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THROW-OFF OF PROJECTILE AT MUZZLE

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Several analyses of the yawing motion of projectiles in the gun and/or during exit from the gun muzzle are examined for their underlying assumptions and the results are discussed. Results of simple, one-dimensional computations at the muzzle are presented parametrically for the muzzle exit conditions of 155 mm projectile M483 and its modifications at zone 4. The results for the basic design are compared with an approximate two-dimensional solution.

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INTRODUCTION

The launch process for projectiles actually begins far behind the muzzle, perhaps even at the origin of rifling. If we assume that the lands and grooves form a helix and are of constant diameter, it is at the origin of rifling that the initial condition for the orientation of the front bourrelet is applied. If there is any clearance, the front bourrelet may not be centered in the tube, and may even rest on the tube wall. The initial deformation of the rotating band takes place at the origin of rifling, although it continues to deform and wear as the projectile travels down the tube. The projectile can be centered in the tube, front and rear, and not experience any malalignment in-bore or at launch.

Unfortunately, the purely axisymmetric case does not appear very likely, and we assume that some perturbing influence will be felt. This leaves us with several possibilities: the projectile could return to the centered position, or it could oscillate without the front bourrelet touching the lands; it could rebound from the lands repeatedly, or lie relatively quiet on the bore, either on a single land or sliding across the lands while sliding down the tube. Furthermore, the rotating band need not engrave symmetrically.

At the muzzle, the front bourrelet is suddenly released (if it were constrained by the lands) while the rotating band is still engaged, causing the component of total spin parallel to the tube axis to be geometrically fixed to the tube. In addition, neither the tube nor the projectile is rigid under these forces; both are flexible, even yielding.

A literature search was undertaken to determine the methods available to calculate the effect of the launch process on the initial yaw and yaw rate of projectile M483. This was done because existing data on the aerodynamic coefficients of the projectile led to the prediction of divergent oscillations in yaw for large initial yaw rates and convergent oscillations for moderate initial yaw rates. It was therefore necessary to make reasonable engineering estimates of the initial yaw rate at the muzzle.

DISCUSSION

The earliest of the analyses uncovered in the literature search is that of Reno (Ref 1) who makes an assumption which effectually locks the plane of yaw to its original direction, rotated by the twist multiplied by the distance traveled to any point in time. He then goes through an extremely general derivation of the Euler equations describing the motion in the tube and, of course, predicts one dimensional angular motion. He does allow for a "bounce" of the forward bourrelet as it hits a land.

An analysis by Thomas (Ref 2) relaxes the requirement that the yaw be constrained to remain in the rotating, original plane, but he maintains the constraint that the rotating band be centered in the tube. Thomas predicts a precession of the symmetry axis of the projectile in a direction opposite to the spin. He states that, for normal projectiles, the effect of clearance on this precession in the tube is to reduce the effective yaw somewhat (Ref 2). He further states that sufficiently great friction between the bourrelet and the lands can cause this precession to be substantial, giving a large effective initial yaw. Without measurements of friction or precession, there is no basis for choosing the magnitude of the friction force. L. C. MacAllister of BRL reports (Ref 3) that the precession rate of artillery is not as great as that of rockets. Rockets fit more loosely in the launching tubes, and their lengths, measured in calibers, are much greater than those of artillery projectiles. A good comparison of Thomas and Reno is provided by Gay (Ref 4), who covers both the analytics and the computational results.

In 1959 a translation (Ref 5) by Hitchcock of Darpas' 1957 publication appeared. In this treatment, the rotating band is not necessarily centered in the tube, and the bourrelet may slide on the lands. It is a two degree-of-freedom analysis (allowing precession), but is restricted to zero static and dynamic imbalance. Furthermore, there are errors which Hitchcock points out (Ref 5), but does not carry through.

Darpas is attempting to write the equation of motion for the center of the projectile's mass by using the normal force, N, as the centripetal acceleration causing the center of mass to describe a circle with uniform speed. For this he needs the radius of the circular motion, OG. We agree with Hitchcock's corrections and have carried them through to a conclusion. In doing so, an approximation is also made to extend the analysis to include the effect of static imbalance. The lever arm of the offset center of gravity (cg) was included in the equations of motion, but the concomitant dynamic inbalance and inertia tensor changes were ignored.

Figure 1 illustrates the situation with the offset cg designated "ε."

Thus, it can be seen that for a balanced projectile, Darpas should have used (when $\sin \delta \approx \delta$, $\cos \delta \approx 1$)

$$OG = \lambda_2 \delta - \varepsilon_2/2 \tag{1}$$

in agreement with Hitchcock's correction. Furthermore, the correct arm length is OG not CG, again, in agreement with Hitchcock. In order to approximately extend the analysis to a cg offset, the arm length is taken as OG´. Again making the small angle approximation,

$$OG' = \lambda_2 \delta - \varepsilon_2/2 + \varepsilon. \tag{2}$$

It is emphasized that this does not include the dynamic imbalance or the cross-coupling that now results from the complete moment of inertia tensor.

The result of this limited extension and Hitchcock's correction is to redefine the geometric constant, k, (Ref 5), and the initial condition, δ_0. The old definition of k was

$$k = \frac{2(1 + \lambda_1/\lambda_2)}{1 + \varepsilon_1/\varepsilon_2} . \tag{3}$$

The new definition is

$$k = \underbrace{\frac{1 + \lambda_1/\lambda_2}{1 + \varepsilon_1/\varepsilon_2}}_{\text{Hitchcock's correction}} + \underbrace{\frac{\varepsilon}{\lambda_2 \delta_0}}_{\text{extension}} \tag{4}$$

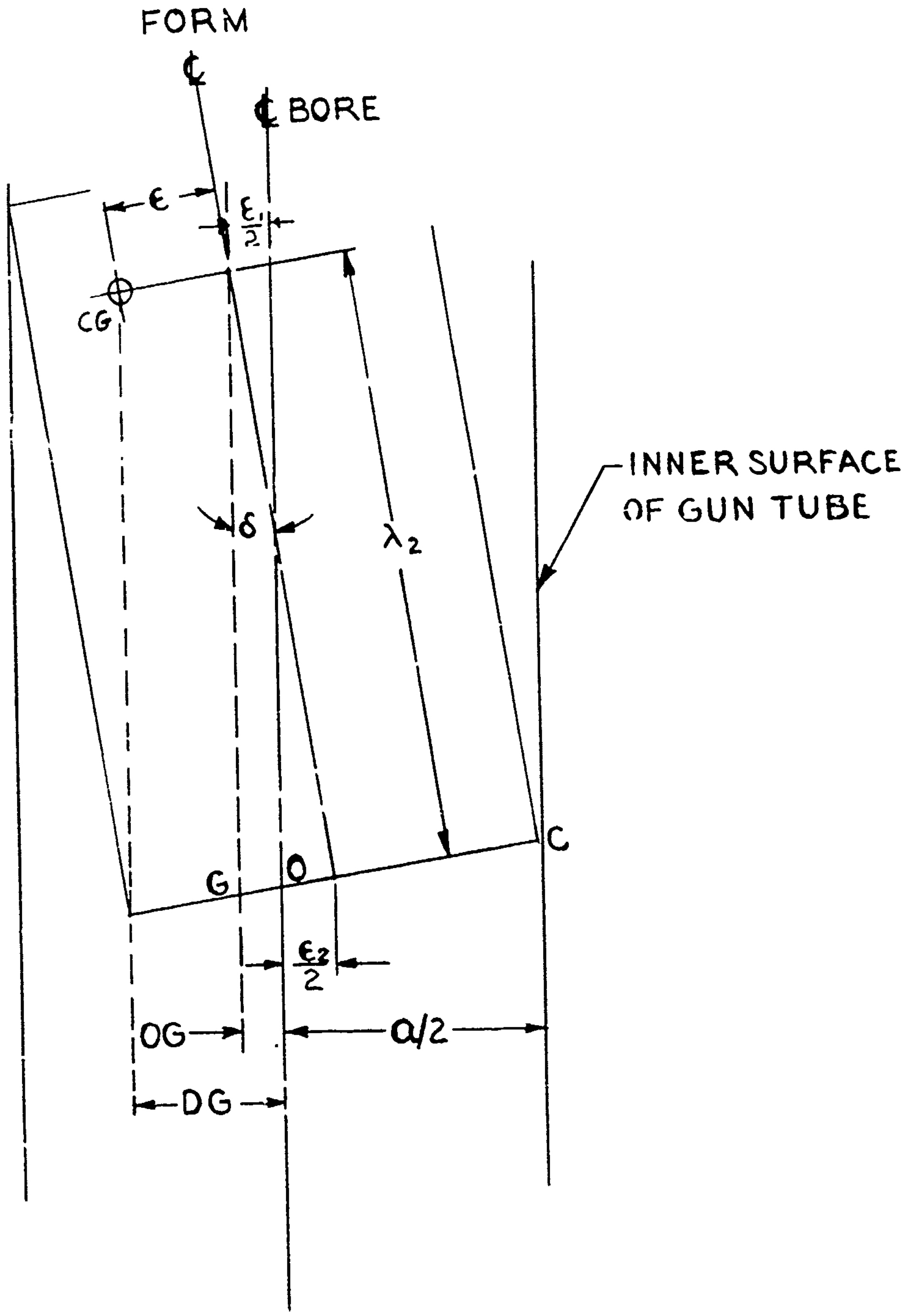

Fig 1 Schematic of in-bore geometry used by Darpas

where now

$$\delta_0 \equiv \frac{\varepsilon_2 \ (1 + \varepsilon_1/\varepsilon_2)}{\lambda_2 \ (1 + \lambda_1/\lambda_2)} \to \frac{\varepsilon}{\lambda_2} \tag{5}$$

Examination of the physical significance of the above changes shows that the mathematically undefined portion of the equation (as $\delta_0 \to 0$) has become well defined. The torques about the contact point (Fig 1, C) move the projectile to the opposite side of the tube and produce the same situation as in the illustration, but with the projectile cocked in the other direction.

The other physical bounds seem to be $\varepsilon_2 = 0$, corresponding to an exact or an interference fit between the rotating band and the rifling, and $\varepsilon_1 = \varepsilon_2$, which is the situation if the rotating band has been worn down during its travel and the clearances are the same at the front and rear bourrelets.

Then, using the corrected and extended equations from Darpas, the throw-off rate is

$$\dot{\delta} = 2\gamma t \tag{6}$$

the throw-off angle is

$$\delta = \delta_0 + 1/2 \ \dot{\delta} t \tag{7}$$

and the precession rate is

$$\dot{\phi} = N + 3\beta t^2 \tag{8}$$

with

$$\beta = \frac{N^3}{6} \left\{ \frac{I_z + m\lambda_2^2 \ (k-1) - I_x}{I_z - m\lambda_2^2 \ (k-1)} \right\} \frac{I_x + 2m\lambda_2^2}{I_z + m\lambda_2^2} \tag{9}$$

and

$$\gamma = \frac{-N^2 \delta_0}{2} \left\{ \frac{I_z - m\lambda_2^2 \ (k-1) - I_x}{I_z + m\lambda_2^2} \right\} \tag{10}$$

where k has the new value developed.

In the case of $\varepsilon_2 = \varepsilon_1$, the predicted precession rate (sometimes called "counter-roll") is small and negative for small static imbalances, but increases to rather large negative values for large offsets (see Fig 2). It is not clear at this time how much of this is due to the failure of the analysis to use the complete inertia tensor and how much to Darpas' expansion solution of the ordinary differential equations, but it would seem that, if the larger values were real, they would have been detected experimentally, and they have not.

Gays's analysis (Ref 6) has essentially the same constraint as Reno's, that is, that the plane of yaw turns with the rifling. However, it is presented in a simple, understandable manner, quite correctly as a one-dimensional problem. Its only difficulty, shared with other analyses, is that no account is taken of dynamic imbalance.

The most recent report reviewed was by Chu and Soechting (Ref 7). They numerically integrated the equations of motion, essentially under Thomas' constraints, but made the friction between the bourrelet and the lands zero. Their work adds to the previous models the mechanics of the bounce of the bourrelet-land interaction. These authors also predict a somewhat large precession of the plane of yaw for a normal projectile.

It appears, with our idealized models of the problem, that if the projectile is free to precess in the tube it will be predicted to do so. However, there appears to be no uniformity as to the conditions under which it will precess. Furthermore, no experimental evidence of this precession or counter-roll is apparent for normal projectiles.

In the absence of a compelling reason to use a more complex analysis, we prefer using a simple calculation like that of Gay, but extended to include the effects of dynamic imbalance during projectile release. This provides a conservative answer by lining up the three effects (static imbalance, dynamic imbalance and yaw in the gun) and by using N as the rate at which the direction of the plane of yaw changes while the rotating band is still in the gun.

To verify this approach, we applied the corrected and extended analysis of Darpas and compared the results with Gay. The simple analysis of Reference 6 has been expanded as follows.

If a pin on the gun axis represents the operation of the rotating band during the time between the exit of the bourrelet and the rotating band from the tube, we define δ as the angle between the gun axis and a line from the effective pin to the center of mass of the

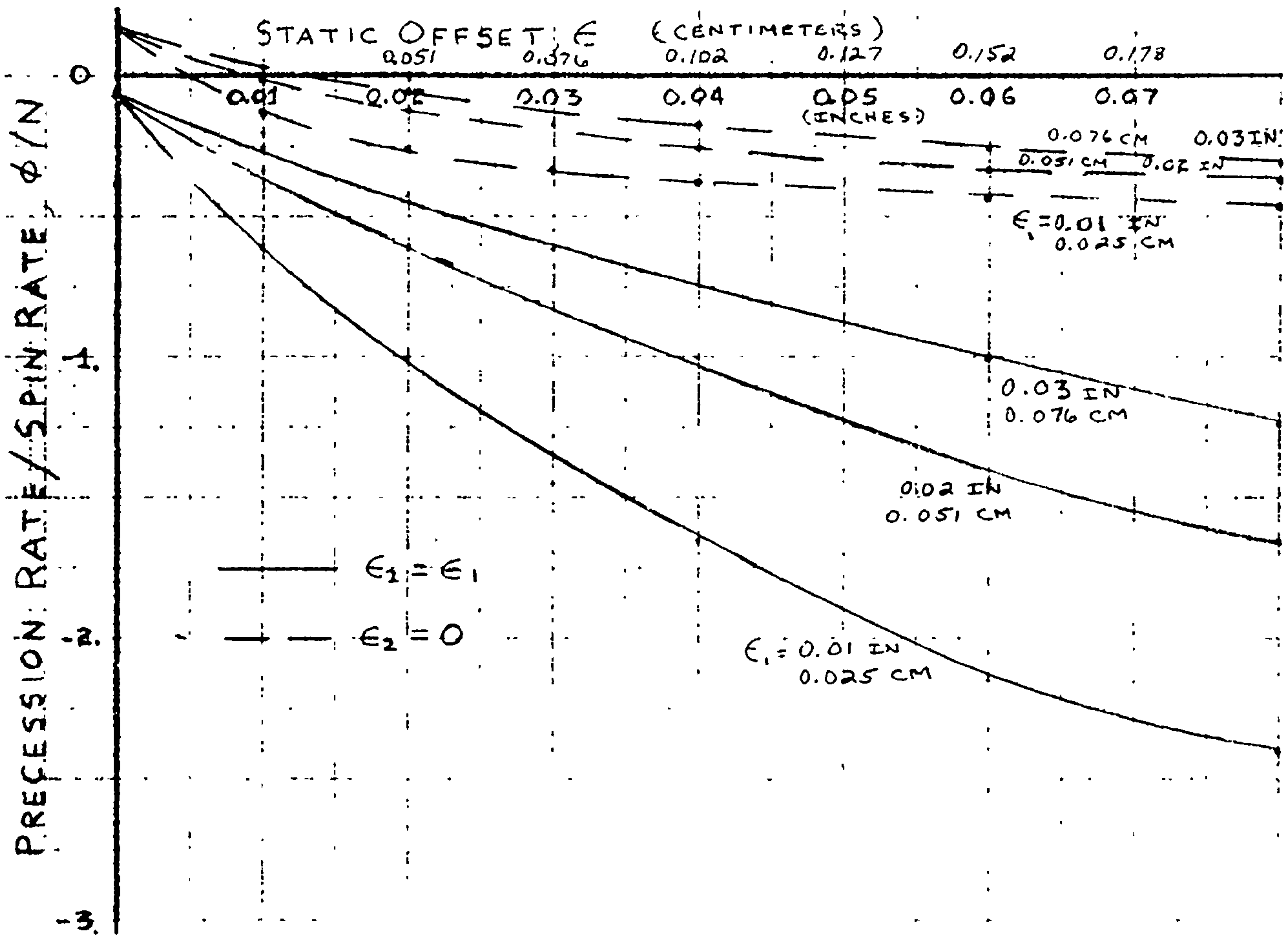

Fig 2 Precession rate of M483 as a function of bore clearance and static imbalance

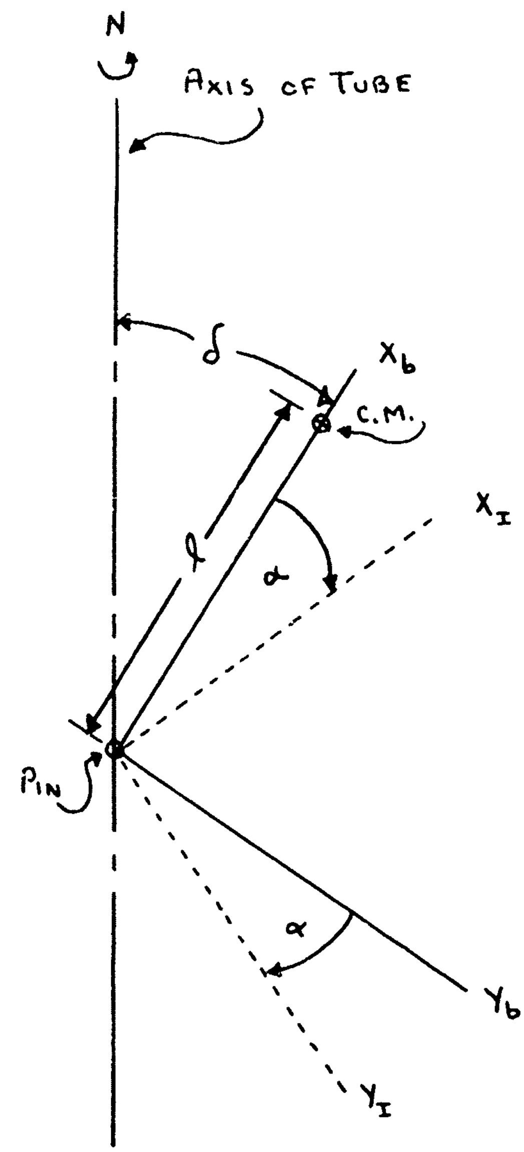

Fig 3 Schematic of axis systems for extension of Gay's analysis

projectile, regardless of the position of the projectile's center of form. We call this line x_b; y_b is normal to it and in the plane of yaw. We then assume that the principal axes of inertia are not in the x_b, y_b directions, but in the directions x_I, y_I, rotated by angle α from x_b, y_b.

In the body frame, the moment of inertia is:

$$\bar{\bar{I}}_b = \begin{pmatrix} I_x \cos^2 \alpha + I_z \sin^2 \alpha & (I_z - I_x) \sin \alpha \cos \alpha & 0 \\ (I_z - I_x) \sin \alpha \cos \alpha & I_z \cos^2 \alpha + I_x \sin^2 \alpha & 0 \\ 0 & 0 & I_z \end{pmatrix} \tag{11}$$

There is no potential energy if we neglect gravity, and the kinetic energy is

$$T = \frac{m}{2} \left\{ (\ell \sin \delta)^2 N^2 + (\ell \dot{\delta})^2 \right\} + \frac{1}{2} \bar{\Omega}_b^{\dagger} \bar{\bar{I}}_b \bar{\Omega}_b \tag{12}$$

where $\bar{\Omega}_b \equiv \begin{pmatrix} N \cos \delta \\ -N \sin \delta \\ \dot{\delta} \end{pmatrix}$ and $\dagger$ denotes transpose.

Then the Lagrangian, L, is equal to the kinetic energy, and writing out the components we have

$$\begin{aligned} L = T = \frac{m}{2} \left\{ \ell^2 \sin^2(\delta) N^2 + \ell^2 \dot{\delta}^2 \right\} + \frac{1}{2} \Big\{ (I_x \cos^2 \alpha + \\ I_z \sin^2 \alpha) N^2 \cos^2 \delta + (I_z \cos^2 \alpha + \\ I_x \sin^2 \alpha) N^2 \sin^2 \delta + I_z \dot{\delta}^2 - 2 (I_z - I_x) \\ \sin \alpha \cos \alpha \sin \delta \cos(\delta) N^2 \Big\} . \end{aligned} \tag{13}$$

Writing Lagrange's equation

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\delta}} \right) - \frac{\partial L}{\partial \delta} = 0 \tag{14}$$

Then

$$(I_z + m\ell^2)\,\ddot{\delta} - \Big\{\sin^2\delta\,(I_z - I_x)\sin\alpha\cos\alpha + \tag{15}$$

$$\sin\delta\,[m\ell^2 + (I_z - I_x)\cos^2\alpha + (I_x - I_z)\sin^2\alpha]\sin\delta\cos\delta +$$

$$(I_x - I_z)\sin\alpha\cos\alpha\cos^2\delta\Big\}\,N^2 = 0$$

It can be noted that if $\alpha = 0$, we obtain

$$(I_z + m\ell^2)\,\ddot{\delta} - \Big\{N^2\,(m\ell^2 + I_z - I_x)\sin\delta\cos\delta\Big\} = 0 \tag{16}$$

which is Gay's result, as it should be.

Since typical artillery projectiles have maximum dynamic imbalance angles of 10^{-2} radians, we make a small angle approximation

$$\alpha << 1 \longrightarrow \sin\alpha \approx \alpha,\ \cos\alpha \approx 1$$

and then from Equation 15

$$(I_z + m\ell^2)\,\ddot{\delta} - N^2\Big\{\sin^2\delta\,(I_z - I_x)\,\alpha + \sin\delta\cos\delta\,\cdot \tag{17}$$

$$[m\ell^2 + (I_z - I_x) - (I_z - I_x)\,\alpha^2] + (I_x - I_z)\,\alpha\cos^2\delta\Big\} = 0$$

We may safely neglect $(I_x - I_z)\,\alpha^2$ with respect to $(I_z - I_x)$. Rearranging, we get

$$(I_z + m\ell^2)\,\ddot{\delta} - N^2\Big\{\sin\delta\cos\delta\,[m\ell^2 + (I_z - I_x)] + (\sin^2\delta - \cos^2\delta)(I_z - I_x)\,\alpha\Big\} = 0 \tag{18}$$

which differs from Gay's analysis by the last term.

We linearize for $\delta << 1$, $\sin\delta \approx \delta$, $\cos\delta \approx 1$ yielding

$$\ddot{\delta} - N^2 \left\{ \frac{\delta(I_z+m\ell^2-I_x)}{I_z+m\ell^2} \right\} + \frac{(I_z-I_x)\alpha N^2}{I_z+m\ell^2} = 0 \qquad (19)$$

which has the solution

$$\delta = \left\{ \delta_0 - \frac{(I_z-I_x)\alpha}{(I_z-I_x+m\ell^2)} \right\} \cosh \left\{ \sqrt{\frac{I_z-I_x+m\ell^2}{I_z+m\ell^2}}\, Nt \right\} + \qquad (20)$$

$$\frac{I_z-I_x}{I_z-I_x+m\ell^2}\ \alpha$$

and, of course,

$$\dot{\delta} = \left\{ \delta_0 - \frac{(I_z-I_x)\alpha}{(I_z-I_x+m\ell^2)} \right\} \sinh. \left\{ \sqrt{\frac{I_z-I_x+m\ell^2}{I_z+m\ell^2}}\, Nt \right\} \cdot \qquad (21)$$

$$\sqrt{\frac{I_z-I_x+m\ell^2}{I_z+m\ell^2}}\ N$$

both as a function of time.

To apply the results to the 155 mm, M483 projectile we use the following nominal description of the projectile:

Transverse moment of inertia	I_z	6,250 lb-in.^{2} (1.83 kg-m^2)
Axial moment of inertia	I_x	527 lb-in.^{2} (0.154 kg-m^2)
Weight	m	103 lb (458 N)
Distance cg to rotating band (uncocked)	ℓ	10 in. (0.25 m)
Launch velocity (zone 4)	V	997 ft/sec (304 m/sec)
Spin rate (zone 4)	N	618 rad/sec
Time, bourrelet release to band release	T	.00113 sec

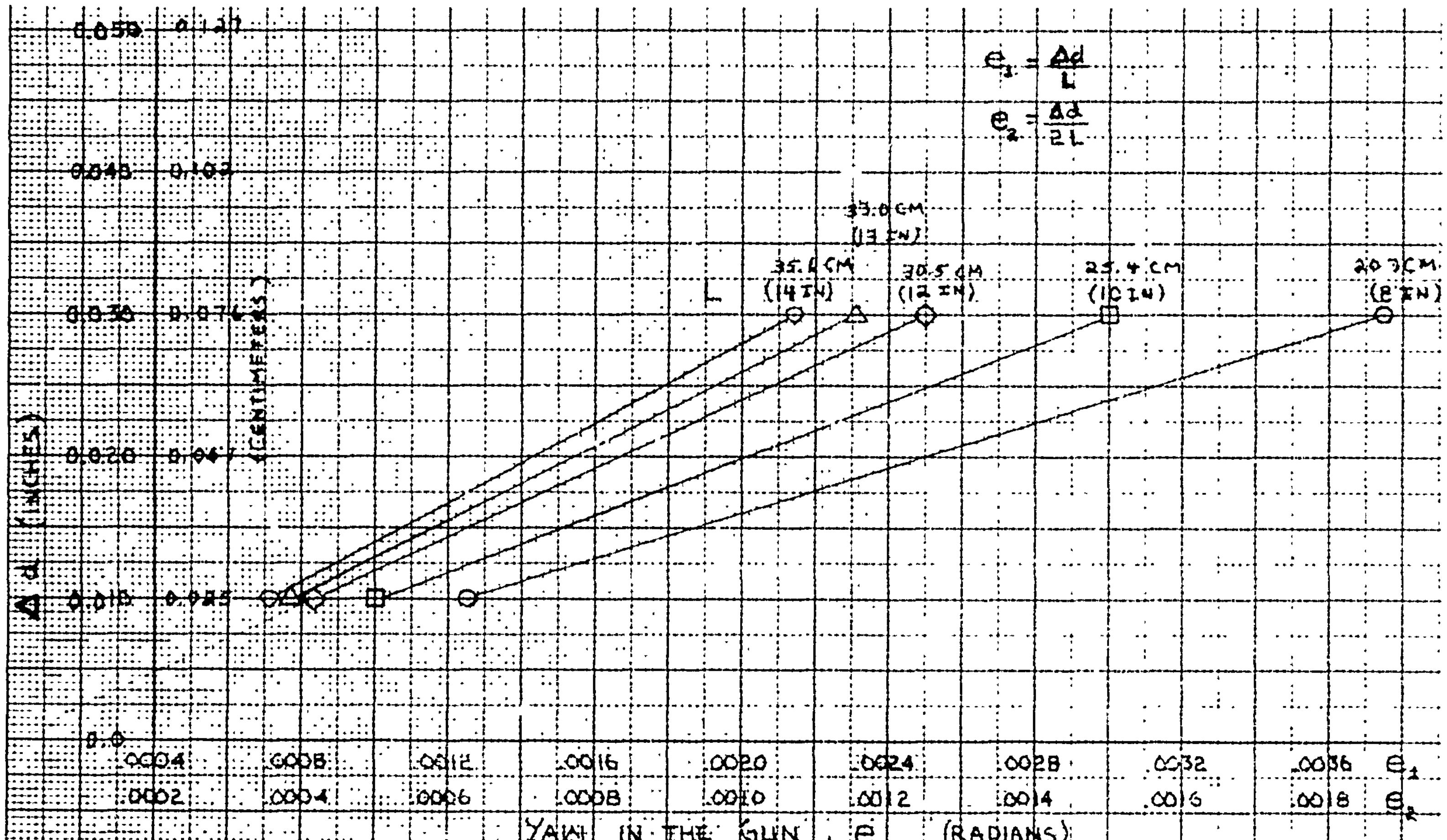

Fig 4 M483 yaw in the tube

For initial conditions, assume that the projectile is cocked in the tube to the limit of the diametral clearance at angle e; and that the static imbalance is ε. Figure 5, which is much exaggerated, illustrates this situation. (The yaw in the gun is given by $e = d/2\ell$ when the rotating band is centered, and by d/ℓ when it rides against one land. The variations with typical values for the M483 are shown in Figure 4.)

Then an angle, β, can be defined between the tube centerline and the center of mass as

$$\beta = e + \varepsilon/\ell \qquad (22)$$

which is the initial condition, δ_0.

Finally, referring to Figure 3, the pin rotates about the axis of the tube at the spin rate, N. The cross spin, the total spin component normal to the projectile's longitudinal axis of form, X_b, at rotating band exit is

$$\Omega = \sqrt{(N \sin \delta_f)^2 + \dot{\delta}_f^2} \qquad (23)$$

The f subscript denotes final conditions, at time T.

Figure 6 shows the results: cross spin of the M483 at zone 4 at the time the rotating band releases, plotted parametrically versus the dynamic imbalance and β, a linear combination of yaw in the tube and a static imbalance angle, ε/ℓ.

For the M483 projectiles whose balance was measured on 4 June 1974 at Aberdeen Proving Ground, the mean dynamic imbalance (α) was found to be .000278 radians and the mean static imbalance (ε) to be 0.0232 cm (0.00913 inches). If the projectiles measured 9 July 1974 at Materiel Test Directorate, APG (Lot LS-DX-3807) are compared to the inside diameter of the M126 Tube No. 20007 at the muzzle after only 184 rounds, the front bourrelet proves to be 0.25 cm (0.1 in.) smaller in diameter than the tube. Therefore, assuming the band is snug in the tube,

$$\beta = \frac{\text{Diametral clearance}}{2\cdot\text{dist fr bour to rot band}} + \frac{\varepsilon}{\ell}$$

$$= \frac{0.01}{2.(13.5)} + \frac{.00913}{10} \approx .0012$$

Under the assumption that the static imbalance, dynamic imbalance and yaw in the gun are coplanar, at least sometimes, Figure 6 can be used to predict a cross spin of approximately one radian per second from the mean values of imbalance, a substantial but not alarming figure. However, that same measured sample of M483 projectiles had a maximum α of .0074 radian and a maximum ε of 0.030 cm (0.012 in.). It is not hard to imagine slightly eccentric rotating band engraving allowing the rear bourrelet to touch the opposite wall, doubling the yaw in the tube, or additional wear increasing clearance much more. Applying these numbers to Figure 6, slightly less than three radians per second can be predicted (without allowing for engraving of the bourrelet or expansion of the tube under pressure) when all the effects are coplanar.

For comparison, the corrected and extended analysis of Darpas was used to produce the results shown in Figure 7, which shows that the case of equal front and rear clearances produces the highest rates. The variables used are different than those used in Gay's method, but the variables in the two methods can be related to one another. When this is done for zero dynamic imbalance, the cross-spin predicted by Darpas' method is somewhat smaller than that predicted by Gay. This is predictable since some of the energy is going into the extra degree of freedom in Darpas' formulation, which can only reduce the yaw rate produced.

Either method predicts possible total initial rates that are in the critical region for the M483 at zone 4 (about 3 rad/sec) for the clearances and imbalances that have been measured for the M483 system, when the various effects are aligned so that they add, rather than cancel.

In order to evaluate the effects of the modifications being made to the M483 projectile, the total initial rates were calculated by Gay's method for the modifications tabulated below. This method gives approximately the same results for our conditions and is much easier to understand and use than Darpas' method.

Since the small effect of dynamic imbalance is evident in Figure 6, the calculations were made for $\alpha = 0$. They were also made for a nominal diametral bore clearance of 0.051 cm (0.020 in.) in order to show the effect of varying wheelbase on the yaw in the gun. The difference in cross-spin at launch between the standard round and the particular modification is shown as a function of static imbalance in Figure 8.

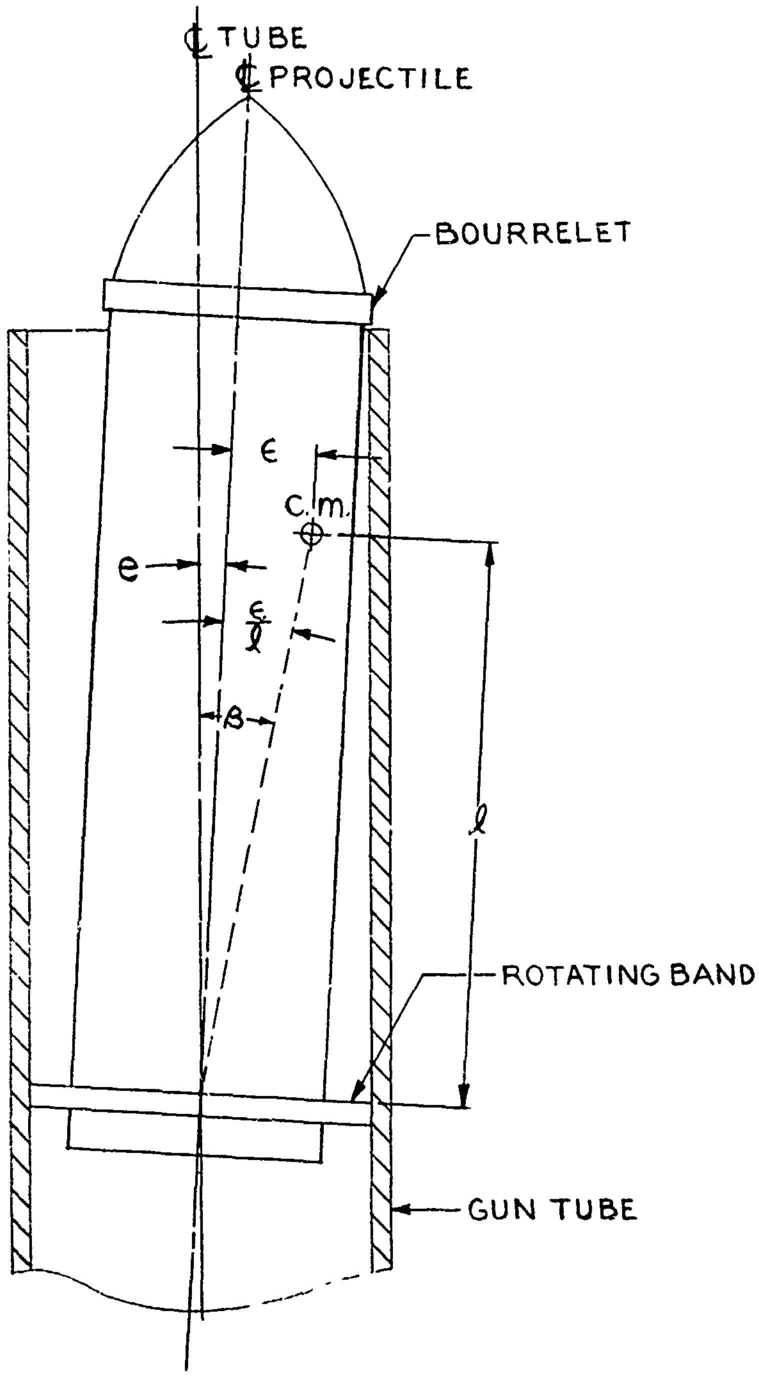

Fig 5 Projectile in the tube at bourrelet release

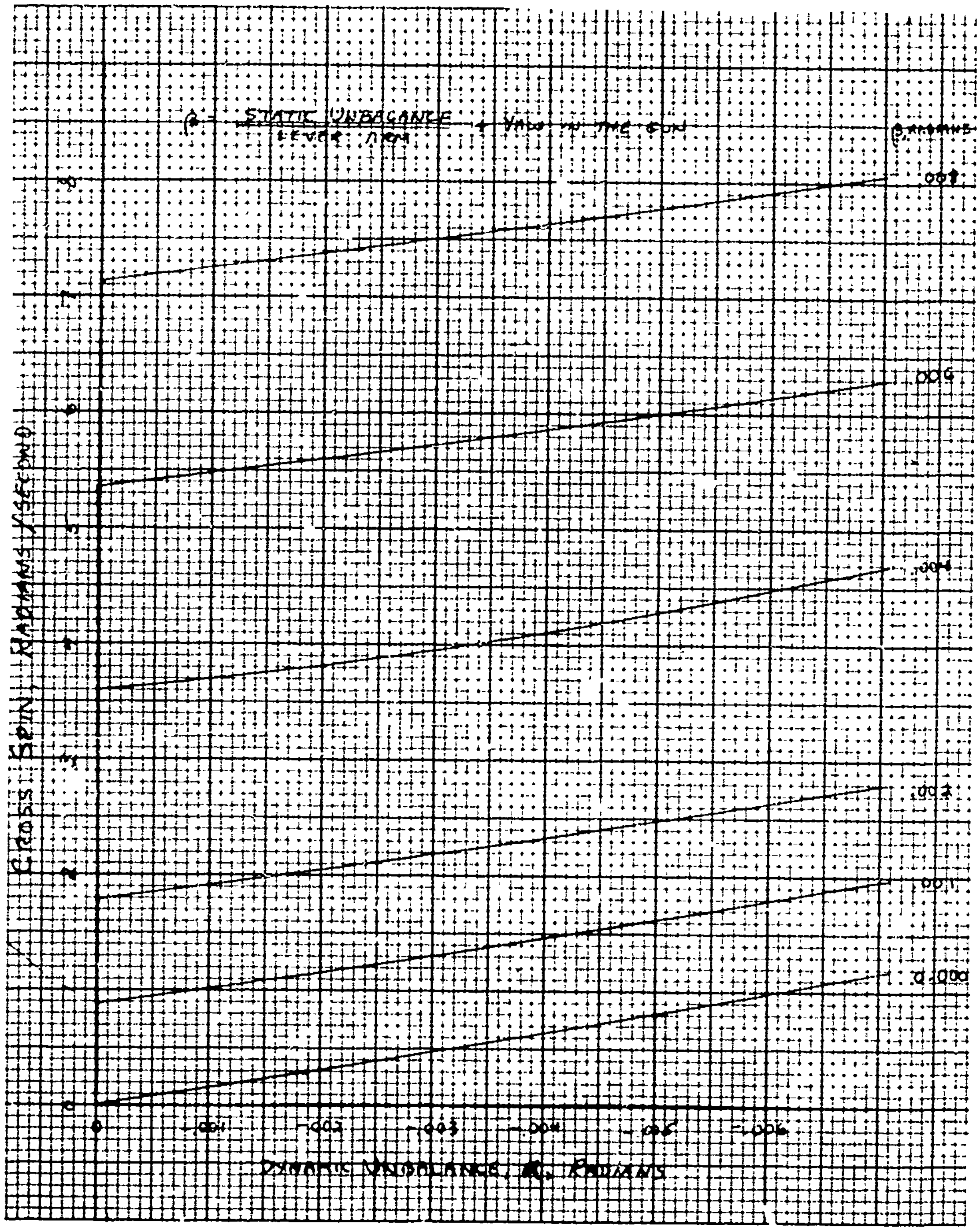

Fig 6   Cross spin at launch versus dynamic unbalance (M483, zone 4)

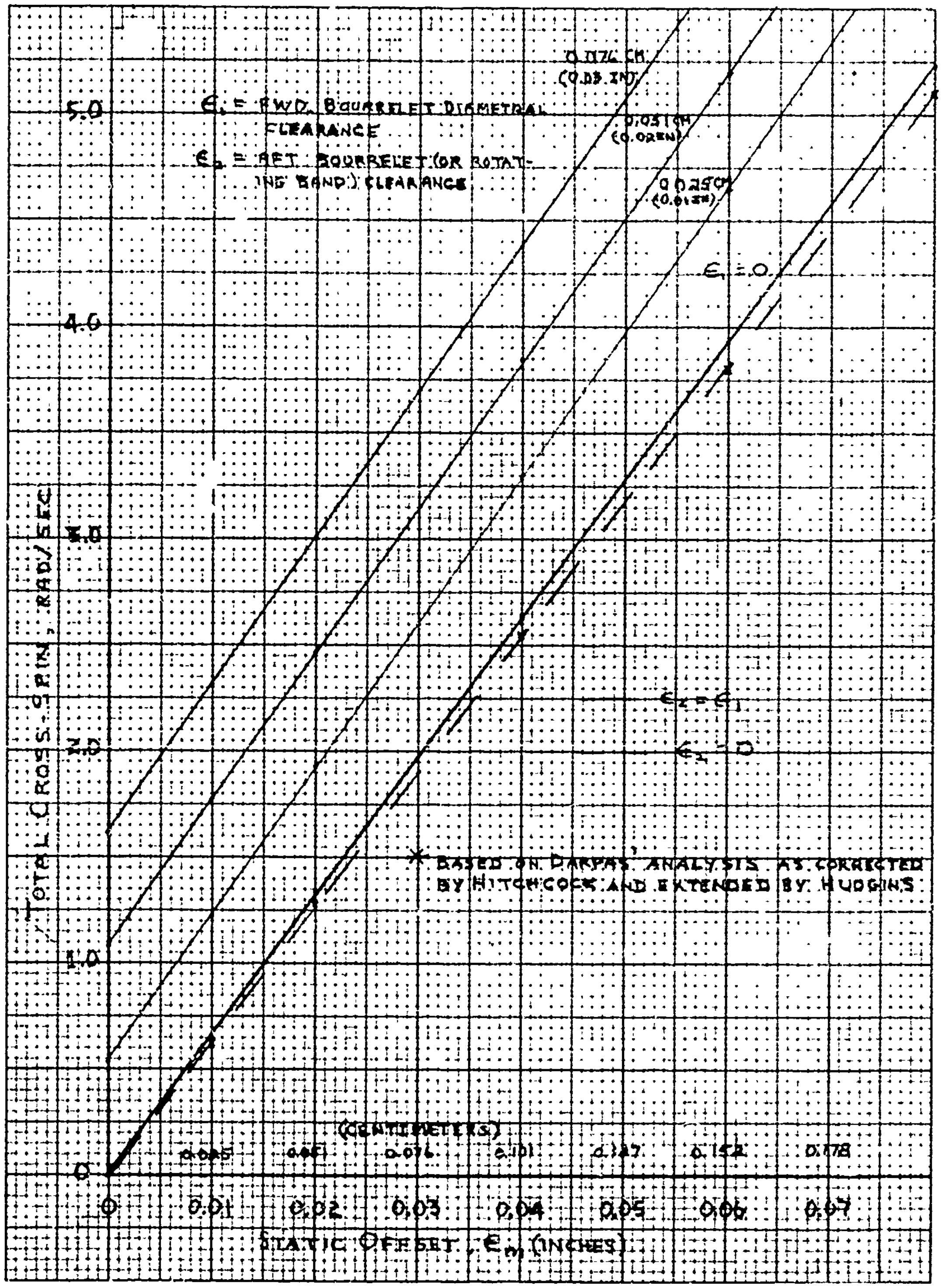

Fig 7 Angular rate at launch for M483 at zone 4

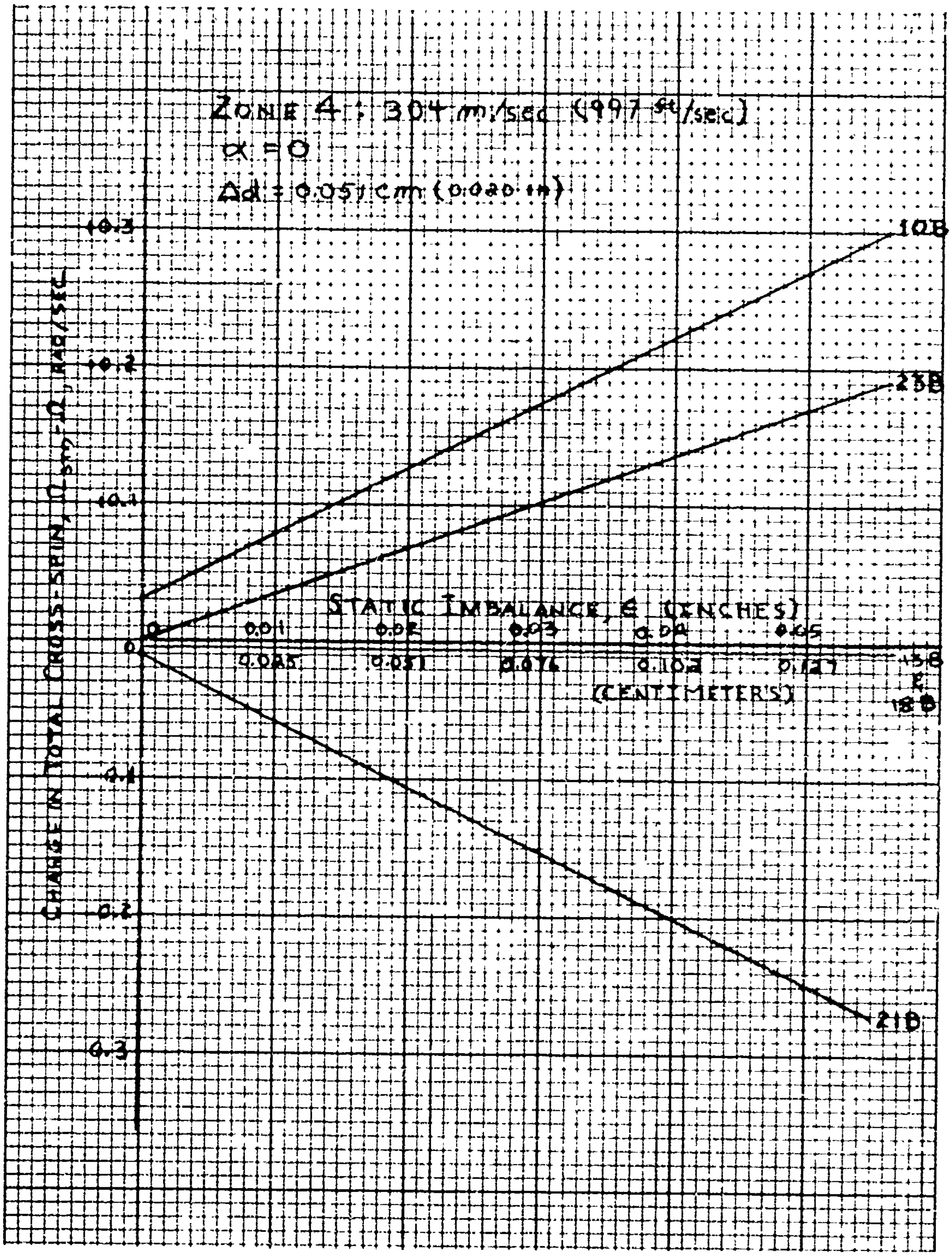

Fig 8 Effect of modifications on angular rate at launch

Those modifications which increase the angular rate do not necessarily produce a worse result, that is, more range dispersion. That decision must be reserved until the effect of the changed gyroscopic and dynamic stability of the projectile is also accounted for. Similarly, decreasing the rate does not necessarily make the situation better.

These cross-spin values are in the axis system subscripted "b" in Figure 3. If the flight behavior of the projectile is to be predicted by means of classical closed-form solutions or numerical integration of the equations of motion of an axisymmetric body, the components of spin used must be those of the axis system subscripted "I" in Figure 3. As an example, since the angle between the "b" and "I" coordinate systems is α, the spin vector

$$\vec{\Omega}\Big|_b = p\hat{X}_b + q\hat{Y}_5 = \binom{p}{q} \tag{24}$$

transformed to principal axes is

$$\vec{\Omega}\Big|_I = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p\cos\alpha - q\sin\alpha \\ q\cos\alpha + p\sin\alpha \end{pmatrix}$$

Obviously, if the technique used to calculate the flight accounts properly for dynamic imbalances, our results are already in the proper coordinate system.

CONCLUSIONS

A one-dimensional analysis, including static and dynamic imbalance, has been used to predict yaw rates at launch on 155 mm projectile M483. This method predicts that rates equal to or greater than the critical value of 3 rad/sec can result from the bore clearances and levels of imbalance actually existing, under certain circumstances. An approximate, two-dimensional analysis including static imbalance and a projectile that is not centered in the bore predicts rates close to, but somewhat smaller than, the one-dimensional analysis. These predicted values can also exceed the critical rate for expected imbalances and bore clearances under similar conditions.

The question of precession or counter-roll is not answerable by one-dimensional methods, and there is a great deal of difference in the assumptions and approximations made to predict it with two-dimensional methods. One can only fall back on the fact that no one seems to have measured any precession experimentally for conventional, spin-stabilized projectiles.

Some of the proposed modifications to the M483 increase the cross-spin at launch, others decrease it. This alone is not sufficient to judge the success or failure of a particular modification, since the gyroscopic and dynamic stability have also been changed. An analytical check can be made using a six-degree-of-freedom trajectory computer program.

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TABLE OF SYMBOLS

a	tube diameter (Darpas)
d	diameter of gun tube
e	yaw in the tube
$\bar{\bar{I}}_b$	moment of inertia tensor in x_b, y_b, z_b frame
I_x, I_y, I_z	principal moments of inertia about the center of mass
k	geometric constant (Darpas and correction)
L	distance from front bourrelet to rear support (pin or rotating band)
ℓ	distance from center of mass of projectile to rear support (pin or rotating band)
m	mass of projectile
N	spin rate about tube centerline
t	time, a running variable
T	time from passage of front bourrelet through the muzzle to the passage of the rotating band through the muzzle
X_I, Y_I, Z_I	axis system centered at the supporting pin in which the moment of inertia tensor is diagonal
X_b, Y_b, Z_b	axis system centered at the supporting pin where x_b- axis passes through the center of mass
α	angle of dynamic imbalance
β	coefficient of expansion of angular equation of motion (Darpas)
γ	coefficient of expansion of angular equation of motion (Darpas)
δ	angle between the bore centerline and a line from the supporting pin through the center of mass of the projectile (Gay and addition)

ε	distance of center of mass from projectile centerline
ε_1	diametral clearance between front support on projectile and tube (Darpas)
ε_2	diametral clearance between rear support on projectile and tube (Darpas)
λ_1	distance from center of mass of projectile to forward support in the tube (Darpas)
λ_2	distance from center of mass of projectile to rear support in the tube (Darpas)
$\bar{\Omega}_b$	total spin vector in x_b, y_b, z_b frame

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