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I. Kupiec

Experimental Verification of the Performance of the Aperture Sampling Technique

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FOR THE COMMANDER

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ABSTRACT

An experimental verification of the Aperture Sampling technique as a means of estimating the angle-of-arrival in the presence of ground reflection interference is reported. The technique is found to perform satisfactorily down to 1/5 of a beamwidth elevation. Beam splitting better than 20:1 is demonstrated. This performance is obtained by using a minimizing search processing that allows use of a larger number of samples than the number required by the closed form solution.

CONTENTS

I. INTRODUCTION

This report describes the initial experiment that was conducted to verify the performance of the Aperture Sampling technique as a means of overcoming low elevation multipath errors in radars. This technique improves the elevation angle-of-arrival estimate from a target in the presence of interfering ground reflections, where large measurement errors occur for conventional radars. The term conventional radar refers here to radars that use monopulse or conical scan techniques to measure the angle-of-arrival.

The previous part of this program was devoted to a critical evaluation of available techniques and development of modifications suitable for ground radar operations and has been reported.[1] It was concluded in that report that the Aperture Sampling technique was superior to other techniques in the presence of a single specular reflection, and that it could be easily extended to handle multiple specular reflections if required.

Questions concerning the actual performance of the Aperture Sampling techniques remained unresolved. In the case of rough terrain, the reflection from the ground includes diffuse reflection in addition to a specular reflection. Some knowledge concerning potential performance was obtained by using a simple modeling of diffuse reflection in a simulation program. While degradation of performance was observed in these simulations, lack of a viable model for ground diffuse reflection prevented definite conclusions.

In these simulations Gaussian noise was used to represent the receiver noise, but for an actual radar environment the thermal noise is not expected to be the performance limiting factor. In the case of realistic ground terrain, the amount of diffuse reflection present in many cases surpasses receiver noise and would constitute the limitation on angular measurement accuracy. Therefore, in addition to analysis and simulations a meaningful experiment is required in order to verify the usefulness of the technique.

In planning such an experiment, the question of tractability is of tantamount importance. It is obvious that if one submits the technique to an extremely rough terrain interpretation of the results, no matter how successful they are, is quite difficult. In such a case one needs a precise characterization of the terrain, from topographical structures to the constituents of the surface grass, soil, trees, etc. In order to avoid such difficulties, it was decided to initially perform the experiment over a flat terrain, as close as possible to an ideal terrain.

The initial experimental results in this report were obtained at the Lincoln Laboratory antenna range. Use of this range for initial tests provides two significant advantages. First, the ground model is simple and reflection from it consists mainly of a specular reflection. Thus, it is possible to describe the environment in terms of few parameters and interpretation of the results is easier. The second advantage of this approach is that measurements over flat terrain set a practical limit on the performance, and thus enable one to assess the value of the technique before carrying out a more complicated experiment.

II. Aperture Sampling Technique

For the sake of completeness, a brief description of the Aperture Sampling technique is presented in this section.

The difficulties in estimating the elevation angle of a target at low elevation stem from the presence of strong ground reflections. This occurs when the target is below 1.5 beamwidths above the ground. In that case, if a conventional angle measurement technique is used, the angular resolution of the antenna is not sufficient to resolve the direct return. Since monopulse and conical scan techniques rely on the presence of a single plane wave, an attempt to use these techniques in the presence of strong multipath yields large errors.

When a single plane wave is incident on an antenna the aperture illumination is uniform in amplitude and linear in phase, in which case the relation between the angle-of-arrival and the illumination is simple. When more than one plane wave is present, this relation is non-linear and complicated. Special techniques must be used in order to derive the angle-of-arrival of the waves from the aperture illumination. In the case of multipath, the situation is even more complicated. In that case, there exists one or more specular reflections, which can be modeled as distinct plane waves. In addition, there will be present diffuse ground reflections, which are represented by a continuous spatial spectrum. At low elevation angles, the diffuse reflection usually is small compared to specular reflection. It can then be considered as a low level interference. Thus, the problem of estimating the elevation angle in the presence of multipath can be viewed as a problem of resolving two or more plane waves in the presence of diffuse reflection and thermal noise interference. Monopulse uses sum and difference of the illumination to determine angle-of-arrival. This turns out to be insufficient for more than one plane wave. The Aperture Sampling technique attempts to accomplish such a goal by sampling the aperture illumination at several points and making use of the illumination structure to resolve in angle a multiplicity of distinct plane waves.

To understand how the technique works, assume that the antenna aperture is sampled at M equidistant points, and let N distinct plane waves incident on the aperture. The voltage V_m at the m^{th} sampling element is given by

$$V_m = \sum_{n=1}^{N} f(\alpha_n)\, A_n \exp(i\,\phi_n)\, \exp\left[ik(m-1)d \sin\alpha_n\right] \tag{1}$$

where α_n, A_n, and ϕ_n are the angle-of-arrival, amplitude and phase of the n^{th} plane wave. The phase is referred to the first sampling element (Fig. 1). $f(\alpha_n)$ is the voltage pattern of the sampling element and k is the free space wave number. d is the spacing between the elements. The amplitude and phase of a plane wave can be combined with the sampling element pattern and represented as an effective complex amplitude $B_n = f(\alpha_n)A_n\exp(i\phi_n)$. The sampled m^{th} voltage can then be written as

$$V_m = \sum_{n=1}^{N} B_n \exp\left[ik(m-1)d \,\mathrm{Sin}\, \alpha_n\right] \tag{2}$$

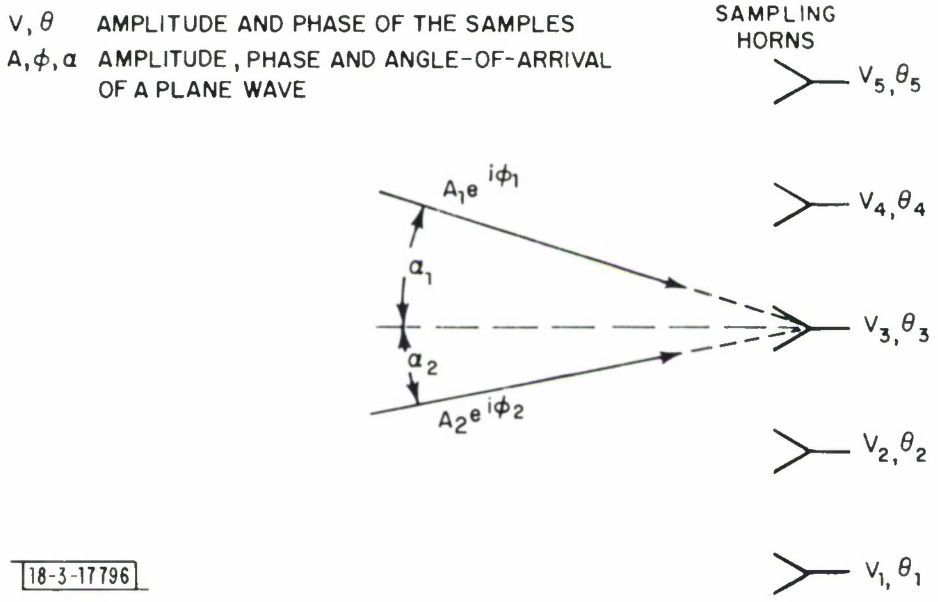

Fig. 1. Aperture sampling of plane waves illumination.

Alternatively, the relation between the column vector $\underline{V}$ representing the M sampled voltages and the column vector $\underline{B}$ representing the effective voltages of the N incident plane waves can be written in a matrix form,

$$\underline{V} = \underline{A}\ \underline{B} \tag{3}$$

Where $\underline{A}$ is a M x N matrix whose entiries are given by

$$A_{mn} = \exp\left[ikd(m-1)\ \mathrm{Sin}\ \alpha_n\right] \tag{4}$$

Eq. (2) can be used to resolve the incoming plane waves and estimate their angle-of-arrival, amplitude and phase. Since each plane wave is characterized by these three unknown parameters, and since each aperture sample yields two independent pieces of information, amplitude and phase, the number of samples needed to resolve N plane waves is $\frac{3N}{2}$.

In general, the presence of thermal noise will cause an error in such an estimation since the measured voltage $\underline{V}_m$ differs from the one predicted by the plane wave model. In the case of multipath the diffuse reflection is an additional source of error, since it too modifies the aperture in a manner unpredictable by the distinct plane wave model. While thermal noise errors may be reduced by increasing signal level, diffuse reflection is proportional to the signal level and increasing the signal level does not reduce this type of errors.

A closed form solution is avialable for Eq. (2), but unfortunately, it does not work all the time. There exist singular conditions under which the aperture illumination is not uniquely related to the incident plane waves. For these conditions very small perturbation in $\underline{V}_m$ results in large parameter estimation errors. [1,2] This difficulty may be overcome by means of a procedure that allows overdetermination (i.e., $M > \frac{3N}{2}$). If the source of error is thermal noise the aperture illumination error $\underline{V}-\underline{V}_m$ is Gaussian and the conditional probability of measuring $\underline{V}_m$ given that N plane waves incident on the aperture is

$$P\,(\underline{V}_m|\,\underline{V}) = (2\pi\sigma^2)^{-M} \exp\left[\frac{1}{2\sigma^2}\,(\underline{V}_m - \underline{V})^*(\underline{V}_m - \underline{V})\right] \tag{5}$$

where σ is the standard deviation of the noise and * denotes the conjugate transpose. For high signal-to-noise ratio this function has a narrow peak at $\underline{V} = \underline{V}_m$. When the errors are due to diffuse reflection, for which case the statistic is unknown, the functional form in (5) is different. However, if the diffuse reflection is small compared to the specular reflection, it will generally possess a narrow peak around $\underline{V} = \underline{V}_m$. Thus, the best estimate is given by the parameters that maximize the likelihood function (5) Or, as conjectured, by the parameters that minimize the product

$$g(\underline{V},\, \underline{V}_m) = (\underline{V}_m - \underline{A}\ \underline{B})^*(\underline{V}_m - \underline{A}\ \underline{B}) \tag{6}$$

Since $g(\underline{V}, \underline{V}_m) \geq 0$ the minimum value is zero, and hence the best parameter estimates are obtained from the

$$\underline{V} = \underline{V}_m = \underline{A}\ \underline{B} \tag{7}$$

However, such an approach requires exactly $\frac{3N}{2}$ samples and fails around certain singular points. We therefore look for a way to minimize (6) when there are more than $\frac{3N}{2}$ samples. An elegant solution to this problem was proposed by Hughes Aircraft Company [3] and was used extensively to process the data in the present experiment. It is particularly useful and convenient because it enables one to address the estimation of angle-of-arrival without explicitly estimating the phase and amplitude of the incoming waves. The function $g(\underline{V}, V_m)$ is minimized first with respect to $\underline{B}$. Since g is a positive-definite quadratic form a necessary and sufficient condition for a minimum [3] is that

$$\frac{d}{d\alpha}\, g\left[\underline{A}(\underline{B} + \alpha\underline{h}),\ \underline{V}_m\right]\Big|_{\alpha=0} = 0 \tag{8}$$

for all $\underline{h}$. Thus,

$$g\left[\underline{A}(\underline{B}+\alpha\underline{h}),\ \underline{V}_m\right] = \left[\underline{V}_m^* - (\underline{B}^* + \alpha\underline{h}^*)\underline{A}^*\right]\left[\underline{V}_m - \underline{A}(\underline{B} + \alpha\underline{h})\right] \tag{9}$$

or

$$\frac{\partial g}{\partial\alpha} = \underline{h}^*\ (\underline{A}^*\underline{A}\underline{B} - \underline{A}^*\underline{V}_m) + (\underline{B}^*\underline{A}^*\underline{A} - \underline{V}_m^*\underline{A})\underline{h} =$$

$$= 2\mathrm{Re}\left[\underline{h}^*(\underline{A}^*\underline{A}\underline{B} - \underline{A}^*\underline{V}_m)\right] = 0$$

Since (9) must be true for all $\underline{h}$ we obtain

$$\underline{A}^*\underline{A}\underline{B} - \underline{A}^*\underline{V}_m = 0 \tag{9a}$$

or

$$\hat{\underline{B}} = (\underline{A}^*\underline{A})^{-1} \underline{A}^*\underline{V}_m \tag{10}$$

Substitution of the estimated $\hat{\underline{B}}$ (amplitude and phase) in $g(\underline{A}\ \underline{B}, \underline{V}_m)$ yields

$$g(\underline{A}\hat{\underline{B}}, \underline{V}_m) = |\underline{V}_m|^2 - \underline{V}_m^* \underline{A}(\underline{A}^*\underline{A})^{-1} \underline{A}^*\underline{V}_m =$$

$$= |\underline{V}_m|^2 \left[1 - \frac{\underline{V}_m^* \underline{A}(\underline{A}^*\underline{A})^{-1} \underline{A}^*\underline{V}_m}{|\underline{V}_m|^2} \right] \geq 0 \tag{11}$$

Thus the minimum is obtained by maximizing

$$q = \frac{\underline{V}_m^* \underline{A}(\underline{A}^*\underline{A})^{-1} \underline{A}^* \underline{V}_m}{|\underline{V}_m|^2} \tag{12}$$

whose maximum value is one. As can be seen, q depends on $\underline{V}_m$ and $\underline{A}$ only and therefore on α_n alone.

For the purpose of evaluation of performance the maximization was conducted numerically. For an assumed number of plane waves q is evaluated successively for many combinations of angle-of-arrival within the unambiguous angle range of the sampling array. The combination that yields the largest q is taken as the estimate. The unambiguous angle interval for the numerical search of the angles that maximize q is determined by the interelement spacing d and the wavelength λ, i.e., $-\frac{\lambda}{2d} < \alpha_n < \frac{\lambda}{2d}$.

The granularity of the search was varied. The rough search, consistent with $\sim$25 dB SNR, was .05°. It was then followed by a local fine search.

III. Aperture Sampling Experimental Considerations

The particular selection of the location and geometry for the experiment was dictated by technical and logistic considerations. The test configuration is shown in Fig. 2. The sampled aperture consisted of six standard gain L-band horns. The target was simulated by a transmitting horn mounted on a remote controlled precision elevator. By accurately measuring the range geometry we were able to determine the true elevation angle of the horn with a precision of .005°. This corresponds to a 500:1 beam splitting since the effective beamwidth of the sampled array was 2.56°. This is well beyond the performance expected from the Aperture Sampling techniques.

The measurement was performed at L-band. CW operation was utilized since proper equipment was available. Careful placement of the equipment to reduce interference from objects on both sides of the range was required as CW operation does not allow the gating out of such interference. The location of the array and target horns was chosen to achieve at least 25 dB rejection by means of the horns' patterns. This was possible because of the relatively short distance between the target horn and the array (Fig. 3).

The use of a short range was advantageous from another consideration. The short range yields a larger angular range without using very high mounting towers. In addition to demonstrating the performance at low elevation corresponding to a small fraction of the effective beamwidth,

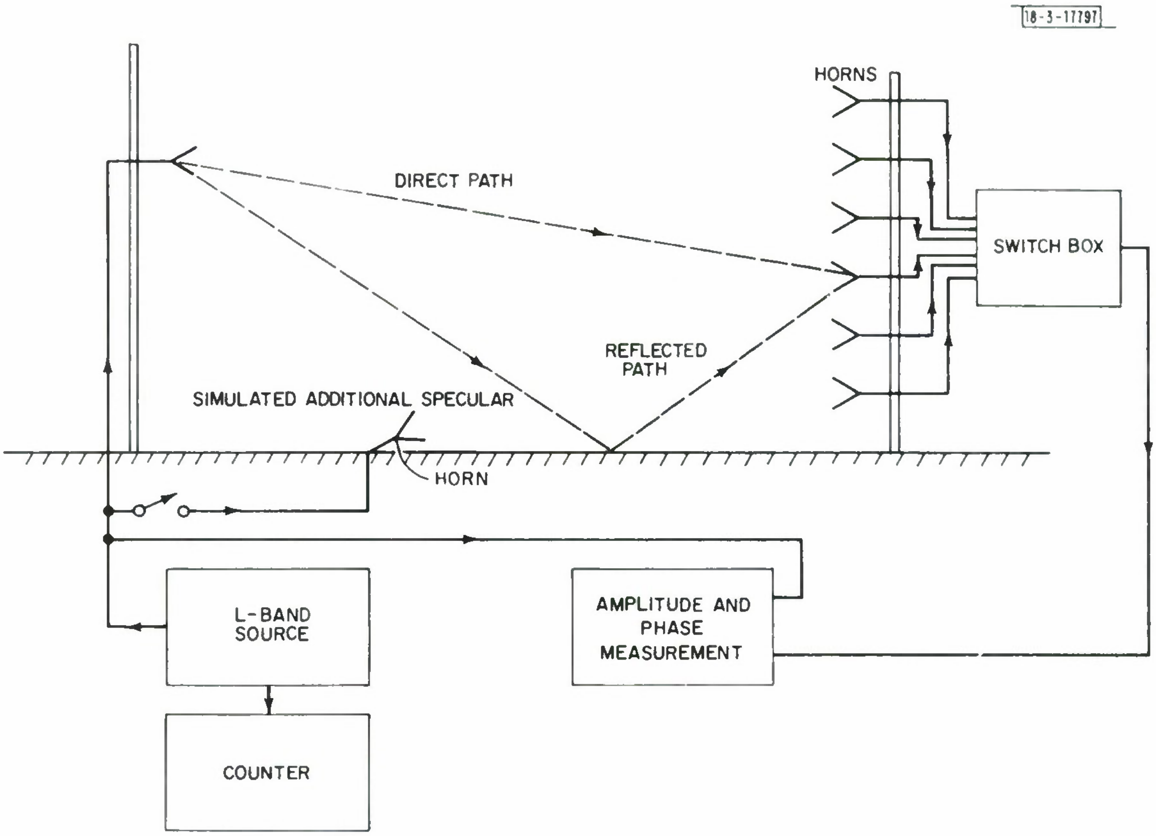

Fig. 2. Experimental setup.

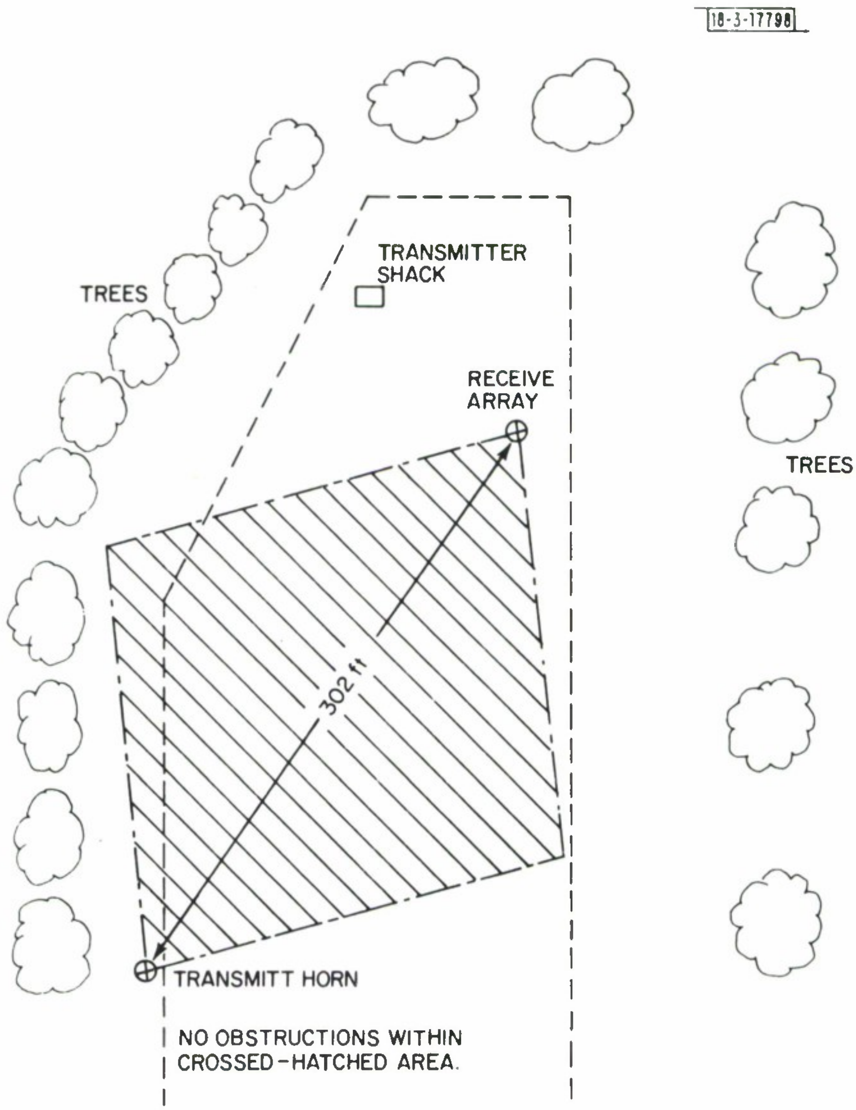

Fig. 3. Range plan.

the performance between one to two beamwidth above the ground was of interest to check the transition from multipath regime to no multipath. The short range enabled us to do that.

The disadvantage in using the short range is that while the target horn was in the far field of an individual sampling horn, it was in the near field of the total sampling aperture. Since the technique was developed for plane waves, a correction for the spherical aberration was necessary. The theory was modified to handle spherical waves from point sources at known ranges. In the case of the multipath experiment such a representation for the direct wave is satisfactory, however, it is questionable for the specular reflection. This representation is suspected of introducing a phase error that bounds the angle estimation precision. The spherical aberration correction was introduced by considering the specular reflection as resulting from an image of the transmitting horn. It will be seen later that this was indeed sufficient for demonstrating that the technique works.

The spherical aberration correction can be derived from Fig. 4 in the following manner:

$$R_n = [R^2 + n^2 d^2 - 2\,Rnd\,\mathrm{Sin}\,E_1]^{1/2} = R\,[1 + \frac{n^2 d^2}{2R^2} - \frac{nd\,\mathrm{Sin}\,E_1}{R}] \tag{13}$$

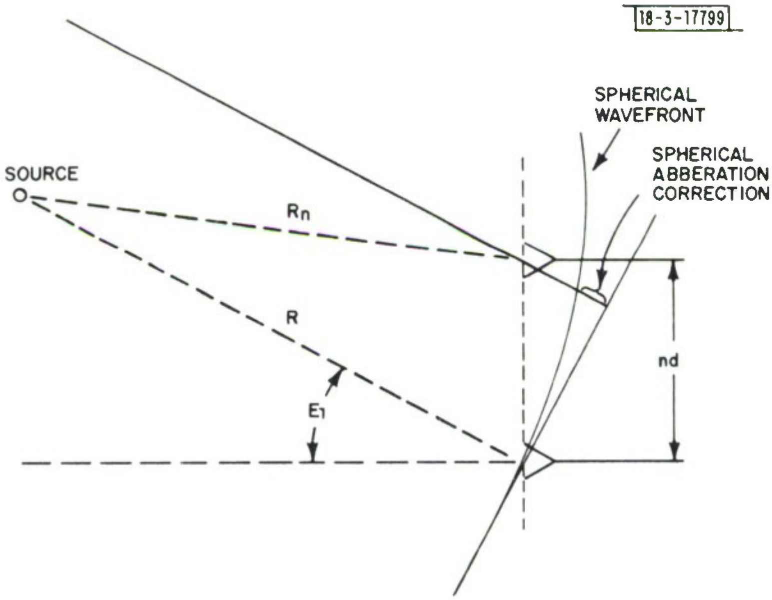

Fig. 4. Spherical abberation correction.

The phase difference is therefore

$$\delta\phi = k(R_n - R) = nkd \sin E_1 - \frac{kn^2d^2}{2R} \tag{14}$$

The first term represents the focused equivalent plane wave. The second term is a parabolic approximation to the spherical aberration. For our case when phase is referred to the lowest element this error could be as much as $\sim 190°$ at the highest horn. This correction was used in processing the measured data.

For the purpose of demonstration and assessment of performance bounds, it is important to have a single specular reflection. The compliance of the range with this requirement was checked prior to aperture sampling data collection. To that end amplitude and phase patterns on each of the array horns were recorded as the transmit horn is continuously moved along its pole. Typical patterns are shown in Fig. 5 and Fig. 6. The amplitude response shows a typical interference between two waves, with alternating constructive and destructive interference. The irregular phase pattern is due to the fact that the waves are spherical and the phase of each at a fixed point varies as a function of elevation. Thus, unlike plane waves the phase between them varies at a fixed point on the array as the transmit horn is moved. The absence of substantial diffuse reflection or multiple specular or any side interference can be observed in Figs. 5 and 6. The presence of such interference would manifest itself by a superimposed

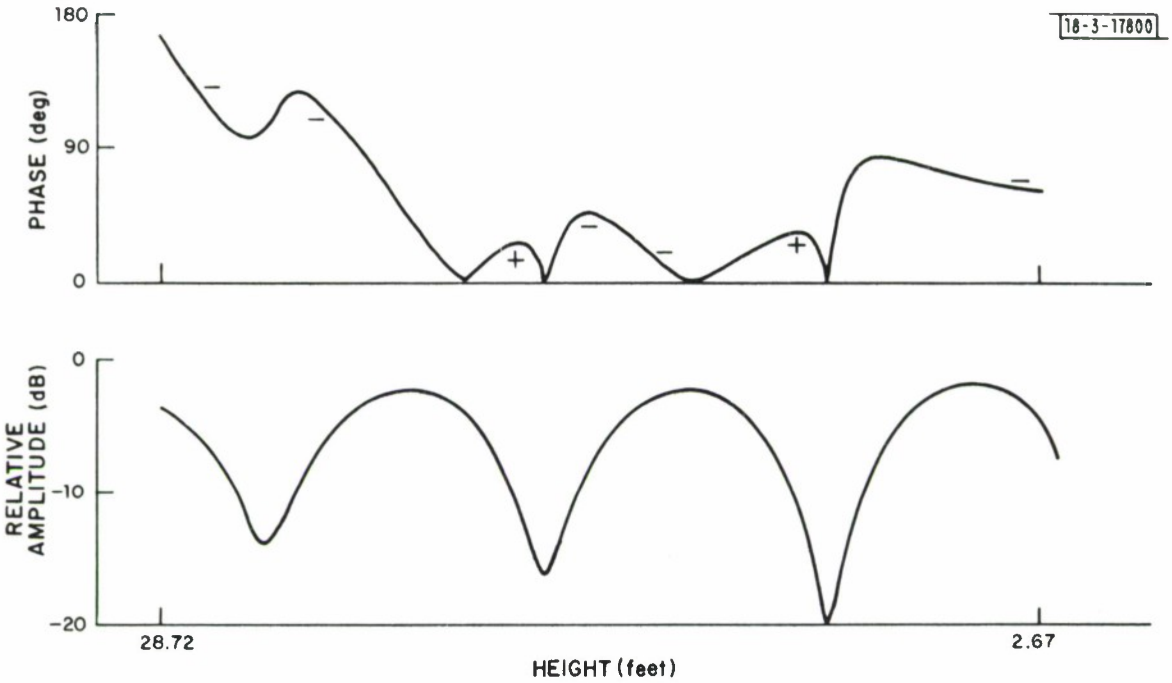

Fig. 5. Amplitude and phase of horn as a function of transmitter height.

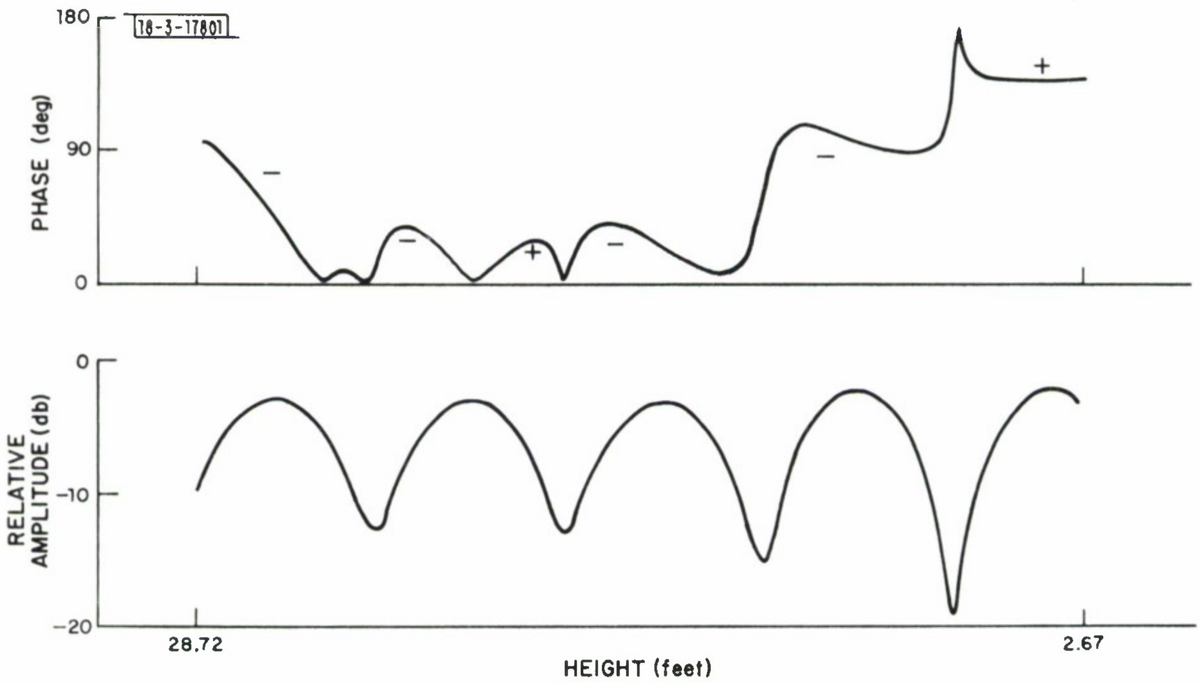

Fig. 6. Amplitude and phase of horn 6 as a function of transmitter height.

ripple on the curves. As can be seen, the ripple is very small and shows that the interference is indeed much smaller than the signal.

The amplitude patterns reveal another shortcoming of the short range, namely, the variation of the magnitude of the reflection coefficient. This departure from a plane wave behavior may be another cause for angle errors which would limit the angle estimation precision. For the same height of transmit horn the magnitude of the reflected wave is seen to be different for different horns in the array. This simply stems from the variation of the reflection coefficient as a function of grazing angle. The grazing angle being different for the various height receive horns. Therefore, rather than having a plane wave with a constant amplitude across the sampling array, the amplitude is non-uniform. The specular reflection may be thought of as having a narrowband spatial spectrum around the specular direction. The finite width of the spatial spectrum sets a bound on the precision of its direction determination. The uncertainty of the direction is of the order of the width of the spectral peak. The pattern corresponding to receive horn number 6 shows variation in the reflection coefficient between .55 - .7.

The measurement instruments are shown in Fig. 2. The CW equipment used has a measurement precision of .1 dB in amplitude and less than 2° in phase. This is sufficient for beam splitting well beyond our expectation from the technique's performance. The length of the cables from the horns to the measuring device was measured precisely in the Laboratory. However, the unequal hanging conditions and exposure to weather result in unknown phase variation. Such errors are estimated to be below 5°. Positioning of the horns and alignment of the array were kept to tolerances that would enable one to determine confidently beam splitting better than 50:1.

Since the waves incident on the sampling horns come from slightly different directions with respect to each horn's boresight, a correction to account for the horn's pattern is required. In the processing of the data we have neglected this effect thereby introducing a small phase and amplitude error. These errors are estimated to be less than 5° in phase and less than .2 dB in amplitude.

The overall equipment and alignment errors are believed to be sufficiently small to enable 40:1 to 50:1 beam splitting. Observation of much better performance is probably fortuitous.

IV. Results of Aperture Sampling Angle Estimation

The performance of the Aperture Sampling technique is represented in terms of angular error as a function of the true elevation angle. Because of the finite range, the reference of zero elevation angle was taken to be at the bottom of the transmit tower. This condition represents the case where the target and its image coalesce. It corresponds to a target on the horizon and at long range for a radar over flat earth. Results of the measurements are shown in Figs. 7, 8, 9. Because of mechanical limitations the lowest transmitter point was 3'. It corresponds to an elevation of .56°, which is about 1/5 of a beamwidth. Measurements were taken for elevations up to 5°, which is approximately two beamwidth.

For radar application the use of this technique should be considered at elevations below two beamwidths. At higher elevation it is certainly possible to use monopulse angle estimation since the normal suppression of the multipath by the antenna beam pattern reduces the multipath error to an acceptable level. Upon switching to the Aperture Sampling technique at low elevations the radar loses the directivity of the total antenna thereby increasing the angular range where multipath interference is significant. This is simply because the sub-arrays used for the purpose of sampling have a broader beam in elevation.

It is therefore important to demonstrate that there exists a smooth transition or an overlap region between the two techniques where either can be used successfully. The transition between the two techniques can be

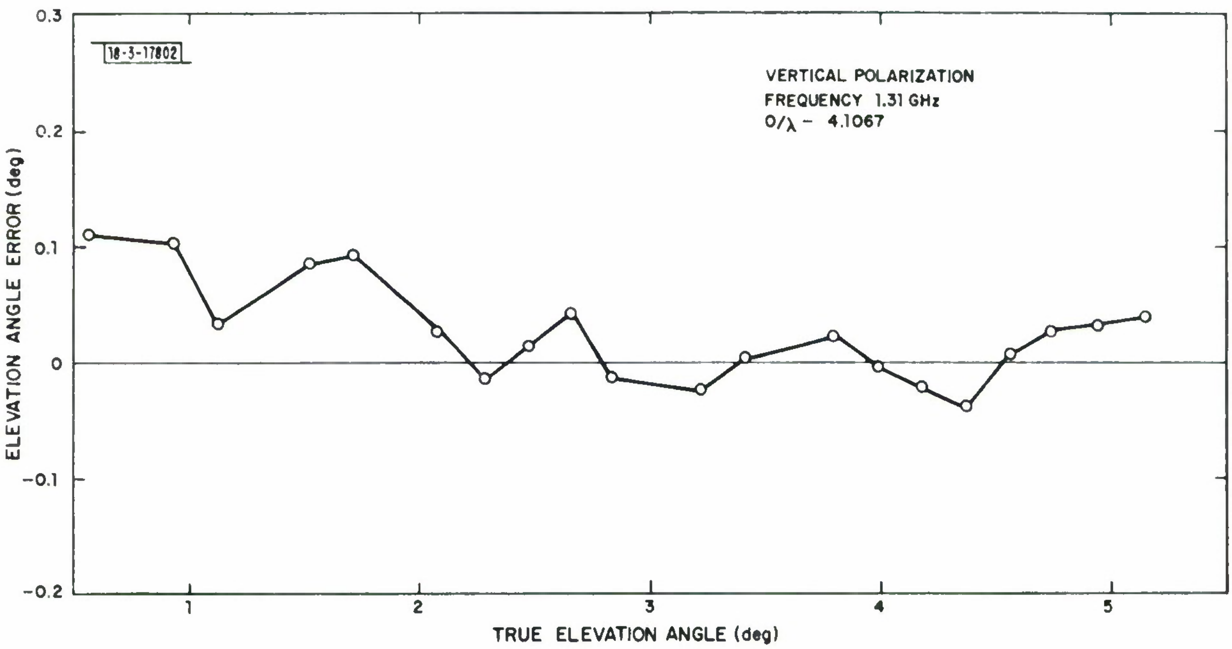

Fig. 7. Direct wave elevation angle errors.

anywhere between 1.5 to 2 beamwidths in elevation. In evaluating the experimental results one has to bear in mind the distinction between these two regions. Below 1.5 beamwidths it is desired to demonstrate the improvement to be gained by use of the Aperture Sampling technique in comparison to monopulse and above 1.5 beamwidths it is sufficient to demonstrate equal performance.

The experiment was conducted with a high signal level in order to exclude thermal noise effects. The signal-to-noise ratio was estimated to be higher than 50 dB. Error bounds for thermal noise indicate that one can expect for such a signal-to-noise ratio even for the lowest elevation angle, an angular error of less than 500th of a beamwidth, i.e., less than .006°. The observed errors are therefore due to the various sources delineated in Section 3.

Angular errors for vertical polarization are shown in Figs. 7 and 8. These figures represent many similar measurements. The more typical data is shown in Fig. 7 indicating errors of less than .05° above 2.5°. Below 2.5° the errors are larger, as can be expected, but still are less than .11° which is better than 20:1 beam splitting. It is sufficient to note that a monopulse under the same condition could yield an error of half a beamwidth. The overall largest peak to peak error is .15°. An average rms error for all elevation angles is approximately .055°.

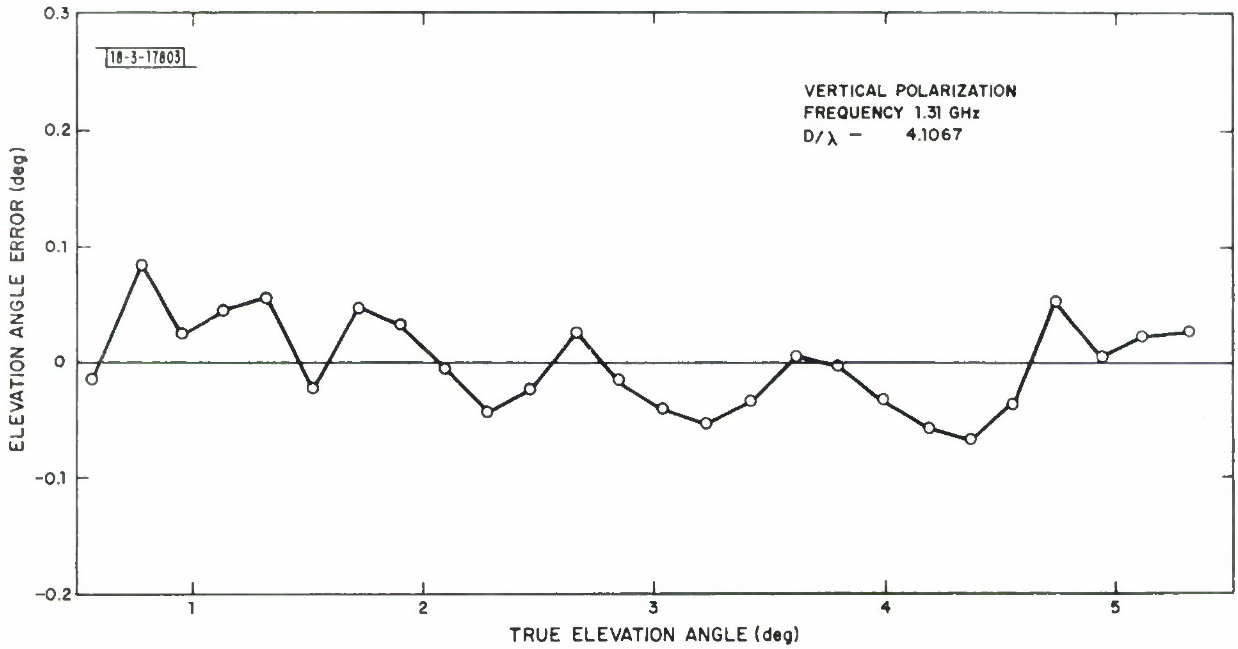

Fig. 8. Direct wave elevation angle errors.

The results from measurements taken a few months later are shown in Figure 8 with similar overall behavior. The variation from one set of data to the other is due to slow seasonal changes in the environment. At the time the data in Figure 8 was acquired the grass was dry and the surrounding deciduous trees had shed their leaves. We don't have a quantitative measure to describe this change. The unique feature of the results in Fig. 8 is the non-typical small error at the lowest elevation. Also, it seems that the array pole developed a tilt or bow that may explain an apparent bias that tends to increase the errors above 3° elevation and decrease the errors below 3°. This tilt introduces an error into our determination of the true elevation angle. The existence of a tilt was verified at a later date when the bias error increased to .15°.

The measurements were repeated for horizontal polarization and the results are shown in Fig. 9. The horizontal polarization measurements required a small reduction in the aperture size due to mechanical limitations. The slight degradation in performance is due to the smaller effective beamwidth. It may also be due to the wider transverse beam of the horn in this polarization, in which case interference from the side of the range could be larger.

The primary purpose of this experiment was to determine the elevation angle of the direct wave. The technique, however, also provides an estimate of the angle-of-arrival of the indirect wave. This output is of interest since it demonstrates the resolving power of the technique. When the second incoming

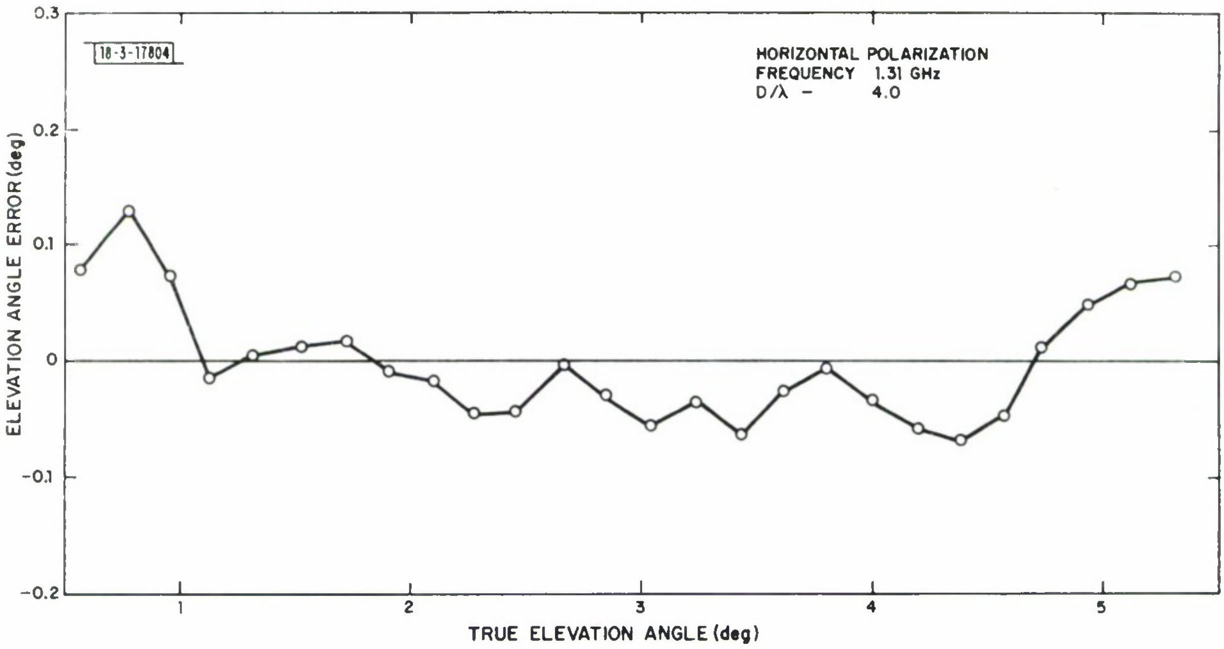

Fig. 9. Direct wave elevation angle errors.

wave is the ground reflection, it is difficult to assess the performance, since there does not exist a good way to determine the true angle-of-arrival of the indirect wave. For the present experiment the true angle was determined by using the range geometry and the assumption that the indirect wave orginates from the transmitter's image. The results are shown in Fig. 10. We observe that the error curve has a bias of about -.3° and larger variation than the estimates for the direct wave. Both results indicate that the indirect wave does not behave as coming from a theoretical image point. The error is not due to failure of the technique but to our inability to determine the true angle of arrival of the plane wave representing the ground reflection.

As the elevation angle changes the Fresnel zone moves, and in essence different parts of the terrain contribute to the indirect wave. For different elevation, the terrain, in effect, appears to reflect from varying directions. In spite of these variations, the performance with respect to the direct wave is essentially invariant. This supports the claim that this technique is independent of terrain conditions and unlike the complex monopulse [4] does not require complicated terrain calibrations.

In most cases the multipath encountered at low elevation by a radar consists of a single specular reflection and a spread of small diffused reflections. The higher the frequency and the rougher the surface, the smaller the magnitude of the specular reflection will be along with an increase in the diffuse reflection. Therefore, the assumption of two plane

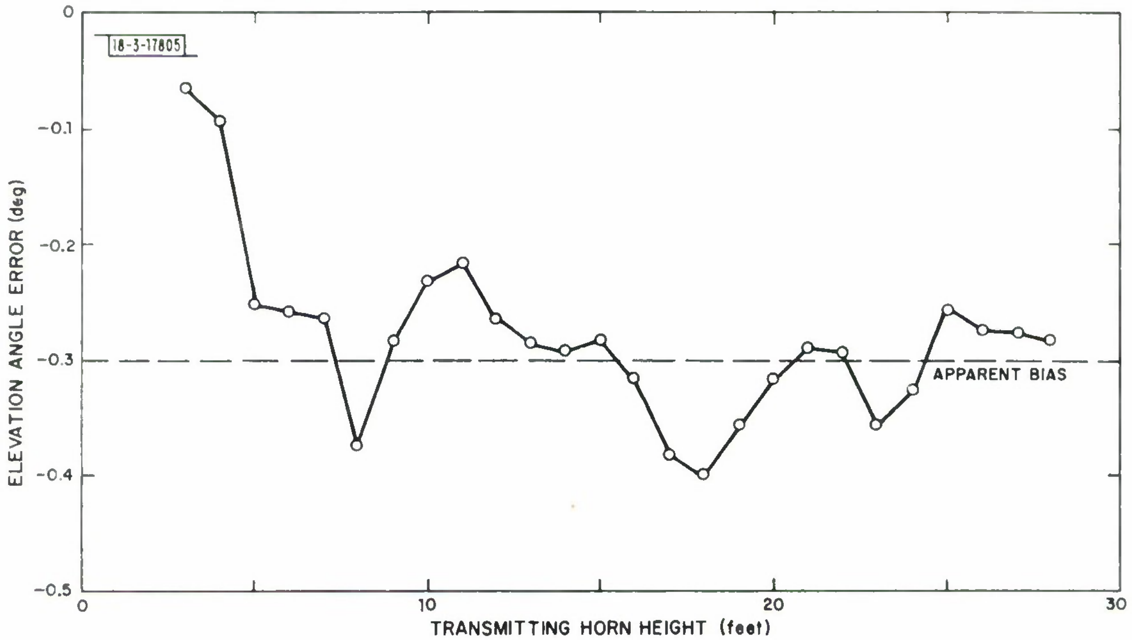

Fig. 10. Errors in estimating the elevation angle of the indirect wave.

waves is reasonable, and as can be seen from the results obtained to date the technique performs well under such conditions. It is, however, possible that in the case of a rough ground terrain there will be present additional specular reflections. If the radar's antenna pattern is not sufficiently narrow to decrease the magnitude of such an additional reflection, there may be a need to process the data by postulating three or more plane waves. Of course, with a limited number of aperture samples the postulated number of plane waves is limited. This question is addressed in Appendix A.

To test the technique in the presence of more than two waves a radiating horn was placed on the grass between the two poles. The results of one such run are shown in Fig. 11. The data was processed assuming both two and three waves. Fig. 11 shows the errors in estimating the elevation angle of the direct wave. The data shows that two waves processing yields smaller errors than three waves processing. When two wave processing is used the ground reflection and the wave originating from the second horn are treated as one wave and the program does not attempt to resolve these two waves. When the data is processed with three waves the program attempts to resolve the waves from the ground and the additional horn. However, the estimation of the direction of the additional horn was unsuccessful and yielded large errors. Apparently there is coupling

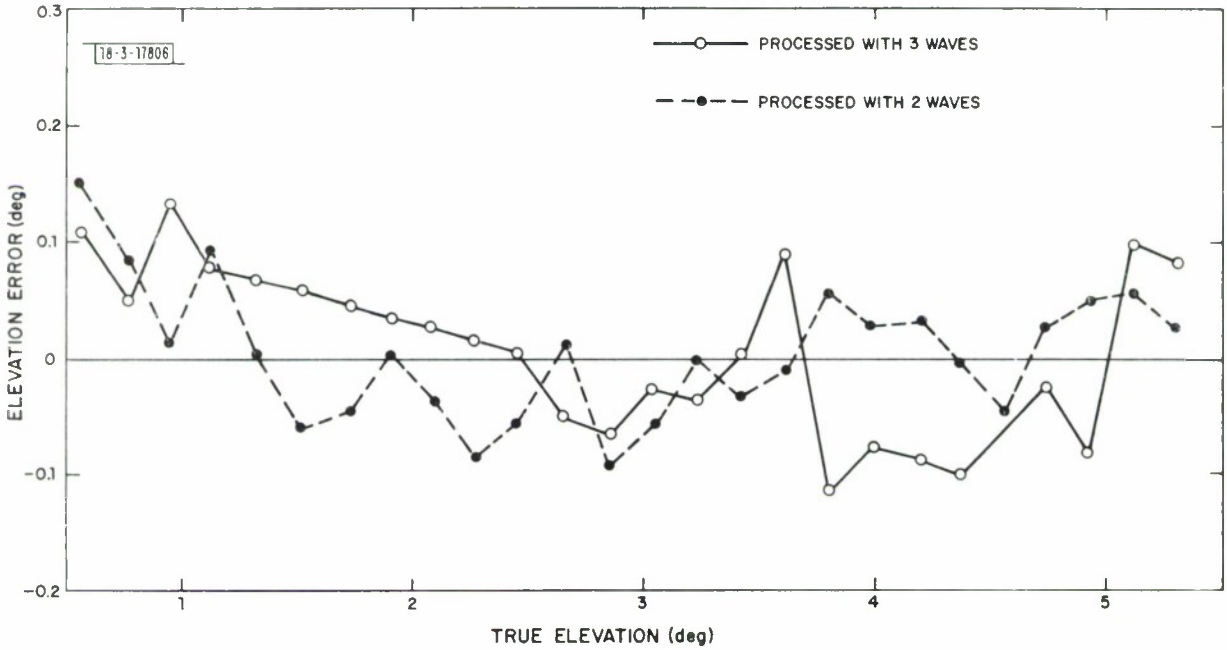

Fig. 11. Errors in estimating the elevation angle of the direct wave in the presence of an additional source.

between the estimations of the angles as the simultaneous estimation of the direct wave shows degradation of performance when compared with the use of two waves. The problem is due to the fact that the reflected wave and the wave from the horn are very close in angle and the technique cannot resolve them. The same problem could arise in the case when only a specular reflection is present if the transmitter horn could be lowered to zero height. In this experiment the lowest height was 3', in which case, assuming the specular arrives from the direction of the image, the angular separation is 1.1°. This is about .4 beamwidth, which apparently is a sufficient separation for the technique to resolve the two waves. The radiation from the additional horn was 5 dB below the direct wave. This is about 1 to 2 below the specular reflection.

Figure 11 suggests that rather than trying to resolve the reflections from the terrain into multiple distinct plane waves one can process the data by using two waves one of which is a resultant of many reflections from the surface.

Comparing the two wave results in Fig. 11 to Fig. 7 shows a small degradation of performance in spite of the presence of an additional wave almost as intense as the specular reflection. In the case of a rough terrain, the ground reflection consists of a cluster of closely spaced plane waves. At a cost of a slight degradation, one may process the data by representing the ground reflection as one plane wave. This simplifies the procedure by eliminating the need for estimating the number of plane waves. The amount of degradation will depend on the spread of the plane wave spectrum of the ground reflection. The obtained precision may be sufficient for a wide variety of terrains. In some cases it may not be satisfactory and a capability to resolve more than two waves may be required. This in turn may require more aperture samples.

V. CONCLUSIONS

The primary objective of the present multipath experiment was to demonstrate the performance of the Aperture Sampling technique in the presence of a strong specular ground reflection.

The experiment has shown that this technique performs satisfactorily down to an elevation of a fifth of a beamwidth. Its ability to resolve the direct return eliminates the typical large periodic error encountered in monopulse radar. The use of the maximum likelihood estimate in processing the data enables one to use a larger number of aperture samples for a given number of plane waves than is required by the closed form solution. This in turn makes it possible to eliminate the large errors associated with the closed form solution at elevation angles for which the phase between the direct and indirect waves at the center of the aperture is a multiple of π .

The experimental results demonstrate that a beam splitting better than 20:1 is possible down to a fifth of a beamwidth. The errors seen in Figs. 7, 8, and 9 are due primarily to the particular experimental arrangement. The various possible sources for errors were described in Section 3 . In particular, the imperfect correction for the spherical abberation and the variations in the magnitude of the reflection coefficient as seen by different horns are unique to the short range. These problems will not be present in a normal radar configuration.

The experimental results in this report are insufficient for the conclusion that the best one can do with this technique is 20:1 beam splitting. Elimination of the spurious error sources peculiar to this experiment will show that at least over a flat ground the angle estimation accuracy can be higher.

A secondary objective was to demonstrate the ability to resolve additional specular reflections. In this regard the determination of the direction of the third wave was not successful. One can only speculate that the presence of spurious error sources made it difficult to resolve the additional wave from reflected wave. However, the results show that the estimation of the direct wave is not very sensitive to the presence of the third wave. It suggests that in the case of multipath it is sufficient to process the data by assuming only two waves. In that case the various reflections from the ground are treated as one resultant wave. The cost is a small degradation in accuracy.

In summary it is felt the experiment provides enough confidence in the technique to warrant further experimentation. The experiment geometry will have to be changed to reduce the measurement errors and to enable measurements at lower elevation. This will provide data for smaller separation between the target and its image. Such a change can be accomplished by using a longer range.

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APPENDIX A

The purpose of this Appendix is to demonstrate that the maximum likelihood processing yields large errors when the number of samples $M = \frac{3N}{2}$, where N is the number of plane waves. Three and five samples for two incident waves were used. For the sake of comparison the aperture size was kept the same in both cases. Therefore, when using three samples the spacing between the samples was doubled. As a result the unambiguous range was halved and we had to limit ourselves to data points below 3° elevation. The dramatic increase in errors when only three samples are used is shown in Fig. A-1.

The analysis of the closed form solution[1] has shown that large errors are to be expected when the phase between the two waves at the center of the aperture is a multiple of π. The large errors result from the fact that the relation between the far field (the incident distinct plane wave spectrum) and the three aperture illumination samples is not unique. It can be shown* that for the above phase condition many combinations of plane waves result in identical voltages at the three symmetrically located sampling elements.

To demonstrate this point suppose the two waves are in phase at the center sampling point of three samples. Let

$$\sigma_1 = kd \operatorname{Sin} \alpha_1 \qquad \text{(A-1)}$$
$$\sigma_2 = kd \operatorname{Sin} \alpha_2 \qquad \text{(A-2)}$$

---

*This fact was brought to the author's attention by Dr. S. Weisbrod of Teledyne Micronetics.

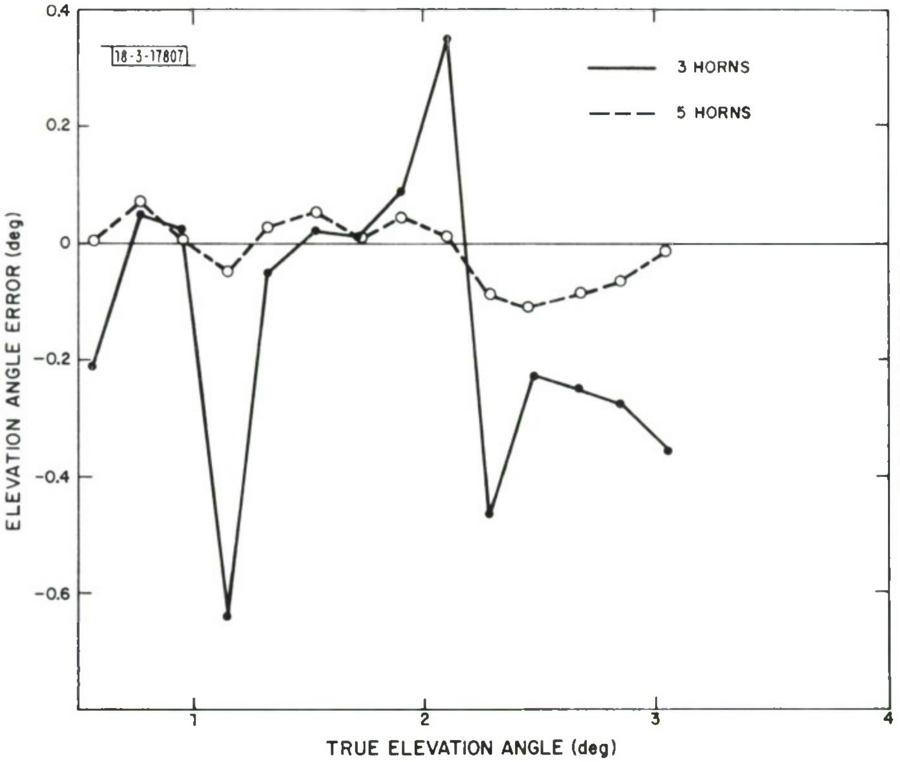

Fig. A-1. Comparison between five and three samples processing.

where d is the spacing between sampling points and α_1, α_2 are angle-of-arrivals of the two waves. If the amplitudes of the plane waves are A_1 and A_2 the sampled voltages are given by

$$V_1 = A_1 \exp(i2\,\sigma_1) + A_2 \exp(i2\,\sigma_2) \tag{A-3a}$$

$$V_2 = A_1 \exp(i\,\sigma_1) + A_2 \exp(i\,\sigma_2) \tag{A-3b}$$

$$V_3 = A_1 + A_2 \tag{A-3c}$$

$$V_4 = A_1 \exp(-i\,\sigma_1) + A_2 \exp(-i\,\sigma_2) \tag{A-3d}$$

$$V_5 = A_1 \exp(-i2\,\sigma_1) + A_2 \exp(-i2\,\sigma_2) \tag{A-3e}$$

A few cases are given in Table A-1.

TABLE A-1

α_1	3.58°	2.388°	1.791°	2.388°
α_2	0°	- .433°	- .734°	- 2.388°
A_1	.5	.6	.75	1.0
A_2	1.5	1.4	1.25	1.0
σ_1	90°	60°	45°	60°
σ_2	0°	- 10.894°	- 18.435°	-60°
V_1	1	1	1	1
V_2	$1.58\angle 18.43°$	$1.88\angle 6.62°$	$1.72\angle 4.5°$	.866
V_3	2	2	2	2
V_4	$1.58\angle 18.43°$	$1.88\angle 6.62°$	$1.72\angle 4.5°$	.866
V_5	1	1	1	1

Table A-1 indicates that the four distinct combinations of plane waves are indistinguishable by observing the samples V_1, V_3, and V_5. Only by virtue of the additional samples V_2 and V_4 the aperture samples can distinguish between the four cases and yield accurate estimation of the angle-of-arrival. This is clearly observed in Fig. A-1. It ought to be emphasized that the continuous illumination is uniquely related to the far field. The sampled illumination when only these particular three samples are taken is not unique. Adding more samples yields a unique relation.

In the experimental set up one cannot determine when the phase relation is exactly a multiple of π. Only after processing the data one may estimate within the experimental errors when this condition occurs. However, in view of the above discussion one can assume that this condition occurs whenever the errors in Fig. A-1 are very large. In this particular case around 1.3° and 2.4°.

In conclusion we would like to emphasize that the advantage of the maximum likelihood approach as compared to the closed form solution is in fact that it allows the use of a larger number of samples, thereby eliminating the large errors around $\phi = n\pi$, where ϕ is the phase between the two plane waves at the center of the aperture.

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An experimental verification of the Aperture Sampling technique as a means of estimating the angle-of-arrival in the presence of ground reflection interference is reported. The technique is found to perform satisfactorily down to 1/5 of a beamwidth elevation. Beam splitting better than 20:1 is demonstrated. This performance is obtained by using a minimizing search processing that allows use of a larger number of samples than the number required by the closed form solution.

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