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MAY 1975

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NOISE IN BROAD-BAND HYDROPHONES

BY

D. STANSFIELD

A.U.W.E. Tech. Note 521/75

ADMIRALTY UNDERWATER WEAPONS ESTABLISHMENT
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by

D. Stansfield

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NOISE IN BROAD-BAND HYDROPHONES

SUMMARY

The sources of noise in underwater electro-acoustic hydrophones are considered with particular reference to piezoelectric piston-type elements. Expressions are derived for the various contributions, and their influence on hydrophone design for broad-band reception is discussed.

It is shown that measurement of ambient noise levels down to Sea State 1/2 equivalent is possible up to 70 kHz with a correctly designed hydrophone and a low noise amplifier with sufficiently high input impedance. Measurements down to this level are more readily carried out with a spherical hydrophone than with a piston hydrophone in a baffle. The hydrophone parameters of greatest importance are the resonance frequency, coupling coefficient, and hydrophone sensitivity. It is important also to avoid mechanical resonances of the hydrophone mounting.

NOISE IN BROAD-BAND HYDROPHONES

I. INTRODUCTION

This report discusses the various sources of noise in the output of pressure hydrophones, and their influence on the performance and design of broad-band receivers. The contributions considered include:

a. thermal noise

b. ambient sea noise

c. amplifier noise

d. noise due to mounting vibration

and these are related to the pressure sensitivity of the hydrophone element itself.

Consideration is given mainly to low-frequency piston-type hydrophones, of a balanced ("acceleration-cancelling") design, but the relationships are also valid for unbalanced designs, and similar principles apply to other forms of construction.

II. THERMAL NOISE

Any resistor at a finite temperature has across it voltage fluctuations which are known as thermal noise. A hydrophone immersed in the sea will respond to pressure fluctuations in the medium which arise similarly from thermal agitation of the medium itself. Mellen (Ref 1) has shown that this thermal noise pressure in a 1 Hz band (p_T) is given by

$$p_T^{\,2} = \frac{4\pi KT\rho c}{\lambda^2} \qquad (1)$$

where K = Boltzmann's constant (1.38×10^{-23} J/deg K)

ρ = density of the medium

c = speed of sound in the medium

λ = wavelength of sound in the medium at the relevant frequency (f)

T = temperature in degrees Kelvin.

It is of interest to express this noise pressure level in terms of the noise voltage output generated by a hydrophone immersed in the sea and subjected to these pressure fluctuations. This requires a knowledge of the hydrophone sensitivity of the device, which may be derived as follows.

The acoustic pressure (p) at a point distant r_o from a small omni-directional source is given by the expression for the total acoustic power:

$$\frac{p^2}{\rho c}\, 4\pi r_o^{\,2} = i^2 R\eta_{ea} \qquad (r_o \gg \lambda) \qquad (2)$$

where $\overline{i^2}$ = mean square input current

R = resistive component of the input impedance

η_{ea} = electro-acoustic efficiency.

Thus, the current projector sensitivity (S_I) is given by

$$S_I^2 = \frac{p^2}{i^2}$$

$$= \frac{\rho c R \eta_{ea}}{4\pi r_o^2} \qquad (3)$$

By the reciprocity theorem, the open-circuit hydrophone sensitivity (M) is related to the projector sensitivity by

$$\frac{M}{S_I} = \frac{2r_o}{\rho f} \qquad (4)$$

where f is the frequency (in Hz)

Thus,

$$M^2 = S_I^2 \frac{4r_o^2}{\rho^2 f^2}$$

$$= \frac{cR\eta_{ea}}{\pi\rho f^2}$$

$$= \frac{R\eta_{ea}}{\pi\rho c} \lambda^2$$

$$\text{i.e.} \qquad M = \left(\frac{R\eta_{ea}}{\pi\rho c} \right)^{1/2} \lambda \tag{5}$$

Therefore, the mean square output voltage (in a 1 Hz band) from the hydrophone is

$$e_T^{\,2} = M^2 p_T^{\,2}$$

$$= \frac{R\eta_{ea}}{\pi\rho c} \lambda^2 \quad \frac{4\pi KT\rho c}{\lambda^2}$$

$$= 4KTR\eta_{ea} \tag{6}$$

For an ideal hydrophone of 100% efficiency (i.e., $\eta_{ea} = 1$), this noise voltage is equal to $(4KTR)^{1/2}$, which is just the noise output to be expected from an electrical resistor of magnitude R. Thus, the electrical impedance of a lossless hydrophone represents correctly the thermal noise output as well as the acoustic impedance due to the medium-a result which may have been expected or hoped for.

Converting the thermal noise pressure given by equation (1) to a spectrum level (denoted by $P_T = 20 \log p_T$), we obtain

$$P_T = 10 \log \frac{4\pi K T \rho}{c} + 20 \log F$$

$$= -115 + 20 \log F \text{ dB re } 1\ \mu b \text{ in Hz band (where } F \text{ is the frequency in kHz)}$$

$$= -15 + 20 \log F \text{ dB re } 1 \mu Pa \text{ in } 1 \text{ Hz band} \qquad (7)$$

This topic has been discussed by various authors, (See e.g., Ref 2, 3, 5, 6); the above treatment is a slightly fuller version of part of an article by Batchelder (Ref 2), who goes on to consider the effect of internal losses in the hydrophone. These losses lead to a reduced efficiency and give rise to additional thermal noise. Batchelder states (without proof) that this added thermal noise degrades the equivalent noise pressure-squared of the real hydrophone to p_T^2/η_{ea}, where η_{ea} is the electro-acoustic efficiency. A derivation and discussion of this relationship is given below.

The electrical characteristics of an ideal piezoelectric hydrophone (with no electrical losses) are often represented by the circuit shown in figure 1(a). Both C_p and R_p depend on frequency, although for frequencies well below resonance C_p becomes almost independent of frequency. It is, of course, also possible (though less usual) to represent the hydrophone impedance by the series circuit of figure 1(b). In these circuits, e_p and e_s represent the sources of thermal noise voltage, and the components are related by the equations: -

$$R_s = \frac{R_p}{1+\omega^2 C_p^2 R_p^2} = \frac{R_p}{1+\beta^2} \quad \text{where } \beta = \omega C_p R_p \tag{8}$$

$$C_s = \left(\frac{1+\beta^2}{\beta^2}\right) C_p \tag{9}$$

The open circuit noise voltage (V_1) is given by

$$V_1^2 = e_s^2 = \frac{e_p^2}{1+\beta^2}$$

$$= 4KTR_s \tag{10}$$

Note also that $\beta = \omega C_p R_p = \frac{1}{\omega C_s R_s}$

Now suppose that another resistor R_2 (with its own associated thermal noise source e_2) is connected in parallel with the hydrophone, to represent the internal electrical losses, as in figure 2(a). After combining R_2 and R_p to form an effective parallel resistor $R_p' = (R_p R_2)/(R_p+R_2)$, the circuit may be converted to the form shown in figure 2(b), the components being given by equations similar to (8) and (9), viz.

$$R_s' = \frac{R_p'}{1+\beta'^2}$$

where $\beta' = \omega C_p R_p'$

The open circuit noise voltage is then given by

$$V^2 = 4KTR_s$$

$$= \frac{4KT\left(\frac{R_p R_2}{R_p+R_2}\right)}{1+\left(\frac{\omega C_p R_p R_2}{R_p+R_2}\right)^2}$$

$$= \frac{4KTR_p\ (1+ R_p/R_2)}{(1+ R_p/R_2)^2 + \beta^2} \qquad (11a)$$

$$= \frac{4KTR_2\ (1+ R_2/R_p)}{(1+ R_2/R_p)^2 + \beta^2\ (R_2/R_p)^2} \qquad (11b)$$

This is the general expression for the thermal noise voltage from this circuit; we now consider how R_s depends on frequency. The variation with frequency of the admittance of an ideal (lossless) hydrophone can be represented by the equivalent circuit shown in figure 3 (in which the components are assumed to be independent of frequency). This gives reasonable agreement with measured admittances for frequencies up to just over the resonance frequency, provided the usual "lumped-mass" approximation is valid. For this circuit, the variation of R_p

(= 1/(conductance)) is given by

$$R_p = R_m \;(1+ Q_m{}^2 \eta_1{}^2)$$

where R_m represents the mechanical loss resistance (in electrical terms)

$$\eta_1 = \frac{\omega}{\omega_1} - \frac{\omega_1}{\omega}$$

$$\omega_1 = \text{mechanical resonance (angular) frequency} = \left(\frac{1}{L_m C_m}\right)^{1/2}$$

$$Q_m = \text{mechanical Q-factor} = \frac{1}{\omega_1 C_m R_m}$$

Since the coupling coefficient of the hydrophone (k_e) is defined by

$$k_e{}^2 = \frac{C_m}{C_o + C_m}$$

and the low frequency capacitance $C_{LF} = C_o + C_m$

we thus obtain

$$R_p = \frac{1+Q_m{}^2 \eta_1{}^2}{\omega_1 k_e{}^2 C_{LF} Q_m} \qquad (12a)$$

$$\approx \frac{Q_m \eta_1{}^2}{\omega_1 k_e{}^2 C_{LF}} \qquad (12b)$$

Equation (12b) is valid at frequencies far enough from resonance for the condition $Q_m^2 n_1^2 >> 1$ to be satisfied; in practice, this means frequencies below the "half-conductance" ($1/Q_m$) band-width around resonance. Substitution of this expression for R_p into equation (11b) would then give the variation with frequency of the noise voltage output.

It is, however, generally more useful to express the noise output in terms of the equivalent noise pressure. For this, we refer to the expression (equation (5)) for the hydrophone sensitivity.

$$\text{i.e.} \qquad M^2 = \frac{R\eta_{ea}}{\pi\rho c} \lambda^2$$

In this expression, R is the resistive component of the total input impedance, i.e., $R = R_s'$

$$\text{Thus,} \qquad M^2 = \frac{\eta_{ea} \lambda^2}{\pi\rho c} R_s'$$

$$= \frac{\eta_{ea} \lambda^2}{\pi\rho c} R_2 \frac{(1+ R_2/R_p)}{(1+ R_2/R_p)^2 + \beta^2(R_2/R_p)^2} \qquad (13)$$

Then, the equivalent noise pressure is given (using equations (11b) and (13) by

$$p^2_{TR} = \frac{V^2}{M^2}$$

$$= \frac{4\pi KT\rho c}{\eta_{ea} \lambda^2} \qquad (14)$$

which is the expression quoted by Batchelder.

In order to calculate the variation with the frequency of p^2_{TR}, we now derive the variation of η_{ea} with frequency, using equation (12). Assume that there are no internal mechanical losses in the hydrophone: i.e., that all the internal losses are electrical and are represented by R_2 (which may itself be a function of frequency). (The following expressions would still represent the variation of η_{ea} with frequency if any internal mechanical losses were a constant fraction of the radiation losses, which would usually be a reasonable assumption.) Thus, we take

$$\eta_{ea} = \frac{1/R_p}{1/R_p + 1/R_2}$$

$$= \frac{1}{1 + R_p/R_2}$$

$$= \frac{1}{1 + \dfrac{(1+Q_m^{\,2}\eta_1^{\,2})}{\omega_1 k_e^{\,2} C_{LF} Q_m R_2}} \qquad (15)$$

The equivalent thermal noise pressure is therefore

$$p^2_{TR} = \frac{4\pi KT\rho c}{\lambda^2}\left(1 + \frac{(1+Q_m^{\,2}\eta_1^{\,2})}{\omega_1 k_e^{\,2} C_{LF} Q_m R_2}\right)$$

$$= \frac{4\pi KT\rho f^2}{c} \left(1 + \frac{(1+Q_m^{\,2}\eta_1^{\,2})}{\omega_1 k_e^{\,2} C_{LF} Q_m R_2} \right) \qquad (16)$$

$$\approx \frac{4\pi KT\rho f^2}{c} \left(1 + \frac{Q_m \eta_1^{\,2}}{\omega_1 k_e^{\,2} C_{LF} R_2} \right) \qquad (17)$$

for $Q_m^{\,2}\eta_1^{\,2} >> 1$.

Now, if R_2 is effectively a constant resistor across the hydrophone - e.g., the input resistance of an amplifier - it is convenient to write

$$\omega_c = \frac{1}{C_{LF} R_2}$$

i.e., ω_c is the roll-off (angular) frequency of the hydrophone capacitance in parallel with R_2. Then,

$$p_{TR}^2 = \frac{4\pi KT\rho f^2}{c} \left(1 + \frac{Q_m}{k_e^{\,2}} \frac{\omega_c}{\omega_1} \eta_1^{\,2} \right) \qquad (18)$$

Substituting the values $K = 1.38 \times 10^{-23}$ J/deg K, $T \approx 280$°K, $\rho = 10^3$ kg/m^3, $c = 1.5 \times 10^3$ m/sec, this gives

$$p_{TR}^2 = 3.24 \times 10^{-14} F^2 \left(1 + \frac{Q_m}{k_e^{\,2}} \frac{F_c}{F_1} \eta_1^{\,2} \right) \ (\mathrm{Pa})^2 \qquad (19)$$

where all the frequencies are in kHz (including Fc, the roll-off frequency and F_1, the resonance frequency).

Thus, the rms noise pressure (in a 1 Hz band) is given by

$$p_{TR}^2 = 1.8 \times 10^{-7} F \left(1 + \frac{Q_m}{k_e^2} \; \frac{F_c}{F_1} \; \eta_1^{\,2} \right)^{1/2} \quad \text{(Pa)} \qquad (20)$$

At low frequencies, where $F << F_1$ (so that $\eta_1 \approx -F_1/F$), and also $F^2 << (Q_m/k_e^{\,2})\, F_c F_1$, this simplifies further to

$$(p_{TR})_o \approx 1.8 \times 10^{-7} \left(\frac{Q}{k_e^2} \; F_c F_1 \right)^{1/2} \quad \text{(Pa)} \qquad (21)$$

Thus, at low frequencies, p_{TR} becomes independent of frequency. Converting these expressions to spectrum levels, we obtain: -

from equation (20), $P_{TR} = -15 + 20 \log F + 10 \log \left\{ 1 + \frac{Q_m}{k_e^2} \frac{F_c}{F_1} \eta_1^{\,2} \right\}$ (22)

dB re 1μPa in 1 Hz band.

from equation (21), $(P_{TR})_o \approx -15 + 10 \log \left\{ \frac{Q_m}{k_e^2} \; F_c F_1 \right\}$ (23)

dB re 1μPa in 1 Hz band.

These thermal noise pressure levels should be compared with background noise levels in the sea due to sources other than thermal fluctuations. For example, typical values of background levels in deep water have been given by Knudsen. For sea state 0, Knudsen's values of ambient noise spectrum levels (P_N) are described approximately by

$$P_N = 45 - 17 \log F \quad \text{dB re } 1\mu\text{Pa in 1 Hz band.} \tag{24}$$

(F in kHz).

These values are fairly well accepted for the range 0.8 to 20 kHz, although with quite wide fluctuations, and are often extrapolated towards the thermal noise limiting curve at higher frequencies

Figures 4(a) and 4(b) show examples of noise levels derived from these equations. Knudsen's curve for deep sea state 0 is shown, extrapolated towards 40 kHz. The line sloping upwards to the right shows the thermal noise pressure for a perfectly efficient hydrophone. The remaining curves in figure 4(a) are examples for more realistic hydrophones of how the equivalent noise pressure levels are affected by values of Q_m and the roll-off frequency F_c, all for a hydrophone resonant at 100 kHz. Figure 4(b) illustrates also the effect of varying the resonance frequency (F_1). These curves show how the thermal noise is increased by the effect of additional electrical losses, so that it may become impossible to measure the sea background noise, particularly in the 30-50 kHz range, if the system is badly designed.

Equations (22) and (24) show that thermal noise will be less than ambient noise (for sea state 0) if

$$-15 + 20 \log F + 10 \log \left\{ 1 + \frac{Q_m}{k_e^{\,2}} \; \frac{F_c}{F_1} \; n_1^{\,2} \right\} < 45 - 17 \log F$$

$$\text{i.e., if} \quad \frac{Q_m}{k_e^{\,2}} F_c F_1 < \frac{10^6}{F^{1.7}} - F^2 \qquad (25)$$

In order to keep thermal noise below sea noise, it is therefore desirable to keep Q_m, F_c and F_1 low, and to have a high value of k_e. Figure 5(a) shows a graph of $10^6/F^{1.7} - F^2$ vs F, together with a nomogram (figure 5(b)) for calculating $Q_m F_c F_1/k_e^{\,2}$. For any set of values of these transducer parameters, calculate $Q_m F_c F_1/k_e^{\,2}$. Sea state 0 noise levels will then be above the thermal noise background for frequencies up to that at which this value of $Q_m F_c F_1/k_e^{\,2}$ is equal to the plotted value of $10^6/F^{1.7} - F^2$.

The parallel resistance R_2 is generally a combination of a resistor representing the dielectric loss in the ceramic and a resistor representing the impedance of any circuit connected to the hydrophone. In the above equations, R_2 is assumed to be independent of frequency. However, if the input impedance of any circuit connected across the ceramic is high compared with the dielectric loss resistor, then R_2 is better represented by a resistor which varies with frequency to keep the dielectric loss per cycle ($\tan \phi$) constant.

For this case, we assume

$$R_2 = \frac{1}{\omega C_{LP} \tan\delta} = \frac{1}{\omega C_{LP}\Delta}, \text{ writing } \tan\delta \equiv \Delta \qquad (26)$$

Then, from equation (17),

$$p_{TR}^2 = \frac{4\pi KT\rho f^2}{c} \left\{ 1 + \frac{Q_m \eta_1^2 \omega\Delta}{\omega_1 k_e^2} \right\}$$

$$= 3.24 \times 10^{-14}\, F^2 \left\{ 1 + \frac{Q_m\Delta}{k_e^2} \frac{\omega}{\omega_1} \eta_1^2 \right\} \quad \begin{matrix}(\mathrm{Pa})^2 \\ (F \text{ in kHz})\end{matrix} \qquad (27)$$

$$\text{i.e.} \quad p_{TR} = 1.8 \times 10^{-7}\, F \left\{ 1 + \frac{Q_m\Delta}{k_e^2} \frac{\omega}{\omega_1} \eta_1^2 \right\}^{1/2} \quad \begin{matrix}(\mathrm{Pa, in\ 1} \\ \mathrm{Hz\ band})\end{matrix} \qquad (28)$$

or,

$$P_{TR} = -15 + 20 \log F + 10 \log \left\{ 1 + \frac{Q_m\Delta}{k_e^2} \frac{F}{F_1} \eta_1^2 \right\} \qquad (29)$$

dB re 1μPa in 1 Hz band

Examples of the shape of some typical curves of this type are shown in figures 6(a) and 6(b). For frequencies well below resonance (F_1), these curves become asymptotic to straight lines given by

$$(P_{TR})_o = -15 + 10 \log F + 10 \log \frac{Q_m\Delta}{k_e^2} F_1 \qquad (30)$$

For this type of loss, equations (24) and (29) show that thermal noise will be less than sea state 0 ambient noise if

$$-15 + 20 \log F + 10 \log \left\{ 1 + \frac{Q_m \Delta}{k_e^2} \frac{F}{F_1} \eta_1^2 \right\} < 45 - 17 \log F$$

$$\text{i.e., if} \quad \frac{Q_m \Delta}{k_e^2} F_1 < \frac{10^6}{F^{2.7}} - F \qquad (31)$$

Figure 7 shows a graph of $(10^6/F^{2.7}) - F$; in this case, thermal noise will be below sea noise for frequencies up to that at which this plotted value becomes equal to $Q_m \Delta F_1/k_e^2$. As before, the range over which ambient sea noise can be measured is increased by making Q_m and F_1 low and k_e high. In this case, the electrical loss factor Δ (= tan δ) should also be low.

Implications for Hydrophone Design

The above relationships lead to the following considerations for hydrophones which are intended for listening over a wide frequency band below resonance - i.e., in the region where the sensitivity varies only slowly with frequency. These considerations are applicable to all the usual forms of piezo-ceramic hydrophones, whatever their construction.

If the desired receiving band does not extend above 30 kHz, the resonance frequency of the hydrophone need not be higher than 50 kHz, and it is then clear from figures 4 and 6 that thermal noise can generally be appreciably below sea noise (for deep sea state 0). This requires only that the efficiency of the hydrophone at resonance is reasonably high, and that thermal noise is not unduly increased by an excessively low value of the parallel resistor R_2. Thus, in this low frequency region, the hydrophone efficiency is not of great significance. [The band for which this is true extends to higher frequencies if the criterion is the measurement of noise in higher sea states, but we consider here only the more usual requirement to measure deep sea state 0. Conversely, the somewhat lower sea noise levels quoted by Wenz (Ref 7) would restrict the band to lower frequencies.]

If the receiving band extends above about 30 kHz, however, thermal noise will generally become significant, even for a perfectly efficient hydrophone. In order to minimise the increase in thermal noise due to hydrophone inefficiency, Q_m and F_1 should be made as low as possible, and k_e as high as possible. If the losses are predominantly due to the ceramic, behaving as tan δ losses (i.e., losses proportional to frequency), the ceramic should have as low a value of tan δ as can be achieved (equation (29)). If the losses are primarily those associated with a constant parallel resistor (R_2), then the value of R_2 should be made high, so that the roll-off frequency (F_c) is low. (Equation (22)). The first consideration is thus to make the resonance frequency as low as is consistent with the highest measuring frequency required. (Although there is little benefit in reducing F_1 below about 40 kHz from this aspect.)

Typically, a resonance frequency 50% above the high end of the frequency range will permit a rise in sensitivity of only 3-4 dB within the band.

It would appear from equation (22) and figure 4 that the thermal noise may always be reduced by making F_c lower, - i.e., by making R_2 or C_{LF} sufficiently high. There are, however, practical difficulties in increasing the hydrophone capacitance indefinitely for a given resonance frequency, and also in making the input impedance of an amplifier indefinitely large. Of these two possibilities, it is preferable to increase the input impedance as much as is feasible, since increasing the hydrophone capacitance is generally associated with reducing its sensitivity. This may then lead to difficulties arising from electronic noise in the amplifier, which has so far been ignored, but which will be considered in the next section.

III. AMPLIFIER NOISE

Noise originating in any receiving amplifier has to be added to the noise output from the hydrophone in determining the background level. Suppose that the noise output of a pre-amplifier is measured, with a (noiseless) capacitor of the same value as the hydrophone capacitance connected across the input in place of the hydrophone. This noise output may be referred to the input by dividing its level by the gain of the pre-amplifier measured by injecting a voltage in series with the capacitor. Let this rms noise spectrum level, referred to the input, be denoted by V_E. Then this may be compared with the hydrophone output, both being functions of frequency.

We have found it convenient so far to refer all noise levels to their equivalent pressure levels at the hydrophone. In order to continue with this approach, we therefore divide the amplifier noise voltage by the hydrophone sensitivity, thus obtaining the rms equivalent noise pressure level (p_E), or its corresponding spectrum level ($P_E = 20 \log p_E$). It is evident that ambient sea noise will only be measurable if the electronic noise is sufficiently low and the hydrophone sensitivity sufficiently high. There are however practical limits on both these factors. Rijnja (Ref 3) has considered the question of input impedance and noise in hydrophone amplifiers, and quotes a typical noise spectrum level for a good amplifier as -160 dB re 1 volt for the range 10 Hz - 100 kHz.(or -170 dB re 1 volt for a very quiet amplifier). This value refers to the noise measured with the input short-circuited - i.e., that part of the noise not arising from the thermal noise of the input resistor, - and was taken to be independent of frequency. Measurements at AUWE on a low-noise pre-amplifier using an FET (Texas Instruments Type E8000/117) input have given the following typical noise spectrum levels, referred to the input: -

Frequency (kHz)	0.5	1.0	2.0	4.0	8.0	16	32
Noise spectrum level (dB re 1 volt in 1 Hz band)	-167	-169	-172	-173	-176	-179	-182

These values were derived from measurements of one-third octave output noise levels, the gain of the complete amplifier being 34 dB for all frequencies in this range. It will be noted that the noise spectrum level generally rises by 2-3 dB per octave as frequency is reduced. These figures represent about the lowest noise levels obtained up to the present at AUWE.

An expression for the pressure sensitivity of a hydrophone may be derived as follows. Consider firstly a balanced piston-type hydrophone, as shown diagrammatically in figure 8. In this type of construction the front piston mass (M_p) is equal to the rear mass, and the element is supported at its "nodal" point half-way along the ceramic stack, in order to reduce the output due to case accelerations. Then, for a design in which all the rings are electrically in parallel, the low-frequency pressure sensitivity (m) of the hydrophone is given by

$$m = \frac{1}{2} \frac{A_p}{A_c} g_{33} t \tag{32}$$

where A_p = area of front piston presented to the water

A_c = cross-sectional area of piezo-ceramic stack

t = thickness of each piezo-ceramic ring

g_{33} = piezoelectric "g-constant" for the ceramic, assumed here to be used in the thickness (or "33") mode.

The low frequency capacitance (C) of the hydrophone is given by

$$C = \varepsilon \frac{A_c}{t} n \tag{33}$$

where ε is the absolute free permittivity of the ceramic (i.e., $\varepsilon = \varepsilon_r \varepsilon_o$), and n is the number of rings in the stack.

If the front and rear halves of the stack are connected electrically in series instead of in parallel, the sensitivity is doubled and the capacitance reduced by a factor of four. Thus the factor m^2C remains constant, and this is true also for other changes in the same basic stack (e.g., dividing the stack into a larger number of rings). In fact, this parameter m^2C is proportional to the stored electrical energy for a given incident acoustic field (and to hhe maximum power output from the hydrophone); from equations (32) and (33), it is given by

$$m^2C = \frac{1}{4} \left(\frac{A_p}{A_c}\right)^2 g_{33}{}^2 \quad t^2 \quad \varepsilon \quad \frac{A_c}{t} \quad n$$

$$= \frac{1}{4} \left(\frac{A_p}{A_c}\right)^2 g_{33}{}^2 \quad \varepsilon \quad V_c \tag{34}$$

where $V_c = A_c nt$ = volume of ceramic stack.

Thus, for geometrically scaled stacks, having the same value of (A_p/A_c), the parameter m^2C is proportional to the ceramic volume. But this is also related to the resonance frequency of the hydrophone, the relationship being readily derived for hydrophones satisfying the "lumped mass" approximation; - i.e., where the dimensions of all parts of the element are small compared with the wavelength of sound in the materials concerned. For such conditions, we may write

$$\omega_r^2 = \frac{E_s A_c}{2ntM_p} \tag{35}$$

where ω_r = angular resonance frequency

and E_s = effective Young's modulus of the stack (including joints etc.)

Then, substituting for A_c,

$$m^2C = g_{33}^2 \ \epsilon \ E_s \ \frac{A_p^2}{8M_p\omega_r^2} \tag{34a}$$

We now use the approximate relationship for the mechanical Q-factor (Q_m) of a balanced element,

$$Q_m = \frac{\omega_r \ 2M_p}{R_r} \tag{36}$$

where R_r = radiation resistance at resonance, and we neglect internal mechanical losses and any radiation reactance. Further, we may write the radiation resistance as

$$R_r = \rho c A_p X$$

where ρ, c are the density and speed of sound in water, and X is a factor which depends only on the ratio of piston diameter to wavelength of sound in water at ω_r. Thus,

$$m^2C \simeq g_{33}{}^2 \; \epsilon \; E_s \; \frac{A_p}{4\rho c X Q_m \omega_r} \qquad (37)$$

Approximate values may be given to some of the constants in this relationship, viz: -

For lead zirconate titanate (e.g. PZT-4):

$$g_{33} = 25 \times 10^{-3} \text{ Vm/N}$$

$$\epsilon = \epsilon_r \epsilon_o = 1300 \times 8.85 \times 10^{-12} = 1.15 \times 10^{-8} \text{ F/m}$$

$$E_s = 6 \times 10^{10} \text{ N/m}^2$$ (≃ 90% of Young's modulus for ceramic, to allow for joints etc.)

$$\rho c = 1.5 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$$

Provided that the piston diameter exceeds 0.4λ at resonance, which is often true, then $0.6 < X < 1.3$. Also, typical values of Q_m lie in the range 10-20 for hydrophones with reasonably light pistons. Thus, to an order of magnitude,

$$m^2C \simeq 8 \times 10^{-10} \frac{A_p}{f_1} \quad (V^2F\ Pa^{-2}) \qquad (38)$$

where m is in V/Pa, C is in Farads, A_p is in m^2, and f_1 is the resonance frequency in Hz. Converting this to more convenient units of

m in $\mu V/Pa$

C in pF

A_p in mm^2

F_1 in kHz

we obtain $(m^2C)_1 \simeq 8 \times 10^5 \; \dfrac{A_p}{F_1} \left\{ \left(\dfrac{\mu V}{Pa}\right)^2 \; pF \right\}$ (38a)

[Note also that m in $\mu V/Pa$ = (m in $\mu V/\mu b$) x 10]

It will often be a design feature that the piston diameter is approximately a constant factor of the wavelength at resonance. In that case, we may write, to a still further degree of approximation,

$$A_p \simeq \frac{K'}{F_1{}^2} \tag{39}$$

where K' is given by

$$K' = \frac{\pi c^2}{4} \left(\frac{D}{\lambda_r}\right)^2 = 1.77 \times 10^6 \left(\frac{D}{\lambda_r}\right)^2 \tag{39a}$$

in which D is the piston diameter, and λ_r is the wavelength of sound in water at F_1. For designs satisfying this criterion, we find

$$m^2C \simeq 8 \times 10^{-10} \; \frac{K'}{F_1{}^3} \tag{40}$$

If $D/\lambda_r \simeq 0.5$, then $K' \simeq 4.4 \times 10^5 \; m^2s^{-2}$, and $m^2C \simeq 3.4 \times 10^{-4}/f_1{}^3$
(If $D/\lambda_r = 1$, $m^2C \simeq 0.14 \times 10^{-2}/f_1{}^3$, i.e. $m\sqrt{C} \simeq 0.04 f_1{}^{-3/2}$ compared with Rijnja's $m\sqrt{C} = 0.1 f_1{}^{-3/2}$)

in more convenient units, as above, equation (40) becomes

$$(m^2C)_1 = 8 \times 10^5 \frac{K'}{F_1^{\,3}} \tag{40a}$$

where m is in μV/Pa, C is in Farads, F_1 is in kHz, K' has the value used in equation (39), and $D/\lambda_r = 0.5$. Table I gives data for several AUWE hydrophones (using LZT ceramic), showing the degree of agreement with the above relationships. Equations (34) and (38a) give reasonable agreement with practical values; agreement with values calculated for $(D/\lambda_r) = 0.5$ is predictably less good, but still well within an order of magnitude.

Equation (38) shows that, for elements satisfying the stated assumptions, the value of m^2C depends on the piston area and resonance frequency, and is independent of the number of rings and the volume of ceramic in the stack. Thus, if the resonance frequency is specified, there is no advantage to be gained from using a larger volume of ceramic. The factor m^2C is maximised by reducing the resonance frequency as much as possible, and by making A_p as large as is consistent with the required application. In general, m^2C is increased by a factor of roughly $(1/F_1^{\,3})$ as F_1 is reduced. (As in equation (40).) Approximate values of $(m^2C)_1$ calculated from equation (38a) are indicated in figure 9.

We have thus determined approximate values of m^2C for balanced piston-type elements. The sensitivity of a hydrophone may be increased, in order to raise the response to acoustic signals above thermal noise, but only at the expense of lowering the capacitance (assuming that A_p/F_1 is as large as possible). This reduction in capacitance may then become harmful for two reasons: -

1. The roll-off frequency (F_c) will be raised, for a given input resistance, and this will cause increased thermal noise, as illustrated in figure 4.

2. Cable capacitance in parallel with the hydrophone causes some loss in sensitivity, which becomes appreciable if the hydrophone capacitance falls to a value comparable with that of the cable. Excessive loss is generally avoided by keeping the hydrophone capacitance at least equal to that of any necessary cable (see Ref 3).

The relevance of (1) above to hydrophone design is illustrated in figure 10. Consider a lead zirconate titanate hydrophone of piston diameter D = 20 mm and resonance frequency F_1 = 100 kHz. Then, from figure 9, its value of m^2C is approximately 2.5×10^6 $(\mu V/Pa)^2 pF$. Suppose firstly that we divide the ceramic in such a way that it has a capacitance of 1000 pF. Then its pressure sensitivity m will be $(2.5 \times 10^3)^{1/2}$ = 50 µV/Pa. The amplifier noise levels may then be converted to equivalent pressure levels, by dividing the noise voltage levels by the above sensitivity figure. The corresponding noise spectrum levels are plotted as the dashed line on figure 10 and are roughly equal to deep sea state 0 noise levels. Suppose also that the resistance (R_2) effectively in parallel with the hydrophone is 0.5 megohm. Then $F_c = 10^{-3} / 2\pi C_{LP} R_2 = 0.32$ kHz. Suppose further that the hydrophone has (typical) values of $Q_m = 15$, $k_e = 0.5$; then

$$\frac{Q_m}{k_e^2} F_c F_1 = 1900$$

Using figure 5a, we see that thermal noise would be less than sea state 0 noise up to about 30 kHz, and the curve plotted in figure 10 for the above parameters shows that the hydrophone thermal noise exceeds ambient sea noise by only a small margin even above 30 kHz. The total background would be obtained by combining the curves for amplifier noise and thermal noise by adding the two power contributions. It is evident from figure 10 that this choice of parameters is such that ambient noise for sea state 0 will be just about measurable from both hydrophone thermal noise and amplifier noise aspects over much of the frequency band, although sea state 1/2 noise would be more readily measured over the whole band.

Suppose, however, that we had chosen to make the hydrophone capacitance 10,000 pF (instead of 1,000 pF). In that case, the pressure sensitivity would be $(2.5 \times 10^2)^{1/2} = 15.8$ µV/Pa, and the amplifier noise equivalent spectrum levels would be increased by 10 dB, thus being significantly above the sea state 0 spectrum level (as shown in figure 10). F_c will be lower than before, and hence hydrophone thermal noise will be even further reduced.

On the other hand, suppose that we had chosen to make the hydrophone capacitance only 100 pF. In that case, the pressure sensitivity would be increased by 10 dB, and the amplifier noise spectrum levels correspondingly reduced. (See figure 10.) F_c would, however, be increased to 3.2 kHz, and this would cause a significant rise in background due to thermal noise over the range 10-70 kHz. In fact, for this example the amplifier noise is generally smaller than the thermal noise for all frequencies down to 1 kHz. The first choice of C = 1000 pF is evidently near the optimum if signals down to sea state 0 are to be measured at frequencies up to nearly 100 kHz. The low capacitance example above could be improved, of course, by increasing the parallel resistor (R_2), if that were possible (e.g., by changing the pre-amplifier design), and a more likely 'optimum' design would therefore probably have a capacitance of 3-500 pF and require an input impedance of the order of 10 megohms.

IV. SPHERICAL HYDROPHONES

Corresponding relationships may be derived for spherical hydrophones, using formulae given in Reference 4. For thin-walled spheres, the sensitivity (m_s) is given by: -

$$m_s = - a.g_{31} \tag{41}$$

where a is the mean radius of the sphere, and g_{31} is the appropriate piezo-electric g-coefficient. The low frequency capacitance (C_s) is:

$$C_s = \frac{4\pi\varepsilon a^2}{t} \quad (1 - t/a) \tag{42}$$

where t is the wall thickness ($t/a << 1$). And the resonance frequency (f_1) is given by

$$f_1 = \frac{1}{2\pi a} \sqrt{\left(\frac{2}{(1-\sigma)} \quad \frac{E_{11}}{\rho_c} \right)} \tag{43}$$

where ρ_c is the density, and σ the Poisson's ratio of the ceramic. Taking a typical value of $(1-t/a) = 0.9$, and material parameter values (for PZT-4) of

$$\varepsilon = 1.15 \times 10^{-8} \quad \mathrm{F/m}$$

$$E_{11} = 8 \times 10^{10} \ \mathrm{N/m^2}$$

$$\sigma = 0.3$$

$$\rho_c = 7.6 \times 10^{3} \quad \mathrm{kg/m^3}$$

$$g_{31} = -11 \times 10^{-3} \quad \mathrm{Vm/N}$$

we obtain the relationship,

$$m_s^{\,2} \ C_s = 1.57 \times 10^{-11} \quad \frac{a^4}{t} \quad \text{(SI units)} \tag{44}$$

or

$$(m_s^{\,2} C_s)_1 = 1.57 \times 10^{4} \quad \frac{a^4}{t} \quad (\mu\mathrm{V/Pa})^2 \quad \mathrm{pF} \tag{44a}$$

(where a and t are in mm)

Similarly, equation (43) becomes

$$f_1 = \frac{870}{a} \quad (\mathrm{Hz}) \tag{45}$$

Thus,

$$m_s{}^2 C_s = \frac{0.010}{f_1{}^3} \left(\frac{a}{t}\right) \quad \left({}^{V}/_{Pa}\right)^2 F \tag{46}$$

where f_1 is in Hz.

Writing F_1 in kHz, and converting to more convenient units as before,

$$(m_s{}^2 C_s)_1 = \frac{1.0 \times 10^{13}}{F_1{}^3} \left(\frac{a}{t}\right) \left(\frac{\mu V}{Pa}\right)^2 \ pF \tag{46a}$$

For example, a spherical hydrophone using PZT-4 ceramic of 22 mm outside diameter and 1.5 mm wall thickness has the typical values shown below: -

	Calculated	Experimental
Resonance frequency (kHz)	84.9	85-90
Capacitance (pF)	8,600	8,500
Sensitivity (μV/Pa)	110	100
$(m_s{}^2 C_s)_1$	11×10^7	8.5×10^7

Approximate values of $(m_s^2C_s)_1$ for thin-walled spheres are plotted in figure 11. This corresponds to figure 9 for a piston-type transducer, and can be used as described above to assess the noise background for various choices of m_s and C_s. For a spherical hydrophone, the only simple re-arrangements of the ceramic are to connect the hemispheres (from which the sphere is generally assembled) either in series or in parallel. Apart from the resonance frequency, the only other parameter to vary is the wall thickness, the value of $m_s^2C_s$ being increased as the thickness is decreased (i.e., as a/t is increased). A limit is usually determined by the pressure which the hydrophone has to survive, or by general mechanical fragility.

V. ACCELERATION SENSITIVITY

Another source of noise arises from the sensitivity of a hydrophone to vibrations of the mounting. In order to consider this, we again envisage a hydrophone satisfying the 'lumped-mass' approximation, and with a mechanical arrangement shown diagrammatically in figure 12.

In this diagram, m_1 represents the front piston, m_2 the rear mass, and m_3 any mass at the mounting position in the element. K_1 is the stiffness of the front portion of the ceramic stack, K_2 the stiffness of the rear portion, and K_3 the stiffness of the mounting arrangement, which is attached to the transducer housing. For example, m_3 and K_3 may represent the mass and stiffness of a 'nodal plate' support of a balanced element. A calculation of the actual values of m_3 and K_3 for any particular structure may not be easy, but it is assumed here that the support may be validly represented as shown, at least for the relevant frequency range. The mechanical radiation impedance at the front piston is represented by Z_1; all other losses are neglected.

It is now assumed that the transducer housing is vibrated sinusoidally (with angular frequency ω) in an axial direction, the displacement of the housing being denoted by x_o. The resulting axial displacements of m_1, m_2, and m_3 are represented (as shown) by x_1, x_2, and x_3, respectively, and the compressive forces in the springs are denoted by F_1, F_2, and F_3.

Then, the equations of motion are: -

For m_1, $$m_1\ddot{x}_1 = F_1 - Z_1\dot{x}_1 \qquad (47a)$$

For m_2, $$m_2\ddot{x}_2 = -F_2 \qquad (47b)$$

For m_3, $$m_3\ddot{x}_3 = F_3 + F_2 - F_1 \qquad (47c)$$

Also, $$F_1 = K_1\,(x_3 - x_1) \qquad (48a)$$

$$F_2 = K_2\,(x_2 - x_3) \qquad (48b)$$

$$F_3 = K_3\,(x_o - x_3) \qquad (48c)$$

Let $x_o = X_o e^{j\omega t}$, $x_1 = X_1 e^{j\omega t}$, $x_2 = X_2 e^{j\omega t}$, $x_3 = X_3 e^{j\omega t}$, where X_1, X_2, and X_3 may be complex.

Then, $\dot{x}_1 = j\omega x_1$, $\ddot{x}_1 = -\omega^2 x_1$, etc.

After some algebra, we obtain: -

$$\frac{x_3}{x_2} = 1 - \frac{\omega^2 m_2}{K_2}$$

$$\frac{x_3}{x_1} = 1 - \frac{\omega^2 m_1}{K_1} + j\frac{\omega Z_1}{K_1}$$

$$\frac{x_3}{x_o} = K_3 \left\{ K_1 + K_2 + K_3 - \omega^2 m_3 - \frac{K_1}{1 - \omega^2 \frac{m_1}{K_1} + j\frac{\omega Z_1}{K_1}} - \frac{K_2}{1 - \frac{\omega^2 m_2}{K_2}} \right\}^{-1}$$

These may be simplified by writing: -

$$\omega_2{}^2 = \frac{K_2}{m_2}, \quad \text{and} \quad n_2{}^2 = \frac{\omega^2}{\omega_2{}^2} \tag{49a}$$

$$\omega_1{}^2 = \frac{K_1}{m_1}, \quad \text{and} \quad n_1{}^2 = \frac{\omega^2}{\omega_1{}^2} \tag{49b}$$

$$Z = \frac{\omega Z_1}{K_1} \tag{49c}$$

Then,
$$\frac{x_2}{x_3} = \frac{1}{1 - n_2^2} \tag{50a}$$

$$\frac{x_1}{x_3} = \frac{1}{1 - n_1^2 + jZ} \tag{50b}$$

$$\frac{x_3}{x_o} = \frac{K_3}{K_1 + K_2 + K_3 - \omega^2 m_3 - \dfrac{K_1}{1-n_1^2+jZ} - \dfrac{K_2}{1-n_2^2}} \tag{50c}$$

The compression of the front part of the stack is given by: -

$$\frac{x_3 - x_1}{x_3} = 1 - \frac{1}{1-n_1^2+jZ}$$

$$= \frac{Z^2 - n_1^2(1-n_1^2) + jZ}{(1-n_1^2)^2 + Z^2} \tag{51a}$$

Similarly, the compression of the rear part is given by: -

$$\frac{x_2 - x_3}{x_3} = \frac{n_2^2}{1-n_2^2} \tag{51b}$$

And the compression of the whole stack is given by: -

$$\frac{x_2 - x_1}{x_3} = \frac{n_2^{\,2}-n_1^{\,2} + jZ}{(1-n_2^{\,2})(1-n_1^{\,2} + jZ)}$$

$$= \frac{(1-n_1^{\,2})(n_2^{\,2}-n_1^{\,2}) + Z^2 + jZ\ (1-n_2^{\,2})}{(1-n_2^{\,2})\ \{(1-n_1^{\,2})^2 + Z^2\}} \tag{51c}$$

These expressions may now be related to the driving excitation x_o and by using equation (50c). To simplify the results, we write

$$\omega_3^{\,2} = \frac{K_3}{m_3} \quad \text{and} \quad n_3^{\,2} = \frac{\omega^2}{\omega_3^{\,2}} \tag{52a}$$

and also $$K_{13} = \frac{K_1}{K_3}, \quad K_{23} = \frac{K_2}{K_3} \tag{52b}$$

Then, after some further algebra, we obtain the following results: -

$$\frac{x_3}{x_o} = \frac{1-n_2^{\,2}}{A^2+Z^2B} \left[\{(1-n_1^{\,2})A - Z^2B\} + jZ\ \{(1-n_1^{\,2})B + A\}\right] \tag{53}$$

where $$A = (1-n_1^{\,2})(1-n_2^{\,2})(1-n_3^{\,2})-n_1^{\,2}(1-n_2^{\,2})K_{13} - n_2^{\,2}(1-n_1^{\,2})K_{23} \tag{54a}$$

and $$B = (1-n_2^{\,2})K_{13} - n_2^{\,2}K_{23} + (1-n_2^{\,2})\ (1-n_3^{\,2}) \tag{54b}$$

The compression of the front part is given by: -

$$\frac{\Delta_f}{x_o} \equiv \frac{x_3 - x_1}{x_o} = \left(\frac{x_3 - x_1}{x_3}\right) \quad \frac{x_3}{x_o}$$

$$= \frac{(1-n_2{}^2)}{(A^2+Z^2B^2)} \left\{ (-n_1{}^2A - Z^2B) + jZ\ (A-n_1{}^2B) \right\} \quad (55)$$

The compression of hhe rear part is given by: -

$$\frac{\Delta_r}{x_o} \equiv \frac{x_2 - x_3}{x_o} = \frac{n_2{}^2}{(A^2+Z^2B^2)} \left[\{(1-n_1{}^2)A - Z^2B\} + jZ\ \{A+(1-n_1{}^2)B\} \right] \quad (56)$$

If we now relate all the frequencies to ω_1, by writing

$$\omega_2 = \frac{\omega_1}{N_{21}} \qquad \text{and} \qquad \omega_3 = \frac{\omega_1}{N_{31}},$$

then

$$n_2 = \frac{\omega}{\omega_2} = \frac{\omega}{\omega_1} N_{21} = N_{21}n_1 \ \text{(i.e. } N_{21}=n_2/n_1\text{) and } n_3=N_{31}n_1 \quad (57)$$

Thus, we obtain,

$$A = (1-n_1{}^2)(1-N_{21}{}^2n_1{}^2)(1-N_{31}{}^2n_1{}^2)-n_1{}^2(1-N_{21}{}^2n_1{}^2)K_{13}$$
$$-N_{21}{}^2n_1{}^2(1-n_1{}^2)K_{23} \quad (58a)$$

$$B = (1-N_{21}{}^2n_1{}^2)K_{13}-N_{21}{}^2n_1{}^2K_{23}+(1-N_{21}{}^2n_1{}^2)(1-N_{31}{}^2n_1{}^2) \quad (58b)$$

(For front part) $$\frac{\Delta_f}{x_o} = \frac{(1-N_{21}{}^2 n_1{}^2)}{(A^2+Z^2B^2)}\left[(-n_1{}^2A-Z^2B)+jZ(A-n_1{}^2B)\right] \quad (59)$$

(For rear part) $$\frac{\Delta_r}{x_o} = \frac{N_{21}{}^2 n_1{}^2}{(A^2+Z^2B^2)}\left[((1-n_1{}^2)A-Z^2B)+jZ(A+(1-n_1{}^2)B)\right] \quad (60)$$

These springs K_1 and K_2 represent the piezo-electric ceramic in the hydrophone element, although the true ceramic parameters may need modification to allow for the effects of joints, non-rigidity of the masses, etc. The two parts are generally connected electrically (either in series or parallel), and the quantity of interest is the total output voltage. If the two parts are in parallel, the voltage is calculated by adding together the piezo-electric charges generated, and dividing by the total capacitance. The charge generated in each part is given by the relationship

$$\frac{\text{Charge}}{\text{Force}} = d_e \quad (61)$$

where d_e is the effective piezo-electric 'd-coefficient'. For the ceramic itself, the coefficient d_{33} relates the charge per unit area to the applied stress (in Coulombs/Newton).

Thus, the charge generated in the front part is proportional to the longitudinal force in the front spring (F_{af}), i.e., to

$$F_{af} = K_1 (x_3 - x_1)$$

And similarly the charge in the rear part is proportional to

$$F_{ar} = K_2 (x_2 - x_3)$$

The total charge generated by the case acceleration is then proportional to $F_a = F_{af} + F_{ar}$, and the voltage output is

$$V_a = F_a \frac{d_e}{C} \tag{61a}$$

where C is the total capacitance of the stack.

(For an acceleration-cancelling design, the connections are such that the charge generated by a compressive force in the front part is opposite to that generated by a tensile force in the rear part.)

$$\text{Thus,} \quad V_a = \frac{d_e}{C} \{K_1 (x_3 - x_1) + K_2 (x_2 - x_3)\}$$

$$= \frac{d_e}{C} K_1 \{(x_3 - x_1) + \frac{K_2}{K_1} (x_2 - x_3)\}$$

$$= \frac{d_e}{C} K_1 \{(x_3 - x_1) + \frac{K_{23}}{K_{13}} (x_2 - x_3)\}$$

$$\text{i.e.,} \quad \frac{V_a}{x_o} = \frac{d_e}{C} K_1 \left\{ \frac{\Delta_f}{x_o} + \frac{K_{23}}{K_{13}} \frac{\Delta_r}{x_o} \right\} \tag{62}$$

The behaviour of the output voltage depends on the factor in brackets, which we denote by

$$\frac{\Delta_t}{x_o} \equiv \frac{\Delta_f}{x_o} + \frac{K_{23}}{K_{13}} \frac{\Delta_r}{x_o} \tag{63}$$

It is instructive to consider the behaviour of these expressions ((59), (60) and (63)) for the acceleration outputs when there is no power dissipation, i.e., $Z = 0$; this would correspond approximately to conditions when the hydrophone is vibrated in air. For this case,

$$\left(\frac{\Delta_f}{x_o}\right)_a = (1-N_{21}{}^2 n_1{}^2)\left(\frac{-n_1{}^2}{A}\right)$$

$$= \frac{n_1{}^2}{A}\,(N_{21}{}^2 n_1{}^2-1) \qquad (64)$$

$$\left(\frac{\Delta_r}{x_o}\right)_a = \frac{N_{21}{}^2 n_1{}^2}{A}\,(1-n_1{}^2) \qquad (65)$$

and for the total output

$$\left(\frac{\Delta_t}{x_o}\right)_a = \frac{n_1{}^2}{A}\,(N_{21}{}^2 n_1{}^2-1) + \frac{K_{23}}{K_{13}}\,\frac{N_{21}{}^2 n_1{}^2}{A}\,(1-n_1{}^2) \qquad (66)$$

The output will become large when $A \to 0$.

i.e., when $(1-n_1{}^2)(1-N_{21}{}^2 n_1{}^2)(1-N_{31}{}^2 n_1{}^2)-n_1{}^2(1-N_{21}{}^2 n_1{}^2)K_{13}$

$$-N_{21}{}^2 n_1{}^2(1-n_1{}^2)K_{23} = 0.$$

For a balanced element (having $N_{21} = 1$ and $K_{13} = K_{23}$), this condition becomes

$$(1-n_1^2)\{(1-n_1^2)(1-N_{31}^2 n_1^2)-n_1^2\ 2K_{13}\} = 0$$

i.e., peak output occurs when $n_1^2 = 1$

$$\text{or when } (1-n_{10}^2)(1-N_{31}^2 n_{10}^2)-n_{10}^2\ 2K_{13} = 0 \tag{67}$$

(writing n_{10} for this particular value of n_1).

$$\text{i.e., when } n_{10}^2 = \frac{(1+N_{31}^2+2\ K_{13}) \pm \{(1+N_{31}^2+2\ K_{13})^2 - 4N_{31}^2\}^{1/2}}{2N_{31}^2} \tag{67a}$$

Unless $K_3 >> K_1$, and the stack and mounting resonances are approximately equal (giving $N_{21} \approx 1$) (i.e., provided only that $1+N_{31}^2+2K_{13} >> 2N_{31}$), the value of n_{10} below the stack resonance ω_1 may be written approximately as

$$n_{10}^2 \approx \frac{1}{N_{31}^2+1+2K_{13}} \tag{68}$$

It can readily be shown that this corresponds to the frequency at which the total mass $(m_1+m_2+m_3)$ resonates on the spring K_3, a frequency at which a peak in the response would indeed be expected. When some dissipation is present, the peak is of course lowered and slightly displaced in frequency.

The value of the radiation impedance factor Z in the equations above may be related to the more convenient mechanical Q-factor (Q_m). Re-writing equation (36) for the Q_m of a balanced hydrophone,

$$Q_m \simeq \frac{\omega_1 \; 2m_1}{R_r}$$

Making the further approximation that the magnitude of Z_1 is equal to its resistive component, we obtain

$$Q_m \simeq \frac{2m_1 \; \omega_1}{Z_1}$$

Then,

$$Z = \frac{\omega Z_1}{K_1} \qquad \text{(from equation (49c))}$$

$$\simeq \frac{\omega \; 2m_1 \; \omega_1}{K_1 \; Q_m}$$

$$= \frac{\omega}{\omega_1} \cdot \frac{2}{Q_m}$$

$$= \frac{2n_1}{Q_m} \qquad (69)$$

This approximate relationship has been used in the following examples, even where the element is not exactly balanced.

The behaviour of Δ_f/x_o, and Δ_r/x_o described by equations (59), (60), and (63) is illustrated for various conditions in figures 13-16. Figure 13 shows how the outputs from front and rear parts oppose each other to give a total output reduced by over 20 dB in the mid-frequency region, for a balanced element with low damping ($Q_m = 100$). For this example, the value of n_{10} given by equation (68) is 0.035, and peaks in the output occur near this value and $n_1 = 1.0$. For lower values of Q_m (which would be more typical for a hydrophone in water), the degree of cancellation is lower, as illustrated in figure 14 for an element having $Q_m = 10$. This arises, of course, because the radiation loading on one end disturbs the balance.

The cancellation may also be spoiled by mechanical unbalance of the front and rear parts of the element. For example, unbalance of the end masses gives $N_{21} \neq 1.0$ (but with $K_{13} = K_{23}$) and figure 15 shows the large increase in output caused by putting $N_{21} = 1.1$ (i.e., masses approximately 20% different) for a high-Q_m element (Curves A and B). There is, however, a much less dramatic effect on the output when the balance has already been spoiled by the larger radiation loading for a lower-Q_m element (see figure 16). Similar effects are caused also by an inequality of the stiffness instead of the masses (Curves C in figure 15 and 16). (In this case, $N_{21}{}^2 = \omega_1{}^2/\omega_2{}^2 = (K_1/m_1)/(K_2/m_2) = K_{13}/K_{23}$, if the masses m_1 and m_2 are equal.)

For a balanced stack in which the dissipation is zero (i.e., $Z = 0$) and the resonances of front and rear portions are equal, (i.e., $N_{21} = 1$), equation (66) for the total power output becomes

$$\left(\frac{\Delta_t}{x_o}\right)_a = \frac{n_1^{\,2}}{A} \quad (1-n_1^{\,2}) \ \left(\frac{K_{23}}{K_{13}} - 1\right) \qquad (66a)$$

Thus, making the resonances equal is not sufficient by itself to give zero output. However, if the stiffnesses are also equal (and hence also the masses), the expression above becomes zero for all frequencies, and the element is then perfectly balanced.

The values of $Q_m = 100$ and $Q_m = 10$ are typical of values which might be found for aluminium piston-type hydrophones in air and water respectively. The general conclusions for broad-band hydrophones to be drawn from the results may be summarised as follows:

(a) A fair degree of cancelling of the output in air due to case accelerations is possible over a range of frequencies below resonance, by using a balanced element mounted at its mid-point.

(b) The efficacy of the cancelling is determined by the accuracy of the balancing. Radiation loading at one end of the element may reduce the cancelling to only about 5 dB in water (for $Q_m = 10$). Unbalance of the masses or stiffnesses of about 10% would then make only small increases in the acceleration output. Unbalance of the masses or stiffnesses of about 10% would cause readily perceptible effects for elements with less damping, and hence makes measurements in air reasonably practicable.

(c) The acceleration response becomes large at a frequency which is near to the resonance of the total mass of the element on the compliance of its mount. This usually represents the low frequency limit of the useful band of the hydrophone, the element's resonance frequency (ω_1) determining the upper limit. It is therefore generally advisable to make the compliance of the 'nodal' mount as large as practicable. It will be seen in the next section that undesirable effects occur also in the element's pressure response in the vicinity of the mounting resonance. For some applications, other constraints such as smoothness of the piston face in its case, or high ambient pressure, demand a relatively low value of the compliance of the mounting plate, and in such cases a compromise must be reached between the conflicting requirements.

VI. PRESSURE SENSITIVITY

The voltage output due to accelerations should always be compared with the voltage output due to the desired pressure signal, since clearly a higher acceleration sensitivity can be tolerated if the pressure sensitivity is also high. In this section, equations are given for the pressure sensitivity of a hydrophone such as that shown diagrammatically in figure 12. In this case, however, the mounting displacement x_o is made zero, and a force F_o sin ωt is applied axially (downwards) to the front piston (m_1). This derivation is intended to investigate particularly the frequency variation of the pressure sensitivity, taking into account the effect of the modal mounting, and its relationship to the earlier expressions (e.g., equation (32)) will be discussed later.

In this case the equations of motion for sinusoidal displacements are: -

$$\text{For } m_1, \quad (K_1 - m_1 \omega^2)\, x_1 = K_1 x_3 - F_o \tag{70}$$

$$\text{For } m_2, \quad (K_2 - m_2 \omega^2)\, x_2 = K_2 x_3 \tag{71}$$

$$\text{For } m_3, \quad (K_1 + K_2 + K_3 - m_3 \omega^2)\, x_3 = K_2 x_2 + K_1 x_1 \tag{72}$$

These simultaneous equations may be solved as in the previous section, or by writing them as: -

$$\begin{aligned}
(K_1 - m_1 \omega^2)\, x_1 \qquad\qquad\qquad\qquad & - K_1 x_3 = -F_o \\
(K_2 - m_2 \omega^2)\, x_2 \qquad & - K_2 x_3 = 0 \\
K_1 x_1 \quad + \quad K_2 x_2 \quad & - (K_1 + K_2 + K_3 - m_3 \omega^2)\, x_3 = 0
\end{aligned}$$

The solution for x_1 is then given by $x_1 = D_1/D$, where

$$D = \begin{vmatrix} K_1-m_1\omega^2 & 0 & -K_1 \\ 0 & K_2-m_2\omega^2 & -K_2 \\ K_1 & K_2 & m_3\omega^2-(K_1+K_2+K_3) \end{vmatrix} \tag{73}$$

and

$$D_1 = \begin{vmatrix} -F & 0 & -K_1 \\ 0 & K_2-m_2\omega^2 & -K_2 \\ 0 & K_2 & m_3\omega^2-(K_1+K_2+K_3) \end{vmatrix} \tag{74}$$

Simplifying, we obtain: -

$$D = K_1K_2K_3[K_{23}n_2^{\,2}(1-n_1^{\,2})+K_{13}n_1^{\,2}(1-n_2^{\,2})-(1-n_1^{\,2})(1-n_2^{\,2})(1-n_3^{\,2})] \tag{75}$$

where $\quad n_1^{\,2} = \omega^2/\omega_1^{\,2}, \qquad n_2^{\,2} = \omega^2/\omega_2^{\,2}, \qquad n_3^{\,2} = \omega^2/\omega_3^{\,2},$

$K_{13} = K_1/K_3, \qquad K_{23} = K_2/K_3, \qquad$ as before.

Also, $D_1 = F_o\ K_2K_3[(1-n_2^{\ 2})(1-n_3^{\ 2})-K_{23}n_2^{\ 2}+K_{13}(1-n_2^{\ 2})]$ (76)

Similarly, $x_2 = D_2/D$, where

$$D_2 = \begin{vmatrix} K_1-m_1\omega^2 & -F & -K_1 \\ 0 & 0 & -K_2 \\ K_1 & 0 & m_3\omega^2-(K_1+K_2+K_3) \end{vmatrix}$$

$$= F_o\ K_1K_2 \tag{77}$$

and $x_3 = D_3/D$, where

$$D_3 = \begin{vmatrix} K_1-m_1\omega^2 & 0 & -F \\ 0 & K_2-m_2\omega^2 & 0 \\ K_1 & K_2 & 0 \end{vmatrix}$$

$$= F_o\ K_1K_2(1-n_2^{\ 2}) \tag{78}$$

The compression of the front spring is then given by: -

$$\frac{x_3 - x_1}{F_o} = \frac{D_3 - D_1}{F_o D}$$

$$= \frac{K_2 K_3}{D} \{K_{23} n_2^2 - (1-n_2^2)(1-n_3^2)\} \tag{79}$$

and the compression of the rear part by

$$\frac{x_2 - x_3}{F_o} = \frac{D_2 - D_3}{F_o D}$$

$$= \frac{K_1 K_2 n_2^2}{D} \tag{80}$$

As before, the charge generated piezo-electrically in the front part is proportional to the force in the front spring (F_{pf}), i.e., to

$$\frac{F_{pf}}{F_o} = \frac{K_1 (x_3 - x_1)}{F_o}$$

$$= \frac{K_{23} n_2^2 - (1-n_2^2)(1-n_3^2)}{K_{23} n_2^2 (1-n_1^2) + K_{13} n_1^2 (1-n_2^2) - (1-n_1^2)(1-n_2^2)(1-n_3^2)} \tag{81}$$

Similarly, the force in the rear spring is: -

$$\frac{F_{pr}}{F_o} = \frac{K_2(x_3-x_2)}{F_o}$$

$$= \frac{K_{23}n_2^{\,2}}{K_{23}n_2^{\,2}(1-n_1^{\,2})+K_{13}n_1^{\,2}(1-n_2^{\,2})-(1-n_1^{\,2})(1-n_2^{\,2})(1-n_3^{\,2})} \quad (82)$$

And the total output voltage is proportional to $F_p = F_{pf} + F_{pr}$,

i.e., to $$\frac{F_p}{F_o} = \frac{2K_{23}n_2^{\,2} - (1-n_2^{\,2})(1-n_3^{\,2})}{K_{23}n_2^{\,2}(1-n_1^{\,2})+K_{13}n_1^{\,2}(1-n_2^{\,2})-(1-n_1^{\,2})(1-n_2^{\,2})(1-n_3^{\,2})} \quad (83)$$

These three equations describe the variation with frequency of the output of the hydrophone. For low frequencies, when n_1, n_2, $n_3 \to 0$, the force in the front spring $F_{pf} \to F_o$, and $F_{pr} \to 0$, as would be expected. As the frequency is increased, the rear part makes a larger contribution. If the element is symmetrical, so that $\omega_1 = \omega_2$ and $K_{13} = K_{23}$, then

$$\frac{F_{pf}}{F_o} = \frac{K_{13}n_1^{\,2}-(1-n_1^{\,2})(1-n_3^{\,2})}{2K_{13}n_1^{\,2}(1-n_1^{\,2})-(1-n_1^{\,2})^2(1-n_3^{\,2})} \quad (84)$$

$$\frac{F_{pr}}{F_o} = \frac{K_{13}n_1^{\,2}}{2K_{13}n_1^{\,2}(1-n_1^{\,2})-(1-n_1^{\,2})^2(1-n_3^{\,2})} \quad (85)$$

$$\frac{F_p}{F_o} = \frac{2K_{13}^2 n_1^2 - (1-n_1^2)(1-n_3^2)}{2K_{13}^2 n_1^2 (1-n_1^2) - (1-n_1^2)^2 (1-n_3^2)}$$

$$= \frac{1}{1-n_1^2} \tag{86}$$

Thus, for a perfectly balanced element, the total pressure response is well-behaved, rising smoothly to a peak at the element's resonance frequency (at $n_1 = 1$), although the contributions from the ceramic in front of and behind the support have a more complicated variation. This is illustrated in figure 17.

However, any unbalance in the element immediately reveals in the total output the effect of the mounting spring. For example, figure 18 shows the effect of unbalanced end-masses for an element similar to that considered for figure 16. An irregularity in the response again occurs near the mounting resonance given by equation (68) (i.e., $n_{10} = 0.035$)

VII. RELATIONSHIP BETWEEN ACCELERATION AND PRESSURE RESPONSES

A high sensitivity to case acceleration is not necessarily too serious, if the pressure sensitivity is also high. Thus, it is generally useful to express the acceleration response in terms of the pressure which would cause an equal output voltage. The resultant factor, expressed in $Pa.s^2/m$ (or more commonly in μb/g) is obtained by dividing the acceleration response by the pressure sensitivity. It is worth noting here that the expressions given above are for the relationship between voltage output and the pressure at the transducer's active surface. This is not exactly equal to the "free-field hydrophone sensitivity", which relates the voltage output to the pressure which would have existed at the position of the transducer surface in the absence of the transducer. For a hydrophone which is small compared with a wavelength, the difference between these two sensitivities is small, but for a hydrophone (however small) in a large rigid baffle, the free-field sensitivity may be twice the pressure sensitivity. In this note, we consider only the pressure sensitivity, and relate that to the acceleration sensitivity measured in air.

Using equations (62) and (66), the voltage output for a case vibration of amplitude x_o is given by

$$\frac{V_a}{x_o} = \frac{d_e}{C} K_1 \left(\frac{\Delta_t}{x_o} \right)$$

Thus, for vibration in air,

$$\frac{V_a}{x_o} = \frac{d_e}{C} K_1 \left(\frac{\Delta_t}{x_o} \right)_a$$

$$= \frac{d_e K_1}{CA} \{n_1^2(n_2^2-1) + \frac{K_{23}}{K_{13}} n_2^2(1-n_1^2)\}$$

The acceleration amplitude is given (for a sinusoidal vibration) by

$$\ddot{x}_o = \omega^2 x_o$$

Thus, the acceleration sensitivity in air is: -

$$\frac{V_a}{\ddot{x}_o} = \frac{d_e K_1}{\omega^2 CA} \{n_1^2(n_2^2-1) + \frac{K_{23}}{K_{13}} n_2^2(1-n_1^2)\} \tag{87}$$

This is related to the equivalent pressure by using equations (61a) and (83), remembering that $A = (1-n_1^2)(1-n_2^2)(1-n_3^2)-n_1^2(1-n_2^2)K_{13}-n_2^2(1-n_1^2)K_{23}$. Thus, the pressure sensitivity is: -

$$\frac{V_p}{(F_o/A_p)} = \frac{d_e A_p}{C} \quad \frac{F_p}{F_o}$$

$$= \frac{d_e A_p}{CA} \{(1-n_2^2)(1-n_3^2)-2K_{23}n_2^2\} \tag{88}$$

Equations (87) and (88) then give the acceleration sensitivity in terms of its equivalent pressure (m_a).

$$\text{i.e.,} \quad m_a = \left(\frac{V_a}{\ddot{x}_o}\right) \Big/ \left(\frac{V_p}{(F_o/A_p)}\right)$$

$$= \frac{K_1}{\omega^2 A_p} \left\{ \frac{n_1^2(n_2^2-1) + (K_{23}/K_{13})n_2^2(1-n_1^2)}{(1-n_2^2)(1-n_3^2) - 2K_{23}n_2^2} \right\} \tag{89}$$

For a balanced element the factor in front of the brackets is approximately: –

$$\frac{K_1}{\omega^2 A_p} = \frac{Q_m \rho c X}{2\omega n_1} \quad \left(\text{using } Q_m = \omega_1 \frac{2m_1}{R_1} = \frac{2K_1}{\omega_1 R_1} = \frac{2K_1}{\omega_1 \rho c X A_p}\right)$$

Here, Q_m is the mechanical Q-factor which the hydrophone will have in water, although the acceleration sensitivity is measured in air. Thus,

$$m_a = \frac{Q_m \rho c X}{2\omega_1 n_1^2} \left\{ \frac{(K_{23}/K_{13}) n_2^2 (1-n_1^2) - n_1^2 (1-n_2^2)}{(1-n_2^2)(1-n_3^2) - 2K_{23} n_2^2} \right\} \tag{89a}$$

Inserting values for a typical example (slightly unbalanced), as before,

i.e., $Q_m = 10$, $X = 1.0$, $K_{13} = K_{23} = 300$, $N_{31} = n_3/n_1 = 15$,

$N_{21} = n_2/n_1 = 1.1$, $F_1 = 100$ kHz, $n_1 = 0.1$,

we find $\quad m_a = 0.30$ Pa s^2/m

$\quad = 30$ μb/g

$\quad = 29$ dB re 1 μb/g

The response will be a maximum when

$$(1-n_2^{\,2})(1-n_3^{\,2}) - 2K_{23}n_2^{\,2} = 0 \tag{90}$$

which for a balanced element is again the same frequency as that given by equation (68).

An example of the behaviour of equation (89a) is shown in figure 19. Experimental values of m_a obtained for the hydrophones listed in table I are given in table II. These examples are hydrophones in which some acceleration cancelling has been deliberately incorporated, and much higher values of acceleration response may be observed for other designs.

It is also of interest to consider the question of how low a value of acceleration sensitivity is necessary. Suppose that a small piston hydrophone is mounted rigidly in a large baffle which is vibrating normal to its surface. If the amplitude of the normal velocity is v_o, then the amplitude of the pressure in the water near to the baffle is $p_o = \rho c v_o$. The normal acceleration ($\ddot{x}_o$) of the baffle is related to its velocity (for sinusoidal excitation) by $\ddot{x}_o = \omega v_o$. Thus,

$$p_o = \frac{\rho c \ddot{x}_o}{\omega} \tag{91}$$

Values of pressure and acceleration for water are shown in figure 20. As an example, at 1 kHz, an acceleration of the baffle of 10^{-4} m/sec^2

($\approx 10^{-5}$ g) gives rise to a pressure of 2.4×10^{-2} Pa (= 0.24 μb) near to the baffle. If the acceleration sensitivity of the hydrophone is m_a, then the voltage output due to acceleration of the baffle has an equivalent pressure of $10^{-4}\ m_a$. At this frequency, there is therefore little point in reducing the output significantly below that due to the acoustic pressure in the water (2.4×10^{-2} Pa): - i.e., m_a does not need to be much less than $2.4 \times 10^{-2}/10^{-4} = 240$ Pa.s^2m (= 2.4×10^4 μb/g). In general, the condition is that the acceleration sensitivity need not be reduced much below the level where

$$\ddot{x}_o m_a = p_o$$

$$= \frac{\rho c \ddot{x}_o}{\omega}$$

i.e.,
$$m_a = \frac{\rho c}{\omega} \tag{92}$$

Values of m_a given by this equation are shown in figure 21. There is little virtue in attempting to make the acceleration sensitivity (measured in air) less than say one-tenth of the value shown in figure 21 for the appropriate frequency.

VIII. CONCLUSION

This report discusses various sources of noise in broad-band hydrophones and their implications on hydrophone design. Thermal noise (discussed in Section II) is reduced by making the electro-acoustic efficiency high - i.e., by making the element capacitance and amplifier input impedance high. The effects of amplifier noise (Section III) are reduced by making the hydrophone pressure sensitivity high, and this often competes with the previous requirement to make capacitance high. Figure 10 illustrates the way in which the parameters may be chosen to optimise the performance. For both cases, the resonance frequency should be made as low as possible, and the coupling coefficient as high as possible.

Another common source of noise in hydrophone systems arises from accelerations of the case, and expressions for the acceleration sensitivity of low frequency thickness-mode piezo-ceramic hydrophones are derived in Section V. Improvements in acceleration sensitivity can be achieved by 'balancing' the element about its mounting, although there is a limit to what is feasible or worthwhile. After balancing, the main feature to control is the resonance frequency of the element on its mounting compliance.

Section VI derives some corresponding expressions for the pressure sensitivity of the hydrophone, and in Section VII the relationship between acceleration and pressure sensitivities is discussed. Again, there is a limit to the degree of acceleration cancelling which is necessary for a hydrophone mounted in a baffle.

A hydrophone's acceleration-cancelling performance is better appreciated by expressing its acceleration sensitivity (in air) in terms of its pressure response, and this is considered in Section VII. Provided frequencies around any resonances are avoided, an acceleration sensitivity of less than 1 $Pa.s^2/m$ (= 100 μb/g) should be readily achievable, and this should be sufficiently below the required level shown in figure 21 for accelerations not to cause serious problems when the hydrophone is mounted in a rigid baffle. However, it should not be deduced that the acceleration 'cancelling' is therefore unnecessary, since experience with hydrophones used in isolation or mounted in more practical baffles suggests that some degree of acceleration cancelling is generally advisable.

An Example

As an example, consider a hydrophone which is desired to receive signals with approximately constant sensitivity over the band 0.1 to 70 kHz. We thus make the fundamental stack resonance 100 kHz, and will assume a typical Q_m of 10. Lead zirconate titanate will be used, in the thickness mode, to achieve a high coupling coefficient (k_e = 0.5). From figure 4, thermal noise generated in the hydrophone is insignificant provided $F_c < 0.1$ kHz, and tan $\delta < 0.01$. If the hydrophone is of the balanced piston type, with a piston diameter of 15 mm (= 1λ at 100 kHz), figure 9 shows that $(m^2C)_1 = 1.4 \times 10^6\ (\mu V/Pa)^2 pF$. Acoustic noise levels equivalent to the amplifier noise values given earlier are obtained by dividing the amplifier noise by the hydrophone sensitivity. If measurements down to low sea states are wanted, a suitable choice of parameters appears to be C = 200 pF, m = 83.7 μV/Pa (i.e., 38.5 dB re 1 μV/Pa, = 18.5 dB re 1 μV/μb). Then equivalent noise levels are always below deep sea state 1/2, as shown in figure 22. In order to make $F_c < 0.1$ kHz, the input resistance of the amplifier must be at least 8 megohms (from $\omega_c = 1/C_{LP}R_2$), and the thermal noise contribution is then small. The total (amplifier and thermal) noise is below sea state 1/2 levels up to 50 kHz, and at least 6 dB below sea state 1/2 up to 20 kHz.

For a spherical hydrophone having its resonance at 100 kHz, the value of $(m^2C)_1$ is given by figure 11 as $5 \times 10^7\ (\mu V/Pa)^2$ pF for a/t = 5. From equation (45), the radius of the sphere is 8.7 mm, and if the two halves of the sphere are connected in series the sensitivity is approximately 190 μV/Pa (i.e., 45.6 dB re 1 μV/Pa) and the capacitance 1250 pF. This sensitivity is higher than for the piston hydrophone example, and the equivalent amplifier noise is thus further suppressed. An input resistance of only 1.3 megohm is now required to give $F_c = 0.1$ kHz, and the resulting total noise level is shown as the heavy dashed curve in figure 22. In this case, the improved value of m^2C permits measurements down to sea state 1/2 over virtually the full range up to 70 kHz.

For the piston-type hydrophone, a balanced design to give a reasonable degree of acceleration cancelling would usually be desirable. For $Q_m \approx 10$, balancing of the two halves to within 10% would generally be adequate. The mounting resonance frequency of the element on its nodal support (given by equation (68)) should be below the low frequency edge of the measuring band (i.e., below 0.1 kHz), in order to avoid unwanted effects in both acceleration and pressure sensitivity curves.

Acknowledgement

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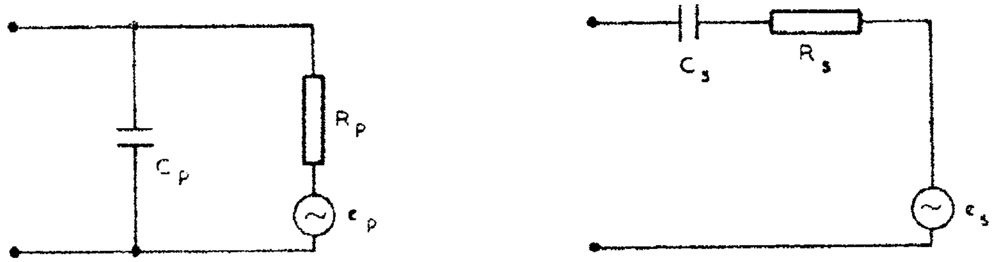
TABLE 1

Hydrophone Type	A	B	C	D
Piston Diameter (mm)	6.36	12.7	25.4	63.5
Resonance Frequency F_1 (kHz)	100	80	60	18
D/λ_r	0.42	0.68	1.0	0.76
A_p (mm^2)	31.7	127	507	3170
Capacitance (pF)	35	150	550	550
Vol. of Ceramic, V_c (mm^3)	320	834	3400	3400
A_p/A_c	1.0	1.11	1.09	6.82
Pressure Sensitivity (exptl) (μV/Pa)	110	100	126	355
$(m^2C)_1$ (exptl) $(\mu V/Pa)^2$ pF	4.2×10^5	1.5×10^6	8.7×10^6	6.9×10^7
$(m^2C)_1$ from eq. (34) $(\mu V/Pa)^2$ pF	5.8×10^5	1.8×10^6	7.3×10^6	28×10^7
$(m^2C)_1$ from eq. (38a) $(\mu V/Pa)^2$ pF	2.5×10^5	1.3×10^6	6.8×10^6	14×10^7
$(m^2C)_1$ from eq. (40a) $(\mu V/Pa)^2$ pF assuming $D/\lambda_r = 0.5$	3.5×10^5	0.7×10^6	1.6×10^6	6×10^7

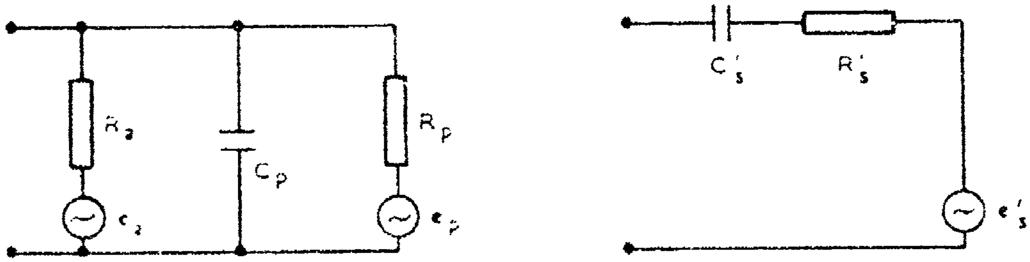
TABLE II

Hydrophone Type	A	B	C	D
Piston Diameter (mm)	6.35	12.7	25.4	63.5
Resonance Frequency (kHz)	100	80	60	18
Acceleration Sensitivity ($\mu V.s^2/m$)	200	700	-	2000
Pressure Sensitivity ($\mu V/Pa$)	110	100	126	355
Accel/Pressure Response m_a ($Pa.s^2/m$)	1.8	7	-	5.6



(a) (b)

FIGS. 1(a)&(b). IDEAL HYDROPHONE IMPEDANCE



(a) (b)

FIGS. 2(a)&(b). HYDROPHONE WITH ADDED LOSS

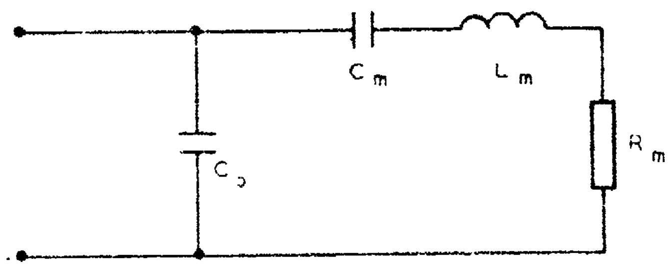
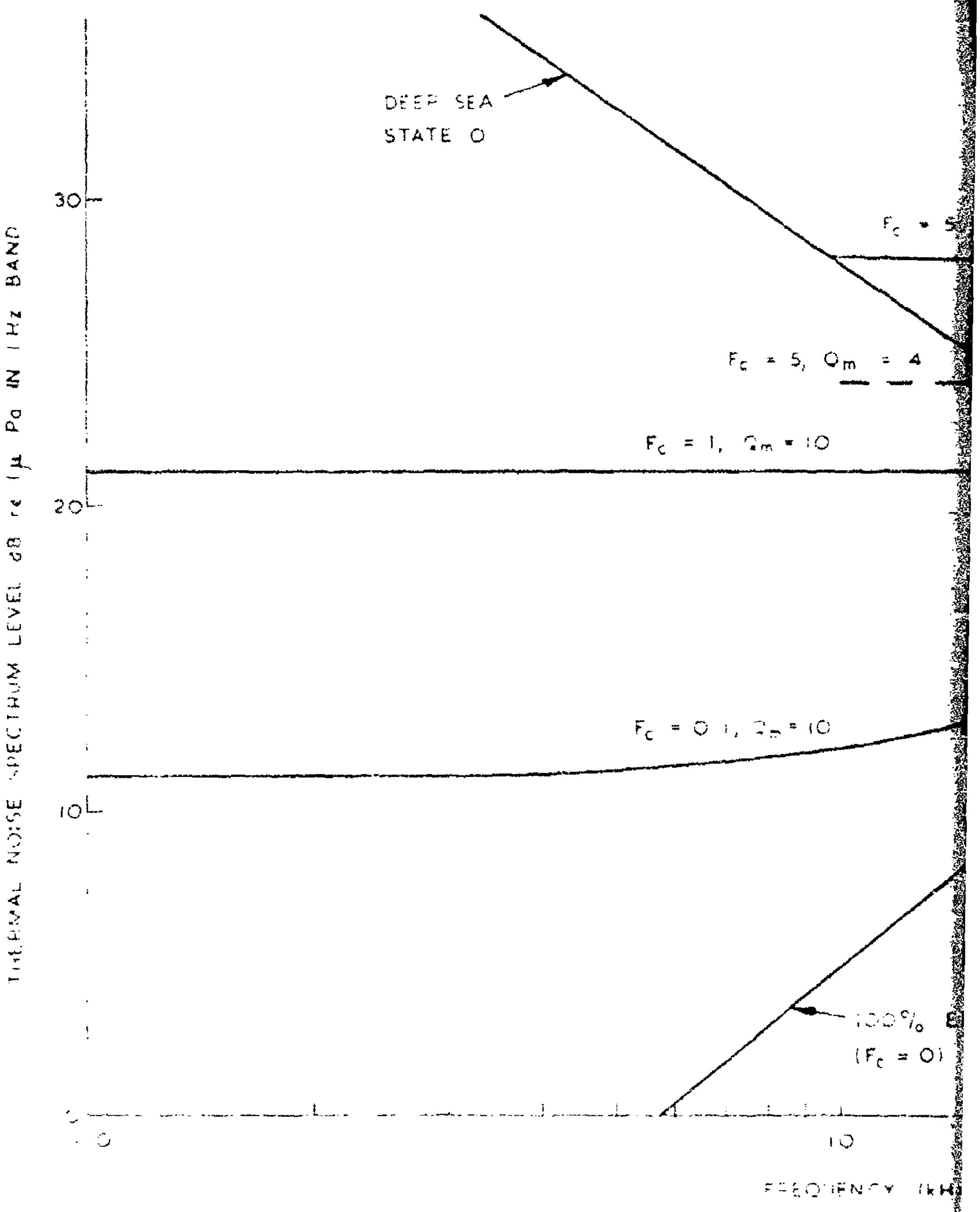


FIG. 3. HYDROPHONE EQUIVALENT CIRCUIT


DEEP SEA
STATE O
THERMAL NOISE SPECTRUM LEVEL dB re 1 μ Pa IN 1 Hz BAND
30
20
10
F_c = 5
F_c = 5, Q_m = 4
F_c = 1, Q_m = 10
F_c = 0.1, Q_m = 10
100%
(F_c = 0)
10
FREQUENCY (kHz

TN 521/75

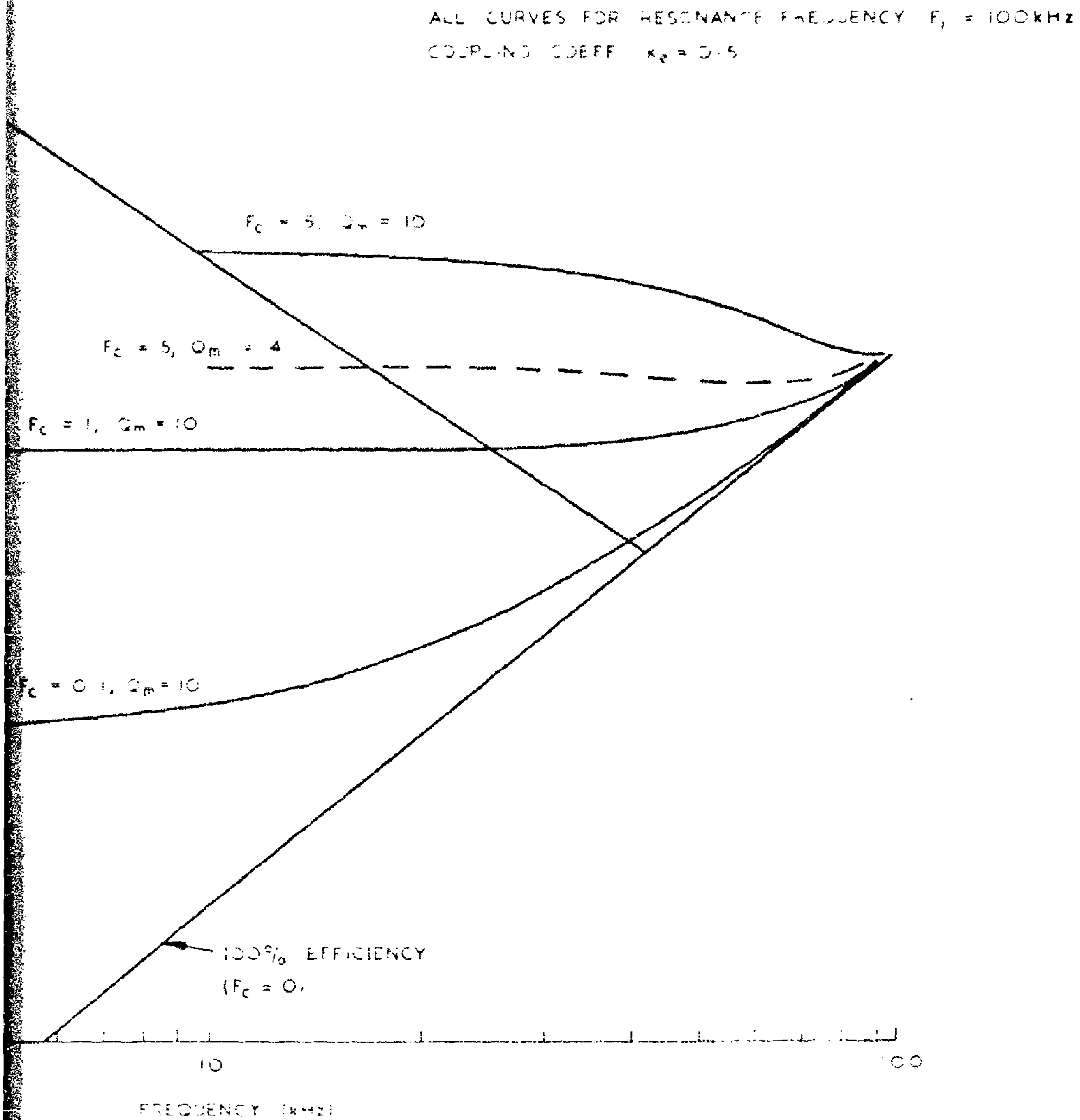


FIG 4(a). EQUIVALENT THERMAL NOISE PRESSURE FOR HYDROPHONE WITH PARALLEL RESISTOR

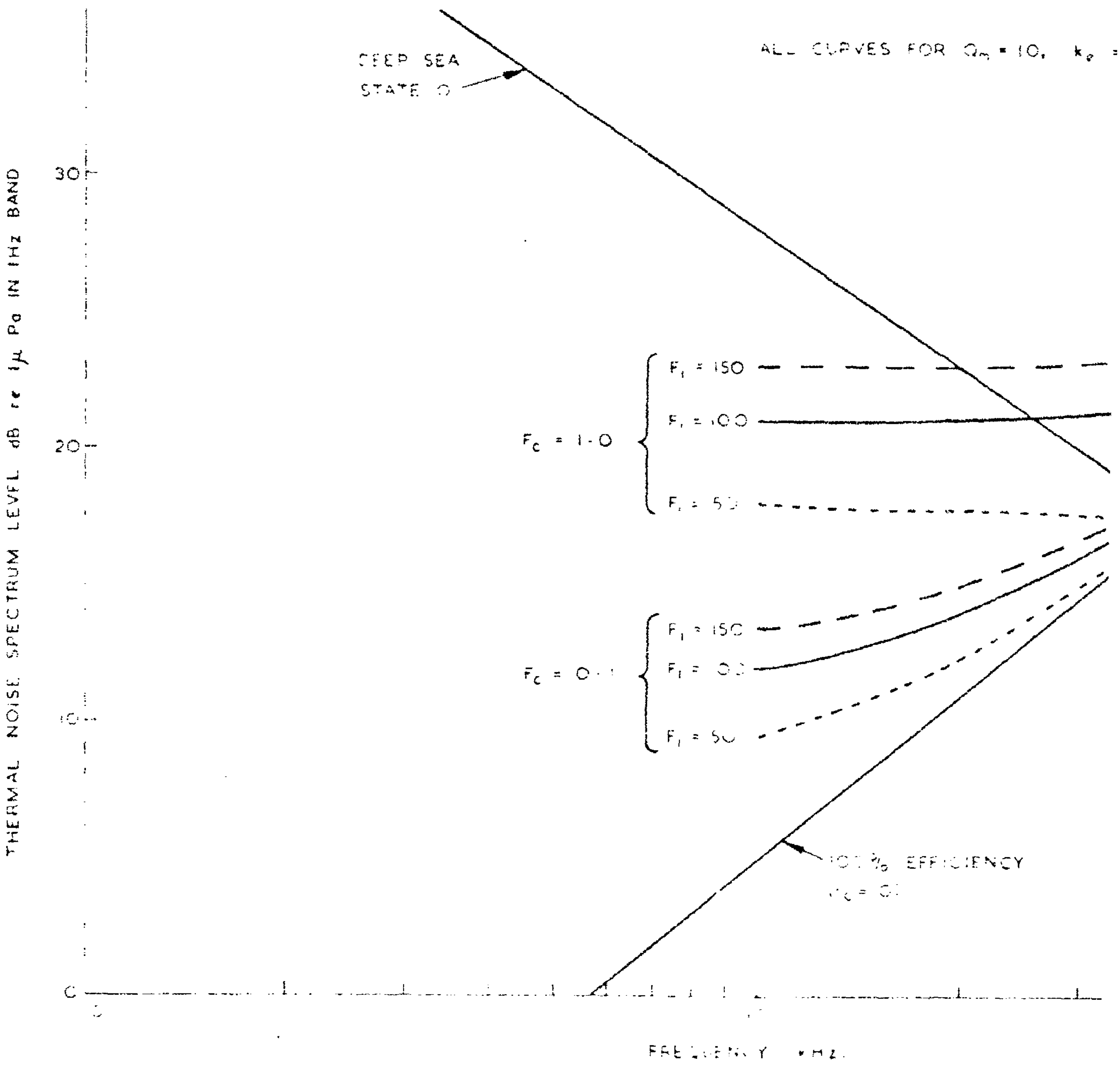


FIG 4(b) EQUIVALE
HYDROPHON

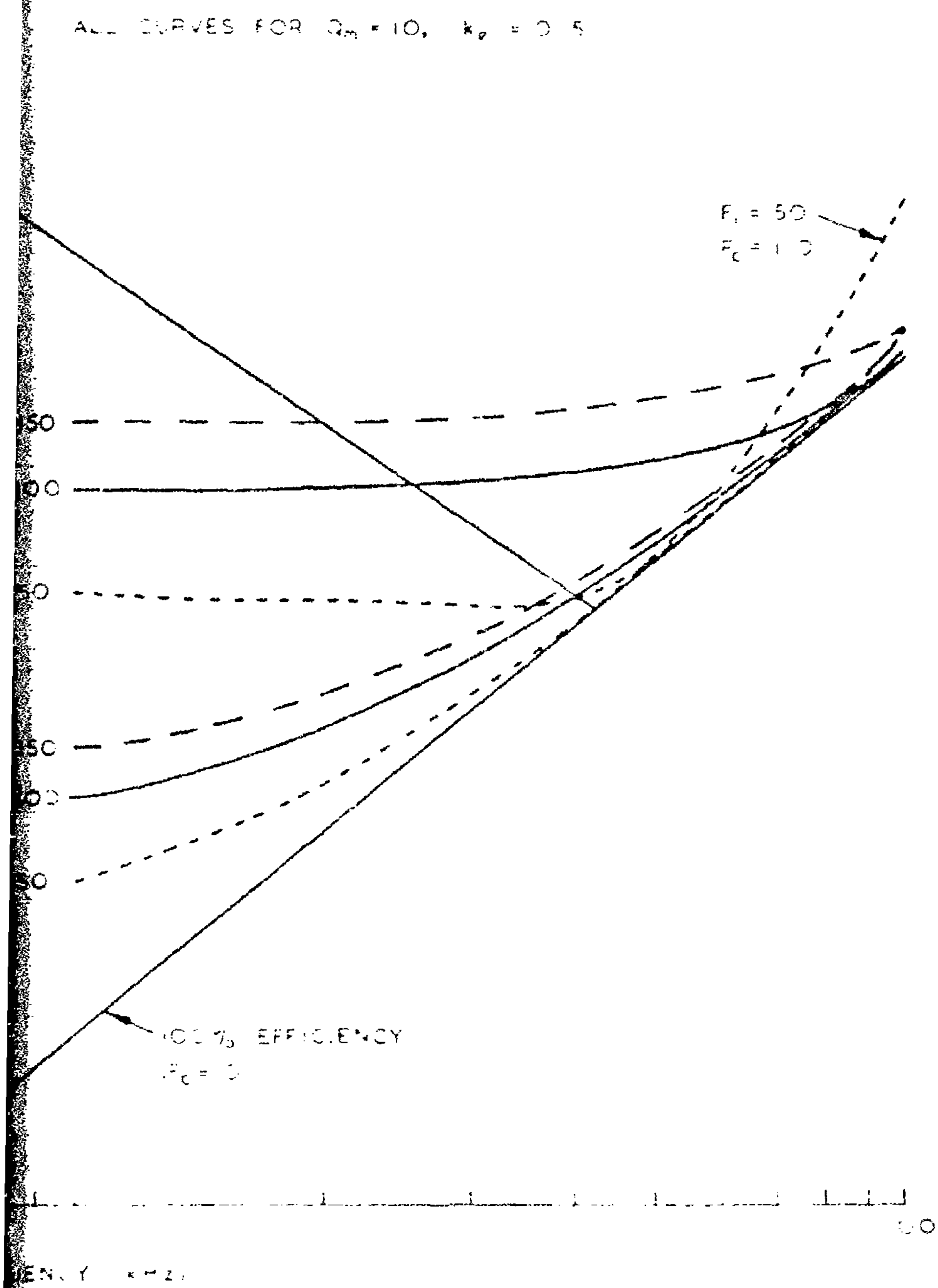


FIG 4(b) EQUIVALENT THERMAL NOISE PRESSURE FOR HYDROPHONE WITH PARALLEL RESISTOR

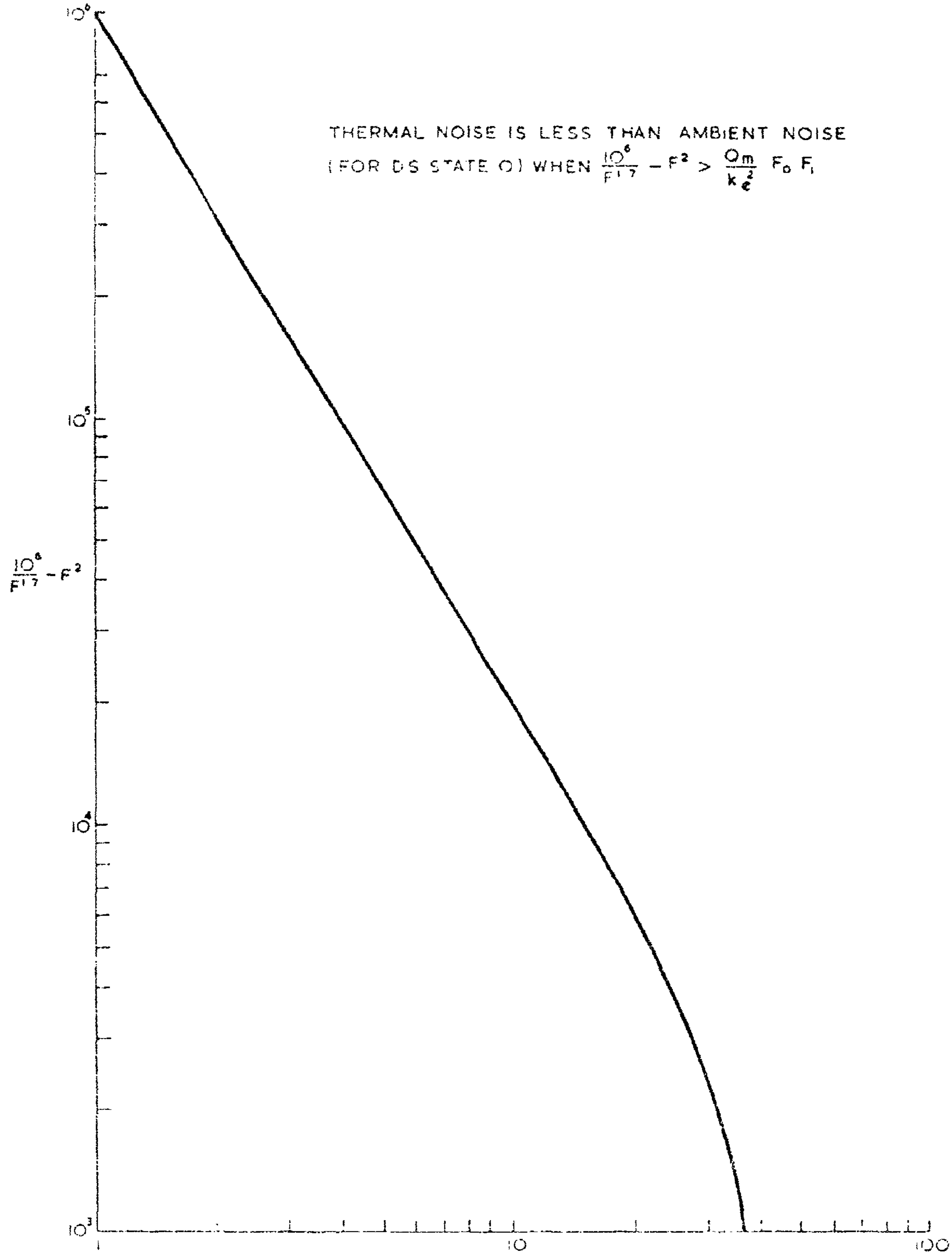
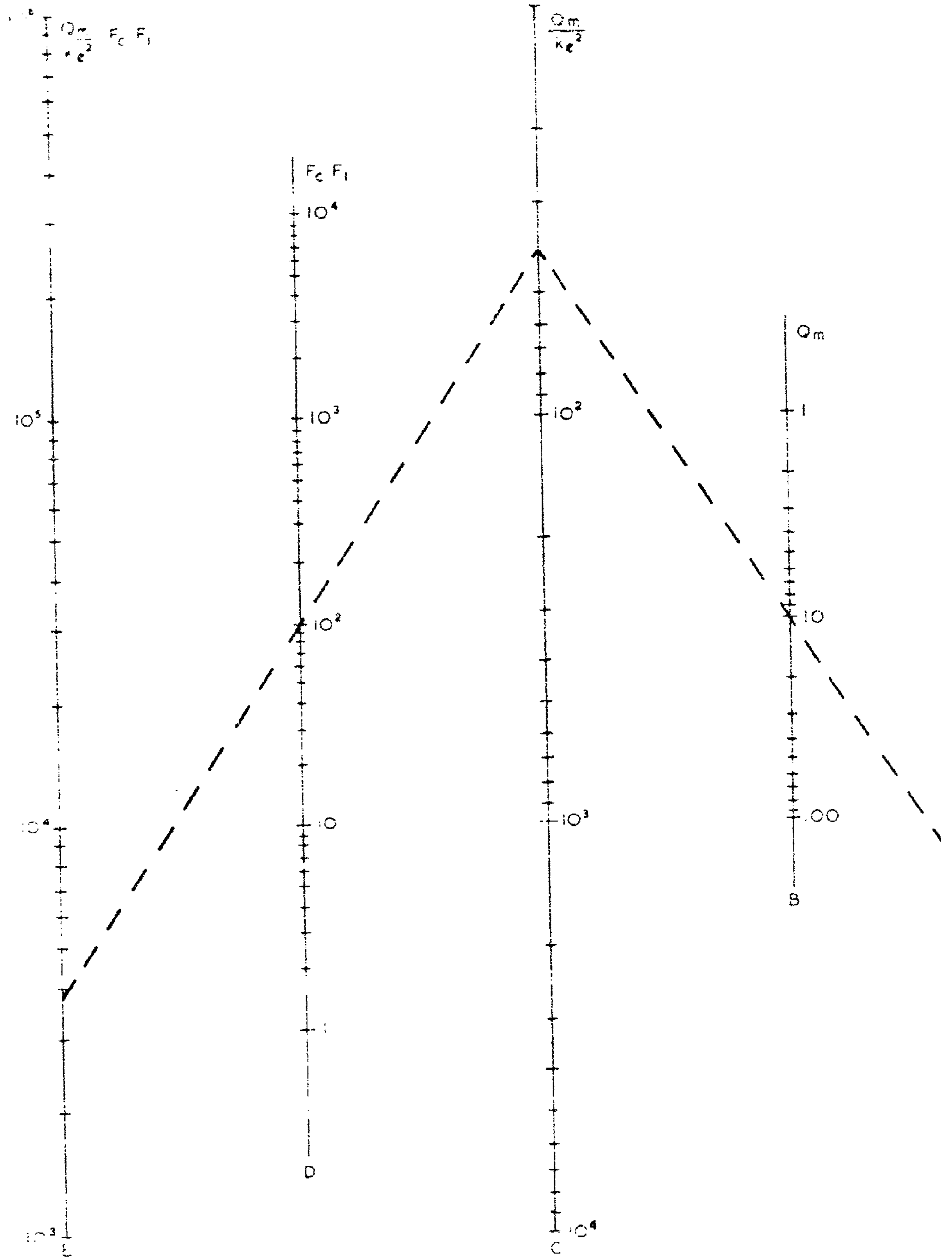


FIG. 5(a) VALUES OF $\frac{10^5}{F^{1.7}} - F^2$ vs F



Qm/κε² Fc F1
10⁵
10⁴
10³
E
Fc F1
10⁴
10³
10²
10
1
D
Qm/κε²
10²
10³
10⁴
C
Qm
1
10
.00
B

USE SCALES A & B AS INDICATED BY DASHED LINE TO FIND $\frac{Q_m}{k_z^2}$ ON SCALE C THEN DRAW LINE FROM THIS VALUE THROUGH CHOSEN VALUE OF $F_c F_1$ ON SCALE D TO DETERMINE VALUE OF $\frac{Q_m}{k_z^2} F_c F_1$ ON SCALE E

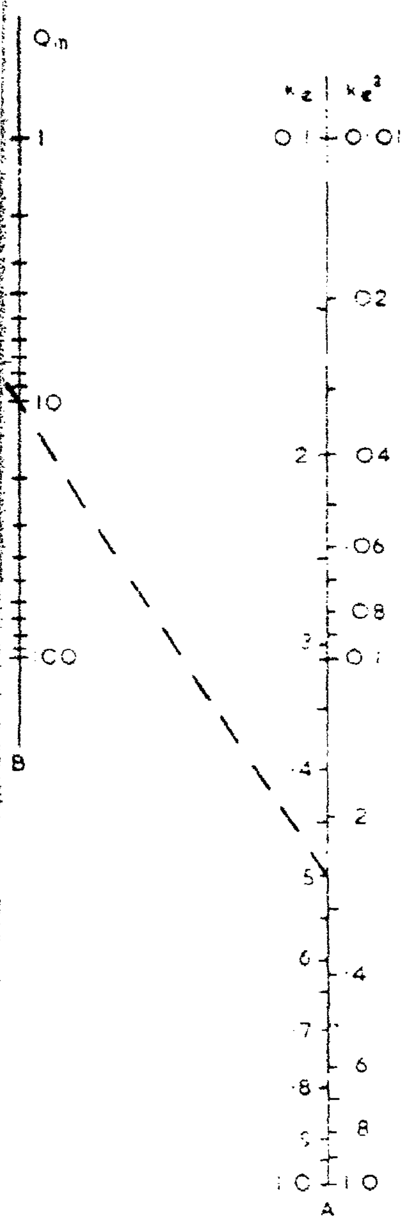
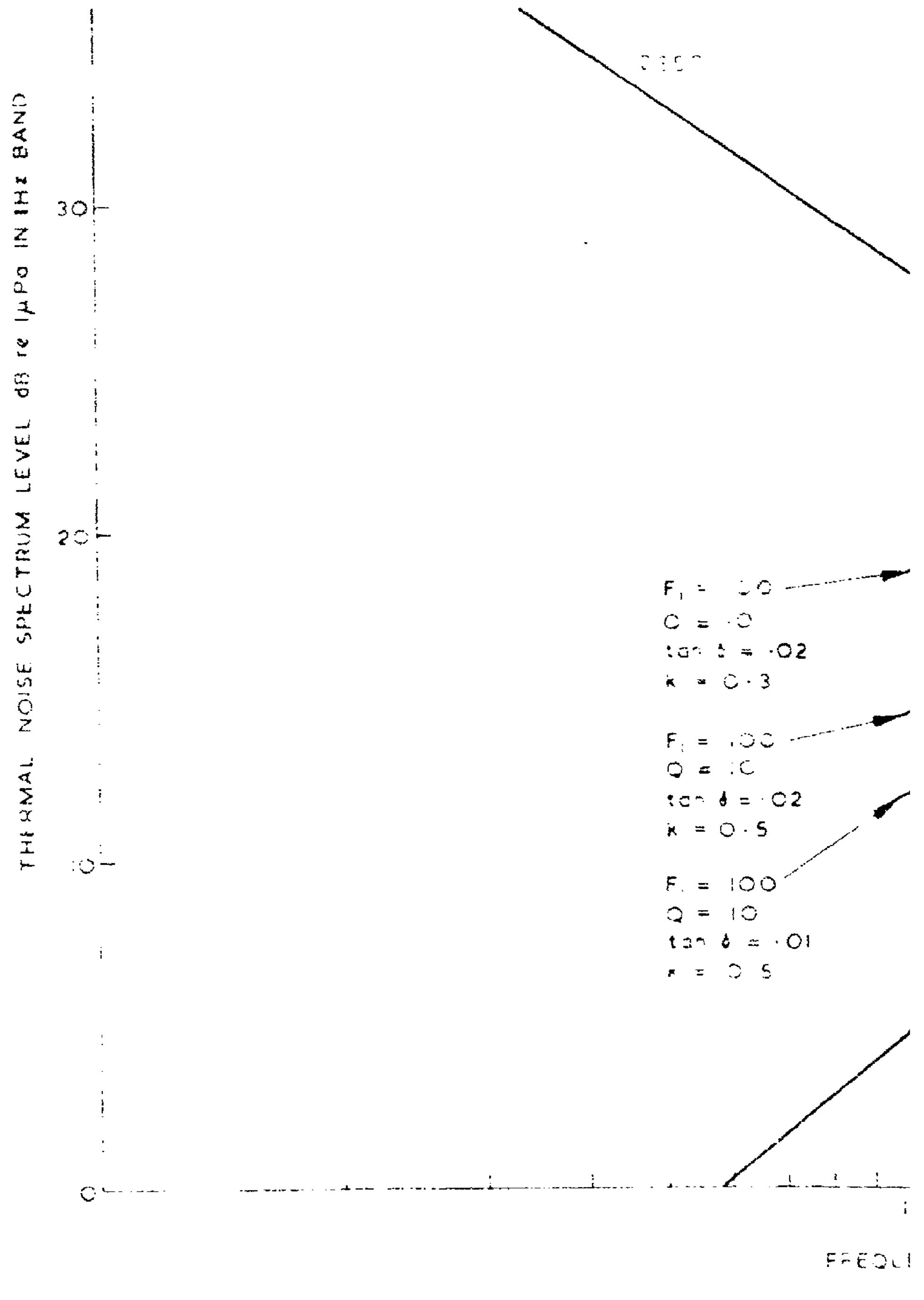


FIG. 5(b) NOMOGRAM FOR CALCULATION OF $\frac{Q_m}{k_z^2} F_c F_1$


THERMAL NOISE SPECTRUM LEVEL dB re 1μPa IN 1Hz BAND
30
20
10
0
F, = 100
Q = 10
tan δ = ·02
k = 0·3
F, = 100
Q = 10
tan δ = ·02
k = 0·5
F, = 100
Q = 10
tan δ = ·01
k = 0·5

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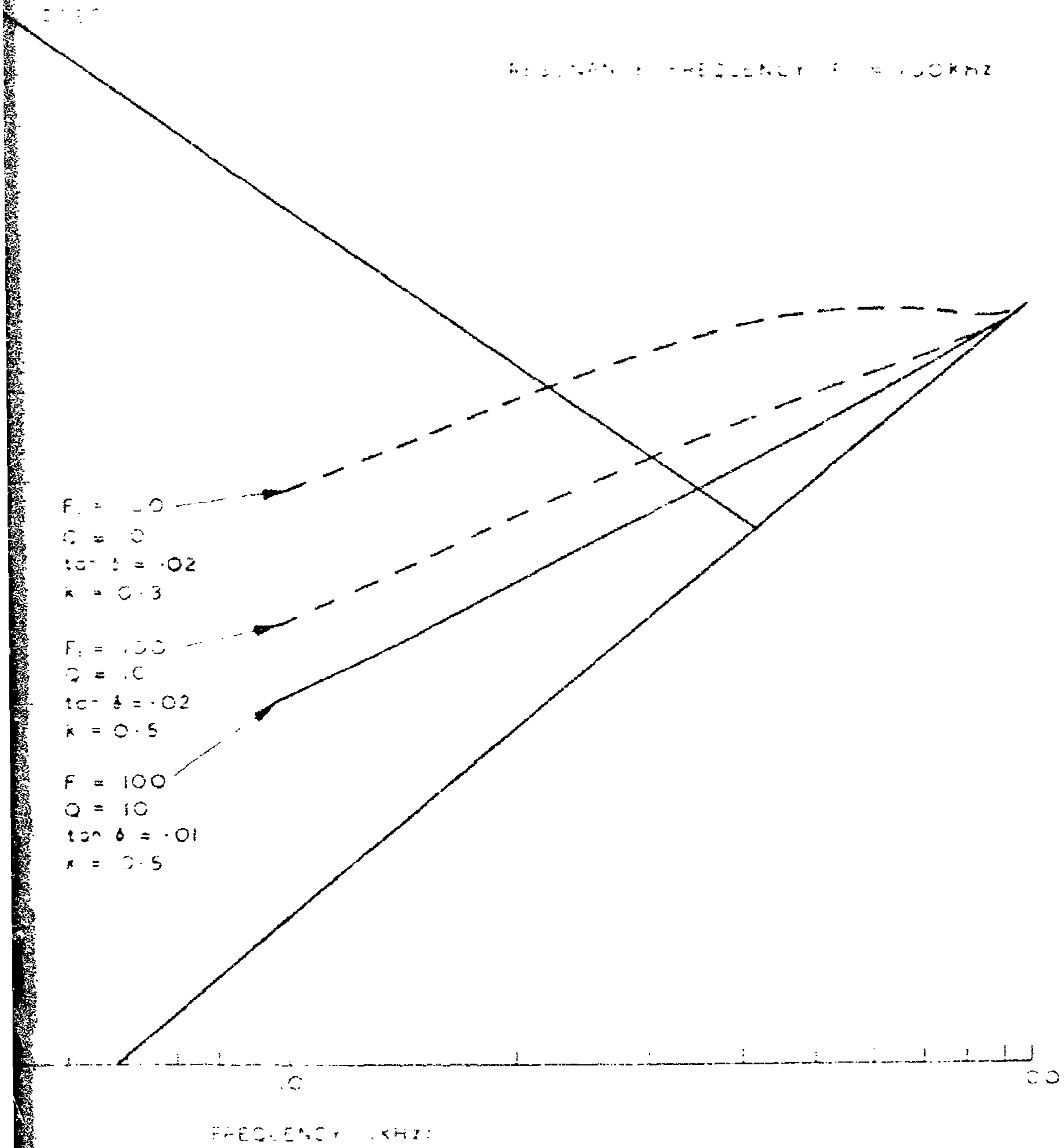

FIG 6a EQUIVALENT THERMAL NOISE PRESSURE FOR HYDROPHONE WITH tan δ LOSSES

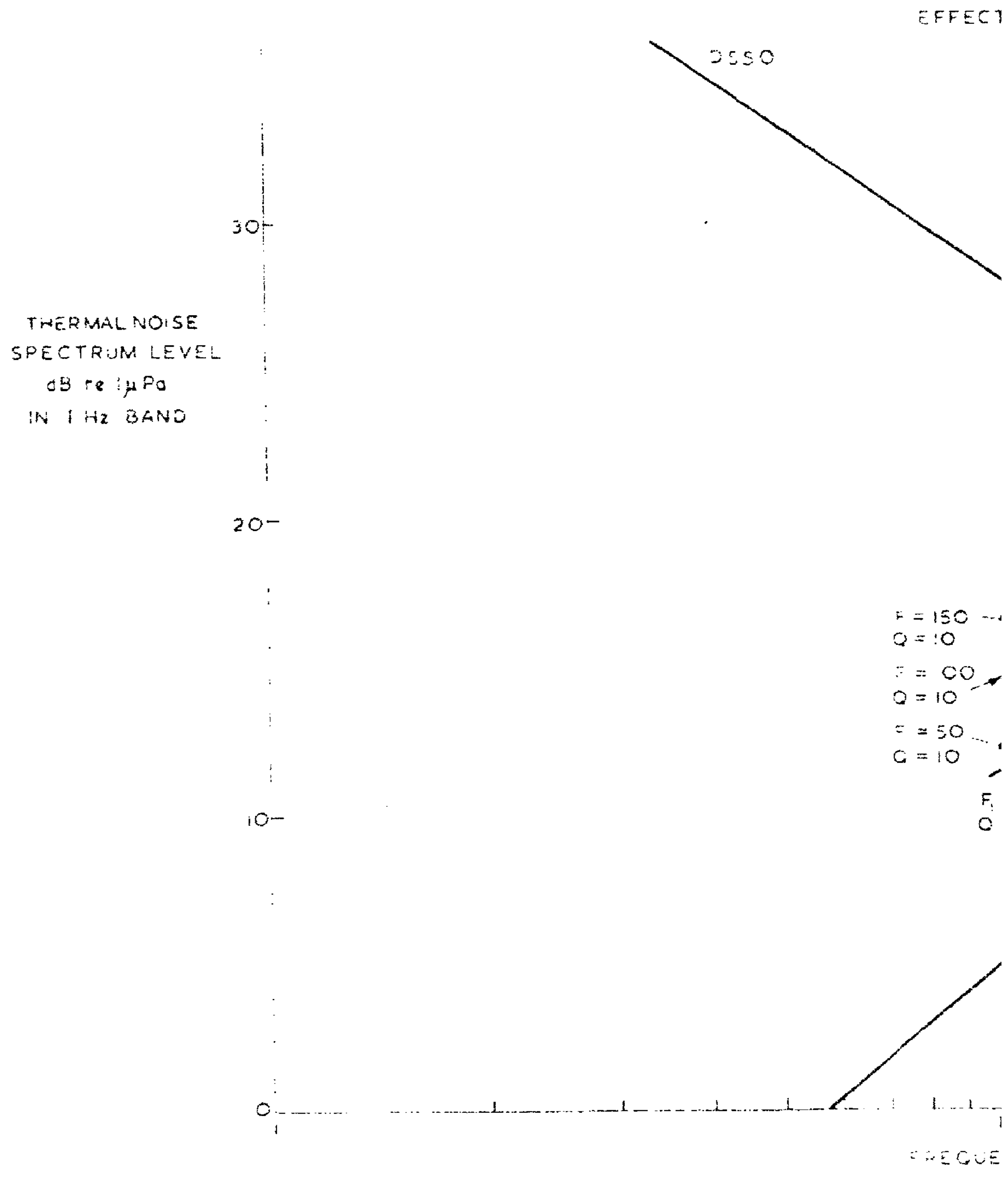

FIG 6

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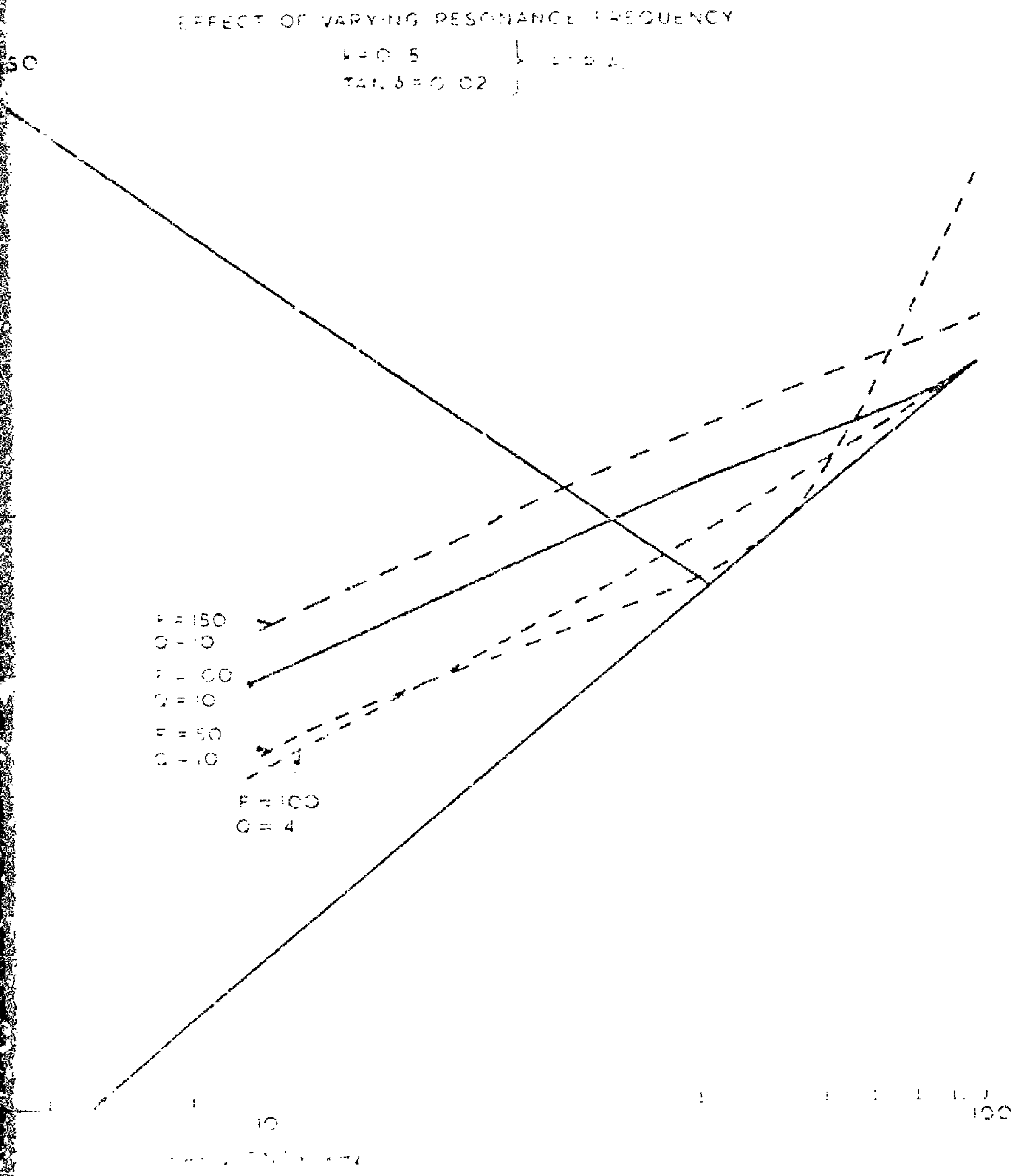

FIG 6(b) EQUIVALENT THERMAL NOISE FOR HYDROPHONE WITH TAN δ LOSSES

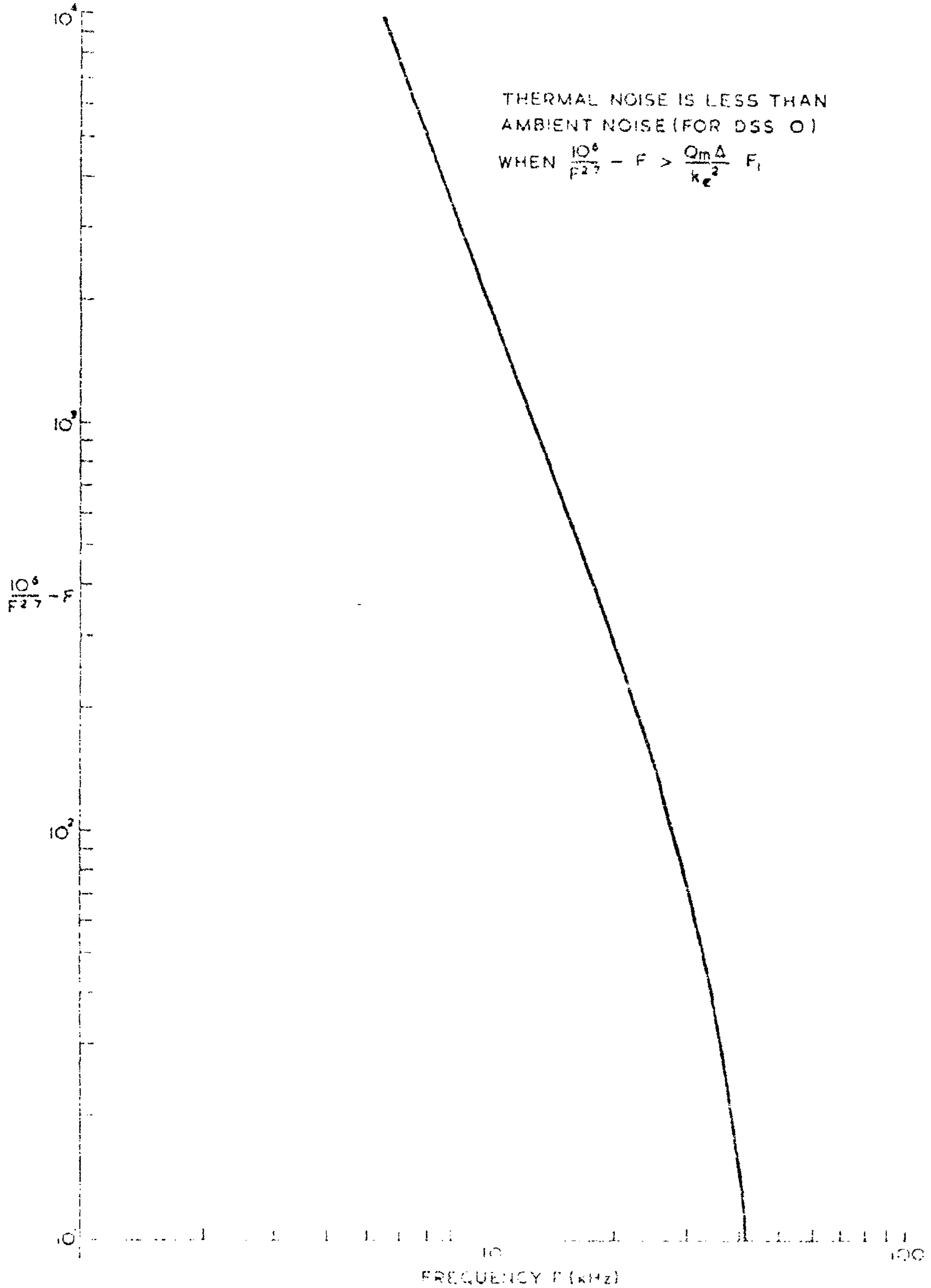

FIG.7. VALUES OF $\left(\frac{10^6}{F^{2.7}} - F\right)$ vs F

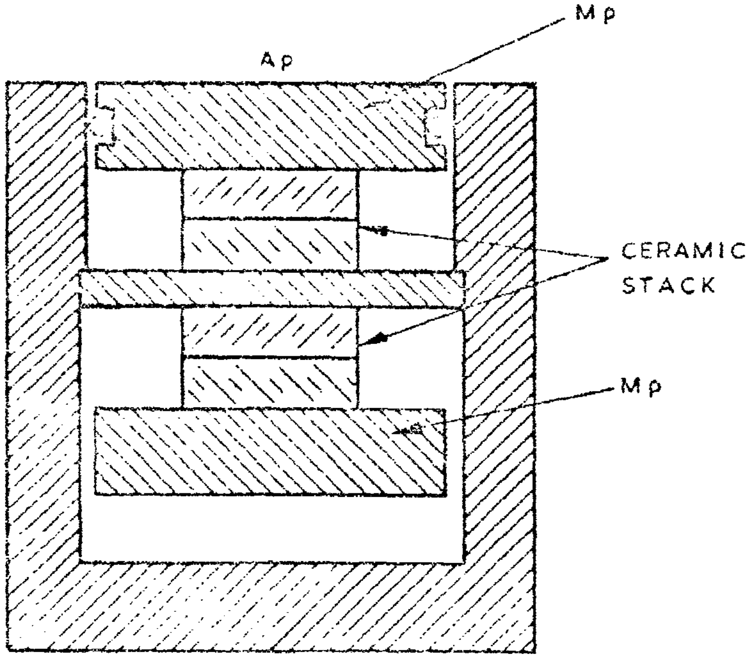

FIG. 8. DIAGRAM OF PISTON-TYPE HYDROPHONE

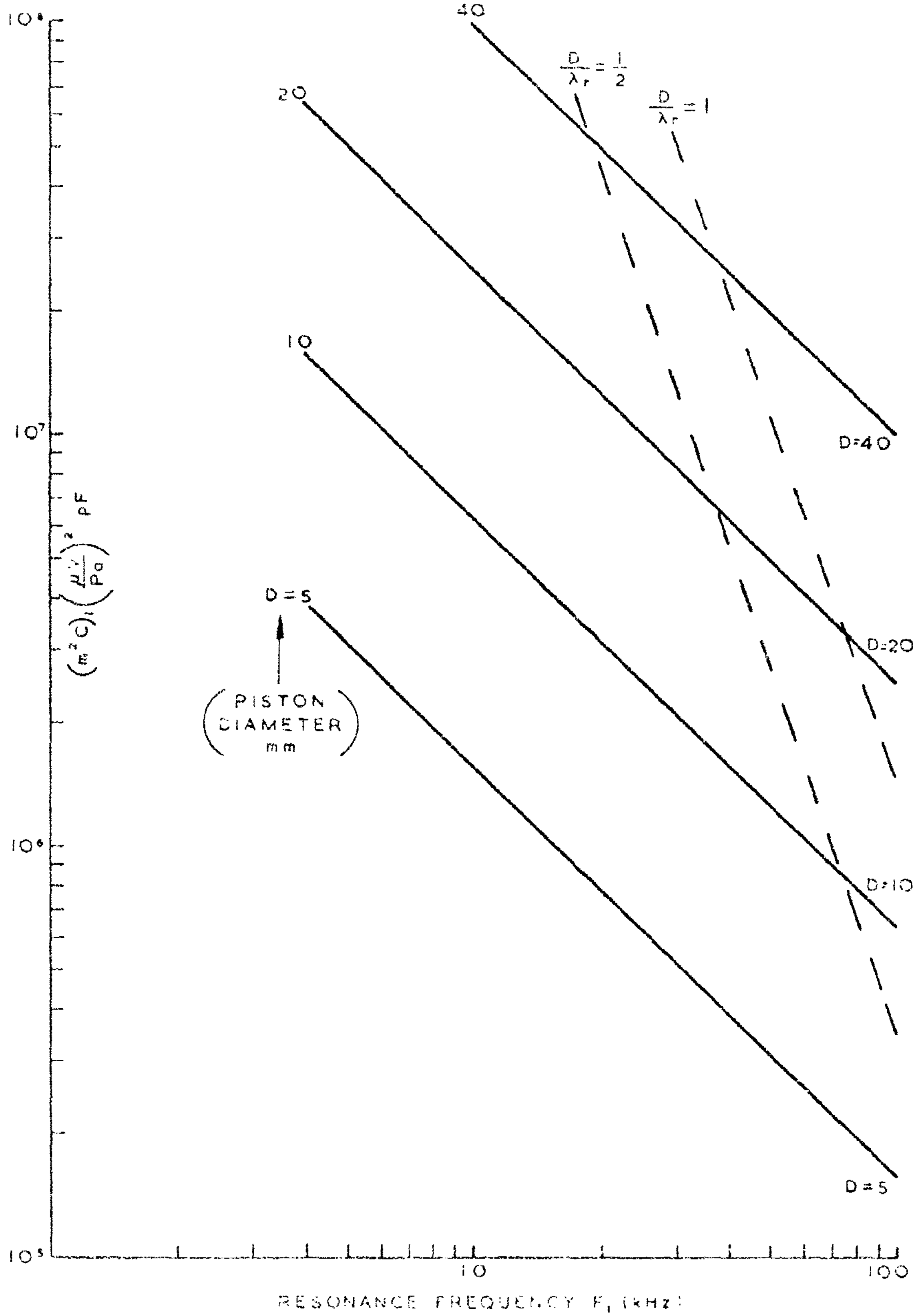

FIG 9. APPROXIMATE VALUES OF $(m^2 C)_1$ vs RESONANCE FREQUENCY FOR PISTON HYDROPHONES

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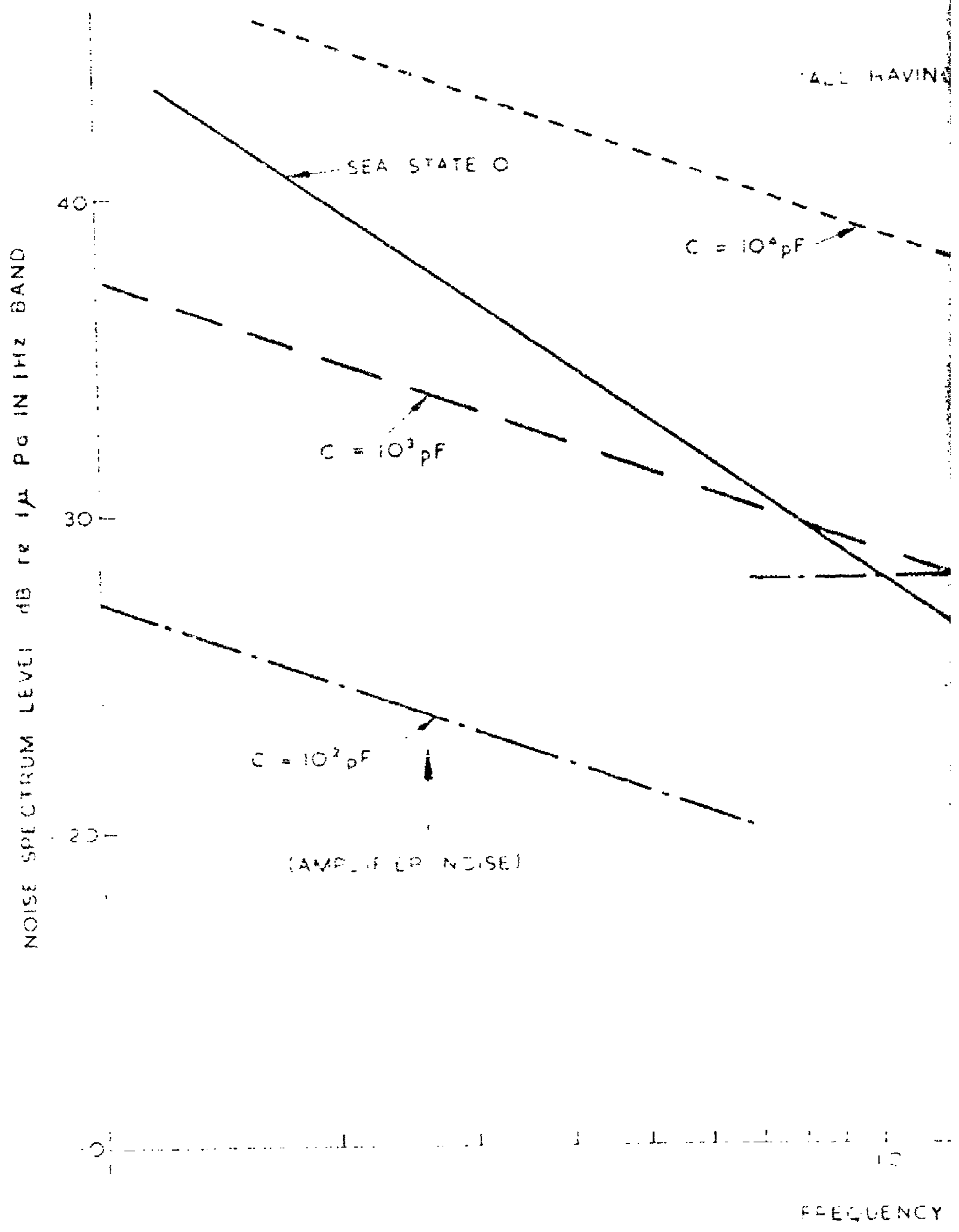

SEA STATE 0
C = 10⁴pF
C = 10³pF
C = 10²pF
(AMPLIFIER NOISE)
NOISE SPECTRUM LEVEL dB re 1μ Pa IN 1Hz BAND
40
30
20
10
FREQUENCY

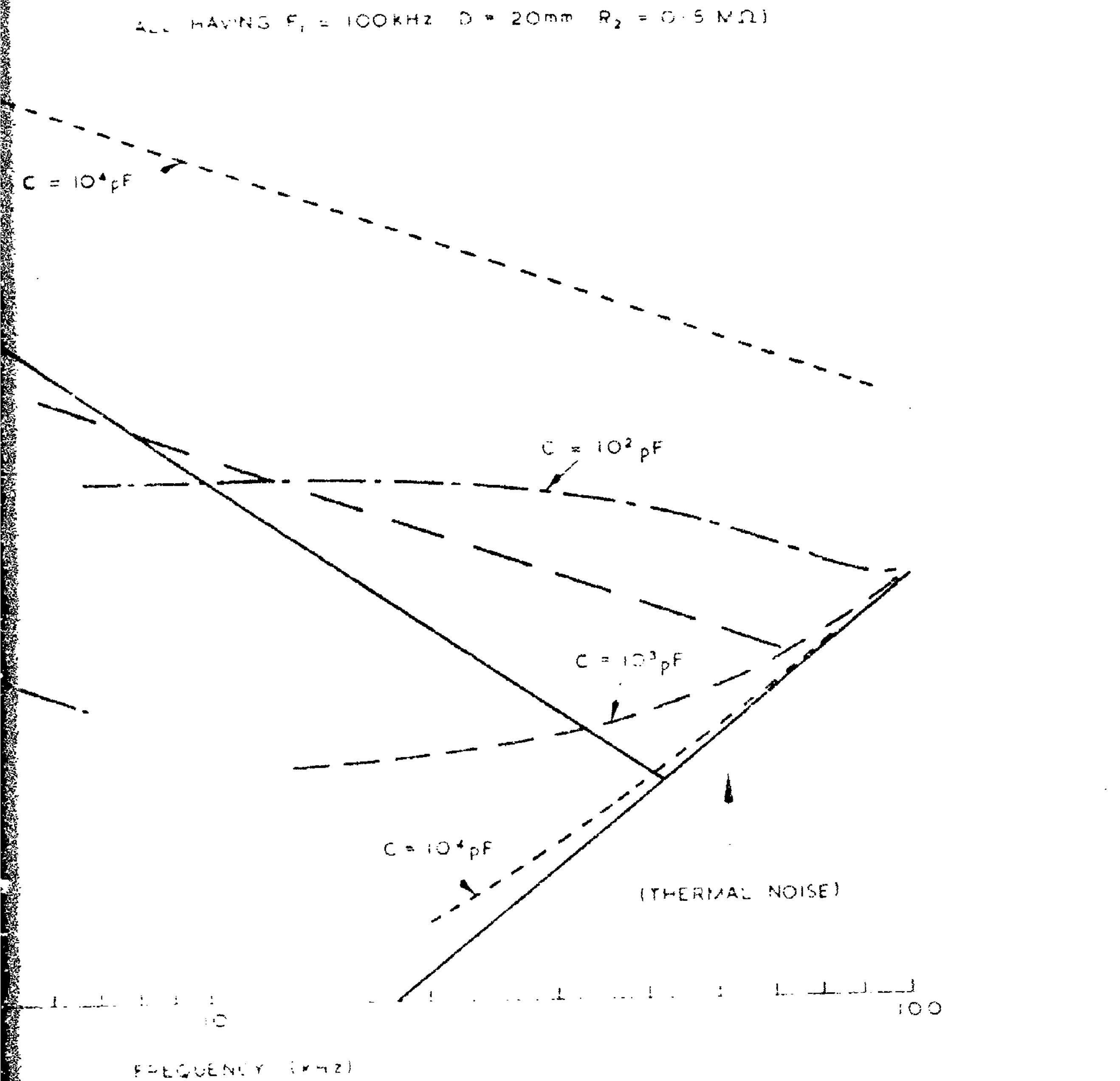


FIG 10 AMPLIFIER & THERMAL NOISE vs FREQUENCY FOR THREE HYDROPHONE DESIGNS

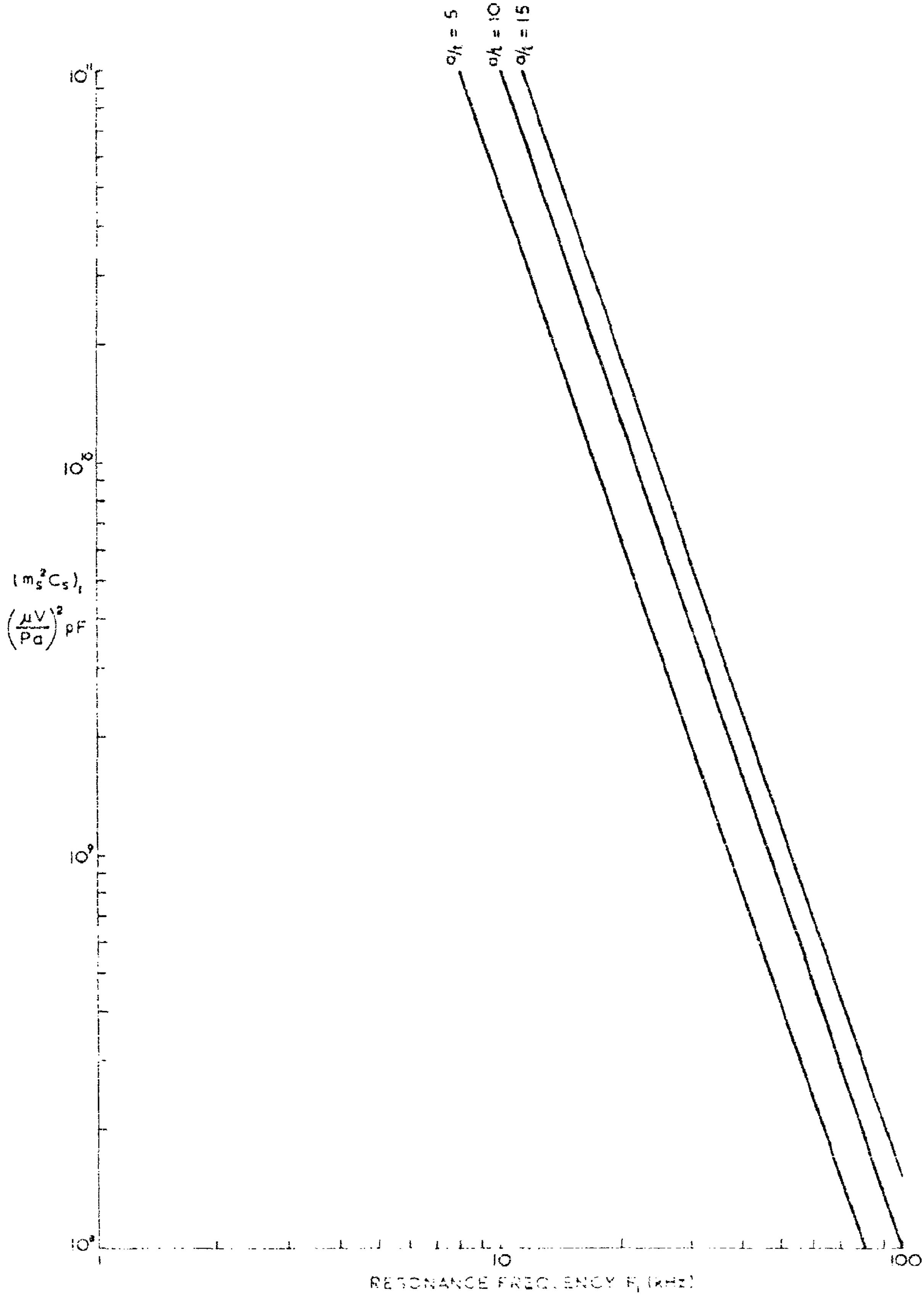


FIG.11 VALUES OF $(m_s{}^2C_s)_1$ vs F_1 FOR SPHERICAL HYDROPHONES

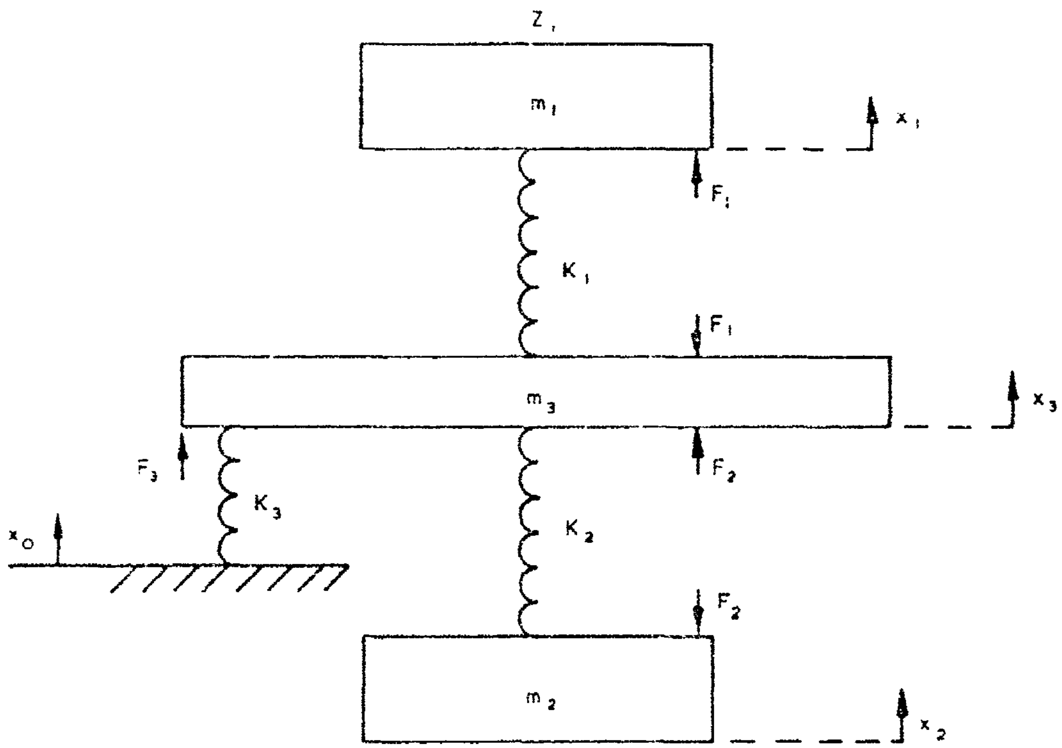


FIG. 12. MECHANICAL REPRESENTATION OF HYDROPHONE ELEMENT

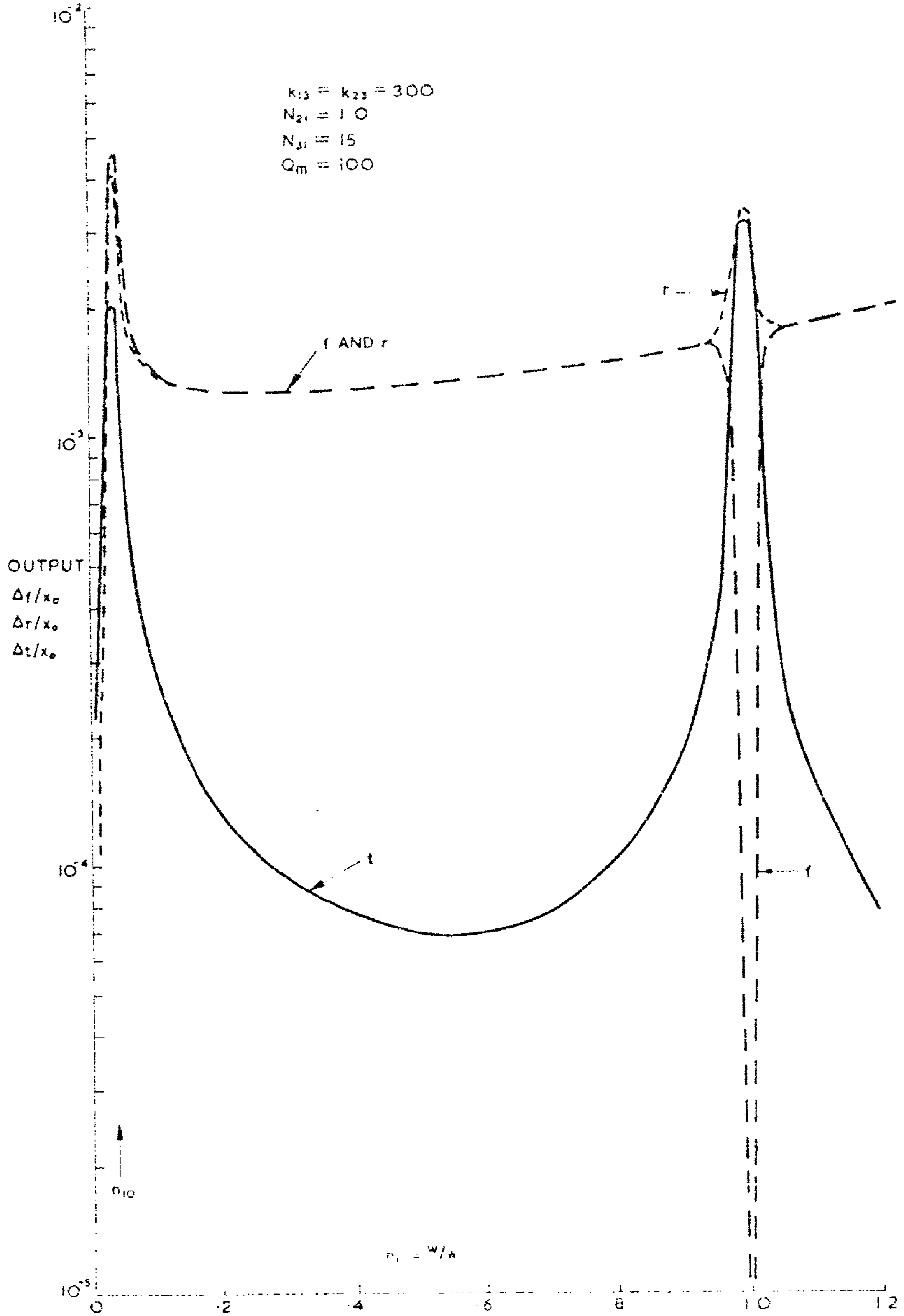


FIG. 13. ACCELERATION RESPONSE OF HYDROPHONE
EFFECT OF CANCELLING FOR NOMINALLY BALANCED HIGH-Q_m ELEMENT

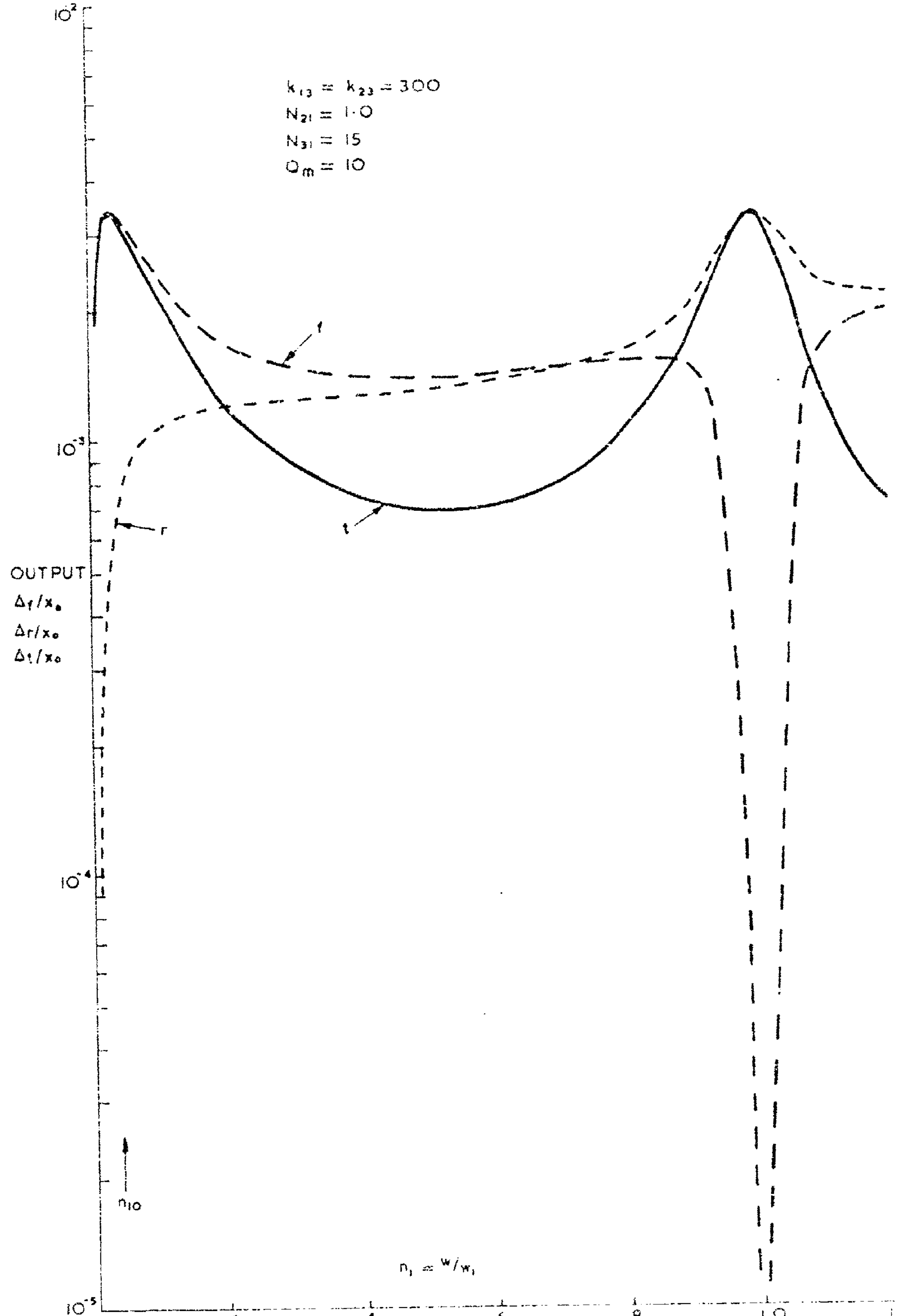


FIG 14 ACCELERATION RESPONSE OF HYDROPHONE
EFFECT OF CANCELLING FOR NOMINALLY BALANCED LOW-Q_m ELEMENT

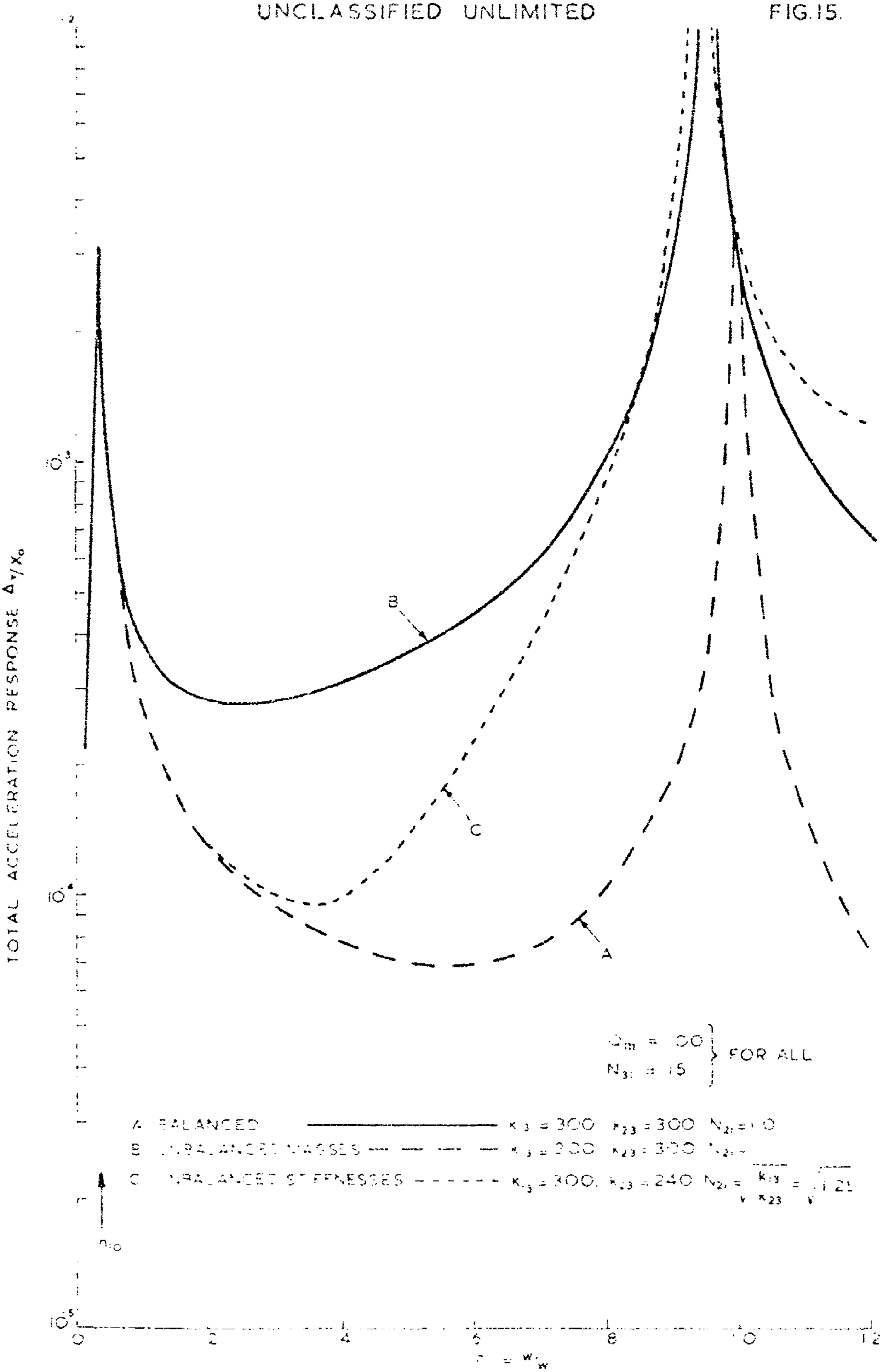


FIG. 15 ACCELERATION RESPONSE
EFFECT OF UNBALANCE FOR HIGH - Q_m ELEMENT

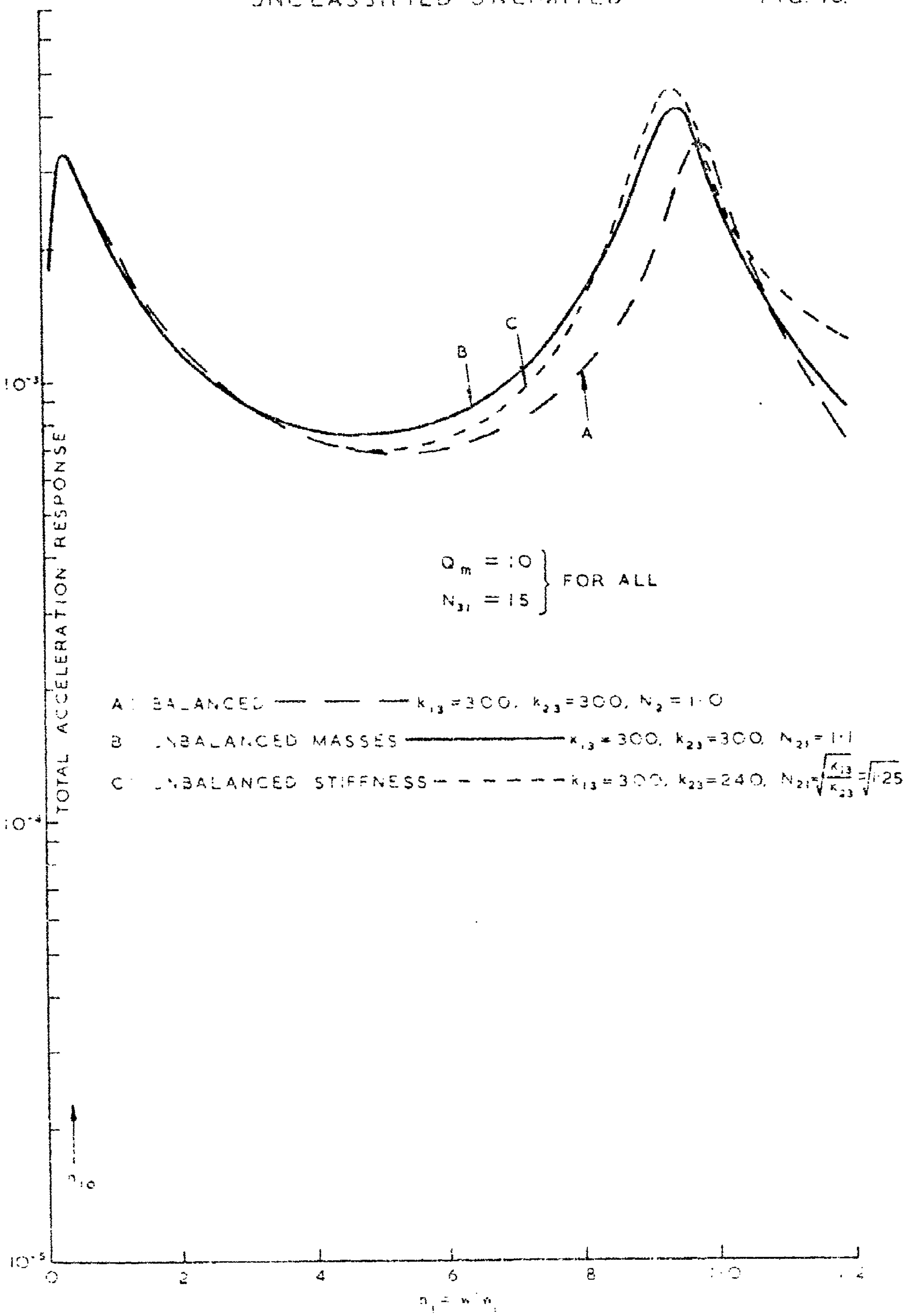


FIG. 16. ACCELERATION RESPONSE
EFFECT OF UNBALANCE FOR LOW Q_m ELEMENT

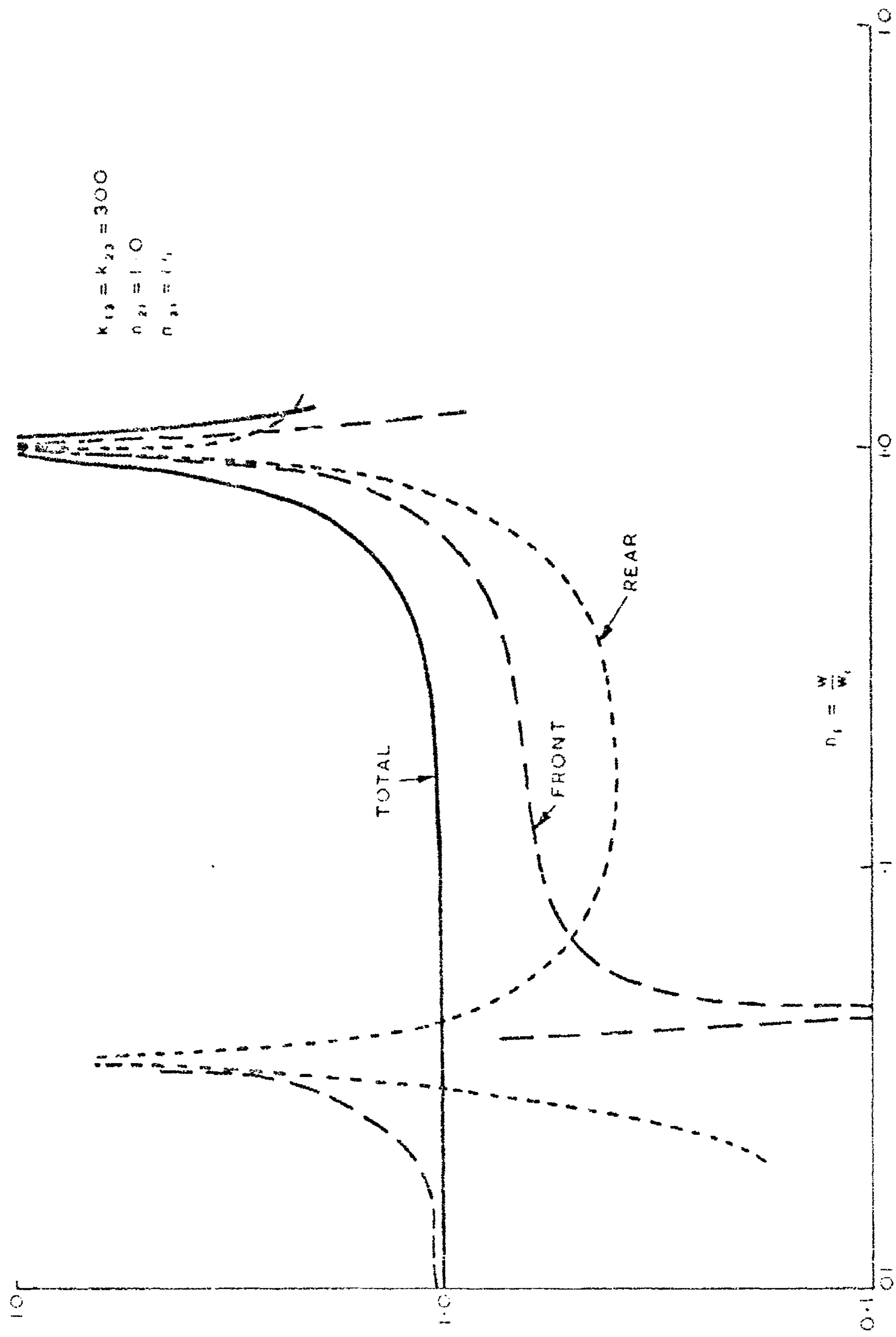


FIG.17. PRESSURE RESPONSE OF BALANCED ELEMENT

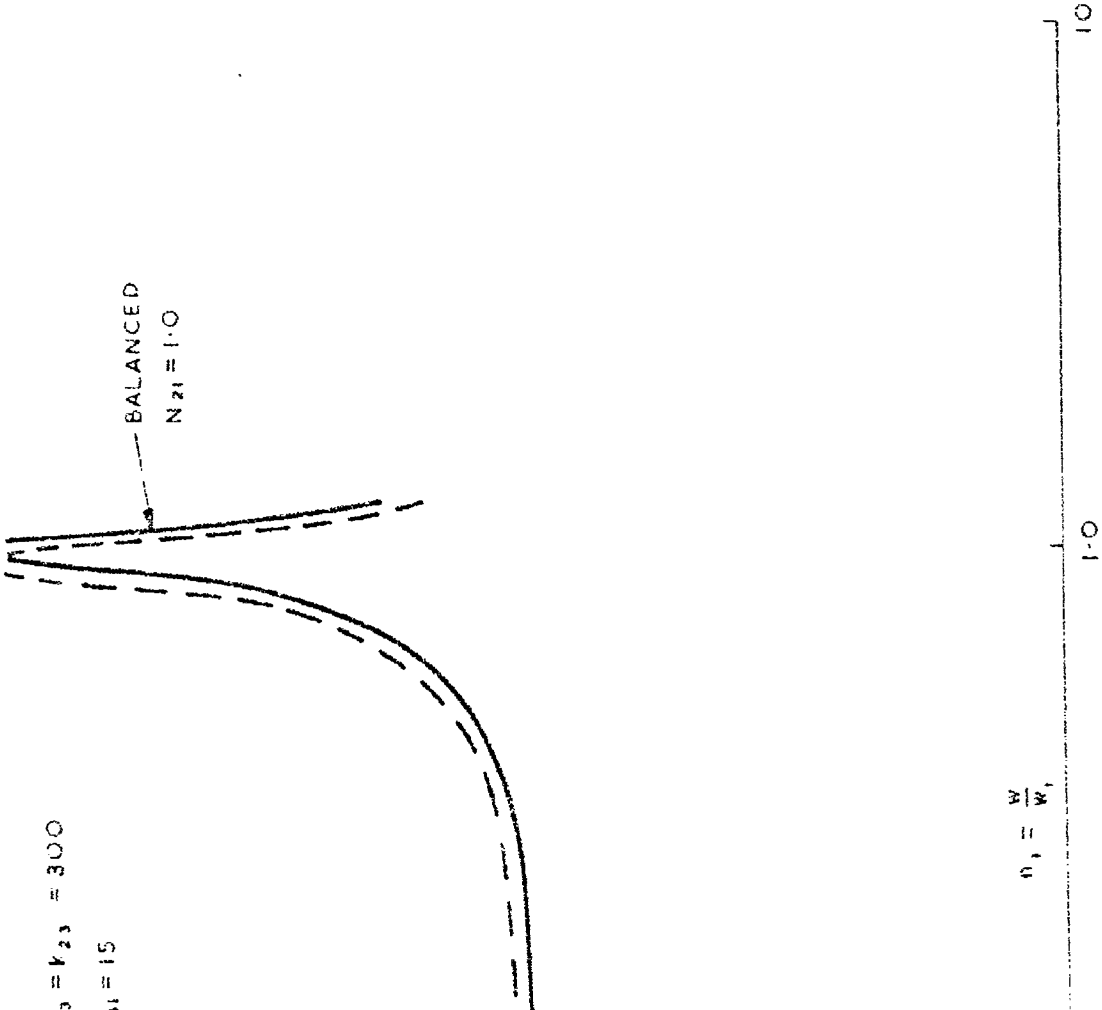


FIG.18. EFFECT OF MASS UNBALANCE ON PRESSURE RESPONSE

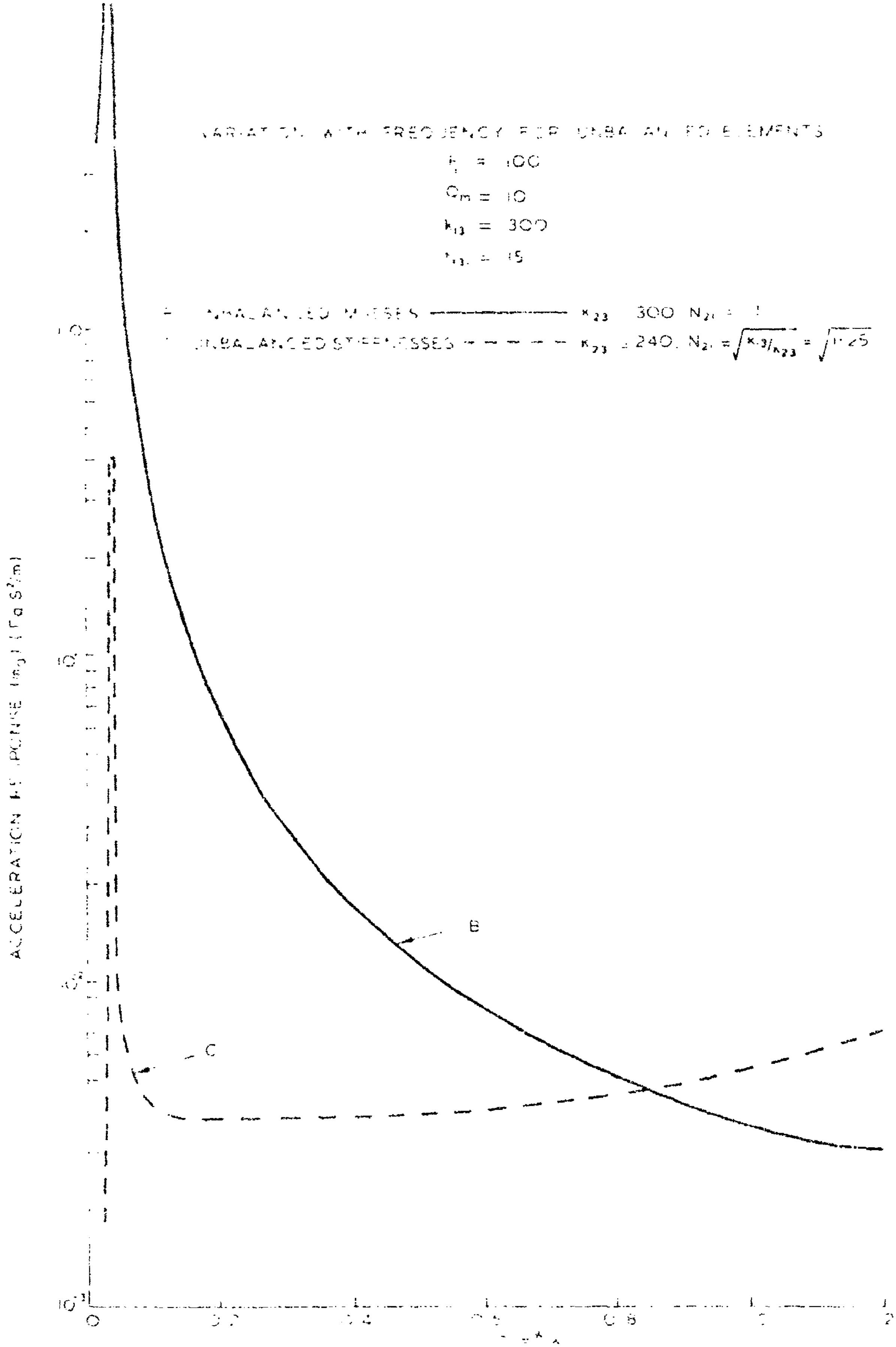
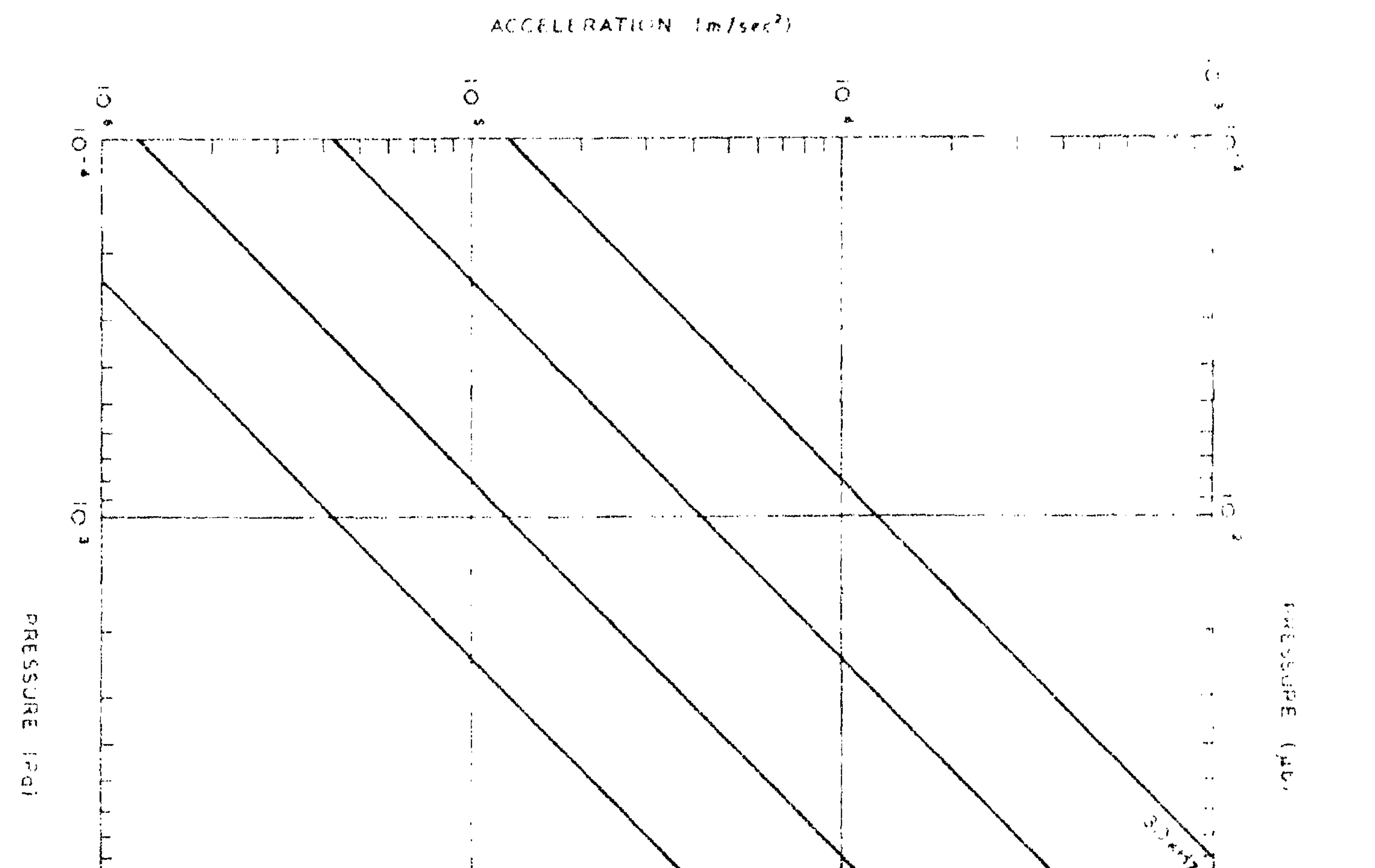


FIG 19 ACCELERATION/PRESSURE RESPONSE


ACCELERATION (m/sec²)
PRESSURE (Pa)
PRESSURE (μb)
3.0 kHz

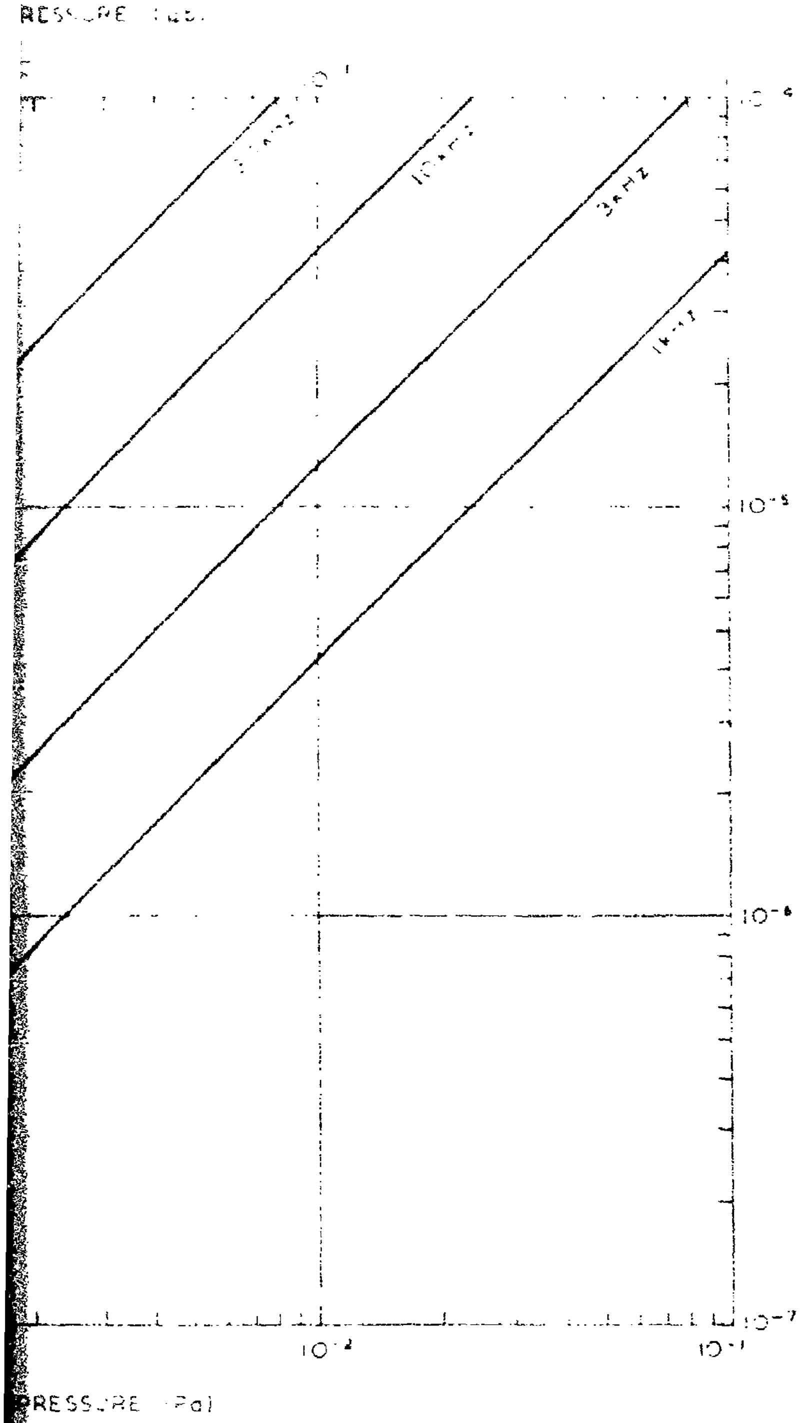
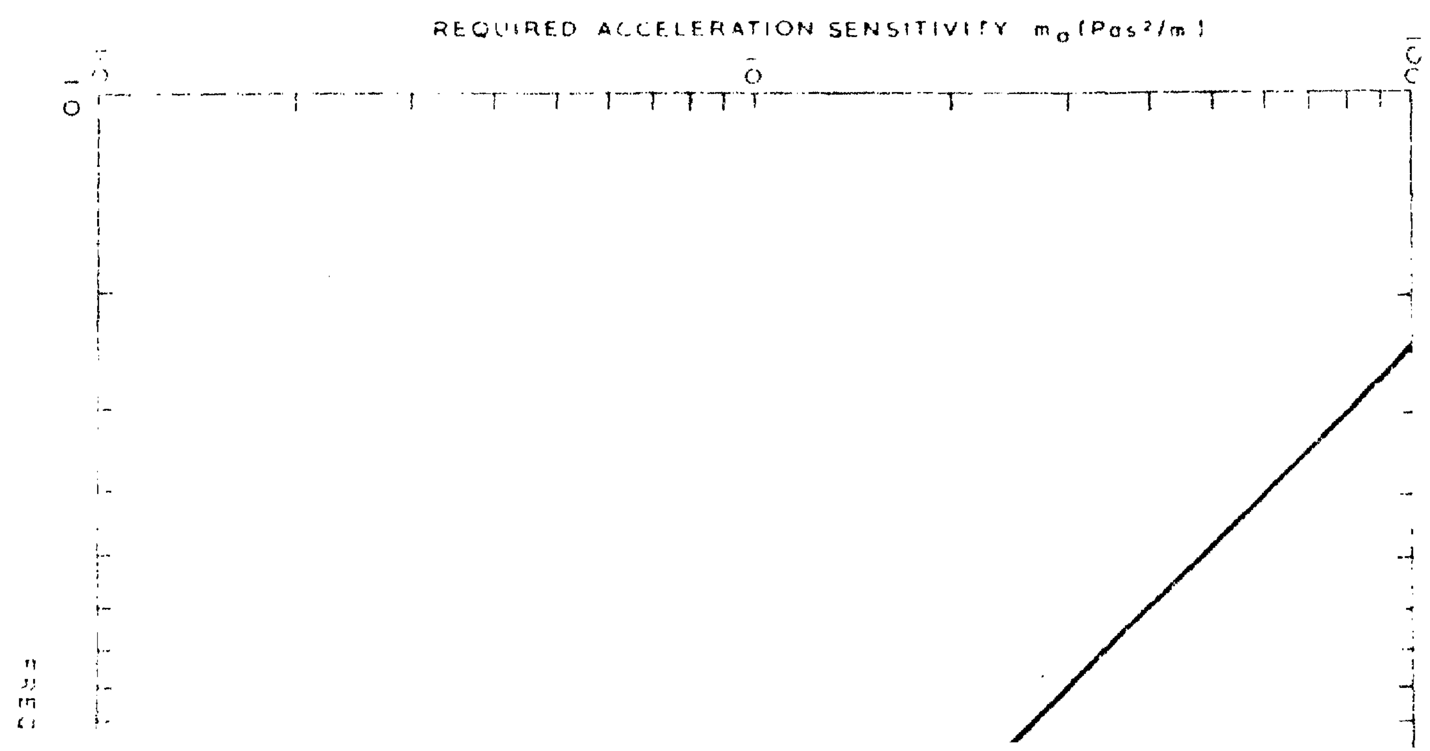


FIG. 20 RELATION BETWEEN PARTICLE ACCELERATION AND PRESSURE IN WATER


REQUIRED ACCELERATION SENSITIVITY m_a (Pa s^2/m)
10
FREQ

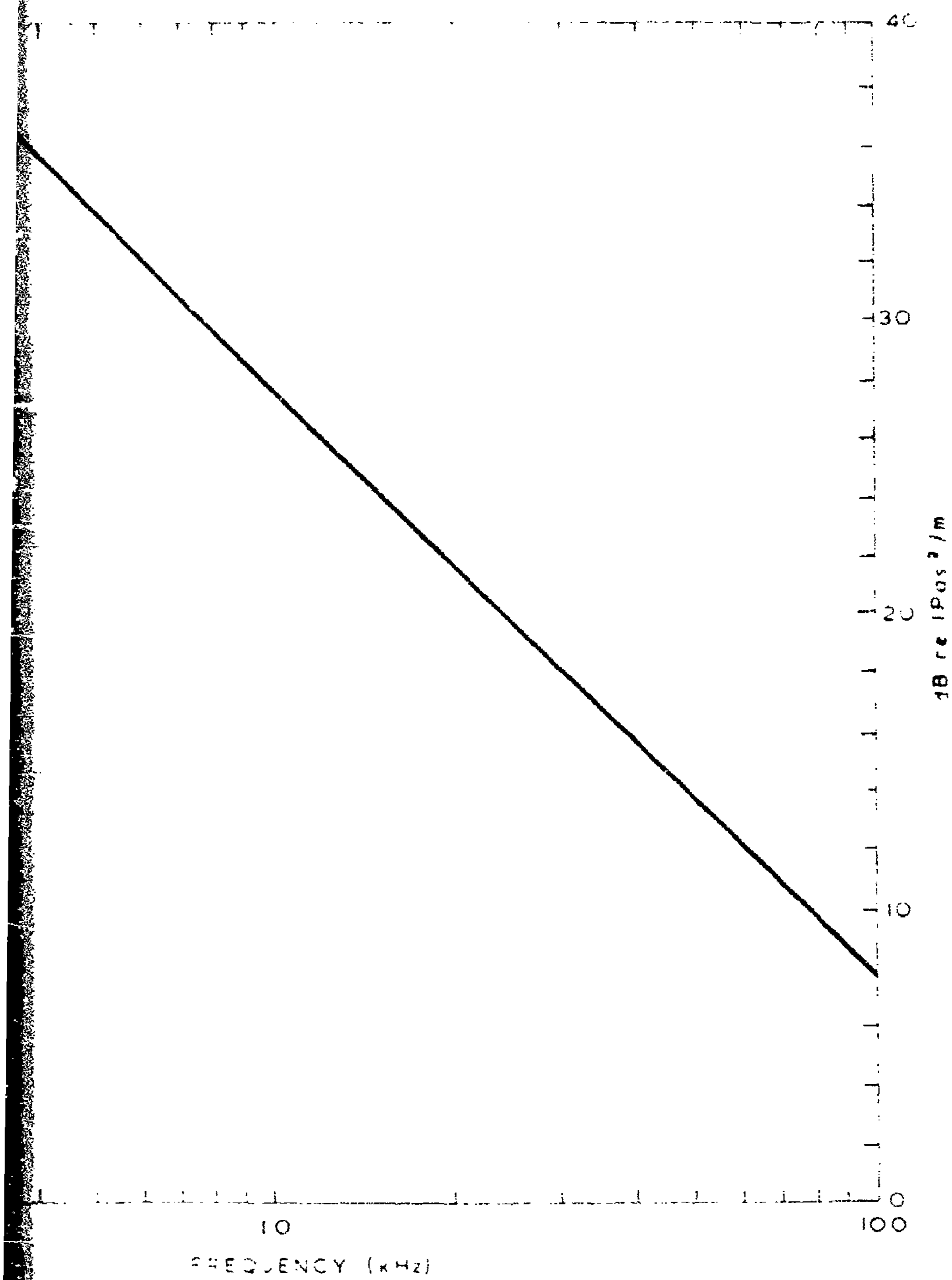
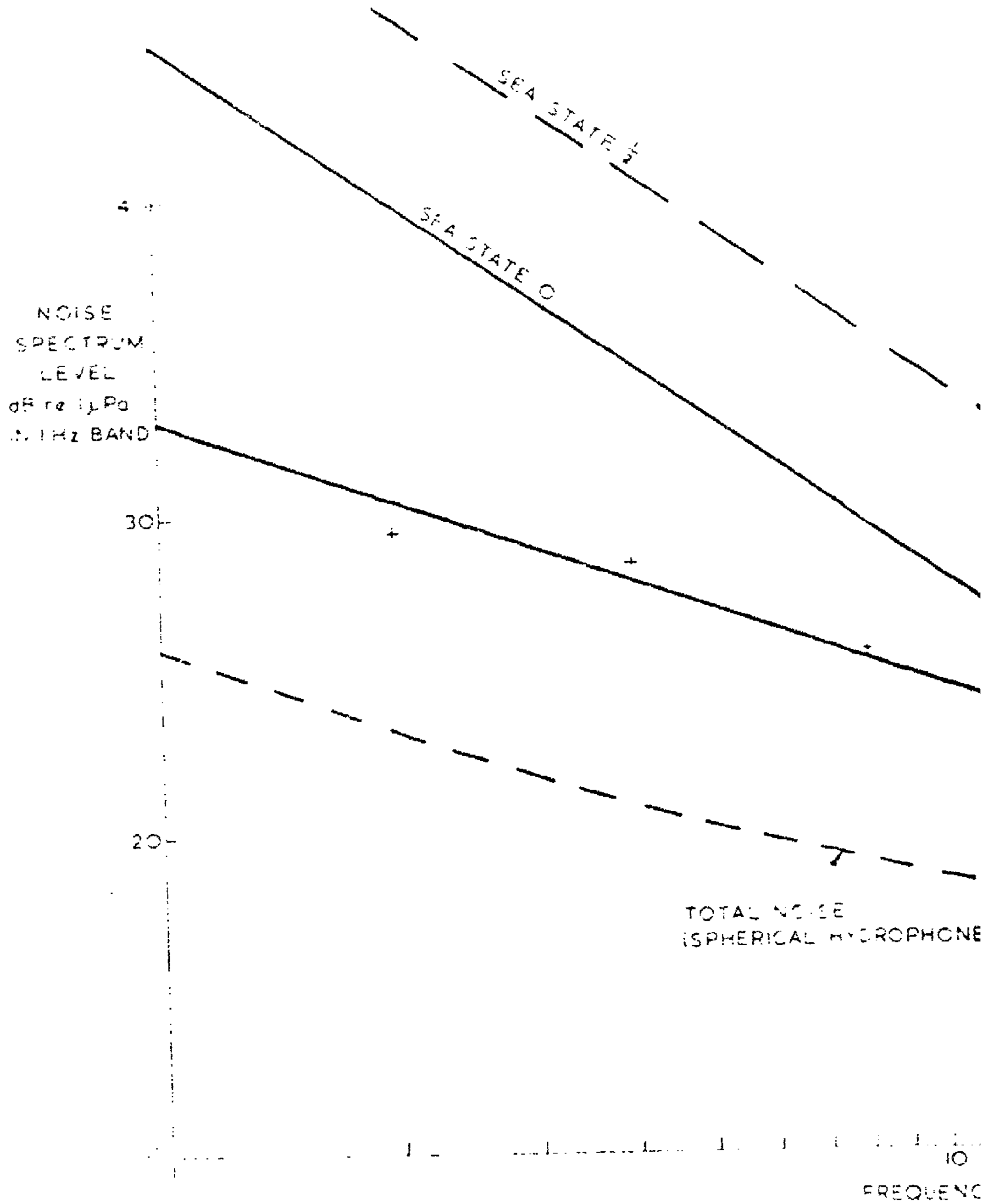


FIG 21. VALUES OF $m_a = \frac{\rho c}{\omega}$


SEA STATE 0
NOISE
SPECTRUM
LEVEL
30
20
10

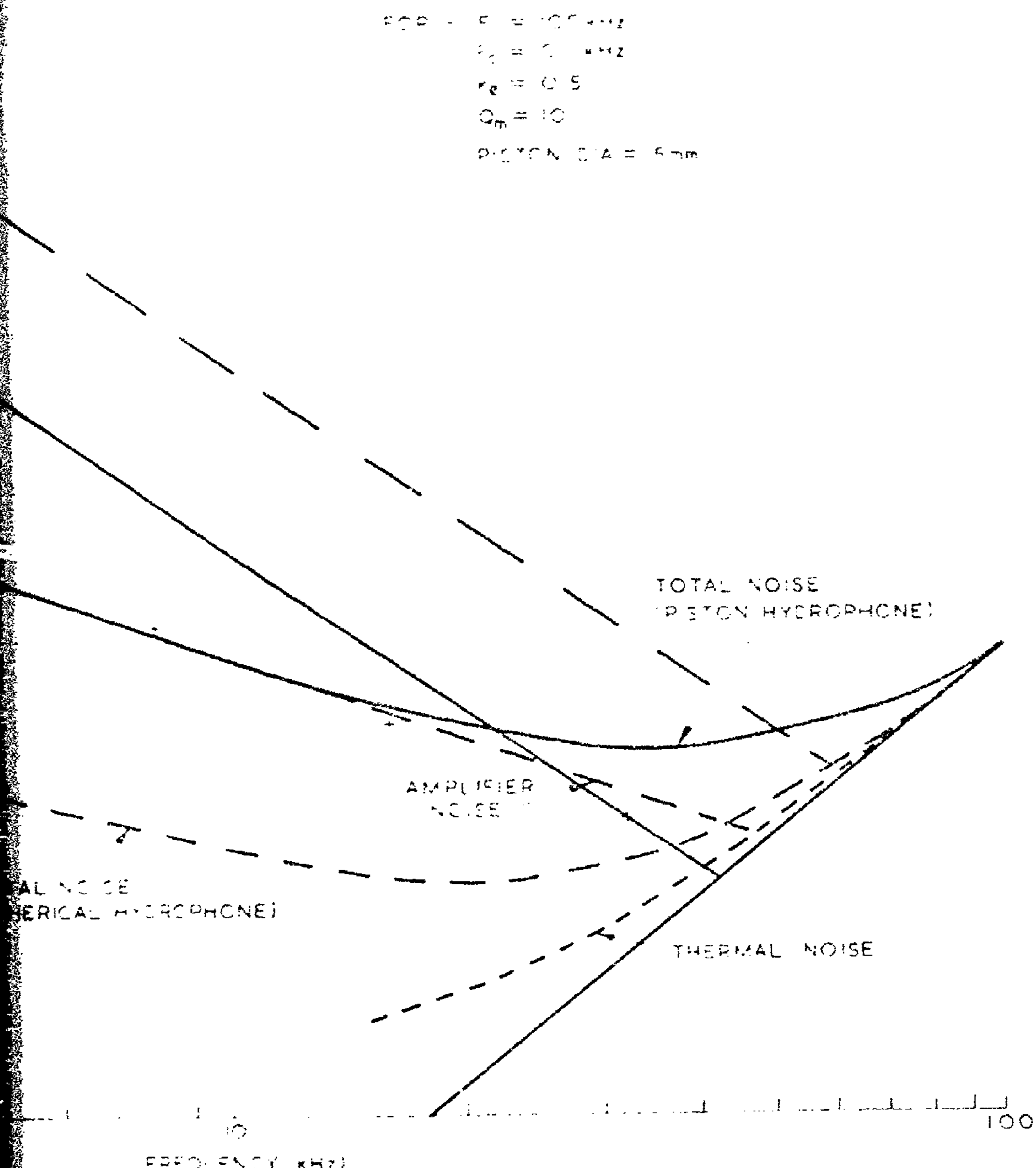


FIG 22 AMPLIFIER & THERMAL NOISE BACKGROUND

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7b. Presented at (for conference papers). Title, place and date of conference

8. Author 1, Surname, initials	9a. Author 2	9b. Authors 3, 4...	10. Date.	pp.	ref.
Stansfield D.			5/75	59	7

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15. Distribution statement

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15. Descriptors (or keywords)

Abstract

The sources of noise in underwater electro-acoustic hydrophones are considered with particular reference to piezoelectric piston-type elements. Expressions are derived for the various contributions, and their influence on hydrophone design for broad-band reception is discussed.

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621.395.62(204.1):
534.837.6

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