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OPTIMAL MID-COURSE MODIFICATIONS OF BALLISTIC MISSILE TRAJECTORIES

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December 1975

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The problem of finding optimal mid-course modifications of ballistic missile trajectories is investigated. A single velocity impulse is applied at the point of transfer from the original orbit to the modified trajectory. This limits the modified trajectory to one which intersects the original trajectory. Optimization of a weighted function of transfer velocity impulse and the time to impact after the modification occurs is accomplished. The function can be weighted to accommodate trade-

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OPTIMAL MID-COURSE MODIFICATIONS

OF BALLISTIC MISSILE TRAJECTORIES

THESIS

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OPTIMAL MID-COURSE MODIFICATIONS OF BALLISTIC MISSILE TRAJECTORIES

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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Graduate Astronautics

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ii

Preface

With the advent of sophisticated ballistic missile early warning and tracking systems, the fact that a missile's trajectory can be measured and predicted allows for a relatively easy high altitude intercept. It has been suggested that a modification from the original trajectory would improve vehicle survivability. My goal was to investigate this type of maneuver and to limit the numerous available modifications by an application of optimization for parameters of interest in the ballistic missile problem.

I would like to thank my advisor, Major Gerald M. Anderson for his helpful suggestions and constructive comments for this study. I would also like to acknowledge Captain Richard M. Potter for introducing me to the numerical methods of ballistic trajectory calculations. Finally, I would like to recognize my wife, Susan, for her patience, understanding, and encouragement during this study.

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Notation

Symbol	
a	semi-major axis
DU	earth canonical distance unit = 20925740 ft
DU/TU	earth canonical velocity unit = 25936.28 ft/sec
ϵ	eccentricity
E	eccentric anomaly
$\mathcal{E}$	specific mechanical energy
F	hit equation
h	specif[illegible]ngular momentum
h_{bo}	burnout altitude
h_{re}	reentry altitude
J	cost function
$\tilde{J}$	augmented cost function
$\overline{P}$	direction to perigee in the perifocal frame
Q	non-dimensional orbit velocity parameter
$\overline{Q}$	perifocal direction perpendicular to $\overline{P}$
r	radius vector magnitude
r_2	radius vector magnitude at the transfer point
TU	earth canonical time unit = 806.8136 sec
T_r	reaction time
T_{f12}	time of flight between burnout and apogee
V	velocity
W	cost function weighting factor
$\overline{X}$	inertial position vector

Symbol

X	inertial $\bar{X}$ direction position component
Y	inertial $\bar{Y}$ direction position component
Z	inertial $\bar{Z}$ direction position component
ΔV	transfer velocity impulse
λ	Lagrange multiplier
μ	gravitational parameter 1 DU^3/TU^2
ν	true anomaly
ν_2	true anomaly at transfer
ϕ_{bo}	flight path angle at burnout
ψ	free flight range angle

Superscripts

-	vector
T	transposed vector

Subscripts

a	apogee
bo	burnout point
f	vehicle at final time
ff	free flight
m	modified orbit
mod	trajectory modification point
n	nominal orbit
p	perifocal $\bar{P}$ direction
p'	modified orbit component, written in nominal perifocal $\bar{P}$ direction
q	perifocal $\bar{Q}$ direction

Subscripts

q'	modified orbit component, written in nominal perifocal $\bar{Q}$ direction
re	reentry point
r	required value
t	target at final time
x	inertial $\bar{X}$ direction component
y	inertial $\bar{Y}$ direction component
z	inertial $\bar{Z}$ direction component
1	vector component
2	vector component
3	vector component

Abstract

The problem of finding optimal mid-course modifications of ballistic missile trajectories is investigated. A single velocity impulse is applied at the point of transfer from the original orbit to the modified trajectory. This limits the modified trajectory to one which intersects the original trajectory. Optimization of a weighted function of transfer velocity impulse and the time to impact after the modification occurs is accomplished. The function can be weighted to accommodate trade-offs between both components.

The study is divided into two parts. Elliptical orbits for a non-rotating, atmosphere free earth are investigated, and then trajectories for an oblate, rotating earth with atmospheric reentry are examined. For the elliptical orbit cases, modified and nominal orbits are coplanar. Pre-apogee, apogee, and post-apogee transfers from the nominal trajectory to a modified orbit at apogee are considered. It is found that a pre-apogee transfer from a near circular nominal trajectory is advantageous for the defined problem.

A method of computing an optimal transfer from a single point on a lofted nominal trajectory for nonplanar nominal and modified trajectories is then presented. An algorithm is derived where a nominal trajectory to a pseudo target is calculated, then transfer to a modified trajectory which impacts the real target is computed. An example problem is shown and the region of a minimum cost transfer is found.

OPTIMAL MID-COURSE MODIFICATIONS OF BALLISTIC MISSILE TRAJECTORIES

I. INTRODUCTION

Background

A long-range ballistic missile trajectory can be divided into three phases, powered flight, free flight, and reentry. After the initial powered flight ceases, the entire trajectory of the vehicle, including impact point can be calculated through use of a series of radar measurements.

Green (Ref 4:6) comments on the anti-ballistic missile intercept problem during the free flight phase: Once the launch has been detected, early warning and tracking information can be supplied to the free flight tracking system of a defender. This mid-course tracking system needs to search only a small space-time zone to acquire the vehicle for trajectory prediction. The long free flight phase is then a disadvantage to the attacking vehicle because intercept is easier.

To counteract this prospect of mission failure, it may be advantageous to make the reentry vehicle change course and target at some time during free flight. However, other factors, such as accuracy, additional mass required, cost, and reliability must be considered. If an acceptible mid-course

modification system is employed, chances of vehicle survival are increased because the missile is not committed to a single trajectory after powered flight ends.

Barnaby (Ref 1:18) suggests that this mid-course modification is a viable penetration aid for use against high altitude intercept of anti-ballistic missiles. Additionally, if this maneuver could be masked, for example, by releasing chaff at the point of trajectory change, all defensive radar contact might be lost.

In any case, the defender has less time for trajectory calculation if the vehicle is again acquired by radar. This reduction in defensive reaction time means that a less accurate measurement of the trajectory is made and the high altitude ABM threat may be circumvented.

Problem Statement

The purpose of this study is to investigate mid-course modifications of long-range ballistic missile trajectories. Optimal orbit changes are found, where the optimality of the new orbit is defined in terms of the total velocity impulse required to change the trajectory, and the warning time given to the defender after the trajectory is modified.

The ballistic trajectory modification, as a specialized case of the general orbital transfer problem, can be accomplished through any number of thrusts. This investigation limits the transfer to a single impulsive thrusting manuever, which in turn, requires that the original and modified trajectories intersect.

Outline

The mid-course modification problem is developed in two stages. First, the two-dimensional transfer, where the nominal and modified orbits are coplanar, is examined.

In Chapter II this type of transfer is defined. The nominal trajectory parameters are developed in Chapter III. Chapters IV, V, and VI present three types of coplanar trajectory modifications, along with the optimum results for each.

Chapter VII poses the general three-dimensional or nonplanar trajectory change problem. Trajectory modifications and optimization for this case are presented in Chapter VIII. This chapter also discusses results of the nonplanar modification case. Finally, the conclusions and recommendations resulting from this study are presented in Chapter IX.

II. PROBLEM DEFINITION - COPLANAR TRAJECTORIES

In this chapter, the basic foundation for the coplanar transfer problem is established. Problem definition includes the assumptions made to analyze the transfer problem, the type of nominal trajectory used in this part of the study, the kinds of orbital transfers considered, and a discussion of optimization pertaining to the trajectory modification problem.

Assumptions

For coplanar nominal and modified trajectories, a spherical, non-rotating earth with no atmosphere is assumed. The equations of a conic, specifically, the equation of an ellipse, then describes the ballistic trajectory. This investigation did not include transfer to parabolic and hyperbolic trajectories since large velocity impulses are needed to establish these types of orbits.

Earth canonical units are used in the analytic expressions describing distance, time, and velocity. These units are defined by Bate (Ref 2:429):

distance unit	1 DU = 20925740 ft
time unit	1 TU = 806.8136 sec
speed unit	1 DU/TU = 25936.28 ft/sec
gravitational parameter	1 $DU^3/TU^2 = 1.407654 \times 10^{16}$ ft^3/sec^2.

Target location and missile altitude at burnout are two basic parameters in the calculations. The target range

is assumed to be a typical ICBM range of 6000 n. mi. Burnout is assumed to occur at .05 DU altitude.

These initial assumptions can be used to solve the transfer problem in closed form, if the correct nominal trajectory is employed.

Nominal Trajectory Definition

There are three different types of ballistic missile trajectories, the low, the high or lofted, and the maximum range trajectory. Both the lofted and the low trajectory can reach a target with the same initial conditions of velocity and altitude at burnout. The distinguishing factor is the direction of the velocity vector at burnout.

The low ballistic trajectory is characterized by its smaller apogee. This type of orbit has the advantage of reaching a target quickly, since less total distance along the trajectory is covered; however, this type of trajectory is not generally useful for ballistic missiles since the vehicle may encounter the atmosphere too quickly. Even though this section does not consider atmospheric reentry, the low trajectory is not used since it is not a realistic situation for ballistic missiles.

The lofted trajectory is produced by having a higher elevation of the velocity vector at burnout. This path has the advantage of being less susceptible to initial errors at burnout. Since this trajectory has a higher apogee, more distance is covered than for a low trajectory to the same target.

As would be expected, the time of flight is longer for this case. Because the final accuracy is better, this type of trajectory is routinely employed for ballistic missiles. The lofted nominal trajectory is used for the second part of this study and is discussed further in Chapter VII.

The third type of trajectory, the one for maximum range, is the lowest energy orbit needed to reach the target. For given initial burnout conditions there is usually a high and low trajectory to a given target range, but there is one target range where only one path to the target exists. This is the maximum range trajectory.

Maximum range trajectories exist only for target ranges below 180°. For a range of 180°, the maximum range trajectory is a circular orbit if burnout occurs above the earth's surface. The vehicle will not impact on the earth's surface for this range. If burnout occurs at sea level, this trajectory is a path along the surface of the earth and cannot be used.

If the maximum range trajectory is assumed symmetrical, i.e., burnout and reentry points are at equal altitudes, then the free flight portion of the orbit can be completely specified by the free flight range, or the range angle subtended at the earth's center, and the burnout altitude.

Because this maximum range trajectory gives the most range for an initial velocity, and since it can be analytically expressed by two parameters, it is selected for use as a nominal trajectory in the coplanar transfer problem. Nominal maximum range trajectory parameters are developed in Chapter III.

Choice of Modified Orbits

In addition to defining the nominal trajectory, the modified orbit which connects a transfer point on the nominal to a target is required. A general transfer at any point in the nominal trajectory includes all types of modified orbits. Several, less complex analytic elliptical cases are examined in this study. Transfer to the apogee of a modified orbit is a logical choice since the velocity magnitude at apogee is the lowest for the modified trajectory. The type of modified trajectories studied are as follows:

1. Transfer from a nominal trajectory at apogee to a new orbit which also has its apogee at the transfer point.

2. Pre-apogee transfer from a nominal trajectory to a new orbit which has its apogee at the transfer point.

3. Post-apogee transfer from a nominal trajectory to a new orbit which has its apogee at the transfer point.

These coplanar transfers are developed in Chapter IV, V, and VI respectively. The equations of an ellipse are used and all orbital parameters are found analytically.

Trajectory Optimization

A number of modified orbits satisfy the problem of impacting upon a target from some point in the nominal trajectory. In order to relate the merits of one trajectory to those of another, a cost function must be defined.

The factors considered in the cost function formulation are the impulse required to transfer to the modified orbit

and the reaction time given to a defender after this change is made. It is assumed that the booster is capable of imparting velocity needed to establish the nominal trajectory so this is not considered in the cost.

The impulse required to transfer from the nominal to the modified trajectory is directly related to the amount of propellant used by the propulsion system. The transfer impulse is defined at the point of transfer as the magnitude of the difference between the two different trajectory velocities:

$$\Delta V = |\bar{V}_m - \bar{V}_n| \tag{1}$$

where $\bar{V}_m$ is the velocity on the modified orbit at the transfer point and $\bar{V}_n$ is the velocity on the nominal trajectory at the transfer point.

Reaction time is included in the cost formulation since vehicle survivability is related to the amount of warning given to the defender. But since both time and ΔV are included in the cost formulation, some type of relation between the two components must be considered.

To produce this relationship, a weighting factor on reaction time is introduced. For high values of this factor, the reaction time contributes most to the cost. Low values of the factor allow transfer ΔV to dominate. The cost function is then defined as

$$J = \frac{1}{2}\Delta V^2 + \frac{W}{2} T_r^2 \qquad (2)$$

where ΔV is the transfer impulse

W is the weighting factor

T_r is the reaction time.

The feature of including W in the cost function allows a trajectory designer to apply the optimization to different mid-course propulsion systems. The effect of varying W was examined and the value of .15 was selected since this gave optimum transfers in a realistic range of transfer ΔV. The trajectories which produce the lowest values of this cost function are the desired solutions to the defined problem.

III. MAXIMUM RANGE NOMINAL TRAJECTORY CALCULATION

Nominal Orbit Parameters

If a maximum range trajectory is assumed to be symmetrical, i.e., burnout and reentry occur at the same altitude, the magnitude of the radius vector at burnout, r_{bo}, and the free flight range angle, ψ_n, describe all elements of the two-dimensional trajectory. This nominal trajectory is shown in fig. 1. It should be noted that ψ_n is the angle between the burnout and reentry radii.

The flight path angle which relates local horizontal to the local velocity vector is defined for the maximum range trajectory (Ref 2:292) as

$$\phi_{bo} = \frac{1}{4}(\pi - \psi_n) \tag{3}$$

where ϕ_{bo} is the flight path angle

ψ_n is the nominal free flight range angle.

The nominal orbit eccentricity can be derived from

$$e_n = \left(1 + \frac{2\mathcal{E}h^2}{\mu^2}\right)^{1/2} \tag{4}$$

where e_n is the nominal orbit eccentricity

$\mathcal{E}$ is the specific mechanical energy of the orbit

h is the specific angular momentum of the orbit

μ is the earth gravitational parameter.

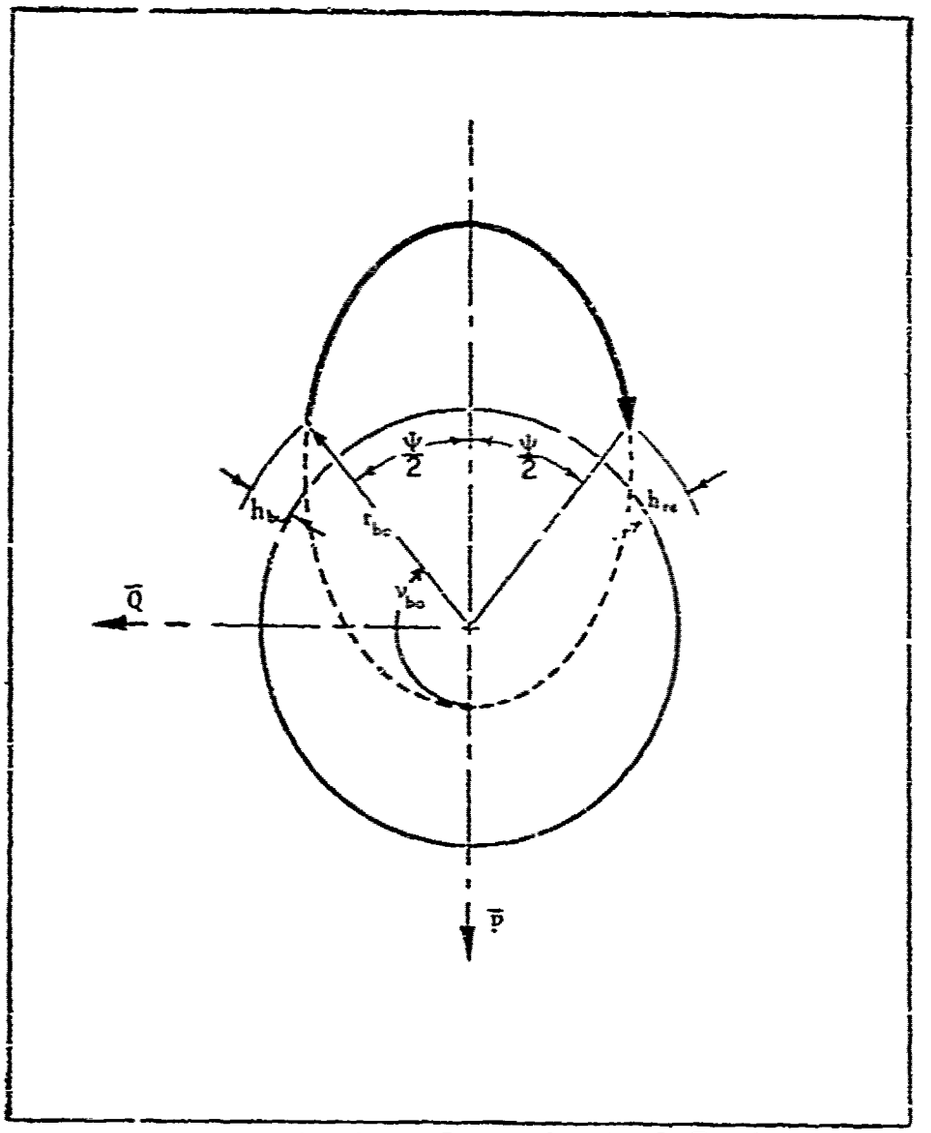

Fig. 1. The Maximum Range Nominal Trajectory

Specific mechanical energy, constant for the orbit, is found from the relation

$$\mathcal{E} = \frac{V_{bo}^2}{2} - \frac{\mu}{r_{bo}} \tag{5}$$

where V_{bo} is the burnout velocity.

The specific angular momentum, also constant for the nominal trajectory, can be found at burnout:

$$h = r_{bo} V_{bo} \cos \Phi_{bo} \tag{6}$$

Equations (5) and (6) require the value of V_{bo} which can be found from the nondimensional orbital parameter, Q_{bo}, defined by Bate (Ref 2:280):

$$Q_{bo} = V_{bo}^2 r_{bo} / \mu \tag{7}$$

This parameter is the squared ratio of the velocity of the vehicle to circular orbital velocity at the burnout point. For a maximum range trajectory (Ref 2:293)

$$Q_{bo} = \frac{2 \sin(\Psi_n/2)}{1 + \sin(\Psi_n/2)} \tag{8}$$

By substituting eqs (5), (6), and (7) into eq (4), the following expression for nominal orbit eccentricity results:

$$e_n = \left[1 + (Q_{bo} - 2) Q_{bo} \cos^2 \phi_{bo}\right]^{1/2} \tag{9}$$

This can be simplified to a function of ψ_n and ϕ_{bo} by substituting eq (8) into eq (9):

$$e_n = \left\{1 - \frac{4 \sin(\Psi_n/2) \cos^2 \phi_{bo}}{\left[1 + \sin(\Psi_n/2)\right]^2}\right\}^{1/2} \tag{10}$$

<u>Nominal Time of Free Flight Calculation</u>

Time of flight on the elliptical orbit is found through use of the Kepler time of flight calculations (Ref 2:185). The true anomaly at burnout for the maximum range trajectory is

$$\nu_{bo} = \pi - (\Psi_n/2) \tag{11}$$

where ν_{bo} is the true anomaly at burnout.

Once knowing ν_{bo}, the eccentric anomaly, E_{bo}, may be found:

$$E_{bo} = \cos^{-1}\left(\frac{e_n + \cos \nu_{bo}}{1 + e_n \cos \nu_{bo}}\right) \tag{12}$$

The nominal orbit semi-major axis, a_n, is found through use of eqs (5), (7), and (8) substituted into

$$\mathcal{E} = -\frac{\mu}{2a_n} \tag{13}$$

This yields the desired result for a_n:

$$a_n = \frac{r_{bo}\left[1+\sin(\Psi_n/2)\right]}{2} \tag{14}$$

Since a_n and E_{bo} are known, the nominal trajectory free flight time can be found. Because the orbit is assumed to be symmetrical, the total time of flight, T_{ff}, is twice the time from burnout to apogee, and by inspection of fig. 1, the eccentric anomaly at apogee is π. Then

$$T_{ff} = 2\,a_n^{3/2}\,(\pi - E_{bo} + e_n \sin E_{bo}) \tag{15}$$

It has been shown that all necessary parameters for the nominal maximum range trajectory can be found as functions of the burnout radius, r_{bo}, and the free flight range angle, Ψ_n.

IV. APOGEE TO APOGEE TRANSFERS

The first case considered is the transfer from a nominal orbit at apogee to a new orbit also at apogee. This type of transfer is depicted in fig. 2. Since the apogee radius of the nominal trajectory defines the transfer point and the equations of an ellipse are employed, the transfer is only a function of the desired final range angle and the eccentricity of the modified orbit.

The required modified range angle employed in all analytic examples is 100°, equivalent to 6000 nautical miles on a spherical earth surface. The nominal orbit used is defined in Chapter III.

Modified Orbit Parameters

The semi-major axis of the modified orbit is defined by

$$a_m = r_a/(1+e_m) \tag{16}$$

where a_m is the modified orbit semi-major axis

r_a is the nominal orbit apogee radius

e_m is the modified orbit eccentricity.

Through the use of the equation of a conic

$$r = \frac{a(1-e^2)}{1+e\cos\nu} \tag{17}$$

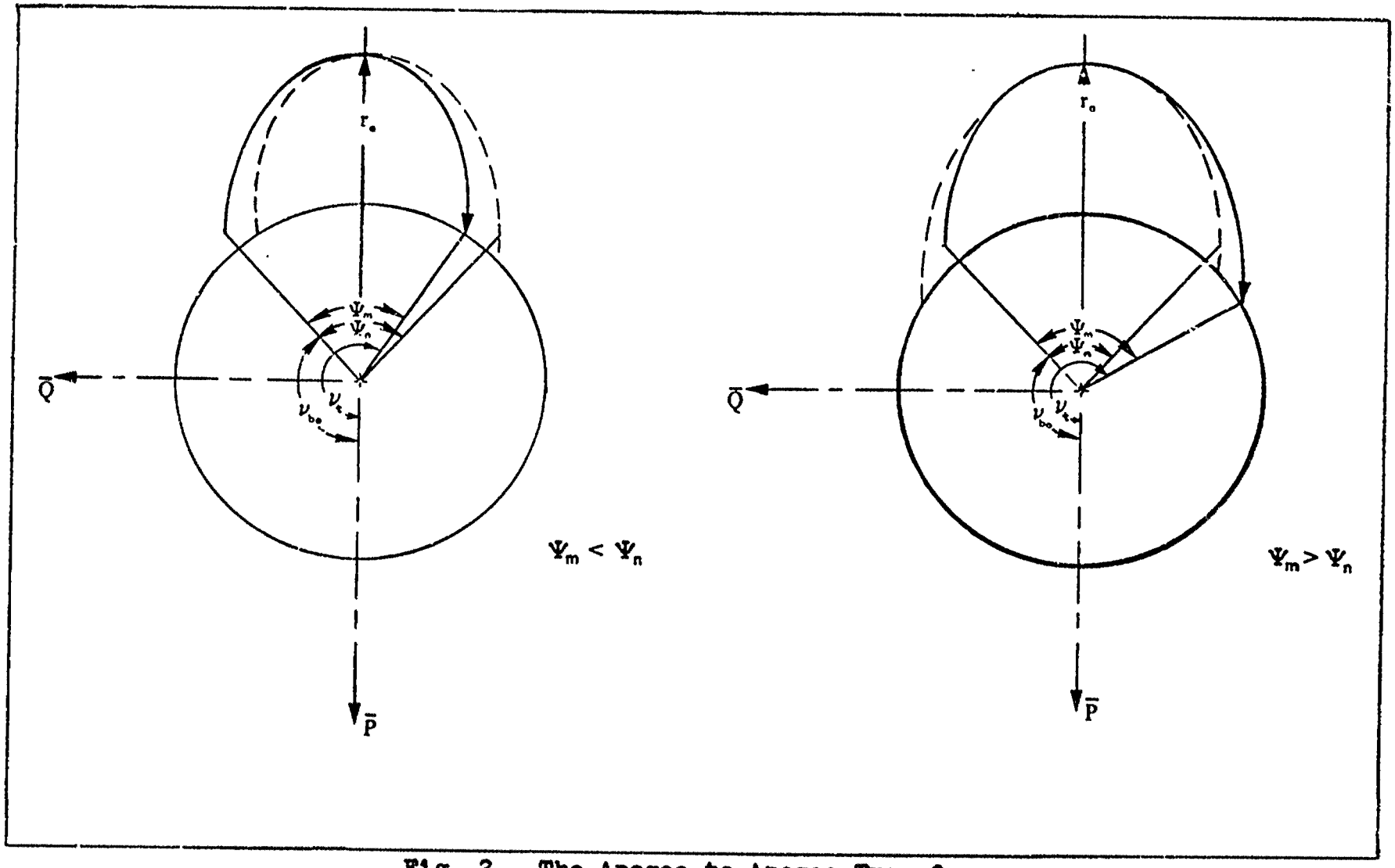

Fig. 2. The Apogee to Apogee Transfer

any true anomaly, ν, can be found in terms of r, a, and e. It is necessary to find the true anomaly at the point where the modified orbit impacts the earth in order to obtain ψ_m, the final range angle resulting from the orbit modification.

Using eq (17) and solving for the true anomaly at the target, ν_t

$$\nu_t = \mathrm{Cos}^{-1}\left\{\frac{1}{e_m}\left[\frac{a_m(1-e_m^2)}{r_t} - 1\right]\right\} \tag{18}$$

where r_t is the radius at the target location. It must be noted that ν_t lies between π and 2π.

For impact at the target on the spherical earth, the value of r_t is 1 DU. Then

$$\nu_t = \mathrm{Cos}^{-1}\left\{\frac{1}{e_m}\left[a_m(1-e_m^2)-1\right]\right\} \tag{19}$$

The final range angle is calculated once the values of true anomaly at the burnout and impact points are known. The range angle traversed on the nominal trajectory is from burnout to apogee or $\pi - \nu_{bo}$. The range angle traversed on the modified orbit is the angular difference between apogee and impact, $\nu_t - \pi$. Combining these two ranges, the resulting range angle, ψ_m, is

$$\Psi_m = \nu_t - \nu_{bo} \tag{20}$$

Cost Function Parameters

The two parameters required to define a cost for each transfer are found analytically. Time of flight calculations are done using the Kepler time of flight equations (Ref 2:186). Time of flight on the nominal trajectory is one-half the nominal time of flight derived in Chapter III:

$$T_{f12} = a_n^{3/2}(\pi - E_{bo} + e_n \sin E_{bo}) \tag{21}$$

where T_{f12} is the time of flight from burnout to nominal orbit apogee (TU).

To find the time of flight from apogee to impact, or the reaction time, it is first necessary to find the eccentric anomaly at impact, E_t:

$$E_t = \cos^{-1}\left(\frac{e_m + \cos \nu_t}{1 + e_m \cos \nu_t}\right) \tag{22}$$

E_t also lies between π and 2π.

Now, the reaction time can be found:

$$T_r = a_m^{3/2}(E_t - e_m \operatorname{Sin} E_t - \pi) \tag{23}$$

where T_r is the reaction time.

To find the velocity impulse at transfer, the apogee velocities of both orbits are calculated and the difference computed. A perifocal coordinate system, illustrated in fig. 2, with $\bar{P}$ in the direction from the orbit focus to perigee, and $\bar{Q}$ perpendicular to $\bar{P}$ in the direction of travel on the orbit, is defined.

Velocities in the perifocal frame at any point can be found, once orbit parameters are known (Ref 2:73):

$$\bar{V} = \left[\frac{1}{r(1 + e \operatorname{Cos} \nu)}\right]^{1/2} \left[-\operatorname{Sin} \nu \bar{P} + (\operatorname{Cos} \nu + e)\bar{Q}\right] \tag{24}$$

At apogee, $\nu = \pi$ which means that $V_p = 0$. Velocity in the $\bar{Q}$ direction is then

$$V_q = -\left(\frac{1 - e}{r_a}\right)^{1/2} \tag{25}$$

The ΔV required at apogee can then be calculated:

$$\Delta V = |V_{qm} - V_{qn}| = \left| -\left(\frac{1-e_m}{r_a}\right)^{1/2} + \left(\frac{1-e_n}{r_a}\right)^{1/2} \right| . \qquad (26)$$

Since ΔV and T_r are known, a cost may be found for each nominal trajectory and its corresponding solution.

Method of Solution

For the apogee to apogee transfer case, it is desired to find a modified orbit eccentricity which, for a given nominal trajectory, hits a specified range angle, ψ_r. The method of solution involves one function of the variable e_m. The function is the "hit" equation, F:

$$F = 0 = \Psi_m - \Psi_r \qquad (27)$$

When F=0, the modified orbit impacts at the required target range, ψ_r. Finding the solution to eq (27) involves an iteration on e_m. A general purpose algorithm, presented by Powell (Ref 7), solves a system of nonlinear equations. This method is used for the iteration on e_m and other iterative formulations derived in this investigation. A further discussion of Powell's algorithm and its use is found in Appendix B.

For a given nominal trajectory, only one solution of eq (27) exists in this type of transfer.* A series of solutions is generated by varying Ψ_n, and a cost function is formed from the set of solutions. The computer program used for this is listed as Program 1 in Appendix C.

Since the cost function is bounded - the lower limit is zero and the upper limit is within the parabolic modified orbit limitation - a minimum cost exists. For a minimum cost solution, an associated nominal trajectory can be found. This nominal trajectory is then used as the initial targeting information for the optimal apogee to apogee transfer. So the minimum cost solution must be found to define the nominal trajectory necessary for an optimal apogee to apogee transfer.

For the problem formulated, an interior minimum is exhibited. The minimum cost value and the associated nominal trajectory can be found by considering a constrained optimization problem. The cost function, eq (2), is augmented with the constraint that the hit equation be solved;

$$\tilde{J} = \frac{1}{2}\Delta V^2 + \frac{W}{2}T_r^2 + \lambda(\Psi_m - \Psi_r) \qquad (28)$$

where λ is the Lagrange multiplier associated with the hit condition.

* The other coplanar transfers considered have an infinite number of solutions for each nominal trajectory.

The minimum of eq (28) is found by solving the first order necessary conditions of a minimum. Three variables, e_m, Ψ_n, and λ are used in the following equations to find the minimum point:

$$\frac{\partial \tilde{J}}{\partial \Psi_n} = 0 = \Delta V \frac{\partial \Delta V}{\partial \Psi_n} + W T_r \frac{\partial T_r}{\partial \Psi_n} + \lambda \frac{\partial \Psi_m}{\partial \Psi_n} \tag{29}$$

$$\frac{\partial \tilde{J}}{\partial e_m} = 0 = \Delta V \frac{\partial \Delta V}{\partial e_m} + W T_r \frac{\partial T_r}{\partial e_m} + \lambda \frac{\partial \Psi_m}{\partial e_m} \tag{30}$$

$$\frac{\partial \tilde{J}}{\partial \lambda} = 0 = \Psi_m - \Psi_n \tag{31}$$

To solve eqs (29), (30), and (31), the partial derivatives of $\tilde{J}$ with respect to Ψ_n and e_m are required. These derivatives are calculated through use of a first difference scheme (Ref 3:217). It was found that regulating the initial perturbations so that first differences are on the order of 10^{-8} allows an accurate minimum point to be calculated. This small first difference is well within the 15 significant digit capability of the computer used.

The first order necessary conditions of a minimum were programmed and solved for this case. A listing of the computer program is found as Program 2 in Appendix C.

Results and Analysis

For a target range of 100°, solutions for transfer from nominal range angles between 70° and 180° are found, using Program 1. An interior minimum in the cost function is exhibited, and the minimum point is found by employing Program 2.

Figure 3 shows the resulting cost function for a weighting factor of .15. Figures 4 and 5 depict the reaction time and velocity which produce the cost function.

A minimum cost transfer for the apogee to apogee transfer case occurs for a nominal range angle of 145.85°. Since the altitude at burnout is assumed to be .05 DU, this nominal range angle is sufficient to define all initial burnout parameters of the nominal trajectory. The optimal apogee to apogee trajectory modification is then produced from this initial trajectory.

The behavior of the cost function for this case can be explained by analyzing figs. 4 and 5. As nominal range increases, a larger modified orbit eccentricity is required in order to hit the target range. The higher eccentric orbits cause a reduction in reaction time, as seen in fig. 4. This effect is shown in eq (23).

Because transfer to a higher eccentric orbit is required for long nominal range angles, the velocity needed to transfer to the modified orbit increases. This is shown in fig. 5.

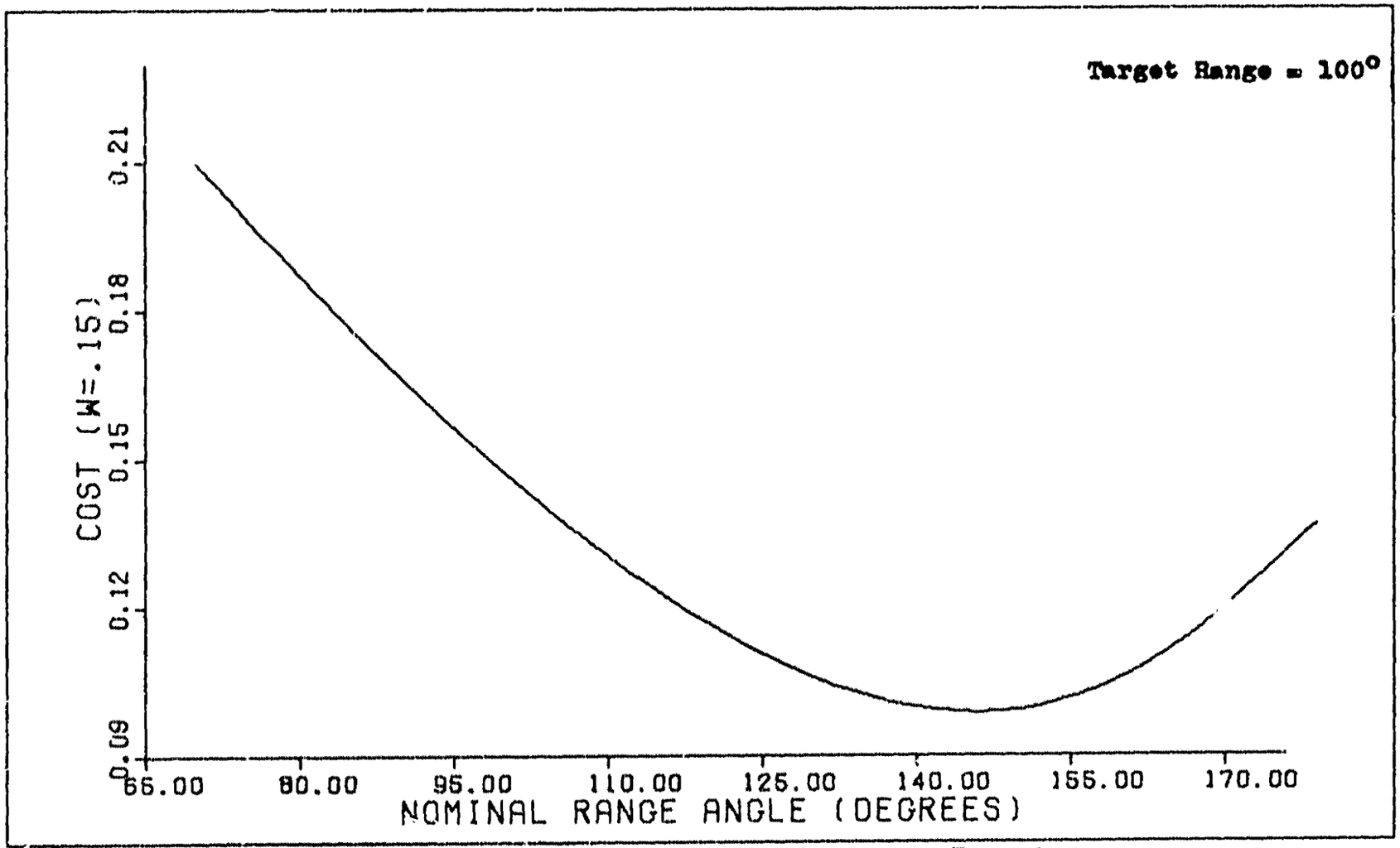

Fig. 3. Cost vs. Range: Apogee to Apogee Transfers

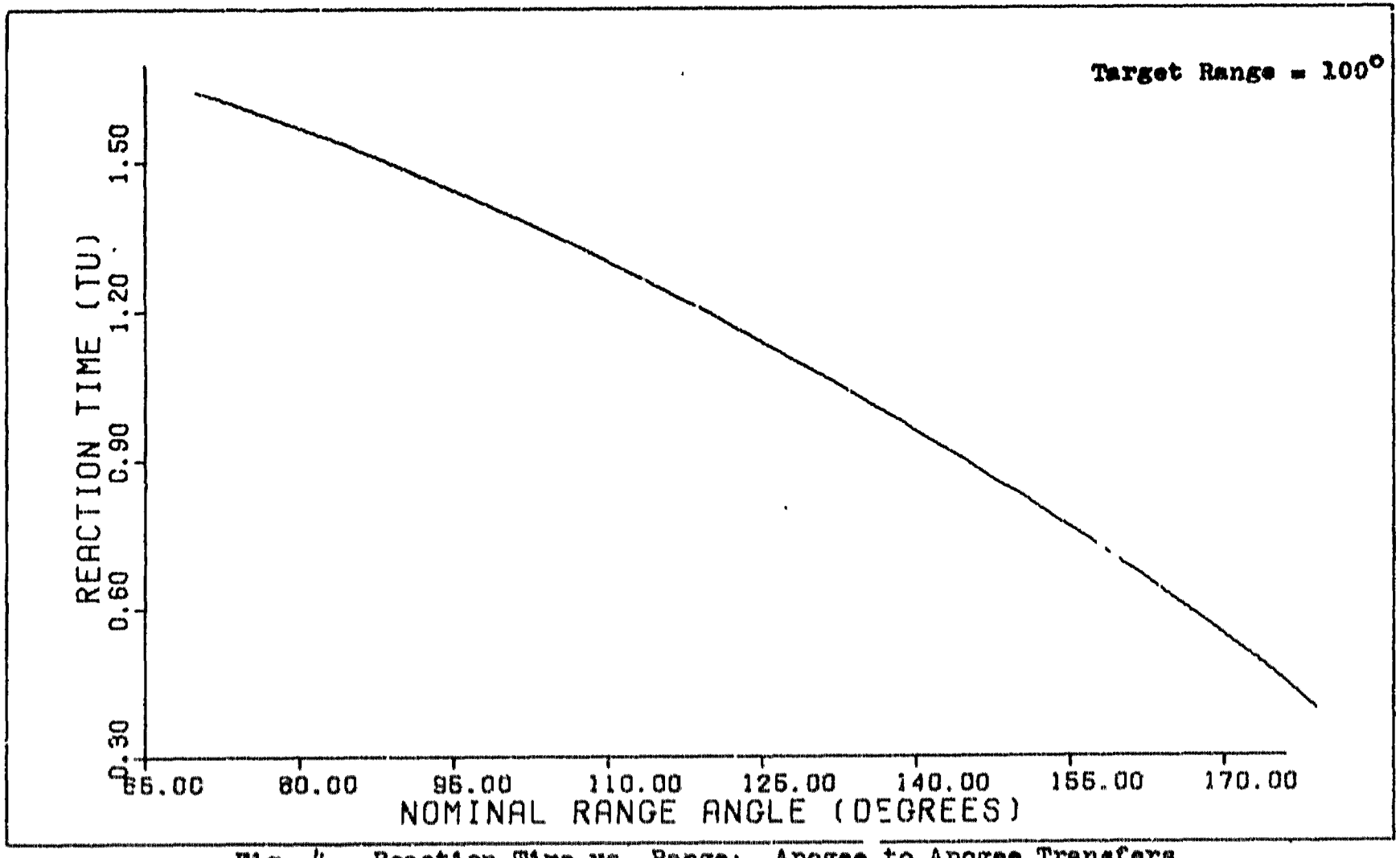

Fig. 4. Reaction Time vs. Range: Apogee to Apogee Transfers

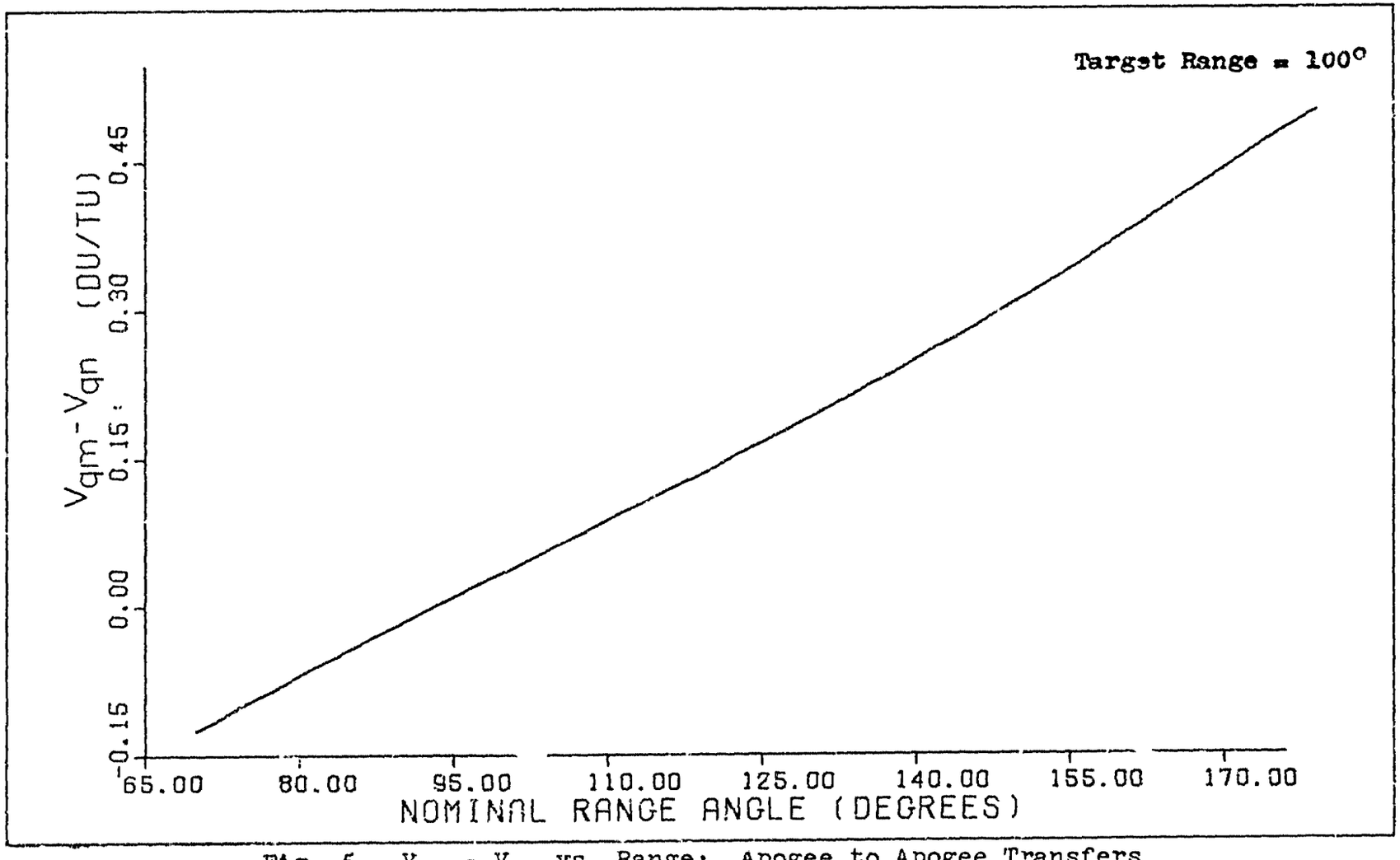

Fig. 5. $V_{qm} - V_{qn}$ vs. Range: Apogee to Apogee Transfers

V. PRE-APOGEE TO APOGEE TRANSFERS

The second case considered is the pre-apogee transfer from a nominal orbit to apogee on the new orbit. A limiting case of this particular transfer is the apogee to apogee transfer described in Chapter IV. The fact that transfer may be accomplished prior to nominal apogee introduces an added degree of freedom from the first case. Three parameters are now required to define the modified orbit. They are the modified orbit eccentricity, e_m, the radius at transfer, r_2, and the desired final range angle, ψ_m. The pre-apogee transfer is shown in fig. 6.

This transfer problem is very similar to the first case considered. The only differences arise in the definition of the transfer point, calculation of the modified range angle, and finding the transfer ΔV. The nominal orbit is the maximum range trajectory described in Chapter III.

Modified Orbit Parameters

The orbital parameters for this case are those found in Chapter IV, with the following exceptions. Definition of the transfer point on the nominal trajectory involves the calculation of a true anomaly at transfer, ν_2. Because the radius at transfer, r_2, is variable, the true anomaly at transfer can be found from eq (17):

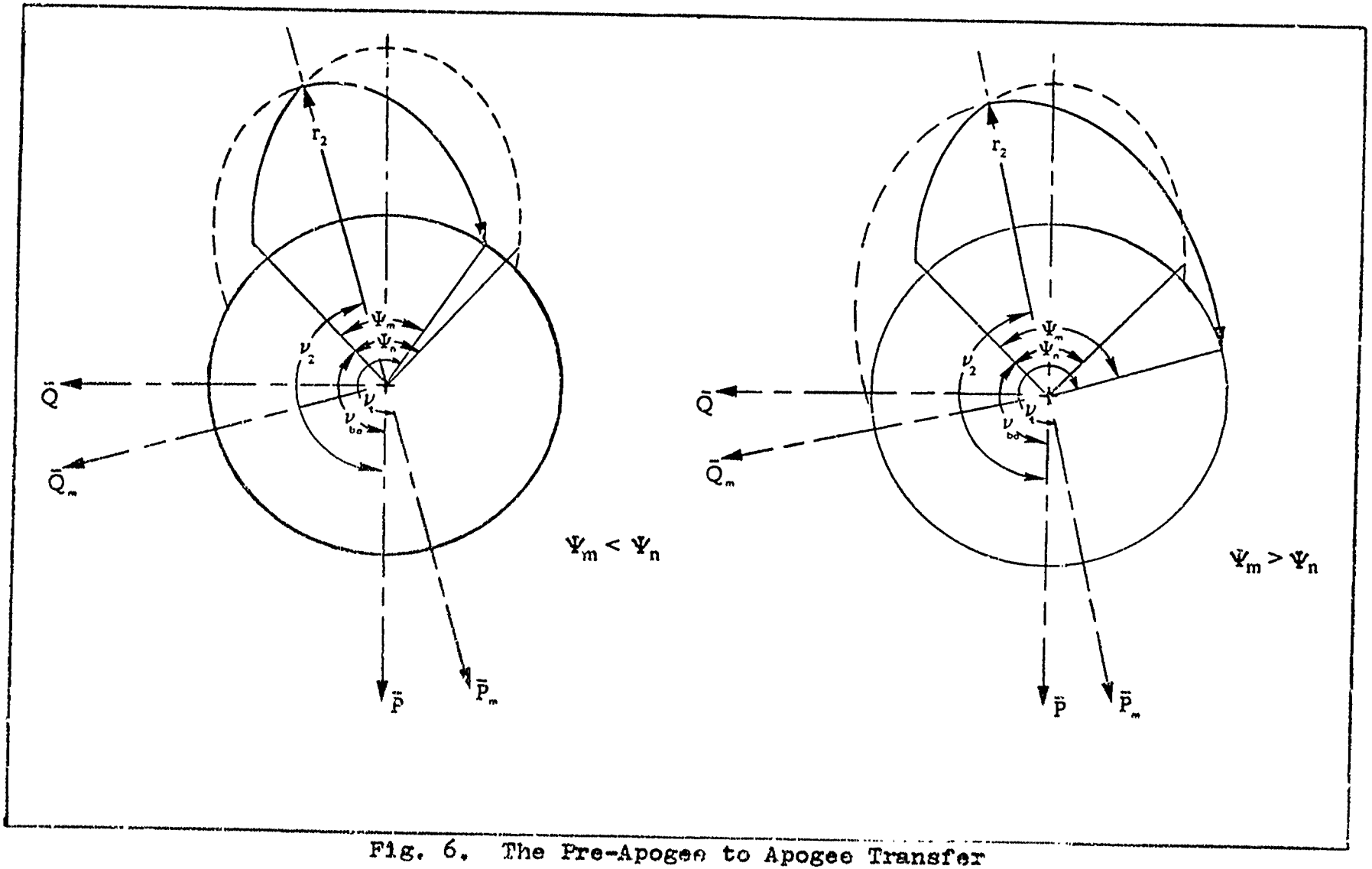

Fig. 6. The Pre-Apogee to Apogee Transfer

$$\nu_2 = \mathrm{Cos}^{-1}\left\{\frac{1}{e_n}\left[\frac{a_n(1-e_n^2)}{r_2} - 1\right]\right\} \tag{32}$$

Since this case considers pre-apogee transfers, ν_2 lies between 0 and π. The variable r_2 is the apogee of the modified orbit.

The range angle traversed on the nominal trajectory is from burnout to transfer, or $\nu_2 - \nu_{bo}$. On the modified orbit, the traversed angle is from apogee to impact, or $\nu_t - \pi$. The resulting range angle, Ψ_m, is

$$\Psi_m = \nu_2 - \nu_{bo} + \nu_t - \pi \tag{33}$$

Cost Function Parameters

Time of flight calculations are a- in Chapter IV. An eccentric anomaly at transfer is defined by using the value of ν_2:

$$E_2 = \mathrm{Cos}^{-1}\left(\frac{e_n + \mathrm{Cos}\,\nu_2}{1 + e_n \mathrm{Cos}\,\nu_2}\right) \tag{34}$$

Since pre-apogee transfers are considered, E_2 also lies between 0 and π.

Using this result, the time of flight on the nominal trajectory is found from the Kepler time of flight equation:

$$T_{f12} = a_n^{3/2}(E_2 - e_n \sin E_2 - E_{bo} + e_n \sin E_{bo}) \qquad (35)$$

where T_{f12} is the time of flight from burnout to trajectory modification. Reaction time, T_r, is found from eq (23).

The transfer ΔV calculations are different from those in Chapter IV. Figure 6 shows that the direction to perigee is different for the modified orbit. A modified orbit perifocal reference frame is defined in order to calculate velocities from eq (24). Velocities on the nominal orbit are found directly from eq (24) in the nominal perifocal reference frame. The relation between the nominal and modified perifocal frames is

$$\begin{bmatrix} V_{p'} \\ V_{q'} \end{bmatrix} = \begin{bmatrix} -\cos \nu_2 & \sin \nu_2 \\ -\sin \nu_2 & -\cos \nu_2 \end{bmatrix} \begin{bmatrix} V_{pm} \\ V_{qm} \end{bmatrix} \qquad (36)$$

as found from the true anomaly at transfer, ν_2, and the geometry of the reference frames. From this relation, the modified orbit velocities, V_{pm} and V_{qm} can be written in nominal perifocal frame as $V_{p'}$ and $V_{q'}$ for direct comparison with the velocities of the nominal trajectory.

From eq (24) the velocity components on the nominal orbit at transfer are

$$V_p = \frac{-\sin \nu_2}{\left[r_2(1+e_n \cos \nu_2)\right]^{1/2}} \tag{37}$$

$$V_q = \frac{e_n + \cos \nu_2}{\left[r_2(1+e_n \cos \nu_2)\right]^{1/2}} \tag{38}$$

where V_p and V_q are the velocities in the nominal perifocal frame.

Because transfer to the modified orbit occurs at apogee, velocity in the modified $\bar{P}$ direction is zero, $V_{pm} = 0$. But

$$V_{qm} = -\left(\frac{1-e_m}{r_2}\right)^{1/2} \tag{39}$$

as before. The radius at transfer, r_2, is the apogee radius of the new orbit.

When the modified orbit velocity is transformed to the nominal perifocal frame by eq (36), the transfer ΔV is found for each nominal perifocal direction:

$$\Delta V_p = V_{p'} - V_p = \sin \nu_2 \, V_{qm} - V_p \tag{40}$$

$$\Delta V_q = V_{q'} - V_q = -\cos \nu_2 V_{qm} - V_q \qquad (41)$$

By combining the two differences, the total ΔV is found:

$$\Delta V = |\bar{V}_m - \bar{V}_n| = \left(\Delta V_p^2 + \Delta V_q^2\right)^{1/2} \qquad (42)$$

Now, since ΔV and T_r are defined, a cost may be found for each nominal trajectory and its corresponding solutions.

Method of Solution

For the pre-apogee to apogee transfer, r_2 may vary between the burnout and apogee radii of each nominal trajectory. An infinite number of modified orbits which hit a required final range angle exist for a single nominal trajectory. The solution of this transfer problem must find the optimal transfer for a given nominal trajectory.

The method of finding this optimal transfer uses two steps. First, the values of r_2 and ψ_n are fixed, and a trajectory which connects the transfer point and the target is computed. For a constant ψ_n, a series of solutions is found by allowing r_2 to vary between burnout and apogee. A set of cost values is then calculated. These cost values are sorted to find the lowest value of cost and the value of r_2 for which it occurs. This solution then represents the region of a minimum cost pre-apogee to apogee transfer

from a given nominal trajectory. Likewise, other transfers are found by calculating solutions for different nominal range angles. The result of this computation is a set of near-optimum pre-apogee to apogee transfers which vary with nominal trajectory. Program 3 of Appendix C is written to accomplish this first step of the solution.

The region of a minimum cost transfer is predicted from the first step. Now, the actual minimum cost transfer for a single nominal trajectory is found by solving the first order necessary conditions of a minimum. The initial estimates for the iterative solution are obtained from the first step.

As before, the augmented cost function, eq (23), is formed. The variables in this problem are e_m, r_2, and λ. The first order necessary conditions are

$$\frac{\partial \tilde{J}}{\partial e_m} = 0 = \Delta V \frac{\partial \Delta V}{\partial e_m} + W T_r \frac{\partial T_r}{\partial e_m} + \lambda \frac{\partial \Psi_m}{\partial e_m} \tag{43}$$

$$\frac{\partial \tilde{J}}{\partial r_2} = 0 = \Delta V \frac{\partial \Delta V}{\partial r_2} + W T_r \frac{\partial T_r}{\partial r_2} + \lambda \frac{\partial \Psi_m}{\partial r_2} \tag{44}$$

$$\frac{\partial \tilde{J}}{\partial \lambda} = 0 = \Psi_m - \Psi_r \tag{45}$$

The partial derivatives of $\tilde{J}$ are found as discussed in Chapter IV. Equations (43), (44), and (45) are solved by an iteration on e_m, r_2, and λ. In this way, the optimum

pre-apogee to apogee transfer for a given nominal range angle is found. A series of solutions, where ψ_n is allowed to vary, then produces the cost function. Program 4 of Appendix C is written to complete this second step.

Results and Analysis

For a target range of 100°, the minimum cost solutions were found with nominal range angles varying between 70° and 180°. A weighting factor of .15 was employed in these calculations. Results are illustrated in figs. 7 through 10.

As shown by fig. 7, the cost function, in this case, decreases up to the maximum nominal range boundary. Figure 8 shows that up to a certain nominal range angle (approximately 138°), the optimum transfer point is the nominal trajectory apogee. Beyond this range angle, the optimum transfer point in the nominal trajectory moves toward burnout.

The change in the cost function, as compared to the first case, is due mostly to the different behavior in the transfer ΔV. Figures 9 and 10 show that as nominal range increases, the reaction time remains a generally decreasing function, but the ΔV differs from the first case. At the point where apogee to apogee transfers become non-optimal, the ΔV begins to decrease. Up to this point, ΔV is an increasing function. This behavior accounts for the decreasing trend in the cost function at the larger nominal range angles.

The initial decrease of ΔV shown in fig. 10 is due to the fact that the magnitude of ΔV is calculated. The

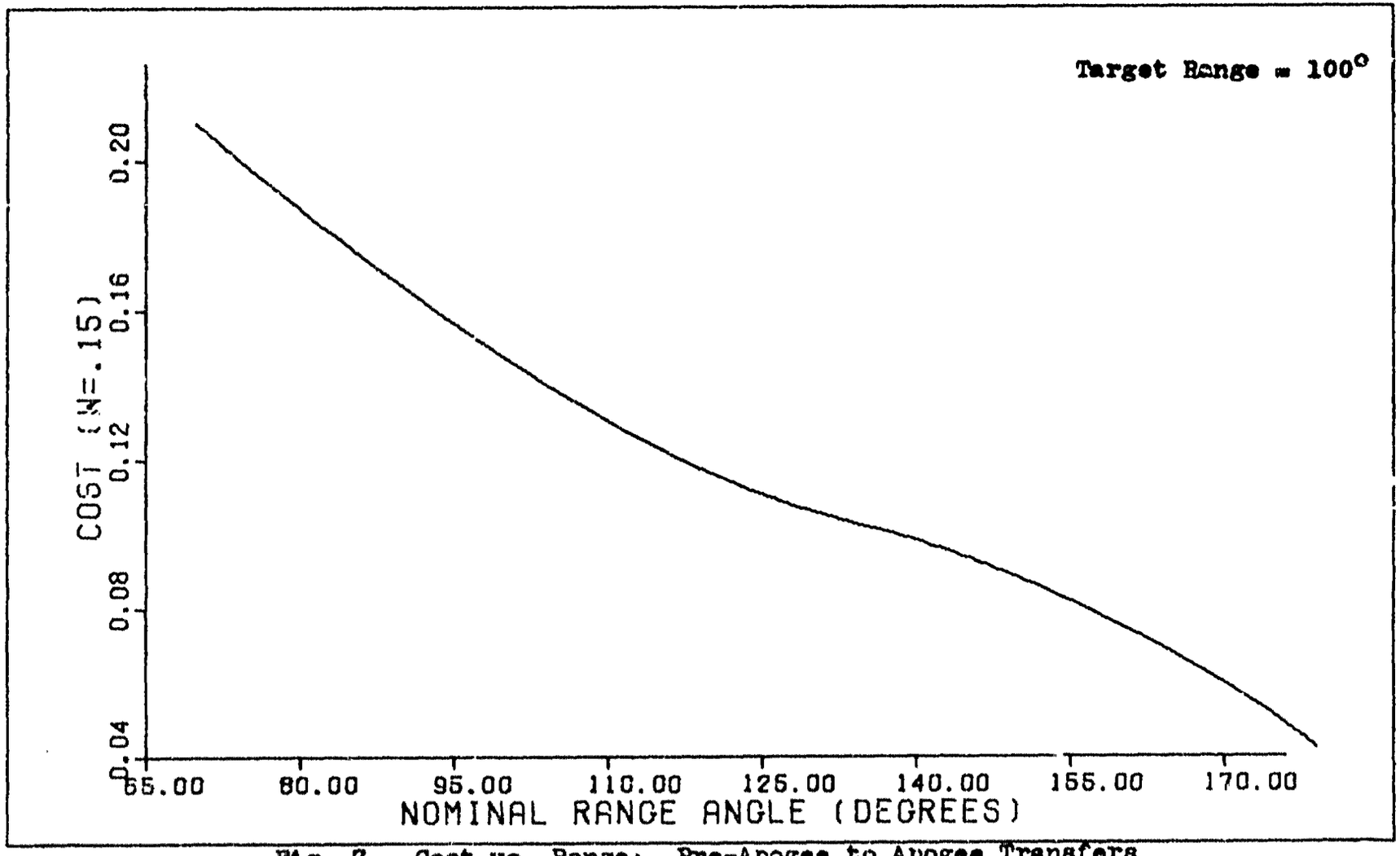

Fig. 7. Cost vs. Range: Pre-Apogee to Apogee Transfers

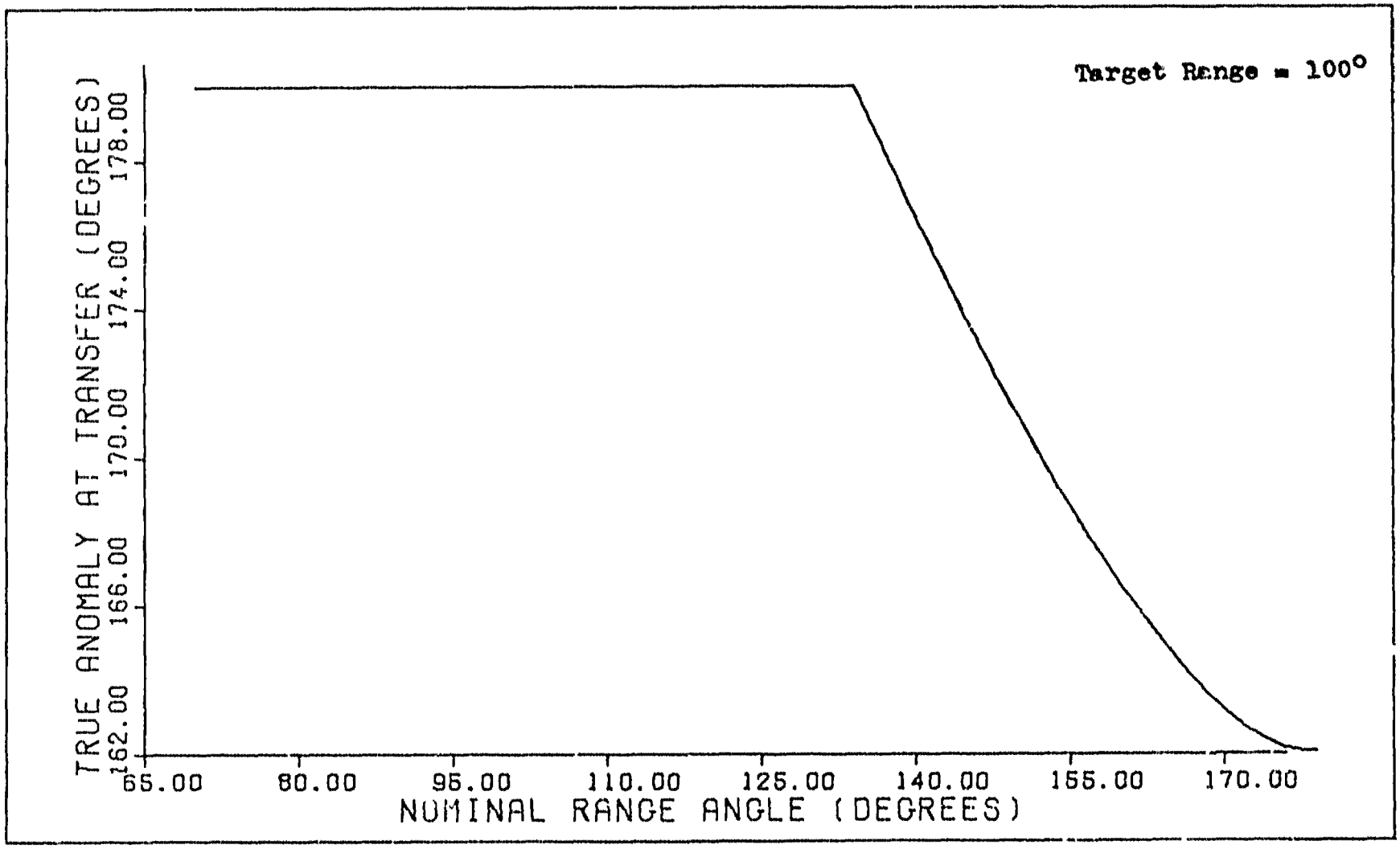

Fig. 8. True Anomaly at Transfer: Pre-Apogee to Apogee Transfers

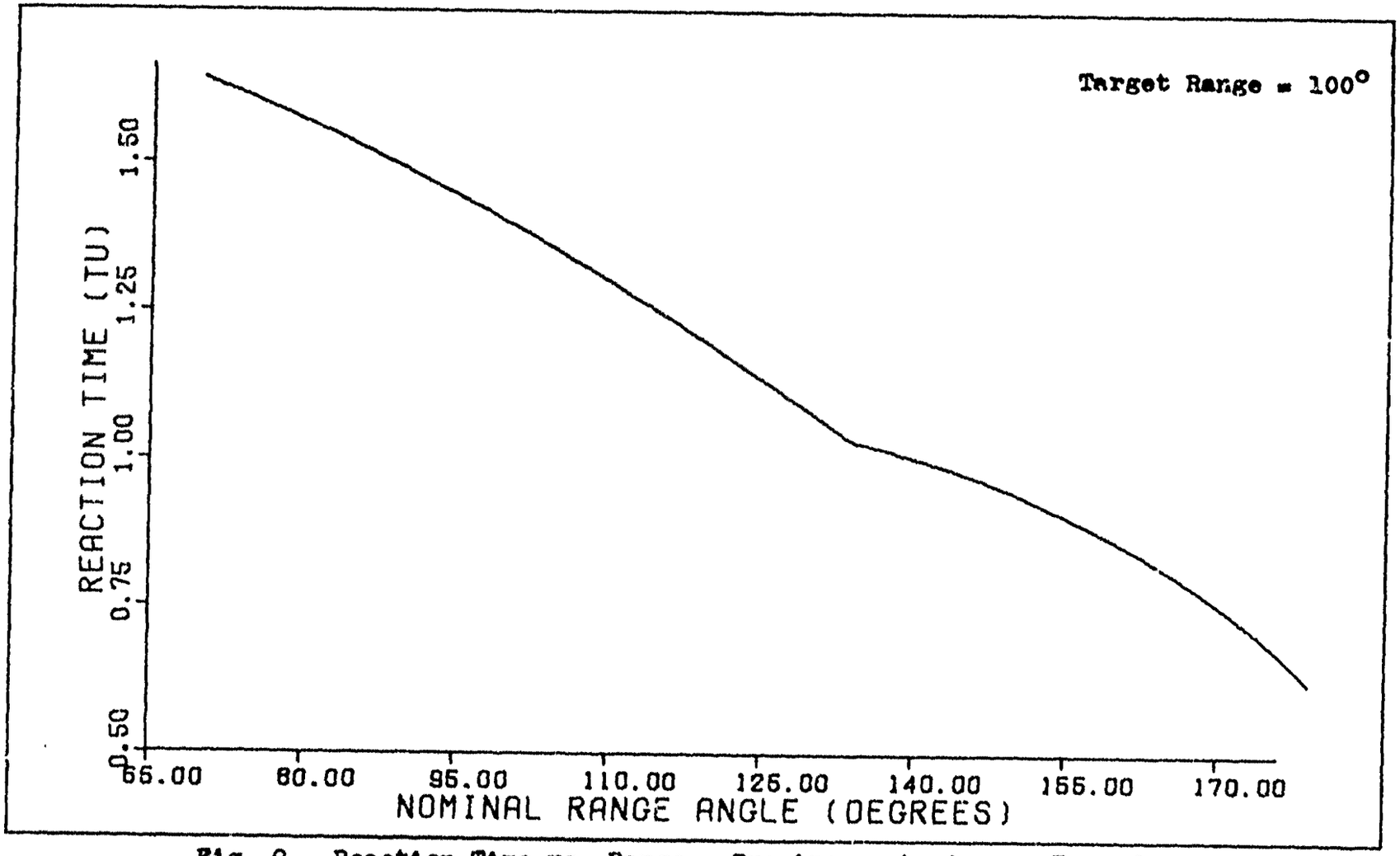

Fig. 9. Reaction Time vs. Range: Pre-Apogee to Apogee Transfers

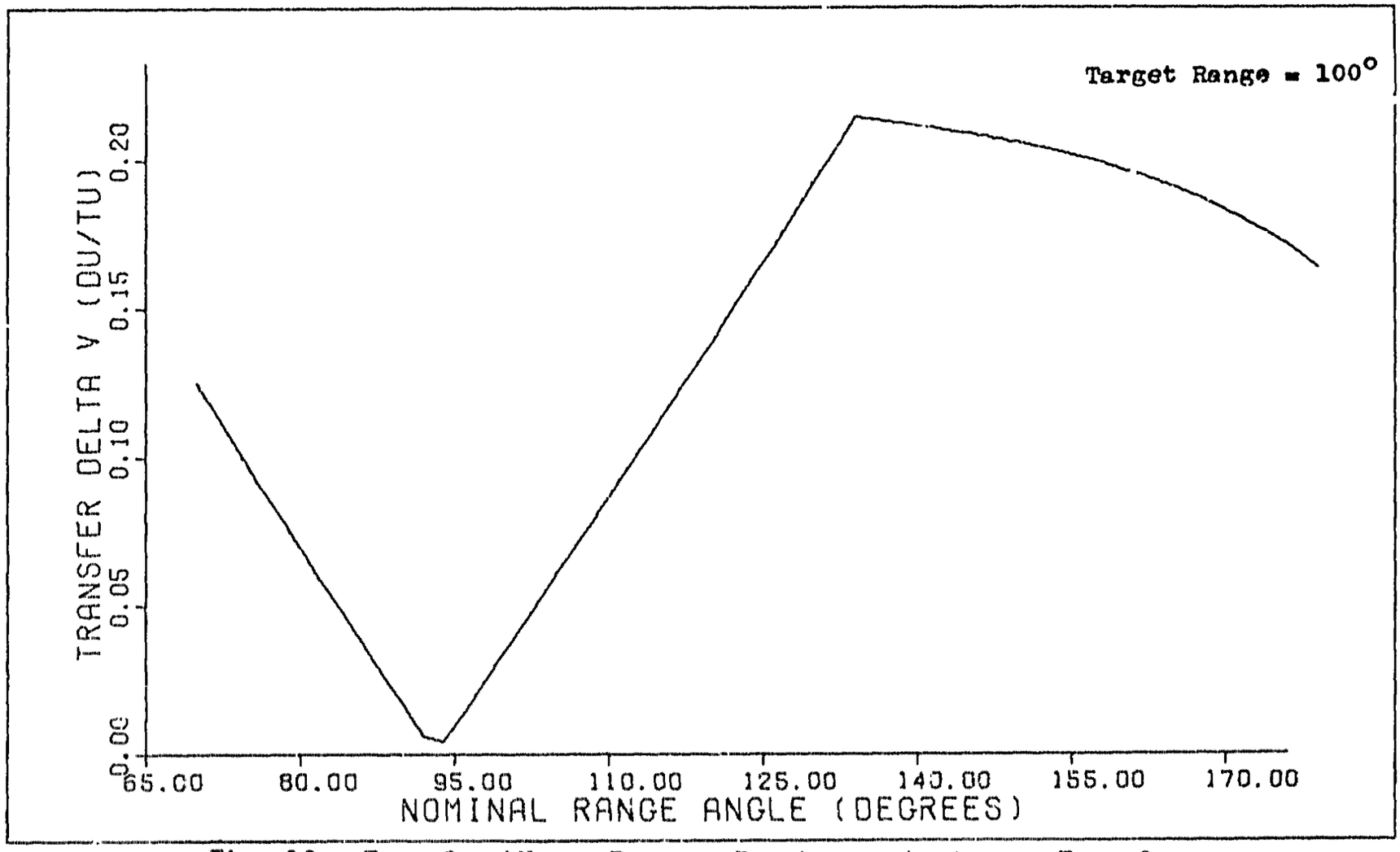

Fig. 10. Transfer ΔV vs. Range: Pre-Apogee to Apogee Transfers

$V_m - V_n$ vs. range plot of the apogee to apogee transfer case, (fig. 5) shows that the ΔV for the lower range angles is applied in the direction opposite to vehicle motion; however, fig. 10 shows velocity magnitude only.

The decrease in ΔV at the larger nominal range angles is explained by intuitive insight into this transfer problem. For $\psi_n > \psi_m$ a relatively high eccentricity of the modified orbit is required for apogee to apogee transfers. If transfer is done prior to the nominal orbit apogee, a lower eccentricity, i.e., a smaller orbit change, is adequate enough to hit the required final range. These facts explain the behavior of this transfer case.

The cost function found for pre-apogee transfers at higher nominal range angles points out a significant advantage in fractional orbital ballistic systems (FOBS). The vehicle is launched into a low, near circular orbit. Before completion of the orbit, a retro rocket slows down the vehicle, causing it to drop on the target. Barnaby (Ref 1:20) discusses this type of system and its basic advatage is that the low orbit remains undetected by ground radar until the range to the target is approximately 1400 km.

VI. POST-APOGEE TO APOGEE TRANSFERS

The third case considered is the post-apogee transfer from a nominal orbit to apogee on the new orbit. Once again, a limiting case of this transfer is the apogee to apogee transfer described in Chapter IV.

This transfer problem is essentially the same as the problem of Chapter V, with the exception that the true anomaly at transfer is after nominal apogee. This transfer is illustrated in fig. 11.

Modified Orbit and Cost Function Parameters

All parameters for this case are the same except those of true anomaly and eccentric anomaly at transfer. Equations (32) and (34) are employed, but it is noted that both ν_2 and E_2 lie between π and 2π.

Method of Solution

The method of computing an optimum cost for each nominal trajectory is as discussed in Chapter V. Both Program 3 and Program 4 of Appendix B are used to find optimum solutions.

Results and Analysis

The cost function found for this transfer case is depicted in fig. 12. Figure 13 shows that for the post-apogee to apogee transfer, the optimum transfer point occurs after nominal apogee for the lower nominal range angles. As range increases, the optimum transfer point moves first

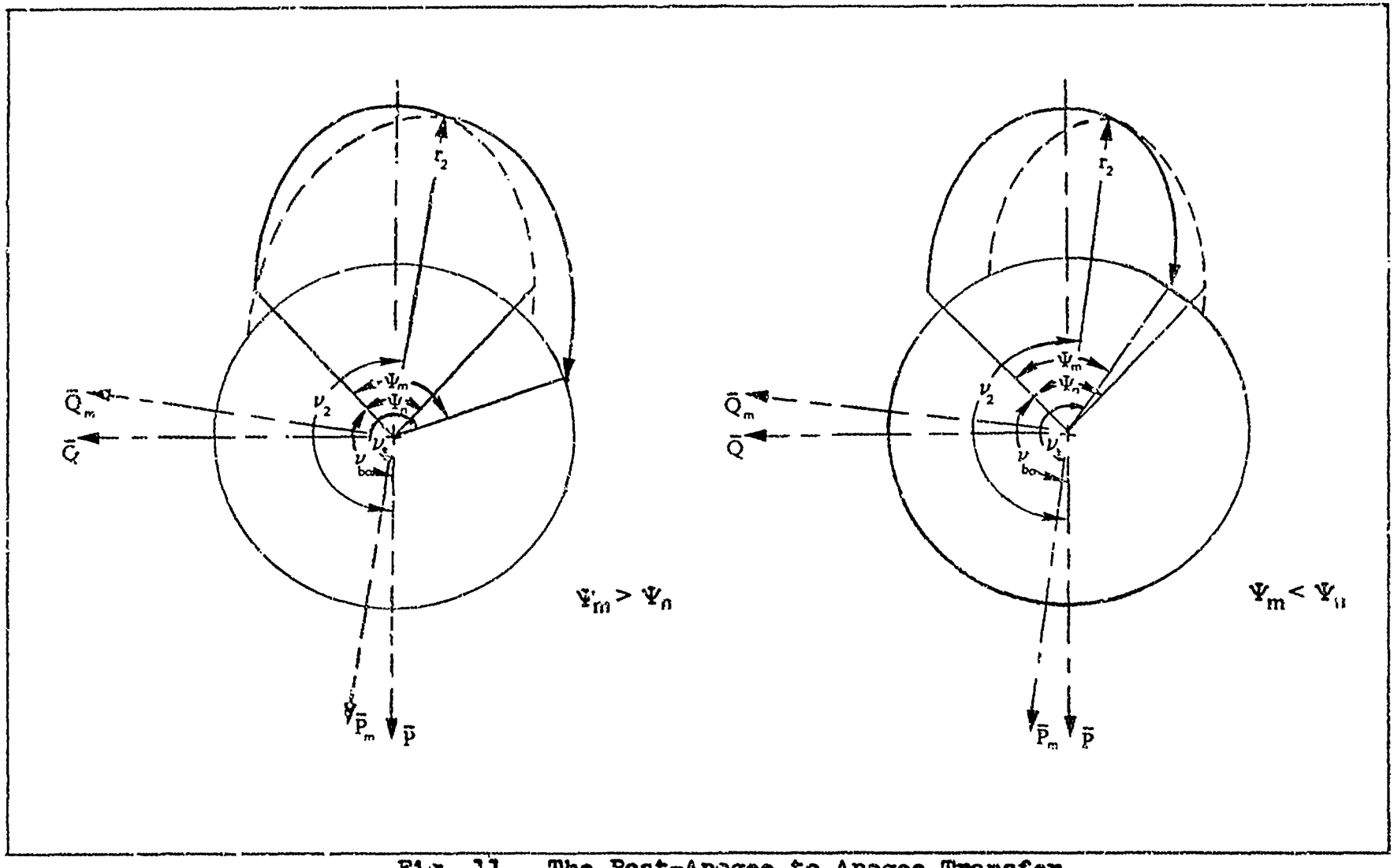

Fig. 11. The Post-Apogee to Apogee Transfer

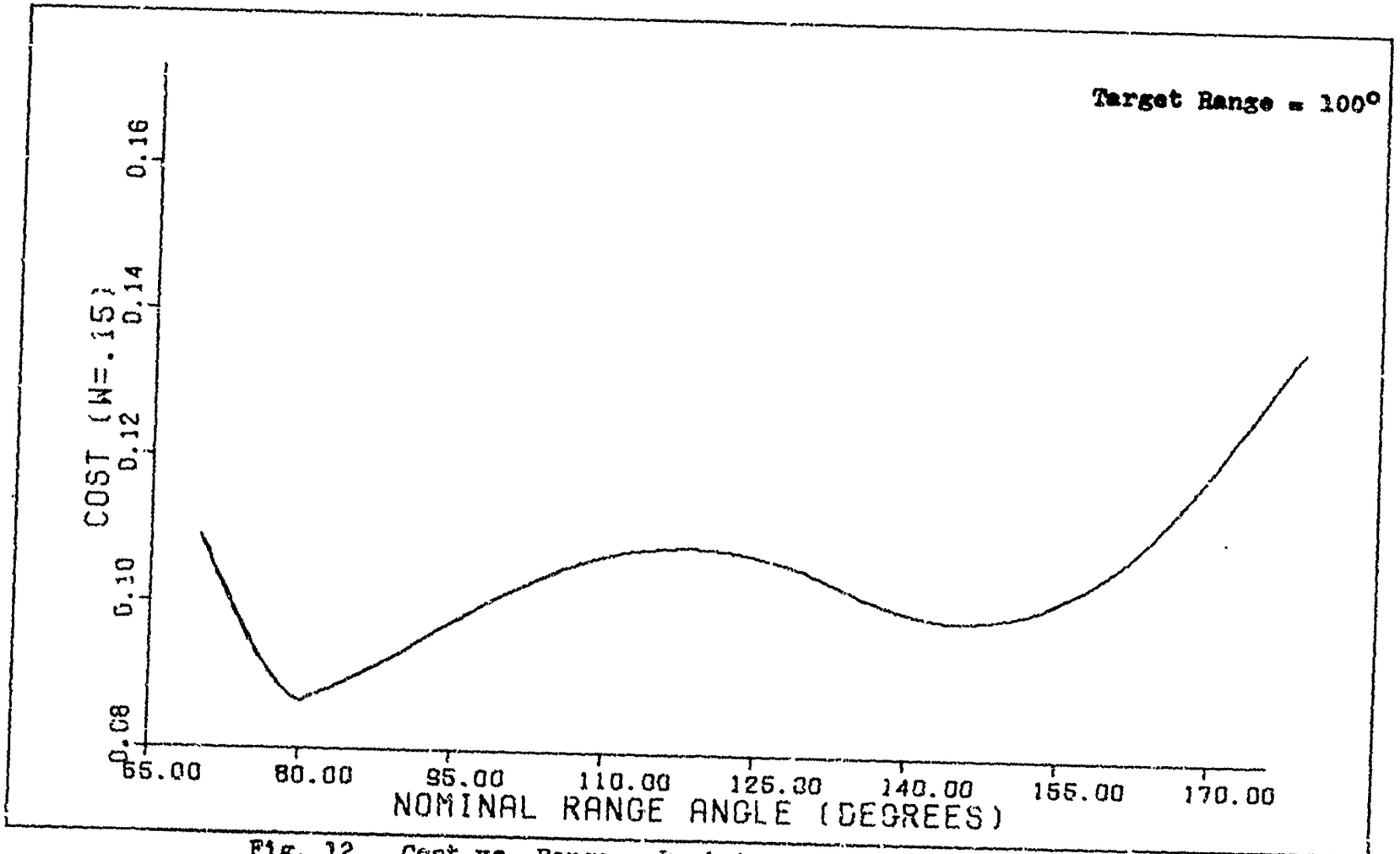

Fig. 12. Cost vs. Range: Post Apogee to Apogee Transfers

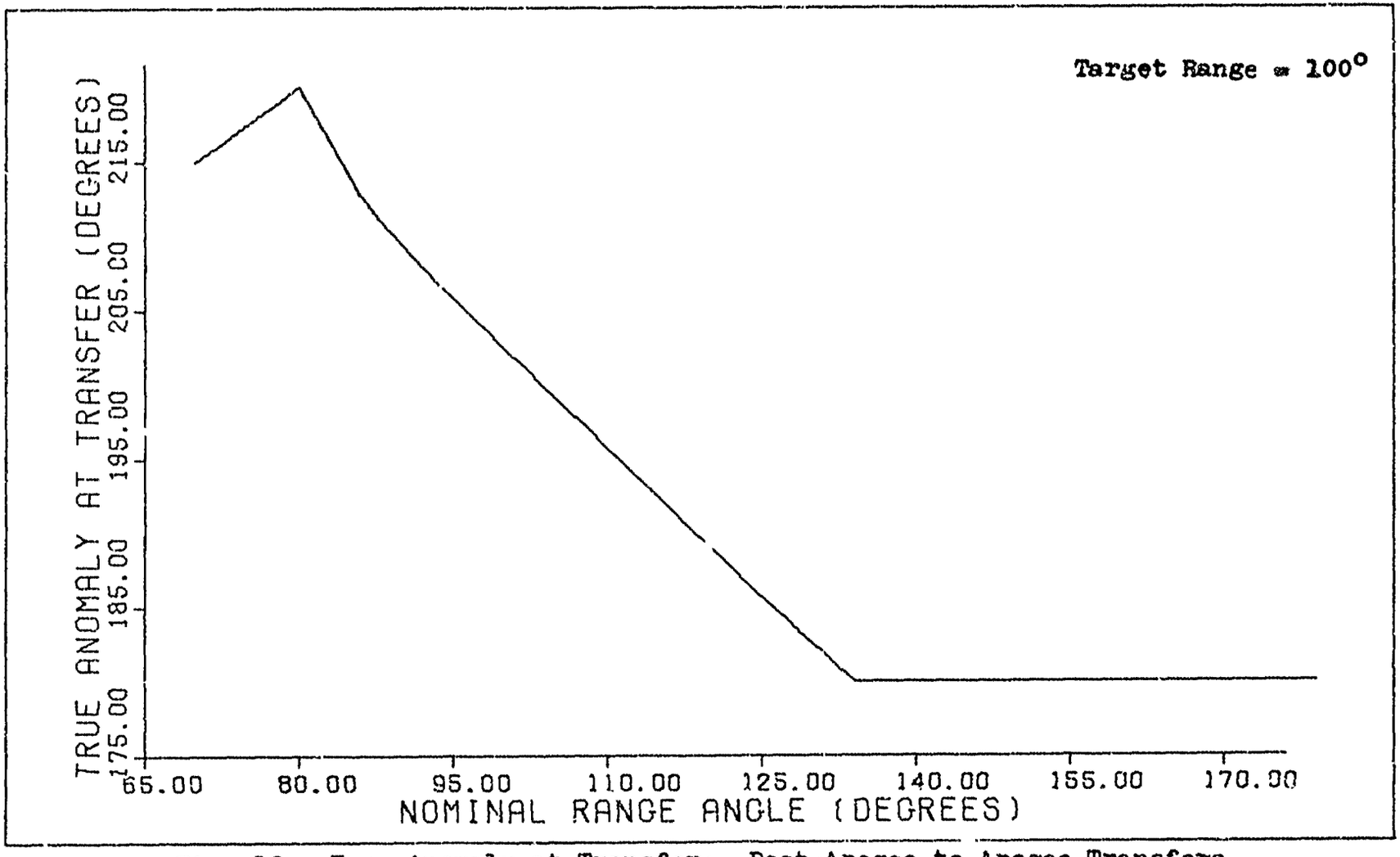

Fig. 13. True Anomaly at Transfer: Post-Apogee to Apogee Transfers

toward the target, then back to apogee, with apogee being the optimum transfer point for large nominal ranges.

Because this is a post-apogee to apogee transfer, the reaction time would be expected to be lower than the first two cases. The reaction time is shorter than for the other two cases, but fig. 14 shows that the reaction time increases up to the point where apogee to apogee transfers become optimal. At this nominal range angle, the reaction time starts decreasing.

Figure 15 shows that the transfer ΔV initially decreases, up to the apogee to apogee transfer point. Beyond this nominal range angle, ΔV increases with nominal range.

These results can also be intuitively explained. As would be expected, the post-apogee to apogee transfers require a higher eccentricity in order to hit the target range. This implies that a higher ΔV than the pre-apogee case is needed. The magnitude of ΔV is higher as expected, but this higher impulse requirement is offset by a decrease in reaction time. The trade-offs between each cost component as determined by the weighting factor, produce the point at which transition from post-apogee transfers to apogee transfers becomes optimal. The weighting factor also produces the behavior exhibited by the true anomaly at transfer in fig. 13.

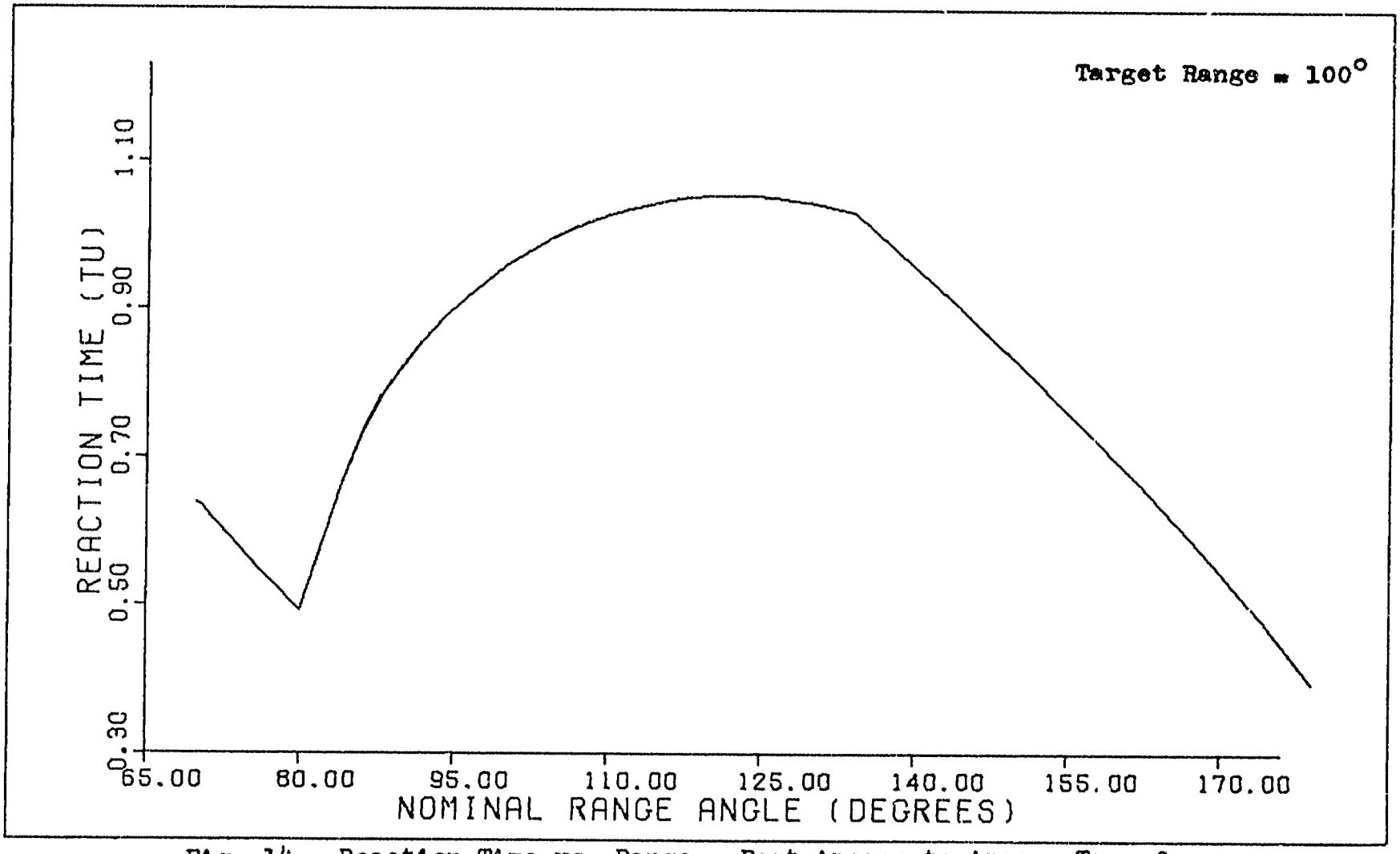

Fig. 14. Reaction Time vs. Range: Post-Apogee to Apogee Transfers

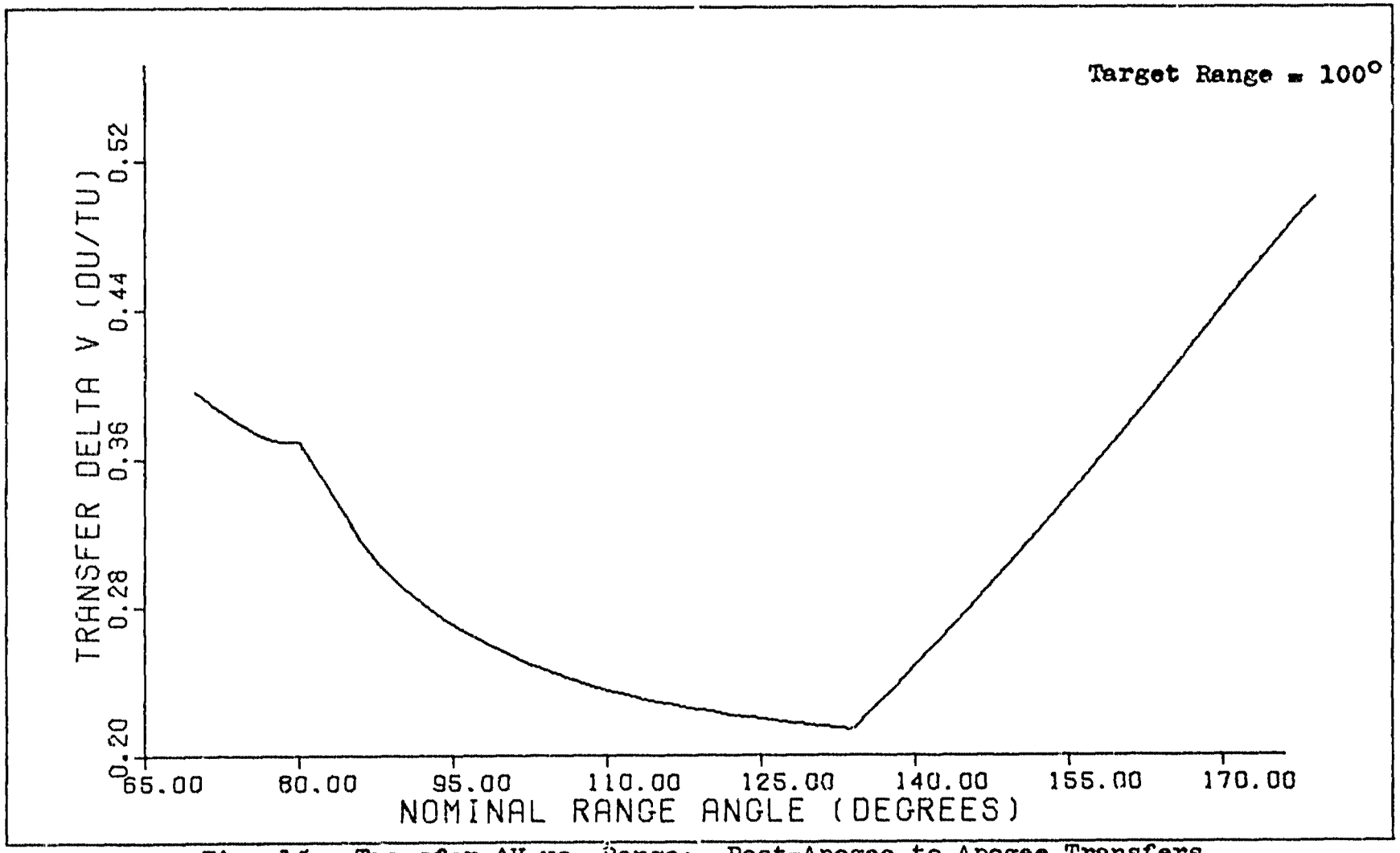

Fig. 15. Transfer ΔV vs. Range: Post-Apogee to Apogee Transfers

VII. PROBLEM DEFINITION - THREE DIMENSIONAL TRANSFERS

Several closed form mid-course transfers have been examined in the preceeding sections to establish the basic behavior of the mid-course modification. To see real world behavior, a more complex model of both the nominal and modified trajectories is needed.

Nominal Trajectory Definition

As previously mentioned, the typical ballistic missile trajectory is lofted. A real world simulation should use this type of nominal trajectory. In addition, earth rotation must be taken into consideration. For a target at 45° latitude, the impact error due to earth rotation is on the order of 400 miles for a normal ballistic missile trajectory (Ref 8:27).

Earth oblateness affects the trajectory by changing the gravitational accelerations from motion in an ideal inverse square gravitational field to accelerations which are dependent on latitude. For ICBM ranges, errors caused by ignoring oblateness effects are on the order of 10 miles (Ref 8:36). Earth oblateness not only affects vehicle accelerations, but initial and final position errors result when locating a point on a spherical earth instead of using an oblate spheroid model. Position errors are latitude dependent and may vary to as much as 21.4 km, as this is the measured equatorial bulge of the earth (Ref 2:95). Therefore, it is imperative that earth oblateness be

considered in a realistic model.

Finally, atmospheric entry of the reentry vehicle must be simulated. Aerodynamic deceleration of the reentry vehicle directly affects the trajectory and must be included. A numerical simulation which accounts for all these real world effects is used in this second part of the investigation.

Assumptions

To generate a trajectory, boundary conditions at burnout and the target location must be established. The position of the missile with respect to the launch site at burnout locates the burnout point in an earth centered inertial frame. Velocity imparted to the missile by the rocket motor is added to the launch site inertial velocity to determine missile inertial velocity at burnout. Several assumptions are made for these initial boundary conditions. The magnitude of a radius vector from the launch site to the missile at burnout is set at 200 n. mi. The elevation, and azimuth from north of the radius vector from the launch site to the burnout point are assumed to be 45° and 20° respectively. The magnitude of the velocity imparted to the missile is assumed to be 25,000 ft/sec, a typical ballistic missile burnout velocity.

The location of the launch site and target on an oblate earth are necessary to establish boundary point positions. These locations are defined in terms of latitude and longitude angles. The launch site and target positions are needed for an earth rotation model: Inertial velocity imparted to the

missile by the launch site varies with launch site latitude. The movement of the target in inertial space is also latitude dependent.

Oblate earth effects upon vehicle motion are considered by including four gravity harmonics (Ref 2:419). These harmonics account only for gravity anomalies symmetrically distributed about the earth's spin axis. Sectoral and tesseral harmonics, dependent on specific longitudes, are not included because their effects vary for different trajectories. Trajectory perturbations caused by the sun and the moon are small and so are not included (Ref 6:42, 8:45).

Adding reentry to the model requires several assumptions. An exponential atmospheric model is used for density calculations. Atmospheric effects are used only when the altitude is below approximately 110 km (Ref 6:44). Altitude computations assume a flat earth in the vicinity of the target because the range covered by the reentry vehicle after reentry is small (Ref 6:44). The altitude is then found by finding the difference between magnitudes of the inertial radius vector to the vehicle and the inertial radius vector to the target. This, in effect, produces an oblate atmospheric model.

For drag calculations, some measure of vehicle streamlining is needed. A ballistic coefficient, relating vehicle mass, frontal area, and drag coefficient to the total drag is employed. The use of this quantity in drag equations is defined in Appendix A. The value of the ballistic coefficient used is 4500 kg/m^2 corresponding to a streamlined or heavy

reentry vehicle.

The fact that the atmosphere rotates in inertial space is also considered in the drag calculations. The rotation of the atmosphere has a net effect of changing the velocity of the vehicle with respect to the air. Atmospheric drag acts as a force opposite to the direction of velocity. Direct entry into the atmosphere is assumed, with no skipping reentry considered. Additionally, no lift due to vehicle attitude is included.

Finally, a time of powered flight, between missile launch and burnout, is needed to determine how far the launch site and target have moved during the powered phase. Time of powered flight is assumed to be 5 minutes.

The assumptions made for the nominal trajectory derivation are summarized in Table I.

TABLE I

LOFTED NOMINAL TRAJECTORY ASSUMPTIONS

Parameter	Assumed Value
Burnout Position Vector*	
Distance	200 n mi
Elevation	45°
Azimuth	20°
Burnout Velocity Magnitude*	25,000 ft/sec
Reentry Altitude	110 km
Vehicle Ballistic Coefficient	4500 kg/m^2
Time of Powered Flight	5 min

* With respect to the launch site.

The use of the preceeding assumptions in defining boundary points and the equations of motion for the numerical simulation of the trajectory is discussed further in Appendix A.

Method of Solution

Because the target is moving, three time varying inertial coordinates determine final conditions. To satisfy the three terminal conditions of the target, three initial conditions at burnout are adjusted. The velocity magnitude is fixed, so only the direction may be varied at burnout. The burnout point is fixed, and only one other parameter, time of flight to impact, remains. The three variables, azimuth of the velocity vector, its elevation, and time of flight, can be varied at burnout so that the vehicle impacts upon the target at the final time.

The three variables are used in Powell's algorithm and the three functions are formed by a three-dimensional "hit" condition at final time:

$$F_1 = 0 = X_f - X_t \tag{46}$$

$$F_2 = 0 = Y_f - Y_t \tag{47}$$

$$F_3 = 0 = Z_f - Z_t \tag{48}$$

where X, Y, and Z are the inertial coordinates

f refers to the vehicle at final time

t refers to the target at final time.

To insure convergence to the lofted trajectory, the initial estimate of the velocity vector elevation is set at 60°. Equations (46), (47), and (48) are then used in Powell's routine where the boundary conditions derived in Appendix A determine trajectory end points. The vehicle behavior in inertial space is modeled by the nonlinear equations of motion, also found in Appendix A. The equations of motion are integrated numerically, using a Runge-Kutta and four point predictor-corrector library routine. The trajectory initial conditions are varied by Powell's routine so that eqs (46), (47), and (48) are satisfied. The solution of a nominal trajectory is incorporated in Program 5 of Appendix C.

VIII. THREE DIMENSIONAL ORBIT TRANSFERS

With a numerical model of the lofted nominal trajectory having been obtained, the mid-course modification simulating real world behavior can be examined. This chapter discusses trajectory optimization for the mid-course modification, the method of solution, and results for a test case.

Trajectory Optimization

The fact that the trajectory is located in three dimensional inertial space must be considered when finding the cost function defined in Chapter II. In order to use a velocity impulse in the function, the magnitude of the inertial velocity change is employed. The cost function is

$$J = \frac{1}{2}\Delta\bar{V}^T\Delta\bar{V} + \frac{W}{2}T_r^2 \tag{49}$$

where $\Delta\bar{V}$ is the velocity vector added at the transfer point.

In order to hit the target and also minimize the cost function, a constrained minimization problem is formulated. As before, an augmented cost function is formed. This cost function is

$$\tilde{J} = \frac{1}{2}\Delta\bar{V}^T\Delta\bar{V} + \frac{W}{2}T_r^2 + \bar{\lambda}^T(\bar{X}_f - \bar{X}_t)$$

$$= \frac{1}{2}(\Delta V_x^2 + \Delta V_y^2 + \Delta V_z^2) + \frac{W}{2}T_r^2 + \lambda_1(X_f - X_t)$$

$$+ \lambda_2(Y_f - Y_t) + \lambda_3(Z_f - Z_t) \qquad (50)$$

where $\bar{X}_f$ is the vehicle location at final time

$\bar{X}_t$ is the target location at final time

$\bar{\lambda}$ is the three dimensional Lagrange Multiplier

X, Y, and Z are inertial coordinates.

By examining the above equation, it can be seen that there are seven unknowns. Seven associated functions are the first order necessary conditions for a minimum:

$$\frac{\partial \tilde{J}}{\partial \Delta V_x} = 0 = \Delta V_x + \lambda_1 \frac{\partial X_f}{\partial \Delta V_x} + \lambda_2 \frac{\partial Y_f}{\partial \Delta V_x} + \lambda_3 \frac{\partial Z_f}{\partial \Delta V_x} \qquad (51)$$

$$\frac{\partial \tilde{J}}{\partial \Delta V_y} = 0 = \Delta V_y + \lambda_1 \frac{\partial X_f}{\partial \Delta V_y} + \lambda_2 \frac{\partial Y_f}{\partial \Delta V_y} + \lambda_3 \frac{\partial Z_f}{\partial \Delta V_y} \qquad (52)$$

$$\frac{\partial \tilde{J}}{\partial \Delta V_z} = 0 = \Delta V_z + \lambda_1 \frac{\partial X_f}{\partial \Delta V_z} + \lambda_2 \frac{\partial Y_f}{\partial \Delta V_z} + \lambda_3 \frac{\partial Z_f}{\partial \Delta V_z} \tag{53}$$

$$\frac{\partial \tilde{J}}{\partial T_r} = 0 = W T_r + \lambda_1 \left(\frac{\partial X_f}{\partial T_r} - \frac{\partial X_t}{\partial T_r} \right) + \lambda_2 \left(\frac{\partial Y_f}{\partial T_r} - \frac{\partial Y_t}{\partial T_r} \right) + \lambda_3 \left(\frac{\partial Z_f}{\partial T_r} - \frac{\partial Z_t}{\partial T_r} \right) \tag{54}$$

$$\frac{\partial \tilde{J}}{\partial \lambda_1} = 0 = X_f - X_t \tag{55}$$

$$\frac{\partial \tilde{J}}{\partial \lambda_2} = 0 = Y_f - Y_t \tag{56}$$

$$\frac{\partial \tilde{J}}{\partial \lambda_3} = 0 = Z_f - Z_t \tag{57}$$

These equations may now be solved.

Method of Solution

The solution to this problem provides conditions at a point of modification in the nominal trajectory. The initial conditions of the modified trajectory define c

trajectory which hits the target and also represents a minimum cost transfer from the point of trajectory change.

The partial derivatives of $\bar{J}$ with respect to ΔV_x, ΔV_y, ΔV_z and T_r are needed. These are found numerically by calculating first differences.

Since there are seven equations and seven unknowns, Powell's algorithm can be employed to satisfy the first order necessary conditions of a minimum. The initial values of ΔV_x, ΔV_y, ΔV_z and T_r can be roughly estimated by generating two nominal trajectories. The first is between the launch site and pseudo target. The second is from the launch site to the real target. A time of flight along the nominal trajectory defines the point of transfer. The initial estimates for ΔV_x, ΔV_y, and ΔV_z are found by calculating the difference of the velocity components of the two trajectories at the time of transfer. The estimate of T_r is the difference between the total time on the second trajectory and the time at modification. The Lagrange multipliers are initially set to zero. The method of finding initial estimates is presented in Table II. These rough initial estimates are within the area of convergence for the algorithm used.

The same equations of motion used for the nominal trajectory generation are employed for the modified orbit calculation. The equations of motion are integrated numerically to solve eqs (51) through (57). This method was programmed and is incorporated in Program 5 of Appendix C.

TABLE II

INITIAL ESTIMATES - NONPLANAR TRAJECTORY MODIFICATION

Variable	Initial Estimate at Time of Modification	
ΔV_x	$V_{xm} - V_{xn}$	modified orbit V_x - nominal orbit V_x
ΔV_y	$V_{ym} - V_{yn}$	modified orbit V_y - nominal orbit V_y
ΔV_z	$V_{zm} - V_{zn}$	modified orbit V_z - nominal orbit V_z
T_r	$T_{ffm}-T_{mod}$	modified orbit free flight time - time at transfer
λ_1	0	
λ_2	0	
λ_3	0	

Results

To test this method, a typical ICBM range to real and pseudo targets is selected. The launch site is located at 37.5° N. latitude and 125° W. longitude. The coordinates of the pseudo and real targets are 54° N. latitude, 3.3° W. longitude and 55.5° N. latitude, 5.4° E. longitude, respectively. The distance between pseudo and real targets is approximately 350 n mi. Modification takes place approximately 32 minutes into the nominal trajectory 62 minutes of flight.

The initial state estimates are scaled to lie between -1 and 1 for better performance of the algorithm (Ref 7:120). In addition, the matrix of $\partial\overline{F}/\partial\overline{X}$, the Jacobian, is examined to check that the scaling of both $\overline{F}$ and $\overline{X}$ do not produce an ill-conditioned matrix. For this reason, the hit conditions, eqs (55), (56), and (57) are also scaled.

After scaling both $\bar{F}$ and $\bar{X}$, the algorithm readily converged to a point where the sum of the squares of $\bar{F}$ is about three. At this point, the algorithm continues to converge, but very slowly. In examining the initial conditions being adjusted, the velocity at modification is changing by about .0001 ft/sec. Time of flight is changing by .001 seconds. Further reduction in the sum of the squares error produces no practical results when noting the initial condition magnitude changes. The fact that the three dimensional hit equation is scaled not only produces better convergence, but also insures that the real target is reached with reasonable accuracy when the algorithm is stopped. For the trajectory under consideration, the miss distances as determined by eqs (55), (56), and (57) in each inertial direction when the algorithm stopped are -497.3 ft, -510 ft, and 415.4 ft in the X, Y, and Z inertial directions respectively. This is translated into a spherical miss at the target of .136 n mi. The magnitude of the transfer ΔV is 1255 ft/sec and the reaction time is 2055.8 seconds. The corresponding cost for this mid-course modification is .488 . These results are summarized in Table III.

An important consideration in selecting a trajectory is its sensitivity to initial errors. Velocity errors in each inertial direction were added, one at a time, and the equations of motion were integrated foreward to final time in order to determine final miss distance due to initial velocity errors. Table IV presents the results of these

calculations. Table IV shows that for this test case, realistic velocity errors at the point of orbit modification produce only small final miss distances.

TABLE III

NONPLANAR TRAJECTORY MODIFICATION RESULTS

Parameter	Value
Launch Site Location	37.5° N.lat. 125° W.long.
Pseudo Target Location	54° N.lat. 3.3° W.long.
Real Target Location	55.5° N.lat. 5.4° E.long.
Velocity Vector at Burnout*	
Azimuth	19°
Elevation	49°
Time of Flight at Transfer	32 min
$\lvert\Delta V\rvert$	1255 ft/sec
ΔV_x	446.3 ft/sec
ΔV_y	1121.8 ft/sec
ΔV_z	342.8 ft/sec
T_r	2055.8 sec
J	.488

* Burnout condition assumptions are found in Table I. These values are the result of the iteration.

TABLE IV

VELOCITY SENSITIVITY - NONPLANAR TRAJECTORY MODIFICATION

Perturbation	Value (ft/sec)	Miss Distance from Calculated Impact Point (n.mi.)
V_x	+.3	.04
	-.3	.06
V_y	+.3	.04
	-.3	.05
V_z	+.3	.02
	-.3	.04

IX. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This study has analyzed the single impulse mid-course ballistic trajectory modification. Methods for examining both coplanar and three-dimensional transfers were presented. Application of the methods shown to real systems is dependent upon knowing trajectory parameters which have been assumed for illustration of the methods.

Coplanar transfers for elliptic orbits with no atmospheric reentry or earth rotation were first examined to define the general behavior of the transfer problem. The nominal orbit parameter of interest is free flight range angle. Because a maximum range nominal trajectory is used, this range angle, along with the altitude of the burnout point, defines all nominal orbit parameters. The optimum transfers for the three cases investigated are plotted against nominal range angle in fig. 16.

As fig. 16 indicates, the pre-apogee transfer to a near circular orbit produces the lowest cost for this formulation. It was assumed that a booster is capable of reaching any range angle up to 180°. If this is not the case, the best type of transfer to use can be found from fig. 16 by noting the maximum attainable nominal range angle.

Optimum solutions for the coplanar orbit transfers were found by solving first order necessary conditions of a minimum in the cost function. This procedure does not assure that a

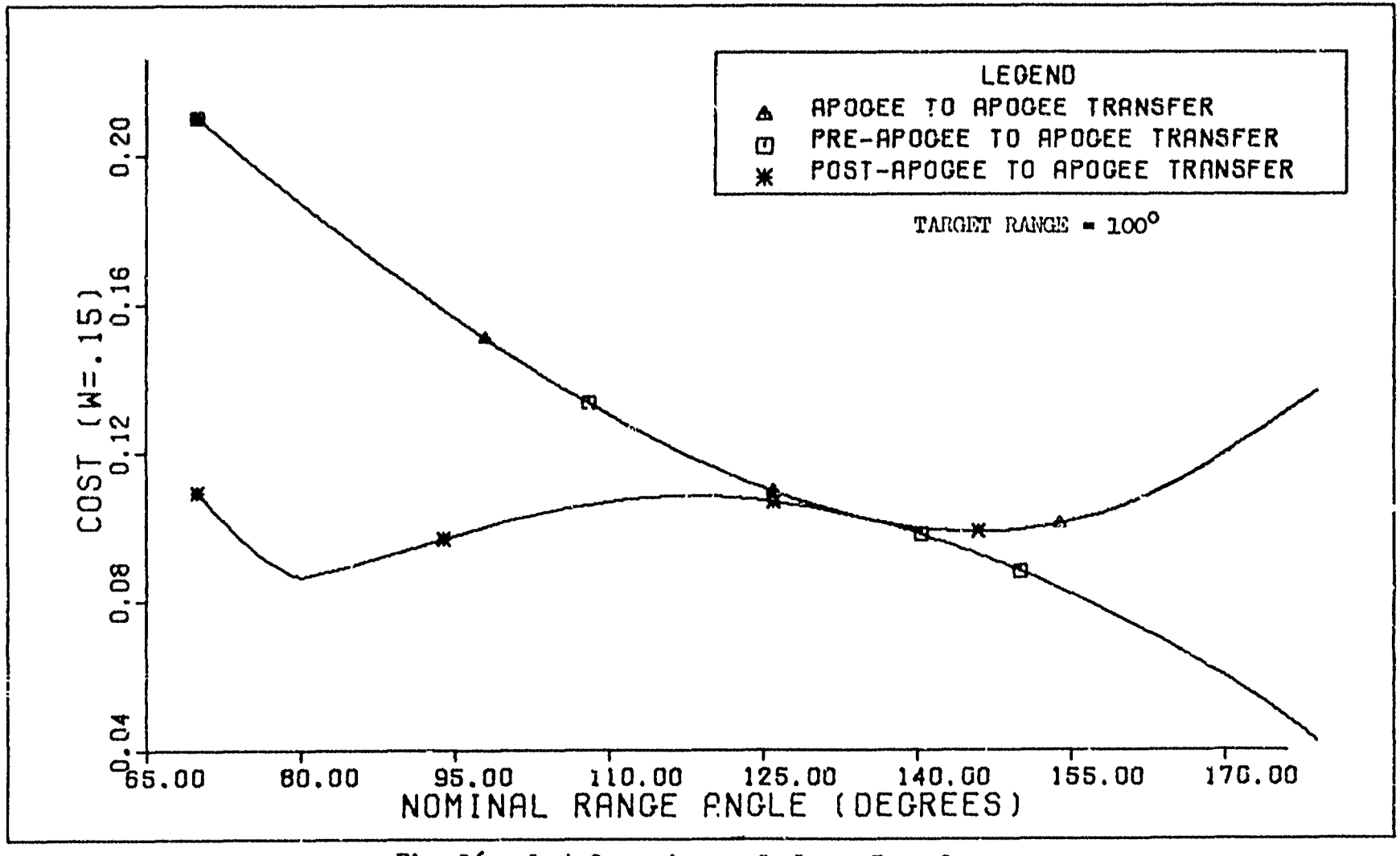

Fig. 16. Cost Comparison - Coplanar Transfers

minimum is actually calculated. This fact was considered and for this reason, the minimum cost solutions of Program 3 were used as a starting point to find the minimum. By starting the algorithm in the area where a series of solutions show the minimum to exist, convergence to the minimum point is insured.

The mid-course transfer was then extended from the coplanar transfer to the case where the nominal and modified orbits are nonplanar. A model which uses a rotating, oblate earth, atmospheric reentry, and a rotating, oblate, exponential atmosphere is employed.

A method of finding the conditions necessary for an optimum trajectory modification was then developed. The region of a minimum cost solution was found by terminating the algorithm before the first order necessary conditions were satisfied exactly. For the example considered, the routine was terminated when velocity change at the point of trajectory modification changed only slightly, beyond reasonable accuracy of control in a realistic system. Final miss distance at the target still remained within an acceptable distance when the algorithm stopped. Practical considerations of computer calculating time also entered into the decision of ' minating the routine. Generation of the one trajectory modification found for this case took approximately 1800 seconds of computer time from the rough initial estimates. Convergence at the termination point was slow.

The sensitivity of the modified trajectory to initial velocity errors was examined by perturbing the transfer velocity impulse. It was found that for the modified trajectory calculated, realistic velocity errors produce no significant final miss at the target.

It should be noted that several assumptions of launch site and target locations, burnout states, and vehicle parameters have been made. These are for illustration purposes and application of the method to a specific problem will change their values.

In addition, the cost function weighting factor was chosen to produce required ΔV in a realistic range of values. The formulation of the problem with a factor which can be changed to accommodate trade-offs between ΔV and reaction time allows a trajectory designer to apply this algorithm to different ΔV ranges.

Recommendations

For the elliptical orbit coplanar transfer, only transfer to a modified orbit apogee was considered. This could be extended to analyze transfer from a nominal orbit to a coaxial elliptic orbit, and then transfer to a generalized orientation elliptical orbit. Because direction of orbit change from nominal to modified trajectory is quite different when the target and nominal ranges vary greatly, it seems that these other two types of transfers could do better in terms of cost for some nominal trajectories.

The effects of changing the cost function weighting factor should be considered. For values near the example weighting factor used, the cost function curves appear to shift as new optimums are defined. However, for different values of W, the cost function appears to change appreciably. This behavior was not examined in detail, and further study is required.

Finally, the performance of an actual system could be estimated using the second part of the investigation. Actual system parameters, which have been estimated for this study would be needed. A set of optimal nonplanar modified trajectories can be generated and the one that performs best, i.e., in error sensitivity and cost, can be chosen.

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Appendix A

NUMERICAL TRAJECTORY CALCULATION

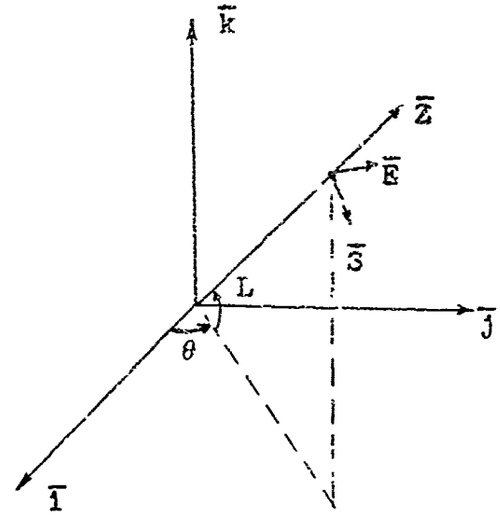
In order to generate a trajectory through use of numerical integration, the boundary conditions of the trajectory must first be defined. Boundary conditions necessary to find a trajectory are the inertial coordinates of the launch site, burnout point, and target. These are functions of time. Inertial burnout velocity is also required. Equations of motion are then derived and a trajectory is fitted between the boundary points.

Reference Frames

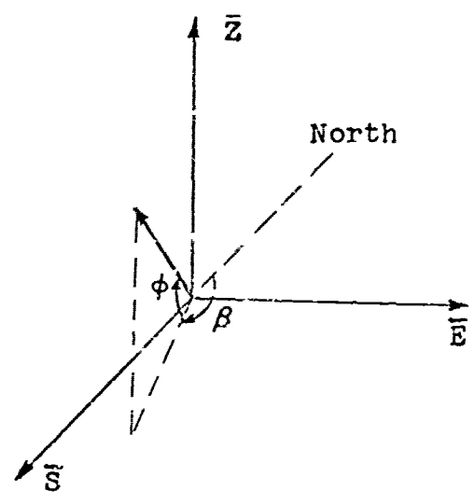
Calculations are done in the earth-centered, non-rotating inertial frame. This reference frame is shown in fig. 17. The frame is defined such that the $\bar{k}$ axis is the earth's spin axis, the $\bar{i}$ axis is coplanar with the $\bar{k}$ axis and the launch site at the time of launch, and the $\bar{j}$ axis completes the $\bar{i}$, $\bar{j}$, $\bar{k}$ axis set.

Local topocentric frames at the launch site and target, shown in fig. 17, are also defined. These sets include an $\bar{S}$ in the direction of local south, $\bar{E}$ in the easterly direction, and a local vertical, $\bar{Z}$.

A vector is located in the topocentric frame through use of an azimuth angle from true north, β, and an elevation angle, ϕ. The topocentric frame is related to the inertial



Earth-Centered Inertial Frame



Local Topocentric Frame

Fig. 17. Reference Frames

frame through a geodetic latitude angle, L, and an equatorial rotation angle, θ, similar to a local sideral time. The reference frames and their relations are shown in fig. 17.

Launch Site and Target Locations

The launch site and target locations in the inertial reference frame are found through use of a geodetic latitude, an equatorial rotation angle, and an oblate earth model, where both target and launch site are assumed to be at sea level.

Launch Site Inertial Position

The oblate earth model of Bate (Ref 2:98) provides $\bar{i}$ and $\bar{k}$ locations of the launch site at time of launch:

$$XLS = \frac{a_e \cos L_L}{(1 - e_e^2 \sin^2 L_L)^{1/2}} \tag{58}$$

$$ZLS = \frac{a_e \sin L_L (1 - e_e^2)}{(1 - e_e^2 \sin^2 L_L)^{1/2}} \tag{59}$$

where XLS is the initial $\bar{i}$ coordinate

ZLS is the initial $\bar{k}$ coordinate

e_e is the eccentricity of the oblate earth (.08181)

L_L is the launch site geodetic latitude

a_e is the equatorial radius of the earth.

If the $\bar{i}$ and $\bar{j}$ coordinates at any time after launch are desired, the rotation angle, θ, is employed. For the launch point

$$\theta_L = \omega_e T \tag{60}$$

where ω_e is the earth's rotation rate

T is the time of missile flight.

The $\bar{k}$ component of the launch site does not change. Therefore the inertial location of the launch point at any time is

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} XLS \cos\theta_L \\ XLS \sin\theta_L \\ ZLS \end{bmatrix} = {}^{LS}\bar{R}_I^{\,I} \tag{61}$$

where ${}^{LS}\bar{R}_I^{\,I}$ is the launch site location with respect to inertial frame, written in the inertial frame.

<u>Target Inertial Position</u>

Similarly, the target location in the inertial reference frame can be found. Define XTS and ZTS such that

$$XTS = \frac{a_e \cos L_T}{(1 - e_e^2 \sin^2 L_T)^{1/2}} \tag{62}$$

$$ZTS = \frac{a_e \sin L_T (1 - e_e^2)}{(1 - e_e^2 \sin^2 L_T)^{1/2}} \tag{63}$$

where L_T is the target geodetic latitude.

The equatorial rotation angle, θ_T, is found once knowing the time and the initial longitude difference between target and launch site, N.

$$\theta_T = \omega_e T + N \tag{64}$$

The angle N is measured eastward from the launch point.

The inertial position of the target at any time is therefore

$$\begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} = \begin{bmatrix} XTS \cos \theta_T \\ XTS \sin \theta_T \\ ZTS \end{bmatrix} = {}^T\bar{R}_I^I \tag{65}$$

where ${}^T\bar{R}_I^I$ is the target location with respect to the inertial frame, written in the inertial frame.

<u>Burnout Point Inertial Location</u>

To find the burnout point in inertial space, it is first necessary to define a vector from the launch point to the burnout point at time of burnout. This vector, written in the

launch site topocentric frame can be defined if a distance, azimuth and elevation (corresponding to a radar measurement) are known. The vector is then written in the launch site topocentric frame at burnout:

$$ {}^{bo}\bar{R}_T^L = \begin{bmatrix} -\cos\beta_R \cos\phi_R \, R_{bo} \\ \sin\beta_R \cos\phi_R \, R_{bo} \\ \sin\phi_R \, R_{bo} \end{bmatrix} \tag{66} $$

where β_R is the azimuth angle from true north of the burnout vector

ϕ_R is the elevation angle of the vector

R_{bo} is the magnitude of the vector.

The topocentric frame at the launch site is related to the defined inertial reference frame by two Euler angle rotations through a latitude angle, L_L, and a rotation angle, θ_L. The resulting transformation matrix from the topocentric to the inertial frame, $[L_{IT}]$, is

$$ [L_{IT}] = \begin{bmatrix} \sin L_L \cos\theta_L & -\sin\theta_L & \cos L_L \cos\theta_L \\ \sin L_L \sin\theta_L & \cos\theta_L & \cos L_L \sin\theta_L \\ -\cos L_L & 0 & \sin L_L \end{bmatrix} \tag{67} $$

where θ_L is defined by eq (60). It is noted that a time from launch to burnout is needed to determine θ_L.

Using this transformation, along with the vectors which describe the burnout point with respect to inertia, eqs (66) and (61), the burnout point is located in the inertial reference frame:

$$ {}^{bo}\bar{R}_I^{I} = [L_{IT}]\, {}^{bo}\bar{R}_T^{L} + {}^{LS}\bar{R}_I^{I} \tag{68} $$

where ${}^{bo}\bar{R}_I^{I}$ is the vector from the earth's center to the burnout point, written in the inertial frame. This vector gives the initial position of the trajectory.

Initial Inertial Velocity

The initial inertial velocity is also required to complete the set of initial conditions of the trajectory. This velocity is found in a method similar to that for the position at burnout.

If a velocity azimuth angle, β_V, elevation angle, ϕ_V, and a magnitude are known, the velocity vector in the topocentric frame can be written as

$$ {}^{bo}\bar{V}_T^{LS} = \begin{bmatrix} -\cos\beta_V \cos\phi_V \, V_{bo} \\ \sin\beta_V \cos\phi_V \, V_{bo} \\ \sin\phi_V \, V_{bo} \end{bmatrix} \tag{69} $$

It is noted that the angles β_V and ϕ_V are not related to the angles β_R and ϕ_R which describe the burnout point with respect to the launch site. However, these angles should be somewhat similar as seen by a physical example: A missile would not reach a burnout point west of a launch site with an easterly velocity.

At this point, it is necessary to note that an initial inertial velocity is imparted to the missile by the launch site. This inertial velocity is due to the earth's rotation and varies with the latitude of the launch site. The velocity is directed along the easterly axis of the local topocentric frame:

$$ {}^{LS}\bar{V}^{I} = V_e \cos L_T \, \bar{e}_E \tag{70} $$

where V_e is the equatorial tangential velocity (1524 ft/sec)

$\bar{e}_E$ is a unit vector in the easterly direction.

By combining eqs (69) and (70) and employing eq (67), the initial velocity can be written in the inertial frame:

$$ {}^{bo}\bar{V}_I^{I} = \left[L_{IT}\right]\left({}^{bo}\bar{V}_T^{LS} + {}^{LS}\bar{V}^{I} \right) \tag{71} $$

The Equations of Motion

The differential equations of motion in the inertial reference frame are required to complete the general trajectory

definition. Two contributions to the acceleration experienced by the vehicle are examined. They are the gravitational accelerations and drag.

Gravitational Accelerations

A gravity model which employs four gravitational harmonics is used, (Ref 2:421):

$$\ddot{X}_g = -\mu \frac{X}{R^3} \left\{ 1 - J_2 \frac{3}{2} \left(\frac{a_e}{R}\right)^2 \left[5 \left(\frac{Z}{R}\right)^3 - 1 \right] \right.$$

$$+ J_3 \frac{5}{2} \left(\frac{a_e}{R}\right)^3 \left[3 \frac{Z}{R} - 7 \left(\frac{Z}{R}\right)^3 \right]$$

$$- J_4 \frac{5}{8} \left(\frac{a_e}{R}\right) \left[3 - 42 \left(\frac{Z}{R}\right)^2 + 63 \left(\frac{Z}{R}\right)^4 \right]$$

$$\left. - J_5 \frac{3}{8} \left(\frac{a_e}{R}\right)^5 \left[35 \frac{Z}{R} - 210 \left(\frac{Z}{R}\right)^3 + 231 \left(\frac{Z}{R}\right)^5 \right] \right\} \qquad (72)$$

$$\ddot{Y}_g = \frac{Y}{X} \ddot{X}_g \qquad (73)$$

$$\ddot{Z}_g = -\mu \frac{Z}{R^3} \left\{ 1 + J_2 \frac{3}{2} \left(\frac{a_e}{R}\right)^3 \left[3 - 5 \left(\frac{Z}{R}\right)^2 \right] \right.$$

$$+ J_3 \frac{3}{2} \left(\frac{a_e}{R}\right)^3 \left[10 \frac{Z}{R} - \frac{35}{3} \left(\frac{Z}{R}\right)^3 - \frac{R}{Z} \right]$$

$$- J_4 \frac{5}{8} \left(\frac{a_e}{R}\right)^4 \left[15 - 70 \left(\frac{Z}{R}\right)^2 + 63 \left(\frac{Z}{R}\right)^4 \right]$$

$$- J_5 \frac{1}{8} \left(\frac{a_e}{R}\right)^5 \left[315 \frac{Z}{R} - 945 \left(\frac{Z}{R}\right)^3 \right.$$

$$\left. \left. + 693 \left(\frac{Z}{R}\right)^5 - 15 \frac{R}{Z} \right] \right\} \qquad (74)$$

where $\ddot{X}$, $\ddot{Y}$, and $\ddot{Z}$ are the inertial accelerations

X, Y, and Z are the inertial distance coordinates

R is the inertial radius magnitude, $(X^2 + Y^2 + Z^2)^{\frac{1}{2}}$

J_2, J_3, J_4, and J_5 are the gravitational harmonic coefficients.

The values used for these coefficients are

$J_2 = 1.08264 \times 10^{-3}$

$J_3 = -2.5 \times 10^{-6}$

$J_4 = -1.6 \times 10^{-6}$

$J_5 = .15 \times 10^{-6}$

This gravitational model accounts only for gravity anomalies symmetrically distributed about the inertial $\bar{k}$ axis. Tesseral and sectorial harmonics, along with the perturbative effects of other heavenly bodies such as the sun and moon are ignored since only small errors result (Ref 6:42, 8:45).

Drag

The acceleration caused by drag is also considered in the equations of motion. It is assumed that burnout occurs above the earth's atmosphere, so only during reentry is drag considered. The atmospheric entry angle was not calculated, and it is assumed that the vehicle enters the atmosphere directly, without skipping.

The acceleration caused by drag (Ref 5:39), may then be written as

$$\bar{d}_I^{\,V_I} = -\frac{\rho C_D A}{2m} V^2 \left(\frac{\bar{V}_I^{\,V_I}}{V} \right) \tag{75}$$

where $\bar{d}_I^{\,V_I}$ is the drag acceleration in the inertial frame

ρ is the local atmospheric density

C_D is the vehicle drag coefficient

A is the projected frontal area

m is the vehicle mass

V is the magnitude of the velocity vector

$\bar{V}_I$ is the inertial velocity vector.

This acceleration acts directly along the flight path and opposite to the velocity vector.

The coefficient $C_D A/m$ is dependent upon the vehicle, and its inverse is called the ballistic coefficient, BC:

$$BC = \frac{m}{C_D A} \tag{76}$$

The ballistic coefficient is a measure of vehicle streamlining. High values of the ballistic coefficient indicate a streamlined vehicle.

An atmospheric model is needed to find the local atmospheric density required in eq (75). An exponential density model (Ref 5:38) is used, and atmospheric effects are considered negligible above an altitude of 110 km. for computational efficiency.

The fact that the earth's atmosphere rotates in inertial space can also be considered in the drag model (Ref 2:424). This phenomena affects the drag by changing the velocity relative to the atmosphere. If the rotating atmosphere is included, the inertial direction drag expressions are

$$\ddot{X}_d = -\frac{\rho_0}{2BC} \exp(-h/23999.3)(V_x + \omega_e Y)V \tag{77}$$

$$\ddot{Y}_d = -\frac{\rho_o}{2BC} \exp(-h/23999.3)(V_y + \omega_e X)V \qquad (78)$$

$$\ddot{Z}_d = -\frac{\rho_o}{2BC} \exp(-h/23999.3)V_z V \qquad (79)$$

where $\ddot{X}_d$, $\ddot{Y}_d$, and $\ddot{Z}_d$ are the inertial drag accelerations (ft/sec^2)

ρ_o is sea level density (slug/ft^3)

h is the local altitude (feet)

V_x, V_y, and V_z are the inertial velocity components (ft/sec)

V is the velocity vector magnitude.

The local altitude, h, is found by assuming a flat earth in the vicinity of the target. Because the range covered during reentry is relatively short, this is a valid assumption (Ref 6:44). Altitude is then

$$h = \left|\overset{V}{\bar{R}}_I\right| - \left|\overset{T}{\bar{R}}_I\right| \qquad (80)$$

where $\overset{V}{\bar{R}}_I$ is the magnitude of the radius vector to the vehicle with respect to the inertial frame

$\overset{T}{\bar{R}}_I$ is the magnitude of the target radius vector in the inertial frame. This magnitude is found from the oblate earth model described by eq (65).

By combining eqs (72), (73), (74), (77), (78), and (79), the differential equations of motion result:

$$\ddot{X} = \ddot{X}_g + \ddot{X}_d \tag{81}$$

$$\ddot{Y} = \ddot{Y}_g + \ddot{Y}_d \tag{82}$$

$$\ddot{Z} = \ddot{Z}_g + \ddot{Z}_d \tag{83}$$

These differential equations of motion can then be integrated numerically to find inertial positions and velocities.

Appendix B

THE NONLINEAR EQUATION SOLVER

The results of this study are dependent upon solving a set of highly nonlinear boundary value problems. The problems may be stated as n functions of n unknowns. In order to solve this set of defined problems, an algorithm for solving systems of nonlinear algebraic equations developed by M. J. D. Powell (Ref 7:115) is employed. This algorithm, named subroutine NSOLA in the computer programs, solves the system of equations $\overline{F}(\overline{X})$, where $\overline{F}$ and $\overline{X}$ are of the same dimension. The Powell algorithm uses a modified Newton iteration where the steepest descent of $\overline{F}(\overline{X})$ is also considered.

Excellent results were obtained in using this routine. Care must be taken, however, to insure that correct scaling of both $\overline{F}$ and $\overline{X}$ is accomplished. With correct scaling, the algorthm usually converges for even poor initial estimates. Both boundary value and minimization problems were solved through use of the routine.

Appendix C

COMPUTER PROGRAM LISTINGS

This section presents the computer programs used in this study. The programs are written in Fortran Extended for use on the CDC 6600 series computer.

```
C   PROGRAM 1 - APOGEE TO APOGEE TRANSFERS

      PROGRAM THESIS(INPUT,OUTPUT,PUNCH)
      COMMON/I/C(12)
      COMMON/II/PSI,PSIR,RBO,KOUNT
      DIMENSION X(2),F(2),AJINV(2,2),W(20)
      PI=ACOS(-1.)
      KOUNT=1
      READ*,PSIRD
 50   READ*,PSID,RBO
      IF(EOF(5LINPUT))2000,60
 60   PSI=PSID*PI/180.
      PSIR=PSIRD*PI/180.
C   PSI IS THE NOMINAL RANGE ANGLE (READ IN DEGREES)
C   PSIR IS THE TARGET RANGE ANGLE (READ IN DEGREES)
C   RBO IS THE GEOCENTRIC BURNOUT RADIUS (DU)
      PRINT*,"REQUIRED RANGE (DEG) IS ",PSIRD
      PRINT*,"RANGE ANGLE (DEGREES) IS ",PSI*180./PI
      PRINT*,"BURNOUT RADIUS (DU) IS ",RBO
C   GUESS INITIAL ECCENTRICITY - X(1)
      X(1)=.5
      X(2)=0.
C   THE FOLLOWING ARE PARAMETERS REQUIRED BY NS01A
      DSTEP=1.E-5
      DMAX=.499
      IPRINT=0
      ACC=1.E-10
      MAXFUN=40
      N=2
      CALL NS01A(N,X,F,AJINV,DSTEP,DMAX,ACC,MAXFUN,IPRINT,W)
      PRINT 1000
      PRINT*," "
      PRINT1010,(C(I),I=1,10)
      KOUNT=KOUNT+1
```

C PROGRAM 1 (CONTINUED)

```
      GOTO 50
 2000 KOUNT=KOUNT-1
 1000 FORMAT("NEW",T14,"RADIUS AT",T26,"NEW",T38,"TOF",T50,"TOF"
     1,T62,"TOTAL",T74,"NOMINAL",T86,"REQUIRED",T98,"NEW RANGE",T110,
     2            /"ECCENTRICITY",T14,"IMPULSE-DU",T26,"PERIGEE-DU",
     3T38,"1-2 SEC",T50,"2-3 SEC",T62,"TOF - SEC",T74,"TOF - SEC",
     4T86,"VEL- DU/TU",T98,"ANGLE-DEG",T110,"  COST",
 1010 FORMAT(1PG11.4,T13,G11.4,T25,G11.4,T37,G11.4,T49,G11.4,T61,G11.4
     1,T73,G11.4,T85,G11.4,T97,G11.4,T109,G11.4)
      END
```

```
C     PROGRAM 1 (CONTINUED)

      SUBROUTINE CALFUN(N,X,F)
      COMMON/I/C(12)
      COMMON/II/PSI,PSIR,RBO,KOUNT
      DIMENSION X(2),F(2)
      PI=ACOS(-1.)
      TU=806.8136
      EC1=X(1)
C     CALCULATE NOMINAL ORBIT PARAMETERS
      PHIBO=(PI-PSI)/4.
      RAD1=4.*SIN(PSI/2.)*(COS(PHIBO)**2)/(1.+SIN(PSI/2.))**2
      ENOM=SQRT(1.-RAD1)
      ANOM=RBO*(1.+SIN(PSI/2.))/2.
      RAPNOM=ANOM*(1.+ENOM)
      GAMABO=ACOS(-COS(PSI/2.))
C     CALCULATE NOMINAL ORBIT TIME OF FLIGHT
      COSE1=(ENOM-COS(PSI/2.))/(1.-ENOM*COS(PSI/2.))
      E1=ACOS(COSE1)
      TFFNOM=2.*SQRT(ANOM**3)*(PI-E1+ENOM*SIN(E1))
      TOFN=TFFNOM*TU
      A1=RAPNOM/(1.+EC1)
      RPER=A1*(1.-EC1)
C     DOES THE MODIFIED ORBIT HIT THE EARTH?
      IF(RPER.GE.1.)GOTO 100
      GAMA3=ACOS(A1*(1.-EC1**2)/(EC1     )-1./EC1)
      GAMA3=2.*PI-GAMA3
C     CALCULATE THE REACTION TIME
      COSE3=(EC1+COS(GAMA3))/(1.+EC1*COS(GAMA3))
      E3=ACOS(COSE3)
      E3=2.*PI-E3
      TFF12=TFFNOM/2.
      TFF23=SQRT(A1**3)*(E3-EC1*SIN(E3)-PI)
      TOF12=TFF12*TU
      TOF23=TFF23*TU
```

```
C     PROGRAM 1 (CONTINUED)

        TTOF=TOF12+TOF23
C     CALCULATE VELOCITIES AT APOGEE AND THE DELTA V REQUIRED
        VNOM=-SQRT((1.-ENOM)/RAPNOM)
        V1=-SQRT((1.-EC1)/RAPNOM)
        DELV=V1-VNOM
        W=.15
C     W IS THE SPECIFIED WEIGHTING FACTOR
        COST=.5*DELV**2+.5*W*TFF23**2
        PSI1R=GAMA2-GAMA00-PI+GAMA3
        F(1)=PSI1R-PSIR
C     F(1) IS THE HIT CONDITION
        F(2)=X(2)
        PSI1=PSI1R*180./PI
        C(1)=EC1
        C(2)=RAPNOM
        C(3)=RPER
        C(4)=TOF12
        C(5)=TOF23
        C(6)=T
        C(7)=TURN
        C(8)=DELV
        C(9)=PSI1
        C(10)=COST
        C(11)=V1
        C(12)=PSI*180./PI
        DO 90 I=1,12
90      B(KOUNT,I)=C(I)
        RETURN
100     PSIR=0.
        F(1)=PSI1R-PSIR
C     ADD PENALTY FOR NOT HITTING THE EARTH
        F(2)=X(2)
        END
```

```
C   PROGRAM 2 - THE OPTIMUM APOGEE TO APOGEE TRANSFER

      PROGRAM THESIS(INPUT,OUTPUT)
      COMMON/I/C(16)
      COMMON/II/PSIR,RBO
      DIMENSION X(3),F(3),AJINV(3,3),W(55)
      PI=ACOS(-1.)
      RBO=1.05
C   RBO IS THE GEOCENTRIC BURNOUT RADIUS (DU)
      READ*,PSIRD
C   PSIR IS THE TARGET RANGE ANGLE (READ IN DEGREES)
      PSIR=PSIRD*PI/180.
      PRINT*," "
      PRINT*," "
      PRINT*,"REQUIRED RANGE (DEG) IS ",PSIRD
      PRINT*,"BURNOUT RADIUS (DU) IS ",RBO
C   GUESS INITIAL ECCENTRICITY - X(1)
      X(1)=.5
C   GUESS OPTIMUM NOMINAL RANGE ANGLE
      X(2)=PSIR
C   GUESS LAGRANGE MULTIPLIER
      X(3)=0.
C   THE FOLLOWING ARE PARAMETERS REQUIRED BY NS01A
      DSTEP=1.E-5
      DMAX=20.
      IPRINT=1
      ACC=1.E-18
      MAXFUN=120
      N=3
      CALL NS01A(N,X,F,AJINV,DSTEP,DMAX,ACC,MAXFUN,IPRINT,W)
      PRINT*," "
      PRINT*,"NOMINAL RANGE ANGLE IS ",X(2)*180./PI
      PRINT*," "
      PRINT 1000
```

```
C   PROGRAM 2 (CONTINUED)

      PRINT*," "
      PRINT1010,(C(I),I=1,10)
 1000 FORMAT("NEW",T14,"RADIUS AT",T26,"NEW",T38,"TOF",T50,"TOF"
     1,T62,"TOTAL",T74,"NOMINAL",T86,"REQUIRED",T98,"NEW RANGE",T110,
     2            /"ECCENTRICITY",T14,"IMPULSE-DU",T26,"PERIGEE-DU",
     3T38,"1-2 SEC",T50,"2-3 SEC",T62,"TOF - SEC",T74,"TOF - SEC",
     4T86,"VEL- DU/TU",T98,"ANGLE-DEG",T110,"  COST")
 1010 FORMAT(1PG11.4,T13,G11.4,T25,G11.4,T37,G11.4,T49,G11.4,T61,G11.4
     1,T73,G11.4,T85,G11.4,T97,G11.4,T109,G11.4)
      END
```

```
C    PROGRAM 2 (CONTINUED)

      SUBROUTINE CALFUN(N,X,F)
      COMMON/I/C(16)
      COMMON/II/PSIR,RBO
      DIMENSION X(3),F(3)
      REAL LAM
      PI=ACOS(-1.)
      SIGNIF=1.E-8
C    SIGNIF IS THE REQUIRED MAXIMUM FIRST DIFFERENCE USED IN FINDING
C    THE PARTIAL DERIVATIVES
      PSIMAX=PI
      ECMAX=.999
      ECMIN=.001
      TU=806.8136
      INDEX=0
 10   DEL=.01
C    DEL IS THE INITIAL STATE PERTURBATION FOR FINDING PARTIAL DERIVATIVE
 11   EC1=X(1)
      PSI=X(2)
      LAM=X(3)
      IF(INDEX.EQ.1)EC1=EC1+DEL
      IF(INDEX.EQ.2)PSI=PSI+DEL
C    INDEX DETERMINES WHICH INITIAL STATE IS PERTURBED
      IF(EC1.GT.ECMAX)EC1=ECMAX
      IF(EC1.LT.ECMIN)EC1=ECMIN
      IF(PSI.GT.PSIMAX)PSI=PSIMAX
      IF(PSI.LT.0.)PSI=0.
C    CALCULATE NOMINAL ORBIT PARAMETERS
      PHIBO=(PI-PSI)/4.
      QBO=(2.*SIN(PSI/2.))/(1.+SIN(PSI/2.))
      VIMP=SQRT(QBO/PBO)
      RAD1=4.*SIN(PSI/2.)*(COS(PHIBO)**2)/(1.+SIN(PSI/2.))**2
      ENOM=SQRT(1.-RAD1)
```

```
C    PROGRAM 2 (CONTINUED)

       ANOM=RBO*(1.+SIN(PSI/2.))/2.
       RAPNOM=ANOM*(1.+ENOM)
       GAMABO=ACOS(-COS(PSI/2.))
C    CALCULATE NOMINAL ORBIT TIME OF FLIGHT
       COSE1=(ENOM-COS(PSI/2.))/(1.-ENOM*COS(PSI/2.))
       E1=ACOS(COSE1)
       TFFNOM=2.*SQRT(ANOM**3)*(PI-E1+ENOM*SIN(E1))
       TOFN=TFFNOM*TU
       GAMA2=PI
       E2=PI
       A1=RAPNOM/(1.+EC1)
       RPER=A1*(1.-EC1)
C    DOES THE MODIFIED ORBIT HIT THE EARTH?
       IF(RPER.GE.1.)GOTO 100
       GAMA3=ACOS(A1*(1.-EC1**2)/(EC1     )-1./EC1)
       GAMA3=2.*PI-GAMA3
C    CALCULATE THE REACTION TIME
       COSE3=(EC1+COS(GAMA3))/(1.+EC1*COS(GAMA3))
       E3=ACOS(COSE3)
       E3=2.*PI-E3
       TFF12=TFFNOM/2.
       TFF23=SQRT(A1**3)*(E3-EC1*SIN(E3)-PI)
       TOF12=TFF12*TU
       TOF23=TFF23*TU
       TTOF=TOF12+TOF23
C    CALCULATE VELOCITIES AT APOGEE AND THE DELTA V REQUIRED
       VNOM=-SQRT((1.-ENOM)/RAPNOM)
       V1=-SQRT((1.-EC1)/RAPNOM)
       DELV=V1-VNOM
       W=.15
C    W IS THE SPECIFIED WEIGHTING FACTOR
       COST=.5*DELV**2+.5*W*TFF23**2
```

```
C   PROGRAM 2 (CONTINUED)

      IF(INDEX.NE.0)GOTO 90
 80   PSI1R=GAMA2-GAMABO-PI+GAMA3
      PSI1=PSI1R*180./PI
      C(1)=EC1
      C(2)=RAPNOM
      C(3)=RPER
      C(4)=TOF12
      C(5)=TOF23
      C(6)=TTOF
      C(7)=TOFN
      C(8)=DELV
      C(9)=PSI1
      C(10)=COST
      C(11)=V1
      C(12)=PSI*180./PI
      C(13)=PSI1R
      C(14)=TFF23
      C(15)=VIMP
      C(16)=C90
      F(3)=PSI1R-PSIR
C   F(3) IS THE HIT CONDITION
 90   CONTINUE
      IF(INDEX.EQ.1)GOTO 200
      IF(INDEX.EQ.2)GOTO 300
      INDEX=INDEX+1
      GOTO 10
 100  PSIR=0.
      GOTO 80
C   CALCULATE THE PARTIALS WRT ECCENTRICITY OF MODIFIED ORBIT
 200  POVEC1=ABS(DELV)-ABS(C(8))
      PTFFC1=TFF23-C(14)
      PPSEC1=PSI1R-C(13)
```

```
C   PROGRAM 2 (CONTINUED)

      VALMAX=AMAX1(PDVEC1,PTFEC1,PPSEC1)
C   IS THE MAXIMUM FIRST DIFFERENCE LT THE SPECIFIED?
      IF(VALMAX.LT.SIGNIF)GOTO 250
C   REDUCE INITIAL PERTURBATION
      DEL=.5*DEL
      GOTO 11
 250  PDVEC1=PDVEC1/DEL
      PTFEC1=PTFEC1/DEL
      PPSEC1=PPSEC1/DEL
      INDEX=INDEX+1
      GOTO 10
C   CALCULATE PARTIALS WRT NOMINAL RANGE ANGLE
 300  PDVSI=ABS(DELV)-ABS(C(8))
      PPSSI=PSI1R-C(13)
      PTFSI=TFF23-C(14)
      VALMAX=AMAX1(PDVSI,PTFSI,PPSSI)
C   IS THE MAXIMUM FIRST DIFFERENCE LT THE SPECIFIED?
      IF(VALMAX.LT.SIGNIF)GOTO 350
C   REDUCE INITIAL PERTURBATION
      DEL=.5*DEL
      GOTO 11
 350  PDVSI=PDVSI/DEL
      PTFSI=PTFSI/DEL
      PPSSI=PPSSI/DEL
      F(1)=C(8)*PDVEC1+W*C(14)*PTFEC1+LAM*PPSEC1
      F(2)=C(8)*PDVSI+W*C(14)*PTFSI+LAM*PPSSI
C   F(1) AND F(2) ARE 1 ST ORDER NECESSARY CONDITIONS
      END
```

```
C    PROGRAM 3 - PRE-APOGEE AND POST-APOGEE TO APOGEE TRANSFERS
C                SORTED SOLUTIONS

      PROGRAM THESIS(INPUT,OUTPUT,PUNCH)
      COMMON/I/C(18)
      COMMON/II/PSI,PSIR,RBO,R2,KOUNT
      COMMON/III/B(100,18)
      COMMON/V/D(100,18)
      DIMENSION X(2),F(2),AJINV(2,2),W(20)
      PI=ACOS(-1.)
      KOUNT=1
      READ*,PSIRD
C    PSIRD IS THE TARGET RANGE ANGLE (READ IN DEGREES)
 50   READ*,PSID,RBO
C    PSID IS THE NOMINAL RANGE ANGLE (READ IN DEGREES)
C    RBO IS THE GEOCENTRIC BURNOUT RADIUS (DU)
      IF(EOF(5LINPUT))2000,60
 60   PSI=PSID*PI/180.
      PSIR=PSIRD*PI/180.
      PRINT*," "
      PRINT*," "
      PRINT*,"REQUIRED RANGE (DEG) IS ",PSIRD
      PRINT*,"RANGE ANGLE (DEGREES) IS ",PSI*180./PI
      PRINT*,"BURNOUT RADIUS (DU) IS ",RBO
      PRINT*," "
      PRINT*," "
C    GUESS INITIAL ECCENTRICITY - X(1)
      X(1)=.5
      X(2)=0.
C    THE FOLLOWING ARE PARAMETERS REQUIRED BY NS01A
      DSTEP=1.E-5
      DMAX=.499
      ACC=1.E-10
      IPRINT=0
```

```
C     PROGRAM 3 (CONTINUED)

      MAXFUN=60
      N=2
C     CALCULATE NOMINAL ORBIT PARAMETERS
      PHIBO=(PI-PSI)/4.
      RAD1=4.*SIN(PSI/2.)*(COS(PHIBO)**2)/(1.+SIN(PSI/2.))**2
      ENOM=SQRT(1.-RAD1)
      ANOM=RBO*(1.+SIN(PSI/2.))/2.
      RAPNOM=ANOM*(1.+ENOM)
      DELR2=(RAPNOM-RBO)/100.
      R2=RAPNOM
C     VARY THE TRANSFER POINT FOR EACH NOMINAL ORBIT
      DO 100 NEC=1,99
      CALL NSO1A(N,X,F,AJINV,DSTEP,DMAX,ACC,MAXFUN,IPRINT,W)
      DO 90 I=1,17
 90   B(NEC,I)=C(I)
 100  R2=R2-DELR2
C     FIND THE LOWEST COST TRANSFER
      CALL SORTBST(KOUNT)
      PRINT*,"THE LOWEST COST FOR THIS TRAJECTORY IS "
      PRINT*," "
      PRINT 1000
      PRINT*," "
      PRINT 1010,(D(KOUNT,I),I=1,10)
C     PUNCH OUT SORTED BEST MODIFIED ORBIT PARAMETERS
C     FOR USE IN PROGRAM 4
      PUNCH 1020,(D(KOUNT,I),I=1,5)
      PUNCH 1020,(D(KOUNT,I),I=6,10)
      PUNCH 1020,(D(KOUNT,I),I=11,15)
      PUNCH 1020,(D(KOUNT,I),I=16,17)
      PRINT*,"NOMINAL APOGEE (DU) IS",D(KOUNT,16)
      PRINT*,"GAMA2 (DEG) IS ",D(KOUNT,17)
      PRINT*," VBO ",D(KOUNT,12),"  TOTAL IMPULSE ",D(KOUNT,13)
```

```
C    PROGRAM 3 (CONTINUED)

      KOUNT=KOUNT+1
      GOTO 50
 2000 KOUNT=KOUNT-1
 1000 FORMAT("NEW",T14,"RADIUS AT",T26,"NEW",T38,"TOF",T50,"TOF"
     1,T62,"TOTAL",T74,"NOMINAL",T86,"REQUIRED",T98,"NEW RANGE",T110,
     2            /"ECCENTRICITY",T14,"IMPULSE-DU",T26,"PERIGEE-DU",
     3T38,"1-2 SEC",T50,"2-3 SEC",T62,"TOF - SEC",T74,"TOF - SEC",
     4T86,"VEL- DU/TU",T98,"ANGLE-DEG",T110,"  COST")
 1010 FORMAT(1PG11.4,T13,G11.4,T25,G11.4,T37,G11.4,T49,G11.4,T61,G11.4
     1,T73,G11.4,T85,G11.4,T97,G11.4,T109,G11.4)
 1020   FORMAT(5(G15.6,1X))
      END
```

```
C     PROGRAM 3 (CONTINUED)

      SUBROUTINE CALFUN(N,X,F)
      COMMON/I/C(18)
      COMMON/II/PSI,PSIR,RBO,R2,KOUNT
      DIMENSION X(2),F(2)
      PI=ACOS(-1.)
      TU=806.8136
      EC1=X(1)
C     CALCULATE NOMINAL ORBIT PARAMETERS
      PHIBO=(PI-PSI)/4.
      QBO=(2.*SIN(PSI/2.))/(1.+SIN(PSI/2.))
      RAD1=4.*SIN(PSI/2.)*(COS(PHIBO)**2)/(1.+SIN(PSI/2.))**2
      ENOM=SQRT(1.-RAD1)
      ANOM=RBO*(1.+SIN(PSI/2.))/2.
      RAPNOM=ANOM*(1.+ENOM)
      GAMABO=ACOS(-COS(PSI/2.))
      COSE1=(ENOM-COS(PSI/2.))/(1.-ENOM*COS(PSI/2.))
      E1=ACOS(COSE1)
C     CALCULATE NOMINAL ORBIT TIME OF FLIGHT
      TFFNOM=2.*SQRT(ANOM**3)*(PI-E1+ENOM*SIN(E1))
      TOFN=TFFNOM*TU
      GAMA21=PI
      E21=PI
 10   A1=R2/(1.+EC1)
      RPER=A1*(1.-EC1)
C     DOES THE MODIFIED ORBIT HIT THE EARTH?
      IF(RPER.GE.1.)GOTO 15
      GOTO 20
 15   EC1=EC1+.001
      X(1)=EC1
      IF(X(1).GT.1.)GOTO 100
      GOTO 10
C     FIND THE TRUE ANOMALY ON THE NOMINAL AT TRANSFER
```

```
C    PROGRAM 3 (CONTINUED)

 20    GAMA2=     (ANOM*(1.-ENOM**2)/(ENOM*R2)-1./ENOM)
       IF(ABS(GAMA2).GT.1.)GOTO 100
       GAMA2=ACOS(GAMA2)
C    FOR PRE APOGEE TRANSFERS, REMOVE THE FOLLOWING LINE
       GAMA2=2.*PI-GAMA2
       COSE2=(ENOM+COS(GAMA2))/(1.+ENOM*COS(GAMA2))
       E2=ACOS(COSE2)
C    FOR PRE APOGEE TRANSFERS, REMOVE THE FOLLOWING LINE
       E2=2.*PI-E2
C    CALCULATE THE TRUE ANOMALY AT IMPACT
       COSGMA3=(A1*(1.-EC1**2))/EC1-1./EC1
       IF(ABS(COSGMA3).GT.1.)GOTO 100
       GAMA3=ACOS(COSGMA3)
       GAMA3=2.*PI-GAMA3
       DEL=5.*PI/180.
       IF(GAMA3-PI.LT.DEL)GOTO 100
       IF(GAMA2.GE.GAMA3)GOTO 100
       COSE3=(EC1+COS(GAMA3))/(1.+EC1*COS(GAMA3))
       E3=ACOS(COSE3)
       E3=2.*PI-E3
C    CALCULATE THE REACTION TIME
       TFF12=SQRT(ANOM**3)*(E2-ENOM*SIN(E2)-E1+ENOM*SIN(E1))
       TFF23=SQRT(A1**3)*(E3-EC1*SIN(E3)-PI)
       TOF12=TFF12*TU
       TOF23=TFF23*TU
       TTOF=TOF12+TOF23
C    CALCULATE VELOCITIES FOR NOMINAL AND MODIFIED ORBITS
C    AT TRANSFER POINT
       VPN=-SIN(GAMA2)/SQRT(R2*(1.+ENOM*COS(GAMA2)))
       VQN=SQRT(1./(R2*(1.+ENOM*COS(GAMA2))))*(ENOM+COS(GAMA2))
       VQ11=-SQRT((1.-EC1)/P2)
       VP11=0.
```

```
C     PROGRAM 3 (CONTINUED)

      VQ1=-SIN(GAMA2)*VP11-COS(GAMA2)*VQ11
      VP1=-COS(GAMA2)*VP11+SIN(GAMA2)*VQ11
C     COMPUTE THE DELTA V REQUIRED
      VPREQ=VP1-VPN
      VQREQ=VQ1-VQN
      VR=SQRT(VPREQ**2+VQREQ**2)
      DELV=VR
      VIMPSQ=QBO/RBO
      VIMP=SQRT(VIMPSQ)
      W=.15
C     W IS THE SPECIFIED WEIGHTING FACTOR
      COST=.5*DELV**2+.5*W*TEF23**2
      PSI1P=GAMA2-GAMABO-PI+GAMA3
      F(1)=PSI1R-PSIR
C     F(1) IS THE HIT CONDITION
      F(2)=X(2)
      PSI1=PSI1R*180./PI
      C(1)=EC1
      C(2)=R2
      C(3)=RPER
      C(4)=TOF12
      C(5)=TOF23
      C(6)=TTOF
      C(7)=TOFN
      C(8)=DELV
      C(9)=PSI1
      C(10)=COST
      C(11)=PSI*180./PI
      C(12)=VIMP
      C(13)=VIMP+C(8)
      C(16)=RAPNOM
      C(17)=GAMA2*180./PI
```

```
C    PROGRAM 3 (CONTINUED)

      RETURN
 100  F(1)=1./EC1
C    ADD PENALTY FOR NOT HITTING THE EARTH
      DO 101 I=1,10
 101  C(I)=0.
      F(2)=X(2)
      C(2)=9999.
      END
```

```
C    PROGRAM 3 (CONTINUED)

       SUBROUTINE SORTBST(KOUNT)
C    THIS SUBROUTINE SORTS OUT THE LOWEST COST MODIFIED ORBIT
       COMMON/I/C(18)
       COMMON/II/PSI,PSIR,RBO,R2
       COMMON/III/B(100,18)
       COMMON/V/D(100,18)
       DEL=.001
       PI=ACOS(-1.)
       J=1
 1     CONTINUE
       NPOS=J
       IF((B(J,9)*PI/180.).LT.PSIR-DEL)GOTO 300
       IF((B(J,9)*PI/180.).GT.PSIR+DEL)GOTO 300
       BEST=B(J,10)
       I=J+1
       DO 100 N=I,100
       IF(B(N,10).LT.BEST)GOTO 10
       GOTO 100
 10    CONTINUE
       IF((B(N,9)*PI/180.).GT.PSIR+DEL)GOTO 100
       IF((B(N,9)*PI/180.).LT.PSIR-DEL)GOTO 100
       NPOS=N
       BEST=B(N,10)
 100   CONTINUE
       DO 200 N=1,17
 200   D(KOUNT,N)=B(NPOS,N)
       DO 5 I=1,100
       DO 5 J=1,18
 5     B(I,J)=0.
       RETURN
 300   J=J+1
       GOTO 1
       END
```

```
C     PROGRAM 4 - PRE-APOGEE AND POST-APOGEE TO APOGEE TRANSFERS
      OPTIMUM SOLUTIONS

        PROGRAM THESIS(INPUT,OUTPUT,PUNCH)
        COMMON/I/C(18)
        COMMON/II/PSI,PSIR,RBO,KOUNT
        COMMON/V/D(100,18)
        DIMENSION X(3),F(3),AJINV(3,3),W(100)
        PI=ACOS(-1.)
        KOUNT=1
        READ*,PSIRD
C     PSIRD IS THE TARGET RANGE ANGLE (READ IN DEGREES)
        PSIR=PSIRD*PI/180.
        RBO=1.05
C     RBO IS THE GEOCENTRIC BURNOUT RADIUS (DU)
 50     CONTINUE
C     READ IN INITIAL GUESSES FOR MODIFIED ORBIT ECCENTRICITY
C     AND RADIUS AT TRANSFER
        READ 1020,X(1),X(2),V1,V2,V3
        IF(EOF(5LINPUT))2000,60
 60     CONTINUE
        READ 1020, V1,V2,V3,V4,V5
C     READ NOMINAL RANGE ANGLE (DEGREES)
        READ 1020,PSID ,V1,V2,V3,V4
        READ 1020,V1,V2
        X(3)=0.
        PSI=PSID*PI/180.
        PRINT*," "
        PRINT*," "
        PRINT*,"REQUIRED RANGE (DEG) IS ",PSIRD
        PRINT*,"RANGE ANGLE (DEGREES) IS ",PSI*180./PI
        PRINT*,"BURNOUT RADIUS (DU) IS ",RBO
        PRINT*," "
        PRINT*," "
```

```
C   PROGRAM 4 (CONTINUED)

C   THE FOLLOWING ARE PARAMETERS REQUIRED BY NS01A
      DSTEP=1.E-06
      ACC=1.E-12
      DMAX=52.
      IPRINT=0
      MAXFUN=200
      N=3
      CALL NS01A(N,X,F,AJINV,DSTEP,DMAX,ACC,MAXFUN,IPRINT,W)
      ERROR=SQRT(F(1)**2+F(2)**2+F(3)**2)
      PRINT*,"SUM OF SQUARES ERROR ",ERROR
      DO 90 I=1,17
 90   D(KOUNT,I)=C(I)
      PRINT*," "
      PRINT*,"THE LOWEST COST FOR THIS TRAJECTORY IS "
      PRINT 1000
      PRINT 1010,(D(KOUNT,I),I=1,10)
      PRINT*,"NOMINAL APOGEE (DU) IS",D(KOUNT,16)
      PRINT*,"GAMA2 (DEG) IS ",D(KOUNT,17)
      PRINT*," V80 ",D(KOUNT,12),"  TOTAL IMPULSE ",D(KOUNT,13)
      PRINT*,"X(3)= ",X(3)
      KOUNT=KOUNT+1
      GOTO 50
 2000 KOUNT=KOUNT-1
 1000 FORMAT("NEW",T14,"RADIUS AT",T26,"NEW",T38,"TOF",T50,"TOF"
     1,T62,"TOTAL",T74,"NOMINAL",T86,"REQUIRED",T98,"NEW RANGE",T110,
     2          /"ECCENTRICITY",T14,"IMPULSE-DU",T26,"PERIGEE-DU",
     3T38,"1-2 SEC",T50,"2-3 SEC",T62,"TOF - SEC",T74,"TOF - SEC",
     4T86,"VEL- DU/TU",T98,"ANGLE-DEG",T110,"  COST")
 1010 FORMAT(1PG11.4,T13,G11.4,T25,G11.4,T37,G11.4,T49,G11.4,T61,G11.4
     1,T73,G11.4,T85,G11.4,T97,G11.4,T109,G11.4)
 1020 FORMAT(5(G15.6,1X))
      END
```

```
C     PROGRAM 4 (CONTINUED)

      SUBROUTINE CALFUN(N,X,F)
      COMMON/I/C(18)
      COMMON/II/PSI,PSIR,RBO,KOUNT
      DIMENSION X(3),F(3)
      REAL LAM
      PI=ACOS(-1.)
      TU=806.8136
C     SPECIFY THE MAXIMUM FIRST DIFFERENCE
      SIGNIF=5.E-08
      ECMAX=.999
      ECMIN=.001
C     INDEX DETERMINES WHICH INITIAL STATE IS PERTURBED
      INDEX=0
 30   DEL=.0001
      IF(INDEX.EQ.2)DEL=-(ABS(DEL))
 31   EC1=X(1)
      R2=X(2)
      LAM=X(3)
      IF(INDEX.EQ.1)EC1=EC1+DEL
      IF(INDEX.EQ.2)R2=R2+DEL
      IF(EC1.GT.ECMAX)EC1=ECMAX
      IF(EC1.LT.ECMIN)EC1=ECMIN
      IF(R2.LT.RBO)R2=RBO
C     CALCULATE NOMINAL ORBIT PARAMETERS
      PHIBO=(PI-PSI)/4.
      QBO=(2.*SIN(PSI/2.))/(1.+SIN(PSI/2.))
      RAD1=4.*SIN(PSI/2.)*(COS(PHIBO)**2)/(1.+SIN(PSI/2.))**2
      ENOM=SQRT(1.-RAD1)
      ANOM=RBO*(1.+SIN(PSI/2.))/2.
      RAPNOM=ANOM*(1.+ENOM)
      GAMABO=PI-PSI/2.
C     CALCULATE NOMINAL ORBIT TIME OF FLIGHT
```

```
C    PROGRAM 4 (CONTINUED)

      COSE1=(ENOM-COS(PSI/2.))/(1.-ENOM*COS(PSI/2.))
      E1=ACOS(COSE1)
      TFFNOM=2.*SQRT(ANOM**3)*(PI-E1+ENOM*SIN(E1))
      TOFN=TFFNOM*TU
      GAMA21=PI
      E21=PI
 10   A1=R2/(1.+EC1)
      RPER=A1*(1.-EC1)
C    LOCATE TRANSFER POINT
 20   GAMA2=     (ANOM*(1.-ENOM**2)/(ENOM*R2)-1./ENOM)
      IF(ABS(GAMA2).GT.1.)GOTO 100
      GAMA2=ACOS(GAMA2)
C    FOR PRE-APOGEE TRANSFERS REMOVE THE FOLLOWING LINE
      GAMA2=2.*PI-GAMA2
      COSE2=(ENOM+COS(GAMA2))/(1.+ENOM*COS(GAMA2))
      E2=ACOS(COSE2)
C    FOR PRE-APOGEE TRANSFERS REMOVE THE FOLLOWING LINE
      E2=2.*PI-E2
      GAMA3=     (A1*(1.-EC1**2)/(EC1     )-1./EC1)
      IF(ABS(GAMA3).GT.1.)GOTO 100
      GAMA3=ACOS(GAMA3)
      GAMA3=2.*PI-GAMA3
      COSE3=(EC1+COS(GAMA3))/(1.+EC1*COS(GAMA3))
      E3=ACOS(COSE3)
      E3=2.*PI-E3
C    CALCULATE THE REACTION TIME
      TFF12=SQRT(ANOM**3)*(E2-ENOM*SIN(E2)-E1+ENOM*SIN(E1))
      TFF23=SQRT(A1**3)*(E3-EC1*SIN(E3)-PI)
      TOF12=TFF12*TU
      TOF23=TFF23*TU
      TTOF=TOF12+TOF23
C    CALCULATE DELTA V
```

```
C    PROGRAM 4 (CONTINUED)

      VPN=-SIN(GAMA2)/SQRT(R2*(1.+ENOM*COS(GAMA2)))
      VQN=SQRT(1./(R2*(1.+ENOM*COS(GAMA2))))*(ENOM+COS(GAMA2))
      VP11=0.
      VQ11=-SQRT((1.-EC1)/R2)
      VQ1=-SIN(GAMA2)*VP11-COS(GAMA2)*VQ11
      VP1=-COS(GAMA2)*VP11+SIN(GAMA2)*VQ11
      VPREQ=VP1-VPN
      VQREQ=VQ1-VQN
      VR=SQRT(VPREQ**2+VQREQ**2)
      DELV=VR
      VIMPSQ=QQO/RBO
      VIMP=SQRT(VIMPSQ)
      W=.15
C    W IS THE SPECIFIED WEIGHTING FACTOR
      COST=.5*DELV**2+.5*W*TFF23**2
      PSI1R=GAMA2-GAMABO-PI+GAMA3
      IF(INDEX.NE.0)GOTO 90
      F(3)=PSI1R-PSIR
C    F(3) IS THE HIT CONDITION
      PSI1=PSI1R*180./PI
      C(1)=EC1
      C(2)=R2
      C(3)=RPER
      C(4)=TOF12
      C(5)=TOF23
      C(6)=TTOF
      C(7)=TOFN
      C(8)=DELV
      C(9)=PSI1
      C(10)=COST
      C(11)=PSI*180./PI
      C(12)=VIMP
```

```
C   PROGRAM 4 (CONTINUED)

      C(13)=PSI1R
      C(14)=TFF23
      C(16)=RAPNOM
      C(17)=GAMA2*180./PI
      C(18)=LAM
      GOTO 90
 100  CONTINUE
      DO 101 I=1,3
 101  F(I)=10.*F(I)
C   FOR NON-CONVERGENCE, THE FOLLOWING IS PRINTED
      PRINT*,"XXX"
      C(7)=0.
      RETURN
 90   CONTINUE
      IF(INDEX.EQ.1)GOTO 200
      IF(INDEX.EQ.2)GOTO 300
      INDEX=INDEX+1
      GOTO 30
C   CALCULATE THE PARTIALS WRT ECCENTRICITY OF MODIFIED ORBIT
 200  PDVEC1=DELV-C(9)
      PPSEC1=PSI1R-C(13)
      PTFEC1=TFF23-C(14)
      T2=ABS(PDVEC1)
      T3=ABS(PTFEC1)
      T4=ABS(PPSEC1)
      VALMAX=AMAX1(T2,T3,T4)
C   IS THE MAXIMUM FIRST DIFFERENCE LT THE SPECIFIED?
      IF(VALMAX.LT.SIGNIF)GOTO 250
C   REDUCE INITIAL PERTURBATION
      DEL=.5*DEL
      GOTO 31
 250  PDVEC1=PDVEC1/DEL
```

```
C    PROGRAM 4 (CONTINUED)

      PTFEC1=PTFEC1/DEL
      PPSEC1=PPSEC1/DEL
      INDEX=INDEX+1
      GOTO 70
C    CALCULATE THE PARTIALS WRT TRANSFER POINT RADIUS
 300  PDVR2=DELV-C(8)
      PPSR2=PSI1R-C(13)
      PTFR2=TEF23-C(14)
      T2=ABS(PDVR2)
      T3=ABS(PTFR2)
      T4=ABS(PPSR2)
      VALMAX=AMAX1(T2,T3,T4)
C    IS THE MAXIMUM FIRST DIFFERENCE LT THE SPECIFIED?
      IF(VALMAX.LT.SIGNIF)GOTO 350
C    REDUCE INITIAL PERTURBATION
 320  DEL=.5*DEL
      GOTO 31
 350  PDVR2=PDVR2/DEL
      PTFR2=PTFR2/DEL
      PPSR2=PPSR2/DEL
      F(1)=C(8)*PDVR2+W*C(14)*PTFR2+LAM*PPSR2
      F(2)=C(8)*PDVEC1+W*C(14)*PTFEC1+LAM*PPSEC1
C    F(1) AND F(2) ARE FIRST ORDER NECESSARY CONDITIONS
      END
```

```
C    PROGRAM 5 NONPLANAR TRAJECTORY MODIFICATION
C              NUMERICAL SOLUTION

      PROGRAM THESIS(INPUT,OUTPUT,PUNCH)
      LOGICAL INATM,MODIFY
      COMMON/I/Y(6),A(2),RBO,VBO
      COMMON/II/SL(8),ST(8),ST1(8)
      COMMON/III/THETA,DT,T,TBO
      COMMON/V/TRJCTRY(1500,7)
      COMMON/VI/TATM,INATM,JPOINTS,MODIFY
      COMMON/XI/Z(7),IPOINT
      COMMON/XIV/W1
      DIMENSION X(7),F(7),AJINV(7,7),W(300)
C    READ LAUNCH SITE LATITUDE AND LONGITUDE (DEGREES)
      READ*,SITLAT,SITLONG
C    READ PSEUDO TARGET LATITUDE AND LONGITUDE (DEGREES)
      READ*,TGTLAT,TGTLONG
C    READ REAL TARGET LATITUDE AND LONGITUDE (DEGREES)
      READ*,TGT1LAT,TGT1LNG
C    NOTE - WEST LONGITUDE IS NEGATIVE
C    READ GUESSES FOR INITIAL STATES OF NOMINAL TRAJECTORY
      READ*,(X(I),I=1,7)
C    NOMINAL STATES ARE -
C      X(1) IS THE AZIMUTH OF THE VELOCITY VECTOR AT BURNOUT
C      X(2) IS THE ELEVATION OF THE VELOCITY VECTOR AT BURNOUT
C      X(3) IS THE TIME OF FLIGHT
C      X(4)=X(5)=X(6)=X(7)=0.   (NOT USED FOR NOMINAL TRAJECTORY)
C    READ GUESSES FOR INITIAL STATES AT MODIFICATION POINT
C    MODIFIED STATES ARE
C      Z(1) IS INERTIAL VX
C      Z(2) IS INERTIAL VY
C      Z(3) IS INERTIAL VZ
C      Z(4) IS REACTION TIME
C      Z(5) IS LAGRANGE MULTIPLIER 1
```

```
C   PROGRAM 5 (CONTINUED)

C     Z(6) IS LAGRANGE MULTIPLIER 2
C     Z(7) IS LAGRANGE MULTIPLIER 3
      READ*,(Z(I),I=1,7)
      READ*,IPOINT
C   IPOINT IS THE INTEGRATION STEP AT WHICH WE WISH TO MODIFY THE
C   TRAJECTORY
      READ*,W1
C   W1 IS THE COST FUNCTION WEIGHTING FACTOR
      PRINT*," INITIAL GUESSES ",(X(I),I=1,7)
      PRINT*,(Z(I),I=1,7)
      PRINT*," "
      PRINT*,"TRAJECTORY FROM"
      PRINT*,SITLAT,SITLONG," DEGREES LATIUDE AND LONGITUDE"
      PRINT*, "          TO    "
      PRINT*,TGTLAT,TGTLONG," DEGREES LATITUDE AND LONGITUDE"
      MODIFY=.FALSE.
      PI=ACOS(-1.)
      SL(1)=SITLAT*PI/180.
      SL(2)=SITLONG*PI/180.
      ST(1)=TGTLAT*PI/180.
      ST(2)=TGTLONG*PI/180.
      ST1(1)=TGT1LAT*PI/180.
      ST1(2)=TGT1LNG*PI/180.
C   SCALE X"S TO RADIANS AND TU"S
      X(1)=X(1)*PI/180.
      X(2)=X(2)*PI/180.
      X(3)=X(3)/806.8136
C   THE FOLLOWING ARE PARAMETERS REQUIRED BY NS01A
      DMAX=500.
      MAXFUN=120
      IPRINT=0
      DSTEP=.001
```

```
C     PROGRAM 5 (CONTINUED)

        N=7
        ACC=10.
C     ACC IS THE REQUIRED TERMINAL ERROR (FEET) NEEDED FROM NS01A
C     SET RADIUS VECTOR WRT LAUNCH SITE AT BURNOUT
        RBO=200.*6076.
C     RBO IS IN FEET
        VBO=25000.
C     VBO IS IN FEET/SECOND
        A(1)=20.*PI/180.
C     A(1) IS THE AZIMUTH OF THE BURNOUT POINT WRT LAUNCH SITE
        A(2)=45. *PI/180.
C     A(2) IS THE ELEVATION OF THE BURNOUT POINT WRT THE LAUNCH SITE
C     A(1) AND A(2) ARE IN RADIANS
C     FIND OPLATE EARTH RADIUS OF PSEUDO TARGET FOR OBLATE ATMOSPHERE MODEL
        CALL LOCTGT(0.,XT,YT,ZT)
        ST(3)=SQRT(XT**2+YT**2+ZT**2)
C     ST(3) IS THE RADIUS TO THE PSEUDO TARGET ON THE OBLATE EARTH
C     FIND NOMINAL TRAJECTORY WHICH HITS PSEUDO TARGET
        CALL NS01A(N,X,F,AJINV,DSTEP,DMAX,ACC,MAXFUN,IPRINT,W)
        PRINT 22, RBO/6076.,VBO
        PRINT 21, X(1)*180./PI,X(2)*180/PI,X(3)*806.8136
        PRINT 20,A(1)*180./PI,A(2)*180./PI
        PRINT*,"  "
        PRINT*,"  "
        PRINT*,"  FINAL CALCULATIONS--- ERRORS IN FEET   "
        PRINT*,"X ERROR ",F(1)
        PRINT*,"Y ERROR ",F(2)
        PRINT*,"Z ERROR ",F(3)
        PRINT*,"   "
        PRINT*,"ERROR RADIUS IS",(SQRT(F(1)**2+F(2)**2+F(3)**2))/6076.," N
      1 MI."
        PRINT*,"   "
```

```
C    PROGRAM 5 (CONTINUED)

      PRINT*,"*****************************************************"
      CALL NEWORBT
 20   FORMAT(" BURNOUT POINT: AZIMUTH (DEG)  ",1PG13.6,5X,"ELEVATION (DE
     1G)  ",G13.6)
 21   FORMAT(" VELOCITY VECTOR AT BURNOUT"/"AZIMUTH (DEG) ",1PG13.6,3X,
     1"ELEVATION (DEG) ",G13.6,/ ,"TIME OF FREE FLIGHT (SEC) ",G13.6)
 22   FORMAT(" BURNOUT RADIUS ",1PG13.6," N. MI.",10X,"BURNOUT VELOCITY
     1",G13.6," FT/SEC")
      END
```

```
C     PROGRAM 5 (CONTINUED)

      SUBROUTINE NEWORB7
C     THIS SUBROUTINE IS USED TO FIND THE MODIFIED TRAJECTORY
      LOGICAL INATM,MODIFY
      INTEGER SCALEX
      COMMON/II/SL(8),ST(8),ST1(8)
      COMMON/V/TRJCTRY(1500,7)
      COMMON/VI/IATM,INATM,JPOINTS,MODIFY
      COMMON/XI/X(7),IPOINT
      COMMON/XIII/SCALEX(7)
      COMMON/XIV/W1
      DIMENSION F(7),AJINV(7,7),W(300)
      AE=2.092567257E7
C     THE FOLLOWING ARE PARAMETERS REQUIRED BY NS01A
      DSTEP=1.E-14
      DMAX=120.
      IPRINT=1
      MAXFUN=200
      ACC=2.37
      N=7
      PI=ACOS(-1.)
      MODIFY=.TRUE.
C     FIND OBLATE EARTH RADIUS OF REAL TARGET FOR OBLATE ATMOSPHERE MODEL
      CALL LOCTGT1(0.,XT,YT,ZT)
      ST1(3)=SQRT(XT**2+YT**2+ZT**2)
C     SCALE GUESSES TO LIE BETWEEN -1 AND 1
      DO 10 I=1,7
 10   SCALEX(I)=5
      DO 20 I=1,7
      IF(ABS(X(I)).LT.1.)GOTO 19
      IF(ABS(X(I)).LT.10.)SCALEX(I)=1
      IF((ABS(X(I)).LT.100.).AND.(SCALEX(I).EQ.5))SCALEX(I)=2
      IF((ABS(X(I)).LT.1000.).AND.(SCALEX(I).EQ.5))SCALEX(I)=3
```

```
C   PROGRAM 5 (CONTINUED)

      IF((ABS(X(I)).LT.10000.).AND.(SCALEX(I).EQ.5))SCALEX(I)=4
      IF(SCALEX(I).EQ.5)PRINT*,"X ",I," NEEDS SCALING IN NEWORBT"
      GOTO 20
 19   SCALEX(I)=0
 20   CONTINUE
      SCALEX(7)=SCALEX(7)-1
 25   CONTINUE
      DO 30 I=1,7
 30   X(I)=X(I)/(10.**SCALEX(I))
C   FIND MODIFIED ORBIT WHICH HITS REAL TARGET
      CALL NSC1A(N,X,F,AJINV,DSTEP,DMAX,ACC,MAXFUN,IPRINT,W)
      PRINT*,"FOR MODIFICATION AT POINT  ",(TRJCTRY(IPOINT,I),I=1,7)
      PRINT*,"DSTEP= ",DSTEP
      PRINT*,"  FINAL CALCULATIONS--- ERRORS IN FEET   "
C   CONVERT HIT EQUATIONS BACK TO FEET
      F(5)=F(5)*AE
      F(6)=F(6)*AE
      F(7)=F(7)*AE
C   THE FOLLOWING ARE USED FOR SCALING PURPOSES ON THE EXAMPLE POINT
      F(7)=F(7)/1.E4
      F(6)=F(6)/1.E4
      F(5)=F(5)/1.E4
      PRINT*,"X ERROR ",F(5)
      PRINT*,"Y ERROR ",F(6)
      PRINT*,"Z ERROR ",F(7)
      PRINT*,"   "
      PRINT*,"ERROR RADIUS IS",(SQRT(F(5)**2+F(6)**2+F(7)**2))/6076.," N
     1 MI."
      PRINT*," INITIAL CONDITIONS"
      DO 40 I=1,7
 40   X(I)=X(I)*(10.**SCALEX(I))
      PRINT*,(X(I),I=1,7)
```

```
C   PROGRAM 5 (CONTINUED)

      PRINT*,"   "
      DELTAV=SQRT(X(1)**2+X(2)**2+X(3)**2)
      PRINT*,"DELTA V (FT/SEC) ",  DELTAV
      PRINT*," REACTION TIME (SEC) ",X(4)
      PRINT*," "
C   FIND COST AND TERMINAL ERROR
      COST=.5*DELTAV**2/2.593628E04**2+.5*W1*(X(4)/806.8136)**2
      PRINT*,"COST ",COST
      PRINT*,"*****************************************************"
      ERR=SQRT(F(1)**2+F(2)**2+F(3)**2)
C   EXAMINE THE JACOBIAN TO SEE IF FURTHER SCALING MIGHT HELP
      PRINT*,"THE JACOBIAN"
      PRINT 120,(W(I),I=1,49)
      PUNCH 1010,IPOINT
      PUNCH 1000,(X(I),I=1,4)
      PUNCH 1000,(X(I),I=5,7),DELTAV
      PUNCH 1000,COST,ERR,F(5),F(6)
      PUNCH 1000,F(7),(TRJCTRY(IPOINT,I),I=1,3)
      PUNCH 1000,(TRJCTRY(IPOINT,I),I=4,7)
120   FORMAT(7(G15.3)/)
1000  FORMAT (4(G20.12))
1010  FORMAT(I3)
100   STOP
      END
```

```
C     PROGRAM 5 (CONTINUED)

      SUBROUTINE CALFUN(N,X,F)
C     THIS SUBROUTINE PROVIDES NSO1A, MAIN, AND NEWORBT WITH A METHOD OF
C     INTEGRATING THE EQUATIONS OF MOTION
      LOGICAL INATM,MODIFY
      INTEGER SCALFX
      COMMON/I/Y(6),A(2),RBO,VBO
      COMMON/II/SL(8),ST(8),ST1(8)
      COMMON/III/THETA,DT,T,TBO
      COMMON/V/TRJCTRY(1500,7)
      COMMON/VI/TATM,INATM,JPOINTS,MODIFY
      COMMON/XI/Z(7),IPOINT
      COMMON/XII/STARTIM
      COMMON/XIII/SCALEX(7)
      COMMON/XIV/W1
      COMMON/XV/XC(4)
      DIMENSION PXF(4),PYF(4),PZF(4),POFTF(3)
      DIMENSION FINALST(3),TGTFNLS(3)
      DIMENSION X(7),F(7)
      REAL LL
      EXTERNAL DFEQ
      PI=ACOS(-1.)
      DUTU=25936.246
      TU=806.8136
      IF(MODIFY)GOTO 100
C     THIS PART OF CALFUN IS USED IN THE CALCULATION OF THE NOMINAL ORBIT
      J1=0
      NIS=200
C     NIS IS THE NUMBER OF INTEGRATION STEPS FROM BURNOUT TO REENTRY
      LL=SL(1)
C     LL IS THE LAUNCH SITE LATITUDE
      V0=1524.*COS(LL)
C     V0 IS THE EASTERLY COMPONENT OF THE INERTIAL LAUNCH SITE VELOCITY
```

```
C     PROGRAM 5 (CONTINUED)

      WEARTH=7.2921152E-05
      TBO=5.*60.
C     TBO IS THE TIME OF POWERED FLIGHT (SECONDS)
      THETA=WEARTH*TBO
C     THETA IS THE ANGULAR ROTATION OF THE LAUNCH SITE DURING THE
C     POWERED PHASE
      AE=2.0925672576E7
      E=.08181
C     THE FOLLOWING COMPENSATE FOR EARTH OBLATENESS
      XLS=AE*COS(LL)/(SQRT(1.-(E**2)*(SIN(LL)**2)))
      ZLS=AE*SIN(LL)*(1.-E**2)/(SQRT(1.-E**2*(SIN(LL)**2)))
C     INITIALIZE ALL STATES AT BURNOUT
C       Y(1) IS INERTIAL X
C       Y(2) IS INERTIAL Y
C       Y(3) IS INERTIAL Z
C       Y(4) IS INERTIAL VX
C       Y(5) IS INERTIAL VY
C       Y(6) IS INERTIAL VZ
      Y(1)=RBO*(-SIN(LL)*COS(THETA)*COS(A(2))*COS(A(1))-SIN(THETA)*
     1COS(A(2))*SIN(A(1))+COS(LL)*COS(THETA)*SIN(A(2)))
     2+XLS*COS(THETA)
      Y(2)=RBO*(-SIN(LL)*SIN(THETA)*COS(A(2))*COS(A(1))+COS(THETA)*COS(A
     1(2))*SIN(A(1))+COS(LL)*SIN(THETA)*SIN(A(2)))
     2+XLS*SIN(THETA)
      Y(3)=RBO*(COS(LL)*COS(A(2))*COS(A(1))+SIN(LL)*SIN(A(2)))
     2+ZLS
      Y(4)=VBO*(-SIN(LL)*COS(THETA)*COS(X(2))*COS(X(1))-SIN(THETA)*
     1COS(X(2))*SIN(X(1))+COS(LL)*COS(THETA)*SIN(X(2)))-VO*SIN(THETA)
      Y(5)=VBO*(-SIN(LL)*SIN(THETA)*COS(X(2))*COS(X(1))+COS(THETA)*
     1COS(X(2))*SIN(X(1))+COS(LL)*SIN(THETA)*SIN(X(2)))+COS(THETA)*VO
      Y(6)=VBO*(COS(LL)*COS(X(2))*COS(X(1))+SIN(LL)*SIN(X(2)))
      T=0.
```

```
C   PROGRAM 5 (CONTINUED)

      DO 20 I=1,6
C   THE ENTIRE NOMINAL TRAJECTORY IS STORED IN THE ARRAY TRJCTRY
 20   TRJCTRY(1,I)=Y(I)
      TRJCTRY(1,7)=T
 1    DT=X(3)*806.8136/FLOAT(NIS)
C   INTEGRATION STEP DT IS IN SECONDS
      DU=0.
      DUM=ABS(DT)
      INATM=.FALSE.
C   INTEGRATE EQUATIONS OF MOTION USING LIBRARY ROUTINE OF 4 TH ORDER
C   RUNGE KUTTA ALGORITHM
      CALL SET(6,T,Y,DT,DFEQ,DU ,.TRUE.,DUM,DUM)
      DO 10J=1,NIS
      CALL STEP(6,T,Y,DT,DFEQ,DU ,.TRUE.,DUM,DUM)
      DO 25 I=1,6
 25   TRJCTRY(J+1,I)=Y(I)
      TRJCTRY(J+1,7)=T
C   HAS THE VEHICLE ENTERED THE ATMOSPHERE?
      IF(INATM.AND.(T.GT.X(3)*806.8136/3.))GOTO 2
 10   CONTINUE
      GOTO 40
 2    NISS=2*NIS
C   INTEGRATE EQUATIONS IN ATMOSPHERE WITH SMALLER STEP SIZE
      TATM=T
      INATM=.TRUE.
      DT=(X(3)*806.8136-T)/FLOAT(NISS)
      DUM=ABS(DT)
      CALL SET(6,T,Y,DT,DFEQ,DU ,.TRUE.,DUM,DUM)
      DO 3 J1=1,NISS
      CALL STEP(6,T,Y,DT,DFEQ,DU ,.TRUE.,DUM,DUM)
      DO 30 I=1,6
 30   TRJCTRY(J+J1,I)=Y(I)
```

```
C    PROGRAM 5 (CONTINUED)

      TRJCTRY(J+J1,7)=T
 3    CONTINUE
      JPOINTS=J+J1-1
C    NOW FIND PSEUDO TARGET LOCATION AT FINAL TIME
 4    CALL LOCTGT(X(3),XT,YT,ZT)
      F(1)=XT-Y(1)
      F(2)=YT-Y(2)
      F(3)=ZT-Y(3)
C    F(1)-F(3) ARE THE THREE DIMENSIONAL HIT EQUATIONS
      F(4)=X(4)
      F(5)=X(5)
      F(6)=X(6)
      F(7)=X(7)
      GOTO 50
 40   CONTINUE
      GOTO 4
 50   RETURN
 100  CONTINUE
C    THIS PART OF CALFUN IS USED IN THE CALCULATION OF THE MODIFIED ORBIT
      NIS=200
      MODIFY=.TRUE.
      IPDERIV=0
C    IPDERIV IS AN INDEX WHICH DETERMINES THE PERTURBED STATE FOR
C    FINDING PARTIAL DERIVATIVES
      SIGNIF=2500.
C    SIGNIF IS THE MAXIMUM FIRST DIFFERENCE IN FEET USED FOR THE PARTIALS
      DO 105 I=1,7
C    CONVERT SCALED GUESSES TO ENGLISH UNITS
 105  X(I)=X(I)*(10.**SCALEX(I))
      DO 110 I=1,6
C    INITIAL CONDITIONS ARE THE STATES AT MODIFICATION IN THE NOMINAL ORBIT
 110  Y(I)=TRJCTRY(IPOINT,I)
```

```
C   PROGRAM 5 (CONTINUED)

      STARTIM=TRJCTRY(IPOINT,7)
C   CHANGE INITIAL CONDITION VELOCITIES (IMPULSIVE ASSUMPTION)
      Y(4)=Y(4)+X(1)
      Y(5)=Y(5)+X(2)
      Y(6)=Y(6)+X(3)
      DT=X(4)/FLOAT(NIS)
      DU=0.
      DUM=ABS(DT)
      T=0.
C   INTEGRATE EQUATIONS OF MOTION FOREWARD TO FINAL TIME
      CALL SET(6,T,Y,DT,DFEQ,DU ,.TRUE.,DUM,DUM)
      DO 120 J=1,NIS
      CALL STEP(6,T,Y,DT,DFEQ,DU ,.TRUE.,DUM,DUM)
C   HAS THE VEHICLE ENTERED THE ATMOSPHERE?
      IF(INATM)GOTO 140
 120  CONTINUE
 130  CONTINUE
      GOTO 150
 140  NISS=2.*NIS
C   INTEGRATE EQUATIONS IN ATMOSPHERE WITH SMALLER STEP SIZE
      DT=(X(4)-T)/FLOAT(NISS)
      DUM=ABS(DT)
      TATM=T
      CALL SET(6,T,Y,DT,DFEQ,DU ,.TRUE.,DUM,DUM)
      DO 145 J1=1,NISS
      J3=J1+1
      CALL STEP(6,T,Y,DT,DFEQ,DU ,.TRUE.,DUM,DUM)
 145  CONTINUE
      JPOINTS=J+J1-1
 150  TFINAL=T/806.8136
C   FIND THE LOCATION OF THE REAL TARGET AT FINAL TIME
      CALL LOCTGT1(TFINAL,XT,YT,ZT)
```

```
C   PROGRAM 5 (CONTINUED)

      DO 160 I=1,3
C   STORE UNPERTURBED FINAL TARGET LOCATION
 160  FINALST(I)=Y(I)
      TGTFNLS(1)=XT
      TGTFNLS(2)=YT
      TGTFNLS(3)=ZT
      IPDERIV=1
 165  CONTINUE
C   NOW PERTURB INITIAL STATES TO FIND PARTIAL DERIVATIVES
      IF(IPDERIV.LT.4)DEL=.5
      IF(IPDERIV.EQ.4)DEL=.3
 167  CONTINUE
      DO 170 I=1,6
 170  Y(I)=TRJCTRY(IPOINT,I)
      STARTIM=TRJCTRY(IPOINT,7)
      IPD=IPDERIV+3
      IF(IPDERIV.LT.4)Y(IPD)=Y(IPD)+DEL
      DO 180 I=4,6
      IPD1=I-3
 180  Y(I)=Y(I)+X(IPD1)
      DT=X(4)/FLOAT(NIS)
      IF(IPDERIV.EQ.4)DT=(X(4)+DEL)/FLOAT(NIS)
      DU=0.
      DUM=ABS(DT)
      T=0.
C   INTEGRATE EQUATIONS OF MOTION FOREWARD TO FINAL TIME
      CALL SET (6,T,Y,DT,DFEQ,DU,.TRUE.,DUM,DUM)
      DO 190 J=1,NIS
      CALL STEP(6,T,Y,DT,DFEQ,DU,.TRUE.,DUM,DUM)
C   HAS THE VEHICLE ENTERED THE ATMOSPHERE?
      IF(INATM)GOTO 200
 190  CONTINUE
```

```
C   PROGRAM 5 (CONTINUED)

      GOTO 220
 200  NISS=2*NIS
      DT=(X(4)-T)/FLOAT(NISS)
      IF(IPDERIV.EQ.4)DT=(X(4)+DEL-T)/FLOAT(NISS)
      DUM=ABS(DT)
C   INTEGRATE EQUATIONS IN ATMOSPHERE WITH SMALLER STEP SIZE
      CALL SET (6,T,Y,DT,DFEQ,DU,.TRUE.,DUM,DUM)
      DO 210 J1=1,NISS
 210  CALL STEP(6,T,Y,DT,DFEQ,DU,.TRUE.,DUM,DUM)
 220  TFINAL=T/806.8136
C   FIND PERTURBED FINAL LOCATION OF REAL TARGET
      CALL LOCTGT1(TFINAL,XT,YT,ZT)
      IF(IPDERIV.GT.3)GOTO 260
C   CALCULATE FIRST DIFFERENCES
      PXF(IPDERIV)=Y(1)-FINALST(1)
      PYF(IPDERIV)=Y(2)-FINALST(2)
      PZF(IPDERIV)=Y(3)-FINALST(3)
      T2=ABS(PXF(IPDERIV))
      T3=ABS(PYF(IPDERIV))
      T4=ABS(PZF(IPDERIV))
      VALMAX=AMAX1(T2,T3,T4)
C   IS THE MAXIMUM FIRST DIFFERENCE LT. SPECIFIED?
      IF(VALMAX.LT.SIGNIF)GOTO 230
C   DECREASE INITIAL PERTURBATION
      DEL=.5*DEL
      PRINT*,"DEL= ",DEL
      GOTO 167
 230  PXF(IPDERIV)=PXF(IPDERIV)/(DEL*TU)
      PYF(IPDERIV)=PYF(IPDERIV)/(DEL*TU)
      PZF(IPDERIV)=PZF(IPDERIV)/(DEL*TU)
      IPDERIV=IPDERIV+1
      GOTO 165
```

```
C   PROGRAM 5 (CONTINUED)

C   CALCULATE FIRST DIFFERENCES
 260  PXF(4)=Y(1)-FINALST(1)
      PYF(4)=Y(2)-FINALST(2)
      PZF(4)=Y(3)-FINALST(3)
      POFTF(1)=XT-TGTFNLS(1)
      POFTF(2)=YT-TGTFNLS(2)
      POFTF(3)=ZT-TGTFNLS(3)
      T2=ABS(PXF(4))
      T3=ABS(PYF(4))
      T4=ABS(PZF(4))
      T5=ABS(POFTF(1))
      T6=ABS(POFTF(2))
      T7=ABS(POFTF(3))
      VALMAX=AMAX1(T2,T3,T4,T5,T6,T7)
C   IS THE MAXIMUM FIRST DIFFERENCE LT. SPECIFIED?
      IF(VALMAX.LT.SIGNIF)GOTO 270
      PRINT*,"FOR TIME DEL= ",DEL
C   DECREASE INITIAL PERTURBATION
      DEL=.5*DEL
      GOTO 167
 270  PXF(4)=PXF(4)/(DEL*DUTU)
      PYF(4)=PYF(4)/(DEL*DUTU)
      PZF(4)=PZF(4)/(DEL*DUTU)
      POFTF(1)=POFTF(1)/(DEL*DUTU)
      POFTF(2)=POFTF(2)/(DEL*DUTU)
      POFTF(3)=PO[illegible]3)/(DEL*DUTU)
C   CONVERT FIRST ORDER NECESSARY CONDITIONS TO EARTH CANONICAL UNITS
      XC(1)=X(1)/DUTU
      XC(2)=X(2)/DUTU
      XC(3)=X(3)/DUTU
      XC(4)=X(4)/TU
      F(1)=XC(1)+X(5)*PXF(1)+X(6)*PYF(1)+X(7)*PZF(1)
```

```
C    PROGRAM 5 (CONTINUED)

      F(2)=XC(2)+X(5)*PXF(2)+X(6)*PYF(2)+X(7)*PZF(2)
      F(3)=XC(3)+X(5)*PXF(3)+X(6)*PYF(3)+X(7)*PZF(3)
      F(4)=XC(4)*W1+X(5)*(PXF(4)-POFTF(1))+X(6)*(PYF(4)-POFTF(2))+X(7)*
     1(PZF(4)-POFTF(3))
      F(5)=FINALST(1)-TGTFNLS(1)
      F(6)=FINALST(2)-TGTFNLS(2)
      F(7)=FINALST(3)-TGTFNLS(3)
      F(5)=F(5)/AE
      F(6)=F(6)/AE
      F(7)=F(7)/AE
C    THE FOLLOWING ARE USED FOR SCALING PURPOSES ON THE EXAMPLE POINT
      F(5)=F(5)*1.E4
      F(6)=F(6)*1.E4
      F(7)=F(7)*1.E4
      DO 280 I=1,7
C    CONVERT GUESSES BACK TO SCALED VALUES
 280  X(I)=X(I)/(10.**SCALEX(I))
      END
```

```
C    PROGRAM 5 (CONTINUED)

       SUBROUTINE LOCTGT(T,XT,YT,ZT)
       COMMON/II/SL(8),ST(8),ST1(8)
       COMMON/III/P,DT,D,TBO
       REAL LATTGT
C    THIS SUBROUTINE CALCULATES INERTIAL POSITION OF THE PSEUDO TARGET
C    GIVEN A TIME OF FREE FLIGHT IN TU
       PI=ACOS(-1.)
       WEARTH=7.2921152E-05
       DELLONG=ST(2)-SL(2)
C    DELLONG IS THE LONGITUDE DIFFERENCE MEASURED EASTERLY FROM THE
C    LAUNCH SITE TO TARGET
 1     CONTINUE
       IF(DELLONG.LT.0.)GOTO 5
       GOTO 10
 5     DELLONG=2.*PI+DELLONG
       GOTO 1
 10    LATTGT=ST(1)
C    LATTGT IS THE TARGET LATITUDE
       THETA=WEARTH*(T*806.8136+TBO)+DELLONG
       AE=2.092567257E7
       E=.08181
C    THESE COMPENSATE FOR EARTH OBLATENESS
       X=AE*COS(LATTGT)/(SQRT(1.-(E**2)*(SIN(LATTGT)**2)))
       ZT=AE*(1.-E**2)*SIN(LATTGT)/(SQRT(1.-(E**2)*(SIN(LATTGT)**2)))
       XT=X*COS(THETA)
       YT=X*SIN(THETA)
       END
```

```
C    PROGRAM 5 (CONTINUED)

      SUBROUTINE LOCTGT1(T,XT,YT,ZT)
      COMMON/II/SL(8),ST1(8),ST(8)
      COMMON/III/P,DT,D,T80
      COMMON/XII/STARTIM
      REAL LATTGT
C    THIS SUBROUTINE CALCULATES INERTIAL POSITION OF THE REAL TARGET
C    GIVEN A REACTION TIME IN TU
      STARTIM=STARTIM/806.8136
C    ADD IN TIME OF FLIGHT ON NOMINAL TRAJECTORY
      T=T+STARTIM
      PI=ACOS(-1.)
      WEARTH=7.2921152E-05
      DELLONG=ST(2)-SL(2)
C    DELLONG IS THE LONGITUDE DIFFERENCE MEASURED EASTERLY FROM THE
C    LAUNCH SITE TO TARGET
 1    CONTINUE
      IF(DELLONG.LT.0.)GOTO 5
      GOTO 10
 5    DELLONG=2.*PI+DELLONG
      GOTO 1
 10   LATTGT=ST(1)
C    LATTGT IS THE TARGET LATITUDE
      THETA=WEARTH*(T*806.8136+T80)+DELLONG
      AE=2.09256725757
      E=.08181
C    THESE COMPENSATE FOR EARTH OBLATENESS
      X=AE*COS(LATTGT)/(SQRT(1.-(E**2)*(SIN(LATTGT)**2)))
      ZT=AE*(1.-E**2)*SIN(LATTGT)/(SQRT(1.-(E**2)*(SIN(LATTGT)**2)))
C   THESE COMPENSATE FOR EARTH OBLATENESS
      XT=X*COS(THETA)
      YT=X*SIN(THETA)
      END
```

```
C     PROGRAM 5 (CONTINUED)

      SUBROUTINE DFEQ(N,X,Y,DY)
C     THIS SUBROUTINE GIVES THE EQUATIONS OF MOTION IN ENGLISH UNITS
C     IT INCLUDES ATMOSPHERE, DRAG, AND GRAVITY MODELS
      LOGICAL INATM,MODIFY
      COMMON/II/SL(8),ST(8),ST1(8)
      COMMON/VI/TATM,INATM,JPOINTS,MODIFY
      DIMENSION Y(6),DY(6)
      REAL MU, J2,J3,J4,J5
      MU=1.407654E+16
      J2=1082.64E-06 $J3=-2.5E-06 $J4=-1.6E-06  $J5=-.15E-06
      BC=28.66 $RHOO=2.3787E-03
C     BC IS THE BALLISTIC COEFFICIENT IN ENGLISH UNITS
      R=SQRT(Y(1)**2+Y(2)**2+Y(3)**2)
C     R IS THE RADIUS VECTOR MAGNITUDE
      V=SQRT(Y(4)**2+Y(6)**2+Y(5)**2)
C     V IS THE VELOCITY VECTOR MAGNITUDE
      H=R-ST(3)
C     H IS THE ALTITUDE
      IF(MODIFY)H=R-ST1(3)
      IF(H.LE.0.)H=0.
      WEARTH=7.2921152E-05
      AE=2.092567257E7
      DY(1)=Y(4)
      DY(2)=Y(5)
      DY(3)=Y(6)
      INATM=.FALSE.
      IF(H/23999.3.LE.15.)INATM=.TRUE.
C     DRAG IS IGNORED IF ABOVE 110 KM
      IF(INATM)10,1
 10   DRAG1=.5*(1/BC)*RHOO*EXP(-H/23999.3)*(Y(4)+WEARTH*Y(2))*V
      DRAG2=.5*(1/BC)*RHOO*EXP(-H/23999.3)*(Y(5)-WEARTH*Y(1))*V
      DRAG3=.5*(1/BC)*RHOO*EXP(-H/23999.3)*(Y(6)*V)
```

```
C    PROGRAM 5 (CONTINUED)

C    DRAG (I) IS THE VALUE OF ATMOSPHERIC DECELERATION IN THE I DIRECTION
      DY(4)=-MU*Y(1)/R**3*(1.-J2*1.5*(AE/R)**2*(5.*Y(3)**3/R**3-1.)+J3*2
     C.5*(AE/R)**3*(3.*Y(3)/R-7.*Y(3)**3/R**3)-J4*5./8.*(AE/R)**4*(3.-42
     C.*(Y(3)/R)**2+63.*(Y(3)/R)**4)-J5*3./8.*(AE/R)**5*(35.*Y(3)/R-210.
     C*(Y(3)/R)**3+231.*(Y(3)/R)**5))
     1-DRAG1
      DY(5)=Y(2)/Y(1)*DY(4)
     1-DRAG2
      DY(6)=-MU*Y(3)/R**3*(1.+J2*1.5*(AE/R)**3*(3.-5.*(Y(3)/R)**2)+J3*1.
     C5*(AE/R)**3*(10.*Y(3)/R-35./3.*(Y(3)/R)**3-R/Y(3))-J4*5./8.*(AE/R)
     C**4*(15.-70.*(Y(3)/R)**2+63.*(Y(3)/R)**4)-J5/8.*(AE/R)**5*(315.*Y(
     C3)/R-945.*(Y(3)/R)**3+693.*(Y(3)/R)**5-15.*R/Y(3)))
     1-DRAG3
      RETURN
 1    DY(4)= MU*Y(1)/R**3*(1.-J2*1.5*(AE/R)**2*(5.*Y(3)**3/R**3-1.)+J3*2
     C.5*(AE/R)**3*(3.*Y(3)/R-7.*Y(3)**3/R**3)-J4*5./8.*(AE/R)**4*(3.-42
     C.*(Y(3)/R)**2+63.*(Y(3)/R)**4)-J5*3./8.*(AE/R)**5*(35.*Y(3)/R-210.
     C*(Y(3)/R)**3+231.*(Y(3)/R)**5))
      DY(5)=Y(2)/Y(1)*DY(4)
      DY(6)=-MU*Y(3)/R**3*(1.+J2*1.5*(AE/R)**3*(3.-5.*(Y(3)/R)**2)+J3*1.
     C5*(AE/R)**3*(10.*Y(3)/R-35./3.*(Y(3)/R)**3-R/Y(3))-J4*5./8.*(AE/R)
     C**4*(15.-70.*(Y(3)/R)**2+63.*(Y(3)/R)**4)-J5/8.*(AE/R)**5*(315.*Y(
     C3)/R-945.*(Y(3)/R)**3+693.*(Y(3)/R)**5-15.*R/Y(3)))
      END
```

C PROGRAM 5 (CONTINUED)

DATA INPUT FOR EXAMPLE POINT

```
37.5,-125.
54.,-3.3
55.5,5.4
19.3562,49.2389,3971.85,0.,0.,0.,0.
-446.32764,-1121.8092,342.86641,2055.8022,.14335176,.80056095,.014013988
98
.15
```

Vita

Matthew P. Gillis III was born on 10 July 1950 in Pittston, Pennsylvania. He graduated from high school in Dallas, Pennsylvania in 1968 and attended the Pennsylvania State University from which he received a Bachelor of Science degree in Aerospace Engineering. Upon graduation in 1972, he was commissioned as a Second Lieutenant through the ROTC program. After completion of missile maintenance officer training at Sheppard AFB, Texas, he served as Sector Maintenance Officer, 381st Missile Maintenance Squadron and then as Chief, Maintenance Training Control Division, 381st Strategic Missile Wing at McConnell AFB, Kansas. He entered the Graduate Astronautics program at the Air Force Institute of Technology in June 1974.

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