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A MODEL FOR TARGET SCINTILLATION

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This report discusses the implementation of an algorithm used to simulate target scintillation. The algorithm takes uniform random noise and modifies it using a simulated low-pass filter to produce estimates of target strength. The actual implementation statistics are compared with the theoretical distribution to ascertain the usefulness of the approach.

CONTENTS

FIGURES

1. INTRODUCTION

Target scintillation is the variation of angular location of the apparent source of the echo signal and is dependent upon relative phase and amplitude of the component echo signals and their apparent angular locations. The motion of the target causes the apparent source to wander back and forth about the physical center of the target. This wander is called angle noise or scintillation [Ref. 1]. Since the noise is random, it is usually calculated using experimental observation data. Few articles have appeared to propose models of the phenomenon.

Recently, we implemented a simulation of target scintillation. The objective of this report is to document this implementation and to define its theoretical basis. These objectives are realized by answering the following questions:

1. What algorithm was actually implemented.
2. What are the statistical characteristics of simulated scintillation?
3. What are the theoretical statistical characteristics of our model?

2. ALGORITHM

The scintillation implementation is based on the model illustrated in Figure 1. Random noise is generated by the algorithm derived in Appendix A The noise is passed through a filter to simulate the correlation of scintillation. The filtered noise then exponentially modifies the nominal target strength which is our output. This arrangement allows for simulation of virtually all types of scintillation because it allows control of both the statistics of the noise and the characteristics of the filter.

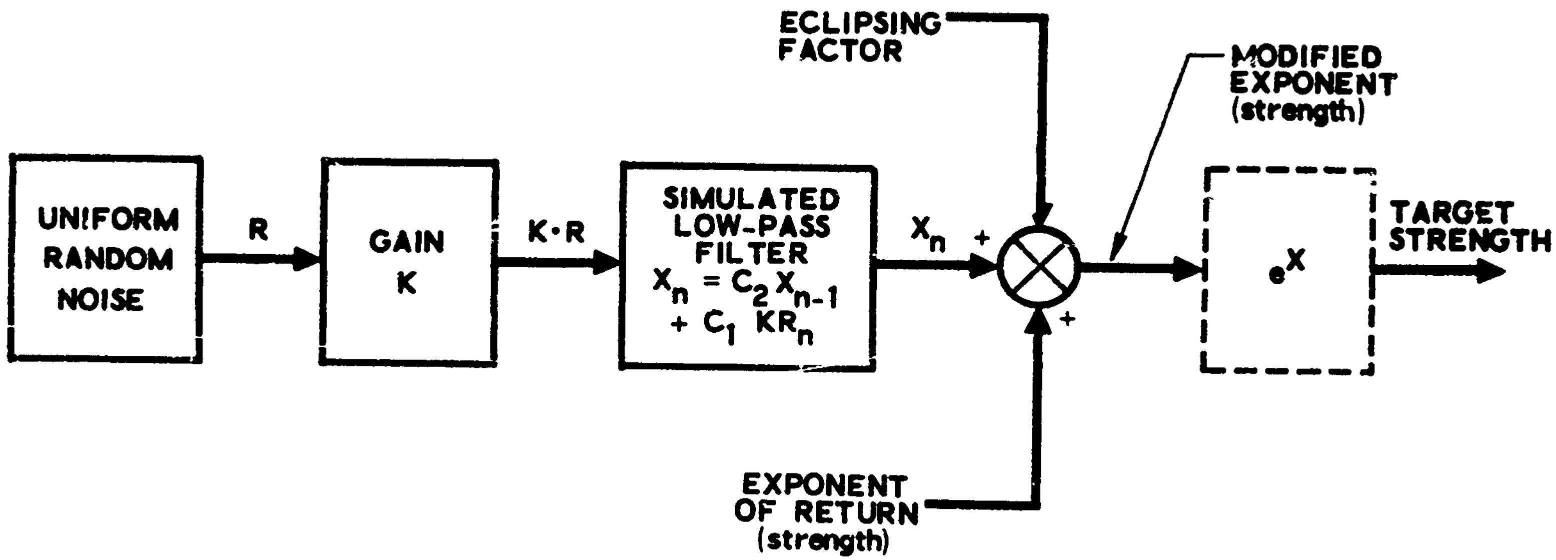

Figure 1. Filter Model

Pseudo-random noise is generated by the algorithm:

$$R_{1(n)} = 65539\, R_{1(n-1)} \text{ (modulo } 2^{31}) \tag{1}$$

$$R_{1(0)} = 12345$$

$$R_n = 2\left(R_{1(n)} - (0.5x2^{31})\right) \tag{2}$$

This noise, magnified by a scale factor K, is filtered by a digital one-pole filter:

$$X_n = C_2 X_{n-1} + C_1 K R_n \tag{3}$$

where:

$$C_1 = 1 - e^{-\frac{\Delta t}{\tau}} \sim \frac{\Delta t}{\tau}$$

$$C_2 = e^{-\frac{\Delta t}{\tau}} \sim 1 - \frac{\Delta t}{\tau}$$

Thus:

$$X_n = \left(1 - \frac{\Delta t}{\tau}\right) \cdot X_{n-1} + \frac{\Delta t}{\tau} \cdot K \cdot R_n \tag{4}$$

The values selected for implementation were

τ = 1.024 second

Δt = 8 milliseconds

$\Delta t/\tau$ = 2/256

K = 128

Therefore:

$$X_n = \frac{254}{256} X_{n-1} + R_n \tag{5}$$

The value of both X_n and the eclipsing constant are added to the exponent of the normal target strength. The range for this exponent is limited to 0 through +63.

3. STATISTICS OF THE ALGORITHM

The random numbers generated in equations (1) and (2) have the following uniform distribution with mean of 0 and variance equal to 1/3.

$$p(R_n) = 1/2 \text{ for } -1 \le R_n < +1$$

where $p(\cdot)$ is the probability of $(\cdot)$.

$$\text{mean} = \mu_R = \int_{-\infty}^{+\infty} R \cdot p(R)\, dR = 0$$

$$\text{variance} = \sigma_R{}^2 = \int_{-\infty}^{+\infty} (R - \mu_R)^2\, p(R)\, dR = 1/3$$

After passing through the filter, the mean, variance, and autocorrelation functions of any one sequence $[X_0, X_1, \ldots X_n]$ as derived in Appendix B are as follows:

$$\mu_X = \mu_R = 0 \tag{6}$$

$$\sigma_X{}^2 = \frac{K^2 C_1{}^2}{1 - C_2{}^2} \sigma_R{}^2 = \frac{K^2 C_1}{2 + C_1} \sigma_R{}^2 = \frac{K^2 C_1}{2 + C_1} \cdot \frac{1}{3} \tag{7}$$

$$\rho(k) = \frac{E(X_n, X_{n-k})}{\sigma_n \cdot \sigma_{n-k}} = C_2^{\ k} \qquad (8)$$

Substituting values:

$$\mu_X = 0$$

$$\sigma_X^{\ 2} = 21.4$$

$$p(k) = (0.9921875)^k \qquad (9)$$

$$\text{Time} = k \cdot \Delta t = 8 \cdot k \text{ milliseconds}$$

where

k = number of samples

The amplitude distribution of X is Gaussian with the above mean and variance. These perturbations are applied to the exponent of target strength so that, in effect, the logarithm of strength is varied. Applying an inverse log transformation, we find:

$$p(X) = \frac{1}{\sqrt{2\pi}\,\sigma_X} e^{-\frac{X^2}{2\sigma_X^{\ 2}}} \qquad (10)$$

$$p(S) = \frac{1}{\sqrt{2\pi} \cdot \sigma_X \cdot S} e^{-\frac{(\ln S - \ln \overline{S})^2}{2\sigma_X^{\ 2}}} \qquad (11)$$

$$\text{for } -\infty < S < +\infty$$

where S = target strength

$\overline{S}$ = mean of S or nominal target strength

This is a log-normal distribution whose

$$\text{mean} = \overline{S}\, e^{\frac{\sigma_X^{2}}{2}} = \overline{S}\, e^{\frac{K^2 C_1^{2}}{1 - C_2^{2}} \frac{\sigma_R^{2}}{2}} \tag{12}$$

$$\text{and variance} = \overline{S}^{2} \cdot e^{\sigma_X^{2}} \left(e^{\sigma_X^{2}} - 1 \right) \tag{13}$$

Note that the average target strength changes from its nominal value $\overline{S}$. Qualitatively, this can be understood by examining a case where the exponent of target strength is alternately changed by +1 and -1. The average value for the exponent for this case is unchanged. But when one inspects the corresponding strengths, they become 1-1/4 rather than 1.

4. THEORETICAL MODEL

Radar scintillation discussions invariably refer back to Swerling's classic paper [Ref. 2]. For his cases I or II, which apply to aircraft, the distribution of reflected energy is:

$$p(S, \overline{S}) = \frac{S}{\overline{S}}\, e^{-\frac{S}{\overline{S}}} \text{ for } x \geq 0 \tag{14}$$

$$= 0 \quad \text{elsewhere}$$

where S = input signal-to-noise ratio (target cross-section)

$\overline{S}$ = average of S over all fluctuations

We must therefore generate a distribution which, when applied to the logarithm of target strength, will result in Eq. (14). If p(x) is this distribution, then

$$x = \ln S \text{ or } S = e^{x}$$

$$dx = \frac{dS}{S} = e^{-x}\, dS$$

$$p(x) = \left.\frac{p(S)}{\left|\frac{dx}{dS}\right|}\right|_{S=e^{x}} = \frac{\frac{1}{\bar{S}} e^{-\frac{S}{\bar{S}}}}{1/S} = \frac{S}{\bar{S}}\, e^{-\frac{S}{\bar{S}}}$$

$$p(x) = \frac{e^{x}}{e^{\bar{x}}}\, e^{-\frac{e^{x}}{e^{\bar{x}}}} \text{ for } -\infty < x < +\infty \qquad (15)$$

The mean and variance of this distribution p(x) as derived in Appendix C are:

$$\text{mean} = \mu_x = \bar{x} - C = \ln \bar{S} - C$$

where $\quad C$ = Euler's constant

$$\text{variance} = \sigma_x^{2} = \frac{\pi}{6}$$

5. COMPARISON OF MODELS

A plot of the theoretical versus the implemented distribution is included as Figure 2. Figure 3 illustrates the input distribution used to generate the distribution in Figure 2. Figure 4 illustrates the transform used to convert theoretically correct noise from the available Gaussian noise. The algorithm closely realizes theoretical scintillation for values greater than 0.6.

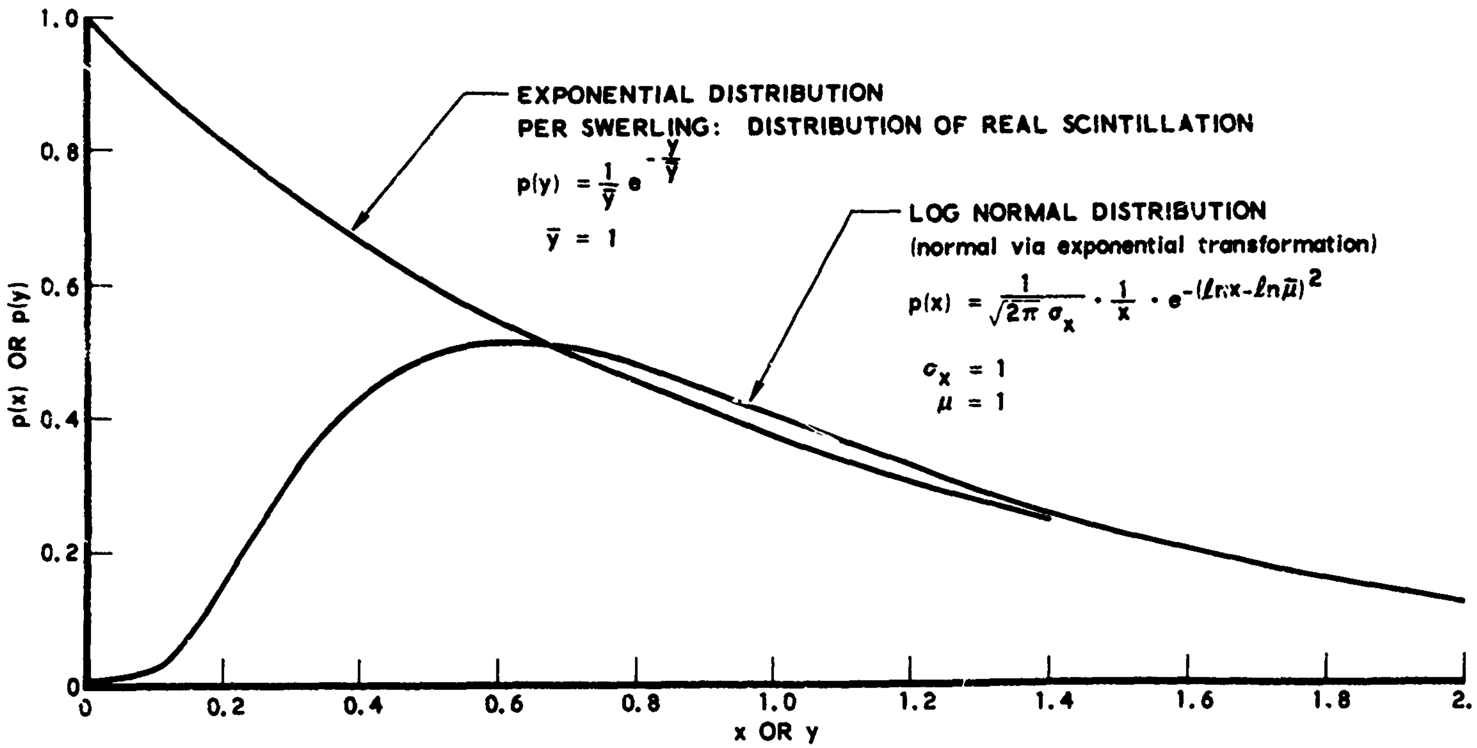

Figure 2. Actual versus Theoretical Distribution of Scintillation

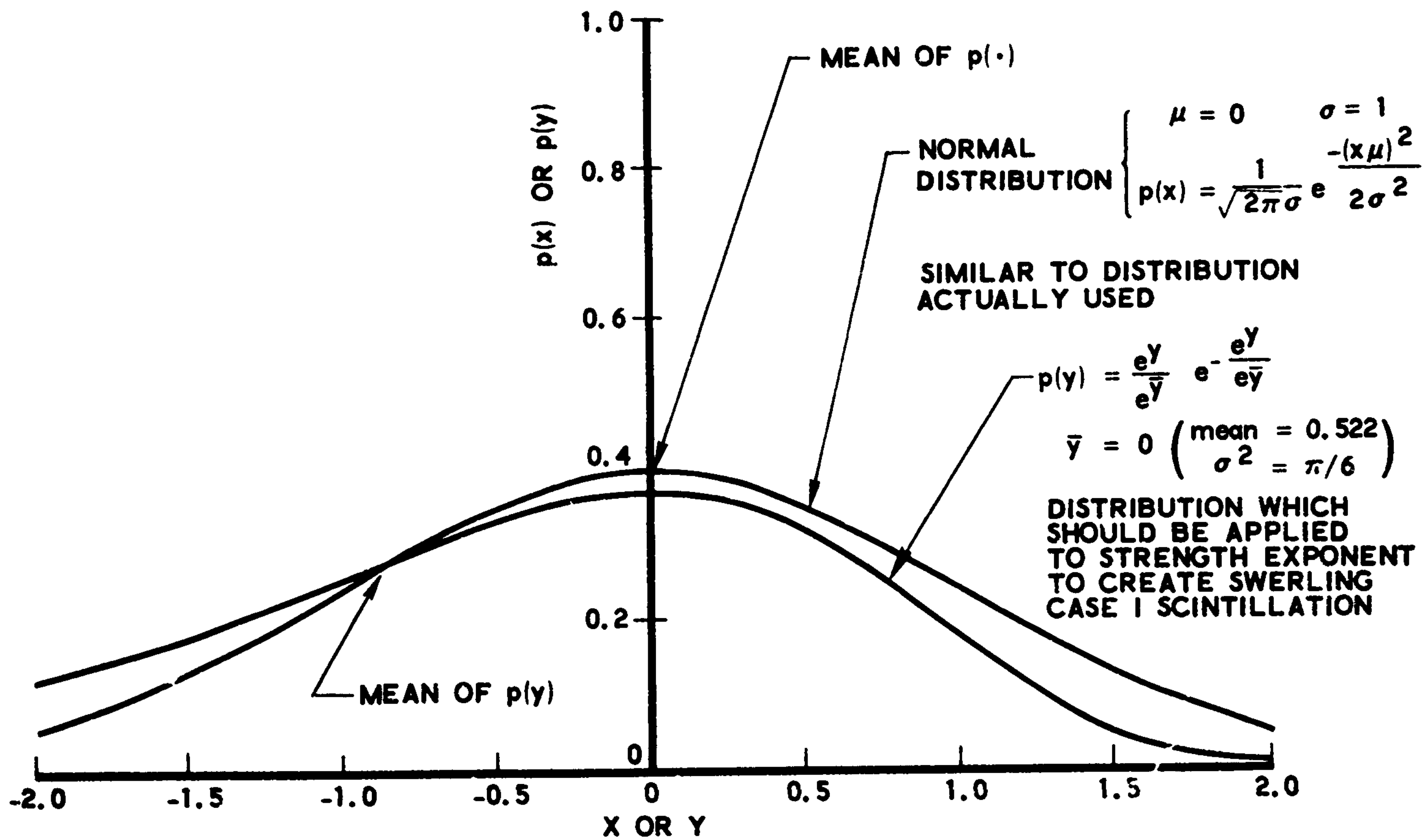

Figure 3. Distributions at Input to Produce Actual or Theoretically Correct Scintillation

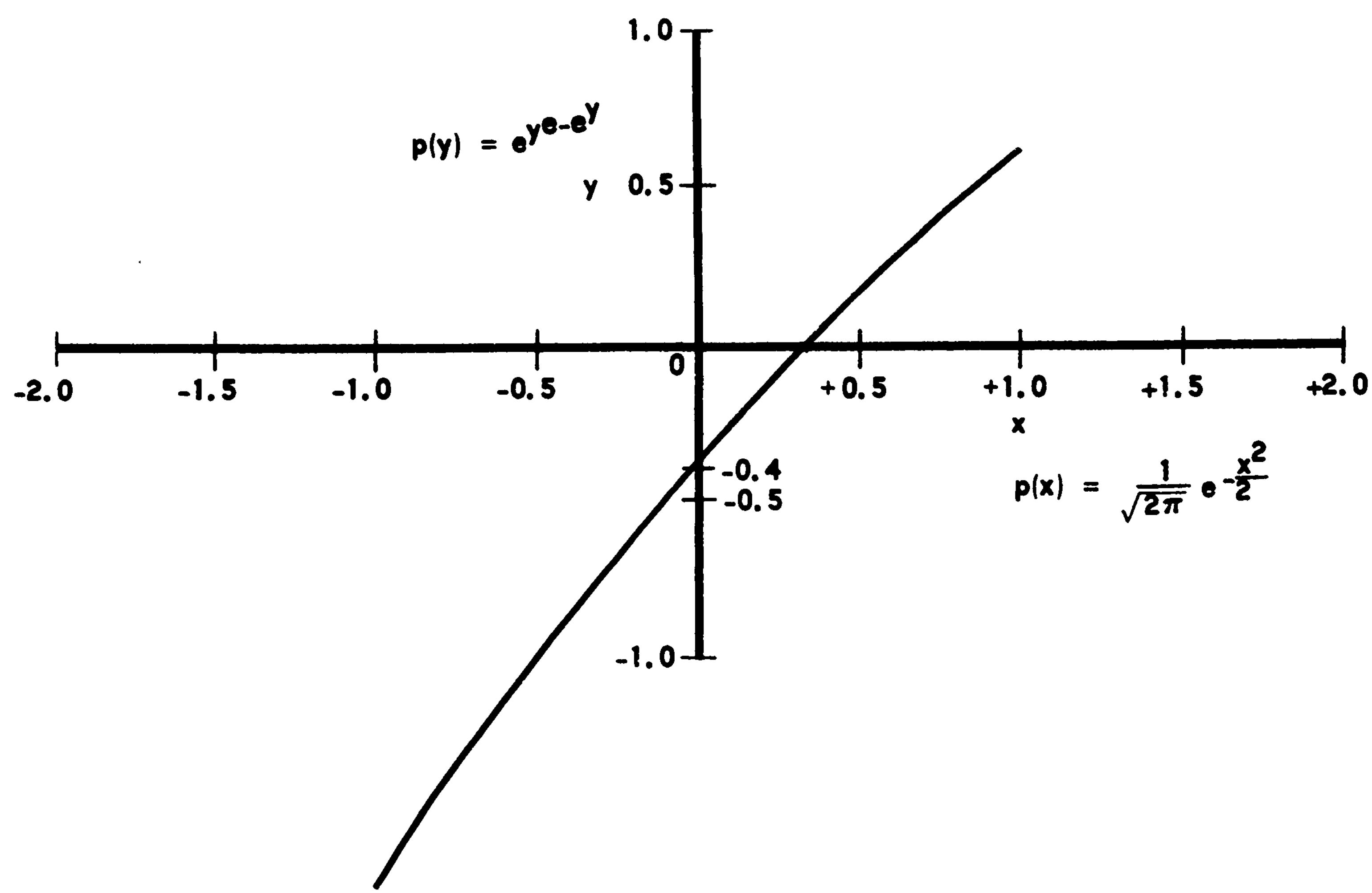

Figure 4. Transformation for Samples from p(x) to Samples from p(y)

APPENDIX A

STATISTICS OF NOISE GENERATOR

The basic random number generator is the same one used in the XDS standard FORTRAN Numerical Subroutine Package. It generates pseudo-random numbers uniformly distributed between 1 and positive full scale (X'7FFFFFFF'). Only odd numbers are generated. Treating the generated sequence R_1 as numbers scaled over 2^{31}

$$\begin{aligned} R_1 &= 1 \qquad \text{for } 0 \leq R_1 < +1 \\ &= 0 \qquad \text{elsewhere} \end{aligned} \tag{A1-1}$$

By equation (2), these are converted to a uniform distribution in the region -1 to +1:

$$\begin{aligned} R &= \frac{1}{2} \qquad \text{for } -1 \leq R < +1 \\ &= 0 \qquad \text{elsewhere} \end{aligned} \tag{A1-2}$$

Hence the mean is:

$$\mu_R = \int_{-\infty}^{+\infty} Rp(R)\, dR = \int_{-\infty}^{+\infty} \frac{1}{2} \cdot R\, dR = \left[\frac{R^2}{4}\right]_{-1}^{+1} = 0 \tag{A1-3}$$

and the variance:

$$\sigma_R^2 = \int_{-\infty}^{+\infty} p(R)(R-\mu_R)^2 dR = \int_{-\infty}^{+\infty} \frac{1}{2} \cdot R^2 dR = \frac{1}{2} \cdot \left[\frac{R^3}{3}\right]_{-1}^{+1} = \frac{1}{3}$$

APPENDIX B

STATISTICS OF SCINTILLATION SEQUENCES

Each sequence of scintillation factors is generated by the recursion relation of Eq.(3). Expanding this equation:

$$X_{0,j} = C_1KR_{0,j}$$

$$X_{1,j} = C_1KR_{1,j} + C_1C_2KR_{0,j}$$

$$X_{2,j} = C_1KR_{2,j} + C_1C_2KR_{1,j} + C_1C_2^2KR_{0,j}$$

$$X_{3,j} = C_1KR_{3,j} + C_1C_2KR_{2,j} + C_1C_2^2KR_{i,j} + C_1C_2^3KR_{0,j}$$

$$\vdots$$

$$X_{n,j} = C_1KR_{n,j} + C_1C_2KR_{n-1,j} + \cdots C_1C_2^nKR_{0,j}$$

or

$$X_{n,j} = C_1K\sum_{i=0}^{n} C_2^{n-i}R_{i,j}$$

The mean of the j^{th} sequence is obtained by summing all terms in the sequence:

$$\mu_j = \underset{n\to\infty}{\frac{1}{n}} C_1K\Big[\left(C_2^0 + C_2^1 + C_2^2 + \cdots C_2^n\right)R_{0,j} + \left(C_2^0 + C_2^1 + C_2^2 + \cdots C_2^n\right)R_{1,j} + \cdots\cdots\Big]$$

$$\mu_j = \underset{n\to\infty}{\frac{1}{n}} C_1K\left[C_2^0 + C_2^1 + \cdots C_2^n\right]\left[R_{0,j} + R_{1,j} + R_{2,j} + \cdots R_{n,j}\right]$$

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The mean of all sequences is obtained by summing across all j:

$$\mu = \lim_{\substack{n \to \infty \\ j \to \infty}} \frac{1}{n} C_1K\left[C_2^0 + C_2^1 + C_2^2 + \cdots C_2^n\right]\left[\frac{1}{j}\sum^{j} R_{0,m} + \frac{1}{j}\sum^{j} R_{1,m} + \cdots \frac{1}{j}\sum^{j} R_{n,m}\right]$$

But

$$\mu = \lim_{\substack{j \to \infty \\ n \to \infty}} \frac{1}{j}\sum_{m=0}^{j} R_{n,m} = \mu_R \text{ (the mean of the random numbers)}$$

and

$$\lim_{n \to \infty} C_2^0 + C_2^1 + \cdots C_2^n = \frac{1}{1 - C_2}$$

Thus

$$\boxed{\mu_X = \frac{C_1K}{1 - C_2}\,\mu_R}$$

However, since μ_R is zero,

$$\boxed{\mu_X = 0}$$

Similarly, we can find the variance:

$$\sigma_X^2 = E\left[(X_n - \mu_X)^2\right] = E(X_n^2) = \lim_{n \to \infty} \frac{1}{n}\sum_{i=0}^{n} X_i^2$$

If we expand a typical term, say $X_{3,j}^2$:

$$X_{3,j}^2 = \left(C_1KR_{3,j} + C_1KC_2R_{2,j} + C_1KC_2^2R_{1,j} + C_1KC_2^3R_{0,j}\right)^2$$

$$\begin{aligned}
&= C_1^2K^2 \Big[C_2^6 \left(R_{0,j}^2\right) \\
&\quad + C_2^5 \left(R_{1,j}R_{0,j} + R_{0,j}R_{1,j}\right) \\
&\quad + C_2^4 \left(R_{2,j}R_{0,j} + R_{1,j}^2 + R_{0,j} \cdot R_{2,j}\right) \\
&\quad + C_2^3 \left(R_{3,j} \cdot R_{0,j} + R_{2,j} \cdot R_{1,j} + R_{1,j} \cdot R_{2,j} + R_{0,j} \cdot R_{3,j}\right) \\
&\quad + C_2^2 \left(R_{3,j} \cdot R_{1,j} + R_{2,j}^2 + R_{1,j} \cdot R_{3,j}\right) \\
&\quad + C_2^1 \left(R_{3,j} \cdot R_{2,j} + R_{2,j} \cdot R_{3,j}\right) \\
&\quad + C_2^0 \left(R_{3,j}^2\right) \Big]
\end{aligned}$$

Summing across all j and recognizing that:

$$\begin{aligned}
E(R_{i,j} \cdot R_{m,j}) &= \sigma_x^2 \qquad i = m \\
&= 0 \qquad i \neq m
\end{aligned}$$

and that:

$$C_2^0 + C_2^2 + C_2^4 + \cdots \rightarrow \frac{1}{1 - C_2^2}$$

We find:

$$\boxed{\sigma_X^2 = \frac{C_1^2K^2}{1 - C_2^2}\sigma_X^2}$$

In an analogous manner we can find the expected value of displaced samples:

$$E(X_n \cdot X_{n-m}) = \frac{C_1^2 K^2 C_2^m}{1 - C_2^2} \sigma_R^2$$

where m is the displacement.

By definition, the correlation coefficient, ρ, is:

$$\rho(m) = \frac{E(X_n \cdot X_{n-m})}{\sigma_{X_n} \cdot \sigma_{X_{n-m}}} = \frac{E(X_n \cdot X_{n-m})}{\sigma_X^2}$$

which, in this case is:

$$\boxed{\rho(m) = C_2^m}$$

The mean and variance of he transformed sequences may be computed from Eq. (11), which is derived in the following manner. Gaussian pertubations characterized by μ_X, and σ_X^2:

$$p(X) = \frac{1}{\sqrt{2\pi}\,\sigma_X} e^{-\frac{(X-\mu_X)^2}{2\sigma_X^2}}$$

are transformed by an inverse log transformation:

$$S = e^X \quad \text{or} \quad X = \ln S$$

$$\frac{dS}{dX} = e^X = \frac{1}{S}$$

where S = transformed target strength

X = log strength = contents of target files

$$p(S) = \left.\frac{p(X)}{\left|\frac{dS}{dX}\right|}\right|_{X = \ln S} = \frac{1}{\sqrt{2\pi}\,\sigma_X} \frac{1}{S} e^{-\frac{(X-\mu_X)^2}{2\sigma_X^2}}$$

$$p(S) = \frac{1}{\sqrt{2\pi}\,\sigma_X} \cdot \frac{1}{S} \cdot e^{-\frac{(\ln S - \ln \overline{S})^2}{2\sigma_X^2}}$$

note:

$$\overline{S} = e^{\mu_X} \quad \text{or} \quad \mu_X = \ln S$$

The mean a d variance of this log-normal distribution are given in Reference 3, Section 10. 14. 2:

$$\text{mean }(S) = \mu_S = e^{\ln \overline{S}} \cdot e^{\frac{\sigma_X^2}{2}} = \overline{S}\, e^{\frac{\sigma_X^2}{2}}$$

$$\text{variance of } S = \sigma_S^2 = e^{2\ln \overline{S}} \cdot e^{\sigma_X^2}\left(e^{\sigma_X^2} - 1\right)$$

$$= \overline{S}^2 \cdot e^{\sigma_X^2}\left(e^{\sigma_X^2} - 1\right)$$

APPENDIX C

STATISTICS OF THEORETICAL DISTRIBUTION

The distribution of the sequence, which, when added to the exponent of target strength will generate theoretically correct scintillation, as derived in Section 4, is:

$$p(x) = \frac{e^{x}}{e^{\bar{x}}} e^{-\frac{e^{x}}{e^{\bar{x}}}} \tag{4-3}$$

$$\text{for } -\infty < x < +\infty$$

The mean, μ_x, and mean square, μ_2, of this distribution are:

$$\mu_x = \int_{-\infty}^{+\infty} x \frac{e^{x}}{e^{\bar{x}}} e^{-\frac{e^{x}}{e^{\bar{x}}}} dx \tag{A3-1}$$

$$\mu_2 = \int_{-\infty}^{+\infty} x^2 \frac{e^{x}}{e^{\bar{x}}} e^{-\frac{e^{x}}{e^{\bar{x}}}} dx \tag{A3-2}$$

To integrate these, we make the substitutions

$$y = e^{x} \text{ for } 0 \leq y < +\infty \tag{A3-3}$$

which implies:

$$dy = de^{x} = e^{x} dx \tag{A3-4}$$

and

$$x = \ln y \tag{A3-5}$$

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Substituting into (A3-2) and (A3-3)

$$\mu_x = \frac{1}{e^{\bar{x}}} \int_0^\infty e^{-\frac{y}{e^{\bar{x}}}} \ln y \, dy \qquad \text{(A3-6)}$$

$$\mu_2 = \frac{1}{e^{\bar{x}}} \int_0^\infty e^{-\frac{y}{e^{\bar{x}}}} (\ln x)^2 \, dy \qquad \text{(A3-7)}$$

The standard forms of these integrals are [from Ref. 4]:

$$\int_0^\infty e^{-px} \ln x \, dx = -\frac{1}{p}(C + \ln p) \qquad \text{(3.711, Ref. 3)}$$

$$\int_0^\infty e^{-px} (\ln x)^2 \, dx = C^2 + \frac{\pi^2}{6} \qquad \text{(3.714, Ref. 3)}$$

where C = Euler's constant = 0.577 ....

Using these and:

$$p = \frac{1}{e^{\bar{x}}}$$

it can be shown that:

$$\mu_x = \left(\frac{1}{e^{\bar{x}}}\right)(-e^{\bar{x}})(C + \ln \frac{1}{e^{\bar{x}}}) \qquad \text{(A3-8)}$$

$$= -(C - \ln e^{\bar{x}}) = \bar{x} - C \qquad \text{(A3-9)}$$

and

$$\mu_2 = \bar{x}^2 - 2\bar{x}C + C^2 + \frac{\pi}{6}$$

$$= (\bar{x} - C)^2 + \frac{\pi}{6} \qquad \text{(A3-10)}$$

The variance is therefore:

$$
\begin{aligned}
\sigma_x^2 &= \mu_2 - \mu_x^2 \\
&= (\bar{x}-C)^2 + \frac{\pi}{6} - (\bar{x}-C)^2 \qquad \text{(A3-11)} \\
&= \frac{\pi}{6}
\end{aligned}
$$

In other words, to generate random variations of target strength for Swerling's Case I or Case II distributions, we apply random perturbations to the exponent of target strength with the distribution of (15), mean of (A3-8), and variance of (A3-11).

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