FORMULATING A PILOT MODEL FOR ENERGY IN RELATION TO THE NATIONAL ECONOMY

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Summary:

This dynamic linear-programming model, on a pilot scale is an attempt
to describe in physical terms many of the technological interactions within
and across the sectors of the economy, including a detailed energy sector.
The general objective of the model is to determine, in the face of changing
energy picture, what the country could achieve in physical terms over the
long term, say 30 years. Preliminary work on the pilot model indicates that
it can be completed within six months and that several useful scenarios can
be run during the ensuing six months. Work on implementation is being done
in close collaboration with Shailendra C. Parikh who prepared the block
diagram of the model found at the end of this paper.

In the model, a 35-sector input-output matrix represents the technology
of various industrial processes of the economy. The net output from the
industry, together with net imports, meets the national bill of goods for
consumption, capital formation and government services. The energy demands
of the economy are met by the activities of the energy sector. The nature
and extent of the capacity expansion in both the energy sector and the rest
of the economy are endogenously determined. Finally, the exogenously given
workforce provides the manpower necessary to sustain industrial production,
energy processing and capacity expansion.

The detailed energy sector in the model includes technological description
of the raw material extraction and energy conversion processes. Oil
and gas exploration and production, oil refining, gas transmission, coal
mining, power generation using coal, oil and gas, and coal gasification
and liquefaction are among the fossil fuel based processes in the model.
Uranium mining, conversion, enrichment and fabrication, light water reactor,
fast breeder reactor, and spent fuel reprocessor are among the nuclear fuel
based processes in the model. The operating levels of the processing units
are limited in one way or the other by the available capacities and proven
reserves in any period. The proven reserves may be augmented by the exploration activity. And, raw material imports/exports make up the difference between the domestic production and usage.

Among the linkages that interconnect the energy sector and the rest of the economy are: energy demands of the economy, total manpower available to all sectors (including energy), favorable balance-of-payments, and bill-of-goods needed for energy processing and capacity expansion.

The model assumes as given some temporal pattern of population growth, workforce availability, and requirements for government services. Initial capacities of various processes are also required. Outputs from the model are a schedule of capacity expansion and the consumption standards achieved.

This descriptive model could be used in conjunction with a linear or a nonlinear objective. The objective could be the maximization of gross national consumption measured in real flows achieved over time. Based on discussions with Robert Wilson, John Shoven and Lars Mathiesen, a promising way has been found to relate a physical flow model with demand functions which depend on monetary prices. The objective could also be to minimize dependence on foreign oil, or to maximize energy output, or to maximize employment. Our intent is to develop on a pilot scale a reasonably accurate general description of the American Economy and more detailed description of the Energy Sector in order to facilitate studies of the physical potential of the economy under (i) alternative objectives, (ii) changing availability of various forms of energy, (iii) changing energy conversion technologies, etc.

Our purpose therefore is to formulate a dynamic planning model for the National Economy including a detailed Energy Sector, all expressed in terms of physical flows and then to develop a system of prices and money flows consistent with it. The general objective of the model is to maximize "gross national consumption" for the population from now into the future.

The model assumes as given some pattern of population growth with perhaps a leveling off of population between the year 2000 and 2050. This growth reflects births, deaths, immigration and emigration. The model could be used to study how energy strategy is effected by changes in population.
Part I: The Physical Flow Model

Physical Flows:

The National Economy and the detailed Energy Sector will be represented in terms of physical flows so as to avoid as much as possible distortions due to price changes and inflation. The input-output coefficient \( a_{ij} \) of an activity (industry) \( j \) is interpreted as the required physical flow of input of product \( i \) per unit physical flow of output of \( j \). The practical question, however, is this: How does one obtain such a coefficient? Let us suppose for the automotive Industry \( (j) \), the input-output coefficients for a given base year (say 1967) are available expressed in dollars. This industry produces a number of different types of vehicles and its total output \( A \), expressed in base-year dollars, is the sum

\[
A = \sum_{i=1}^{k} p_i v_i
\]

where \( p_i \) is the price of the \( i \)-th type vehicle produced and \( v_i \) the number of such vehicles produced. In this case the actual physical flows are the \( v_i \). For the National Economy, industries (like the automotive sector) will, in general, be a heterogeneous aggregate. Therefore some scheme is necessary to combine the physical units. It is proposed that we simply use the base-year prices \( p_i \) to weight the physical units into an aggregate total. In any future year after the base year, we use these same factors, \( p_i \), interpreting them, however, as weights and not as prices. The implicit assumption is that in some future period \( t \), the aggregated amount of production \( A \cdot x_j(t) \) for the automotive industry is made up, in detail, of physical amounts

\[
v_1 x_j(t), v_2 x_j(t), \ldots, v_k x_j(t),
\]

i.e., the same proportional breakdown of physical flows as the base year. A price \( p_t \) for the aggregated automotive unit in period \( t \) is interpreted to mean that the breakdown of the new price is
i.e., the same proportional breakdown of prices as in the base year. For
the base year \( x_t = 1 \) and \( x_j(t) = 1 \). The input coefficient \( a_{jk} \), the
purchases of industry \( k \) from (say) the automotive industry \( j \) will, in
fact, have a different proportionate breakdown of vehicles but it is assumed,
on the average, when weighted over all \( k \), that the proportionate breakdown
is the same as that for the base year.

Maximizing "Gross National Consumption":

The approach used in this section is based on discussions with
Robert Wilson, John Shoven, and Lars Mathiesen. The objective of the
model, broadly speaking, is to maximize the vector sum of the bill-of-
goods received per person summed over time. The time horizon, \( T \), will
be the year 2000 (or perhaps the year 2050). If the undiscounted sum above
leads to solutions in which people are very poor in the immediate future
in order to be overly rich later on, then discount factors \( \lambda_t \) (or some
equivalent device) will be used to obtain alternative solutions. To avoid
end effects at time \( T \), a steady growth factor \( g \) will be assumed from \( T \)
to \( \infty \), i.e., the inventory-capacity state at end of time \( T+1 \) divided by \( g \)
is made the same as time \( T \). This means that in the objective function the
gross national consumption for period \( T \) receives a higher weight \( \lambda_{T+1} \)
relative to other periods.

Suppose in the base year that the physical bill-of-goods for people
with consumption level \( M_k \) (income less taxes and savings in base-year
dollars) is

\[
(4) \quad b^k = \begin{pmatrix}
    b_{1k} \\
    b_{2k} \\
    \vdots \\
    b_{nk}
\end{pmatrix}, \quad M_1 < M_2 < \ldots < M_k, \\
k = (1, 2, \ldots, K).
\]

Let \( u_k^t \) be the number of people in period \( t \) that receive \( b^k \). Then
(5) \[-u_0^t + u_1^t + u_2^t + \ldots + u_K^t = P(t),\]
(6) \[\text{Work Force}(t) + u_0^t - u_1^t - u_2^t - \ldots - u_K^t \leq 0,\]

where \(P(t)\) is the population at time \(t\) and \(u_0^t\) measures the number who receive more than that specified by their consumer level. The total bill-of-goods for time \(t\) is

(7) \[b_1^t u_1^t + b_2^t u_2^t + \ldots + b_K^t u_K^t = b(t).\]

and the gross national consumption in period \(t\) is

(8) \[M_1^t u_1^t + M_2^t u_2^t + \ldots + M_K^t u_K^t = \text{GNC}(t).\]

The overall objective could be to maximize gross national consumption discounted over time, i.e.,

(9) \[
\max_{t=1}^{T+1} \sum_{t=1}^{T+1} \lambda_t \text{GNC}(t),
\]

where \(\lambda_t\) is a discount factor and \(\lambda_{T+1}\) is a special weight for period \(T+1\).

The demand distribution \(b_k^t\) for a fixed consumption level \(M_k\), of course, depends on prices \(\pi_k\) where \(\sum_k b_{ik} = M_k\) are derived by solving the linear program. Jorgenson and Lau in their paper "The Structure of Consumer Preferences" (November 1974) have investigated various functional relations for fixed incomes. The system consisting of the negative of the logarithm of the direct utility function and the indirect demand functions is dual to the system consisting of the logarithm of the indirect utility function and the direct demand functions. They suggest as an approximation of the latter
Assuming the price on labor constraint (6) is the same as the population constraint (5), yields

\[
\sum_j x_{ij} b_{ik} = M_k.
\]

Such demand functions, if available, will be used to modify the assumed \( b_{ik} \). It can be shown because of relation (11), that the introduction of alternative columns with higher utility within a consumption class \( M_k \) will not effect \( x_i \). A generalized linear programming approach thus can be used to maximize (as a secondary objective) utility within the consumer classes while holding gross national consumption at a maximum.

In a similar manner production functions, if available, can be used to generate alternative inputs per unit of output which are more profitable to the producer. It is easy to prove that the introduction of the latter into the basis also raises the gross national consumption.

The general objective thus is to maximize the gross national consumption over time with adjustment of consumer expenditures within an income class to price changes.

Equations:

There are \( i = (1, \ldots, n) \) equations corresponding to \( n \) industries, the last two \( n-1 \) and \( n \) are electric and non-electric power. Equation \( n+1 \) will be used to calculate labor requirements. Equation \( n+2 \) will be used to impose a general-capacity constraint over all capacity expansion activities. There are also additional constraints for the detailed energy sector and there are equations relating capacity in one period with that of the next period by industry.

Capacity (Expansion and Depreciation):

Except for the energy area, capacity of an industry \( j \) in period \( t \) will be denoted by the variable \( l_j(t) \). To adjust for depreciation (and obsolescence), the amount of capacity carried over from period \( t \) to \( (t+1) \) will be the discounted amount:

\[
\alpha_i = \sum_j \beta_{ij}, \quad \beta_{ij} = \sum_j \beta_{ij}.
\]
where $0 < d_j \leq 1$ can depend on $t$. Capacity in period $t$ can be increased by a capacity expanding activity whose inputs are a vector of consumer items $j = (1, \ldots, n-2)$ plus labor $(j = n+1)$ plus the use of certain tools produced by a general-capacity expanding industry, $j = n+2$. The aggregation of tools used constitute the amount of the capacity utilized from the general capacity expanding industry. This industry also can use part of its capacity for its own expansion. Industrial activity $X_j(t)$ in general must satisfy

$$X_j(t) \leq \xi_j(t).$$

The same considerations hold for the Energy Sector except they are applied in detail to all the alternative ways to generate processed energy, and to the supporting activities including those that affect the availability of unprocessed energy stock: (crude oil, uranium ore, etc.).

**Government Requirements:**

These are assumed as exogeneously given by time period in terms of physical flows of consumer goods and government employment.

**Export-Imports:**

The quantity of exports and imports depends on prices and on short or long term balance of payments. Unfortunately we do not know these prices until we introduce money and these depend on how much (in physical terms) is exported or imported. We propose an iterative process for determination. Initially we take historical trends of costs. Later on corrected prices for commodities as determined by the model could be used in its place. Based on historical trends and base year dollars let $V_i(e_i)$ denote the expected dollar sales if a quantity $e_i$ in physical units of commodity $i$ is exported. Similarly let $W_i(f_i)$ denote the expected dollar costs if a quantity $f_i$ of commodity $i$ is imported. Assuming a favorable balance of payments,

$$\Sigma V_i(e_i) \geq \Sigma W_i(f_i),$$

for each period $t$ (or perhaps less restrictively over a period of several years).
The amount of sales $V_i(e_i)$ from exports of commodity $i$ can be expected to be a concave function of $e_i$ while the costs $W_i(f_i)$ of import of commodity $i$ can be expected to be a convex function of $f_i$. [If estimates for these functions are not available, then the sum of prices in the base year for commodity $i$, times quantities imported (or exported) in period $t$ could be used to estimate the balance of payments.]

**Determining Production Levels to Maximize "Gross National Consumption":**

Our purpose now will be to summarize the mathematical system of relations discussed so far. Later we will introduce a system of prices, profits, investments and monetary flows consistent with maximum utility. Let:

- $X = X^t$ be the vector of levels of industrial production of consumer items in period $t$.
- $E = E^t$ be the vector of expansion of capacity of the industries. In order to simplify the discussion, new capacity will be assumed to become available one period after the inputs required to build the new capacity. In practice these inputs may be required several time periods earlier.

Let

$$(GOV) = (GOV)^t$$

$$(IMP) = (IMP)^t$$

$$(EXP) = (EXP)^t$$

$$(DEM) = (DEM)^t$$

Then for each period:

$$(I-A)X - B^tE - (GOV)^t - (EXP)^t + (IMP)^t = (DEM)^t$$

where $L = I-A$ is a Leontief Matrix for the base year (interpreted for future years as physical flows). The last two columns of $(I-A)$ will be replaced later by several alternative ways to generate electric and non-electric power. $B$ is the matrix of inputs for the capacity expansion.
industries. The latter contains \((n-2) + k + 1\) columns because there are \(n-2\) columns corresponding to the expansion of each industry \(j = (1, ..., n-2)\), \(k\) columns corresponding to expansion of the detailed energy sector and one column corresponding to the expansion of the general-capacity expansion industry.

Total labor requirements are given by:

\[
(I-2) \quad A^y_{n+1} X + B^E_{n+1} E + (GOV)_{n+1} = (DEM)_{n+1}
\]

and capacity limitations are given by

\[
(I-3) \begin{align*}
X_j &\leq \ell_j, & \text{for } j = (1, ..., n-2) \\
Y_j &\leq \ell_{nj}, & \text{for } j = (1, ..., k) \\
B_{n+2} E &\leq \ell_{n+k-1}
\end{align*}
\]

where \(Y_j\) are the detailed activities of the energy sector, and \(\ell_{n+k-1}\) is the capacity of the general-capacity expansion industry.

Restrictions on import-exports take the form of favorable balance of payments (expressed initially in base-year dollars and prices) for each year (or perhaps less restrictively over a period of several years)

\[
(I-4) \quad \Sigma V_i(EXP_i) \geq \Sigma W_i(IMF_i)
\]

where \(V_i\) are concave functions and \(W_i\) are convex functions of the amounts exported and imported.

The total "gross national consumption" associated with period \(t\) is

\[
(I-5) \quad \text{CNC}(t) = M_1 u_1^t + M_2 u_2^t + \cdots + M_K u_K^t
\]

where the total population is

\[
(I-6) \quad P(t) = u_1^t + u_2^t + \cdots + u_K^t, \quad u_i^t \geq 0
\]

and the total bill of goods is

\[
(I-7) \quad b(t) = b_1^t u_1^t + \cdots + b_K^t u_K^t
\]
where \( b^k \) is the bill of goods of income-class \( M_k \) can change as a function of prices as described in the earlier section "Maximizing Gross National Consumption", see equations (4) and (9).

The various periods are interrelated by the carrying of capacity from one period to the next,

\[
0 = \xi^1 + E^1 - \xi^2
\]
\[
0 = + 5 \cdot \xi^2 + E^2 - \xi^3
\]
\[
0 = - 5 \cdot \xi^3 + E^3 - \xi^4
\]
\[
(1-8)
\]
\[
0 = 5 \cdot \xi^T + E^T - \xi^{T+1}
\]

where \( \delta \) is a diagonal matrix of the rate of depreciation of capacity from one period to the next. The general objective of the physical flow model is to maximize total "gross national consumption":

\[
(1-9) \quad \text{Max} \sum_{t=1}^{T+1} \lambda_t \text{GNC}(t)
\]

The Detailed Energy Sector:

For any period the energy sector is expanded to account for the alternative ways to produce electric and non-electric power. Therefore the \( [I-A] \) term in (1) is replaced by

\[
\left[ \begin{array}{cc}
I_{n-2} - A_{11} & - \bar{A}_{21} \\
- A_{21} & I - \bar{A}_{22}
\end{array} \right] \left[ \begin{array}{c}
\bar{x}_N \\
\bar{x}_E
\end{array} \right]
\]

where the upper left refers to the non-energy sector. The substitute columns are denoted by bar symbols. \( \bar{I} \) means that output \( I \) associated
with columns of a substitution class all appear in the same row. \( \mathbf{X}_E \) now represents the detailed breakdown of levels of activities of the energy sector summarized in terms of types of processed energy, e.g., hydroelectric, electricity from coal, synthetic gasoline, atomic power, etc. Let \( \mathbf{Y} \) be the vector of activity levels of the detailed energy model:

\[
\mathbf{D\mathbf{Y}} = \mathbf{X}_E, \quad Y \geq 0, \ D \geq 0
\]

(1-10)

\[
\mathbf{C\mathbf{Y}} = \mathbf{b}
\]

\[
Y_j \leq \varepsilon_{n-2+j}, \quad j = (1, 2, \ldots, k)
\]

where \( D \geq 0 \) aggregates the detailed activities of the energy sector into the total amounts of hydroelectric energy generated, coal-electric generated, etc., which we have denoted by \( \mathbf{X}_E \).

**Broken Line Fits to Concave Functions such as the Import-Export Exchange Functions:**

A well known technique can be used for fitting the favorable balance of trade relations in any period (or groups of periods):

\[
\sum_{i=1}^{n} V_i(\text{EXP}_i) + \sum_{i=1}^{n} [-W_i(\text{IMP}_i)] \geq 0.
\]

Note that each term of the left hand side above is a concave function of its arguments. Since it is also separable, we can make a broken line fit for each commodity:

- **Revenues (base year prices)**
- **NET IMPORT of COMMODITY i**
- **NET EXPORT of COMMODITY i**
- **Purchases (base year prices)**

\( s_1 > s_2 > \cdots > s_k > \cdots \)
Let \( s_1 > s_2 > \ldots \) be the successive decreasing slopes of the broken line segments and \( \alpha_1, \alpha_2, \ldots \) the widths of the projections of the segments.

It is not difficult to show that the introduction of auxiliary variables \( v_k \) and the replacement of net imports of item \( i \) and net revenues (base year prices) by

\[
\text{Net IMPORTS}_i = \text{IMP}_i + v_1 + v_2 + \ldots + v_k + \ldots, \quad 0 \leq v_k \leq \alpha_k
\]

\[
\text{Net REVENUES}_i = \text{REV}_i + s_1 v_1 + s_2 v_2 + \ldots + s_k v_k + \ldots, \quad s_{k-1} < s_k < s_k
\]

where \((\text{IMP}_i, \text{REV}_i)\) are the coordinates of the first point on the curve, will always underestimate the net revenues, except if the increments \( v_k \) are chosen so as to maximize \( v_1 \) first, then \( v_2, \ldots \). Any feasible solution to the approximating linear program will therefore have the property that the corresponding non-linear program will also have a favorable balance of trade using base-year prices. Our procedure will be to iteratively correct these prices with the endogenously determined prices associated with the optimum solution of the linear program. [In a more sophisticated version complementary pivot theory could be used instead of iterations to obtain the equilibrium solution.]

Part II: Prices and the Monetary Flow Model

From Part I: The Physical Flow Model, we determine for any given period \( t \) the levels \( X = X^t \) of activity of the consumer item industries and services. We also determine \( \theta P = \theta^t P^t \) which is that part of the population (which we will refer to as \( P_1 \)) directly employed by industry or by government. The remaining part \((1-\theta)P\) (which we will refer to as \( P_2 \)) consists of those working for government, schools, research, welfare, those living on interest, principal or dividends. In Part II we set up a linear program for determining optimal monetary policy.

From the physical model we can determine the vector bill of goods \( B \) per person. We can also determine a vector \( B_{n+1} E_j \), the bill of goods of consumer items and labor requirements \( B_{n+1} E_j \) of each capacity expansion industry; also \( G \) the vector of Government demands + Export-Import demands for consumer items, and \( G_{n+1} \) the government requirements for labor. Also known is INT interest on Savings and LI and LO which are the repayments.
of interest and principal by industry and government on loans contracted in periods prior to \( t \). The latter are in monetary units, all others are in physical units.

The solution obtained in Part I for detailed energy sector is summarized into inputs of consumer goods for electric power \((j = n-1)\) and for non-electric power industries \((j = n)\) and for the corresponding capacity expansion industries \(E_{n-1}, E_n\). This implicitly reconstitutes the Leontief substitution matrix into a square matrix \( L = I - A \) where the last two columns reflect the proper proportions of the detailed energy sector inputs.

The Table (page 12) gives the monetary flows in the economy by activity (columns) for major categories of expenditure (rows). These net-out to zero by row and column. The unknowns in the Table are:

\[ \begin{align*}
\pi &: \text{prices for consumer goods (vector)} \\
\pi_{n+1} &: \text{wage rates} \\
p &: \text{profit rates for consumer industries (vector)} \\
q &: \text{profit rates for the capacity expansion industries (vector)} \\
k &: \text{tax rate on profits} \\
t_{n+1} &: \text{tax rate per person} \\
s &: \text{saving rate on personal income} \\
e &: \text{investment rate on consumer industries (vector)} \\
I &: \text{investments by the private sector (vector)} \\
GE &: \text{government expenses to meet its own requirements, subsidy of construction and equipment purchases} \\
GS &: \text{government subsidies of salaries of schools, research, welfare, health, etc.} \\
LI &: \text{(Vector) payback by industry and LG payback by government of loans made in periods prior to } t \text{. LI, LG and INI are assumed known at time } t \text{ but would be treated as unknowns if we are optimizing overtime. INT is interest on saving.} \\
BG &: \text{loans to government in period } t.
\end{align*} \]

It is assumed (in constructing the Table) that the general-capacity expansion activity is dropped and is indirectly accounted for by increasing the input coefficients \( E_{1j} \) for each \( j = 1, \ldots, n \).
<table>
<thead>
<tr>
<th>Activities</th>
<th>Consumer Industries J = (1, 2, ..., n)</th>
<th>Private Sector P1</th>
<th>Private Sector P2</th>
<th>Capacity Expansion Industries J = (1, 2, ..., n)</th>
<th>Government Demands + Exports - Imports</th>
<th>Government Returns to Private Sector P2 + Banks</th>
<th>Bank Loans to Gov't Interest on Deposit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumer Item</strong></td>
<td>Inter-Industry Transactions</td>
<td>Consumer Item Purchases</td>
<td>Consumer Item Purchases</td>
<td>Consumer Item Purchases</td>
<td>Net Consumer Item Purchases</td>
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</tr>
<tr>
<td>1 = (1, 2, ..., n)</td>
<td>$X_j$</td>
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<tr>
<td><strong>Income Private Sector P1</strong></td>
<td>Payments to Labor</td>
<td>Income of P1</td>
<td></td>
<td>Payments to Labor</td>
<td>Payments to Labor</td>
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<td>Interest to P</td>
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<td></td>
<td>$P_{n+1}^{1} A_{n+1, j} X_j$</td>
<td>$P_{n+1}^{1} \theta P$</td>
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<td>$P_{n+1}^{1} \theta P$</td>
<td>$P_{n+1}^{1} \theta P$</td>
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<td><strong>Government</strong></td>
<td>Taxes</td>
<td>Taxes</td>
<td>Taxes</td>
<td>Taxes</td>
<td>Government Exp. for the above</td>
<td>Government Borrowing</td>
<td></td>
</tr>
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<td></td>
<td>$-P_{j} X_j$</td>
<td>$-P_{j} \theta P$</td>
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<tr>
<td><strong>Income Private Sector P2 + * Banks and Other Investors</strong></td>
<td>Profits + Payback on Loans</td>
<td>Savings</td>
<td></td>
<td>Private Investment</td>
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<td>$P_{j} X_j - L_{j}$</td>
<td>$P_{j} \theta P$</td>
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<tr>
<td><strong>Industrial Investment J</strong></td>
<td>Investment in new capacity</td>
<td></td>
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<td>Industrial Investment</td>
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</tbody>
</table>

**Known quantities:**
- Input-Output Coefficients, $L = (I-A)$, $A_{n+1}$
- Population $P$
- Private Sectors $\theta P$, $(1-\theta) P$
- Consumer Bill of Goods, $\theta P$
- Bill of Goods for Capacity Expansion $B_{i} F_j$
- Labor $B_{n+1,i} F_j$
- Government Bill of Goods + Exports-Imports and Labor $G$, $C_{n+1}$
- Interest + Principal LI, LG, INT paid by banks to P1 and P2.

**Unknowns:**
- Prices $(e, e_{n+1})$
- Profit rates $(p, q)$
- Investment rates $e$
- Tax rates $t_{n+1}$
- on profits, $s$ saving rate, $\Delta$ government surplus.
We will now show that there are some freedoms in monetary policy. For example suppose we arbitrarily fix $b$, the vector of the gross incomes of the consumer industries greater than $L_I$ and fix the tax rate per person in the private sectors. Then the relations

$$\pi[L_{ij} X_j] - \pi_{n+1}[A_{n+1,j} X_j] = b,$$

$$L = (I-A)$$

$$-\pi[\beta_j \theta P] + \pi_{n+1}(1-\theta) P = (t_{n+1}-\text{INT}) \cdot P$$

will yield $(\pi, \pi_{n+1}) > 0$ because of the properties of Leontief matrices unless the saving rate or $\text{INT}$ is too high. From the relationship

$$GE = \pi G + \pi_{n+1} G_{n+1}$$

$$GS = \pi_{n+1}(1-\theta) P$$

we obtain $GE > 0$, $GS > 0$. From the relationships

$$e_j X_j + (k+1) p_j X_j = b_j - LI_j$$

$$e_j X_j - (k+1) q_j E_j + I_j = \sum_i \pi B_{ij} E_j + \pi_{n+1} B_{n+1,j} E_j$$

we can determine $e_j$ and $p_j$ under the assumption that industry $j$ will want to maximize its investment in order to avoid taxes on profits and future payments of interest to the private sector for investments made by the private sector $P_2$. This is done by solving the two equations as a linear program in unknowns $e_j \geq 0$, $p_j \geq 0$, $q_j \geq 0$, $I_j \geq 0$ with the objective to maximize $e_j$. This will determine $e_j$ and $p_j$ and the sum $I_j - (k+1)q_j E_j$. The profit rate $q_j$ for construction can now be arbitrarily chosen and $I_j$ determined.

All terms in the table have now been determined except the breakdown of the sum: $\Delta - BG$. If positive, $\Delta$ is the government surplus which is shown as loaned to banks for investment. If negative, $BG$ is bank loans to government to cover the government deficit.

This completes the discussion that there is some freedom in choosing a monetary system consistent with the optimal physical flows.
Inflation:

The presence of required loan repayments, LI and LG, means that inspite of non-unique choice of prices, it is not possible, in general, (because of the condition \( b > LI \)) to deflate the arbitrarily chosen gross incomes for industry and PI by a deflator \( p \) so that

\[
p \pi b^* = \pi_{\text{base year}} b^*
\]

where \( b^* \) is some standard bill-of-goods (bread basket) used to weight prices in year \( t \) in order to develop a comparative index with base-year prices. (By our earlier discussion in Part I, the prices of each consumer item in the base year is unity.)

Determining the Choice of Prices, Investments, Profits, Taxes by Means of a Linear Program:

Note that the relations between these non-negative unknowns, as displayed in the Table, are linear once the physical flows are determined as discussed in Part I. Moreover one period is linearly linked to the next by the payback of interest and principal on loans. A linear program could thus be set up and solved over the time horizon \( (1, 2, \ldots, T) \). The objective of the linear program could be the minimization, for example, of interest on loans. Relations for each period, similar to the one above, could be included to see if the value of money could be maintained the same as in the base-year. If some inflation is necessary, then the smallest inflation rate could be determined.

We have thus, obtained our objective. We have outlined a procedure in Part I for activity in Industry and in particular the detailed activities of the Energy Sector in terms of physical flows. In Part II we have outlined how to determine a monetary system and optimal amounts of investment necessary to achieve the levels of activities specified in Part I.

However, computing prices, wages, taxes, investments subsidies, interest and loans in this way may not reflect properly the short run dynamic inertias, of the economy. It is suggested therefore that the physical flows of Part I and perhaps some data from Part II be used as the target trajectory for a detailed economic simulation.
EXHIBITS

PILOT: A Model Integrating Total Economy and Energy

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