STRESS ANALYSIS OF PLASTIC ROTATING BANDS

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S. W. Tsaï
Project Monitor

FOR THE DIRECTOR

S. W. Tsaï, Chief
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Dynamic stress analysis of plastic rotating bands, as used on projectile nose, has been obtained by means of finite-difference computer code. The first phase of the interaction within the barrel, involving the axisymmetric convergence of the plastic bands into the barrel, has been investigated in this study. The numerical solutions for three rotating band problems were obtained. The results of those solutions showed that (a) shear stresses acting along the barreller interface due to interior ballistics rotating bands, (b) dynamic stress analysis finite-difference code.
Friction can have a major effect on rotating band performance, and (b) that with a current band design, the sharply-beveled rear section of the band becomes grossly distorted and develops a large annular lip protruding from the rear of the band (i.e., feathering). The third case considered a design modification made to alleviate the extrusion problem. These initial analyses show that finite-difference code solutions of the dynamics of rotating bands are practical and can provide a useful tool to investigate a number of key factors bearing on plastic band performance, such as material properties, stress-strain conditions experienced, and band geometries.
PREFACE

This report describes a research program to analyze the dynamic stresses and deformations in plastic rotating bands (such as used on projectiles) as they are forced into a converging forcing cone at high speed. A two-dimensional finite-difference computer code was used for the analyses. The program was conducted during the period April through September 1975, at California Research & Technology, Inc. 6269 Variel Ave., Woodland Hills, CA 91364, under Contract F33615-75-C-5206, initiated under Task No. ILIR0073. The research was performed by M. H. Wagner and K. N. Kreyeugen. W. S. Goerke, C. K. Wilson, and C.C. Fulton provided assistance in code development and programming. The Project Engineer for the Air Force Materials Laboratory was S. W. Tsai, Chief, Mechanics & Surface Interactions Branch, Nonmetallic Materials Division (AFML/MBM). The authors submitted this report in October 1975.
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SECTION I
INTRODUCTION, SUMMARY, AND RECOMMENDATIONS

A. BACKGROUND

Rotating bands are used on projectiles to impart spin (by transmitting torque from rifling to the projectile), to improve the propellant gas seal, and to reduce wear of the barrel (by minimizing direct contact between the hard projectile and the walls). Rotating bands must generally be of pliable and/or ductile material, since they must withstand substantial plastic flow in accommodating to the barrel and to the rifling. The material must also have sufficient strength to resist the propellant gas pressure, to transmit torque to the projectile, and to remain attached to the rotating projectile as it leaves the muzzle.

This sketch (not to scale) shows a typical rotating band. The projectile is somewhat smaller than the minor diameter of the bore, but the OD of the rotating band is larger, so that there is dimensional interference. When the projectile is fired, it "tunnels" into a forcing cone wherein the rotating band is squeezed down until it fits into the barrel. To accomplish this squeezing, the rotating band material:
(1) Close circumferentially into the rifling grooves

(2) Is compressed radially under the high confining pressure of the barrel

(3) Close axially to relieve the volumetric compression (i.e., the rotating band becomes longer).

Within the barrel, the rifling can have either a fixed or a variable twist (pitch) along the barrel length. This twist, acting through the rotating band, imparts angular acceleration as the projectile itself undergoes linear acceleration down the barrel. As the projectile leaves the muzzle, the radial compression of the band is relaxed. The band remains attached to the projectile until it reaches the target.

Some nylon-type plastics are desirable materials for rotating bands, and they are successfully used in this application. However, failures of nylon rotating bands have been experienced, especially in projectiles which are designed to reach higher muzzle velocities. Information about the timing of such failures (i.e., the location within the barrel at which failure occurs) and the cause thereof is not available, nor is it readily attainable through experiments. Such information is needed in order to design better rotating band-rifling systems, and in order to select or develop better materials for rotating bands.

Numerical hydro-elastic-plastic code analyses of the dynamics of rotating band distortion as the projectile tunnels into the forcing cone provide an approach for obtaining information about the dynamic stresses and deformations in rotating bands. This report presents the results of three such analyses.
P. OBJECTIVES

The objectives of this preliminary program were:

(1) To adapt a two-dimensional numerical code for analysis of the axisymmetric stresses and distortions of rotating bands entering the forcing cone,

(2) Through analyses of a representative, or "baseline" rotating band design, to show the utility of numerical dynamic stress and deformation analyses as a tool for designing, evaluating, and improving rotating bands, and

(3) To identify the stress and deformation histories experienced in critical regions in the representative rotating band design.

C. NUMERICAL APPROACH FOR DYNAMIC STRESS ANALYSIS

When the projectile enters the barrel, the significant interference fit requires that the rotating band be severely compressed and that it be deformed in a very short interval of time. To accommodate the nonlinear, dynamic processes involved, a finite-difference code was used, specifically, the CRT WAVE-L code. This is a Lagrangian code based on Wilkins' HEMP method. The WAVE-L code has been applied to a wide range of impulsive loading problems, including hypersonic impact, projectile penetration, and dynamic stress analysis.

WAVE-L is a "first principle" code which obtains "time-marching" solutions of the conservation equations and the
Constitutive equations describing the properties (elastic, plastic, and fracture) of the materials involved. The code is two-dimensional, i.e., problems must be described in two space dimensions. This includes 2-D plane strain problems, as well as 3-D problems which are axisymmetric.

The geometry of the projectile/rotating band tunneling into a smooth-walled forcing cone and traveling through a smooth bore is axisymmetric, and can thus be described using two space dimensions. When rifling grooves in the barrel are encountered, however, the problem is no longer strictly axisymmetric, in that 2-D non-symmetrical effects occur (i.e., circumferential flow). In the present study, we have decoupled the problem, treating only the axisymmetric aspects of the initial tunneling processes.

(A separate but coupled plane strain analysis can be made of the non-symmetric interactions between the rotating band and the rifling in the barrel. Recommendations for this type analysis, together with a suggested approach, are given in Section I E1 of this report.)

Figure 1 is a schematic defining the computational field for analysis of a projectile/rotating band tunneling into a forcing cone. It is assumed that neither the projectile nor the barrel deform significantly, so they are treated as rigid boundaries which confine and deform the nylon rotating band. (This is a reasonable assumption; the distortions in the barrel and projectile due to stress levels imposed as the nylon compresses are very small compared to the rotating band distortions.) The actual field of analysis can thus be confined to the rotating band itself.

For the code analysis, the geometry of the rotating band slot (i.e., the slot in the projectile containing the
rotating band) and the geometry of the forcing cone are specified, as well as the velocity at which the projectile is moving. For the present analysis, this was specified to be 500 fps. (For convenience in possible future comparisons with experiments, the projectile was assumed to be stationary, with the barrel moving over it at 500 fps. This transformation has no effect on the stresses in the rotating band.)

The rotating band itself is described by a Lagrangian network of computational cells, and by a set of equations (based on measured properties) which characterize the elastic and plastic stress-strain behavior of the rotating band material (in the cases treated here, Nylon 6/12). No failure criteria were used in these analyses, although the capability to treat certain types of failure exists in the WAVE-L code.

D. SUMMARY OF RESULTS

Numerical solutions were performed for three rotating band problems as defined in Figure 2. The baseline design (Cases 1 and 2) approximately corresponds with the rotating band for a 20mm M-56 projectile. Representative major and minor diameters for a 20mm barrel are .817 in. and .786 in., respectively. The nominal barrel diameter (D_0 = .809 in.) chosen for these axisymmetric analyses gives the same overall volumetric compression of the rotating band entering the forcing cone as occurs in a rifled barrel.

The presence of friction (coefficient \( \mu = 0.1 \)) in Case 1 produced almost immediate locking between the nylon and the forcing cone. This was because the normal stresses, \( \sigma_n \), across the converging interface build up very quickly to the level where \( \sigma_n \) exceeds the shear strength of the nylon. Nylon near the interface thereafter distorts at
essentially the full projectile-barrel relative velocity. This clearly shows that friction can be very important in rotating band design. However, we consider the results of Case 1 to be an exaggerated indication of the effects of friction, at least in a plastic material like nylon. Initial friction and plastic distortion will produce significant heating of the surface, thereby softening or melting the material such that the surface cannot thereafter support large frictional stresses.

We did not have properties to quantitatively define this softening and weakening of the surface; it appeared reasonable for the purposes of these analyses to assume that the friction becomes so small as to be negligible. In Case 2, we therefore repeated the analysis of the baseline rotating band design, with no friction. Figure 3 indicates the way in which the frictionless basic band distorts as it tunnels into the forcing cone. The nylon is initially squeezed inward. The stress on the surface of the nylon builds up to about 40,000 psi. This compression causes the nylon to begin to flow rearward, producing gross distortion of the sharply-beveled trailing surface, as seen in Figure 3. As a result, nylon extrudes out of the rotating band slot and into the converging region between the projectile and forcing cone walls. Eventually, this will lead to a very thin, probably irregular annulus of nylon between the aft end of the projectile and the barrel (i.e., feathering). Portions of this badly extruded annulus may break off, either in the barrel or after the projectile leaves the muzzle. Presumably this irregular extrusion will have undesirable effects on the in-flight aerodynamics of the projectile.
To avoid, or at least lessen, the rearward extrusion of nylon out of the rotating band slot, a modified design was considered (Case 3). As seen in Figure 2, this design substituted a more gradual slope for the trailing surface. Figures 4 and 5 show the pattern of distortion which develops in the modified design. The gradually-sloping trailing surface bulges out towards the forcing cone. Some rearward extrusion of nylon out of the rotating band slot eventually occurs, but it is greatly reduced, as compared to the basic design.

Figure 6 compares the axial force history applied to the projectile by the basic and the modified rotating band designs. The force is higher in the modified case because the more gradual trailing surface closes against the forcing cone, thereby increasing the surface area of the rotating band which acts on the forcing cone. The increased force does not appreciably affect the projectile dynamics, however; the impulse transmitted to the projectile with the modified rotating band as it tunnels into the forcing cone will affect its velocity by only about 1 fps.

Detailed information about the stresses and distortions in the basic design (Case 2) and the modified design (Case 3) is given in the text and in the appendices. This includes time histories of the stresses at several points within the rotating bands.

E. CONCLUSIONS AND RECOMMENDATIONS

1. Utility of Numerical Analysis of Dynamic Stresses and Distortions in Rotating Bands

The three analyses which were performed show that finite-difference solutions of rotating bands tunneling
into forcing cones arc practical, and are specifically useful for these purposes:

- To identify material properties which have major effects on design and performance
- To identify stress-strain conditions which are experienced by rotating band materials
- To identify design weaknesses
- To suggest methods for correcting such weaknesses
- To evaluate design modifications.

Examples of these uses are seen in the three cases which were analyzed:

In Case 1, a code solution was used to identify a critical material property (friction), showing that it can have a major effect on rotating band design and performance. (The lack of data on frictional properties for candidate materials under pertinent conditions of stress, temperature, and sliding velocity is a deficiency which needs to be remedied. The pertinent conditions for these and other property measurements can be determined from numerical solutions.)

In Case 2, a code solution was used to identify a design weakness in the baseline rotating band geometry, and to suggest a means for correcting the problem. (The sharply-beveled trailing surface of the baseline geometry leads to gross distortion and eventually to extrusion of material out of the rotating band slot and into the narrowing region between the projectile and the barrel. A more gradual slope for the trailing surface may alleviate this problem.)
In Case 3, a code solution was used to evaluate a possible design improvement in the baseline geometry. (This improvement involved the gradually sloping trailing surface. It greatly reduced, but did not wholly eliminate, the extrusion problem.)

In all the cases, stress and strain levels, and strain rates were determined at several stations. (Maximum compressive stresses were of the order of 2.5 kb, or about 38,000 psi. Maximum strain rates in these 500-fps cases were of the order of \(10^4 - 10^5/\text{sec.}\))

2. Recommendations

Applications. The above uses of the code can be of substantial value in design and evaluation, and in material selection or development. These applications should be pursued.

- Candidate new designs, as well as existing designs which are not performing well, should be analyzed to identify critical regions where failures may occur. Possible design modifications should then be numerically evaluated.

- A systematic study should be made to determine the desirable mechanical property attributes of rotating band materials.

Code Validation. Although the basic WAVEL numerical method has been validated in a number of prior experimental comparisons for other applications, it would be desirable to verify the ability of the code (and material models) to predict
rotating band processes. A possible experiment would involve reverse ballistic firing of a "barrel" over a fixed "projectile" which is supported on a rod.* Force-time measurements could be made in the support rod as the barrel pushes over the projectile.

**Interactions with Rifling.** In addition to the axisymmetric processes occurring as the rotating band is squeezed into the barrel (which are analyzed in the three cases treated so far), non-symmetric processes are involved as the rifling lands force deep grooves into the rotating band, and as the projectile travels down the barrel. These processes involve circumferential flow of material as the rifling lands impinge into the band, and the application of tangential forces to the rotating band by the twist of the rifling.

Analyses of the stresses and deformations in rotating bands due to these asymmetric processes should be made. A possible approach would be to treat the problem in plane strain, as indicated in Figure 7. In this approach, the projectile would be represented by the rigid-body central core, and the barrel would also be represented as a rigid body (concentric to the core). The rotating band (between the rigid projectile and barrel) would be described with the elastic-plastic rotating band material properties.

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*This experiment has been suggested by H. Swift of UDRI*
There would be a sliding interface between the band and the barrel. Friction could be specified in the z-direction (perpendicular to the plane of solution), thereby allowing the effects of sliding friction in the third dimension (z-direction) to be incorporated in the plane strain solution. The band would be locked to the projectile body.

Initially, the rifling lands, as represented by the periodic protrusions around the rigid barrel, would be nonexistent (i.e., the barrel surface would be smooth). The flat surfaces would thereafter be gradually forced into the rotating band by prescribing their displacement, \( r(t) \) a function of time. The angular acceleration produced by the rifling would be represented by prescribing rotating of the barrel by \( \omega(t) \), as determined by the projectile velocity and by the pitch of the rifling.

The same types of information could be obtained from the plane strain solutions as from the axisymmetric solutions which are described in the report; i.e., field plots of distortions, particle velocities, and principal stresses, and parameter vs time plots at selected stations.
SECTION II
NUMERICAL SOLUTIONS

A. PROBLEM DEFINITION

A representative "baseline" rotating band of interest was chosen as an initial vehicle to test and demonstrate the finite-difference code technique of analysis. This is illustrated in Figure 1 (page 21). The specific baseline dimensions, and the computational grid used to represent the band, are shown in Figure 8. The band design shown is one which has been considered for a 20mm M-56 projectile.

The analyses which were performed were confined to the axisymmetric convergence of the band as it engaged with the forcing cone, and therefore did not explicitly treat the rifling. Instead, a uniform barrel diameter (.809 in.), between the major and minor diameters of the barrel (.817 in. and .786 in., respectively), was chosen which provides the same overall volumetric compression of the rotating band as occurs in the rifled barrel. The initial .829-in. OD of the rotating band is thus subjected to a maximum radial convergence of .010 in. in the forcing cone.

A nominal constant 500 fps was chosen for the velocity at which the projectile enters the forcing cone. In the actual gun firing, the projectile is accelerated from an initial rest position during the tunneling process. This mode of engagement can be handled by the code, but we did not have information defining a typical acceleration-time history in the forcing cone. This problem is not felt to be particularly
sensitive to velocity, so long as the velocity is anywhere in the low subsonic range (which cover: the range of velocities of interest here), and so long as the material properties are not rate sensitive (as was the case with the material model used in these analyses).

In the code solution, the forcing cone and barrel were set up to move at the prescribed velocity, and the projectile was fixed. This was done for convenience and efficiency, since it immediately gives the flow in the band due to the interaction, rather than centering the flow about the projectile velocity. This arrangement also corresponds to a reverse ballistic laboratory test (in which the band is mounted on rigid rod, and a heavy ring simulating the barrel is projected against the band), which could be used to confirm the validity of the code solutions.

B. MATERIAL PROPERTIES

A material of current interest, Nylon 6/12, was chosen for the plastic band. The following initial value properties were employed in the code solution for this material:

- Density, \( \rho_0 = 1.08 \text{ gm/cm}^3 \)
- Young's modulus, \( E = 300 \text{ ksi} = 20.7 \text{ kb} \)
- Poisson's ratio, \( \nu = 0.4 \)

These values imply the following other properties:

- Bulk modulus, \( K = 500 \text{ ksi} = 34.5 \text{ kb} \)
- Shear modulus, \( G = 107 \text{ ksi} = 7.4 \text{ kb} \)
- Dilatational wave speed, \( c_p = 6640 \text{ ft/sec} = 2026 \text{ m/sec} \).
The equation of state was assumed to be linear elastic. An elastic-plastic constitutive model was used, employing the von Mises yield criterion and a non-associated flow rule. The value of yield strength used was 16.4 ksi (1.13 kb). Tensile stress was limited by imposing a minimum value for hydrostatic tension, \( P_{\text{min}} = -2.93 \) ksi (-.202 kb). Under uniaxial stress, this gives a maximum permissible tensile stress of 8.8 ksi (.61 kb), corresponding to the tensile strength of the material.

The steel projectile and barrel were treated as rigid bodies. The plastic band was assumed to be locked to the projectile all along the mating interface.

In the Case 1 analysis, shearing forces due to friction were generated at the band/barrel interface, in addition to the normal forces developed as the band is deformed. A nominal constant value for the coefficient of friction of 0.1 was used. Cases 2 and 3 were run with no friction, both for comparison and to provide a lower bound on the shear deformation of the band surface. (The solution results from Case 1 indicated that the frictional effects resulting from a friction coefficient of 0.1 were excessive.)

**C. Numerical Method**

The WAVE-L code was employed for these calculations. WAVE-L is a two-dimensional code which solves the equations of motion for elastic-plastic bodies by means of a finite-difference Lagrangian-cell technique. The mathematical formulation is basically the same as that described by Wilkins. An important feature of the code is its provision for sliding interfaces. (In a normal Lagrangian grid, the cells are locked together, and no sliding can occur.)
A sliding interface was used for the band/barrel interface. Another important capability of WAVE-L is its ability to treat moving rigid bodies (in this case, the barrel). The code computes the loading forces on the rigid body and thus the deceleration can be included in the calculations, given the mass of the rigid body. This rigid-body formulation was previously developed to treat the penetration dynamics of projectiles. Some additional development of the code was required to handle the plastic band interaction geometry.

D. NUMERICAL SOLUTION OF CASE 1 - PLASTIC BAND INTERACTION WITH FRICTION

The initial Lagrangian grid configuration set up for the problem described above is shown in Figure 8. The plastic band was resolved with 8 cells across its width and 53 cells along its length. One hundred six lattice points were used along the barrel to compute the distribution of stresses and force components acting on the barrel.

The code results of this first case, which included frictional effects, are depicted in Figures 9 to 12, which are plots of the grid configuration and velocity field in the band for times of 1.5 and 3.6 usec.

The plots of the grid also show which material is currently undergoing plastic deformation; i.e., is on the yield surface. This is denoted by cells containing an \( x \) or \( + \), with \( x \) indicating a compressive pressure, \( P > 0 \), and \( + \) indicating hydrostatic tension, \( P < 0 \).

The velocity vector field plots show the direction and magnitude of the particle velocity at each lattice point in the computing grid.
The results show that a strong upward axial flow is induced in the band next to the contact surface. Note that the band surface appears to be dragged along by the barrel, causing severe distortion of the material next to the band. The frictional forces built up across the interface were sufficient to lock a portion of the nylon surface onto the barrel, so that the surface was actually moving upward at the full barrel velocity. This clearly shows that friction is important, and indicates that frictional properties under high stress, high velocity conditions need to be defined. However, we consider the results obtained in Case 1 to be an exaggeration of the actual frictional effects. Since there is a significant heating of the surface due to friction and plastic deformation, it is likely that the surface material softens such that it could not support frictional stresses as large as that computed with a coefficient of friction of 0.1. Thus we decided to discontinue the running of Case 1 and to instead run a second case, with no friction.

Additional results of Case 1 are given in the next section in comparative plots.

E. NUMERICAL SOLUTION OF CASE 2 - PLASTIC BAND INTERACTION WITHOUT FRICTION

The problem conditions for Case 2 were kept exactly the same as for Case 1, except that the band/barrel interface was assumed to be frictionless. Thus no shearing forces are generated at the interface.
The results of the code solution for this case are depicted in Figures 13 to 14, which show plots of the grid configuration and/or velocity field for times of 1.5, 3.1, 7.9, and 11.4 μsec. Note that the early time flow is now radially inward, as opposed to the strong axial flow seen in the case with friction. At later times, however, a strong axial flow occurs, as the upper end of the band is extruded into the open cone.

By 7.9 μsec, the extrusion action at the rear of the band had produced such severe grid distortion that the solution could not be continued. To allow continued integration, a rezone of the grid was performed. Rezones are used to reposition the computational grid in a distorted region so as to give more regular cell shapes; a comprehensive rezone processor correctly redistributes the cell variables among the new cells. Code modifications required to treat rezones of the plastic band problem were developed and checked out for this first rezone and will be available in the future as needed.

Following the rezone, the solution was continued out to a time of 11.4 μsec. The extrusion action noted earlier in the solution continued in this later phase of the solution, causing formation of a pronounced annular lip protruding out from the rear of the band. The presence of such a lip would presumably degrade the aerodynamic stability of the bullet. The solution was terminated at this point, since the behavioral pattern of this band design was clear (for this material) and because the axial forces were significantly diminished. An enlarged view of the extruded rear section of the band at the end of the solution is shown in Figure 20. As the projectile moves
further into the converging forcing cone, this extruded annulus would become thinner and longer.

Comparative time histories of the axial and radial forces acting at the band/barrel interface for Cases 1 and 2 (with and without friction), are shown in Figures 21 and 22. Figure 21 shows the increase in axial force at early times caused by friction. However, by 5.5 usec, the axial force in the case without friction surpasses that with friction. Significantly higher normal stresses build up in the case without friction, since the band material next to the barrel does not flow up (axially) as much, and is thus subjected to greater radial compression by the forcing cone. Complete time histories of the axial and radial forces for Case 2 are shown in Figures 23 and 24. The peak axial force was 840 lb, occurring at 5.6 usec, and the peak radial force was 1670 lb/rad, occurring at 6.7 usec. At 11.4 usec, at the end of the solution, the axial force had dropped off to 13% of its peak value.

Stress distributions on the forcing cone are shown in Figures 25 and 26 for the two cases.

F. NUMERICAL SOLUTION OF CASE 3 - MODIFIED PLASTIC BAND DESIGN

The solution results of Case 2 indicated that the band distortion was concentrated at the junction of the 45° beveled rear section and the projectile. For the purpose of alleviating the severe distortion occurring at the rear of the band, it was decided to evaluate the behavior of a modified design, having a longer, more gradual sloping rear section. The modified design is shown in Figure 27. It has a long, 10° sloping rear section replacing the 45°
beveled section of the original design. The flat section and the 6° sloping forward section making initial contact with the forcing cone were kept the same as in the baseline design so as to isolate the effect of the modified rear section. The front of the band ahead of the start of the forcing cone was shortened, since it has little effect on the problem dynamics. The overall length of the modified band was .32 in., compared with .28 in. for the baseline design.

The initial Lagrangian grid for the code solution of Case 3 is also indicated in Figure 27. The zoning was kept the same as in the previous problem; i.e., 8 cells across the width of the band. Fifty-nine cells were used along its length.

The results of the Case 3 solution were very interesting: with the long, shallow-sloping rear section, the response of the band was much more regular, smoothly bowing out as it was forced back. The solution was carried out to 27.6 μsec, by which time the axial relative displacement of the band/barrel was .166 in. As an indication of the uniform deformation behavior, no rezones of the grid were required during this period. (In the previous problem, severe distortion necessitated a rezone at 7.9 μsec.) A sequence of the band/barrel configurations and/or velocity fields for times of 7.7, 15.5, 20.7, and 27.6 μsec are shown in Figures 28 to 33. By 27.6 μsec, some distortion has built up, at the rear of the band, but it is less than seen with the previous design.

Time histories of the axial and radial forces at the band/barrel interface in the modified design (Case 3) are
shown in Figures 6 and 31, plus those for the baseline
and in Fig. 21 for comparison. At early times, the re-
response is seen to be the same, confirming that the geometry
and the initial contact surface was the same between the
problems. The modified design produces a significantly
larger force pulse, corresponding to the increased length
of band behind the forcing tip (.27 in. compared with .17
in.). Also, the band bows out until it reaches the forcing
cone. The resulting high pressure between the band and
the cone produces a large axial force. Assuming a projec-
tile weight of ~4 lb, the peak deceleration imparted to
the projectile due to convergence in the modified design
is ~2000 g's. This deceleration for 20 μsec would reduce
the projectile velocity by 1.3 fps.

Distributions of the normal stress along the band/
substrate interface for several times are shown in Figure 35.

The peak stress levels experienced within the band
during the interaction are indicated in Figure 36. The
mm compressive stresses along the inner and outer edge
about midway across the band vs axial distance are
plotted. Also shown for comparison are the corresponding
lines for the baseline band design.

Additional results from the numerical solutions are
shown in the Appendices.
Projectile and barrel are considered to be rigid materials.

Figure 1. Field of Analysis for Computation of a Projectile/Rotating Band Tunneling into a Potting Cone.
Figure 2. Problem Conditions for Numerical Solutions

CASE 1: Baseline Design,
Material - Nylon 6/12
Friction across nylon-barrel interface ($\mu = 0.1$)

CASE 2: Baseline Design,
Material - Nylon 6/12,
No friction

CASE 3: Modified Design,
Material - Nylon 6/12
No friction

Same dimensions as Case 1, but shape of rotating band is changed
Figure 3. Deformation and Flow in Plastic Band at 11.4 μsec, Case 2
(Baseline design, no friction)
Figure 4. Deformation and Flow in Plastic Band at 7.7 μsec, Case 3 (Modified design)
Figure 5. Deformation of Plastic Band at 20.7 and 27.6 μsec, Case 3 (Modified Design)
Figure 6. Axial Force at Band/Barrel Interface vs Time for Baseline and Modified Plastic Band Designs
Rotation by rifling represented by rotating outer rigid body (barrel) according to specified $w(t)$.

Inner surface is smooth at $t = 0$

Rifling bands impinge into rotating band as specified by $r(t)$.

Friction in $z$-direction specified by $\sigma_z(\sigma_r)$.

Barrel is rigid body.

Rotating band is described by lagrangian grid and by elastic-plastic properties of material.

Figure 7. Suggested Method for Plane Strain Analysis of Rotating Band-Rifling Interaction
Figure 8. Baseline Plastic Band Geometry and Computational Grid (Cases 1 and 2 - Baseline Design)
Figure 9. Grid Configuration at 1.5 usec, Case with Friction (Case 1 - Baseline Design)
Figure 10. Velocity Field at 1.5 μsec, Case with Friction
(Case 1 - Baseline Design)
Figure 11. Grid Configuration at 3.6 μsec, Case with Friction (Case 1 - Baseline Design)
Figure 12. Velocity Field at 3.6 µsec, Case with Friction
(Case 1-Baseline Design)
Figure 13. Velocity Field at 1.5 usec, Case without Friction (Case 2-Gasline Design)
Figure 14: Grid Configuration at 3.1 μsec, Case without Friction (Case 2 - Baseline Design)
Figure 15: Velocity Field at 3.1 usec, Case without Friction (Case 2-Baseline Design).
Figure 16. Grid Configuration at 7.9 μsec, Case without Friction (Case 2 - Baseline Design)
Figure 17. Velocity Field at 7.9 usec, Case without Friction
(Case 2 - Baseline Design)
Figure 18. Grid Configuration at 11.4 μsec, Case without Friction
(Case 2 - Baseline Design)
Figure 19. Velocity Field at 11.4 μsec, Case without Friction (Case 2 - Baseline Design)
Figure 20. Grid Configuration at Rear Section of Plastic Band at 11.4 usec, Case without Friction (Case 2 - Baseline Design)
Figure 21. Comparison of Axial Forces on Barreled with and without Surface Friction (Cases 1 and 2 - Baseline Design)
Figure 22. Comparison of Radial Forces on Barrel with and without Surface Friction (Cases 1 and 2 - Baseline Design)
Figure 23. Axial Force at Band/Barrel Interface vs Time, Case without Friction (Case 2 - Baseline Design)
Figure 24. Radial Force at Bond/Barrel Interface vs Time, Case without Friction (Case 2 - Baseline Design).
Figure 25. Normal Stress ($\sigma_n$) and Tangential Stress ($\tau$) Distributions on Barrel. Case with Friction (Case 1 - Baseline Design)
Figure 26. Normal Stress ($\sigma_n$) Distributions on Barrel, Case without Friction (Case 2 - Gasoline Design).
Figure 27. Modified Plastic Band Geometry and Computational Grid
(Case 3 - Modified Design)
Figure 28. Grid Configuration at 7.7 μsec, Modified Plastic Sand Design  
(Case 3)
Figure 29. Velocity Field at 7.7 usec, Modified Plastic Bond Design (Case 3)
Figure 30. Grid Configuration at 15.3 usec, Modified Plastic Band Design (Case 3)
Figure 31. Grid Configuration at 20.7 μsec, Modified Plastic Band Design (Case 3)
Figure 32. Grid Configuration at 27.6 μsec, Modified Plastic Band Design (Case 3)
Figure 33. Velocity Field at 27.6 usec, Modified Plastic Band Design (Case 3)
Figure 34. Radial Force at Band/Barrel Interface vs. Time for Baseline and Modified Plastic Band Designs
Figure 35. Distributions of Normal Stress along Band/Barrel Interface,
Modified Plastic Band Design
Figure 36. Peak Compressive Stresses within Band; Baseline and Modified Plastic Band Designs (Cases 2 and 3)
APPENDIX A

PRINCIPAL STRESS FIELD PLOTS

In addition to the plots of the grid configuration and the velocity field presented in Section II of the report, plots of the principal stress field occurring at several times during each of the solutions were obtained. A representative selection of these for Cases 1, 2, and 3 are contained in this Appendix.

In the stress field plots, the principal components of the stress tensor for each cell are shown as follows: The magnitude of the two principal stresses in the r-z plane are plotted in their corresponding principal directions. The third principal stress (in the azimuthal direction) is plotted along the line bisecting the other two principal directions. Vectors pointed to the right are compressive, to the left, tensile. An example of how a stress tensor is plotted is sketched below:

The scale of the stress vectors is given at the top of each plot. The plots listed below are given in the following set of figures:

Case 1: 1.5, 3.5 μsec
Case 2: 2.1, 7.9, 11.6 μsec
Case 3: 7.7, 15.5, 27.6 μsec
Figure A-1. Principal Stress Field at 1.5 μsec, Case with Friction (Case 1 - Baseline Design)
Figure A-2. Principal Stress Field at 3.6 μsec, Case with Friction (Case 1 - Baseline Design)
Figure A-3. Principal Stress Field at 3.1 usec, Case without Friction (Case 2 - Baseline Design)
Figure A-4. Principal Stress Field at 7.9 μsec, Case without Friction
(Case 2 - Baseline Design)
Figure A-5. Principal Stress Field at 11.4 usec, Case without Friction (Case 2 - Baseline Design)
Figure 4-6. Principal Stress Field at 7.7 μsec; Modified Plastic Band Design  
(Case 3)
Figure A-7. Principal Stress Field at 15.5 usec, Modified Plastic Band Design (Case 3)
Figure A-8. Principal Stress Field at 27.6 μsec, Modified Plastic Band Design (Case 3)
APPENDIX B
TIME HISTORIES OF STRESS AND VELOCITY

Parameter-time histories were recorded at several locations in the plastic band. The locations of the stations are shown in Figure B-1 for Cases 1 and 2 and in Figure B-2 for Case 3. The stations for Case 3 were placed at analogous positions to those used in the first design so that the response of the two bands could be compared. The following parameters are plotted:

a. Radial stress \( \sigma_r \)
   Axial stress \( \sigma_z \)
   Hoop stress \( \sigma_0 \)
   Shear stress \( \sigma_{rz} \)

b. Radial particle velocity \( \dot{r} \)
   Axial particle velocity \( \dot{z} \)

Positive stresses are compressive. Positive \( \dot{r} \) is radially outward. Positive \( \dot{z} \) is axially upward (in the direction of the barrel velocity).

The first set of figures (B-3 to B-9) are comparative plots of the response for Cases 1 and 2 (the basic plastic band geometry with and without friction) at selected stations. The previously noted "locking-on" of the surface of the band to the barrel due to friction during the Case 1 solution is clearly shown in Figure B-8, the velocity plot at Station 8. The second set of figures (B-10 to B-19) are comparative plots from Cases 2 and 3 (the basic and modified plastic band geometry). Note that the extrusion action that occurred at Station 9 in the original design did not occur with the revised design (Figure B-19).
Figure B-1. Locations of Parameter-Time Stations (Cases 1 and 2-Baseline Design)
Figure 5-2. Locations of Parameter-Time Stations, Modified Plastic Sand Design (Case 3)
Figure B-3. Stress Components at Station 2 - Baseline Design
Figure B-4. Stress Components at Station 5 - Baseline Design
Figure 3-5. Velocity Components at Station 5 - Baseline Design
Figure B-6. Velocity Components at Station 7 - Baseline Design
Figure B-7. Stress Components at Station 8 - Baseline Design
Figure B-8. Velocity Components at Station 8 - Baseline Design
Figure B-8. Velocity Components at Station 9 - Baseline Design
Figure B-10. Stress Components at Station 2, Baseline and Modified Plastic Bond Designs
Figure B-11. Stress Components at Station 3, Baseline and Modified Plastic Beam Designs.
Figure B-12. Stress Components at Station 4, Baseline and Modified Plastic Band Designs
Figure B-13. Stress Components at Station 5, Baseline and Modified Plastic Bend Designs
Figure 5-14. Stress Components at Station 6, Baseline and Modified Plastic Bond Designs.
Figure 3-15. Velocity Components at Station 6. Baseline and Modified Plastic Band Designs
Figure E-16. Stress Components at Station E: Baseline and Modified Plastic Wind Designs
Figure B-17. Velocity Components at Station 8: Baseline and Modified Plastic Band Designs.
Figure B-18. Stress Components at Station 9, Baseline and Modified Plastic Band Designs
Figure B-19. Velocity Components at Station 9, Baseline and Modified Plastic Band Designs.
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