ANALYSIS OF AIRCRAFT EVASION STRATEGIES IN AIR-TO-AIR MISSILE EFFECTIVENESS MODELS

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G. Carpenter and M. Falco

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This report presents a new methodology with which to quantify missile effectiveness and aircraft vulnerability. The approach is a blend of applications of optimal control theory, stochastic learning theory, and simulation. This methodology permits an evaluation of aircraft evasive maneuvering and countermeasures deployment strategy as an integral part of the effectiveness/vulnerability measurements. The strategy determination is a form of feedback control policy based upon a discretized set of information thresholds in the relative coordinate space as would be available to an evading aircraft pilot. The optimization criteria of an evading aircraft is that of maximizing the survival probability for all relative coordinates. The representative model chosen for illustration is evasion from a close range air-to-air IR guided missile. The effectiveness/vulnerability results as well as sample trajectories illustrating the optimal evasive maneuvering are given for several studies conducted with the model. One study illustrates how optimal maneuvering can degrade IR missile effectiveness by taking advantage of the close-range, narrow-field-of-view "saturation" properties of the seeker sensor. Other studies concern the effects of optimal maneuvering on off-boresight launch effectiveness and on effectiveness sensitivity with warhead lethality variations.
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LIST OF SYMBOLS

\( a \) Aircraft longitudinal acceleration

\( C_{D_0} \) Zero-lift drag coefficient

\( C_{D_I} \) Induced drag coefficient

\( v_{\text{LIM}} \) Maneuvering g limit due to structural constraint

\( K_{\text{ap}} \) Autopilot closed loop gain

\( K_g \) Guidance gain

\( m \) Missile mass

\( n_{yc} \) Yaw acceleration command

\( P_K \) Kill probability

\( P_{ij} \) Decision table element - probability of selecting the \( j^{th} \) control in the \( i^{th} \) observed state

\( R \) Missile-to-aircraft range

\( R_A \) Aircraft turn radius

\( R_{\text{min}} \) Minimum relative range

\( R_S \) Seeker saturation range

\( r \) Missile/autopilot yaw rate

\( r_l \) Line-of-sight turning rate

\( r_s \) Seeker line-of-sight turning rate

\( r_{s\text{LIM}} \) Seeker rate limit

\( S \) Aerodynamic reference area

\( T \) Thrust

\( t_B \) Engine burn time

\( t_{DL} \) Control activation delay time
\( t_G \)  
Total guidance time

\( u_i \)  
Elemental maneuver controls

\( v_i \)  
Elemental countermeasure controls

\( V_A \)  
Aircraft velocity

\( V_M \)  
Missile velocity

\( V_{SS} \)  
Aircraft steady-state maneuver velocity

\( V_x, V_y \)  
Relative velocity components

\( x_i \)  
Relative states determined by aircraft observable decomposition

\( x_A, y_A \)  
Inertial components of aircraft position

\( x_M, y_M \)  
Inertial components of missile position

\( c \)  
Outcome description

\( \beta \)  
Track crossing angle

\( \alpha \)  
Aircraft control parameter

\( \psi \)  
Missile inertial heading

\( \theta \)  
Off-boresight angle (gimbal angle)

\( LIM \)  
Seeker gimbal angle limit

\( \lambda(\cdot) \)  
Learning algorithm reinforcement weights

\( \rho \)  
Atmospheric density

\( \tau_p \)  
Autopilot time constant

\( \tau_s \)  
Seeker time constant

\( \phi \)  
Aircraft inertial heading

\( \gamma \)  
Inertial line-of-sight angle

\( \psi \)  
Missile angle-off relative to aircraft velocity vector
I. INTRODUCTION

Quantifying missile effectiveness and aircraft vulnerability by using air-to-air and surface-to-air simulation models is of considerable value in 1) developing more lethal missile systems and improving their operational deployment and 2) determining operational tactics to enable an evading aircraft to increase its survivability. However, the use of such simulation models is frequently criticized because:

1) The missile capability results (which are given by launch envelope data predicted by simulation models with limited maneuvering targets) are overly optimistic in view of real-world statistics.

2) Few approaches exist that can determine an evading aircraft's best maneuvering and countermeasures deployment strategies (in terms of maximizing survivability) as a "closed-loop" feedback control over all relative coordinates.

Our approach, described later in this introduction, overcomes these objections.

References 1 and 2 outline the methodology that has been adopted to date by many investigators concerned with missile effectiveness. This methodology is characterized by an extensive model that simulates in detail missile trajectory, warhead detonation, and associated aircraft structure damage to determine aircraft survivability. The evading aircraft generally performs prescribed "open loop control" maneuvers such as maximum "g" turns or sinusoidal weaving maneuvers during the trajectory portion of the simulation. The missile effectiveness determination is then
made by a statistical sampling over many possible launch conditions as in Ref. 1, or a selected set of conditions as in Ref. 2. The methodology in Ref. 3 is a departure from that of Refs. 1 and 2 in that aircraft evasive maneuvers that maximize the missile miss distance are computed by gradient methods for selected initial conditions. This approach moves in the direction of optimizing the aircraft maneuvering strategy, but has associated with it many mathematical and computational restrictions. An important restriction is that the payoff function must be a continuously differentiable function of the relative coordinates. This mathematical difficulty implies a computational problem in solving for the optimal feedback control for the aircraft over all relative coordinates. Thus, only an open loop control maneuver can be computed for the aircraft for each launch condition.

The approach taken in this report provides a new methodology with which to consider the effectiveness/vulnerability quantification problem. The approach is basically a blend of applications of optimal control theory and stochastic learning theory. The first application of this methodology to one-on-one dogfight game models with gun weapons was reported in Ref. 4. In this setting the best aircraft maneuvering strategies (as a discretized feedback control which maximizes kill probability) for each combatant were computed by a learning algorithm. The domains of effectiveness of each system were then quantified by simulation. That same approach can easily be extended to the missile combat model considered here, with the two-sided game model aspect being replaced by a one-sided control problem for the evading aircraft. This methodology permits model realism not generally considered in other optimization approaches: developing maneuver and countermeasure strategies based upon the pilot's available "information thresholds" in the relative
coordinate space and using survival probability in place of miss
distance criteria as an optimization criteria. Thus the methods
shown here provide one means of resolving criticism No. 2 and offer
at the same time a means of accounting for the real world versus
computational model differences of criticism No. 1 by having the
optimal evasive strategy as an integral part of the effectiveness
determination.

A representative evasion model chosen for study in this re-
port is the close-range air-to-air IR guided missile in its launch
and postlaunch flight phases employed against aircraft with per-
formance attributes of current day fighters. The problem has been
arbitrarily limited to two dimensional flight in the horizontal
plane with the evading aircraft employing only its maneuvering con-
trol capability to increase its survival probability. The choice
of model was quite arbitrary as the methodology can easily accommo-
date radar guided missiles, countermeasures strategy analysis, and
three dimensional flight. In the last case, computer running times
can be expected to increase four fold over the two dimensional case
for corresponding levels of model detail. The surface-to-air missile
evasion problem would be as easily resolved by the methodology as the
air-to-air case.

Four sets of results involving the IR close-range missile model
are obtained to illustrate the effect of aircraft employing optimal
maneuvering versus nonmaneuvering control on missile effectiveness
and aircraft vulnerability measures. The first result set essen-
tially provides an effectiveness/vulnerability data baseline for a
standardized missile configuration. The second result set deals
with the saturation of the missile seeker field of view for very
close target ranges. In this case the seeker field of view is satu-
rated and guidance neutralized at relative target ranges determined
by the track crossing angle between the missile/aircraft trajectories. In the first result set the saturation range is a constant independent of track crossing angle. The third set shows effectiveness/vulnerability sensitivity to target off-boresight error at launch. The last result set shows the sensitivity to improvements in warhead lethality. Sample trajectories depicting the optimal maneuvering are given along with the effectiveness/vulnerability results for each of the four cases.

The authors wish to acknowledge the computational assistance rendered by Mr. Arthur Kaercher of the Grumman Data Systems Corporation in the course of obtaining the numerical results.
II. AIR-TO-AIR MISSILE MODEL

This section presents the mathematical models for the evading aircraft and attacking missile employed in the maneuvering tactics determination and system capability analyses. For the models described, we have limited the analyses to combat in the horizontal plane to simplify presentation. The same modeling technique without alteration in solution methodology can be straightforwardly applied to three dimensional combat problems without reservation.

AIRCRAFT MANEUVERING STRATEGY DEVELOPMENT

The maneuvering strategy construction relies on the specification of which relative coordinates (positions, rates) and what threshold levels for these coordinates will approximate the aircraft pilot's observable information during the engagement. The motivation for the construction and interpretation of the aircraft observable data in terms of relative coordinate decompositions has been described in Section 4 of Ref. 1. The decompositions lead to a finite number of relative coordinate contingencies or regions for consideration in the strategy development. The horizontal plane model considered here assumes the aircraft can observe range (R) and angle-off (ω) information with the thresholds shown in the decomposition of Fig. 2-1.

When crossing a threshold for any of the regions making up the decomposition, the aircraft will be permitted to select an elemental control as outlined in the hypothetical case below. In the vectorgram of Fig. 2-2 the elemental controls \( u_1 \) and \( u_5 \) could correspond to maximum performance turns that decelerate the aircraft to a low steady-state velocity; \( u_2 \) and \( u_4 \) are maximum performance turns at a high steady-state velocity; and \( u_3 \) is a
Fig. 2-1 Aircraft Observable State, R, w
straight ahead dash policy. In the case of countermeasures deployment, the elemental control choices might be:

- \( v_1 \) - jettison flare decoy
- \( v_2 \) - illuminate arc-lamp
- \( v_3 \) - jettison and ignite fuel charge
- \( v_4 \) - no countermeasures deployment

In this illustration the aircraft would select a pair of controls \((u, v)\) within each region. The specification of a \((u, v)\) pair for all regions as shown in Fig. 2-1 would constitute a maneuvering/countermeasures strategy in our terminology.
DYNAMICAL MODEL

The dynamical equations for a representative air-to-air infrared guided missile in its postlaunch maneuvering phase is presented in this section. In addition, the evading aircraft dynamical equations are also given. The missile systems sensor characteristics, guidance constraints, aerodynamic constraints, and warhead properties as incorporated in the model are explained.

The coordinate nomenclature employed with the trajectory dynamics description is given in Fig. 2-3. The symbol nomenclature is presented in the front of this report.

![Coordinate Nomenclature](image-url)
Missile Equations of Motion in Inertial Coordinates

\[ \dot{x}_M = V_m \cos \phi \]
\[ \dot{y}_M = V_m \sin \phi \]
\[ \dot{z} = \tau \]
\[ V_M = -\frac{V_m^2}{2m} \frac{SC_D}{D_0} - \frac{2m}{\rho SC_D} (n_y c)^2 + \frac{T}{m} \]

Aircraft Equations of Motion in Inertial Coordinates

\[ \dot{x}_A = V_A \sin \phi \]
\[ \dot{y}_A = V_A \cos \phi \]
\[ \dot{z} = \frac{V_A}{R_A} \]
\[ \dot{V}_A = a \]

The values for the control variable \( u \) and the steady-state velocity \( V_{SS} \) associated with five representative elemental maneuver choices are as follows:

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( u_4 )</th>
<th>( u_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maneuver Control</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>( V_{SS} )</td>
<td>( V_{T1} )</td>
<td>( V_{T2} )</td>
<td>( V_D )</td>
<td>( V_{T2} )</td>
<td>( V_{T1} )</td>
</tr>
<tr>
<td>( R_A )</td>
<td>( R_{T1} )</td>
<td>( R_{T2} )</td>
<td>( - )</td>
<td>( R_{T2} )</td>
<td>( R_{T1} )</td>
</tr>
</tbody>
</table>
The longitudinal acceleration/deceleration parameter, $a$, is determined by the following scheme:

1) Determine $V_{SS}$ according to the selected maneuver $u_1$ from the above table.

2) If

\[
\begin{align*}
V_A > V_{SS} & \quad a = a^- \\
V_A < V_{SS} & \quad a = a^+ \\
V_A = V_{SS} & \quad a = 0
\end{align*}
\]

The dynamical capability of the aircraft can be specified then by the set of parameters $(V_T, V_{T2}, V_D, R_{T1}, R_{T2}, a^+, a^-)$, which are given for a representative aircraft in Table 4-1. The aircraft trajectory dynamics also requires the quantities $R$ and $\omega$ for the strategy development. These quantities are made available by the following computations:

\[
R = [ (x_A - x_M)^2 + (y_A - y_M)^2 ]^{\frac{1}{2}}
\]

\[
\omega = \frac{3\pi}{2} - \psi
\]

where

\[
\psi = \tan^{-1} \left( \frac{y_A - y_M}{x_A - x_M} \right)
\]

**Missile Guidance Considerations**

The equations corresponding to the line of sight geometry as employed in the missile guidance are as follows:
Relative velocity components

\[ V_x = V_A \sin \theta - V_M \cos \phi \]
\[ V_y = V_A \cos \theta - V_M \sin \phi \]

Line of sight yaw rate

\[ r_1 = \frac{(V_x \sin \phi - V_y \cos \theta)}{R} \]

with \( R \) and \( \phi \) as computed on page 10.

The equations corresponding to the seeker and autopilot response and associated constraints are:

Seeker line of sight rate response

\[ \dot{r}_s = \frac{(r_1 - r_s)}{r_s} \]

with the seeker rate limited by \( |r_s| \leq r_{sLIM} \)

Missile/autopilot yaw rate response

\[ \dot{r} = \frac{1}{ap} \left( \frac{K_{ap} n_{yc}}{V_M} - r \right) \]

where \( n_{yc} = \frac{K_r V_M}{g_s} \) is the commanded acceleration.

The commanded acceleration is restricted in magnitude to correspond to a maximum allowable maneuvering \( g \) limit required by structural considerations. In this case the constraint \( n_{yc} \leq g_{LIM} \) is imposed.

Other guidance constraints are considered in the model:

A control activation delay at launch of \( t_{DL} \) seconds.

In this case \( n_{yc} = 0 \) when \( 0 \leq t \leq t_{DL} \); \( n_{yc} \) as above for \( t > t_{DL} \).
A maximum guidance time of \( t_G \) seconds after which \( n_{yc} = 0 \)

A gimbal angle limit corresponding to seeker line of sight and boresight axis limitations. In this case if \( \theta > \theta_{\text{LIM}} \), then the commanded acceleration \( n_{yc} = 0 \) corresponds to centering the controls. The seeker is modeled to require the correct line of sight when excursions lead back to within the allowable angle range. Two limits are employed: \( \theta_{\text{LIM}_L} \), a lower limit angle value at launch, and \( \theta_{\text{LIM}_F} \), a higher value of flight.

The thrust \((T)\) and burn rate \((\dot{m})\) are specified as functions of time in the model, and data for the representative problem considered are given in Section IV. Along with this the aerodynamic drag coefficients \( C_{D_0} \) and \( C_{D_1} \) are incorporated as functions of Mach number and the air density computed from an exponential atmosphere model. These aerodynamic data for a representative missile configuration operated at a specific altitude are also given in Section IV.

MISSILE SEEKER PROPERTIES

It is important to note that the above model developed for simulation of the seeker response has made use of the assumption that perfect geometric line of sight information (with a time constant of \( \tau_s \)) was made available to the autopilot whenever the target was in the gimbal angle range of the seeker head. Line of sight rate errors due to detector signal/noise properties at various ranges, atmospheric properties, multiple sources, countermeasures, etc. can easily be incorporated in the model in a straightforward manner without any alteration of the basic solution methodology. An example showing how a specific type of line of sight
rate information error can be modeled is treated in this report. Briefly, the error modeled deals with the seeker detectors field of view being "filled" by the nonzero size source at very close target ranges in comparison to the generally assumed point source property. This blooming error is dependent upon the target aircraft track relative to the missile boresight axis as well as relative range. A more detailed discussion of this error model is given in Section IV.

MISSILE WARHEAD DETONATION AND KILL DETERMINATION

The problem of terminating the missile trajectory with detonation of the warhead and subsequent damage assessment of the target aircraft is tremendously complex and is generally analyzed separately from the maneuvering problem. These analyses seek the determination of kill\textsuperscript{1} probability given relative flight conditions sufficient to activate the fusing and detonation sequence. This probability is empirically derived by detailed simulation of the fusing and detonation procedures and study of the warhead blast and/or fragment interaction with all known vulnerable areas of the specific target aircraft. Moreover, this assessment is made for all the various dynamical aspects, range, azimuth angle, elevation angle, velocity magnitude, and orientation at detonation. As this report is concerned with aircraft evasive tactic methodology, the kill determination phase is much simplified in scope, but sufficiently realistic to be representative of the lethality data typically associated with small close-range infrared-missile warheads detonated near current fighter aircraft.

\textsuperscript{1}By "kill" one assumes at least to disable the aircraft so that its primary mission capability is eliminated, but not necessarily to render catastrophic destruction.
The simplification employed relies upon computing the minimum relative range $R_{\text{min}}$ (from the equation on page 10) between the centers of mass for any missile/aircraft trajectory pair. The minimum range value is used to determine a kill probability from a lethality plot, as shown in Fig. 2-4. This typical function represents "average" data over all missile/aircraft orientations and relative velocities at detonation. It is assumed that detonation occurs at precisely the minimum relative range point. The numerical $R_{\text{min}}$ value is utilized to specify the kill/no-kill outcome distribution employed subsequently in the learning and statistics phases of the computational approach.

![Graph](image)

**Fig. 2-4 Warhead Lethality**
III. COMPUTATIONAL APPROACH

The computational method is presented in two phases: a "learning" phase, which is associated with the optimal strategy development for the aircraft, and a statistics phase associated with the determination of the aircraft vulnerability and missile effectiveness measures.

LEARNING PHASE

Strategy Initialization

A stochastic learning algorithm has been developed for strategy resolution in two-player, one-on-one duels in Ref. 1. That methodology is applied to the aircraft/missile problem in a similar manner. In this case one player (missile) employs a prespecified guidance policy while the second player (aircraft) has its strategy resolved by the algorithm in accordance with a survival goal. Section II outlines the aircraft maneuvering strategy definitions in terms of the observable relative coordinate data and elemental controls. The countermeasures strategy development, although not described in this report, would be treated in a manner similar to the maneuvering case. One initially begins with the aircraft strategy represented by the decision table:

\[
\begin{array}{ccccccc}
 & & & & & & u_j \\
 & & & & & & \\
 & & & & & & \\
 & & & & & & \\
 & x_i & & & & p_{ij} \\
 & & & & & & \\
 & & & & & & \\
\end{array}
\]
where the \( x_i \), \( i = 1, \ldots, N \) represent the observable coordinate decomposition given in Fig. 2-1 and where \( u_j, j = 1, \ldots, 5 \) represent the elemental maneuver choices as shown in Fig. 2-2. Initially the table begins with \( p_{ij} = 0.2 \) for all \( i = 1, \ldots, N \) and \( j = 1, \ldots, 5 \); that is, the elemental control choices are selected at random in an equally likely manner for each visited coordinate region \( x_i \).

**Trajectory and Outcome Simulation**

With the initial strategy for the aircraft selected as given previously, the simulation of a trajectory, outcome determination, and strategy modification cycle of the learning phase are begun. The trajectory simulation begins with an initial condition, relative range \( R \), angle-off \( \omega \), and boresight angle \( \theta \) being chosen at random in a uniform manner over the allowable space of initial conditions. A control choice \( u_j \) is then selected for the aircraft in accordance with the assumed starting strategy for the specific region \( x_i \) given by the range \( R \) and angle-off \( \omega \) thresholds. The missile and aircraft trajectories are then integrated until the next aircraft threshold is crossed and a new aircraft control decision selected. This process continues until one of the following events occurs, a minimum miss distance \( R_{\text{min}} \) is obtained, \( R < 48,000 \) ft, or \( t > t_G \). The region/control sequence employed by the aircraft meanwhile is temporarily stored for reference.

The outcome of any trajectory sample is determined in the following manner: For a given \( R_{\text{min}} \) a kill or miss is obtained by consulting the distribution for \( P_K \) given as a function of \( R_{\text{min}} \), as shown in Fig. 2-4. If \( R > 48,000 \) or \( t > t_G \), the outcome is classified as a miss.

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Strategy Modification

For each of the two outcomes \((\gamma)\) we employ the following weightings \(u(\gamma)\) in the strategy modification portion of the learning phase.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>(u(\gamma))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kill</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>Miss</td>
<td>2</td>
</tr>
</tbody>
</table>

These weightings are consistent with a survival rationale as a goal in the combat for the aircraft (see Chapter 4 of Ref. 1). If the aircraft is in region \(i\) using control \(u_k\) and outcome \(\gamma\) occurs, then the strategy table is modified from \(p_{ij}\) to \(\tilde{p}_{ij}\) by first modifying \(p_{ij}\) to \(\tilde{p}_{ij}\) where

\[
\tilde{p}_{ij} = u(\gamma)p_{ij}
\]

\(j = k\)

\[
\tilde{p}_{ij} = p_{ij}
\]

\(j \neq k\)

The \(\tilde{p}_{ij}\) is then renormalized to form \(\tilde{p}_{ij}\)

\[
\tilde{p}_{ij} = \frac{\tilde{p}_{ij}}{\sum_{j} \tilde{p}_{ij}}
\]

This process is carried out over all \(x_i, u_j\) pairs temporarily stored for that trajectory resulting in an updated strategy table for the aircraft, as follows:
Approximately 100 trajectories with initial conditions selected at random within each region are simulated to obtain a converged decision table that represents the "optimized" survival strategy for the aircraft.

STATISTICS PHASE

The statistics phase of the computations now fixes the converged decision table and computes the aircraft vulnerability and missile effectiveness measures in Monte Carlo fashion. For the vulnerability data approximately 100 trajectory computations with initial conditions in each observable region were obtained. The quantitative measure of vulnerability is the kill probability of the missile as a function of all range (R) and angle-off (θ) thresholds relative to the aircraft as outlined in the observable set description of Fig. 2-1. The missile effectiveness data are obtained in a similar way, except here the initial conditions are chosen at random from an arbitrarily chosen set of launch regions that comprise the totality of all possible launch coordinates for the missile (see Fig. 3-1). The effectiveness data is given as a kill probability for target range (R) and target heading relative to missile boresight axis (θ) for launches within the off-boresight requirement of the missile seeker head for each region.

Without any loss in generality we have assumed that θ = π/2 at launch; otherwise the aircraft heading relative to missile boresight axis nomenclature on each effectiveness chart should be θ - π/2.
Fig. 3-1 Missile Launch Coordinates
of the decomposition in Fig. 3-1. Within each region the $P_K$ value represents an average of all launches that satisfy the off-boresight requirements $LIM_L = \pm 20^\circ$. 
IV. COMPUTATIONAL RESULTS

SUMMARY

The main ingredient in this methodology development, when compared to other missile effectiveness analyses, is the evading aircraft's employment of a maneuvering and/or countermeasures feedback control policy that minimizes the kill probability of the missile over-all launch initial conditions. (Only the maneuvering case is treated in this report.) These optimized policies can in some cases (depending on the specific missile/target aircraft systems design data) seriously degrade missile system effectiveness when compared with system evaluations made with nonmaneuvering or open-loop maximum 3 turn policies for the target aircraft.

The specific missile and target aircraft data given in Table 4-1 and Fig. 4-1 and employed to obtain the results presented are representative of current-day IR missile systems deployed for close-in air

![Graph showing longitudinal deceleration parameter over time](image-url)
### TABLE 4-1 REPRESENTATIVE MISSILE AND AIRCRAFT DATA

#### MISSILE DATA

<table>
<thead>
<tr>
<th>Aerodynamic Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reference Area</strong></td>
<td>$S$ (ft$^2$)</td>
<td>0.01644</td>
</tr>
<tr>
<td><strong>Zero-Lift Drag Coefficient</strong></td>
<td>$C_{D_0}$</td>
<td>0.8 for $M &lt; 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.3 for $M &gt; 1$</td>
</tr>
<tr>
<td><strong>Induced Drag Coefficient</strong></td>
<td>$C_{D_i}$</td>
<td>80.0 for $M &lt; 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$80.0 - (13.6)(M - 1)$ for $M &gt; 1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Engine Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thrust (15,000 ft altitude)</strong></td>
<td>$T$ (lb)</td>
<td>2820.0</td>
</tr>
<tr>
<td><strong>Engine Burn Time</strong></td>
<td>$t_B$ (sec)</td>
<td>5.2</td>
</tr>
<tr>
<td><strong>Missile Mass at Launch</strong></td>
<td>$m_0$ (slugs)</td>
<td>6.211</td>
</tr>
<tr>
<td><strong>Engine Mass Flow Rate</strong></td>
<td>$m$ (slugs/sec)</td>
<td>-0.3584</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seeker Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Field of View</strong></td>
<td>(deg)</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>Gimbal Limits during flight</strong></td>
<td>(deg)</td>
<td>±40.0</td>
</tr>
<tr>
<td><strong>Gimbal Rate Limit</strong></td>
<td>(deg/sec)</td>
<td>20.0</td>
</tr>
<tr>
<td><strong>Seeker Output Gain</strong></td>
<td>$K_g$</td>
<td>4.0</td>
</tr>
<tr>
<td><strong>Seeker Time Constant</strong></td>
<td>$\tau_s$ (1/sec)</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidance and Control Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Launch Delay Time</strong></td>
<td>$t_{DL}$ (sec)</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>Autopilot Gain</strong></td>
<td>$K_{ap}$</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>Autopilot Time Constant</strong></td>
<td>$\tau_{ap}$ (1/sec)</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Commanded Turning Acceleration</strong></td>
<td>$n_{yc}V_M$</td>
<td>30.0 g's</td>
</tr>
<tr>
<td><strong>Limit</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Guidance Time</strong></td>
<td>$t_G$ (sec)</td>
<td>20.0</td>
</tr>
</tbody>
</table>

#### AIRCRAFT DATA

<table>
<thead>
<tr>
<th>Maneuver Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dash Velocity</strong></td>
<td>$V_D$ (fps)</td>
<td>900.0</td>
</tr>
<tr>
<td><strong>Steady State Turn Velocity</strong></td>
<td>$V_{T_1}$ (fps)</td>
<td>900.0</td>
</tr>
<tr>
<td><strong>Deceleration Turn Limit</strong></td>
<td>$V_{T_2}$ (fps)</td>
<td>500.0</td>
</tr>
<tr>
<td><strong>Steady-State Turn Radius</strong></td>
<td>$R_{T_1}$ (ft)</td>
<td>5000.0</td>
</tr>
<tr>
<td><strong>Deceleration Turn Radius</strong></td>
<td>$R_{T_2}$ (ft)</td>
<td>3555.0</td>
</tr>
<tr>
<td><strong>Longitudinal Acceleration</strong></td>
<td>$a^+$ (ft/sec$^2$)</td>
<td>+22.0</td>
</tr>
<tr>
<td><strong>Longitudinal Deceleration</strong></td>
<td>$a^-$ (ft/sec$^2$)</td>
<td>see Fig. 4-1</td>
</tr>
</tbody>
</table>
duels and current-day fighter aircraft maneuvering capability. The representative launch condition selected is at a 15,000 ft altitude with the target and launch aircraft flying at the initial speed of 900 ft/sec.

The results obtained in this report, which illustrate the levels of degradation achievable by optimal maneuvering control, fall essentially into four study categories as outlined in Table 4-2. Studies 1 and 2 focus on the blooming error property of narrow field of view sensors at close target ranges. Study 1 examines the case where the sensor field of view becomes saturated at a nominal relative range $R_s = 1000$ ft from the target independent of the missile/target aspect specified by the track crossing angle $\alpha_c$. In this case, the missile guidance command is neutralized ($\alpha_c = 0$) at the $R_s = 1000$ ft point until detonation. The missile warhead lethality model as assumed for study 1 is given in Table 4-2. In this case, the maximum $P_K$ value of 0.7 is maintained for a minimum miss distance $R_{\text{min}}$ of between zero and 5 ft, and quadratically decreases to $P_K = 0$ at a distance of 20 ft. Study 2 considers the case where the saturation range $R_s$ is dependent upon the track crossing angle $\alpha_c$. The functional dependence on $\alpha_c$ for this case is shown in Table 4-2. This particular $R_s$ versus $\alpha_c$ relationship is derived in a geometric way from a simplified afterburner plume radiance pattern and sensor field of view limit consideration, as shown in Fig. 4-2. The 21$\frac{1}{2}$ field of view limit gives an $R_s = 250$ ft for the tail aspect, increasing to 2000 ft for the beam aspect for a typical afterburner plume pattern. The 2000 ft value of $R_s$ is maintained constant throughout the beam to head-on aspects for the sake of simplicity.†

†The methodology is not limited to these simplified saturation effect cases as presented in this section, and, in fact, if extensive model detail in terms of seeker characteristics is available it can be easily incorporated.
<table>
<thead>
<tr>
<th>Study No.</th>
<th>Description</th>
<th>Results</th>
<th>Electro-Seeker Saturaton Model</th>
<th>Electro-Seeker Lethality Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Constant Sation Range Seeker</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>2</td>
<td>Aspect Dependent Sation Range Seeker</td>
<td><img src="image4" alt="Graph" /></td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>3</td>
<td>Off-axis Launch Sensitivity</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
<td><img src="image9" alt="Graph" /></td>
</tr>
<tr>
<td>4</td>
<td>Warhead Lethality Sensitivity</td>
<td><img src="image10" alt="Graph" /></td>
<td><img src="image11" alt="Graph" /></td>
<td><img src="image12" alt="Graph" /></td>
</tr>
</tbody>
</table>
Study 3 deals with quantifying the off-boresight launch angle impact on aircraft vulnerability. Two special cases are considered in this study: the on-boresight case and the 15-20° off-boresight case representative of launches at the edge of the boresight launch envelope. In this study, the saturation property of the seeker and warhead lethality model are identical to those employed in study 2.

Study 4 deals with quantifying the change in kill effectiveness with increased warhead lethality. In this study, the saturation property of the seeker is maintained as employed in studies 2 and 3.

The computational results for studies 1 through 4 are presented with the following format: In each of the studies the missile effectiveness results are presented first, the aircraft vulnerability measures second, followed by sample evasion trajectories. The effectiveness and vulnerability results are given for both the non-manuever and "optimal" learned maneuver cases.
STUDY NO. 1 - CONSTANT SATURATION RANGE SEEKER

The missile effectiveness results for study 1 are presented in Fig. 4-3. Three levels of $P_K$ have been arbitrarily selected as shown to simplify the results presentation. The actual $P_K$ values, however, are available in the computational results for more detailed appraisals. The diagram at left shows the kill effectiveness in the launch coordinate space nomenclature of Fig. 3-1, when the aircraft does not employ evasive maneuvering but is in straightline flight. The diagram at the right shows the degradation in effectiveness made possible with optimal learned maneuvers. Note

*All regions shown in white, in all missile effectiveness, and aircraft vulnerability results, correspond to levels of $P_K \leq 0.1$. 

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that no missile launches were considered at ranges $R < 5000$ ft in any of the results reported here. This was an arbitrarily imposed criterion in light of the missile guidance delay parameter $t_{DL} = 2$ sec and was implemented only for the purpose of reducing the computer running time by limiting the simulation to fewer initial conditions. One can see the degree of degradation in kill effectiveness in the tail aspect ($\phi = \pm 60^\circ$) and head-on aspect ($\phi = 120$ to $240^\circ$) brought about by the learned maneuvers. Figure 4-4 shows the missile effectiveness comparison between an aircraft employing

![Diagram showing missile effectiveness comparison]

**Fig. 4-4** Missile Effectiveness Results, Constant Saturation
Range = 1000 Feet ($5g$ Turn vs. Optimal)
a 5 g right turn and one employing the learned maneuvers. The
5 g right turn case represents the kind of "open loop" maneuvering
usually considered in missile effectiveness analyses (see Refs. 1
and 2). For relative headings in the right hemisphere, $\theta = 0^\circ$
to $180^\circ$, the learned maneuvers markedly reduce the $P_K \leq 0.3$
level for all but one region in contrast to the 5 g turn. In
the left hemisphere, the reduction is less marked. The head-on and
tail aspect cases where $P_K \geq 0.3$ for the 5 g turn case have
been reduced to levels $< 0.3$ for the maneuvering case. On the
other hand, some regions now appear in the $\geq 0.1$ range for the
learned maneuvers as contrasted with the $\leq 0.1$ values in the 5 g
turn case. This particular situation is brought on by the discrete
levels chosen for plotting; e.g., the 5 g turn case has regions
for which the $P_K$ values are between 0.09 and 0.1, while the
corresponding regions in the maneuvering case have values between
0.10 and 0.11. Moreover, the sensitivity of the learning algo-
rithm is such that it could not differentiate between maneuver se-
quences that produced $P_K$ levels having those small differences.
At the risk of these small data abnormalities in some of the re-
sults, the $P_K$ levels were plotted as computed without adjustment
for solution variability.

The aircraft vulnerability results are presented in Fig. 4-5
for the nonmaneuvering versus learned maneuvering cases. In these
cases, the $P_K$ data are given with respect to the aircraft ob-
servable state description in Fig. 2-1. One can see the marked $P_K$
degradation possible in the head-on and tail aspects by employing
the learned maneuvers as contrasted with the straight ahead flight
policy.

We now turn to the illustrations of strategy and trajectory
sample results. The results were generated from hard copies of
solutions made available by a computer-driven CRT display. Two forms of trajectory plot are illustrated for each trajectory sample. The first shows motion in coordinates relative to the evading aircraft and is useful in demonstrating the learned aircraft maneuvering strategy. The second is the normally drawn plot in absolute coordinates showing the motion of both vehicles during the engagement. The optimal learned strategy data, though available, is not presented for each result in its entirety (viz, with regard all regions in Fig. 2-1) because the general characteristics of the derived strategies are more easily inferred from selected sample trajectories. Table 4-3 supplies an outline for the sample results.
### TABLE 4-3 SAMPLE RESULTS - EVASIVE STRATEGIES AND TRAJECTORIES

<table>
<thead>
<tr>
<th>Study No.</th>
<th>Study Description</th>
<th>Sample Result No.</th>
<th>Figure No.</th>
<th>Aircraft Maneuvering Comparisons</th>
<th>Launch Initial Conditions*</th>
<th>Minimum Miss Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Constant Seeker Saturation Range</td>
<td>1A 4-6 and 4-7</td>
<td>Nonmaneuvering Versus Learned Maneuvering</td>
<td>11900 200 0</td>
<td>Versus</td>
<td>4.0 Versus 22.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1B 4-8 and 4-9</td>
<td>Nonmaneuvering Versus Learned Maneuvering</td>
<td>20000 10 0</td>
<td>Versus</td>
<td>2.3 Versus 24.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1C 4-10 and 4-11</td>
<td>5g Right Turn for Counterpart Initial Conditions</td>
<td>14000 255 Versus 105 Versus 0</td>
<td>Versus</td>
<td>47.1 Versus 6.7</td>
</tr>
<tr>
<td>2</td>
<td>Aspect Dependent Seeker Saturation Range</td>
<td>2A 4-14 and 4-15</td>
<td>Nonmaneuvering Versus Learned Maneuvering</td>
<td>15000 145 0</td>
<td>Versus</td>
<td>0.8 Versus 28.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2B 4-16 and 4-17</td>
<td>Nonmaneuvering Versus Learned Maneuvering</td>
<td>7500 110 10</td>
<td>Versus</td>
<td>11.4 Versus 2400</td>
</tr>
<tr>
<td>3</td>
<td>Off-Boresight Launch</td>
<td>3A 4-20 and 4-21</td>
<td>On Boresight</td>
<td>15000 -40 0</td>
<td>Versus</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3B 4-22 and 4-23</td>
<td>Boresight Error Lag Versus Lead</td>
<td>15000 40 18 Versus 18</td>
<td>Versus</td>
<td>22.9 Versus 7.4</td>
</tr>
<tr>
<td>4</td>
<td>Warhead Lethality</td>
<td>None</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

*Aircraft Relative Coordinates*
corresponding to studies 1 through 3. For study 1, the constant saturation range case, a typical "tail shot" example is given by result 1A in Figs. 4-6 and 4-7. Figure 4-6 shows the motion in coordinates relative to the aircraft and illustrates the weaving maneuver (control reversals at sequential range thresholds) developed by the learning algorithm to minimize the kill probability against the constant saturation range type missile seeker. For this specific launch initial condition the minimum miss distance comparison is 4 and 23 ft for the straight and learned strategy, respectively. Figure 4-7 shows the absolute coordinate trajectory comparison for the same initial condition. Result 1B, shown in Figs. 4-8 and 4-9, depicts the case "head-on" initial conditions for the nonmaneuvering and learned maneuvering comparisons. Figure 4-8 again shows the weaving maneuver developed by the learning technique in relative coordinates, and Fig. 4-9 shows the maneuvering comparison in absolute coordinates. Result 1C shows the effect of a steady 5 g right turn for two counterpart initial conditions representative of a "beam" launch aspect for the constant saturation range seeker. The dashed trajectory in Figs. 4-10 and 4-11 shows the turn into the missile approach heading, and the solid trajectory shows the turn away for the counterpart initial condition. These typify the kinds of "open loop" maneuvering control given the evading aircraft in most effectiveness evaluations (see Ref. 1 and 2). One can see from the difference in the miss distance results of 6.7 and 47 ft for the counterpart initial conditions that a single open loop control policy (whether a maximum g turn or straight flight) may not furnish the evading aircraft with its best defensive policy when compared to closed loop control over all relative coordinates. The effectiveness results associated with the open loop cases tend to overestimate missile capability.
Evasive Strategies and Trajectories

Parametric Case: Study No. 1, Result 1A

Initial Conditions:

- **Range**: 10,000.0 ft.
- **Time**: 4.1 sec.
- **Angle-Off**: 200.0 deg.
- **BoreSight**: 0.0 deg.
- **Time**: 4.0 ft.
- **Time**: 3.9 sec.
- **Scale**: 3000 ft.
- **Amin**: 22.7 ft.

Fig. 4-6 Evasive Strategies and Trajectories...

Study No. 1, Result 1A (Relative)
Fig. 4-7 Evasive Strategies and Trajectories,
Study No. 1, Result 1A (Absolute)

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Fig. 4-8 Evasive Strategies and Trajectories, Study No. 1, Result 1B (Relative)
Evasive Strategies and Trajectories

Parametric Case: Study No. 1, Result 1B

Initial Conditions:
- Range = 20000.0 ft.
- Heading = 170.0 deg.
- Bore sight = 0.0 deg.
- Time = 6.6 sec.
- Rmin = 2.3 ft.
- Scale 3000 ft.
- Time = 6.9 sec.
- Rmin = 24.9 ft.

Fig. 4-9 Evasive Strategies and Trajectories, Study No. 1, Result 1B (Absolute)
Evasive Strategies and Trajectories

Parametric Case: Study No. 1, Result 1C

Initial Conditions:
- Range: 14000.0 ft.
- Angle-off: 255.0 deg.
- Bore sight: 0.0 deg.
- Time: 10.1 sec.
- \( R_{\text{MIN}} \): 47.0 ft.

Initial Conditions:
- Range: 14000.0 ft.
- Angle-off: 105.0 deg.
- Bore sight: 0.0 deg.
- Time: 6.6 sec.
- \( R_{\text{MIN}} \): 6.7 ft.

Scale: 3000 ft.

Fig. 4-10 Evasive Strategies and Trajectories, Study No. 1, Result 1C (Relative)
Fig. 4-11 Evasive Strategies and Trajectories, Study No. 1, Result 1C (Absolute)
STUDY No. 2 - ASPECT DEPENDENT SATURATION RANGE SEEKER

The missile effectiveness results for study 2, the aspect dependent seeker case, are presented in Fig. 4-12. The diagram at the left shows the kill effectiveness in the launch coordinate space for a nonmaneuvering aircraft. The diagram at the right shows the degradation in effectiveness possible with the optimal learned maneuvers. A marked reduction exists in the tail and beam launch orientations in the 5-16 K ft ranges and elimination of the head-on capability in the 16 to 48 K ft ranges. The aircraft vulnerability results for this study are presented in Fig. 4-13 in coordinates relative to the aircraft. One can see by comparison

Fig. h-12 Missile Effectiveness Results, Aspect Dependent Saturation Range = 250-2000 Feet
with the results in Fig. 4-5, the constant saturation range case, that optimal maneuvering plays a stronger role in achieving vulnerability reductions for the aspect dependent case. Two sets of comparison trajectory results are given to illustrate the optimal maneuvering for study 2 (see Table 4-3). Sample result 2A given in Figs. 4-14 and 4-15 illustrates the nonmaneuvering/maneuvering comparisons for a tail shot initial condition at medium range. One can see the maneuvering strategy of the aircraft is to turn hard to obtain a beam aspect at the longer relative ranges followed by a turn reversal at the saturation range threshold of 2000 ft
Fig. 4-14 Evasive Strategies and Trajectories, Study No. 2, Result 2A (Relative)
Evasive Strategies and Trajectories

Parametric Case: Study No. 2 Result 2A

Initial Conditions:
- Range = 15000.0 ft.
- Heading = 35.0 deg.
- Bore Sight = 0.0 deg.
- Time = 11.5 sec.
- Time = 10.7 sec.

Scale: 3000 ft.

Fig. 4-15 Evasive Strategies and Trajectories, Study No. 2, Result 2B (Absolute)
to open the miss distance. It is important to note that in the interpretation of the optimal learned maneuvers the criterion for "optimality" is not maximizing the minimum miss distance for the evader, but maximizing the survival probability. This criterion, for example, has the effect of rating control sequences that achieve miss distances of say 20 ft and 2500 ft for this warhead lethality as equally desirable. Therefore, the learning technique may show optimal maneuvering sequences that defy intuition for certain portions of the relative coordinate space, yet satisfy the stated criteria for optimality. Sample result 2B illustrates the maneuvering comparisons for a close range beam launch initial condition. The trajectory result comparisons are depicted in Figs. 4-16 and 4-17 for both the nonmaneuvering and learned maneuver cases. The guidance delay parameter $t_{DL}$ is a particularly important missile characteristic for this launch condition, and the execution of a proper maneuver sequence by the evading aircraft can definitively degrade missile effectiveness at these close ranges.

STUDY No. 3 - OFF-BORESIGHT LAUNCH SENSITIVITY

The results for study 3, which is aimed at illustrating the sensitivity of target aircraft vulnerability to off-boresight launch conditions, are presented in Fig. 4-18. The plot on the left shows the vulnerability for the on-boresight launch case. One may compare this plot with the averaged result obtained for -20° to +20° boresight error spread as shown in the right hand plot of Fig. 4-13. The comparison shows the vulnerability to be basically the same for tail aspect launches and slightly reduced in the beam and head-on situations for the on-boresight launch case. These vulnerabilities can be compared with data for the 15° to 20° off-boresight errors at launch are restricted to the +15 to +20° edge of boresight condition; therefore the right half plane depicts results for
Fig. 4-16 Evasive Strategies and Trajectories, Study No. 2, Result 2B (Relative)
EVASIVE STRATEGIES AND TRAJECTORIES

PARAMETRIC CASE: STUDY NO 2 RESULT 2B

INITIAL CONDITIONS:
- RANGE = 7500.0 FT.
- HEADING = 90.0 DEG.
- BORESIGHT = 10.0 DEG.
- TIME = 6.4 SEC.
- RMIN = 11.4 FT.
- TIME = 6.1 SEC.
- RMIN = 2400.0 FT.

SCALE: 3000 FT.

Fig. 4-17 Evasive Strategies and Trajectories, Study No. 2, Result 2B (Absolute)
a 15° to 20° lag, and the left half plane a 15° to 20° lead with respect to target line of sight (see Fig. 4-19). One can see that when compared to the on-boresight data that the lead effect, generally, increases target vulnerability, while the lag effect diminishes it. It is important to note that the optimized maneuvers for these statistical results were restricted to be the same as those computed for the -20° to +20° off-boresight study, thereby allowing straightforward comparison between all three boresight launch cases. Sample trajectories depicting the evasive
STUDY No. 4 - WARHEAD LETHALITY SENSITIVITY

We now move on to study 4 and show the sensitivity of aircraft vulnerability with change in warhead lethality. Table 4-2 describes the improved warhead model of study 4 as assumed in the comparison with the baseline warhead of studies 1 through 3. The improved warhead described as the $P_K(18,60)$ warhead is as lethal as the baseline warhead called $P_K(6,20)$ at three times the minimum range. Figure 4-24 shows the vulnerability results for the baseline and improved warhead for both the nonmaneuvering and learned maneuvering
Evasive Strategies and Trajectories

Parametric Case: Study No. 3 Result 3A

Initial Conditions:
- Range = 15000.0 ft.
- Time = 5.6 sec.
- Angle-off = -40.0 deg.
- BoreSight = 0.0 deg.
- Scale = 3000 ft.

Fig. 4-20 Evasive Strategies and Trajectories, Study No. 3, Result 3A (Relative)
Fig. 4-21 Evasive Strategies and Trajectories,
Study No. 3, Result 3A (Absolute)
Evasive Strategies and Trajectories

Parametric Case: Study No. 3 Result 3B

Initial Conditions:
- Range = 15000.0 ft.
- Time = 5.0 sec.
- Angle-off = -40.0 deg.
- BoreSight = -10.0 deg.

Initial Conditions:
- Range = 15000.0 ft.
- Time = 5.7 sec.
- Angle-off = -40.0 deg.
- BoreSight = 10.0 deg.

Scale 3000 ft.

Fig. 4-22 Evasive Strategies and Trajectories, Study No. 3, Result 3B (Relative)
Evasive Strategies and Trajectories

Parametric Case: Study No. 3

Result 3B

Initial Conditions:

- Range = 15000.0 ft.
- Time = 5.0 sec.
- Heading = 202.0 deg.
- Bore sight = -10.0 deg.

Initial Conditions:

- Range = 15000.0 ft.
- Time = 5.7 sec.
- Heading = 238.0 deg.
- Bore sight = 10.0 deg.

Scale: 3000 ft.

Fig. 4-23 Evasive Strategies and Trajectories, Study No. 3, Result 3B (Absolute)

50
Fig. 4-24 Missle Effectiveness Results, Warhead Lethality Study
cases. The improved warhead model, as the results show, allows much less of a vulnerability reduction with optimal maneuvering than did the baseline warhead. With such an improved system, a tactics analyst would perhaps divert more of his effort into the countermeasures policy area rather than rely upon maneuvering to achieve vulnerability reductions.

This study concludes the computational results portion of this report. The studies illustrated were intended to furnish a small sampling of the type of result that is available to the system designer and tactics analyst in system effectiveness studies using the described methodology.
V. CONCLUSIONS

The important relationship of aircraft evasive maneuvering within the analysis of missile effectiveness has been made clear with the numerical studies conducted here. The first three result sets clearly show marked reductions in missile effectiveness achieved by the optimal "closed loop" maneuvering over the non-maneuvering and other open loop control cases for typical close range IR missile combat. In the constant seeker saturation range case, the optimal evasive policy for most launch coordinates is essentially a weaving maneuver whose control reversal points are dependent upon the relative range coordinate thresholds. In the aspect dependent case, the aircraft policy is essentially a maximum rate turn to present a beam aspect to the oncoming missile followed by a maximum rate turn reversal at the seeker saturation range threshold. The off-boresight launch results show that optimal maneuvering can achieve considerable reductions in missile effectiveness at close ranges when launch off-boresight angle lags the line of sight. The last of the results configured the improved missile warhead to be sufficiently lethal so as to obtain negligible effectiveness reductions with optimal maneuvering. It indicates that the major burden of the evasion problem should be placed in the domain of countermeasures deployment. The optimal maneuvering data (particularly in the first three result sets for the baseline missile warhead lethality) when averaged over all of the launch coordinates corresponds numerically to those statistical levels achieved in actual combat firings. The computational costs are reasonable, requiring one-half hour for the complete solution set for the 2-D problem with projected running times of two hours for comparably detailed 3-D models on IBM 370/168 type computers.
The technique as described can be incorporated in existing simulation programs where the evasive maneuver optimization can be programmed to become an integral part of the trajectory simulation process and where the fusing, detonation, and warhead damage assessment portion would be maintained and computed in its original detail. On the other hand, the methodology here can be easily augmented to include summary statistics of the detailed warhead damage effects to provide a self-contained program in its own right, which might offer shorter computer running times per problem than the former modification. The methodology also is easily extendable to countermeasures strategy determination and to problems of three-dimensional flight. If necessary, the methodology can permit analysis of missile effectiveness to be achieved with in-flight alteration of missile guidance or counter-countermeasures policies. This is tantamount to reverting back to a two-sided game model for the combat in place of the one-sided control problem as analyzed here.
VI. REFERENCES


APPENDIX

COMPUTER PROGRAM LISTING
CONTINUOUS TWO DIMENSIONAL MISSILE VS. AIRCRAFT CONFLICT MODEL

OCTOBER 1974

C.COMMON
MLK=MAX,ACC1,T1Z,V1Z,V1F,U1,VI,DI,MIN,S1L=10

DIMENSION DEC(150,5),TEM(10),RT1(150,5),KTEM(10),
1U(500,2),SWU(500),M(11,12,1)

2X(12),XINTA(12),VFI(15),PS1(5),ACC1(2,2)

INTEGER

DATA P1/1.014159265359/9793/0.080/1.900/0.45/0.25/

DATA TST/5.9E-04/

DATA WC/1.4E0/, SMAX/0.5E0/

C.INITIAL STATE
WM,VM,GAMMA,VM,RANGE,PS1,RS,R,TIME,XT,YT,HEADING/

DATA XINTA/0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
1 0.0,0.0,0.0 /

DATA NOREG/132/

TWODI = 2.0*141
TIME=ICHON(3)
KNTUT = 1
ND = 0.0

DO 100 1=1,1,2156

C.CALL IN1, ORDERS REGIONS 1/2

100 YXY = RKM(DUM)

C.CALL IN(MI)

10 READ(S,99)IDEN

99 FORMAT(110)

89 FORMAT(110)

READ(S,98)ACC1(1,1),ACC1(1,2),ACC1(2,1),ACC1(2,2)

READ(S,98)VEF1(1),VEF2(1)

VEF1(1) = VEF2(1)

READ(S,98)DS,DT,

READ(S,98)WIN,LOSS

READ(S,98)DT,TMAX

READ(S,97)LOOP,MODE,KB1,KB2,NPLS

97 FORMAT(115)

IF(MODE,EO,1)NPLS = 1

IDEN IS RUN NUMBER FOR IDENTIFICATION PURPOSE ONLY
ACC1 ARE ACCELERATION COEFF FOR PLAYER 1
VEF1 ARE STEADY STATE VELOCITIES FOR PLAYER 1
DS, DT ARE THE TWO TURE RADII FOR PLAYER 1
WIN,LOSS ARE WIGHTS FOR PLAYER 1 FOR WIN AND LOSS
TMAX IS THE MAXIMUM TIME ALLOWED FOR ANY TRAJECTORY
SMAX IS WEIGHT FACTOR, USUALLY AVG. NO. SAMPLES PER REGION PER RUN

MODF = 1 FOR LEARNING MODE = 2 FOR STATISTICS (NO LEARNING) RUN

KB1 = 1 DECISION TABLE WILL BE INPUT, KB1 = 0 IT IS NOT INPUT

KB2 = 1 STATISTICAL TABLE WILL BE INPUT, KB2 = 0 IT IS NOT INPUT

NOTE THE ENTIRE TABLE (376 CARDS) WILL BE INPUT OR NOT, THE ENTRIMD
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C TABLE WILL BE PRINTED AT THE END OF THE RUN
C WHEN MODE = 2 THE STATISTICS CAN BE GENERATED RELATIVE TO EITHER
C THE AIRCRAFT OR MISSILE BY SETTING NPLS
C NPLS = 1 STATISTICS USING THE AIRCRAFT INITIAL CONDITION SPACE
C NPLS = 2 STATISTICS USING THE MISSILE INITIAL CONDITION SPACE
C
0029 WRITE(6,96) IDEN, ACC1(1,1), ACC1(1,2), ACC1(2,1), ACC1(2,2)
0030 1(VEL(1)) = 1.5
0031 DISC DIF
0030 96 FORMAT(23H INPUT FOR RUN NUMBER*/100/
0031 127H ACCELERATION COEFFICIENTS */1P4E18.6 /
0032 225H STEADY STATE VELOCITIES /
0033 311H PLAYER 1 */1P5E18.6 /
0034 511H TURN RAD11 */1P3E18.6)
0035 WRITE(6,95) WINWLOSS
0036 95 FORMAT(31H WEIGHTS FOR PLAYER 1 WIN, LOSS, 2F10.4)
0037 94 FORMAT(10H MODELTA T =, F12.4, 9H Tmax =, F12.4 /
0038 17H SMAX =, F12.4 )
0039 93 FORMAT( 1H0, 11.0, 26H SAMPLES REQUESTED MODE =, 15,
0040 110H KB1, KB2 =, 215, 9H NPLS =, 15,/
0041 267H MODE = 1 FOR LEARNING, MODE = 2 FOR STATISTICS ONLY
0042 39N /)
0043 4 524H KB1 = 1 DEC. TABLE WILL BE INPUT, = 0 IT WILL NOT /
0044 5 534H KB2 = 1 STAT. TABLE WILL BE INPUT, = 0 IT WILL NOT ///)
0045 IF(KB1 = EQ. 0) GO TO 135
0046 115 CONTINUE
0047 READ(S=92) X(TEM(L), L=1,5)
0048 92 FORMAT(15, 5F4.6)
0049 IF(K = LE. 0) GO TO 125
0050 120 DEC1(K, J) = TEM(J)
0051 120 DEC1(K, J) = TEM(J)
0052 GO TO 115
0053 125 CONTINUE
0054 WRITE(6, 91) IDEN
0055 91 FORMAT(45H DECISION TABLE INPUT
0056 91 PRINT(45H DECISION TABLE INPUT
0057 DO 130 I=1, NREG
0058 130 WRITE(6, 90) (DEC1(I, J), J=1, 5)
0059 90 FORMAT(15, 1P5E14.7)
0060 IF(K = LE. 0) GO TO 160
0061 135 IF(KB2 = EQ. 0) GO TO 160
0062 137 CONTINUE
0063 READ(S=81) X(TEM(L), L=1,5)
0064 81 FORMAT(15, 1P5E14.7)
0065 IF(K = LE. 0) GO TO 145
0066 89 FORMAT(6I5)
0067 DO 140 J=1, 5
0068 140 RT12(K, J) = TEM(J)
0069 GO TO 137
0070 145 CONTINUE
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(0061 WRITE(6,*),I10
0062 FORMAT(34H1STATISTIC TABLE INPUT FOR RUN NO., 110
0063 IF(NPLS*EO,1) WRITE(6,*70)
0064 IF(NPLS*EO,2) WRITE(6,*71)
0065 FORMAT(5X,24HRELATIVE TO THE AIRCRAFT, //)
0066 FORMAT(5X,24HRELATIVE TO THE MISSILE, //)
0067 DO 150 I=1,NOREG
0068 150 WRITE(6,811),RT12(I,J),J=1,5
0069 87 FORMAT(15,2X,5I5)
0070 IF(KHI*NEO,0) GO TO 175
0071 DO 170 I=1,NOREG
0072 170 J=1,5
0073 DECJ(I,J) = 0.2E0
0074 CONTINUE
0075 IF(KB2*NEO,0) GO TO 190
0076 DO 180 J=1,NOREG
0077 DO 180 J=1,5
0078 IF(KH1*NEO,0) GO TO 190
0079 CONTINUE
0170 CONTINUE
0175 IF(KB2*NEO,0) GO TO 190
0176 DO 180 J=1,NOREG
0177 DO 180 J=1,5
0178 IF(KH1*NEO,0) GO TO 190
0179 CONTINUE
0175 IF(KB2*NEO,0) GO TO 190
C MAIN LOOP FROM 1 TO NUMBER OF SAMPLES STARTS HERE
0080 DO 500 JXJL = 1,LOOP
0081 FRAN = 73 + 60*ROM(DUM)
0082 KTRAN = FRAN
C ROT WILL DEFINE INITIAL CONDITION WHICH WILL BE RANDOMLY CHOSEN
0083 CALL ROT1RAN,M1,ROE,OME,TME)
0084 INCR = 1
0085 IF(MODE*EQ,1) GO TO 200
0086 IF(NPLS*EQ,1) GO TO 200
0087 DO 195 I=1,12
0088 195 X1(I) = XINTAL(I)
0089 PH1 = OME/ROE
0090 TME = TME/ROE
0091 X3(I) = P1/2,E0
0092 X5(I) = ROE
0093 X6(I) = (P1/2,E0) - THE
0196 IF(X6(I)GT,TWOP1) X6(I) = X6(I) - TWOP1
0195 IF(X6(I)LT0,E0) X6(I) = TWOP1 + X6(I)
0196 IF(X6(I)LT0,E0,OR*X6(I),GT,TWOP1) GO TO 196
0197 X1(I) = X5(I)*COS(X6(I))
0198 X1(I) = X5(I)*SIN(X6(I))
0199 X12(I) = PHI
0100 V1 = VEL1(I3)
0101 XR = X1(I0) - X1(I)
0102 YR = X1(I1) - X12(I)
0103 RC = SORT(XR**2 + YR**2)
0104 VXL = V1*SIN(X12(I)) - X4*COS(X1(I))
0105 VYL = V1*COS(X12(I)) - X4*SIN(X1(I))

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MIS00860
MIS00970
MIS00980
MIS00990
MIS01000
MIS01010
MIS01020
MIS01030
MIS01040
MIS01050
MIS01060
MIS01070
MIS01080
MIS01090
MIS01100
MIS01110
MIS01120
MIS01130
MIS01140
MIS01150
MIS01160
MIS01170
MIS01180
MIS01190
MIS01200
MIS01210
MIS01220
MIS01230
MIS01240
MIS01250
MIS01260
MIS01270
MIS01280
MIS01290
MIS01300
MIS01310
MIS01320
MIS01330
MIS01340
MIS01350
MIS01360
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0106 PSIC = ATAN2(YR,XR)
0107 PSIDOT = (-VXL*SIN(PSIC) + VYL*COS(PSIC))/RC
0108 X(7) = PSIDOT
0109 GO TO 215

0110 200 CONTINUE
0111 DO 201 J=1,12
0112 201 X(I) = W#TAL(I)
0113 OME = OME/RC
0114 THE = THE/RC
0115 X(6) = 3.E0*PI/2.E0 - OME
0116 205 IF(X(6)+GT.TWOP1) X(6) = X(6) - TWOP1
0117 IF(X(6)+LT.0.E0) X(6) = TWOP1 + X(6)
0118 IF(X(6)+LT.0.E0.OR.X(6)+GT.TWOP1) GO TO 205
0119 X(5) = RDE
0120 X(3) = X(6) + THE
0121 210 IF(X(3)+GT.TWOP1) X(3) = X(3) - TWOP1
0122 IF(X(3)+LT.0.E0) X(3) = TWOP1 + X(3)
0123 IF(X(3)+LT.0.E0.OR.X(3)+GT.TWOP1) GO TO 210
0124 X(10) = X(5)*COS(X(6))
0125 X(11) = X(5)*SIN(X(6))
0126 XR = VEL1(3)
0127 YR = X(11) - X(2)
0128 RC = SQRT(XR**2 + YR**2)
0129 220 IF(XR+GT.0.E0) XR = X(11) - X(2)
0130 VXL = V1*SIN(X(12)) - X(4)*COS(X(3))
0131 VYL = V1*COS(X(12)) - X(4)*SIN(X(3))
0132 PSIC = ATAN2(YR,XR)
0133 PSIDOT = (-VXL*SIN(PSIC) + VYL*COS(PSIC))/RC
0134 X(7) = PSIDOT
0135 DO 225 J=1,600
0136 SWU(J) = 0.
0137 U(J+1)=0
0138 ( 225 U(J+2)=0
0139 C SWU WILL STORE WEIGHTS, U(1,J) REGION, STRATEGY FOR PLAYERS
0140 C NU IS A COUNTER FOR U SUBSCRIPT I
0141 NU = 0
0142 T = 0.
0143 KILL = 0
0144 230 CONTINUE
0145 XR = X(10) - X(1)
0146 YR = X(11) - X(2)
0147 N1 = SQRT(XR**2 + YR**2)
0148 01 = ((3.E0*PI/2.E0) - X(12) - X(6))*RD
0149 235 IF(01+LT.360.E0) 01 = 01 - 360.E0
0150 IF(01+LT.0.E0.OR.01+GT.360.E0) GO TO 235
0151 242 CONTINUE
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C REGI SUBROUTINE PACKS THE SUBSCRIPTS OF M1(11,12,13)

C INTO NI FOR PLAYER 1
C NOTE THE REGION NUMBER FOR PLAYER 1 IS ACTUALLY REGION 2/1
C IF REGION FOR PLAYER 1 HAS NOT CHANGED NS1 = NI.
C NEW REGIS OBTAINED USING NI, M1, THEN A NEW STRATEGY IS PICKED AND
C REGION NO. AND STRATEGY ARE STORED

0152 CALL REG(N1,R1,01,T1)
0153 IF (N1 .EQ. NS1) GO TO 275
0154 NI = NS1
0155 NU = NU+1

C TEST FOR CHATTER - MAX. OF 450 REG. MAY BE VISITED BY PLAYER 1
C NO REINFORCEMENT FOR CHATTER

0156 IF (NU .GE. 450) GO TO 500
0157 I1 = N1/1000
0158 NN = NI - I1*1000
0159 I2 = NN/10
0160 I3 = NN - I2*10
0161 IF (I3 .EQ. 0) I3 = 1
0162 I = M1(I1,12,13)
0163 U(NU,1) = 1
0164 R = RDM(DUM)
0165 T1Z = T

C KS1 = 1 SLOW LEFT, = 2 FAST LEFT, = 3 STRAIGHT, = 4 FAST RIGHT, = 5 SLOW

0166 KS1 = 5
0167 I1 = DIS
0168 VIF = VEL1(5)
0169 UI = 1.0
0170 IF (R .LE. DEC1(I1,1)) GO TO 260
0171 TEMP = DEC1(I1,1)*DEC1(I1,2)
0172 IF (R .LE. TEMP) GO TO 261
0173 TEMP = TEMP + DEC1(I1,3)
0174 IF (R .LE. TEMP) GO TO 262
0175 TEMP = TEMP + DEC1(I1,4)
0176 IF (R .LE. TEMP) GO TO 263
0177 GO TO 270
0178 260 KS1 = 1
0179 DI = DIS
0180 UI = -1.0
0181 VIF = VEL1(1)
0182 GO TO 270
0183 KS1 = 2
0184 DI = DIS
0185 UI = -1.0
0186 VIF = VEL1(2)
0187 GO TO 270
0188 KS1 = 3
0189 DI = DIS
0190 UI = 0.0


```fortran
VIF = VEL1(3)
GO TO 270
263 KSI = 4
264 D1=D1F
265 U1 = 1.0
266 VIF = VEL1(4)
270 U(NU,2)=KSI
C THIS RUN INITIALIZES V TO VMAX
1 0198 IF(INCHR .EQ. 1) V1Z=VEL1(3)
1 0199 IF(INCHR .EQ. 1) V1=V1Z
1 0200 IF(INCHR .EQ. 1) V1Z = V1
1 0201 INCHR = 2
1 0202 IZ0=2
1 0203 IF(V1 .GT. VIF) IZ0=1
1 0204 275 TF = T + DT
1 0205 CALL RK1(X,T1,TF,DT)
1 0206 T = TF
1 0207 C DYNAMIC EQUATIONS
1 0208 IF(KILL.EQ.0) GO TO 230
1 0209 IF(KILL .EQ. 2) GO TO 475
1 0210 IF(KILL .EQ. 3) CALL BIG1(SWU,WWIN,NU,WC)
1 0211 IF(KILL .EQ. 3) CALL SMALL1(SWU,WLOSS,NU,WC)
1 0212 DO 400 I=1,NU
1 0213 IR = U(I,1)
1 0214 IU = U(I,2)
1 0215 DEC1(IR,1)=DEC1(IR,1)+SWU(1)
1 0216 DEN = DEC1(IR,1)+DEC1(IR,2)+DEC1(IR,3)
1 0217 + DEC1(IR,4) + DEC1(IR,5)
1 0218 DO 395 J=1,5
1 0219 DEC1(IR,J)=DEC1(IR,J)/DEN
1 0218
c c this level is defined in data statement
1 0219 IF(DEC1(IR,1) .LT. TST .OR. DEC1(IR,2) .LT. TST
1 0220 .OR. DEC1(IR,3) .LT. TST .OR. DEC1(IR,4) .LT. TST) GO TO 396
1 0220 IF(DEC1(IR,5) .LT. TST) GO TO 396
1 0221 GO TO 406
1 0222 396 KD = 1
1 0223 IF(DEC1(IR,2) .GE. DEC1(IR,1)) KD=2
1 0224 IF(DEC1(IR,3) .GE. DEC1(IR,KD)) KD=3
1 0225 IF(DEC1(IR,4) .GE. DEC1(IR,KD)) KD=4
1 0226 IF(DEC1(IR,5) .GE. DEC1(IR,KD)) KD=5
1 0227 DO 397 J=1,5
1 0228 IF(DEC1(IR,J) .GE. TST) GO TO 397
```
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0229 TMI = DST - DEC1(IR, J)
0230 DEC1(IR, J)=TST
0231 DEC1(IR, KD)=DEC1(IR, KD)-TMI
0232 397 CONTINUE
0233 400 CONTINUE
0234 475 CONTINUE
0235 IU = KRN

C RT12(I, 1) ----- RECORD OF THE NO. OF WINS DUE TO TMAX
C RT12(I, 2) ----- RECORD OF THE NO. OF WINS DUE TO MISS
C RT12(I, 3) ----- RECORD OF THE NO. OF LOSSES DUE TO A HIT
C RT12(I, 4) ----- THE AVERAGE MISS DISTANCE FOR RT12(I, 2)
C RT12(I, 5) ----- THE AVERAGE MISS DISTANCE FOR RT12(I, 3)

0236 RT12(IU, KILL) = RT12(IU, KILL) + 1.0
0237 IF(KILL * EQ. 2) RT12(IU, 4) = ((RT12(IU, 2) - 1.0) * RT12(IU, 4) +

1.0) / RT12(IU, 2)
0238 IF(KILL * EQ. 3) RT12(IU, 5) = ((RT12(IU, 3) - 1.0) * RT12(IU, 5) +

1.0) / RT12(IU, 3)

0239 ITIME=ICHRON(0)
0240 FTIME=ITIME
0241 FTIME=FTIME/600.
0242 IF(FTIME * GE. 50.) GO TO 502
0243 IF(IJKL * EQ. 00.) GO TO 500
0244 IF(IJKL * NE. 2000) KOUT=2000
0245 K=2000*KOUT
0246 KOUT=KOUT+1
0247 WRITE(6, 79, K)
0248 WRITE(6, 78, K)

79 FORMAT(10H1 DEC TABLES AFTER 17.9H SAMPLES //)
78 FORMAT(25H1 STATISTIC TABLES AFTER 17.9H SAMPLES //)

0250 DO 505 I=1, NOWEG
0251 WRITE(6, 90) (DEC1(I, J), J=1, 5)
0252 WRITE(6, 78)

0253 DO 507 I=1, NOWEG
0254 WRITE(6, 81) (RT12(I, J), J=1, 5)
0255 500 CONTINUE

501 FORMAT(30H1 TIME EXCEEDED NO. SAMPLES = 17)

0257 GO TO 503
0258 WRITE(6, 50) IJKL
0259 503 CONTINUE
C KRT2 CONTAINS COUNT OF THE OUTCOMES WIN(TMAX), WIN(MISS), LOSS

0260 FOR PLAYER 1 PER INITIAL REGION

0261 WRITE(6, 86) I
0262 RUN 110/45HOSLEEF, FLFFT, STRAIGHT, FRIGHT, SRIGHT // 0263 510 WRITE(6, 92) I, (DEC1(I, J), J=1, 5)
0264 WRITE(6, 85) I

0265
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85 FORMAT(9i6) STATISTIC TABLE
110 AT THE END OF RUN

1 IF(NPLS.EQ.0) WRITE(6,70)

IF(NPLS.EQ.2) WRITE(6,71)

1 DO 520 I=1,NOREG

520 WRITE(7,81)I,(RT12(I,J),J=1,5)

520 WRITE(7,81)I,(RT12(I,K),K=1,5)

522 WRITE(6,44) I(DEN1)-DEN1

541 FORMAT(54H1 PROBABILITIES WIN(MAX),WIN(MISS),WIN(TOTAL),LOSS)

11100///

74 FORMAT(3H FOR PLAYER 1)

//

75 DO 550 K=1,NOREG

76 DEN1 = 0.0

77 DO 535 I=1,3

79 DEN1 = DEN1 + RT12(K,I)

550 WRITE(6,83)K,(PBS(I),I=1,4)

83 FORMAT(15,4F9.5)

555 CONTINUE

550 WRITE(6,96) I(DEN1),ACC1(1,1),ACC1(1,2),ACC1(2,1),ACC1(2,2),

1((VEL1(I)),I=1,5), DIS,DIF

589 WRITE(6,96) WIN, LLOSS

0280 CALL EXIT

END
SUBROUTINE OUTCOM(X,T,IF,IOC)

COMMON /BLK1/TMAX,ACC1,T12,V12,V1F,U1,V1,D1,MIN,KILL,120

DIMENSION X(12),ACC(12)

D.T A=0.70E0, R1/00.0000, R2/12.00/

IOC = 0

IF(ABS(X(7)).GT.3.4907E-1) X(7) = ABS(X(7))/X(7)*3.4907E-1

VXL = V1*SIGNUM(X(12)) - X(4)*COS(X(3))

VYL = V1*COS(X(12)) - X(4)*SIGNUM(X(3))

XR = X(10) - X(1)

YR = X(11) - X(2)

RC = SORT(XR**2 + YR**2)

PSIC = ATAN2(YR,XR)

RDOT = (VXL*COS(PSIC) + VYL*SIN(PSIC))

C KILL = 0 DEFAULT

C KILL = 1 AIRCRAFT WINS (EXCEEDED TMAX)

C KILL = 2 AIRCRAFT WINS (WARHEAD MISS)

C KILL = 3 AIRCRAFT LOSS (WARHEAD HIT)

KILL = 0

IF(RC.LT.40.03) GO TO 4

KILL = 1

GO TO 900

IF(TL.TLT.TMAX) GO TO 5

KILL = 1

GO TO 900

IF(RC.LT.0.04 AND RDOT.GT.0.00) GO TO 10

GO TO 900

CONTINUE

DEN = VXL**2 + VYL**2

IF(DEN.EQ.0.00) GO TO 20

TMIN = (-VYL*YR - VXL*X1)/DEN

XM = X(1) + TMIN*X(4)*COS(X(3))

YM = X(2) + TMIN*X(4)*SIN(X(3))

XT = X(10) + TMIN*V1*SIN(X(12))

YT = X(11) + TMIN*V1*COS(X(12))

RMIN = SORT((XT-XM)**2 + (YT-YM)**2)

TMIN = T + TMIN

IF(RMIN.LT.R1) PK = A

IF(RMIN.GE.R1 AND RMIN.LE.R2) PK = (A*(R2-RMIN)**2)

/((R2-R1)**2)

IF(RMIN.GT.R2) PK = 0.00

RNUM = RDM(DUM)

KILL = 2

IF(RNUM.LE.PK) KILL = 3

GO TO 900

KILL = 1

GO TO 900

WRITE(18,200)

200 FORMAT(5x,*NO COLLISION*)

PARALLEL COURSES WITH EQUAL SPEED*1)

IF(KILL.NE.0) IOC = 1

900

PARALLEL COURSES WITH EQUAL SPEED*1)

IF(KILL.NE.0) IOC = 1

900
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0001 SUBROUTINE FUNC(F,X,T)
0002 COMMON /ALK/THX,AACCI,TIZ,VIF,U1,V1,D1,RMIN,KILL,I2O
0003 DIMENSION F(12),X(12),ACCI(2,2)
0004 DATA OLDC/0.0/0.
0005 DATA P1.T.1415926E0/,TWOPI/6.283153E0/
0006 DATA RHO/1.0518E-3/, S/0.1644E0/
0007 TIME = T
0008 AMACH = X(4)/1.085E3
0009 TME = T-T17
0010 VI= 55ACCI(IZO1)TYME2+ACCI(IZO2)TYME+V1Z
0011 IF(I1ZO.EQ.1) end. VI = GT.V1F) VI = V1F
0012 IF(IIZO.EQ.1) AND. VI = LT. V1F) VI = V1F
0013 V1D1 = VI/D1
0014 TERM1 = U1 * V1U1
0015 XR = X(10) - X(1)
0016 YR = X(11) - X(2)
0017 RC = SORT(XR**2 + YR**2)
0018 VXL = V1*SIN(X(12)) - X(4)*COS(X(3))
0019 VYL = V1*COS(X(12)) - X(4)*SIN(X(3))
0020 PSIC = ATAN2(YR,XR)
0021 PSIDOT = (-VXL*SIN(PSIC) + VYL*COS(PSIC))/RC
0022 X(5) = RC
0023 X(6) = PSIC
0024 X(7) = PSIDOT
0025 THRUST = 2820.0E0
0026 IF(TIME.EQ.5.2E0) THRUST = 0.0E0
0027 AMASS = 0.211E0 - 0.35B6E0*TME
0028 IF(TIME.EQ.5.2E0) AMASS = 4.347E0
0029 CD0 = 8.E-1
0030 IF(AMACH.GT.1.5E0) CD0 = 1.5E0
0031 CDI = 60.0E0
0032 IF(AMACH.GT.1.0E0) CDI = 80.E0 - 13.6E0*(AMACH-1.0E0)
0033 IF(ABS(X(7)).GT.3.4907E-1) X(7) = ABS(X(7))/X(7)*3.4907E-1
0034 CACEL = 4.4E0*X(7)
0035 CACELY = CACEL*X(4)
0036 IF(TIME.EQ.2E0) CACEL = 0.0E0
0037 IF(PSIC.LT.0.0E0) PSIC = PSIC + TWOPI
0038 IF(PSIC.GT.1.0E0) GAMMA = X(3)
0039 10 IF(GAMMA.GT.TWOPI) GAMMA = GAMMA - TWOPI
0040 IF(GAMMA.LT.0.0E0) GAMMA = TWOPI + GAMMA
0041 IF(GAMMA.GT.0.0E0 OR.GAMMA.LT.TWOPI) GO TO 10
0042 AOFF = ABS(GAMMA - PSIC)
0043 IF(AOFF.GT.PI) AOFF = TWOPI - AOFF
0044 IF(AOFF.GT.0.69813E0) CACEL = 0.0E0
0045 PHI = X(12)
0046 IF(PHI.GT.TWOPI) PHI = PHI - TWOPI
0047 IF(PHI.LT.0.0E0) PHI = TWOPI + PHI
0048
```
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1
0049 IF(PHI.LT.0.*E0.OR.PHI.GT.TWOPI) GO TO 15
0050 IF(ALPHAC.GT.TWOPI) ALPHAC = ALPHAC - TWOPI
0052 IF(ALPHAC.LT.0.*E0) ALPHAC = TWOPI + ALPHAC
0053 IF(ALPHAC.LT.0.*E0.OR.ALPHAC.GT.TWOPI) GO TO 20
0054 SAT = 1125.*E0 + 875.*E0*SIN(2.*E0*(ALPHAC-(PI/4.*E0)))
0055 IF(ALPHAC.GT.1.*5708E0.AND.ALPHAC.LT.4.*71239E0) SAT = 2000.*E0
0056 IF(RC.LT.SAT) CACFL = 0.*E0
1
0057 OLDC = CACEL
0058 F(1) = X(4)*COS(X(3))
0059 F(2) = X(4)*SIN(X(3))
0060 F(3) = X(8)
0061 F(4) = (-RHO*S*X(4)**2*CD0/(2.*E0*AMASS)) -(2.*E0*AMASS*CACEL)**2
0062 1/(RHO*S*CD1)) + (THUST/AMASS))
0063 F(5) = 0.*E0
0064 F(6) = 0.*E0
0065 IF(ABS(X(7)).GT.3.4907E-1) F(7) = 0.*E0
0066 F(8) = (1.*E0/0.3E0)*(1.0E0*CACEL-X(8))
0067 F(9) = 1.*E0
0068 F(10) = V1*SIN(X(12))
0069 F(11) = V1*COS(X(12))
0070 F(12) = TERM1
0071 RETURN
END
```
SUBROUTINE RK1(X,T1,TF,HSTEP)

C THIS SUBROUTINE INTEGRATES THE FOLLOWING EQUATIONS
C
C XDOT = F(X,T) = F
C WHICH PROVIDES THE VALUE OF THE FUNCTIONS AT THE
C FINAL TIME TF,
C
C X(TF) SOLUTION TO THE NONLINEAR EQU.
C THE FUNCTION F MUST BE SUPPLIED BY A SUBROUTINE WITH
C THE NAME SUBROUTINE FUNC(F,X,T)
C
C DEFINITION OF THE ARGUMENTS IN THE CALL STATEMENT
C
C X = X(T1)
C T1 = INITIAL TIME
C TF = FINAL TIME
C HS = INTEGRATION STEPSIZE
C
C AFTER THE INTEGRATION IS COMPLETED THE OUTPUT IS
C
C X = X(TF)
C
C THE LENGTH OF THE STATE VECTOR MUST BE SPECIFIED BY
C THE STATEMENT ML = OR PUT IN THE CALI. STATEMENT.
C
C DIMENSION F(12), QX(12,4), X(12), YX(12)
C
ML = 12
H = HSTEP
H2 = H/2.0E0
T = TF - T1
IF(T.EQ.0.0E0) GO TO 900
TNS = ABS(T/H) + 1.0E-4
JS = TNS
TJ = JS
TT = TI
KZ = 0
IF(JS.EQ.0) GO TO 200
12 KZ = KZ + 1
TIME = TT
DO 70 IA=1,4
KA = IA - 1
GO TO (20,30,40,1A
70 HA = 0.0E0
FORTAN IV G LEVEL     21

0020 KA = 1
0021 GO TO 50
0022 30 HA = H2
0023 GO TO 50
0024 40 HA = H
0025 50 DO 60 I=1,ML
0026 60 YX(I) = X(I) + HA*QX(I,KA)
0027 TT = TIME + HA
0028 CALL FUNC(F+YX,TT)
0029 DO 70 I=1,ML
0030 70 QX(I+1A) = F(I)
0031 DO 80 I=1,ML
0032 80 F(I) = (QX(I,1)+2.0E0*QX(I,2)+2.0E0*QX(I,3)
0033 1 +QX(I,4))/6.0F0
0034 DO 100 I=1,ML
0035 100 X(I) = X(I) + HA*F(I)
0036 IF(IQC.EQ.1) GO TO 900
0037 IF(KZ.EQ.JS+1) GO TO 900
0038 IF(KZ.EQ.JS) GO TO 12
0039 200 H = T - 1-H
0040 H2 = H/2.0E0
0041 GO TO 12
0042 900 RETURN
0043 END
**FORTRAN IV G LEVEL 21**

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
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<tbody>
<tr>
<td>0001</td>
<td>SUBROUTINE SMALLI(S,W,N,WC)</td>
</tr>
<tr>
<td>0002</td>
<td>DIMENSION S(500)</td>
</tr>
<tr>
<td>0003</td>
<td>IF(N .LE. 0) GO TO 30</td>
</tr>
<tr>
<td>0004</td>
<td>C = N</td>
</tr>
<tr>
<td>0005</td>
<td>C = W/C</td>
</tr>
<tr>
<td>0006</td>
<td>C = C*WC</td>
</tr>
<tr>
<td>0007</td>
<td>DO 100 I = 1, N</td>
</tr>
<tr>
<td>0008</td>
<td>F = I</td>
</tr>
<tr>
<td>0009</td>
<td>IF(W .EQ. 1.) C = 0.</td>
</tr>
<tr>
<td>0010</td>
<td>100 S(I) = 1. - F*C</td>
</tr>
<tr>
<td>0011</td>
<td>99 FORMAT(10H SMALL S =/(1PE20.7))</td>
</tr>
<tr>
<td>0012</td>
<td>30 RETURN</td>
</tr>
<tr>
<td>0013</td>
<td>END</td>
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</tbody>
</table>

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MIS04240
MIS04250
MIS04260
MIS04270
MIS04280
MIS04290
MIS04300
MIS04310
MIS04320
MIS04330
MIS04340
MIS04350
MIS04360
MIS04370
SUBROUTINE BIGI(S,W,N,WC)

DIMENSION S(500)

IF(N.LE.0)GO TO 30

C = N
C = (W-1.)/C
C = C*WC
DO 100 I=1,N
F(I) = 1
100 S(I) = 1. + F(I)

99 FORMAT( 9H BIG S =/(1P6E20.7) )

N=1=1)I(S(I)99.6(ETI

30 RETURN

END
SUBROUTINE ROTIN(M,ROE,OMG,THE)

DIMENSION M(11,12,1),R(12),O(13),T(2)

DATA R/0...250...500...1000...200...300...500...800.../

DATA O/0...30...60...90...120...150...180...210...240...270.../

DATA T/0...0.../

DO 100 I=1,11

K=M(11,12,1)

100 CONTINUE

IF(K.EQ.1)GO TO 110

ROE = R(1I) + RDM(DUM) * (R(I+1)-R(I))

OMG = O(12) + RDM(DUM) * (O(I+1)-O(I))

THE = T(I3) + RDM(DUM) * (T(I3+1)-T(I3))

RETURN

END
SUBROUTINE REG(N,ROE,OMG,TME)
DIMENSION R(I2),O(I3)
DATA R/0.250,0.500,1.000,2.000,3.000,4.000,8.000,
112.000,1.600,2.400,4.800,
DATA O/0.300,0.90,1.20,1.50,1.80,2.10,2.40,2.70,
3.00,3.30,3.60,
IF(R(I) .GT. ROE .OR. ROE .GT. R(I+1)) GO TO 90
IF(ROE .LT. 0.0) GO TO 900
NR = 1000
GO TO 110
DD 100 1=1,11
IF(ROE .LE. R(I) .OR. ROE .GT. R(I+1)) GO TO 140
NR = I + 1000
GO TO 110
100 CONTINUE
110 IF(OMG .NE. 0(I)) GO TO 115
NC = 10
GO TO 130
115 DD 120 1=1,12
IF(OMG .LE. O(I) .OR. OMG .GT. O(I+1)) GO TO 120
NO = 1 + 10
GO TO 130
120 CONTINUE
130 NT = 1
N = NR + NO + NT
130 RETURN
024 WRITE(6,99)ROE,OMG
025 FORMAT(10I8,1X,1AH,ERROR IN REG,ROE =.1PE18.5, OMG =.1PE18.5)
027 CALL EXIT
END
SUBROUTINE INI
DIMENSION (11, 12, 13)
I3 = 1
K = 0
DO 100 I1 = 1, 11
DO 100 I2 = 1, 12
DO 100 I3 = 1, 13
100 M(I1, I2, I3) = K
IF (K .NE. 0) GO TO 120
WRITE(6, 99) K
99 FORMAT(52H) SUBROUTINE INI DEFINES THE FOLLOWING ORDER FOR THE
114, 10H ROWS OF M //29H 11, 12, 13 M(11, 12, 13) //)
DO 110 I1 = 1, 11
DO 110 I2 = 1, 12
110 WRITE(6, 98) I1, I2, I3, M(I1, I2, I3)
98 FORMAT(10X, 3I5, , 5X, 15)
120 RETURN
END