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1. Introduction

The design of structures requires that members of the structure be selected with the capacity to perform under the anticipated loading conditions. The selection of member geometry is often fairly simple, once the critical loading conditions have been established. However, there are cases, particularly where the member geometry is complex, that the selection of an optimum member configuration is difficult. In these cases it is of interest to investigate the possibility of using optimization techniques which can predict the optimum member configuration directly. The use of direct optimization techniques could also be particularly advantageous if the maximum permitted stresses are a function of the member geometry and loading. Probably the most common situation where this interaction between allowable stress and member geometry occurs is in column design, where the critical buckling stress is a function of length and the radius of gyration of the section. For most applications the optimum configurations are tabulated in handbooks and the design engineer can quickly select optimum sections.

When new design criteria are developed it is often necessary to develop new information which will ultimately be tabulated for the practicing engineer who wishes to avoid the tedious calculations required to select optimum sections. In this paper we will investigate the possibility of using geometric programming methods to optimize the geometry of a pitched tapered beam subjected to uniformly distributed loads. The beam geometry is shown in Figure 1 and is characterized by the span, 2L, the width b, the radius of curvature R, the heights at the center H and H_c, the height at the support H_s, the roof angle β, and the slope of the lower surface φ.

In this paper we have chosen to optimize the design of the pitched tapered beam with respect to volume. Fox [1] presented a computer program for minimizing the volume of beams using a technique that required a detailed understanding of the constraints and employed a rather arbitrary fixed-step search technique to move along constraint surfaces until a minimum volume was achieved. This method provided optimal designs in many cases but because of the arbitrary step sizes used in the program it can be shown that the attaining of minimum volumes is not always ensured. Since this program was developed it has been shown by Barrett et al. [2] that the tensile strength perpendicular to grain of timber is not independent of member geometry as has been previously assumed. In particular, given a set of design parameters x_i which characterize the beam geometry, the stresses and deflection are checked using formulae normally specified in building codes. Stresses and deflections so calculated must not exceed the maximum (or allowable) values. These relationships can, therefore, be expressed in the form of inequality constraints

$$\sigma_j(x_i) \le F_j \tag{1.1}$$

where σ_j is the computed stress component and F_j is the maximum allowable value for that stress component. In addition to stress constraints, deflection constraints are usually employed to keep the deflection D of the member below a specified fraction of the span and accordingly the constraint will have the form

$$D(x_i) \le \alpha S \tag{1.2}$$

where α is the appropriate fraction and S is the beam span.

In general, there will be other relations which control beam geometry. For example in beam design often the ratio of beam height to width is restricted to satisfy lateral stability requirements. In the case of the pitched tapered beam there will be requirements for a minimum length of tangent on the beam lower surface. These constraints can be written in the form

$$f_j(x_i) \le 1 \ . \tag{1.3}$$

The allowable tensile strength perpendicular to grain for pitched tapered beams is given by

$$F_r = \frac{324}{\sqrt[5]{bHR_m\varphi}} \qquad (1.4)$$

where b is the beam width, H the height at the centerline, R_m the radius to midheight and φ the angle in radians between the centerline and the tangent point. The allowable stress according to Eq. (1.4) is generally lower than the 65 psi used previously, thereby requiring modification of the beam configurations which have been tabulated in [3].

The optimal beam geometry will be found by formulating the optimization problem as a generalized geometric (signomial) programming problem. In the next section the detailed engineering formulation of the problem will be presented. The signomial programming formulation is derived in Section 3 and in Section 4 the results of a few sample designs are given.

2. Engineering Considerations

Given specifications for the uniformly distributed design dead load ω_0 and snow load ω_s, the total span 2L and the roof angle β, the design engineer must develop a beam of adequate capacity. The capacity of a beam is assessed by evaluating stress, deflection and geometric criteria which are normally specified by building code authorities or dictated by manufacturing requirements. For this paper the design criteria for allowable shear and bending stress, F_s and F_B and the deflection constraints are those specified by the Canadian Standard

Association Standard CSA 086-1970 [4]. Currently the allowable tension perpendicular to grain stress F_r, permitted in Canada for Douglas fir, dry service condition and normal load duration is 65 psi. It has been shown that this allowable stress is not adequately conservative for large beams and for this paper the allowable stress shall be that recommended by Barrett et al. [2].

In design the engineer strives to produce a minimum cost structure consistent with the requirements for public safety and protection of property. This may be accomplished by minimizing the volume of material used in the members. Accordingly, in our optimization problem the objective function to be minimized is the volume of the beam subject to the design constraints developed below.

There are constraints on the bending stresses at the beam centerline and tangent point (TP, Figure 1), the shear stress at the support (A, Figure 1) and tension perpendicular to grain stress at the centerline. The specific form of these constraints for the pitched tapered beam subjected to a uniformly distributed load is as follows.

For the bending stresses σ_B, we require $\sigma_B \leq F_B$ where F_B is the given allowable bending stress. Bending stresses must be checked at two positions, at the centerline and at the tangent point. Therefore at the centerline the constraint has the following form

$$\sigma_B = \frac{6M_c}{bH^2} [1 + 2.7 \tan\beta] \leq F_B \tag{2.1}$$

where M_c, the bending moment at the centerline is given by

$$M_c = \frac{\omega L^2}{24} . \tag{2.2}$$

Similarly, the bending stress at the tangent point is given by

$$\sigma_B = \frac{6M_T}{bH_T^2} [1 + 2\cdot7 \tan \beta] \leq F_B \tag{2.3}$$

where M_T, the bending moment at the tangent point is given by

$$M_T = M_c - \omega R^2 \sin^2 \varphi/24 . \tag{2.4}$$

Here,

H, H_T = beam heights at centerline and tangent point, respectively, inches,

L = half-span, inches,

b = beam width, inches,

β = roof angle, degrees,

ω = uniformly distributed load, pounds per lineal foot.

The beam loading tends to increase the radius of curvature of the beam, thereby introducing stresses in the radial direction. The magnitude of the tension perpendicular to grain stress σ_r is given by

$$\sigma_r = K_r \frac{6M_c}{bH^2} \leq F_r \tag{2.5}$$

where

$$K_r = A + B\left(\frac{H}{R_m}\right) + c\left(\frac{H}{R_m}\right)^2 \tag{2.6}$$

and

R_m = radius to midheight $(R + H/2)$, inches

A,B,C = functions of β, tabulated in [4].

The shear stresses τ at the support are required to satisfy the relation $\tau \leq F_s$ and this constraint is formulated as follows

$$\tau = \frac{3}{2} \frac{v}{bH_s} \leq F_s \tag{2.7}$$

where

$$v = \frac{\omega L}{12} \tag{2.8}$$

and

H_s = beam height at the support, inches

v = shear force, pounds

A constraint on the midspan deflection of beams is normally imposed to prevent excessive deflection which could damage ceiling materials. The maximum allowable deflection δ_{max}, is usually expressed as a function of the total span. The corresponding constraint in our case is

$$\delta = \frac{5\omega L^4}{288EI} Y \leq \delta_{max}$$

where

$$Y = 0.2 + 0.8\, H_c/H_s, \quad \text{(see [4])} \tag{2.10}$$

$$I = bH_c^3/12 \tag{2.11}$$

and

E = modulus of elasticity, psi

H_c = height at centerline for the double-tapered component of the beam, inches.

The final constraints are constraints on geometry. The first is introduced to ensure that the tangent point (T.P., Figure 1) is positioned at an adequate distance from the end of the beam. This constraint is required to prevent springback when the beam is released from the clamps after curing of the glue is complete. For convenience the constraint is formulated as follows

$$L - R \sin \varphi \geq \alpha H \tag{2.12}$$

where α is a given constant.

To complete the engineering formulation it is necessary to specify the allowable values to be used in the stress and deflection constraints. These values depend on the species of wood used in the beam. If Douglas fir is used the allowable values are as follows

$$F_B = 2760(1 - 2000(t/R)^2) \qquad \text{(psi)} \tag{2.13}$$

where

t = lamination thickness, inches.

$$F_r = K(bHR_m\varphi)^{-0.2} \qquad \text{(psi)} \tag{2.14}$$

$$F_s = 190 \qquad \text{(psi)} \tag{2.15}$$

where

$K = 324$ (if φ is in radians)

$$\delta_{max} = \frac{2L}{\xi} \tag{2.16}$$

where we assume $\xi = 180$.

The above formulas for F_B and F_S include a 15% increase in allowable stress for snow load conditions.

Finally, the radius of curvature R is constrained to be greater than or equal to 330 inches so that excessive stresses will not be introduced when the beams are fabricated.

The variable portion of the half-beam volume, to be minimized, is given by (see Figure 1)

$$V = Lb(H_S + H_C) + bR^2(\tan \varphi - \varphi) . \tag{2.17}$$

The total half-beam volume is equal to $V - L^2 b \tan \beta$.

This concludes the engineering formulation of the optimal design problem.

3. Signomial Programming Formulation

In this section we present a signomial programming formulation of the optimal beam design problem. The main difficulty in reformulating this problem as a signomial program is that some of the relations appearing in the preceding section are equations, whereas signomial programming constraints must be inequalities. As will be shown below, some of the equations of the engineering formulation are used to eliminate variables and others are converted into inequalities in such a way that hopefully they hold as equations at an optimum. Another minor problem arises from the appearance of trigonometric functions in the design formulas. Taylor series approximations of these functions are used below to obtain generalized polynomials as required.

Note that converting an equality constraint $g(x) = 1$ into two inequalities $g(x) \leq 1$, $g(x) \geq 1$ is not practical, since the algorithm used for the numerical solution is based on the assumption that the interior of the constraint set is nonempty. For this reason, equalities can only be converted into one-sided inequalities. The sense of the inequalities is usually determined by physical or design considerations (see, for example, [5]). Unfortunately, there are cases where these considerations are quite complex and cannot be observed by simple inspection of the constraints. Consequently, a trial-and-error approach is necessary. Simple examples can, however, be constructed showing that not every equality constrained problem can be solved by converting equations into single inequalities.

Let us derive now the signomial program for the beam design in detail. The volume of the beam as given in the engineering formulation is

$$V = Lb(H_s + H_c) + bR^2(\tan\varphi - \varphi) \;. \tag{3.1}$$

Using the identity

$$H_c + L\tan\varphi = H_s + L\tan\beta \tag{3.2}$$

and a three-term Taylor expansion of $\tan\varphi$

$$\tan\varphi \cong \varphi + \frac{1}{3}\varphi^3 + \frac{2}{15}\varphi^5 \tag{3.3}$$

we obtain the volume to be minimized

$$\begin{aligned} V = 2LbH_c - L^2b\tan\beta + L^2b\varphi + \frac{1}{3}L^2b\varphi^3 \\ + \frac{2}{15}L^2b\varphi^5 + \frac{1}{3}bR^2\varphi^3 + \frac{2}{15}bR^2\varphi^5 \end{aligned} \tag{3.4}$$

where H_s, the beam height at the support, has been eliminated.

The variable H_c is defined by the identity

$$H_c = H + R - \frac{R}{\cos \varphi} . \qquad (3.5)$$

In converting this relation into an inequality we can ensure that in an optimal solution the inequality will hold as an equation by writing

$$H_c \geq H + R(1 - \frac{1}{\cos \varphi}) . \qquad (3.6)$$

Since in (3.4) we try to lower the value of H_c as much as possible, the inequality in (3.6) will be tight in an optimal solution. Substituting

$$\frac{1}{\cos \varphi} - 1 \cong \frac{1}{2} \varphi^2 + \frac{5}{24} \varphi^4 + \frac{61}{720} \varphi^4 \qquad (3.7)$$

into (3.6) and rearranging yields

$$H_c + \frac{1}{2} R\varphi^2 + \frac{5}{24} R\varphi^4 + \frac{61}{720} R\varphi^6 \geq H \qquad (3.8)$$

or

$$2R^{-1}\varphi^{-2}H - 2R^{-1}\varphi^{-2}H_c - \frac{10}{24} \varphi^2 - \frac{122}{720} \varphi^4 \leq 1 . \qquad (3.9)$$

The bending stress constraint at centerline is given by (2.1), (2.2) and (2.13) as

$$\frac{6M_c(1 + 2.7\tan \beta)}{bH^2} \leq 2760[1 - 2000 (\frac{t}{R})^2] \qquad (3.10)$$

or

$$\Gamma L^2 H^{-2} + 2000t^2 R^{-2} \leq 1 \qquad (3.11)$$

where

$$\Gamma = \frac{(1 + 2.7 \tan \beta)\omega}{4(2760)b} \quad . \tag{3.12}$$

Turning now to the bending stress constraint at tangent point we have from (2.2), (2.3), (2.4) and (2.13)

$$6\left(\frac{\omega L^2}{24} - \frac{\omega R^2 \sin^2\varphi}{24}\right)\frac{(1 + 2.7 \tan \beta)}{bH_T^2} \leq 2760[1 - 2000(\frac{t}{R})^2] \tag{3.13}$$

or

$$\Gamma L^2 H_T^{-2} - \Gamma R^2 \sin^2\varphi \ \ H_T^{-2} + 2000t^2R^{-2} \leq 1 \ . \tag{3.14}$$

The beam height at tangent point H_T, is related to the other beam heights by

$$H_T = (1 - \frac{R \sin \varphi}{L}) \ H_c + (\frac{R \sin \varphi}{L}) \ H_s \ . \tag{3.15}$$

From (3.2) and (3.5) we obtain

$$H_T = H + R - R \cos \varphi - R \tan \beta \sin \varphi \ . \tag{3.16}$$

Instead of converting (3.16) into an inequality and guessing its sense, we substitute (3.16) into (3.14) and by letting

$$\sin \varphi \cong \varphi - \frac{1}{6} \varphi^3 + \frac{1}{120} \varphi^5 \tag{3.17}$$

$$\cos \varphi \cong 1 - \frac{1}{2} \varphi^2 + \frac{1}{24} \varphi^4 \tag{3.18}$$

we write the bending stress constraint at tangent point as

$$\Gamma L^2 H^{-2} + \frac{1}{12} H^{-1} R \varphi^4 + 2 \tan\beta\, H^{-1} R \varphi + \frac{\tan\beta}{60} H^{-1} R \varphi^5 + 2000 t^2 R^{-2}$$

$$+ 500 t^2 \tan\beta\, H^{-2} \varphi^5 + \frac{2(1 - \Gamma - \tan^2\beta)}{45} H^{-2} R^2 \varphi^6 + \tan\beta\, H^{-2} R^2 \varphi^3$$

$$+ 2000 t^2 H^{-1} R^{-1} \varphi^2 + \frac{2000 t^2 \tan\beta}{3} H^{-1} R^{-1} \varphi^3 + \tan^2\beta\, 2000 t^2 H^{-2} \varphi^2$$

$$+ \frac{(3/4 - \tan^2\beta)\, 2000 t^2}{3} H^{-2} \varphi^4 - H^{-1} R \varphi^2 - \frac{\tan\beta}{3} H^{-1} R \varphi^3$$

$$- 2000 t^2 \tan\beta H^{-2} \varphi^3 - (\Gamma + \tan^2\beta) H^{-2} R^2 \varphi^2 - \left(\frac{3/4 - \Gamma - \tan^2\beta}{3}\right) H^{-2} R^2 \varphi^4$$

$$- \frac{\tan\beta}{4} H^{-2} R^2 \varphi^5 - \frac{2000 t^2}{12} H^{-1} R^{-1} \varphi^4 - 4000 t^2 \tan\beta\, H^{-1} R^{-1} \varphi$$

$$- \frac{2000 t^2 \tan\beta}{60} H^{-1} R^{-1} \varphi^5 - \frac{2(1 - \tan^2\beta)\, 2000 t^2}{45} H^{-2} \varphi^6 \le 1. \qquad (3.19)$$

Next we consider the constraint on tension perpendicular to grain stress. We have by (2.5), (2.6) and (2.14)

$$\frac{6M_c A}{b} H^{-2} + \frac{6M_c B}{b} H^{-1} R_m^{-1} + \frac{6M_c C}{b} R_m^{-2} \le \frac{K}{(bHR_m\varphi)^{0.2}} \qquad (3.20)$$

or

$$\frac{6M_c A}{b^{0.8} K} H^{-1.8} R_m^{0.2} \varphi^{0.2} + \frac{6M_c B}{b^{0.8} K} H^{-0.8} R_m^{-0.8} \varphi^{0.2} + \frac{6M_c C}{b^{0.8} K} H^{0.2} R_m^{-1.8} \varphi^{0.2} \le 1$$

$$(3.21)$$

where

$$R_m = R + \frac{1}{2} H . \qquad (3.22)$$

Direct substitution of (3.22) into (3.21) would yield a nonsignomial constraint and, therefore, should be avoided. Consequently, we must treat (3.22) as an inequality that has to hold as an equation at optimum.

Since R_m has both positive and negative exponents in (3.21) the sense of the inequality to replace (3.22) cannot be determined in advance. Multiplying, however, the left hand side of (3.21) by the identity $(R + H/2)/R_m$ yields the new tension-perpendicular-to grain stress constraint

$$\begin{aligned} &\frac{6M_cA}{b^{0.8}K} RH^{-1.8}R_m^{-0.8}\varphi^{0.2} + \frac{6M_cB}{b^{0.8}K} RH^{-0.8}R_m^{-1.8}\varphi^{0.2} \\ &+ \frac{6M_cC}{b^{0.8}K} RH^{0.2}R_m^{-2.8}\varphi^{0.2} + \frac{3M_cA}{b^{0.8}K} H^{-0.8}R_m^{-0.8}\varphi^{0.2} \\ &+ \frac{3M_cB}{b^{0.8}K} H^{0.2}R_m^{-1.8}\varphi^{0.2} + \frac{3M_cC}{b^{0.8}K} H^{1.2}R_m^{-2.8}\varphi^{0.2} \leq 1 \end{aligned} \tag{3.23}$$

where all the terms on the left hand side are positive and the exponents of R_m are all negative. Now we convert (3.22) into the inequality

$$R_m \leq R + \frac{1}{2} H \tag{3.24}$$

or

$$R_m R^{-1} - \frac{1}{2} HR^{-1} \leq 1 \ . \tag{3.25}$$

Note that the above considerations are valid if the inequality (3.23) is tight in the optimal solution. It may happen, however, that both (3.23) and (3.25) are strict inequalities at optimum. In this case the sense of (3.25) must be reversed (such a reversal was in fact necessary in one of the cases solved).

The shear stress constraint is formulated from (2.7) by using (3.5) and (3.3). We obtain

$$\left(\frac{\omega L}{8bF_s} + L \tan \beta\right)H_c^{-1} - L\varphi H_c^{-1} - \frac{1}{3} L \varphi^3 H_c^{-1} - \frac{2}{15} L \varphi^5 H_c^{-1} \leq 1 . \tag{3.26}$$

The deflection constraint is formulated from (2.9), (2.10), (2.11) and (2.16) as

$$\eta H_c^{-3} + 4\eta H_c^{-2} H_s^{-1} \leq 1 \tag{3.27}$$

where

$$\eta = \frac{0.208\xi\omega L^3}{Eb} . \tag{3.28}$$

Multiplying both sides of (3.27) by H_s and substituting (3.2) yields

$$\eta H_c^{-2} - \eta L \tan \beta \; H_c^{-3} + \eta L \tan \varphi \; H_c^{-3} + 4\eta \; H_c^{-2}$$

$$\leq H_c - L \tan \beta + L \tan \varphi \tag{3.29}$$

and by (3.3) we obtain

$$5\eta \; H_c^{-3} + \eta L\varphi H_c^{-4} + \frac{1}{3} \eta L\varphi^3 H_c^{-4} + \frac{2}{15} \eta \; L\varphi^5 H_c^{-4} + L \tan \beta \; H_c^{-1}$$

$$- \eta \; L \tan \beta \; H_c^{-4} - L\varphi H_c^{-1} - \frac{1}{3} L\varphi^3 H_c^{-1} - \frac{2}{15} L\varphi^5 H_c^{-1} \leq 1 . \tag{3.30}$$

The constraint on the geometry of the beam given by (2.12) is rearranged to

$$\frac{\alpha}{L} H + \frac{1}{L} R \sin \varphi \leq 1 \tag{3.31}$$

and by (3.17) it becomes

$$\frac{\alpha}{L} H + \frac{1}{L} R\varphi - \frac{1}{6L} R\varphi^3 + \frac{1}{120L} R\varphi^5 \leq 1 . \tag{3.32}$$

The last geometry constraint, $R \geq 330$, is not treated explicitly, since the numerical algorithm for the solution of the beam design problem requires upper and lower bounds on all the design variables, thus the value 330 will be used as the lower bound on R.

The optimal beam design is obtained (after specifying the appropriate constants) by solving the signomial program of minimizing (3.4) subject to the constraints (3.9), (3.11), (3.19), (3.23), (3.25), (3.26), (3.30) and (3.32). The variables to be determined by the optimization are H, H_c, R, R_m, V and φ. Note that an optimal solution to the signomial programming formulation of the design problem is acceptable only if the inequalities (3.9),and (3.25) hold as equations at optimum.

A few sample problems of optimal beam designs were solved by the computer code GGP, based on the generalized geometric (signomial) programming algorithm of Avriel, Dembo and Passy [5]. These optimal design solutions are presented in the next section.

4. Sample Designs

Optimal beam configurations are sought for three different spans and loading conditions. The specified beam parameters are shown in Table 1.

Table 1

Input Parameters for Beam Optimization

Case	Roof Angle β (degrees)	Half-Span L (inches)	Width b (inches)	Lamination Thickness t (inches)	Load ω(lb/ft)
1	9.46	360	8.75	1.5	1200
2	9.46	240	6.75	1.5	1200
3	9.46	120	3.00	1.5	400

For the above roof angle the corresponding constants are $A = 0.0367$, $B = 0.0794$, $C = 0.213$. In addition, the modulus of elasticity is assumed to be $E = 1.93 \times 10^6$ psi and a value of $\alpha = 1.5$ is taken in (2.12).

Optimal solutions were obtained by the computer code GGP in less than 10 seconds of CPU time on an IBM 370/168 computer. The optimal design variables are listed in Table 2.

Table 2

Optimal Design Variables

Case	Volume ft^3	φ degrees	H inches	H_c inches	R inches	R_m inches
1	184.32	3.84	70.7	68.3	1063	1099
2	71.05	5.01	50.3	47.0	860	886
3	5.93	6.16	19.5	17.6	330	340

It is interesting to observe the binding design constraints at optimum for the above cases (in addition to those which must be tight because they were originally equations).

Table 3

Binding Design Constraints at Optimum

Case	Binding Design Constraints
1	Tension $\perp$ grain stress (3.23); Shear stress (3.26)
2	Tension $\perp$ grain stress (3.23); Shear stress (3.26)
3	Bending at tangent point (3.19); Shear stress (3.26)

In Cases 1 and 2 constraint (3.23) is tight at optimum and consequently (3.25), the defining relation for R_m, is also satisfied as an equation. In Case 3, however, (3.23) is no longer binding and at first we obtained a solution in which both (3.23) and (3.25) were strict inequalities. We, therefore, reversed the sense of the inequality in (3.24) and (3.25) to

$$R_m \geq R + \frac{1}{2} H \tag{4.1}$$

and

$$RR_m^{-1} + \frac{1}{2} HR_m^{-1} \leq 1 \tag{4.2}$$

respectively, and (3.25) was replaced by (4.2) in the program. This change resulted in the above listed optimal solution for Case 3 in which, of course, (4.2) held as an equation.

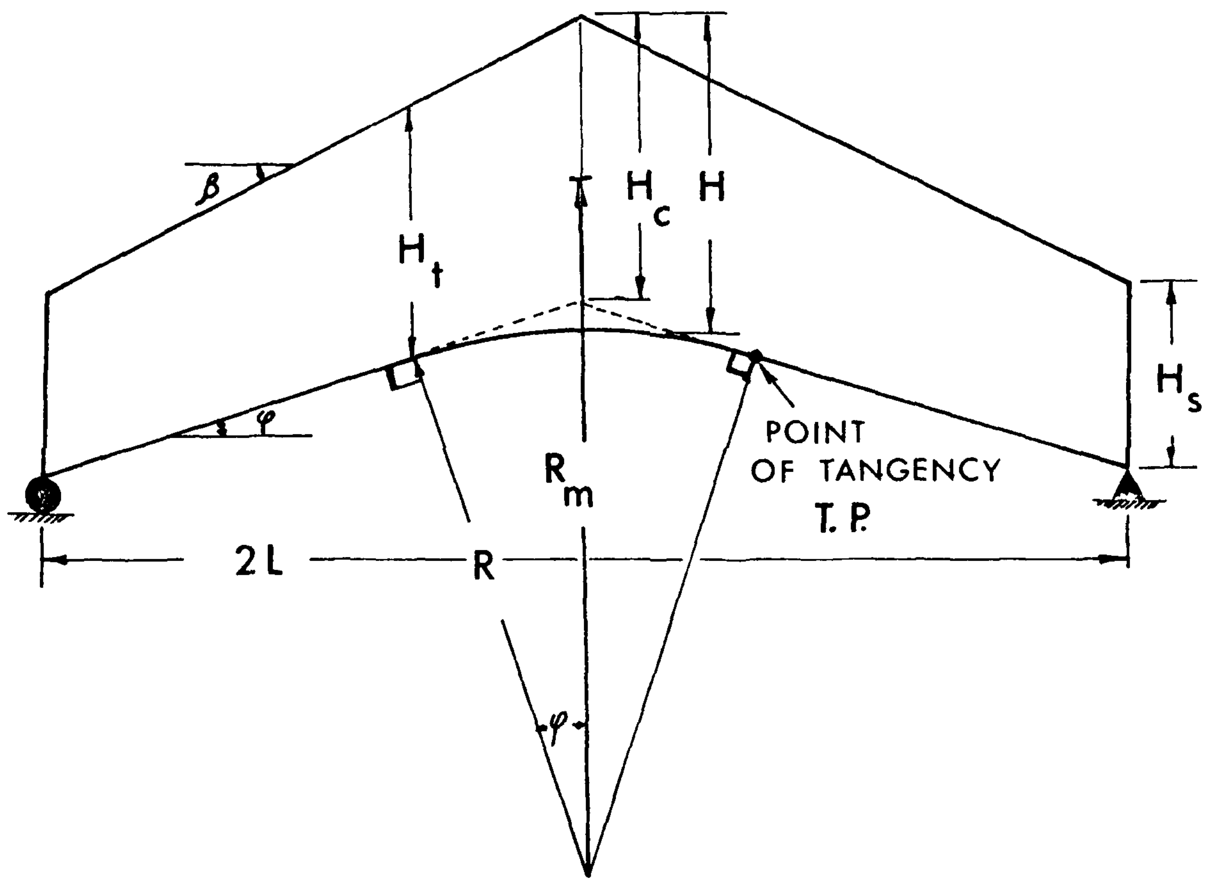

FIGURE 1. Pitched Tapered Beam

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The optimal design of a pitched tapered laminated wood beam is considered. An engineering formulation is given in which the volume of the beam is minimized. The problem is then reformulated and solved as a generalized geometric (signomial) program. Sample designs are presented.

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