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UTILIZATION OF THE BEND TEST FOR DETERMINING TENSILE PROPERTIES OF A BRITTLE MATERIAL

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ABSTRACT

A limited number of 'end as well as tension tests were performed at room temperature on specimens made of Norton HS-130 grade silicon nitride. By application of the two-parameter Weibull analysis for a material governed by volumetric flaw distribution, tensile properties of the specimen, based upon the data of both types of testing, were determined and compared. The results show, at least for the specimens tested, that the bend test tends to predict fracture stresses of the tension specimens approximately 8% higher than those obtained in the actual tension tests. (Author)

CONTENTS

I. INTRODUCTION

Interest in the bend test has gained considerable attention in recent years because of the greater use of high strength materials with little ductility and because of the development and exploitation of such brittle materials as ceramics and carbides.

If tensile properties of a material are sought, it is only natural to think of manufacturing and testing a tension specimen made of the material in question. Because of size limitations, however, even when the material is easily machinable, the task of manufacturing a tension specimen sometimes becomes impossible. On the other hand, even when sufficient material is available, the cost of manufacturing a tension specimen may become prohibitive as, for example, machining a dumbbell-shaped tension specimen made of an extremely brittle material as those mentioned above. In such cases, a bend test with its major advantage of employing a simple specimen with a rectangular cross section becomes a welcome substitute provided, of course, that such a test does yield reliable predicted tensile results.

It has been shown by Nadai[1] that it is theoretically possible to apply a bend test to determine the tensile and compressive stress-strain curves of a material, and experimental verification of this hypothesis has been accomplished with 4340 steel heat treated to various strength levels.[2] Results indicate that close agreement exists, at least to strains to 1-1/2 to 2%, between the stress-strain curves predicted from the bend tests and those determined from the actual tension and compression tests.

In order to include the gamut of material variation, i.e., ductile to brittle, bend specimens were designed and tested according to Reference 2 and tension specimens were designed and tested essentially according to Reference 3. Sufficient expressions have been derived and presented from which properties of tension specimens can be predicted from bend test results. Predicted and actual properties of tension specimens were subsequently obtained and compared.

In determining fracture stresses of ceramic specimens, recourse was made to a statistical approach, and in this study the Weibull statistical theory of fracture,[4] the most widely accepted theory, was chosen. This theory uses two basic criteria of failure: size and normal stress. In the Weibull three-parameter analysis, fracture is predicted in terms of the three-material parameters: zero probability strength, flaw density exponent, and a scale parameter. In the Weibull two-parameter analysis, on the other hand, the first of these material parameters is assumed to be zero, and fracture is predicted in terms of the two remaining parameters.

1. NADAI, A. *Plasticity*. McGraw-Hill Book Company, Inc., New York, 1931.
2. CAMPO, J. *Utilization of the Bend Test for Determining Stress-Strain Curves*. Army Materials and Mechanics Research Center Technical Report, AMMRC TR 71-13, July 1971.
3. DRISCOLL, G. W., and BARATTA, F. I. *Modifications to an Axial Tension Tester for Brittle Materials*. Army Materials and Mechanics Research Center Product Technical Report, AMMRC PTR 71-3, August 1971.
4. WEIBULL, W. *A Statistical Theory of the Strength of Materials*. Ingeniors Vetenskaps Akadamien-Handlingar, no. 151, 1939.

II. PROBABILITY OF FRACTURE

Three-Parameter Analysis

For a stress field in a homogeneous isotropic material governed by volumetric flaw distribution, the probability of fracture at a given stress σ is given:[5]

$$P_f = \begin{cases} 1 - \exp\left[-\int_V \left(\frac{\sigma - \sigma_u}{\sigma_o} \right)^m dV \right] = 1 - \frac{1}{e^R}, & \sigma \geq \sigma_u \\ 0, & \sigma < \sigma_u \end{cases} \tag{1}$$

where

$$R = \int_V \left(\frac{\sigma - \sigma_u}{\sigma_o} \right)^m dV \tag{2}$$

is the risk of rupture, and

σ_u = zero probability strength (strength below which there is no fracture)

m = Weibull modulus or flaw density exponent

σ_o = scale parameter.

The last three values are material parameters only.

Two-Parameter Analysis

If it is assumed that $\sigma_u = 0$ (and certainly there can be no fracture at zero stress level), then Equations (1) and (2) become:

$$P_f = 1 - \exp\left[-\int_V \left(\frac{\sigma}{\sigma_o} \right)^m dV \right] = 1 - \frac{1}{e^{R'}} \tag{3}$$

where

$$R' = \int_V \left(\frac{\sigma}{\sigma_o} \right)^m dV \tag{4}$$

is the modified risk of rupture.

Application of the two-parameter analysis yielded values of material parameters that described the test data very well and the results of this analysis are herein reported.

5. WEIL, N. A., and DANIEL, I. M. *Analysis of Fracture Probabilities in Nonuniformly Stressed Brittle Materials.* Journal of the American Ceramic Society, v. 47, June 1964, p. 268-74.

III. MODIFIED RISK OF RUPTURE

It should be apparent that the value of modified risk of rupture of a specimen — in terms of maximum tensile stress, for example, within the specimen — is the result of integration of Equation (4) throughout the total volume. In addition, if the stress throughout the specimen is not constant, this integration may become rather cumbersome.

Generally, however, a specimen is so designed that the greatest risk of rupture occurs in the middle or gage length section, and, therefore, the value of modified risk of rupture determined by integrating throughout the volume of the gage length section will be only negligibly smaller than that determined by integrating throughout the total volume.

Equation (4) may always be expressed as:

$$R' = KV \left(\frac{\sigma_{max}}{\sigma_o} \right)^m \tag{5}$$

where

K = a load factor determined by carrying out portions or all of the integration indicated in Equation (4), $K = K(m)$.

V = volume of all or gage length section only of the specimen, as selected

σ_{max} = maximum stress (tensile in this study) in specimen

σ_o, m = material parameters.

Shown below are listed values of R' for the cases pertinent to this study (see Appendixes A and B for actual derivations):

a. Third-Point Loading Bend Specimen

1. Total volume considered

$$R' = \frac{V_{b_{total}} (m+3)}{6(m+1)^2} \left(\frac{\sigma_b}{\sigma_o} \right)^m \tag{6}$$

where in Equation (5) K is taken as

$K = K_b = \dfrac{(m+3)}{6(m+1)^2}$ and subscript b refers to bend specimen.

2. Only volume of gage length section considered

$$R' = \frac{V_{b_{GL}}}{2(m+1)} \left(\frac{\sigma_b}{\sigma_o} \right)^m \tag{7}$$

where in Equation (5) K is taken as

$K = K_b = \frac{1}{2(m+1)}$ and again subscript b refers to bend specimen

and where

$V_{b_{total}}$ = total volume of bend specimen

$V_{b_{GL}}$ = volume of gage length section of bend specimen

σ_b = maximum tensile stress in bend specimen

= tensile stress in outer fiber in gage length section of bend specimen.

b. Dumbbell-Shaped Tension Sp

1. Total volume considered

$$R' = K_t \, V_{t_{GL}} \left(\frac{\sigma_t}{\sigma_o} \right)^m \tag{8}$$

where in Equation (5) K is taken as

$K = K_t$ = a function of m (see Table 3 for pertinent values of K_t) and subscript refers to tension specimen.

2. Only volume of gage length section considered

$$R' = V_{t_{GL}} \left(\frac{\sigma_t}{\sigma_o} \right)^m \tag{9}$$

where in Equation (5) K is taken as

$K = K_t = 1.0$ and again subscript t refers to tension specimen

and where

$V_{t_{GL}}$ = volume of only gage length section of tension specimen

σ_t = maximum tensile stress in tension specimen

= tensile stress in gage length section of specimen.

IV. WEIBULL MODULUS

According to Reference 6, the value of m, Weibull modulus or flaw density exponent, may be determined by use of the expression listed below:

$$\frac{\Gamma\left(1+\frac{1}{m}\right)}{\left[\Gamma\left(1+\frac{2}{m}\right)-\Gamma^2\left(1+\frac{1}{m}\right)\right]^{\frac{1}{2}}} = \frac{\bar{\sigma}_{fract}}{S_{\bar{\sigma}_{fract}}} \qquad (10)$$

where

$\bar{\sigma}_{fract}$ = mean fracture stress

$$= \frac{1}{N}\sum_{n=1}^{N}\sigma_{fract}$$

$S_{\sigma_{fract}}$ = standard deviation of mean fracture stress

$$= \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\sigma_{fract}-\bar{\sigma}_{fract}\right)^2}$$

Γ = gamma function

and

σ_{fract} = individual fracture stress

= $\sigma_{b_{fract}}$ for a bend specimen

= $\sigma_{t_{fract}}$ for a tension specimen.

As may be seen, the value of m is determined from test results.

For all bend and tension specimens in this work the fracture stresses were recorded, and these values are listed in Tables 1 and 2. Details of the actual test systems, procedures, and methods of computation, are contained in References 2 and 3.

6. LENOE, E. M. Army Materials and Mechanics Research Center, unpublished research.

However, it will be noted here that fracture stress in bending is defined as the maximum tensile stress in the gage length section at fracture, and these values were determined from:

$$\sigma_{b_{fract}} = \frac{M_{max} c}{I} \tag{11}$$

where

M_{max} = maximum bending moment at fracture

c = 1/2 depth of specimen

I = moment of inertia of cross section of specimen.

Fracture stress in the dumbbell-shaped tension specimen is also defined as the maximum tensile stress in the gage length section at fracture and these values were determined from:

$$\sigma_{t_{fract}} = k \; p_{i_{fract}} \; \frac{r_o^2 - r_i^2}{r_i^2} \tag{12}$$

where

k = constant of calibration = 0.97

$p_{i_{fract}}$ = internal pressure at fracture

r_o = outside (largest) diameter of specimen

r_i = inside (smallest) diameter of specimen.

Finally, Equation (10) was used to determine the values of m for all bend as well as tension tests, and these values are shown in Table 3.

V. SCALE PARAMETER

The value of σ_o, the scale parameter, may be determined as indicated by:[6]

$$\sigma_o = \frac{\bar{\sigma}_{fract} \, (KV)^{1/m}}{\Gamma\left(1 + \frac{1}{m}\right)} \tag{13}$$

where all terms have already been defined.

For the specimens involved in this work, Equation (13) becomes:

a. Third-Point Loading Bend Specimens

1. Total volumes considered

$$\sigma_o = \frac{\bar{\sigma}_{b_{fract}} \left[\frac{V_{b_{total}} (m+3)}{6(m+1)^2} \right]^{1/m}}{\Gamma \left(1+\frac{1}{m} \right)} \tag{14}$$

2. Only volumes of gage length sections considered

$$\sigma_o = \frac{\bar{\sigma}_{b_{fract}} \left[\frac{V_{b_{GL}}}{2(m+1)} \right]^{1/m}}{\Gamma \left(1+\frac{1}{m} \right)} \tag{15}$$

b. Dumbbell-Shaped Tension Specimens

1. Total volumes considered

$$\sigma_o = \frac{\bar{\sigma}_{t_{fract}} \left[K_t \, V_{t_{GL}} \right]^{1/m}}{\Gamma \left(1+\frac{1}{m} \right)} \tag{16}$$

where values of K_t are those shown in Table 3.

2. Only volumes of gage length sections considered

$$\sigma_o = \frac{\bar{\sigma}_{t_{fract}} \left(V_{t_{GL}} \right)^{1/m}}{\Gamma \left(1+\frac{1}{m} \right)} \tag{17}$$

Values of σ_o, determined by Equations (14) and (15) for bend specimens and by Equations (16) and (17) for tension specimens, are also listed in Table 3.

VI. RELATIONSHIPS BETWEEN BEND AND TENSILE STRESSES

The relationships between bend and tensile stresses depend upon the expressions representing the values of modified risks of rupture. By equating the expressions for modified risks of rupture for either the total or gage length volumes the following relationships are obtained:

a. Total Volumes of Specimens Considered

Equating the value of R' shown in Equation (6) to that shown in Equation (8) leads to:

$$\sigma_t = \sigma_b \left[\frac{(m+3)\ V_{b_{total}}}{6(m+1)^2\ K_t\ V_{t_{GL}}} \right]^{1/m} \tag{18}$$

where values of K_t again are those shown in Table 3.

b. Only Volumes of Gage Length Sections Considered

Equating the value of R' shown in Equation (7) to that shown in Equation (9) leads to:

$$\sigma_t = \sigma_b \left[\frac{V_{b_{GL}}}{2(m+1)\ V_{t_{GL}}} \right]^{1/m} \tag{19}$$

Note that in both expressions the values of σ_o factors are out.

VII. DISCUSSION AND RESULTS

For convenience, the individual fracture stresses have been presented for all test specimens, both bend and tension. The bend test values have been presented in three groups of 8, 17, and 19, as shown in Table 1, and the tension test values in two groups of 6 and 11, as shown in Table 2. In either type of testing, the smaller number of test results is also included in the larger number.

Based on these experimentally determined values of fracture stresses and the expressions presented in the text, various properties have been determined and shown in Table 3. These properties include mean fracture stresses, standard deviations, coefficients of variance, load factors, Weibull moduli, and scale parameters. The Weibull moduli and scale parameters were determined both by expressions in which consideration was given to total volumes as well as expressions in which consideration was given to volumes of only gage length sections of the specimens.

Values of mean fracture stresses of tension specimens – both those determined from the actual tension tests as well as those predicted from the bend tests – have been listed in Table 4. Percentage discrepancies, i.e., measures of disagreement between predicted and actual mean fracture stresses, are also shown in this table.

Table 1. BEND TEST RESULTS OF NORTON HS-130 GRADE SILICON NITRIDE SPECIMENS TESTED AT ROOM TEMPERATURES

Fractures Near or Under Rollers Omitted		Fractures Under Rollers Omitted		All Fractures Included	
n	$\sigma_{b\,fract}$ (psi)	n	$\sigma_{b\,fract}$ (psi)	n	$\sigma_{b\,fract}$ (psi)
1	74,167	1	72,222	1	72,222
2	82,778	2	74,167	2	74,167
3	95,277	3	82,778	3	82,778
4	95,556	4	85,111	4	85,111
5	97,222	5	95,277	5	88,889
6	100,000	6	95,556	6	88,889
7	101,389	7	95,833	7	95,277
8	106,667	8	96,444	8	95,556
		9	97,222	9	95,833
		10	98,889	10	96,484
		11	100,000	11	97,222
		12	100,722	12	98,889
		13	101,389	13	100,000
		14	103,194	14	100,722
		15	106,667	15	101,389
		16	107,611	16	103,194
		17	109,444	17	106,667
				18	107,611
				19	109,444

Note: All fractures occurred in gage length sections only.

n = number of specimen when fracture stresses are listed in ascending order of magnitude.

$\sigma_{b\,fract}$ = fracture stress of bend specimen
= maximum tensile stress (in outer fiber of gage length section) of bend specimen at fracture.

Table 2. TENSION TEST RESULTS OF NORTON HS-130 GRADE SILICON NITRIDE SPECIMENS TESTED AT ROOM TEMPERATURE

Fractures in Gage Length Sections Only		All Fractures (Inside and Outside Gage Length Sections)	
n	$\sigma_{t\,fract}$ (psi)	n	$\sigma_{t\,fract}$ (psi)
1	62,880	1	61,820
2	63,030	2	62,880
3	65,460	3	63,030
4	73,540	4	63,620
5	76,770	5	65,390
6	78,400	6	65,460
		7	69,820
		8	73,540
		9	76,770
		10	78,400
		11	85,410

Legend

n = number of specimen when fracture stresses are listed in ascending order of magnitude.

$\sigma_{t\,fract}$ = fracture stress of tension specimen
= maximum tensile stress (in gage length section) of tensile specimen at fracture.

Table 3. VALUES OF MEAN FRACTURE STRESSES, STANDARD DEVIATIONS, COEFFICIENTS OF VARIANCE, WEIBULL MODULI, LOAD FACTORS, AND SCALE PARAMETERS FOR NORTON HS-130 GRADE SILICON NITRIDE SPECIMENS TESTED AT ROOM TEMPERATURE.

						Based on Total Volumes			Based on Volumes of Only Gage Length Section		
Type of Test	Number of Tests	$\bar{\sigma}_{fract}$ (psi)	$S_{\bar{\sigma}_{fract}}$ (psi)	$V_{\bar{\sigma}_{fract}}$ ( )	m	K_b	K_t	σ_0 (psi)	K_b	K_t	σ_0 (psi)
Bend	8	94,132	9,903	10.52	11.57	0.01544	1.2319	61,137	0.03994	1.0	60,356
	17	95,443	10,542	11.05	10.94	.01623	1.2386	60,868	.03864	1.0	60,013
	19	94,753	10,173	10.74	11.27	.01580	1.2348	61,072	.04075	1.0	60,259
Tension	6	70,013	6,440	9.20	13.27	.01332	1.2146	58,679	.03504	1.0	57,823
	11	69,649	7,479	10.73	11.29	.01577	1.2345	56,626	.04068	1.0	55,579

Legend

$\bar{\sigma}_{fract}$ = mean fracture stress

$S_{\bar{\sigma}_{fract}}$ = standard deviation of mean fracture stress

$V_{\bar{\sigma}_{fract}}$ = coefficient of variance of mean fracture stress = $\dfrac{S_{\bar{\sigma}_{fract}}}{\bar{\sigma}_{fract}} \times 100$ — See Equation (10)

m = Weibull modulus or flaw parameter

K_b = load parameter of bend specimen

= $\dfrac{m+3}{6(m+1)}$ when total volumes are considered, see Equation (A19)

= $\dfrac{1}{2(m+1)}$ when volumes of only gage length sections are considered, see Equation (21)

K_t = load factor of tension specimen

= values indicated in Equation (B14) when total volumes are considered.

= 1.0 when volumes of only gage length sections are considered, see Equation (B11)

σ_0 = scale parameter, see Equations (14) through (17)

Plots of modified risk of rupture of tension specimens versus maximum tensile stress are shown in Figure 1, and plots of probability of fracture of tension specimens versus maximum tensile stress are shown in Figures 2 and 3. These figures offer a means of comparison of tensile properties predicted from bend tests to those determined from actual tension tests. In addition, Figure 1 shows the effects of consideration of volumes of gage length sections in lieu of total volumes of specimens; Figure 2 shows how well a fit exists between the predicted and experimental probabilities of fracture values, i.e., how well the predicted values fit the data; and both Figures 1 and 3 show the effects of a number of tests on the validity of test data.

In Section III only the expressions for determining values of modified risks of rupture have been listed, but the actual derivations of these expressions are shown in the appendixes.

Although it was planned to test 20 each of both bend as well as tension specimens, it should be noted that 19 bend but only 11 tension test results are reported. Extreme difficulty in machining the tension specimens is the primary cause for this difference. Some tension specimens broke during machining and never could be tested. Those that were completed had machining lines in the

Table 4. VALUES OF MEAN FRACTURE STRESSES OF TENSION SPECIMENS DETERMINED BY TENSION TESTS, MEAN FRACTURE STRESSES OF TENSION SPECIMENS DETERMINED BY BEND TESTS, AND PERCENTAGE DISCREPANCIES FOR NORTON HS-130 GRADE SILICON NITRIDE SPECIMENS TESTED AT ROOM TEMPERATURE.

			Volumes of Only Gage Length Sections Considered		Total Volumes Considered	
Type of Test	Number of Tests	$\bar{\sigma}_{t_{fract_{act}}}$ (psi)	$\bar{\sigma}_{t_{fract_{pred}}}$ (psi)	Discrepancy* ( )	$\bar{\sigma}_{t_{fract_{pred}}}$ (psi)	Discrepancy· ( )
Tension	6	70,013				
	11	69,649				
Bend	8		75,287	7.5	74,894	7.5
	17		75,765	8.2	75,356	8.2
	19		75,444	7.9	75,144	7.9

*Discrepancy between predicted value and actual value of 70,013 psi.
·Discrepancy between predicted value and actual value of 69,649 psi.

Legend

$\bar{\sigma}_{t_{fract_{act}}}$ = actual mean fracture stress of tension specimens determined from tension test results (Equation (12)).

$\bar{\sigma}_{t_{fract_{pred}}}$ = predicted mean fracture stress of tension specimens determined from bend test results. (Equation (18) when total volumes are considered and Equation (19) when volumes of only gage length sections are considered.)

Discrepancy = . disagreement between actual and predicted mean fracture stresses of tension specimens.

$$\frac{\bar{\sigma}_{t_{fract_{pred}}} - \bar{\sigma}_{t_{fract_{act}}}}{\bar{\sigma}_{t_{fract_{act}}}} \times 100$$

circumferential direction and the finishes were poor. Some of these were tested and broke below the expected tensile strength. It was theorized that the low strength values were due to direction and degree of surface finish. Recourse was made to machine lapping each of the remaining tension specimens to a 4 rms finish in the longitudinal direction, the same direction, and degree of surface finish as any of the bend specimens. The effect of this operation was to increase the tensile strength substantially. The only bend test result not reported was one in which premature failure occurred because strain gage wires were inadvertently placed between the specimen and one of the steel rollers (load surfaces) of the test fixture. Needless to add, the cost of machining the tension specimens was unusually high.

An important question arises concerning differences in results obtained from expressions involving total volumes to those involving volumes of only gage length sections. The answer to this question may be observed from the plots of modified risks of rupture versus stress level based on both types of expressions as shown in Figure 1. Since for any stress level the lower the value of modified risk of rupture, the lower the value of probability of failure and, conversely, the greater the chances of survival; a plot of this type is useful for comparison purposes. For the results herein reported, it may be seen that little or no differences exist between values of modified risks of rupture determined from

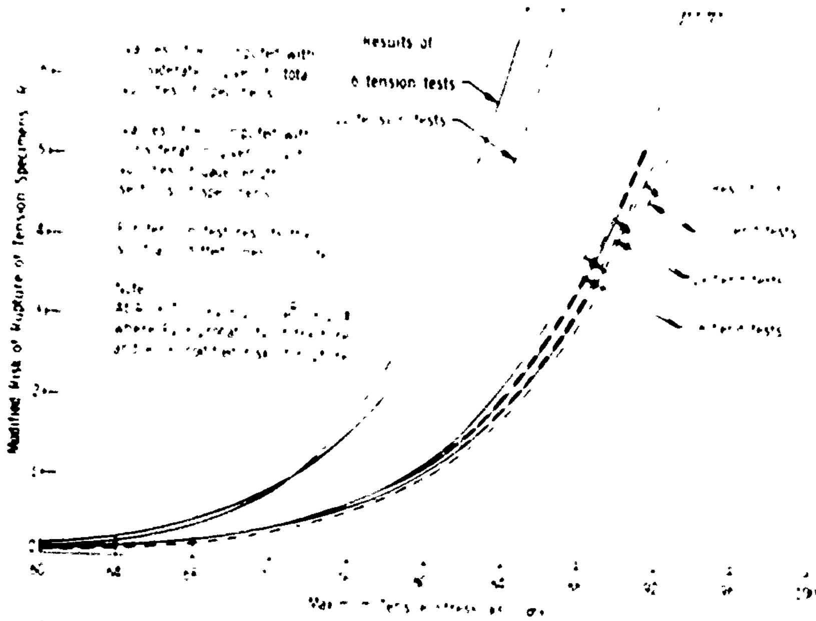


Figure 1. Modified risk of rupture of tension specimens versus maximum tensile stress of Norton HS-130 grade silicon nitride specimens tested at room temperature.

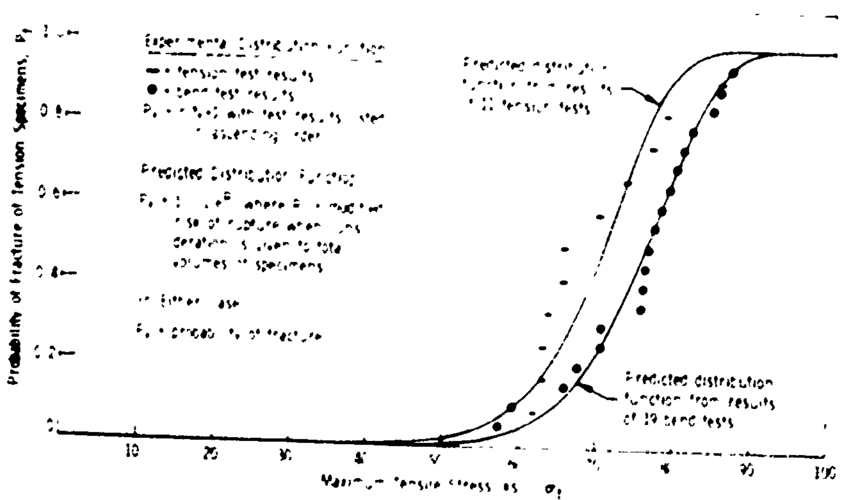


Figure 2. Probability of fracture of tensile specimens versus maximum tensile stress of Norton HS-130 grade silicon nitride specimens tested at room temperature.

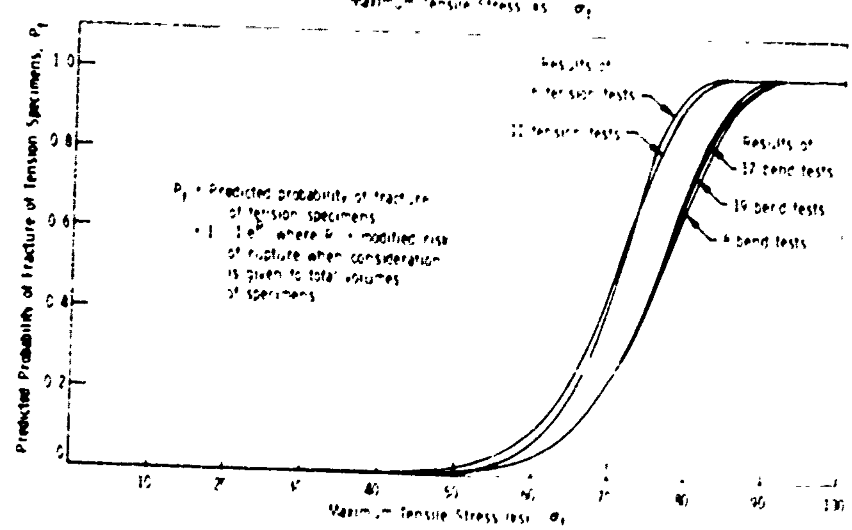


Figure 3. Predicted probability of fracture of tension specimens versus maximum tensile stress of Norton HS-130 grade silicon nitride specimens tested at room temperature.

either set of expressions, i.e., those involving total volumes and those involving volumes of only gage length sections. Where the slight differences do exist (bend test values only), the results derived when consideration is given to total volumes are more conservative, i.e., at any given stress level, the probability of failure is greater. When there is any doubt, therefore, as to whether or not the contributions to the value of modified risk of rupture of a specimen due to the omitted sections really are negligible, the expressions involving total volumes should be used.

Another important question arises concerning the number of tests that should be made before valid data can be expected, and this question may be answered with the help of Figures 1 and 3. The spread between the plots in either group (results based on both bend as well as tension tests) indicates differences due to the number of tests selected while the spread between groups of plots indicates differences between values predicted from bend tests and those determined from tension tests, i.e., how well the bend test replaces the tension test. The latter will be discussed shortly.

From Figure 1, plots of modified risk of rupture versus maximum tensile stress within the tension specimen, it may be seen that considerable spread does exist when the number of tension tests is increased from 6 to 11 and when the number of bend tests is increased from 8 to 17 or from 8 to 19. What is more important is that considerable spread (in the reverse direction in this case) exists even when a relatively high number of tests is increased by only 2 more, from 17 to 19.

These spreads are to be expected, however, since R', modified risk of rupture, really depends upon m (see Equations (8) and (9) where K_t and σ_0 are both functions of m), and m in turn depends upon $\bar{\sigma}_{fract}$ and $S_{\bar{\sigma}_{fract}}$ [see Equation (10)]. These last two terms are experimentally determined, and it should be obvious that the smaller the number of tests involved in determining these terms the greater will be the effect on their values when one or more test results, either excessively high or low, are added.

Fortunately, however, the indicated spreads in the probability of fracture plots of Figure 3 are not too bothersome. Only 3% disagreement exists when the tension tests are increased from 6 to 11 or when the bend tests are increased from 8 to 17. Even less disagreement exists when the bend tests are increased from 8 to 19 or from 17 to 19. The reason for these close agreements is due to the insensitivity of the value of modified risk of rupture on the value of probability of fracture as indicated in Equation (3).

To answer the question of required number of tests for valid data, the greater the number of tests, the more valid are the results likely to be, and certainly 20 tests are not too many.

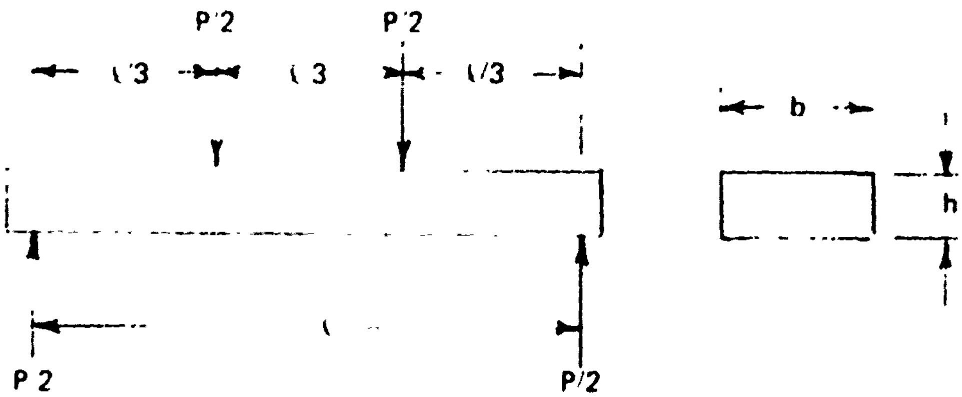
One final question remains concerning substitution of the bend test for the tension test. As mentioned earlier, the spreads between groups of plots of Figures 1 and 3 are indications of the answer to this question. The smaller the spread in any one group of plots, the closer the agreement. In addition, the percentage discrepancies between mean fracture stresses predicted from bend tests to those determined from tension tests, indicated in Table 4, also bring out the answer. It may be seen from these values that the bend test may be substituted for the tension test, at least for the specimens tested, within an accuracy of about 8%.

APPENDIX A. DERIVATION OF MODIFIED RISK OF RUPTURE FOR THIRD-POINT LOADING BEND TEST SPECIMENS

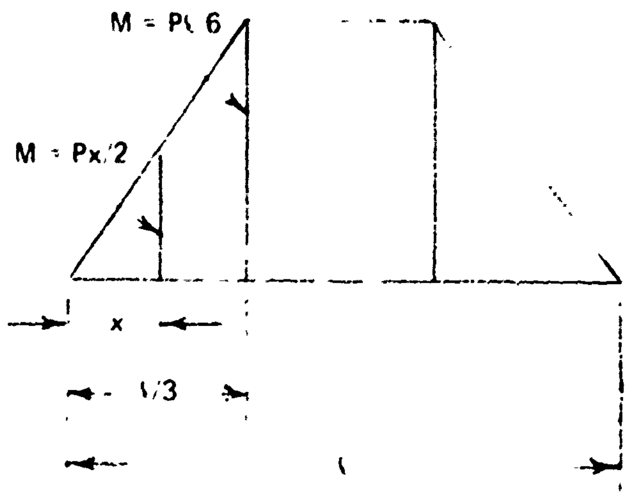
Determination of the value of modified risk of rupture may be made by carrying out the integration indicated by Equation (4) of the text which is repeated here:

$$R' = \int_V \left(\frac{\sigma}{\sigma_0} \right)^m dV \qquad \text{(A1)}$$

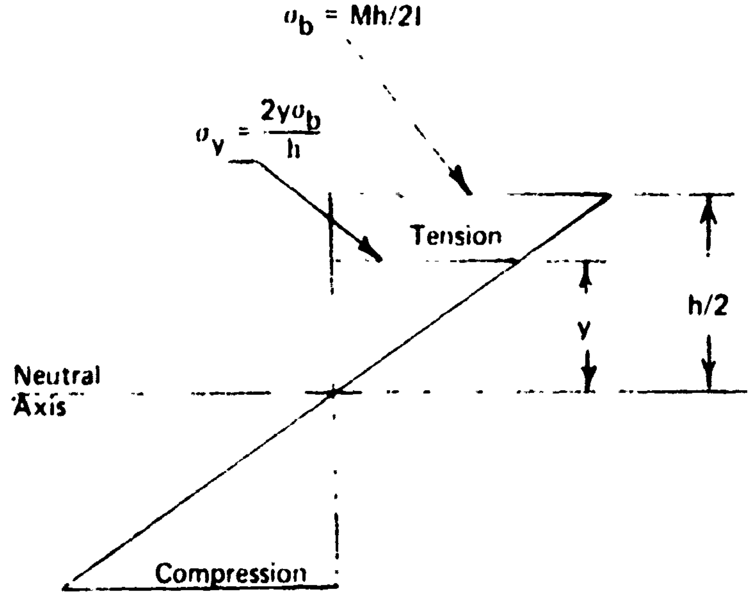
Diagrams indicating type of loading and distribution of both moment and stress will help in carrying out the indicated integration:



THIRD-POINT LOADING BEND TEST



MOMENT DISTRIBUTION



STRESS DISTRIBUTION

In the gage length section (middle third of specimen in this case)

$$M = \frac{Px}{2} = \frac{P}{2}\frac{\ell}{3} = \frac{P\ell}{6} \tag{A2}$$

$$\sigma_b = \frac{Mh}{2I} = \frac{P\ell}{6}\frac{h}{2I} = \frac{Ph\ell}{12I} \tag{A3}$$

$$\sigma_b = \frac{Ph\ell}{12}\frac{12}{bh^3} = \frac{P\ell}{bh^2} \tag{A4}$$

$$\sigma_y = \frac{2y\ \sigma_b}{h} \tag{A5}$$

and in the end sections,

$$M = \frac{Px}{2} \tag{A6}$$

$$\sigma_b = \frac{Mh}{2I} = \frac{Px}{2}\frac{h}{2I} = \frac{Phx}{4I} \tag{A7}$$

$$\sigma_b = \frac{Phx}{4}\frac{12}{bh^3} = \frac{3Px}{bh^2} \tag{A8}$$

$$\sigma_y = \frac{2y}{h}\frac{3Px}{bh^2} = \frac{6Pxy}{bh^3} \tag{A9}$$

Carrying out the integration throughout the total volume of the specimen leads to:

$$R' = b\left[\int_0^{\ell/3}\int_0^{h/2}\left(\frac{2y\ \sigma_b}{h\ \sigma_o}\right)^m dydx + 2\int_0^{\ell/3}\int_0^{h/2}\left(\frac{6Pxy}{bh^3\sigma_o}\right)^m dydx\right] \tag{A10}$$

where σ_b = maximum tensile fiber stress (in gage length section) of specimen

or

$$\sigma_b = \frac{3P}{bh^2}\,\frac{\ell}{3} = \frac{P\ell}{bh^2}.$$

Continuing with the integration leads to:

$$R' = b\left[\left(\frac{2\sigma_b}{h\sigma_o}\right)^m \int_0^{\ell/3}\int_0^{h/2} y^m\,dydx + (2)\left(\frac{6P}{bh^3\sigma_o}\right)^m \int_0^{\ell/3}\int_0^{h/2} x^m y^m\,dydx\right] \cdot \quad \text{(A11)}$$

$$R' = b\left[\frac{2^m}{h^m}\,\frac{\sigma_b^m}{\sigma_o^m}\,\frac{\ell}{3}\int_0^{h/2} y^m dy + \frac{2^{m+1}\;3^m\;P^m}{b^m\;h^{3m}\;\sigma_o^m}\,\frac{(\ell/3)^{m+1}}{m+1}\int_0^{h/2} y^m\,dy\right] \quad \text{(A12)}$$

$$R' = \frac{b\left(\frac{h}{2}\right)^{m+1}}{m+1}\left[\frac{2^m\,\sigma_b^m\,\ell}{3h^m\,\sigma_o^m} + \frac{2^{m+1}\;3^m\;P^m\;\ell^{m+1}}{3^{m+1}\;b^m\;h^{3m}\;\sigma_o^m\;(m+1)}\right] \quad \text{(A13)}$$

$$R' = \frac{b\;h^{m+1}}{2^{m+1}\;\sigma_o^m\;(m+1)}\left[\frac{2^m\;\sigma_b^m\;\ell}{3h^m} + \frac{2^{m+1}\;3^m\;P^m\;\ell\;\ell^m}{3^{m+1}\;b^m\;h^m\;h^{2m}\;(m+1)}\right] \quad \text{(A14)}$$

$$R' = \frac{b\;h^{m+1}}{2^{m+1}\;\sigma_o^m\;(m+1)}\left[\frac{2^m\;\sigma_b^m\;\ell}{3h^m} + \frac{2\;2^m\;\ell\;\sigma_b^m}{3h^m\;(m+1)}\right] \quad \text{(A15)}$$

$$R' = \frac{b\;\ell\;h}{(2)(3)(m+1)}\left(\frac{\sigma_b}{\sigma_o}\right)^m\left[1 + \frac{2}{m+1}\right] \quad \text{(A16)}$$

(Contribution from gage length section) (Contribution from the two ends)

$$R' = \frac{b\;\ell\;h}{6(m+1)}\left(\frac{\sigma_b}{\sigma_o}\right)^m\left[\frac{m+3}{m+1}\right] \quad \text{(A17)}$$

$$R' = \frac{V_{b_{total}}\;(m+3)}{6\,(m+1)^2}\left(\frac{\sigma_b}{\sigma_o}\right)^m \quad \text{(A18)}$$

and finally

$$R' = K_b V_{b_{total}} \left(\frac{\sigma_b}{\sigma_o} \right)^m \tag{A19}$$

where

$$K_b = \frac{m+3}{6(m+1)^2}.$$

And listing only the first half of the expression shown in Equation (A16) leads to that portion contributed to the value of R' from only the gage length section of the specimen:

$$R' = \frac{b \ell h}{(3)(2)(m+1)} \left(\frac{\sigma_b}{\sigma_o} \right)^m \tag{A20}$$

$$R' = \frac{V_{b_{GL}}}{2(m+1)} \left(\frac{\sigma_b}{\sigma_o} \right)^m \tag{A21}$$

or

$$R' = K_b V_{b_{GL}} \left(\frac{\sigma_b}{\sigma_o} \right)^m \tag{A22}$$

where

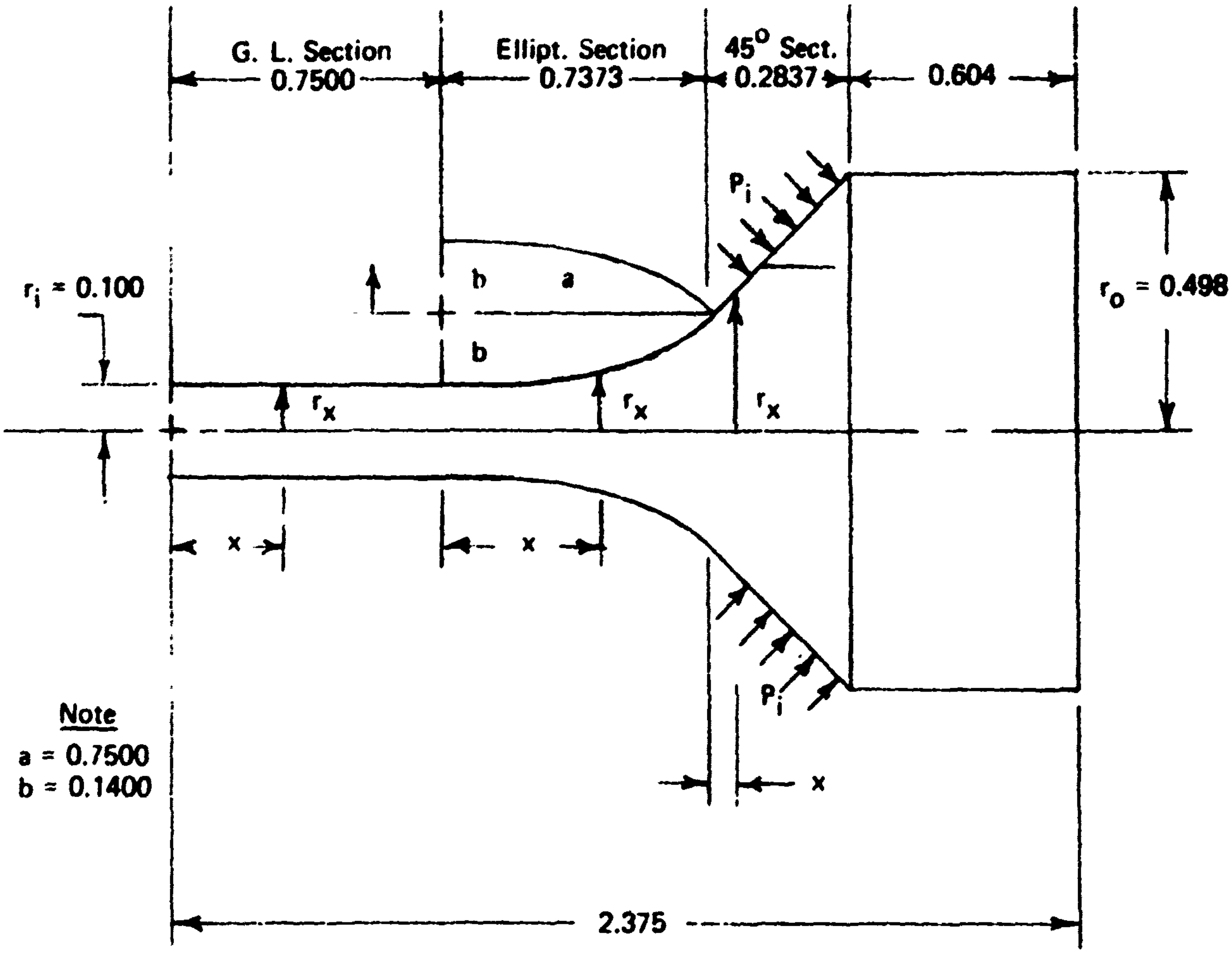
$$K_b = \frac{1}{2(m+1)}.$$

APPENDIX B. DERIVATION OF MODIFIED RISK OF RUPTURE FOR DUMBBELL-SHAPED TENSION TEST SPECIMENS

Determination of the modified risk of rupture of any specimen may be made by carrying out the integration indicated by Equation (4) of the text, and again this expression is repeated here:

$$R' = \int_V \left(\frac{\sigma}{\sigma_o}\right)^m dV \tag{B1}$$

A loading diagram and information concerning stress distribution that will help in carrying out the indicated integration are:



LOADING DIAGRAM

1. Gage Length Section

$$r_x = r_i = 0.100 \tag{B2}$$

$$\sigma_x = \sigma_t = \frac{(0.498^2 - 0.100^2)\ P_i}{0.100^2} = 23.80\ P_i \tag{B3}$$

2. Elliptical Section

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{0.75^2} = \frac{y^2}{0.14^2} = 1$$

$$r_x = 0.24 - 0.14 \sqrt{1 - \left(\frac{x}{0.75}\right)^2} \tag{B4}$$

$$\frac{\sigma_x}{23.80\ P_i} = \frac{0.01}{r_x^2}$$

$$\sigma_x = \frac{0.238\ P_i}{r_x^2} \tag{B5}$$

3. 45° Section

$$r_x = x + 0.2143 \tag{B6}$$

$$\sigma_x = \frac{0.238\ P_i}{r_x^2} \tag{B7}$$

With the above information, the integration may now be carried out in three parts:

$$R' = 2\pi \int_0^{0.7500} \left(\frac{\sigma_x}{\sigma_o}\right) r_x^2\, dx + 2\pi \int_0^{0.7573} \left(\frac{\sigma_x}{\sigma_o}\right)^m r_x^2\, dx + 2\pi \int_0^{0.2837} \left(\frac{\sigma_x}{\sigma_o}\right)^m r_x^2\, dx \tag{B8}$$

where the values of σ_x and r_x are those listed in the stress distribution information.

Carrying out the required integration leads to:

1. Contribution to R' from Gage Length Section

$$R' = (2\pi)\left(\frac{23.80P_i}{\sigma_o}\right)^m \int_0^{0.7500} (0.0100)\ dx \tag{B9}$$

$$R' = (0.015\pi)\left(\frac{23.80P_i}{\sigma_o}\right)^m \tag{B10}$$

or

$$R' = K_t\ V_{t_{GL}}\left(\frac{\sigma_t}{\sigma_o}\right)^m \tag{B11}$$

i.e., the value of $K_t = 1.0$ regardless of the value of m.

2. Contribution to R' from Elliptical Sections

$$R' = (2\pi)\left(0.238P_i\right)^m \int_0^{0.7373} r_x^{2-2m}\ dx \tag{B12}$$

$$R' = 0.015\pi\left(\frac{23.80P_i}{\sigma_o}\right)^m \left[\frac{(2)(0.238)^m}{(0.015)(23.80)^m}\right] \int_0^{0.7373} r_x^{2-2m}\ dx \tag{B13}$$

or

$$R' = (K_t)\left(V_{t_{GL}}\right)\left(\frac{\sigma_t}{\sigma_o}\right)^m \tag{B14}$$

and by numerical integration for the pertinent values of m the values of K_t are:

m	K_t
10.94	0.2386
11.27	.2348
11.29	.2345
11.52	.2319
13.27	.2146

3. Contributions to R' by 45° Sections

$$R' = (2\pi)\ (0.238)^m \left(\frac{P_i}{\sigma_o}\right)^m \int_0^{0.2837} (x + 0.2143)^{2-2m}\, dx \tag{B15}$$

$$R' = (0.015\pi) \left(\frac{23.80 P_i}{\sigma_o}\right)^m \left[\frac{(2)(0.238)^m}{(0.015)(23.80)^m} \int_0^{0.2837} (x + 0.2143)^{2-2m}\, dx\right] \tag{B16}$$

but

$$\int_0^{0.2837} (x + 0.2143)^{2-2m} = \left|\frac{x + 0.2143}{3-2m}\right|_0^{0.2837} \tag{B17}$$

$$\therefore\ R' = (0.015\pi) \left(\frac{23.80 P_i}{\sigma_o}\right)^m \left[\frac{(2)(0.238)^m}{(0.015)(23.80)^m}\right] \left|\frac{(x + 0.2143)^{3-2m}}{3-2m}\right|_0^{0.2837} \tag{B18}$$

$$R' = (0.015\pi) \left(\frac{23.80}{\sigma_o}\right)^m \left[\frac{(2)(0.238)^m}{(0.015)(23.80)^m}\right] \left[\frac{(0.498)^{3-2m}}{3-2m} - \frac{(0.2143)^{3-2m}}{3-2m}\right] \tag{B19}$$

$$R' = (0.015\pi) \left(\frac{23.80 P_i}{\sigma_o}\right)^m \left[\frac{2}{(0.015)(3-2m)}\right] \left[\frac{0.498^{3-2m} - 0.2143^{3-2m}}{100^m}\right] \tag{B20}$$

or

$$R' = (K_t) \left(V_{t_{GL}}\right) \left(\frac{\sigma_t}{\sigma_o}\right)^m \tag{B21}$$

where for the pertinent values of m the values of K_t are:

m	K_t	
10.94	3.974×10^{-9}	$\equiv 0$
11.27	2.322	$\equiv 0$
11.29	2.247	$\equiv 0$
11.52	1.547	$\equiv 0$
13.27	0.091	

Contribution to R' from Total Specimen

The values of R', then, for the total specimen is simply the sum of the values contributed by its parts, or

$$R' = K_t V_{t_{GL}} \left(\frac{\sigma_t}{\sigma_o} \right)^m \tag{B22}$$

where for the pertinent values of m the values of K_t are:

m	K_t
10.94	1 + 0.2386 = 1.2386
11.27	1 + .2348 = 1.2348
11.29	1 + .2345 = 1.2345
11.52	1 + .2319 = 1.2319
13.27	1 + .2146 = 1.2146