AD A-007 056

DYNAMIC CALIBRATION FOR DELCO'S CAROUSEL VB IMU

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Prepared for:

Space and Missile Systems Operations
Air Force Weapons Laboratory

23 September 1974

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: SAMSO-TR-74-246	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER: AD/A-007056	
4. TITLE (and Subtitle): DYNAMIC CALIBRATION FOR DELCO'S CAROUSEL VB IMU	5. TYPE OF REPORT & PERIOD COVERED: Final. September 1974
	6. PERFORMING ORG. REPORT NUMBER: TR-0075(5413-02)-1
7. AUTHOR(s): A. C. Liang and J. L. LeMay	8. CONTRACT OR GRANT NUMBER(s): F04701-74-C-0075
9. PERFORMING ORGANIZATION NAME AND ADDRESS: The Aerospace Corporation, El Segundo, California 90245	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Space and Missile Systems Organization, Air Force Systems Command, Los Angeles Air Force Station, Ca. 90045	12. REPORT DATE: 23 September 1974
	13. NUMBER OF PAGES: 8
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release. Distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Filtering, IMU calibration, Carousel IMU, Optimal estimation

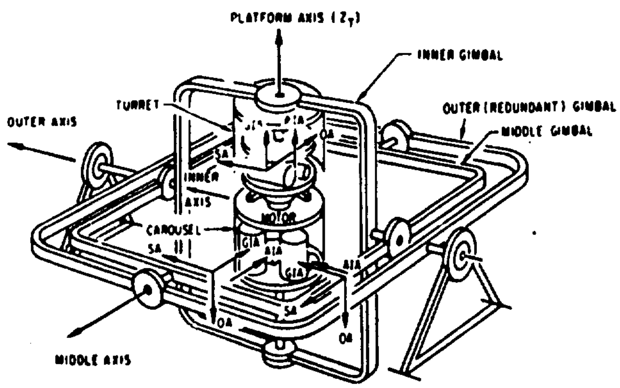
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper addresses the application of filtering theory to the problem of IMU dynamic calibration and, in particular, to the Carousel VB system. In the calibration procedure, accelerometer outputs are summed over long time intervals before processing. For this reason, the standard K-B filter needs to be modified as presented in this paper. Filter derivation and calibration results from processing real accelerometer data are presented herein.

1. INTRODUCTION

The Carousel VB IMU is an all-attitude, four-gimbal inertial platform in which two orthogonal gyros and accelerometers are mounted on a carouseling platform which rotates at 1 rpm. The third set of instruments remains inertial along the carouseling axis. For a more detailed description of the IMU, refer to Ref. (1) and Figure 1. This IMU is currently used in the T IIIC launch vehicle. The calibration process extracts optimum estimates of the IMU parameters for IMU compensation using accelerometer output data.

Because the measurement data are summed over a long time interval compared to the system dynamics update cycle, the standard extended K-B filter has to be reformulated. The filter formulation, as well as filter estimates from processing actual IMU output data, are presented.



GIA - GYRO INPUT AXIS, OA - OUTPUT AXIS, SA - SPIN AXIS
AIA - ACCELEROMETER INPUT AXIS

Figure 1. Carousel VB Gimbal and Platform Configuration

2. SYSTEM BEHAVIOR OF CAROUSEL VB IMU

2.1 COORDINATE AND TRANSFORMATION DEFINITIONS

E-N-U: An inertial orthogonal system which coincides with east-north-up at "go-inertial".

Turret: (X_T, Y_T, Z_T) Orthogonal system fixed to drifting turret (Z_T along gravity).

Platform: (X_p, Y_p, Z_p) Orthogonal system fixed to carousel platform. X-Y, gyro, and accelerometers are referenced to this coordinate system. X_p, Y_p along the ideal X, Y accelerometer and gyro input axes, Z_p along Z_T.

E: E-N-U $\longrightarrow$ Turret (3 × 3)

T: Turret $\longrightarrow$ Platform (3 × 3)

$$T = \begin{bmatrix} \cos(\theta YN) & \sin(\theta YN) & 0 \\ -\sin(\theta YN) & \cos(\theta YN) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where θYN = Carousel angle; function of time only.

Let ψ, θ, ϕ be the three Eulerian angles that define the turret axes with respect to the E-N-U system. E can be expressed as

$$E = \begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\sin\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix} \quad (2)$$

REFERENCE

(1) Final Error Analysis Report, USGS Program, EP 2291, Delco Electronics, General Motors Corp. (15 August 1972).

2.2 CAROUSEL VB MODELING

There are 29 parameters that characterize the performance of the IMU. They are gyro drifts (3), gyro unbalances (9), gyro misalignments (4), gyro scale factors (2), accelerometer biases (3), accelerometer scale factors (3), and misalignments (5).

The drift rates along the turret axes (X_T, Y_T, Z_T) are composed of applied torques, gyro drifts, and unbalance drifts. Note that all drifts are resolved into the turret system.

$$\boxed{\underline{\omega}_T \quad [T]^{-1}[\tau]\left\{\underline{R} + [TSF]\,\underline{t}_g + [U][\tau]^{-1}[T][E]\,\underline{g}\right\}} \qquad (3)$$

where

$$\underline{\omega}_T = \begin{bmatrix} \omega_{XT} \\ \omega_{YT} \\ \omega_{ZT} \end{bmatrix} = \text{platform drift rates}$$

$$\tau = \begin{bmatrix} 1 & \tau_{21} & \tau_{31} \\ \tau_{12} & 1 & \tau_{32} \\ 0 & 0 & 1 \end{bmatrix} = \text{gyro misalignment matrix}$$

and T is as defined by Eq. (1)

$$TSF = \begin{bmatrix} TSF_1 & 0 & 0 \\ 0 & TSF_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{torquer scale factors}$$

$$\underline{t}_g = \begin{bmatrix} t_{gx} \\ t_{gy} \\ 0 \end{bmatrix} = \text{applied torque (along x, y gyro input axes): functions of time only}$$

$$U = \begin{bmatrix} U_{11} & \cdots & U_{13} \\ \vdots & & \\ U_{31} & & U_{33} \end{bmatrix} = \text{gyro unbalance matrix}$$

$$\underline{R} = \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} = \text{gyro drifts}$$

T = Carousel matrix [see Eq. (1)]

E = 3×3 transformation matrix [see Eq. (2)]

$\underline{g}$ = local gravity vector in E-N-U system (function of time only due to earth rotation)

The sensed acceleration as measured by the accelerometers can be expressed as follows:

$$\boxed{\underline{Z} = \underline{b} + [K]\cdot[\gamma][T][\beta][E]\,\underline{g}} \qquad (4)$$

where

$$\underline{b} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \text{accelerometer bias}$$

$$[K] = \begin{bmatrix} K_x & 0 & 0 \\ 0 & K_y & 0 \\ 0 & 0 & K_z \end{bmatrix} = \text{reciprocal of accelerometer scale factors}$$

$$[\beta] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta_1 & \beta_2 & 1 \end{bmatrix} = \text{z-accelerometer misalignments matrix}$$

[T] = Carousel matrix [see Eq. (1)]

$$[\gamma] = \begin{bmatrix} 1 & \gamma & \alpha_1 \\ 0 & 1 & \alpha_2 \\ 0 & 0 & 1 \end{bmatrix} = \text{x, y accelerometer misalignments matrix}$$

[E] = as defined in Eq. (2)

$\underline{g}$ = gravity vector in E-N-U coordinates

The integrating accelerometer output over a fixed time interval is simply the integral of the "measured" sensed acceleration over that interval. This output is used as the "measurement" data for the filter.

The components of E are functions of the Euler angles ψ, θ, ϕ. The Euler angles are governed by the following set of differential equations:

$$\left\{\begin{aligned} \dot{\psi} &= (\omega_{XT}\sin\phi + \omega_{YT}\cos\phi)/\sin\theta \\ \dot{\theta} &= \omega_{XT}\cos\phi - \omega_{YT}\sin\phi \\ \dot{\phi} &= -(\omega_{XT}\sin\phi\cos\theta + \omega_{YT}\cos\phi\cos\theta)/(\sin\theta) + \omega_{ZT} \end{aligned}\right. \qquad (5)$$

where ω_{XT}, ω_{YT}, ω_{ZT} are as defined in Eq. (3).

3. FILTER FORMULATION

The "states" of the filter are the 29 parameters to be estimated and the Euler angles which define the orientation of the IMU (turret system). Equation (5) describes the dynamics of the attitude; the parameters are constants with white Gaussian-state noise. Equation (4) is the measurement equation for the filter. Equations (4) and (5) are then linearized for the purposes of computing filter gains.

The linearized system can be described as

$$\begin{cases} \underline{x}_n = \phi_n \underline{x}_{n-1} + \underline{u}_n \\ \underline{z}_n = H_n \underline{x}_n + \underline{v}_n \end{cases} \tag{6}$$

where

- $\underline{x}_n$ = states (32-dimensional vector containing 29 parameters and 3 attitude errors)
- $\underline{z}_n$ = measurements (3-dimensional vector)
- $\underline{u}_n$ = state noise
- $\underline{v}_n$ = measurement noise
- ϕ_n = state transition matrix over one cycle (1 sec)
- H_n = measurement matrix relating accelerometer counts (velocities) to the states at each cycle (1 sec)

with

$$E[\underline{u}_n \underline{u}_m] = \delta_{nm} Q_n$$

$$E[\underline{v}_n \underline{v}'_m] = \delta_{nm} R_n$$

Both ϕ_n, H_n are updated over 1-sec intervals.

In the factory calibration problem, the accelerometer data are processed once every 135 steps (seconds) with the accelerometers accumulating counts over the entire 135 sec. Therefore, the system model can be written as follows:

$$\begin{aligned} \underline{X}_N &= \phi^*_N \underline{X}_{N-1} + \underline{U}^*_N \\ \underline{Z}_N &= H^*_N \underline{X}_{N-1} + \underline{V}^*_N \end{aligned} \tag{7}$$

where

$$\phi^*_N = \phi_n \phi_{n-1} \cdots \phi_{n-k+1} \tag{8}$$

$$\begin{aligned} H^*_N = H_n \phi_n \phi_{n-1} \cdots \phi_{n-k+1} \\ + H_{n-1} \phi_{n-1} \cdots \phi_{n-k+1} \\ + \cdots + H_{n-k+1} \phi_{n-k+1} \end{aligned} \tag{9}$$

$$\begin{aligned} \underline{U}^*_N = \underline{n}_n + \phi_n \underline{u}_{n-1} + \cdots \\ + \phi_n \phi_{n-1} \cdots \phi_{n-k+2} \underline{u}_{n-k+1} \end{aligned}$$

$$\begin{aligned} \underline{V}^*_N = {} & H_n \underline{u}_n + (H_n \phi_n + H_{n-1}) \underline{u}_{n-1} \\ & + (H_n \phi_n \phi_{n-1} + H_{n-1} \phi_{n-1} + H_{n-2}) \underline{u}_{n-2} + \cdots \\ & + (H_n \phi_n \phi_{n-1} \cdots \phi_{n-k+1} \\ & + H_{n-1} \phi_{n-1} \phi_{n-2} \cdots \phi_{n-k+1} \\ & + H_{n-k+1}) \cdot \underline{u}_{n-k} + \underline{v}_n + \underline{v}_{n-1} + \cdots + \underline{v}_{n-k+1} \end{aligned}$$

Each N step is 135 sec, whereas each n step is 1 sec. The state and measurement noise terms $\underline{U}^*_N$ and $\underline{V}^*_N$ are no longer uncorrelated. The correlation functions become

$$\begin{aligned} M^*_N = E[\underline{U}^*_N \underline{V}^{*\prime}_N] = {} & Q_n H'_n + \phi_n Q_{n-1}(\phi'_n H'_n + H'_{n-1}) \\ & + \phi_n \phi_{n-1} Q_{n-2}(H_n \phi_n \phi_{n-1} + H_{n-1} \phi_{n-1} + H_{n-2})' \\ & + \cdots + \\ & + \phi_n \phi_{n-1} \cdots \phi_{n-k+2} Q_{n-k+1}(H_n \phi_n \cdots \phi_{n-k+2} \\ & + H_{n-1} \phi_{n-1} \cdots \phi_{n-k+2} + \cdots + H_{n-k+1})' \end{aligned} \tag{10}$$

$$\begin{aligned} Q^*_N = E[\underline{U}^*_N \underline{U}^{*\prime}_N] = {} & Q_n + \phi_n Q_{n-1} \phi'_n + \cdots \\ & + \phi_n \phi_{n-1} \phi_{n-k+2} Q_{n-k+1} \phi'_{n-k+2} \cdots \phi'_{n-1} \phi'_n \end{aligned} \tag{11}$$

$$\begin{aligned} R^*_N = E[\underline{V}^*_N \underline{V}^{*\prime}_N] = {} & R_n + R_{n-1} + \cdots + R_{n-k+1} \\ & + H_n Q_n H'_n + (H_n \phi_n + H_{n-1}) Q_{n-1}(H_n \phi_n + H_{n-1})' \\ & + \cdots + (H_n \phi_n \phi_{n-1} \cdots \phi_{n-k+2} + H_{n-1} \phi_{n-1} \cdots \\ & \quad \phi_{n-k+2} + \cdots + H_{n-k+1}) Q_{n-k+1} \\ & \quad (H_n \phi_n \phi_{n-1} \cdots \phi_{n-k+2} + \cdots + H_{n-k+1})' \end{aligned} \tag{12}$$

Let

$$\hat{\underline{X}}_{N/K} = E[\underline{X}_N | Z_1, Z_2, \ldots Z_K]$$

$$\overline{P}_N = E[(\underline{X}_N - \hat{\underline{X}}_{N/N})(\underline{X}_N - \hat{\underline{X}}_{N/N})']$$

Given the system of Eq. (2), the filtering equations can be derived as follows:

$$\hat{\underline{X}}_{N/N} = E[\underline{X}_N | Z_1, \ldots, \underline{Z}_{N-1}] + E[\underline{X}_N | \underline{Z}_N - H_N \hat{\underline{X}}_{N-1/N-1}] \qquad (13)$$

Let

$$\tilde{\underline{Z}}_N = \underline{Z}_N - H_N \hat{\underline{X}}_{N-1/N-1}$$

Then Eq. (13) can be written as

$$\hat{\underline{X}}_{N/N} = \hat{\underline{X}}_{N/N-1} + E[\underline{X}_N \tilde{\underline{Z}}'_N] E[\tilde{\underline{Z}}_N \tilde{\underline{Z}}'_N]^{-1} \tilde{\underline{Z}}_N \qquad (14)$$

But,

$$\begin{aligned} E[\underline{X}_N \tilde{\underline{Z}}'_N] &= E[(\phi^*_N \underline{X}_{N-1} + \underline{U}^*_N) \{H^*_N(\underline{X}_{N-1} - \hat{\underline{X}}_{N-1/N-1}) + \underline{V}^*_N\}'] \\ &= \phi^*_N \overline{P}_{N-1} H^{*\prime}_N + E[\underline{U}^*_N \underline{V}^{*\prime}_N] \\ &= \phi^*_N \overline{P}_{N-1} H^{*\prime}_N + M^*_N \end{aligned} \qquad (15)$$

where M^*_N is as defined in Eq. (10).

Furthermore,

$$\begin{aligned} E[\tilde{\underline{Z}}_N \tilde{\underline{Z}}'_N] &= E[\{H^*_N(\underline{X}_{N-1} - \hat{\underline{X}}_{N-1/N-1}) + \underline{V}^*_N\} \{H^*_N(\underline{X}_{N-1} - \hat{\underline{X}}_{N-1/N-1}) + \underline{V}^*_N\}'] \\ &= H^*_N \overline{P}_{N-1} H^{*\prime}_N + E[\underline{V}^*_N \underline{V}^{*\prime}_N] \\ &= H^*_N \overline{P}_{N-1} H^{*\prime}_N + R^*_N \end{aligned} \qquad (16)$$

Combining Eqs. (14), (15), and (16), the filtering equation can be written as

$$\hat{\underline{X}}_{N/N} = \phi^*_N \hat{\underline{X}}_{N-1/N-1} + K_N[\underline{Z}^*_N - H^*_N \hat{\underline{X}}_{N-1/N-1}] \qquad (17)$$

with

$$K_N = (\phi^*_N \overline{P}_{N-1} H^{*\prime}_N + M^*_N) [H^*_N \overline{P}_{N-1} H^{*\prime}_N + R^*_N]^{-1} \qquad (18)$$

K_N is the filter gain with M^*_N and R^*_N defined in Eqs. (10), and (12), respectively.

The covariance update equations become

$$\overline{P}_N = E[(\underline{X}_N - \hat{\underline{X}}_{N/N})(\underline{X}_N - \hat{\underline{X}}_{N/N})'$$

but

$$\underline{X}_N - \hat{\underline{X}}_{N/N} = \phi^*_N \underline{X}_{N-1} + \underline{U}^*_N - [\phi^*_N \hat{\underline{X}}_{N-1/N-1} + K_N\{H_N(\underline{X}_{N-1} - \hat{\underline{X}}_{N-1/N-1}) + \underline{V}^*_N\}]$$

$$\therefore \overline{P}_N = (\phi^*_N - K_N H^*_N) \overline{P}_{N-1} (\phi^*_N - K_N H^*_N)' + Q^*_N + K_N R^*_N K'_N + M^*_N + M^{*\prime}_N \qquad (19)$$

where M^*_N, Q^*_N, R^*_N are as defined in Eqs. (10), (11), and (12), respectively.

Equations (17), (18), and (19) constitute the complete set of filtering equations, keeping in mind, in Eqs. (7) through (12), that the subscripts n are 1-sec steps; the subscripts N are 135-sec steps.

4. FILTER PERFORMANCE

Plots of filter estimates of representative IMU parameters are shown in Figure 2. The data were taken on 5 June 1972 from Carousel V IMU production unit No. 2. The abcissa of the plots is in cycles, where each cycle is equivalent to 2.25 min of real time. The criteria for filter performance were the convergence of filter covariance and the measurement residual. The residuals were unbiased and of low magnitude (about 0.3% of measurement magnitude).

5. CONCLUSION

The optimum filter derived in this paper produced optimum IMU parameter estimates and showed shorter convergence time than suboptimal filters.

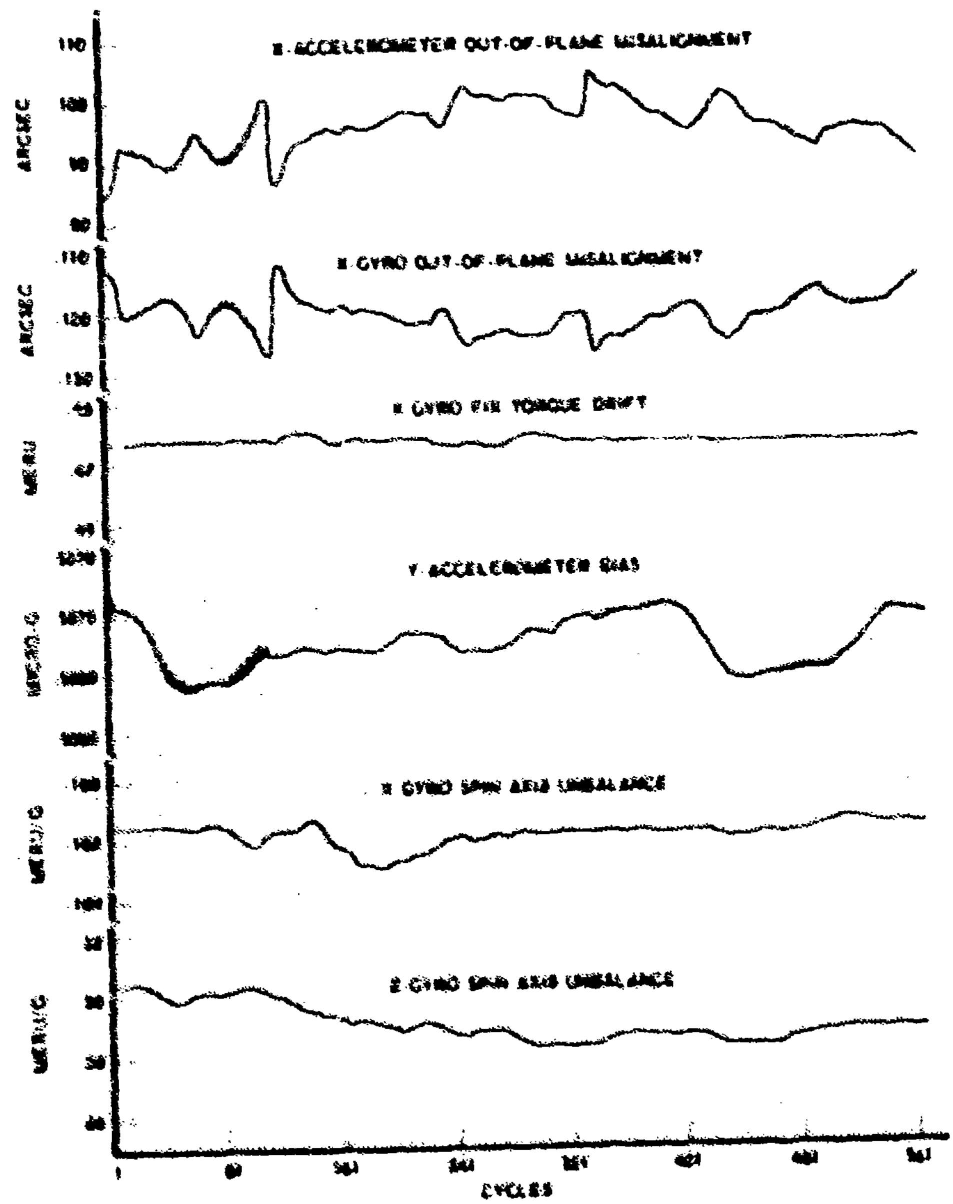


Figure 2. Filter Estimates of Selected IMU Parameters