA.
      ENTRY INIT 88
      OMEGA = 2.0*PI*FREQ*1000.0
      WAVE NR = OMEGA/VEL LT
      NGSQ = EPSREL-I*SIGMA/OMEGA/EPSLNO
C
      ARG = 2.0/(WAVE NR*RE)
      AVRKOT = EXP(ALOG(ARG)/3.0)
      KVRACT = 1.0/AVRKOT
      KVRATT = 1.0/AVRKOT**2
      KK = I*AVRKOT
      KX = I/(WAVE NR*RE)
C
      EMZR = 2.0*(RALT-H)/RE
      EMZT = 2.0*(TALT-H)/RE
C
      EMD = 2.0*(D-H)/RE
      NDSQ = 1.0+EMD
      EMO = -2.0*H/RE
      NOSQ = 1.0-2.0*H/RE
      RETURN
C
      END
```

```
      SUBROUTINE MDHNKL (Z,H1,H2,H1P,H2P,E)
C   FOR MS
C   THIS SUBROUTINE COMPUTES VALUES OF VARIABLES WHICH MAY BE COMBINED
C                 TO FORM MODIFIED HANKEL FUNCTIONS AS DESCRIBED
C                 IN 'TABLES OF THE MODIFIED HANKEL FUNCTIONS OF ORDER
C                 ONE THIRD AND OF THEIR DERIVATIVES', BY THE STAFF OF
C                 THE COMPUTATION LABORATORY, HARVARD UNIVERSITY
C                 PRESS, 1945.  THE NOTATION IS SUCH THAT THE
C                 ACTUAL H1=H1*EXP(-E), THE ACTUAL H2=H2*EXP(E).
C                 ACTUAL DERIVATIVES WRT ARGUMENT, Z, ARE H1P*EXP(-E)
C                 AND H2P*EXP(E).
C
      COMPLEX Z,H1,H2,H1P,H2P,E,
     $          R,
     $          I,PARTS,
     $          ZPOWER,TERM,
     $          SUMA,SUMB,SUMC,SUMD,
     $          MZ3,
     $          F,G,FP,GP,T,TP,
     $          IPI16T,I2THRD,IPI512,
     $          SUM1,SUM2,SUM1P,SUM2P,
     $          RTZ,OVRZ,OVRM4Z,Z3H,M3HOVZ,IRTZ,MIZ3H,M1OVZ3,
     $          CSUM,ESUM,OSUMP,ESUMP,
     $          EDIFF
      DIMENSION A(22),B(22),CAP(14),PART(2)
      EQUIVALENCE (PART(1),PARTS)
      DATA A0/0.93043671693/
      DATA B0/0.67829872514/
      DATA CAP/
     $   1.04166666666667D-01,8.35503472222222D-02,1.28226574556327D-01,
     $   2.91849026464140D-01,8.81627267443758D-01,3.32140828186277D 00,
     $   1.49957629868626D 01,7.89230130115870D 01,4.74451538866000D 02,
     $   3.20749009100000D 03,2.40865496000000D 04,1.99923120000000D 05,
     $   1.79190200000000D 06,1.74843770000000D 07/
      DATA I/(0.0,1.0)/
      DATA ROOT3/1.73205080756888/
      DATA ALPHA/0.85366721883895 1/
      DATA PI/3.14159265358979/
      DATA IIST/1/
      DATA IFIRST/1/
C
C
      ZMAG=CABS(Z)
      IF (ZMAG .GT. 4.2) GO TO 50
C
C   MACLAURIN'S SERIES FORM FOR MAGNITUDE(Z) NOT GREATER THAN 4.2
      IF (IIST .EQ. 0) GO TO 12
      A(1) = 1.0/6.0
      B(1) = 2.0/24.0
      DO 11 M=2,22
      A(M) = A(M-1)*(3.0*M-2.0)/((3.0*M)*(3.0*M-1.0)*(3.0*M-2.0))
      B(M) = B(M-1)*(3.0*M-1.0)/((3.0*M+1.0)*(3.0*M)*(3.0*M-1.0))
   11 CONTINUE
```

```
      IIST = 0
   12 SUMA = 1.0
      SUMB = 1.0
      SUMC = 0.0
      SUMD = 0.0
      ZPOWER = 1.0
      MZ3 = -Z**3
      DO 15 M=1,22
      ZPOWER = ZPOWER*MZ3
      TERM = A(M)*ZPOWER
      SUMA = SUMA+TERM
      SUMC = SUMC+M*TERM
      PARTS = TERM
      IF (ABS(PART(1))+ABS(PART(2)) .LT. 1.0E-10) GO TO 16
      TERM = B(M)*ZPOWER
      SUMB = SUMB+TERM
      SUMD = SUMD+M*TERM
      PARTS = TERM
      IF (ABS(PART(1))+ABS(PART(2)) .LT. 1.0E-10) GO TO 16
   15 CONTINUE
   16 F = A0*SUMA
      G = Z*B0*SUMB
      FP = 3.0*A0*SUMC/Z
      GP = (3.0*SUMD+SUMB)*B0
      T = I*(G-F-F)/ROOT3
      TP = I*(GP-FP-FP)/ROOT3
      H1 = G+T
      H1P = GP+TP
      H2 = G-T
      H2P = GP-TP
      E = 0.0
      RETURN
C
C  ASYMPTOTIC SERIES FOR MAGNITUDE(Z) GREATER THAN 4.2
   50 IF (IFIRST .EQ. 0) GO TO 51
      IPI16T = 16.0*PI*I/12.0
      I2THRD = I*2.0/3.0
      IPI512 = 5.0*PI*I/12.0
      ALPHSQ = ALPHA**2
      IFIRST = 0
   51 RTZ = CSQRT(Z)
      OVRZ = 1.0/Z
      OVRM4Z = -0.25*OVRZ
      Z3H = Z*RTZ
      M3HOVZ = -1.5*OVRZ
      IRTZ = I*RTZ
      MIZ3H = -I*Z3H
      MIOVZ3 = -OVRZ**3
      F = -I2THRD*Z3H+IPI512
      R = CSQRT(ALPHSQ/RTZ)
C
      OSUM = 0.0
      OSUMP = 0.0
```

```
      ESUM = 0.0
      ESUMP = 0.0
      ZPOWER = 1.0
      DO 52 M=2,14,2
      ZPOWER = ZPOWER*M10VZ3
      TERM = CAP(M-1)*ZPOWER
      OSUM = CSUM+TERM
      OSUMP = CSUMP+(M-1)*TERM
      TERM = CAP(M)*ZPOWER
      ESUM = ESUM+TERM
      ESUMP = ESUMP+M*TERM
      PARTS = TERM
      IF (ABS(PART(1))+ABS(PART(2)) .LT. 1.0E-8) GO TO 53
   52 CONTINUE
   53 CONTINUE
      OSUM = MIZ3H*OSUM
      OSUMP = MIZ3H*M3HOVZ*OSUMP
      ESUMP = M3HOVZ*ESUMP
      SUM1 = 1.0-OSUM+ESUM
      SUM2 = 1.0+OSUM+ESUM
      SUM1P = -OSUMP+ESUMP
      SUM2P = OSUMP+ESUMP
C
      H1 = SUM1
      H2 = SUM2
      H1P = (CVRM4Z+IRTZ)*SUM1+SUM1P
      H2P = (CVRM4Z-IRTZ)*SUM2+SUM2P
      PARTS = Z
      IF (PART(1) .GT. 0.0) GO TO 70
C
      IF (PART(2) .GT. 0.0) GO TO 63
      EDIFF = CEXP(E+E-IPI16T)
      H1 = H1+EDIFF*H2
      H1P = H1P+EDIFF*H2P
      GO TO 70
   63 EDIFF = CEXP(-E-E+IPI16T)
      H2 = H2+EDIFF*H1
      H2P = H2P+EDIFF*H1P
C
   70 H1 = R*H1
      H2 = R*H2
      H1P = R*H1P
      H2P = R*H2P
      RETURN
C
      END
```

```
      SUBROUTINE FS INTG
C   FOR MS
C   THIS SUBROUTINE PERFORMS AN INTEGRATION OF THE DIFFERENTIAL
C                 EQUATIONS FOR THE IONOSPHERE REFLECTION MATRIX
C                 THROUGH A FREE SPACE REGION OVER A CURVED EARTH.
C                 THE INTEGRATION MAY BE PERFORMED IN EITHER A POSITIVE
C                 OR NEGATIVE HEIGHT DIRECTION, BUT IN THIS PROGRAM THE
C                 INTEGRATION IS ALWAYS UPWARD.  THE INTEGRATION VARIABLES
C                 ARE THE ELEMENTS OF THE MATRIX (R+1.0)/C WHERE R IS THE
C                 REFLECTION MATRIX DESCRIBED BY BUDDEN.  THE SOLUTION
C                 IS BASED ON BUDDEN, RADIO WAVES IN THE IONOSPHERE,
C                 IN PARTICULAR THE MATERIAL ON PP.118,327-329,
C                 330-338,AND 343-345.  IF IDERIV
C                 IS SET NON-ZERO, THE DERIVATIVES OF (R+1.0)/C ELEMENTS
C                 WRT C=COS(THETA) ARE ALSO INTEGRATION VARIABLES.
C                 COMPUTATION AT THETA=90 IS NOT EXCLUDED.  ALSO,
C                 THE EQUATIONS ARE FORMULATED IN SUCH A WAY THAT A
C                 SMOOTH TRANSITION IS MADE FROM THE 'CURVED EARTH' FORM
C                 TO THE 'FLAT EARTH' FORM FOR APPPOPRIATE VALUES OF
C                 THETA.  THE INITIAL AND FINAL VALUES OF
C                 THE INTEGRATION VARIABLES ARE STORED IN THE COMMON
C                 AREA 'INTEGR'.
C
      COMMON/XPRNT/XPRINT
      COMMON/INTEGR/R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC
      COMMON/THETA C/THETA
      COMMON/C COM/C
      COMMON/H COM/H
      COMMON/WN/WAVE NR
      COMMON/IDERV C/IDERIV
      COMPLEX THETA,C,
     $          R11,R21,R12,R22,
     $          DR11DC,DR21DC,DR12DC,DR22DC,
     $          I,KK,KX,CSQ,
     $          EXPON,
     $          Z,H1,H2,H1P,H2P,E0,E,
     $          H1PP,H2PP,DZDC,
     $          A11,A12,A21,A22,
     $          DA11DC,DA12DC,DA21DC,DA22DC,
     $          F1,G1,A1,B1,F2,G2,A2,B2,
     $          DF1DC,DG1DC,DA1DC,DB1DC,DF2DC,DG2DC,DA2DC,DB2DC,
     $          EP1,EP2,EY1,EY2,
     $          DEP1DC,DEP2DC,DEY1DC,DEY2DC,
     $          DEN,V1,V2,
     $          DDENDC,DV1DC,DV2DC
      REAL KVRATT,NOSQ,NZSQ
      INTEGER XPRINT
      DATA I/(0.0,1.0)/
      DATA RE/6369.427/
      DATA RTD/57.29578/
C
C
      IF (XPRINT .NE. 0) PRINT 100,R11,R21,R12,R22
```

```
  100 FORMAT ('0','AT HT=ZO R =',4(2X,2F12.5))
      IF (IDERIV .NE. 0 .AND. XPRINT .NE. 0)
     $          PRINT 101,DR11DC,DR21DC,DR12DC,DR22DC
  101 FORMAT ('0','DERIVS =',4X,4(2X,1P2E12.5))
C
      C = COS(THETA/RTD)
      CSQ = C**2
C
C  COMPUTATION OF HEIGHT-GAIN COEFFICIENTS F,G,A,B FOR TWO CONDITIONS
C                ON THE UPGOING WAVE AT HEIGHT=ZO,  NAMELY, E11=1,EY=0
C                AND E11=0,EY=1.  EACH OF THE COEFFICIENTS F,G,A,B IS
C                DIVIDED BY C=COS(THETA).
      Z = KVRATT*(CSQ+EMO)
      CALL MDHNKL (Z,H1,H2,H1P,H2P,EO)
      A11 = H1
      A12 = H2
      A21 = C*H1+(KK*H1P+KX*H1)/NOSQ
      A22 = C*H2+(KK*H2P+KX*H2)/NOSQ
      DEN = A11*A22-A21*A12
      F1 = (R11*A22-2.0*A12)/DEN
      G1 = (2.0*A11-R11*A21)/DEN
      F2 = R12*A22/DEN
      G2 = -R12*A21/DEN
C
      IF (IDERIV .EQ. 0) GO TO 25
      DZDC = KVRATT*2.0*C
      H1PP = -Z*H1
      H2PP = -Z*H2
      DA11DC = H1P*DZDC
      DA12DC = H2P*DZDC
      DA21DC = H1+(C*H1P+(KK*H1PP+KX*H1P)/NOSQ)*DZDC
      DA22DC = H2+(C*H2P+(KK*H2PP+KX*H2P)/NOSQ)*DZDC
      DDENDC = A11*DA22DC+DA11DC*A22-A21*DA12DC-DA21DC*A12
      DF1DC = (R11*DA22DC+DR11DC*A22-2.0*DA12DC)/DEN-F1*DDENDC/DEN
      DG1DC = (2.0*DA11DC-R11*DA21DC-DR11DC*A21)/DEN-G1*DDENDC/DEN
      DF2DC = (R12*DA22DC+DR12DC*A22)/DEN-F2*DDENDC/DEN
      DG2DC = (-R12*DA21DC-DR12DC*A21)/DEN-G2*DDENDC/DEN
C
   25 A11 = H1
      A12 = H2
      A21 = C*H1+KK*H1P
      A22 = C*H2+KK*H2P
      DEN = A11*A22-A21*A12
      A1 = R21*A22/DEN
      B1 = -R21*A21/DEN
      A2 = (R22*A22-2.0*A12)/DEN
      B2 = (2.0*A11-R22*A21)/DEN
C
      IF (IDERIV .EQ. 0) GO TO 30
      DA11DC = H1P*DZDC
      DA12DC = H2P*DZDC
      DA21DC = H1+(C*H1P+KK*H1PP)*DZDC
      DA22DC = H2+(C*H2P+KK*H2PP)*DZDC
```

```
      DDENDC = A11*DA22DC+DA11DC*A22-A21*DA12DC-DA21DC*A12
      DA1DC = (R21*DA22DC+DR21DC*A22)/DEN-A1*DDENDC/DEN
      DB1DC = (-R21*DA21DC-DR21DC*A21)/DEN-B1*DDENDC/DEN
      DA2DC = (R22*DA22DC+DR22DC*A22-2.0*DA12DC)/DEN-A2*DDENDC/DEN
      DB2DC = (2.0*DA11DC-R22*DA21DC-DR22DC*A21)/DEN-B2*DDENDC/DEN
C
C  COMPUTATION OF UPGOING FIELDS E11 AND EY AT HEIGHT=ZZ FOR THE TWO
C                 CONDITIONS DESCRIBED ABOVE.
   30 Z = KVRATT*(CSQ+EMZ)
      CALL MEHNKL (Z,H1,H2,H1P,H2P,E)
      EXPON = CEXP(E-E0)
      H1 = H1/EXPON
      H2 = H2*EXPON
      H1P = H1P/EXPON
      H2P = H2P*EXPON
C
      A21 = C*H1+(KK*H1P+KX*H1)/NZSQ
      A22 = C*H2+(KK*H2P+KX*H2)/NZSQ
      EP1 = (F1*A21+G1*A22)/2.0
      EP2 = (F2*A21+G2*A22)/2.0
C
      IF (IDERIV .EQ. 0) GO TO 35
      H1PP = -Z*H1
      H2PP = -Z*H2
      DA21DC = H1+(C*H1P+(KK*H1PP+KX*H1P)/NZSQ)*DZDC
      DA22DC = H2+(C*H2P+(KK*H2PP+KX*H2P)/NZSQ)*DZDC
      DEP1DC = (F1*DA21DC+DF1DC*A21+G1*DA22DC+DG1DC*A22)/2.0
      DEP2DC = (F2*DA21DC+DF2DC*A21+G2*DA22DC+DG2DC*A22)/2.0
C
   35 A21 = C*H1+KK*H1P
      A22 = C*H2+KK*H2P
      EY1 = (A1*A21+B1*A22)/2.0
      EY2 = (A2*A21+B2*A22)/2.0
C
      IF (IDERIV .EQ. 0) GO TO 40
      DA21DC = H1+(C*H1P+KK*H1PP)*DZDC
      DA22DC = H2+(C*H2P+KK*H2PP)*DZDC
      DEY1DC = (A1*DA21DC+DA1DC*A21+B1*DA22DC+DB1DC*A22)/2.0
      DEY2DC = (A2*DA21DC+DA2DC*A21+B2*DA22DC+DB2DC*A22)/2.0
C
C  COMPUTATION OF REFLECTION COEFFICIENTS AT HEIGHT=ZZ.
   40 V1 = F1*H1+G1*H2
      V2 = F2*H1+G2*H2
      DEN = EP1*EY2-EP2*EY1
      R11 = (V1*EY2-V2*EY1)/DEN
      R12 = (V2*EP1-V1*EP2)/DEN
C
      IF (IDERIV .EQ. 0) GO TO 45
      DV1DC = (F1*H1P+G1*H2P)*DZDC+DF1DC*H1+DG1DC*H2
      DV2DC = (F2*H1P+G2*H2P)*DZDC+DF2DC*H1+DG2DC*H2
      DDENDC = EP1*DEY2DC+DEP1DC*EY2-EP2*DEY1DC-DEP2DC*EY1
      DR11DC = (V1*DEY2DC+DV1DC*EY2-V2*DEY1DC-DV2DC*EY1)/DEN
     $          -R11*DDENDC/DEN
```

```
      DR12DC = (V2*DEP1DC+DV2DC*EP1-V1*DEP2DC-DV1DC*EP2)/DEN
     $          -R12*DDENDC/DEN
C
   45 V1 = A1*H1+B1*H2
      V2 = A2*H1+B2*H2
      R21 = (V1*EY2-V2*EY1)/DEN
      R22 = (V2*EP1-V1*EP2)/DEN
C
      IF (IDERIV .EQ. 0) GO TO 50
      DV1DC = (A1*H1P+B1*H2P)*DZDC+DA1DC*H1+DB1DC*H2
      DV2DC = (A2*H1P+B2*H2P)*DZDC+DA2DC*H1+DB2DC*H2
      DR21DC = (V1*DEY2DC+DV1DC*EY2-V2*DEY1DC-DV2DC*EY1)/DEN
     $          -R21*DDENDC/DEN
      DR22DC = (V2*DEP1DC+DV2DC*EP1-V1*DEP2DC-DV1DC*EP2)/DEN
     $          -R22*DDENDC/DEN
C
   50 IF (XPRINT .NE. 0) PRINT 500,R11,R21,R12,R22
  500 FORMAT ('0','AT HT=ZZ R =',4(2X,2F12.5))
      IF (IDERIV .NE. 0 .AND. XPRINT .NE. 0)
     $          PRINT 501,DR11DC,DR21DC,DR12DC,DR22DC
  501 FORMAT ('0','DERIVS = ',4X,4(2X,1P2E12.5))
      RETURN
C
C
C  COMPUTATION WHICH IS INDEPENDENT OF THETA.  THE INTEGRATION WILL
C                PROCEED FROM HEIGHT Z0 TO HEIGHT ZZ WHEN FSINTG IS
C                CALLED.
      ENTRY INIT FS(Z0,ZZ)
      ARG = 2.0/(WAVE NR*RE)
      AVRKOT = EXP(ALOG(ARG)/3.0)
      KVRATT = 1.0/AVRKOT**2
      KR = I*AVRKOT
      KX = I/(WAVE NR*RE)
C
      EMO = 2.0*(Z0-H)/RE
      NOSQ = 1.0+EMO
      EMZ = 2.0*(ZZ-H)/RE
      NZSQ = 1.0+EMZ
      RETURN
C
      END
```

```
      SUBROUTINE FCT VAL (THETAZ,F)
C   FOR MS.
C   ROUTINE FOR COMPUTING F FUNCTION VALUES AT MESH POINTS, CALLED ON
C                 PRINCIPALLY BY SUBROUTINES FZEROS AND NO MESH.  THIS
C                 IS THE MODIFIED F FUNCTION WHICH HAS NO POLES AND WHICH
C                 HAS NO ZEROS AT THETA=90.
C
      COMMON/IDERV C/IDERIV
      COMMON/IXACT C/IEXACT
      COMMON/THETA C/THETA
      COMMON/INTEGR/R11,R21,R12,R22
      COMMON/RB COM/NUM11,NUM22,DEN11,DEN22
      COMPLEX THETAZ,F,
     $          THETA,
     $          R11,R21,R12,R22,
     $          NUM11,NUM22,DEN11,DEN22
C
C
      THETA = THETAZ
      IDERIV=0
      IF(IEXACT .EQ. 0) GO TO 11
      CALL INTFG
      CALL FS INTG
      GO TO 12
   11 CALL LAGRNG
   12 CALL RBARS
      F = (NUM11-R11*DEN11)*(NUM22-R22*DEN22)
     $          -R12*R21*DEN11*DEN22
      RETURN
C
      END
```

```
      SUBROUTINE F DFDT (THETAZ,F,DFDT)
C  FOR FS
C  ROUTINE FOR COMPUTING F FUNCTION VALUES AND THEIR DERIVATIVES
C                 WRT THETA AT ARBITRARY VALUES OF THETA.  CALLED ON
C                 PRINCIPALLY BY SUBROUTINES NO MESH AND FINAL.  THIS
C                 IS THE MODIFIED F FUNCTION WHICH HAS NO POLES AND WHICH
C                 HAS NO ZEROS AT THETA=90.
C
      COMMON/IDERV C/IDERIV
      COMMON/IXACT C/IEXACT
      COMMON/KXACT C/ KEXACT
      COMMON/THETA C/THETA
      COMMON/INTEGR/R11,R21,R12,R22,
     $              DR11DC,DR21DC,DR12DC,DR22DC
      COMMON/RB COM/NUM11,NUM22,DEN11,DEN22,
     $              DNM1DC,DNM2DC,DDN1DC,DDN2DC
      COMPLEX
     $              THETAZ,F,DFDT,
     $              THETA,
     $              DFDC,S,
     $              R11,R21,R12,R22,
     $              DR11DC,DR21DC,DR12DC,DR22DC,
     $              NUM11,NUM22,DEN11,DEN22,
     $              DNM1DC,DNM2DC,DDN1DC,DDN2DC,
     $              FACTR1,FACTR2,R12R21,DN1DN2,
     $              DFDR11,DFDR21,DFDR12,DFDR22,
     $              DFDNM1,DFDNM2,DFDDN1,DFDDN2
      DATA RTD/57.295  /
C
C
      THETA = THETAZ
      IDERIV=1
      IF(KEXACT .EQ. 0 .AND. IEXACT .EQ. 0) GO TO 11
      CALL INTEG
      CALL FS INTG
      GO TO 12
   11 CALL LAG DER
   12 CALL RBARS
C
      FACTR1 = NUM11-R11*DEN11
      FACTR2 = NUM22-R22*DEN22
      R12R21 = R12*R21
      DN1DN2 = DEN11*DEN22
      F = FACTR1*FACTR2-R12R21*DN1DN2
C
      DFDNM1 = FACTR2
      DFDNM2 = FACTR1
      DFDDN1 = -R11*FACTR2-R12R21*DEN22
      DFDDN2 = -R22*FACTR1-R12R21*DEN11
      DFDR11 = -DEN11*FACTR2
      DFDR21 = -R12*DN1DN2
      DFDR12 = -R21*DN1DN2
      DFDR22 = -DEN22*FACTR1
```

```
      DFDC  =  DFDR11*DR11DC+DFDR21*DR21DC
     $               +DFDR12*DR12DC+DFDR22*DR22DC
     $               +DFDNM1*DNM1DC+DFDNM2*DNM2DC
     $               +DFDDN1*DDN1DC+DFDDN2*DDN2DC
C
      S = DSIN(THETA/RTD)
      DFDTL = -S*DFDC/RTD
      RETURN
C
      END
```

```
      SUBROUTINE LAGRNG
C   FOR MS
C   ROUTINE FOR PERFORMING LAGRANGE INTERPOLATION OF REFLECTION
C                 COEFFICIENTS IN THE COS(THETA) PLANE.
C
      COMMON/THETA C/THETA
      COMMON/INTEGR/R(4),DRDC(4)
      COMPLEX THETAG(1),R GIVEN(4,1),
     $          THETA,R,DRDC,
     $          C,PROD,SUM(4),COEF(4,100),CG(100),DEN(100)
      DATA RTD/57.29578/
C
C
C   LAGRANGE INTERPOLATION.
      C = CCOS(THETA/RTD)
   21 DO 22 I=1,4
   22 SUM(I) = 0.0
      DO 24 JG=1,NG
      PROD = 1.0
      DO 23 J=1,NG
      IF (J .EQ. JG) GO TO 23
      PROD = PROD*(C-CG(J))
   23 CONTINUE
      DO 24 I=1,4
   24 SUM(I) = SUM(I)+COEF(I,JG)*PROD
      DO 25 I=1,4
   25 R(I) = SUM(I)
      RETURN
C
C
C   INITIALIZATION OF LAGRANGE INTERPOLATION USING INFORMATION ON
C                 LOCATION OF GIVEN VALUES IN COS(THETA) PLANE.
      ENTRY INIT LG (THETAG,NG)
      DO 31 JG=1,NG
   31 CG(JG) = CCOS(THETAG(JG)/RTD)
C
      DO 34 JG=1,NG
      PROD = 1.0
      DO 33 J=1,NG
      IF (J .EQ. JG) GO TO 33
      PROD = PROD*(CG(JG)-CG(J))
   33 CONTINUE
   34 DEN(JG) = PROD
      RETURN
C
C
C   FURTHER INITIALIZATION OF LAGRANGE INTERPOLATION USING INFORMATION
C                 ON GIVEN VALUES.
      ENTRY SET LAG (NG,R GIVEN)
      DO 41 JG=1,NG
      DO 41 I=1,4
   41 COEF(I,JG) = R GIVEN(I,JG)/DEN(JG)
      RETURN
```

```
C
C
      ENTRY LAG DER
C  COMPUTATION OF DERIVATIVES OF INTERPOLATED VALUES WRT COS(THETA).
C                 NO EXACT VALUES OF DRDC ARE USED.
C                 THE LAGRANGE INTERPOLATION FORMULA ITSELF
C                 IS DIFFERENTIATED WRT COS(THETA).
      C = COS(THETA/RTD)
      DO 62 I=1,4
   62 SUM(I) = 0.0
      NGM1 = NG-1
      DO 64 JA=1,NGM1
      JAP1 = JA+1
      DO 64 JB=JAP1,NG
      PROD = 1.0
      DO 63 J=1,NG
      IF (J .EQ. JA .OR. J .EQ. JB) GO TO 63
      PROD = PROD*(C-CG(J))
   63 CONTINUE
      DO 64 I=1,4
   64 SUM(I) = SUM(I)+PROD*(COEF(I,JA)+COEF(I,JB))
      DO 65 I=1,4
   65 DRDC(I) = SUM(I)
      GO TO 21
C
      END
```

```
      SUBROUTINE FZEROS (TLEFT,TRIGHT,TBOT,TTOP,TMESH,TOL,MPRINT,
     $           ZEROS,NR Z)
C  FOR ROOT-FINDER
C  FZEROS IS A ROUTINE FOR FINDING THE ZEROS OF A COMPLEX FUNCTION, F,
C                WHICH LIE WITHIN A SPECIFIED RECTANGULAR REGION OF THE
C                COMPLEX (THETA) PLANE, PROVIDED THE FUNCTION HAS NO
C                POLES IN THE VICINITY OF THE RECTANGLE.
C  EXPLANATION OF PARAMETERS--
C        TLEFT - VALUE OF REAL PART OF THETA AT LEFT EDGE OF RECTANGLE.
C        TRIGHT - VALUE OF REAL PART OF THETA AT RT EDGE OF RECTANGLE.
C        TBOT - VALUE OF IMAG PART OF THETA AT BOTTOM EDGE OF RECTANGLE.
C        TTOP - VALUE OF IMAG PART OF THETA AT TOP EDGE OF RECTANGLE.
C        TMESH - SET EQUAL TO ABOUT HALF THE AVERAGE SPACING BETWEEN
C                ZEROS WITHIN THE RECTANGLE.  A SMALLER VALUE MAY BE USED
C                AS A SAFETY MEASURE, BUT TOO SMALL A VALUE WILL RESULT
C                IN EXCESSIVE RUN TIME.
C        TOL - TOLERANCE TO WHICH ZEROS ARE TO BE FOUND.  IF TWO
C                ZEROS ARE CLOSER THAN 'TOL", THE ROOT-FINDER WILL STOP
C                WITH AN ERROR MESSAGE.
C        MPRINT - NORMALLY SET TO ZERO.  A NON-ZERO VALUE LEADS TO
C                PRINT-OUT FOR DEBUGGING.
C        ZEROS - OUTPUT LIST OF (COMPLEX) VALUES OF THETA AT WHICH
C                ZEROS ARE FOUND.
C        NR Z - THE NUMBER OF ZEROS FOUND.
C  SUBROUTINES TO BE PROVIDED--
C        FCT VAL (THETA,F) - TO RETURN THE VALUE OF THE FUNCTION, F,
C                AT THE POINT IN THE COMPLEX PLANE SPECIFIED BY 'THETA'.
C        F DFDT (THETA,F,DFDT) - SAME AS 'FCT VAL' EXCEPT THAT THE
C                DERIVATIVE, DFDT, OF THE FUNCTION WRT THETA MUST ALSO
C                BE RETURNED.
C
      COMMON/ZLS COM/NR ZT,NR ZLS(100)
      COMMON/TMC COM/TMESHC
      DIMENSION ZEROS(2,1)
      COMPLEX F,PREV F,F00,F10,F01,F11,PARTS
      DIMENSION PART(2),SOL(2),THETA(2)
      DIMENSION K EDGE1(50),K EDGE2(50),K EDGE3(50),K EDGE4(50)
      EQUIVALENCE (PART(1),PARTS)
C
C
      IF (MPRINT .NE. 0) PRINT 906
  906 FORMAT ('1')
C
      TMESHC = TMESH
C
C  SIDES OF RECTANGLE IN TMESH UNITS
      JLT = TLEFT/TMESH
      IF (TLEFT .GE. 0.0) JLT = JLT-1
      IF (TLEFT .LT. 0.0) JLT = JLT-2
C
      JRT = TRIGHT/TMESH
      IF (TRIGHT .GT. 0.0) JRT = JRT+2
      IF (TRIGHT .LE. 0.0) JRT = JRT+1
```

```
C
      JBOT = TBOT/TMESH
      IF (TBOT .GE. 0.0) JBOT = JBOT-1
      IF (TBOT .LT. 0.0) JBOT = JBOT-2
C
      JTOP = TTOP/TMESH
      IF (TTOP .GT. 0.0) JTOP = JTOP+2
      IF (TTOP .LE. 0.0) JTOP = JTOP+1
C
C  INITIALIZATION OF PARAMETERS.
      KI = JTOP
      KR = JLT
      KEDGE = 1
      CALL FIND F (KR,KI,F)
      NR E1 = 0
      NR E2 = 0
      NR E3 = 0
      NR E4 = 0
      NR ZT = 0
      NR ZL = 0
      NR Z = 0
   10 IF (NR ZL .EQ. 0) GO TO 20
      NR ZLS(NR Z) = NR ZL
      NR ZL = 0
C
   20 PREV F = F
      IF (NR ZL .GT. 1) PRINT 920,NR ZL
  920 FORMAT ('0',I3,1X,'MODES FOUND ON SAME PHASE LINE')
      NR ZL = 0
      GO TO (21,26,31,36),KEDGE
C
C  SEARCH ALONG LEFT EDGE OF RECTANGLE FOR SIGN CHANGES IN IMAG(F).
   21 IF (KI .EQ. JBOT) KEDGE = 2
      IF (KEDGE .EQ. 2) GO TO 26
      KI = KI-1
      CALL FIND F (KR,KI,F)
      IF ((AIMAG(PREV F) .GT. 0.0 .AND. AIMAG(F) .GT. 0.0)
     $          .OR. (AIMAG(PREV F) .LT. 0.0 .AND. AIMAG(F) .LT. 0.0))
     $          GO TO 20
      IF (NR E1 .EQ. 0) GO TO 25
      DO 22 K=1,NR E1
      IF (KI .EQ. K EDGE1(K)) GO TO 20
   22 CONTINUE
   25 FO1 = PREV F
      FOO = F
      LI = KI
      LR = JLT
      GO TO 43
C
C  SEARCH ALONG BOTTOM EDGE OF RECTANGLE FOR SIGN CHANGES IN IMAG(F).
   26 IF (KR .EQ. JRT) KEDGE = 3
      IF (KEDGE .EQ. 3) GO TO 31
      KR = KR+1
```

```
      CALL FIND F (KR,KI,F)
      IF ((AIMAG(PREV F) .GT. 0.0 .AND. AIMAG(F) .GT. 0.0)
     $          .OR. (AIMAG(PREV F) .LT. 0.0 .AND. AIMAG(F) .LT. 0.0))
     $          GO TO 20
      IF (NR E2 .EQ. 0) GO TO 28
      DO 27 K=1,NR E2
      IF (KR .EQ. K EDGE2(K)) GO TO 20
   27 CONTINUE
   28 F00 = PREV F
      F10 = F
      LI = JBOT
      LR = KR-1
      GO TO 48
C
C  SEARCH ALONG RIGHT EDGE OF RECTANGLE FOR SIGN CHANGES IN IMAG(F).
   31 IF (KI .EQ. JTOP) KEDGE = 4
      IF (KEDGE .EQ. 4) GO TO 36
      KI = KI+1
      CALL FIND F (KR,KI,F)
      IF ((AIMAG(PREV F) .GT. 0.0 .AND. AIMAG(F) .GT. 0.0)
     $          .OR. (AIMAG(PREV F) .LT. 0.0 .AND. AIMAG(F) .LT. 0.0))
     $          GO TO 20
      IF (NR E3 .EQ. 0) GO TO 33
      DO 32 K=1,NR E3
      IF (KI .EQ. K EDGE3(K)) GO TO 20
   32 CONTINUE
   33 F10 = PREV F
      F11 = F
      LI = KI-1
      LR = JRT-1
      GO TO 53
C
C  SEARCH ALONG TOP EDGE OF RECTANGLE FOR SIGN CHANGES IN IMAG(F).
   36 IF (KR .EQ. JLT) GO TO 80
      KR = KR-1
      CALL FIND F (KR,KI,F)
      IF ((AIMAG(PREV F) .GT. 0.0 .AND. AIMAG(F) .GT. 0.0)
     $          .OR. (AIMAG(PREV F) .LT. 0.0 .AND. AIMAG(F) .LT. 0.0))
     $          GO TO 20
      IF (NR E4 .EQ. 0) GO TO 38
      DO 37 K=1,NR E4
      IF (KR .EQ. K EDGE4(K)) GO TO 20
   37 CONTINUE
   38 F11 = PREV F
      F01 = F
      LI = JTOP-1
      LR = KR
      GO TO 58
C
C  ENTER MESH SQUARE FROM LEFT SIDE OR EXIT RECTANGLE AT RIGHT EDGE.
   41 LR = LR+1
      IF (LR .LE. JRT-1) GO TO 42
      NR E3 = NR E3+1
```

```
      K EDGE3(NR E3) = LI+1
      IF (MPRINT .NE. 0) PRINT 990
      GO TO 10
   42 F01 = F11
      F00 = F10
   43 CALL FIND F (LR+1,LI+1,F11)
      CALL FIND F (LR+1,LI,F10)
      LEDGE = 1
      ENTER R = 0.0
      ENTER I = -AIMAG(F00)/AIMAG(F01-F00)
      GO TO 60
C
C  ENTER MESH SQUARE FROM BOTTOM SIDE OR EXIT RECTANGLE AT TOP EDGE.
   46 LI = LI+1
      IF (LI .LE. JTOP-1) GO TO 47
      NR E4 = NR E4+1
      K EDGE4(NR E4) = LR
      IF (MPRINT .NE. 0) PRINT 990
      GO TO 10
   47 F00 = F01
      F10 = F11
   48 CALL FIND F (LR,LI+1,F01)
      CALL FIND F (LR+1,LI+1,F11)
      LEDGE = 2
      ENTER R = -AIMAG(F00)/AIMAG(F10-F00)
      ENTER I = 0.0
      GO TO 60
C
C  ENTER MESH SQUARE FROM RIGHT SIDE OR EXIT RECTANGLE AT LEFT EDGE.
   51 LR = LR-1
      IF (LR .GE. JLT) GO TO 52
      NR E1 = NR E1+1
      K EDGE1(NR E1) = LI
      IF (MPRINT .NE. 0) PRINT 990
      GO TO 10
   52 F11 = F01
      F10 = F00
   53 CALL FIND F (LR,LI+1,F01)
      CALL FIND F (LR,LI,F00)
      LEDGE = 3
      ENTER R = 1.0
      ENTER I = -AIMAG(F10)/AIMAG(F11-F10)
      GO TO 60
C
C  ENTER MESH SQUARE FROM TOP SIDE OR EXIT RECTANGLE AT BOTTOM EDGE.
   56 LI = LI-1
      IF (LI .GE. JBOT) GO TO 57
      NR E2 = NR E2+1
      K EDGE2(NR E2) = LR+1
      IF (MPRINT .NE. 0) PRINT 990
      GO TO 10
   57 F01 = F00
      F11 = F10
```

```
   58 CALL FINC F (LR,LI,F00)
      CALL FINC F (LR+1,LI,F10)
      LEDGE = 4
      ENTER R = -AIMAG(F01)/AIMAG(F11-F01)
      ENTER I = 1.0
      GO TO 60
C
C  FOR DEBUGGING ONLY, PRINT CO-ORDINATE OF LOWER LEFT CORNER
C                OF CURRENT MESH SQUARE.  RESULTING SET OF PRINTED
C                CO-ORDINATES GIVES TRACE OF EACH LINE ALONG WHICH
C                IMAG(F)=0.0 FROM ITS ENTRY ON THE EDGE OF THE
C                RECTANGLE TO ITS EXIT AT ANOTHER POINT ON THE
C                RECTANGLE.
   60 IF (MPRINT .NE. 0) PRINT 960,LR,LI
  960 FORMAT (' ',20X,2I5)
C  TEST FOR THERE BEING TWO (HYPERBOLIC) LINES ENTERING AND LEAVING
C                THE MESH SQUARE ALONG EACH OF WHICH IMAG(F)=0.0
C                IF SO, SET 'LTWO' NON-ZERO.
      LTWO = 0
      IF ((AIMAG(F00) .GT. 0.0 .AND. AIMAG(F11) .GT. 0.0
     $          .AND. AIMAG(F10) .LT. 0.0 .AND. AIMAG(F01) .LT. 0.0)
     $          .OR. (AIMAG(F00) .LT. 0.0 .AND. AIMAG(F11) .LT. 0.0
     $          .AND. AIMAG(F10) .GT. 0.0 .AND. AIMAG(F01) .GT. 0.0))
     $          LTWO = 1
C  TEST FOR THERE BEING AT LEAST ONE (HYPERBOLIC) LINE ENTERING AND
C                LEAVING THE MESH SQUARE ALONG WHICH REAL(F)=0.0
C                IF NOT, SET 'I90' TO ZERO.
      I90 = 1
      IF ((REAL(F00) .GT. 0.0 .AND. REAL(F10) .GT. 0.0
     $          .AND. REAL(F01) .GT. 0.0 .AND. REAL(F11) .GT. 0.0)
     $          .OR. (REAL(F00) .LT. 0.0 .AND. REAL(F10) .LT. 0.0
     $          .AND. REAL(F01) .LT. 0.0 .AND. REAL(F11) .LT. 0.0))
     $          I90 = 0
      IF (LTWO .EQ. 0 .AND. I90 .EQ. 0) GO TO 70
C
C  COMPUTATION OF COEFFICIENTS TO BE USED IN DESCRIBING THE VARIATION
C                OF THE   FUNCTION WITHIN A MESH SQUARE GIVEN THE
C                VALUES AT THE CORNERS OF THE SQUARE AND LINEAR
C                VARIATION ALONG ITS EDGES.  ALSO, THE POSITION OF
C                CROSSING OF THE HYPERBOLIC ASYMPTOTES (WHICH ARE
C                PARALLEL TO  THE SIDES OF THE SQUARE) FOR THE LINES
C                IMAG(F)=0.0 ARE COMPUTED IF BOTH LINES (BRANCHES)
C                ENTER AND LEAVE  THE SQUARE.
      PARTS = F00
      AR = PART(1)
      AI = PART(2)
      PARTS = F01-F00
      BR = PART(1)
      BI = PART(2)
      PARTS = F10-F00
      CR = PART(1)
      CI = PART(2)
      PARTS = F00+F11-F01-F10
```

```
      DR = PART(1)
      DI = PART(2)
C
      IF (LTWO .EQ. 0) GO TO 61
      CENTR R = -BI/DI
      CENTR I = -CI/DI
      IF (I90 .EQ. 0) GO TO 70
C
   61 AB = AR*BI-AI*BR
      AC = AR*CI-AI*CR
      AD = AR*DI-AI*DR
      BC = BR*CI-BI*CR
      BD = BR*DI-BI*DR
      CD = CR*DI-CI*DR
C
      IF (ABS(BD) .LT. ABS(CD)) GO TO 64
C  SOLUTION FOR TWO POSSIBLE POINTS AT WHICH THERE MAY BE CROSSINGS
C                 OF THE (HYPERBOLIC) LINES REAL(F)=0.0 AND IMAG(F)=0.0.
C                 A CROSSING POINT IS CHOSEN TO BE A ZERO OF THE
C                   FUNCTION IF IT LIES WITHIN THE CURRENT MESH SQUARE
C                 AND IF IT LIES ON THE BRANCH OF IMAG(F)=0.0 CURRENTLY
C                 BEING FOLLOWED.  MULTIPLE CROSSINGS ALONG THIS BRANCH
C                 (ACTUALLY A SERIES OF CONNECTING HYPERBOLIC BRANCHES)
C                 IN THE SAME OR IN DIFFERENT SQUARES ARE NOTED SINCE
C                 THESE MUST LATER BE RESOLVED.
      A = CD
      B = (AD-BC)*0.5
      C = AB
      CALL QUAD (A,B,C,SOL,NR SOL)
      IF (NR SOL .EQ. 0) GO TO 70
      DO 63 N=1,NR SOL
      UREAL = SOL(N)
      IF (UREAL .LT. 0.0 .OR. UREAL .GT. 1.0) GO TO 63
      D1 = BR+CR*UREAL
      D2 = BI+CI*UREAL
      IF (ABS(D1) .GE. ABS(D2)) UIMAG = -(AR+CR*UREAL)/D1
      IF (ABS(D2) .GE. ABS(D1)) UIMAG = -(AI+CI*UREAL)/D2
      IF (UIMAG .LT. 0.0 .OR. UIMAG .GT. 1.0) GO TO 63
      IF (LTWO .EQ. 0) GO TO 62
      IF ((ENTER R-CENTR R)*(U REAL-CENTR R) .LT. 0.0
     $          .OR. (ENTER I-CENTR I)*(U IMAG-CENTR I) .LT. 0.0)
     $          GO TO 63
   62 THETA(1) = (LR+U REAL)*TMESH
      THETA(2) = (LI+U IMAG)*TMESH
      IF (MPRINT .NE. 0) PRINT 962,LR,LI,THETA(1),THETA(2)
  962 FORMAT (' ',2I4,3X,'THETA =',2F9.4)
      NR ZT = NR ZT+1
      ZEROS(1,NR ZT) = THETA(1)
      ZEROS(2,NR ZT) = THETA(2)
      IF (NR ZL .EQ. 0) NR Z = NR Z+1
      NR ZL = NR ZL+1
   63 CONTINUE
      GO TO 67
```

```
C
C  ALTERNATE SOLUTION FOR THE ABOVE.  TWO FORMS ARE NEEDED SINCE,
C                IN ANY GIVEN CASE, EITHER FORM (BUT NOT BOTH FORMS)
C                MAY BE INDETERMINATE.
   64 A = BD
      B = (AD+BC)*0.5
      C = AC
      CALL QUAD (A,B,C,SOL,NR SL
      IF (NR SOL .EQ. 0) GO TO 70
      DO 66 N=1,NR SOL
      UIMAG = SOL(N)
      IF (UIMAG .LT. 0.0 .OR. UIMAG .GT. 1.0) GO TO 66
      D1 = CR+DR*UIMAG
      D2 = CI+DI*UIMAG
      IF (ABS(D1) .GE. ABS(D2)) UREAL = -(AR+BR*UIMAG)/D1
      IF (ABS(D2) .GE. ABS(D1)) UREAL = -(AI+BI*UIMAG)/D2
      IF (UREAL .LT. 0.0 .OR. UREAL .GT. 1.0) GO TO 66
      IF (LTWO .EQ. 0) GO TO 65
      IF ((ENTER R-CENTR R)*(U REAL-CENTR R) .LT. 0.0
     $          .OR. (ENTER I-CENTR I)*(U IMAG-CENTR I) .LT. 0.0)
     $          GO TO 66
   65 THETA(1) = (LR+U REAL)*TMESH
      THETA(2) = (LI+U IMAG)*TMESH
      IF (IMPRINT .NE. 0) PRINT 965,LR,LI,THETA(1),THETA(2)
  965 FORMAT (' ',2I4,3X,'THETA =',2F9.4)
      NR ZT = NR ZT+1
      ZEROS(1,NR ZT) = THETA(1)
      ZEROS(2,NR ZT) = THETA(2)
      IF (NR ZL .EQ. 0) NR Z = NR Z+1
      NR ZL = NR ZL+1
   66 CONTINUE
   67 CONTINUE
C
   70 CONTINUE
C  TEST FOR EXIT FROM LEFT EDGE OF MESH SQUARE.
      IF (LEDGE .EQ. 1) GO TO 72
      IF ((AIMAG(F00) .GT. 0.0 .AND. AIMAG(F01) .GT. 0.0)
     $          .OR. (AIMAG(F00) .LT. 0.0 .AND. AIMAG(F01) .LT. 0.0))
     $          GO TO 72
      IF (LTWO .EQ. 0) GO TO 51
      EXIT R = 0.0
      EXIT I = -AIMAG(F00)/AIMAG(F01-F00)
      IF ((ENTER R-CENTR R)*(EXIT R-CENTR R) .LT. 0.0
     $          .OR. (ENTER I-CENTR I)*(EXIT I-CENTR I) .LT. 0.0)
     $          GO TO 72
      GO TO 51
C
C  TEST FOR EXIT FROM BOTTOM EDGE OF MESH SQUARE.
   72 IF (LEDGE .EQ. 2) GO TO 73
      IF ((AIMAG(F00) .GT. 0.0 .AND. AIMAG(F10) .GT. 0.0)
     $          .OR. (AIMAG(F00) .LT. 0.0 .AND. AIMAG(F10) .LT. 0.0))
     $          GO TO 73
      IF (LTWO .EQ. 0) GO TO 56
```

```
      EXIT R = -AIMAG(F00)/AIMAG(F10-F00)
      EXIT I = 0.0
      IF ((ENTER R-CENTR R)*(EXIT R-CENTR R) .LT. 0.0
     $          .OR. (ENTER I-CENTR I)*(EXIT I-CENTR I) .LT. 0.0)
     $          GO TO 73
      GO TO 56
C
C  TEST FOR EXIT FROM RIGHT EDGE OF MESH SQUARE.
   73 IF (LEDGE .EQ. 3) GO TO 74
      IF ((AIMAG(F10) .GT. 0.0 .AND. AIMAG(F11) .GT. 0.0)
     $          .OR. (AIMAG(F10) .LT. 0.0 .AND. AIMAG(F11) .LT. 0.0))
     $          GO TO 74
      IF (LTWO .EQ. 0) GO TO 41
      EXIT R = 1.0
      EXIT I = -AIMAG(F10)/AIMAG(F11-F10)
      IF ((ENTER R-CENTR R)*(EXIT R-CENTR R) .LT. 0.0
     $          .OR. (ENTER I-CENTR I)*(EXIT I-CENTR I) .LT. 0.0)
     $          GO TO 74
      GO TO 41
C
C  TEST FOR EXIT FROM TOP EDGE OF MESH SQUARE.
   74 IF (LEDGE .EQ. 4) GO TO 90
      IF ((AIMAG(F01) .GT. 0.0 .AND. AIMAG(F11) .GT. 0.0)
     $          .OR. (AIMAG(F01) .LT. 0.0 .AND. AIMAG(F11) .LT. 0.0))
     $          GO TO 90
      IF (LTWO .EQ. 0) GO TO 46
      EXIT R = -AIMAG(F01)/AIMAG(F11-F01)
      EXIT I = 1.0
      IF ((ENTER R-CENTR R)*(EXIT R-CENTR R) .LT. 0.0
     $          .OR. (ENTER I-CENTR I)*(EXIT I-CENTR I) .LT. 0.0)
     $          GO TO 90
      GO TO 46
C
   90 IF (NR Z .NE. 0) CALL NO MESH (TMESH,TOL,MPRINT,ZEROS,NR Z)
      RETURN
C
   90 PRINT 909,LR,LI
  909 FORMAT ('0','NO EXIT FROM MESH SQUARE',2I6)
      STOP
C
  990 FORMAT('0')
C
      END
```

```
      SUBROUTINE QUAD (A,B,C,SOL,NR SOL)
C   FOR ROOT-FINDER
C
C   SOLUTION FOR THE REAL ROOTS OF A QUADRATIC EQUATION OF THE
C                 FORM A*X**2+2.0*B*X+C=0.0, WHERE X IS CALLED 'SOL'
C                 IN THIS ROUTINE.  THE NUMBER OF REAL ROOTS FOUND IS
C                 GIVEN BY 'NR SOL'.  A VALUE OF 1 FOR 'NR SOL' RESULTS
C                 FROM THE QUADRATIC EQUATION APPROACHING LINEARITY.
C                 USED BY SUBROUTINES FZEROS AND NO MESH.
C
      DIMENSION SOL(2)
C
C
      ACOBSQ = A*C/B**2
      IF (ABS(ACOBSQ) .LT. 0.3) GO TO 20
C
      ARG = B**2-A*C
      NR SOL = 0
      IF (ARG .LT. 0.0) RETURN
      NR SOL = 2
      ROOT = SQRT(ARG)
      SOL(1) = (-B+ROOT)/A
      SOL(2) = (-B-ROOT)/A
      RETURN
C
   20 TERM = 1.0
      SUM = 1.0
      DO 21 K=1,50
      TERM = TERM*((K-0.5)/(K+1.0))*ACOBSQ
      SUM = SUM+TERM
      IF (ABS(TERM) .LT. 1.0E-10) GO TO 22
   21 CONTINUE
   22 SOL(1) = -C/(2.0*B)*SUM
      NR SOL = 1
      IF (ABS(A/(2.0*B)) .LT. 1.0E-30) RETURN
      NR SOL = 2
      SOL(2) = -2.0*B/A-SOL(1)
      RETURN
C
      END
```

```
      SUBROUTINE FIND F (JR,JI,F)
C   FOR ROOT-FINDER
C
      COMMON/TMC COM/TMESH
      DIMENSION PART(2)
      COMPLEX F,THETA,PARTS
      EQUIVALENCE (PART,PARTS)
C
C
      PART(1) = JR*TMESH
      PART(2) = JI*TMESH
      THETA = PARTS
      CALL FCT VAL (THETA,F)
      RETURN
C
      END
```

```
      SUBROUTINE NO MESH (TMESH,TOL,MPRINT,ZEROS,NR Z)
C   FOR ROOT-FINDER
C   ROUTINE FOR FINDING EXACT (IN THE SENSE OF NO MESH APPROXIMATION)
C                 LOCATIONS OF ZEROS OF THE FUNCTION, F, FOR WHICH
C                 A COMPLETE, BUT APPROXIMATE, SET WAS FOUND IN
C                 SUBROUTINE FZEROS.
C                 TWO DEFICIENCIES REMAIN IN THIS ROUTINE.  ONE IS
C                 THAT ZEROS CLOSER THAN THE FINITE VALUE 'TOL' CANNOT
C                 BE RESOLVED.  THIS HAS CAUSED A PROBLEM A FEW TIMES
C                 IN USAGE TO DATE, BUT HAS ALWAYS BEEN RESOLVED BY
C                 USING A SMALLER VALUE OF 'TOL'.  THE SECOND PROBLEM
C                 IS THAT THE RESOLUTION OF 'MULTIPLE CROSSINGS',
C                 WHICH OCCUR BECAUSE OF THE NON-ANALITICITY OF THE
C                 MESH APPROXIMATION, IS SUCH THAT IN PRINCIPLE A
C                 ZERO CAN BE MISSED.  THERE IS NO EVIDENCE THIS HAS
C                 EVER OCCURRED IN PRACTICE.  IT MAY OR MAY NOT BE
C                 COST EFFECTIVE TO DEVELOP THE PROGRAM FURTHER TO
C                 CORRECT THESE TWO PROBLEMS.
C
      COMMON/ZLS COM/NR ZT,NR ZLS(100)
      DIMENSION PART(2),SOL(2)
      COMPLEX ZEROS(1),
     $          THETA,
     $          F,DFDT,DELT,
     $          PARTS,
     $          ZEROSO(100),TMIN,
     $          ATEMP,BTEMP,CTEMP,DTEMP,
     $          FARRAY(4,4),F00,F01,F10,F11
      EQUIVALENCE (PART(1),PARTS)
C
C
      IF (MPRINT .NE. 0) PRINT 900
  900 FORMAT ('1','ITERATIONS TO REMOVE MESH')
C
      NS = 1
C
C  NEWTON-RAPHSON (FIRST-ORDER) ITERATION TO FIND EXACT LOCATIONS
C                 OF THE ZEROS OF F.
      DO 29 JZ=1,NR ZT
      ZEROSO(JZ) = ZEROS(JZ)
      IF (MPRINT .NE. 0) PRINT 901,JZ
  901 FORMAT (' ',I8)
   21 IF (MPRINT .NE. 0) PRINT 902,ZEROS(JZ)
  902 FORMAT (' ',10X,2(F9.4))
      THETA = ZEROS(JZ)
      CALL F DFDT (THETA,F,DFDT)
      DELT = -F/DFDT
      ZEROS(JZ) = ZEROS(JZ)+DELT
      IF (CABS(DELT) .GT. 0.3*TOL) GO TO 21
   29 CONTINUE
C
C  NUMBER OF UNIQUE ZEROS COMPARED TO TOTAL NUMBER MINUS
C                 MULTIPLICITY DUE TO MULTIPLE CROSSINGS.
```

```
   30 IF (NR ZT .EQ. 1) RETURN
      JZ = 1
      DO 33 J=2,NR ZT
      MATCH = 0
      JM1 = J-1
      DO 32 JJ=1,JM1
      IF (CABS(ZEROS(J)-ZEROS(JJ)) .GT. TOL) GO TO 32
      IF (MPRINT .NE. 0) PRINT 905,J,JJ
  905 FORMAT ('0','ZEROS NR',I3,1X,'AND',I3,1X,
     $            'ARE THE SAME')
      MATCH = 1
   32 CONTINUE
      IF (MATCH .NE. 0) GO TO 33
      JZ = JZ+1
      ZEROS(JZ) = ZEROS(J)
   33 CONTINUE
C
      IF (JZ .EQ. NR Z) RETURN
      IF (JZ .LT. NR Z) GO TO 50
      PRINT 947,JZ,NR Z
  947 FORMAT ('0','WARNING--',I3,1X,'ZEROS FOUND ON',I3,1X,
     $            'PHASE LINES')
      NR Z = JZ
      RETURN
C
C  THE FOLLOWING LOGIC IS NEEDED ONLY OCCASIONALLY, NAMELY, WHEN
C                TEST INDICATES THAT A ZERO HAS BEEN MISSED IN THE
C                INITIAL NEWTON-RAPHSON ITERATION. IT ALLOWS FOR
C                STEP-WISE CHANGE IN THE FORM OF THE FUNCTION FROM
C                THE MESH APPROXIMATION TO THE EXACT FORM.
   50 NS = NS*2
      IF (NS .GT. 16) GO TO 90
C
      DO 59 JZ=1,NR ZT
      IF (MPRINT .NE. 0) PRINT 901,JZ
      ZEROS(JZ) = ZEROS0(JZ)
      DO 58 K=1,NS
      FN FRCT = FLOAT(K)/FLOAT(NS)
      FM FRCT = 1.0-FN FRCT
   51 THETA = ZEROS(JZ)
      IF (MPRINT .NE. 0) PRINT 902,ZEROS(JZ)
      CALL F DFDT (THETA,F,DFDT)
      IF (K .EQ. NS) GO TO 56
C
      JRO = REAL(THETA)/TMESH
      IF (REAL(THETA) .LT. 0.0) JRO = JRO-1
      JIO = AIMAG(THETA)/TMESH
      IF (AIMAG(THETA) .LT. 0.0) JIO = JIO-1
      DO 52 JRLOOP=1,4
      DO 52 JILOOP=1,4
      JR = JRO+JRLOOP-2
      JI = JIO+JILOOP-2
   52 CALL FIND F (JR,JI,FARRAY(JRLOOP,JILOOP))
```

```
C
      DMIN = 9.9E9
      DO 67 JRLOOP=1,3
      DO 67 JILOOP=1,3
      F00 = FARRAY(JRLOOP,JILOOP)
      F01 = FARRAY(JRLOOP,JILOOP+1)
      F10 = FARRAY(JRLOOP+1,JILOOP)
      F11 = FARRAY(JRLOOP+1,JILOOP+1)
      JR = JR0+JRLOOP-2
      JI = JI0+JILOOP-2
      REFR = REAL(ZEROS(JZ))/TMESH-JR
      REFI = AIMAG(ZEROS(JZ))/TMESH-JI
      ATEMP = F00
      BTEMP = F01-F00
      CTEMP = F10-F00
      DTEMP = F00+F11-F01-F10
      PARTS = (ATEMP+BTEMP*REFI+CTEMP*REFR+DTEMP*REFR*REFI)*FM FRCT
     1          +F*FN FRCT
      AR = PART(1)
      AI = PART(2)
      PARTS = (BTEMP+DTEMP*REFR)*FM FRCT+DFDT*(0.0,1.0)*TMESH*FN FRCT
      BR = PART(1)
      BI = PART(2)
      PARTS = (CTEMP+DTEMP*REFI)*FM FRCT+DFDT*TMESH*FN FRCT
      CR = PART(1)
      CI = PART(2)
      PARTS = DTEMP*FM FRCT
      DR = PART(1)
      DI = PART(2)
C
      AB = AR*BI-AI*BR
      AC = AR*CI-AI*CR
      AD = AR*DI-AI*DR
      BC = BR*CI-BI*CR
      BD = BR*DI-BI*DR
      CD = CR*DI-CI*DR
C
      IF (ABS(BD) .LT. ABS(CD)) GO TO 64
      A = CD
      B = (AD-BC)*0.5
      C = AB
      CALL QUAD (A,B,C,SOL,NR SOL)
      IF (NR SOL .EQ. 0) GO TO 67
      DO 62 N=1,NR SOL
      UREAL = SOL(N)
      IF (UREAL+REFR .LT. -0.030 .OR. UREAL+REFR .GT. 1.030) GO TO 62
      D1 = BR+DR*UREAL
      D2 = BI+DI*UREAL
      IF (ABS(D1) .GE. ABS(D2)) UIMAG = -(AR+CR*UREAL)/D1
      IF (ABS(D2) .GE. ABS(D1)) UIMAG = -(AI+CI*UREAL)/D2
      IF (UIMAG+REFI .LT. -0.003 .OR. UIMAG+REFI .GT. 1.003) GO TO 62
      PART(1) = (JR+UREAL+REFR)*TMESH
      PART(2) = (JI+UIMAG+REFI)*TMESH
```

```
      THETA = PARTS
      DIST = CABS(THETA-ZEROS(JZ))
      IF (DIST .GT. DMIN) GO TO 62
      DMIN = DIST
      TMIN = THETA
   62 CONTINUE
      GO TO 67
C
   64 A = BD
      B = (AD+BC)*0.5
      C = AC
      CALL QUAD (A,B,C,SOL,NR SOL)
      IF (NR SOL .EQ. 0) GO TO 67
      DO 66 N=1,NR SOL
      UIMAG = SOL(N)
      IF (UIMAG+REFI .LT. -0.030 .OR. UIMAG+REFI .GT. 1.030) GO TO 66
      D1 = CR+DR*UIMAG
      D2 = CI+DI*UIMAG
      IF (ABS(D1) .GE. ABS(D2)) UREAL = -(AR+BR*UIMAG)/D1
      IF (ABS(D2) .GE. ABS(D1)) UREAL = -(AI+BI*UIMAG)/D2
      IF (UREAL+REFR .LT. -0.003 .OR. UREAL+REFR .GT. 1.003) GO TO 66
      PART(1) = (JR+UREAL+REFR)*TMESH
      PART(2) = (JI+UIMAG+REFI)*TMESH
      THETA = PARTS
      DIST = CABS(THETA-ZEROS(JZ))
      IF (DIST .GT. DMIN) GO TO 66
      DMIN = DIST
      TMIN = THETA
   66 CONTINUE
   67 CONTINUE
C
      IF (DMIN .GT. 9.0E9) GO TO 95
      DELT = TMIN-ZEROS(JZ)
      ZEROS(JZ) = TMIN
      IF (CABS(DELT) .LT. 0.03*TMESH) GO TO 57
      GO TO 51
   56 DELT = -F/DFDT
      ZEROS(JZ) = ZEROS(JZ)+DELT
      IF (CABS(DELT) .LT. 0.3*TOL) GO TO 57
      GO TO 51
   57 IF (MPRINT .NE. 0) PRINT 907
  907 FORMAT ('0')
   58 CONTINUE
   59 CONTINUE
      GO TO 30
C
   90 PRINT 990,JZ,NR Z
  990 FORMAT ('0','ONLY',I3,1X,'ZEROS FOUND ON',I3,1X,'PHASE LINES')
      STOP
C
   95 PRINT 995
  995 FORMAT ('0','PROBLEMS IN SUBROUTINE NO MESH')
      STOP
```

END

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