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JET NOISE:
A SURVEY AND A PREDICTION
FOR SUBSONIC FLOWS

ENGINE TEST FACILITY
ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
ARNOLD AIR FORCE STATION, TENNESSEE 37389

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DIRECTORATE OF TECHNOLOGY
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The state-of-the-art of the prediction of turbulent jet noise is surveyed. This survey includes a description of the available experimental data on subsonic and supersonic, cold and hot jets, and of present theoretical treatments of the mechanisms of turbulent jet noise production. A detailed analysis of the production of subsonic cold jet noise, based on the acoustic analogy formulation, is described, and results of computations using this analysis

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20. ABSTRACT (Continued)

and a turbulent kinetic energy analysis of the jet flow field are presented and compared with representative experimental data.

PREFACE

The work reported herein was conducted by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). The results were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee, under Program Element 65807F. The work was done under ARO Project Numbers RF438 and R32P-52A. The author of this report was Philip T. Harsha, ARO, Inc. The manuscript (ARO Control No. ARO-ETF-TR-74-115) was submitted for publication on November 22, 1974.

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1.0 INTRODUCTION

Jet noise is a phenomenon which is "to a greater or lesser extent, inseparable from turbojet and rocket propulsion" (Ref. 1). Its existence in aerospace applications of turbojet propulsion has generated the greatest amount of public comment and research effort, but jet noise sources exist in other, more commonplace areas. For example, the hiss of steam or compressed air escaping from a pressure relief valve is an industrial illustration of jet noise quite common at AEDC, as is the noise of air or liquid streams in valves. In the latter case, the jet noise problem is compounded by the existence of separated flow and interaction of the acoustic field with enclosing pipes or ducts. An even more common example of jet noise is the hiss of the escaping spray from aerosol cans.

However, it is in aerospace applications that the most serious research into the mechanisms and reduction of jet noise is being carried out. This work began in earnest with the advent of the commercial jet transport, for although such aircraft provided great gains for the airlines and the airline passenger, they were, and are, undeniably noisy. The first attempts at reducing jet noise centered on the development of suppressors, which were designed to break up the hot, high-speed exhaust jet and promote its mixing with the ambient airflow. Such devices worked, often in spite of aeroacoustic theory, but they both increased the weight of the engine and reduced its performance.

A better answer, both technically and from the economic viewpoint of the user of the transport, was the introduction of the turbofan engine. Theory indicates that the overall sound power output of a turbulent jet is proportional to the eighth power of one-dimensional jet velocity (U_j^8) (Ref. 2), and so decreasing the average jet velocity by surrounding the central high-speed jet with a lower speed cold annular jet from the fan should reduce the noise. Another way of looking at this phenomenon is to note that the outer, lower velocity stream serves to shield the central jet, so that it is primarily the slower, cooler, outer stream which contributes to the noise signature.

The advent of the high-bypass-ratio fan jet, such as the TF39/CF-6 family used in the C-5A and DC-10, the JT9D family used in the 747, and the RB211 used in the L-1011, reduced the jet noise of these transports to the point where the major noise contributions were caused by the fan itself. Research emphasis then shifted to efforts to reduce this source of noise by suitable design and the use of sound absorbing materials in the fan duct. Two factors have recently combined to generate a resurgence of interest in the subject of jet noise. One is that the efforts to quiet the rotating machinery noise in high-bypass-ratio turbofan engines have been successful to the point that jet noise is again a major contributor to the overall noise of a turbofan engine. In addition, a new family of quiet short-haul aircraft is in active development, and successful operation of these aircraft will require a further reduction in engine noise power output.

This report is divided into two main parts. In the first part, a survey of the present state of knowledge of aeroacoustic phenomena will be presented. This survey will cover both theoretical and experimental work on subsonic jets, supersonic jets (including the phenomena of screech and shock-associated noise), and combustion noise. In it, both the acoustic analogy and convected wave equation approaches to the analysis of turbulent noise will be discussed, as will the issue of large-scale structure in turbulent jets. The second part of this report will include a discussion of the application to the aeroacoustic problem of the AEDC-developed turbulent kinetic energy analysis technique for the prediction of free turbulent jets. It will be shown that this method provides a reasonably accurate technique for the prediction of subsonic, cold jet noise contours at angles greater than 20 deg from the jet axis, without the need for empirical input information. Full details of this theoretical development, which proceeds from the acoustic analogy theory, will be given.

2.0 SURVEY OF JET NOISE LITERATURE

2.1 GENERAL FEATURES OF JET NOISE

The subject of aeroacoustics, i.e., the study of noise produced aerodynamically, can fairly be said to have begun with the work of Lighthill, in 1952 (Ref. 2). As Doak (Ref. 3) has pointed out, other workers had touched on the edges of this subject, but it is with the publication of Ref. 2 that the modern work begins. Thus, the subject of aeroacoustics is only some twenty years old, and serious controversy still exists over many of its aspects. However, the general features of aeroacoustic phenomena are generally agreed upon and have been described in several review papers which have been published since Lighthill's early work (Refs. 4-14).

In all turbulent jets, turbulent fluctuations are a major source of sound. For subsonic, cold, steady, turbulent jets, the turbulent fluctuations are the only source of sound. Lighthill's "acoustic analogy" formulation, put forward in his 1952 paper (Ref. 2) shows that the far-field noise produced by a turbulent jet can be related to the Reynolds stress term in the equation of motion describing a turbulent jet and can be identified as an acoustic quadrupole. An acoustic quadrupole is a collection of four equal and opposite sources arranged as shown in Fig. 1. These sources would exactly cancel if it were not for the small distance between them. Because there is a small distance between them, the contribution of each source is heard at slightly different times at the far-field position of an observer, and so cancellation is not complete. Such an arrangement is obviously an inefficient producer of sound; even for a high-temperature, high-speed jet, the acoustic efficiency, defined as the ratio of the acoustic power radiated to the jet power, is on the order of 0.5 percent (Ref. 1). But, for example, at the liftoff of a Saturn V rocket, this represents 2 x 10^8 watts of radiated acoustic power (Ref. 12), so that even though turbulent noise production is an inefficient process, prodigious amounts of energy may still be generated.

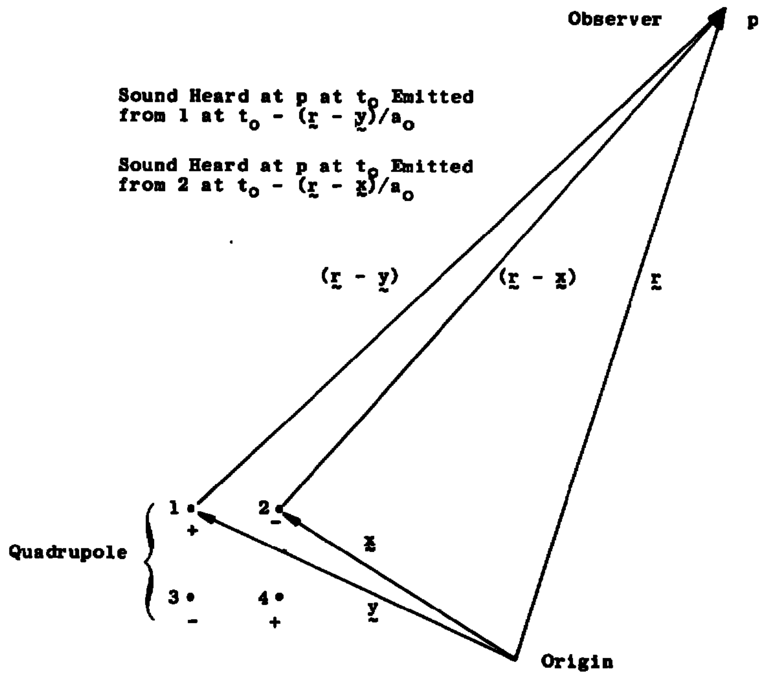

Figure 1. Schematic of quadrupole noise radiation.

An alternate way of looking at turbulent jet noise production involves consideration of fluid dilatations (Refs. 9, 10, 15, and 16). In this view, as described, e.g., in Ref. 15, fluctuating accelerations in the flow are associated with unsteady pressures within the flow or nearby. In response to the pressures, the fluid elements contract or expand very slightly. These dilatations act essentially like simple sources (Refs. 15 and 16) to radiate sound to the far field. The fluctuating pressure field includes both the radiated sound and "pseudosound." The latter is the incompressible pressure fluctuation that balances the velocity fluctuation and represents no loss of energy from the jet (Ref. 13); at low speeds, the pseudosound field is the dominant part of the pressure "fluctuation," and only a very small proportion of the pressure field represents radiated sound.

In recent years, a number of investigators have become concerned with the possible existence of a large scale underlying structure in free turbulent flows. This interest was stimulated by some remarkable high-speed motion pictures of a two-dimensional shear layer (Ref. 17), which appear to show the existence of a wave-like coherent structure within the turbulent flow, and by experiments carried out to investigate the effects of forcing a periodic motion on a circular jet (Ref. 18). While such structures, if they exist in a general turbulent flow, would not, by themselves, generate turbulent noise, since noise generation demands a time-rate-of-change of the turbulence structure, they could be responsible for providing the coherence between the small eddies necessary for noise to be produced (Ref. 19). Intensive investigations of large-scale phenomena in turbulent jets are currently under way.

In aeroacoustic studies, a turbulent jet is commonly divided into three regions, as shown in Fig. 2. Immediately downstream of the nozzle lip a quasi-two-dimensional "shear layer" regime develops, as the boundary layers on the nozzle wall adjust to the sudden removal of the wall constraint. In engineering practice, the boundary layers on the walls are commonly turbulent, but in small scale jet experiments, these boundary layers may be laminar or transitional. This difference between laboratory experiments and full-scale jets may be significant; at the very least, there are substantial differences in the initial rate of development of the shear layer (Ref. 20). Although an initially laminar shear layer will quickly become turbulent at the Reynolds numbers of interest in aeroacoustic studies, the transition process is poorly understood and may introduce large-scale flow instabilities which persist longer or are stronger than those which would develop in an initially turbulent shear layer. This point needs further investigation, and the obvious and less-obvious differences between full-scale and laboratory jets should be kept in mind when comparing results and predictions.

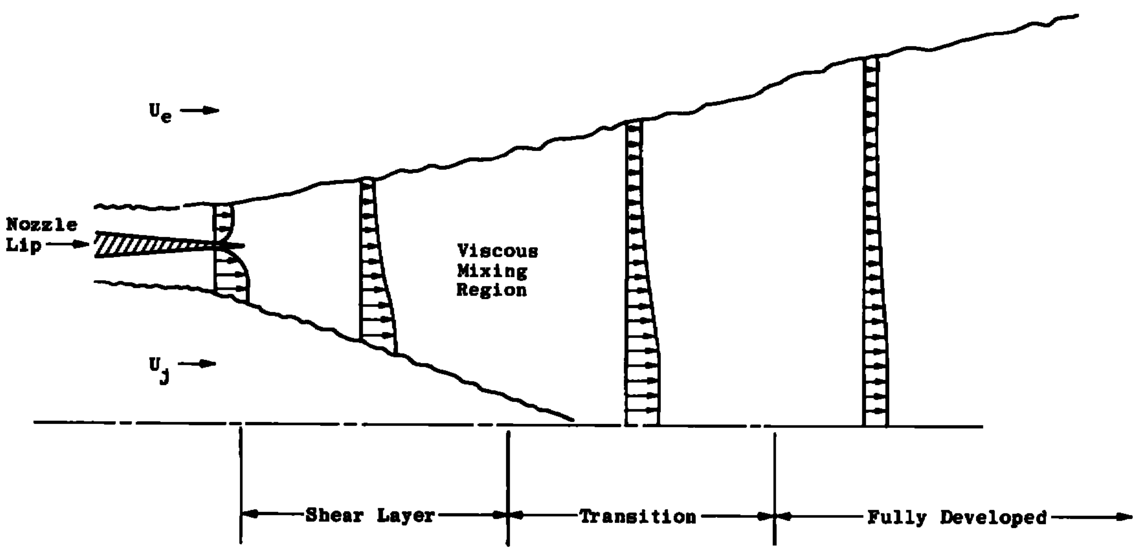

Figure 2. Jet flow field.

Downstream of the shear layer region (Fig. 2), the jet flow passes through a region in which the shear layers reach the jet centerline and an axisymmetric flow field develops. This is generally termed the "transition" region, although the label is inappropriate in that it does not refer to the more common use of the word transition in fluid mechanics, denoting the change from a laminar to a turbulent flow. However, since the use of the word "transition" to describe the region shown in Fig. 2 is widespread in the aeroacoustic literature, this usage will be retained in this report.

The third region of interest in a turbulent jet is the "fully developed" region which follows the transition region, at an axial distance depending on the parameter used to characterize the nature of the flow. Thus, if the mean velocity profile is used to characterize the jet, the fully developed region will be found to begin closer to the nozzle exit than if the turbulent shear stress profile is used to describe the flow. In either case, the fully-developed region is considered to exist when the profiles of velocity or shear stress, suitably normalized, no longer change with distance.

Several investigations, both theoretical and experimental, have been concerned with the noise power output of the three regions of a turbulent jet. For example, Lighthill (Ref. 1), from dimensional arguments based on his acoustic analogy theory, reasoned that in the shear layer the sound power output per unit length should be constant (i.e., follow an "x^0" law) while in the fully developed region, it should vary as $x^{-6}\ell^{-1}$, where x is the distance from the nozzle and ℓ is a correlation length scale proportional to the jet width. Far enough into the fully developed region that $\ell \propto x$, this yields an x^{-7} law for sound power output per unit length for subsonic turbulent jets. A similar conclusion was reached by Ribner (Ref. 21) and by Lilley (Ref. 22). While experimental confirmation of this result is difficult, since it involves the location of sources in a turbulent jet, the "x^0, x^{-7}" law has been supported by several experiments, notably those of Lee (Ref. 23) and Dyer (Ref. 24). However, for <u>supersonic</u> jets, experiment has shown that the turbulent sound power output is proportional to x^1 in the shear layer region (Ref. 25).

It is generally accepted that most of the sound power output of cold, subsonic turbulent jets occurs in the shear layer or possibly in the transition regime. For hot subsonic jets, this also appears to be true, although the simple scaling appropriate to cold subsonic jets is not appropriate (Ref. 26). Supersonic jets introduce new phenomena, including screech (Refs. 27 and 28) and shock noise (Refs. 29 and 30), in any flow in which the jet is not perfectly expanded. These phenomena alter the axial distribution of the noise sources, introducing, in the case of screech, discrete frequency spikes to the noise signature, while shock noise produces a broad-band increase in the noise output in the region of shocks embedded in the imperfectly expanded jet.

One of the most obvious features of jet noise is its directivity. If a noise contour observed far from a subsonic turbulent jet is plotted on a contour plot ("far" is defined as the region whose distance from a noise source substantially exceeds a wavelength, in a given narrow band of frequencies, Ref. 7), the curve shown schematically in Fig. 3 will be obtained. Characteristically, there will be a marked reduction in the forward quadrant (opposite to the direction of motion of the jet flow) and a "dimple" along the jet axis in the direction of motion of the jet. A considerable amount of theoretical and experimental work has been involved with the prediction and understanding of the effects which produce the directivity of jet noise. The reduction opposite to the direction of motion is known to be caused by the motion of the noise sources in the jet; this is the "convection" effect first described by Lighthill (Ref. 2) and corrected by Ffowcs Williams (Ref. 31). The "dimple" in the rear quadrants has been extensively investigated by Ribner and his students, and has been shown to be caused by refraction of the sound by velocity gradients within the jet (Refs. 32 to 35). Whether refraction merely modifies the noise distribution or has an effect on the source strengths themselves is a matter of some controversy (see, e.g., Refs. 3 and 36). The convection effect involves, in addition to the directivity effect shown in Fig. 3, an amplification of the noise produced and a Doppler shift in its frequency (Ref. 31).

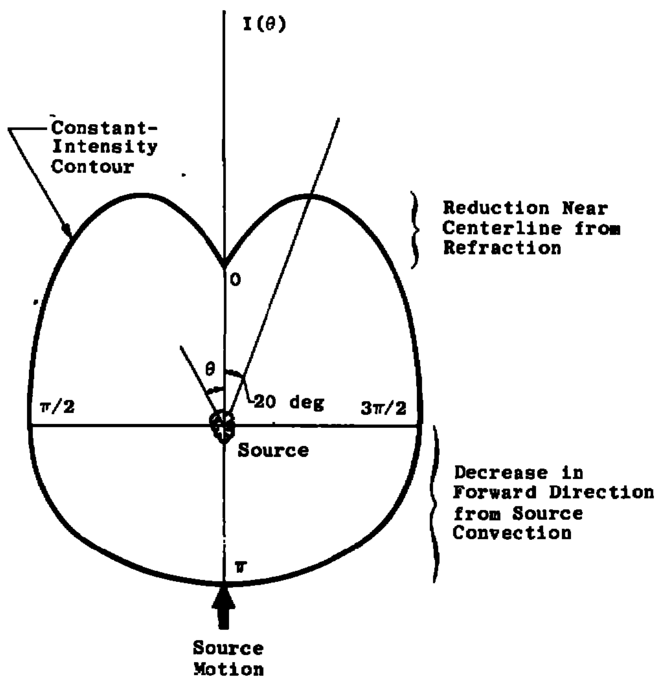

Figure 3. Directivity of a subsonic jet noise.

The introduction of additional noise sources in supersonic jets has already been touched upon. Another question of great technological importance involves the effects of density and temperature gradients such as exist in hot subsonic jets. Apart from the obvious effects on directivity that a gradient in acoustic velocity would produce, do hot jets involve additional source mechanisms such as density and entropy fluctuations? This subject is still embroiled in controversy. Although a broad and fundamentally correct formulation has been presented by Doak (Ref. 3), which makes it clear that additional source mechanisms do exist, the importance of these mechanisms has not been verified, and indeed the ability of the acoustic analogy theory to explain some of the effects observed in hot jets without introducing additional sources has been defended (e.g., Ref. 37). Despite the intensive analytical work now in progress on this subject, at present only empirical rules for the effect of density gradients have been deduced (Refs. 26 and 38), and these rules are quite complicated. Finally, investigation of the technologically important phenomenon of combustion noise has only recently been begun (Refs. 39 to 42).

In the parts of this section to follow, a fairly detailed survey of the currently published experimental and analytical work on turbulent jet noise will be presented. An attempt has been made in this survey to include all of the readily available papers and reports on turbulent jet noise in order to provide a complete bibliography for future research. This review includes material published through June 1974.

2.2 EXPERIMENTAL WORK

Experimental studies of aeroacoustic phenomena have been carried out almost continuously since the early 1950's, and as in many technological fields, there has been a near-exponential increase in the number of papers published per year since the late 1960's. The subsonic turbulent jet into still air has received the most detailed investigation, including studies of the overall sound power level of the noise produced, its frequency spectrum, and investigations of the location of the aeroacoustic sources in the jet. Other subsonic turbulent jet experiments have been concerned with propagation effects such as source convection and refraction and with the basic mechanisms of jet noise production.

Hot subsonic and cold and hot supersonic jets have also received much attention, although not in the detail which characterizes most subsonic jet experiments. This is, of course, a function of the complexity of hot subsonic and cold and hot supersonic jet flows, as well as the difficulty involved in instrumenting such flow fields. Thus, supersonic jet experiments and experiments on subsonic hot jets have primarily been concerned with measurements of the far-field noise produced by such jets, although some studies of basic mechanisms in supersonic jets have been carried out.

2.2.1 Experiments in Subsonic Cold Flow

Subsonic cold jets are the easiest flow fields to set up in a typical laboratory situation which still exhibit many of the more important aeroacoustic features of "real" (non-laboratory) flows. Because of this, they have been extensively investigated, with the investigations aimed at three major goals: noise measurements, investigations of propagation effects, and the investigation of basic noise mechanisms.

2.2.1.1 Overall Noise Measurements

Fairly typical of early detailed measurements on turbulent jet noise is the work of Gerrard (Ref. 43) in which the far-field noise intensity from a set of jets produced by a fully developed pipe flow was investigated. Several Mach numbers were studied, and intensity contours at several frequencies were measured for each Mach number. These measurements qualitatively substantiated the U^8 law put forward by Lighthill. Further confirmation of the Lighthill overall sound power level scaling law was presented in Ref. 44 from measurements obtained in a reverberation chamber.

Total sound power (i.e., summed over all frequencies) was presented by Howes (Ref. 45) who collected data from a number of sources and correlated them against the so-called "Lighthill parameter." This parameter, which is a result of dimensional reasoning based on Lighthill's acoustic analogy theory (Ref. 2) states that the total sound power (w) of a subsonic jet is given by the relation:

$$w = K_L \rho_o A U^8 / a_o^5 \qquad (1)$$

where K_L is the Lighthill constant, ρ_o and a_o are the density and acoustic velocity in the free-field, respectively, A is the jet nozzle exit area, and U is the (one-dimensional) jet exit velocity.

Although most early experiments supported the U^8 law (Eq. (1)), under some conditions other investigators found that the U^8 relationship was not followed even for subsonic cold jets. Thus, Bushell (Ref. 46) found that the overall sound pressure level (OASPL) of subsonic turbulent jets deviated from the eighth power law for velocities below approximately 500 ft/sec; he attributed this to the noise produced by fluctuations in the flow at the nozzle exit which, as pointed out, e.g., in Ref. 47, could follow a U^6 law. Other evidence of deviation from the U^8 law were observed by Gruschka and Schrecker (Ref. 48) in two dimensional jets and at some observation angles in circular jets by Ahuja and Bushell (Ref. 36). Other measurements of overall sound power levels are presented in Refs. 36, 38, 49, and 50. Measurements of the noise radiated by subsonic coaxial jets were presented by Williams, Ali, and Anderson (Ref. 51).

2.2.1.2 Frequency Spectra

While the OASPL of a jet may be the noise quantity most easily measured, the frequency spectrum that the jet produces is of interest both because of its impact on the annoyance created by a turbulent jet and because of the information it provides with regard to the location of noise sources and refraction and source convection phenomena. The effect of frequency on the annoyance factor is caused by the fact that different frequencies of sound are perceived as more or less annoying by a human observer. Thus, two jets of equal OASPL (integrated over all frequencies) may be perceived as being more or less objectionable depending on their frequency content. Further, since the effects of refraction depend on the ratio between the wavelength of sound and the thickness of the region of the jet over which significant gradients occur, different frequencies will be affected differently by refraction.

Investigations of the frequency content of subsonic jet noise have been carried out primarily since 1969. In that year, Krishnappa and Csanady (Ref. 52) reported a series of measurements of measured sound pressure levels in various 1/3-octave frequency bands, as a function of observation angle, for two subsonic jet Mach numbers (M_j). They found that most of the noise produced by the jet at $M_j = 0.63$ could be interpreted as caused by the interaction of the turbulence in the flow field with the mean velocity gradient; called "shear noise," this interpretation of the noise production mechanism, which follows from an interpretation of the source term in the Lighthill theory (Refs. 2, 53, and 54), will be described in the next section of this report. Krishnappa and Csanady also investigated the influence of vortex generators at the nozzle exit on jet noise production, finding that the presence of such devices does not radically modify the frequency composition of the noise from the jet.

A careful series of measurements, made in an anechoic chamber on a circular jet, 25 mm in diameter, over a jet velocity range of from 90 to 300 m/sec were reported by Lush (Ref. 49) in 1971. In these experiments, noise measurements were made as functions both of frequency and angle of observation. Lush found that the Lighthill eighth power law held for both acoustic power and intensity, and that for angles from the jet axis $\theta > 45$ deg (see Fig. 3), the center frequency of the 1/3-octave band at which the peak noise radiation occurs scales with the Strouhal number fD_0/a_0, where f is the center frequency at the point of noise emission, D_0 is the jet diameter, and a_0 is the ambient speed of sound. For angles less than 45 deg, the appropriate scaling parameter was found to be the Strouhal number based on the center frequency measured at the observer's location. This frequency is different from that at the source since the sources in the jet are moving, and thus the frequency is Doppler shifted.

If the Lighthill theory is compared with measured noise in different frequency bands at different angles to the jet axis, Lush finds that the theory fits the experimental data well at low frequencies ($fD_o/a_o < 0.15$) at most observation angles and at higher frequencies at θ = 90 deg; however, the theory overestimates the measured sound power level at high frequencies close to the axis. This discrepancy occurs at a critical frequency which is inversely proportional to the jet diameter and increases with emission angle but is independent of jet velocity. At jet Mach numbers >0.2, this critical frequency is lower than the peak frequency, resulting in a reduction in the noise from the level that would be predicted by a straightforward application of subsonic jet noise theory.

Other detailed investigations of the frequency spectra of turbulent jet noise are presented in Refs. 26, 36, 50, and 55 to 59. Of these, Ahuja (Ref. 50) and Ahuja and Bushell (Ref. 36) carried out the most extensive measurements of cold jet noise; these experiments complement the work of Lush (Ref. 49). Ahuja and Bushell (Ref. 36) made measurements of the far-field noise of three subsonic cold air jets, with diameters of 1.52, 2.40, and 2.84 in.; the velocity range was from 200 to 1000 ft/sec. As did Lush, Ahuja and Bushell found that at 90 deg to the jet axis the overall sound power levels of the jets were proportional to U_j^8, as expected. However, they found a higher power law for $\theta < 90$ deg, reaching a peak of U_j^9 at 20 deg to the jet axis. Again in agreement with Lush, they found that the center frequency at peak emission at the angle of peak emission could be predicted by $ST_P = f_P D_o/a_o = 0.2$, and that Lighthill's theory agrees with noise measurements for $ST < 0.2$ and $\theta > 45$ deg; at higher frequencies, it overestimates the noise emission, and at lower frequencies it underestimates the noise emission.

2.2.1.3 Noise Source Location

One of the major areas of recent interest in the jet noise problem is the development of techniques to establish the location of the noise sources in a jet. The problem has obvious relevance, since establishment of the source locations would also help to establish the source mechanisms and to define the portion of the jet in which suppression techniques might be used to best effect. However, attempts to define noise source locations have been recently criticized on theoretical grounds (Ref. 60). This criticism is based on Kirchoff's theorem, stating the equivalence of the exterior field generated by a source distribution interior to a closed surface and that generated at the surface by a suitable distribution of sources. This effect can be interpreted (Ref. 60) to mean that the interior source distribution of the jet cannot be determined from measurements in the exterior field. The source distribution can only be determined to within an arbitrary function with zero normal gradient at the imagined closed surface surrounding the noise generating region (the jet).

Source location measurements in subsonic jets have been reported by several investigators (Refs. 23 and 61 to 68), using a number of different experimental techniques. Lee (Refs 23 and 68) used a technique in which the signals of a microphone probe in the far field and a hot-film probe in the jet were cross-correlated. He interpreted his results to mean that there were approximately 2500 independent sources in the mixing region of a subsonic jet. Grosche, Jones, and Wilhold (Ref. 61) used an acoustic mirror technique to isolate noise sources within subsonic and supersonic jets, and Maestrello (Ref. 62) used an acoustic image plane technique to perform similar measurements. Both of these techniques indicated that the bulk of the noise radiated was generated in the transition regime of the jet (Fig. 2); a similar result was obtained by measuring density fluctuations using an infrared absorption cross-beam technique and relating the density fluctations to the noise source distribution (Refs. 64 and 65). Further work with the hot film probe and microphone technique, termed the "causality correlation" technique by Siddon (Ref. 66) has been reported in Refs. 63, 66, and 67.

2.2.1.4 Refraction and Convective Amplification

Propagation effects include the effects of refraction and convective amplification, which modify the directivity and also the apparent source strength in turbulent jet noise. Refraction effects arise because the wavelengths of jet noise are, for high frequencies, comparable to the scale of the jet, so that distortion of the sound waves can be produced on passage through the jet. Under some circumstances, the phenomena which produce refractive effects may also alter the strength of the noise sources themselves. Convective amplication arises in the Lighthill acoustic analogy because the noise sources are assumed to be in motion in an equivalent acoustic medium at rest. This results in an apparent increase in the strength of the eddy emitters which is a function of observation angle and jet Mach number and which, thus, in part, produces the observed directivity of jet noise. Another measurement of interest in jet noise work is the measurement of the convection velocity, i.e., the velocity of the frame of reference moving with the eddies in the turbulent jet. In the Lighthill theory and other acoustic analogy theories, such a convection velocity is assumed to exist, and this velocity is used to define the convective amplification in a particular region of the jet.

The effects of refraction in a cold jet were first observed by Gerrard (Ref. 43), who, however, explained the directivity he measured entirely by the convective amplification effect. The isolation of the refraction effect and the proof that refraction was the cause of the "dimple" in the typical directivity curve for a subsonic jet was the subject of considerable effort by Ribner and his students (Refs. 32, 33, 34, and 69). These experiments involved the placement of a harmonic acoustic point source on the centerline of a jet and observation of the directivity of the source with and without the jet flowing.

The results showed conclusively that refraction created the intensity minimum on the centerline of the jet, and that the minimum deepens as either the jet velocity or sound frequency increases. These investigations also showed that both mean velocity and temperature gradients produce refraction effects, which become more important as the wavelength of the sound approaches the scale of the gradient.

The basic directivity of jet noise, i.e., the directivity that remains if the effects of refraction and convection are factored out of the measurement, is one important clue to the form of the noise sources. For example, by postulating a particular form for the quadrupoles that provide the basic noise sources in his model of the turbulent noise mechanism, Lighthill (Ref. 5) arrived at a "four-leaf-clover" shape for the basic directivity. Chu (Ref. 70) investigated basic directivity by evaluating the moving frame noise source autocorrelation coefficient. He obtained a basic directivity $\propto 1 + 2.6|\cos\theta|$; Ribner (Ref. 71), in a later analysis based on Chu's experimental work, modified this result to indicate a basic directivity of the form $A + B(\cos^2\theta + \cos^4\theta)/2$. Since the analysis indicated that A and B were of comparable magnitude, this expression shows that the basic noise pattern in any plane through the jet axis resembles an ellipse of modest eccentricity. Further experiments on the basic directivity of turbulent jet noise have been carried out by MacGregor, et al. (Ref. 34), extending the previous results to measurements at various frequencies. The "flattened ellipse" shape is recovered, but variations at different frequencies are noted.

Investigations of the apparent directivity of jet noise, i.e., that observed in typical anechoic chamber measurements of model jets, have been reported for subsonic jets (e.g., in Refs. 29, 34, 36, 49, 52, 58, 72 and 73). Characteristically, these reproduce the heart-shaped pattern shown schematically in Fig. 3.

For the acoustic analogy theory of Lighthill to be valid, the "compactness" condition $\omega\ell/a_o \ll 1$ must be satisfied, where ω is the circular frequency of a turbulent pressure or density fluctuation and ℓ is a measure of the correlation length. As Lighthill shows (e.g., Ref. 7), this condition is much more readily satisfied if the turbulent fluctuations are viewed in a coordinate frame moving with the convection velocity of the turbulence, in which the frequency ω is much reduced over that which is observed in a fixed frame. This reduction in frequency is easy to observe in the case of a completely frozen (in time) pattern: in this case $\omega = 0$ in the moving frame, but in a fixed frame $\omega \propto U_c/\ell$ where U_c is the convection velocity. The application of the acoustic analogy theory then requires the definition of a convection velocity; measurements of the convection velocity have been obtained by Davies, Fisher, and Barratt (Ref. 74), Bradshaw, Ferriss, and Johnson (Ref. 20), and Wooldridge and Wooten (Ref. 75). In general, the convection velocity can be deduced by forming an envelope of the velocity fluctuation autocorrelation curves

$\overline{u(t)u(t + \tau)}$ versus τ at various separations in the direction of the jet flow (see, e.g., Ref. 74). Using such a technique, Davies, et al., found $U_c \approx 0.6U_j$ where U_j is the velocity at the jet centerline, with the measurement of U_c being valid over the larger part of the mixing layer region of the jet. Bradshaw, et al., gave U_c a somewhat higher value, varying across the mixing layer at $x/D_o = 2$ from 0.8 U_j near the inner edge to 0.1 U_j near the outer edge. Wooldrige and Wooten (Ref. 75) found that if narrow-band autocorrelations were studied, a relatively strong variation of convection velocity with signal frequency was observed, with the convection velocity observed at the higher frequencies being very close to the broadband value. Davies, et al. (Ref. 74), also estimated ω in terms of the rms velocity fluctuation (u'); obtaining $\omega\ell \approx 1.1$ u', from which it can be seen that the compactness condition is well satisfied at least for low subsonic jets.

2.2.1.5 Basic Mechanisms of Jet Noise

Lighthill's acoustic analogy theory showed that the noise source term for a turbulent jet could be written in terms of correlation of the velocity fluctuations in a turbulent jet (Refs. 1, 2, 5, 7, and 8); the Meecham (Ref. 16) and Ribner (Ref. 15) acoustic analogy theories showed that equivalently the source could be written in terms of correlations of the fluctuating pressure. However, neither theory could be used to predict noise production without some measurements of these correlations. But, there is little hope of experimentally evaluating the full aeroacoustic source because, in each case, fourth time derivatives of a product of two two-point correlation functions are required to define the source term. Experiments can lead to scaling and approximating rules, however, and a number of experimental studies of circular jets has been carried out to investigate aspects of the jet noise problem from the standpoint of basic mechanisms. Both Davies, et al. (Ref. 74), and Bradshaw, et al. (Ref. 20), report measurements of the turbulent velocity fluctuation correlation scales and spectra in the jet mixing region. Further measurements of these quantities have been reported in Refs. 76 to 78.

Both the Meecham (Ref. 16) and Ribner (Ref. 15) theories describe the far-field noise radiation in terms of correlations of fluctuating pressures in the source field. One of the advantages claimed for this approach is that the pressure, a scalar, seems to be a more accessible quantity than the velocity, a vector. However, serious difficulties have been encountered in attempts to measure the fluctuating pressure field in the jet. Hot-wire measurements by Ko and Davies (Ref. 79) were interpreted as reflecting density fluctuations in the potential core of a circular jet, which are related to pressure fluctuations in the mixing layer. Fuchs (Ref. 80) used a microphone probe to measure the fluctuating pressure field in the mixing zone (but not the pressure-fluctuation source term itself) and found that the pressure field was very highly correlated over the jet cross section for the first 6 to 8 diameters of the jet. This implies that the assumption of small correlation volumes (i.e., "compact" sources) is not valid. Further work on the development of fluctuating pressure instrumentation is described in Refs. 63, 73, 78, 81, and 82. Several of these experiments (e.g. Ref. 78) suggest that a large slowly varying pressure field structure exists in the jet flow, slowly convecting downstream. The convection velocity of this pressure structure is approximately constant across the mixing layer, so that the evolution of the structure in a convected frame is very slow.

2.2.1.6 The Question of Large Scale Structure

The apparent existence of a large-scale structure in the pressure field is perhaps related to the large-scale velocity field structures that are currently being observed in a variety of flows. Evidence for a large-scale structure in a turbulent jet mixing layer was described by Bradshaw, et al. (Ref. 20), although they described this structure in terms of Townsend's large eddies (see Grant, Ref. 83) rather than in terms of the currently popular coherent structures. In a later experiment, Wooldridge, et al. (Ref. 77), obtained power spectral density curves of turbulent intensity measurements which showed a peak at a frequency which scaled as a Strouhal number; they found also that the energy represented by this peak increased as the flow moved downstream. Crow and Champagne (Ref. 18) were able to show, by forcing oscillations in a jet flow, that the jet acts as an amplifier, so that the fundamental wave generated by the forcing grows in amplitude downstream. This growth continues until nonlinearity generates a harmonic, which then retards the fundamental until the two attain a saturation intensity independent of the intensity of the forcing wave. This saturation intensity is largest for a Strouhal number of 0.3, leading to the interpretation that the wave at ST = 0.3 is in some sense the wave least capable of generating a harmonic and thus the most capable of reaching a large amplitude before saturating. Large-scale structures or periodicities in subsonic jet flows have also been observed in Refs. 80 and 84. However, Siddon (Ref. 66) argues that the number of uncorrelated sources observed in "causality correlation" measurements made in his and Ribner's laboratories implies little effect of the large scale structure, if it exists, on turbulent noise production. (Note though that the measurements made by Fuchs (Ref. 80) indicate essentially no uncorrelated sources in a jet.) Another objection to the large scale structure hypothesis is that no measurements indicating its existence have been carried out at Reynolds numbers typical of engineering practice, and the damping inherent in turbulent diffusion and dissipation at these Reynolds numbers may destroy a coherent structure before it can develop to the scale observed at lower Reynolds numbers. In all likelihood, the hypothesis put forward by Arndt and George (Ref. 19) will prove to be the correct one: at high Reynolds numbers the underlying structure lies too deeply buried within the fine-scale structure of the turbulence to be extracted except by sophisticated spectral means (see the "orthogonal decomposition" procedure described in Ref. 19), and it does not produce noise of itself but provides the necessary coherence for the noise-producing smaller eddies.

2.2.2 Experiments on Hot Subsonic Jets

Subsonic cold jets provide a relatively simple laboratory flow and still display many of the features of the noise from more complex flow configurations. However, turbojet engine jets are generally hot and high speed, and these factors introduce additional noise sources as well as complicating the analysis of the noise from turbulent fluctuations. In subsonic hot jets, temperature or entropy fluctuations can provide an additional source of noise, while the existence of temperature gradients in the flow can increase the effects of refraction. When the jet flow becomes supersonic, additional mechanisms, such as screech, shock noise, and eddy Mach wave radiation, become apparent, adding to the noise production of a jet. Finally, as anyone who has listened to a jet of combustible gas before and after ignition can attest, the noise produced by combustion can represent a formidable additional source of noise in some flows.

The effects of heating a subsonic jet on the noise produced by the jet have been extensively studied by Hoch, et al. (Ref. 26). These experiments, carried out both at the SNECMA and the NGTE, involved comprehensive far-field noise measurements on heated jets from convergent nozzles over a range of velocities from 100 to 800 m/sec. Typical jet noise correlation parameters in present use introduce a factor $(\rho_j/\rho_o)^W$ into the Lighthill correlating parameter (such as Eq. (1)) to account for the effect of jet temperature (and thus density) difference from the ambient, and one of the aims of the research reported in Ref. 26 was to develop an expression for the value of W necessary to correlate the sound power level at the peak angle of emission, using the Lighthill parameter. They found that the noise radiation decreases as the jet density decreases (relative to the ambient) at high jet velocities, but at lower velocities, it increases with decreasing density. It was also observed that the low velocity effect was observed only up to the peak frequency (i.e., the frequency at which the peak sound power is generated), whereas the high velocity effect was found at all frequencies. From this, they concluded that the proper expression for W had to be velocity-dependent, at least in the case where the jet density is changed by heating the jet gas.

Additional data on the effect of density ratio on jet noise have been presented by Tanna, et al. (Ref. 38). Other investigations of heated jets have been concerned with the location of sources in such jets (Ref. 63), the effects of refraction (Refs. 32 and 69) and the measurement of convection velocity in hot jets (Ref. 85). The latter experiment is interesting also from the standpoint of the instrumentation used: the technique was to characterize the fluctuating density in a high temperature subsonic jet using a laser schlieren system. This technique yields, among other things, a convection velocity for the density fluctuations, which was found to be $0.8U_j$ to $0.9U_j$, somewhat greater than the convection velocity for velocity fluctuations.

2.2.3 Experiments on Supersonic Jets

Strong interest in the noise radiated from supersonic jets has resulted from the use of afterburning turbojets in supersonic transport designs, and to a lesser extent from interest in the noise produced by large rocket engines. Numerous measurements of the noise radiated from supersonic jets have been made, both in the laboratory and in engine tests (e.g., Refs. 11, 25, 29, 38, 45, 57, 73, 81, and 86). These experiments established that the noise from jets issuing from convergent-divergent nozzles at the design pressure ratio is lower than that from a convergent nozzle at the same pressure ratio (Ref. 86); that the peak noise intensity appears farther downstream in a supersonic jet than in subsonic jets (Refs. 11, 72, and 81); that in supersonic jets the noise intensity increases with distance from the nozzle exit in the shear layer region rather than remaining constant as for subsonic jets (Ref. 25); and that shock waves in the jet alter the directivity of the noise emitted (Ref. 29). In several of these reports (e.g., Refs. 29, 38, 73, and 81), frequency spectra of the noise radiated by supersonic model jets and supersonic jets from engine configurations are presented for comparison with future theoretical predictions.

2.2.3.1 Source Location

One experiment, carried out by Bishop, et al. (Ref. 87), was concerned with locating the noise sources in a supersonic jet. Reasoning that the noise sources in a high-speed jet are eddies moving supersonically relative to the ambient, for which the conventional acoustic near field does not exist (Ref. 88), Bishop, et al., concluded that the noise sources could be located by exhausting the jet through a wall which could be moved relative to the nozzle exit. Since the near field is negligible in this case, the wall would effectively cut off sound emission from upstream. Measurements carried out in this manner indicated that the primary noise source was at $x/D = 4$, well within the shear layer region, which conflicts with other experimental evidence that the major source is located near the end of the potential core ($x/D \approx 15$ at $M_j = 2.45$ as in the jet of Bishop, et al.). Of considerable interest is the conclusion that Bishop, et al., reach with regard to the size of the structure producing the noise in this supersonic convergent nozzle experiment. Eddies moving supersonically relative to the ambient speed of sound are thought to generate noise through the generation of an eddy Mach wave field. For such a field, the wave numbers of both the sound field and the source field are equal (Ref. 31), so that both the scale of the wave and the scale of the eddy are equal. The experiment shows that the peak frequency at $x/D = 7$ is 2 kHz which corresponds to an eddy scale of 0.5 ft at a point where the shear layer thickness is 0.25 ft. This implies a dominant eddy scale twice the scale of the shear layer which in turn implies that the motion is coherent on this scale, i.e., that a large-scale coherent structure exists in this flow field on a scale considerably larger than the basic turbulent eddies. Bishop, et al., go on to interpret these eddies as the result of an instability mechanism based on the eddy viscosity Reynolds number. As the eddy viscosity Reynolds number is lower than the Reynolds number based on the physical viscosity, the instability limits for the flow are relatively narrow, and only disturbances in a narrow band of frequencies are amplified. The existence of a coherent structure generated by this relatively narrow-band instability mechanism offers the possibility of control of the noise generation mechanism by control of the coherent structure.

2.2.3.2 Basic Mechanisms

There are three basic mechanisms of noise production in a supersonic turbulent jet, in addition to the quadrupole radiation generated by the turbulent velocity fluctuations. These include screech, which is a discrete frequency source, shock noise, which is a broad band source, and eddy Mach wave radiation.[1] The first two of these mechanisms occur in underexpanded jets, while the third, created by eddies moving supersonically relative to the ambient, is found in all jets above transonic velocities but dominates when the jet is correctly expanded and its Mach number is high enough that the convection velocity throughout the shear layer is supersonic relative to the ambient speed of sound (Ref. 87).

2.2.3.3 Screech

The first of these mechanisms to be explained was screech, which was studied exhaustively by Powell (Refs. 4, 27, 28, 89, and 90) in both two-dimensional and axisymmetric jets. Screech is a powerful, discrete frequency source observed in underexpanded supersonic jets. Powell showed that an edge tone mechanism was responsible for this noise source. An eddy, passing through a shock wave in the underexpanded jet emits sound on interacting with the shock. The sound wave so produced travels upstream in the subsonic portion of the shear layer and causes a stream disturbance near the jet exit, which then travels downstream to interact with the shock structure, repeating the process. The mechanism suggested by Powell was confirmed by other investigators (Refs. 91 and 92); Westley and Wooley (Ref. 93) in an extensive study of the phenomenon found that the screech frequency is dependent on the nozzle pressure ratio, decreasing for increasing pressure ratio. They also found that two modes of screech could be observed in spark Schlieren photographs, one of which was axisymmetric and the other spiraling. The spiral mode was not observed for nozzle pressure ratios greater than 3.5. Further, they observed that, for axisymmetric jets, the screech frequency versus pressure ratio curves are discontinuous, with sudden drops in screech frequency observed at certain pressure ratios.

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[1]The phenomenon of screech observed in supersonic jets differs from combustion roughness or screech observed in turbojet combustor cans. However, some similarities in basic mechanisms may exist.

2.2.3.4 Shock Noise

Shock noise also occurs in incorrectly expanded jets, and is also caused by the interaction between turbulence and the shock structure. However, in the case of shock noise, no feedback mechanism exists, and a broadband noise results. Unlike screech, the frequency of the broadband shock noise is a function of position in the jet (Ref. 93). Like screech, shock-associated noise is principally a function of the jet pressure ratio, but independent of angle of observation or jet stagnation temperature (Refs. 30 and 94). In at least one case, the manipulation of the shock structure in a coaxial jet apparatus has been successfully used to eliminate shock-associated noise (Ref. 95).

2.2.3.5 Eddy Mach Waves

The eddy Mach wave mechanism for supersonic jet noise production has only been observed inferentially on schlieren photographs of supersonic jets, and at least one writer has questioned its existence as a mode of turbulent noise production (Ref. 96). From a theoretical standpoint, Ffowcs Williams (Ref. 31) showed that, at some speeds, the quadrupole source generated by turbulent velocity fluctations degenerates into equivalent simple sources because cancellation of the outputs of the sources making up the quadrupole (Fig. 1) cannot occur, and he likened the simple source radiation so produced to an eddy Mach wave. Other theoretical treatments have utilized the concept of eddy Mach waves, as will be discussed in the following section, but no experimental evidence for this mechanism has been observed other than the waves observed on schlieren photographs (e.g., by Powell, Ref. 4).

2.2.4 Combustion Noise

The technologically important problem of combustion noise has received emphasis only recently, for as the other noise sources in a turbojet engine have been reduced, the contribution due to combustion has become a larger portion of the total noise emitted. The first experiment in combustion noise is that by Smith and Kilham (Ref. 97), who studied combustion noise from open, premixed, turbulent, hydrocarbon-air flames, interpreting it as caused by a distribution of monopole sources. They found that the peak frequency occurred at a constant value of the Strouhal number, higher than the peak Strouhal number for a cold jet, and that the acoustic intensity was proportional to U^4. Hurle, et al. (Ref. 98), on the other hand, interpreted their measurements of open, premixed, turbulent flames as, acoustically, a distribution of multipole sources. They found that the radiated sound was proportional to the rate of increase of gas volume during combustion and thus proportional to the rate of combustion. More recent measurements made by Shivashankara, et al. (Ref. 42), have shown that the noise produced by open premixed turbulent flames is a function of the gas speed, the burner diameter, the fuel mass fraction, and the laminar flame speed; their data correlation indicates that the acoustic intensity is proportional to $U^{5.36}$. Noise emission from the subsonic jet from a combustor can has been studied by Abdelhamid, et al. (Ref. 41); they found that combustion generated noise is dominant at low frequencies. Comparison of the combustor exhaust jet with a cold jet at the same Mach number shows that the combustor jet is some 10 to 20 db noisier at the same Mach number. Note, however, that a cold jet at a given Mach number would have a lower velocity than the corresponding hot jet, and thus would be expected to produce less noise.

2.3 THEORETICAL STUDIES

The development of theoretical approaches to the problem of the prediction of turbulent jet noise has proceeded in much the same manner as the experimental work. As new experiments are reported, their conclusions stimulate the development of new theories to explain the effects observed, and so there has been as much increase in the number of theoretical predictions in recent years as there has been in the number of experimental studies. In general, theoretical treatments can be divided into two categories, acoustic analogy models and direct solution procedures; the latter category generally involves the solution of a convected wave equation to obtain the acoustic field. Acoustic analogy theories avoid the solution of a convected wave equation by solving the "analogous" problem of moving sources in a stationary, constant-property, equivalent acoustic medium, rather than the actual problem of acoustic radiation within and from a moving acoustic medium. The effect of the transformation is to lump all of the effects that occur in the real jet, when a sound field propagates through a moving variable property acoustic region, into an equivalent source term, so that approximations made to evaluate the source (which now contains the unknown density or pressure field) must involve propagation as well as production terms. Despite the obvious difficulties inherent in this approach, the resulting economies of solution are extremely valuable, and by far the majority of aeroacoustic predictions made to date have used some form of the acoustic analogy approach. Work currently underway is primarily directed to convected wave equation models, because of the greater generality they appear to offer (e.g., the capability to handle subsonic and supersonic jets with the same formalism, including the effects of refraction), and some workers are attempting to exploit the large-scale coherence phenomenon to obtain predictions of jet noise.

2.3.1 Acoustic Analogy Theories

The acoustic analogy model was developed by Lighthill in 1952 (Ref. 2) and has been used by a great many investigators since its development. Its development begins with the fundamental equations of motion, expressed in terms of mass density (ρ) and the momentum density (ρv_i):

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_i)}{\partial x_i} = 0 \qquad (2)$$

$$\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_i v_j + P_{ij}) = 0 \qquad (3)$$

where the summation convention has been assumed. The term P_{ii} is the stress between adjacent fluid elements, the sum of a pure pressure term ($P\delta_{ij}$), and a viscous stress. Equation (3) can be written in the form of a wave equation for the density by adding $a_o{}^2\,\partial\rho/\partial x_i$ to both sides and rearranging:

$$\frac{\partial(\rho v_i)}{\partial t} + a_o{}^2\frac{\partial \rho}{\partial x_i} = -\frac{\partial}{\partial x_j}\,(\rho v_i v_j + P_{ij}) - a_o{}^2\rho\delta_{ij}) \qquad (4)$$

By defining the stress term as $T_{ij} = \rho v_i v_j + P_{ij} - a_o{}^2\rho\delta_{ij}$, Eq. (4) can be rewritten

$$\frac{\partial(\rho v_i)}{\partial t} + a_o{}^2\frac{\partial \rho}{\partial x_i} = -\frac{\partial T_{ij}}{\partial x_j} \qquad (5)$$

Physically, Eq. (5) states that a fluctuating flow of gas, such as a turbulent jet, generates in the atmosphere outside it the same fluctuation of density as would be produced in a classical stationary acoustic medium by a system of externally applied stresses Tij (Ref. 8). By eliminating the momentum density from Eqs. (5) and (2), a wave equation for the density fluctuation is obtained in the form

$$\frac{\partial^2 \rho}{\partial t^2} - a_o{}^2\frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \qquad (6)$$

which is the fundamental form of Lighthill's acoustic analogy theory. Eq. (6) is exact; no approximations have been made in deriving it from the fundamental equations of motion. Note that the forcing term in Eq. (6) is a sum of second derivatives, which indicates that the atmospheric sound field can be regarded as caused by a continuous distribution of acoustic quadrupoles of strength Tij per unit volume (Refs. 2, 7, and 8).

However, Tij in Eq. (6) contains the unknown (ρ), and it is this problem that complicates the use of the acoustic analogy model. If Tij is assumed to be in some fashion known, then the solution to Eq. (6) may be formally written

$$a_o{}^2(\rho - \rho_o) = \frac{\partial^2}{\partial x_j \partial x_i}\int_v \frac{[T_{ij}]}{4\pi r}\,dr \qquad (7)$$

where v stands for the region of turbulent flow, and r represents the distance from the source to the point x_i where $a_o^2(\rho - \rho_o)$ is measured. In the acoustic approximation, $a_o^2(\rho - \rho_o)$ is approximately the sound pressure. The brackets around Tij indicate that it is evaluated at the time $t - r/a_o$, the instant at which the source quadrupole emitted the sound heard at the observer at the the time t. It might be noted that v ought to refer to all space; however, outside the region of turbulent flow, Tij approaches zero rapidly.

Two approaches to the use of Eq. (7) can be taken. Without knowledge of the actual form of Tij, Eq. (7) can be manipulated to obtain scaling laws for the behavior of the far-field noise in different circumstances. This type of manipulation led, for example, to the famous U^8 scaling law for overall sound power level (Ref. 2). Other work involved with manipulation of the source integral to study the characteristics of jet noise is reported in Refs. 5, 31, 99, and 100. The second approach to the use of Eq. (7) is to attempt to model the Tij term in terms of known or calculable turbulent stresses in a jet. Most of this work, which is quite extensive, has been involved with the analysis of the subsonic jet, and techniques have been developed to obtain models both for Tij (e.g., Refs. 19, 22, 53 to 56, 71, 76, and 101 to 106) and for the equivalent source term in the Meecham-Ribner theory (Refs. 15, 16, and 107 to 109).

2.3.1.1 Scaling Laws for Cold Jets

By assuming Tij to be known and studying the behavior of the solution (Eq. (7)), a substantial amount of general information can be obtained. The well-known U^8 law (Eq. (1)) was obtained by Lighthill (Ref. 2) from dimensional reasoning; it is also possible to obtain from Eq. (7) an indication of the magnitude of convective amplification effect (Refs. 5 and 31). This effect is explained physically by Lighthill (Ref. 2) as an increase in the apparent noise level for moving eddies caused by an apparent increase in the size of the noise-producing eddy and an increase in the time over which a given eddy contributes to the far field noise at a point. Convective amplification is observed to decrease the noise in the upstream quadrants (opposite to the direction of motion of the noise sources) and increase it in the downstream quadrants (in the direction of motion of the noise sources), with no effect at 90 deg to the jet axis. Lighthill obtained a convective amplification factor of $|1 - M_c \cos \Theta|^{-6}$ for subsonic flow (Ref. 5); Ffowcs Williams (Ref. 31) corrected this to $|1 - M_c \cos \Theta + (\omega \ell/a_o)^2|^{-5}$, where in both cases M_c is the convection Mach number, related to the ambient speed of sound, i.e., $M_c = U_c/a_o$. The factor $(\omega \ell/a_o)$ was introduced by Ffowcs Williams to account for the fact that as $M_c \rightarrow 1$, neither the volume of the emitting eddy nor the time difference across the eddy becomes infinite, so that the convective amplification cannot become infinite. In the particular case of emission at the Mach angle, $\Theta = \cos^{-1}(1/M_c)$ both the emitting volume and the emission time differences increase by the factor $a_o/\omega \ell$, where ω is a typical circular frequency in the moving frame, rather than by the factor $1/(1 - M_c \cos \Theta)$. Generalizing this reasoning, Ffowcs Williams (Ref. 31) arrived at the convection factor shown here. Ffowcs Williams also showed that, at low supersonic speeds, the appropriate convection factor is $|M_c \cos \Theta|^{-5}$; this combines with the basic U^8 law to yield an overall noise intensity in supersonic flow $\propto U^3$.

The Meecham-Ribner theory produces an expression for the far field density fluctuation of the form (Ref. 15):

$$P^{(1)} = \frac{1}{4\pi a_o{}^2 r} \int_v \frac{\partial^2}{\partial t^2} P^{(0)}(\tau,\hat{t})d\tau \tag{8}$$

where $P^{(1)}$ is the far field acoustic pressure and $P^{(0)}$ is the quasi-incompressible "pseudosound" pressure fluctuation in the flow field. The symbol $\hat{t}$ represents the retarded time $t - r/a_o$. In general, the pressure fluctuation within the flow field is the sum of the turbulent pressure fluctuations, the acoustic pressure fluctuation, and this quasi-incompressible pseudosound fluctuation. For $M^2 << 1$, the pseudosound dominates, and this approximation has been made in writing Eq. (8). In the far field, $P - P_o = P^{(1)}$, since the pseudosound contribution diminishes rapidly away from the source region, and since in the acoustic approximation $P - P_o = a_o{}^2 (\rho - \rho_o)$, Eq. (8) can be written as

$$a_o{}^2(\rho - \rho_o) = \int_v \frac{[-\ddot{P}^{(0)}/a_o{}^2]}{4\pi r} d\tau \tag{9}$$

where $\ddot{P}^{(0)}$ represents $\partial^2/\partial t^2(P^{(0)})$.

Ribner (Ref. 15) has shown that Eq. (9) is formally equivalent to Eq. (7), at least for cold subsonic jets, and thus the same type of scaling laws can be deduced from manipulation of Eq. (9) as from manipulations of Eq. (7). Such scaling law development has been carried out by Ribner (Ref. 15) and by Meecham and Ford (Ref. 16) and Meecham (Ref. 107), yielding the same U^8 law as obtained from the Lighthill analysis and the convection factor $|1 - M_c \cos \Theta|^{-5}$ which is also obtained from the Lighthill acoustic analogy theory.

2.3.1.2 Source Evaluation for Cold Jets

Considerable effort has been involved in the evaluation of the source term appearing in Eq. (7), i.e., in obtaining models for

$$T_{ij} = \rho v_i v_j + P_{ij} - a_o{}^2 \rho \delta_{ij} \tag{10}$$

One of the first of these approaches is that reported by Proudman (Ref. 101) in which the acoustic output of a finite region of decaying isotropic turbulence in an infinite compressible fluid was considered. However, this sort of model is somewhat oversimplified with respect to the jet noise problem, and Lilley (Ref. 22) generalized the approach to consider a region of isotropic turbulence superimposed on a shear flow. To use this model, Lilley first broke the integrand in Eq. (7) down into two parts, after writing Tij in terms of the turbulent velocity fluctuations. The first part of the source expression represents noise emitted by the turbulence in the presence of strong mean shear-"shear noise"- and the second represents turbulence-turbulence interactions - "self noise." Expressions for the noise intensity generated by both types of interaction were derived by modeling the flow field as made up of small eddy volume emitters. Using similarity laws to evaluate the parameters that contribute to the noise intensity (e.g., for the mean velocity, turbulent shear, turbulence intensity), Lilley was able to obtain expressions for the axial and lateral distribution of turbulent noise emission, yielding, for example, the conclusion that the noise sources remain of constant strength with x in the mixing layer and decrease as x^{-7} in the fully developed region. Although many detail refinements have been made to the model proposed by Lilley, many of the ideas found in the later work (such as Refs. 55 and 103) appear to have originated here.

The division of the Lighthill source into shear noise and self-noise components is a feature of a number of models of the noise production in turbulent jets (Refs. 53 to 56, 76, 102, and 103). In several of these models (Refs. 54 to 56), different convection factors are used for the self-noise and shear-noise terms following the analysis of Jones (Ref. 54); however, the use of different convection factors for the different sources is not universally followed. An interesting feature of recent modeling work using the self-noise, shear-noise concept is the approach of assigning a given frequency to a group of eddy volume emitters: Benzakein, Chen, and Knott (Ref. 103) assume that at each axial station the noise emission is at one dominant frequency, given by the relationship proposed by Davies, et al. (Ref. 74), between turbulent fluctuation intensity and frequency ($\omega \ell = 1.1u'$), evaluated at one characteristic point in each cross section. Moon (Refs. 55 and 56) generalized this hypotheses to define two characteristic frequencies, an octave apart, for each eddy volume emitter, so that a distribution of frequencies is obtained both axially and laterally. The octave spacing between the self-noise frequency and the shear noise frequency was chosen to agree with the experimental results described in e.g., Ref. 9.

Several investigators have explored the modeling of the source term in the Lighthill expression without dividing it into self-noise and shear-noise contributions; for example, in Ref. 104, a Fourier analysis of the Lighthill source term is undertaken, assuming axisymmetric geometry (i.e., coordinates r, ϕ, z) and expanding the autocorrelations that make up the turbulent contributions to the Lighthill source in expansions with respect to ϕ. The assumption of a generalized turbulence model leads in this analysis to the convective amplification factor derived by Ffowcs Williams (Ref. 31) without recourse to a moving frame representation. The special case of axisymmetric turbulence is assumed in Ref. 105 in modeling the Lighthill source term, replacing the more common assumption of isotropic turbulence. A special form of the Lighthill source, in terms of vorticity, is used in Ref. 106 to evaluate the far-field noise output of a jet modeled as a train of toroidal vortices; this model is an attempt to use some of the observed large scale behavior of jets to predict jet noise radiation. Finally, Arndt and George (Ref. 19) redefine shear noise and self noise in terms of the large-eddy structure they hypothesize lies beneath the fine scale turbulence in high Reynolds number jets. In their model, "self noise" production involves the interation between like large eddies and "shear noise" the interaction of unlike ones.

Only a small amount of work has been done to date on evaluating the source term in the Meecham-Ribner pressure fluctuation theory. One reason for this is that, although the pressure fluctuations that form the source term in the Meecham-Ribner theory are conceptually simple to understand, little work has been done on the turbulence modeling necessary to theoretically predict the pressure fluctuations in a turbulent jet. An additional problem is that the source field for the pressure fluctuation theory is somewhat larger than that for the Lighthill source; as pointed out by Ribner (Ref. 15) and others (Refs. 1 and 13) the integration over the source in the Meecham-Ribner model must be taken at least 2/3 of a typical sound field wavelength beyond the source boundaries. Because of these problems, most of the attempts at evaluating the source term in the Meecham-Ribner theory have concentrated on relating the source field in an unknown jet flow to that in a known jet flow. Such an approach was used, e.g., by Scharton and Meecham (Ref. 108): a theory is developed to obtain an expression for the far-field noise frequency spectrum in terms of the spectrum of the pressure fluctuations measured in the jet. The approach used by Pinkel (Ref. 109) is intended to allow theoretical predictions of jet noise radiation following the method of Scharton and Meecham (Ref. 108). Here, however, a simple theory is developed to relate the pressure fluctuations in "cells" in a jet to the jet total kinetic energy (i.e., mean flow and turbulent kinetic energy). The basic rationale for this approach is to develop scaling laws for multiple jet nozzles, based on single jet results.

2.3.1.3 Scaling Laws for Supersonic and Hot Jets

In his 1963 paper, Ffowcs Williams (Ref. 31) extended the Lighthill analysis to supersonic flows, and investigated noise power scaling laws at both subsonic and supersonic velocities. As was described in a previous section, this extension showed that the convective amplification factor for supersonic speeds was $|M_c \cos\theta|^{-5}$, and this combined with the basic U^8 law to produce an overall sound power level variation proportional to U^3. At sonic speeds the sound radiation process was likened to an eddy Mach wave mechanism, whose strength is proportional to U^3. (Note that in this and subsequent discussions the words supersonic and sonic are relative to the speed of sound in the ambient fluid.) Lush, Fisher, and Ahuja (Ref. 99) and Lilley (Ref. 100) investigated scaling laws for the noise from hot jets, both subsonic and supersonic, from the standpoint of the Lighthill source mechanism. These investigations were occasioned by the experimental work of Hoch, et al. (Ref. 26), who showed that, contrary to earlier reasoning based on Lighthill's scaling laws (Refs. 1 and 7), heated jets are quieter than unheated jets at the same velocity only above a certain critical velocity. Below this velocity, noise levels increase with increasing jet temperature. To interpret this result, Lush, Fisher, and Ahuja begin with the complete Lighthill source term (Eq. (10)), and divide the fluctuating density ρ' into two parts, one related to pressure fluctuations at constant entropy and the other related to entropy fluctuations at constant pressure. By assuming that Reynolds stress and entropy fluctuations are uncorrelated, this leads to an expression for the overall noise intensity of the form

$$I = A(U_j/a_o)^8 + B(U_j/a_o)^4$$

with

$$A \propto (T_j/T_o)^{-1}, \; B \propto (T_j/T_o)^{-1} \; (\ln T_j/T_o)^2$$

in which T_j is the jet stagnation temperature and T_0 the ambient fluid temperature. The results of this analysis show that the noise contribution from the Reynolds stress fluctuations decreases with the jet stagnation temperature increase, as expected, but an additional source appears for hot jets, for which $I \propto U^4$, and which increases with jet stagnation temperature increase.

Lilley's analysis (Ref. 100) is somewhat more fundamental than that performed by Lush, Fisher, and Ahuja (Ref. 99), but reaches similar conclusions. Lilley notes that the Lighthill source term includes effects of refraction and diffraction, and unless these are properly evaluated, the results for far-field sound intensity obtained with the use of the Lighthill theory are incorrect. He then proceeds to derive the proper expression for the solution to Lighthill's equation for the case of a subsonic jet with a general temperature variation. This analysis shows that the source term includes contribution of heat release, enthalpy fluctuations, and kinetic energy fluctuations. The first term of the generalized source expression produces the usual result for a volume distribution of quadrupoles, i.e., $I \propto U^8$, but a second term arises, proportional to U_j^4, multiplied by the square of the fluctuating enthalpy, which is negligible for cold subsonic jets.

2.3.1.4 Source Evaluation for Hot and Supersonic Jets

Evaluation of the Lighthill source term in supersonic and hot jets has, except for the eddy Mach wave mechanism, in general followed the approaches used for subsonic noise source evaluation. The eddy Mach wave mechanism has been investigated by Ffowcs Williams (Refs. 13 and 88). At sonic velocities (relative to the ambient speed of sound), Ffowcs Williams and Maidanik (Ref. 88) find that the only remaining source is the temporal gradient of pressure in the presence of mean shear. This yields, after several further approximations, an expression for the far-field noise intensity in terms of $\overline{(dp/dt)}^2$ measured within the flow field, which is shown to agree with the trends shown by measurements concerned with two-dimensional boundary layer noise.

Applications of the subsonic flow analysis of the Lighthill source term developed by Benzakein, Chen, and Knott (Ref. 103) to the supersonic jet noise problem have been demonstrated by Knott and Chen (Ref. 110) and Knott and Benzakein (Ref. 111). Reasonable success was achieved; however, the severe effects of refraction in supersonic flows are not in general correctly modeled with this approach. A further analysis of the noise from supersonic jets in the near field was performed by Chen, Knott, and Benzakein (Ref. 112); in the near field, the distances between the noise sources becomes an important parameter.

2.3.1.5 Combustion Noise

Combustion noise can be interpreted as caused by a distribution of acoustic monopole (Ref. 97) or multipole (Ref. 98) sources. Strahle (Ref. 39) adopted the former view and analyzed the Lighthill source expression for the situation in which monopole sources dominate. As Eq. (10) shows, the Lighthill source expression is made up of three terms, involving $\rho v_i v_j$, ρ, and p; the latter two can be considered monopole sources. Strahle shows that, in general, the source represented by the term P_{ij} (Eq. (10)) is much smaller than $a_o{}^2 \rho \delta_{ij}$ and writes the far-field noise intensity for the case of negligible quadrupole radiation in terms of the density fluctuation portion of Lighthill's source expression. Expressing the far-field noise intensity in this manner allows the development of scaling laws for combustion noise, which are dependent on the model used for the turbulent flame. If a wrinkled-laminar model is used for the flame, the far-field sound pressure is given by an expression of the form

$$P \propto U^{4-3q} \, S_L{}^{r+3q} \, \ell^{2+r} \tag{11}$$

where U represents the gas velocity, S_L the laminar flame speed for the mixture, ℓ a correlation length scale, and $o < q < 2/3$, $o < r < 1$. For the distributed reaction model,

$$P \propto U^2 \, S_L{}^3 \, \ell^3 \tag{12}$$

In Ref. 113, Strahle extends his combustion noise analysis to show that for burner-type flows (in which upstream density fluctuations are attenuated) the source term can be rewritten as the second time derivative of a fluctuating reaction rate correlation. This work, and the previous work on combustion noise, are summarized in Ref. 114.

2.3.1.6 Other Studies

The convective amplification formulation devised by Lighthill (Refs. 1 and 2) and Ffowcs Williams (Ref. 31) can, for high subsonic speeds, yield noise power laws much higher than the eighth-power law associated with Lighthill's name. Yet experimental results indicate that the eighth-power law is valid over a wide range of jet velocities. This paradox was known at the earliest stages of jet noise research; Lighthill himself (Ref. 5) proposed that there is a reduction in turbulence intensity at higher speeds, compensating for the convective amplification effect. However, the experiments carried out by Lush (Ref. 49) indicate that Lighthill's explanation is not correct. These experiments have been considered by some as evidence that the acoustic analogy formulation is incapable of modeling the jet noise source even for subsonic jets, since it may be interpreted as a change in source strength with velocity that is not included in the model formulation. However, in two recent papers Mani has shown (Refs. 37 and 115) that at least conceptually the observed effects can be explained by the acoustic analogy theory.

One possible explanation for the reduction in the convective amplification effect at high speeds, as proposed by Mani (Ref. 115) and others (Refs. 53 and 116) is that in a high-speed jet the noise sources are relatively deep within the jet where they are moving considerably more slowly with respect to their local surroundings than they are with respect to the ambient. To investigate this explanation, Mani (Ref. 115) studied an idealized problem of a monopole source in a uniform slug flow jet and found for this idealized problem that the convective amplification factor becomes a function of the source frequency and the jet Mach number, as observed by Lush (Ref. 49). In Ref. 37, Mani extended this analysis to consider both the effects of source convection and sound refraction by density gradients. The results of Ref. 115 are repeated for the case of the convective amplification effect, and the simple analysis also shows that the exponent on the density ratio in the Lighthill overall sound power expression is a function of jet velocity and source frequency, in agreement with the experimental results of Hoch, et al. (Ref. 26).

2.3.2 Direct Models

The shortcomings of the acoustic analogy approach are well known and become more important as more complex phenomena are investigated. In essence, the difficulties inherent in the application of the acoustic analogy model in complicated flow fields arise from the inclusion of the unknown (ρ) in the source term on the right-hand side of the Lighthill equation for far-field acoustic density fluctuations. For subsonic flows, the assumption that $\rho = \rho_0$ (in the source term) is commonly made, but in general, any assumption regarding ρ is tantamount to assuming a solution to the problem. Ffowcs Williams (Ref. 13) has pointed out that the assumption that $\rho = \rho_0$ in the evaluation of Tij is essentially the first term in a series expansion for Tij, and as such is a permissible approximation when density gradients are small, while Mani (Ref. 37) notes that if Tij is written as the sum of mean and fluctuating parts and it is noted that only the fluctuating part contributes to the noise field, then the assumption that $\rho = \rho_0$ becomes an assumption that the correlation $\overline{\rho' u' v'}$ is small relative to $\rho_0 \overline{u' v'}$. The latter assumption is commonly made in turbulence theory and may be justifiable in many flows.

Nonetheless, there are numerous circumstances in which the economy of solution offered by the acoustic analogy approach cannot be used. One obvious circumstance is in the analytical study of refraction effects; another is in the study of sonic and high supersonic speed jets in which the assumptions basic to the analysis of the Lighthill source term break down. In these cases, the "direct" problem must be solved, rather than the acoustic analogy model, and the problem generally involves the solution of a convected wave equation for density or pressure fluctuations.

2.3.2.1 Refraction and Reflection Studies

The subject of refraction and reflection of sound waves by velocity gradients or discontinuities is one which is not treated at all by the acoustic analogy model, yet is of prime importance in the application of jet noise research to aircraft engine noise suppression and other technological problems. Thus, considerable interest developed early in the theoretical examination of these phenomena. Two approaches were taken in this work: the examination of the behavior of an assumed acoustic waveform on encountering a velocity field discontinuity and the solution of a convected wave equation to determine the propagation of a given disturbance through a disturbed acoustic medium. Examples of the first approach are described in Refs. 117 to 121, and solutions for particular cases of convected wave equation descriptions of the acoustic flow field are given, e.g., in Refs. 35, 122, and 123.

Miles (Ref. 117) and Gottlieb (Ref. 118) both studied the behavior of acoustic waves incident on an interface of relative motion, modeled in the analysis as a vortex sheet. In Miles' work, a plane wave was assumed, while Gottlieb considered a cylindrical or spherical wave from a line or point source. Similar results were obtained by both investigators, namely that the incident wave could be strongly distorted by the presence of the velocity discontinuity. Yeh (Ref. 119) considered the problem of reflection and transmission of a sound wave incident upon a finite moving layer, modeled as a uniform stream separated from quiescent surroundings by two vortex sheets. Solution of the problem is, of course, strongly dependent on the boundary conditions assumed, and Yeh notes that incorrect boundary conditions were used in earlier work. In this and a comparable analysis by Graham and Graham (Refs. 120 and 121), similar conclusions are reached; sharp variations in reflection and transmission coefficients are observed for particular values of U/a, where U and a are the velocity and speed of sound, respectively, in the moving layer. Graham and Graham (Ref. 121) further show that under certain circumstances a waveguide effect occurs in their model of a shear layer (a linear-velocity profile parallel-flow layer separated by vortex sheets from the ambient fluid).

Moretti and Slutsky (Ref. 122) considered the problem of the acoustic far field of a harmonic singularity embedded in a parallel-flow model of a jet. Two cases were considered, in one of which the observer and source were considered fixed, but the intervening medium was considered to be in motion, and in the other of which the medium and observer were considered to be at rest and the source in motion. In order to evaluate these two cases, they formulated the problem by writing small-pertubation equations for the inviscid medium and jet, representing the boundary between the two flows as a velocity discontinuity. The jet was divided into regions in which the velocity was considered to be uniform, building up a stepwise representation of the jet flow. This work is a forerunner of the current analyses of the homogeneous form of the convected wave equation derived by Lilley (Ref. 100), which will be described in a subsequent section.

Further work on the convected wave equation approach to the problem of refraction of acoustic waves by a jet was carried out by Slutsky and Tamagno (Ref. 123) who used the formulation developed by Moretti and Slutsky (Ref. 122) to address the problem of the shielding effect of a jet on noise sources external to it and the effect on the far-field noise of velocity and temperature nonuniformity in the jet. They found that the effect of the jet as a shield is not large, and that temperature nonuniformities of the jet do not exact a major effect except at higher frequencies. Schubert (Refs. 35 and 124) also considered the problem of refraction, for a case similar to the experiments of Atvars, Schubert, and Ribner (Ref. 32) - a harmonic source on the centerline of the jet. To perform the analysis Schubert developed a convected wave equation for the acoustic pressure fluctuations, similar to the equation derived by Phillips (Ref. 125) for acoustic pressure fluctuations in the near field of a hypersonic jet. It is extremely difficult to solve the equation Schubert obtained, but his one result did confirm that the depression in the overall sound power level noted on the jet centerline by Atvars, et al. (Ref. 32), was indeed caused by refraction. An unexpected prediction of this work is that the effects of refraction are felt some 100 diameters downstream in the jet.

2.3.2.2 Supersonic Jet Noise

Because of the complex nature of the supersonic jet flow aeroacoustic phenomena, a number of theoretical approaches for dealing with aspects of supersonic jet noise have been developed. These can conveniently be divided into four types: feedback mechanisms, represented by the analyses of screech (Ref. 27) and shock noise (Ref. 94), eddy Mach waves (Ref. 126), convected wave equation approaches (Refs. 125, 127, and 128), and analyses based on a shear layer instability equation (Refs. 96, 129, and 130). None of the analyses of itself is capable of predicting the entire range of aeroacoustic behavior observed in a supersonic turbulent jet.

Screech and shock noise have been touched on in an earlier section. In a series of papers, Powell (Refs. 4, 27, 28, 89, and 90) established a feedback mechanism as the cause of the phenomenon of screech and used a simple theory based on this feedback mechanism to estimate the screech wavelength as a function of excess pressure ratio for a convergent nozzle. (Excess pressure ratio is defined by Powell (Ref. 27) as $(P_e\text{-}P_{crit})/P_o$, where P_{crit}/P_o is the sonic pressure ratio and P_e is the nozzle exit static pressure.) No prediction of the strength of the noise sources was attempted. In a similar analysis, Harper-Bourne and Fisher (Ref. 94) developed a theory to account for the noise contribution caused by interactions of the turbulence with shockwaves in the jet; again, no estimate of source strength is obtained in this approach.

Eddy Mach wave radiation has not been extensively analyzed from the standpoint of source term modeling. A crude model for the supersonic shear layer was developed by Ribner (Ref. 126) in which it is treated as an assemblage of square eddies convected at supersonic speed. Unbalanced pressure fluctuations in the eddies are envisaged to cause ripples in the interface between the shear layer and the ambient fluid. The requirement that pressures match across the interface results in pressure fluctuations in the exterior fluid, which are seen as eddy Mach waves. Ribner notes that this model produces results similar to those obtained from acoustic analogy models using the modified convective amplification factors developed by Ffowcs Williams (Ref. 31) and Ribner (Ref. 9).

The convected wave equation approaches developed to attack the problem of noise radiation from a shear layer are of more general interest than models developed for particular supersonic jet phenomena such as screech or eddy Mach wave radiation. Powell (Ref. 125) noted that, for $M >> 1$, the Lighthill approach cannot be used since the necessary analysis of the source term Tij cannot be carried out without neglecting terms which are important. In addition, the retarded time effect, which is neglected within an eddy in the analysis of Lighthill's source term (Ref. 5), cannot be neglected for $M >> 1$, and the assumption that $\rho \approx \rho_o$ in the source term is not in general true.

To overcome these problems Phillips derives from the continuity and momentum equations, along with an expression of the second law of thermodynamics, and assuming the perfect gas law applies, an equation for the fluctuating pressure field around a supersonic jet. This equation takes the form

$$\frac{D^2}{Dt^2} \ln\left(\frac{P}{P_o}\right) - \frac{\partial}{\partial x_i} \left\{ a^2 \frac{\partial}{\partial x_i} \ln(P/P_o) \right\} =$$

$$\gamma \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \gamma \frac{D}{Dt}\left(\frac{1}{C_p} \frac{Ds}{Dt}\right) - \gamma \frac{\partial}{\partial x_i} \left\{ \frac{1}{\rho} \frac{\partial}{\partial x_j} [\mu(e_{ij} - \tfrac{2}{3} \Theta \delta_{ij})] \right\} \quad (13)$$

in which P_o is a reference pressure, γ the ratio of specific heats, s the entropy, C_p the specific heat at constant pressure, e_{ij} the strain rate tensor $(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$, and Θ is the dilatation $\partial u_k/\partial x_k$. The left-hand side of Eq. (13) represents a convected wave equation for the variable $r = \ln(P/P_o)$, with a relatively complex forcing function on the right-hand side. Phillips interprets the forcing terms, reading from left to right, as, first, generation of pressure fluctuations by velocity fluctuations, second, generation of pressure fluctuations by entropy fluctuations, and, third, diffusion and dissipation of pressure fluctuation by viscosity.

Phillips obtained a solution to Eq. (13) for the near field of a shear layer in the asymptotic case $M \rightarrow \infty$, using a Fourier transform technique. The solution shows that $I = (P - P_o)^2 \propto M^{3/2}$ for $M >> 1$, and that the direction of the sound radiation approaches the perpendicular to the (plane-parallel) shear layer as $M \rightarrow \infty$.

The extension of the Phillips formulation to low-supersonic and transonic Mach numbers has been undertaken by Pao (Refs. 127 and 128), incorporating both Mach wave radiation and a self-noise mechanism as acoustic sources. To obtain a solution to the convected wave equation (i.e., the left-hand side of Eq. (13)), Pao assumes a generalized form for the right-hand side (the source term). Solutions are obtained using a Fourier transform technique; as yet no general technique for the solution of the complete Phillips equation has been described.

The interest in the existence of large-scale coherent structures in jets and the mathematical techniques which have been developed for the solution of instability problems in two-dimensional shear flows have led to attempts to describe the production of noise by a supersonic flow in terms of an instability mechanism in the flow. One of the first of these analyses was carried out by Berman and Ffowcs Williams (Ref. 129), using a two-dimensional vortex sheet model of a compressible jet. The idea is advanced that a jet acts as a broad band amplifier of high gain, so that disturbances in the flow field can grow rapidly to a size where nonlinear effects bring about a significant interaction with the mean flow. In the phenomenon of screech, this mechanism provides the essential element in the feedback cycle: the gain in strength of a downstream traveling disturbance necessary to allow it to overcome radiation and viscous losses. To test this idea, a linearized instability analysis was performed on a model of a compressible two-dimensional jet. The results show (as spectacularly confirmed in low-speed axisymmetric flow by the experiments of Crow and Champagne (Ref. 18)) that a jet can indeed act as a high-gain amplifier of certain disturbances.

A similar approach has been taken by Tam (Refs. 96 and 130) in an attempt to explain the noise radiation from supersonic shear layers without recourse to an eddy Mach wave mechanism. Tam notes (Ref. 96) that the eddy Mach wave mechanism has never had a satisfactory physical explanation, although Ribner (Ref. 126) considered the flow field to be analogous to the well-known supersonic flow over a wavy wall. Indeed, Tam argues that if it exists, eddy Mach wave radiation would be found mostly in the initial region of the jet, where the flow is most highly sheared and where the convection velocity is highest, but numerous experiments (e.g., Refs. 11, 73, and 81) show that the peak noise radiation from supersonic jets occurs in the transition region. Further, the eddy Mach wave radiation frequency inferred from the wave spacing observed on shadowgraph and schlieren photographs is much higher than the dominant frequencies measured with a microphone in the far field.

To replace the eddy Mach wave mechanism, Tam postulates that the noise radiated from the shear layer of a nearly ideally expanded supersonic jet is produced by a large-scale instability structure. From a simple model of the instability phenomenon in a two-dimensional shear layer, he finds that two large-scale unstable waves are preferentially amplified. The rapid growth of these waves causes oscillations to penetrate the mixing layer at two locations and interact strongly with the ambient fluid, producing intense noise radiation. But because only two instability wave numbers are involved in the mechanism postulated by Tam (Refs. 96 and 130), only the peak frequency of the radiated noise is predicted. Thus, the instability theory as proposed by Tam cannot represent a complete theory of noise radiation from supersonic shear layers. The influence and importance of the instability mechanism awaits experimental confirmation.

2.3.2.3 General Convected Wave Equation Formulations

Convected wave equation models should incorporate the effects observed from the solution of the acoustic analogy formulation in simple flows, yet provide the generality necessary to handle more complex flow fields as well. Indeed, as pointed out by Doak (Ref. 3), all theories should tend toward a one-to-one correspondence with the relevant special formulations at the proper limits. The search for a consistent formulation of the problem of turbulent jet noise, valid in general, and providing a clear separation of the models for noise production and its propagation through a generalized medium, has been the subject of a sustained research effort over the past several years.

In a long, informative, and entertaining review article, Doak (Ref. 3) has critically examined the foundations of most of the current theories of aeroacoustic noise. He applies two principal criteria to the theories he reviews: first, that in the proper limit, the theories should reduce to the Rayleigh formulation for acoustic motion in the limit of small amplitudes, and second, that the equations should separate effects of propagation through variable-property media from the true source terms. The major reason for the review Doak presents is to attempt to develop a uniform theoretical model for aeroacoustic sound generation and propagation which will be valid throughout the spectrum of flows in which aeroacoustic phenomena occur. To be this widely applicable, a general aeroacoustic model must be capable of handling subsonic and supersonic, cold and hot flows.

Doak notes that, while the study of jet noise <u>per se</u> dates more or less from the work of Lighthill (Ref. 2), the basic study of acoustics dates from the work of Stokes, Helmholtz, Kirchoff, and Rayleigh, summarized in the book by Rayleigh (Ref. 131). The fundamental feature of this work is the discovery of three types of motion in an acoustic medium: acoustic wave motion, vorticity diffusion motion, and thermal diffusion motion. All three of these modes coexist in small amplitude fluctuations. All three of these modes should, therefore, be present in a general aeroacoustic formulation.

In his review, Doak (Ref. 3) derives a general acoustic wave equation for a general medium in motion. The only assumptions made in the derivation are that the medium is an ideal gas, with Stokesian viscous and heat conduction properties. The result is a general, fourth-order, nonlinear partial differential equation for the logarithmic pressure r, ($r = \ell n\ P$), which is a generalized form of Rayleigh's equation for acoustic motion. It is, therefore, a convected, inhomogeneous, scalar wave equation for a viscous, heat-conducting, nonuniform acoustic medium. By taking the curl of this equation, a generalized form of the governing equation for the vortical mode of motion can be obtained, while the energy equation written in terms of r provides an equation for the thermal mode. If the generalized equations for the acoustic, vortical, and thermal modes are specialized to the case of small-amplitude disturbances in a fluid of otherwise uniform thermal and flow properties, and only the linear terms are retained, the Rayleigh linearized acoustic equations are obtained. In this limit, the equations are uncoupled (except through their boundary conditions), and thus the acoustic problem in the limit of small amplitudes reduces to a superposition of acoustic, vortical, and thermal effects. The generalized equations also satisfy the requirement that they reduce to the proper formulation in the limit of small amplitudes.

The Phillips formulation for the supersonic shear layer problem (Ref. 125) results in a convected wave equation for $r = \ell n\ P/P_o$ (Eq. (13)). Doak (Ref. 3) shows that this equation reduces to one of Rayleigh's equations in the small-amplitude limit, satisfying one of the basic criteria he sets forth; further, it can be obtained from Doak's generalized formulation in the special case of no body forces or heat sources in the flow field. However, Doak shows that the term

$$\gamma \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}$$

appearing in the source expression of Eq. (13) and defined by Phillips (Ref. 125) as "generation of pressure fluctuations by velocity fluctuations," is not entirely a source term. Instead, it can be shown that, even when viscous and thermal conduction effects, external forcing, and heating of the acoustic medium are all zero, the material derivative of Eq. (13) results in an additional term in the variable $r = \ell n\ P$ which appears from the source term

$$\gamma \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}$$

The resulting equation is

$$\frac{D}{Dt}\left[\frac{\partial}{\partial x_j}\left(a^2 \frac{\partial r}{\partial x_i}\right) - \frac{D^2 r}{Dt^2}\right] - 2\frac{\partial u_i}{\partial x_i}\frac{\partial}{\partial x_j}\left(a^2 \frac{\partial r}{\partial x_i}\right) = 2\gamma \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_k}\frac{\partial u_k}{\partial x_i} \tag{14}$$

in which the second term on the left-hand side, which arose from the generation term of Phillips' equation, is in reality a propagation term, which Doak calls "shear refraction."

Equation (14) is the starting point for the convected wave equation analysis put forward by Lilley (Refs. 100 and 132). For his analysis, Lilley specializes Eq. (14) to consider a mixing region modeled as a transversely sheared unidirectional mean flow on which is superimposed a small velocity perturbation, so that $u_i = \overline{u_1} + v_i$. For such a flow field, the right-hand side of Eq. (14) becomes at least quadratic in the perturbation velocities:

$$2\gamma \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_k}\frac{\partial u_k}{\partial x_i} = 6\gamma \frac{\partial \overline{u}_1}{\partial x_2}\frac{\partial v_2}{\partial x_k}\frac{\partial v_k}{\partial x_1} + 2\gamma \frac{\partial v_i}{\partial x_j}\frac{\partial v_j}{\partial x_k}\frac{\partial v_k}{\partial x_i} \tag{15}$$

so that, to first order, the homogeneous convected wave equation for the logarithmic pressure perturbation r' is

$$\frac{\overline{D_1}}{Dt}\left[\frac{\partial}{\partial x_i}\left(\overline{a^2} \frac{\partial r'}{\partial x_i}\right) - \frac{\overline{D^2} r'}{Dt^2}\right] - 2\frac{\partial \overline{u_1}}{\partial x_2}\frac{\partial}{\partial x_1}\left(\overline{a^2} \frac{\partial r'}{\partial x_2}\right) = 0 \tag{16}$$

in which

$$\frac{\overline{D}_1}{Dt} = \frac{\partial}{\partial t} + \overline{u}_1(x_2)\frac{\partial}{\partial x_1}$$

and the overbars represent mean values.

The fact that Eq. (14) is quadratic in the perturbation velocities allows the reduction to a homogeneous equation necessary for the proper behavior of the formulation in the limit of small amplitudes to be achieved. However, this reduction occurs only for the special case of unidirectional, transversely sheared flow. In other flows, the velocity perturbation source term remains, and since it still contains gradients of the irrotational part of the particle velocity. which can be related to the acoustic and thermal types of motion through the mass balance (continuity) equation, in these other flows the problem of separation of source terms and propagation terms remains.

To achieve a separation of the production and propagation terms, Doak proposes a formulation in which the primary variable, replacing the velocity (v_i) is the linear momentum density ρv_i (Refs. 3, 133, and 134). Such a formulation, specialized to an inviscid, nonheat-conducting gas, is described in Ref. 133. In the course of the development, Doak defines pyknodynamic and pyknostatic field variables, by analogy to electrodynamic and electrostatic fields; the root "pykno-" is defined as "pertaining to mass density." Both the mass density fluctuations and the scalar potential of the linear momentum density are pyknodynamic field variables, while the vector potential components that enter the problem are pyknostatic. The significance of this separation is that, at least in some limiting cases, the acoustic pressure fluctuations can be identified with the pyknodynamic variables. However, in general, this is not so; the difficulty lies in the Reynolds stress terms which do not, in general, allow a decomposition into dynamic and static parts.

For flow fields in which external forces and heat addition are negligible and all fluctuations are small, the formulation proposed by Doak produces a set of six simultaneous equations for the acoustic and thermal parts of the scalar potential of the linear momentum density fluctuations, the fluctuating mass density, and the components of the vector potential of the linear momentum density fluctuation. This formulation succeeds in unambiguously defining the acoustic, thermal, and turbulent (vortical) components of the motion, but the resulting equations for the vector linear momentum density (the pyknostatic part of the field) are fourth order, while the other equations are second order. For fairly uniform flows, without large mean vorticity gradients, the formulation can be further specialized, the result is a convected wave equation with a Lighthill-type source, which Doak (Ref. 133) takes as evidence that the convected wave equation form is necessary even in the small gradient case.

2.3.2.4 Solutions for Convected Wave Equation Formulations

To this point, little has been said with regard to solution of the convected wave equation models of aeroacoustic phenomena. In part, this is because solutions to the general formulation have not yet been attempted. With the exception of the supersonic jet solutions of the Phillips form of the convected wave equation approach (Refs. 125, 128, and 129), only the Lilley equation (Eq. (14)) has been studied in any detail.

In order to obtain a general solution of Eq. (14), a solution to the homogeneous form of Eq. (14) must be obtained for source-free regions of the flow, and the source term in Eq. (14) and particular solutions obtained for regions containing noise sources. Such an approach was considered by Lilley, Morris, and Tester (Ref. 135). In this work, the shear layer in the flow was replaced by a vortex sheet, and inner and outer solutions to the equation obtained and matched across the vortex sheet. Both the homogeneous equation and a nonhomogeneous, simple-source formulation were used for the inner equation. The mathematical model is quite similar to that used by Moretti and Slutsky (Ref. 122) and Slutsky and Tamagno (Ref. 123), and the results, that the generation of high frequency components is not controlled by gradients of the mean velocity and temperature in the flow and that in a given frequency band only certain regions of the flow contribute to the radiated sound, compare with the earlier work as well.

Solutions to the Lilley equation have also been investigated by Berman (Ref. 136) in terms of the Fourier transform of the source term for unidirectional transversely sheared layers (Eq. (15)) and by Tester and Burrin (Ref. 137) for a point-source plug flow model and a Lighthill-type source. Berman notes that one philosophical problem with the Lilley formulation is that the homogeneous Lilley equation has the same form as the equation used to study the stability of free shear layers. Since the equation represents an unstable system, how can it represent a stable phenomenon such as jet noise propagation? In answer to this dilemma, Berman (Ref. 136) notes that the equation admits two sets of solutions, and in the case of noise sources in a flow field the stable set is the one of interest.

Berman's solution technique involves using a three-dimensional Fourier transform, in terms of frequency and the directions normal to the flow. An analytical solution in terms of the assumed source is obtained for the limits of very low and very high frequencies, while a boost in sound level is evident at very low frequencies, lending theoretical support to the contention that the presence of a flow field about a source can significantly alter its sound field.

The phenomenon of "low-frequency lift" and "high frequency attenuation" is also found in the solution to the Lilley equation obtained by Tester and Burrin (Ref. 137) in terms of the Fourier transform of a source term similar to the Lighthill source. They also obtain a solution to the Lilley equation in terms of a point source-plug flow model; a uniform radius jet is assumed, with the wake of the point-source-producing probe replaced by a zero-radius vortex sheet. The theoretical model is intended to reproduce the experimental setup used by MacGregor, Ribner, and Lamb (Ref. 72). Theoretical results do show the reduction of sound pressure level near the axis observed by MacGregor, et al., and related by them to refraction, but the results obtained for the sound pressure level reduction as a function of frequency are not in good agreement with experiment.

Improvement in the overall agreement with experiment obtained using Lilley's convected wave equation formalism is dependent on obtaining good models for the generation mechanism involved in the equation. Work aimed at improving models for the source term has been reported by Morris (Ref. 138). The model, which includes the organized jet structure, is obtained by dividing the jet velocity and pressure fields into three parts - a time average part, a time-dependent organized fluctuation, and a disorganized background fluctuation. The latter part of the field is accounted for by an eddy viscosity model. A Fourier decomposition of the equation describing the organized motion is carried out, and the resulting structure is found to be dominated by spatially unstable modes. It is assumed that the most preferred mode, at any given location dominates. The downstream amplitude of the organized motion and its effect on the mean flow are obtained as the solution to a set of integral equations for mean momentum, mean mechanical energy, and mean fluctuation intensity. Noise radiation is found to be caused by axial variation in amplitude of any single frequency component.

At this writing, no coupling of the source modeling proposed by Morris (Ref. 138) with any solution techniques for the Lilley form of the acoustic equations has been reported.

2.3.2.5 Large Structure (Instability) Models

Attempts to model the production of turbulent jet noise in terms of the large-scale instability structure observed in at least some jet flows have occupied several investigators. Michalke (Ref. 104) considered the Fourier analysis of the Lighthill source, writing the source terms in a Fourier series with respect to the azimuthal coordinate in a cylindrical coordinate system (r, z, ϕ). The results showed that the azimuthal modes, which represent correlation across the jet, and thus are a form of large-scale structure, become important for Helmholtz numbers (i.e., Strouhal number times Mach number) greater than one, and that these modes lead to a refraction-like effect on sound pressure level near the jet axis.

Lau, Fisher, and Fuchs (Ref. 139), Lush (Ref. 140), and Hardin (Ref. 106) all consider a model of a turbulent jet as made up of a train of vortices. In Lau, et al. (Ref. 139), and Lush (Ref. 140), a two-dimensional shear layer model is considered. Lau, et al., show that such a model gives good agreement when compared with far-field velocity and pressure time-histories, but Lush points out that the Lau, et al., results indicate that the far-field velocity and pressure are in phase, which is not in accord with experiment. This objection can be overcome by modeling the flow field as a convected train of vortex pairs.

In his circular jet model, Hardin (Ref. 106) conceives of the jet as made up of a train of toroidal vortex rings, which propagate downstream. Using Lighthill's far-field acoustic expression in terms of vorticity, Hardin evaluates the noise produced by this model of a circular jet. These results indicate that noise production in this model occurs mainly near the jet exit and depends on temporal changes in the toroidal radii.

A model such as developed by Hardin is also put forward by Laufer, Kaplan, and Chu (Ref. 141) as a mechanism for jet noise production, except that the determining factor in this model is the rate at which the toroidal structures pair with each other. In Ref. 141, Laufer, et al., define the interaction process as a simultaneous acceleration and deceleration of vorticity-containing coherently moving regions followed by a pairing process. Because a pair of vortices behaves so that its motion produces a zero net change of momentum, it can be considered to behave as a dipole with a combined instantaneous strength of zero. In the far field, such closely spaced dipoles would appear to degenerate to a quadrupole source.

The analysis of turbulent jet sources by Morris (Ref. 138) was described in the preceeding section. A similar analysis, in which the instantaneous quantity q is written as a sum, $q = \bar{q} + q' + q''$, of a time average, $\bar{q}$, a periodic wavelike component, q', and a random turbulent quantity, q'', has been presented by Liu (Ref. 142). As in Ref. 138, the quantity q' is assumed to be governed by an instability kinetic energy equation. The results of Ref. 142 are similar to those of Ref. 138, although the work described in Ref. 142 is limited to the near field as there is no attempt made to analyze the source term in Lighthill's expression for the far-field noise.

2.3.2.6 Excess Noise

"Excess noise" is that noise produced by turbojet engines in excess of that which would be predicted by Lighthill's U^8 law. Crighton (Refs. 8 and 143) has postulated that it is produced by shear layer instabilities which lead to correlated thrust and mass flux disturbances at the engine exhaust nozzle exit. In Ref. 143, Crighton solves an idealized axisymmetric vortex sheet instability problem to estimate the acoustic efficiency of this mechanism and finds that the efficiency is great enough ($\sim 10^{-6} u_1^3$) to be a significant source of excess noise.

2.4 SUMMARY

From the discussion of theory and experiment in this survey, it is clear that there is as yet no generally accepted complete theory for turbulent noise generation in jets, nor is there a wide enough background of experiments on which to build semi-empirical formulations. Indeed, there are aspects of both the theoretical and experimental work that are highly controversial. The existence of a large-scale coherent structure has been proved in certain flows, but its overall importance at any Reynolds number is still a subject of controversy. The theoretical basis of aeroacoustics is rapidly changing, with attempts underway to develop a broad, general formulation which is suitable for prediction over a wide range of Mach numbers, and for flows with strong velocity and temperature gradients. Even as these newer theoretical approaches, based on solution of a convected wave equation, are being developed, the ability of the acoustic analogy theory to predict many of the features of aerodynamic noise which convected wave equation models have been developed to predict is being defended.

There is clearly a need to experimentally define the limits of existence, if there are any, of large scale coherent structures at Reynolds numbers of practical interest, and to perform far-field noise measurements simultaneously. Both the aerodynamic and aeroacoustic phenomena need to be documented simultaneously in all appropriate experiments in order to provide a data base for development of models of the turbulent noise sources. On the other hand, the search for the individual sources within a jet would seem to be unrewarding, since the measurements are extremely difficult to make, and to interpret once made. From the standpoint of attempts to predict aeroacoustic phenomena, a consistent set of flow field and aeroacoustic data are of more immediate interest than a distribution of rather arbitrarily designated sources.

Although the utility of the acoustic analogy theory has been seriously attacked, it is still the only theoretical framework that has been used for predictions of the noise from turbulent jets. Its ability to predict the noise produced by a subsonic jet has often been demonstrated, and it may yet be possible to develop empirical corrections for refraction and other phenomena to extend the usefulness of the theory to flows in which the assumptions used to evaluate the source term are not quite valid. The extension of the theory presupposes that there is little effect of the flow field on the strength of the acoustic sources themselves, and under many circumstances this is no doubt true.

Convected wave equation formulations are, or should be, of considerably more general applicability than acoustic analogy formulations. However, it seems clear that in general they too suffer ambiguities in the definition of acoustic sources. While this may not be as severe a problem as for the acoustic analogy theories, solution of the equations that result from convected wave equation formulation is extremely difficult, making it difficult to apply these theories. Considerable work needs to be done to develop convected wave equation approaches to the point where they have even the limited utility that acoustic analogy theories have today.

While approaches which attempt to link noise production to a coherent structure model of a turbulent jet are of great interest and in some cases great elegance, such mechanisms cannot by themselves explain turbulent noise production. Thus, except in special cases (such as supersonic shear layers) such models alone are unlikely to be of much utility in predicting overall noise radiation, or in devising the means to control turbulent jet noise.

3.0 AN APPLICATION OF THE ACOUSTIC ANALOGY THEORY

3.1 INTRODUCTION

In the preceding section, it was pointed out that the Lighthill acoustic analogy theory (Refs. 1, 2, 5, and 7) is much the most widely used formulation for predictions of turbulent noise from jets. It was, however, also clear that the applications of the theory suffer from a number of difficulties, not the least of which is an inability to handle the effects of refraction. The fact that this is a defect of applications of the theory must be kept in mind; the formulation proposed by Lighthill is exact, but contains the unknown within the source expression. Thus, in general, the sound field must be known in order for the equation governing it to be solved. Although this problem would seem to be suited for iterative techniques, iterative methods of solution have thus far not been very successful (Ref. 135), so that the normal method of solution of the turbulent jet noise problem using the acoustic analogy formulation is to approximate the source term in some manner. It is in this approximation that the discrepancies between the theory and experiment develop.

Nonetheless, the approximations used to evaluate the source term in the Lighthill formulation are illustrative of the interaction between turbulent jet predictive methods and the turbulent noise problem. Therefore, in this section the analysis required to write the source term in the Lighthill formulation in terms of quantities which can be calculated using a turbulent kinetic energy (TKE) model for the flow-field development will be displayed in detail. The approximation used for the source term is based on that developed in Refs. 55 and 56, which is, in turn, based on several earlier analyses.

In his predictions, Moon (Refs. 55 and 56) used both experimental data for the velocity fluctuations and the predictions of a turbulent flow-field model as inputs to his acoustic formulation. He found that good agreement with measured overall sound pressure level and frequency spectrum data could be obtained using experimental fluctuation data as inputs, but that the turbulent intensities predicted by the theoretical analysis of the jet that he used had to be increased by an arbitrary factor to obtain good agreement. Clearly this is not satisfactory in a model designed to predict jet noise production. In the work described here, a more sophisticated model has been used to obtain a theoretical prediction of the turbulent intensities in the jet, in order to obtain a truly predictive model. Certain modifications to the constants used by Moon in the acoustic formulation were also made in the course of this work, and the formulation of the convection Mach number was corrected.

3.2 DEVELOPMENT OF SOURCE FIELD APPROXIMATION

The starting point for the analysis of turbulent jet noise using the acoustic analogy theory is Eq. (6) for the far-field acoustic density fluctuation in the "analogous" acoustic medium at rest:

$$\frac{\partial^2 \rho}{\partial t^2} - a_o{}^2 \frac{\partial^2 \rho}{\partial x_j \partial x_j} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

where (Eq. (10))

$$T_{ij} = \rho v_i v_j + P_{ij} - a_o{}^2 \rho \delta_{ij}$$

In these equations, ρ is the density, v_i, i = 1, 2, 3, the components of the velocity vector, a_o the speed of sound in the undisturbed medium, x_i, i = 1, 2, 3, are space coordinates, and P_{ij} represents the stress tensor. If the velocity field is divided into a mean and a fluctuating part, i.e., $v_i = U_i + u_i$, where u_i represents the fluctuating velocity component in the i-direction, and Tij is expanded, Eq. (6) becomes

$$\frac{\partial^2 \rho}{\partial t^2} + 2\overline{U}_i \frac{\partial^2 \rho}{\partial t \partial x_i} + \overline{U}_i\overline{U}_j \frac{\partial^2 \rho}{\partial x_i \partial x_j} - a_o{}^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} - \rho\left(\frac{\partial \overline{U}_j}{\partial x_i}\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_i}{\partial x_i}\frac{\partial \overline{U}_j}{\partial x_j}\right) = S \tag{17}$$

where

$$S = \frac{\partial^2}{\partial x_i \partial x_j}\left(\rho u_i u_j + P_{ij} - a_o{}^2 \rho \delta_{ij}\right) + 2\frac{\partial \overline{U}_i}{\partial x_j}\frac{\partial(\rho u_i)}{\partial x_i} + 2\frac{\partial}{\partial x_j}\left(\rho u_j \frac{\partial \overline{U}_i}{\partial x_i}\right) \tag{18}$$

The continuity equation in the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho \overline{U}_j) = 0 \tag{19}$$

has been used in obtaining Eq. (17), and it has also been assumed that the density fluctuations are negligible.

To simplify the theoretical formulation, it is assumed that $\overline{U}_1 = \overline{U}(x_2, x_3)$ and $\overline{U}_2 = \overline{U}_3 = 0$. In order to properly evaluate the noise sources, they must be determined in a moving system, since a frozen pattern of convected turbulence produces no noise. Thus the coordinate transformation,

$$t' = t,\ x_1' = x_1 - \overline{U}_1(x_2,x_3)t,\ x_2' = x_2,\ x_3' = x_3 \tag{20}$$

is performed. Under this transformation

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \overline{U}_1 \frac{\partial}{\partial x_1'} \tag{21}$$

and

$$\frac{\partial}{\partial x_1'} = \frac{\partial}{\partial x_1} \tag{22}$$

so that

$$\frac{\overline{D}}{Dt} = \frac{\partial}{\partial t} + \overline{U}_1 \frac{\partial}{\partial x_1} = \frac{\partial}{\partial t'} - \overline{U}_1 \frac{\partial}{\partial x_1'} + \overline{U}_1 \frac{\partial}{\partial x_1'} = \frac{\partial}{\partial t} \tag{23}$$

Now

$$\frac{\partial}{\partial x_2} = \frac{\partial}{\partial x_1'}\frac{\partial x_1'}{\partial x_2} + \frac{\partial}{\partial x_2'}\frac{\partial x_2'}{\partial x_2} + \frac{\partial}{\partial x_3'}\frac{\partial x_3'}{\partial x_2} + \frac{\partial}{\partial t'}\frac{\partial t'}{\partial x_2} = \frac{\partial}{\partial x_1'}\left(-t\frac{\partial \overline{U}_1}{\partial x_2}\right) + \frac{\partial}{\partial x_2'} \tag{24}$$

so

$$\frac{\partial}{\partial x_2'} = \frac{\partial}{\partial x_2} \tag{25}$$

and similarly

$$\frac{\partial}{\partial x_3'} = \frac{\partial}{\partial x_3} \tag{26}$$

By using Eqs. (21), (22), (25), and (26) and specializing to an idealized mixing layer (i.e., $\overline{U}_2 = \overline{U}_3 = 0$), Eq. (17) becomes

$$\frac{\partial^2 \rho}{\partial t'^2} - a_o^2 \frac{\partial^2 \rho}{\partial x_i' \partial x_j'} = \frac{\partial^2}{\partial x_i' \partial x_j'} (\rho u_i u_j) + 2 \frac{\partial \overline{u}_1}{\partial x_j'} \frac{\partial}{\partial x_i'} (\rho u_j) = S' \tag{27}$$

with the term (P_{ij} - $a_o^2\ \rho\delta_{ij}$) neglected. Equation (27) is the ordinary wave equation, and the solution can be obtained by Green's function techniques. Because this portion of the analysis is quite often skipped in turbulent jet noise calculations, it will be described in detail in this section. The basic approach follows that laid out in Morse and Ingard (Ref. 144)[2].

3.2.1 Self Noise

In this section, the general solution to Eq. (27) will be obtained. The conversion from a spatial derivative of the source expression to a time derivative, which is not often treated in detail will be displayed; this conversion will allow a treatment of the velocity fluctuations that enter the source term in terms of characteristic frequencies, which will then be related to the characteristic frequencies of the noise produced by the jet.

Consider, for purposes of illustration, only the first term on the right-hand side of Eq. (27). This term. which represents the interaction of the turbulence with itself, is commonly referred to as the self noise term Under the assumption that in the radiated field $P - a_o^2\rho = 0$, Eq. (27) for self noise becomes

$$\frac{\partial^2 P_1}{\partial x_i^2} - \frac{1}{a_o^2}\frac{\partial^2 P_1}{\partial t^2} = -\frac{\partial^2(\rho u_i u_j)}{\partial x_i \partial x_j} = -\frac{\partial^2 S_{ij}^1}{\partial x_i \partial x_j} \tag{28}$$

where the primes have been dropped and the symbol 1 refers to the self noise contribution. Take the Fourier transform of Eq. (28), where

$$\widetilde{P}_1(\underset{\sim}{r},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P_1(\underset{\sim}{r},t)e^{i\omega t}\,dt = \mathcal{F}\{P_1(\underset{\sim}{r},t)\} \tag{29}$$

so

$$P_1(\underset{\sim}{r},t) = \int_{-\infty}^{\infty} \widetilde{P}_1(\underset{\sim}{r},\omega)e^{-i\omega t}\,d\omega = \mathcal{F}^{-1}\{\widetilde{P}_1(\underset{\sim}{r},\omega)\} \tag{30}$$

For convenience in the remaining development, the subscript and superscript 1 will be dropped. The radius $\underset{\sim}{r}$ is the distance from the origin of coordinates to the observer (see Fig. 4). Also note that

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[2]The author thanks Dr. James Maus of the University of Tennessee Space Institute for bringing this solution technique to his attention.

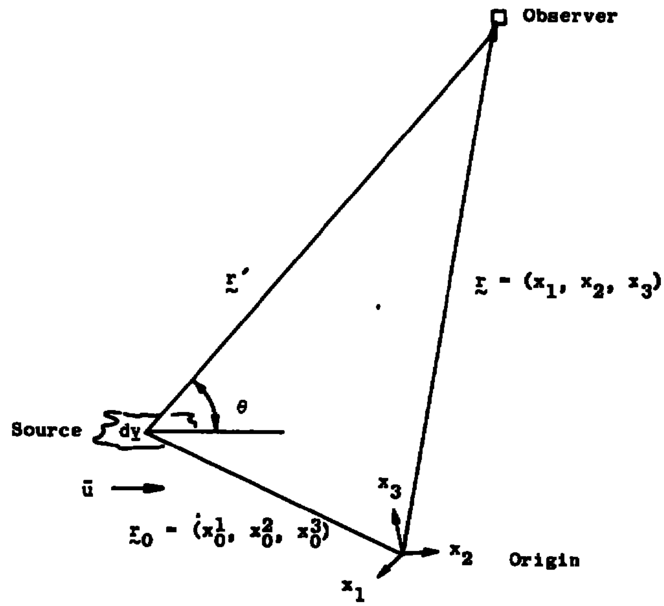

Figure 4. Source-observer geometry.

$$\mathcal{F}\left\{\frac{\partial P}{\partial t}\right\} = -i\omega\tilde{P}(\underset{\sim}{r},\omega) \tag{31}$$

$$\mathcal{F}\left\{\frac{\partial^2 P}{\partial t^2}\right\} = -\omega^2\tilde{P}(\underset{\sim}{r},\omega) \tag{32}$$

In the Fourier transformed plane, the solution technique in effect is to obtain a solution at one frequency; a superposition of solutions obtained through the inversion integral defines the far-field acoustic pressure over all frequencies. The Fourier transform of Eq. (28) is, using Eqs. (29) to (32),

$$\frac{\partial^2\tilde{P}}{\partial x_i^2} + k^2\tilde{P} = -\frac{\partial^2\tilde{S}_{ij}}{\partial x_i \partial x_j} \tag{33}$$

where $k = \omega/a_o$. If the right-hand side is assumed to be known, the solution can be obtained by a Green's function technique using the Green's function for the nonlinear Helmholtz equation:

$$g_\omega(\underset{\sim}{r}/\underset{\sim}{r}_o) = e^{ikR}/4\pi R \tag{34}$$

where $R = |\underset{\sim}{r} - \underset{\sim}{r}_o|$; Eq. (34) is the solution of the equation

$$\frac{\partial^2\tilde{P}}{\partial x_i^2} + k^2\tilde{P} = -\delta(\underset{\sim}{r} - \underset{\sim}{r}_o)$$

Thus, the solution to Eq. (33) is

$$\widetilde{P}(\underset{\sim}{r},\omega) = \int_{v_o} \left[\frac{\partial^2 \widetilde{S}_{ij}}{\partial x_o{}^i \partial x_o{}^j}\right] g_\omega \; (\underset{\sim}{r}/\underset{\sim}{r}_o) dv_o \tag{35}$$

Consider now the expression

$$\frac{\partial \widetilde{S}_i}{\partial x_o^i} g_\omega (\underset{\sim}{r}/\underset{\sim}{r}_o) = \frac{\partial}{\partial x_o^i} (\widetilde{S}_i g_\omega) - \widetilde{S}_i \frac{\partial g_\omega}{\partial x_o^i} \tag{36}$$

so

$$\int_{v_o} \frac{\partial \widetilde{S}_i}{\partial x_o^i} g_\omega \, dv_o = -\int_{v_o} \widetilde{S}_i \frac{\partial g_\omega}{\partial x_o^i} dv_o + \int_{v_o} \frac{\partial}{\partial x_o^i} (\widetilde{S}_i g_\omega) \, dv_o \tag{37}$$

Applying the divergence theorem to the last term and assuming that V_o encompasses the entire disturbance region result in

$$\int_{v_o} \frac{\partial}{\partial x_o^i} (\widetilde{S}_i g_\omega) dv_o = \int_s \widetilde{S}_i g_\omega \eta_i \cdot dS \tag{38}$$

But everywhere outside the disturbance region, $\widetilde{S}_i = 0$, so

$$\int_{v_o} \frac{\partial \widetilde{S}_i}{\partial x_o^i} g_\omega (\underset{\sim}{r}/\underset{\sim}{r}_o) dv_o = -\int_{v_o} \widetilde{S}_i \frac{\partial g_\omega}{\partial x_o^i} dv_o \tag{39}$$

Now, let $Z_i = \partial \widetilde{S}_{ij}/\partial x_o{}^j$, therefore

$$\int_{v_o} \frac{\partial^2 \widetilde{S}_{ij}}{\partial x_o^j \partial x_o^i} g_\omega (\underset{\sim}{r}/\underset{\sim}{r}_o) dv_o = \int_{v_o} \frac{\partial Z_i}{\partial x_o^i} g_\omega (\underset{\sim}{r}/\underset{\sim}{r}_o) dv_o = -\int_{v_o} \frac{\partial \widetilde{S}_{ij}}{\partial x_o^j} \frac{\partial g_\omega}{\partial x_o^i} dv_o \tag{40}$$

By application of the divergence theorem a second time, one gets

$$\int_{v_o} \frac{\partial^2 \widetilde{S}_{ij}}{\partial x_o{}^i \partial x_o^j} g_\omega \; (\underset{\sim}{r}/\underset{\sim}{r}_o) dv_o = \int_{v_o} S_{ij} \frac{\partial^2 g_\omega}{\partial x_o^i \partial x_o^j} dv_o \tag{41}$$

and thus, Eq. (35) becomes

$$\widetilde{P}(\underset{\sim}{r},\omega) = \int_{v_o} \widetilde{S}_{ij} \frac{\partial^2 g_\omega}{\partial x_o^i \partial x_o^j} dv_o \tag{42}$$

If, now, the inequality

$$k\ell = \frac{\omega\ell}{a_o} << 1, \text{ or } \ell << \lambda$$

applies where ℓ is a scale appropriate to the source region, it can be assumed that the Green's function and its derivatives are approximately constant through the source region. Then,

$$\tilde{P}(\underset{\sim}{r},\omega) = \frac{\partial^2 g_\omega}{\partial x_o^i \partial x_o^j} \int_{v_o} \tilde{S}_{ij}\, dv_o \tag{43}$$

Now

$$g_\omega = e^{ikR}/4\pi R$$

where

$$R = |\underset{\sim}{r} - \underset{\sim}{r}_o| = [(x_1 - x_o^1)^2 + (x_2 - x_o^2)^2 + (x_3 - x_o^3)^2]^{½} \tag{44}$$

so that

$$\frac{\partial g_\omega}{\partial x_o^i} = \frac{e^{ikR}}{4\pi R^3}(1 - ikR)(x_i - x_o^i) = f(R)(x_i - x_o^i) \tag{45}$$

then

$$\frac{\partial^2 g_\omega}{\partial x_o^i \partial x_o^j} = -f(R)\delta_{ij} + (x_i - x_o^i)f'(R)\frac{\partial R}{\partial x_o^j} \tag{46}$$

Since

$$f'(R) = \frac{-e^{ikR}}{4\pi R^2}\left[(ik)^2 - \frac{3ik}{R} + \frac{3}{R^2}\right] \tag{47}$$

and

$$\frac{\partial R}{\partial x_o^j} = -\frac{(x_j - x_o^j)}{R} \tag{48}$$

then

$$\frac{\partial^2 g_\omega}{\partial x_o^j \partial x_o^i} = -\frac{-e^{ikr}}{4\pi R^3}(1 - ikR)\delta_{ij} + \frac{(x_i - x_o^i)(x_j - x_o^j)}{R}\frac{e^{ikR}}{4\pi R^2}\left[(ik)^2 - \frac{3ik}{R} + \frac{3}{R^2}\right] \tag{49}$$

Evaluating Eq. (49) at $r_o = 0$, $x_o^i = x_o^j = 0$ gives

$$\left.\frac{\partial^2 g_\omega}{\partial x_o^j \partial x_o^i}\right]_{r_o=0} = \frac{-e^{ikr}}{4\pi r^3}(1 - ikr)\delta_{ij} + \frac{x_i x_j}{r}\frac{e^{ikr}}{4\pi r^2}\left[(ik)^2 - \frac{3ik}{r} + \frac{3}{r^2}\right]. \tag{50}$$

By introducing the far-field approximation

$$r >> \lambda >> a \qquad (kr >> 1) \tag{51}$$

and retaining only the leading term, noting that $x_i x_j = O(r)$, Eq. (50) becomes

$$\frac{\partial^2 g_\omega}{\partial x_o^i \partial x_o^j} = \frac{x_i x_j}{4\pi r^3} e^{ikr} (ik)^2$$

so that Eq. (35) may be written

$$\tilde{P}(\underset{\sim}{r},\omega) \simeq \frac{x_i x_j}{4\pi r^3} e^{ikr}(ik)^2 \int_{v_o} \tilde{S}_{ij}\, dv_o \tag{52}$$

Taking the inverse Fourier transform of Eq. (52) and reintroducing the subscript and superscript 1 notation to label the self-noise term gives

$$P_1(\underset{\sim}{r},t) \simeq \int_{-\infty}^{\infty} \frac{x_i x_j}{4\pi r^3} e^{ikr}(ik)^2 \int_{v_o} \tilde{S}_{ij}^{1}\, dv_o\, e^{-i\omega t}\, d\omega \tag{53}$$

$$\simeq \frac{x_i x_j}{4\pi r^3 a_o^2} \int_{v_o} \int_{\omega} (i\omega)^2\, \tilde{S}_{ij}^{1} e^{-i\omega(t - \underset{\sim}{r}/a_o)}\, d\omega dv_o \tag{54}$$

noting that $k = \omega/a_o$, and thus

$$P_1(\underset{\sim}{r},t) \simeq \frac{x_i x_j}{4\pi r^3 a_o^2} \int_{v_o} \left[\frac{\partial^2 S_{ij}^{1}}{\partial t^2}\right] dv_o \tag{55}$$

where the bracket in Eq. (55) represents evaluation at time $t - r/a_o$. The intensity of the sound at a point where the pressure is P is $1/\rho_o a_o$ times the mean-square fluctuation of P. Thus, in general, the intensity of the sound, i_{se}, is given by

$$i_{se} = \frac{1}{\rho_o a_o} \overline{P_1(r_1, t^+) P_1(r_2, t^+, \tau)} \tag{56}$$

where $t^+ \doteq t - r/a_o$. The term τ represents the emission time delay between the two sources that contribute to the correlation. Thus,

$$i_{se} = \frac{x_i x_j x_k x_\ell}{16\pi^2 r^6 a_o^5} \frac{\rho^2}{\rho_o} \int_v \overline{\left[\frac{\partial^2 u_i u_j(r_1,t)}{\partial t^2}\right]_{t^+} \left[\frac{\partial^2 u_k u_\ell}{\partial t^2}(r_2,t,\tau)\right]_{t^+ + \tau}} dv_o \tag{57}$$

In Ref. 14, it is shown that, for stationary turbulence, Eq. (57) may be rewritten

$$i_{se} = \frac{x_i x_j x_k x_\ell}{16\pi^2 r^6 a_o^5} \frac{\rho^2}{\rho_o} \int_v \frac{\partial^4}{\partial \tau^4} \overline{\{u_i u_j(r_1,0)] \; [u_k \; u_\ell(r_2,\tau)]} \, dv_o \tag{58}$$

Proudman showed (Ref. 101) that the velocity fluctuation may be written in the form

$$x_i x_j \; u_i u_j = |x|^2 u_x^2 = r^2 u_x^2 \tag{59}$$

where u_x is the fluctuation in the direction of the observation point ($\underset{\sim}{r}$). Thus, Eq. (58) can finally be put in the form

$$i_{se} = \frac{\rho^2}{16\pi^2 r^2 \rho_o a_o^5} \int_{V_o} \frac{\partial^4}{\partial \tau^4} \overline{u_x^2(\underset{\sim}{r_1},0) u_x^2(\underset{\sim}{r_2},\tau)} \, dv_o \tag{60}$$

3.2.2 Shear Noise

In a similar fashion, an expression can be derived for the second term on the right-hand side of Eq. (27). This term, representing the interaction of turbulence with the mean flow, is commonly referred to as shear noise. The resulting expression, assuming pressure and velocity correlation terms may be neglected, is shown by Moon (Ref. 55) to be

$$i_{sh} = \frac{\rho^2 \cos^4\theta}{4\pi^2 r^2 \rho_o a_o^5} \left(\frac{\partial \bar{U}_1}{\partial r}\right)^2 \int_{V_o} \frac{\partial^2}{\partial \tau^2} \overline{u_1 u_r(r_1,0) u_1 u_r(r_2,\tau)} \, dv_o$$

$$+ \frac{\rho^2 \sin^2\theta \cos^2\theta}{4\pi^2 r^2 \rho_o a_o^5} \left(\frac{\partial \bar{U}_1}{\partial r}\right)^2 \int_{V_o} \frac{\partial^2}{\partial \tau^2} \overline{u_r u_r(r_1,0) u_r u_r(r_2,\tau)} \, dv_o \tag{61}$$

where θ represents the angle between the radius vector $\underset{\sim}{r}^1$ (Fig. 4) between the source and observer and the axis (recall that $\underset{\sim}{r_o} = 0$) was assumed in the development of Eq. (55) and thus $\underset{\sim}{r}^1 = \underset{\sim}{r}$, and only contributions from the "x - r" and "r - r" quadrupoles (i.e., quadrupoles whose axes are oriented along axial and cross-sectional planes) are admitted. Equations (60) and (61) form the basis for calculating turbulent jet noise intensity through the present model, neglecting the effects of refraction. Note that the radiation due to self noise depends on the second time derivative of the fluctuating velocity correlation, while the shear noise contribution depends on the first time derivative; thus self noise should radiate at a higher characteristic frequency than shear noise. The directivity of the shear noise contribution evidenced in Eq. (61) arises from the geometric relationships required to write the $(\partial \overline{U}_1 | \partial x_j)$ term that arises in the general formulation of the shear noise expression in terms of the radial velocity gradient.

Following Moon (Refs. 55 and 56), different convection factors are applied to the shear noise and self noise parts of the expression for noise intensity. Thus, for shear noise the Lighthill, Ffowcs Williams convection factor (Refs. 1 and 31)

$$C_{se} = [(1 - M_c \cos\theta)^2 + \alpha_{se}^2 M_c^2]^{-5/2} \tag{62}$$

is applied, while the convection factor for shear noise is taken to be that given by Jones (Ref. 54):

$$C_{sh} = [(1 - M_c \cos\theta)^2 + \alpha_{sh}^2 M_c^2]^{-3/2} \tag{63}$$

With these convection factors defined, the expression for the sound intensity in the far field due to a unit volume of turbulence becomes

$$\begin{aligned} i_t = \frac{\rho^2}{4\pi^2 r^2 \rho_o a_o^5} \Bigg\{ & \frac{1}{4} \int_{v_o} \frac{\partial^4}{\partial \tau^4} \overline{u_x^2(\underset{\sim}{r_1},0) u_x^2(r_2,\tau)} dv_o \frac{1}{[(1 - M_c \cos\theta)^2 + \alpha_{se}^2 M_c^2]^{5/2}} \\ & + \left(\frac{\partial \overline{U}_1}{\partial r}\right)^2 \Bigg[\int_{v_o} \frac{\partial^2}{\partial \tau^2} \overline{u_1 u_r(\underset{\sim}{r_1},0) u_1 u_r(r_2,\tau)} dv_o \frac{\cos^4\theta}{[(1 - M_c \cos\theta)^2 + \alpha_{sh}^2 M_c^2]^{3/2}} \\ & + \int_{v_o} \frac{\partial^2}{\partial \tau^2} \overline{u_r u_r(\underset{\sim}{r_1},0) u_r u_r(r_2,\tau)} dv_o \frac{\sin^2\theta \cos^2\theta}{[(1 - M_c \cos\theta)^2 + \alpha_{sh}^2 M_c^2]^{3/2}} \Bigg] \Bigg\} \end{aligned} \tag{64}$$

3.2.3 Modeling of Noise Sources

Despite the numerous approximations and simplifications that have been made in developing Eq. (64), a numerical evaluation of this expression still requires a detailed knowledge of the second and fourth time derivatives of two-point velocity correlations. In general, such detailed data are not available, and no technique exists for making an analytical prediction of these correlations. Therefore, further simplifications are necessary to recast Eq. (64) in terms of quantities for which reliable predictive techniques exist.

Following Moon (Ref. 55) the covariances appearing in Eq. (64) are rewritten in terms of fourth-order correlation coefficients, defined in general by the expression,

$$R_{ijk\ell}(\eta,\xi,t) \equiv \frac{\overline{u_iu_j(\eta,0)u_ku_\ell(\xi,t)}}{\overline{u_iu_j(\eta,0)u_ku_\ell(\eta,0)}} \tag{65}$$

Then

$$\frac{\partial^2}{\partial\tau^4}\overline{u_x{}^2(\underset{\sim}{r_1},0)u_x{}^2(\underset{\sim}{r_2},\tau)} = \frac{\partial^4}{\partial\tau^4}\overline{(u_x{}^2)^2}\ R_{xxxx}\ (V_o,\tau) \tag{66}$$

$$\frac{\partial^2}{\partial\tau^2}\overline{u_1u_r(\underset{\sim}{r_1},0)u_1u_r(\underset{\sim}{r_2},\tau)} = \frac{\partial^2}{\partial\tau^2}\overline{(u_1u_r)^2}\ R_{1r}R_{1r}\ (V_o,\tau) \tag{67}$$

$$\frac{\partial^2}{\partial\tau^2}\overline{u_r{}^2(\underset{\sim}{r_1},0)u_r{}^2(r_2,\tau)} = \frac{\partial^2}{\partial\tau^2}\overline{(u_r{}^2)^2}\ R_{rrrr}\ (V_o,\tau) \tag{68}$$

At this point, Moon (Ref. 55) makes a fundamental assumption. Lighthill (Ref. 2) suggested that the time dependence implicit in Eqs. (66) to (68) could be written in terms of a characteristic eddy frequency (ω). Moon argues that, ignoring any correlation between shear and self noise, and noting that shear noise radiates at a lower frequency than self noise, it is reasonable to assume that each eddy volume radiates sound at only two characteristic frequencies, one corresponding to quadrupole radiation (ω_{se}) and one to dipole radiation (ω_{sh}). The broadband spectrum characteristic of jet noise is then built up by the contribution of a continuous spectrum of eddies each radiating at a local characteristic frequency. Under this assumption, which is the key assumption in the jet noise modeling developed by Moon (Ref. 55), Eqs. (66) to (68) become

$$\frac{\partial^4}{\partial\tau^4}\overline{(u_x{}^2)^2}\ R_{xxxx}\ (v_o,\tau) = \omega_{se}^4\ \overline{(u_x{}^2)^2}\ R_{xxxx}(v_o) \tag{69}$$

$$\frac{\partial^2}{\partial\tau^2}\overline{(u_1u_r)^2}\ R_{1r}R_{1r}(v_o,\tau) = \omega_{sh}^2\ \overline{(u_1u_r)^2}\ R_{1r}R_{1r}\ (v_o) \tag{70}$$

$$\frac{\partial^2}{\partial\tau^2}\overline{(u_r{}^2)^2}\ R_{rrrr}\ (v_o,\tau) = \omega_{sh}^2\ \overline{(u_r^2)^2}\ R_{rrrr}\ (v_o) \tag{71}$$

In general, both the correlations ($\overline{u_x^2}$, $\overline{u_1u_r}$ and $\overline{u_r^2}$) and the correlation coefficients are functions of the space coordinates within the source region. However, if it is assumed that each eddy volume emitter occupies a volume measured by the orthogonal length scales $\ell_1 \ell_2 \ell_3$, that within the volume the correlation coefficients are unity and beyond its edges the correlation coefficients are zero, and that the velocity fluctuation correlations are sensibly constant within an eddy, then the integrals in Eq. (64), using Eqs. (69) to (71), may be approximated by expressions of the form

$$\int_{v_o} \overline{(u_x^2)^2}\, R_{xxxx}\, dv_o = \beta_{xx} \overline{(u_x^2)^2} \ell_1 \ell_2 \ell_3$$

where $\beta_{xx} \ell_1 \ell_2 \ell_3$ defines the volume of the "xx" eddy emitter. Under these assumptions, Eq. (64) becomes, since, as Moon shows (Ref. 55), for an assumed normal distribution of Reynolds stress terms

$$\overline{(u_i u_j)^2} = 2(\overline{u_i u_j})^2$$

$$i_t = \frac{\beta_{xx} \rho^2 \omega_{se}^4 (\overline{u_x^2})^2 \ell_1 \ell_2 \ell_3}{8\pi^2 r^2 \rho_o a_o^5} \; \frac{1}{[(1 - M_c \cos\theta)^2 + \alpha_{se}^2 M_c^2]^{5/2}}$$

$$+ \frac{\beta_{1r} \rho^2 \omega_{sh}^2 (\overline{u_1 u_r})^2 \ell_1 \ell_2 \ell_3}{2\pi^2 r^2 \rho_o a_o^5} \left(\frac{\partial \overline{U}_1}{\partial r}\right)^2 \frac{\cos^4\theta}{[(1 - M_c \cos\theta)^2 + \alpha_{se}^2 M_c^2]^{3/2}}$$

$$+ \frac{\beta_{rr} \rho^2 \omega_{sh}^2 (\overline{u_r^2})^2 \ell_1 \ell_2 \ell_3}{2\pi^2 r^2 \rho_o a_o^5} \left(\frac{\partial \overline{U}_1}{\partial r}\right)^2 \frac{\sin^2\theta \cos^2\theta}{[(1 - M_c \cos\theta)^2 + \alpha_{se}^2 M_c^2]^{3/2}} \quad (72)$$

In order to use Eq. (72) for the prediction of noise intensity, models must be developed for ω_{se} and ω_{sh}, $\overline{u_x^2}$, $\overline{u_1 u_r}$, $\overline{u_r^2}$, and the eddy length scales ℓ_1, ℓ_2, and ℓ_3. Following Davies, et al. (Ref. 74), the characteristic eddy frequencies are written in terms of the local mean velocity and the eddy length scales,

$$\omega_{se} = \gamma_{se} \frac{\overline{U}_1}{\ell_2}, \qquad \omega_{sh} = \gamma_{sh} \frac{\overline{U}_1}{\ell_1} \quad (73)$$

where $\ell_2 \approx 0.5\ell_1$, so that at a given point the frequency of the shear noise is half that of the self noise, ensuring an octave frequency separation between the sources. The eddy length scales themselves can be obtained from measurements of the growth rate of the shear layer in the first regime of a circular jet; Moon (Ref. 55) chooses these to be

$$\ell_1 = 0.716\ \ell_s,\ \ell_2 = \ell_3 = 0.358\ \ell_s \quad (74)$$

where ℓ_s is the width of the shear layer at a given axial station.

To complete the modeling of the far-field noise intensity produced by a subsonic jet, it is necessary only to develop a means for predicting the correlation $\overline{u_x{}^2}$, $\overline{u_1 u_r}$, and $\overline{u_r{}^2}$, and to assign values to the constants β_{xx}, β_{1r}, β_{rr}, γ_{sh}, γ_{se}, a_{sh}, and a_{se}. In the present work, the correlations $\overline{u_x{}^2}$, $\overline{u_1 u_r}$, and $\overline{u_r{}^2}$ are all modeled in terms of the turbulent kinetic energy (TKE) at each point in the flow field, and the TKE field is obtained by the technique described in detail in Ref. 145. It is assumed that $u_x \approx u_1$ (recall that u_x represents the component of the turbulent fluctuation in the direction of the observer), and that all three expressions ($\overline{u_x{}^2}$, $\overline{u_1 u_r}$, and $\overline{u_r{}^2}$) can be approximated by the local value of the turbulent kinetic energy, $k = 1/2\overline{(u_1{}^2 + u_2{}^2 + u_3{}^2)}$. This assumption is of course not correct in detail, since it has been shown (e.g. Ref. 145) that $\overline{u_1 u_r} \simeq 0.3$ k; however, it is not crucial to the analysis since the bulk of the noise production is predicted to originate from the normal stress terms of Eq. (72). Indeed, substitution of the correct proportionality for the correlation $\overline{u_1 u_r}$ degraded the prediction of the noise field. The shear noise term affects primarily the low frequencies, and in this region, Eq. (72) substantially underpredict the observed noise intensities even under the assumption that $\overline{u_1 u_r} = k$.

The terms of the form $\beta\ell_1\ell_2\ell_3$ represent the volumes of the assumed eddy emitters, and if a cylindrical model is used for each eddy emitter as used by Moon (Ref. 55), then for an axially oriented emitting volume $\ell_2 = \ell_3$ and $\beta = \pi/4$. In the present work, the value for β_{xx} is assumed to be $\pi/4$, but $\beta_{1r} = \beta_{rr} = \pi$. The argument used to justify this assumption is that the eddies which produce "shear" noise must be larger than those responsible for "self" noise, since, in a turbulent flow, the shear stress appears to be tied up in the larger eddies of the flow. If it is further assumed that the ratio β_{sh}/β_{se} has approximately the same value as the ratio of the turbulence macroscale to the turbulence microscale, L/λ, then the measurements of microscale and macroscale reported in Townsend (Ref. 146) in a two-dimensional wake imply $\beta_{sh}/\beta_{se} \simeq 4$.

Three further parameters remain to be defined in order to use Eq. (72) to produce a prediction of the overall noise level of a subsonic circular jet. The convection Mach number (M_c) was taken by Moon (Refs. 55 and 56) to be equal to the local velocity (u_1) divided by the ambient speed of sound (a_o). However, there is considerable evidence (Refs. 20 and 75) that the lateral variation of the convection Mach number is less marked than that of the mean axial velocity in the circular jet. Because of this evidence, a more gradual variation of the convection Mach number has been assumed in this analysis, taking the form of a linear lateral variation from $M_c = 0.7\,\bar{U}_1/a_o$ at the inner edge of the mixing region to $M_c = 0.3\,\bar{U}_1/a_o$ at the outer edge. The constants a_{se} and a_{sh} which appear in the convective amplification expression in Eq. (72) are both taken to be 0.55, following

Moon. Finally, values of γ_{se} and γ_{sh} were selected by comparison of theory and experiment; the numerical values obtained in this manner are $\gamma_{sh} = \gamma_{sh'} = 0.35$.

In summary, the overall noise production in the far field from a unit volume of turbulence in a jet is given by Eq. (72), in which

$$\omega_{se} = \gamma_{se}\overline{U}_1/\ell_2$$

$$\omega_{sh} = \gamma_{se}\overline{U}_1/\ell_1$$

$$\ell_1 = 0.716\ \ell_s$$

$$\ell_2 = 0.358\ \ell_s$$

where ℓ_s is the shear layer width from $\overline{U}_1 = 0.95\ \overline{U}_{CL}$ to $U_1 = 0.05\ \overline{U}_{CL}$,

$$M_c = \frac{\overline{U}_a}{a_o}\ [0.7 - 0.4(y - y_I)/(y_E - y_I)]$$

where y_E and y_I are the radii to the outer and inner edges of the shear layer, respectively, and a_o is the ambient speed of sound,

$$\overline{u_1{}^2} = \overline{u_1 u_r} = \overline{u_r{}^2} = k$$

$$\gamma_{se} = \gamma_{sh} = 0.35$$

$$a_{se} = a_{sh} = 0.55$$

$$\beta_{xx} = \pi/4$$

and

$$\beta_{1r} = \beta_{rr} = \pi$$

To obtain the total OASPL radiated from the jet at a given distance, r, and angle, ψ (see Fig. 5), $a_o\rho_o$ times Eq. (72) is integrated over the sound producing region,

$$\text{OASPL}\ (r,\psi) = 2\pi a_o \rho_o \int_o^{z_o} \int_{y_I}^{y_E} i_t\ r dr dz \tag{75}$$

Since both ω_{se} and ω_{sh} represent circular frequencies, Eq. (72) can also be used to obtain the predicted noise spectra as a function of frequency at a given observation angle and radius.

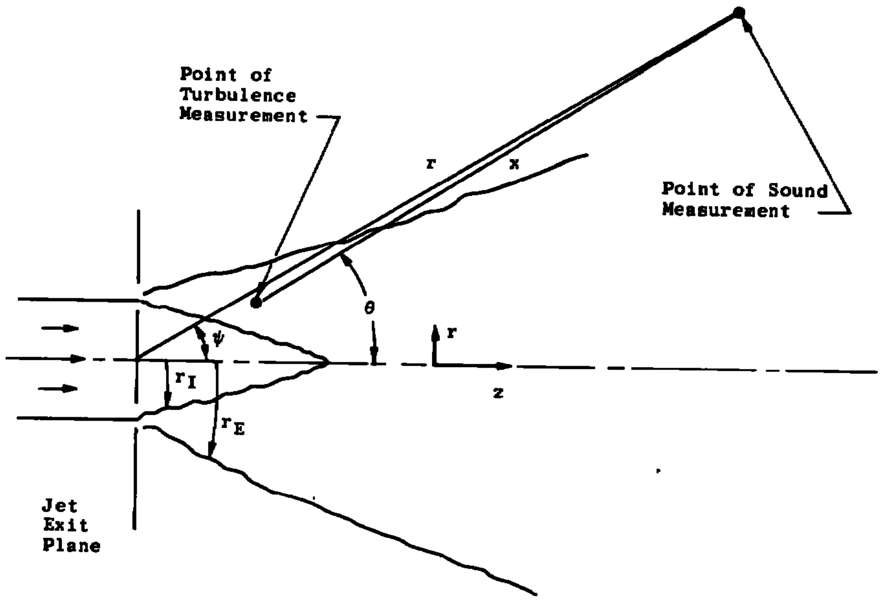

Figure 5. Axisymmetric jet and observer geometry.

3.3 PREDICTION OF JET NOISE

In order to establish the capabilities of the analysis of the jet noise production mechanism described in this section, computations of the noise field of 3/8-in.-diam subsonic jet were carried out. The experimental results were those obtained by Moon (Refs. 55 and 56). This particular set of experiments was chosen primarily for convenience; the data compare favorably with other investigations of the noise from subsonic jets. Data for three values of jet centerline velocity are available in Refs. 55 and 56; U_j = 300, 500, and 700 ft/sec. In addition to measurements of the overall sound pressure level (OASPL) as a function of observation angle for all three jets, the data also include 1/3-octave frequency spectra for the three jets as a function of observation angle. All measurements were carried out at an observation distance, R, from the jet of 64.5 in.

The jet flow field was analyzed using the turbulent kinetic energy (TKE) analysis reported in Ref. 145. As is customary in using this analysis, no changes to the constants and empirical functions used were made. A sample prediction of the centerline velocity decay for the 500-ft/sec case is shown in Fig. 6, compared with the experimental centerline velocity data. From Fig. 6, it is clear that the overall prediction of the fluid mechanics of this jet is satisfactory.

Prediction of the far-field noise intensity using Eq. (72) requires the prediction of mean velocity and turbulent kinetic energy profiles, both of which are available from the TKE calculations. Distributions of the OASPL for all three jets are shown in Fig. 7. Data were not taken for angles to the axis of less than 20 deg (except in one case) nor more than 110 deg. Predictions are not shown below 20 deg because the effects of refraction of sound by the jet flow became important in this region; effects of refraction are not included in the model. In the range over which data are available, the prediction obtained agrees quite well with the experiment, for all jet velocities.

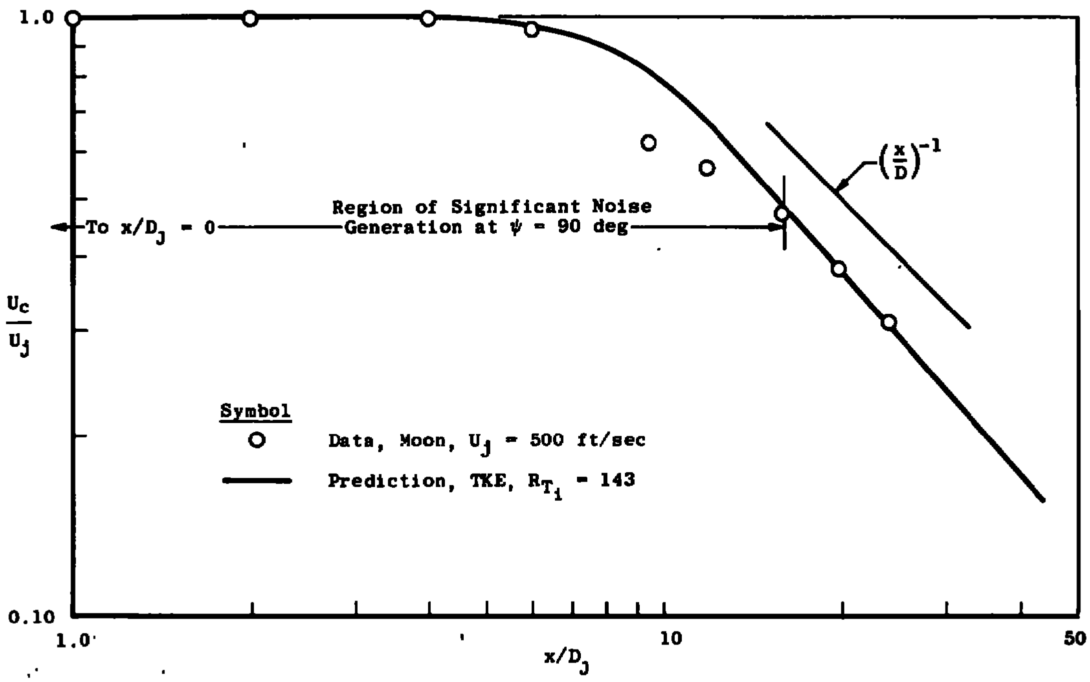

Figure 6. Jet centerline velocity prediction.

An indication of the predicted sound pressure level per "jet slice" can be obtained by evaluating the inner integral in Eq. (73) over small intervals of the axial coordinate (z). Such an indication is shown in Fig. 8, for the 500-ft/sec jet at various observation angles. While there are no direct data available from these experiments for comparison with the calculations shown in Fig. 8, it is of interest to note that the model predicts that the peak noise production occurs near the jet exit, that at ψ = 90 deg, the noise production in the core is essentially constant with x, and that the noise production is essentially complete fairly near the end of the velocity potential core.

Comparisons of predicted and measured frequency spectra for the 300-, 500-, and 700-ft/sec jets are shown in Figs. 9, 10, and 11, respectively. Considerably more deviation from experiment is observed in these figures than in the prediction of overall sound pressure level (i.e., integrated over all frequencies) shown in Fig. 7. At low frequencies, the predictions fall in general below the data, while at high frequencies, they are above the

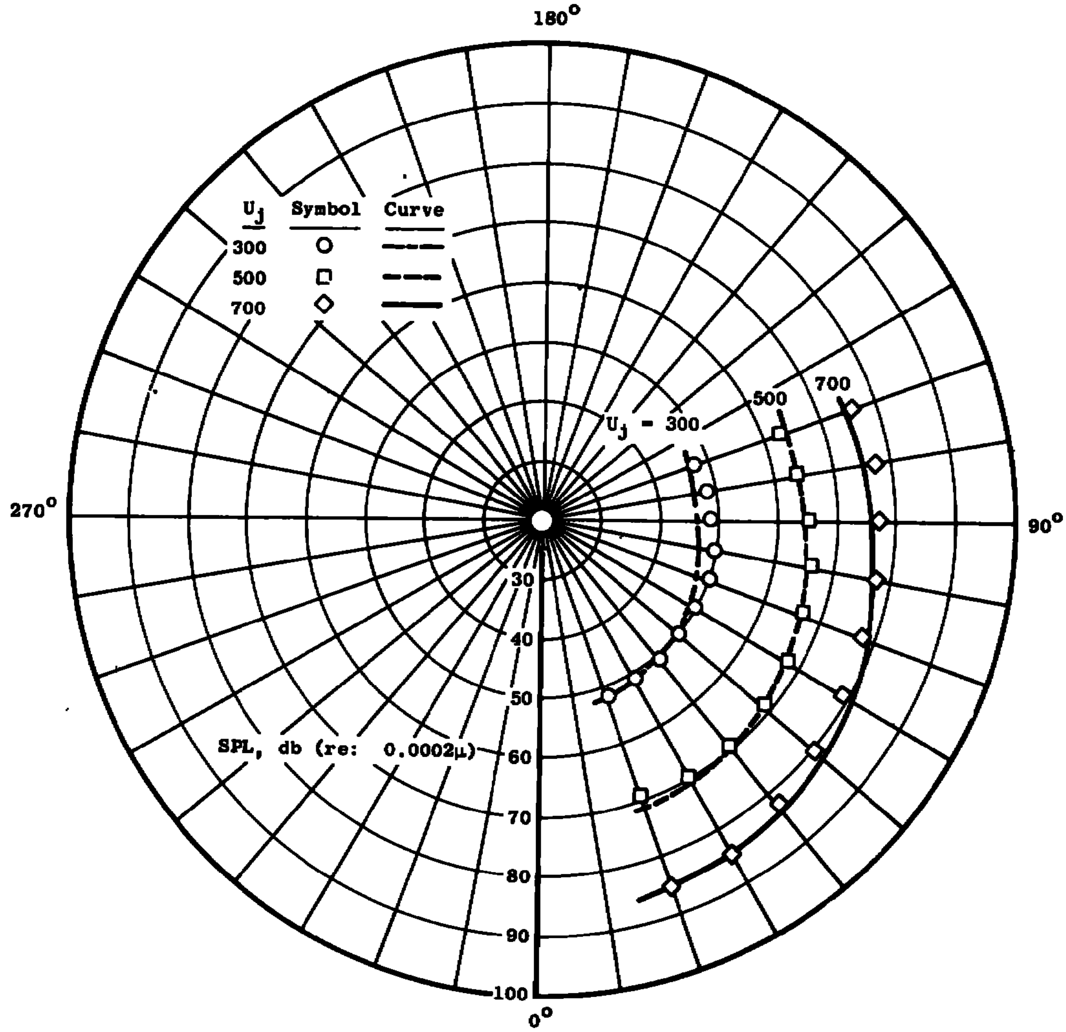

Figure 7. Comparison of predicted and experimental distribution of OASPL.

experiment. However, it must be recalled that this analysis does not include the effects of refraction, and solution to convected wave equation formulations have shown (e.g. Ref. 136) that refraction causes an increase in the apparent sound pressure level at low frequencies and a decrease at high frequencies ("low-frequency lift" and "high-frequency attenuation"). The failure of the acoustic analogy model may thus be expected in these regions.

Figures 9 through 11 also show a dip in the frequency spectrum which always occurs at relatively high frequencies. This reduction in the predicted sound pressure level is a numerical artifice which can be reduced or removed by increasing the number of

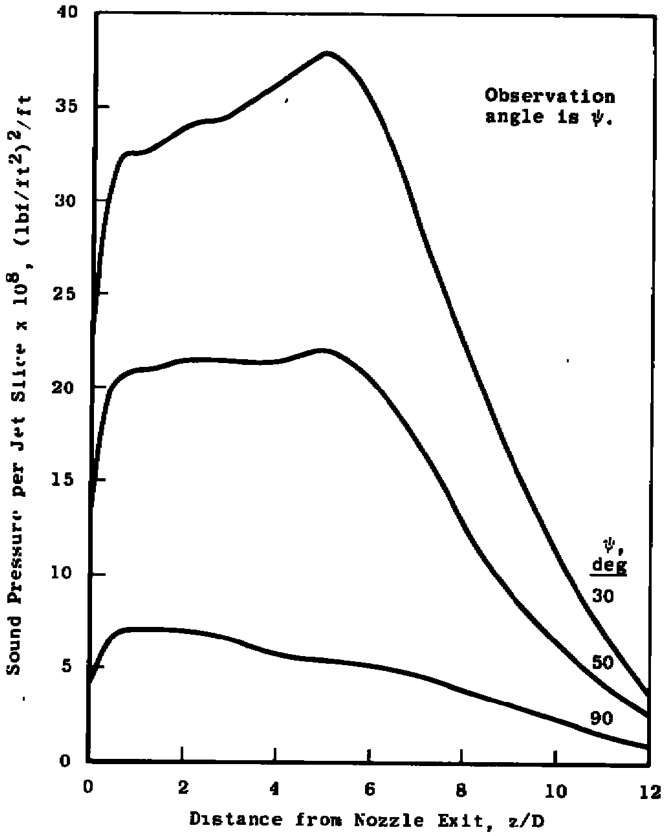

Figure 8. Sound pressure level per jet slice, U_j = 500 ft/sec.

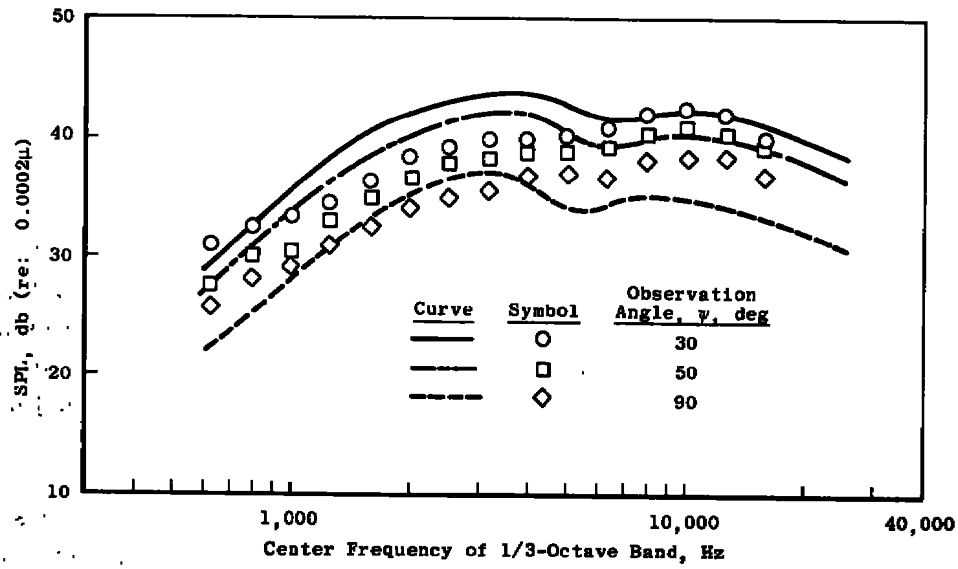

Figure 9. Frequency spectra at 300 ft/sec.

cross-stream points in the finite-difference formulation of the jet flow field. However, since the results of the acoustic analogy model are at best incomplete and possibly invalid at high frequencies, it was not considered to be worthwhile to attempt to improve this aspect of the prediction.

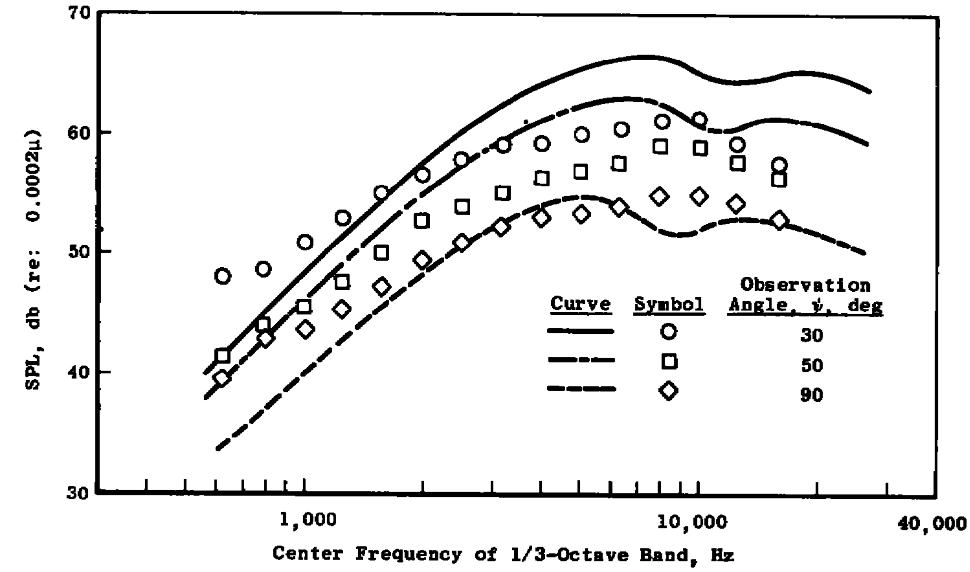

Figure 10. Frequency spectra at 500 ft/sec.

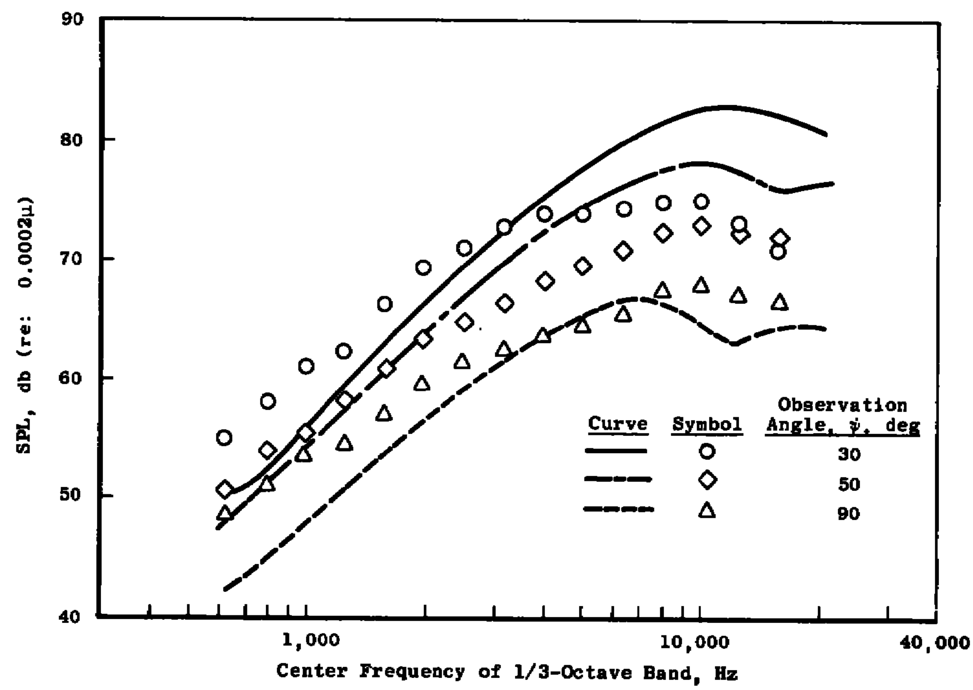

Figure 11. Frequency spectra at 700 ft/sec.

3.4 SUMMARY AND CONCLUSIONS

In this section, an expression for the acoustic power output of a subsonic jet has been derived in terms of obtainable correlations of the turbulent velocity fluctuations and applied to the prediction of subsonic jet noise. Over a limited range of observation angle and frequency, this approach produces reasonably good results. However, the model does not explicitly include the effects of refraction, and thus it cannot be applied near the jet flow axis. These calculations do show that, given a model for the relationship between turbulent velocity correlation and jet noise currently available, flow field predictive methods can be used to generate the information needed in the model.

Improvements to the prediction shown in this report can be obtained in two ways: through an empirical formulation for the effects of refraction in the acoustic analogy model, or through the development of convected wave equation techniques for modeling jet noise which include, among other things, the effects of refraction. The former approach would appear to be cumbersome, as experiments have shown that refraction is a function of frequency as well as angle to the jet axis, so that a general empirical formulation for refraction would appear to be an elusive goal. Unfortunately, the latter approach, while intellectually satisfying, is vexed with problems, since the solution of the noise propagation equation is extremely difficult, even with assumed simple sources. When general solutions to the problems of convected wave equation formulations are achieved, it does appear feasible to couple them to flow-field analyses in the manner described herein.

The number of assumptions used in the analytical development should be kept in mind in interpreting the results shown in this report. If it is true that, given enough adjustable constants, one can reasonably expect to fit any set of experimental data - "give me enough constants and I can fit an elephant" - then an analysis such as is presented here could be properly interpreted as a complicated (and obscure) data fitting technique. The fact that the final result is presented on a logarithmic scale is a tremendous aid as well: a 100-percent error in sound pressure level represents "only" about 6 db. Yet even with the tolerance for error inspired by a decided scale, the predictions made with the method described in this report are by no means exact, nor are they applicable over a broad range of flow conditions.

In Section 2.0 of this report, a number of different approaches to the prediction of aeroacoustic noise phenomena were described. These approaches involve several different postulated mechanisms for the production of noise by turbulent jets, and for the propagation of noise through the jet and the surrounding medium. Despite the differences in the postulated mechanisms, most of these theories are capable of predicting at least some of the features of turbulent jet noise correctly. In part, this paradox may be explained by the tolerance for error implied by a decibel scale, but in general, it must indicate that the proper general theoretical formulation for the aeroacoustic source has not been achieved. While perhaps providing some information about overall noise levels to be expected from certain jets, the currently available theories cannot be used to shed light on the most important technological problem in jet noise, the isolation and suppression of the noise producing mechanisms in a turbulent jet.

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