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OPTIMAL REPLACEMENT POLICY FOR THE F-15 AIRCRAFT ENGINE MODULES

James A. Forbes, et al

Air Force Institute of Technology
Wright-Patterson Air Force Base, Ohio

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SLSR 7-75B

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OPTIMAL REPLACEMENT POLICY FOR THE F-15 AIRCRAFT ENGINE MODULES

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics Management

By

James A. Forbes, BA
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August 1975

This thesis, written by

Captain James A. Forbes

and

Captain Phillip P. Wyatt

has been accepted by the undersigned on behalf of the faculty of the School of Systems and Logistics in partial fulfillment of the requirements for the degree of

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The F-100 engine, used on the F-15 aircraft, differs from previous engines in that it is modularized. Certain of the engine modules must be removed in order to perform maintenance on others. As an example, the fan module must be removed to repair the core module. If an unfailed module is removed to facilitate maintenance on another module, it may be worthwhile to replace the unfailed module rather than reinstall it. Such a replacement is termed opportunistic maintenance. For instance, if less than one operating hour remained before the unfailed module reached its maximum operating time (MOT), opportunistic replacement would be attractive. However, any early replacement results in lost service life. The purpose of the study was to determine if an algorithm could be developed which would determine when opportunistic replacement is optimal and when it is not. An algorithm was developed and programmed in FORTRAN. Variables addressed include transportation, packing, manpower, parts, depot overhaul costs and module failure tests. Data were obtained from Edwards Air Force Base F-15 Joint Test Force. Sensitivity of the model to changes in variables was investigated. The replacement decision was generally insensitive to changes in the input variables. 182 Pages.

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INTRODUCTION

Statement of the Problem

The F-15 aircraft, currently entering the Air Force inventory, is equipped with Pratt and Whitney Aircraft F-100-PW-100 augmented turbofan engines (8:2). The F-100 engine differs from older engines in the Air Force inventory in two major respects: 1. the design incorporates state-of-the-art technology and 2. the engine employs modular construction. An optimal, field level replacement policy for the engine modules is needed (9).

The F-100 engine is composed of five modules (11:48-49). As shown in Figure 1.1, they are the fan module, core module, fan drive turbine module, augmentor/exhaust module, and the gearbox module. In addition to the modules, a number of external accessories (e.g., plumbing, wiring, actuators, probes, valves, pumps, etc.) are required to complete the engine (18:9-2A--9-5). When performing maintenance on the F-100 engine, the technician has certain sequences he must follow. These sequences result from the physical order of the modules on the engine. For instance, to remove the fan drive turbine

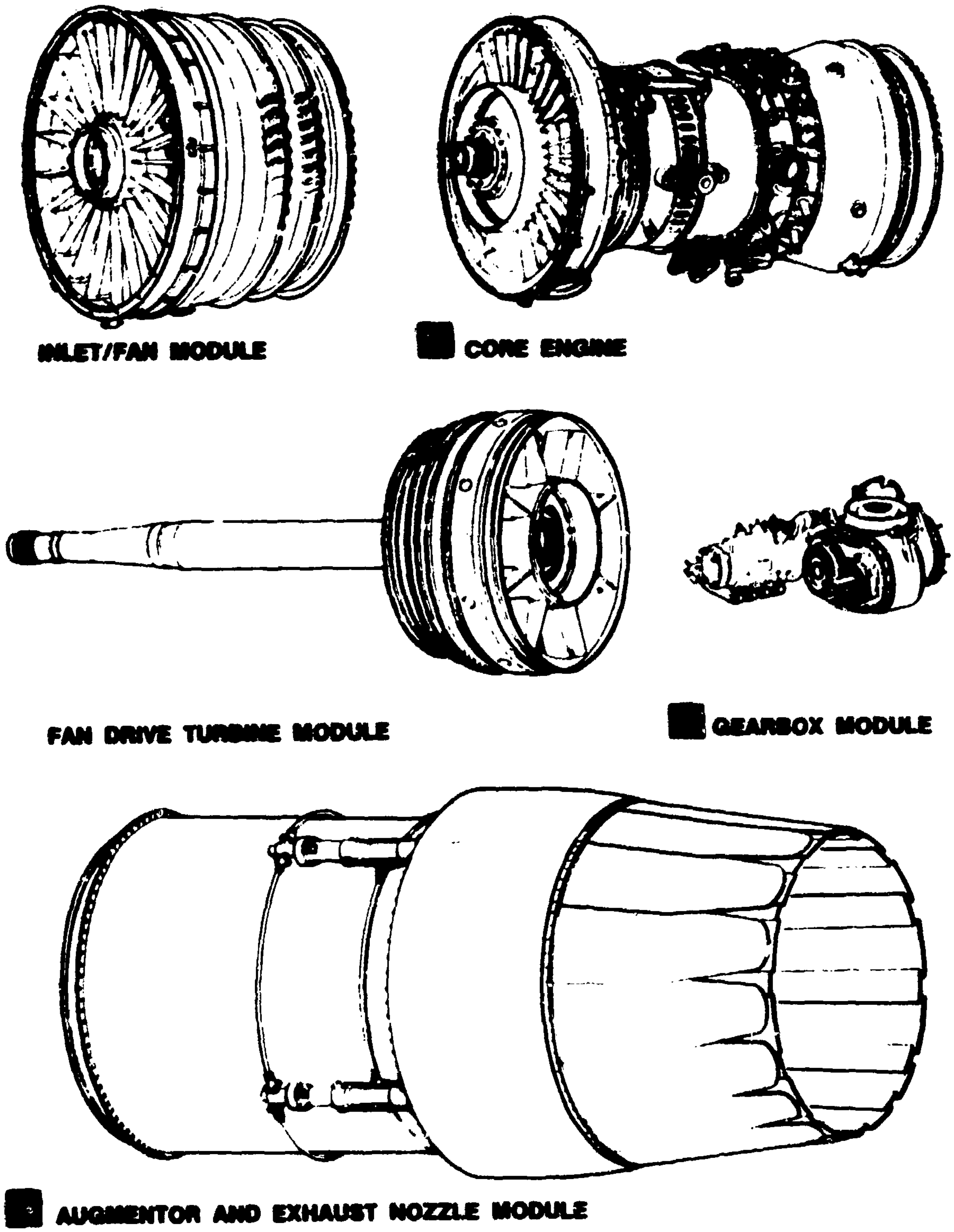

Figure 1.1

F-100 Engine Modules
(Courtesy Pratt and Whitney Aircraft Company)

module, the augmentor/exhaust module must be removed first (18:3-39). Similarly, in order to remove and replace the core module, both the fan module and fan drive turbine modules must first be removed. If the core module required replacement and there were nothing wrong with the fan drive turbine module, the latter module would be removed solely to facilitate the maintenance on the core module and then be reinstalled. There may be opportunities, however, when it is advisable to replace rather than reinstall the fan drive turbine module when it is already removed from the engine. To look at the extreme case, if less than one operating hour remained before the fan drive turbine module were due mandatory replacement, reinstallation of the same module would be of doubtful merit. At the opposite extreme, if several thousand hours of life remained, other things being equal, replacement of the module would be unwise. Thus, at one end of the spectrum, one would expect to incur excessive maintenance manhours and downtime because the opportunity to accomplish concurrent maintenance is foregone; at the other, much useful service life is foregone. Somewhere along the continuum an optimal tradeoff between maintenance cost and lost service life exists. A policy which optimizes the replace-not-replace decision for the F-100 engine modules has not yet been developed (7;9).

Importance of the Problem

The acquisition cost of an F-100 engine is approximately $1.7 million. F-100 engine replacement parts are correspondingly expensive (7). The high cost of the F-100 engine and its sub-components has spurred certain efforts to optimize logistic support for the engine. For instance, the Air Force Logistics Command (AFLC) uses a model called MOD-METRIC to provide improved depot management of inventory levels for both the basic engine and its modules (13:472).[1] MOD-METRIC utilizes historical consumption data and forecast flying hours in order to compute required engine and module stocks. The end objective is to determine the smallest stock level (and, therefore, cost) which will provide required support (13:472). MOD-METRIC facilitates decision making at the depot level. Further reduction in cost should be possible if the base level decision on when to replace modules can be facilitated (9). Further development of the example given previously should clarify this point. If work is required on the engine core module (which requires removal of the fan drive turbine

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[1]MOD-METRIC is an extension of the METRIC (Multi-Echelon-Technique-for-Recoverable-Item-Control) model developed for the USAF by the RAND Corporation as a method of determining stock levels for recoverable items (13:472). METRIC addressed a multi-echelon (e.g., both depot and field level), multi-item inventory system. MOD-METRIC extends the METRIC model to include an indentured inventory system. The F-100 engine modules, for instance, are an indenture of the basic engine (13:472).

module), under what situations should the technician replace, rather than simply reinstall, the fan drive turbine module? Under the current operating criteria, each module (except the augmentor/exhaust module) is given a maximum operating time (MOT) at the end of which a time-change is required (7). In our example, if one hour remained on the fan drive turbine module until time-change was required, the technician would probably decide to change the fan drive turbine module now rather than reinstall the same module and then, one operating hour later, remove it again. These factors lead to the following questions: where, in the continuum of possibilities, is the breakpoint between replace and not replace which will result in minimal cost and ?. is the effort required to determine the breakpoint worth the savings which result from its determination? Determination of the replace-not-replace breakpoint is of considerable current interest in the F-15 System Program Office (SPO), at Air Force Logistics Command (AFLC) Headquarters, at Tactical Air Command (TAC) Headquarters, and at the field level (7;9).

Objectives

The objectives of this thesis were to develop an algorithm which would locate the economical replace-not-replace breakpoint for the five modules comprising the F-100-PW-100 engine installed in the F-15 aircraft and to investigate sensitivity of this breakpoint to the algorithm's input variables.

Scope

It should be recognized that deciding whether or not to replace an unfailed part is only one of several similar decisions faced by the maintenance technician. For example, USAF Technical Order 2J-1-31 provides guidance on when to field repair and when to return to depot components which have already failed (31). Technical Order 2J-1-27 provides guidance on when to minor overhaul at depot level and when to major overhaul at depot level components which have failed (30). This thesis will address only the decision on whether or not to replace an unfailed module at the field level. A statistical model rather than inspection criteria will be used to anticipate failure.

CHAPTER II

LITERATURE REVIEW AND RESEARCH QUESTIONS

Air Force Jet Engine Maintenance

The Air Force engine maintenance program consists of two echelons--Jet Engine Intermediate Maintenance (JEIM) and depot overhaul/repair (2:vii). JEIM is base level repair. Depot overhaul/repair refers to extensive tear-down and renewal performed at either an Air Logistics Center or contractor facility.

Periodic maintenance of jet engines consists of scheduled inspections at intermediate level and scheduled major overhaul at the depot level. In addition to periodic or scheduled maintenance, unscheduled maintenance can and does occur. Unscheduled maintenance requirements may be generated for either the base level or depot level. As an example, catastrophic failure of a component would generate an unscheduled maintenance requirement at the base and/or depot level.[1] Usually unscheduled maintenance is more costly than scheduled maintenance (10:223). Unscheduled maintenance is basically corrective, whereas scheduled maintenance is basically preventive (29:1-1).

---

[1]By catastrophic failure is meant failure characterized by sudden, unexpected damage or loss.

<u>Standard jet engine maintenance cycle</u>. Figure 2.1 illustrates the standard Jet Engine Maintenance Cycle. The Standard Jet Engine Maintenance Cycle is the most common (11). When an aircraft engine is reported operating outside of established performance parameters or does not meet some specified inspection criterion an unscheduled maintenance action is generated. A maintenance technician, or team of maintenance technicians is dispatched to the aircraft to analyze the discrepancy. Normally the engine will be analyzed or "trouble shot" to determine the reason for the discrepancy. Depending upon several factors (such as estimated repair time, type of failure, inability to determine cause for failure, availability of a repair asset required, etc.) a decision will be made to attempt repair on the aircraft or remove the engine from the aircraft and take it into the shop. Assuming the engine was removed and is operable, the next step, normally, is to perform a test cell run to isolate the malfunction.[1] Next, the engine is routed through the jet engine repair shop and the defective components replaced. For excessively damaged engines requiring large manhour expenditures, a decision may be made to ship the entire engine to a depot

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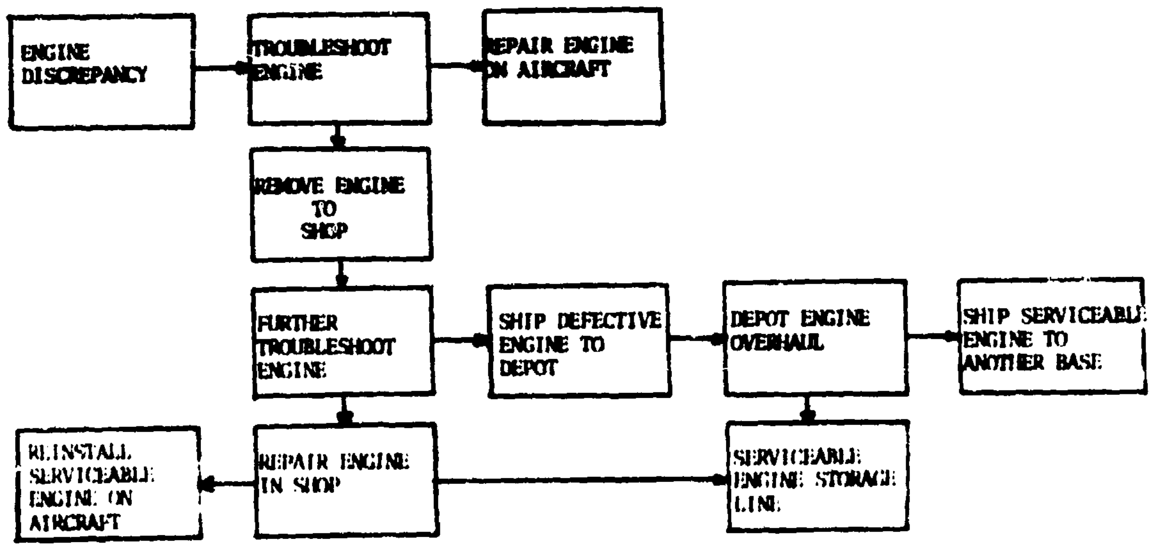
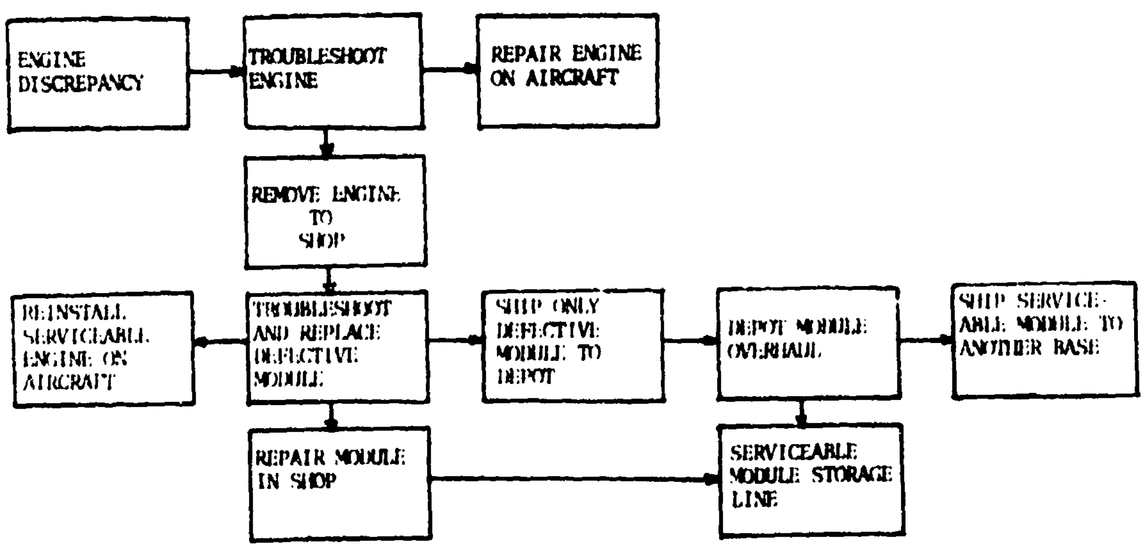
[1]If the engine is not operable, the malfunction is known with a high degree of certainty, or in order to save time the engine may be routed directly to the jet engine repair shop (9).



Standard Jet Engine Maintenance Cycle



Modular Jet Engine Maintenance Cycle

Figure 2.1

Typical Jet Engine Maintenance Cycle

overhaul facility. Thus, the engine becomes an unscheduled requirement at the depot.

Engines in the Air Force inventory typically have an established maximum operating time (MOT). On reaching its MOT, an engine is removed from the aircraft in which it is installed and returned to the depot for major overhaul. Removal on reaching MOT is a scheduled maintenance action.

F-100 engine maintenance cycle. Figure 2.1 also illustrates the F-100 engine maintenance cycle. The F-100 engine maintenance cycle differs from the standard jet engine maintenance cycle because of the modular construction of the F-100 engine (11).

Unscheduled intermediate level maintenance is essentially the same until the F-100 engine is removed from the aircraft. Normally, a test cell run is made prior to engine disassembly.[1] On the test cell run, the defect which caused engine removal is isolated to a specific module. In the repair shop, the defective module is removed and replaced with a serviceable module from the stock maintained in the repair shop (13:473). The intent of removing and replacing modules is to minimize the time

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[1] As was true with the standard jet engine maintenance cycle, the engine may be routed directly to the repair shop.

required to return the engine to serviceable condition (7). Engine modularization was one of the design steps toward minimizing repair cycle time on the F-15 aircraft (16). Repair of modules is scheduled separately from repair of the entire engine and does not delay returning the engine to serviceable condition. If base level repair is infeasible, the module is returned to the depot for overhaul or repair.

Each of the F-100 engine modules (except the augmentor/exhaust module) is assigned a maximum operating time (MOT). When a module reaches its MOT, the engine of which the module is a part is removed from the aircraft and brought into the intermediate level repair shop. The module which reached MOT is removed, replaced with a serviceable module from the shop stock, and returned to the depot for overhaul.

Optimal Maintenance Theory

Jorgenson, McCall and Radner (10:20-77) in a RAND Corporation report entitled, Optimal Maintenance of Stochastically Failing Equipment provide a comprehensive treatment of the mathematical determination of optimal maintenance policies. As developed in their report, maintenance problems may be divided into two classes--deterministic and stochastic (10:1). Deterministic problems are those where the requirements and outcomes of every maintenance action are known with certainty. For stochastic problems, on the other hand, the requirements and outcomes of maintenance actions are random in nature (10:1). That is, the amount of service life produced by a unit of equipment between the time when a maintenance action is performed on it and the time of failure is random in nature rather than known before hand. Jet engine failures are stochastic in nature; in fact, the stochastic nature of aircraft engine failures underlies the adoption by the USAF of the actuarial method of predicting aggregate engine failures (29:1-1--1-4).

The question formulated in the statement of the problem, earlier in this thesis, was when to replace an engine module which had already been removed from the engine but which, itself, had not yet failed or reached MOT. Replacement and not-replacement were the only alternatives considered. This limitation is reasonable since the field level maintenance echelon does not have the option of repairing an engine module removed by reason of accumulated time, but must return it to the depot (7;9).

The three independent variables which determine the optimal average expenditure per unit time are replacement cost before failure, replacement cost after failure, and the hazard rate (21:71-74). Jorgenson, McCall and Radner note that:

> If replacement costs more after a failure than before, in the absence of uncertainty the equipment

> will always be replaced *just* before it fails. For stochastically failing equipment, replacement *just* before failure is impossible due to uncertainty about when failure will occur [10:205].

For the stochastic situation it may or may not pay to replace the equipment before failure, i.e., establish a maximum operating time or opportunistic replacement policy. Replacement is justifiable if two conditions are satisfied: 1. the time to failure distribution for the piece of equipment must demonstrate wearout, and 2. it must cost more to replace the piece of equipment after failure than before (10:221). Replacement prior to failure, however, results in the loss of some unused life. Thus, the optimal decision depends on the tradeoff between the value of unused life and the cost of the avoided failure (10:207).

A generally used technique for determining if an equipment item demonstrates wearout is examination of the hazard rate of the equipment item (24:170-171). Hazard rate is defined as the ratio of the number of failures occurring in a time interval to the number of equipment units which survived until the beginning of the interval, divided by the length of the time interval (24:161). Figure 2.2 illustrates a typical hazard rate function known as a "bathtub" curve. In this general case the hazard rate initially decreases with age, remains constant for a period of time, and then increases. The equipment item demonstrates wearout when the hazard rate increases (24:171).

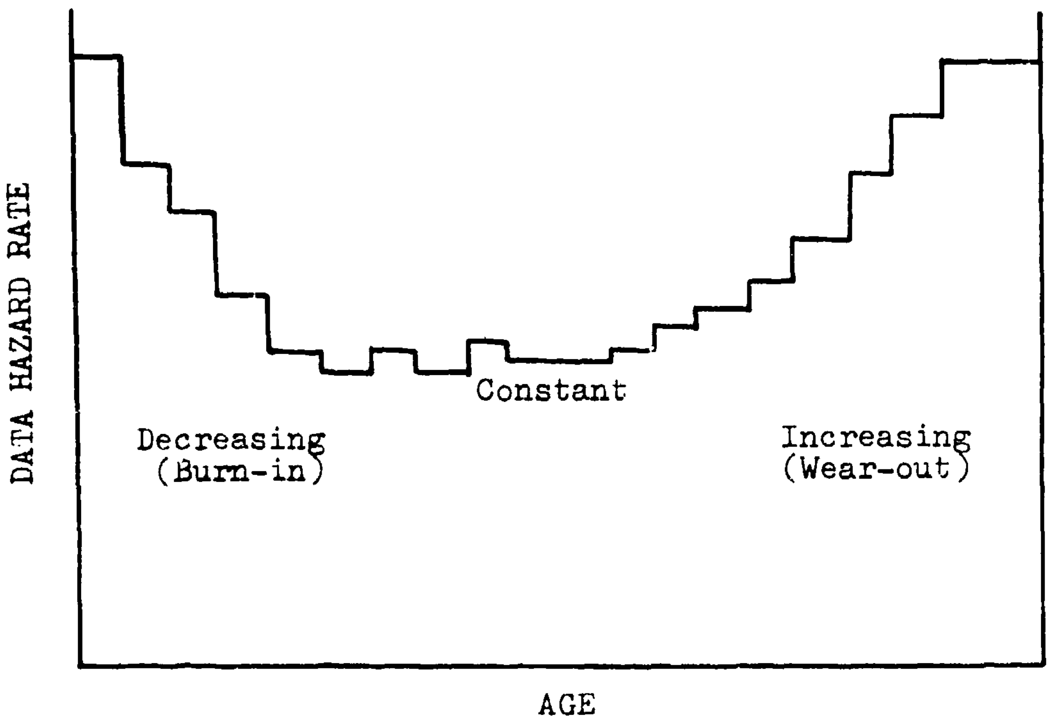

Figure 2.2

Typical Component Failure History

Even if the equipment item exhibits wearout, replacement before failure would still not be justified unless it costs more to replace the item after failure than before (10:223). Replacement cost, either before or after failure would generally include the opportunity cost of the downtime while the equipment is out of service and the actual dollar replacement cost (10:224). Opportunity cost normally is a measure of lost revenue (22:472-473). National defense, the product of military operations, is by nature not priceable (29:36-37). Thus, an amount of lost revenue cannot be determined. This is not to say, however, that the opportunity cost of incurred downtime must necessarily be ignored. Jorgenson, McCall and Radner have suggested that the purchase price of an equipment item be amortized over its total expected life. In this manner, a cost per unit of operation (e.g., hour, cycle, etc.) can be determined (10:224). The value of the amount of equipment operating time foregone by some replacement action is an opportunity cost.

Intuitively, one would expect that a replacement before failure would result in less downtime than a replacement after failure (10:222). For instance, a replacement after failure is generally an unscheduled maintenance action. As an unscheduled action is unexpected, it generally incurs greater queue (waiting) time before the replacement action is begun. Jorgenson, McCall and Radner (10:223) point out that:

> . . . a second factor determining replacement costs is the amount of resources needed to perform the action. Usually, more resources are needed for an unscheduled replacement for three reasons: (1) generally a more complex maintenance operation is involved, and a failed part may have considerably lower trade-in value than an unfailed one; (2) additional resources are needed to repair parts damaged by the inservice failure; and (3) it is often necessary to transport maintenance resources to the failed system.

Under certain circumstances, replacement before failure may cost less for at least one additional reason. To use the F-100 engine as an example, when core module failure or removal for maximum operating time (MOT) is the only reason for engine disassembly, all costs associated with removing and replacing the engine in the aircraft, transporting the engine to and from the repair shop, and removing and replacing the inlet fan module (in order to gain access to the core module) are sunk costs against the core module.[1] In this event, if the inlet fan module were replaced with a new module rather than simply being reinstalled, then the additional engine removal and replacement, engine transportation, and module removal and replacement costs would be avoided. Replacement of the inlet fan module (or any other module)

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[1]The inlet fan module, gearbox module, fan drive turbine module and augmentor module must be removed to gain complete access to the core module (18:5-2--5-71).

given that its removal were required for another reason is an opportunistic replacement (10:224).

In summary, the following factors can be expected to result in different costs for replacement before failure and replacement after failure:

1. Differences in downtime required for the replacement action.

2. Differences in resources required for the replacement action.

3. Opportunistic replacement when removal is required to support mandatory maintenance of another component.

Interviews with representatives of the Product Support Division, Pratt and Whitney Aircraft Company (27), the prime Air Logistics Center (ALC) for the F-100 engine (26), and personnel of the Edwards Air Force Base F-15 Joint Test Force (3;5;12;20) were conducted to determine if it is possible to currently measure the difference in resources or downtime for replacement before and after failure. Although the personnel interviewed acknowledged that there are probably such differences, the data to determine such differences are not currently available. The difficulties in determining resource differences can be illustrated by examining depot overhaul costs. Whereas depot overhaul of older technology engines involves literally complete teardown of the engine, this is not true for the F-100 engine. Currently, the depot overhaul approach for the F-100 engine is to look at each engine individually and only replace: 1. parts which have met or nearly met cycle limitations and 2. failed parts.[1] Thus, the resources required to overhaul an F-100 engine at depot are dependent not only on the reason precipitating overhaul (e.g., MOT, damage beyond field level capability to repair, or opportunistic replacement) but also on the age, condition, and accrued cycles of the engine components. Secondly, data available on F-100 engine depot overhauls are very limited. The San Antonio ALC (the prime ALC for the F-100 engine) received the first F-100 engine for overhaul in January 1975. As a result of the uncertainty over what maintenance actions a typical engine will require and the lack of a developed data base, a distinction cannot as yet be made between the cost to depot overhaul a failed engine or module and one which is unfailed (26).

A similar difficulty exists when one attempts to determine differences in resources or downtime required for intermediate level engine maintenance. Manhour and

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[1] Cycle in this context is used in a different sense than when speaking of a maintenance cycle (see page 8). Many of the components on the F-100 engine have cycle limitations, where a cycle is generally considered an exercise of the engine throttle from idle to an advanced power setting and back to idle. The concept of cycle limitations is undergoing much discussion at the current time (7;9) and is not addressed in this thesis.

clockhour data on F-100 intermediate level engine maintenance are obtainable from two sources: 1. the Pratt and Whitney Aircraft Company F-100-PW-100 Qualitative, Quantitative, Personnel Requirements Information (QQPRI) (17) and 2. from data maintained by the Human Factors Test Office of the F-15 Joint Test Force, Edwards Air Force Base (12). Neither the QQPRI nor the Human Factors Test Office currently makes a distinction between the manhours or clockhours to perform maintenance on unfailed modules and similar requirements for failed modules. First, there is no effort at this time to make such a distinction and secondly, it is questionable if such a distinction could be made at this time considering the formative state of available data.

Of the three factors which can result in a cost difference between replacement of a failed module and unfailed module, only the savings through opportunistic replacement of a module which is already removed is tractable at the current time. It is this savings resulting from opportunistic replacement of an unfailed removed module which is explored in this thesis. As possible savings resulting from less resources or less downtime required for replacement of an unfailed module are not considered, we believe that total savings from opportunistic replacement are understated. The effect of uncertainty about the total savings resulting from opportunistic replacement was explored through sensitivity analysis and is discussed later.

Jorgenson, McCall and Radner have shown how to find an optimal opportunistic maintenance policy for a system composed of two components, one of which is constant hazard (10:244-251). The Jorgenson, McCall and Radner model is developed in terms of decision rules. Over the interval $0 < n \leq N$ where N is the maximum operating time (MOT) and n is the module age beyond which opportunistic replacement is worthwhile, a module is replaced at failure in the interval $0 < n$, replaced at failure or opportunistically in the interval $n \leq N$, and mandatorily on reaching age N. The values of n and N which will result in least cost can be determined analytically for any given combination of cost to replace before failure, cost to replace after failure and hazard rate.

Once values for n and N are established, they, in effect, form a replacement policy. The object of this thesis is to develop an algorithm which will enable managers to find the optimal replacement policy for the modules of the F-100 engine. For the F-100 engine, maximum module operating times (N) have been established (7). The breakpoint (n) between replacement at failure and opportunistic replacement has not been established. Thus, the algorithm developed in this thesis solves only for the optimal value of n, given a fixed value for N. In the literature reviewed on optimal preventative maintenance policy determination (6:271-283;10:205-268; 19:229-249;21:61-67;35:267-280) a general algorithm which can be directly applied to the F-100 engine module was not found.

Research Questions

1. What algorithm can be developed for a five module system which, for any combination of module operating hours and hazard rates, will determine the optimal opportunistic replacement policy?

2. How sensitive is the optimal opportunistic replacement policy to uncertainty about the underlying failure distribution?

3. How sensitive is the optimal opportunistic replacement policy to cost estimate uncertainty?

4. What is the magnitude of the savings which can be realized by an opportunistic replacement policy when compared with a replace at failure policy?

CHAPTER III

METHODOLOGY

Introduction

This chapter describes the method used to answer the research questions. The procedures used to determine hazard rates and cost data are described. Next, the structure of an optimal replacement policy for the five module system is discussed. Finally, procedures to determine sensitivity of the optimal policy to failure rate uncertainty and cost uncertainty are described.

Hazard Rate Models

In Chapter II, the hazard rate was described as a tool for determining if a component exhibited wearout and as an input to determining the optimal replacement policy. This section will discuss how hazard rates may be modeled.

Shooman (24:160-170) has shown how to develop a hazard rate function from failure data. If the time scale in a hazard rate graph is divided into intervals, the data hazard rate for each interval may be calculated as the fraction of components surviving until the beginning of the interval, but failing during the interval, divided by the interval length. Algebraically the data hazard rate is defined as

$$Z_d(t) = \frac{[n(t_i) - n(t_i + \Delta t_i)] / n(t_i)}{\Delta t_i} \qquad (3.1)$$

for $t_i < t \leq t_i + \Delta t_i$ where $Z_d(t)$ is the data hazard rate, $n(t_i)$ is the number of survivors at the beginning of period t_i, $n(t_i + \Delta t_i)$ is the number of survivors at the end of the period t_i, Δt_i is the width of the period in time units, and t is the total observation time. As an example, if there were 113 survivors at the beginning of the period, 24 failures during the period, and the period were 1000 hours

$$Z_d(t) = \frac{[113 - (113 - 24)] / 113}{1000} = .0002124 \qquad (3.2)$$

Although the hazard rates for each interval may simply be plotted on a histogram, Shooman (24:185) points out that in order to generalize from sample data to the population of similar components it is essential to fit the failure data with a mathematical model. Of the wide range of models available, Shooman (24:195) suggests that the piecewise linear, exponential, and Weibull models are sufficiently inclusive that virtually all hazard rates may be described by them. Figure 3.1 illustrates each of these models. Included with each illustration is the general algebraic form of the model.

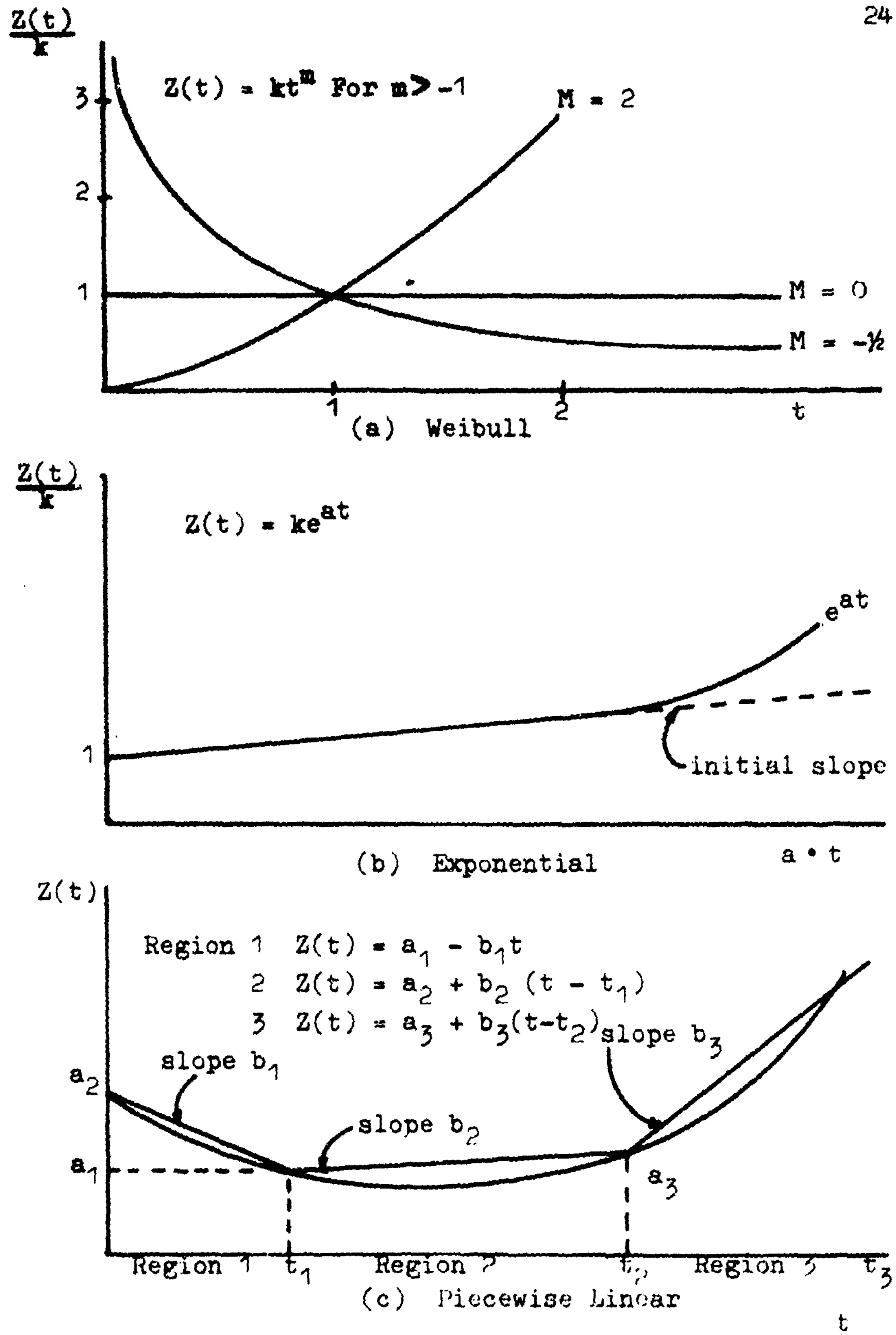

Figure 3.1

Hazard Rate Models

Fitting failure data with a mathematical model is a two step process: 1. choosing the appropriate model and 2. determining model parameters (24:457). As Shooman (24:457) indicates: "The initial procedure in the choice of a model is to plot the histogram for $Z_d(t)$ from the failure data." A judgement is then made from inspection of the $Z_d(t)$ function. If inspection is not adequate to determine the appropriate model, more powerful analytical and graphical techniques are available (24:458-462). For instance, if the underlying time to failure distribution is exponential (i.e., the hazard rate is constant), a plot of the number of operating hours at which a component failed vs. the natural logarithm of $\frac{i}{N+1}$ where i is the sequence number of the ith failure and N is the number of components in the original population, will be a straight line (24:459-460). Similar techniques are applicable to the Weibull model and piecewise linear model (24:460-463).

Estimation of model parameters, once the appropriate model has been selected, can be accomplished through either least squares estimates, moment estimates, or maximum-likelihood estimates (24:464). Shooman (24:464) recommends the maximum-likelihood estimator and Jorgenson, McCall and Radner (10:251) also employ the maximum likelihood estimator. It is used in this thesis because its validity and applicability is widely accepted by researchers.

Engine Failure Data

Universe description. An F-100 engine prototype was first operated in flight on 27 July 1972 (8:2). Since that time, the F-100 engine and its modules have undergone a number of modifications to resolve various after-burner, fuel control, and other component malfunctions. As a result, the F-100 engine inventory consists of a number of different configurations.

Engine Modules Studied

On the advice of the Engine Project Office, F-15 System Program Office (9), the population of F-100 engine modules studied was limited to those modules originally installed on engines serial numbered 023 and subsequent, with the exception of engine number 050. At the current time, engines serially numbered 023 and above, except for engine number 050, are of like configuration and are the operational configuration.

The data which must be collected in order to determine a particular module's (e.g., fan module, core module, etc.) hazard rate are the times at failure of those modules which have failed. Time at failure is a discrete, infinite, ratio level random variable. Time at failure data are available through two sources:

1. the Engine Status Reporting System, DO24, for engines and modules assigned to operational units and 2. through the F-15 System Program Office for engines and modules assigned to Air Force and Contractor Research, Development, Test and Evaluation (RDT&E) units (9).

Reporting instructions for the Engine Status Reporting System are contained in Air Force Manual 400-1, "Selective Management of Propulsion Units" (34:4-5). The source document utilized in recording the collection of propulsion unit data is the Air Force Form 1534 (34:5). Subsequent to completion by responsible personnel at an Air Force Base, AF Forms 1534 are key punched and the data transmitted by Automatic Digital Network (AUTODIN) to Oklahoma City Air Logistics Center (OCALC/ACDT), Tinker Air Force Base, Oklahoma where it is monitored, processed and maintained (34:12). Engine or module operating time at failure and serial number are specifically collected by this system.

The Base Engine Manager, designated in accordance with Air Force Manual 400-1, is responsible for auditing and controlling AF Forms 1534 submitted from his base (34:10). He develops local procedures in accordance with which an initial check of the accuracy of AF Form 1534 data is made. A second check on data validity is accomplished by edit routines within the DO24 system (14:19). Finally, at the end of each month, OCAMC/ACDT provides the Base Engine Manager with a reconciliation listing (DO24AEH1A) which the engine manager compares with his original AF Forms 1534 and verifies for accuracy (34:5).

As discussed above, data on RDT&E engines are available through the F-15 System Program Office. The data used in this thesis were maintained in mechanized form by the F-15 Joint Test Force at Edwards Air Force Base, California. Although the data are not subject to validity checks as visible as those provided by the DO24 system, F-15 System Program Engine Project Office personnel express confidence in the validity of their data (9).

Certain engine and module removals are precipitated by events other than engine or module failure. Specifically, it was necessary to purge the collected data of removals to facilitate other maintenance on an aircraft, or removal caused by other management decisions. The Engine Status Reporting System includes codes to distinguish between reasons for removal. Computer programs have previously been developed to sort engine failure data by removal code (14:78-82). For RDT&E engines, the data were screened utilizing the same programs developed for DO24 data after reformatting.

As the F-100 engine is just entering the inventory, total operating hours and the total number of module removals are quite low. An initial computer tape containing all D024 reporting transactions for F-100 modules through 30 March 1975 contained only one reported module failure. A subsequent tape containing data through June 1975 revealed three additional removals. Data obtained from the F-15 Joint Test Force were more productive though still containing a limited number of data points. The total number of failures and maximum time removals by module type were as follows:[1]

Table 3.1

Maximum Time Removals and Failures by Module Type Reported by F-15 Joint Test Force[2]

Module Type	Failures	Maximum Time Removals
Fan	16	2
Core	5	8
Fan Drive Turbine	12	4
Gearbox	8	6

We believe that the number of reported module failures and maximum operating time removals are insufficient to reliably determine the underlying hazard rate. Shooman (24:457), for instance, notes that statistical techniques

[1]The augmentor module is not assigned a MOT and for this reason was not examined.

[2]Methods used to screen the raw data on module transactions are described in Chapter V.

of analysis begin to be of significant benefit when at least 20 components have been life tested. None of the modules listed in Table 3.1 experienced a total number of removals (failures plus maximum time removals) greater than 18. However, data which was available was analyzed to determine a best initial estimate of the hazard rate function for each module. Hazard rate functions thus determined were used to find an initial optimal value of n (the breakpoint between replace at failure and opportunistic replacement) and to investigate sensitivity of this value to changes in the hazard rates.

Cost Factors and the Structure of an Optimal Policy

As previously pointed out, in addition to knowing the hazard rate, one must also know the relative costs of replacement before failure and replacement after failure to determine if replacement before failure is warranted. In Chapter II, the factors which could be expected to result in less cost for replacement before failure were found to be: 1. fewer required resources, 2. less downtime for replacement and 3. opportunistic replacement when an equipment item was already removed. Further, in Chapter II, the area of interest for this thesis was delimited to determining an optimal policy for opportunistic replacement. As the reader will recall, this limitation was occasioned by the, as yet, unsettled state of the data on resource costs for F-100 engine maintenance actions. Opportunistic replacement policies have been explored by Jorgenson, McCall and Radner (10:123-126) and others (6:271-283;21:61-71;38:267-270), the most complete treatment being by Jorgenson, McCall and Radner. A specific approach which recognized the savings realizeable through replacement of a component which has already been removed to facilitate other maintenance was not found in the literature. Such an approach is developed here as an extension of the Jorgenson, McCall and Radner model.

Following Jorgenson, McCall and Radner (10:244-251) one may divide the cycle from depot overhaul to the next subsequent depot overhaul of a single module into two regions as illustrated in Figure 3.2.

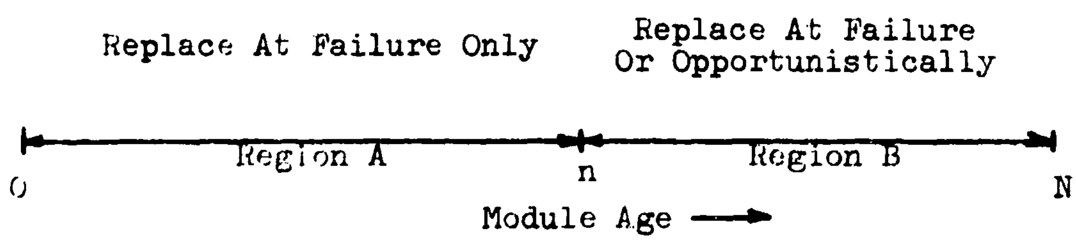

Figure 3.2

Module Depot Overhaul Cycle

In region A (i.e., age less than n) a module would be replaced and returned to the depot only at failure.[1] In region B, a module would be replaced both at failure and opportunistically. Opportunistic replacement is meant to be the replacement of a module when it is removed to facilitate maintenance but is still operable. N is the maximum operating time where modules are mandatorily removed from service. For modules of age less than n, opportunistic replacement before failure costs more than reinstalling the same module. For modules of age greater than n, opportunistic replacement before failure costs less than reinstalling the same module.

The mechanics of Figure 3.2 may be understood by considering the underlying costs involved. During a single cycle from depot to overhaul, certain costs are always incurred and others may or may not be incurred. Conceptually, the problem is similar to the analysis of fixed and variable costs discussed in economics (22:463-466). For a single overhaul cycle, the fixed costs are:

---

[1]Specifically, the module would be returned to the depot only if it experienced a failure beyond the capability of an intermediate level maintenance facility to repair. When gathering data on module times at failure in support of this thesis, only failures precipitating a return to a depot level facility were counted.

1. The cost to pack a module for shipment to the depot and to unpack a module upon receipt at the intermediate level.

2. Transportation cost from the intermediate maintenance level to the depot and back.

3. The cost to overhaul the module at depot. As will be explained in Chapter V this cost will always be assumed to be the same. Further, this cost includes packing and unpacking costs at the depot.

In Chapter II, it was argued that how one assigns the costs to remove and replace the engine in the aircraft, transport the engine to and from the intermediate maintenance shop, and remove and replace a module on the engine depends on the circumstances causing module removal. Given that, for instance, the fan module failed, then these costs would be attributable to the fan module. On the other hand, if the core module had failed and removal of the unfailed fan module were required for core module repair, then these costs would be attributable to the core module. Subsequent replacement of the fan module, given that the engine were already removed from the aircraft, transported to the intermediate maintenance shop, and disassembled, would not generate additional costs to perform these actions. The expected engine removal and replacement, transportation, and disassembly and reassembly costs attributable to the fan module depend on the probability of the fan module being replaced either at failure or alternately upon it being replaced opportunistically.

One may note by reference to Figure 3.2 that the probabilities of replacement at failure and opportunistic replacement could be expected to bear some relationship to the sizes of region A and region B. Although the specific relationship remains to be examined, it does not seem unreasonable that as region A shrinks in relation to region B, the probability of opportunistic replacement of the fan module would increase. Expected expenditures on engine removal, reinstallation, transportation and disassembly/assembly attributable to the fan module are equal to the dollar cost to perform these tasks multiplied by the probability of incurring them. Thus, as region B increases in proportion to region A, expected expenditures on engine removal, reinstallation, transportation, disassembly and assembly attributable only to the fan module would decrease.

There is, however, a penalty for increasing the size of region B. In region A the fan module is replaced only if it fails. In region B the fan module is replaced for failure or opportunistically. As region B increases (n moves towards zero) the probability of early opportunistic replacement increases and one would expect that cycle length, i.e., the mean time at removal (MTAR) would decrease. Figure 3.3 illustrates the general relationship between cost and cycle length. As n moves from N toward O, total cost, which is composed of fixed cost and variable cost, would decrease. The cycle length over which we would spread these costs, however, would also decrease.

Thus, it is not immediately obvious if a decrease in total cost per overhaul cycle is advantageous or not. One must look further and determine cost per operating hour (10:247). The optimal value of n would be that n which results in minimum cost per operating hour.

Following our discussion thus far, suppose that total cost is some function f(n) and that cycle length is also a function g(n); both f(n) and g(n) are monotonic and increasing in the interval (O, N). Let h(n) be cost per hour as a function of n where

$$h(n) = \frac{f(n)}{g(n)} \qquad (3.3)$$

Following accepted methods of marginal analysis (18:50-83), the point of minimal total cost per operating hour could be found by setting

$$\frac{d}{dn} h(n) = \frac{d}{dn} \frac{f(n)}{g(n)} = 0 \qquad (3.4)$$

and solving for the value of n which would make this relationship true. Analytic development of equation (3.3) and an algorithm for solving for n are discussed in

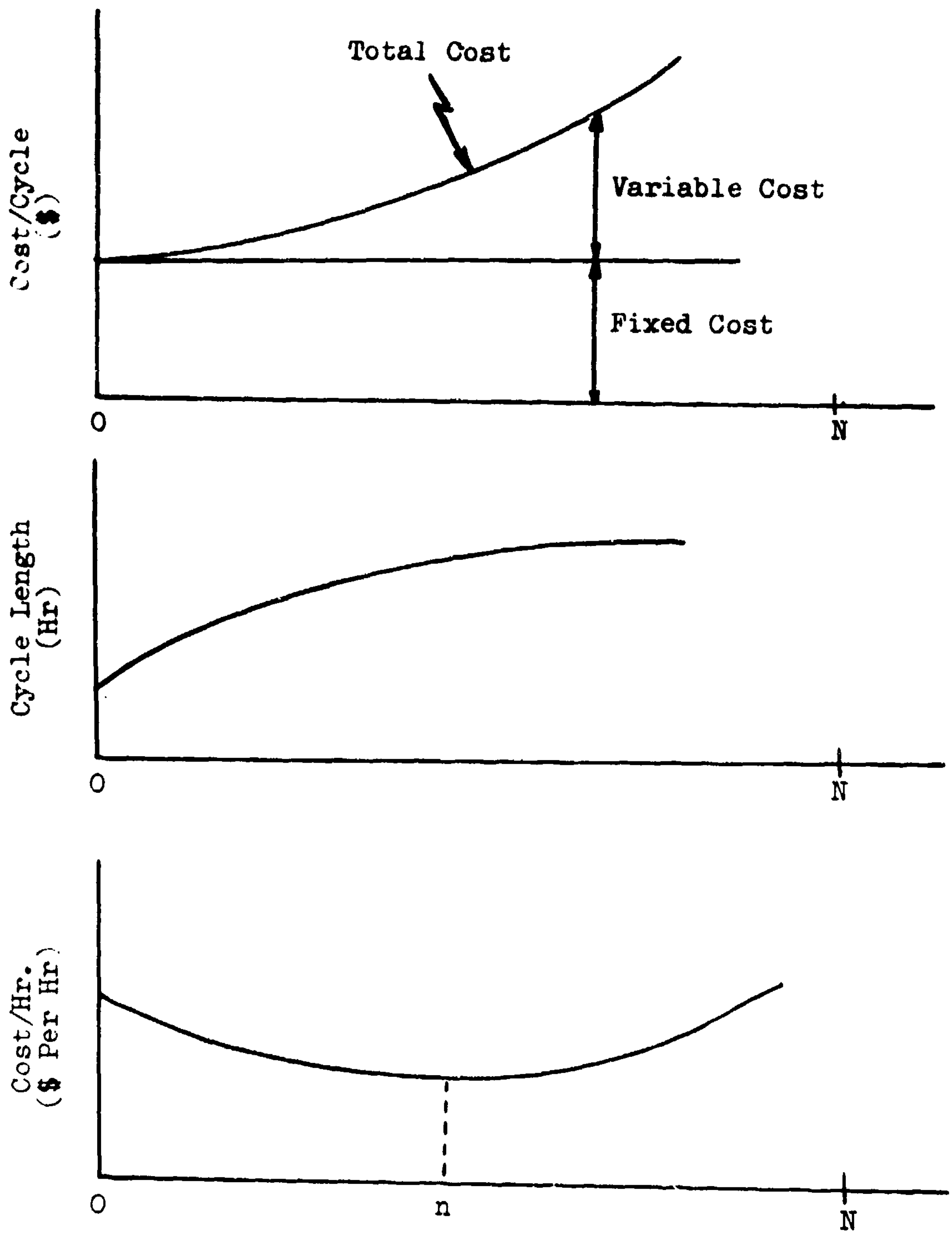

Figure 3.3

Cost and Cycle Length As A Function of n

Chapter IV, Mathematics of the Algorithm.

Sensitivity of the Optimal Policy to Uncertainties About Hazard Rate Function Parameters and Cost Factors

In practice the parameters of the hazard rate function are not generally known with certainty (10:237). Furthermore, the total cost under an optimal policy may not differ significantly from the cost under a replace-at-failure policy. As an example, Jorgenson (10:228) described a system whose optimal policy produced only a 2.5 percent savings over the replace-at-failure policy. Therefore, before implementing an optimizing policy, it would be worthwhile to determine the policy's sensitivity to uncertainty in hazard rate parameters and cost parameters.

In order to determine sensitivity of the optimal value of n and the optimal policy cost to uncertainty about the hazard rate parameters, all input variables except one of the hazard parameters were held fixed while that parameter was varied over a range on either side of the initial estimate. This procedure was then repeated for each of the other parameters. Sensitivity to cost uncertainty was also examined in the same manner by varying one cost input at a time while all other inputs were held constant.

Finally, a comparison between cost of operating the system until failure and the cost of optimal replacement was made to determine the significance of savings, if any.

Summary List of Assumptions

1. The engine failure data collected for this thesis are valid. For operational engines this assumption is based on an examination of the process by which data are generated, recorded, and accumulated as described earlier in this chapter. For RDT&E engines, this assumption is based on discussions with the Engine Project Office, F-15 System Program Office.

2. The hazard function of each of the five modules is independent of the other modules. This assumption is based on discussion of the two-component system by Jorgenson, McCall and Radner (10:244-251).

3. Transportation costs and depot overhaul costs for modules returned to the overhaul facility are assumed to be the same whether the module has failed or not.

Summary List of Limitations

1. The algorithm developed in this thesis to determine an optimal replacement policy is applicable only to a five component system. Similar methodology can be used in other systems. Further, the only decision the policy will facilitate is whether or not to replace a component which has not yet failed.

2. Generalizations derived from the data gathered by this research can be made only to the population of

F-100 engine modules originally installed on F-100 engines 023 through 049 and engine 051, and to subsequent modules meeting the same design specifications.

3. Opportunistic policies for unfailed modules that are not removed to facilitate other maintenance were not considered.

4. Conceptually, it is optimal economically to solve for n and N simultaneously but in this thesis N is given. Logically, in practice we grope toward an appropriate N through careful observation of failures.

CHAPTER IV

MATHEMATICS OF THE ALGORITHM

Introduction

This chapter presents the mathematics of the algorithm used to find an optimal value of n for a given set of hazard function parameters and cost factors. In essence, the algorithm is an extension of a model developed by Jorgenson, McCall and Radner (10:244-251) for a two component system. The current formulation assumes continuous underlying time at failure distributions, permits the second component (in this case the core module) to have a general hazard function, and solves for n; the Jorgenson, McCall and Radner model on which it is based assumed discrete distributions, restricted one system component to a constant hazard, and solved simultaneously for n and N.

The order of presentation is as follows: first, mathematical expressions are derived to calculate expected cost per cycle; second, expected cycle length is addressed; third, calculation of conditional probabilities is discussed; fourth, the method of incorporating core module age is introduced and finally, the minimization technique is discussed.

Table 4.1 indicates which modules must be removed to facilitate removal of other modules. It can be seen that only core module and fan drive turbine module removal precipitate removal of other modules. It will be recalled that our interest is in when to replace an unfailed module which has already been removed to facilitate maintenance on another module. It can be seen from Table 4.1 that the fan, fan drive turbine, augmentor/exhaust and gearbox modules under certain circumstances will require prior removal to facilitate maintenance on another module. The augmentor/exhaust module, however, does not have an established depot overhaul interval (7). All repair is accomplished at the field level and, from the standpoint of the algorithm developed in this chapter, the augmentor/exhaust module will not be considered. The fan module, fan drive turbine module and gearbox modules are removed to facilitate maintenance on the core module. Thus we need only explore when to replace unfailed fan modules, fan drive turbine modules and gearbox modules when they are removed to facilitate maintenance on the core module. The algorithm is developed below in terms of the fan module but is equally applicable to the fan drive turbine and gearbox modules.

Table 4.1

Engine Module Removal Sequence

Module Requiring Removal	Other Modules Which Must Be Removed In Support
Fan	None
Core	Fan, fan drive turbine, augmentor/exhaust, gear-box
Fan Drive Turbine	Augmentor/exhaust
Augmentor/Exhaust	None
Gearbox	None

Opportunistic replacement before failure is warranted only if 1. opportunistic replacement costs less than reinstalling the module and 2. the module has an increasing hazard rate in the region where opportunistic policies are considered. Whether or not a module satisfies the second criterion is determined by examining its hazard rate as discussed in Chapters III and V. The algorithm developed in this chapter would be exercised only if the hazard criterion were met.

Determination of Expected Cost

Figure 3.2 from Chapter III is redrawn here for ready reference. The reader will recall that in region A, the fan module is replaced and returned to the depot only if it sustains a failure requiring depot overhaul. In region B, the fan module is replaced and returned to the depot if it either fails or reaches its maximum operating time or if the core module is removed for any reason.[1]

---

[1]It should be noted that the possibility of intermediate level reparable failures exists in both regions. Thus, it is necessary to screen data from which hazard parameters are determined for the fan module, fan drive turbine module and gearbox module in such a manner that only depot reparable failures are included. The simplifying assumption is made here that all failures for module age > n are depot reparable only. Strictly, this is not likely to be true. The direction of probable bias resulting from this simplification is discussed in Chapter VI.

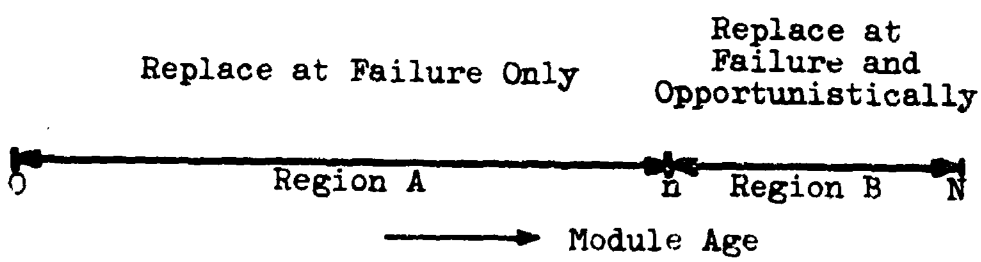

Figure 4.1

Module Depot Overhaul Cycle

Define:

N — Age of the fan module at which mandatory replacement of the fan module occurs given survival until that age,

n — Earliest age of the fan module at which opportunistic replacement is permitted: n is the leading edge of region B,

$Q(fc)$ — Probability that an engine requires field level replacement of the fan module and the core module simultaneously between n and N given survival of both until n,

$Q(f\bar{c})$ — Probability that an engine requires field level replacement of the fan module before the core module between n and N given survival of both until n,

$Q(\bar{f}c)$ — Probability that an engine requires field level replacement of the core module before the fan module between n and N given survival of both until n,

$q(f)$ — Probability of fan replacement before age n,

$q_i(fc)$ — Probability that an engine requires field level replacement of the fan module and core module simultaneously in the i-th interval between n and N given survival of both until n,

$q_i(f\bar{c})$ — Probability that an engine requires field replacement of the fan module first in the i-th interval between n and N given survival of both until n,

$q_i(\bar{f}c)$ — Probability that an engine requires field replacement of the core module first in the i-th interval between n and N given survival of both until n,

$QFAN(t_{k_{Fan}} \mid t_{j_{Fan}})$

Probability of fan module failure between fan module age t_j and fan module age t_k given survival until fan module age t_j,

$QCORE(t_{k_{core}} \mid t_{j_{core}})$

Probability of core module failure between core module age t_j and core module age t_k given survival until core module age t_j,

$f_{Fan}(t)$	Fan module failure probability density function,
$f_{core}(t)$	Core module failure probability density function,
FANREP	Fan module field level replacement cost,
FANDEP	Fan module depot overhaul cost,
FANPACK	Cost to package the fan module for shipping and unpackage upon receipt,
FANSHIP	Round trip shipping cost for the fan module between the field level repair shop and the depot,
FANMOT	Fan module maximum operating time,
COREMOT	Core module maximum operating time.

The probability of fan module removal for failure in the interval (0, n) is simply

$$q(f) = \int_0^n f_{Fan}(t)\, dt \qquad (4.1)$$

Consider an interval of width δt between n and N where $\delta t = t_{i+1} - t_i$ and δt is sufficiently small that only one of the possibilities $q_i(fc)$, $q_i(f\bar{c})$, or $q_i(\bar{f}c)$ can occur in the interval. During the i-th interval, the probability that the fan module, the core module, or both are removed given both are installed at the same time, both are the same age, and given survival of both until n is

$$q_i(f \cup c \mid n) = q_i(fc) + q_i(\overline{f}c) + q_i(f\overline{c}). \qquad (4.2)$$

When $n < t_i \leq N - \delta t$, then

$$\begin{aligned} q_i(fc) = {} & \left[1 - \mathrm{QFAN}(t_i \mid n)\right] \bullet \\ & \left[1 - \mathrm{QCORE}(t_i \mid n)\right] \bullet \\ & \mathrm{QFAN}(t_{i+1} \mid t_i) \bullet \\ & \mathrm{QCORE}(t_{i+1} \mid t_i), \end{aligned} \qquad (4.3)$$

and when $N - \delta t < t_i \leq N$ and N = COREMOT

$$\begin{aligned} q_i(fc) = {} & \left[1 - \mathrm{QFAN}(N - \delta t \mid n)\right] \bullet \\ & \left[1 - \mathrm{QCORE}(N - \delta t \mid n)\right] \bullet \\ & \mathrm{QFAN}(N \mid N - \delta t) \end{aligned} \qquad (4.4)$$

since core removal would then be certain.[1] Similarly, if $N - \delta t < t_i \leq N$ when N = FANMOT then

$$\begin{aligned} q_i(fc) = {} & \left[1 - \mathrm{QFAN}(N - \delta t \mid n)\right] \bullet \\ & \left[1 - \mathrm{QCORE}(N - \delta t \mid n)\right] \bullet \\ & \mathrm{QCORE}(N \mid N - \delta t) \end{aligned} \qquad (4.5)$$

The terms $q_i(f\overline{c})$ and $q_i(\overline{f}c)$ are developed similarly. When $n < t_i \leq N - \delta t$, then

---

[1]The conditional probabilities developed in this chapter may not be intuitively clear. A more complete mathematical treatment is found in Appendix H.

$$q_i(f\bar{c}) = \left[1 - QFAN(t_i \mid n)\right] \cdot \left[1 - QCORE(t_{i+1} \mid n)\right] \cdot QFAN(t_{i+1} \mid t_i); \quad (4.6)$$

and, when $N - \delta t < t_i \leq N$ and N = COREMOT $q_i(f\bar{c}) = 0$, but if N = FANMOT then

$$q_i(f\bar{c}) = \left[1 - QFAN(N - \delta t \mid n)\right] \cdot \left[1 - QCORE(N \mid n)\right]. \quad (4.7)$$

Also, when $n < t_i \leq N - \delta t$,

$$q_i(\bar{f}c) = \left[1 - QFAN(t_{i+1} \mid n)\right] \cdot \left[1 - QCORE(t_i \mid n)\right] \cdot QCORE(t_{i+1} \mid t_i) \quad (4.8)$$

but, when $N - \delta t < t_i \leq N$ and N = COREMOT

$$q_i(\bar{f}c) = \left[1 - QFAN(N \mid n)\right] \cdot \left[1 - QCORE(N - \delta t \mid n)\right], \quad (4.9)$$

but $q_i(\bar{f}c) = 0$ if N = FANMOT.

Then, where $M = (N - n)/\delta t$ such that M is an integer,

$$Q(fc) = \sum_{i=1}^{M} q_i(fc) \quad (4.10)$$

$$Q(f\bar{c}) = \sum_{i=1}^{M} q_i(f\bar{c}), \text{ and} \quad (4.11)$$

$$Q(\bar{f}c) = \sum_{i=1}^{M} q_i(\bar{f}c) \qquad (4.12)$$

It should be noted that $q_i(fc)$, $q_i(\bar{f}c)$, and $q_i(f\bar{c})$ are approximations of $\frac{d}{dt} Q(fc)$, $\frac{d}{dt} Q(\bar{f}c)$, and $\frac{d}{dt} Q(f\bar{c})$ respectively. The accuracy of the approximation depends on how many segments the interval (n, N) is broken into--that is on the size of δt. A more complete discussion of approximation accuracy is found in Appendix F.

FANPACK, FANSHIP and FANDEP were previously defined as the cost to package and unpackage the fan module, ship the module to and from the depot and accomplish fan module depot repair. FANPACK, FANSHIP and FANDEP are experienced with certainty during a single overhaul cycle. FANREP, the cost of field level removal, however, is experienced only if the fan module fails in the interval (O, n) or is removed before the core module in the interval (n, N). It would not be experienced if the core module failed before the fan module in the interval (n, N). Therefore, the expected cost in the interval (O, N) is

$$E(cost) = \qquad (4.13)$$

$$FANPACK + FANSHIP + FANDEP + FANREP \bullet \{q(f) + [1 - q(f)] \bullet [Q(fc) + Q(f\bar{c})]\}^{1}$$

[1] We will use the convention that if both the fan module and core module require removal at the same time (e.g., simultaneous failure) FANREP is chargeable to the fan module.

Determination of Expected Cycle Length

The expected value of a discrete probability distribution is

$$E(k) = \mu_k = \sum kP(k) \qquad (4.14)$$

where k is a discrete random variable and P(k) is the probability of observing the value k (37:148). The equivalent expression for a continuous distribution is

$$E(x) = \mu_x = \int_a^b xf(x)\,dx \qquad (4.15)$$

where x is a continuous random variable, f(x) is its density function and a and b are the limits of integration (37:193).

In the interval (0, n), the probability of fan module replacement was

$$q(f) = \int_0^n f_{Fan}(t)\,dt \qquad (4.1)$$

and in the interval (n, N) the probability of fan module replacement is

$$P(\text{replacement in interval } (n, N)) = \qquad (4.16)$$

$$[1 - q(f)] \bullet \sum_{i=1}^{M} [q_i(fc) + q_i(\bar{f}c) + q_i(f\bar{c})]$$

The expected time at replacement of the fan module then is

E(time at replacement) = (4.17)

$$\int_0^n t \bullet f_{Fan}(t)\,dt$$

$$+ [1 - q(f)] \bullet$$

$$\sum_{i=1}^{M-1} \left\{ \frac{(t_i + t_{i+1})}{2} [q_i(fc) + q_i(f\bar{c}) + q_i(\bar{f}c)] \right\}$$

$$+ [1 - q(f)] \bullet [1 - QCORE(N \mid 0)],^1$$

where the term

$$[1 - q(f)] \bullet [1 - QCORE(N \mid 0)]$$

is the probability of fan module and core module survival until N. The expected time at replacement is also the expected cycle length since the fan module is returned to the depot on replacement.

Conditional Probabilities

$QFAN(t_{k_{Fan}} \mid t_{j_{Fan}})$ and $QCORE(t_{k_{core}} \mid t_{j_{core}})$ are conditional probabilities. Using a derivation by Papoulis (15:179), the conditional probability of module removal between t_j and t_k given survival until t_j can be expressed as

$$P(\text{removal} \mid \text{survival until } t_j) = \frac{F(t_k) - F(t_j)}{1 - F(t_j)} \quad (4.18)$$

---

[1]It is assumed that replacement occurs at the mid-point of the interval (t_i, t_{i+1}).

where F(t) is the cumulative distribution function evaluated at t.

Core Module Age and Difference in MOT's

It would not normally be true that both the fan module and core module would have the same number of accumulated hours. To illustrate with an obvious example, if at some fan module age t a new (0 time) core module were installed on the engine, the difference between core module and fan module age would be t hours. As core module age can have significant effect on the probability of core module removal, it is important to provide for core module age as an input to the algorithm. Further, although at the current time both the core module and the fan module have the same maximum operating time (MOT) this will not necessarily be true in the future. A difference in fan module and core module MOT is also an important input to the algorithm. To illustrate, if the fan module and core module were of the same age at the time of fan module opportunistic replacement and the core module had the earlier MOT, the fan module cycle could not last longer than the amount of time remaining on the core module. Core module removal at its MOT given survival to its MOT is a certainty. Therefore, we would, with certainty, ship the fan module to the depot on achievement of core module MOT under an opportunistic policy.

The following method will be used to incorporate core module age and a difference between fan module MOT and core module MOT into the algorithm. Let

$$\text{FANTIME} \equiv \text{Age of fan module}$$

$$\text{CORETIME} \equiv \text{Age of core module}$$

$$\Delta\,\text{time} \equiv \text{FANTIME} - \text{CORETIME}$$

and

$$\Delta\,\text{MOT} \equiv \text{FANMOT} - \text{COREMOT}$$

Then, alternatively, we may express CORETIME and COREMOT in terms of FANTIME and FANMOT as

$$\text{CORETIME} = \text{FANTIME} - \Delta\,\text{time} \tag{4.19}$$

and

$$\text{COREMOT} = \text{FANMOT} - \Delta\,\text{MOT}. \tag{4.20}$$

FANMOT could be reached first if

$$\text{FANMOT} - \text{FANTIME} < \text{COREMOT} - \text{CORETIME}$$

which, by definition, is the same as

$$\text{FANMOT} - \text{FANTIME} < \text{FANMOT} - \Delta\,\text{MOT} - (\text{FANTIME} - \Delta\,\text{time})$$

or, more simply

$$\Delta\,\text{time} > \Delta\,\text{MOT}.$$

Then, if $\Delta\,\text{time} > \Delta\,\text{MOT}$

$$N = \text{FANMOT}, \tag{4.21}$$

and if $\Delta\,\text{time} = \Delta\,\text{MOT}$

$$N = \text{FANMOT} = \text{COREMOT} \tag{4.22}$$

but if $\Delta\,\text{time} < \Delta\,\text{MOT}$

$$N = \text{FANMOT} - \Delta\,\text{time} + \Delta\,\text{MOT}. \tag{4.23}$$

The conditional probability QCORE($t_{k_{core}} | t_{j_{core}}$) is read as the probability of core module failure between core module age t_j and core module age t_k given survival until core module age t_j. When the difference between FANTIME and CORETIME is Δ time, the conditional probability QCORE($t_{k_{core}} | t_{j_{core}}$) can be expressed in terms of FANTIME as QCORE($t_{k_{fan}} - \Delta$ time $| t_{j_{fan}} - \Delta$ time) where this expression appears in Equations (4.3), (4.4), (4.5), (4.6), (4.7), (4.8), (4.9) and (4.17).

Minimization Technique

Expected cost per hour is determined as the ratio of expected cost per cycle to expected cycle length. Expected cost per cycle and expected cycle length are expressed by Equations (4.13) and (4.17) respectively. Minimization of this ratio is tractable through numerical techniques (10:123). The technique used in this thesis was to calculate the expected cost per hour as the value of n was stepped in 10 hour increments from 0 to 250 hours. This process was then repeated as core age was increased in ten hour increments from zero to 250 hours. For each value of core age, the optimal value of n was that value which resulted in least expected cost per hour. Choice of ten hour increments for n and core age was in large measure arbitrary. Obviously, one could make the interval as sma as desired. Smaller intervals, however, exact a penalty in computer processing time. Using ten hour intervals, 1.2 processing hours were required to run the program listed in Appendix F in the batch mode on the computer available at Wright-Patterson Air Force Base.

CHAPTER V

DATA ACQUISITION AND ANALYSIS

Cost Data

Several elements make up the costs involved in modular engine repair actions (5). The costs addressed in this research effort were: 1. module depot overhaul costs, 2. base-to-depot and return transportation costs, 3. module packing and unpacking costs and 4. intermediate (field) level repair costs. Each of these cost elements will be discussed separately in this section. Detailed calculations and the data used are found in Appendix B. For consistency, all base level repair costs are assumed to take place at Edwards Air Force Base, California.

Depot costs. Overhaul at depot level is accomplished to return a module to like-new configuration. Several factors such as depot material cost, depot manhour cost and depot overhead are elements of the single cost referenced in this thesis as depot overhaul cost. In Chapter II it was noted that depot overhaul cost per engine or module can be highly variable. Thus, reduction of depot overhaul cost to a single figure ignores the considerable uncertainty inherent in this cost. More detailed cost data, however, is not yet available. For this reason, the officially accepted module depot overhaul costs were utilized as a starting point to determine an optimal policy and to investigate sensitivity of the policy. The officially accepted module overhaul costs were taken from San Antonio ALC/MAWWF letter of 12 March 1975 entitled Module Depot Overhaul Costs (7).

Base to depot transportation costs. Engine modules for the F-100 engine are shipped by truck and classified as machinery, parts, steel (25). Charges for this movement are based solely upon weight. AFLC/MMP provided a chart showing the weight of each module when properly packed in its shipping container (7). A shipping rate schedule (Edwards Air Force Base, California to San Antonio Air Logistics Command, Texas) which showed cost per hundred-weight was supplied by the Office of the Chief of Transportation/DSTRP, Kelly Air Force Base, Texas. Multiplication of the appropriate rate schedule by the module shipping weight gave the cost of one way movement of each individual module. Since overhaul action requires a new module to be sent to the base, as well as shipment of the old one to depot, complete transportation cost involves two-way shipment which doubles the cost thus calculated.

Intermediate (field) level costs. Several costs are incurred at field level, consisting of manpower charges and, if appropriate, material charges. All manpower costs were figured based upon standard team sizes of four, three or two technicians as required for the action. Costs per clockhour were calculated for each size standard team based upon wage rates found in AFM 177-101. Three sources were used to obtain estimates for the clockhours required for each task. The three potential sources were: 1. Qualitative and Quantitative Personnel Requirements Information (QQPRI) (17), a document provided by Pratt and Whitney to predict manhour requirements, 2. time and motion studies from videotape films produced at Edwards Air Force Base during technical order validation work and 3. subjective estimates based upon a supervisor's actual experience on the F-100 engine. A comparison of the three estimates is presented in Table E.11. The QQPRI figures were predictions made based upon engineering design. Validation of the figures is as yet incomplete but preliminary results were inconclusive (12). Time and motion studies from videotape films were found to be poor estimates since technical order validation action requires considerable time to stop and document difficulties found. Subjective estimates based upon experience were declared to be the most consistent, valid figures available at this time (12;20). Since these estimates reflect 100 percent utilization of personnel, an allowance factor of 1.67 provided by the F-15 JTF at Edwards Air Force Base was applied to convert these estimates to expected average performance. Manhour estimates are available in Appendix B for comparison purposes. Estimated clockhours multiplied by team costs per clockhour established the manhour costs for each task. Material costs were determined by pricing the materials consumed in each task. The field level tasks necessary for this analysis were: 1. module removal and replacement/reinstallation, 2. module packing for shipment, 3. engine removal and replacement/reinstallation and 4. base transportation to engine shop. Each task will be discussed individually below.

Module removal and replacement/reinstallation. Since this thesis addressed opportunistic replacement at the module level, only removal and replacement/reinstallation at the aggregate module level were considered. Field level module repair costs were not considered. Expendable material required for module removal and replacement was included to provide a total field level module removal and replacement/reinstallation cost.

Module packing for shipment and unpacking upon receipt. Only manpower cost was involved in packaging the module for shipment once removed from the engine; material is not consumed during this task.

Engine removal and replacement/reinstallation. The engine change operation essentially consists of three operations: . removal, 2. reinstallation and 3. trim.

Team composition cost multiplied by clockhours required determined manhour costs. Interestingly, an engine change has been made in as little as 20 minutes, showing the accomplishments possible when a highly trained team is used and all tools and equipment are prepositioned (7;9). No consumable material was found to be required for engine change.

Base transportation. The time and team size required to move the engine from the aircraft to the engine shop for teardown and repair constitute the basis for calculating base transpc ation cost. For the present situation, this cost is almost neglible but is included for completeness. If a Queen Bee operation, currently under discussion, is adopted for the F-100 engine, base transportation costs as defined would become an inter-base transportation cost for the complete engine and could be substantial.

Module Failure Data

Two sources were available for module failure data--operational engine data obtained through the standard engine status reporting system and RDT&E engine data obtained through the F-15 Joint Test Force at Edwards Air Force Base, California. A complete computer program package written in FORTRAN is provided in Appendix C to manipulate standard engine status reporting system (AFM 400-1) data. Acknowledgement must be made to the thesis team of Pansza and Woods (14) for development of many of the programs included herein. Operational flying has produced too few engine failures to date for any significant results using this reporting system.

RDT&E engine data provided the only other source of engine removals for the F-100 engine. A tailor made reporting system is in use at Edwards Air Force Base, California which provides data in a format incompatible with AFM 400-1 formating. Data was obtained in paper output form from the F-15 Joint Test Force YF-100 Engine Module Report. Data was keypunched onto punch cards for input into the CREATE system, an AFLC Honeywell 635 Dual Processor computer at Wright-Patterson Air Force Base. A final check on data conversion from report form to punch card form was accomplished by verifying all cards. This operation involves essentially retyping all data using the punched card just produced and the original worksheets. Computer programs were developed to structure the large volume of data into a readable format and screen it for module removals due only to failure or expiration of established operating hours. Since these programs were developed for only one time use, only the logic of their operation is presented in Appendix C. As future failure data on the F-100 engine will be recorded in the standard engine status reporting system; programs which will screen this data are presented in some detail. A listing of RDT&E removal times upon which the failure distribution parameters were computed is presented in Appendix D. Recalling the discussion of potential hazard rate models from Chapter III, the researchers were next faced with the decision as to which model was most appropriate to model the failure data produced. The models, as recommended by Shooman are: 1. exponential hazard, 2. piecewise linear hazard and 3. Weibull hazard (24:194). Shooman further notes that "a good way to treat these (component failure) data is to compute and plot either the failure density function or the hazard rate as a function of time (24:160)." The development of the data hazard rate was presented in Chapter III of this document. Shooman recommends the use of "engineering judgment" to select the model most appropriate for the data being analyzed (24:457). Selection of the Weibull model was based upon its acceptable general fit when plotted against data hazard computations and its wide usage in reliability work (24:190;36:293). Furthermore, the Weibull permits modeling both increasing and decreasing hazard rates with the same mathematical formula (24:190). This was of considerable value when performing sensitivity analysis.

Plots of the data failure rates are found in Figures 5.1 thru 5.4. The Weibull functions determined to fit the data by the program contained in Appendix E are shown on the same graphs. One notes the relative "noiseness" of engine failure data as described by Hansza

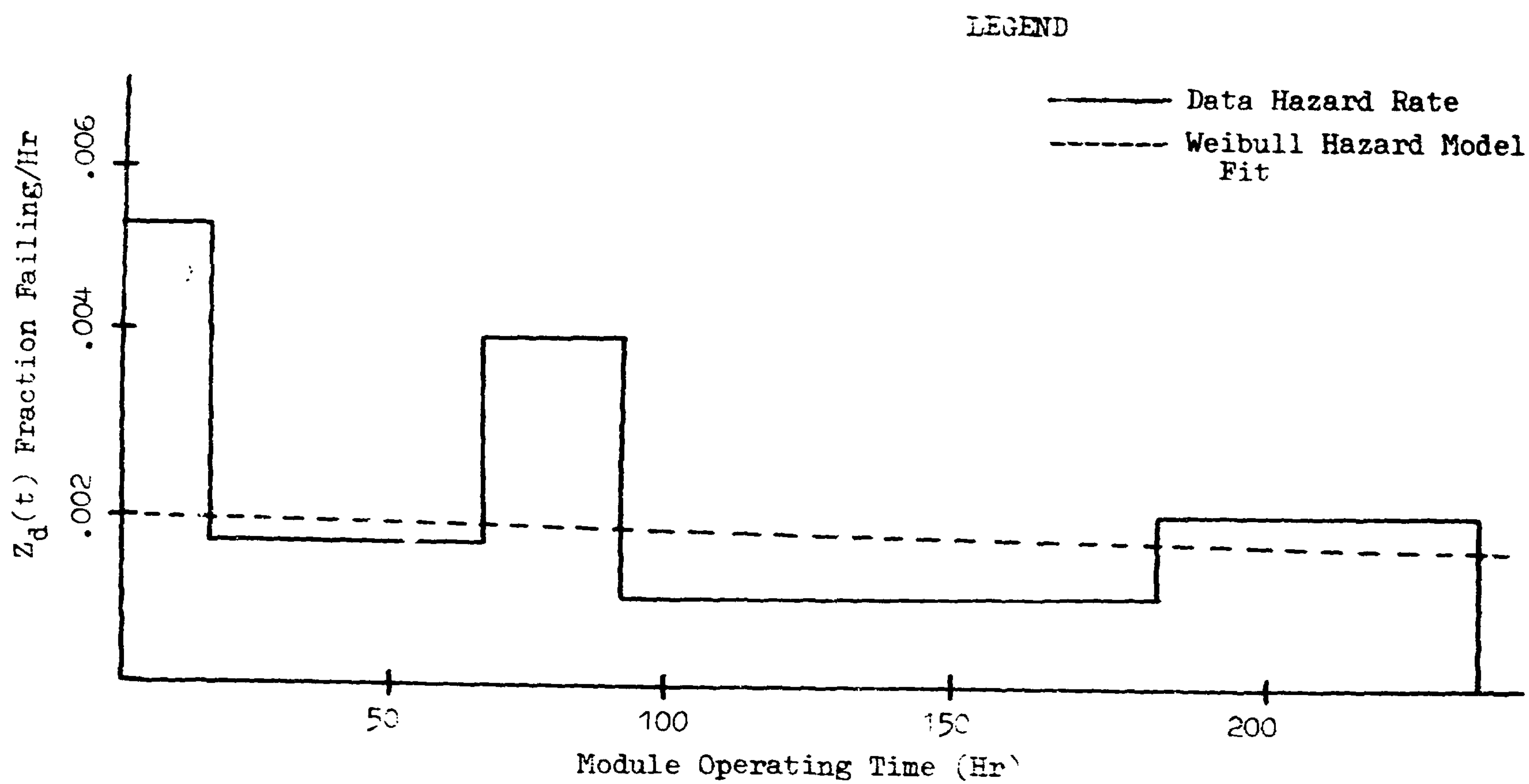

Figure 5.1

Core Module Data Hazard Rate

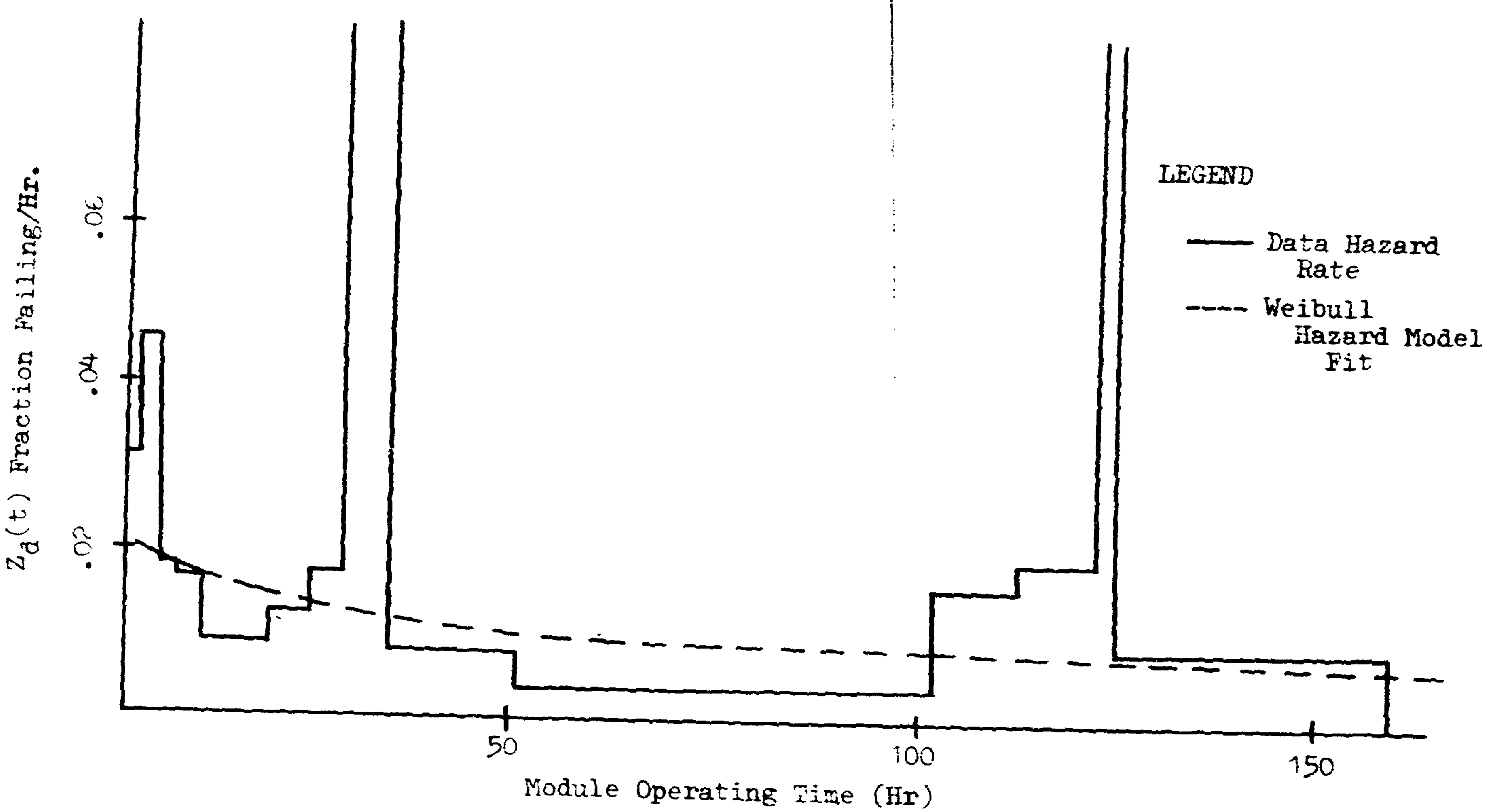

Figure 5.2

Inlet/Fan Module Data Hazard Rate

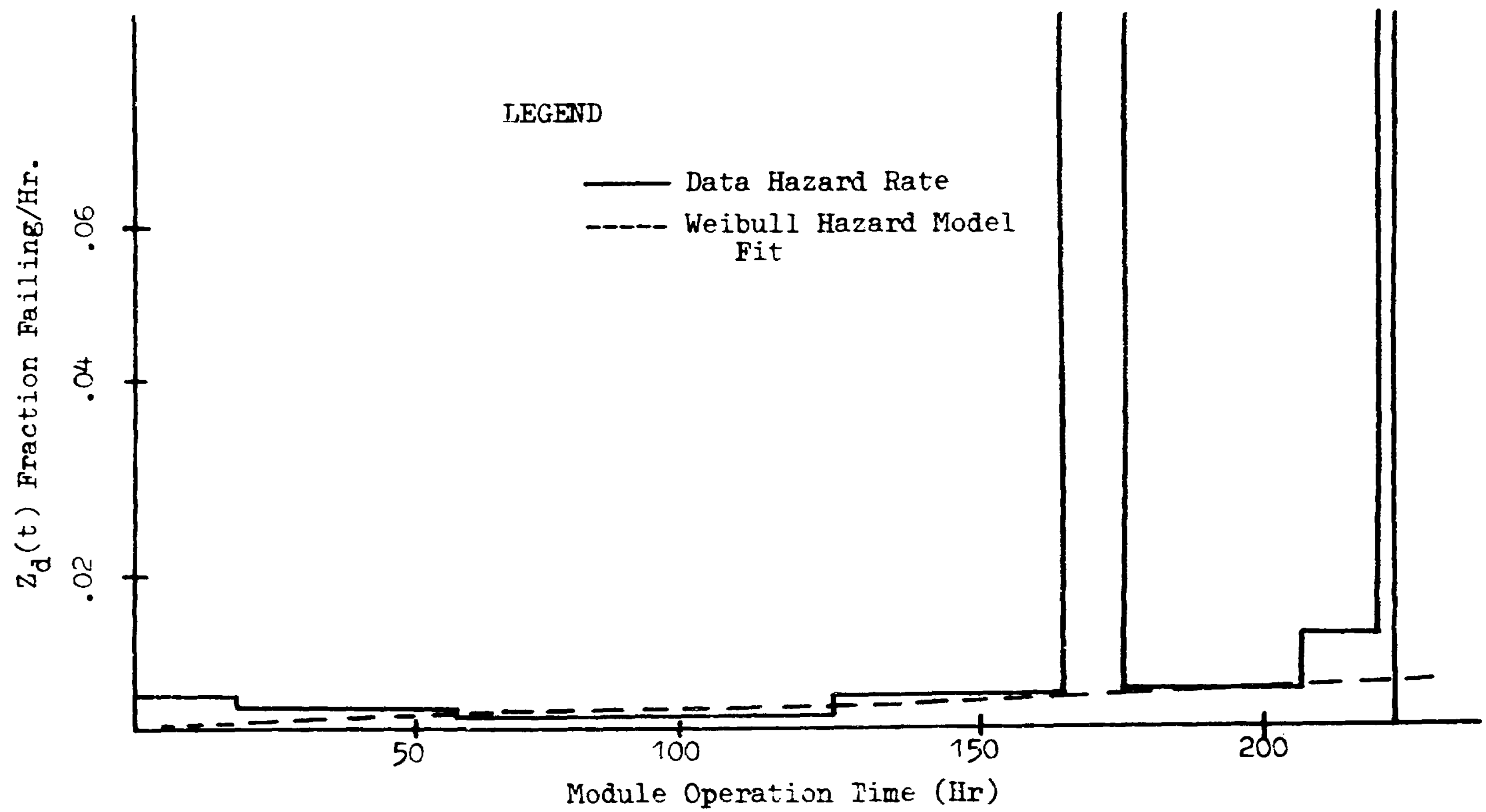

Figure 5.3

Gearbox Module Data Hazard Rate

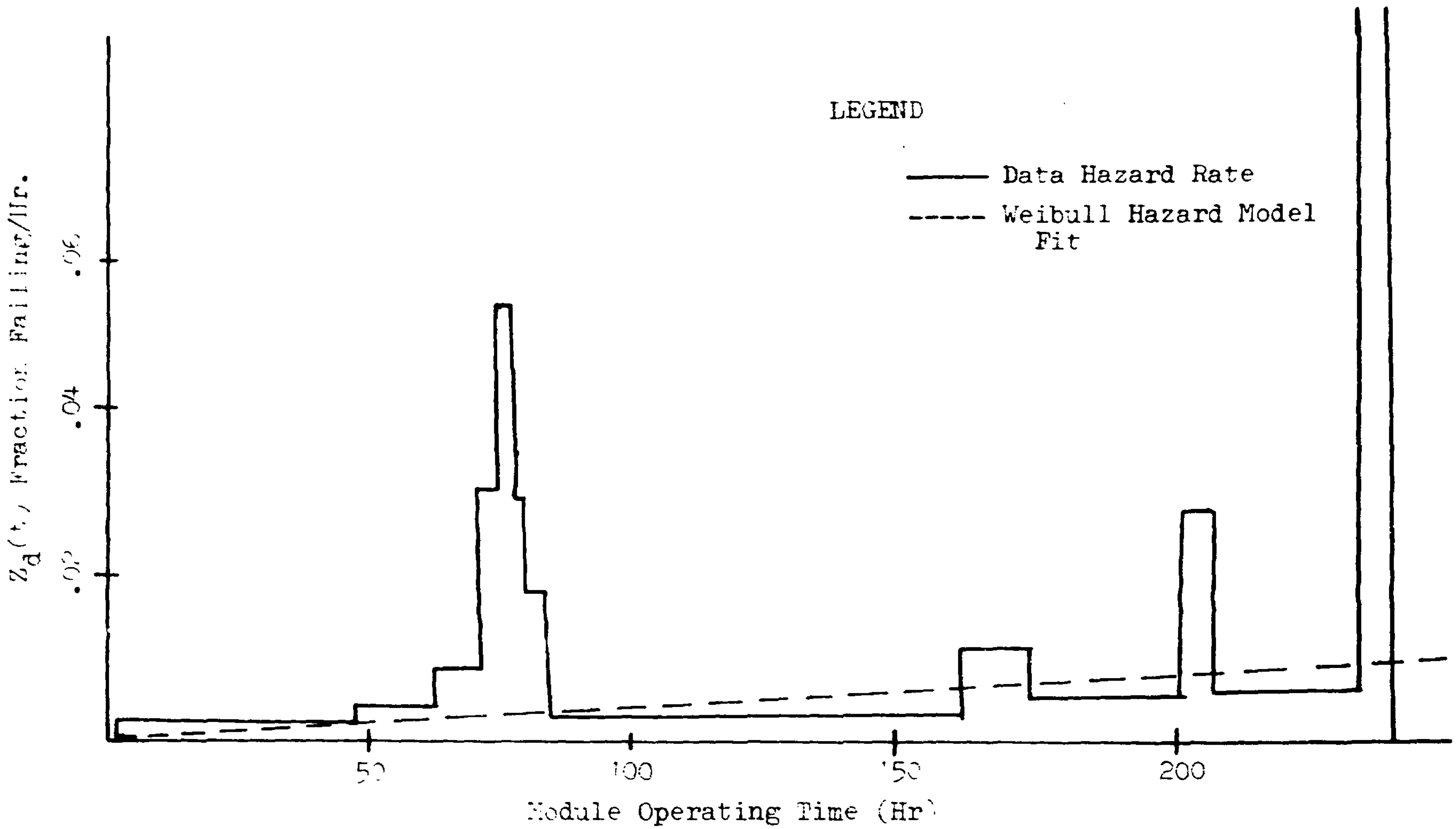

Figure 5.4

Fan Drive/Turbine Module Data Hazard Rate and Woods previously (14:45). This wide variation requires additional caution when using a small number of failure points to establish a statistical distribution describing module failures.

CHAPTER VI

RESULTS AND CONCLUSIONS

Results

Initial data output. Figure H.1 in Appendix H is a sample of the data output provided by the FORTRAN program described in Appendix F. There is, as anticipated, a point of minimum cost per hour. For the baseline cost and failure rate parameters developed in Appendices B and D, minimum cost per hour occurred at approximately 230 hours. The optimal cost per hour under an opportunistic replacement policy was approximately $.03 less than the cost under a replace at failure policy.

Figure 6.1 is a plot of cost per hour versus the location of n for the fan drive turbine module when the replacement core module is of age zero. Figure 6.2 is a similar plot when the core module is of age 240 hours. The effect of core module age is quite dramatic when n is located early in the fan drive turbine overhaul cycle. In both the case of a zero time core module and a 240 hour old core module, however, the curves are relatively flat in the region near 250 hours. Figure 6.3 is a plot of cost per hour versus the location of n for the gearbox, given replacement core age of zero.

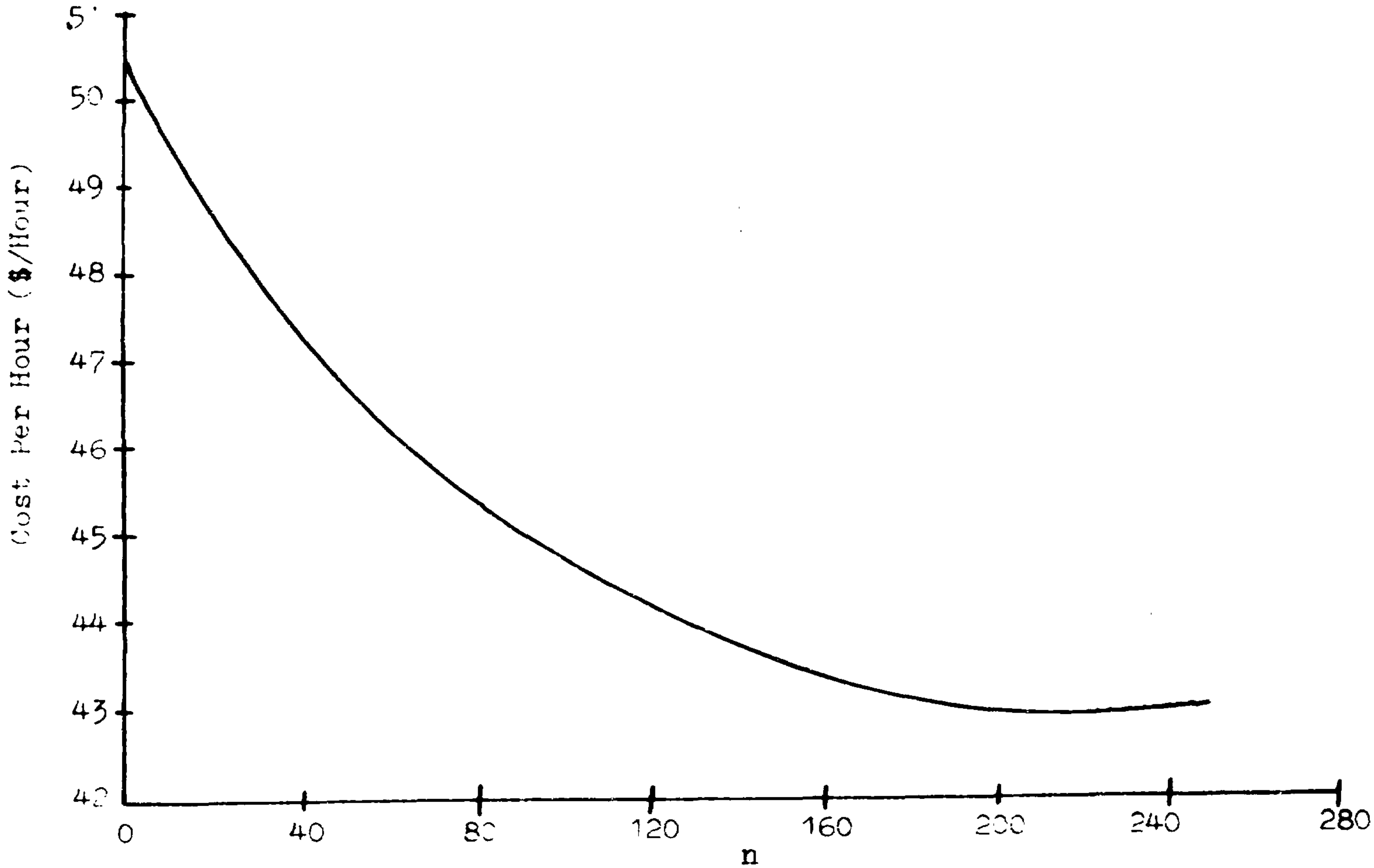

Figure 6.1

Location of n Versus Cost Per Hour for Fan Drive Turbine
When Core Module Age = 0 Hours

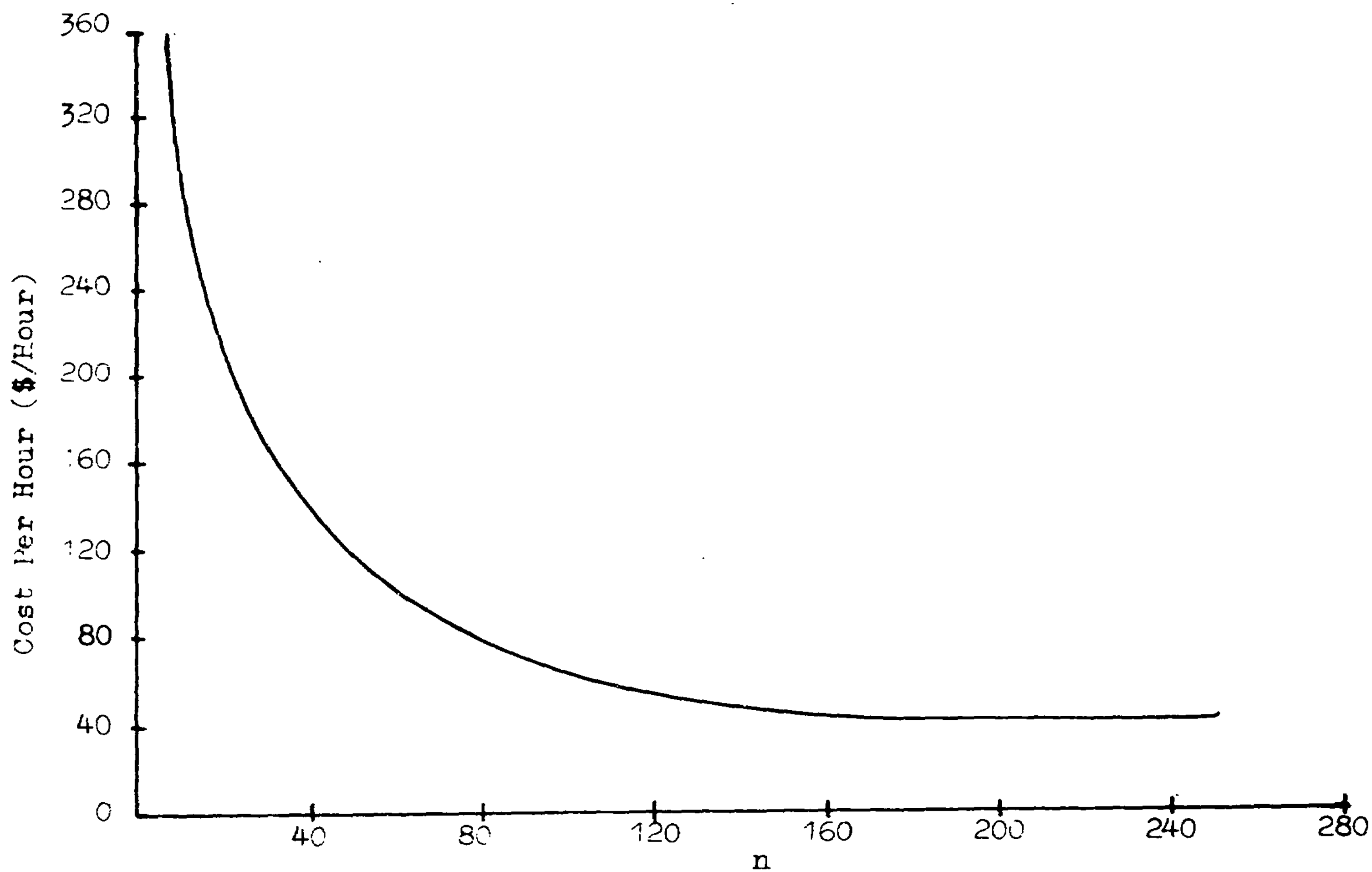

Figure 6.2

Location of n Versus Cost Per Hour for Fan Drive Turbine
When Core Module Age = 240 Hours

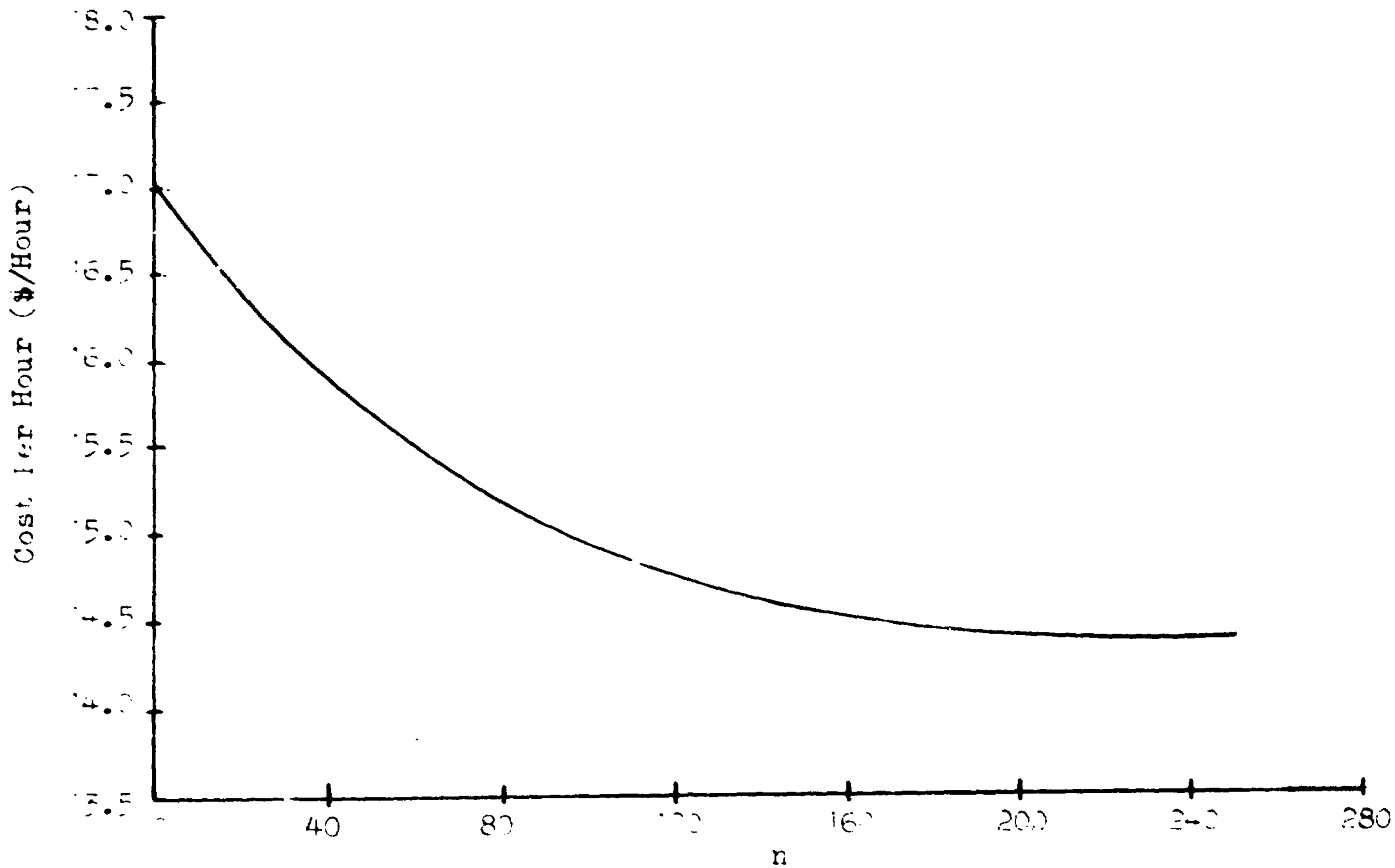

Figure 6.3

Location of n Versus Cost Per Hour for Gearbox
When Core Module Age = 0 Hours

<u>Sensitivity analysis</u>. The fan drive turbine module was chosen arbitrarily for sensitivity analysis. It is anticipated that trends observed for the fan drive turbine would also be true for the gearbox module although the values of savings per hour, cycle length and other output variables would of course be different.

Tables G.1 and Figure G.1 in Appendix G contain a summary of changes in the location of the optimal n, cost per operating hour, expected savings per operating hour under an opportunistic policy and expected cycle length as the algorithm input parameters were varied. The results are summarized briefly in Figure 6.4.

For the range of cost estimates and hazard parameters explored when cost inputs or hazard parameters were changed one at a time, expected savings under an opportunistic policy only exceeded \$.10 per operating hour when the replacement core module Weibull m parameter was 1.0 or when the replacement core module k parameter was 0.10. With k = .10, the expected savings were \$.1438 per hour. With m = 1.0, the expected savings were \$.4317 per hour. The Weibull m parameter determines the shape of the distribution while the k parameter determines scale. In general, the m parameter primarily affects the rate of change of the hazard rate while the k parameter for a given value of m primarily influences the number of failures per time unit. An m parameter value

Change:	Effect On: Optimal Value of n	Effect On: Savings Per Hour For Optimal Opportunistic Policy
Increase in Variable Costs	Earlier	More
Increase in Constant Costs	Later	Less
Increase in Core Module Weibull m or k Parameter	Later	Less
Increase in Fan Drive Turbine Weibull k Parameter	No Change	Significantly More
Increase in Fan Drive Turbine Weibull m Parameter	Earlier	More
Core Module Age	Variable*	Less

*Optimal value of n varied either earlier or later depending on the hazard rate parameters and cost inputs.

Figure 6.4

Influence of Cost and Failure Rate Changes of 1.0 and k parameter value of 0.10 were the highest tested during sensitivity analysis.

Very limited sensitivity analysis was conducted varying two parameters at a time. We were interested in observing the effect on cost per operating hour when both variable costs were increased and the core module experienced either a relatively high m or k parameter value. With the core module k parameter equal to 0.10 and MOD1REP (field level replacement cost) increased $100 over the base line (a 12 percent increase) savings per operating hour under an opportunistic replacement policy were approximately $.17. When the core module m parameter was increased to 1.0, and MOD1REP simultaneously increased by $100, saving per operating hour increased to $.54.

The mean value of savings per operating hour, for those algorithm input parameter combinations tested, was approximately $.06 per operating hour. Translating this figure into savings per year using the fiscal year 1981 flying program of 175,000 flying hours (which requires approximately 350,000 operating hours) when F-15 fleet acquisition will be complete, savings on the order of $21,000 per year might be anticipated under an opportunistic maintenance policy. We wish, however, to stress that the output data summarized in Appendix H and discussed in this chapter are no better than the hazard parameter

estimates and cost inputs used. Hazard parameter estimates and cost estimates are, in our opinion, fraught with considerable uncertainty at present. The main interest of the authors of this thesis was in developing a method to find an optimal replacement policy. The data output provided is meant to illustrate only the general range of values which may be observed. With better hazard rate parameter estimates and better cost inputs, the relative magnitude of savings under an opportunistic replacement policy could change significantly. Illustration of this possibility is found in the behavior of cost per operating hour when more than one input to the algorithm was changed at one time. Further, there are several inherent assumptions and limitations in the algorithm which must be understood. These assumptions and limitations are discussed in the next section.

Review of Assumptions and Limitations

The algorithm developed in Chapter IV will determine the optimal opportunistic replacement policy given the assumptions and limitations which are built into it. The assumptions and limitations which were initially identified as pertinent to the research are listed in Chapter II. Certain of these relate primarily to data gathered in support of the research. Those which are incorporated into the algorithm are:

Module hazard rate functions were assumed independent of one another. This is an assumption which seldom holds true in practice. Certain types of engine failures will quite frequently result in damage to more than one component of an engine. As an example, foreign object damage (resulting from ingestion of a bird or other solid object) may cause portions of the inlet fan to process through the core module causing damage to the high speed compressor. The probable bias of the assumption of independent hazard rates would be to increase the number of replacement actions in the interval (n, N) which result from joint failure of two or more modules. The convention used in this thesis was to charge the full value of the replacement cost to the module replaced opportunistically (e.g., fan module or fan drive turbine module) in the event of this occurence. Thus, relaxing this assumption (which would require a considerable increase in the complexity of the algorithm) would probably result in decreased savings per operating hour and a shift of the optimal location of n towards N.

Transportation costs and depot costs were assumed to be the same for failed and unfailed modules. As discussed in Chapter V, depot overhaul costs are not yet well defined. A single cost for failed and unfailed modules was used due to lack of a more explicit cost information. As Jorgenson, McCall and Radner (10:222) point out, however, one would normally expect the cost to repair an unfailed module to be lower since fewer components should require replacement and overhaul actions should be facilitated.

A more subtle question, related to overhaul costs, was overlooked when developing the algorithm. We did not include the impact of changes in n on the spare modules required to fill the various segments of the depot repair and transportation pipelines. Note that as n approaches 0, the expected cycle length would decrease, which would cause required spare levels to increase. The probable bias from this omission would require study to ascertain with confidence, for as cycle length decreased (and the number of modules depot overhauled per year increased) depot overhead costs would be spread over a wider base. Given these conditions, use of a single cost estimate for depot overhaul of a module would be even more questionable. We suggest, however, without any justification other than intuition, that the cost due to increased spares requirements would probably predominate. In this case, the bias would be to increase fixed costs, decrease savings per operating hour under an opportunistic policy, and shift the optimal location of n towards N. Explicit consideration of the effects of a change in n on spares requirements and depot overhead will, again, entail an increase in the complexity of the algorithm.

When developing the algorithm, an additional assumption which became necessary was that all failures in the interval (n, N) of the module for which opportunistic replacement was being studied result in shipment to the depot. This is probably not an unreasonable assumption for values of n close to an optimally located N, where wearout would be significant. The assumption is essentially indefensible as n approaches 0, however. If the assumption were valid, there would be scant justification for an intermediate level module repair capability. We suggest, however, based on results of the sensitivity analysis, that the region of interest where the optimal value of n is most likely to be found is near N rather than near 0. The probable bias resulting from the assumption that module failures in the interval (n, N) are depot reparable only would be to increase expected cycle length under an opportunistic policy and decrease the probability of a field level replacement action. Both of these effects would result in a decrease in cost per operating hour under an opportunistic replacement policy.

There is an inherent limitation in solving for n independent of N. To illustrate, during sensitivity analysis the maximum operating times for the fan drive turbine and core modules were experimentally increased to 500 hours. When this change was incorporated, optimal cost per operating hour dropped to $36.91 which is eight percent less than that achieved with MOT's equal to 250 hours. Based on the results of the study by Jorgenson, McCall and Radner (10:225-235), it is economically optimal to solve for n and N simultaneously. Only a moderate change to the FORTRAN program in Appendix F--including an additional do-loop to scan over the range of interest for maximum operating time and additional statements to store minimum costs as the do-loops were executed--would be required, but the hazard function must be clearly defined over the relevant region.

At the time interviews were conducted to gather cost data (20), the subject of test cell procedures for the F-100 engine was not well settled. The original concept was to accomplish engine trouble shooting on the aircraft prior to in shop maintenance. There is at the present time a trend towards engine trouble shooting on the test cell prior to an engine undergoing maintenance. Test cell cost was not included in the calculation of base level replacement cost. One would expect inclusion of this cost to result in increased savings per hour under an opportunistic replacement policy, since it is a variable cost, and movement of the optimal value of n away from N.

Conclusions

With the limitations and assumptions outlined above, an algorithm has been developed to determine the optimal location of the breakpoint between the replace at failure region and the opportunistic replacement region of the module overhaul to overhaul cycle. This answers research question number one. Sensitivity of the optimal location of the breakpoint to changes in hazard rate parameters and changes in cost inputs was examined through sensitivity analysis. The optimal location of the break-point varied from the maximum operating time to 30 hours less than the maximum operating time for those values of input hazard rate parameters and cost inputs studied. This answers research question number two. As the inputs to the algorithm were varied, expected savings possible under an opportunistic policy varied from less than $.01 per operating hour to slightly over $.50 per operating hour. Only in the case of a core module Weibull m parameter equal to 1.0 did expected savings exceed $.15 per hour. Thus, expected savings per hour under an opportunistic policy is relatively insensitive to changes in cost and hazard rate parameter inputs. This answers research question number three. The average magnitude of expected savings under an opportunistic policy was $.06 per operating hour for those hazard rate parameters and cost inputs studied. $.06 per operating hour represents an approximate 0.1 percent savings over a replace at failure policy. In terms of the FY-81, F-15 flying program, savings on the order of $21,000 might be expected. This answers research question number four.

Recommendations

Although the authors would like to believe that this thesis reflects a moderate amount of research effort, we recognize that we have barely scratched the proverbial surface in terms of the amount of work yet to be done in the area of aircraft engine module opportunistic replacement policy. In particular, we would suggest that effort be directed toward: 1. improving hazard rate parameter estimates and cost estimates as the F-15 aircraft and F-100 engine accrue more operational experience, 2. incorporating necessary changes into the algorithm so that some of the restrictive assumptions employed in this thesis can be relaxed, 3. exploring the effect of changes in cycle length on spares requirements and depot overhead charges and 4. performing additional sensitivity analysis by varying more than one input variable at a time and expanding the range within which variables are tested.

APPENDIX A

DEFINITIONS

APPENDIX A

DEFINITIONS

Augmented Turbofan Engine - A basic turbofan jet engine with an augmentor attached to discharge end of turbine section. An augmentor mixes the hot turbine discharge gases and the relatively cool fan bypass air. The mixture thus obtained is burned in the after-burner segments.

Catastrophic Failure - A failure characterized by sudden, unexpected damage or loss.

Concurrent Maintenance - Accomplishment of two or more independent maintenance actions at the same time.

Cycle Time - The length of time from installation of a module until its removal for maximum operating time (MOT), failure, or opportunistic replacement.

Deterministic - The result of a given action is known with complete certainty.

Downtime - Any time period in which an aircraft or component is not available for use. This is normally classed as Not Operationally Ready-Maintenance (NORM) or Not Operationnaly Ready-Supply (NORS).

Echelons - Levels of the maintenance organizational hierarchy.

End Item - An item selected for specific configuration and accounting control. (e.g., aircraft engines and airframes).

Fixed Costs - Those costs known to occur with certainty during a single cycle. For this research, fixed cost included engine removal and installation costs, module packing and unpacking costs, and transportation costs for a module between base and overhaul facility.

Lost Service Life - The sacrificed, otherwise available, service life of a component due to replacement before failure.

Module Construction - The concept where functionally and physically associated parts are removable as units.

Opportunistic Replacement - The replacement of an end item specifically during a time when the item must be removed to perform other maintenance (e.g., replacement of the removed inlet/fan module given mandatory core replacement).

Queen Bee Operation - The maintenance concept of assigning intermediate level maintenance capability for specific type, model and series of aircraft engine to a centralized location which, in turn, provides support to other bases. Under this concept, supported bases would basically have only remove and replace or minor repair capability.

Stochastic Failure - The time at which failure occurs is not known with certainty, i.e., the time at failure is governed by a probabilistic mechanism. Only the expected time of failure can be determined.

Test Cell Run - Operation of a jet engine on a specially designed fixture (cell) after removal from an aircraft but prior to teardown, specifically accomplished to isolate a defective component or components. Test cell runs also are made after build-up or repair to verify maintenance actions.

Trouble-shoot - The maintenance actions necessary to isolate a defective component or components. This may be accomplished on the aircraft or after removal.

Variable Costs - Costs which vary in amount or may or may not occur during a single cycle. As an example, in the event of opportunistic replacement, certain costs are not incurred which would be incurred in the event of a replacement at failure.

Wearout - The characteristic where hazard rate increases with age.

APPENDIX B

COST DATA

Table B.1

Depot Overhaul Costs and Base-to-Base Transportation Costs

Module	Depot Overhaul Estimate[1]	Module Size[2] (Inches)	Module[2] Wt. (lb)	Rate Per Hundred-Wt.	Transportation Cost	
					One Way	Two Way
Inlet/Fan	$7,825.00	43x55x55	1180	$11.28	$135.36	$270.72
Gearbox	$2,315.00	[illegible]5x33x35	265	$12.55	$ 37.65	$ 75.30
Fan Drive/Turbine	$6,174.00	5[illegible]x46x92	1388	$11.28	$157.92	$315.84

NOTES:

[1]Source: San Antonio ALC/MAWWF, Letter, dated 12 March 1975 (7)

[2]Source: AFLC/MMP chart (7)

[3]Rate schedule (Edwards Air Force Base to San Antonio ALC) furnished by DSTRP/Kelly Air Force Base, Texas (25) as follows:

Weight (lb)	Charge (per hundred wt)	Weight (lb)	Charge (per hundred wt)
Up to 499	$12.55	2,000 - 4,999	$10.45
500 - 1,999	$11.28	5,000 - 9,999	$ 9.90

Table B.[illegible]

Standard Four Man Team Costs

Number Required	Rank	Wage Rate[1]	Personnel Cost Per Clockhour
1	SSgt	$4.75	$ 4.75
[illegible]	Sgt	$3.92	$ 3.92
[illegible]	A1C	$3.29	$ 6.58
Total Team Cost/Hour			$15.25

[1] Per hour worked. Source: AFM 177-101

Table B.3

Standard Three Man Team Costs

Number Required	Rank	Wage Rate[1]	Personnel Cost Per Clockhour
[illegible]	Sgt	$3.92	$ 3.92
[illegible]	A1C	$3.29	$ 6.58
Total Team Cost/Hour			$10.50

[1] Per hour worked. Source: AFM 177-101

Table B.4

Standard Two Man Team Costs

Number Required	Rank	Wage Rate[1]	Personnel Cost Per Clockhour
1	Sgt	$3.92	$3.92
1	A1C	$3.29	$3.29
Total Team Cost/Hour			$7.21

[1]Per hour worked. Source: AFM 177-101

Table B.5

Module Packing and Unpacking Costs

Module	Pack			Unpack			Total Cost
	Size Team	Clock Hrs	Cost	Size Team	Clock Hrs	Cost	
Inlet/Fan	2	1.5	$18.06	2	1.5	$18.06	$36.12
Gearbox	[illegible]	.33	$ 4.01	[illegible]	.25	$ 3.01	$ 7.02
Fan Drive/Turbine	[illegible]	1.25	$15.05	2	1.25	$15.05	$30.10

SAMPLE COMPUTATION:

Packing an inlet/fan module requires a two (2) man team for 1.50 hours.

(clock hours required) X (allowance factor) X (team cost/hour) = task cost

(1.5 hours) X (1.6[illegible]) X ($[illegible]) = $18.06

An allowance factor of 1.6[illegible] was recommended for use by Mr. Rodrigues ([illegible]), Edwards F-15 Joint Task Force to allow for expected average task time. His estimates all based upon no lost motion and a highly skilled team.

Table B.6

Module Removal and Reinstallation/Replacement Manhour Costs

Module	Removal			Reinstallation/Replacement			Total Cost
	Team Size	Clock Hrs	Cost	Team Size	Clock Hrs	Cost	
Inlet/Fan	3	5.0	$ 87.68	3	7.0	$122.75	$210.43
Gearbox	2	5.0	$ 60.20	2	7.5	$ 90.30	$150.50
Fan Drive/Turbine							
Steps Required							
1 Aug	2	1.5	$ 18.06	2	1.75	$ 21.07	$ 39.13
2 Outer Fan Ducts	4	9.0	$229.21	4	12.0	$305.61	$534.82
3 FD/T	2	1.0	$ 12.04	2	1.5	$ 18.06	$ 30.10
Total Cost for FD/T Module R&R							$604.05

<u>SAMPLE COMPUTATION:</u>

Removing an inlet/fan module requires a three (3) man team for 5.0 hours.

(clock hours) X (allowance factor) X (team cost/hour) = task cost

(5.0) X (1.67) X ($10.50) = $87.68

Table B.7

Module Removal and Replacement/Reinstallation Material Costs

Module	Quantity	P/N	Item	Unit Price	Total Price
Inlet/Fan	1	MS9880-10	washer	$ .33	$ .33
	1	429326	washer-key	$ .18	$ .18
	2	MS9967-215	packing	$4.54	$9.08
	1	MS9388-020	packing	$ .22	$ .22
Total					$9.81
Gearbox	2	MS9966-12	packing	$1.07	$ 2.14
	2	MS9966-10	packing	$ .72	$ 1.44
	1	613949	washer-key	$ .13	$ .13
	2.3 gals.	MIL-L-7808	oil change	$2.29/Qt.	$21.07
Total					$24.78

Table B.7 (Cont'd)

Module	Quantity	P/N	Item	Unit Price	Total Price
Fan Drive/Turbine	[illegible]	505398	gasket	$ 1.10	$ 2.20
	[illegible]	MS9967-214	packing	$ 4.13	$ 8.26
	[illegible]	MS9967-129	packing	$ 1.49	$ 2.98
	[illegible]	[illegible]	gasket	$ .44	$ .44
	[illegible]	2155306	gasket	$ 3.64	$ 3.64
	[illegible]	2156866	gasket	$ 3.48	$ 3.48
	[illegible]	4000644	gasket	$13.33	$26.66
	[illegible]	MS9386-133	packing	$ .58	$ .58
	[illegible]	ST1000-169	packing	$ 2.26	$ 2.26
	[illegible]	ST1000-150	packing	$ 1.35	$ 2.70
	5	[illegible]	packing	$ .29	$ 1.45
	5	[illegible]	gasket	$ .29	$ 4.35
	5	[illegible]	gasket	$ .30	$ 1.50

Table B.7 (Cont'd)

Module	Quantity	P/N	Item	Unit Price	Total Price
	4	2162927	gasket	$ .44	$ 1.76
	4	4030916	washer	$ .34	$ 1.36
	10	4023115	nut	$ .30	$ 3.00
	5	4023116	nut	$ .60	$ 3.00
	6	4016855	nut	$2.33	$ 13.98
	6	4006921	bolt	$4.02	$ 24.12
Total					$107.72

[1]Includes material to remove and replace the augmentor and outer fan ducts.

Source: PWA FRDC Form 63, dated 11 July 1975. DD 1149/GBL listing (9)

Table B.8

Engine Change Costs[1]

Task	Team Size	Clockhours	Cost
Removal	4	1.5	$ 38.20
Installation	4	1.5	$ 38.20
Trim	4	3.0	$ 76.40
Total Cost			$152.80

[1]No material consumed in this task (20)

SAMPLE CALCULATION:

Engine removal requires a four (4) man team for 1.5 hours.

(clockhours) x (allowance factor) x (team cost/hour)

= task cost

(1.5) x (1.67) x ($15.25) = $38.20

Table B.9

Base Transportation Cost

Task	Team Size	Clock Hrs	Cost
Movement from acft. to shop	2	0.375[1]	$4.52
Movement from shop to acft.	2	0.375[1]	$4.52
Total Cost			$9.04

[1]Estimates were 15-30 minutes so the average was used.

Total R&R Engine Costs = $161.84 (which is the sum of total costs from Table 8 and 9).

Table B.10

Aggregated Costs

Module	Packing Cost (MOD1PACK)	Depot Overhaul Cost (MOD1DEP)	Shipping Cost (MOD1SHP)	Module R&R Cost (MOD1REP)
Inlet/Fan	$36.12	$7,825	$270.72	$382.08
Gearbox	$ 7.02	$2,315	$ 75.30	$337.12
Fan Drive/Turbine	$30.10	$6,174	$315.84	$873.61

SAMPLE CALCULATION OF MODULE R&R COST:

For Inlet Fan--

manhour cost + material cost + engine change cost = total R&R cost

$210.43 + $9.81 + $161.84 = $382.08

Table B.11

Comparison of Personnel Requirement Estimates
For Module Removals and Installations

Module	QQPRI Estimate[1]		PSTE Demonstration[2]		Expert Opinion[3]	
	Team Size	Clock Hrs	Team Size	Clock Hrs	Team Size	Clock Hrs
Inlet/Fan	2	2.28	2	5.5	2	12.0
Gearbox	2	8.40	2	7.0	2	12.5
Fan Drive/Turbine						
Steps Required						
1 Aug	3	5.04	*	*	2	3.25
2 Outer Fan Ducts	4	16.30	*	*	4	21.0
3 FD/T Module	2	13.83	2	6.9	2	2.5

[1]Source: Pratt and Whitney Aircraft (17)

[2]Source: Human Factor Office, Edwards Air Force Base, California (12)

[3]Source: Engine Shop, Edwards Air Force Base, California (20)

*Data not collected

APPENDIX C

PROGRAMS USED TO SCREEN
MODULE FAILURE DATA

APPENDIX C

PROGRAMS USED TO SCREEN MODULE FAILURE DATA

This appendix contains a copy of each different type of computer program used in the module failure data screening portion of the research effort. The programs included here are representative of the programs used; i.e., some programs used were duplicates of the programs here with the exception of the data files processed.

The manner of presentation used for this appendix will be to list the variables and/or files used in each program, followed by an actual listing of the programs. Listing the programs was made possible by use of a computer program, NICELIST, developed by Major Jim Abbott, Computer Support Section, School of Systems and Logistics, whom the authors sincerely thank.

Many of the computer programs listed in this appendix and used in the research effort were developed by Pansza and Woods (14:95-121) and are used with their permission.

A1. <u>Program TAPEFILE Variables and/or Files Used.</u>

NOTE: This CARDIF Program converts data from BCD tape to BCD permanent disk file.

WYATT - The magnetic tape containing the transaction history of all operational F-100 engines through 30 June 1975.

NEWTAPE - The permanent disk file containing the data generated by the program TAPEFILE. This data was obtained from the magnetic tape WYATT.

A2. Program TAPEFILE Listing

```
5##N,R(SL)
40$:IDENT:WP5502,AFITSL   ***FORBES/WYATT***
50$:FORTY:NFORM,NLNO
60$:SELECTA:FILE200
70$:OPTION:FORTRAN,NOMAP
80$:EXECUTE
90$:LIMITS:14,8K
95$:FFILE:11,NOSRLS
100$:TAPE:11,AID,,74875,,WYATT
110$:PRMFL:21,R/W,S,P1938/NEWTAPE
120$:ENDJOB
```

B1. **Program FILE200 Variables and/or Files Used.**

R – Used in reading one entire data record into memory from magnetic tape WEAFT.

A1 – Module Serial Number.

A2 – Station (Base) Name.

A3 – Date of Transaction.

A4 – Module Transaction and Module Condition.

A5 – Module Removal Reason and Module Hours Since Overhaul.

A6 – Engine Designation and Engine Serial Number.

N – A counter used to indicate the number of records processed.

B2. Program FILE200 Listing

```
C  THIS TSS PPOGRAM (FILE200) IS USED IN PROGRAM
C  TAPEFILE
CHARACTER R*200,A1*8,A2*12,A3*5,A4*2,A5*7,A6*15
20 READ(11,501,END=99)R
DECODE(R,502)A1,A2,A3,A4,A5,A6
N=N+1
WRITE(21,503)A1,A2,A3,A4,A5,A6
GOTO 20
99 WRITE(06,504)N
501 FORMAT(A200)
502 FORMAT(14X,A8,11X,A12,2X,A5,6X,A2,54X,A7,6X,A15,52X)
503 FORMAT(1H ,A8,A12,A5,A2,A7,A15)
504 FORMAT(1H1,"# OF RECORDS = ",I12)
STOP
END
```

C1. Program SPLIT-OC Variables and/or Files Used.

A - Variable used in reading the first two data elements of the data file NEWTAPE2. These data elements have no significance in this research effort but are retained to simplify formatting compatibility with other programs.

TYPEMOD - Variable used to specify to which of the five modules this data element refers. FA refers to a fan module entry. FB refers to a core module entry. FC refers to a fan drive turbine module. FD refers to an augmentor/exhaust module. FE refers to a gearbox module. 69 refers to an entire engine entry.

B - Variable used in reading the last 45 data elements in the data file NEWTAPE2. These data elements contain information such as data of report, reason for report, reason for removal (if appropriate), engine operating hours as of this date, engine serial number.

I - A counter used to indicate the total number of data records processed.

J - A counter used to indicate the number of inlet fan module records processed.

K - A counter used to indicate the number of core module records processed.

L - A counter used to indicate the number of fan drive/turbine module records processed.

M - A counter used to indicate the number of augmentor/exhaust modules records processed.

N - A counter used to indicate the number of gearbox module records processed.

NENG - A counter used to indicate the number of whole engine records processed.

II - Variable used to sum the number of records written to module files and the engine file.

C2. Program SPLIT-OC Listing

```
10C  THIS TSS PROGRAM (SPLIT-OC) READS DATA IN FROM A FILE,
20C  SPLITS THE DATA INTO INDIVIDUAL MODULE FILES AND
30C  WRITES THE MODULE DATA TO THE APPROPRIATE FILE.
40 CHARACTER A*2,TYPEMOD*2,B*45
50 PRINT,"COMPILATION IS COMPLETE"
60 CALL ATTACH(1,"P1938/NEWTAPE2;",3,0,,)
70 CALL ATTACH(2,"P1938/OCAMA2/DATA1/INFAN;",3,0,,)
80 CALL ATTACH(3,"P1938/OCAMA2/DATA1/CORE;",3,0,,)
90 CALL ATTACH(4,"P1938/OCAMA2/DATA1/FANDRIVE;",3,0,,)
100 CALL ATTACH(7,"P1938/OCAMA2/DATA1/AUG;",3,0,,)
110 CALL ATTACH(8,"P1938/OCAMA2/DATA1/GEARBOX;",3,0,,)
120 CALL ATTACH(9,"P1938/OCAMA2/DATA1/ENGINE;",3,0,,)
130 CALL FMEDIA(2,5)
140 CALL FMEDIA(3,5)
150 CALL FMEDIA(4,5)
160 CALL FMEDIA(7,5)
170 CALL FMEDIA(8,5)
180 CALL FMEDIA(9,5)
190 I=0
200 J=0
210 K=0
220 L=0
230 M=0
240 N=0
250 NENG=0
260 DO 100 IK=1,5000
270 READ(1,101,END=99)A,TYPEMOD,B
280 IF(IK.EQ.1)PRINT,"FILE READIN BEGUN"
290 101 FORMAT(1X,A2,A2,A45)
300 IF(TYPEMOD.NE."FA")GO TO 1
```

```
310 J=J+1
320 IF(J.EQ.1)PRINT,"FOUND THE FIRST INLET FAN MODULE"
330 WRITE(2,101)A,TYPEMOD,B
340 1 CONTINUE
350 IF(TYPEMOD.NE."FB")GO TO 2
360 K=K+1
370 IF(K.EQ.1)PRINT,"FOUND THE FIRST CORE MODULE"
380 WRITE(3,101)A,TYPEMOD,B
390 2 CONTINUE
400 IF(TYPEMOD.NE."FC")GO TO 3
410 L=L+1
420 IF(L.EQ.1)PRINT,"FOUND THE FIRST FAN DRIVE MODULE"
430 WRITE(4,101)A,TYPEMOD,B
440 3 CONTINUE
450 IF(TYPEMOD.NE."FD")GO TO 4
460 M=M+1
470 IF(M.EQ.1)PRINT,"FOUND THE FIRST AUG MODULE"
480 WRITE(7,101)A,TYPEMOD,B
490 4 CONTINUE
500 IF(TYPEMOD.NE."FE")GO TO 5
510 N=N+1
520 IF(N.EQ.1)PRINT,"FOUND THE FIRST GEARBOX MODULE"
530 WRITE(8,101)A,TYPEMOD,B
540 5 CONTINUE
550 IF(TYPEMOD.NE."68")GO TO 6
560 NENG=NENG+1
570 IF(NENG.EQ.1)PRINT,"FOUND THE FIRST WHOLE ENGINE"
580 WRITE(9,101)A,TYPEMOD,B
590 6 CONTINUE
600 IF(TYPEMOD.NE."01")GO TO 7
610 PRINT,101,A,TYPEMOD,B
620 7 CONTINUE
630 100 CONTINUE
640 99 CONTINUE
```

```
I=IK-1
PRINT,"SORTING COMPLETE"
PRINT,""
PRINT,""
PRINT,""
PRINT,"NUMBER ORIGINAL DATA ELEMENTS IS ",I
PRINT,""
PRINT,"# OF DATA ELEMENTS IN  INFAN FILE IS ",J
PRINT,""
PRINT,"# OF DATA ELEMENTS IN CORE FILE IS ",K
PRINT,""
PRINT,"# OF DATA ELEMENTS IN FANDRIVE FILE IS ",L
PRINT,""
PRINT,"# OF DATA ELEMENTS IN AUG FILE IS ",M
PRINT,""
PRINT,"# OF DATA ELEMENTS IN GEARBOX FILE IS ",N
PRINT,""
PRINT,"# OF WHOLE ENGINE TRANACTIONS IS ",NENG
PRINT,""
PRINT,""
II=J+K+L+M+N+NENG
PRINT,"THE SUM OF MODULE FILES IS ",II
PRINT,""
IF(II-I+1)11,12,13
13 PRINT,"ERROR...ERROR...MODULE FILES PICKED UP EXTRA DATA"
GO TO 50
12 PRINT,"ALL SYSTEMS GOOD.........FILE SPLIT PERFECT!!!!!"
GO TO 50
11 PRINT,"ERROR...ERROR...NOT ALL DATA ELEMENTS TRANSFERRED"
50 CONTINUE
STOP
END
```

D1. Program SNOLST Variables and/or Files Used.

A - The file line number associated with one record.

E - The Module Serial Number. In this format the manufactor's code and all leading zeros have been stripped off.

F - Defined all the data in one record except that defined by the variables A and E above.

SEN - Module serial number. A mechanism used to determine if the transaction just read by the program SNOLST was related to a Module Serial Number that was different from the one previously read.

Z - A counter used to indicate the number of records processed.

OCAMA1/DATA1/INFAN - ASC II permanent file structure containing the transaction history of the inlet/fan modules. OCAMA1 is a catalogue of all data from the D024 Engine Status Reporting System. DATA1 is a subcatalogue containing all data prior to screening for errors. INFAN is the file name.

D2. Program SNOLST Listing

```
10C  THIS TSS PROGRAM (SNOLST) SELECTS EACH DIFFERENT
20C  MODULE SERIAL NUMBER & LISTS THEM
30 CHARACTER A*6,E*6,F*37,SEN*6
40 Z=0
50 200 FORMAT(A6,A6,A37)
60 CALL ATTACH(22,"P1938/OCAMA1/DATA1/INFANT",1,0,,)
70 10 READ(22,200,END=99)A,E,F
80 Z=Z+1
90 IF(Z.EQ.1)GOTO 30
100 IF(E.EQ.SEN)GOTO 10
110 SEN=E
120 GO TO 20
130 30 SEN=E
140 20 PRINT 201,A,E,F
150 201 FORMAT(1X,A6,A6,A37)
160 GO TO 10
170 99 PRINT:"# OF RECORDS USED",Z
180 STOP
190 END
```

E1. **Program ZEROPT Variables and/or Files Used.**

A - The file line number associated with one data record.

B - Module Serial Number.

C - Engine designator and Module Removal Reason.

D - Module Operating Time.

E - Defined all the data in one record except that defined by the variables A, B, C and D above.

SEN - Module Serial Number. A mechanism used to determine if the transaction just read by the program ZEROPT was related to a Module Serial Number that was different from the one previously read.

N - A counter used to indicate the number of records processed.

OCAMA1/DATA1/INFAN - ASC II file: structure as defined in Program SNOLST.

E2. Program ZEROPT Listing

```
10C  THIS TSS PROGRAM (ZEROPT) LISTS THE FIRST SERIAL
20C  NUMBER & EACH ZERO OPERATION TIME PERIOD WITHIN
30C  THAT SERIAL NUMBER SEQUENCE
40 CHARACTER A*6,B*6,C*6,D*4,E*27,SEN*6
50 N=0
60 100 FORMAT(A6,A6,A6,A4,A27)
70 CALL ATTACH(11,"P1938/OCAMAI/DATAI/INFANS",1,0,,)
80 10 READ(11,100,END=99)A,B,C,D,E
90 N=N+1
100 IF(N.EQ.1)GO TO 30
110 IF(B.EQ.SEN)GO TO 40
120 30 SEN=B
130 50 PRINT 200,A,B,C,D,E
140 200 FORMAT(1X,A6,A6,A6,A4,A27)
150 GO TO 10
160 40 IF(D.EQ."0000")GO TO 50
170 GO TO 10
180 99 PRINT:"THE NUMBER OF RECORD USED WAS",N
190 STOP
200 END
```

F1. Program BRCHECK Variables and/or Files Used.

A- Line Number, Module Serial Number and Module Designator.

B - Module Removal Reason.

C - Defined all the data in one record except that defined by the variables A and B above.

RM - A vector used to store each different module removal reason during the execution of that part of the program BRCHECK which identified these values. Also used to provide a listing of the different values.

N - A counter used to indicate the number of records processed.

J - A counter used to index the vector RM.

RFAIRC - ASC II permanent file containing the transaction history of those sample modules.

F2. Program RRCHECK Listing

```
C  THIS TSS PROGRAM (RRCHECK) GIVES A LIST OF REMOVAL
C  REASONS USED IN THE TRANSACTIONS
CHARACTER A*16,B*2,C*31,RM*2(100)
DATA(RM(I),I=1,100)/100*"  "/
N=0
J=0
201 FORMAT(A16,A2,A31)
CALL ATTACH(22,"P1938/RFAIRC;",1,0,,)
10 READ(22,201,END=99)A,B,C
N=N+1
IF(B.EQ."  ")GO TO 10
IF(RM(1).NE."  ")GO TO 50
RM(1)=B
J=J+1
GO TO 10
50 DO 30 I=1,100
IF(B.EQ.RM(I))GO TO 10
30 CONTINUE
RM(J+1)=B
J=J+1
GO TO 10
99 DO 40 I=1,100
IF(RM(I).EQ."  ")GO TO 60
PRINT 202,RM(I)
202 FORMAT(1X,A2)
40 CONTINUE
60 PRINT:"# OF RECORDS WAS",N
STOP
END
```

G1. **Program ZEROCH Variables and/or Files Used.**

B - The file line number associated with one record.

C - Type module code.

D - Zeroes.

E - Module Serial Number

F - Station (Base) Name.

G - Date of Transaction.

H - Module Transaction and Module Condition.

J - Module Removal Reason.

K - Zero.

M - Module Hours Since Overhaul.

N - Blank.

P - Engine Type.

S and T - Engine Serial Number.

Z - A counter used to indicate the number of records processed.

G2. Program ZEROCKN Listing

```
C  THIS TSS PROGRAM (ZEROCKN) CHECKS FOR ERRONEOUS
C  INSERTIONS WHERE ZEROS & BLANKS WERE REQUIRED
C  IN THE DATA FORMAT
 CHARACTER B*5,C*1,D*2,E*6,F*12,G*5,H*2,K*1,J*2,
&M*4,N*1,P*2,Q*2,R*2,S*7,T*1
 Z=0
 200 FORMAT(A5,A1,A2,A6,A12,A5,2A2,A1,A4,A1,3A2,A7,A1)
 CALL ATTACH(22,"P1938/WYATT8",1,0,,)
 10 READ (22,200,END=99) B,C,D,E,F,G,H,J,K,M,N,P,Q,R,S,T
 Z=Z+1
 IF(K.NE."0".AND.K.NE." ") GO TO 20
 IF(Q.NE."00".AND.Q.NE."  ") GO TO 20
 IF(D.NE."00".OR.N.NE." ") GO TO 20
 GO TO 10
 20 PRINT 201,B,C,D,E,F,G,H,J,K,M,N,P,Q,R,S,T
 201 FORMAT (1X,A5,A1,A2,A6,A12,A5,2A2,A1,A4,A1,3A2,A7,A1)
 GO TO 10
 99 PRINT:"# OF RECORDS USED",Z
 STOP
 END
```

H1. **Program TIMECK Variables and/or Files Used.**

A - Line number associated with one file record.

B - Module Serial Number.

BB - Module Designator.

C - Module Removal Reason.

D - Module Operating Time.

E - Defined as all the data on one record except those defined by the variables A, B, BB, C, and D above.

SN - Module Serial Number. A mechanism used to determine when all of the transactions pertaining to a given Module Serial Number have been processed.

OPT - Module Operating Time. A mechanism used to detect an erroneous decrease in operating time.

N - A counter used to indicate the total number of records processed.

WYATT - ASC II permanent disk file that contained the screened module transaction histories.

H2. Program TIMECK Listing

```
C  THIS TSS PROGRAM (TIMECK) CHECKS TO DETERMINE IF
C  MODULE OPERATING TIME ERRONEOUSLY DECREASES
CHARACTER A*6,B*6,BB*4,C*2,D*4,E*27,SN*6,OPT*4
N=0
100 FORMAT(A6,A6,A4,A2,A4,A27)
CALL ATTACH(11,"P1938/WYATT;",1,0,,)
200 FORMAT(1X,A6,A6,A4,A2,A4,A27)
10 READ(11,100,END=99)A,B,BB,C,D,E
N=N+1
IF(N.EQ.1)GO TO 20
IF(SN.EQ.B)GO TO 30
20 SN=B
OPT=D
GO TO 10
30 IF(C.EQ."  ".AND.D.NE."0000")GO TO 10
IF(D.EQ.OPT)GO TO 10
IF(D.EQ."    ")GO TO 10
IF(D.EQ."0000")GO TO 50
IF(D.LT.OPT)GO TO 60
OPT=D
GO TO 10
50 OPT=D
GO TO 10
60 PRINT 200,A,B,BB,C,D,E
GO TO 10
99 PRINT*,"THE NUMBER OF RECORDS USED WAS",N
STOP
END
```

I1. <u>Program BRANDTM Variables and/or Files Used</u>.

A - Line Number, Module Serial Number and the Module Designator.

B - Module Removal Reason.

C - Module Operating Time.

D - Defined as all the data elements in one record except those defined in the variables A, B and C above.

N - A counter used to indicate the total number of records processed.

WYATT - Permanent disk file that contained the transaction history records of all modules.

I2. Program RRANDTM Listing

```
10C  THIS TSS PROGRAM (RRANDTM) CHECKS FOR A MODULE
11C  REMOVAL REASON AND FOR NO MODULE OPERATION TIME
20 CHARACTER A*16,B*2,C*4,D*27
30 N=0
40 100 FORMAT(A16,A2,A4,A27)
50 200 FORMAT(1X,A16,A2,A4,A27)
60 CALL ATTACH(11,"P193B/MYATT;",1,0,,)
70 10 READ(11,100,END=99)A,B,C,D
80 N=N+1
90 IF(B.NE."  ".AND.C.EQ."    ")GO TO 30
100 GO TO 10
110 30 PRINT 200,A,B,C,D
120 GO TO 10
130 99 PRINT,"THE NUMBER OF RECORDS USED WAS",N
140 STOP
150 END
```

J1. <u>Program DUPCK Variables and/or Files Used</u>.

A - Line Number, Module Serial Number and Module Designator.

B - Module Removal Reason.

C - Module Operating Time.

D - Defined as all the data in one record except that defined by the variables A, B and C above.

N - A counter used to indicate the total number of records processed.

M - Used as an indication to the program, that the first transaction with a removal reason had been processed.

TIME - Module Operating Time. A mechanism used to detect duplication of module operation time points between successive module removal transaction.

WYATT - Permanent disk file that contained the transaction history records of all modules.

J2. **Program DUPCK Listing**

```
C  THIS TSS PROGRAM (DUPCK) CHECKS FOR DUPLICATION
C  OF MODULE OPERATION TIME POINTS BETWEEN SUCCESSIVE
C  MODULE REMOVAL TRANSACTIONS
   CHARACTER A*16,B*2,C*4,D*27,TIME*4
   CALL ATTACH(11,"P1938/MYATT;",1,0,,)
   N=0
   M=0
 1 READ(11,100,END=99)A,B,C,D
   N=N+1
 100 FORMAT(A16,A2,A4,A27)
   IF(B.EQ."  ")GO TO 1
   M=M+1
   IF(M.NE.1)GO TO 20
   TIME=C
   GO TO 1
 20 IF(C.EQ.TIME) GO TO 30
   TIME=C
   GO TO 1
 30 PRINT:A,B,C,D
   GO TO 1
 99 PRINT:"# OF RECORDS WAS  ",N
   STOP
   END
```

K1. Program REMOVALS Variables and/or Files Used.

FILENAME - Variable name used for the name of the data file to be read from.

RITENAME - Variable name used for the name of the data file to be written to.

A - Defined all the data in one record for the first 23 spaces.

REMOVAL - Variable used to contain Removal Code entries from data file.

B - Defined all the data in one record for the last 23 spaces.

I - Counter used to count number of records read from FILENAME.

J - Counter used to count number of records written to RITEFILE.

K2. Program Removals Listing

```
10C  THIS TSS PROGRAM (REMOVALS) FINDS EACH MODULE REMOVAL
20C  ACTION AND PRINTS OUT EACH REMOVAL AT THE TERMINAL.
30C  AN OPTION IS AVAILABLE IN THE PROGRAM TO WRITE THESE
40C  REMOVALS TO ANOTHER FILE RATHER THAN HAVE THEM PRINTED OUT
50 CHARACTER FILENAME*40,RITENAME*40
60 CHARACTER ANS*3
70 CHARACTER A*23,REMOVE*1,B*23
80 69 CONTINUE
90 PRINT 66
100 PRINT,"DO YOU DESIRE OUTPUT WRITTEN AUTOMATICALLY TO A FILE"
110 PRINT,"RATHER THAN PRINTED OUT AT A TSS TERMINAL?"
120 PRINT,"ANSWER YES OR NO."
130 READ,ANS
140 IF(ANS.NE."YES")GO TO 77
150 PRINT,""
160 PRINT,"TYPE THE FILENAME YOU HAVE ALREADY CREATED TO WHICH"
170 PRINT,"THIS REMOVAL DATA IS TO BE WRITTEN.  IF NOT CREATED YET,"
180 PRINT,"ABORT THIS PROGRAM NOW AND CREATE UNDER ACCESS."
190 PRINT,"EXAMPLE OF CORRECT INPUT IS:"
200 PRINT,"/P1938/OCAMAI/REMOVALS/INFANR;"
210 READ,RITENAME
220 77 CONTINUE
230 PRINT,""
240 PRINT,"ENTER THE CATALOGUE/FILENAME TO BE READ. FOLLOW WITH"
250 PRINT,"A SEMICOLON--EXAMPLE OF CORRECT INPUT IS:"
260 PRINT,"/P1938/OCAMAI/DATAI/INFAN;"
270 READ,FILENAME
280 II=0;J=0
290 PRINT 66
300 66 FORMAT(///)
```

```
310 CALL ATTACH(13,FILENAME,3,0,,)
320 IF(ANS.NE."YES")GO TO 24
330 CALL ATTACH(1,RITENAME,3,0,,)
340 CALL FMEDIA(1,5)
350 24 CONTINUE
360 DO 1 I=1,50000
370 READ(13,101,END=2)A,REMOVE,B
380 101 FORMAT(3X,A23,A1,A23)
390 IF(REMOVE.NE."L") GO TO 1
400 J=J+1
410 IF(ANS.NE."YES")GO TO 3
420 WRITE(1,101)A,REMOVE,B
430 GO TO 1
440 3 PRINT 101,A,REMOVE,B
450 1 CONTINUE
460 2 II=I-1
470 PRINT,""
480 PRINT,""
490 PRINT,"NUMBER OF RECORDS IN ",FILENAME," IS ",II
500 PRINT,"NUMBER OF MODULE REMOVALS IS ",J
510 PRINT,"ARE ADDITIONAL CONVERSIONS REQUIRED?"
520 READ,ANS
530 IF(ANS.EQ."YES")GO TO 69
540 STOP
550 END
```

Programs to handle Edwards Air Force Base RDT&E Engine Data were developed to manipulate the data provided into the proper format for usage beginning with Program SPLIT-OC. The programs listed in this appendix were then used to screen all files of obvious errors. Since the initial manipulation programs were designed for one time usage only, the authors have not included them in this document.

APPENDIX D

MODULE TIMES AT FAILURE

APPENDIX D

MODULE TIMES AT FAILURE

	Module			
	Inlet/Fan	Core	Gearbox	Fan Drive Turbine
Data Produced	1.6 2.9 6.1 9.8 18.2 23.9 27.6 29.5 32.6 32.7 49.0 101.8 112.1 122.0 133.6 159.2	14.6 64.0 86.7 180.5 237.5	21.7 60.8 127.1 164.7 174.4 207.4 220.4 220.7	47.5 64.0 72.9 75.4 78.5 84.1 163.7 174.8 203.0 208.2 238.6 239.4
Number of MOT Rmvls Recorded	2	8	6	4
Total No. of DATA Points	18	13	14	16
m Parameter estimate[1]	-0.2407	-0.0132	0.6601	0.71019
Std Error of Estimate[1]	0.0829	1.1106	0.1250	0.0800
k Parameter Estimate[1]	0.0279	0.00205	0.00014	0.000163
Std Error of Estimate[1]	0.0070	0.00092	0.00005	0.000043

[1] As determined using LIKELY2, Appendix E.

APPENDIX E

DEVELOPMENT OF A COMPUTER PROGRAM TO PERFORM MAXIMUM LIKELIHOOD ESTIMATES (MLE) OF THE PARAMETERS OF A TRUNCATED WEIBULL DISTRIBUTION

APPENDIX E

DEVELOPMENT OF A COMPUTER PROGRAM TO PERFORM MAXIMUM LIKELIHOOD ESTIMATES (MLE) OF THE PARAMETERS OF A TRUNCATED WEIBULL DISTRIBUTION

In working with data from component life cycle testing, it is usually desirable to generalize from sample data to the entire population of similar components. Shooman (24:195) points out the essentiality of fitting failure data to a statistical distribution for this purpose. Shooman further urges use of maximum likelihood estimators for parameter estimation, once a specific distribution is selected, since they offer "the most flexible and powerful of modern estimation techniques (24:472)." As discussed in Chapter V, the authors chose the Weibull distribution for this research effort. The specific form of the Weibull distribution used is:

Probability Density Function :

$$f(t) = Kt^{m} e^{-Kt^{m+1}/(m+1)} \qquad (E.1)$$

$$m > -1$$
$$K \geq 0$$

where m and K are the parameters of the distribution and t is the variable of interest—in this situation—time.

Once failure data was screened and data points established (See Appendix C for procedure used), the task became to determine the appropriate parameters for the Weibull distribution to model each module hazard rate. One notes the existence of an established maximum operating time (MOT) for each module, except the augmentor/exhaust module. This MOT essentially truncates the distribution and this effect must be addressed in order to have valid parameter estimates.

Maximum likelihood estimates (MLE) of the shape and scale factors of a truncated, Weibull distribution have been developed by Shooman (24:477). The expressions for the shape factor, m, and scale factor, K, where r is the number of failures in time T, n is the number of components in the original population and t_1, t_2, . . ., t_r are the failure times of the r components are:

$$K = \frac{r(m+1)}{\sum_{i=1}^{r} t_i^{m+1} + (n-r)T^{m+1}} \qquad (E.2)$$

$$\frac{1}{r}\sum_{i=1}^{r} \ln t_i = \frac{\sum_{i=1}^{r} t_i^{m+1} \ln t_i + (n-r)T^{m+1} \ln T}{\sum_{i=1}^{r} t_i^{m+1} + (n-r)T^{m+1}} - \frac{1}{m+1} \qquad (E.3)$$

Equation (E.3) cannot be solved explicitly for m. Shooman (24:477) recommends graphical solution, that is plotting the left side of Equation (E.3) against the right side. Although this is certainly an alternative, the current authors decided to take advantage of the computational power of the computer and develop a FORTRAN program which would search for a value of m which would satisfy Equation (E.3). It can be noted that the left side of Equation (E.3), not containing m, is a constant. This simplified the programming in that only the right side of the equation needed to be repetitively recalculated. A graphical chart depicting the computer search operation is included in Figures E.1 and E.2. A copy of the program, LIKELY2 is included later in this appendix. In order to verify that the program functioned properly, a number of data files with known Weibull shape and scale parameters were created using the inverse transform method. One file of 1000 data elements was created with a positive shape parameter and one of the same size with a negative shape parameter. With n set equal to r and the truncation point set just above the value of the largest data element, LIKELY2 was used to estimate the parameters of the underlying distribution. Finally, the positive shape parameter file was truncated by simply splitting the file into two smaller files, one containing values greater than the truncating point and one smaller than the truncating point. LIKELY2

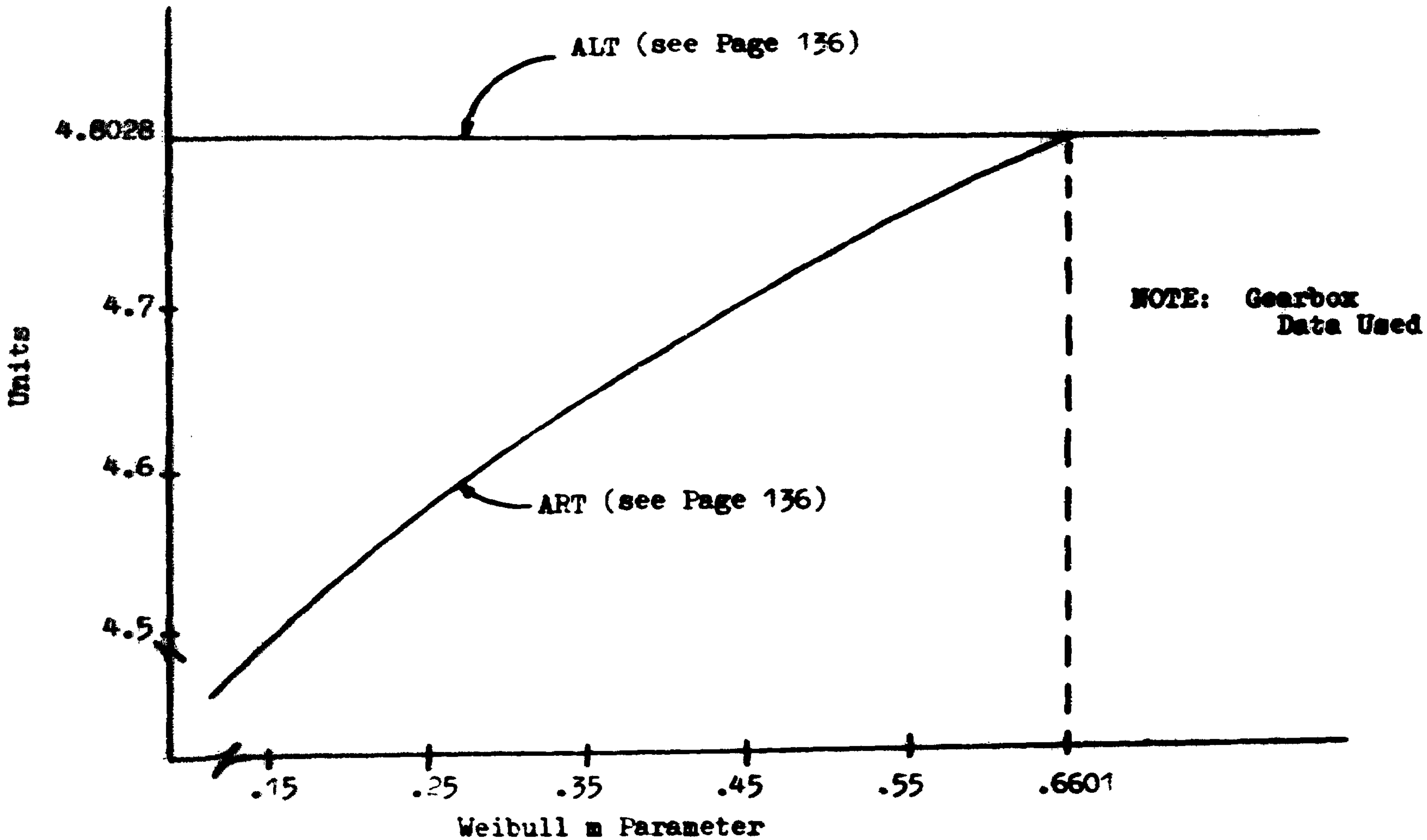

Figure E.1

Graphical Depiction of LIKELY2 for Positive m Search

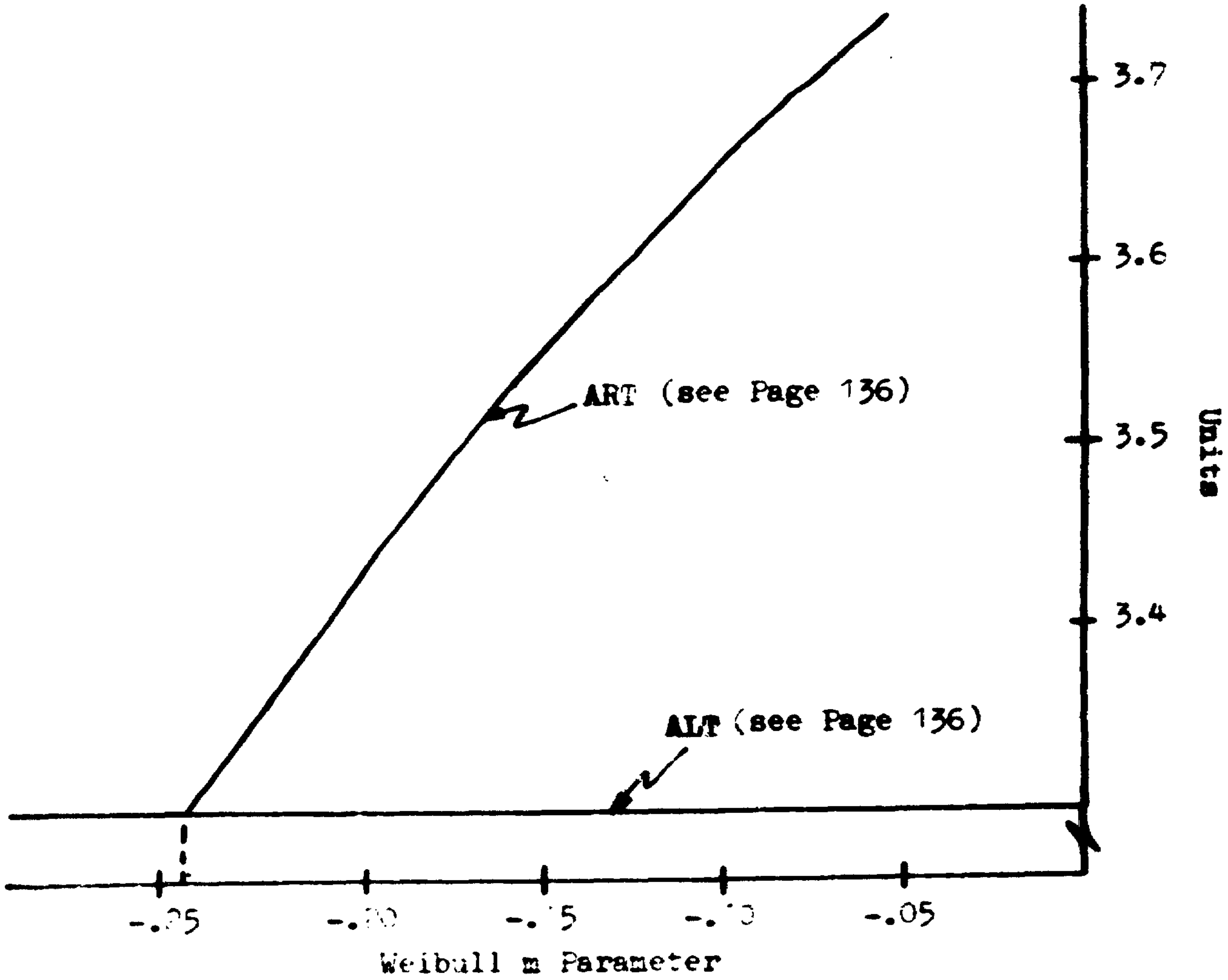

Figure E.[illegible]

Graphical Depiction of LIKELY2 for Negative m Search was then used to estimate parameters from the truncated data file. Results returned by LIKELY2 were found to be statistically accurate at the 95 percent confidence level when compared to the parameters used originally to write the data points.

Formulae are also available (24:482-483) to find the amount of variance involved in each estimate of the parameters m and K. The expressions for parameter estimation variance for a truncated Weibull hazard model where m is the estimate of the shape parameter, K is the estimate of the scale parameter, n is the number of components in the original population, r is the number of observed failures in the test period T, and $t_1, t_2, \ldots, t_r$ are the failure times for the r components are:

$$\mathrm{Var}\ K \approx K^2/r \qquad (E.4)$$

$$\mathrm{Var}\ m \approx \frac{1/K}{\frac{2}{(m+1)^3}\cdot\left[\sum_{i=1}^{r} t_i^{m+1} + (n-r)T^{m+1}\right]} \qquad (E.5)$$

$$- \frac{2}{(m+1)^2}\left[\sum_{i=1}^{r} t_i^{m+1} \ln t_i + (n-r)T^{m+1} \ln T\right]$$

$$+ \frac{1}{m+2}\left[\sum_{i=1}^{r} t_i^{m+1} (\ln t_i)^2 + \sum_{i=1}^{r} t_i^{m} + T^{m+1} (\ln T)^2\right]$$

Using Chebyshev's inequality (24:483) which states that:

$$P(|\Theta - \mu| \geq N\sigma) \leq \frac{1}{N^2} \quad \text{(E.6)}$$

where:

- Θ random variable with an arbitrary distribution
- μ expected value of Θ or $E(\Theta)$
- N number of standard deviations
- standard deviation of Θ or σ_Θ

one may determine appropriate confidence bands for the parameter estimates.

The computer program developed in FORTRAN to find MLE's and their variances is shown in Figure E.4. Basically, the program consists of a Main Program and seven function subprograms. The main program initializes all variables, reads data elements in, determines when m has been estimated to an accuracy of .00001 and provides output. Inspection of the right hand side of Equation (E.3) revealed its monotonic behavior. Because of this behavior, the interval bisection offered considerable computational efficiency and was incorporated. Function subprograms were used to perform summing operations and computation of the variance of the m parameter due to its complexity. Table E.1 lists the variable names used in the main program with their associated meanings. Figure E.3 is a simplified flowchart of the main program.

Table E.1

VARIABLES USED IN LIKELY2

ANS	Character variable used to determine if full explanation of program usage is desired.
FILENAME	Character variable used to contain the name of the file containing data to be analyzed.
JR	Counter used in summing subprograms.
J	Counter used in do-loop.
T(J)	Vector used to store failure data points.
XM	The value for the Weibull parameter "m" for which this program was developed.
BIGT	The truncation point specified.
N	Number of elements in original population.
X	Individual data points, as read from file.
I	Counter used to determine number of data points read in.
ALT	Variable used to store the computed value of the left side of Equation (E.3).
ART	Variable used to store the computed value of the right side of Equation (E.3).
XK	The value for the Weibull parameter "K" for which this program was developed.
ZZ	Dummy variable used in reading the line numbers on the data file (once read these numbers are discarded).
XLOW	Variable used to store the value of XM for interval bisection computations.

XHIGH	Variable used to store the value of XM for interval bisection computations.
XDIF	Difference between XHIGH and XLOW.
XKVAR	Variance of Weibull "K" parameter estimate computed using MLE's.

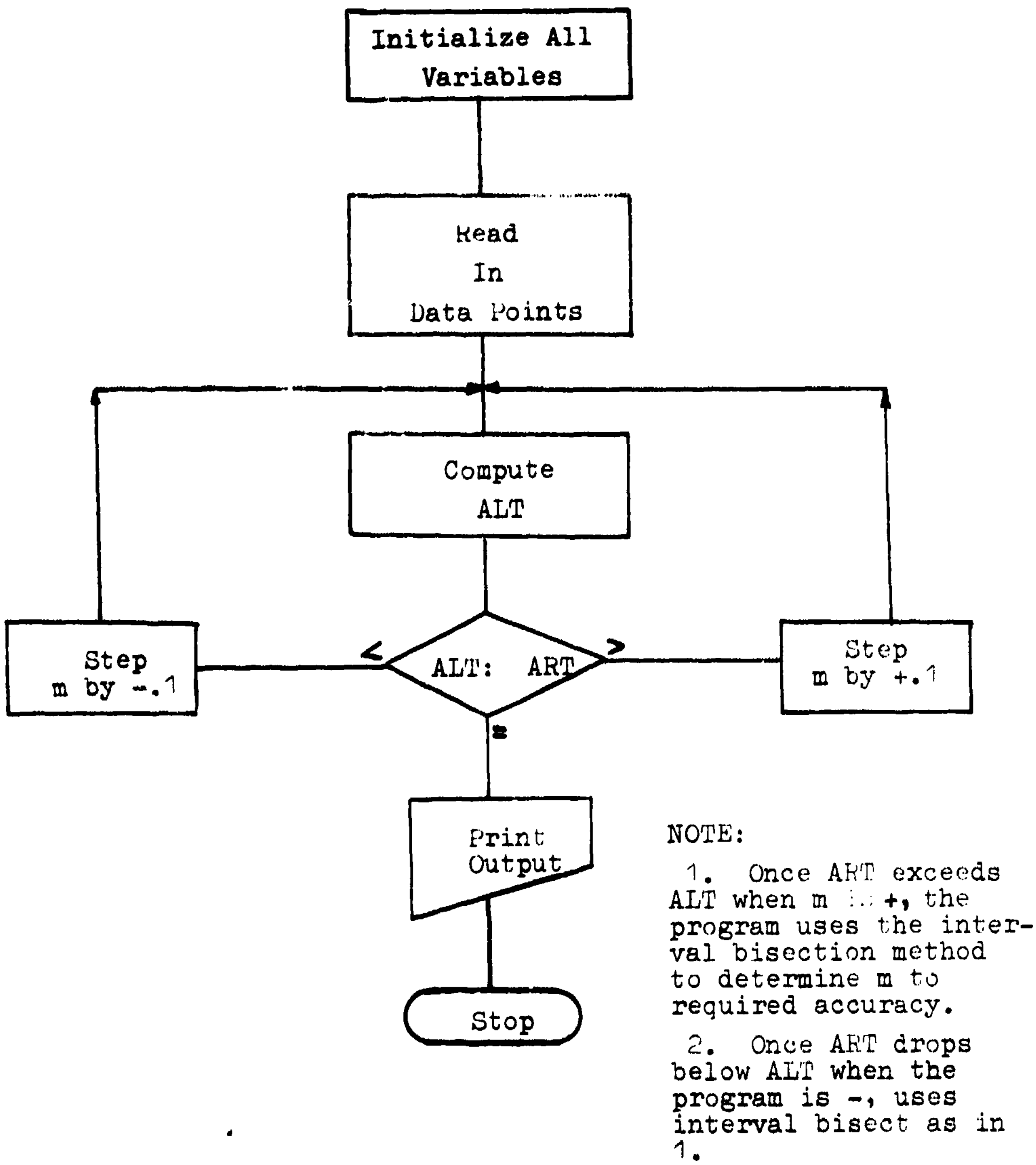

Figure E.3

Simplified Flowchart of Main Program

Figure E.4

Program LIKELY2 Listing

```
10 CHARACTER ANS*8
20 PRINT, "THIS PROGRAM READS IN VALUES FROM A FILE"
30 PRINT, "AND COMPUTES MAXIMUM LIKELIHOOD ESTIMATES"
40 PRINT, "OF THE PARAMETERS OF A WEIBULL DISTRIBUTION"
50 PRINT, "THE DATA MAY BE FROM INCOMPLETE LIFE TESTING OR"
60 PRINT, "ANY OTHER TRUNCATED DISTRIBUTION"
70 PRINT, "THE PROGRAM IS DIMENSIONED TO ACCEPT UP TO"
80 PRINT, "1000 DATA ELEMENTS"
90 PRINT, "DO YOU WANT AN EXPLANATION OF HOW TO USE THE PROGRAM:"
100 PRINT, "ANSWER YES OR NO."
110 READ, ANS
120 IF(ANS.EQ."NO") GOTO 76
130 PRINT, " "
140 PRINT, " "
150 PRINT, "  IN ORDER TO USE THIS PROGRAM, THE DATA FILE:"
160 PRINT, "     1)  MUST HAVE LINE NUMBERS"
170 PRINT, "EXAMPLE:      LINE NUMBER  DATUM"
180 PRINT, "     2)  MUST HAVE A  9999.0  FOLLOWING THE LAST DATUM"
190 PRINT, "EXAMPLE       LINE NUMBER     9999.0"
200 PRINT, "     3)  MUST HAVE THE VALUES FOR THE TRUNCATION POINT"
210 PRINT, "         AND THE NUMBER OF ELEMENTS IN THE ORIGINAL"
220 PRINT, "         POPULATION ON THE FIRST LINE FOLLOWING THE 9999.0."
230 PRINT, "         IF THE DISTRIBUTION IS NOT TRUNCATED, THE TRUNCATION"
240 PRINT, "         POINT MAY BE ANY POSITIVE VALUE AS IT WILL NOT ENTER"
250 PRINT, "         THE CALCULATIONS.  THE NUMBER OF ELEMENTS IN THE "
260 PRINT, "         ORIGINAL POPULATION, IN THE CASE OF NO TRUNCATION, "
270 PRINT, "         HOWEVER, MUST EQUAL THE TOTAL NUMBER OF DATA ELEMENTS IN"
280 PRINT, "         YOUR DATA FILE."
290 PRINT, "         EXAMPLE OF LAST DATA FILE LINE"
300 PRINT, "         LINE NUMBER    TRUNCATION POINT    ORIGINAL POP."
```

```
PRINT, " "
PRINT,"              1.920                    4063.9              580"
PRINT, " "
PRINT, "  "
76 PRINT, " "
DIMENSION T(1000)
CHARACTER FILENAME*25
PRINT, "ENTER THE NAME OF THE FILE WHICH CONTAINES YOUR DATA"
PRINT, "EXAMPLE"
PRINT, "      75B04/DATA;"
PRINT, "BE SURE TO END YOUR DATA FILE NAME WITH A SEMICOLON(;)"
READ, FILENAME
CALL ATTACH(13,FILENAME,3,0,,)
PRINT, "  "
PRINT, "  "
JR = 0
DO 66 J = 1,1000
T(J) = 0
66 CONTINUE
XM = 0
BIGT = 0
N = 0
Y = 0
I = 0
ALT = 0
ART = 0
XK = 0
1 READ(13,200) ZZ,X
200 FORMAT(V)
IF(X-9999.0) 14,2,14
14 I = I + 1
T(I) = X
JR = I
GOTO 1
```

```
2 READ(13,200) ZZ,BIGT,N
C
C
PRINT, "THE NUMBER OF ELEMENTS READ WAS", JR
PRINT, "THE TRUNCATION POINT IS", BIGT
PRINT, "THE NUMBER OF ELEMENTS IN THE ORIGINAL POPULATION WAS", N
C
C
3 ALT = SUM1(JR,T)
ART = RIGHT(JR,T,XM,BIGT,N)
IF(ART.LT.ALT) GOTO 5
C
C  *****INITIAL APPROXIMATION OF MINUS M*****
C
78 XM = XM - .1
ART = RIGHT(JR,T,XM,BIGT,N)
IF(ART.LE.ALT) GOTO 79
GOTO 78
C
C
C
C  *****INITIAL APPROXIMATION OF PLUS M*****
C
5 IF(ART.LT.ALT) GOTO 8
STOP "ALT GREATER THAN ALT AT LINE 470"
8 XM = XM + .1
ART = RIGHT(JR,T,XM,BIGT,N)
IF(ART.GT.ALT) GOTO 9
IF(XM.LE.10.0) GOTO 8
STOP "M EXCEEDED 10.0 AT LINE 530"
C
C
C
C  *****DETERMINE M TO 5 DECIMAL PLACES USING*****
```

```
990C  *****INTERVAL BISECTION METHOD                       *****
1000 79 XLOW = XM
1010 XDIF = .1
1020 XM = XM + .1
1030 GOTO 21
1040 9 XLOW = XM - .1
1050 XDIF = .1
1060 21 XHIGH = XM
1070 XM = (XM+XLOW)/2.
1080 23 ART = RIGHT(JR,T,XM,BIGT,N)
1090 IF(ART.LT.ALT) GOTO 22
1100 XDIF = XHIGH - XLOW
1110 IF(XDIF.LT. .00001) GOTO 11
1120 GOTO 21
1130 22 XLOW = XM
1140 XM = (XHIGH+XLOW)/2.
1150 XDIF = XHIGH - XLOW
1160 IF(XDIF.LT. .00001) GOTO 11
1170 GOTO 23
1180C
1190C
1200C
1210C
1220 11 XK = (JR*(XM+1.0))/(SUM3(JR,T,XM)+(N-JR)*BIGT**(XM+1.))
1230 PRINT, "THE VALUES OF M AND K ARE"
1240 PRINT 102, XM,XK
1250 VAR = VARM(JR,T,XM,BIGT,N,XK)
1260 XKVAR = (XK**2.)/JR
1270 PRINT, "THE VARIANCES OF M AND K ARE"
1280 PRINT 104, VAR,XKVAR
1290 PRINT, "THE STANDARD DEVIATIONS OF M AND K ARE"
1300 PRINT 102, SQRT(VAR),SQRT(XKVAR)
1310 101 FORMAT(F14.4,4X,F14.4)
1320 102 FORMAT(F14.5,4X,F14.8)
```

```
103 FORMAT(//)
104 FORMAT(E15.8,4X,E15.8)
STOP
END
C
C
FUNCTION RIGHT(JR,T,XM,BIGT,N)
DIMENSION T(1000)
RIGHT = (SUM2(JR,T,XM)+(N-JR)*((BIGT**(XM+1.))*
&ALOG(BIGT)))/(SUM3(JR,T,XM)+(N-JR)*(BIGT**
&(XM+1.)))-1./(XM+1.)
RETURN
END
C
C
FUNCTION SUM1(JR,T)
DIMENSION T(1000)
SUM1 = 0
DO 10 I = 1,JR
SUM1 = SUM1+ALOG(T(I))
10 CONTINUE
SUM1 = SUM1/JR
RETURN
END
C
C
FUNCTION SUM2(JR,T,XM)
DIMENSION T(1000)
SUM2 = 0
DO 10 I = 1,JR
SUM2 = SUM2+(T(I)**(XM+1.))*(ALOG(T(I)))
10 CONTINUE
RETURN
END
```

```
1670C
1680C
1690 FUNCTION SUM3(JR,T,XM)
1700 DIMENSION T(1000)
1710 SUM3 = 0
1720 DO 10 I = 1,JR
1730 SUM3 = SUM3 + T(I)**(XM+1.)
1740 10 CONTINUE
1750 RETURN
1760 END
1770C
1780C
1790 FUNCTION SUM4(JR,T,XM)
1800 DIMENSION T(1000)
1810 SUM4 = 0
1820 DO 10 I = 1,JR
1830 SUM4 = SUM4 + (T(I)**(XM+1.))*(ABS(ALOG(T(I)))**2.)
1840 10 CONTINUE
1850 RETURN
1860 END
1870C
1880C
1890 FUNCTION VARM(JR,T,XM,BIGT,N,XK)
1900 DIMENSION T(1000)
1910 DIVA=(2./((XM+1.)**3.))*(SUM3(JR,T,XM)
1920&+(N-JR)*BIGT**(XM+1.))
1930 DIVB=(2./((XM+1.)**2.))*(SUM2(JR,T,XM)
1940&+(N-JR)*(BIGT**(XM+1.))*ALOG(BIGT))
1950 DIVC=(1./(XM+1.))*(SUM4(JR,T,XM)+SUM5(JR,T,XM)
1960&+(BIGT**(XM+1.))*((ALOG(BIGT))**2.))
1970 VARM = (1./XK)/(DIVA - DIVB + DIVC)
1980 RETURN
1990 END
2000C
```

```
C
FUNCTION SUM5(JR,T,XM)
DIMENSION T(1000)
SUM5 = 0
DO 10 I = 1,JR
SUM5 = SUM5 + T(I)**(XM)
10 CONTINUE
RETURN
END
```

APPENDIX F

FORTRAN FORM OF THE ALGORITHM

APPENDIX F

FORTRAN FORM OF THE ALGORITHM

The algorithm developed in Chapter IV was programmed in FORTRAN and is shown in Figure F.2. Basically, the program consists of a Main Program and five function subprograms. The main program initializes failure distribution parameters and cost inputs, contains two do-loops which increment the value of n and the age of the core module, manipulates probabilities and expectations returned by the subprograms, and provides data output. The function subprograms perform the detailed calculations of probabilities and expected values.

Table F.1 lists the variable names used in the main program with their associated meanings. Throughout the program, MOD1 refers to the module for which the optimal opportunistic replacement policy is being determined. MOD2 refers to the core module. The program could be used for any two component system where it is desired to find the optimal opportunistic replacement policy for one of the components given the maximum operating times and failure distribution parameters. For this reason, the symbols MOD1 and MOD2 were used rather than, for instance, fan and core.

Numerical solution for the optimal value of n is performed in the main program. Figure F.1 is a simplified flow chart of the main program. The value of MOD2AGE is initially set at 0 at line 400. The value of TSTAR is initially set to 0 at line 600. Lines 610 through 640 determine the difference, if any, between MOD2AGE and TSTAR, calculate the maximum remaining hours on MOD1, and determine if the value of N represents MOD1MOT or MOD2MOT. Lines 660 through 840 make the necessary calls to the function subprograms to determine values of CYCLE, TCOST and HOURCOST. These three values are printed out along with MOD2AGE and TSTAR. The program then loops back, increments the value of TSTAR by MOD1MOT/25 hours and determines CYCLE, TCOST and HOURCOST once more. TSTAR is incremented in MOD1MOT/25 hour steps until MOD1MOT is reached. Once MOD1MOT is reached, the program loops back to line 440, increments MOD2AGE by the value of SCAN, sets the value of TSTAR to zero, and increments TSTAR in MOD1MOT/25 hour steps once more. This pattern of incrementing TSTAR from 0 hours to MOT hours and then incrementing the value of MOD2AGE is continued until MOD2AGE equals 25 times the value of SCAN. If SCAN is set at MOD2MOT/25, the final value of MOD2AGE will equal MOD2MOT.

As currently written, the program prints one line of output for each value of MOD2AGE and TSTAR. By deleting line 970 and inserting the following statement between lines 930 and 940, the program will print the optimal value of TSTAR (and the associated values of CYCLE, TCOST and HOUR-COST) for each value of MOD2AGE:

PRINT 5, MOD2AGE,XSTAR,HOURCOST,CYCLE,TCOST

Function subprogram SINGMEAN performs the integral

$$XMU1 = \int_0^{TSTAR} t \bullet f_{MOD1}(t)\, dt \qquad (F.1)$$

numerically using Simpson's rule (23:14). Numerical evaluation was necessary since (F.1) has a closed form for the Weibull PDF only when the limits of integration are 0 and infinity. In order to insure that SINGMEAN functioned properly, a special "test" Main Program was designed with just sufficient statements to pass arguments to SINGMEAN and print results. The Weibull PDF was sketched for m = .71019 and k = .000163 which were the maximum likelihood estimates of the parameters of the fan drive turbine module. The value of the PDF is approximately zero for values of t beyond 500 hours. Mean time to failure was analytically determined as (24:221)

$$\mu = \Gamma\left(\frac{1}{m+1}\right)\left[(m+1)\left(\frac{k}{m+1}\right)^{1/(m+1)}\right]^{-1} \qquad (F.2)$$

The analytical mean of a Weibull distribution with m = .71019 and k = .000163 is 200.172 hours. A value of t = 6000 hours which is well beyond 500 hours was passed to SINGMEAN. The absolute error of the value of XMU1 returned by SINGMEAN is sensitive to the number of segments used in numerical approximation of the integral. With 200 segments, error was over six percent. By increasing the number of segments to 2000, the mean returned by SINGMEAN was 201.683 which is approximately .8 percent error. The relatively large number of segments used in SINGMEAN is the primary reason the algorithm requires 1.2 hours of processing time. Further increasing the number of segments in SINGMEAN would require even longer processing time. For this reason, .8 percent error was accepted. The complete algorithm was run once with 5000 segments to determine if the optimal value of TSTAR was sensitive to residual error in SINGMEAN. The optimal value of TSTAR was the same as that found using 2000 segments in SINGMEAN. HOURCOST was approximately .07 percent higher using 5000 segments. This difference is felt to be insignificant.

Function subprogram QMOD calculates the value of

$$F(t \mid t_j \leq t \leq t_k) = \frac{F(t) - F(t_j)}{F(t_k) - F(t_j)} \qquad (F.3)$$

for the Weibull PDF. No approximations are used in this function subprogram. The subprogram was checked individually, however, in the same manner as SINGMEAN and returned the analytically correct probability.

Function subprograms QFCBAR, QFBARC and QQFC are essentially similar. The mechanics of the three function subprograms are illustrated here by reference to function QFCBAR. In Chapter IV, the probability of the fan module requiring replacement before the core module in the interval (n, N) given fan module survival until n and core module survival until $n - \Delta t$ was

$$Q(f\bar{c}) = \sum_{i=1}^{M} q_i(f\bar{c}) \qquad (F.4)$$

when $n < t_i \leq N - \delta t$,

where

$$q_i(f\bar{c}) = \left[1 - QFAN(t_i \mid n)\right] \cdot \left[1 - QCORE(t_{i+1} \mid n)\right] \cdot QFAN(t_{i+1} \mid t_i) \qquad (F.5)$$

and when $N - \delta t < t_i \leq N$ $q_i(f\bar{c}) = 0$ if N = COREMOT but if N = FANMOT then

$$q_i(f\bar{c}) = \left[1 - QFAN(N - \delta t \mid n)\right] \cdot \left[1 - QCORE(N \mid n)\right]. \qquad (F.6)$$

Function QFCBAR performs this summation. Three values are returned by the subprogram. In addition to the value of (F.3) the subprogram also returns

$$QFCBARL = \sum_{i=1}^{M-1} \frac{(t_i + t_{i+1})}{2} q_i(f\bar{c}) \qquad (F.7)$$

and

$$YMU1 = \sum_{i=1}^{M-1} \frac{(t_i + t_{i+1})}{2} q_i(f\bar{c}) \qquad (F.8)$$

QFCBARL is used to pass the value of X4 to the main program. YMU1 is used to pass the contribution of Q(fc) to the calculation of

$$\sum_{i=1}^{M-1} \frac{(t_i + t_{i+1})}{2} \left[q_i(fc) + q_i(\bar{f}c) + q_i(f\bar{c})\right] \qquad (F.9)$$

required by Equation (4.17) in Chapter IV. Function subprograms QQFC and QFBARC differ from function QFCBAR only in that they perform calculations involving $q_i(fc)$ and $q_i(\bar{f}c)$ respectively.

Function subprograms QFCBAR, QFBARC and QQFC were checked by writing a short main program with just sufficient statements to pass the necessary arguments and print results. The value of

$$1 - \sum_{i=1}^{M} q_i(fc) + q_i(\bar{f}c) + q_i(f\bar{c}) \qquad (F.10)$$

over the interval (0, t) is equivalent to

$$\left[1 - F_{MOD1}(t)\right] \bullet \left[1 - F_{MOD2}(t)\right] \qquad (F.11)$$

Thus comparing the value of (F.10) with the analytic two component system reliability for some time t provides a check on the function subprograms. This check was performed for a value of XM1 = .66, XK1 - .000141, XM2 = -.1316 and XK2 = .002051. The analytic reliability is .334. With 200 segments in each of the function subprograms, the value of (F.10) was .336 which is .6 percent error.

A second check of subprograms QFCBAR, QFBARC and QQFC is possible by comparing the mean time at failure of a two component system returned by the subprograms with the analytically determined mean time at failure.

For a system of Weibull components, mean time at failure may be calculated analytically over the interval $(0, \infty)$ when the component m parameters are equal (23:221).[1] For $m = .66004$ and k values of .000141 and .002051, analytic mean time at failure is 48.38 hours. The m and k values chosen are in the range relevant to distributions studied in this thesis. The mean determined by QFCBAR, QFBARC and QQFC was 48.48 hours which is an error of approximately .2 percent. Functions QFCBAR, QFBARC and QQFC, like SINGMEAN, require an upper bound for the interval over which approximation is to be accomplished. For the m or k values chosen, the PDF's are approximately 0 beyond $t = 500$ hours. An upper bound of 2000 was used to check the subprograms.

---

[1] The formula for calculating system mean time to failure is

$$\mu = \Gamma\left(\frac{1}{m+1}\right) \bullet \left[(m+1)\left(\frac{k_1 + k_2}{m+1} \right)^{1/(m+1)} \right]^{-1}.$$

Table F.1

PROGRAM SYMBOLOGY

N	Maximum possible age at replacement of module 1. Refer to Equation (4.21) in Chapter IV.
MOD2AGE	Age of module 2.
MOD1PACK	Dollar cost to pack and unpack module 1 at the field level.
MOD1DEP	Dollar cost to depot overhaul module 1.
MOD1REP	Dollar cost to remove and replace module 1. This cost includes the cost to remove the engine from the aircraft, transport the engine to the repair shop, perform the in shop module removal and replacement, transport the engine from the shop to the aircraft, reinstall and trim check the engine.
MOD1SHIP	Cost, in dollars, to transport module 1 between the field repair shop and the depot. Shipping cost is round trip.
MOD1MOT	Module 1 maximum operating time.
MOD2MOT	Module 2 maximum operating time.
ZM1	Weibull m parameter for module 1.
XK1	Weibull k parameter for module 1.
XM2	Weibull m parameter for module 2.
XK2	Weibull k parameter for module 2.
SCAN	Amount by which module 2 age is incremented in do-loop.
DELTAM	Difference between module 1 MOT and module 2 MOT. Refer to Equation (4.20) in Chapter IV.

TSTAR	Breakpoint between replace at failure region and opportunistic replacement region. TSTAR is equivalent to n as used in Chapter IV.
TCOST	Total cost per cycle in dollars.
CYCLE	Cycle length in hours.
HOURCOST	Cost per hour in dollars per hour.
XLOW	Variable used to store the lowest value of hourcost.
XSTAR	The value of TSTAR associated with XLOW.
I, J	Counters used in do-loops.
UL	Maximum hours remaining on module 1 before replacement.
DELTAT	Difference between module 1 age and module 2 age. Refer to Equation (4.19) in Chapter IV.
NFIX	Switch variable used to pass information on which module will reach its MOT first.
X1	Probability of module 1 requiring replacement before module 2 in the interval $n < t \leq N$ given survival of both until TSTAR and TSTAR - DELTAT respectively.
X2	Probability of module 2 requiring replacement before module 1 in the interval TSTAR $< t \leq$ N given survival of both until TSTAR and TSTAR-DELTAT respectively.
X3	Probability of module 1 and module 2 requiring simultaneous replacement in the interval TSTAR $< t \leq$ N given survival of both until TSTAR and TSTAR- DELTAT respectively.
X4	Defined analogously to X1 over the interval TSTAR $< t <$ N.
X5	Defined analogously to X2 over the interval TSTAR $< t <$ N.
X6	Defined analogously to X3 over the interval TSTAR $< t <$ N.
X7	Probability of module 1 failure in the interval $0 \leq t \leq$ TSTAR.

X8	Reliability of module 1 in the interval $0 \leq t \leq TSTAR$.
X11	Probability that module 1 requires replacement before module 2 in the interval $TSTAR < t \leq N$ given module 2 survival until TSTAR-DELTAT.
X22	Probability that module 2 requires replacement before module 1 in the interval $TSTAR < t \leq N$ given module 2 survival until TSTAR-DELTAT.
X33	Probability that module 1 and module 2 require simultaneous replacement in the interval $TSTAR < t \leq N$ given module 2 survival until TSTAR-DELTAT.
X44	Defined analogously to X11 over the interval $TSTAR < t < N$.
X55	Defined analogously to X22 over the interval $TSTAR < t < N$.
X66	Defined analogously to X33 over the interval $TSTAR < t < N$.
XMU1	The contribution to mean time to removal of module 1 by the interval $0 \leq t \leq TSTAR$.
XMU3	The contribution to mean time to removal of module 1 by the interval $TSTAR < t \leq N$.

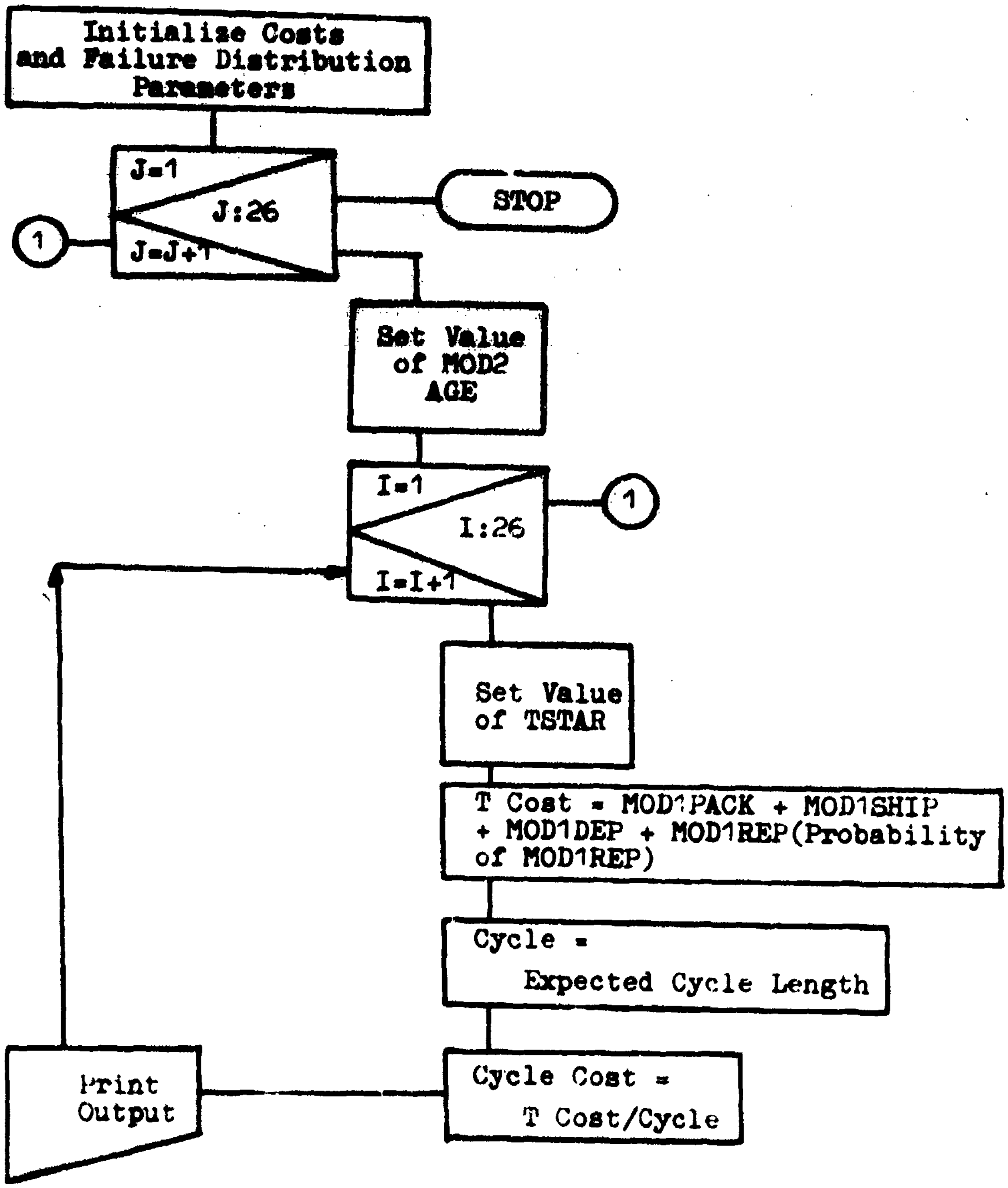

Figure F.1

Simplified Flowchart of Main Program NEWL-MOD

Figure F.2

Listing of NEWL-MOD

```
REAL N,MOD2AGE
REAL MODIPACK,MODIREP,MODISHIP,MODIDEP
REAL MOD1MOT,MOD2MOT
DOUBLE PRECISION OMOD
C
C
C *****MODULE UNDER STUDY: FAN DRIVE TURBINE
C
C
C
C *****FAILURE DISTRIBUTION PARAMETERS
C
XM1= 0.71019
XK1 = .000163
XM2=-.01316
XK2 = .002051
C
C *****MODULE MAXIMUM OPERATING TIMES
C
MOD1MOT = 250.
MOD2MOT = 250.
C
C *****COST DATA
C
MODIPACK = 36.10
MODIDEP = 6174.
MODISHIP = 315.34
MODIREP = 273.61
C
C *****ITERATION INTERVAL FOR MOD2AGE
```

```
310C
320 SCAN = 10.
330C
340C ********************
350C *****BEGIN ALGORITHM
360C ********************
370C
380 PRINT 3
390 PRINT 3
400 MOD2AGE = 0.
410 SCAN = 10.
420 DELTAM = MOD1MOT - MOD2MOT
430 TSTAR = 0.
440 DO 4 J = 1,26
450 PRINT 11
460 PRINT, "MOD2AGE            TSTAR          HOURCOST        CYCLE            TCOST"
470 MOD2AGE = SCAN*(FLOAT(J) - 1.)
480 XLOW = 10000.
490 PRINT 3
500 PRINT 2
510 2 FORMAT(/)
520 3 FORMAT(//)
530 5 FORMAT(F9.4,4(F14.4))
540 11 FORMAT(3H1    )
550 UL = MOD1MOT
560 IF(MOD1MOT.GT.MOD2MOT-MOD2AGE) UL =
570&(MOD2MOT - MOD2AGE)
580 DD = MOD1MOT/25.
590 DO 1 I = 1,26
600 IF(I.GT.2.AND.I.LT.13) GOTO 1
610 TSTAR = (FLOAT(I - 1))*DD
620 DELTAT = TSTAR - MOD2AGE
630 IF((MOD1MOT-TSTAR).LT.(MOD2MOT-MOD2AGE)) N = MOD1MOT
640 NFIX = 0.
```

```
IF((MOD1MOT-TSTAR).GE.(MOD2MOT-MOD2AGE)) GOTO 8
9 CONTINUE
X1 = QFCBAR(XM1,XM2,XK1,XK2,TSTAR,N,NFIX,DELTAT,QFCBARL,YMU1)
X2 = QFBARC(XM1,XM2,XK1,XK2,TSTAR,N,NFIX,DELTAT,QFBARCL,YMU2)
X3 = QQFC(XM1,XM2,XK1,XK2,TSTAR,N,NFIX,DELTAT,QQFCL,YMU3)
X4 = QFCBARL
X5 = QFBARCL
X6 = QQFCL
X7 = QMOD(XM1,XK1,0.,TSTAR)
X8 =1. - X7
X11 = X1*X8
X22 = X2*X8
X33 = X3*X8
X44 = X4*X8
X55 = X5*X8
X66 = X6*X8
YMU1 = SINGMEAN(XM1,XK1,TSTAR)
YMU3 = X8*(YMU1+YMU2+YMU3)
TCOST = MOD1PACK+MOD1SHIP+MOD1DEP+ MOD1REP*(X11+X33+X7)
CYCLE = XMU1+XMU3 + N*(1. -(X44+X55+X66+X7))
HOURCOST = TCOST/CYCLE
IF(HOURCOST.LE.XLOW) XSTAR = TSTAR
IF(HOURCOST.LE.XLOW) XLOW = HOURCOST
PRINT 5,MOD2AGE,TSTAR,HOURCOST,CYCLE,TCOST
GOTO 1
6 N = TSTAR + JL
NFIX = 1
IF((MOD1MOT-TSTAR).EQ.(MOD2MOT-MOD2AGE)) NFIX = 2
GOTO 9
1 CONTINUE
4 CONTINUE
STOP
END
```

```
C
C
C
FUNCTION SINGMEAN(XM,XK,TT)
DOUBLE PRECISION F
F = 0
D = TT/2000.
DO 1 I = 2,2000
TI = (FLOAT(I) - 1.)*D
F = F + TI*(XK*TI**XM)*EXP((-XK*TI**(XM+1.))/(XM+1.))
1 CONTINUE
F = F*2.
F = F + 1. + TT*(XK*TT**XM)*EXP((-XK*TT**(XM+1.))/(XM+1.))
SINGMEAN = .5*D*F
RETURN
END
C
FUNCTION QFCBAR(XM1,XM2,XK1,XK2,TSTAR,N,NFIX,DELTAT,QFCBARL,YMU1)
REAL N
DOUBLE PRECISION QMOD,XMUL1,XMUL2,XMUL3,SQI,SSQI
SQI = 0$XMUL1 = 0$XMUL2 = 0$XMUL3 = 0
SSQI = 0.
D = (N-TSTAR)/200.
DO 1 I = 1,200
T1 = TSTAR + (FLOAT(I - 1))*D
T2 = T1 + D
XMUL1 = 1. - QMOD(XM1,XK1,TSTAR,T1)
XMUL2 = 1. - QMOD(XM2,XK2,TSTAR-DELTAT,T2-DELTAT)
XMUL3 = QMOD(XM1,XK1,T1,T2)
SQI = SQI + XMUL1*XMUL2*XMUL3
SSQI = SSQI + XMUL1*XMUL2*XMUL3*.5*(T1+T2)
1 CONTINUE
YMU1 = SSQI
QFCBARL = SQI
```

```
IF(NFIX.EQ.1.OR.NFIX.EQ.2) GOTO 2
XMUL1 = 1. - QMOD(XM1,XK1,TSTAR,T1)
XMUL2 = 1. - QMOD(XM2,XK2,TSTAR-DELTAT,N-DELTAT)
SQI = SQI + XMUL1*XMUL2
2 QFCBAR = SCI
RETURN
END
C
C
FUNCTION QFBARC(XM1,XM2,XK1,XK2,TSTAR,N,NFIX,DELTAT,QFBARCL,YMU2)
REAL N
DOUBLE PRECISION QMOD,XMUL1,XMUL2,XMUL3,SQI,SSQI
SQI = 0$SSQI = 0$XMUL1 = 0$XMUL2 = 0$XMUL3 = 0
D = (N - TSTAR)/200.
DO 1 I = 1,200
T1 = TSTAR + (FLOAT(I - 1))*D
T2 = T1 + D
XMUL1 = 1. - QMOD(XM1,XK1,TSTAR,T2)
XMUL2 = 1. - QMOD(XM2,XK2,TSTAR-DELTAT,T1-DELTAT)
XMUL3 = QMOD(XM2,XK2,T1-DELTAT,T2-DELTAT)
SQI = SQI + XMUL1*XMUL2*XMUL3
SSQI = SSQI + XMUL1*XMUL2*XMUL3*.5*(T1+T2)
1 CONTINUE
YMU2 = SSQI
QFBARCL = SQI
IF(NFIX.EQ.2.OR.NFIX.EQ.0) GOTO 2
XMUL1 = 1. - QMOD(XM1,XK1,TSTAR,N)
XMUL2 = 1. - QMOD(XM2,XK2,TSTAR-DELTAT,T1-DELTAT)
SQI = SQI + XMUL1*XMUL2
2 QFBARC = SQI
RETURN
END
C
C
```

```
FUNCTION QQFC(XM1,XM2,XK1,XK2,TSTAR,N,NFIX,DELTAT,QQFCL,YMU3)
REAL N
DOUBLE PRECISION QMOD,XMUL1,XMUL2,XMUL3,XMUL4,SQI,SSQI
SQI = 0$XMUL1 = 0$XMUL2 = 0$XMUL3 = 0$XMUL4 = 0
SSQI = 0.
D = (N - TSTAR)/200.
DO 1 I = 1,200
T1 = TSTAR + (FLOAT(I - 1))*D
T2 = T1 + D
XMUL1 = 1. - QMOD(XM1,XK1,TSTAR,T1)
XMUL2 = 1. - QMOD(XM2,XK2,TSTAR-DELTAT,T1-DELTAT)
XMUL3 = QMOD(XM1,XK1,T1,T2)
XMUL4 = QMOD(XM2,XK2,T1-DELTAT,T2-DELTAT)
SQI = SQI + XMUL1*XMUL2*XMUL3*XMUL4
SSQI = SSQI + XMUL1*XMUL2*XMUL3*XMUL4*.5*(T1+T2)
1 CONTINUE
YMU3 = SSQI
QQFCL = SQI
T1 = T1 + D
T2 = N
XMUL1 = 1. - QMOD(XM1,XK1,TSTAR,T1)
XMUL2 = 1. - QMOD(XM2,XK2,TSTAR-DELTAT,T1-DELTAT)
IF(NFIX.EQ.2) GOTO 4
IF(NFIX.EQ.1) GOTO 2
XMUL3 = QMOD(XM2,XK2,T1-DELTAT,T2-DELTAT)
SQI = SQI + XMUL1*XMUL2*XMUL3
GOTO 3
2 XMUL3 = QMOD(XM1,XK1,T1,T2)
SQI = SQI + XMUL1*XMUL2*XMUL3
3 QQFC = SQI
RETURN
4 SQI = SQI + XMUL1*XMUL2
GOTO 3
END
```

```
C
C
FUNCTION CMOD(XM,XK,TT1,TT2)
DOUBLE PRECISION CMOD,XNUMB,DENOMB
XNUMB = 1. - EXP((-XK*TT2**(XM + 1.))/(XM + 1.))
& - (1. - EXP((-XK*TT1**(XM + 1.))/(XM + 1.)))
DENOMB =  EXP((-XK*TT1**(XM + 1.))/(XM + 1.))
CMOD = XNUMB/DENOMB
RETURN
END
```

APPENDIX G

PROGRAM OUTPUT

APPENDIX G

PROGRAM OUTPUT

This appendix contains output data from the FORTRAN program listed in Appendix F. Figure G.1 is a sample of the output provided by the program. Table G.1 summarizes the results from sensitivity analysis and is presented in the following format:

1. Column 1 indicates the input variable under study.

2. Column 2 contains the value of the input variable under study.

3. The value in column 3 is the optimal value of n (the breakpoint between the replace at failure region and the opportunistic replacement region) for the input parameters used. Precision is ± 10 hours.

4. The value in column 4 is the expected cost per hour under the optimal opportunistic replacement policy for the input parameters used.

5. The value in column 5 is the expected cost per hour under the corresponding replace at failure policy.

6. The value in column 6 is the difference in expected cost between the replace at failure policy and the opportunistic replacement policy.

7. The value in column 7 is the expected cycle length under the optimal opportunistic policy for the input

parameter used.

8. The value in column 8 is the expected cycle length under the corresponding replace at failure policy. The baseline values of the program input variables were:

XM1	.71019
XK1	.000163
XM2	-.01316
XK2	.002051
MOD1REP	$862.52
MOD1DEP	$6174.00
MOD1PACK	$30.10
MOD1SHIP	$315.84
MOD1MOT	250.0 hours
MOD2MOT	250.0 hours
SCAN	10.0 hours

Table G.1

Output Sample

1 Variable	2 Value	3 Optimal t (hours)	4 Minimum Hour Cost ($)	5 Replace at Fail-Hour Cost ($)	6 Difference ($)	7 Opportunistic Cycle Length (hours)	8 Replace at Fail Cycle Length (hrs)
(Base-line)	—	230	43.0768	43.1077	.0306	170.3914	170.5224
MOD1REP	662.52	230	41.9177	41.9348	.0171	170.3914	170.5224
	762.52	230	42.4973	42.5212	.0239	170.3914	170.5224
	962.52	230	43.6563	43.6941	.0378	170.3914	170.5224
	1062.52	230	44.2358	44.2805	.0448	170.3914	170.5224
	1262.52	220	45.3882	45.4534	.0652	170.2268	170.5224
MOD1DEF	3087	210	25.4796	25.5498	.0702	169.99[illegible]3	170.5224
	12,348	240	79.8406	79.8495	.0188	170.4888	170.5224
XM1	.005	220	29.8910	29.9825	.0915	244.08[illegible]	244.9366
	.05	220	30.0157	30.1064	.0907	243.08[illegible]	243.9290
	.1	220	30.1867	30.2761	.0895	241.7239	242.5613
	.2	220	30.6674	30.7535	.0861	237.9882	238.7962

Table G.1 (Cont'd)

Variable	2 Value	3 Optimal n (hours)	4 Minimum Hour Cost ($)	5 Replace at Fail-Hour Cost ($)	6 Difference ($)	7 Opportunistic Cycle Length (hours)	8 Replace at Fail Cycle Length (hrs)
	.5	230	34.5262	34.5883	.0621	212.0531	212.3209
	1.0	250	—	74.8675	—	—	98.0908
	1.2	250	—	109.8719	—	—	66.8399
XK[illegible]	.0005	220	33.3809	33.4468	.0659	218.9336	219.5669
	.0002	230	46.2432	46.266[illegible]	.0229	158.6278	158.7299
	.0003	230	54.8518	54.8[illegible]	.0093	133.8089	133.8620
	.0010	250	—	105.8853	—	—	69.3564
X[illegible]	-.50	240	43.0539	43.0666	.0127	170.4932	170.5224
	-.[illegible]	230	43.0478	43.0666	.0188	170.4187	170.5224
	.[illegible]	23[illegible]	43.[illegible]352	43.0666	.0314	170.3890	170.5224
	.[illegible]5	[illegible]	4[illegible]	43.0666	.0600	170.0346	170.5224
	.50	[illegible]	[illegible]	43.0666	.1306	169.7024	170.5224
	.[illegible]	[illegible]	[illegible]349	43.0666	.4317	168.2183	170.5224
X[illegible]	.[illegible]5	[illegible]30	43.[illegible]	43.0666	.005[illegible]	170.4864	170.5224

Table G.1 (Cont'd)

1 Variable	2 Value	3 Optimal n (hours)	4 Minimum Hour Cost ($)	5 Replace at Fail-Hour Cost ($)	6 Difference ($)	7 Opportunistic Cycle Length (hours)	8 Replace at Fail Cycle Length (hrs)
	.001	230	43.0526	43.0666	.014	170.4555	170.5224
	.005	230	42.9917	43.0666	.0749	170.2159	170.5224
	.010	230	42.9233	43.0666	.1433	169.9329	170.5224
{MOD1MOT, MOD2MOT}	500, 500	480	36.9076	36.9088	.0012	198.9586	198.9721
{MOD1REP, XM2}	962.52, 1.0	230	43.1109	43.6530	.5421	168.2183	170.5224

MOD2AGE	TSTAR	[illegible]RCOST	CYCLE	TCOST
10.0000	0.	49.8[illegible]	139.1934	6938.0248
10.0000	10.0000	49.55[illegible]	143.8107	7125.9541
10.0000	20.0000	48.791[illegible]	146.3348	7139.8677
10.0000	30.0000	48.092[illegible]	148.7167	7152.1602
10.0000	40.0000	47.46[illegible]	150.9566	7164.4039
10.0000	50.0000	46.88[illegible]	153.0554	7176.5323
10.0000	60.0000	46.3730	155.0147	7188.4919
10.0000	70.0000	45.9091	156.8368	7200.2351
10.0000	80.0000	45.4929	158.5243	7211.7211
10.0000	90.0000	45.1206	160.0803	7222.9142
10.0000	100.0000	44.7889	161.5084	7233.7845
10.0000	110.0000	44.4948	162.8124	7244.3069
10.0000	120.0000	44.2355	163.9964	7254.4614
10.0000	130.0000	44.0084	165.0648	7264.2319
10.0000	140.0000	43.8111	166.0220	7273.6074
10.0000	150.0000	43.6416	166.8725	7282.5801
10.0000	160.0000	43.4978	167.6212	7291.1464
10.0000	170.0000	43.3779	168.2725	7299.3057
10.0000	180.0000	43.2803	168.8312	7307.0607
10.0000	190.0000	43.2034	169.3020	7314.4171
10.0000	200.0000	43.1458	169.6892	73[illegible].3826
10.0000	210.0000	43.1064	169.9974	7327.9684
10.0000	220.0000	43.0838	170.2308	7334.1871
10.0000	230.0000	43.0771	170.3935	7340.0546
10.0000	240.0000	43.0853	170.4895	7345.5899
10.0000	250.0000	43.1077	170.5224	7350.8206

Figure G.1

Sample Program Output[1]

---

[1] Baseline hazard rate and cost inputs for fan drive turbine; Core Age = 10 hours.

APPENDIX H

CONDITIONAL PROBABILITIES

APPENDIX H

CONDITIONAL PROBABILITIES

Chapter IV made extensive use of conditional probabilities of the form $q_i(fc)$, $q_i(\bar{f}c)$ and $q_i(f\bar{c})$. The reader will recall, for instance, that

$$q_i(fc) = \left[1 - QFAN(t_i \mid n)\right] \bullet \left[1 - QCORE(t_{i+1} \mid t_i)\right] \bullet QFAN(t_{i+1} \mid t_i) \bullet QCORE(t_{i+1} \mid t_i) \tag{H.1}$$

in the internal $n < t_i \leq N - \delta t$,

where

$q_i(fc)$ ≡ Probability that an engine requires field level replacement of the fan module and core module simultaneously in the i-th interval between n and N given survival of both until n, and

$QFAN(t_{k_{Fan}} \mid t_{j_{Fan}})$ ≡

Probability of fan failure between t_j and t_k given survival until t_j

$QCORE(t_{k_{core}} \mid t_{j_{core}})$ ≡

Probability of core failure between t_j and t_k given survival until t_j.

Let

$f_{Fan}(t)$ ≡ Fan module failure density function.

$f_{core}(t)$ ≡ Core module failure density function.

Then

$$\left[1 - \mathrm{QFAN}(t_i \mid n)\right] \bullet \left[\mathrm{QFAN}(t_{i+1} \mid t_i)\right] \qquad \text{(H.2)}$$

$$= \left[1 - \frac{\int_n^{t_i} f_{Fan}(t)\,dt}{1 - \int_0^n f_{Fan}(t)\,dt}\right] \bullet \left[\frac{\int_{t_i}^{t_{i+1}} f_{Fan}(t)\,dt}{1 - \int_0^{t_i} f_{Fan}(t)\,dt}\right],$$

which is equivalent to

$$\left[\frac{\left(1 - \int_0^n f_{Fan}(t)\,dt\right) - \int_n^{t_i} f_{Fan}(t)\,dt}{1 - \int_0^n f_{Fan}(t)\,dt}\right] \bullet$$

$$\left[\frac{\int_{t_i}^{t_{i+1}} f_{Fan}(t)\,dt}{1 - \int_0^{t_i} f_{Fan}(t)\,dt}\right],$$

which reduces to

$$\frac{\int_{t_i}^{t_{i+1}} f_{Fan}(t)\,dt}{1 - \int_0^n f_{Fan}(t)\,dt}\ .$$

Similarly,

$$\left[1 - \mathrm{QCORE}(t_i \mid n)\right] \bullet \left[\mathrm{QCORE}(t_{i+1} \mid t_i)\right]$$

$$= \frac{\int_{t_i}^{t_{i+1}} f_{core}(t)\,dt}{1 - \int_0^n f_{core}(t)\,dt}\ ,$$

and Equation (H.1) becomes

$$q_i(fc) = \frac{\left(\int_{t_i}^{t_{i+1}} f_{Fan}(t)\,dt\right)\cdot\left(\int_{t_i}^{t_{i+1}} f_{core}(t)\,dt\right)}{\left(1-\int_0^n f_{Fan}(t)\,dt\right)\cdot\left(1-\int_0^n f_{core}(t)\,dt\right)} \qquad (H.3)$$

Equation (H.3) represents the probability of FAN/CORE system failure in the interval (t_i, t_{i+1}) given survival until n. The devisor in Equation (H.3) insures that the probability of failure over the interval (n, ∞) is equal to 1. A geometric interpretation of the conditional probabilities in Equation (H.3) is shown in Figure H.1. The PDF has a spike at N where mandatory removal occurs. $q_i(\bar{f}c)$ and $q_i(f\bar{c})$ are developed analogously with the exception that one module survives through the interval (t_i, t_{i+1}) while the other fails in it.

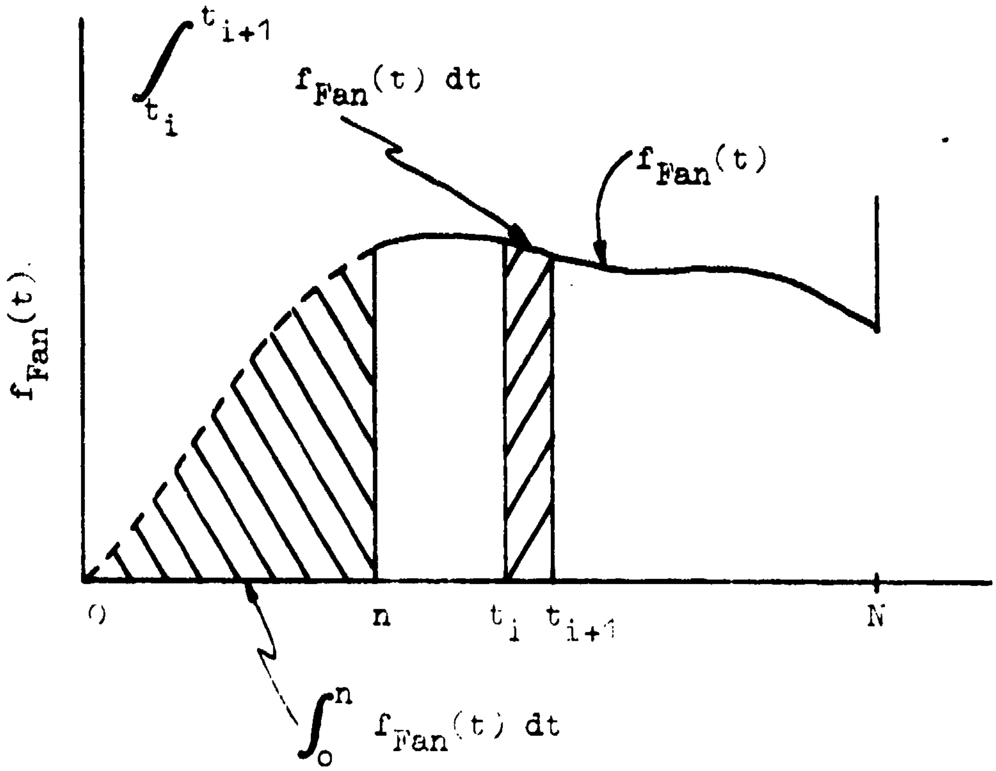

Figure H.1

Geometric Interpretation of Conditional Probabilities in Equation (H.3)

SELECTED BIBLIOGRAPHY

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A. References Cited

1. Anthony, Robert N. Management Accounting Principles. Revised Ed., Homewood, Illinois: Richard D. Irwin, Inc., 1970.

2. ARINC Research Corporation. A Study of the Air Force Jet Engine Maintenance Program. May 1970.

3. Ballou, Alfred, Captain, USAF. Avionics Intermediate Shop Logistics Manager, F-15 Joint Test Force. Personal interview. Edwards Air Force Base, California, 25 April 1975.

4. Barlow, Richard E., Frank Proscham, and Larry C. Hunter. Mathematical Theory of Reliability. New York: John Wiley & Sons, Inc., 1967.

5. Davis, Dempsey, Lt. Colonel, USAF. Chief, F-15 Logistics Evaluation and Support Team, F-15 Joint Test Force. Personal interview. Edwards Air Force Base, California, 25 April 1975.

6. Duncan, John, and Lawrence S. Schalnick. "Interrupt and Opportunistic Replacement Strategies for Systems and Deteriorating Components," Operational Research Quarterly, XXIV (June 1973), pp. 271-283.

7. Harruff, Thomas R. Deputy Chief, Directorate of Propulsion and Auxiliary Power Systems, Headquarters Air Force Logistics Command. Personal interviews. Wright-Patterson Air Force Base, Ohio, conducted intermittently from September 1974 through July 1975.

8. Henry, Pat. "Powering the Eagle: An Overview of the P&WA F-100 Engine Development and Status," McDonnell Aircraft Company Product Support Digest, XXI (3rd Quarter, 1974), pp. 2-4.

9. Johnson, E. A., Lt. Colonel, USAF. Chief, F-100 Engine Project Office, F-15 Systems Program Office. Personal interviews. Wright-Patterson Air Force Base, Ohio, conducted intermittently from September 1974 through July 1975.

10. Jorgenson, D. W., Jr. J. McCall, and R. Radner. *Optimal Maintenance of Stochastically Failing Equipment*. Santa Monica: The RAND Corporation, 1966.

11. McDonnell Aircraft Company. *F-15 Eagle: Maximum Availability--Minimum Maintenance*, PS 838, 10 September, 1973, pp. 42-51.

12. Metzler, Tom, Captain, USAF. Chief, Human Factors Test Office, F-15 Joint Test Force. Personal interview. Edwards Air Force Base, California, 25 April 1975.

13. Muckstadt, John A. "A Model for a Multi-Item, Multi-Echelon, Multi-Indenture Inventory System," *Management Science*, XX (December 1973), pp. 472-481.

14. Panazza, Anthony J., and Don F. Woods. "Fighter Aircraft Engine Dependability Versus Reconnaissance Aircraft Engine Dependability When Both Aircraft Use the Same Engine." Unpublished Master's Thesis, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson Air Force Base, Ohio, 1974.

15. Papoulis, A. *Probability, Random Variables and Stochastic Processes*. New York: McGraw-Hill, 1965.

16. Pratt and Whitney Aircraft Company. Briefing presented to Air Force Association 1973 Annual National Convention, Washington, D. C., September 1973.

17. ________. *F-100-PW-100 Qualitative, Quantitative, Personnel Requirements Information (QQPRI)*. PWA FR-4870, 1 March 1972.

18. ________. *Intermediate Maintenance Instructions USAF Model YF-100-PW-100 Aircraft Engine*. Development Program Manual DPM 2J-F-100-6, 1 February 1972.

19. Reynolds, David S., and I. Richard Savage. "Random Wear Models and Reliability Theory," *Advanced Applied Probability*, III (1971), pp. 229-248.

20. Rodrigues, Guadalupe, Logistics Management Specialist (Engine), F-15 Joint Test Force. Personal interview. Edwards Air Force Base, California, 25 April 1975.

21. Roll, Y., and P. Naor. "Preventive Maintenance of Equipment Subject to Continuous Deterioration and Stochastic Failure," Operational Research Quarterly, XVIV (March 1968), pp. 61-71.

22. Samuelson, Paul A. Economics. 9th Edition, New York: McGraw-Hill Book Company, 1973.

23. Selby, Samuel M. ed. Standard Mathematical Tables. 22nd Edition, Cleveland, Ohio: CRC Press, Inc., 1974.

24. Shooman, Martin L. Probabilistic Reliability: An Engineering Approach. New York: McGraw-Hill Book Company, 1968.

25. Silvaman, Gloria, Rate Clerk, Office of Chief of Transportation. Telephone interview. Kelly Air Force Base, Texas, July 7, 1975.

26. Steppenback, Clifford, Engineering Planning Section, Technological Repair Center. Telephone interview. San Antonio Air Logistics Center, Texas, January 27, 1975.

27. Stough, F. E., Chief, Product Support Division, Florida Research and Development Center, Pratt and Whitney Aircraft Company. Telephone interview. West Palm Beach, Florida, February 21, 1975.

28. Taliaferro, Richard T. "Economic Analysis of Public Programs." Unpublished AFIT Text used in QS 5.32, Department of Quantitative Studies, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson Air Force Base, Ohio, 1973.

29. U. S. Department of the Air Force. Maintenance Management. AF Manual 66-1. Volume I. Washington, D. C.: Government Printing Office, 1974.

30. ________. Maintenance Instructions: Engines and Engine Controls, Instrument, and Fire Protection Systems; USAF Series F-15-A 73-085 Aircraft and Up, TF-15-A 73-108 Aircraft and Up. AF Technical Order 1F-15A-2-9. Washington, D. C.: Government Printing Office, 15 November 1974.

31. ________. Minimum Engine Operating Hours for Economy in Continued Field Repair. AF Technical Order 2J-1-31. Washington, D. C.: Government Printing Office, 1972.

32. ________. Minor Overhaul of Gas Turbine Engines. AF Technical Order 2J-1-27. Washington, D. C.: Government Printing Office, 1970.

33. ________. Procedures for Determining Aircraft Engine (Propulsion Unit) Failure Rates, Actuarial Engine Life, and Forecasting Monthly Engine Changes by the Actuarial Method. AF Technical Order 00-25-128. Washington, D. C.: Government Printing Office, 1972.

34. ________. Selected Management of Propulsion Systems. AF Manual 400-1. Volume I (Draft). Undated.

35. Wagner, Harvey M. Principles of Management Science. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1970.

36. Weibull, Waloddi. "A Statistical Distribution Function of Wide Applicability." Journal of Applied Mechanics, September 1951, pp. 293-297.

37. Willis, Raymond E., and Normal L. Chervany. Statistical Analysis and Modeling for Management Decision—Making. Belmont, California: Wadsworth Publishing Company, Inc., 1974.

38. Woodman, R. C. "Replacement Policies for Components that Deteriorate." Operational Research Quarterly, XVIII (September 1967), pp. 267-270.

B. Related Sources

Campbell, William Giles, and Stephen Vaughn Ballou. Form and Style: Theses, Reports, Term Papers. 4th Edition, Boston: Houghton Mifflin Company, 1974.

Drennen, J. E., R. A. Yereance (Ydeas), and J. L. Easterday. Reliability Program Management Guide. Technical Report, AFAL-TR-73-285, Air Force Avionics Laboratory (AFSC), Wright-Patterson Air Force Base, Ohio, 1971.

Fishman, George A. Military and Economic Consequences of Alternative Spare Engine Policies. Memorandum RM-4474-PR, Santa Monica: The RAND Corporation, March 1965.

Kuhl, Red L. "Analysis of Factors Influencing Spare Engine Management." Unpublished Master's Thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson Air Force Base, Ohio, 1970.

Kruse, Karl W. DoD Aircraft Engine Requirements Study. Institute for Logistics Research, United States Army Logistics Management Center, March 1972.

Munford, A. G., and A. K. Shahani. "A Nearly Optimal Inspection Policy." Operational Research Quarterly, XXIII (September 1972), pp. 270-275.

Research and Communicative Studies Department. "Style and Guidelines Manual for Theses and Technical Reports." Unpublished manual, (Revised May 1974), School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson Air Force Base, Ohio, 1974.

Wilson, Michael George. "A Stochastic Repairable Item Inventory Model." Unpublished Master's Thesis, United States Naval Postgraduate School, Monterey California, 1971.