Report 2135

PROGRAMMING LANGUAGE FOR THE SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS USING HYBRID COMPUTERS
PHASE I

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PROGRAMMING LANGUAGE FOR THE SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS USING HYBRID COMPUTERS—PHASE I

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This first attempt has been successful and provides a hybrid-computer scheme which is at least 50 times faster than the comparable purely digital approach. The program makes use of hybrid-computer graphics, with the input applied directly to the Tektronix 4010 Graphics Terminal and the solution curves presented to the graphics terminal on-line.
 PREFACE

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PROGRAMMING LANGUAGE FOR THE SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS USING HYBRID COMPUTERS

PHASE I

I. INTRODUCTION

1. Objective. The objective of this report is twofold. The first objective is to provide a progress report on hybrid-computer solution techniques for partial differential equations. The second objective is to provide documented details of the solution mechanics and to illustrate the power and speed of the hybrid computer when solving partial differential equations (PDE). A solution-speed comparison between the hybrid and digital techniques shows the hybrid to be the faster of the two.

2. Background. The Electrical Equipment Division, U.S. Army Mobility Equipment Research and Development Center (USAMERDC), is involved in the research, development, and engineering of electromagnetic machinery, power conditioners, and power electronics components (SCR’s, transistors, and rectifiers). These efforts require the solution of partial differential equations in order to provide flux plots and equipotential plots. When digital-computer techniques are used, these problem solutions are slow and costly. However, by using hybrid-computer techniques, we can reduce these computing costs by a factor of 15 to 25, with a corresponding increase in computing speed by a factor of between 15 and 100. The Electrical Equipment Division has a powerful, interactive hybrid-computer facility (Figure 1), which is part of the CAD-E facility (Figure 2). The hybrid computer is a Digital Equipment Corporation PDP-15/ Applied Dynamics AD-4 hybrid computer coupled to a Tektronix 4010 Graphic Terminal. Figure 3 shows the PDP-15/76 digital processor which has a unichannel, 1.2-milion-word disk and 16K of core. The AD-4 analog processor (Figure 4) has 96 amplifiers as well as an autopatch capability. The technical paper Hybrid Computer Solution Techniques for Laplace’s Equations, by the authors of this report, has helped immensely in preparing this report.*

3. Organization. This report is divided into five parts: Introduction, Program Philosophy, Computer-Solution Mechanics, Examples, and Conclusions and Future Work. Additional material is given in the three appendixes. The Program Philosophy section describes the philosophy of program development. The section on Computer-Solution Mechanics presents the details of problem setup for the hybrid-computer solution. The Examples section and the appendixes present sample problem solutions and special considerations. This report will provide the basis for comparing the interactive hybrid-computer solution of partial differential equations to the digital-computer approach.

Figure 1. Interactive hybrid-computer facility.
Figure 2. Computer-aided design engineering facility.
Figure 3. PDP-15/76 digital processor.
Figure 4. AD-4 analog processor.
II. PROGRAM PHILOSOPHY

This report describes a hybrid-computer solution approach to the solution of partial differential equations. However, to understand the reasoning for this method, the pure analog-computer approach to the solution of partial differential equations must be discussed. The technical background for this effort also will be useful to the full understanding of the program philosophy.

4. Technical Background. The background of the present work, typical equations, and their method of solution are discussed below.

a. Status in this Area of Work. During the early 1960’s, much work was accomplished for the solution of partial differential equations on analog computers. With the expected use of hybrid computers, the emphasis was shifted to their utilization. However, the efforts since then have been small, with little to show but theory. In the digital area, work has progressed, mainly because of the easier man/machine interface and because of the efforts of universities and the large computer companies.

b. Types of Problems. The Electrical Equipment Division is involved in the solution of partial differential equations for heat transfer and magnetic flux in electric and electronic equipment. As a result, the first problem to be examined and set up will be the diffusion problem and its associated equations. The solution of this type of equation will provide immediate benefits to the Electrical Equipment Division.

c. Types of Partial Differential Equations. There are three types of partial differential equations which are representative of a large number of engineering problems encountered:

\[ K \frac{\partial^2 \phi}{\partial t^2} = \nabla^2 \phi + f \]  
(heat equation or diffusion equation),

\[ K \frac{\partial^2 \phi}{\partial t^2} = \nabla^2 \phi + f \]  
(wave equation), and

\[ K \frac{\partial^2 \phi}{\partial t^2} = \nabla^4 \phi + f \]  
(dynamic structural equation (biharmonic equation)),

where \( \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \) and \( \nabla^4 \phi = \frac{\partial^4 \phi}{\partial x^4} + \frac{\partial^4 \phi}{\partial y^4} + \frac{\partial^4 \phi}{\partial z^4} + 2 \left\{ \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial x^2 \partial z^2} + \frac{\partial^4 \phi}{\partial y^2 \partial z^2} \right\} \).
d. **Usual Methods of Solution.** There are three major techniques of solution: (1) **separation of variables,** (2) **finite difference,** and (3) **stochastic.** Generally, we will use the finite-difference technique because it can handle time-varying boundary conditions and nonlinearities easily. The separation-of-variables technique assumes linearity. For the digital solution, one reduces the partial differential equation to a set of algebraic equations using the finite-difference technique. This means that iterative techniques must be employed to obtain solutions. For the analog solution, one obtains a set of ordinary differential equations using the finite-difference technique.

5. **Analog Approach.** The general approach to be used to solve the two-dimensional Laplace equation, \( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0, \) is to use finite differences for one of the space variables and to solve the other variable continuously. On the analog computer, this means we have two choices. We can divide the space such that we solve for a continuous solution as a function of \( y \) at each of a series of \( x \)-stations, or we can solve for a continuous solution as a function of \( x \) at each of a series of \( y \)-stations. Basing our calculations on engineering considerations for accuracy, we will try to use only a few stations. This also will reduce the number of analog components. In order to demonstrate solution accuracy and to identify mechanization problems, the first test problem is one that has an exact solution and that is a special case of the more general problem which will be solved as the approach is refined into a programing language.

The interesting general problem for the electrical engineer designing military generators and motors is one which provides the flux or flowline patterns and the equipotential-line patterns of the magnetostatic field in a section of the air gap of the machine. Figure 5 is a diagram of this complicated geometry. Here we need to be able to take care of a complicated geometry with different types of iron and with various boundary conditions. The overall objective is to provide a language which allows the design engineer to draw this picture on the graphic screen, to input the required boundary conditions, to solve the problem on the hybrid computer, and to provide a picture of the desired distributions of flux and potential, displayed on the graphic screen. The first test case is a simplified example, that will allow for an exact solution, which can be used for a comparison of results. Figure 6 is a diagram of a rectangular space used for the first test case.

III. **COMPUTER-SOLUTION MECHANICS**

6. **Solution Mechanics.** For the test case, we have a rectangular region, and we will investigate the field inside this region when three boundaries are at \( \psi=0 \) and one is at \( \psi=f(x) \). The exact solution for this case is \( 100 \psi(x,y)=100 \sin \frac{\pi x}{a} \cdot \frac{\sinh \left[ \pi (b-y)/a \right]}{\sinh \left[ \pi b/a \right]} \).
Figure 5. Typical electromagnetic machine geometry.
where \( a \) and \( b \) are as defined in Figure 6. This solution has been mechanized on the PDP-15 section of the hybrid to provide \( \psi(x,y) \) for comparisons. Two analog solutions have been studied (6a and 6b, below).

### a. Continuous x, Discrete y

In this solution, the analog computer simultaneously solves a set of differential equations at each of a series of \( y \)-stations to provide \( \psi(x)|_{y_a} \), where \( \alpha \) is the station number/location, which will give the value of \( \psi(x,y) \) at all points if \( y \) is on a station line. Some extrapolation means is assumed: of course, if \( \Delta y \) is small enough, it will not matter. For this solution-method example, we will use six stations in the \( y \)-direction. For boundary conditions, we have even derivates (\( y_o \) is considered even) specified at the boundaries \( y_o = 100 \sin \frac{\pi x}{a} \), and \( y_5 = 0 \) for all \( x \). Also, \( y_1, y_2, y_3, \) and \( y_4 \) have a boundary condition of 0 for \( x=0 \) and \( x=a \). In Hausner's rules for mechanization (Appendix A), rule 2 states that we should arrange the grid stations so that an integer station \((y_o, y_5)\) appears at the boundary since we have even derivatives specified at the boundary. The next Hausner rule (rule 3) says that we should generate high-order derivatives with first-order approximations, mechanizing all lower order derivates as summational outputs.

Thus, we let \( D_j = \psi''_j \approx \frac{\psi_{j-1} - 2\psi_j + \psi_{j+1}}{h^2} \) and \( \phi_{j,\frac{5}{2}} = \psi'_{j,\frac{5}{2}} \approx \frac{\psi_{j-1} + \psi_{j+1}}{h} \), where \( h \) is and \( j \) is \( \phi_{j,\frac{5}{2}} = \psi'_{j,\frac{5}{2}} \approx \frac{-\psi_{j-1} + \psi_{j+1}}{h} \), so \( D_j \approx \frac{-\phi_{j,\frac{5}{2}} + \phi_{j+\frac{5}{2}}}{h} \). Thus, we generate five intermediate solutions \((\phi_{1/2}, \phi_{3/2}, \phi_{5/2}, \phi_{7/2}, \phi_{9/2})\) and use eight integrators (Figure 7).

Setting \( \frac{\partial^2 \psi_n}{\partial y^2} = \frac{\phi_{n+\frac{5}{2}} - \phi_{n-\frac{5}{2}}}{(\Delta y)^2} \), a finite-difference equation for \( y \), in the \( \frac{\partial^2 \psi_n}{\partial x^2} = \frac{\partial^2 \psi_n}{\partial y^2} \) equation gives us: \( \frac{\partial^2 \psi_n}{\partial x^2} \bigg|_{y_n} = -\left[ \frac{\phi_{n+\frac{5}{2}} - \phi_{n-\frac{5}{2}}}{(\Delta y)^2} \right] \). Then we can solve for \( \psi(x)|_{y_n} \) by using the unscaled equations:

\[
\frac{d^2 \psi_1}{dx^2} = \frac{\phi_{1/2} - \phi_{3/2}}{(\Delta y)^2}
\]
\[
\frac{d^2 \psi_2}{dx^2} = \frac{\phi_{3/2} - \phi_{5/2}}{(\Delta y)^2}
\]
\[
\frac{d^2 \psi_3}{dx^2} = \frac{\phi_{5/2} - \phi_{7/2}}{(\Delta y)^2}
\]
Figure 6. Rectangular space.
Figure 7. Grid for continuous x, discrete y.
\[
d\frac{\psi_4}{dx^2} = \frac{\phi_{7/2} - \phi_{9/2}}{(\Delta y)^2}
\]

\[
\phi_{1/2} = \psi_1 - 100 \sin \frac{\pi x}{a}
\]

\[
\phi_{3/2} = \psi_2 - \psi_1
\]

\[
\phi_{5/2} = \psi_3 - \psi_2
\]

\[
\phi_{7/2} = \psi_4 - \psi_3
\]

\[
\phi_{9/2} = 0 - \psi_4
\]

For mechanization purposes, we replace \( t \) by \( x \) in a one-to-one replacement (i.e., 1 second = 1 unit of distance in \( x \)).

In the unscaled equation, \( \Delta y = \frac{b}{(\text{No. of Stations} - 1)} \), so we have a way to incorporate \( a \) and \( b \) in the solution. For scaling use the values given in the table are typical.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Est. Max. Value</th>
<th>Scale Factor</th>
<th>Scaled Computer Variable</th>
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<tr>
<td>( \phi )</td>
<td>100 v</td>
<td>( \frac{100}{100} )</td>
<td>( [\phi] )</td>
</tr>
<tr>
<td>( \psi )</td>
<td>100 v</td>
<td>( \frac{100}{100} )</td>
<td>( [\psi] )</td>
</tr>
<tr>
<td>( \psi' )</td>
<td>100 v/s</td>
<td>( \frac{100}{100} )</td>
<td>( [\psi'] )</td>
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(In our problem as it is set up, the X-generator (Integrator 271) is generating 10 v/s or 0.1 s/v. When we measure 10 volts on X at 10 v/s, we had 1 second, or 1 unit of distance in X, which corresponds to \( a \).) For this problem, we used the initial-condition (IC) pots on the \( \psi' \)-integrator to obtain the proper boundary condition for \( \psi_1 \) through \( \psi_4 \) at \( x=a \). In this problem, we used these pots to make \( \psi_1, \psi_2, \psi_3, \) and \( \psi_4=0 \) at \( x=a \).

b. Continuous \( y \), Discrete \( x \). This method is identical to the continuous \( x \), discrete \( y \) method except that the problem space is divided into stations in the \( x \)-direction. The problem is solved continuously in the \( y \)-direction. This method is discussed in more detail in the examples (section IV).
7. Special Techniques. Two special techniques for problem solution may be mentioned.

a. Dividing Problem Space. In an effort to minimize equipment and to provide an easy conversion to autopatch, we will divide the problem space into three fixed stations and one variable station. Using symmetry (special case), we get mirror-image solutions in the right half and in the left half of the rectangular space. Therefore, by this consideration, we get 2n-3 solutions for n stations. Using the hybrid-solution control, we will set the variable station at a specified ΔX-spacing from the center station, and a solution will be obtained. Then ΔX will be increased, and the problem will be solved again. This iterative process will be repeated until all specified stations are used. This method allows for linear or nonlinear spacing.

b. Approaching Boundary Value by Varying IC-Pots. Another iterative process found to be useful occurs in satisfying the boundary equations. By varying the IC-pots on the ψ-integrators one at a time and in station order from left to right, we can iteratively approach the required boundary value. This method requires that the first pot be varied until the ψ-variable equals zero at the prescribed location on the x-axis (x=a) while all other pots are fixed. Then, the second pot is varied until ψ=0 at the same location. This process is repeated sequentially until all variables (ψ₁, ψ₂, ψ₃, and ψ₄) are zero at the same point. This method will be illustrated clearly by the examples, which follow in the next section. Both of the iterative processes described above are performed rapidly by the PDP-15 digital computer.

IV. EXAMPLES

8. Laplace Equations for Two-Dimensional Solution. The geometry of this problem dictates use of the continuous y, discrete x solution method. Based on trial solutions, it was determined that six stations are adequate (five fixed and one variable station). Two stations are at the boundaries, x=0 and x=a, where ψ₀ = ψ₅ = 0. Figure 8 is a diagram of the space, with the variable station shown as a broken line.

For this mechanization, X₀, X₁, X₂, X₃, and X₅ are fixed locations, and X₄ varies. Because of symmetry, X₁ and X₂ will have mirror-image solutions in the right half-space, and X₄ will have mirror-image solutions in the left half-space. Point X₃ is located at $\frac{3a}{6}$, while X₁ is at $\frac{a}{6}$ and X₂ is at $\frac{2a}{6}$. By symmetry conditions, there will be an identical solution to X₂ at $\frac{4a}{6}$, to X₁ at $\frac{5a}{6}$, and to X₄ at $\frac{(3 + K₄)a}{6}$, with K₄ being specified by the user. For initial conditions along the y=0 boundary,
Figure 8. Problem space.
\[ \psi_0 = 100 \sin \left( \frac{\pi}{a} \left( 0 \right) \right) ; \psi_1 = 100 \sin \left( \frac{\pi}{a} \left( a/6 \right) \right) ; \psi_2 = 100 \sin \left( \frac{\pi}{a} \left( \frac{2a}{6} \right) \right) ; \psi_3 = 100 \sin \left( \frac{\pi}{a} \left( \frac{3a}{6} \right) \right) ; \psi_4 = 100 \sin \left( \frac{\pi}{a} \left( \frac{4a}{6} \right) \right) ; \text{where} \]
\[ K = 3 + K_4. \]

When the method described previously was used, it was possible to solve the equations:

**Equation**  
\[ \psi_1'' = \frac{\left( \phi_{1/2} - \phi_{3/2} \right)}{\Delta x_{11}} \]
\[ \psi_2'' = \frac{\left( \phi_{3/2} - \phi_{5/2} \right)}{\Delta x_{21}} \]
\[ \psi_3'' = \frac{\left( \phi_{5/2} - \phi_{7/2} \right)}{\Delta x_{31}} \]
\[ \psi_4'' = \frac{\left( \phi_{7/2} - \phi_{9/2} \right)}{\Delta x_{41}} \]

**Definition**  
\[ \Delta x_{11} = \frac{a}{6} \]  \( (1) \)
\[ \Delta x_{21} = \frac{a}{6} \]  \( (2) \)
\[ \Delta x_{31} = \frac{1}{2} \left( \frac{a}{6} \right) + \frac{1}{2} \left( K_4 \right) \]  \( (3) \)
\[ \Delta x_{41} = \frac{1}{2} \left( K_4 \right) + \frac{1}{2} \left( \frac{3a}{6} - K_4 \right) \]  \( (4) \)
\[ \Delta x_{12} = \frac{a}{6} \]  \( (5) \)
\[ \Delta x_{22} = \frac{a}{6} \]  \( (6) \)
\[ \Delta x_{32} = \frac{a}{6} \]  \( (7) \)
\[ \Delta x_{42} = K_4 \]  \( (8) \)
\[ \Delta x_{52} = \frac{3a}{6} - K_4 \]  \( (9) \)

Variable \( K_4 \) is defined as follows:
\[ K_4 = KR \left( \frac{3a}{6} \right), \]  \( (10) \)

where \( KR \) is the spacing factor.
Changing the equation form, we obtain the following $\psi$ and $\phi$ values:

\[ \psi_1 = \frac{1}{(\Delta x_{11})} \left( \phi_{1/2} - \phi_{3/2} \right) \]  
\[ \psi_2 = \frac{1}{(\Delta x_{21})} \left( \phi_{3/2} - \phi_{5/2} \right) \]  
\[ \psi_3 = \frac{1}{(\Delta x_{31})} \left( \phi_{5/2} - \phi_{7/2} \right) \]  
\[ \psi_4 = \frac{1}{(\Delta x_{41})} \left( \phi_{7/2} - \phi_{9/2} \right) \]

\[ \phi_{1/2} = \frac{1}{(\Delta x_{12})} \left( \psi_1 - \psi_o \right) \]  
\[ \phi_{3/2} = \frac{1}{(\Delta x_{22})} \left( \psi_2 - \psi_1 \right) \]  
\[ \phi_{5/2} = \frac{1}{(\Delta x_{32})} \left( \psi_3 - \psi_2 \right) \]  
\[ \phi_{7/2} = \frac{1}{(\Delta x_{42})} \left( \psi_4 - \psi_3 \right) \]

\[ \phi_{9/2} = \frac{1}{(\Delta x_{52})} \left( \psi_5 - \psi_4 \right) \]

Continuing to change the equation form (since $\psi_o = \psi_5 = 0$), we obtain the following:

\[ (\Delta x_{12}) \phi_{1/2} = \psi_1 \]  
\[ (\Delta x_{22}) \phi_{3/2} = \psi_2 - \psi_1 \]  
\[ (\Delta x_{32}) \phi_{5/2} = \psi_3 - \psi_2 \]  
\[ (\Delta x_{42}) \phi_{7/2} = \psi_4 - \psi_3 \]  
\[ (\Delta x_{52}) \phi_{9/2} = (-\psi_4) \]
\[0.01 \psi''_1 = \left( \frac{0.01}{\Delta x_{11}} \right) \phi_{1/2} - \left( \frac{0.01}{\Delta x_{11}} \right) \phi_{3/2}\]  \hspace{1cm} (25)

\[0.01 \psi''_2 = \left( \frac{0.01}{\Delta x_{21}} \right) \phi_{3/2} - \left( \frac{0.01}{\Delta x_{21}} \right) \phi_{5/2}\]  \hspace{1cm} (26)

\[0.01 \psi''_3 = \left( \frac{0.01}{\Delta x_{31}} \right) \phi_{5/2} - \left( \frac{0.01}{\Delta x_{31}} \right) \phi_{7/2}\]  \hspace{1cm} (27)

\[0.01 \psi''_4 = \left( \frac{0.01}{\Delta x_{41}} \right) \phi_{7/2} - \left( \frac{0.01}{\Delta x_{41}} \right) \phi_{9/2}\]  \hspace{1cm} (28)

\[(\Delta x_{12}) \phi_{1/2} = (P_{224}) \psi_1\]  \hspace{1cm} (29)

\[(\Delta x_{22}) \phi_{3/2} = (P_{226}) \psi_2 - (P_{223}) \psi_1\]  \hspace{1cm} (30)

\[(\Delta x_{32}) \phi_{5/2} = (P_{244}) \psi_3 - (P_{236}) \psi_2\]  \hspace{1cm} (31)

\[(K_1 \Delta x_{42}) \phi_{7/2} = (K_1) (P_{266}) \psi_4 - (K_1) (P_{247}) \psi_3\]  \hspace{1cm} (32)

\[K_1 = \frac{\Delta x_{32}}{\Delta x_{42}}\]  \hspace{1cm} (33)

\[(K_2 \Delta x_{52}) \phi_{9/2} = -(K_2) P_{256} \psi_4\]  \hspace{1cm} (34)

\[K_2 = \frac{\Delta x_{32}}{\Delta x_{52}}\]  \hspace{1cm} (35)

Continuing the rearrangements:

\[P_{232} (\Delta x_{12}) \phi_{1/2} = \left( \frac{0.01}{\Delta x_{11}} \right) \phi_{1/2}\]  \hspace{1cm} (36)

\[P_{245} (\Delta x_{22}) \phi_{3/2} = \left( \frac{0.01}{\Delta x_{21}} \right) \phi_{3/2}\]  \hspace{1cm} (37)

\[P_{227} (\Delta x_{22}) \phi_{3/2} = \left( \frac{0.01}{\Delta x_{21}} \right) \phi_{3/2}\]  \hspace{1cm} (38)

\[P_{233} (\Delta x_{32}) \phi_{5/2} = \left( \frac{0.01}{\Delta x_{21}} \right) \phi_{5/2}\]  \hspace{1cm} (39)

\[P_{243} (\Delta x_{32}) \phi_{5/2} = \left( \frac{0.01}{\Delta x_{31}} \right) \phi_{5/2}\]  \hspace{1cm} (40)
\[ K_1 P_{253} (\Delta x_{42}) \phi_{7/2} = \left( \frac{0.01}{\Delta x_{31}} \right) \phi_{7/2} \]  
\[ (41) \]

\[ K_1 P_{265} (\Delta x_{42}) \phi_{7/2} = \left( \frac{0.01}{\Delta x_{41}} \right) \phi_{7/2} \]  
\[ (42) \]

\[ K_2 P_{276} (\Delta x_{52}) \phi_{9/2} = \left( \frac{0.01}{\Delta x_{41}} \right) \phi_{9/2} \]  
\[ (43) \]

Finally, we obtain the following pot settings:

\[ P_{224} = 1 \]  
\[ (44) \]

\[ P_{226} = 1 \]  
\[ (45) \]

\[ P_{223} = 1 \]  
\[ (46) \]

\[ P_{244} = 1 \]  
\[ (47) \]

\[ P_{236} = 1 \]  
\[ (48) \]

\[ P_{266} = \frac{\Delta x_{32}}{\Delta x_{42}} \]  
\[ (49) \]

\[ P_{247} = \frac{\Delta x_{32}}{\Delta x_{42}} \]  
\[ (50) \]

\[ P_{256} = \frac{\Delta x_{32}}{\Delta x_{52}} \]  
\[ (51) \]

\[ P_{232} = \frac{0.01}{(\Delta x_{11}) (\Delta x_{12})} \]  
\[ (52) \]

\[ P_{245} = \frac{0.01}{(\Delta x_{11}) (\Delta x_{22})} \]  
\[ (53) \]

\[ P_{227} = \frac{0.01}{(\Delta x_{22}) (\Delta x_{21})} \]  
\[ (54) \]

\[ P_{233} = \frac{0.01}{(\Delta x_{32}) (\Delta x_{21})} \]  
\[ (55) \]

\[ P_{243} = \frac{0.01}{(\Delta x_{32}) (\Delta x_{31})} \]  
\[ (56) \]
\[ P_{253} = \frac{0.01}{(\Delta x_{31}) (\Delta x_{42}) (K_1)} \]  \hspace{1cm} (57)

\[ P_{265} = \frac{0.01}{(\Delta x_{41}) (\Delta x_{42}) (K_1)} \]  \hspace{1cm} (58)

\[ P_{276} = \frac{0.01}{(\Delta x_{41}) (\Delta x_{52}) (K_2)} \]  \hspace{1cm} (59)

The program will scan the space as previously described, and with four different positions for station \( X_4 \) we actually obtain data for 15 equivalent stations as is shown by Figure 9.

In order to obtain the desired plots, it is necessary to perform a core search for a specified \( \psi \)-value:

a. Check out the specified X-station and its equivalent image.

b. Use straight-line interpolation between data points.

For example: \( y \) value = ITM/10,000, where ITM = b

\[ x \text{-value} = x \text{-station location} \]

For a specified X:

a. Start at the maximum \( \psi \)-value until \( \psi \) in core is less than the specified \( \psi \).

b. Back up one space and check discrete \( y \)-values; use linear extrapolation to get specified value \( x,y \) data.

9. Hybrid-Computer Solution.

a. General. The hybrid-computer solution may be illustrated graphically. The problem-space geometry is shown in Figure 8 and the space with the solution grid is shown in Figure 9. The finite-difference equations are shown in Figure 10, and the computer patching diagram is given as Figure 11. A program control flow chart is shown in Figure 12, and the patchboard is shown in Figure 13. Figure 14 shows the logic patchboard.

b. Computer Program PDR2B. The computer program is stored in the execute file, PDR2B, in the RMM file on disk. Program listings and subroutines are
BASIC FINITE DIFFERENCE SCHEME
FOR HYBRID COMPUTER

A CHANGE TO AN ORDINARY 2nd ORDER DIFFERENTIAL
EQUATION AT EACH X-STATION

\[
\begin{align*}
\dot{\psi}_1 &= \frac{1}{\Delta x_{11}} (\phi_{1/2} - \phi_{3/2}) \\
\dot{\psi}_2 &= \frac{1}{\Delta x_{21}} (\phi_{3/2} - \phi_{5/2}) \\
\dot{\psi}_3 &= \frac{1}{\Delta x_{31}} (\phi_{5/2} - \phi_{7/2}) \\
\dot{\psi}_4 &= \frac{1}{\Delta x_{41}} (\phi_{7/2} - \phi_{9/2}) \\
\phi_{1/2} &= \left(\frac{1}{\Delta x_{12}}\right) (\psi_1 - \psi_0) \\
\phi_{3/2} &= \left(\frac{1}{\Delta x_{22}}\right) (\psi_2 - \psi_1) \\
\phi_{5/2} &= \left(\frac{1}{\Delta x_{32}}\right) (\psi_3 - \psi_2) \\
\phi_{7/2} &= \left(\frac{1}{\Delta x_{42}}\right) (\psi_4 - \psi_3) \\
\phi_{9/2} &= \left(\frac{1}{\Delta x_{52}}\right) (\psi_5 - \psi_4)
\end{align*}
\]

Figure 10. Finite-difference equations.
Figure 11. Computer patching diagram.
Figure 12. Program flow chart.
Figure 13. Analog patchboard.
Figure 14. Logic patchboard.
given in Appendix B. The large size of this problem requires "chaining," and the program details are in Appendix C. The following is a description of the use and response of PDR2B. With the PDP-15/AD-4 hybrid up and running, the PDP-15 executive supplies a "$" to indicate user input. To the "$" on the Tektronix 4010, the user types in "E PDR2B." The computer prompting response is a statement for input:

"Input A, B, DEL1, DEL2, DEL3, DEL4, DEL5: F5.2, 5F5.4." This allows the user to provide the x and y space dimensions (A and B, respectively). The spacing for the variable grid line, referenced from the center line, is not used. Once this spacing is input, the computer responds with the prompting: "Specify Number of Lines LT16." This allows the user to vary the number of stations for trial solutions. The computer prints the value of DEL (as measured from the center x-station) and the IC-pot values, which are required to satisfy the boundary conditions through the closed-loop, analog iterative process, described in Appendix B. Figure 15 shows the computer prompting. Program solution output is shown by Figures 16 and 17. Figure 18 illustrates the solution with a grid, while Figure 19 depicts the solution without a grid. Normally, for production runs, the problem grid would be well specified; but, for this problem, it was not. Several linear and nonlinear spacings were investigated. It should be noted that the nonlinear grid helps to clarify solution slopes in specific areas of interest. The use of nonlinear grid is optional (i.e., it can be selected as needed). The 16K core of the present PDP-15 digital subsection of the hybrid unit limits us to about 20 grid stations (40 with symmetry), but more would be available if we had written the solution to disk or tape storage and had performed the graphics with another program. Also, the graphics display uses a simplified, point-to-point plotting routine, which could be refined for smoother curves.

The b/a-ratio limits for this method as it is presently programed are between 0.1 and 0.3, mainly because of the assumed scaling. This limitation will be eliminated later, but it is not serious enough to warrant a change for the trial example. Figures 15 through 19, which depict the solution on the Tektronix 4010 Graphic Terminal screen, were used to demonstrate the problem I/O and do not describe accurate solutions. The next set of figures, which is hardcopy output for the Tektronix graphics display, is used to provide the comparison of accuracy between the exact and hybrid solutions for this example. The exact solution uses a mathematical solution subroutine in place of the hybrid subroutine set PDE, MCON, and PDE2 (see Appendix C for more details). All other input and output subroutines stay the same. Using the problem definition parameters (A=1, B=.1) and 10 lines (stations), we can compare results. Note that the computer uses nine lines to divide the right-hand space of the problem into 10 spaces. Figure 20 is the hardcopy output for the hybrid solution, and Figure 21 is the hardcopy output for the exact solution. Appendix C contains $\psi(y)$-data for each X-station generated by the exact and hybrid solutions.
Figure 15. Program computer prompting.
Figure 17. Program solution output (completed).
Figure 18. Solution with grid.
Figure 20. Hardeopy output of hybrid solution.

Figure 21. Hardeopy output of exact solution.
In clock time, each hybrid-computer solution set took 30 seconds. (A 33-grid solution, including the symmetry, took about 7 minutes.) The hybrid-computer solution runs 100 times faster than real time and is faster than the exact solution provided by the PDP-15 only. Figures 16 and 17 verify our original assumption: that we could scan the space, while maintaining five stations fixed and one moving, because the first three pot settings (two stations are at the boundary, where \( \psi=0 \)) always return to the same value at solution; however, the grid station, being moved, changes the pot value.

V. CONCLUSIONS AND FUTURE WORK

10. Conclusions and Future Work. So far, we have shown a technique for solving partial differential equations on hybrid computers which is at least 50 times faster than the digital solution. This speed of solution occurs because we solve the problem in a continuous, closed-loop, analog process. Also, we have established an iterative solution technique, which converges rapidly and allows us to maintain overall, simplified digital control over the closed-loop, analog solution process. The comparison of the hybrid solution to the exact analytical solution demonstrates the accuracy of this approach.

The next steps are to generate the problem menus and to solve the field problem for a slot geometry and, then, for other complex geometries. The progress demonstrated to date offers an optimistic outlook for complete success in the future planned work of this project.
HAUSNER'S* RULES FOR MECHANIZATION

The following is a list of Hausner's Rules used in this project:

Rule 1 — To obtain a kth-order solution, all approximations must be kth order, including those accounting for boundary conditions.

Rule 2 — If only even derivatives of a dependent variable (such as u, u'', u''', etc.) are specified at a boundary, arrange the grid stations so that an integer station (say, X₀ or X₁) appears at a boundary. If at least one odd derivative (u', u''', etc.) is specified at a boundary, a half-integer station (say, X₁/₂) should be placed at a boundary.

Rule 3 — Generate high-order derivatives with first-order-derivative approximations, mechanizing all lower order derivatives as summational outputs.

A brief discussion of the analog control routines used to reach solutions is given in this appendix.

B-1. Differentiation with Respect to $y$. The analog computer actually performs $\frac{dy}{dt}$ as $\frac{d\psi}{dt}$, where $y$ is represented as $\tau$ on a one-to-one basis. The time-base (or $y$-base) generator, integrator 271, normally is providing 10 v/s; thus, we get 0.1 s/v as the output. Since 1 unit of $y$ is equivalent to 1 second, it takes 0.2 second to provide 0.2 unit of $y$. This means that the integrator output is 2 volts in 0.2 second (0.1 s/v \times 2 volts = 0.2 second). In order to provide the proper output rate for integrator 271, pot 273 is set to 0.01 with 100 volts input. The normal integrator rate is 10 v/s in quadrant two of the analog patchboard.

B-2. Closed-Loop Analog Solution. The fastest possible solution is obtained when the analog computer operates in a closed-loop fashion. The solution control is accomplished as follows: (1) The user provides input parameters to the digital unit; (2) the digital computer uses these parameters to automatically scale the problem, to set the analog comparator pot settings for time (or b) value in order to place the computer in hold, and to set the pots and start the solution; (3) the digital unit waits a sufficient time in order to allow the analog unit to go to “hold,” checks end-point values for convergence, resets the computer to run again, and repeats this until convergence occurs; (4) once convergence occurs, the digital unit resets the computer and causes the analog unit to operate for a set number of predetermined increments, at which points the analog comparator places the computer into the “hold” mode and the digital unit samples and stores $\psi$, $y$, and $x$-data; (5) this process is repeated until all specified x-stations have been used; (6) once all x-stations have been used, the digital unit asks the user to specify $\psi$, $\Delta \psi$, and the number of lines to plot; and (7) the digital unit uses these data to search its stored $\psi$, $y$, and $x$-data and to provide the plot. The digital unit is programmed to provide many variations of the plotting, once the hybrid unit has finished computing, in order to keep from having to recompute each time a new plot variation is needed.

B-3. Analog Comparator Logic. The logic and analog patching needed to accomplish the time (or $y$-) control is shown by Figure 11. The output of integrator 271 is fed through pot 277 to amplifier 233. The output of amplifier 233 goes to comparator
231 on the analog patchboard. The reference voltage (equivalent to y=b) comes from amplifier 223, which is the other input to comparator 231. When the sum of the inputs goes positive (occurs at the instant y becomes infinitesimally larger than b), a logic 1 is generated by the out-point on the logic patchboard. Since “out” on comparator 231 is connected to SYS Hold, it receives a logic 1, which places the analog unit in the “hold” mode, thus stopping computation. In order to reset properly, the digital unit overrides the patched “hold” mode by a “hold” command, reads the desired \( \psi \)-value, places the computer in the IC-mode, and resets the comparator output to logic 0 by setting pot 237 to 0. For the sampling of \( \psi \)-, x-, and y-data after convergence tests are met, pot 237 is incremented to the preset values, thus stopping the computer at the desired points of y, reading the data, and continuing to the next point as soon as pot 237 is updated. This process is limited to 12 data points because of the dimension statement, which reflects present core limits. Methods that would allow more points could be used but are not required for the test example.

B-4. Iteration Control. The computer is programed to set all four IC-pots (for the first derivative of \( \psi \)) initially and, then, to go through a preset sequence to set the first IC-pot until \( \psi_1 \) goes to 0 at y=b. The computer then goes to the second IC-pot and changes it until \( \psi_2 = 0 \) at y=b. Each time, all \( \psi \)'s are sampled to see if they are simultaneously 0 at y=b. This process continues to \( \psi_3, \psi_4, \psi_1, \psi_2, \psi_3, \psi_4, \) etc. until \( \psi_1 = \psi_2 = \psi_3 = \psi_4 = 0 \) at y=b. This process generally converges in less than 30 seconds (about 10 iterations at most).
C-1. Introduction. This appendix gives a listing of the problem source code, the chaining routine, and the several programs used in this study and depicts the program flowcharts.

The hybrid-computer program consists of a main program (designated subroutine POT) and eight subroutines: PDE, MCON, CON, READSI, PDE2, DISK, DRW, and DRWA. The hybrid-computer program also requires the hybrid routines and the Tektronix routines. The problem requires "chaining" on the 16K core configuration of the PDP-15. The chaining routine produces the XCT and XCU files and allows the program to be run by using E_____PDR2B.

C-2. Hybrid Program Listings. The following listings are the routines used for the hybrid-computer solution.
SUBROUTINE POT

THIS WILL ACT AS THE MAIN PROGRAM

DIMENSION TST(2)
COMMON/W/Y(18), IPSI
COMMON/DRWY/XLOC(18), IY(18), ITM(12), ISTA(20, 12), IV(4), DELT(15)
COMMON/POTV/P(20), DEL, A
COMMON/DIM/B, IFR
COMMON/P1/N, JK, KZ
COMMON/GRD/NPSIG, NTF
DATA TST(1), TST(2)/3MTST, 4H SRC/
JK=4
NA=1
CALL STIND(IE, 2237, 0)
WRITE(4, 601)
READ(4, 600) A
READ(4, 600) B
IBR=IFIX(10000.*B)
WRITE(4, 2051)
2050 FORMAT(1X, 25H SPECIFY NO OF LINES LT 16)
READ(4, 6004) NLines
6004 FORMAT(I2)
DELTX=.5/(FLOAT(NLINES))
DELTA(1)= DELTX
NTF=NLINES-1
DO 2050 NT=2, NTF
DELTA(NT)=DELTA(NT-1)+DELTX
2050 CONTINUE
DELTA(1)=1./18.
DELTA(2)=2./18.
DELTA(14)=8./18.
DELTA(3)=4./18.
DELTA(4)=5./18.
DELTA(15)=8./18.
DELTA(5)=7./18.
DELTA(6)=7.3/18.
DELTA(7)=7.6/18.
DELTA(8)=8./18.
DELTA(9)=8.1/18.
DELTA(10)=8.2/18.
DELTA(11)=8.3/18.
DELTA(12)=8.4/18.
DELTA(13)=8.5/18.
601 FORMAT(1X, 18H INPUT A, B, DEL1, DEL2, DEL3, DEL4, DEL5 : F5.2, 5F5.4)
900 CONTINUE
697 FORMAT(1X, 23H INPUT: A, DEL: F5.2, F5.4)
WRITE(4, 11) DELTA(NA)
DEL=DELTA(NA)
11 FORMAT(1X, 18H DEL = ', F10.4)
CR=2.*DEL
C4=CR+(3./6.)
DX11=A/6.
DX21=A/5.
DX31=(A+(3.*C4))/12.
DX41=(C4/2.)*(3.*A/6.-C4)/2.
DX12=A/6.
DX22=A/6.
DX32=A/6.
DX42=C4
DX52=(H/2. )-C4
C1=DX32/DX42
C2=DX32/DX52

602 FORMAT(1X, 6(1X, F10.4))

P(1)=1.
P(2)=1.
P(3)= .01/(DX11*DX12)
P(4)=SIN(3. 14159/6.)
P(5)= .01/(DX11*DX22)
P(6)=1.
P(7)=SIN(3. 14159*2./6.)
P(8)=1.
P(9)=.01/(DX22*DX21)
P(10)= .01/(DX32*DX21)
P(11)=SIN(3. 14159*3./6.)
P(12)= .01/(DX32*DX31)
P(13)=1.
P(14)=DX32/DX42
P(15)= .01/(DX31*DX42+C1)
P(16)=DX32/DX52
P(17)=SIN(3. 14159*(1.+CR)/2.)
P(18)= .01/(DX41*DX42+C1)
P(19)=DX32/DX42
P(20)= .01/(DX41*DX52+C2)
IF(P(14).LT.1.5)GO TO 698
PT0T1=P(14)*P(15)
P(14)=1.
P(15)=PT0T1
PT0T2=P(19)*P(18)
P(19)=1.
P(18)=PT0T2

698 CONTINUE
IF(P(16).LT.1.5)GO TO 699
PT0T3=P(16)*P(20)
P(16)=1.
P(20)=PT0T3

699 CONTINUE
DO 700 NP=1, 20
C WRITE(4, 6010)P(NP)
700 CONTINUE
600 FORMAT(F5.2, F5.4)
6010 FORMAT(1X, F10.4)
CALL PDE
CALL MCON
CALL PHE2
NA=NA+1
JK=JK+1
IF(NA.GT.NTF)GO TO 3000
GO TO 900
3000 CONTINUE
CALL DISK
2021 FORMAT(1X, T4, 'ITM', T14, 'X1', T22, 'X2', T30, 'X3', T38,
1 'X4', T46, 'X5', T54, 'X6', T62, 'X7', T70, 'X8')
C SEARCH FOR SPECIFIED PSI FOR EQUIPOT PRINTOUT
4003 CONTINUE
WRITE(4, 2006)
FORMAT(1X, 11HSPECIFY PSI)
READ(4, 1009) PSI, PSID
READ(4, 1021) NPSI
FORMAT(12)
NPSIG = 0
CONTINUE
DO 4000 IZR = 1, NPSI
        PSI = IFIX(PSI*100.)
FORMAT(1X, T9, 'PSI', T20, 'Y', T31, 'X', T42, 'XI')
K = 1
NTF3 = NTF + 3
DO 1000 NA = 1, NTF3
    CONTINUE
    DO 10 IP$ = IFIX(PI*100.)
        NA = 1, KZ
        IF(ISTA(N, NA).LT. IPSI) GO TO 1002
CONTINUE
1001 NB = NA
    NC = NB - 1
    DELSTA = FLOAT(ISTA(N, NC) - ISTA(N, NB))
    DELTM = FLOAT(ITM(NC) - ITM(NB))
    Y(K) = (FLOAT(ITM(NC)) - (DELTM*FLOAT(ISTA(N, NC) - IPSI)/DELSTA))
    X = XLOC(N)
    XI = A - XLOC(N)
    K = K + 1
CONTINUE
1000 CONTINUE
1009 FORMAT(2F10.3, i2)
1010 FORMAT(1X, 4(1X, F10.3))
    IF(IZR.GT.1) GO TO 4001
    IF(NPSIG.GT.0) GO TO 4001
    CALL DRW,
    CONTINUE
    CALL DRWA
    PSI = PSI + PSID
    CONTINUE
    PSI = PSI - (FLOAT(NPSI)*PSID)
    NPSIG = 1
    WRITE(4, 1011)
    WRITE(4, 1012)
CONTINUE
1011 FORMAT(1X, 18HN0 GRID OR RESTART)
1012 FORMAT(1X, 18HTYPE 2 FOR RESTART)
READ(4, 1021) MST
    IF(MST.EQ.2) GO TO 4003
    GO TO 4002
STOP
END
SUNDEROUTINE PIE

C PROGRAM PIE**122573**
DIMENSION IPT(20), IPTV(20), ADEL(18)
COMMON/P1/NA, JK, K2
COMMON/POV/P(20), DEL, A
COMMON/DRUP/XLOC(18), IY(18), ITM(12), ISTA(20, 12), IV(4), DELTA(15)
DATA IPT/2224, 2223, 2232, 2235, 2245, 2236, 2230, 2226, 2227, 2233,
1 2242, 2243, 2244, 2247, 2253, 2256, 2264, 2265, 2266, 2275/
CALL LEX(IE, 1)
CALL TSCAL(IE, 0)
CALL LOAD(IE)

600 FORMAT(F5.2, 5F5.4)
405 CONTINUE
CALL LEX(IE, 1)
DEL=DELTA(NA)
XLOC(1)=A/6.
XLOC(2)=2.*A/6.
XLOC(3)=3.*A/6.
ADEL(JK)=DEL
XLOC(JK)=3.*A/6. +DEL
DO 750 IPV=1, 20
IPTV(IPV)=IFIX(10000. *P(IPV))

750 CONTINUE
CALL INITAUE(IE, 0)
CALL CONSO(IE, 0)
CALL LEX(IE, 1)
CALL TSCAL(IE, 0)
CALL LOAD(IE)

5 CONTINUE
CALL STIND(IE, 2277, 10000)
CALL STIND(IE, 2275, 0)
DO 10 K=1, 20
CALL STIND(IE, IPT(K), IPTV(K))

10 CONTINUE
C SET TIME BASE
CALL STIND(IE, 2273, 1000)
CALL READ(IE, 0200, IDUM)
CALL LOAD(IE)
C WRITE(4, 2000)
2000 FORMAT(IX, 27HSET IC POTS 260, 261, 262, 263)
RETURN
END
SUBROUTINE MCON
INTEGER PSI(100)
COMMON IJ, IK, IIJ, IDELX
CALL INITA(IE, 0)
CALL CONSO(IE, 0)
CALL TSCAL(IE, 2)
K=1
200    IJ=2225
       IK=2234
       IL=2246
       IM=2274
       IIJ=0201
       IF(K.GT.1) GO TO 206
       K=1
       IX=5000
       CALL STIND(IE, IJ, IX)
       CALL STIND(IE, IK, IX)
       CALL STIND(IE, IL, IX)
       CALL STIND(IE, IM, IX)
206    CALL CDN(IX, PSI, I, J)
       LIX=IX
       IJ=2234
       IIJ=0221
       IF(K.EQ.1) GO TO 207
       GO TO 208
207    IX=5000
       GO TO 209
208    IX=LX2
       CALL CDN(IX, PSI, I, J)
       LX2=IX
       IJ=2246
       IIJ=0241
       IF(K.EQ.1) GO TO 210
       GO TO 211
210    IX=5000
       GO TO 212
211    IX=LX3
       CALL CDN(IX, PSI, I, J)
       LX3=IX
       IJ=2274
       IIJ=0261
       IF(K.EQ.1) GO TO 213
       GO TO 214
213    IX=5000
       GO TO 215
214    IX=LX4
       CALL CDN(IX, PSI, I, J)
       LX4=IX
       K=K+1
       IJ=2225
       IIJ=0201
       IX=LX1
       CALL REDSI(IX, PSI, I, J, IB)
       IF(PSI(I).GE.-100. AND. PSI(I).LE.100) GO TO 220
       GO TO 226
220    IX=LX2
       IJ=0234
       IIJ=0221
       IX=LX2
CALL READSI(IX, PSI, I, J, IB)
IF(PSI(I), GE. -100. AND. PSI(I), LE. 100) GO TO 225
GO TO 203
225
IJ=2246
IIJ=0241
IX=LX3
CALL READSI(IX, PSI, I, J, IB)
IF(PSI(I), GE. -100. AND. PSI(I), LE. 100) GO TO 235
GO TO 211
226
GO TO 206
235 CONTINUE
C PAUSE
CALL IC(IE)
CALL STIND(IE, 2277, 0)
CALL READ(IE, 0200, IVDUM)
CALL READ(IE, 2222, IVDUM)
CALL WAIT(200)
RETURN
C STOP
END
SUBROUTINE CON(IX, PSI, I, J)
INTEGER PSI(100)
COMMON I, IK, IIJ, IDELX
CALL READ(IE, 2225, IX325)
CALL READ(IE, 2225, IX335)
CALL WAIT(70)
CALL READ(IE, 2210, IX315)
CALL WAIT(70)
CALL WAIT(70)
I = 1
CALL READSI(IX, PSI, I, J, IB)
IF(I .LE. 1) GO TO 50
IF(PSI(I).EQ.PSI(J)) GO TO 900
IF(PSI(I).GE.-100. AND.PSI(I).LE.100) GO TO 999
IF(I.GT.1) GO TO 15
IF(PSI(I).GT.100) GO TO 20
GO TO 100
IF(I.EQ.1) GO TO 20
IF(PSI(I).LT.0) GO TO 999
IF(PSI(I).LT.PSI(J)) GO TO 20
GO TO 100
IX = IX + IDELX
I = I + 1
J = I - 1
CALL READSI(IX, PSI, I, J, IB)
IF(I .LE. 1) GO TO 51
IF(PSI(I).EQ.PSI(J)) GO TO 900
IF(PSI(I).GE.-100. AND.PSI(I).LE.100) GO TO 999
IF(PSI(I).LT.100) GO TO 25
GO TO 20
IX = IX - IDELX
I = 1
CALL READSI(IX, PSI, I, J, IB)
IF(I .LE. 1) GO TO 52
IF(PSI(I).EQ.PSI(J)) GO TO 900
IF(PSI(I).GE.-100. AND.PSI(I).LE.100) GO TO 999
IX = IX - IDELX
I = I + 1
J = I - 1
CALL READSI(IX, PSI, I, J, IB)
IF(I .LE. 1) GO TO 53
IF(PSI(I).EQ.PSI(J)) GO TO 900
IF(PSI(I).GE.-100. AND.PSI(I).LE.100) GO TO 999
IF(PSI(I).LT.100) GO TO 25
GO TO 31
IX = IX - IDELX
I = I + 1
J = I - 1
CALL READSI(IX, PSI, I, J, IB)
IF(I .LE. 1) GO TO 54
IF(PSI(I).EQ.PSI(J)) GO TO 900
IF(PSI(I).GE.-100. AND.PSI(I).LE.100) GO TO 999
IF(PSI(I).GT.100) GO TO 999
IF(PSI(I).GT.PSI(J)) GO TO 100
54
IX=IX+IDELX
I=1
110 CALL READSI(IX, PSI, I, J, IB)
IF(I.LE.1)GO TO 55
IF(PSI(I).EQ.PSI(J))GO TO 900
55 IF(PSI(I).GE.-100. AND. PSI(I).LE.100)GO TO 999
111 IX=IX+IDELX
I=I+1
J=I-1
CALL READSI(IX, PSI, I, J, IB)
IF(I.LE.1)GO TO 56
IF(PSI(I).EQ.PSI(J))GO TO 900
56 IF(PSI(I).GE.-100. AND. PSI(I).LE.100)GO TO 999
IF(PSI(I).GE.100)GO TO 999
IF(PSI(I).LT.PSI(J))GO TO 100
GO TO 111
900 WRITE(4,901)
901 FORMAT(1X,2HFU)
999 CONTINUE
RETURN
END
SUBROUTINE READSI(IX, PSI, I, J, IB)
INTEGER PSI(100)
COMMON IJ, IK, IIJ, IDELX
COMMON DIM/B, IBR
IB=9000
CALL IC(IE)
CALL WAIT(70)
CALL STIND(IE, IJ, IX)
CALL STIND(IE, 2237, IBR)
CALL WAIT(10)
CALL READ(IE, 0200, IDZ)
CALL WAIT(70)
CONTINUE
C
C ANALOG CONTROL LOOP
C USES ANALOG COMPARATOR, 331
CALL OP(IE)
CALL WAIT(1000)
CALL HOLD(IE)
CALL WAIT(70)
CALL READ(IE, IJ, IPSI)
CALL WAIT(70)
CALL IC(IE)
CALL WAIT(30)
CALL STIND(IE, 2237, 0)
PSI(I) = IPSI
CALL READ(IE, IJ, IXP)
CALL WAIT(70)
WRITE(4, 1006) IJ, IX, IXP, IPSI, PSI(I)
CALL WAIT(100)
IDELX=10
IF(IABS(PSI(I)).GE.4000)GO TO 10
IF(IABS(PSI(I)).GE.2000)GO TO 9
IF(IABS(PSI(I)).GE.1000)GO TO 8
IF(IABS(PSI(I)).GE.500)GO TO 7
IF(IABS(PSI(I)).GE.350)GO TO 6
IDELX=IDELX
GO TO 125
6
IDELX=2*IDELX
GO TO 125
7
IDELX=3*IDELX
GO TO 125
8
IDELX=6*IDELX
GO TO 125
9
IDELX=9*IDELX
GO TO 125
10
IDELX=14*IDELX
125 RETURN
END
SUBROUTINE PDE2
C      PROGRAM PDE*'122673**
DIMENSION IPT(20), IPTV(20), ADEL(18)
COMMON/P1/HK, JN, KZ
COMMON/POT/PS(20), DEL, A
COMMON/DRUP/XLOC(18), IY(18), ITM(12), ISTA(20, 12), IV(4), DELTA(15)
COMMON/DIM/B, IBR
DATA IPT/2224, 2223, 2232, 2235, 2245, 2236, 2230, 2226, 2227, 2233,
1  2242, 2243, 2244, 2247, 2253, 2256, 2264, 2265, 2266, 2276/
600     FORMAT(F5.2, 5F5.4)
405     CONTINUE
750     CONTINUE
CALL INITA(IE, 0)
CALL CONSO(IE, 0)
CALL LEXIE, 0
5       CONTINUE
CALL STINDIE, 2277, 10000)
CALL STINDIE, 2275, 0)
C       SET TIME BASE
CALL STINDIE, 2273, 1000)
ITM(1)=0
CALL WAIT(70)
C CALL STINDIE, 2275, 10000)
CALL WAIT(70)
K=1
MR=1
300     CONTINUE
DO 3000 K=1, 12
Y=B*FLOAT(K)/11
IYAS=IFIX(10000.*Y)
CALL WAIT(70)
CALL HOLD(IE)
CALL STINDIE, 2237, IYAS)
CALL WAIT(70)
CALL READIE, 0200, IVDUM)
CALL WAIT(100)
CALL READIE, 0241, ISTA(3, K))
CALL WAIT(70)
CALL READIE, 0221, ISTA(2, K))
CALL WAIT(70)
CALL READIE, 0201, ISTA(1, K))
CALL WAIT(70)
CALL READIE, 0251, ISTA(JK, K))
CALL WAIT(70)
CALL READIE, 0271, ITM(K))
CALL WAIT(70)
C IF(ITM(K).GE IBR)GO TO 102
J1=ISTA(1, K)
J2=ISTA(2, K)
J3=ISTA(3, K)
J4=ISTA(4, K)
IX=ITM(K)
CALL WAIT(100)
400     CONTINUE
IF(K.EQ.12)GO TO 102
CALL OP(IE)
CALL WAIT(1000)
CALL HOLD(IE)
C
GO TO 400
C
IF(MR.EQ.1)K=0
MR=MR+1
C
IF(K.GE.12)GO TO 102
C
K=K+1
C
GO TO 300
3000 CONTINUE
102 CONTINUE
CALL WAIT(100)
KZ=K
CALL IC(IE)
CALL WAIT(200)
200 FORMAT(1X,5(1X,I7))
CALL IC(IE)
CALL WAIT(1000)
CALL STIND(IE, 2275, 0)
CALL WAIT(100)
I=2225
DO 2001 NI=1, 4
   229 GO TO (231, 227, 228, 229), NI
   228 I=2274
   227 GO TO 231
   226 I=2246
   225 I=2234
231 CALL WAIT(100)
CALL READ(IE, I, IV(NI))
CALL READ(IE, I, IV(NI))
CALL WAIT(70)
2001 CONTINUE
CALL WAIT(70)
WRITE(4,2005) IV(1), IV(2), IV(3), IV(4)
2005 FORMAT(1X, 21HPOTS 225, 234, 246, 274, 4(1X, I7))
   CALL WAIT(70)
500 CONTINUE
2020 FORMAT(1X, 9(1X, I7))
2021 FORMAT(1X, T4, 'ITM', T14, 'X1', T22, 'X2', T30, 'X3', T38,
   1 X4', T46, 'X5', T54, 'X6', T62, 'X7', T70, 'X8')
RETURN
END
SUBROUTINE DISK
DIMENSION TST(2)
COMMON/W/Y(18), IPSI
COMMON/DRWP/XLOC(18), IY(18), ITM(12), ISTA(20, 12), IV(4), DELTA(15)
COMMON/POTV/P(20), DEL, A
COMMON/DIN/B, IBR
COMMON/P1/NA, JK, KZ
COMMON/GRD/NPSIG, NTF
DATA TST(1), TST(2)/3HTST, 4H SRC/
CALL ENTER(7, TST)
NTF3=NTF+3
DO 500 M=1, NTF3
DO 500 NZ=1, KZ
WRITE(7, 2020) ITM(NZ), ISTA(M, NZ)
500 CONTINUE
2020 FORMAT(1X, 9(1X, I7))
CALL CLOSE(7)
RETURN
END
SUBROUTINE DRW
COMMON/W/Y(18), IPSI
COMMON/DR/Z(38), ZY(38)
COMMON/DRW/XLOC(:6), IV(18), ITM(12), ISTA(28,12), IV(4), DELTA(15)
COMMON/POTV/P(20), DEL, A
COMMON/DIM/E, IBR
COMMON/GRD/NPSIG, NTF
NTF3=NTF+3
DO 99 N=1,NTF3
C READ(4,98)XLOC(N), Y(N)
99 CONTINUE
98 FORMAT(2F10.5)
CALL INITT(0)
CALL ERASE
CALL MOVABS(100,100)
CALL DRWABS(100,700)
CALL DRWABS(100,700)
CALL DRWABS(100,100)
CALL DRWABS(100,100)
NLY=48
LY=100
NLYT=IFIX(10.*B)+48
DO 251 N=1,10
CALL MOVABS(100,LY)
CALL DRWABS(90,LY)
CALL MOVABS(50,LY)
CALL ANCHO(48)
CALL ANCHO(46)
CALL ANCHO(NLY)
NLY=NLY+1
IF(NLY.GT.NLYT)GO TO 261
LY=(600/(NLYT-48))LY
251 CONTINUE
261 CONTINUE
DO 200 MT=1,NTF3
XI=A-XLOC(MT)
KL=IFIX(XLOC(MT)*(900./A))+100
KLI=IFIX(XI*(900./A))+100
CALL MOVABS(KL,90)
CALL DRWABS(KL,700)
CALL MOVABS(KLI,700)
CALL DRWABS(KLI,90)
CALL MOVABS(KLI-10,80)
XLOCX=XLOC(MT)
ID1=IFIX(XLOCX*10.)
ID2=IFIX(XLOCX*10.)-(10*ID1)
XLOC=XLOC+X/A/8.
IXC2=48
IXC1=48
DO 252 NR=1,9
IF(ID1.EQ.NR)IXC1=IXC1+NR
IF(ID2.EQ.NR)IXC2=IXC2+NR
252 CONTINUE
CALL ANCHO(46)
CALL ANCHO(IXC1)
CALL ANCHO(IXC2)
CONTINUE
CALL MOVABS(20,700)
CALL ANCHO(66)
CALL MOVABS(1000,30)
CALL ANCHO(65)
CONTINUE
CALL MOVABS(50,50)
CALL HOME
CALL ANMODE
RETURN
END
SUBROUTINE DRWA
COMMON/W/Y(18), IPSI
COMMON/DR/Z(36), ZY(36)
COMMON/DRW/P/XLOC(18), IV(18), ITM(12), ISTA(20, 12), IV(4), DELTA(15)
COMMON/POTY/P(20), DEL, A
COMMON/DIM/B, IBP
COMMON/GRD/NPSIG, NTF
IF(NPSIG. NE. 1) GO TO 100
CALL MOVABS(100, 100)
CALL DRWABS(100, 700)
CALL DRWABS(100, 700)
CALL DRWABS(100, 100)
CALL DRWABS(100, 100)
100 CONTINUE
NTF3=NTF+3
DO 100 N=1, NTF3
Z(N)=XLOC(N)
Z(N+NTF3)=A-XLOC(N)
ZY(N)=Y(N)
ZY(N+NTF3)=Y(N)
100 CONTINUE
ITOT=1
300 CONTINUE
IF(ITOT. GT. 2000) GO TO 400
NTFR=(2*NTF3)-1
DO 220 N=1, NTFR
IF(Z(N+1). LT.Z(N)) GO TO 598
GO TO 220
598 ZV1=Z(N+1)
ZV2=Z(N)
ZY1=ZY(N+1)
ZY2=ZY(N)
Z(N)=ZV1
Z(N+1)=ZV2
ZY(N)=ZY1
ZY(N+1)=ZY2
N=1
ITOT=ITOT+1
220 GO TO 300
400 CONTINUE
301 FORMAT(1X, T5, 'X', T15, 'Y')
302 FORMAT(1X, 2F10.5)
CALL HOME
PSI=FLOAT(IPSI)/100.
X=0.
DO 498 NP=1, 1000
TPSI=100.*SIN((3.14159*X)/A)
IF(TPSI.GE.PSI) GO TO 497
X=X+.005
498 CONTINUE
497 IZ=IFIX(X*<900./A)>+100
XEND=A-X
CALL MOVABS(IZX, 100)
NTFRA=NTFR+1
DO 411 NO=1, NTFRA
IF(NO. EQ. 1) GO TO 473
ZDEL=Z(NQ)-Z(NQ-1)
IF(ZDEL.LT.(.001))GO TO 411
473 CONTINUE
KLX=IFIX(Z(NQ)*(900. /A))+100
IF(KLX.LT.IZX)GO TO 411
IZXE=IFIX(XEND*(900. /A))+100
IF(KLX.GT.IZX)GO TO 411
KLY=IFIX(ZY(NQ)*(600. /B))+100
C IF(NQ.EQ.9)GO TO 413
465 CONTINUE
CALL DRWABS(KLX,KLY)
GO TO 411
413 CONTINUE
ID1=IPSI/1000
ID2=(IPSI-(ID1*1000))/100
ICX2=48
ICX1=48
DO 414 N=1,9
IF(ID1.EQ.N)ICX1=ICX1+N
IF(ID2.EQ.N)ICX2=ICX2+N
414 CONTINUE
C CALL ANCHO(ICX1)
C CALL ANCHO(ICX2)
C CALL MOVREL(-20.0)
GO TO 465
411 CONTINUE
IZXA=IFIX(XEND*(900. /A))+100
CALL DRWABS(IZXA,100)
CALL ANMODE
C STOP
RETURN
END
C-3. Exact Solution Listings. The following listings are used for the exact solution, which is run using “E-IDEA” since the exact solution also required chaining.

```
$ E  IDEA

SUBROUTINE POT
    DIMENSION TST(2)
    COMMON/Y/IV(10), IPSI
    COMMON/IRUP/XLOC(18), IV(18), ITM(12), ISTA(20, 12), IV(4), DELTA(15)
    COMMON/POTY/P(20), DEL, A
    COMMON/BIM/B, IBR
    COMMON/F1/NA, JK, KZ
    COMMON/GRD/NPSIG, NTF
    DATA TST(1), TST(2)/3HTST, 4H SRC/
    JK=4
    NA=1
    WRITE(4,601)
    READ(4,600)A
    READ(4,600)B
    IBR=IFIX(10000.*B)
    WRITE(4,2051)
2051 FORMAT(1X,25HSpecify no of lines lt 16)
    READ(4,6004)NLINES
6004 FORMAT(I2)
    DELTX=.5/(FLOAT(NLINES))
    DELTA(1)=DELTX
    NTF=NLINES-1
    DO 2050 NT=2,NTF
        DELTA(NT)=DELTA(NT-1)+DELTX
2050 CONTINUE
    CONTINUE
    DELTA(1)=1./18.
    DELTA(2)=2./18.
    DELTA(3)=4./18.
    DELTA(4)=5./18.
    DELTA(5)=7./18.
    DELTA(6)=7.3/18.
    DELTA(7)=7.6/18.
    DELTA(8)=8./18.
    DELTA(9)=8.1/18.
    DELTA(10)=8.2/18.
    DELTA(11)=8.3/18.
    DELTA(12)=8.4/18.
    DELTA(13)=8.5/18.
601 FORMAT(1X, 'Input A, B, DEL1, DEL2, DEL3, DEL4, DEL5 : F5.2, 5F5.4')
900 CONTINUE
697 FORMAT(1X, 'Input: A, DEL: F5.2, F5.4')
    WRITE(4,11)DELTA(NA)
    DEL=DELTA(NA)
11 FORMAT(1X, 'DEL=', F10.4)
    CR=2.*DEL
    C4=CR*A*(3./6.)
    DX11=A/6.
    DX21=A/6.
    DX31=(A+(6.*C4))/12.
```
DX41 = C4/2.
DX12 = H / 6.
P<4> = SIN (3.14159 / 6.)
P<5> = 01 / (DX11 * DX22)
P<6> = 1.
P<7> = SIN (3.14159 * 2.6.)
P<8> = 1
P<9> = 01 / (DX22 * DX21)
P<10> = 01 / (DX21 * DX22)
P<11> = SIN (3.14159 * 3.6.)
P<12> = 01 / (DX22 * DX32)
P<13> = DX32 / DX42
P<14> = 1
P<15> = 01 / (DX31 * DX42 * C1)
P<16> = DX32 / DX52
P<17> = SIN (3.14159 * 1. + CR) / 2.
P<18> = 01 / (DX41 * DX42 * C1)
P<19> = DX32 / DX42
P<20> = 01 / (DX41 * DX52 * C2)

IF (P<14>.LT.1.) GO TO 698
PT0T1 = P<14> * P<15>
P<14> = 1
P<15> = PT0T1
PT0T2 = P<18> * P<19>
P<19> = 1
PT0T3 = P<16> * PT0T2

IF (P<16>.LT.1.5) GO TO 699
PT0T3 = P<16> * PT0T2

DO 700 NP = 1, 20
CURITE<4,6010>P<NP>
700 CONTINUE
600 FORMAT (F5.2, F5.4)
6810 FORMAT (I10, I10, 2E16.4)
CONTINUE

FORMAT(1X,9(1X,I7))
FORMAT(1X,T4, 'ITM', T14, 'X1', T22, 'X2', T30, 'X3', T38, 'X4', T46, 'X5', T54, 'X6', T62, 'X7', T70, 'X8')

SEARCH FOR SPECIFIED PSI FOR EQUIPOT PRINTOUT

CONTINUE

WRITE(4,2006)
FORMAT(1X,11HSPECIFY PSI)
READ(4,1009) PSI
READ(4,1009) PSID
READ(4,1021) NPSIG
CONTINUE

DO 4000 IIR=1, NPSIG
   PSI=IFIX(PSI+100.)
4000 CONTINUE

FORMAT(1X,T9, 'PSI', T20, 'Y', T31, 'X', T42, 'XI')
K=1
NTF3=NTF+3
DO 1000 N=1, NTF3
   DO 1001 NA=1, K2
      IF(ISTA(N,NA).LT.IPSI) GO TO 1002
   CONTINUE
   NB=NA
   NC=NB+1
   DELSTA=FLOAT(ISTA(N,NC)-ISTA(N,NB))
   DELTM=FLOAT(ITM(NC)-ITM(NB))
   Y(K)=FLOAT(ITM(NC))-(DELTM*(FLOAT(ISTA(N,NC)-IPS)/DELSTA))/10000.
   X=XLOC(N)
   XI=A-XLOC(N)
   K=K+1
1000 CONTINUE

FORMAT(2F10.3,12)
FORMAT(1X,4(1X,F10.3))
IF(IZR.GT.1) GO TO 4001
IF(NPSIG.GT.0) GO TO 4001
CALL DRW

CONTINUE

CALL DRWA
PSI=PSI+PSID
CONTINUE

PSI=PSI-(FLOAT(NPSIG)*PSID)
NPSIG=1
WRITE(4,1011)
WRITE(4,1012)

FORMAT(1X,18HNO GRID OR RESTART)
FORMAT(1X,18HTYPE 2 FOR RESTART)
READ(4,1021) MST
IF(MST.EQ.2) GO TO 4003
GO TO 4002
STOP
END
SUBROUTINE EXACT
COMMON/DRWP/XLOC(18), IY(18), ITM(12), ISTA(20, 12), IV(4),
1 DELTA(15)
COMMON/POTV/P(20), DEL, A
COMMON/DIM/B, IBR
COMMON/P1/NA, JK, KZ
PI=3.1415926
XLOC(1)=A/6.
XLOC(2)=2.*A/6.
XLOC(3)=3.*A/6.
DO 5 I=4, 18
XLOC(I)=XLOC(3)+DELTA(I-3)
5 CONTINUE
K2=12
DO 10 IX=1, 18
DO 20 IY=1, 12
Y=B*FLOAT(IYA-1)/11.
ITM(IYA)=IFIX(Y*10000.)
Q1=PI*XLOC(IX)/A
Q2=PI*B/A
Q3=PI*(B-Y)/A
PSI=100.*SIN(Q1)*(EXP(Q3)-EXP(-Q3))/(EXP(Q2)-EXP(-Q2))
ISTA(IX, IYA)=IFIX(PSI*100.)
20 CONTINUE
10 CONTINUE
RETURN
END
SUBROUTINE DRW
COMMON/W/Y(18), IPSI
COMMON/DR/Z(30), ZY(30)
COMMON/DRU/P/XLOC(18), IY(18), ITM(12), ISTA(20, 12), IV(4), DELTA(15)
COMMON/POTV/P(20), DEL, A
COMMON/DIM/B, IBR
COMMON/GRD/NPSIG, NTF
NTF3=NTF+3
DO 99 N=1, NTF3
READ(4,98)XLOC(N), Y(N)
99 CONTINUE
98 FORMAT(2F10.5)
CALL INITT(0)
CALL ERASE
CALL MOVABS(100, 100)
CALL DRWABS(100, 700)
CALL DRWABS(1000, 700)
CALL DRWABS(1000, 100)
CALL DRWABS(100, 100)
NLY=48
LY=100
NLYT=IFIX(10.*B)+48
DO 251 N=1, 10
CALL MOVABS(100, LY)
CALL DRWABS(90, LY)
CALL MOVABS(50, LY)
CALL ANCHO(48)
CALL ANCHO(46)
CALL ANCHO(NLY)
NLY=NLY+1
IF(NLY.GT.NLYT)GO TO 261
LY=(600/(NLYT-48)) + LY
251 CONTINUE
261 CONTINUE
DO 200 MT=1, NTF3
XI=A-XLOC(MT)
KL=IFIX(XLOC(MT)*900./A)+100
KLI=IFIX(XI*900./A)+100
CALL MOVABS(KL, 90)
CALL DRWABS(KL, 700)
CALL MOVABS(KLI, 700)
CALL DRWABS(KLI, 90)
CALL MOVABS(KL-10, 80)
XLOC=XLOC(MT)
ID1=IFIX(XLOC*10.)
ID2=IFIX(XLOC*100.)-(10*ID1)
XLOC=XLOC+A/8.
IXC2=48
IXC1=48
DO 252 NR=1, 9
IF(ID1.EQ.NR)1XC1=IXC1+NR
IF(ID2.EQ.NR)1XC2=IXC2+NR
252 CONTINUE
CALL ANCHO(46)
CALL ANCHO(IXC1)
CALL ANCHO(IXC2)
CONTINUE
CALL MOVABS(20, 700)
CALL ANCHO(65)
CALL MOVABS(1000, 30)
CALL ANCHO(65)

CONTINUE
CALL MOVABS(50, 50)
CALL HOME
CALL ANMODE
RETURN
END
SUBROUTINE DRWA
COMMON/U/Y(18), IPSI
COMMON/DP/Z(36), ZY(36)
COMMON/DRP/XLOC(18), IY(18), ITM(12), ISTA(20, 12), IV(4), DELTA(15)
COMMON/POTV/P(20), DEL, A
COMMON/IDM/I, IBR
COMMON/GRD/NPSIG, NTP
IF(NPSIG.NE.1)GO TO 1000
CALL MOVABS(100,100)
CALL DRWABS(100, 700)
CALL DRWABS(1000,700)
CALL DRWABS(1000,100)
CALL DRWABS(100,100)
1000 CONTINUE
NTF3=NTF+3
DO 100 N=1,NTF3
Z(N)=XLOC(N)
Z(N+NTF3)=A-XLOC(N)
ZY(N)=Y(N)
ZY(N+NTF3)=Y(N)
100 CONTINUE
ITOT=1
300 CONTINUE
IF(ITOT.GT.2000)GO TO 400
NTFR=(2*NTF3)-1
DO 220 N=1,NTFR
IF(Z(N+1).LT.Z(N))GO TO 598
GO TO 220
598 ZV1=Z(N+1)
ZV2=Z(N)
ZY1=ZY(N+1)
ZY2=ZY(N)
ZV1=ZV1
ZV2=ZV2
ZY1=ZY1
ZY2=ZY2
N=1
ITOT=ITOT+1
GO TO 300
220 CONTINUE
400 CONTINUE
301 FORMAT(1X,T5, 'X', T15, 'Y')
302 FORMAT(1X, 2F10.5)
CALL HOME
PSI=FLOAT(IPSI)/100.
X=0.
DO 498 NP=1,1000
TPSI=100.*SIN((3.14159265/(A)
IF(TPSI .GE. PSI)GO TO 497
X*X+.005
497 CONTINUE
498 CONTINUE
IZX=IFIX(X*(900./A))+100
XEND=X-X
CALL MOVABS(IZX,100)
NTFR=NTFR+1

DO 411 NO=1, NTFRA
IF(No.EQ.1) GO TO 473
ZDEL=Z(N0)-Z(N0-1)
IF(ZDEL.LT.(.001)) GO TO 411

473 CONTINUE
Klx=IFIX(Z(NO)*<900, /A)>+100
IF(Klx.LT.IZX) GO TO 411
IZXE=IFIX(XEND*(900, /A)>+100
IF(Klx.GT.IZX) GO TO 411
KLy=IFIX(ZY(NO)*<600, /B)>+100
C IF(No.EQ.9) GO TO 413

465 CONTINUE
CALL DRWABS(KLX, KLY)
GO TO 411

413 CONTINUE
ID1=IPSI/1000
ID2=(IPSI-ID1*1000)/100
ICX2=48
ICX1=48
DO 414 N=1, 9
IF(ID1.EQ.N) ICX1=ICX1+N
IF(ID2.EQ.N) ICX2=ICX2+N

414 CONTINUE
C CALL ANCHO(ICX1)
C CALL ANCHO(ICX2)
C CALL MOVREL(-20, 0)
GO TO 465

411 CONTINUE
IZXA=IFIX(XEND*(900, /A)>+100
CALL DRWABS(IZXA, 100)
CALL 'ANMODE
C STOP
RETURN
END
C-4. Stored Data for $\psi(x, y)$. The $\psi(y)$-data taken for each x-station during the exact and hybrid solutions are provided as comparison data between solutions.

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C-5. Program Control Flow Charts. The hybrid program is shown in Figure C-1, while the exact program is depicted in Figure C-2.
Figure C-1. Block diagram – hybrid program.
Figure C-2. Block diagram — exact program.
C-6. Chaining Routine. The chaining routine is as follows:

```plaintext
CLOGIC END
SRK RN -5
SMK ON
SX ON
SCHAIN
CHAIN VGA

"#HE XCT FILE
>PDRZL
LIST OPTIONS & PARAMETERS
>EX, 1G5, S2
DEFINE RESIDENT CODE
>POT, ASTRU, GMMUX, #DDR, #RUN, #CON0, #INIT*
DESCRIBE LINKS & STRUCTURE
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>L2=POT/CON,READSI
>L3=POT
>L4=DISK
>L5=REW
>L6=DECVA
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37533-37636 02124

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WAIT 34356-34496 23331
SET 33745-33435 03411
IC 33377-33374 23346
KYSPD 33125-33376 02272
ISTAT 32764-33124 03121
ADDR 32677-32763 23965
1ICPG 32521-32576 33156
GEMARD 32577-32528 02222
SETLIZ 32252-32276 33705
INITA 32111-32051 02941
FLOAT 32022-32021 02921
F1NX 31765-31777 33913
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