PRESSURE RECOVERY AND RELATED PROPERTIES IN
SUPersonic DIFFUSERS: A REVIEW

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Yale University

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IN
SUPERSONIC DIFFUSERS:

A REVIEW

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DEPARTMENT OF ENGINEERING
AND APPLIED SCIENCE

YALE UNIVERSITY
This report covers a literature survey on the status of present knowledge on supersonic diffusers. Three types of diffusers are examined: wind tunnel diffusers, ramjet inlets, and supersonic ejectors. No systematic study on wind tunnel diffusers has been found. The results obtained in specific studies, made in conjunction with the design and installation of a number of wind tunnels, show that 100% of normal shock recovery is obtained for M<10. At higher test section Mach numbers, the pressure recovery...
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ABSTRACT

This report covers a literature survey on the status of present knowledge on supersonic diffusers. Three types of diffusers are examined: wind tunnel diffusers, ramjet inlets, and supersonic ejectors. No systematic study on wind tunnel diffusers has been found. The results obtained in specific studies, made in conjunction with the design and installation of a number of wind tunnels, show that 100% of normal shock recovery is obtained for M<10. At higher test section Mach numbers, the pressure recovery becomes much poorer than the normal shock value. Some systematic work exists for ramjet diffusers, with most of the attention being paid to external compression configurations. Boundary layer effects, including methods for its control, have been studied to some extent. For supersonic ejectors, comprehensive studies have been made, and useful empirical design methods are available. We find that nonequilibrium flows in diffusers or ejectors have not been studied. Since the flows in the test section of hypersonic wind tunnels, and in the cavity of gasdynamic laser systems, are known to be significantly out of equilibrium, knowledge of nonequilibrium effects in diffusers is particularly important in these situations. Therefore, a systematic study on nonequilibrium flows in supersonic diffusers should be made. Finally, a bibliography with over ninety entries and a summary of the principal features of idealized flow in diffusers are included in the appendices.
ACKNOWLEDGEMENT

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The importance of supersonic diffusers to a wide variety of flow processes in research, development, flight and applied science is well known. Efficient pressure recovery is a dominant consideration in establishing the design and power requirements of wind tunnels. Rotating flow machinery may encounter supersonic flow fields with the requirement of providing efficient supersonic diffusers. Flying articles with supersonic inlets require high and well controlled pressure recovery for stable operation and reliable performance. The duration and strength of the high intensity output from gas dynamic lasers are critically sensitive to predictable and high pressure recovery associated with high mass flow rate.

Because of this importance and because it has seemed that fundamental questions remain unanswered, we have undertaken a survey of the current literature on the status of present knowledge on supersonic diffusers. To this end, we have sought to obtain some relatively unavailable industrial, governmental, and private laboratory reports as well as papers published in the usual archival journals. To assist this undertaking a formal library-computer search has been made of appropriate titles. In addition, direct contacts, correspondence, and other private communications between ourselves and other workers in the field have been included in our efforts as we attempt to achieve completeness. Altogether, over 160 reports and abstracts were studied, of these about ninety were considered relevant to the subject being reviewed and listed in the Bibliography.

This report summarizes our results to date. We have performed no new experimental or theoretical work; we can
only evaluate and analyse the work of others at this time. However, our efforts in this regard are felt to be a necessary and timely prerequisite to much needed systematic improvement in the state-of-the-art.

II. REVIEW OF RESEARCH ON DIFFUSERS

A. Wind Tunnels

Diffusers for supersonic and hypersonic wind tunnels have been studied at a number of wind tunnel installations.* These studies were usually made in order to determine the overall pressure ratio required for the operation of a particular wind tunnel configuration. This information is, in turn, used to estimate the power requirements for the wind tunnel. Thus, virtually all the wind tunnel diffuser studies known to us are design-oriented and they are optimized for a single facility. Most of the diffusers were of standard configuration, with a converging entrance, a constant area throat section, and a diverging exhaust section. The converging and diverging sections were simply frustrums of a cone, pyramid, or a wedge. The lengths of each section, the throat area and the entrance angle were varied and an optimum combination of these, giving the most favorable operating condition, were noted. The studies tend to be very specific; usually only the diffuser entrance and exit pressures have been measured. The Reynolds number and Mach number effects in these flows were not systematically investigated.

* In Appendix I, we summarize the principal features of idealized wind tunnel diffuser flow.
In 1953, Lukasiewicz made an extensive study of the test results of a number of supersonic wind tunnel diffusers then in existence. They included the 18 cm x 18 cm (7.1" x 7.1") tunnel (Diggins 1951)* and the 12 cm x 12 cm (4.7" x 4.7") tunnel (Wegener and Lobb 1953) of the Naval Ordnance Laboratory, the 2" x 2.5" tunnel of Cal Tech (Heppe 1947), the 11" hypersonic tunnel at NASA - Langley (Bertram 1950), and the 1.3" x 1.3" tunnel of MIT (Neumann & Lustwerk 1951). From these data, Lukasiewicz concluded that for test section Mach numbers of 1 < M < 10, the static pressures achievable at the exit of fixed-geometry diffusers where M=0 were close to those computed from normal shock recovery theory. With variable-geometry diffusers, a maximum pressure recovery of almost twice the normal shock recovery could be attained. The test-section Reynolds numbers based on nozzle exit height ranged from 2 x 10^5 to 3 x 10^6 in these wind tunnels. All of the test sections had closed-jets.

A number of studies have been made on wind tunnel diffusers since the time of Lukasiewicz' summary. The Mach number range has been extended to twenty-six, and low Reynolds number data have become available. In addition, some results have been obtained for the monatomic gases argon and helium. The optimum diffuser pressure recoveries, defined as the ratio of nozzle supply pressure to diffuser exit pressure, \( p_{to}/p_{t2} \), realized in these wind tunnels are compared to the theoretical normal shock recovery in Figs.1 and 2, for \( \gamma = 1.4 \) and 5/3 respectively. Unless otherwise

* Complete reference to papers cited may be found in Appendix II, the Bibliography. This includes all relevant diffuser literature known to us, even though not cited in the text.
stated, the results were obtained in "clean" wind tunnels (i.e., without models). The results compiled by Lukasiewicz (1953) are also plotted in Fig.1. It is clear from Figure 1 that the conclusion reached by Lukasiewicz on pressure recovery holds for $M < 10$. At higher Mach numbers, the recovery is significantly poorer than the normal shock recovery. However, since the Reynolds numbers in these high Mach number tunnels are lower than those for $M < 10$, it is not clear that this deterioration in pressure recovery is due exclusively to higher Mach numbers.

The results reported by Hastings (1954), (1955), by Midden and Cocke (1964), and by Austin (1966) all fall in the Mach number range of $1 < M < 10$. The test section Reynolds numbers in these wind tunnels ranged from $2 \times 10^5$ (Austin 1966) to $8 \times 10^6$ (Hastings 1955), i.e. they varied by a factor of forty. These conditions are roughly the same as those of Wegener and Lobb (1953), Heppe (1947) and Neumann and Lustwerk (1949, 1951). It is not surprising that the results of these new studies agree with the normal shock recovery theory which was found in the older studies. The diffusers used by Hastings (1954,55) are of the two-dimensional, adjustable wall type, similar to that described by Wegener and Lobb (1953). Hastings (1954) has moreover studied the effects of suction applied to the wind tunnel test chamber, to remove about 10% of the test gas before entering the diffuser. The limits of pressure recovery achieved with and without suction are indicated by the length of the bar in Fig.1. The application of suction improved the diffuser pressure recovery by as much as a factor of two at $M = 5$. It should be remembered that this improvement in recovery is obtained at the expense of additional pumping
power. Thus, the usual economic benefits associated with improved pressure recovery are compromised in this case. In addition to pressures at the nozzle and diffuser exits, Hastings (1954) also made continuous static pressure surveys along the diffuser sidewall and took some schlieren photographs. These provide useful information for an understanding of the flow and shock patterns in diffuser flows. He found that the larger amount of static pressure rise was accomplished in the supersonic part of the diffuser. Not shown on Fig. 1 is another study by Hastings and Roberts (1957) using the same 18 cm x 18 cm wind tunnel where they found that nearly 100% of normal shock recovery was obtained at $M = 2.86$ and $4.92$ with atmospheric supply conditions. The wind tunnels used by Austin (1966) and Midden and Cocke (1964) are both axisymmetric open-jet systems with fixed geometry diffusers. Midden and Cocke found that 10% blocking of test sectional area (11" diameter) by models may be tolerated in their wind tunnel.

Clark (1965), using the Langley 12" hypersonic "ceramic heated tunnel", measured the pressure recovered in the fixed geometry, axisymmetric diffuser. The Mach number in the free jet test section is about 13.6, and the Reynolds number based on the nozzle exit diameter ranged from $2 \times 10^4$ to $1.8 \times 10^5$ depending on the nozzle supply pressure. This nine-fold variation in Reynolds number is the widest range covered in a single study. Effects of diffuser blocking (not shown in Fig. 1) were also studied with various models by Clark. Diffuser performance at much lower Reynolds numbers ($Re \sim 10^3$) was investigated by Boylan (1964). Here, diffusers of the same basic design, a constant area duct with cones fitted at both ends, were used. The lengths and cross-sectional
areas of the constant area and conical sections were systematically varied. Boylan's (1964) setup permits the free jet length to be varied from zero (i.e., closed jet configuration) to fourteen nozzle exit diameters. He found that optimum results were obtained with a free jet length of roughly six nozzle exit diameters. Argon was also used by Boylan as the test gas, and results for argon are plotted in Fig.2.

A model blockage study of a 4" diameter Mach 10 to 14 free jet wind tunnel was made at Ohio State University by Scaggs and Petrie (1961). Diffuser performance with models (1/8" D-5/8"D) installed in the test section was investigated. At Mach numbers of 10 and 14.3, pressure recoveries of 44% to 60% of the normal shock value were obtained, depending on the size of the model. The diffuser efficiencies reported by Thomas, Lee and Von Eschen (1957, 1959) of the Ohio State University 8" hypersonic tunnel and by Scaggs, Burggraf, and Gregorek (1963) of the ARL-30" tunnels are roughly 100% of the normal shock recovery, as quoted by Clark (1965). The highest Mach numbers studied were in the range 22 < M < 26 at Princeton University (Vas and Koppenwallner 1964, Vas and Allegre 1966). The Reynolds numbers in both tunnels are about 3 x 10^4. The static pressure recovery (from 5% of normal shock value at M = 17.7 to 46% at M = 25.7) was rather poor compared with other wind tunnels. The hypersonic nitrogen tunnel at the Aerodynamic Research Facility at Göttingen described by Koppenwallner (1966) is of practically the same design as the Princeton tunnel (Vas and Koppenwallner 1964). With a nozzle exit diameter of 25 cm, the Reynolds number in the test section is 3 x 10^4 < Re_D < 4.7 x 10^4. The pressure recovery
achieved in the diffuser in this tunnel is better than that obtained by Vass and Koppenwallner. Still, only about 50% of the normal shock recovery was attained in the Göttingen tunnel.

It is not clear at this time why the pressure recovery is so much poorer than the normal shock recovery for these hypersonic wind tunnels. Both viscous and nonequilibrium effects may be responsible for this deterioration in diffuser efficiency. Not only is the Reynolds number in these tunnels much lower than the tunnels at moderate Mach numbers, but also the high Mach number flow in the test section is known to be significantly out of equilibrium.

B. Ramjets

The ramjet diffuser serves to decelerate air entrained at supersonic speeds down to speeds appropriate to the static pressure requirements for internal combustion. Faro and Keirsey (1967) have reviewed ramjet diffuser performance parameters as a part of a series of reports in ramjet technology. We shall use and follow their treatment of ramjet diffusers and summarize the principal results contained in their paper. Although Faro and Keirsey (1967) restricted their attention to axisymmetric configurations, the principal design considerations regarding external-compression limitations, compression surface design and inlet design are fundamental to all types. For both internal and external diffusion, the three basic characteristics determining the diffusing effectiveness were held to be total pressure recovery (the ratio of total pressure in the free stream to total pressure at the diffuser's exit, as previously defined), the capture-area (or mass flow) ratio, and the total drag of the diffuser.
The idealized ramjet with internal compression is discussed in Appendix I. The fixed throat, single-shock diffuser has as advantages its low external drag and simplicity of construction. However, the area contraction ratio required during starting imposes low pressure recovery and the overall performance deteriorates rapidly at off-design Mach numbers once the shock is swallowed. Further, the length of the convergent section required for the most satisfactory results (typically, a throat length four times the throat diameter) gives rise to thick throat boundary layers. Thus, this configuration, which has had a more successful application to wind tunnels than to ramjets, is usually limited to flight Mach numbers less than about 1.6. On the other hand, variable geometry devices alleviate many of the adverse effects associated with the fixed throat area configuration at a cost of increased mechanical complexity. Flexible plates in two-dimensional devices, including ones with boundary layer bleed, and translating centerbodies in axisymmetric diffusers give consistent improvement in pressure recovery over the fixed inlets as shown in Figure 3.

By externally compressing the flow through oblique shocks, greater flexibility can be achieved in the ramjet's operation. As discussed in Appendix I, the recovered stagnation pressure increases, neglecting boundary layer effects, as the number of oblique shocks increases; the limit, of course, is achieved when the compression is accomplished through an infinite number of very weak waves, a procedure possible in principle with an isentropic shock-free surface. The primary design variables for oblique-shock diffusers are found to be:
1. The shape of the external compressing surface
2. The position of the cowl lip with respect to the innerbody tip
3. The position and shape of the “shoulder” of the innerbody with respect to the cowl lip
4. The geometry of the cowl lip
5. The geometry of the diffuser-centerbody configuration downstream of the inlet.

These parameters are adjusted so as to control boundary-layer development and to give stable, unseparated flow in the duct as well as minimum external drag. Although practical considerations (such as inlet length, viscous effects, the limit of external compression dictated by a consistent shock solution at the point of coalescence, and reasonable cowl lip drag) preclude the occurrence of a completely isentropic compression, several practical inlets have nonetheless been developed based on the principle of isentropic compression. Figure 4 shows maximum theoretical pressure recovery for the basic diffuser types; in each case it is first assumed that there is only a simple normal shock at the lip (external compression) and secondly that the maximum internal contraction allowed by the entering Mach number is followed by a simple normal shock wave at the associated throat.

Boundary layer problems, even in the absence of flow separation, can profoundly affect air capture pressure recovery and inlet drag. Beneficial results have been obtained, insofar as boundary layers on the compression surface are concerned, through the use of surface roughness or trip rings to assure transition to turbulence and through
suction. Bleeding off the boundary layer immediately down-
stream of the sharp turn has been found effective in reduc-
ing problems associated with boundary layers at the throat. In addi-
tion, pressure recovery losses associated with bound-
ary layer-shock wave interactions in the throat have been 
alleviated by elongating the throat and by using "vortex 
traps"; in both cases, improved pressure recovery is 
achieved at the expense of other previously well-controlled 
parameters. Diffuser "buzz" is another profoundly important 
effect. In this case, a shock wave oscillating at the dif-
fuser's entrance produces fluctuating internal pressures 
which cause, under extreme cases, a cycle of flame-out and 
re-ignition and at the very least a heavy penalty in gross 
thrust in the ramjet's performance. Since the buzz theories 
and the one-dimensional flow analysis are not sufficiently 
developed to predict the performance and stability limits 
accurately, results from wind tunnel and free jet tests are 
used in matching the inlet and combustor characteristics in 
attempts to ensure stable and efficient behavior.

C. Ejectors

Ejector-diffuser configurations are important to studies 
on rocket and jet engine design. Figure 5 shows flow patterns 
in a typical supersonic ejector after it has been properly 
started. Here, as in the other diffusers just reported, 
optimum stable operating condition is represented by a shock 
wave in the diverging section of the diffuser. Many addi-
tional complications arise in the case of ejector-diffusers, 
not the least of which is the extent to which the variations 
in exit pressure over a relatively short range of values can 
dramatically alter the nature of the flow in the test section 
as well as the flow in the air settling chamber so critical
to effective aerodynamic simulation. In addition, the ejector performance characteristics are very sensitive to friction influences explicitly and implicitly in their sensitivity to diffuser length and to stagnation temperatures.

Ginoux (1972) has edited a comprehensive summary of the status of research and development on supersonic ejectors. This summary includes articles by Uebelhack, Taylor, Addy, and Peters with nearly 250 references to both American and European literature on the subjects reported. Topics treated are (1) a one-dimensional inviscid analysis of supersonic ejectors, (2) an analysis and design method for ejector systems with second throat diffusers, (3) the analysis of supersonic ejector systems, (4) ejector design for a variety of applications, and (5) an analysis of ducted mixing and burning of coaxial streams.

Taylor's article (in Ginoux 1972) paid particular attention to the various empirical ejector design methods developed from experimental results which are used for practical applications; comparisons with one-dimensional theory are made where possible. Ejector-diffuser inlet geometry effects were found to be significant; the use of truncated conical inlets in the cylindrical diffusers produced as much as a 600-percent improvement in diffuser performance. Although the compression shock system in a long duct is a series of lambda shocks resulting from an interaction between the boundary shock and the boundary layer on the duct wall, the one-dimensional normal shock relationship used with the duct inlet Mach number will predict the pressure rise across the shock system within approximately 5 percent. Second throat contraction ratio and length
of minimum area had the greatest influence on the starting and operating pressure ratios. The significant parameter involving nozzle total pressure level was found to be the unit Reynolds number at the nozzle exit times the nozzle throat diameter; for values of this parameter of less than one million, significant variations in the minimum cell pressure ratio occurred. In addition, Taylor studied the effects of different driving fluids on ejector-diffuser performance. Five different gases were used. The known variations in specific heat with temperature were included in a one-dimensional isentropic calculation of pressure recovery. However, observed values of pressure recovery were anomalously below the theoretical predictions. These results have emphasized the importance of real gas phenomena.

D. Conclusion

Many studies have been made for diffusers in conjunction with the design and installation of new supersonic and hypersonic wind tunnels since the time of Lukasiewicz's review in 1953. However, no systematic study has been found. The available results indicate that about 100% of the normal shock pressure recovery is obtained in almost all of the wind tunnels with Mach numbers less than ten. The pressure recovery deteriorates as the Mach number increases—to only 20% of the normal recovery for Mach number about 24. This result is not explained at this time.

There is some systematic work done for ramjet diffusers. Despite their successful application in wind tunnels, variable area or translating centerbody diffusers have been found impractical for ramjets. Most ramjet diffuser studies focused on the configuration of external compression inlets, in which pressure recovery is obtained through a series of weak
oblique shocks rather than a single normal shock with accompanying higher entropy increase. Boundary layer effects have been studied to some extent. Methods of controlling boundary layer growth and separation have been tested. Diffuser "buzz" remains to be a phenomenon insufficiently studied.

Comprehensive studies have been made for supersonic ejectors. Many useful experimental results and theoretical formulations are available in the literature. In particular, empirical design methods applicable to a wide variety of uses exist, and they have already achieved much success. Nonetheless, for problems requiring careful concern for real gas phenomena, much is yet to be done before satisfactory solutions can be obtained.

III. FUTURE WORK

From these results, there are at least three areas where more systematic research on diffuser flows is needed:

1. Reynolds number effects should be studied as follows: (a) Re<10⁵ for M<10 and (b) Re>10⁵ for M>10. This should include theoretical studies on viscous effects and diffuser stability for the wide range of diffuser configurations as well as experimental effects.

2. Nonequilibrium phenomena in diffuser flows should be studied. This should include real gas effects revealed in variations in specific heats and in condensation as well as in the usual effects due to chemically relaxing processes, (such as disocia-
tion and vibrational excitation).

3. New geometries and flow decelerating mechanisms should be studied. Grid nozzles and boundary layer suction and blowing are two examples of devices and techniques worthy of further examination. Grid diffuser should be tried.

In addition, a more comprehensive range of studies on the usual configuration would be useful. The emphases would be on those investigations which can be conducted so as to maximize the prospects for generalizations. This could include, for example, studies of the systematics of pressure recovery in straight channels so as to extend the results of Baker (1965) and studies on recovery in gases other than air and $\text{N}_2$. 
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Author(s) and year</th>
<th>Test section conditions</th>
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<tr>
<td>○</td>
<td>Heppe (1947)</td>
<td>M: 2.5-3.5, Re: 1.6x10^6, Dimension: 2&quot;x2.5&quot;</td>
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<td>Neumann &amp; Lustwerk (1949-51)</td>
<td>M: 2-3, Re: 2x10^5, Dimension: 1&quot;D &amp; 1.25&quot;x1.25&quot;</td>
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<td>Thomas &amp; Lee (1957)</td>
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<td>X</td>
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<td>M: 19, Re: 2x10^5, Dimension: 30&quot;D</td>
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<td>Scaggs, Burggraf &amp; Gregorek (1963)</td>
<td>M: 12-13 (b), Re: 1.7-2.5x10^3, Dimension: 5.8&quot;D</td>
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<td>◊</td>
<td>Boylan (1964)</td>
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<td>□</td>
<td>Midden &amp; Cocke (1964)</td>
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<td>Vas &amp; Koppenwallner (1964)</td>
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<td>Austin (1966)</td>
<td>M: 3.5 &amp; 7, Re: 2x10^5, Dimension: 8&quot;D</td>
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<td>Koppenwallner (1966)</td>
<td>M: 21-23, Re: 3-4.7x10^4, Dimension: 10&quot;D</td>
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(a) Helium test gas.
(b) Nitrogen test gas.
(c) Argon test gas.
(d) Quoted by Clark (1965)
NORMAL SHOCK RECOVERY, $\gamma = 1.4$

FIGURE 1
FIGURE 3
FIGURE 4
Diagram of a supersonic ejector and reference stations

Supersonic regime normal shock wave in the diffuser

Supersonic portion

Supersonic regime close to transition

Mixed flow regime

Mixed flow regime, supersonic jet attached to one side

Mixed flow regime with separation in primary nozzle

FIGURE 5
APPENDIX I

ELEMENTS OF DIFFUSER FLOW

The purpose of a supersonic diffuser is to (a) reduce the flow of a supersonic stream to subsonic \(0 < M < 1\) speeds while (b) retaining the highest possible total or stagnation pressure. In principle, this can always be accomplished with 100% efficiency through an isentropic compression. In this ideal situation we note that the kinetic energy of the flow is converted to pressure free of losses. In fact, numerous effects arising from the details in starting procedures, geometric design, shock waves, viscous effects, etc. conspire to provide loss mechanisms, with associated entropy increase, and a corresponding total pressure loss. Theoretical work and experiments on diffuser configurations are aimed at minimizing these losses. However, owing to the complications, no simple theory is available. In this appendix, we briefly discuss three examples of simple diffuser flow: wind tunnel flow, supersonic inlet flow, and ramjet flow. We limit ourselves to a review, with some illustrations, of the simplest elements in each case.

Let us characterize the pressure recovery as the ratio of total pressure after the diffuser, \(p_{t2}'\), to total pressure before the diffuser, \(p_{t0}'\). For isentropic compressor:

(i) \(\frac{p_{t2}'}{p_{t0}'} = \frac{p_{t0}'}{p_{t0}'} = 1\)

by definition. For normal shocks and with ideal gas flow

(ii) \(\frac{p_{t2}'}{p_{t0}'} = \left(1 + \frac{2}{\gamma + 1} \left(M_1^2 - 1\right)\right)^{1/(\gamma-1)} \left(\frac{\gamma-1}{\gamma+1}M_1^2 + 2\right)^{\gamma/(\gamma-1)}\)

where \(M_1\) is the Mach number immediately upstream from the shock wave. In this latter case, at \(M_1 = 1\), \(\frac{p_{t2}'}{p_{t0}'} = 1\) and \(\frac{p_{t2}'}{p_{t0}'}\) decreases with increasing \(M_1\) beyond \(M_1 = 1\). Equations (i) and (ii) represent the two limiting cases for an idealized diffuser. Further, deviations from idealized conditions as a result of
viscous effects, etc. will reduce the actual pressure recovery available below the values given in equation (ii) for the normal shock recovery.

In a given flow environment, the overall goal is that of minimizing the "strength" of the single normal shock wave since the major component of total pressure loss is derived from the increase in entropy across this discontinuity. The ultimate presence of some shock wave phenomena is dictated by the inherently nonisentropic nature of supersonic flow starting processes in ducts and inlets. Thus, for an initially supersonic stream at $M_{∞}$, if the flow upstream of the shock wave can be decelerated loss free to a new lower Mach number, $M_s < M_{∞}$, then as the value of $M_s$ approaches 1, the pressure recovery approaches 100%.

Converging ducts decelerate supersonic flow; hence, the converging-diverging duct with appropriately chosen throat area should improve on the recovery in equation (ii). Boundary layer evolution is known to eventually decelerate shock waves in shock tube flow; hence long constant area test section-diffuser configurations inevitably weaken the shock waves when they are formed far downstream of the supersonic nozzle. Further, deceleration across a single normal shock wave always involves a greater entropy increase than the same overall deceleration across more than one oblique shock wave. Thus, spikes generating oblique shock waves are expected to improve the recovery when they are introduced into the diffuser inlet.

With this background, the onset of supersonic flow in a simple diffuser duct, a case of internal compression, may now be considered. Our discussion will follow that given in Chapman and Walker (1971). If a wind tunnel with the two-nozzle-configuration shown in Figure I-1 is started from rest, the nozzle (1) (or the nozzle N) throat area $A_{Nt}$ must be less than the nozzle (2) (or the diffuser D) throat area $A_{Dt}$ in order to avoid choking the test section. That is, when $A_{Nt} = A_{Dt}$ and as $M = 1$ is achieved throughout, further reduction of the compressor suction pressure will not produce any changes upstream from the diffuser throat. When $A_{Nt} < A_{Dt}$ and
when M = 1 is obtained at the nozzle's throat, continued reduction in the downstream pressure establishes supersonic flow in the divergent portion of the nozzle. As the normal shock appears, taking successive positions downstream in the nozzle, a loss in the stagnation pressure occurs. The diffuser throat area must be large enough to accommodate the stagnation pressure loss of the strongest possible shock wave, namely, one occurring at the test section Mach number $M_T$. The proper area ratio in this case is also given by equation (ii) since here $A_{Dt}/A_{Nt} = (p_{t2}/p_{t0})^{-1}$. When this minimum area is satisfied, a starting pressure ratio given by equation (ii) (where $M_1 = M_T$) allows the shock wave to travel from the nozzle into the test section. Since friction effects cause all positions in the test section to be unstable, the shock wave continues through the test section to a point in the diverging section of the diffuser with an area equal to that of the test section. Once the starting shock is swallowed, the compressor suction pressure can be increased, causing the shock wave to move back upstream to positions of lower Mach numbers and hence less stagnation pressure loss. When the shock wave returns to the diffuser throat, the condition of minimum stagnation loss is achieved. This recovery is given by equation (ii) where $M_1$ is given by $M_{Dt}$, the Mach number of the flow at the diffuser throat.

For example, for $M_T = 3.0$, $p_T = 10$ psia, $T_T = 483.6^\circ R$, $A_{Nt} = 0.236 \text{ ft}^2$, $A_T = 1.0 \text{ ft}^2$, one finds:

$$\frac{p_{t2}}{p_{t0}} \bigg|_{M = 3.0} = 0.3283 = \frac{A_{Nt}}{A_{Dt}}$$

for the "swallowing" condition. Therefore

$$A_{Dt} = 0.720 \text{ ft}^2$$

and we note

$$r_{ps} \equiv \frac{p_{t0}}{p_{t2}} = 1/0.3283 = 3.05$$
as the starting pressure ratio for the wind tunnel with a starting compressor inlet or suction pressure of 3.28 psia. Isentropic expansion from $A_{NT}$, where $M = 1$, to $A_{Dt}$ gives

$$M_{Dt} = 2.655$$

and

$$\frac{P_{t2}}{P_{t0}} \bigg|_{M = 2.655} = 0.440$$

i.e., a final compressor inlet pressure of 4.40 psia for maximum pressure recovery.

The starting properties of a fixed geometry supersonic inlet diffuser (our next case) are illustrated in Figure I-2. This is also a case of internal compression. The fact that inlets are started from rest usually dictates the details of the shock wave phenomena in the starting processes. There are two possible sequences. In this discussion, let us use $M_{\infty d}$ as the design Mach number and $A_C/A_t$ as the diffuser area ratio. If the diffuser is sized so that a shock wave stands at the entrance when $M_{\infty} = M_{\infty d}$, then for $1 < M_{\infty} < M_{\infty d}$ a curved, detached shock stands in front of the diffuser. As $M_{\infty}$ increases to $M_{\infty} = M_{\infty d}$, the shock wave reaches the diffuser's lips, is swallowed and assumes a position in the diverging part of the diffuser where $A = A_C$ (assuming the appropriate back pressure). With a subsequent increase in the diffuser back pressure, maximum pressure recovery is achieved when the shock wave can be returned to the throat. If, on the other hand, the diffuser for $M_{\infty d}$ is sized so that isentropic compression to sonic speed is possible in principle between the lip and the throat, then the flow in the supersonic freestream must first be accelerated until the subsonic flow downstream from the external shock wave in the diffuser lips is appropriate for sonic flow at the throat. This will be true at $M_{\infty} = M_{\infty b} > M_{\infty d}$. Then the shock wave can be swallowed, the flow decelerated to
\( M_\infty = M_{\text{ad}} \) and the shock wave at the throat vanishes giving 100% pressure recovery. Both of these sequences for the supersonic inlet are shown in Figure I-2.

Let us now design a supersonic inlet for \( M_{\text{ad}} = 3.0 \) and examine two alternative two-dimensional configurations. First, assume that the flow deceleration takes place at the inlet through one normal shock wave. For \( M_1 = 3.0 \), equation (ii) gives

\[
P_{tD}/P_\infty = 0.328.
\]

Next assume a diffuser with a wedge shaped spike containing two successive \( 8^\circ \) turning angles. In this case, the deceleration occurs through two weak oblique shock waves followed by a normal shock wave. For \( M_\infty = 3.0 \), \( M_{w1} = 2.56 \) and \( M_{w2} = 2.20 \) with wave angles \( \theta_{w1} = 25.8^\circ \) and \( \theta_{w2} = 29.5^\circ \). Using these wave angles, \( M_{w,n} = 1.305 \) and \( M_{w1,n} = 1.26 \) gives

\[
P_{wl}/P_\infty = 1.82; \quad P_{w2}/P_{wl} = 1.686.
\]

From these:

\[
\frac{P_{tw1}}{P_{t\infty}} = \frac{P_{tw1}}{P_{wl}} \times \frac{P_{wl}}{P_\infty} \times \frac{P_\infty}{P_{t\infty}} = 0.930
\]

\[
\frac{P_{tw2}}{P_{tw1}} \times \frac{P_{wl}}{P_{w2}} \times \frac{P_{w2}}{P_{tw1}} = 0.960
\]

For \( M_{w2} = 2.20 \), \( P_{tD}/P_{tw2} = 0.628 \) and

\[
\frac{P_{tD}}{P_{t\infty}} = \frac{P_{tD}}{P_{tw2}} \times \frac{P_{tw2}}{P_{tw1}} \times \frac{P_{tw1}}{P_{t\infty}} = 0.560
\]
Thus, the overall pressure recovery is greater and the advantage of diffusing through several oblique shock waves is seen. This is an example of external compression. Theoretically, if the flow is allowed to pass through a large number of oblique shock waves, each turning the flow through a very small angle, the inlet flow conditions should approach those of an isentropic compression. This discussion leads us to our final case, the external "spike" diffuser, or ramjet diffuser, shown in Figure 1-3.

There are three different modes of operation for the spike diffuser. Critical operation occurs with the diffuser operating at design speed and the downstream engine conditions appropriately chosen. If, for some reason such as changing turbine speed, fuel flow rate, etc., the flow resistance downstream of the inlet increases, the normal shock wave moves ahead of the inlet. Hence, mass flow is reduced by spillover, and the pressure recovery is unfavorable. If, on the other hand, the downstream resistance decreases, the normal shock reaches an equilibrium position inside the diffuser. The mass flow rate is not affected; however, the pressure recovery is again reduced since the normal shock wave occurs at a higher Mach number in the diverging channel. In actual operation, the supercritical mode is preferred since it offers the greatest Mach number flexibility without loss of mass flow to the engine.
Figure 1-1. The sequence of flow states in the process of starting a supersonic wind tunnel. From Chapman and Walker (1971).
Figure I-2. Fixed Geometry Supersonic Inlet. (a) $A_t/A_c = (A_t/A_c)_c$; (b) Operating characteristics; (c) $A_t/A_c = (A_t/A_c)_a$. From Chapman and Walker (1971).
Figure 1-3. Ramjet Modes of Operation. 
(a) Critical; (b) Subcritical; (c) Supercritical
APPENDIX II

BIBLIOGRAPHY

Anafirev, V.M., Kozlov, G.I., Roitenberg, D.I. 1969  
Application of a Shock Wind Tunnel to a Study of  
Diffusers. Akademiia Nauk. SSSR, Izvestiia, Mekhaniki  
Zhidkosti i Gaza. March-April, 117-120.
Arnold, F. 1969  An Investigation of Diffuser Operation in  
Austin, R.F. 1966  An Open-Jet Wind Tunnel Investigation of  
A Fixed-Geometry Diffuser System at Mach Numbers 3.6  
and 7.0. AEDC-TR-66-11.
Azzouz, D. and Pratt, N.H. 1968  Supersonic Turbulent Mixing  
II. Design and Construction of a Mach 3 Blow-Down Wind  
from Turbulent Boundary Layers Interacting with Shock  
Waves and Expansion Waves. AIAA Journal 8, No. 10  
1871-1873.
Baker, P.J. 1965  Heat Transfer in a Supersonic Parallel  
Baker, P.J. and Martin, B.W. 1965  Some Operating Character-  
ististics of the Supersonic Axi-symmetric Parallel Diffuser.  
Baker, P.J. and Martin, B.W. 1966  Heat Transfer in Super-  
sonic Separated Flow Over a Two-Dimensional Backward-  
Barton, D.L. and Taylor, D. 1959  An Investigation of Ejectors  

Bertolo, C. and Bussi, G. 1967 Studio di Diffusori Super-sonici per Impianti di Prova di Propulsori. Pubblicazione N. 93 N69-35575 Instituto di Macchine e Motori per Aeromobili

Bertram, M.H. 1950 Investigation of the Pressure-Ratio Requirements of the Langley 11-inch Hypersonic Tunnel with a Variable-Geometry Diffuser. NACA RM L50113


Davidson, V.Ye. and Chukhalo, N.A. 1968 On the Starting of Supersonic Nozzles with the Aid of Cylindrical Diffusers.


Hermann, R. 1956 Supersonic Inlet Diffusers and Introduction to the Internal Aerodynamics. Honeywell Co. (Minneapolis, Minn.).


Lobb, R.K. 1952 NOL Hypersonic Tunnel No. 4 Results III: Diffuser Investigation with Models and Supports. NOL NAVORD Rept. 2435.


Neumann, E.P. and Lustwerk, F. 1951 High Efficiency Supersonic


Smith, R.T. 1964 Experimental Studies of High Temperature Hypersonic Diffusers. Air Force Flight Dynamics
Laboratory TDR-64- 4.
Wegener, P.P. and Lobb, R.K. 1952 NOL Hypersonic Tunnel No. 4 Results II - Diffuser Investigation. NOL NAVORD Report 2376.