RMC Research Corporation

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Report UR-226

IMPLEMENTATION OF A MODEL OF REQUISITION PROCESSING FOR THE SHIPS SUPPLY SUPPORT STUDY

Project Director: George M. Lady

Consultant: Carl M. Harris

October 23, 1973

Prepared for

Naval Supply System Command

7910 Woodmont Avenue, Bethesda, Maryland 20014

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INTRODUCTION

This report presents the results of implementing a methodology to relate Navy Supply System throughput time associated with requisition processing and materials handling to the resources dedicated to those activities. This effort on the part of the RMC Research Corporation is in support of the S^4 (Ships Supply Support Study) project which is a general simulation of supply system activities. Consideration of alternative methodologies and selection of a particular method were accomplished by RMC Research Corporation personnel prior to the study effort reported on in this document.[1]

In developing the methodology, supply system activities were studied in the areas of communications, transportation, inventory management, and stock point operations. Pilot applications of the method and other data revealed the throughput times associated with communications activities were not large enough to warrant an expensive modeling effort. Transportation activities could be sufficiently modeled on the basis of an enumeration of transport vehicles, schedules, and various administratively determined policies. The interesting and seemingly important portions of throughput time for modeling purposes were associated with requisition processing and materials handling at inventory control points and stock point facilities.

Further study of materials handling operations revealed that resource level/ throughput time relationships could be reasonably explained by simple queueing

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1. See RMC, Inc. Final Report, UR-176, Methodology for the Measurement of the Relationship Between Naval Supply System Resources and Supply System Throughput Time, June 5, 1972.

theoretic formalisms and an incorporation of pickup and delivery schedules. Generally, the data describing materials handling operations revealed very modest backlogs. Confidence in the throughput times predicted by an extrapolation of materials handling models was enhanced by the fact that the models could be designed to closely replicate the materials handling process. Only that portion of throughput time associated with requisition processing appeared to offer complexities which would not submit to main force analytic techniques.

MODELING REQUISITION PROCESSING

Study requisition processing at inventory control points (ICPs) and stock points revealed long waiting times not apparently related to servicing the requisitions. Certain of these times could be explained by policy and administrative procedures not related to resource levels. However, in the initial modeling efforts waiting times due to backlogs could not be explained by the relatively simple queueing theory models attempted.[1] Several factors could explain the failure to model backlogs:

- the queueing models attempted did not take into account the system of priorities under which requisitions were processed,
- the resources dedicated to requisition processing are often used in a number of other capacities,
- the assumptions about work load arrival rate and service time distributions used by the queueing model might be unrealistic, and
- the backlogs might be contrived by supply system personnel for any of a number of reasons ranging from a desire to efficiently use personnel to foster job security.

The emphasis of the methodology implementation reported on here was to more intensively study and model requisition processing especially at inventory management facilities. A serious difficulty was the lack of a suitable supporting data base. For reasons of economy the study was constrained to secondary data sources. An investigation of information contained in the Military Standard

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1. A highly detailed simulation, SIMCOM, conducted by FMSO personnel to model requisition processing at SPCC also failed to predict waiting times due to backlogs.

Requisitions and Issue Procedures (MILSTRIP) reporting system revealed that sufficient data were available although such data were not currently reported in a usable fashion. The automatic data processing arrangements at both Ships Parts Control Center (SPCC) and Electronics Supply Office (ESO) maintain what is called a "requisition status file." The purpose of this file is to enable the inventory management facility to determine the location (in its administrative structure) and status of a requisition as needed, usually in response to an inquiry from the originator of the requisition. Included among the data contained in this file are the "date of receipt" by the facility, the particular branch or division of the facility in which the requisition is located at a given point in time (the "local routing code"), the "date of last action" which usually denotes its initial receipt in its current location, and its status (e.g., under manual review, on back order, under procurement, etc.). The sum total of these data would document the following:

- the particular administrative subdivisions of the facility which serviced a given requisition,
- the numbers of requisitions received by each subdivision per day,
- the distribution of times a requisition spent in each administrative subdivision,
- the frequency with which requisitions experienced alternative final actions (e.g., the percent sent to stock points, placed on backorder, placed under procurement, etc., . . .),
- the frequency with which requisitions received by a given subdivision were sent on to each other subdivision, and
- the distribution of total times from initial receipt of the requisition until its final disposition.

The problem with the status file records as kept was that the prior "history" of a requisition was erased when the file was updated.

An investigation by Fleet Material Support Office (FMSO) personnel revealed that all entries to the status file were retained for 30 days at the inventory control points in a concentrated data format. A very extensive and ambitious programming task was undertaken at FMSO to extract these data and generate the outputs enumerated above. A program was successfully completed and a data extraction and data analysis effort was implemented for data describing 30 days of operations at both SPCC and ESO. The data analysis was segmented with respect to the priority of the requisition (Issue Group I, II, or III), the extended price of the requisitioned items (more or less than $2,200.00), and whether it was an FSN versus part numbered requisition. A presentation of the results of modeling such data from SPCC is presented in Chapter 3.

In addition to satisfying data needs, RMC prepared a detailed analytic model utilizing a queueing theoretic approach which specified elapsed time distributions for requisitions being serviced under a multipriority system. Although the analytic principles employed were general to any number of priorities, programs were written for the specific cases of no priorities, two priorities, and three priorities. A discussion of how priorities were taken into account is given in Chapter 3 with a detailed description of the mathematics provided in Appendix A.

Other portions of the analysis were programmed and placed in on-line status. Tests were provided to determine if the work unit arrival rate distributions and the service time distributions that were actually observed sufficiently resembled those distributions which were assumed the case in order to parameterize the multipriority queueing model. Convolution routines were programmed to assist in combining several branch throughput time distributions into a single "path" throughput time distribution. Finally, automated procedures were developed which estimate the proportion of requisitions leaving a given subdivision that eventually reach each other subdivision and which enumerate the various "paths" through a facility that requisitions might take.

CAPACITY FACTORS

An input to the multipriority model is an estimate of the capacity (work units per time period) of each functional subdivision of a facility to process requisitions. This estimate plus the assumed work load are used by the multipriority queueing model to generate throughput time distributions for each such subdivision. In the case of stock points the MUACS[1] data base provides work unit standards per dedicated man-hour for many tasks performed within the facility, on the basis of engineering estimates. Standards for tasks for which engineering data do not exist are extrapolated from current observations. In addition the DIMES[2] reporting system provides monthly observations of work units completed and man-hours expended which can readily be used to estimate capacity factors.

The data situation at inventory control points was found to be less straightforward. Since the resources of the ICP are utilized in a number of functions other than requisition processing, performance data are not available for requisition processing per se. However, investigation by RMC Research Corporation personnel revealed that the cost accounting categories maintained at SPCC and ESO would permit a fair estimate of the man-hours dedicated (charged) to requisition processing within each functional subdivision. From other records FMSO personnel were able to achieve estimates of the number of requisitions processed by each subdivision. In combination, these data were used to estimate man-hour/capacity relationships.

OUTLINE OF THIS REPORT

Part of the effort to implement the methodology discussed here entailed consulting support. In particular, the RMC Research Corporation was charged with helping FMSO personnel to use the method in modeling supply system operations other than ICP requisition processing and in planning and designing inputs and outputs of data extraction and analysis of records in the ICP status file. The performance of these tasks entailed numerous working sessions and presentations at the FMSO facility in Mechanicsburg, Pennsylvania and at NAVSUP[3] headquarters.

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1. Manpower Utilization and Control System.
2. Defense Integrated Management Engineering System.
3. Naval Supply Systems Command.

This report is organized as follows. Chapter 2 presents the methodology utilized and some illustrations of its application to data describing operations at NSC Norfolk. Chapter 3 presents analysis of a portion of the data collected at SPCC with an extrapolation of throughput times under various changes in capacity assumptions. Chapter 4 presents a discussion of the insights gained from the methodology implementation. Appendix A is a mathematical discussion of the multipriority queueing model and other statistical procedures developed for the methodology. Appendix B contains listings of the relevant computer programs.

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METHODOLOGY

The method chosen to study the resource level/throughput time relationship is as follows:

- deconsolidate the facility under study into subdivisions consistent with data availability and the various tasks performed by the facility;
- document the "paths" a requisition may follow among these subdivisions and estimate the frequency with which each such "path" occurs;
- for each subdivision estimate a distribution of throughput times taking into account available resources and their associated capacity, work load, and waiting times associated with administratively determined policies;
- amend the subdivision throughput time distributions as dictated by scheduling factors associated with the movement of requisitions within the facility and convolute the appropriate subdivision distributions to achieve a "path" throughput time distribution; and
- take the average of the "path" throughput time distributions weighted by the frequency with which each path occurs to achieve a facility throughput time distribution.[1]

Examining the steps comprising the methodology reveals that throughput time is sensitive to

- servicing times,
- waits due to backlogs,

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1. The subject matter could well be segmented for throughput times with respect to the alternative disposition of the requisition (e.g., for an ICP: the distribution of times associated with, being sent to a stock point, placed on back-order, canceled, etc.).

- pickup and delivery schedules,
- the "paths" the requisitions follow, and
- administratively determined waits.

The service time portion of throughput time is not at issue in this study. That is, the service rate of an individual is not under study; rather, the capacity of an administrative subdivision is a function of the number of man-hours assumed available given the service rate per man-hour.[1] Waits due to backlogs are (presumably) precisely that portion of throughput time which is to be explained by the queueing theory portion of the method, given the assumptions about capacities and work load levels. It is the variation in waiting times due to backlogs predicted by the queueing theory on the basis of variations in capacities that constitutes the central resource level/throughput time relationship of the methodology. In practice, the queueing theory predicted very modest backlogs, as observed capacities were very large compared to work loads. Some discussion of the implications of these results is contained in Chapter 4. Pickup and delivery schedules very often tend to dominate the throughput time predicted by the methodology. If capacities are large compared to work loads, then the queueing theory will tend to predict that a requisition is ready to go on the next scheduled pickup after its delivery to a servicing station. Indeed even a cursory view of supply system operations leads to the conclusion that requisition processing throughput times can be very significantly influenced by variations in messenger service.

The various "paths" a requisition may follow through a facility are related to resources indirectly. Obviously if requirements to service a requisition are amended to include greater or fewer servicing units, then the resources utilized in processing a requisition are accordingly greater or fewer. If the number of man-hours dedicated to each servicing unit remains constant, then changing the "paths" utilized or the frequency with which a "path" occurs will change the work load requirement for the servicing units involved. Such changes will impact on

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1. This service rate might be from an engineering standard as with MUACS or more normally imputed from observation.

throughput time by implying longer or shorter waits due to queues. The "paths" followed by requisitions are determined by policy decisions. As a result, experiments concerning the influence on throughput time of changing servicing requirements have not been attempted. However, the methodology is easily able to support such experiments if desired. Administratively determined waits (e.g., waits due to the competitive requirements of the procurement process) provide additive constants to the relevant throughput time distributions. Experiments with administratively determined waiting times are beyond the scope of this methodology implementation.

STEPS IN THE METHODOLOGY

The analysis of requisition processing accepts the following data inputs:

- a definition of the elements or subdivisions of the system under study;
- the frequency distribution of work unit flows among the elements of the system, e.g., the proportion of work units leaving each element that immediately go to each other element;
- the work units per time period capacity of each system's element; and
- the work load (in each priority as appropriate) of each element.

The automated analytical portions of the method are as follows in the order of their usage.

Tests for Distributional Characteristics

The multipriority queueing model was implemented under the assumptions that the underlying input stream for each priority is Poisson and that service times are exponentially distributed. Procedures have been automated which evaluate the degree to which observed arrivals and service times are described by these assumptions.

Analysis of Work Load Distributions

A procedure has been programmed which accepts the frequency distribution of flows among system elements and extrapolates them to predict the work load of each systems element as a percent of the requisitions leaving any other given systems element. The gross work load of an element may be calculated by multiplying this percent times the number of requisition arriving in the system at each originating element and adding the products together.

Estimation of Throughput Time Distributions for Each System Element

The multipriority queueing model accepts data describing average work loads for each priority considered and capacity for each element. The model outputs the expected throughput time distribution for each element by priority.

Path Analysis

A procedure has been automated which enumerates the possible "paths" a requisition may follow through a facility and computes the frequency with which each path occurs. This analysis is based on the data describing the frequency distribution of requisition flows among system's elements.

Convolution Routines

The throughput time distribution for a "path" followed by a requisition is determined by convoluting the throughput time distributions for each of the constituent elements of the path with adjustments included for pickup and delivery schedules and administratively determined waiting times.

Using the data and automated procedures described above, the steps of the analysis described below are as follows:

- eliminate cycles from flow matrix,
- establish work load factors,
- establish capacity factors,
- implement multipriority queueing model,
- path analysis,

- establish frequency distribution of throughput times per path, and
- calculate distribution of system throughput times.

STEP I: ELIMINATE CYCLES FROM FLOW MATRIX

Data input: Frequency distribution of flows among elements (flow matrix).

Program: Manual; circumstances such that a work unit flows from one element to another and returns are eliminated by treating the originating element differently (i.e., as a distinct, new element) upon its receipt of the returning work unit.

Output: Amended flow matrix with cycles eliminated.

Remark: The flows among elements are organized in a matrix format. If a work unit is sent from one facility element to another (say from the customer service branch to the technical branch of a stock point) and returns a cycle is established which the automated analytic procedures would treat endlessly. In order to avoid the "error" such a process would introduce into the analysis, "dummy" elements are introduced which receive requisitions returning to an element and distribute them to all but that element from which they have returned thus eliminating the cycle.

STEP II: ESTABLISH WORK LOAD FACTORS

Data input: Amended frequency distribution of flows among elements (amended flow matrix).

Program: Matrix power series.

Output: Work units reaching any given element as a proportion of those leaving any given element.

Remark: Of primary interest is the work load per element. This is given for each element by taking the work units assumed to be received by each initial element and multiplying by the proportion of those receipts which were received by each other element. However, for the purposes of special studies the program output displays receipts by an element with respect to work units leaving any other element. If desired, work load distinctions can be maintained with respect to a number of criteria. As discussed in Chapter 1, the data describing SPCC and ESO were segmented with respect to issue group, price, and FSN versus part numbered requisitions.

STEP III: ESTABLISH CAPACITY FACTORS

The determination of an estimate of the size of a system subdivision work unit capacity as a function of dedicated resources is difficult. Several approaches are possible; one way is to represent rather carefully the "technology" or process involved at a fine level of detail and on the basis of the identification of bottlenecks or other criteria establish capacities. Such a procedure is expensive and requires a level of detail not otherwise usable to the analysis. The DIMES supplemental data report for stock point operations includes "standards" for work unit completions per man-hours expended. Unfortunately these "standards" have not been established for all tasks involved in stock point operations. Further, an inspection of the data reveals that actual performance is often very different (by a factor of 100 percent or more) than that which would be predicted by an application of the standards. Individuals on the spot testified that such variance could be explained in part by the fact that many of the standards were out of date and no longer descriptive of current practices. In any event, such standards were not available for ICPs.

An alternative procedure is to observe the work units which are in fact processed and compare this figure to the man-hours dedicated to requisition processing. Assuming that data are available which provide this information, some lower bound to capacity can be inferred (e.g., what is observed is possible). A complicating difficulty in the assessment of capacity is that of "multi-use" resources. The manpower resources of an ICP are utilized in a number of activities in addition to requisition processing. Cost accounting categories reveal (to a degree) the man-hour split among various activities by the same individual. Of principle concern, however, is whether or not a change in work load will result in a change in waiting times because a given man-hour must cope with more or fewer requisitions, or will result in a change in man-hours dedicated. That is, will more requisitions lead to longer waiting times due to queues or will man-hours be released from other tasks to accommodate the increased work load? A number of studies of requisition processing have recently been conducted.[1] A uniform observation of these studies is that peak work loads are much larger than average (or "normal") work loads. The general conclusion from a study of these observations is that requisition processing resources are utilized well below their apparent capacity to complete work units. When such low levels of utilization are extended into models which include an analysis of queueing discipline, the results of the modeling usually fail to explain waits due to backlogs which are actually observed. Further discussion of this problem will be offered in Chapter 4.

For purposes of illustrating the method, data describing NSC Norfolk will be presented. FMSO personnel collected cost account data from the DIMES supplemental data report for the months of July, August, and September 1972. These data permitted the calculation of work units per man-hour and man-hours per calendar hour (e.g., the "apparent" number of individuals working per hour) for each system's element for each month. The measurement of capacity is calculated by taking the product of the maximum observed rate of processing per man-hour and the maximum man-hour allocations. The processing rate and man-hour allocations used may occur in different time periods. Given capacity per hour and given the proportion of requisitions received by NSC Norfolk that reach each given element from Table 2-1 (work units per work unit),[2] the maximum number of requisitions received by NSC Norfolk per hour which each element can support is calculated by dividing hourly capacity by the work units per work unit proportion (e.g., if "purchase" can process 34.06 work units per hour and if 12 requisitions per thousand received by NSC Norfolk go to "purchase," then "purchase" can support a gross arrival rate of 34.06/.012 = 2838.3 work units per hour).

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1. Some data sources are: the SIMCOM model of SPCC developed by FMSO in 1971-72; MUACS data collected by RMC Research Corporation personnel describing operations at NSC Norfolk during 1971-72; the DIMES supplemental data report for NSC Norfolk for July, August, and September 1972; SPCC cost accounting data for 1972 and SPCC performance data for January and February 1973; and Lynch and Verich, Requisition Throughput Time Simulation at NSC San Diego, March 1973.

2. e.g., work units received by the element per work unit received by the facility.

Table 2-1

DATA DESCRIBING CAPACITY FACTORS FOR NSC NORFOLK

Stock Point Division	(1) Processing Rate/ Hour	(2) Man-Hours/ Calendar Hours	(3) Capacity/ Hour	(4) Work Units Rec'd at Div./ Work Units Rec'd at Stock Point	(5) Maximum Supportable Work Units/ Hour	(6) Relative Work Load	(7) Observed Work Load As % of Capacity	(8) Work Units Arriving Per Man-Hour
Purchase	.91	37.43	34.06	.012	2838.3	.70	.39	.35
Customer Service	22.22	71.02	1578.06	.340	4641.6	.43	.24	5.33
Technical	2.81	19.35	54.37	.020	2718.5	.73	.41	1.15
Issue	9.34	189.57	1770.58	.705	2511.5	.79	.44	4.11
Packing	6.97	177.16	1234.80	.620	1991.6	1.00	.56	3.90

Going through this calculation for each element reveals substantial differences in the gross arrival rates that each element can support. Of the five elements modeled by the queueing formalism the maximum supportable arrival (that associated with "customer service") is 2.38 times the minimum such rate (that associated with "packing"). As a result, "packing" comprises the bottleneck in the processing system described by the data. Relative to its capacity "customer service" receives only 43 percent of what "packing" receives per hour. These relative weightings were used as appropriate for each element.

The data for NSC Norfolk is summaried in Table 2-1. The columns of Table 2-1 are as follows:

<u>Column 1</u>: The largest of the three numbers* calculated by dividing "work units completed" by "man-hours expended."

<u>Column 2</u>: The largest of the three numbers calculated by dividing "man-hours expended" by "man-hours per man per month (i.e., 8 x "work days in month").

<u>Column 3</u>: (Column 1) x (Column 2).

<u>Column 4</u>: The number of work units received by an element per work units received by NSC Norfolk, from Column 1 of Table 1.

<u>Column 5</u>: (Column 3)/(Column 4).

<u>Column 6</u>: A rescaling of Column 5 such that each entry is multiplied by $(1991.6)^{-1}$. Such indicates the relationship of the maximum work units received to that allowed by the bottleneck capacity of "packing."

<u>Column 7</u>: The data describing operations at NSC Norfolk showed "packing" receiving an average of 56 percent of its capacity. The implied workload of each other element is scaled accordingly (i.e., Column 7 = (Column 6) x (.56)).

<u>Column 8</u>: The steady state work units per hour per hour as governed by "packing" receiving 56 percent of capacity = (Column 1) x (Column 7).

(N.B., the computer and the "communications" aspects of stock point operations were not modeled. For the range of work loads to be considered, it was determined that requisitions received were always ready to move on per the pickup and delivery schedule.)

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*i.e., for July, August, and September.

STEP IV: IMPLEMENT MULTIPRIORITY QUEUEING MODEL

Data input: For each subdivision of the system, work load rates per time period for each priority from STEP II above; element capacities from STEP III.

Program: Any of a no-priority, two-priority, and three-priority programs are available. Although issue group distinctions usually provide a context of three priorities, it is sometimes the case that issue groups are combined.

Output: For each priority,

- the average number of work units in the system,
- the average wait in the system,
- the variance of the system wait, and
- the distribution of throughput times.

STEP V: PATH ANALYSIS

Data input: Amended frequency distribution of flows among systems elements.

Program: A path analysis program traces through the system and documents the various sequences of systems subdivision that a requisition can encounter in moving through the system.

Output: For each terminating event (e.g., manner of completing the requisition),

- an enumeration of the alternative sequences of systems subdivisions that a requisition can encounter, and
- the frequency with which each "path" occurs.

Remark: A path is a sequence of elements that a requisition or corresponding material may feasibly encounter. A path is always initiated by an element which receives the work unit from outside the system (e.g., communications) and terminated by an element which passes the work unit on outside the system or otherwise satisfies the requisition (e.g., the "customer service" subdivision of a stock point sends a requisition to an ICP or the "purchase" subdivision of an ICP places a requisition on back order). Given the frequency with which a requisition flows between all pairs of systems elements, the frequency with which a path occurs is measured by taking the product of the frequency values of the flows which comprise the sequence.

STEP VI: ESTABLISH FREQUENCY DISTRIBUTION OF THROUGHPUT TIMES PER PATH

Data input: Frequency distribution of throughput times for each element from Step IV; an enumeration of path elements from Step V.

Program: Convolution routine; the distribution of throughput times per path is computed by convoluting the throughput time distributions of the path elements; an average waiting time per path is also calculated.

Output: Frequency distribution of throughput times and average throughput time per path.

Remark: Institutional factors with respect to batching or other system's waits not relating to the actual servicing of a work unit are taken into account at this point. The waiting times per work unit are amended to account for such waits as appropriate. (e.g., If requisitions leave an element every two hours and the waiting time distribution is with respect to each hour, then an amended distribution is constructed showing zero probability on the odd hours and the sum of the current and the hour previous probability on the even hours; if work units must wait in an element for a fixed average time independent of servicing, then the entire waiting time distribution is "shifted" forward in time by that average wait.)

STEP VII: CALCULATE DISTRIBUTION OF SYSTEM THROUGHPUT TIMES

Data input: Probability of path occurrence from Step V, frequency distribution of throughput times per path from Step VI.

Program: Manual; the frequency distributions for the system as a whole is calculated by taking the weighted average of the path throughput time distributions with the path probabilities serving as weights. In addition, the frequency distribution of throughput times for any given terminating event (e.g., throughput times in a stock point for requisitions sent to an ICP) can be determined by taking the weighted average of the relevant paths using normalized path probabilities as weights (i.e., multiplying the relevant path probabilities by a factor such that they sum to one).

Output: Frequency distributions of throughput times and average throughput time for the system as a whole and for each manner in which a work unit leaves the system as desired.

Remark: A waiting time frequency distribution can of course be calculated in response to any criterion which identifies a subset of paths in the system (e.g., all paths that use a given element, a given two elements, etc., . . .).

ILLUSTRATION OF THE METHODOLOGY

Data describing requisition processing and materials handling at NSC Norfolk were collected by FMSO personnel and will be used here to illustrate the various techniques and procedures of the methodology.

STEP I: ELIMINATE CYCLES

Consideration of data availability and the nature of the tasks performed at a stock point suggested that stock point operations should be represented by seven subdivisions of the stock point "system." These are

- Communications,
- Customer Service,
- "Computer,"
- Technical,
- Purchase,
- Issue, and
- Packing.

These administrative subdivisions were chosen in order to use existing cost account data as reported in the DIMES supplemental data report (i.e., a finer detail is not readily accessible). An analysis of the flows among these elements revealed that requisitions flowing from the "computer" to "Technical" would in part flow back to the "computer." To eliminate this cycle, the "computer" was represented by two elements: "Computer A" which accounts for requisitions initially received and "Computer B" which accounts for requisitions returned to the "computer" from "Technical." Requisitions returning to the stock point from DLSC[1] were ignored as they would comprise only about 1 percent of the total (as a result total throughput time for the stock point as a whole will fail to take into account the long waiting times associated with

1. Defense Logistics Services Center.

filling a requisition cycling through DLSC; however, throughput time for each separate terminating circumstance is unbiased). The following flow diagram, Figure 2-1, summarizes the amended flow matrix.

STEP II: ESTABLISH WORK LOAD FACTORS

Conceptually there are a number of ways to determine the work load of each portion of the stock point as a function of the total number of requisitions arriving at the facility per same time period. Simple averaging or regression procedures suffer from the need to take into account the time lags between a requisition arrival at the stock point and its eventual receipt by some subdivision of the stock point. The procedure used here is that of tracing through the network illustrated in Figure 2-1 and determining for each system's element the proportion of work units it receives. This would be a very tedious process even for only moderately large systems.

As a result a computer program was prepared which computes the proportion of requisitions reaching any given systems element relative to those leaving each other systems element. This program utilizes the frequency distributions of work unit flows between pairs of systems elements such as those displayed in Figure 2-1. For a stock point all requisitions are represented as arriving at the "communications" subdivision. As a result the work load of each other subdivision may be calculated by determining the proportion of requisitions leaving "communications" which reaches each other subdivision and multiplying that proportion by the number of requisitions assumed to be arriving at "communications." The calculation of proportions is displayed in Table 2-2 for NSC Norfolk based upon the frequency distributions of flows given in Figure 2-1. Generally, the numbers in the table indicate the number of requisitions reaching the row coordinate as a proportion of the number leaving the column coordinate. As "communications" is always encountered first, the first column of Table 2-2 contains the proportions of particular interest. For example, the first entry reveals that 34 percent of the requisitions received by "communications" are sent on to "customer service," and so on. It should be noted that the analysis takes the entire network into account and includes all possible paths between "communications" and each other subdivision when computing the relevant proportion.

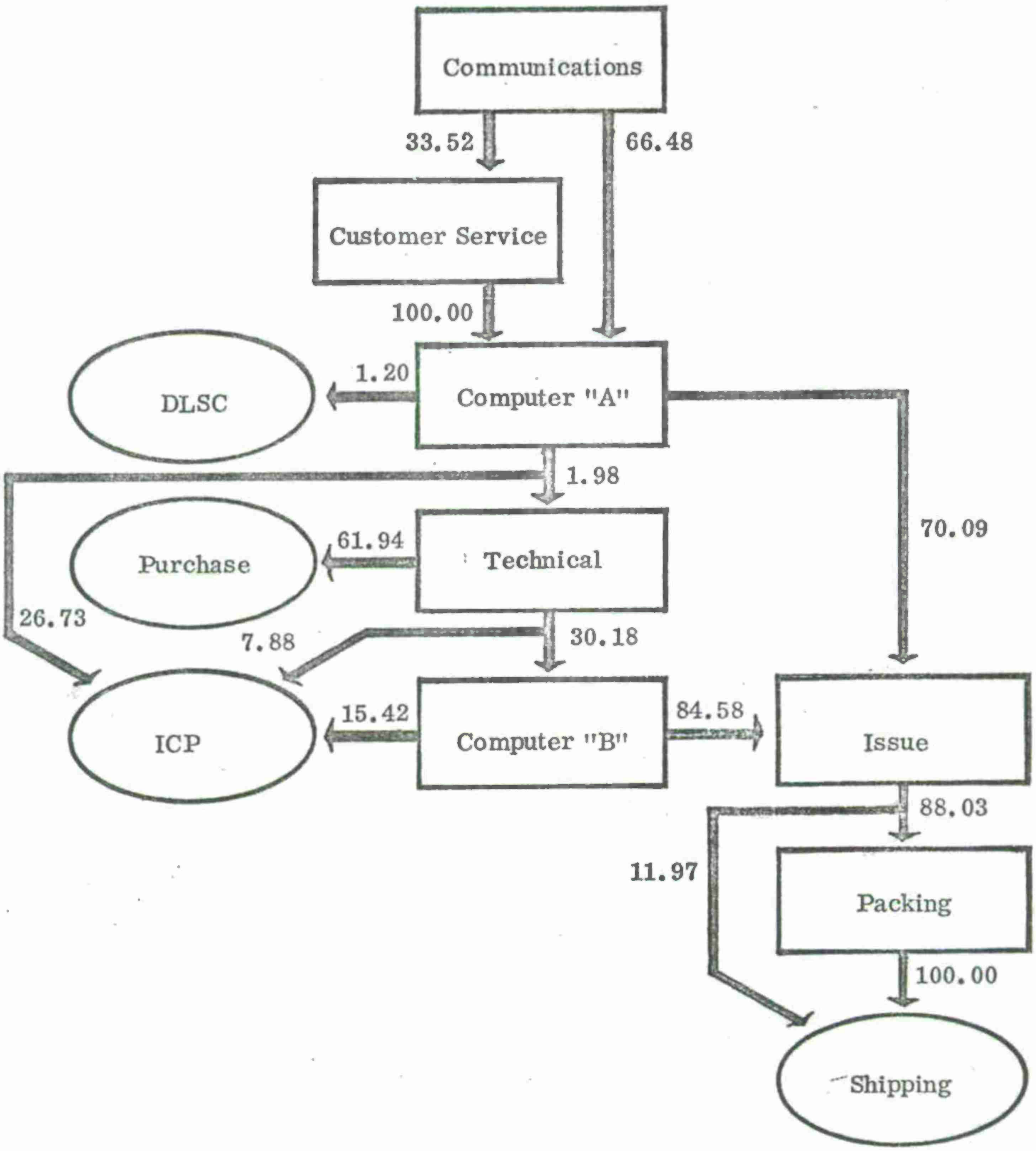

Figure 2-1: NETWORK OF REQUISITION AND MATERIAL FLOWS AT NSC NORFOLK

Table 2-2

FLOW PROPORTIONS AMONG STOCK POINT DIVISIONS

	CCMU	CUSSRV	CCMPA	TECH	COMPB	ISSUE	PACK
CUSSRV	0.3400						
CCMPA	1.0000	1.0000					
TECH	0.0200	0.0200	0.0200				
CCMPB	0.0060	0.0060	0.0060	0.3000			
ISSUE	0.7051	0.7051	0.7051	0.2550	0.8500		
PACK	0.6205	0.6205	0.6205	0.2244	0.7480	0.8800	
DLSC	0.0100	0.0100	0.0100				
PURCH	0.0124	0.0124	0.0124	0.6200			
ICP	0.2725	0.2725	0.2725	0.1250	0.1500		
SHIP	0.7051	0.7051	0.7051	0.2550	0.8500	1.0000	1.0000

STEP III: ESTABLISH CAPACITIES

As discussed above an accurate estimate of an element's capacity based on available secondary data is difficult to achieve. Even when engineering estimates are available they are often incomplete or out of date. As a result, it is necessary to estimate capacity factors from observations of performance. Presumably the capacity per man-hour of an element is no smaller than observed work rates. In Table 2-1, work unit capacities per man-hour are provided based on the maximum per hour work rate and maximum quantity of man-hours per calendar hour expended over a three-month period. In the multipriority queueing model output displayed below (starting on page 2-17) for the issue and packing elements, the capacities are given respectively by:

capacity for issue = 9.34 per man-hour, and

capacity for packing = 6.97 per man-hour.[1]

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1. These measurements were derived from data collected for July, August, and September 1972.

STEP IV: IMPLEMENT MULTIPRIORITY QUEUEING MODEL[1]

An issue not firmly resolved by this study is the choice of a unit of analysis for the queueing theoretic model. That is, what is to comprise the basic servicing unit (e.g., how many man-hours per calendar hour comprise a "servicing unit hour")? A study of intra-element structure was deemed beyond the scope of the analysis and no other statistical means of approaching this issue was developed. Generally, the choice of a unit of analysis was governed by the goal of fitting the output of the multi-priority queueing model to observed behavior. Further discussion of this problem is provided in connection with the modeling of SPCC operations discussed in Chapter 3.

The multipriority queueing model was applied to the NSC Norfolk data for the elements "issue" and "packing" using the man-hour as the basic unit of analysis. This choice is recommended by the fact that issue and packing operations have a very simple structure similar to many parallel servicing units. The capacity and arrival rate data were not further scaled as comprehensive performance data were not available for the stock point at the time the example was formulated. Using the capacity estimates given above and the arrival rates given above in Table 2-1 the queueing model was implemented for three cases:

- the base case per the observed data, and
- two "experimental cases assuming a 20 and a 40 percent increase in work load.

The arrivals were split among the priorities as follows:

- IG-I, 10 percent;
- IG-II, 40 percent; and
- IG-III, 50 percent.

The outputs of the model are self-explanatory. Statistics for the several priorities, high to low, are displayed from left to right, or from top to bottom as appropriate.

---

1. The multipriority queueing model is given in Appendix A.

ISSUE: BASE CASE

```
THE AVERAGE # OF UNITS OF PRIORITIES 1,2,&3 IS
  .492769E-01  .631339E-01       .107799

THE TOTAL AVERAGE WAIT IN THE SYSTEM IS (IN HRS.)
  .535789E-01

THE AVG SYSTEM WAIT FOR EACH PRIORITY IS
       .120188  .384963E-01  .523294E-01

THE VARIANCE OF THE SYSTEM WAIT FOR EACH PRIORITY IS
  .223273E-01  .255265E-01  .695990E-01

DISTRIBUTIONS FOR 3 PRI-S WAITING TIMES
            NUMBER OF HOURS                     PROBABILITY
     0         THROUGH            1                 1

            NUMBER OF HOURS                     PROBABILITY
     0         THROUGH            1                 1

            NUMBER OF HOURS                     PROBABILITY
     0         THROUGH            1                .978759
     1         THROUGH            2             .212411E-01


DISTRIBUTION TAKEN OVER ALL PRIORITIES
     0         THROUGH            1                .989354
     1         THROUGH            2             .106464E-01
     2         THROUGH            3                 0
THE WEIGHTED AVERAGE OF THE PRI-I DISTRIBUTION IS
     1


THE WEIGHTED AVERAGE OF THE PRI-II DISTRIBUTION IS
     1


THE WEIGHTED AVERAGE OF THE PRI-III DISTRIBUTION IS
     1.02124


THE WEIGHTED AVERAGE FOR ALL 3 DISTRIBUTIONS IS
     1.01065
```

ISSUE: +20 PERCENT ARRIVALS

THE AVERAGE # OF UNITS OF PRIORITIES 1,2,&3 IS
.596426E-01 .809684E-01 .162488

THE TOTAL AVERAGE WAIT IN THE SYSTEM IS (IN HRS.)
.614804E-01

THE AVG SYSTEM WAIT FOR EACH PRIORITY IS
.12172 .411007E-01 .657844E-01

THE VARIANCE OF THE SYSTEM WAIT FOR EACH PRIORITY IS
.247270E-01 .329654E-01 .126687

DISTRIBUTIONS FOR 3 PRI-S WAITING TIMES

NUMBER OF HOURS			PROBABILITY
0	THROUGH	1	1

NUMBER OF HOURS			PROBABILITY
0	THROUGH	1	1

NUMBER OF HOURS			PROBABILITY
0	THROUGH	1	.944934
1	THROUGH	2	.550658E-01

DISTRIBUTION TAKEN OVER ALL PRIORITIES

0	THROUGH	1	.972411
1	THROUGH	2	.275887E-01
2	THROUGH	3	0

THE WEIGHTED AVERAGE OF THE PRI-I DISTRIBUTION IS
1

THE WEIGHTED AVERAGE OF THE PRI-II DISTRIBUTION IS
1

THE WEIGHTED AVERAGE OF THE PRI-III DISTRIBUTION IS
1.05507

THE WEIGHTED AVERAGE FOR ALL 3 DISTRIBUTIONS IS
1.02759

ISSUE: +40 PERCENT ARRIVALS

```
THE AVERAGE # OF UNITS OF PRIORITIES 1,2,&3 IS
  .701975E-01       .101336       .247554

THE TOTAL AVERAGE WAIT IN THE SYSTEM IS (IN HRS.)
  .728847E-01

THE AVG SYSTEM WAIT FOR EACH PRIORITY IS
      .123153  .440592E-01  .859561E-01

THE VARIANCE OF THE SYSTEM WAIT FOR EACH PRIORITY IS
  .272130E-01  .423561E-01       .244051

DISTRIBUTIONS FOR 3 PRI-S WAITING TIMES
            NUMBER OF HOURS                   PROBABILITY
       0        THROUGH           1               1

            NUMBER OF HOURS                   PROBABILITY
       0        THROUGH           1                .996296
       1        THROUGH           2            .370364E-02

            NUMBER OF HOURS                   PROBABILITY
       0        THROUGH           1                .862938
       1        THROUGH           2                .122138
       2        THROUGH           3            .149245E-01


DISTRIBUTION TAKEN OVER ALL PRIORITIES
       0        THROUGH           1                .929868
       1        THROUGH           2            .626565E-01
       2        THROUGH           3            .747524E-02
       3        THROUGH           4              0
THE WEIGHTED AVERAGE OF THE PRI-I DISTRIBUTION IS
       1


THE WEIGHTED AVERAGE OF THE PRI-II DISTRIBUTION IS
       1.0037


THE WEIGHTED AVERAGE OF THE PRI-III DISTRIBUTION IS
       1.15199


THE WEIGHTED AVERAGE FOR ALL 3 DISTRIBUTIONS IS
       1.07761
```

PACKING: BASE CASE

```
THE AVERAGE # OF UNITS OF PRIORITIES 1,2,&3 IS
  .850366E-01       .118069       .253059

THE TOTAL AVERAGE WAIT IN THE SYSTEM IS (IN HRS.)
     .116965

THE AVG SYSTEM WAIT FOR EACH PRIORITY IS
     .218043  .756851E-01       .129774

THE VARIANCE OF THE SYSTEM WAIT FOR EACH PRIORITY IS
  .408704E-01  .679436E-01       .288458

DISTRIBUTIONS FOR 3 PRI-S WAITING TIMES
          NUMBER OF HOURS                 PROBABILITY
     0       THROUGH          1              .985076
     1       THROUGH          2           .149245E-01

          NUMBER OF HOURS                 PROBABILITY
     0       THROUGH          1              .978759
     1       THROUGH          2           .212411E-01

          NUMBER OF HOURS                 PROBABILITY
     0       THROUGH          1               .82111
     1       THROUGH          2              .155163
     2       THROUGH          3           .237271E-01


DISTRIBUTION TAKEN OVER ALL PRIORITIES
     0       THROUGH          1              .900566
     1       THROUGH          2           .875705E-01
     2       THROUGH          3           .118636E-01
     3       THROUGH          4             0
THE WEIGHTED AVERAGE OF THE PRI-I DISTRIBUTION IS
     1.01492


THE WEIGHTED AVERAGE OF THE PRI-II DISTRIBUTION IS
     1.02124


THE WEIGHTED AVERAGE OF THE PRI-III DISTRIBUTION IS
     1.20262


THE WEIGHTED AVERAGE FOR ALL 3 DISTRIBUTIONS IS
     1.1113
```

PACKING: +20 PERCENT ARRIVALS

```
THE AVERAGE # OF UNITS OF PRIORITIES 1,2,&3 IS
      .1033          .155508        .441397

THE TOTAL AVERAGE WAIT IN THE SYSTEM IS (IN HRS.)
      .149616

THE AVG SYSTEM WAIT FOR EACH PRIORITY IS
      .219787  .831591E-01        .188631

THE VARIANCE OF THE SYSTEM WAIT FOR EACH PRIORITY IS
    .465448E-01  .924375E-01       .695239

DISTRIBUTIONS FOR 3 PRI-S WAITING TIMES
          NUMBER OF HOURS                 PROBABILITY
     0       THROUGH          1               .981041
     1       THROUGH          2            .189595E-01

          NUMBER OF HOURS                 PROBABILITY
     0       THROUGH          1               .963719
     1       THROUGH          2            .362812E-01

          NUMBER OF HOURS                 PROBABILITY
     0       THROUGH          1               .421656
     1       THROUGH          2               .483059
     2       THROUGH          3            .688418E-01
     3       THROUGH          4            .216555E-01
     4       THROUGH          5            .478710E-02


DISTRIBUTION TAKEN OVER ALL PRIORITIES
     0       THROUGH          1               .694427
     1       THROUGH          2               .257931
     2       THROUGH          3            .344209E-01
     3       THROUGH          4            .108277E-01
     4       THROUGH          5            .239355E-02
     5       THROUGH          6                0
THE WEIGHTED AVERAGE OF THE PRI-I DISTRIBUTION IS
     1.01896


THE WEIGHTED AVERAGE OF THE PRI-II DISTRIBUTION IS
     1.03628


THE WEIGHTED AVERAGE OF THE PRI-III DISTRIBUTION IS
     1.70486


THE WEIGHTED AVERAGE FOR ALL 3 DISTRIBUTIONS IS
     1.36883
```

PACKING: +40 PERCENT ARRIVALS

```
THE AVERAGE # OF UNITS OF PRIORITIES 1,2,&3 IS
      .122018      .20082      .852805

THE TOTAL AVERAGE WAIT IN THE SYSTEM IS (IN HRS.)
      .215276

THE AVG SYSTEM WAIT FOR EACHPRIORITY IS
      .221851  .920101E-0      .312383

THE VARIANCE OF THE SYSEM WAIT FOR EACH PRIORITY IS
  .524528E-01      .126009     2.10788

DISTRIBUTIONS FOR 3 PRI-S WAITING TIMES
          NUMBER OF HOURS                   PROBABILITY
     0       THROUGH           1                 .973557
     1       THROUGH           2             .264426E-01

          NUMBER OF HOURS                   PROBABILITY
     0       THROUGH           1                 .938776
     1       THROUGH           2             .612245E-01

          NUMBER OF HOURS                  PROBABILITY
     0       THROUGH           1                 .421656
     1       THROUGH           2                 .177096
     2       THROUGH           3                 .264186
     3       THROUGH           4             .758377E-01
     4       THROUGH           5             .318081E-01
     5       THROUGH           6             .162806E-01
     6       THROUGH           7             .943225E-02
     7       THROUGH           8             .370364E-02


DISTRIBUTION TAKEN OVER ALL PRIORITIES
     0       THROUGH           1                 .68372
     1       THROUGH           2                 .115656
     2       THROUGH           3                 .132093
     3       THROUGH           4             .379188E-01
     4       THROUGH           5             .159040E-01
     5       THROUGH           6             .814029E-02
     6       THROUGH           7             .471613E-02
     7       THROUGH           8             .185182E-02
     8       THROUGH           9                 0
THE WEIGHTED AVERAGE OF THE PRI-I DISTRIBUTION IS
     1.02644


THE WEIGHTED AVERAGE OF THE PRI-II DISTRIBUTION IS
     1.06122


THE WEIGHTED AVERAGE OF THE PRI-III DISTRIBUTION IS
     2.22413


THE WEIGHTED AVERAGE FOR ALL 3 DISTRIBUTIONS IS
     1.63918
```

STEPS V AND VI: PATH ANALYSIS AND PATH FREQUENCY DISTRIBUTIONS

For purposes of example only the single sequence of events leading to the issue and packing of stocked material in response to stock numbered requisitions was considered. Referring to the diagram given in Figure 2-1, this sequence entails the receipt of the requisition by the communications element; sending the requisition to the computer whether or not via the customer service element; and then sending the requisition to the issue element and the corresponding material to the packing element. The general procedure is to estimate the throughput time distribution for the sequence of events by convoluting the throughput time distributions for each element in the sequence.

For the issue sequence in this illustration the elements "communications," "customer service," and "computer A" were consolidated into a single element. Generally it was supposed that a requisition would be processed by each of these elements and sent on in a fashion governed by pickup and delivery schedules. For the sake of the example these were assumed to be

IG-I: once per hour,
IG-II: twice a day, and
IG-III: once a day.[1]

The distributions were then convoluted with the throughput time distributions for issue and packing for each Issue (group and for each of the three cases: base case, +20 percent arrivals, and +40 percent arrivals. These throughput time distributions are displayed on the following pages.

---

1. In fact, batching requisitions for the warehouse occurs once a day for both IG-II and IG-III although at different times. IG-I requisitions are not sent quite as often as once every work day hour. In addition, IG-I requisitions are batched several times during the night.

ISSUE/PACKING THROUGHPUT TIMES: IG-I

BASE CASE

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 6 MAX DAYS= .75

HOURS	PROB-HOURS	DAYS	PROB-DAYS
4	.663		
5	.3334		
6	.330000E-02		
		1	1

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
4.34

+20 PERCENT ARRIVALS

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 6 MAX DAYS= .75

HOURS	PROB-HOURS	DAYS	PROB-DAYS
4	.6566		
5	.3368		
6	.660000E-02		
		1	1

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
4.35

ISSUE/PACKING THROUGHPUT TIMES: IG-I (Continued)

+40 PERCENT ARRIVALS

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 6 MAX DAYS= .75

HOURS	PROB-HOURS	DAYS	PROB-DAYS
4	.6499		
5	.3402		
6	.990000E-02		
		1	1

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
4.36

ISSUE/PACKING THROUGHPUT TIMES: IG-II

BASE CASE

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 11 MAX DAYS= 1.375

HOURS	PROB-HOURS	DAYS	PROB-DAYS
6	.6566		
7	.134000E-01		
8	0		
		1	.67
10	.3234		
11	.660000E-02		
		2	.33

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
7.34

ISSUE/PACKING THROUGHPUT TIMES: IG-II (Continued)

+20 PERCENT ARRIVALS

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 11 MAX DAYS= 1.375

HOURS	PROB-HOURS	DAYS	PROB-DAYS
6	.6432		
7	.268000E-01		
8	0		
		1	.67
10	.3168		
11	.132000E-01		
		2	.33

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
7.36

+40 PERCENT ARRIVALS

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 12 MAX DAYS= 1.5

HOURS	PROB-HOURS	DAYS	PROB-DAYS
6	.623502		
7	.460960E-01		
8	.402000E-03		
		1	.67
10	.307098		
11	.227040E-01		
		2	.33

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
7.39

ISSUE/PACKING THROUGHPUT TIMES: IG-III

BASE CASE

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 21 MAX DAYS= 2.625

HOURS	PROB-HOURS	DAYS	PROB-DAYS
8	0		
		1	0
10	.675024		
11	.145488		
12	.191520E-01		
16	0		
		2	.84
18	.128576		
19	.277120E-01		
20	.364800E-02		
		3	.16

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
11.5

+20 PERCENT ARRIVALS

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 22 MAX DAYS= 2.75

HOURS	PROB-HOURS	DAYS	PROB-DAYS
8	0		
		1	0
10	.331632		
11	.400176		
12	.794640E-01		
13	.272160E-01		
14	.151200E-02		
16	0		
		2	.84
18	.631680E-01		
19	.762240E-01		
20	.151360E-01		
21	.518400E-02		
		3	.16

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
12.05

ISSUE/PACKING THROUGHPUT TIMES: IG-III (Continued)

+40 PERCENT ARRIVALS

DISTRIBUTION THRU ELEMENT 3

MAX HOURS= 26 MAX DAYS= 3.25

HOURS	PROB-HOURS	DAYS	PROB-DAYS
8	0		
		1	0
10	.303408		
11	.172368		
12	.213024		
13	.870240E-01		
14	.341040E-01		
15	.188160E-01		
16	.374400E-02		
		2	.838488
17	.134400E-02		
18	.579600E-01		
19	.328320E-01		
20	.405760E-01		
21	.165760E-01		
22	.649600E-02		
23	.358400E-02		
24	.185600E-02		
		3	.161224
		4	.288000E-03

SUM OF PROB-DAYS= 1

THE AVERAGE NUMBER OF HOURS IS
12.66

Inspection of this illustration reveals that even an increase of 40 percent in work load leads to a projection by the model of a very small increase in throughput time. As mentioned above, this conclusion is typical of studies of this kind and can be attributed to the (apparently) low levels of utilization at supply system activities. In the analysis of SPCC data provided in the next chapter more comprehensive performance data were available and the data were scaled such that the multipriority queueing model more closely fitted observed throughput time distributions. In these instances the projections by the model were more sensitive to changes in the capacity-arrival rate relationship.

STEP VII: SYSTEM THROUGHPUT TIMES

If more than one path is studied, the throughput time distribution for the system is achieved by taking the weighted average of the path distributions. The weight associated with a path is the frequency with which the path occurs as calculated by the product of the frequency of the individual flows which make up the path. For the issue sequence studied the path frequency would be .617 (e.g., 61.7 percent of the requisitions received by NSC Norfolk follow this path) as calculated by the flow frequencies given in Figure 2-1.

3

MODELING REQUISITION PROCESSING AT SPCC

The steps in the modeling procedure presented in Chapter 2 were arranged in the sequence in which they arose in the model building process. This chapter reports on the implementation of the model. As it turned out the model was applied with the steps occurring in a somewhat different sequence. As a result the steps are presented in this chapter in the sequence of their application. The numbering of the steps presented in Chapter 2 is retained.

The application of the methodology to inventory management practices was handicapped by a lack of supporting secondary data. The general requirement was a documentation of where requisitions flowed within an organization, how long they remained at each of the various elements which processed them, and an accounting of the man-hours expended for requisition processing. No currently collected data base explicitly responded to these needs. RMC Research Corporation personnel with close support and cooperation from personnel of the Administrative Management Division of SPCC determined that the currently maintained "requisition status file" at SPCC appeared to contain data sufficient to support the analysis, though its reporting format would require some amendment to generate precisely the data required.

Generally, it was desired to use the status file to document the "histories" of requisitions within SPCC both in terms of where the requisitions flowed and how long they remained at the various subdivisions of SPCC which processed them. It was also hoped that man-hour allocations to requisition processing could be determined at the same time from cost accounting data maintained separately. The difficulty in using the status file stemmed from the fact that each time the file was updated with respect to a requisition's location or status, all information about its previous history was erased. Some 400 requisitions were selected and their descriptions in the status file were extracted once per day for a 30-day period. As a result of this exercise it was determined that the data contained in the status file were otherwise sufficient to support the analysis if some means could be found to preserve them.

Accordingly, FMSO personnel studied the problem of using the status file at SPCC to generate the needed requisition histories. It was determined that all entries to the status file were maintained in a concentrated format data file for roughly four weeks. A very substantial computer programming task was undertaken by FMSO personnel to extract the needed data from the concentrated file and perform the required data analysis to generate the inputs for the methodology. The extraction and analysis programs were successfully applied to files maintained at both SPCC and ESO for data describing a 30-day period documenting all requisitions being processed by the inventory control points. The data analysis program generated the following descriptors of ICP operations:

- for each subdivision of the facility the number of requisitions received per day over the period;
- for each subdivision the number of requisitions completed on each day;
- the proportion (and number) of requisitions which flowed directly from each subdivision to each other subdivision;
- for each subdivision the frequency distributions of final actions, e.g., the proportionate breakdown of what eventually happened to requisitions which reached each given subdivision;
- the distributions of total elapsed times required to complete requisitions broken down with respect to type of final action; and
- the distributions of times requisitions spent in each subdivision.

The data were segmented with respect to each of the three issue groups and with respect to unit prices above and below $2,200.00. Only requisitions not referred automatically were included.

STEP I: ELIMINATE CYCLES FROM FLOW MATRIX

The local routing codes used in the SPCC status file enabled the study of operations at SPCC at the branch level. Data extraction and analysis were performed for the following branches:

Code	(Division) Branch
494	"Computer"
(720)	(Financial Control)
722	Material Accounting
(770)	(Purchase)
771	Selected Items Purchasing
773	Buying
774	Contract Management
778	Purchase Services
(810)	(Support Determination)
813	Ordnance Support
814	Technical Support
(840)	(Stock Control)
841	Special Support
842	Customer Requirements
844	Safety and Electrical
845	Machinery and Engine
846	Weapons
(870)	(Nuclear Equipment Support)
871	Nuclear Weapons
872	Nuclear Propulsion
873	Nuclear Propulsion Material
(890)	(Strategic Systems Support)
891	Support Determination
892	Material Management
893	Program Management

To reduce the scope of the data modeling effort it was decided to limit the modeling effort to federal stock numbered requisitions with unit cost below $2,200. An examination of requisition flows among branches revealed that a requisition could flow through a given branch a second time in a number of instances. As a result a number of "dummy" branches were introduced as discussed above. Figure 3-1 displays the pattern of flows among branches for federal stock numbered requisitions with unit price below $2,200 at SPCC as revealed by the data extraction and analysis program. The "dummy" branches are designated by adding a letter to the branch code. Branch 81 is a consolidation of branches 813 and 814, 84 a consolidation of 844, 845, and 846, and 89 a consolidation of 891, 892, and 893.

The flow matrix, Figure 3-1, was then analyzed to document the total flows. Computer software was developed which determined the proportion of requisitions leaving any given branch which arrive at each other branch. This proportion and the number of requisitions entering the system can be combined to provide a "steady state" estimate of work load per branch. However, since work load data were collected from the status file, this steady state estimate was not used in the multipriority queueing model. Table 3-1 summarizes the results of this flow analysis for the various final actions:

back order (BKORD),
sent to ICP (ICP),
referred (REFRD),
purchased (SPOTBUY),
cancelled (CANC), and
a substitute found (SUB).

The final actions are the row coordinates and the branch codes the column coordinates. Each entry gives the proportion of requisitions received by a branch that is completed by each of the actions indicated. Since essentially all requisitions are processed through the computer the entries for branch "494" display the distribution of final actions for the ICP as a whole. Below this display is an enumeration of the proportion of requisitions received by SPCC (and not referred automatically) that goes to each branch.

STEP V: PATH ANALYSIS

Since the number of branches which could potentially be studied is large, a path analysis was conducted next to identify those branches appearing prominently

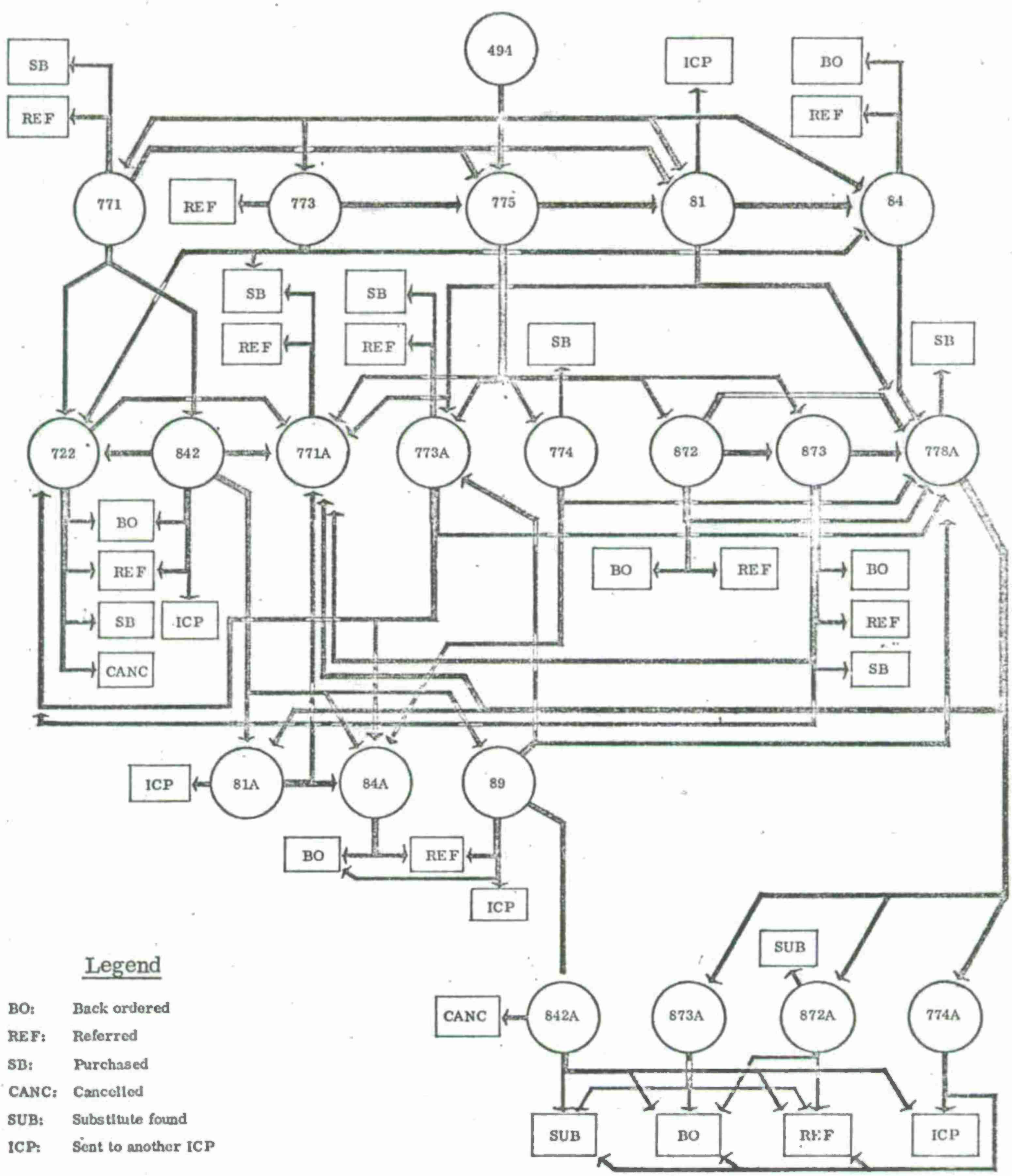

Figure 3-1: SPCC: FLOW OF REQUISITIONS, IG-II, FSN, UNIT PRICE < $2,200

Table 3-1

PROPORTION OF REQUISITIONS PROCESSED PER BRANCH PER FINAL ACTION

BKORD	494	722	771	773	773A	774	774A	778	778A	81	81A	84	84A
	0.1274	0.1700	0.0657	0.1128	0.1405	0.1838	0.2500	0.0865	0.1094	0.1522	0.2275	0.5350	0.6500
BKORD	842	842A	872	872A	873	873A	89						
	0.2464	0.1300	0.2215	0.3800	0.1616	0.2000	0.3534						
ICP	494	771	773	773A	774	774A	778	778A	81	81A	84	842	842A
	0.0196	0.0226	0.0133	0.0229	0.0199	0.0500	0.0166	0.0272	0.0986	0.1400	0.0087	0.1096	0.1600
ICP	872	873	89										
	0.0126	0.0054	0.1197										
REFRD	494	722	771	771A	773	773A	774	774A	778	778A	81	81A	84
	0.1618	0.0612	0.1546	0.1400	0.1681	0.1887	0.1770	0.1500	0.1521	0.1658	0.1585	0.1939	0.2331
REFRD	84A	842	842A	872	872A	873	873A	89					
	0.3500	0.2990	0.5600	0.3241	0.5500	0.1567	0.2700	0.0407					
SPOTBUY	494	722	771	771A	773	773A	774	774A	778	778A	81	81A	84
	0.6348	0.2488	0.7059	0.8600	0.6419	0.6034	0.6162	0.5500	0.7121	0.6934	0.5828	0.4386	0.2219
SPOTBUY	842	842A	872	873	873A	89							
	0.3127	0.1000	0.3929	0.4623	0.5300	0.1853							
CANC	494	722	771	773	773A	778	842	842A	872	873	89		
	0.0037	0.0700	0.0049	0.0069	0.0056	0.0026	0.0035	0.0500	0.0063	0.0287	0.0040		
SUB	773A	774	778A	84	872	872A							
	0.0020	0.0031	0.0042	0.0013	0.0018	0.0700							

PROPORTION OF TOTAL REQUISITIONS PROCESSED BY EACH BRANCH

494	722	771	771A	773	773A	774	774A	778	778A	81	81A	84	84A
	0.0672	0.5200	0.2946	0.2000	0.0953	0.0420	0.0292	0.3000	0.1674	0.2020	0.0337	0.1624	0.0259
494	842	872	872A	873	873A	89	BKORD	ICP	REFRD	SPOTBUY	CANC		
	0.0312	0.0150	0.0066	0.0183	0.0066	0.0022	0.1274	0.0196	0.1618	0.6348	0.0037		

in requisition processing histories. The methodology includes a computer routine which utilizes the flow matrix to enumerate the various possible paths and computes the frequency with which each path occurs. Inspection of Figure 3-1 reveals that the number of all possible paths, however infrequent, would be very large. As a result the routine was amended to enumerate only those paths which occur at least once per thousand requisitions. These paths were organized by final action and are displayed at the end of the chapter. Paths occurring at least once per thousand requisitions accounted for 83 percent of all requisitions. If only paths occuring once per hundred are counted, 55 percent of all requisitions are accounted for. The very large number of paths generated by the flow matrix indicates that the level of detail achieved using the branches of SPCC as a unit of analysis is somewhat finer than was originally supposed.[1] Paths which occur at least once per hundred requisitions are enumerated in Table 3-2. It was decided to go forward only with an analysis of branches appearing in the paths given in Table 3-2. These are 771, 773, 778, 81, and 84.

STEP III: ESTABLISH CAPACITY FACTORS

Since work load estimates were taken directly from the data extraction and analysis efforts, rather than inferred from the flow analysis, observed man-hour allocations were used to determine work load per (potentially) available man-hour. As a result it is appropriate to consider performance data on which to base capacity estimates. Cost accounting data for the year 1972 were examined to document branch performance. For each month two figures were calculated (when possible):

- the average number of requisitions completed per man-hour charged to requisition processing, and
- the average number of man-hours charged to requisition processing per calendar hour.

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1. There are 104 paths occurring once per thousand requisitions which fail to account for 17 percent of the requisitions processed (of those not automatically referred by the computer). A more complete application of the method to so large a system would require automation of additional aspects of the path analysis: a straightforward task.

Table 3-2

REQUISITION PROCESSING PATHS
OCCURRING AT LEAST ONCE PER HUNDRED REQUISITIONS

Path No.	Frequency	Branches	Final Action
1	.050	494, 84	Back Order
2	.042	494, 778, 771A	Referred
3	.012	494, 773	Referred
4	.018	494, 84	Referred
5	.229	494, 771	Spot Buy
6	.031	494, 778, 771A	Spot Buy
7	.032	494, 771, 778, 771A	Spot Buy
8	.030	494, 773, 778, 771A	Spot Buy
9	.013	494, 84, 778A, 771A	Spot Buy
10	.029	494, 81, 771A	Spot Buy
11	.028	494, 771, 81, 771A	Spot Buy
12	.044	494, 773	Spot Buy

As it happened these calculations were performed for all branches and the results are presented in Table 3-3 below. The servicing rate per man-hour of a branch is chosen as the maximum observed rate in Table 3-3 and the available man-hours (e.g., man-hour capacity) per calendar hour are chosen as the maximum observed allocation in Table 3-3.

STEP II: ESTABLISH WORK LOAD FACTORS

Arrivals at each branch in each priority were provided by the data extraction and analysis program. It was provisionally decided to express arrivals in terms of the (potentially) available man-hours as determined from the 1972 performance data given in Table 3-3. This was achieved by dividing monthly arrivals by an assumed 176 calendar hours in the month and dividing the quotient by the maximum observed man-hour allocation for 1972. As discussed, this procedure treats the branch intra-structure as essentially independent. The results of these calculations are provided in Table 3-4.

STEP IV: IMPLEMENT MULTIPRIORITY QUEUEING MODEL

At this stage in the analysis the multipriority queueing model is applied. The model accepts parameter values measuring the average arrival rate and servicing rate per time unit for each of three priorities and outputs throughput time distributions and other statistics as displayed in Chapter 2. In order to implement the model some assumption must be made about branch capacities (as discussed) and the "time unit of analysis" must be selected (e.g., arrivals and capacity per hour? per day? . . .). The goal was to implement the model using parameters descriptive of the time period viewed by the data extraction and analysis effort and compare the model results with observed throughput time distributions. The observed throughput time distributions revealed average times of several days magnitude and throughput time distributions covering over 10, sometimes over 20, days.

As discussed it was provisionally decided to model the branch man-hour, that is, express capacities and arrivals in terms of the projected available man-hours

Table 3-3

PROCESSING RATES AND MAN-HOUR ALLOCATIONS PER BRANCH FOR 1972

Branches	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
771	1.93 36.99	- 36.12	3.06 41.82	1.00 39.64	2.56 39.60	1.68 37.61	1.88 33.44	2.01 34.20	1.64 37.53	1.71 34.82	1.80 33.29	1.76 29.38
773	.37 77.10	.35 78.84	.36 89.67	.31 84.29	.32 73.06	.31 75.20	.36 73.64	.43 79.61	.40 78.66	.38 76.29	.19 82.00	.20 75.69
774	.31 58.02	.48 56.57	.54 66.49	.45 58.75	.49 52.90	.52 52.57	.51 49.52	.64 53.22	.56 54.10	.46 52.77	.36 52.47	.33 44.26
778	.63 64.94	.73 62.18	.60 71.72	.46 73.39	.68 66.22	.62 60.06	.70 59.16	.81 60.00	.67 60.77	.72 59.35	.87 60.73	.69 52.21
813	1.10 20.79	.73 19.14	1.23 16.65	1.16 16.81	1.24 14.01	1.32 15.48	1.62 16.89	1.25 15.26	1.55 17.53	1.51 14.61	1.67 14.29	1.78 11.87
814	.78 75.23	.81 70.05	.80 68.97	1.12 61.22	.71 55.95	.74 50.81	.71 54.75	.84 55.19	.72 55.95	.73 55.67	.70 58.43	.74 47.47
841	.92 2.79	.96 2.05	.66 1.89	.98 4.94	3.99 5.73	- 4.40	4.08 5.41	2.42 6.64	3.64 5.53	3.02 5.52	2.58 6.84	3.18 6.14
842	2.64 57.05	3.53 54.65	3.79 61.63	2.96 56.60	4.18 52.65	4.94 49.41	2.83 47.21	2.39 54.64	4.37 53.65	2.40 47.92	4.63 51.56	4.54 48.19

Format:

requisitions per man-hour
----
man-hours per calendar hour

Table 3-3 - Processing Rates and Man-Hour Allocations Per Branch for 1972 (Continued)

Branches	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
844	2.38 26.82	2.70 24.37	2.57 25.68	3.76 23.55	4.05 21.47	3.82 18.06	3.93 15.98	4.25 16.33	3.43 16.96	5.67 17.52	2.85 20.08	4.32 19.90
845	1.39 23.96	1.66 22.73	1.47 26.06	3.48 22.67	2.87 20.77	1.83 19.20	1.94 20.20	1.86 20.74	1.40 23.43	2.85 21.31	1.51 21.21	1.45 18.30
846	2.28 31.01	3.42 29.25	- 30.02	3.79 24.92	4.20 22.86	3.93 20.72	3.60 22.08	3.03 21.27	- 24.20	3.15 23.38	3.91 23.83	3.26 21.29
871	1.07 1.87	1.60 1.58	- -	1.71 1.95	1.58 1.50	1.65 0.95	- 0.58	1.26 0.65	1.24 0.45	1.10 0.62	1.24 0.65	1.44 0.44
872	1.41 10.58	1.64 10.54	- -	1.90 9.98	1.72 10.61	2.01 7.83	1.97 8.39	1.76 8.96	1.45 9.24	2.39 7.55	1.85 7.90	1.61 7.38
873	- 5.38	- 5.28	- -	- 5.41	- 5.30	- 5.26	- 5.00	- 5.01	0.13 5.84	0.24 6.35	0.15 7.01	0.33 6.40
891	3.03 12.34	3.35 12.86	3.06 11.15	2.54 9.77	8.41	3.21 9.52	4.73 11.09	2.37 9.57	2.41 8.80	3.58 9.23	3.45 8.94	2.09 8.26
892	1.26 20.23		0.93 24.74	1.04 23.35	1.06 22.07	0.95 19.09	1.11 19.05	1.02 22.52	0.92 20.32	1.06 19.69	1.23 19.73	0.76 16.38
893	- -	- -	- -	- -	- -	- 0.40	- 0.05	- -	- -	- -	- -	- -

Format: requisitions per man-hour
----
man-hours per calendar hour

Table 3-4

OBSERVED REQUISITION ARRIVALS PER MONTH AND REQUISITION ARRIVALS PER AVAILABLE MAN-HOUR

Branch	771			773			778			813		
Issue Group	I	II	III	I	II	III	I	II	III	I	II	III
Arrivals per month	2,318	4,218	1,697	671	1,779	4,573	1,607	3,803	3,145	262	586	773
Arrivals per branch man-hour available	.315	.573	.231	.043	.113	.290	.124	.294	.243	.072	.160	.211

Branch	814			844			845			846		
Issue Group	I	II	III	I	II	III	I	II	III	I	II	III
Arrivals per month	942	1,386	1,553	318	1,350	841	208	767	394	258	1,199	974
Arrivals per branch man-hour available	.071	.105	.117	.067	.286	.178	.045	.167	.086	.047	.220	.178

per calendar hour. The best observed servicing rate in 1972 was chosen as the capacity measure and the maximum (monthly) allocation during that period as the measure of available man-hours. Servicing rates were set equal for all priorities. Arrivals per man-hour were set at those average rates observed (in January, February 1973) using the data extraction and analysis program. The results were disappointing though not entirely unexpected. Expressed in hourly terms the multipriority queueing model depicted throughput times of only a few hours. In no instance did the model generate distributions of times which stretched over many days as did the observed data. These results confirmed earlier findings, even though now the queueing model was additionally taking into account the priority system created by the three issue groups.

When the inability to model long throughput times was first encountered in the methodology development phase of the study, deficiencies in model assumptions were immediately suspected. Accordingly, statistical procedures were found which tested the assumptions of a Poisson input stream and exponentially distributed servicing times (it was not in any case supposed that discrepancies in distributional assumptions alone could explain the very large waiting times observed). These procedures are discussed in Appendix A. As it turned out servicing times (as opposed to elapsed times in a branch) were not directly observed and the test of their distributional characteristics was not conducted. However, arrivals were documented and the sizes of daily arrivals were tested against daily arrival sizes projected under the assumption that they were Poisson distributed. The Kolmogorov-Smirnov test was employed. The distributions of arrivals were tested at all but two branches at SPCC (data problems eliminated two branches which were not in any case modeled). At the 5-percent level of significance the Poisson assumption was not rejected for any of the branches tested.

Next the capacity estimates employed were called into question. Generally, the branches under study conduct a number of activities in addition to requisition processing. Perhaps the use of the best observed performance entailed, practically speaking, an over-estimate of servicing ability. In the spirit of using the mathematical formalism descriptively, it was decided to search for capacity assumptions which would give descriptive results when incorporated in the mathematical model. Further, it was realized that the special handling procedures for IG-I requisitions implied a smaller capacity than that for IG-II and IG-III requisitions. The refinement of alternative capacities was also incorporated into the model. The effort to fit capacities failed to produce realistic results and was abandoned. In general the problem was the need to specify extremely high processing rates for IG-II and IG-III requisitions in order to compensate for the lower capacity for IG-I requisitions.

Although it was known that many of the waiting times experienced in requisition processing are not directly related to available processing resources, it was difficult to accept that the independence between resources and elapsed times was so extreme that the multipriority queueing model was totally unable to document any, even tenuous, connection. As a result, a final fitting of the model was implemented. If the servicing and arrival rates are scaled down with respect to shorter time units, the throughput time distributions depicted by the model become fuller and extend over more (smaller) periods. This characteristic of the multipriority queueing model was used to replicate observed throughput time distributions as follows. The IG-II and IG-III servicing rate was set at the maximum observed rate for 1972. The IG-I servicing rate was set at one-half that value. The observed arrivals were expressed per available branch man-hour per calendar hour using the maximum man-hour allocation observed in 1972. The output of the multipriority queueing model was relabeled in terms of days. The observed throughput time distribution for IG-II requisitions was studied and the number days required for 90 percent of the requisitions to be completed was noted. A scalar was then sought such that if the arrival and servicing rates of the queueing model were multiplicatively scaled by that scalar, the IG-II throughput time distribution would predict all requisitions completed in the number of days within which 90 percent was actually observed to be completed. For purposes of comparison the observed distribution was then normalized (e.g., rescaled proportionately to show 100 percent rather than 90 percent completed in the observed number of days).

Several remarks are immediately appropriate. The degree to which a mathematical model generated descriptively can then be used predictively is an empirical issue for which this study was not designed and which it could not afford to investigate if such an investigation was to be expensive in time. It was hoped that the mathematical model could depend in large part on structural (causal) insights and a sufficient verification be achieved from data collected over several months. No use of the model could be so verified. Further, the procedure to "fit" the model to observations outlined above was chosen because it was inexpensive in time and other resources. The effort to fit the model was undertaken at a late hour in the contract schedule. It would have been no less (and no doubt more) appropriate to scale the data towards making the observed and modeled average throughput times as nearly equal as possible.[1] Alternatively the absolute deviation between the observed and modeled distributions could provide a measure of "fit." The Kilmogorov-Smirnov test statistic was computed for several of the branch distributions. An alternative fitting procedure would be the reach for that scale factor which minimized that statistic.

Stated simply the analytic thrust of the study was to model requisition processing based on a fair estimate of the characteristics of that process with the resulting advantage of an _a priori_ focus on the appropriate mathematical structures. Observed data failed to verify the usefulness of the model so constructed. The further contention that a substantial portion of requisition processing is in fact modeled, but that the model (for whatever reason) requires a certain adjusting to accurately describe reality, goes beyond what the study resources had been allocated to deal with. Hence, the following use of the model is not offered as the best means of fitting the model to the available data.

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1. It would have taken more time however.

OUTPUTS OF THE MULTIPRIORITY QUEUEING MODEL

Using the data and procedures outlined above the multipriority queueing model was run. The outputs are presented in the following tables. Displays are given for branches 771, 773, 778, and 81 (the weighted average of branches 813 and 814). In addition to the base case described above, available capacities were changed by +10 percent and -10 percent and the model was rerun. There were two exceptions. Branch 778 capacities could not be scaled down by as much as 10 percent so the run for reduced capacity was made at 95 percent base case capacity. In addition, Branch 773 achieved a work load performance which exceeded the maximums observed in 1972. In this instance the assumed capacities were scaled up to those (relatively) of the most highly utilized branch (778). As a result, the reduced capacity run for this branch was also made at 95 percent.

Branch 771, IG-I

Days	Probabilities			
	Observed	Model: Base Case	Model: +10 Percent Capacity	Model -10 Percent Capacity
1	.281	.211	.211	.211
2	.185	.211	.300	.211
3	.059	.186	.089	.089
4	.071	.186	.280	.089
5	.052	.121	.066	.245
6	.077	.040	.031	.080
7	.101	.023	.014	.036
8	.069	.013	.009	.012
9	.022	.007		.008
10	.014	.002		.004
11	.020			
12	.034			
13	.014			
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
26				
Average Days	4.24	3.13	2.88	3.42

Branch 773, IG-I

Days	Probabilities			
	Observed	Model: Base Case	Model: +10 Percent Capacity	Model -10 Percent Capacity
1	.219	.105	.105	.084
2	.212	.105	.105	.084
3	.049	.105	.105	.084
4	.024	.105	.167	.084
5	.042	.061	.061	.144
6	.103	.061	.061	.059
7	.079	.061	.061	.059
8	.054	.061	.128	.059
9	.042	.128	.085	.059
10	.012	.069	.036	.103
11	.006	.052	.030	.058
12	.012	.024	.019	.036
13	.073	.017	.010	.024
14	.018	.012	.010	.017
15	.037	.009	.005	.012
16	.012	.007	.006	.009
17	.006	.005	.003	.007
18		.006	.003	.005
19		.003		.004
20		.003		.003
21				.003
22				
23				
24				
25				
26				
Average Days	5.46	6.28	5.64	6.66

Branch 778, IG-I

Days	Probabilities			
	Observed	Model: Base Case	Model: +10 Percent Capacity	Model -10 Percent Capacity
1	.419	.211	.211	.211
2	.426	.380	.422	.333
3	.025	.169	.211	.122
4	.040	.155	.107	.227
5	.029	.052	.033	.062
6	.032	.021	.014	.026
7	.029	.011	.003	.013
8		.001		.006
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
26				
Average Days	2.05	2.57	2.38	2.76

Branch 81, IG-I

Days	Probabilities			
	Observed	Model: Base Case	Model: +10 Percent Capacity	Model -10 Percent Capacity
1	.246	.141		
2	.184	.141		
3	.303	.141		
4	.087	.098		
5	.062	.098		
6	.050	.098		
7	.054	.126		
8	.014	.061		
9		.033		
10		.019		
11		.016		
12		.009		
13		.007		
14		.004		
15		.003		
16		.001		
17				
18				
19				
20				
21				
22				
23				
24				
25				
26				
Average Days	2.96	4.71		

Branch 771, IG-II

Days	Probabilities			
	Observed	Model: Base Case	Model: +10 Percent Capacity	Model -10 Percent Capacity
1	.159	.141	.211	.141
2	.091	.141	.211	.141
3	.034	.265	.186	.141
4	.053	.124	.186	.099
5	.048	.124	.111	.099
6	.104	.085	.046	.099
7	.198	.053	.020	.102
8	.142	.028	.014	.072
9	.046	.014	.009	.031
10	.020	.012	.006	.026
11	.023	.006		.013
12	.039	.006		.013
13	.018	.003		.007
14	.025			.007
15				.004
16				.003
17				.003
18				
19				
20				
21				
22				
23				
24				
25				
26				
Average Days	5.96	3.85	3.16	4.81

Branch 773, IG-II

Days	Probabilities			
	Observed	Model: Base Case	Model: +10 Percent Capacity	Model -10 Percent Capacity
1	.226	.105	.105	.084
2	.169	.105	.105	.084
3	.041	.105	.105	.084
4	.050	.105	.180	.084
5	.052	.044	.074	.120
6	.054	.044	.074	.035
7	.127	.044	.074	.035
8	.034	.044	.103	.035
9	.028	.161	.058	.035
10	.019	.084	.036	.161
11	.045	.035	.030	.061
12	.015	.036	.015	.058
13	.021	.024	.011	.026
14	.039	.017	.010	.027
15	.021	.008	.006	.013
16	.004	.010	.005	.015
17	.025	.007	.004	.008
18	.009	.006	.004	.009
19	.009	.003		.005
20	.011	.004		.006
21		.003		.003
22		.002		.003
23				.003
24				.002
25				.002
26				
Average Days	5.93	6.64	5.49	7.42

Branch 778, IG-II

Days	Probabilities			
	Observed	Model: Base Case	Model: +10 Percent Capacity	Model -10 Percent Capacity
1	.345	.141	.211	.141
2	.268	.141	.211	.141
3	.024	.222	.148	.141
4	.026	.081	.148	.081
5	.040	.081	.161	.081
6	.027	.155	.052	.081
7	.033	.072	.032	.128
8	.035	.039	.015	.085
9	.025	.023	.011	.036
10	.030	.015	.007	.030
11	.020	.013	.003	.015
12	.031	.007		.014
13	.038	.005		.007
14	.028	.004		.007
15	.030	.001		.004
16				.004
17				.003
18				
19				
20				
21				
22				
23				
24				
25				
26				
Average Days	4.38	4.29	3.34	4.98

Branch 81, IG-II

Days	Probabilities			
	Observed	Model: Base Case	Model: +10 Percent Capacity	Model -10 Percent Capacity
1	.135	.190	.190	.141
2	.174	.190	.190	.141
3	.075	.098	.209	.155
4	.092	.080	.130	.074
5	.090	.173	.106	.074
6	.122	.117	.062	.126
7	.076	.053	.034	.107
8	.081	.029	.018	.049
9	.029	.018	.010	.033
10	.028	.011	.008	.018
11	.030	.007	.006	.013
12	.033	.005	.003	.010
13	.013	.002	.001	.007
14	.011	.001		.005
15	.011	.001		.002
16				.001
17				.001
18				.001
19				
20				
21				
22				
23				
24				
25				
26				
Average Days	5.02	3.90	3.38	4.55

The Kolmogorov-Smirnov test statistic was computed for each base case against the observed distribution. The results are given below.

Table 3-5

KOLMOGOROV-SMIRNOV TEST STATISTIC

IG \ Branch	771	773	778	81
IG-I	5.97	2.88	7.62	7.81
IG-II	11.84	3.98	9.01	4.21

The indicated fits are not too good; however, there are several extenuating considerations. The mathematical model included a smoothing routine which would show badly in a test of this kind against more irregular distributions. Some of these irregularities would be neutralized in the convoluted distributions so that it may be expected that the path throughput time distributions "fit" the model better.

STEP VI: ESTABLISH FREQUENCY DISTRIBUTIONS OF THROUGHPUT TIMES PER PATH

The convolution of the branch throughput time distributions for a number of paths are displayed below. The weighted averages of path distributions were not taken.

Path: 494, 778, 771A, Referred

Frequency: .0416

Days	Probabilities					
	IG-I		IG-II			
	Observed	Model: Base	Observed	Model: Base	Model: +10 Percent	Model: -10 Percent
Average Days	6.28	5.70	10.34	8.12	6.59	9.77
2	.114	.044	.055	.020	.045	.020
3	.197	.125	.074	.040	.089	.040
4	.111	.155	.040	.089	.115	.060
5	.071	.178	.034	.097	.141	.065
6	.069	.171	.040	.116	.154	.071
7	.073	.130	.059	.111	.133	.076
8	.095	.086	.108	.110	.108	.091
9	.085	.053	.117	.107	.082	.096
10	.049	.029	.073	.081	.052	.088
11	.026	.016	.043	.065	.035	.076
12	.025	.008	.039	.049	.023	.064
13	.031	.003	.046	.036	.013	.055
14	.027	.001	.046	.026	.008	.048
15	.011		.043	.018	.004	.039
16	.003		.035	.013	.002	.029
17	.003		.024	.008		.022
18	.002		.021	.005		.016
19	.001		.021	.003		.012
20			.021	.002		.009
21			.019	.001	.006	
22			.015			.005
23			.009			.003
24			.005			.002
25			.003			.001
26			.003			
27			.002			
28			.001			
29						
30						
31						
32						
33						
34						
35						
36						
37						
38						
39						
40						

Path: 494, 773, 778, 771A, Spotbuy

Frequency: .0297

Days	Probabilities					
	IG-I		IG-II			
	Observed	Model: Base	Observed	Model: Base	Model: +10 Percent	Model: -10 Percent
Average Days	11.74	11.98	16.26	14.68	12.12	17.10
2	0	0	0	0	0	0
3	.026	.005	.013	.002	.005	.002
4	.068	.018	.023	.006	.019	.005
5	.072	.034	.024	.016	.026	.010
6	.051	.053	.020	.026	.044	.015
7	.045	.069	.029	.037	.062	.022
8	.057	.077	.030	.046	.076	.028
9	.075	.079	.051	.052	.085	.035
10	.080	.077	.064	.057	.092	.040
11	.070	.075	.057	.061	.092	.044
12	.054	.077	.044	.064	.088	.050
13	.044	.076	.045	.069	.082	.057
14	.044	.074	.042	.069	.073	.057
15	.052	.067	.056	.068	.062	.059
16	.051	.057	.054	.065	.051	.059
17	.035	.045	.048	.061	.041	.057
18	.033	.033	.048	.056	.031	.057
19	.027	.024	.039	.048	.024	.055
20	.023	.017	.039	.040	.017	.051
21	.021	.013	.037	.033	.013	.046
22	.017	.009	.036	.027	.009	.040
23	.017	.007	.033	.022	.007	.031
24	.010	.005	.029	.017	.005	.031
25	.007	.003	.022	.013	.003	.027
26	.005	.002	.021	.010	.002	.022
27	.004	.001	.019	.008	.001	.018
28	.004		.017	.006		.015
29	.002		.015	.004		.012
30	.001		.011	.003		.009
31			.009	.002		.007
32			.007	.002		.006
33			.006	.001		.005
34			.005			.003
35			.004			.003
36			.003			.002
37			.002			.001
38			.002			
39			.001			
40			.001			

Path: 494, 771, 778, 771A, Spotbuy

Frequency: .0310

Days	Probabilities					
	IG-I		IG-II			
	Observed	Model: Base	Observed	Model: Base	Model: +10 Percent	Model: -10 Percent
Average Days	10.52	8.83	16.30	11.99	9.64	14.58
2	0	0	0	0	0	0
3	.030	.009	.009	.003	.009	.003
4	.077	.036	.017	.008	.028	.008
5	.074	.067	.015	.023	.051	.017
6	.063	.102	.014	.039	.079	.022
7	.059	.131	.017	.061	.105	.033
8	.064	.142	.025	.075	.120	.042
9	.081	.135	.042	.089	.116	.052
10	.099	.117	.061	.097	.100	.062
11	.075	.091	.055	.098	.080	.069
12	.058	.065	.047	.095	.061	.070
13	.058	.043	.047	.085	.044	.073
14	.053	.027	.057	.074	.030	.077
15	.054	.016	.068	.061	.020	.070
16	.047	.009	.071	.050	.013	.065
17	.030	.005	.062	.039	.008	.059
18	.021	.002	.048	.030	.005	.052
19	.018	.001	.042	.022	.003	.045
20	.015		.042	.016	.003	.037
21	.013		.041	.012	.001	.031
22	.008		.038	.008		.026
23	.004		.032	.005		.020
24	.003		.026	.004		.016
25	.002		.021	.002		.013
26	.002		.019	.001		.010
27	.001		.017			.007
28			.015			.005
29			.012			.004
30			.009			.003
31			.006			.002
32			.005			.001
33			.004			
34			.003			
35			.002			
36			.001			
37						
38						
39						
40						

Path: 494, 771, 81, 771A, Spotbuy

Frequency: .0282

Days	Probabilities					
	IG-I		IG-II			
	Observed	Model: Base	Observed	Model: Base	Model: +10 Percent	Model: -10 Percent
Average Days	11.44	10.90	17.11	11.33	9.40	13.66
2	0	0	0	0	0	0
3	.015	.006	.003	.004	.008	.003
4	.040	.019	.008	.011	.025	.008
5	.060	.036	.010	.027	.050	.017
6	.070	.056	.011	.044	.078	.026
7	.062	.076	.014	.063	.102	.034
8	.062	.090	.020	.077	.117	.042
9	.074	.099	.031	.091	.113	.056
10	.080	.102	.042	.097	.117	.063
11	.073	.099	.046	.098	.095	.070
12	.071	.092	.046	.091	.072	.073
13	.065	.080	.051	.082	.058	.072
14	.054	.066	.059	.070	.047	.071
15	.052	.052	.066	.057	.030	.068
16	.041	.039	.070	.045	.020	.062
17	.036	.028	.067	.034	.014	.055
18	.030	.019	.061	.025	.009	.048
19	.023	.013	.058	.019	.005	.041
20	.019	.009	.056	.013	.003	.034
21	.017	.006	.052	.009	.002	.028
22	.012	.003	.046	.006	.001	.022
23	.008	.002	.038	.004		.017
24	.005	.001	.031	.003		.013
25	.004		.026	.002		.010
26	.003		.022			.008
27	.002		.018			.006
28	.001		.014			.004
29			.010			.003
30			.007			.002
31			.005			.002
32			.003			.001
33			.003			
34			.002			
35			.001			
36						
37						
38						
39						
40						

Path: 494, 81, 771A, Spotbuy

Frequency: .0288

Days	Probabilities					
	IG-I		IG-II			
	Observed	Model: Base	Observed	Model: Base	Model: +10 Percent	Model: -10 Percent
Average Days	7.21	7.78	11.15	7.56	6.34	9.04
2	.069	.030	.021	.028	.040	.020
3	.097	.060	.040	.054	.080	.040
4	.134	.086	.032	.091	.120	.062
5	.131	.103	.035	.098	.142	.066
6	.077	.111	.041	.108	.145	.071
7	.087	.109	.051	.114	.128	.082
8	.089	.110	.074	.118	.100	.094
9	.084	.098	.094	.097	.073	.094
10	.070	.083	.083	.076	.050	.084
11	.049	.067	.069	.059	.034	.071
12	.034	.049	.070	.044	.021	.060
13	.032	.032	.071	.030	.014	.058
14	.028	.021	.067	.021	.009	.043
15	.022	.015	.056	.014	.005	.032
16	.011	.010	.042	.009	.003	.024
17	.006	.006	.032	.006	.001	.018
18	.004	.004	.028	.004		.013
19	.003	.002	.025	.002		.010
20	.001	.001	.019	.001		.007
21			.012			.004
22			.009			.003
23			.005			.002
24			.004			.002
25			.002			
26			.002			
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PATH ENUMERATION

The following is the enumeration of paths occurring more than once per thousand requisitions. The paths are displayed as follows:

- the final action is given as "user ending node";
- all paths start at "494," the computer;
- the path is then enumerated backwards starting with the "user ending node," the branches are shown pairwise followed by the frequency of the flow between the pair; and
- the frequency of the path is displayed as "freq. prod." which is the product of the pairwise frequencies.

Table 3-6

AN ENUMERATION OF REQUISITION PROCESSING PATHS

```
 USER ENDING NODE IS:   22  BKORD
STARTING NODE IS:    1  494      NO. OF PAIRS:   3
  22  BKORD          2  722         0.170   2  722          3  771          0.070
   3  771            1  494         0.520
FREQ. PROD. =      0.0061880
STARTING NODE IS:    1  494      NO. OF PAIRS:   3
  22  BKORD          2  722         0.170   2  722          5  773          0.080
   5  773            1  494         0.200
FREQ. PROD. =      0.0027200
STARTING NODE IS:    1  494      NO. OF PAIRS:   4
  22  BKORD          8  774A        0.250   8  774A        10  778A         0.180
  10  778A          13  84          0.320  13  84           1  494          0.100
FREQ. PROD. =      0.0014400
STARTING NODE IS:    1  494      NO. OF PAIRS:   2
  22  BKORD         13  84          0.500  13  84           1  494          0.100
FREQ. PROD. =      0.0500000
STARTING NODE IS:    1  494      NO. OF PAIRS:   3
  22  BKORD         13  84          0.500  13  84           5  773          0.090
   5  773            1  494         0.200
FREQ. PROD. =      0.0090000
STARTING NODE IS:    1  494      NO. OF PAIRS:   3
  22  BKORD         13  84          0.500  13  84          11  81           0.220
  11  81             1  494         0.080
FREQ. PROD. =      0.0088000
STARTING NODE IS:    1  494      NO. OF PAIRS:   4
  22  BKORD         13  84          0.500  13  84          11  81           0.220
  11  81             3  771         0.150   3  771          1  494          0.520
FREQ. PROD. =      0.0085800
STARTING NODE IS:    1  494      NO. OF PAIRS:   4
  22  BKORD         13  84          0.500  13  84          11  81           0.220
  11  81             5  773         0.070   5  773          1  494          0.200
FREQ. PROD. =      0.0015400
STARTING NODE IS:    1  494      NO. OF PAIRS:   4
  22  BKORD         13  84          0.500  13  84          11  81           0.220
  11  81             9  778         0.100   9  778          1  494          0.100
FREQ. PROD. =      0.0011000
STARTING NODE IS:    1  494      NO. OF PAIRS:   5
  22  BKORD         13  84          0.500  13  84          11  81           0.220
  11  81             9  778         0.100   9  778          3  771          0.200
   3  771            1  494         0.520
FREQ. PROD. =      0.0011440
STARTING NODE IS:    1  494      NO. OF PAIRS:   5
  22  BKORD         13  84          0.500  13  84          11  81           0.220
  11  81             9  778         0.100   9  778          5  773          0.480
   5  773            1  494         0.200
FREQ. PROD. =      0.0010560
STARTING NODE IS:    1  494      NO. OF PAIRS:   4
  22  BKORD         14  84A         0.650  14  84A          6  773A         0.090
   6  773A           9  778         0.210   9  778          1  494          0.100
FREQ. PROD. =      0.0012285
STARTING NODE IS:    1  494      NO. OF PAIRS:   5
  22  BKORD         14  84A         0.650  14  84A          6  773A         0.090
   6  773A           9  778         0.210   9  778          3  771          0.200
   3  771            1  494         0.520
FREQ. PROD. =      0.0012776
STARTING NODE IS:    1  494      NO. OF PAIRS:   5
  22  BKORD         14  84A         0.650  14  84A          6  773A         0.090
   6  773A           9  778         0.210   9  778          5  773          0.480
   5  773            1  494         0.200
FREQ. PROD. =      0.0011794
STARTING NODE IS:    1  494      NO. OF PAIRS:   4
  22  BKORD         14  84A         0.650  14  84A          7  774          0.160
   7  774            9  778         0.140   9  778          1  494          0.100
FREQ. PROD. =      0.0014560
STARTING NODE IS:    1  494      NO. OF PAIRS:   5
  22  BKORD         14  84A         0.650  14  84A          7  774          0.160
   7  774            9  778         0.140   9  778          3  771          0.200
   3  771            1  494         0.520
FREQ. PROD. =      0.0015142
STARTING NODE IS:    1  494      NO. OF PAIRS:   5
  22  BKORD         14  84A         0.650  14  84A          7  774          0.160
   7  774            9  778         0.140   9  778          5  773          0.480
   5  773            1  494         0.200
FREQ. PROD. =      0.0013978
STARTING NODE IS:    1  494      NO. OF PAIRS:   5
  22  BKORD         14  84A         0.650  14  84A         12  81A          0.350
  12  81A           15  842         0.190  15  842          3  771          0.060
   3  771            1  494         0.520
FREQ. PROD. =      0.0013486
STARTING NODE IS:    1  494      NO. OF PAIRS:   4
  22  BKORD         14  84A         0.650  14  84A         15  842          0.150
  15  842            3  771         0.060   3  771          1  494          0.520
FREQ. PROD. =      0.0030420
```

Table 3-6 (Continued)

```
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  22  BKORD         15  842             0.050   15  842          3  771            0.060
   3  771            1  494             0.520
FREQ. PROD. =      0.0015600
    USER ENDING NODE IS:,    23   ICP
STARTING NODE IS:    1  494        NO. OF PAIRS:   2
  23  ICP           11  81              0.090   11  81           1  494            0.080
FREQ. PROD. =      0.0072000
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  23  ICP           11  81              0.090   11  81           3  771            0.150
   3  771            1  494             0.520
FREQ. PROD. =      0.0070200
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  23  ICP           11  81              0.090   11  81           5  773            0.070
   5  773            1  494             0.200
FREQ. PROD. =      0.0012600
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  23  ICP           15  842             0.070   15  842          3  771            0.060
   3  771            1  494             0.520
FREQ. PROD. =      0.0021840
    USER ENDING NODE IS:,    24   REFRD
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  24  REFRD          2  722             0.050    2  722          3  771            0.070
   3  771            1  494             0.520
FREQ. PROD. =      0.0018200
STARTING NODE IS:    1  494        NO. OF PAIRS:   2
  24  REFRD          3  771             0.080    3  771          1  494            0.520
FREQ. PROD. =      0.0416000
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  24  REFRD          4  771A            0.140    4  771A         9  778            0.360
   9  778            1  494             0.100
FREQ. PROD. =      0.0050400
STARTING NODE IS:    1  494        NO. OF PAIRS:   4
  24  REFRD          4  771A            0.140    4  771A         9  778            0.360
   9  778            3  771             0.200    3  771          1  494            0.520
FREQ. PROD. =      0.0052416
STARTING NODE IS:    1  494        NO. OF PAIRS:   4
  24  REFRD          4  771A            0.140    4  771A         9  778            0.360
   9  778            5  773             0.480    5  773          1  494            0.200
FREQ. PROD. =      0.0048384
STARTING NODE IS:    1  494        NO. OF PAIRS:   4
  24  REFRD          4  771A            0.140    4  771A        10  778A           0.460
  10  778A          13  84              0.320   13  84           1  494            0.100
FREQ. PROD. =      0.0020608
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  24  REFRD          4  771A            0.140    4  771A        11  81             0.420
  11  81             1  494             0.080
FREQ. PROD. =      0.0047040
STARTING NODE IS:    1  494        NO. OF PAIRS:   4
  24  REFRD          4  771A            0.140    4  771A        11  81             0.420
  11  81             3  771             0.150    3  771          1  494            0.520
FREQ. PROD. =      0.0045864
STARTING NODE IS:    1  494        NO. OF PAIRS:   2
  24  REFRD          5  773             0.060    5  773          1  494            0.200
FREQ. PROD. =      0.0120000
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  24  REFRD          6  773A            0.060    6  773A         9  778            0.210
   9  778            1  494             0.100
FREQ. PROD. =      0.0012600
STARTING NODE IS:    1  494        NO. OF PAIRS:   4
  24  REFRD          6  773A            0.060    6  773A         9  778            0.210
   9  778            3  771             0.200    3  771          1  494            0.520
FREQ. PROD. =      0.0013104
STARTING NODE IS:    1  494        NO. OF PAIRS:   4
  24  REFRD          6  773A            0.060    6  773A         9  778            0.210
   9  778            5  773             0.480    5  773          1  494            0.200
FREQ. PROD. =      0.0012096
STARTING NODE IS:    1  494        NO. OF PAIRS:   2
  24  REFRD         13  84              0.180   13  84           1  494            0.100
FREQ. PROD. =      0.0180000
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  24  REFRD         13  84              0.180   13  84           5  773            0.090
   5  773            1  494             0.200
FREQ. PROD. =      0.0032400
STARTING NODE IS:    1  494        NO. OF PAIRS:   3
  24  REFRD         13  84              0.180   13  84          11  81             0.220
  11  81             1  494             0.080
FREQ. PROD. =      0.0031680
STARTING NODE IS:    1  494        NO. OF PAIRS:   4
  24  REFRD         13  84              0.180   13  84          11  81             0.220
  11  81             3  771             0.150    3  771          1  494            0.520
```

Table 3-6 (Continued)

```
FREQ. PROD. =         0.0030888
STARTING NODE IS:      1  494           NO. OF PAIRS:   4
  24  REFRD           14  84A              0.350   14  84A              15  842              0.150
  15  842              3  771              0.060    3  771               1  494              0.520
FREQ. PROD. =         0.0016380
STARTING NODE IS:      1  494           NO. OF PAIRS:   3
  24  REFRD           15  842              0.140   15  842               3  771              0.060
   3  771              1  494              0.520
FREQ. PROD. =         0.0043680
STARTING NODE IS:      1  494           NO. OF PAIRS:   3
  24  REFRD           17  872              0.220   17  872               9  778              0.050
   9  778              1  494              0.100
FREQ. PROD. =         0.0011000
STARTING NODE IS:      1  494           NO. OF PAIRS:   4
  24  REFRD           17  872              0.220   17  872               9  778              0.050
   9  778              3  771              0.200    3  771               1  494              0.520
FREQ. PROD. =         0.0011440
STARTING NODE IS:      1  494           NO. OF PAIRS:   4
  24  REFRD           17  872              0.220   17  872               9  778              0.050
   9  778              5  773              0.480    5  773               1  494              0.200
FREQ. PROD. =         0.0010560
STARTING NODE IS:      1  494           NO. OF PAIRS:   4
  24  REFRD           18  872A             0.550   18  872A             10  778A             0.060
  10  778A            13  84               0.320   13  84                1  494              0.100
FREQ. PROD. =         0.0010560
   USER ENDING NODE IS:     25   SPOTBUY
STARTING NODE IS:      1  494           NO. OF PAIRS:   3
  25  SPOTBUY          2  722              0.180    2  722               3  771              0.070
   3  771              1  494              0.520
FREQ. PROD. =         0.0065520
STARTING NODE IS:      1  494           NO. OF PAIRS:   3
  25  SPOTBUY          2  722              0.180    2  722               5  773              0.080
   5  773              1  494              0.200
FREQ. PROD. =         0.0028800
STARTING NODE IS:      1  494           NO. OF PAIRS:   2
  25  SPOTBUY          3  771              0.440    3  771               1  494              0.520
FREQ. PROD. =         0.2287999
STARTING NODE IS:      1  494           NO. OF PAIRS:   4
  25  SPOTBUY          4  771A             0.860    4  771A              2  722              0.080
   2  722              3  771              0.070    3  771               1  494              0.520
FREQ. PROD. =         0.0025043
STARTING NODE IS:      1  494           NO. OF PAIRS:   4
  25  SPOTBUY          4  771A             0.860    4  771A              2  722              0.080
   2  722              5  773              0.080    5  773               1  494              0.200
FREQ. PROD. =         0.0011008
STARTING NODE IS:      1  494           NO. OF PAIRS:   3
  25  SPOTBUY          4  771A             0.860    4  771A              9  778              0.360
   9  778              1  494              0.100
FREQ. PROD. =         0.0309600
STARTING NODE IS:      1  494           NO. OF PAIRS:   4
  25  SPOTBUY          4  771A             0.860    4  771A              9  778              0.360
   9  778              3  771              0.200    3  771               1  494              0.520
FREQ. PROD. =         0.0321984
STARTING NODE IS:      1  494           NO. OF PAIRS:   4
  25  SPOTBUY          4  771A             0.860    4  771A              9  778              0.360
   9  778              5  773              0.480    5  773               1  494              0.200
FREQ. PROD. =         0.0297216
STARTING NODE IS:      1  494           NO. OF PAIRS:   5
  25  SPOTBUY          4  771A             0.860    4  771A             10  778A             0.460
  10  778A             6  773A             0.480    6  773A              9  778              0.210
   9  778              1  494              0.100
FREQ. PROD. =         0.0039876
STARTING NODE IS:      1  494           NO. OF PAIRS:   6
  25  SPOTBUY          4  771A             0.860    4  771A             10  778A             0.460
  10  778A             6  773A             0.480    6  773A              9  778              0.210
   9  778              3  771              0.200    3  771               1  494              0.520
FREQ. PROD. =         0.0041471
STARTING NODE IS:      1  494           NO. OF PAIRS:   6
  25  SPOTBUY          4  771A             0.860    4  771A             10  778A             0.460
  10  778A             6  773A             0.480    6  773A              9  778              0.210
   9  778              5  773              0.480    5  773               1  494              0.200
FREQ. PROD. =         0.0038281
STARTING NODE IS:      1  494           NO. OF PAIRS:   5
  25  SPOTBUY          4  771A             0.860    4  771A             10  778A             0.460
  10  778A             6  773A             0.480    6  773A             11  81               0.160
  11  81               1  494              0.080
FREQ. PROD. =         0.0024306
STARTING NODE IS:      1  494           NO. OF PAIRS:   6
  25  SPOTBUY          4  771A             0.860    4  771A             10  778A             0.460
  10  778A             6  773A             0.480    6  773A             11  81               0.160
  11  81               3  771              0.150    3  771               1  494              0.520
FREQ. PROD. =         0.0023698
STARTING NODE IS:      1  494           NO. OF PAIRS:   5
  25  SPOTBUY          4  771A             0.860    4  771A             10  778A             0.460
  10  778A             7  774              0.730    7  774               9  778              0.140
   9  778              1  494              0.100
```

Table 3-6 (Continued)

```
FREQ. PROD. =          0.0040430
STARTING NODE IS:       1  494        NO. OF PAIRS:   6
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A              7  774            0.730     7   774              9   778               0.140
   9   778               3  771            0.200     3   771              1   494               0.520
FREQ. PROD. =          0.0042048
STARTING NODE IS:       1  494        NO. OF PAIRS:   6
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A              7  774            0.730     7   774              9   778               0.140
   9   778               5  773            0.480     5   773              1   494               0.200
FREQ. PROD. =          0.0038813
STARTING NODE IS:       1  494        NO. OF PAIRS:   4
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A             11  81             0.110    11   81               1   494               0.080
FREQ. PROD. =          0.0034813
STARTING NODE IS:       1  494        NO. OF PAIRS:   5
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A             11  81             0.110    11   81               3   771               0.150
   3   771               1  494            0.520
FREQ. PROD. =          0.0033942
STARTING NODE IS:       1  494        NO. OF PAIRS:   4
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A             13  84             0.320    13   84               1   494               0.100
FREQ. PROD. =          0.0126592
STARTING NODE IS:       1  494        NO. OF PAIRS:   5
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A             13  84             0.320    13   84               5   773               0.090
   5   773               1  494            0.200
FREQ. PROD. =          0.0022787
STARTING NODE IS:       1  494        NO. OF PAIRS:   5
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A             13  84             0.320    13   84              11   81                0.220
  11   81                1  494            0.080
FREQ. PROD. =          0.0022280
STARTING NODE IS:       1  494        NO. OF PAIRS:   6
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A             13  84             0.320    13   84              11   81                0.220
  11   81                3  771            0.150     3   771              1   494               0.520
FREQ. PROD. =          0.0021723
STARTING NODE IS:       1  494        NO. OF PAIRS:   5
  25   SPOTBUY           4  771A           0.860     4   771A            10   778A              0.460
  10   778A             15  842            0.220    15   842              3   771               0.060
   3   771               1  494            0.520
FREQ. PROD. =          0.0027154
STARTING NODE IS:       1  494        NO. OF PAIRS:   3
  25   SPOTBUY           4  771A           0.860     4   771A            11   81                0.420
  11   81                1  494            0.080
FREQ. PROD. =          0.0288960
STARTING NODE IS:       1  494        NO. OF PAIRS:   4
  25   SPOTBUY           4  771A           0.860     4   771A            11   81                0.420
  11   81                3  771            0.150     3   771              1   494               0.520
FREQ. PROD. =          0.0281736
STARTING NODE IS:       1  494        NO. OF PAIRS:   4
  25   SPOTBUY           4  771A           0.860     4   771A            11   81                0.420
  11   81                5  773            0.070     5   773              1   494               0.200
FREQ. PROD. =          0.0050568
STARTING NODE IS:       1  494        NO. OF PAIRS:   4
  25   SPOTBUY           4  771A           0.860     4   771A            11   81                0.420
  11   81                9  778            0.100     9   778              1   494               0.100
FREQ. PROD. =          0.0036120
STARTING NODE IS:       1  494        NO. OF PAIRS:   5
  25   SPOTBUY           4  771A           0.860     4   771A            11   81                0.420
  11   81                9  778            0.100     9   778              3   771               0.200
   3   771               1  494            0.520
FREQ. PROD. =          0.0037565
STARTING NODE IS:       1  494        NO. OF PAIRS:   5
  25   SPOTBUY           4  771A           0.860     4   771A            11   81                0.420
  11   81                9  778            0.100     9   778              5   773               0.480
   5   773               1  494            0.200
FREQ. PROD. =          0.0034675
STARTING NODE IS:       1  494        NO. OF PAIRS:   5
  25   SPOTBUY           4  771A           0.860     4   771A            12   81A               0.510
  12   81A              10  778A           0.130    10   778A            13   84                0.320
  13   84                1  494            0.100
FREQ. PROD. =          0.0018246
STARTING NODE IS:       1  494        NO. OF PAIRS:   5
  25   SPOTBUY           4  771A           0.860     4   771A            12   81A               0.510
  12   81A              15  842            0.190    15   842              3   771               0.060
   3   771               1  494            0.520
FREQ. PROD. =          0.0026000
```

Table 3-6 (Continued)

STARTING NODE IS:	1 494	NO. OF PAIRS: 4			
25 SPOTBUY	4 771A	0.860	4 771A	15 842	0.060
15 842	3 771	0.060	3 771	1 494	0.520
FREQ. PROD. =	0.0016099				
STARTING NODE IS:	1 494	NO. OF PAIRS: 2			
25 SPOTBUY	5 773	0.220	5 773	1 494	0.200
FREQ. PROD. =	0.0440000				
STARTING NODE IS:	1 494	NO. OF PAIRS: 3			
25 SPOTBUY	6 773A	0.220	6 773A	9 778	0.210
9 778	1 494	0.100			
FREQ. PROD. =	0.0046200				
STARTING NODE IS:	1 494	NO. OF PAIRS: 4			
25 SPOTBUY	6 773A	0.220	6 773A	9 778	0.210
9 778	3 771	0.200	3 771	1 494	0.520
FREQ. PROD. =	0.0048048				
STARTING NODE IS:	1 494	NO. OF PAIRS: 4			
25 SPOTBUY	6 773A	0.220	6 773A	9 778	0.210
9 778	5 773	0.480	5 773	1 494	0.200
FREQ. PROD. =	0.0044352				
STARTING NODE IS:	1 494	NO. OF PAIRS: 3			
25 SPOTBUY	6 773A	0.220	6 773A	11 81	0.160
11 81	1 494	0.080			
FREQ. PROD. =	0.0028160				
STARTING NODE IS:	1 494	NO. OF PAIRS: 4			
25 SPOTBUY	6 773A	0.220	6 773A	11 81	0.160
11 81	3 771	0.150	3 771	1 494	0.520
FREQ. PROD. =	0.0027456				
STARTING NODE IS:	1 494	NO. OF PAIRS: 3			
25 SPOTBUY	7 774	0.110	7 774	9 778	0.140
9 778	1 494	0.100			
FREQ. PROD. =	0.0015400				
STARTING NODE IS:	1 494	NO. OF PAIRS: 4			
25 SPOTBUY	7 774	0.110	7 774	9 778	0.140
9 778	3 771	0.200	3 771	1 494	0.520
FREQ. PROD. =	0.0016016				
STARTING NODE IS:	1 494	NO. OF PAIRS: 4			
25 SPOTBUY	7 774	0.110	7 774	9 778	0.140
9 778	5 773	0.480	5 773	1 494	0.200
FREQ. PROD. =	0.0014784				
STARTING NODE IS:	1 494	NO. OF PAIRS: 6			
25 SPOTBUY	8 774A	0.550	8 774A	10 778A	0.180
10 778A	6 773A	0.480	6 773A	9 778	0.210
9 778	3 771	0.200	3 771	1 494	0.520
FREQ. PROD. =	0.0010378				
STARTING NODE IS:	1 494	NO. OF PAIRS: 5			
25 SPOTBUY	8 774A	0.550	8 774A	10 778A	0.180
10 778A	7 774	0.730	7 774	9 778	0.140
9 778	1 494	0.100			
FREQ. PROD. =	0.0010118				
STARTING NODE IS:	1 494	NO. OF PAIRS: 6			
25 SPOTBUY	8 774A	0.550	8 774A	10 778A	0.180
10 778A	7 774	0.730	7 774	9 778	0.140
9 778	3 771	0.200	3 771	1 494	0.520
FREQ. PROD. =	0.0010522				
STARTING NODE IS:	1 494	NO. OF PAIRS: 4			
25 SPOTBUY	8 774A	0.550	8 774A	10 778A	0.180
10 778A	13 84	0.320	13 84	1 494	0.100
FREQ. PROD. =	0.0031680				
STARTING NODE IS:	1 494	NO. OF PAIRS: 2			
25 SPOTBUY	9 778	0.090	9 778	1 494	0.100
FREQ. PROD. =	0.0090000				
STARTING NODE IS:	1 494	NO. OF PAIRS: 3			
25 SPOTBUY	9 778	0.090	9 778	3 771	0.200
3 771	1 494	0.520			
FREQ. PROD. =	0.0093600				
STARTING NODE IS:	1 494	NO. OF PAIRS: 3			
25 SPOTBUY	9 778	0.090	9 778	5 773	0.480
5 773	1 494	0.200			
FREQ. PROD. =	0.0086400				
STARTING NODE IS:	1 494	NO. OF PAIRS: 4			
25 SPOTBUY	10 778A	0.110	10 778A	6 773A	0.480
6 773A	9 778	0.210	9 778	1 494	0.100
FREQ. PROD. =	0.0011088				
STARTING NODE IS:	1 494	NO. OF PAIRS: 5			
25 SPOTBUY	10 778A	0.110	10 778A	6 773A	0.480
6 773A	9 778	0.210	9 778	3 771	0.200
3 771	1 494	0.520			
FREQ. PROD. =	0.0011532				

Table 3-6 (Continued)

```
STARTING NODE IS:    1  494       NO. OF PAIRS:   5
  25  SPOTBUY       10  778A         0.110   10  778A        6  773A        0.480
   6  773A           9  778          0.210    9  778         5  773         0.480
   5  773            1  494          0.200
FREQ. PROD. =       0.0010644
STARTING NODE IS:    1  494       NO. OF PAIRS:   4
  25  SPOTBUY       10  778A         0.110   10  778A        7  774         0.730
   7  774            9  778          0.140    9  778         1  494         0.100
FREQ. PROD. =       0.0011242
STARTING NODE IS:    1  494       NO. OF PAIRS:   5
  25  SPOTBUY       10  778A         0.110   10  778A        7  774         0.730
   7  774            9  778          0.140    9  778         3  771         0.200
   3  771            1  494          0.520
FREQ. PROD. =       0.0011692
STARTING NODE IS:    1  494       NO. OF PAIRS:   5
  25  SPOTBUY       10  778A         0.110   10  778A        7  774         0.730
   7  774            9  778          0.140    9  778         5  773         0.480
   5  773            1  494          0.200
FREQ. PROD. =       0.0010792
STARTING NODE IS:    1  494       NO. OF PAIRS:   3
  25  SPOTBUY       10  778A         0.110   10  778A       13  84          0.320
  13  84             1  494          0.100
FREQ. PROD. =       0.0035200
STARTING NODE IS:    1  494       NO. OF PAIRS:   4
  25  SPOTBUY       20  873A         0.530   20  873A       10  778A        0.060
  10  778A          13  84           0.320   13  84          1  494         0.100
FREQ. PROD. =       0.0010176
   USER ENDING NODE IS:   26   CANC
STARTING NODE IS:    1  494       NO. OF PAIRS:   3
  26  CANC           2  722          0.070    2  722         3  771         0.070
   3  771            1  494          0.520
FREQ. PROD. =       0.0025480
STARTING NODE IS:    1  494       NO. OF PAIRS:   3
  26  CANC           2  722          0.070    2  722         5  773         0.080
   5  773            1  494          0.200
FREQ. PROD. =       0.0011200
```

4

CONCLUDING REMARKS

The purpose of the research effort reported on here was to apply a methodology which would document the relationship between requisition processing throughput times and the resources dedicated to requisition processing. The major result of the study is a reinforcement of an insight gained earlier in the development of the method (and concluded by other studies):

> that requisition processing throughput times are sensitive to many factors other than simply manpower available to do the processing.

As a result, available data reflect so many influences on elapsed processing times that those portions of throughput time distributions which are attributable to available man-hours are difficult to document. The design of a statistical model to estimate the influence of resource constraints would presumably require a substantial amount of longitudinal performance data as well as data describing the perhaps complex priority structure governing the allocation of resources between requisition processing and other tasks. It is not clear that existing data sources would support such a statistical model.

Further, it is not clear that, practically speaking, requisition processing as currently practiced is in fact resource constrained at all. It can be argued that a legitimate and necessary task of the Navy Supply System is the maintenance of excess capacity against potential emergency requirements. If this is the case, then current performance data will fail to reveal a sensitivity of throughput time distributions to changes in the work load/capacity relationship. The study of resources assigned to processing and billing requisitions might appropriately be directed to the size and expense of excess capacity. Additionally, structural issues might arise. If excess capacity is indeed being maintained, it would appear to be unevenly maintained. Requisition processing resources appear to be under-utilized to a greater degree than materials handling resources. In fact the issue of the supply system's maintaining a capability might lead to performance concepts somewhat different from those imposed on the system from the standpoint of its processing of requisitions.

The results of this research provide a model of requisition processing which is not firmly verified from observed requisition processing practices. Ignoring, generically speaking, the tasks and the training in tasks which comprise requisition processing, it does not appear that the distributions of throughput times associated with requisition processing arise in general as the result of resource constraints. However, the ability to parameterize the model provides great flexibility in replicating those subportions of processing practices which might be determined to be resource-constrained, or in extrapolating the implications of work load/capacity relationships far beyond circumstances currently observed.

APPENDIX A

- multipriority queueing model
- Kolmogorov-Smirnov test
- results for nonpreemptive queues

This paper presents the mathematical models used to study the queueing theoretic aspects of the throughput time associated with requisition processing and materials handling. A model was prepared for circumstances of each of one, two, or three priorities. The underlying input stream for each priority is assumed to be Poisson in all of the models. This is clearly not an exact representation of reality, but it can be shown that the more realistic assumption that inputs are at constant intervals in batches whose random sizes are Poisson can be well-approximated by a Poisson process. This is demonstrated through the following argument:

Pr $\{$ a total of n units have arrived on or before the end of the k^{th} uniformly spaced interval $\}$

= Pr $\{$ k modules of random and Poisson-distributed size lead to a total of n $\}$

= Pr $\{$ sum of k Poissons totals n $\}$.

But it is well known that k identical Poissons with parameter a sum to another Poisson, this one with parameter ka. Hence,

Pr $\{$ sum of k Poissons totals n $\}$

$$= e^{-ka} (ka)^n / n!,$$

that is, Poisson with mean ka. In view of the facts that the system will not be able to react immediately to the input and that the time between batches can be lowered, the overall process (not just that evaluated at a point of arrival) will largely act as a Poisson process even though the proved result is not valid for times between batches.

A routine has, in fact, been provided (under the name of POIS) for testing whether any particular set of data are Poisson distributed. In addition, since the models also assume that service times are exponentially distributed, a second routine (names EXP) has been written for the testing of exponentiality.

Generally, the easiest and most familiar way to test for Poisson or exponential character is to use a X^2 goodness-of-fit test on the data presented in block histogram form against a theoretical distribution with each parameter replaced by its maximum-likelihood estimator, which is

$\hat{\lambda} = n / \sum_{i=1}^{n} t_i$ for both the exponential $\lambda e^{-\lambda t}$ and Poisson $(\lambda t)^n e^{-\lambda t}/n\,!$,

where t_i is the time between the $(i-1)^{st}$ and i^{th} occurrences. The resulting statistic is then[1]

$$X_k^2 = \sum_{i=1}^{n} [(o_i - e_i)^2 / e_i],$$

where o_i is the number observed in the i^{th} frequency class (out of a total of between 10 and 20 classes), e_i the number expected in the i^{th} frequency class if the hypothesized distribution were correct, and k the number of degrees of freedom, always equal to the total number of classes less one and then minus one for each parameter estimated. Of course, the usual precautions must be taken to keep the number in any class from being too small (a rule of thumb being less than five).

Great care should always be exercised in doing X^2 goodness-of-fit tests and the analyst would, of course, be well advised to search for a definitive exposition on the subject in the statistical literature. The basic weaknesses of the X^2 test are its requirement for large samples, its heavy dependence on the choice of the number and position of the time-axis intervals, and its possibly very high Type II error (this is expressed in terms of the probability of accepting a false hypothesis) for feasible alternative distributions. In view of these difficulties, we are instead going to suggest two tests for use in our context: the Kolmogorov-Smirnov (K-S) for Poisson fits and the F-test for exponentials.

The K-S test compares deviations of the empirical CDF from the theoretical CDF, and uses as its test statistic the maximum absolute deviation, that is,

$$E = \text{maximum} \left| n_j - F(N_j) \right|,$$

where n_j is the j^{th} ordered (ascending) observation, and $F(N_j)$ is the Poisson CDF,

---

1. X^2 critical values can be found in almost any statistics text.

$\sum_{i=0}^{n_j} [e^{-\lambda} \lambda^i / i\,!]$, with $\lambda = 1/\bar{t}$, $\bar{t}$ the sample mean. The 5 percent and 1 percent critical values for the K-S are stored in the computer, and the hypothesis of "Poissonness" is rejected then if the value of E exceeds the tabulated critical value.

Statistics are given in Table 1.

This test was originally derived for fitting continuous CDFs, but can be used as a slightly more conservative test in discrete cases, with its power improving with increasing sample sizes.

For the exponential F test, the first r ($\doteq n/2$) and then (n - r) of a set of n hypothesized exponential interoccurrence times $\{t_i\}$ are grouped, and S_i is used to denote the i^{th} normalized spacing, that is,

$$S_i = (n = i + 1)\ (t_i - (t_i - 1))\ (t_o \equiv 0).$$

Then the $\{S_i\}$ are independent and identically distributed exponentials with exactly the same mean as the underlying distribution. Thus it follows that the quantity

$$F = \frac{\sum_{i=1}^{n} S_i / r}{\sum_{i=r+1}^{n} S_i / (n - r)}$$

is the ratio of two gammas and is distributed as an F distribution with 2 r and 2 (n - r) degrees of freedom where the hypothesis of exponentiality is true. Therefore, a two-tailed F test would be performed on the F statistic calculated from the data in order to determine whether the stream is indeed truly exponential. The left and right F critical points for a and b degrees of freedom at the 5 percent level of significance (say, $F_{.025}$ (a, b) and $F_{.975}$ (a, b), respectively) can be found from the following approximate formula (tested to be within 0.6 percent accuracy of exact values):

$$\begin{cases} F_{.975}(a, b) = \dfrac{a + 1.739}{.1197\,a + .1108} - \dfrac{b - 3.986}{.1414\,b - .2864} \\ \qquad\qquad + .145 - .00170\,a + \dfrac{.06150\,a - 2.706}{b + 30} \\ F_{.025}(a, b) = 1/F_{.975}(b, a). \end{cases}$$[1]

This approximate approach is a great help in computer applications since the storage of a complete F table can be replaced by the use of these equations and, further, no interpolation formula is needed as might be required by the F-table-storage method.

---

1. See Table 1.

Table 1

KOLMOGOROV-SMIRNOV CRITICAL VALUES

Sample Size	5 Percent	1 Percent
5	.56	.67
6	.53	.63
7	.50	.60
8	.47	.56
9	.44	.53
10	.41	.49
11	.40	.47
12	.38	.45
13	.37	.43
14	.35	.41
15	.34	.40
16	.33	.39
17	.32	.38
18	.31	.37
19	.30	.36
20	.29	.36
21	.29	.35
22	.29	.34
23	.28	.33
24	.28	.32
25	.27	.32
26	.27	.31
27	.26	.31
28	.26	.30
29	.25	.30
30	.25	.29
≥ 30	$1.36/\sqrt{n}$	$1.63/\sqrt{n}$

So, for the marginal queueing model implemented at each channel, if the Kolmogorov-Smirnov and F statistics do not lead to rejection, it is assumed that work units arrive as a Poisson process to a single exponential channel and that upon arrival to the system each unit is designated to be a member of one of three priority classes (or less as circumstances dictate). The usual convention is to number the priority classes so that the smaller the number, the higher the priority. Let it further be assumed that the arrivals of the first or highest priority have mean arrival rate of a_1 work units per unit time, that the second or middle priority units have mean rate a_2 work units per unit time, and that the third or lowest priority units have mean a_3 work units per unit time, such that their sum is called a. The corresponding service rates shall then be u_1, u_2, and u_3 work units per time unit for priorities 1, 2, and 3, respectively. Let it further be supposed that the first priority items have the right to be served ahead of the others, but that once a service of a priority 2 or 3 work unit is begun, it cannot be interrupted by preemption.

In light of these assumptions, it has been shown [see Cobham (1954) or Morse (1958)] that the expected number of work units in the queueing system for each priority can be fairly easily found in terms of the input and service parameters. If Q_1, Q_2, and Q_3 are used to denote these averages, then we have [see Equation (A.4) in the Appendix]:

$$Q_1 = \frac{a_1 \sum_{k=1}^{3} (a_k/u_k^2)}{1 - a_1/u_1} + \frac{a_1}{u_1}$$

$$Q_2 = \frac{a_2 \sum_{k=1}^{3} (a_k/u_k^2)}{(1 - a_1/u_1)(1 - a_1/u_1 - a_2/u_2)} + \frac{a_2}{u_2} \qquad (1)$$

$$Q_3 = \frac{a_3 \sum_{k=1}^{3} (a_k/u_k^2)}{(1 - a_1/u_1 - a_2/u_2)(1 - a_1/u_1 - a_2/u_2 - a_3/u_3)} .$$

The mean system waiting times, say W(1), W(2), and W(3), are then found by applying Little's formula, $Q = aW$, on Equation (1), so that $W(1) = Q_1/a_1$, $W(2) = Q_2/a_2$, and $W(3) = Q_3/a_3$. The total average system wait can then be obtained by the weighted average of W(1), W(2), and W(3), namely,

$$W = (a_1/a)\, W(1) + (a_2/a)\, W(2) + (a_3/a)\, W(3).$$

The variances of the system delays for the three priorities can be determined using results of Kesten and Runnenburg (1957) which were derived in a manner very similar to the work of Cobham. Without going into the details of the required results, suffice it to say that the key ones are those which give the second moments of the waits in line as

$$W_2(1) = \frac{2\sum_{k=1}^{3}(a_k/u_k^3)}{1 - a_1/u_1} + \frac{2\,(a_1/u_1^2)\sum_{k=1}^{3}(a_k/u_k^2)}{(1 - a_1/u_1)^2}$$

$$W_2(2) = \frac{2\sum_{k=1}^{3}(a_k/u_k^3)}{(1 - a_1/u_1)^2\,(1 - a_1/u_1 - a_2/u_2)} + \frac{2\left[\sum_{k=1}^{3}(a_k/u_k^2)\right]\left[\sum_{k=1}^{2}(a_k/u_k^2)\right]}{(1 - a_1/u_1)^2\,(1 - a_1/u_1 - a_2/u_2)^2} + \frac{2\,(a_1/u_1^2)\sum_{k=1}^{3}(a_k/u_k^2)}{(1 - a_1/u_1)^3\,(1 - a_1/u_1 - a_2/u_2)}\,.$$

$$W_2(3) = \frac{2\sum_{k=1}^{3}(a_k/u_k^3)}{(1 - a_1/u_1 - a_2/u_2)^2 (1 - a_1/u_1 - a_2/u_2 - a_3/u_3)}$$

$$+ \frac{2\left[\sum_{k=1}^{3}(a_k/u_k^2)\right]^2}{(1 - a_1/u_1 - a_2/u_2)^2 (1 - a_1/u_1 - a_2/u_2 - a_3/u_3)^2}$$

$$+ \frac{2\left[\sum_{k=1}^{3}(a_k/u_k^2)\right]\left[\sum_{k=1}^{2}(a_k/u_k^2)\right]}{(1 - a_1/u_1 - a_2/u_2)^3 (1 - a_1/u_1 - a_2/u_2 - a_3/u_3)}$$

The variances of the system delays are thus

$$V(1) = W_2(1) - [W(1) - 1/u_1]^2 + 1/u_1^2,$$

$$V(2) = W_2(2) - [W(2) - 1/u_2]^2 + 1/u_2^2,$$

and

$$V(3) = W_2(3) - [W(3) - 1/u_3]^2 + 1/u_3^2.$$

The well-known inequality due to Chebyshev, namely,

$$\Pr\left\{ \left| X - E[X] \right| \geq k\sigma \right\} \leq 1/k^2, \tag{2}$$

is employed to get the probability distribution governing the waiting times for each of the priorities, and then the system distribution is achieved by mixing according to the proper proportions. The use of the inequality will give conservative bounds instead of exact expressions, but these bounds are sufficiently tight for modeling purposes and any final answer should be reasonably robust with respect to the approximation, especially in view of the fact that many such queueing systems will eventually be combined and any errors will tend to neutralize each other in the end.

To be more exact, it is assumed that the right-hand inequality in (2) is binding and thus that

$$\Pr \left\{ \left| X - E[X] \right| \geq k\sigma \right\} = 1/k^2.$$

Now assuming further that the probability distribution has equal probability on each side of the mean,

$$\Pr \left\{ X - E[X] \geq k\sigma \right\} = 1/(2k^2)$$

and

$$\Pr \left\{ E[X] - X \geq k\sigma \right\} = 1/(2k^2).^{1}$$

So, given the mean $E[X]$ and the variance σ^2 (or, equivalently, the standard deviation σ), the distribution function may be reconstructed by varying k in reasonably small steps over an appropriate range. In the program written for the analysis this is done automatically for each subsystem and then, for any specific values of the input parameters, summary information about the queue is printed out in the form of the average number of units of each priority in the system, the total average system wait, the variance of the system wait for each priority, the (approximate) probability distribution for the three system delays.

The distributions for the three priorities must then be combined in order to obtain the probabilities for the total process. This is done by the usual mixing procedure as follows. If the individual probabilities for the k^{th} priority are denoted by $\left\{ p_i(k),\ 1 \leq i \leq 20 \right\}$, and the combined distribution by $\left\{ C_i,\ 1 \leq i \leq 40 \right\}$, then

$$C_i = (a_1/a)\, p_i(1) + (a_2/a)\, p_i(2) + (a_3/a)\, p_i(3).$$

For circumstances under which a two-priority or a no-priority service protocol is observed, appropriately amended programs have been provided NAVSUP. In the following appendix may be found additional technical discussions of queues with many priorities.

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1. This assumption may be changed by altering the value of an appropriate parameter in the program.

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RESULTS FOR NONPREEMPTIVE EXPONENTIAL QUEUES
WITH MANY PRIORITIES

RESULTS FOR NONPREEMPTIVE EXPONENTIAL QUEUES WITH MANY PRIORITIES

As was noted earlier in the previous discussion, the determination of the mean system sizes and waiting times can be found via a direct expected-value procedure such as that due to Cobham [1954] or through the more classical differential-difference-equation method such as that found in Morse [1958]. Cobham's approach to the mean delay for display was selected since it is more straightforward and can, in fact, be just as well used to handle multiple priorities as two priorities.

To begin, suppose that items of a k^{th} priority (the smaller the number, the higher the priority) arrive before a single channel according to a Poisson distribution with parameter a_k ($k = 1, 2, \ldots, p$) and that these work units wait on a first-come, first-served basis within their respective priorities. Let the service distribution for the k^{th} priority be exponential with mean $1/u_k$. Whatever the priority of a unit in service, it completes its service before another item is admitted.

Begin by defining

$$r_k = a_k/u_k \quad (1 \leq k \leq p)$$

and

$$R_k = \sum_{i=1}^{k} r_i \quad (R_0 \equiv 0,\ R_p \equiv R).$$

The system is stationary for $R_p = R < 1$.

Then suppose that a work unit of priority i arrives at the system at time t_0 and enters service at time t_1. Its line wait is thus $T_q = t_1 - t_0$. At t_0 assume that there are n_1 work units of priority 1 in the line ahead of this new arrival, n_2 of priority 2, n_3 of priority 3, etc. Let S_0 be the time required to finish the item already in service and S_k be the total time required to serve n_k. During the new work unit's waiting time T_q, (say) n_k' items of priority $k < i$ will arrive and go to service ahead of this current arrival. If S_k' is the total service time of all the n_k', then it can be seen that

$$T_q = \sum_{k=1}^{i-1} S_k' + \sum_{k=1}^{i} S_k + S_0.$$

If expected values are taken on both sides of the foregoing, then we find that

$$W_q^{(i)} \equiv E[T_q] = \sum_{k=1}^{i-1} E[S_k'] + \sum_{k=1}^{i} E[S_k] + E[S_0].$$

Since $R_{i-1} < R_i$ for all i, $R < 1$ implies that $R_{i-1} < 1$ for all i.

To find $E[S_0]$, observe that the combined service distribution is the mixed exponential, which is formed from the law of total probability as

$$B(t) = \sum_{k=1}^{p} a_k \left(1 - e^{-u_k t}\right)/a,$$

where

$$a = \sum_{k=1}^{p} a_k.$$

The random variable "remaining time of service," S_0, has the value 0 if the system is idle and hence

$$E[S_0] = \Pr \{\text{system is busy}\}\ E[S_0 \mid \text{busy system}].$$

But the probability that the system is busy is

$$a \cdot (\text{expected service time}) = a \sum_{k=1}^{p} (a_k/a)\,(1/u_k)$$

$$= R,$$

and

$$E[S_0 \mid \text{system busy}]$$

$$= \sum_{k=1}^{p} E[S_0 \mid \text{system busy with k type work unit}] \cdot \Pr\{\text{work unit has priority } k\}$$

$$= \sum_{k=1}^{p} (1/u_k)\,(r_k/R).$$

Therefore

$$E[S_0] = R \sum_{k=1}^{p} (1/u_k)\,(r_k/R)$$

$$= \sum_{k=1}^{p} (r_k/u_k). \qquad \text{(A. 1)}$$

Since n_k and the service times of individual work units, $S_k^{(n)}$, are independent,

$$E[S_0] = E[n_k\, S_k^{(n)}]$$

$$= E[n_k]\; E[S_k^{(n)}]$$

$$= E[n_k]/u_k.$$

Utilizing Little's formula then gives

$$E[S_k] = a_k\, W_q^{(k)}/u_k$$

$$= r_k\, W_q^{(k)}.$$

Similarly,

$$E[S_k'] = E[n_k']/u_k,$$

and then utilizing the uniform property of the Poisson we have

$$E[S_k'] = a_k W_q^{(i)}/u_k.$$

Therefore

$$W_q^{(i)} = W_q^{(i)} \sum_{k=1}^{i-1} r_k + \sum_{k=1}^{i} r_k W_q^{(k)} + E[S_0],$$

or

$$W_q^{(i)} = \frac{\sum_{k=1}^{i} r_k W_q^{(k)} + E[S_0]}{1 - R_{i-1}}. \tag{A.2}$$

The solution to Equation (A.2) was found by Cobham, after whom much of this analysis follows, by induction on i, after a general pattern emerged upon iteration. That solution is

$$W_q^{(i)} = \frac{E[S_0]}{(1 - R_{i-1})(1 - R_i)}.$$

Using Equation (A.1) finally gives

$$W_q^{(i)} = \frac{\sum_{k=1}^{p} (r_k/u_k)}{(1 - R_{i-1})(1 - R_i)}. \tag{A.3}$$

Note that (A.3) holds as long as $R = \sum_{k=1}^{p} r_k < 1$. Of course, the individual mean system delay for priority i is therefore

$$W^{(i)} = W_q^{(i)} + 1/u_i.$$

Therefore, from Little's formula, the mean number of work units of priority i present in the system is given by

$$Q_i = a_i W_q^{(i)} + a_i/u_i$$

$$= \frac{a_i \sum_{k=1}^{p} (r_k/u_k)}{(1 - R_{i-1})(1 - R_i)} + \frac{a_i}{u_i} \qquad \text{(A.4)}$$

and that the total expected system size is

$$Q = \sum_{i=1}^{p} [L_q^{(i)} + a_i/u_i]$$

$$= \sum_{i=1}^{p} \left[\frac{a_i \sum_{k=1}^{p} (r_k/u_k)}{(1 - R_{i-1})(1 - R_i)} + \frac{a_i}{u_i} \right].$$

Expressions very similar to that of Equation (A.3) were found for the higher moments of the line delays for each priority by Kesten and Runnenburg (1957). These results can also be found in the more readily available reference Cohen (1969). The formulas are a bit lengthy and will not be directly noted here but instead may be found within the program in the calculations leading to the system waiting time variance.

APPENDIX B

PROGRAM LISTINGS

- Multipriority Queueing Model
- Kolmogorov-Smirnov Test Statistic for Poisson Fit
- General Kolmogorov-Smirnov Test Statistic Routine
- Convolution Routine
- Exponential Fit Test

MULTIPRIORITY QUEUEING MODEL

```
1*  THIS IS PROGRAM QUEPRI FOR THE 3 PRIORITY, 3 ARRIVAL RATES,
2*  3 SERVICE RATE PROBLEM
3   DIM X(41),Y(3,41),M(3,41),N(3,41),P(3,41),S(81),T(3,41)
4   DIM U(3,41),V(3),W(3),A(3),Q(3,41)
5   DIM Z(3,41)
8 * U1,U2,&U3 ARE THE SERVICE RATES, LINES 19,20,&21.
10* IF THERE ARE NO PRIORITIES YOUSHOULD USE PROGRAM QUEONE
18 U4=1.305
19 U1=1.1*U4
20 U2=2*U1
21 U3=U2
22* A1,A2,&A3 ARE THE ARRIVAL RATES, LINES 23,24,&25
23 A1=.372
24 A2=.882
25 A3=.729
26  R1=A1/U1 + A2/U2 + A3/U3
27  IF R1>1. GO TO 290
28  A=A1+A2+A3
29 S1=1.-A1/U1
30  S2=S1-A2/U2
31  S3=S2-A3/U3
32  Y=A1/(U1↑2) + A2/(U2↑2) + A3/(U3↑2)
33  PRINT "THE AVERAGE # OF UNITS OF PRIORITIES 1,2,&3 IS"
34  Q1=A1*Y/S1 + A1/U1
35  Q2=A2*Y/(S1*S2) + A2/U2
36  Q3=A3*Y/(S2*S3) + A3/U3
37  W(1)=Q1/A1
38  W(2)=Q2/A2
39  W(3)=Q3/A3
40  PRINT Q1,Q2,Q3
41  W=A1*W(1)/A + A2*W(2)/A + A3*W(3)/A
42  PRINT
43 PRINT "THE TOTAL AVERAGE WAIT IN THE SYSTEM IS (IN DAYS)"
44  PRINT W
45  PRINT
46  X=A1/(U1↑3)+A2/(U2↑3)+A3/(U3↑3)
47  Y=A1/(U1↑2)+A2/(U2↑2)+A3/(U3↑2)
48  A(1)=2*X/S1 + 2*Y*(A1/(U1↑2))/(S1↑2)
49  V(1)=A(1) - (W(1) - (1/U1))↑2 + U1↑(-2)
50  A(2)=2*X/((S1↑2)*S2) + 2*Y*(Y-A3/(U3↑2))/((S1*S2)↑2)
51  A(2)=A(2)+2*Y*(A1/(U1↑2))/((S1↑3)*S2)
52V(2)=A(2)-(W(2)-(1./U2))↑2 + U2↑(-2)
53  A(3)=2*X/((S2↑2)*S3) + 2*(Y↑2)/((S2*S3)↑2)
54  A(3)=A(3)+2*Y*(Y-A3/(U3↑2))/((S2↑3)*S3)
55 *
56V(3)=A(3)-(W(3)-(1./U3))↑2 + U3↑(-2)
57  PRINT "THE AVG SYSTEM WAIT FOR EACH PRIORITY IS"
58  PRINT W(1),W(2),W(3)
59  PRINT
60  PRINT "THE VARIANCE OF THE SYSTEM WAIT FOR EACH PRIORITY IS"
61  PRINT V(1),V(2),V(3)
62  PRINT
```

```
63  PRINT "DISTRIBUTIONS FOR 3 PRI-S WAITING TIMES"
64  FOR K=1 TO 3
65  IF W(K) < 40. GO TO 68
66  L(K,40)=1
67  GO TO 179
68 IF W(K)+(V(K))↑.5 > 40. GO TO 177
69  FOR I=1 TO 40
70  X(I)=1.-50./((10+I)↑2)
71  Y(K,I)=W(K)+((V(K)↑.5)*(10+I)/10.)
72  IF Y(K,I) >40. GO TO 82
73 *
75  Z(K,I)=INT(Y(K,I))+1
76  S(41-I)=1.-X(I)
77  T(K,41-I)=W(K)-((V(K)↑0.5)*(10+I)/10.)
78  IF T(K,I)>=0. GO TO 80
79  T(K,I)=0.
80  U(K,I)=INT(T(K,I))+1
81  NEXT I
82  FOR I = 1 TO 40
83  N(K,I)=0
84  NEXT I
85  FOR I = 1 TO 40
86  L=U(K,I)
87  J=Z(K,I)
88  N(K,L)=N(K,L)+1
89 N(K,J)=N(K,J)+1
90  NEXT I
91  M(K,0)=0
92  P=0.
93  FOR I = 1 TO 40
94  M(K,I)=0
95  FOR J=1 TO I
96  M(K,I)=M(K,I)+N(K,J)
97  NEXT J
98  M=M(K,I)
99 L=M(K,I-1)
100 FOR N = 1 TO 40
101 S(40+N)=X(N)
102 NEXT N
103 P(K,I)=S(M)-S(L)
104 P=P+P(K,I)
105 NEXT I
106 PRINT"              NUMBER OF DAYS","                    PROBABILITY"
107 G=0.
108 Z=0.
109 FOR I=1 TO 40
110 P(K,I)=P(K,I)/P
111 Q(K,I)=P(K,I)
112 NEXT I
113 P=0
114 FOR I=1 TO 40
115 P=P+P(K,I)
```

```
116 IF F(K,I)>0 GØ TØ 119
117 Z=Z+1.
118 IF F(K,I+1)>0 GØ TØ 121
119 IF F=1 GØ TØ 170
120 NEXT I
121 J=INT((Z+1)/2)
122*
125 IF INT(Z/2)=Z/2 GØ TØ 162
129*
130 FØR I=1 TØ 40
131 IF F(K,I)*F(K,I+1)>0. GØ TØ 160
132 *
134 FØR N= 1 TØ J+1
135 G(K,I+N-1)=F(K,I)/(J+1)
136 NEXT N
137 G(K,I+1+Z)=F(K,I+1+Z)/(J+1)
138 FØR N= 1 TØ J
140 G(K,I+J+N-1)=G(K,I+J+N-1)+G(K,I+1+Z)
141*
145 NEXT N
148*
149 *
150 GØ TØ 170
155 *
160 NEXT I
162 FØR I=1 TØ 40
163 IF F(K,I)*F(K,I+1)>0. GØ TØ 169
164 FØR N=1 TØ Z/2 + 1
165 G(K,I+N-1)=F(K,I)/(J+1)
166 G(K,I+Z/2+N)=F(K,I+Z+1)/(Z/2+1)
167 NEXT N
168 GØ TØ 170
169 NEXT I
170 FØR I=1 TØ 40
171 PRINT I-1,"THRØUGH",I,G(K,I)
172 G=G+G(K,I)
173 IF G>1.-10↑(-5) GØ TØ 175
174 NEXT I
175 PRINT
176 GØ TØ 180
177 FØR I=1 TØ 40
178 G(K,I)=1/40
179 NEXT I
180 NEXT K
181 DIM C(41)
185 C(1) = 0.
190 PRINT
195*
200 FØR I=1 TØ 40
201 C(I)= (A1*G(1,I) + A2*G(2,I) + A3*G(3,I))/A
202*
```

```
203  IF C(I-1)<10.↑(-5) GO TO 220
205*
210 NEXT I
215*
220  R=0
221 FOR I =1 TO 40
222 R=R+I*G(1,I)
223 NEXT I
224 PRINT"THE WEIGHTED AVERAGE OF THE PRI-I DISTRIBUTION IS"
225 PRINT R
226 PRINT
227 PRINT
230 S=0
231 GO TO 301
232*
233*
234*
235*
236 PRINT
237 PRINT
240 T=0
241 FOR I=1 TO 40
242 T=T+I*G(3,I)
243 NEXT I
244 PRINT"THE WEIGHTED AVERAGE OF THE PRI-III DISTRIBUTION IS"
245 PRINT T
246 V=0.
247 FOR I=1 TO 40
248 V=V+I*C(I)
249 NEXT I
250 PRINT
251 PRINT
252 PRINT"THE WEIGHTED AVERAGE FOR ALL 3 DISTRIBUTIONS IS"
253 PRINT V
255 GO TO 300
290 PRINT "HELP-THE SYSTEM IS OVERLOADED!!"
300 STOP
301 FOR I = 1 TO 40
302 S=S+I*G(2,I)
303 NEXT I
304 PRINT " THE WEIGHTED AVERAGE OF THE PRI-II DISTRIBUTION IS"
305 PRINT S
306 PRINT
307 PRINT
308 GO TO 240
2231*
```

KOLMOGOROV-SMIRNOV TEST STATISTIC FOR POISSON FIT

```
1 * THIS IS PRØGRAM PØIS FØR TESTING PØISSØN FITS FØR SAMPLE SIZES
2 * BETWEEN N=5 & 100 AT A 5% LEVEL ØF SIGNIFICANCE
3   DIM P(251),Q(251),R(251),S(251),T(101),F(251),D(251),A(251)
4   N=10
5   DATA 35,14,49,53,57,21,30,37,42,48
6 *
7 *
8 *
9 *
10*
15  FØR I=1 TØ N
20  READ A(I)
25  NEXT I
30  A=0.
35  FØR I=1 TØ N
40  A=A+A(I)/N
45  NEXT I
50  F(0)=1.
55  FØR I=1 TØ 250
60  F(I)=A*F(I-1)/I
65  NEXT I
70  P(0)=EXP(-A)
75  Q(0)=P(0)
80  FØR I=1 TØ 250
85  P(I)=P(0)*F(I)
90  Q(I)=Q(I-1)+P(I)
95  NEXT I
100 R(14)=1/10
105 R(21)=1/10
110 R(30)=1/10
115 R(35)=1/10
120 R(37)=1/10
125 R(42)=1/10
130 R(48)=1/10
135 R(49)=1/10
140 R(53)=1/10
145 R(57)=1/10
150 *
155 *
160 *
165 *
170 *
175 *
180 *
185 *
190 *
195 *
200 *
205 S(0)=R(0)
210 FØR I=1 TØ 250
```

```
215 S(I)=S(I-1)+R(I)
220 NEXT I
225 D(0)=ABS(C(0)-S(0))
230 FØR I=1 TØ 250
235 D(I)=ABS(C(I)-S(I))
240 NEXT I
245 E=D(0)
250 FØR I=1 TØ 250
255 IF D(I)<E GØ TØ 265
260 E=D(I)
265 NEXT I
270 PRINT "BRANCH 774:  CØNTRACT MANAGEMENT"
275 PRINT E
280 DATA .56,.53,.50,.47,.44,.41,.40,.38,.37,.35,.34,.33,.32,.31
285 DATA .30,.29,.29,.29,.28,.28,.27,.27,.26,.26,.25,.25,.24
290 FØR I=5 TØ 30
295 READ T(I)
300 NEXT I
305 IF N<=30 GØ TØ 330
310 T(N)=1.36/(N↑.5)
330 IF E>T(N) GØ TØ 350
340 PRINT "YES-THE DATA ARE PØISSØN!"
345 GØ TØ 400
350 PRINT "NØ-THE DATA ARE NØT PØISSØN AT THE 5% LEVEL!"
351 DATA .67,.63,.60,.56,.53,.49,.47,.45,.43,.41,.40,.39,.38
352 DATA .37,.36,.36,.35,.34,.33,.32,.32,.31,.31,.30,.30,.29
353 FØR I=5 TØ 30
354 READ T(I)
355 NEXT I
360 IF N<=30 GØ TØ 370
365 T(N)=1.63/(N↑.5)
370 IF E>T(N) GØ TØ 375
372 PRINT "BUT THEY ARE AT THE 1% LEVEL"
373 GØ TØ 400
375 PRINT "THE DATA ARE STILL NØT PØISSØN!"
400 STØP
```

```
1 * THIS IS ROUTINE "KØS" FØR TESTING EQUALITY ØF DISTRIBUTIØNS.
2*
3 * K IS THE NUMBER ØF CLASS INTERVALS ØR RESPØNSE VALUES.
5 N1=1781
10  N2=N1
15 K=15
20  DIM L(30),M(30),F(30),G(30)
21  DIM D(30)
25 DATA 623,478,42,47,72,48,58,63,45,52,35,55,69,51,52
26*
30  FØR I=1 TØ K
35  READ L(I)
40  F(I)=L(I)/N1
45  NEXT I
50 DATA 251,251,395,144,144,276,128,69,41,27,23,12,9
51 DATA 7,2
55  FØR I=1 TØ K
60  READ M(I)
65  G(I)=M(I)/N2
70  NEXT I
75  F(0)=0.
80  G(0)=0.
85  FØR I=1 TØ K
90  F(I)=F(I)+F(I-1)
94*
95  G(I)=G(I)+G(I-1)
96  NEXT I
97*
98*
100*
105 FØR I=1 TØ K
110 D(I)=ABS(F(I)-G(I))
115 NEXT I
116*WE HAVE JUST CØMPUTED ALL THE ABSØLUTE DIFFERES BETWEN
117*THE TWØ CUMULATIVE DISTRIBUTIØN FUNCTIØNS.
118*WE MUST NEXT DETERMINE WHICH ØF THESE ABSØLUTE DIFFERENCES
119*IS THE LARGEST-WHEN MULTIPLIED BY AN APPRØPRIATE CØNSTANT,
120*THIS WILL BE ØUR TEST STATISTIC.
121 E=D(1)
125 FØR I=1 TØ K
130 IF D(I)<E GØ TØ 140
135 E=D(I)
140 NEXT I
145 F=(E*(2↑(1/2)))/((1/N1+1/N2)↑(1/2))
150 PRINT "THE VALUE ØF THE TEST STATISTIC IS",F
151 PRINT
152 PRINT "THEREFØRE,"
155 IF F>1.36 GØ TØ 190
156*1.36 IS THE CRITICAL 5% KØLMØGØRØV-SMIRNØV VALUE.
160 PRINT "THE TWØ PØPULATIØNS HAVE PRØVIDED RESPØNSES WHICH ARE"
161 PRINT "STATISTICALLY IDENTICAL!"
165 STØP
190 PRINT "THE TWØ DISTRIBUTIØNS CAN BE CØNSIDERED TØ BE"
191 PRINT "SIGNIFICANTLY DIFFERENT!"
200 STØP
```

CONVOLUTION ROUTINE

```
10 * THIS PROGRAM IS USED TO CONVOLUTE PROBABILITY DISTRIBUTIONS THRU
15 *    ANY NUMBER OF PATH ELEMENTS.
20 *
25 *   THE FOLLOWING VARIABLES ARE USED:
30 *       N1=NUMBER OF PATH ELEMENTS
40 *       M1,M2=NUMBER OF HOURS IN A PARTICULAR DISTRIBUTION
45 *       D(I),E(I),F(I)=PROBABILITY DISTRIBUTION AFFECTING AN ELEMENT
50 *             D(I)=INPUT DISTRIBUTION
55 *             E(I)=INTERNAL DISTRIBUTION
60 *             F(I)=OUTPUT DISTRIBUTION
65 *
70 *   THE FOLLOWING DATA LINES ARE REQUIRED:
75 *       1000 DATA N1
80 *       2000 DATA M1,E(1),E(2),.....,E(M1)
90 *            REPEAT ABOVE LINE FOR ALL ELEMENTS
92 *
94 *
100 DIM A(100),D(100),E(100),F(100)
106 LET G3=0
110 READ N1
160 READ M1
190 FOR J=1 TO M1
200 READ D(J)
220 NEXT J
230 FOR I=2 TO N1
240 READ M2
250 FOR J=1 TO M2
260 READ E(J)
280 NEXT J
290 LET M3=M1+M2
300 FOR J=2 TO M3
310 LET F(J)=0
320 FOR K=I-1 TO M1
330 IF J-K<1 THEN 360
340 IF J-K>M2 THEN 360
350 LET F(J)=F(J)+D(K)*E(J-K)
360 NEXT K
362 NEXT J
364 IF I=N1 THEN 380
371 FOR J=I TO M3
372 LET D(J)=F(J)
373 NEXT J
374 LET M1=M3
375 NEXT I
380 PRINT
382 PRINT
384 PRINT
390 PRINT
400 PRINT "THE NUMBER OF BRANCHES IN THIS PATH ARE"; I
410 PRINT
```

```
420 PRINT "MAX DAYS ="; M3
430 PRINT
460 PRINT "DAYS", "PROB-DAYS"
470 LET Y=I-1
480 LET Q1=0
490 FOR J=I TO M3
500 LET Q1=Q1+F(J)
510 LET Y=Y+1
520 IF Y<8 THEN 570
530 PRINT J,F(J)
532 LET Q2=J/8
536*
537 LET Q3=Q3+Q1
539*
540 LET Y=0
550 LET Q1=0
560 GOTO 590
570 IF F(J)<.001 THEN 590
580 PRINT J,F(J)
590 LET D(J)=F(J)
600 NEXT J
603 IF Y=8 THEN 610
605 LET Q2=Q2+1
606 PRINT " "," ",Q2,Q1
608 LET Q3=Q3+Q1
610 PRINT
612 PRINT "                              SUM OF PROB-DAYS=",Q3
700 Z=0
710 FOR J=I TO M3
720 Z=Z + J*F(J)
725 NEXT J
726 PRINT
730 PRINT "THE AVERAGE NUMBER OF DAYS IS"
740 PRINT Z
1000 DATA 2
2000*
2001*
2002*
2003*
2004 DATA 15,.135,.174,.075,.092,.09,.122,.076
2005 DATA .081,.029,.028,.03,.033,.013,.011
2006 DATA .011
2007*
2008 DATA 14,.159,.091,.034,.053,.048,.104,.198
2009 DATA .142,.046,.02,.023,.039,.018,.025
2010*
2011*
2999*
9999 END
```

EXPONENTIAL FIT TEST

```
1     REM  THIS IS PRØGRAM EXP FØR TESTING EXPØNENTIAL
2     REM  FITS FØR SAMPLE SIZES BETWEEN 5 AND 500
3     REM  AT A 5% LEVEL ØF SIGNIFICANCE.
4     REM  N WILL DENØTE THE TØTAL NUMBER ØF PØINTS.
5     REM  DATA IS INPUTED FØR T(I)="CLØCK" TIMEØF ITH ARRIVAL.
7     N=10
8     DIM S(501),T(501)
10     DATA 10,20,30,40,50,60,70,80,90,100
11    REM
12    REM
13    REM
14    REM
15    REM
16    REM
17    REM
18    REM
19    REM
20    REM
25    FØR I=1 TØ N
30    READ T(I)
35    NEXT I
40    T(0)=0.
45    FØR I=1 TØ N
50    S(I)=(N-I+1.)*(T(I)-T(I-1))
55    NEXT I
60     L=INT(N/2)
65    Y1=0.
70     FØR I=1 TØ L
75    Y1=Y1+S(I)
77     NEXT I
80     Y1=Y1/L
85    Y2=0.
90     FØR I=L+1 TØ N
95    Y2=Y2+S(I)
100   NEXT I
102    Y2=Y2/(N-L)
105    Q=Y1/Y2
109    PRINT "BRANCH 774:  CØNTRACT MANAGEMENT"
110    PRINT Q
115    L1=2*L
116    L2=2*(N-L)
120    PRINT "DEGREES ØF FREEDØM ARE" L1 "AND" L2
121    D1=L1
122    D2=L2
125    F2=(D1+1.739)/(.1197*D1+.1108)  -(D2-3.986)/(.1414*D1-.2864)
126    F2=F2+(.145-.0017*D1)  + (.0615*D1-2.706)/(D2+30.)
130    G1=(D2+1.739)/(.1197*D2+.1108)  - (D1-3.986)/(.1414*D2-.2864)
131    G1=G1+(.145-.0017*D2)  + (.0615*D2-2.706)/(D1+30.)
135    F1=1./G1
140    IF Q<F1 THEN 200
145    IF Q>F2 THEN 200
150   PRINT "YES, THE DATA ARE EXPØNENTIAL!"
175   GØ TØ 300
200   PRINT "NØ, THE DATA ARE NØT EXPØNENTIAL!"
300   END
```

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13. ABSTRACT

Network theory and the theory of queues under several servicing priorities are combined in a mathematical simulation of requisition processing. Data describing requisition processing at NSC Norfolk and the Ships Parts Control Center are presented. Throughput time distributions which are observed are compared to throughput time distributions predicted by the simulation model. Generally, the model failed to explain requisition waiting times due to backlogs.

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