AD/A-004 956

SURFACE EFFECTS AN ANALYSIS AND REVIEW OF THE PROCESSES INVOLVED AND PHENOMENA OBSERVED

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Prepared for:

Office of Naval Research

December 1974

AD/A-004 956

HYDRONAUTICS, Incorporated

TECHNICAL REPORT 7211-9

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By

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NR 062-472

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER T. R. 7211-9	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Surface Effects an Analysis and Review of the Processes Involved and Phenomena Observed		5. TYPE OF REPORT & PERIOD COVERED Technical 6. PERFORMING ORG. REPORT NUMBER 7211-9
7. AUTHOR(s) O. M. Phillips		8. CONTRACT OR GRANT NUMBER(s) N00014-72-C-0509 NR 062-472
9. PERFORMING ORGANIZATION NAME AND ADDRESS HYDRONAUTICS, Incorporated 7210 Pindell School Road, Howard County, Laurel, Maryland 20810		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 062-472
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Department of the Navy Arlington, Virginia 22217 (Code 438)		12. REPORT DATE December 1974 13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; its distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Wind Waves
Internal Waves
Drift Current
Wave-Current Interactions
Surface Films

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Processes involved in the generation of modulations in short wave structure by internal waves are analyzed with emphasis on field conditions. The applicability or otherwise of theoretical analyses and laboratory experiments to field situations is discussed, the problem being the variety of interacting phenomena found under natural conditions. The mean distribution

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S/N 0102-014-6601

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20. in the vertical of the horizontal current consists of a surface wind drift, generally about 3 percent of the wind speed, and below this, a current profile that is highly variable in time and in location. The variation in surface current produced by internal waves in the presence of such a mean current distribution are considered and is shown that, for a given internal wave, the variations are amplified by a surface current in the same direction as the internal wave propagation, reduced if opposed.

Direct interaction between a surface current distribution and surface waves are also, considered. Except in very unusual circumstances (very fast internal waves, surface current in the opposite direction) the wave lengths involved in direct resonance are so short that energy input from the wind will mask this process - resonance is not considered to be a significant process in generating short wave modulations. Longer, faster surface waves, whose time constant for energy input from the wind is much greater, also experience modulations in their propagation through an internal wave field, though to a considerably smaller extent. However, these modulations do influence strongly the structure that short waves can maintain against breaking. The process of wave breaking is considered and estimates are made of the modulations in short wave energy density resulting from this indirect interaction in the presence of wind.

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ABSTRACT

This report analyzes and reviews the processes involved in the generation of modulations in short wave structure by internal waves. The emphasis is on field conditions where, in general, there is wind blowing, a mean current with vertical structure, long and short waves at different directions to the internal wave and possibly surface films. The applicability or otherwise of theoretical analyses and laboratory experiments to field situations is discussed, the problem being the variety of interacting phenomena found under natural conditions. At the end of each main section of the report a summary is given of the principal conclusions of that section.

In Section 2, the mean distribution in the vertical of the horizontal current is discussed. In the top few millimeters there is a surface wind drift, generally about 3 percent of the wind speed. Below this, the current profile is highly variable in time and in location, but is essentially horizontal, with a much greater horizontal length scale than vertical scale. Response times are estimated. Section 3 considers the variation in surface current produced by internal waves in the presence of such a mean current distribution. It is shown that, for a given internal wave, the variations are amplified by a surface current in the same direction as the internal wave propagation, reduced if opposed.

The formation of surface films in the presence of internal waves, currents, winds, surface waves, etc., is discussed in Section 4 and limits are given for conditions under which saturated films should be found.

Section 5 reviews known results concerning the direct interaction between a surface current distribution and surface waves and considers their applicability to field situations. Except in very unusual circumstances (very fast internal waves, surface current in the opposite direction) the wave lengths involved in direct resonance are so short that energy input from the wind will mask this process — resonance is not considered to be a significant process in generating short wave modulations. Longer, faster surface waves, whose time constant for energy input from the wind is much greater, also experience modulations in their propagation through an internal wave field, though to a considerably smaller extent. However, these modulations do influence strongly the structure that short waves can maintain against breaking. The process of wave breaking is considered in Section 6 and estimates are made of the modulations in short wave energy density resulting from this indirect interaction in the presence of wind. The results are summarized in Section 6.4.

1. INTRODUCTION AND OUTLINE

An understanding of the processes involved in the production of surface effects by internal gravity waves or other subsurface disturbances is essential for reliable prediction, but this understanding has remained elusive. A variety of physical phenomena are potentially involved — non-resonant coupling between short surface waves and the moving disturbance pattern; resonant interactions which are more selective in their occurrence but, under idealized conditions at least, potentially stronger; indirect coupling between short waves and the moving pattern, either via longer waves or swell or via sub-surface wind-induced currents; modulations in the pattern or intensity of small scale wave breaking again either directly or enhanced by an intermediary such as surface wind drift; modulations in the density of surface material with consequent variations in energy loss rate and energy density of short waves. Under natural conditions some or all of these may occur in varying combinations; what is demonstrable in the laboratory under carefully controlled conditions and justified theoretically by a correctly argued analysis may be overwhelmed in the field by a quite different process where there is a wide range of perturbations of many kinds. Certainly some of the early observations of LaFond (1962) on the occurrence and movements of slicks pointed rather definitely to an association with internal waves, but under other sets of natural conditions, other balances obtain and the associations are no longer clear.

The aim of this report is to summarize what we know about the processes that occur near the sea surface that may be relevant to these questions and to evaluate their interplay and their combined consequences insofar as they produce an observable variation in surface properties as a result of a sub-surface disturbance. What theoretical models do we have that offer guidance to observation in the laboratory and in the field? What physical processes are responsible for the phenomena already observed in the laboratory? Which of these are relevant in the field and what are the optimum conditions for detectability in the field — which property of what provides modulations under the widest range of conditions and what is the best way of measuring it? This is the range of questions to which this report is addressed in the hope that it will place much of the excellent work already done into context and indicate directions where future work may be most fruitful.

The wind is generally blowing over the ocean. The surface drag produces a thin surface drift layer and, if it has been blowing for a substantial time, sub-surface wind-induced currents are present also. Waves exist over a continuous range of scales that can extend from capillaries to long swells; in an active wind-generated sea, waves are breaking sporadically, sometimes entraining air and forming a visible white cap but sometimes on such a small scale that no air entrainment takes place. Generally evaporation produces a cooled surface 'skin';

when wave breaking occurs this skin is locally and abruptly disrupted so that the local surface temperature, measured radiometrically, is also disrupted abruptly. Surface films, usually of organic origin, are present almost ubiquitously. Superimpose on all this a very small amplitude, large-scale disturbance generated by an internal wave — what are the consequent large scale modulations?

These phenomena will be considered in turn with what we know about their self- and mutual interactions. First is the wind-induced drift, both in the thin surface layer and on a larger scale that may extend to depths of many meters. What are the response times of these to changes in surface stress? Secondly, how do they respond to variations in the underlying current, whether steady currents or the moving orbital current patterns of internal waves? Third are the surface films, their properties and the variations in film density induced by surface motion, whether produced by surface waves or internal waves, possibly augmented by wind-drift currents. What is the back reaction on the waves themselves, either to variations in the damping of short components or to an influence of the density on the vigor of wave breaking? Fourth, we consider the possible modulations in surface wave energy density produced by near-surface current distributions, either by resonance (with maybe 'blockage') or non-resonance. What influence does the wind have on these modulations? What effect does the wind drift in the thin surface layer have on wave propagation and the occurrence of these modulations? Fifth, what about wave breaking? We show that this is influenced strongly by the thin wind-induced surface layer and that the short wave energy density that can be maintained against breaking depends on the combined influence of longer waves and the thin surface layer.

In all of these questions, the aim is to draw together the results from theory, from the laboratory and from the field to try to develop a consistent picture or to pinpoint where inconsistencies lie. The emphasis will be on the surface effects themselves, rather than on the mode of generation of the subsurface disturbance that produces them. We will assume that, by one means or another, we have a disturbance at a given depth; what are its surface manifestations? But these are enough questions, and it is time to look for answers.

2. MEAN VELOCITY STRUCTURE ABOVE THE THERMOCLINE

2.1 The Surface Drift

When the wind blows over the surface of the sea, the stress in the air or the water is communicated by the Reynolds stresses associated with the almost invariably turbulent motion. In the absence of wave breaking, however, the tangential stress across the interface must be communicated from the air to the water by molecular viscosity, since the Reynolds stress at the interface vanishes. This requires a substantial velocity gradient at the surface in both the air and the water:

$$\frac{dU}{dz} = \frac{\tau_o}{\rho\nu} \qquad [2.1]$$

where τ_o is the tangential stress at the interface and ρ and ν are the density and kinematic viscosity of either the air or water. With increasing distance on either side of the interface, the Reynolds stress $-\rho\overline{uw}$ increases rapidly and soon dominates so that the mean velocity gradient reduces rather abruptly. This viscous sublayer is well known in aerodynamics; over a smooth plate its thickness is approximately $10\nu(\rho/\tau_o)^{\frac{1}{2}}$ and the velocity at its outer edge relative to the plate is about $10(\tau_o/\rho)^{\frac{1}{2}} = 10u_*$, where u_* is the friction velocity of the air flow. Even a smooth water surface is mobile in the sense that tangential motions are possible so that the build up of the Reynolds stress above and below the air-water interface would be expected to be more rapid and the viscous sublayer consequently thinner than over a flat plate. Nevertheless on dimensional grounds, the scaling is the same; the thickness of this layer in the air is of order ν_a/u_* and the velocity scale is u_*, where ν_a is the viscosity of the air. The stress τ_o is continuous across the interface so that the velocity scale in the water motion

$$w_* = \left(\frac{\tau_o}{\rho_w}\right)^{\frac{1}{2}} = \left(\frac{\rho_a}{\rho_w}\right)^{\frac{1}{2}} u_* \simeq 3.4 \times 10^{-2} u_* \qquad [2.2]$$

where ρ_a and ρ_w represent the air and water densities respectively. The thickness of the viscous sublayer is proportional to ν_w/w_*. Relative to the speed of the interface itself, the velocity profiles in the air and the water have an anti-symmetry but with different length and velocity scales. The ratio of the length scales may be uncertain if the flow is aerodynamically rough, but the ratio of the velocity scales in water to air is simply $w_*/u_* = 3.4$ percent. Consequently, the speed q of the water at the interface relative to the water below (where the mean velocity profile is quite flat) is predicted to be about 3 percent of the air speed as usually measured at, say, 10 meters where again the mean profile is relatively flat.

In summary, then, we would expect on these grounds a thin wind-drift layer occupying the top few millimeters of the water surface across which the velocity difference is of the order 3 percent of the wind speed. The existence of this layer has only a minor influence on the propagation speed of waves whose wavelength is large compared with the layer depth (Phillips, 1973), since the dispersion relation is the result of an integral balance over the whole wave motion. However, as will be seen later, it does influence very strongly the condition for wave breaking that intimately involves the motion right at the water surface. If the waves do break, the viscous sublayer is disrupted locally and must be re-established by the surface stress. The time to accomplish this can be estimated by equating the impulse (force times time) supplied per unit area by the surface wind stress to the momentum (water density times depth times velocity) that has to be acquired by the layer. Thus,

$$\rho_a u_*^2 T_e \approx \rho_w \frac{\nu_w}{w_*} w_*$$

so that the re-establishment time

$$T_e \sim \frac{\rho_w \nu_w}{\rho_a u_*^2} \qquad [2.3]$$

For wind speeds of 10 knots or greater this characteristic time is very short, a fraction of a second, and only in very light winds (of order 2 knots) is the re-establishment time of the order 1 second or greater.

However, for small <u>changes</u> in surface stress or in u_*, the response time for readjustment may be substantially greater. If u_* increases to $u_* + \delta u_*$, the readjustment time is

$$T_r \sim \frac{\rho_w \nu_w}{2\rho_a u_* \delta u_*} \qquad [2.4]$$

or greater than [2.3] by a factor $(u_*/25u_*)$. Consequently in a non-breaking wave, when the tangential wind stress at the crests is greater by a few percent (Brooke Benjamin, 1959) than in the troughs, the response of the thin surface layer to the stress variation is negligible for waves of one second period or less, provided the wind speed is not more than about 15 or 20 knots. On the other hand, Equation [2.3] shows that if the sublayer is disrupted entirely, it is essentially re-established very quickly by the wind stress.

Measurements of the surface wind drift in the field were made some years ago by Keulegan (1951); he reported a surface drift of approximately 3.3 percent of the wind speed. Wu (1968) in a careful laboratory study measured not only the surface drift but, by using spherical floats of different sizes, was able to confirm the sharp sub-surface velocity gradient and its variation with wind speed. Figure 1 shows his measurements of dU/dz as a function of U_o^2, the square of the centerline wind speed in his tunnel, which is very closely proportional to the surface stress. Over most of the range the two are proportional, confirming that the viscous balance of Equation [2.1] is indeed relevant.* When, at the highest wind speeds, the

*Incidentally, the mean viscous stress in the wind drift layer can be estimated from the best fit line of Figure 1 to be $0.8 \times 10^{-3}\ \rho_a U_o^2$ in the absence of wave breaking, somewhat greater than Wu estimates for the total stress from his air flow measurements. Since there is also momentum flux to the waves, it should be less — the reason for this discrepancy is unknown.

waves were breaking, the mean velocity gradient was reduced primarily as a result of the sporadic patches of vigorous turbulence and the locally increased Reynolds stresses. Wu's results for the surface drift are shown in Figure 2, together with others obtained by Phillips and Banner (1974) in a different laboratory. These observations and experiments certainly confirm the existence of this wind-drift layer, both in the laboratory and the field, the magnitude of the surface drift q being about 3 to 4 percent of the wind speed as indeed the simple theory predicts.

2.2 Sub-Surface Currents

The wind also generates sub-surface currents on a larger scale of course. The vertical gradient of Reynolds stress is balanced by the horizontal acceleration of the fluid and/or by the Coriolis acceleration. In the open sea, with a gradually developing wind over a large area, the mean horizontal current distribution can approach the classical Ekman spiral, with a depth scale of the order w_*/Ω (perhaps 100 meters or so), where Ω is the vertical component of the earth's rotation. The conditions necessary for this, however, are extreme and are not generally to be expected. The vertical profile of mean horizontal current in the mixed layer over depths from 1 meter, say, to the thermocline certainly depends on the past history in space and time of winds in the vicinity, on whether or not the point of interest is near a shoreline, a current system or a detached eddy, and must be expected to be highly variable. Curiously enough, few detailed field measurements of this kind seem to have been taken except for a limited number in particular current systems like the Pacific Equatorial System (Montgomery and Stroup, 1962). Experiments such as MODE concentrate on greater depths and have sparse coverage in this range. Nevertheless, it is probably a safe assertion that there is no reason to expect that the vertical profile of horizontal current in the mixed layer is either zero, uniform or a classical Ekman spiral; we must allow for considerable variability.

Suppose, then, that there is a current distribution in the mixed layer in which the Reynolds stress gradients are balanced by the mean acceleration and by Coriolis accelerations. Suppose now that the flow field is disturbed by an internal wave or other perturbation in the neighborhood of the thermocline. Both the distribution of Reynolds stresses and of mean current will be perturbed; if the perturbation is instantaneous, the altered Reynolds stresses have no time to react back on the mean flow. But the perturbation is not instantaneous and the question arises: Under what conditions will the perturbation in Reynolds stress influence the induced perturbation in the current?

The question can be posed analytically as follows: We take a frame of reference moving with the internal wave train as in Figure 3 and define phase averaged quantities < > as averages along the y-direction, parallel to the internal wave crests. The mean velocity field defined in this way is $U(x,z)$, $V(x,z)$, $W(x,z)$ and the total velocity, including turbulent and surface wave fluctuations is $U(x,z) + u(x,y,z,t)$, etc. The upper mixed layer above the thermocline at a depth $z = -d$ is generally of almost uniform density and the time scale of the internal waves of concern are short compared with one pendulum day, so that in this region, Coriolis and buoyancy effects can reasonably be neglected. The momentum equations in the x- and z-directions are:

$$\left.\begin{aligned} U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial z} + \frac{\partial}{\partial x}\langle u^2\rangle + \frac{\partial}{\partial z}\langle uw\rangle &= -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x} \\ U\frac{\partial W}{\partial x} + W\frac{\partial W}{\partial z} + \frac{\partial}{\partial x}\langle uw\rangle + \frac{\partial}{\partial z}\langle w^2\rangle &= -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial z} \end{aligned}\right\} \qquad [2.5]$$

whence, by cross-differentiation

$$U\frac{\partial \Omega}{\partial x} + W\frac{\partial \Omega}{\partial z} = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2}\right)\langle uw\rangle - \frac{\partial^2}{\partial x\partial z}\left(\langle u^2\rangle + \langle w^2\rangle\right) \qquad [2.6]$$

where

$$\Omega = \frac{\partial U}{\partial z} - \frac{\partial W}{\partial x} \qquad [2.7]$$

is the mean vorticity in the flow. The right hand side of [2.6] represents the influence on the flow of the perturbations in Reynolds stresses that result from the internal wave induced disturbance. In the absence of the internal wave but with the horizontal current structure, $U = U(z)$, $W = 0$, $\langle uw\rangle$ = const and $\langle u^2\rangle$, $\langle w^2\rangle$ = functions of z alone; all terms vanish identically.

The Reynolds stress perturbations cannot be estimated precisely from analytical considerations. It has been argued in the context of air flow over surface waves (Lighthill, 1962; Phillips, 1968) that provided U in this frame of reference does not vanish at some level (i.e. provided the mean flow in this frame does not reverse, in which case we have a critical layer) the magnitude of these terms is negligible compared with, say, the first term of [2.6]. In this circumstance, the streamlines of the mean flow are the same as they would be if the flow were non-turbulent (though not, of course, irrotational because of the mean shear) and the variations in surface velocity could be calculated ignoring the effects of the turbulence.

Some experimental results shed light on this question. The same Equation [2.6] is relevant to the air flow over surface waves where also there is shear, turbulence and an imposed wave perturbation. Laboratory measurements of the rate of decrease with height of the wave induced fluctuations have been made by Karaki and Hsu (1968). In these measurements, there is a critical or matched layer (where the wind speed U equals the wave speed c) quite close to the water surface so that the amplitude and phase relations are distorted between the surface profile and the induced air flow disturbance. In the region above the critical layer however, Karaki and Hsu found that the (U,W) perturbations are, within experimental accuracy, in quadrature (as they are in an inviscid, non-turbulent flow). Furthermore, the decrease of the amplitude of the perturbations with height is reasonably consistent (Figure 4) with the Lighthill-Miles approximation for the perturbation stream function

$$\psi \sim (\overline{U}(z) - c)\, e^{-kz} \qquad [2.8]$$

which is valid away from the critical layer in a non-turbulent flow.

These results suggest that insofar as the wave-induced fluctuations are concerned, the turbulence maintained and generated by the mean flow has no substantial influence, at least in the air flow over waves. Is the same true for a current distribution in the upper ocean moving relative to an internal wave field supported by the thermocline below? The principal dynamical difference between the two cases concerns the ratio of two time scales — the readjustment time T_a of the turbulence to a perturbation, relative to the time that a parcel of fluid takes to move from one wave crest to the next, $\lambda/[\overline{U}(z) - c]$ where λ is the wave length. If this ratio is large in both situations, then the turbulence has little opportunity to adjust to a local disturbance and its structure is modified only slightly. If it is small, then the turbulence is approaching equilibrium with the locally disturbed mean field at all stages.

The readjustment time of the turbulence can, very plausibly, be identified with the integral time scale of the energy-containing eddies in a frame of reference moving with the mean flow — this is the appropriate measure of the "memory" of the turbulent energetic (and Reynolds stress) structure. There is now good evidence (Davies, Fisher and Barratt, 1963) that in shear flow this time scale is proportional to and of the order of $(\partial\overline{U}/\partial z)^{-1}$, the inverse of the local mean shear, so that the ratio of the turbulent adjustment time to the time of passage over one wave length is

$$\gamma = \frac{\overline{U}(z) - C}{\lambda(\partial\overline{U}/\partial z)} \qquad [2.9]$$

This ratio, of course, varies in each flow but representative values are of interest. In air flow over waves in the laboratory, such as in Karaki and Hsu's experiments, above the critical layer $\overline{U}(z) - C \sim 5$ m/sec, $\lambda \sim 0.5$ m and $\partial\overline{U}/\partial z \sim 50$ sec^{-1}, decreasing with height. With these figures, $\gamma \sim 0.2$ (increasing with height) so that in this region the turbulence is adjusting reasonably rapidly to the disturbed flow. One would therefore expect <u>larger</u> variations in Reynolds stress variations $\langle uw \rangle$, $\langle u^2 \rangle$, etc., than when $\gamma \gg 1$, but in spite of this, the wave-induced perturbations seem not to be influenced substantially (c.f. Figure 4). In the ocean, with a velocity difference U - C between the internal waves and the water near the surface of 0.5 m/sec, an internal wave wavelength of 200 m and a mean current shear of 10^{-2} sec^{-1}, $\gamma = 0.25$! Representative oceanic values of γ are apparently of the same order as laboratory values; we conclude tentatively that the perturbations to the current induced by the internal waves can be calculated neglecting the influence of the turbulence to an accuracy exemplified by the comparison of Figure 4.

2.3 Summary

The ocean above the thermocline will have mean motions on at least two distinct scales:

1. Very near the surface, in the top few millimeters, there is a wind drift of about 3-4 percent of the wind speed — laboratory and field measurements are consistent with theoretical expectations. It is disrupted locally by wave breaking, but is re-established rapidly by the wind. It responds less rapidly to small changes in wind stress and will tend to be augmented somewhat near wave crests by the additional wind stress (though, as we will see later, other dynamical wave-interaction effects are usually dominant).

2. On a larger depth scale, of the order of tens of meters, the horizontal current is likely to be variable with depth and irregular; the distortions of this current distribution produced by an internal wave do depend on the current distribution itself but are not strongly dependent on the ambient turbulence level.

3. THE VARIATIONS IN SURFACE VELOCITY RESULTING FROM INTERNAL WAVES

We are interested in the variations in the velocity distribution near the ocean surface that are associated with internal waves. However, the small scale configuration and properties of the surface are coupled to longer waves that are influenced by the variations in the phased averaged velocity to depths of, say, 10 m. Consequently, we need to estimate the variations in horizontal current near the surface (in this sense) that are induced by internal waves, but we are not here concerned with the variations at greater depths.

3.1 The Irrotational Solution

When there is no mean current, or the current is uniform with depth, the solution is elementary and well known. If d is the depth to the top of the thermocline (the bottom of the mixed layer) and a the amplitude of the wave disturbance there, then in a sinusoidal wave train (or a group with more than two or three waves) the velocity induced at the surface is in the direction of wave propagation and is of the form (Phillips, 1968, p. 167)

$$U = \frac{-an}{\sinh kd} \cos (kx-nt) \qquad [3.1]$$

when the vertical displacement at depth d is $\zeta = a \cos (kx-nt)$. The internal wave speed C depends on the wave length, the structure of the thermocline and the mode number of the internal wave, but [3.1] remains valid for the velocity in the mixed layer. For a sharp thermocline in deep water, for the lowest mode,

$$C = \frac{n}{k} = \left\{ \frac{g\,\delta\rho}{\rho} \; \frac{1}{k(1 + \coth kd)} \right\}^{\frac{1}{2}} \qquad [3.2]$$

For internal waves that are long compared with the thermocline depth d, kd << 1 and

$$C \simeq \left(\frac{\delta\rho}{\rho}\, gd \right)^{\frac{1}{2}} \qquad [3.3]$$

Characteristically, for a thermocline depth of 50 m and a density jump $\delta\rho/\rho \sim 10^{-3}$, $C \sim 0.7$ m/sec for internal wave lengths of about 200 m or more; smaller for shorter wave lengths. The magnitude of the orbital velocity differences induced by the internal wave near the surface is 2an; if the wave amplitude is 5 m then (with the figures above) this difference is only 0.2 m/sec for a 200 m internal wave, decreasing in inverse proportion as the internal wave length λ increases. The maximum strain rate near the surface is $ank = 3 \times 10^{-3}$ sec^{-1} for a 200 m wave; this is proportional to λ^{-2}.

Fluid velocities associated with surface wind-generated waves are usually larger than the value calculated above and (on account of the smaller scales of surface waves) the strain rates are <u>very</u> much larger. Clearly, the motions induced by the internal waves are generally a very small perturbation (but of relatively large scale) on an active and noisy background; the possibility of detecting the signal depends on (a) whether the velocity variations induced near the surface by the internal wave can, under certain natural conditions, be augmented and (b) the existence of one or more indicators (or dynamical processes) that are sufficiently responsive to these variations.

3.2 Interactions with Surface Currents

The expression [3.1] holds only in the absence of any vertical variation in the mean horizontal current. When there is structure in the current, the surface velocity variations produced by internal waves can be modified substantially as the following analysis demonstrates.

In Section 2 of this work, it was shown that the wave induced perturbations appear to be substantially unaffected by the turbulence provided there is no critical layer where the component of the mean current velocity in the direction of propagation of the internal wave is equal to the internal wave speed. Under these circumstances, [2.6] reduces to

$$U \frac{\partial \Omega}{\partial x} + W \frac{\partial \Omega}{\partial z} = 0 \qquad [3.4]$$

for the component of phase averaged vorticity (the mean plus the wave-induced variation) normal to the direction of wave propagation. Since $\Omega = \partial U/\partial z - \partial W/\partial x$, this equation is

$$U \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial z} - \frac{\partial W}{\partial x}\right) + W \frac{\partial}{\partial z}\left(\frac{\partial U}{\partial z} - \frac{\partial W}{\partial x}\right) = 0$$

or, after a little manipulation and use of the incompressibility condition $\partial U/\partial x + \partial W/\partial z = 0$ (note $\partial/\partial y = 0$),

$$\frac{\partial}{\partial z}\ U \frac{\partial U}{\partial x} + \frac{\partial}{\partial z}\left\{W\left(\frac{\partial U}{\partial z} - \frac{\partial W}{\partial x}\right)\right\} - \frac{\partial}{\partial x}\left(U \frac{\partial W}{\partial x}\right) = 0 \qquad [3.5]$$

Integrate this equation vertically from the top of the disturbed thermocline at $z = -d(x)$ to the surface:

$$\frac{\partial}{\partial x}\tfrac{1}{2}U^2\Big|_0 - \frac{\partial}{\partial x}\tfrac{1}{2}U^2\Big|_{-d} - W\left(\frac{\partial U}{\partial z} - \frac{\partial W}{\partial x}\right)\Big|_{-d} - \int_{-d}^{0}\frac{\partial}{\partial x}\left(U\frac{\partial W}{\partial x}\right)dz = 0 \qquad [3.6]$$

and compare the order of magnitude of these terms.

In the frame of reference moving with the waves, the mean velocity U is of the order C in magnitude and the variations in it are of order Ca/λ. By the incompressibility condition, for internal waves appreciably longer than the thermocline depth d, $W \sim C\ (a/\lambda)(d/\lambda)$. Now the third term contains the quantity dU/dz which is either of order $Ca/\lambda d$ or dominated by the ambient mean shear at the depth -d. In order to refer the mean surface current to the current speed (if any) at the thermocline, we will need to choose d at a level where the mean shear is small so that the difference is clearly defined; the second alternative is therefore excluded. Consequently the orders of magnitude are:

		Relative Order
1st two terms	C^2a/λ^2	1
Third term	$C^2(a/\lambda)^2(d/\lambda)^2 . 1/\lambda$	$(a/\lambda)(d/\lambda)^2$
Fourth term	$C^2(a/\lambda)(d/\lambda)^2\ 1/\lambda$	$(d/\lambda)^2$

Now $a/\lambda << 1$ (generally less than 0.2) and in many cases of interest $d/\lambda < 0.3$ approximately. In the long wave length limit as $(d/\lambda)^2 \to 0$, Equation [3.6] reduces simply to

$$\frac{\partial}{\partial x}\tfrac{1}{2}U^2\Big|_o - \frac{\partial}{\partial x}\tfrac{1}{2}U^2\Big|_{-d} = 0$$

or

$$U_o^2 - U_{-d}^2 = \text{const w.r.t. } x. \qquad [3.7]$$

It can also be shown, by taking the component of the vorticity equation in the direction of wave propagation that the surface current transverse to the direction of wave propagation is unaffected by the internal waves:

$$v_o = \text{const.} \qquad [3.8]$$

The detailed calculation can be extracted from Phillips and Banner (1974).

Note that the expression [3.7] is valid only in the long wave length domain, when $(\lambda/d)^2 >> 1$. The case of shorter wave lengths can be considered numerically by integration of Equation [3.4] but this work is not yet complete. Note also that [3.7], relating the variations in surface current to the velocities at the thermocline depth, does not involve the details of the current structure in between. It is an integrated form of the vorticity conservation equation; to this order in (λ/d) the vorticity is simply the mean shear whose integral is the velocity difference across the layer.

3.3 Surface Current Modulations

The velocities U_o and U_{-d} are taken in a frame of reference moving with the internal wave, so that if $u(x)$ represents the orbital velocity at the thermocline and $U_s(x)$ the surface current (relative to the datum against which C is measured), then [3.7] becomes

$$[U_s(x) - C]^2 - [u(x) - C]^2 = \text{const} \qquad [3.9]$$

Now in a periodic internal wave, $u(x) = 0$ at the phase of the wave where the thermocline displacement vanishes and if U_s is the surface current at this phase point, then

$$[U_s(x) - C]^2 - [u(x) - C]^2 = [U_s - C]^2 - C^2$$

and

$$U_s(x) = C - [(C-u(x))^2 - U_s\ (2C - U_s)]^{\frac{1}{2}} \qquad [3.10]$$

the negative sign being chosen since $U_s - C < 0$ when there is no critical layer. Note that when the surface current vanishes, $U_s = 0$ and [3.10] reduces to $U_s(x) = u(x)$, the long wave length limit for internal waves in an irrotational flow. Since

$u = -(C\,a/d)\cos\chi$ for long internal waves, where χ is the phase of the wave, $\chi = 0$ corresponding to an internal wave crest, [3.10] can be written

$$\frac{U_s(x)}{C} = 1 - [(1 + (a/d)\cos\chi)^2 - r(2-r)]^{\frac{1}{2}} \qquad [3.11]$$

where $r = U_s/C$. If U_s and therefore r are positive, i.e. when the mean surface current has a component in the same direction as that of the internal wave propagation, the modulations in surface current are greater than in a purely irrotational wave. $U_s(x)$ is greatest (in the positive x-direction) when $\cos\chi = -1$, above the internal wave troughs.

Figure 5 shows some representative distributions of surface current with respect to phase of the internal wave for various values of r when $(a/d) = 0.1$. Note that the amplitude of the modulations in surface current increases for a fixed a/d as the mean surface current increases; when $r = U_s/C = 0.5641$ (for $a/d = 0.1$) the magnitude of the surface current above the internal wave trough becomes equal to the phase speed of the internal wave. For larger values of U_s/C the water here is in fact surging ahead faster than the wave and, in a frame of reference moving with the wave, there is a closed eddy in the mean flow near the surface. The detailed calculation becomes unreliable under these circumstances (Reynolds stress variations being important) but the pattern of mean streamlines becomes qualitatively as in Figure 6. When U_s/C is substantially greater than 1, there is a submerged critical layer, with energy exchange between the current and the internal waves.

The magnitude of the modulations in surface current as functions of r and a/d is shown in Figure 7; the amplifying effect of a surface current moving with the internal waves is evident. The curves for various values of a/d terminate at the point where the velocity of the water at the surface above the internal wave trough is equal to the phase speed of the wave. As Figure 5 shows, the forward surging motion is both more localized and more intense than the backwards surge over the internal wave crests.

Similar information is given in Figure 8. Here, for example, an internal wave with amplitude a = 0.075 d in a surface current with mean speed 0.48 C has surface current modulations twice that of an irrotational wave with the same amplitude and speed, the velocity difference at the surface being 0.3 C between points over the internal wave trough and crest. Points that lie in the upper right, outside the contours, identify conditions in which a closed eddy exists at the surface with fluid over the trough surging ahead faster than the wave profile.

When the surface current is inclined to the direction of propagation of the internal wave, the component of the current normal to the propagation direction is not affected (Phillips and Banner, 1974). In a frame of reference moving with the waves, the pattern of surface streamlines is as shown in the upper part of Figure 9 for the case $(U_s)_x/C = 0.5$, $a/d = 0.1$

with an angle of 45° between the surface current and the propagation direction of the waves. When $(U_s)_x/C$ becomes larger (or the wave amplitude a/d increases) the flow reverses in the detached eddy and as indicated qualitatively in Figure 9b, a divergence line and a convergence line appears. The latter (which will be seen later is significant for slick formation) lies between an internal wave crest and the following trough. As the x-component of U_s increases, the two lines separate towards adjacent crests until, at a greater value of $U_s/C > 1$, they coalesce at the crest and disappear — at all phases of the internal wave, the surface current is then overtaking the wave. Figure 10 illustrates the various domains of behavior; the right-hand branch being rather unreliable since a critical layer is then developing and the essentially non-turbulent calculation here becomes unreliable.

3.4 Summary

1. The modulations in surface current produced by internal waves are amplified when a mean surface current is moving in the direction of internal wave propagation, suppressed when moving against.

2. The amplification ratio becomes large when the component of mean current speed along C, U_s, is close to the propagation speed C of the internal waves and the internal wave slope is small.

3. When U_s/C is near unity, the surface current reverses direction relative to the internal wave pattern, even for internal waves of small slope. This forms convergence lines between a crest and the following trough for slicks and other surface material. Above the internal wave trough, the water near the surface moves faster than the internal wave pattern.

4. These effects are strongest when the internal wave length is much greater than the thermocline depth. Calculations for cases when the two are comparable are presently being done.

4. SURFACE FILMS

4.1 General Properties

Surface films are very widespread on the ocean surface, whether as dispersed or saturated monolayers or thicker films, and seem generally to be of biological origin. Anyone who has worked with precise wave experiments in the laboratory can attest to the difficulty of getting rid of surface films, even on "clean" chlorinated and filtered water. A general account of the phenomena of surface films is given by Davies and Rideal (1963). The thickness of a monolayer is of the order 10^{-7} cm; it has a surface viscosity of 10^{-3} to 1 c.g.s. units (gm sec^{-1}) and its compressibility is very non-linear with film density.

Conservation equations can be written for the surface film as follows. If $\underset{\sim}{v}$ is the film velocity over the surface, γ the film density (mass/area), then by mass conservation

$$\frac{\partial \gamma}{\partial t} + \nabla \cdot (\gamma \underset{\sim}{v}) = 0 \qquad [4.1]$$

or

$$\frac{d\gamma}{dt} + \gamma \, \nabla \cdot \underset{\sim}{v} = 0 \qquad [4.2]$$

where the divergence is, of course, on the surface. The momentum equation for the film is

$$\frac{d}{dt} \quad (\gamma \, \underset{\sim}{v}) = - \nabla p_f + \mu \nabla^2 \underset{\sim}{v} + \underset{\sim}{F} \qquad [4.3]$$

where $p_f = p_f(\gamma)$ is the film pressure, μ the surface viscosity and F the drag force per unit area resulting from (a) the wind stress at the top and (b) the lower stress from any relative motion of the film in the underlying fluid. Since F may be of the order 1 dyne/cm² while $\gamma \sim 10^{-7}$ gm cm² the acceleration terms in [4.3] are negligible and the stress balance is, in essence, static:

$$-\frac{\partial p_f}{\partial \gamma} \nabla\gamma + \mu\nabla^2 \underset{\sim}{v} + \underset{\sim}{F} = 0 \qquad [4.4]$$

The quantity $\partial p_f/\partial\gamma$ is related to the two-dimensional compressibility modulus c_s^{-1} (Davies and Rideal, p. 265)

$$c_s^{-1} = \gamma \, \partial p_f/\partial\gamma \qquad [4.5]$$

For a very disperse film, $c_s^{-1} \rightarrow p_f \rightarrow 0$ and $\mu \rightarrow 0$ as $\gamma \rightarrow 0$. For tightly packed monolayers, $c_s^{-1} \sim 10^2$ dynes cm^{-1} for liquid films and $c_s^{-1} \sim 10^3$ for fatty acids. Other figures and experimental results are given in the book cited.

In the ocean, dispersed films are not usually visible to the eye while concentrated films can be seen generally by their influence on short wave breaking (as will be discussed later). Films showing interference coloring are much thicker than monolayers -- generally oil spills.

4.2 Influence of Short Waves on Average Film Density

The most vigorous motions at the ocean surface result from surface waves, but the length scale of this motion is small compared with internal wave lengths of interest. Accordingly, let $\underset{\sim}{V}(x,t)$ represent the film velocity averaged over several wave lengths of the short surface wave field, and $\underset{\sim}{v}'(x,t)$ represent the fluctuations about this average produced by small scale surface waves. Similarly, let $\Gamma(x,t)$ and $\gamma'(x,t)$ represent the mean and fluctuating parts of the film density.

Equation [4.1], when locally averaged in this way, becomes

$$\frac{\partial \Gamma}{\partial t} + \nabla \cdot (\Gamma \underset{\sim}{V} + \overline{\gamma' v'}) = 0 \qquad [4.6]$$

Since the root mean square wave-induced velocity $(\overline{v^2})^{\frac{1}{2}}$ is generally large compared with V, the influence of the second term on the mean film density must be estimated.

Consider a wave moving across the surface where there is a diffuse film of density γ_o. If the wave profile is

$$\zeta = a \cos (\underset{\sim}{k} \cdot \underset{\sim}{x} - nt)$$

then the surface velocity in deep water is

$$\underset{\sim}{v} = \ell \, an \cos (\underset{\sim}{k} \cdot \underset{\sim}{x} - nt) \qquad [4.7]$$

where ℓ is a unit vector in the direction of k and the frequency $n = (gk)^{\frac{1}{2}}$. In a frame of reference moving with speed $c = \ell(g/k)^{\frac{1}{2}}$ of the wave

$$v = an \cos kx - c \qquad [4.8]$$

where the x-direction is taken in the direction of wave travel. The conservation Equation [4.1] becomes

$$\frac{\partial}{\partial x}\left\{\gamma[an \cos kx - c]\right\} = 0$$

since in this frame of reference, the motion is steady. Thus

$$\gamma\,[an \cos kx - c] = \text{const} = -\Gamma_o c \qquad [4.9]$$

Now for surface waves $c \gg an$, so that

$$\gamma = \Gamma_o\left\{1 - \frac{an}{c}\cos kx\right\}^{-1} \approx \Gamma_o\left\{1 + \frac{an}{c}\cos kx + 0\left(\frac{an}{c}\right)^2\right\}$$

and the convariance between the fluctuations γ' and v'

$$\overline{\gamma' v'} = \tfrac{1}{2}\Gamma_o a^2 n^2/c \qquad [4.10]$$

With a spectrum of short waves $\Phi(n)$, essentially uni-directional

$$\overline{\gamma' v'} = \frac{\Gamma_o}{g}\int n^3 \Phi(n)\,dn \qquad [4.11]$$

$$= \Gamma_o V_w \text{ , say} \qquad [4.12]$$

where V_w is the effective surface film flux velocity induced by the waves — in fact, the Stokes drift. In a saturated spectrum, $\Phi(n) = \beta g^2 n^{-5}$, $n > n_o$,

$$V_w \approx \beta g / n_o \qquad [4.13]$$

where n_o (rad/sec) is the frequency of the spectral peak and $\beta \sim 1.1 \times 10^{-2}$. The drift velocity V_w is thus approximately 1 percent of the speed of the highest waves. The mean local film density Γ is therefore governed by

$$\frac{\partial \Gamma}{\partial t} + \nabla \cdot \left\{ \Gamma(\underset{\sim}{V} + \underset{\sim}{V}_w) \right\} = 0 \qquad [4.14]$$

4.3 Variations in Average Film Density Produced by Internal Waves

In a dispersed film, c_s^{-1} and μ are both small and [4.4] reduces to $\underset{\sim}{F} = 0$; the local wind stress at the upper surface of the film balances the drag of the water below. The film simply moves with the free surface at a velocity that is the vector sum of the 'surface' current $\underset{\sim}{U}_s$ discussed in Section 3, the Stokes drift $\underset{\sim}{V}_w$ and microscale surface wind drift $\underset{\sim}{q}$ the last of which is in the direction of the wind stress with magnitude approximately 3 or 4 percent of the wind speed U_o. The conservation equation for the mean film density [4.14] is therefore

$$\frac{\partial \Gamma}{\partial t} + \nabla \cdot [\Gamma(\underset{\sim}{U}_s + \underset{\sim}{V}_w + \underset{\sim}{q})] = 0 \qquad [4.15]$$

Take coordinates with x in the direction of wave propagation and moving with the speed C of the internal waves, in which frame [4.15] becomes

$$\frac{\partial \Gamma}{\partial t} + \frac{\partial}{\partial x}\left[\Gamma(U_s + q + V_w - C)_x\right] = 0 \qquad [4.16]$$

involving the components of the velocities in the x-direction. In a very long internal wave train a steady state may be set up in which

$$\frac{\partial}{\partial x}\left[\Gamma(U_s + q + V_w - C)_x\right] = 0 \qquad [4.17]$$

so that the average local film density varies inversely with the x-component of the surface velocity including the wind drift observed as we move with the internal wave. If at some phase of the wave

$$U_s(x) + q + V_w - C \rightarrow 0 \qquad [4.18]$$

the solution fails; $\Gamma \rightarrow \infty$ and the film is clearly no longer diffuse.

The development of patches of high film concentration when a pulse of internal waves enters a region of diffuse film can be shown simply. From [4.16]

$$\frac{1}{\Gamma}\frac{\partial \Gamma}{\partial t} + (U_s - C + q + V_w)\frac{1}{\Gamma}\frac{\partial \Gamma}{\partial x} = -\frac{\partial U_s(x)}{\partial x} \qquad [4.19]$$

since C and q are constant; we neglect the variation in V_w on the ground that it is dominated by the longest waves (c.f. 4.13). Thus

$$\frac{D}{Dt} \ln \frac{\Gamma}{\Gamma_o} = - \frac{\partial U_s}{\partial x} \qquad [4.20]$$

where D/Dt represents the derivative following the motion of the film and Γ_o is the initial film density. If at some phase of the wave [4.17] is satisfied, a convergence line exists in which $\partial U_s/\partial x < 0$; material points of the film will move towards this line and (from 4.20) the film density increases exponentially

$$\Gamma(t) \rightarrow \Gamma_o \exp \int^t s(t)dt$$

where s(t) is the convergence $- \partial U/\partial x$ experienced by the material point. As t increases, $s(t) \rightarrow s(0)$, the convergence <u>at</u> the convergence line and

$$\Gamma(t) \rightarrow \Gamma_o \exp [s(0)t]. \qquad [4.21]$$

The film density therefore increases exponentially with time until it saturates, even if the initial film is very disperse — the lower the initial film density the longer the film will take to saturate and the narrower the ultimate saturated region, except at the leading edge of an internal wave group where material will continue to be collected.

The development of concentrated slicks is therefore dependent on [4.18] being satisfied at some phase of the internal wave. The condition is illustrated in Figure 11. If $V_w + q = 0$ (say if the wind is blowing along the crests of the internal waves) slicks are formed for values of a/d and U_s/C that correspond to points inside the bowl-shaped curve. If there is no mean surface current component in the direction of wave travel, the amplification effect on the surface speed modulations disappears and slicks are formed only inside the V-shaped region shown. In any event, for slicks to be associated with internal waves, it is necessary that the vector sum of the average current, the wind drift (about 3 percent of the wind velocity) and the wave drift (about 1 percent of the speed of the highest waves) have a component in the direction of internal wave travel that is of the same order as (though not necessarily close to) the speed of the internal waves. The range of conditions under which they will be seen increases with the ratio of internal wave amplitude to thermocline depth.

If the average density of surface film is very low, the material gathered at the convergence lines will lie in a thin strip and may be dispersed again by breaking waves incident on this region. The influence of saturated or nearly saturated slicks on short wave breaking will be considered in Section 6 of this report. Their influence on the damping of unbroken waves is well known (see, for example, Phillips, 1968, p. 37). The suppression of short wave breaking together with the increased attenuation of capillary waves gives the surface of an extended saturated slick a glassy (or oily!) appearance. LaFond's (1962) early observations on the associations between internal waves and moving slicks were concerned with extended saturated slicks, easily visible by eye. The associations between the two phenomena are clear in his observations, though his simple explanation is untenable.

If the condition [4.18] is not satisfied, that is, if the oceanic conditions are represented by points outside the domains of Figure 11, variations in film density will be generated by internal waves, but whether or not a slick is formed depends upon the average ambient density of the surface film. For, from [4.17] the local mean film density $\Gamma(x)$ is given by

$$\Gamma(x)\ [U_x(x) + q + V_w - C] = \overline{\Gamma}\ (\overline{U}_s + q + V_w - C] \qquad [4.22]$$

If $\overline{U}_s + q + V_w - C > 0$, the maximum film density will occur when U_s is least, above the internal wave crest (see Figure 5). On the other hand, if $\overline{U}_s + q + V_w - C < 0$, the internal wave is overtaking the surface material everywhere and the maximum film density will occur where U_s is greatest, above an internal wave trough. We see later that if the local mean surface film is unsaturated (not tightly packed, but fairly close to it) variations in mean film density can have a significant effect on wave breaking.

4.4 Summary

1. Internal waves modulate the density of naturally occurring surface films; this in turn can influence the breaking of small-scale waves (see Section 6.2) even when the average film density is such that no slick is visible.

2. Visible slicks are formed when the vector sum of the surface current, the wind drift (about 3 percent of the wind velocity) and the wave drift about 1 percent of the speed of the highest waves) has a component in the direction of wave travel that is of the same order as (though not necessarily very close to) the speed of the internal wave. The range of conditions under which visible slicks will be seen increases with the ratio of internal wave amplitude to thermocline depth. The convergence line associated with the slick lies between an internal wave crest and the following through, if $\overline{U}_s + q + V_w < C$; between a trough and the following crest if $\overline{U}_s + q - V_w > C$.

5. ENERGY MODULATIONS IN A SURFACE WAVE TRAIN PRODUCED BY DIRECT INTERACTION

The direct interaction between surface waves and much longer internal waves produces modulations in energy density of the surface waves that can certainly be observed under ideal conditions in the laboratory. Whether or not modulations are generated in the field in this way depends on whether other effects with a shorter time scale than the surface-internal wave interaction will not, or will overwhelm this process. There are two ways of considering the dynamics of the simple direct interaction; they are complimentary and each useful in appropriate circumstances. It can be considered from the point of view of a wave train on a slowly varying current or from the viewpoint of resonant interactions among internal and surface waves.

5.1 Waves on a Slowly Varying Current

The energy density E of a wave train in a slowly varying current U(x,t) is specified by the equation

$$\frac{\partial E}{\partial t} + \nabla \cdot \left\{ E(\underset{\sim}{U} + \underset{\sim}{c}_g) \right\} + \underset{\approx}{S} \cdot \nabla \underset{\sim}{U} = 0 \qquad [5.1]$$

where $\underset{\sim}{c}_g$ is the local group velocity and S the radiation stress tensor (integrated Reynolds stress), which in deep water takes the form

$$\underset{\approx}{S} = \begin{pmatrix} \frac{1}{2}E & 0 \\ 0 & 0 \end{pmatrix} \qquad [5.2]$$

where the 1-direction is that of the local wave propagation. The pattern of wave-numbers is given by the kinematic equation

$$\frac{\partial \underset{\sim}{k}}{\partial t} + \nabla\,(\sigma + \underset{\sim}{k} \cdot \underset{\sim}{U}) = 0 \qquad [5.3]$$

where $\sigma = \sigma(k)$ is the intrinsic frequency; for surface waves $\sigma = (gk)^{\frac{1}{2}}$. The derivation of these equations is given by Phillips (1968).

When a pattern of internal waves (either a continuous train or a steady pulse) is propagating with speed C beneath the surface, a distribution of surface current of the form $U_s(x-Ct)$ is set up (with mean $\overline{U}_s$) and this interacts with the surface waves. In the absence of effects such as wind input, such as wave breaking or attenuation (which will be considered later) the distribution of surface wave energy can be found by integration of [5.1] and [5.3]. In some circumstances, if we move with the internal waves, a steady state disturbance of surface wave energy is established and solutions for $\overline{U}_s = 0$, no surface current, have been given by Phillips (1971) and Gargett and Hughes (1972). These results are generalized below. When the internal and surface waves are collinear, the distribution of local surface wave intrinsic speed $c = (g/k)^{\frac{1}{2}}$ is given by the solution to the quadratic

$$\frac{c^2}{c_o} = \frac{c + U_s(x) - C}{c_o + \overline{U}_s - C} \qquad [5.4]$$

where c_o is the speed at the point where $U_s = \overline{U}_s$. Consequently

$$\frac{c(x)}{c_o} = \frac{c_o + \{c_o^2 - 4(U_s(x) - C)(c_o + \overline{U}_s - C)\}^{\frac{1}{2}}}{2(c_o + \overline{U}_s - C)} \qquad [5.5]$$

The alternative root is eliminated since as $U_s \to \overline{U}_s$, $c \to c_o$. The distribution of energy density in the surface wave is

$$\frac{E(x)}{E_o} = \frac{c_o\ (\overline{U}_s - C + \frac{1}{2}c_o)}{c(x)\ (U_s(x) - C + \frac{1}{2}c(x))} \qquad [5.6]$$

Note the singularity in this solution if

$$U_s(x) + \tfrac{1}{2}c(x) - C = 0 \qquad [5.7]$$

when the energy flux velocity $U_s + (1/2)c$ of the surface wave is just equal to the phase speed of the internal waves. At this point the surface wave speed can be shown (Phillips, 1971) to become complex and waves can no longer propagate past this point. This is the phenomenon of blockage — surface wave energy tends to accumulate at particular phase points of the internal wave, and the way that it accumulates can be described by an unsteady analysis based on [5.1]. The modulations in surface velocity required to produce blockage are illustrated in Figure 12. Note that, at the points where the curve touches the horizontal axis, a very small perturbation in a uniform current will produce "blockage" — this is the phenomenon of resonance with internal and surface waves of small slope, described by Lewis, Lake and Ko (1974) for $U_s = 0$ (no mean surface current). The condition for resonance is that

$$\overline{U}_s + \tfrac{1}{2}c_o - C = 0 \qquad [5.8]$$

For a surface current in the direction of travel of the wave, resonance occurs at a shorter wave length than in the absence of a current; if it is in the opposite direction, resonant surface waves are longer. It will be seen later that at short wave lengths, about 15 centimeters or less, resonance or blockage in the presence of wind under natural conditions is probably not a significant dynamical process being overwhelmed by other effects such as wind energy input and wave breaking.

5.2 Non-Resonance Modulations in Longer Waves

Longer waves or swell that move considerably faster than the internal waves may not suffer blockage or undergo resonance, but they are modulated by the internal wave, particularly when a surface current is involved. It will be found later that important effects can result from these modulations by further interaction with short breaking waves.

Consider first the case when the surface current, surface wave and internal wave are all collinear. From [5.4] for differential increments δU in surface current

$$\frac{2\delta c}{c_o} = \frac{\delta c + \delta U}{c_o + \overline{U}_s - C}$$

so that

$$\delta c = \frac{c_o}{c_o + 2\overline{U}_s - 2C}\,\delta U \qquad [5.9]$$

and from [5.6]

$$Ec\ \{U_s(x) + \tfrac{1}{2}c(x) - C\} = \text{const}$$

$$\frac{\delta E}{E} + \frac{\delta c}{c} + \frac{\delta U_s + \frac{1}{2}\delta c}{\overline{U}_s + \frac{1}{2}c_o - C} = 0$$

With the aid of [5.9] and after a little algebra, it is found that

$$\frac{\delta E}{E} = -4\ \frac{\delta U_s}{c_o} \left\{\frac{1 + \frac{3}{2}\,y}{(1 + 2y)^2}\right\} \qquad [5.10]$$

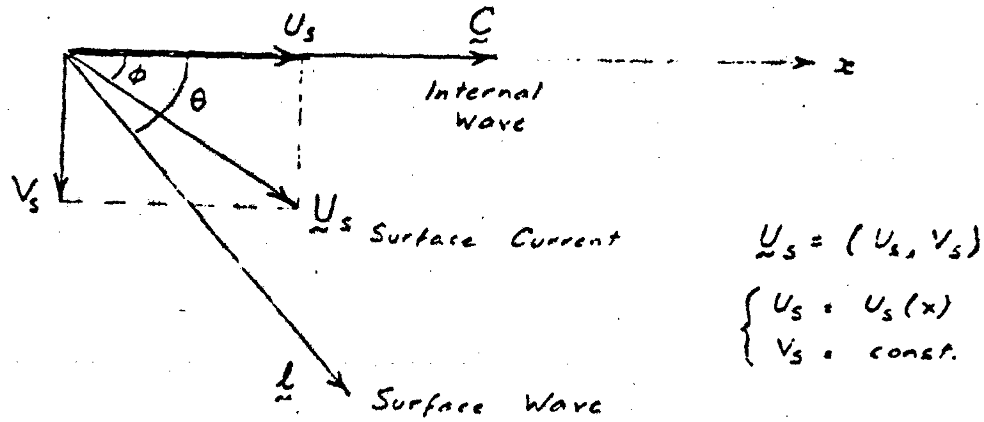
where

$$y = \frac{\overline{U}_s - C}{c_o}$$

For long surface waves travelling much faster than the internal waves, $|y| << 1$ and the modulations in energy density are approximately

$$\frac{\delta E}{E} \approx -4\ \frac{\delta U_s}{c_o} \qquad [5.11]$$

These results can be generalized to situations in which the internal wave motion, the surface current and the surface wave are not all collinear. Consider the geometry specified by the following diagram



Taking a reference frame moving in the x-direction with the internal wave (as before) the kinematics of the surface wave are specified by [5.3] as

$$\frac{\partial}{\partial x}\{\sigma + \underset{\sim}{k}\cdot(\underset{\sim}{U}_s - \underset{\sim}{C})\} = 0$$

or

$$\frac{\partial}{\partial x}\{(gk)^{\frac{1}{2}} + k[U_s(x)\cos\theta + V_s\sin\theta - C\cos\theta]\} = 0 \qquad [5.12]$$

and from $\nabla \times \underset{\sim}{k} = 0$, we have

$$\frac{\partial}{\partial x}(k\sin\theta) = 0 \qquad [5.13]$$

Accordingly, the <u>variations</u> in wave pattern are given by

$$\tfrac{1}{2}c\,\delta k + \delta k\,\{[U_s(x) - C]\cos\theta + V_s\sin\theta\} + \delta U_s\,k\cos\theta$$
$$- k\,[(U_s(x) - C)\sin\theta - V_s\cos\theta]\,\delta\theta = 0$$

and

$$\delta k \sin \theta + \delta\theta\, k \cos \theta = 0$$

Solving, the variation in wave number magnitude is given by

$$\frac{\delta k}{k} = -\frac{\cos \theta}{\frac{1}{2}c + U_s - C}\,\delta U_s \qquad [5.14]$$

and the variation in direction by

$$\delta\theta = \frac{\sin \theta}{\frac{1}{2}c + U_s - C}\,\delta U_s \qquad [5.15]$$

involving only the component of surface current in the x-direction. Since $\delta c/c_o = -\frac{1}{2}(\delta k/k)$, this reduces to [5.9] when $\theta = 0$.

The energy density distribution is given by

$$\frac{\partial}{\partial x}\{Ec\,(-C + U_s + \tfrac{1}{2}c \cos \theta)\} = 0 \qquad [5.16]$$

(This generalization of [5.6] is obtained most readily from Bretherton and Garrett's action conservation principle.) Thus variations are given by

$$\frac{\delta E}{E} + \frac{\delta c}{c} + \frac{\delta U_s + \frac{1}{2}(\delta c) \cos \theta - \frac{1}{2}c \sin \theta\, \delta\theta}{\frac{1}{2}c \cos \theta - C + U_s} = 0$$

From this, and with the aid of [5.14] and [5.15] it is found after some algebra that

$$\frac{\delta E}{E} = -2\left(\frac{\delta U_s}{c}\right)\frac{1 + \cos 2\theta + y(2 + \cos \theta)}{(\cos \theta + 2y)\,(1 + 2y)} \qquad [5.17]$$

where $y = (U_s - C/c)$. When $\theta = 0$, this reduces to [5.10]. For long surface wave travelling much faster than either the internal waves or the surface current, $y \ll 1$ and

$$\frac{\delta E}{E} \approx -2 \frac{(1 + \cos 2\theta)}{\cos \theta} \frac{\delta U_s}{c} = -4 \cos \theta \frac{\delta U_s}{c} \qquad [5.18]$$

The simplicity of this generalization of [5.11] is rather surprising. These results will be used later.

5.3 Resonance

Three wave trains of small slope can undergo resonant interactions, exchanging substantial amounts of energy amongst themselves if their wave-numbers k_i and frequencies n_i simultaneously obey the relations

$$\begin{aligned} \underset{\sim}{k}_1 - \underset{\sim}{k}_2 &= \underset{\sim}{k}_3 \\ n_1 - n_2 &= n_3 \end{aligned} \qquad [5.19]$$

Suppose the wave trains 1 and 2 represent surface waves of almost the same wavelength, so that $\underset{\sim}{k}_1 = \underset{\sim}{k}_2 + \delta\underset{\sim}{k}$, and let the wave train 3 represent an internal wave. With a surface current U_s, assumed constant, the frequencies $n_2 = \sigma_2 + \underset{\sim}{k}_2 \cdot \underset{\sim}{U}_s$, $n_1 = \sigma_2 + \delta\sigma + (\underset{\sim}{k}_2 + \delta\underset{\sim}{k}) \cdot \underset{\sim}{U}_s$, so that the conditions [5.13] become

$$\begin{aligned} \delta\underset{\sim}{k} &= \underset{\sim}{k}_3 \\ \delta\sigma + \delta\underset{\sim}{k} \cdot \underset{\sim}{U}_s &= n_3 \end{aligned} \qquad [5.20]$$

where, as before, σ represents the intrinsic frequency. From the first of [5.20] resonance can occur only if the difference between the wave numbers of the surface waves, $\delta \underset{\sim}{k}$, is equal to the wave-number of the internal wave. Furthermore, from the second equation, if $\delta k = |\delta \underset{\sim}{k}| = k_3$,

$$\frac{\delta\sigma}{\delta k} + \frac{\delta \underset{\sim}{k} \cdot \underset{\sim}{U}_s}{\delta k} = \frac{n_3}{k_3}$$

or, in terms of the group velocity of the surface waves,

$$c_g(k) + U_s \cos \varphi = C \qquad [5.21]$$

where φ is the angle between the surface current and the internal waves. From the theory of resonant interactions, if initially the energy resides in one of the surface wave trains (say 1) and the internal wave (3), the amplitude of the other surface wave train (2) at a neighboring wave number will grow — the initially uniform surface wave train will develop modulations or groups that grow in time. This is the phenomenon of resonance that has been investigated in this context by Lewis et al. (1974) for the case $U_s = 0$.

It is important to realize that resonance and blockage are two different ways of describing the same physical effect, but each description has its own limitations. The idea of resonance assumes small amplitude waves and small disturbances — it cannot cope with surface velocity variations of the same order as C, the speed of the internal wave. On the other hand, there is no restriction on the relative wave lengths involved. The analysis of Section 5.1 demands that the surface wave length be short compared with the internal wave length but imposes no restriction on the magnitude of the variation in U_s. It is not intrinsically a steady state approach. When (a) the surface wave length is indeed much less than the internal wave length (true under oceanic conditions) and (b) the variation in surface current is very small (sometimes true, sometimes not under natural conditions), the two approaches overlap: the condition [5.21] for resonance is identical to [5.7] for blockage (note that in [5.7] only the case $\phi = 0$ is considered).

The important work by Lewis et al. (1974) has (a) provided a detailed analysis for the resonance when $U_s = 0$, calculating the interaction coefficients and (b) verified by laboratory measurements the accuracy of the theory, showing that under resonant conditions the modulations in energy density initially develop such that

$$\frac{\delta E}{E} \approx 5 k_i u_o t \qquad [5.22]$$

where u_o is the maximum orbital velocity of the internal wave and k_i its wave number and finally (c) as in other resonant interaction experiments (e.g. McGoldrick, Phillips, Huang and Hodgson, 1966) the width of the resonance band decreases as the interaction time increases, provided the internal wave slope is small. When $(U_s - \overline{U}_s)/C$ is not negligibly small, the "band width" of the response is not small (see Figure 12).

5.4 Resonance and Blockage in the Field

The work of Gargett and Hughes, Phillips, Lewis, Lake and Ko has established a good theoretical foundation for these phenomena (if not a complete one in all respects) and the experiments of Lewis et al. have shown that the resonance phenomenon can be reproduced in the laboratory. Gargett and Hughes described some suggestive field phenomena, but qualitatively only. Zonation of small scale waves in the field has also been established under some conditions (Perry and Schimke, 1965): can this be attributed directly to resonance or blockage?

Consider first of all the wave lengths involved. The phase speed of internal waves in the ocean is of the order 1 m/sec at most; the estimate of Section 3.1 gives 0.7 m/sec. The wave length in meters and frequencies in cycles per second (in brackets) of surface waves resonant with internal waves are given in the table below for various values of the surface current:

TABLE 1

	C = 0.75	0.50	0.25 m/sec.
$U_s \cos \phi$ = 0.75	-	-0.15 (3.33)	-0.60 (1.7)
0.50	0.15 (3.33)	-	0.15 (3.33)
0.25	0.60 (1.7)	0.15 (3.33)	-
0	1.35 (1.1)	0.60 (1.7)	0.15 (3.33)
-0.25	2.40 (.83)	1.35 (1.1)	0.60 (1.7)
-0.50	3.75 (.67)	2.40 (.83)	1.35 (1.1)
-0.75 (m/sec)	5.4 (.55)	3.75 (.67)	2.40 (.83)

Wavelengths, in meters, and frequencies, bracketed, in hz of surface waves that are resonant with internal waves with phase speed C (m/sec.) in the presence of surface drift U_s (m/sec.) at an angle ϕ to the direction of motion of the internal waves. The positive direction is taken in the direction of motion of the internal waves; negative wavelengths indicate surface waves moving in the opposite direction.

Recall also that for short surface waves their amplitude in an actively wind generated wave field is determined by their condition for local stability — increased energy input will result not in increased amplitude but in increased wave breaking. (For wave components near the spectral peak, non-linear conservative energy exchanges are also important (Hasselmann et al., 1973). An energy loss will, however, result in a decreased amplitude provided the local rate of loss of energy resulting from the interaction exceeds the rate of energy input from the wind.

If these short wave components are already saturated by the wind in the absence of an internal wave disturbance, under what conditions will such a disturbance produce modulations? Clearly, when the wind is unable to regenerate locally the wave energy in the regions where it is decreasing as a result of the resonant or blockage interaction. From [5.22] the maximum rate of decrease of wave energy is given by

$$\frac{1}{E}\frac{dE}{dt} \simeq -5u_o k_i \qquad [5.23]$$

where k_i is the internal wave number and u_o is the magnitude of the surface velocity variations, described in Section 3. Under the most favorable circumstances for resonance, with a mean surface drift nearly equal to the internal wave speed*, $u_o \to C$, the speed of the internal wave, so that

$$\frac{1}{E}\frac{dE}{dt} < -5n_i$$

*Resonance theory is then unreliable for such large surface currents.

where n_i is the radian frequency of the internal wave. A very high frequency internal wave on the thermocline has a period of 5 minutes or so (this corresponds to the conditions calculated in Section 3.1) so that $n_i < 2 \times 10^{-2}$. Consequently, for the most favorable conditions for resonance in the ocean,

$$\frac{1}{E}\frac{dE}{dt} < -10^{-1}\ \text{sec}^{-1} \qquad [5.24]$$

The information available on the rate of energy input from the wind has been reviewed; recent relevant measurements have been made by Wu (1973). Contours of the growth rate of surface waves in wind are shown in Figure 13. For surface wave lengths less than about 15 centimeters that influence radar backscattering measurements, the growth rate for the wind exceeds 10^{-1} sec^{-1} except for relatively light winds ($u_* < 25$ cm/sec, $U < 5$ m/sec – 10 knots); if the wind is comparable to, or greater than this limit, modulations in the short wave lengths will not develop directly from resonant interactions.

Longer wave lengths, whose growth rates under the influence of the wind are slower, may develop modulations, either resonant or non-resonant, but these will not be observable directly using surface scanning radar with wave lengths in the C, X or P bands. Nevertheless, it will be seen in Section 6 that such modulations may in turn influence much shorter components which are directly observable in this way.

Wave-wave interactions will also serve to make more uniform the distribution of surface wave energy in the face of internal wave modifications. These have been considered by Zachariasen (1973) under the assumption that energy input from the wind and energy dissipation by breaking and so forth are linear functions of the wave energy density. It is difficult to draw from this analysis any simple conclusions except that (1) if the interactions are weak, under non-resonant conditions the perturbations in energy density are in phase with the surface current as in a single wave train (see equation 5.12); (2) under resonant conditions there will be perturbations in energy density in phase with the rate of surface strain rather than the surface current, as the simple (no wind, no interaction) theory indicates. The occurrence of these modulations in the theory is, however, a consequence of the "softness" of the dissipation function assumed (linear in energy density E); it is more likely that the rate of dissipation increases more rapidly with E than linearly and this would tend to suppress the modulations. The simple equilibrium range idea is essentially equivalent to zero dissipation when $E < E_{max}$ with breaking and very rapid dissipation when $E > E_{max}$.

5.5 Summary

1. Resonance and blockage are clearly established phenomena both theoretically and experimentally, in the laboratory.

2. In the field, significant resonance effects are confined to relatively long wave lengths (1-5 m or so) and may occur, the optimum conditions being (a) relatively large internal wave speeds in the direction of surface wave travel (b) a surface current opposed to the direction of surface wave travel and (c) relatively light winds. Such modulations will not be directly observable using conventional surface-scanning radar.

3. Except in very light winds, resonant modulations in short waves (wave lengths of 15 centimeters or less) will be masked by energy input from the wind, and any modulations that are observed by radar must be produced by some other, indirect effect, in which resonance may play a part, but not the sole part.

4. Amplitude modulations are produced by non-resonant interactions between the internal wave and surface waves moving much more rapidly than the internal wave. Detailed results are given in Section 5.2. Again, these modulations are not directly observable by surface scanning radar but, as will be seen in Section 6, can produce variations in small scale surface wave structures that are.

6. WAVE BREAKING

A breaking wave is defined simply as one in which some fluid elements near the crest are moving forward faster than the wave profile. Wave breaking occurs on waves over a considerable range of scales; at the larger scales, breaking is more vigorous, air is entrained and the breaking is clearly visible as the formation of white caps. Small scale breaking (wave lengths 5-20 cm) is however widespread in an active wind-generated wave field (Banner and Phillips, 1974); air is not usually entrained, the flow being characterized by the occurrence of surface stagnation points and the profile by the appearance of irregular 'steps' at the edge of the small breaking zone.

6.1 Incipient Breaking

The maximum amplitude that a wave train (or a local group of waves) can attain without breaking depends strongly on the wind drift in the upper few millimeters below the air-water interface. If the local surface drift is q_o (the value at the mean water level $\zeta = 0$; very closely the local mean value) and the phase speed of the wave is c, then the maximum amplitude ζ_m that the wave can have without breaking is given by

$$\zeta_m = \frac{c^2}{2g}\left(1 - \frac{q_o}{c}\right)^2 \qquad [6.1]$$

(Banner and Phillips, 1974), as illustrated in Figure 14. In the absence of surface drift, $\zeta_m = c^2/2g$ the Stokes value, but under natural conditions the maximum wave amplitude is substantially less. The effect is particularly significant at those short wavelengths that are responsible for radar backscattering. In a wind of 10 m/sec, $q_o \sim 30$ cm/sec. A surface wave length of 10 centimeters has $c = 40$ cm/sec and a maximum amplitude of only 0.03 times the Stokes limit. If the locally averaged value of the surface drift varies by an amount δq, the maximum wave amplitude varies as

$$\frac{\delta \zeta_m}{\zeta_m} = \frac{-2\delta q}{c - q_o} \qquad [6.2]$$

so that for wave speeds only slightly greater than q_o, even small increases in q will produce substantial reductions in ζ_m.

If the local wave amplitude is forced to exceed the limit [6.1], water at the crest tumbles forward, the wave breaks, losing energy until the local energy density is sufficiently reduced that breaking ceases. It is suspected, but not known with certainty, that there is a hysteresis involved in breaking; waves that begin to break may continue to do so until their amplitude is significantly less than [6.1]. The onset of breaking in a particular wave crest is generally quite abrupt; there is a nonlinear feedback process resulting from the compression of the vortical layer that rapidly augments the forward surface surge as the breaking condition is approached.

Once waves on a small scale begin to break, as in Figure 16, the wave profile is far from sinusoidal. The breaking region ahead of the crest has an irregular area of high slope where the water is falling forward; in terms of the Fourier components, many higher harmonics are generated. The high frequency unsteady motion in the small tumbling region also generates many capillary components with wave lengths of a few millimeters that may not be excited directly by the wind.

The presence of a surface slick that is saturated or close to saturated has a very strong influence on wave breaking. Compression of the slick is resisted by the gradient of the film pressure (see Section 4), so that the forward surge near the short wave crest that leads to breaking is resisted and breaking is delayed. Energy dissipation is, of course, augmented even without breaking, though it seems to be the inhibition of short wave breaking that gives the dramatic difference in surface appearance between a concentrated slick and a surrounding cleaner surface. No longer are small scale capillary wavelets generated by the breaking and the attenuation rate is generally sufficient to prevent direct generation from the wind.

6.2 The Suppression of Short Waves by Swell in the Presence of Wind Drift

The surface wind drift is not uniform in the presence of a wave field because of the surface straining associated with the waves. When waves are propagating generally in the same direction as the drift, the drift is augmented in the vicinity of the wave crest. The wave breaks when the augmentation is such that the surface speed at the wave crest exceeds the phase speed of the wave, but the amplification is present even in non-breaking waves. The maximum value of the surface drift at a long wave crest is given by Phillips and Banner (1974) as

$$q = (C_\ell - u) - \left\{(C_\ell - u)^2 - q_o\,(2C_\ell - q_o)\right\}^{\frac{1}{2}} \qquad [6.3]$$

where C_ℓ is the long wave speed, u its orbital speed at the surface (= $C_\ell ak$ where a = long wave amplitude, k = wave-number) and q_o is the value of the surface drift at points where $\zeta = 0$ on the mean water level, or approximately the average value of the surface drift over the whole surface. For wave periods greater than about 2 seconds ($\lambda > 10$ m) variations in wind stress may influence the distribution (see Section 2 of this report); the effect should be calculated numerically but it is expected that the increased wind stress at the wave crests may even augment further the increased drift there produced by the dynamics of the strained surface layer. The neglect of this further effect should represent an under-estimate of the effects on short waves.

When short waves are being overtaken by a longer one, near the long wave crest the short waves experience an augmented wind drift q_o, so that the maximum height that they can have at incipient breaking is reduced in accordance with equation [6.1]. In fact, in an active wind with continuing energy supplied to the short waves, they can be expected to actually break on a small scale as they approach the long wave crest and the local mean drift q_o increases so that [6.1] becomes more stringent. Once the long wave crest has passed they may be regenerated by the wind but only to break at the next long wave crest when the increased drift again limits the amplitude. Accordingly, the presence of longer waves and wind drift reduces the amplitude of short waves, the amount of the reduction being dependent on the long wave speed and slope. By assuming that the average energy density in a short wave field, constantly regenerated by the wind, is proportional to (in fact, roughly equal to) the energy density when the short waves are at the point of incipient breaking at the long wave crest, Phillips and Banner (1974) were able to account quantitatively for reductions in short wave energy density produced by long waves in a variety of experimental situations. Early measurements by Mitsuyasu (1966) were also very consistent with the theoretical predictions (see Figure 17) though experimental verification is still rather sparse.

For short waves superimposed on long ones, the maximum amplitude at incipient breaking is given by

$$\zeta_{max} = (2g')^{-1} \; (c_c - q)^2 \qquad [6.4]$$

where q represents the locally averaged surface drift [6.3] induced by the long wave at its crest. This can be expressed alternatively as

$$\frac{q}{C_\ell} = (1 - ak) - \left\{(1 - ak)^2 - \frac{q_o}{C}\left(2 - \frac{q_o}{C}\right)\right\}^{\frac{1}{2}} \qquad [6.5]$$

where q_o is the overall average surface drift and ak the long wave slope. c_c is the short wave speed at the long wave crest (also modified by the straining of the long wave from its undisturbed value c_o):

$$c_c = c_o(1 - ak) \qquad [6.6]$$

and g' is the apparent gravitational acceleration experienced by the short wave

$$g' = g(1 - ak). \qquad [6.7]$$

6.3 Short Wave Modulations Associated with Internal Waves

Assume that internal waves produce a distribution of surface current whose variations may or may not be augmented by a mean surface current as in Section 3 of this report. Assume that the wind is blowing in some direction generating waves over a range of scales; let us (rather simplistically) represent these waves initially as a train of long waves moving much faster than the internal wave superimposed on which are short wavelets (say in the 2-10 cm range). The characteristic growth time for the long waves is taken to be large (much larger than the time to propagate from one internal wave crest to the next) so that the direct effect of wind re-generation can be neglected during these time intervals. On the other hand, the growth time for the short waves is small; they are generally saturated and if their amplitude is severely restricted at a long wave crest, they will rapidly be regenerated once the crest is passed. The quantitative form of these assumptions is considered later.

Since the response time of the wind-drift layer (in the upper few milimeters) is short (see Section 2) the mean wind drift, averaged over an area small compared with an internal wave length but large compared with a long surface wave length is, in essence, constant. The direct straining of the surface wind-drift layer by the internal wave is too slow and of too small a magnitude (by several orders) to modify the structure of the wind drift layer directly. The short time scales appropriate to this layer indicate that, so far as the internal waves are concerned, the surface drift layer is everywhere in local equilibrium with the wind and shorter waves.

The long waves propagate relatively rapidly through the variable current pattern produced by the internal waves. They are not resonant and they are not blocked, but (relatively small) modulations in long wave amplitude are produced as described in Section 5. The long waves are smallest when the surface drift component in the direction of internal wave travel is largest (see Eq. 5.18) if $\theta < \pi/2$. The small modulations in long wave slope produce modulations in the maximum values of surface drift at the long wave crests, where it is augmented by the convergence in the long waves. An important point is that the surface drift generated at the long wave crest is dependent on, and a substantial fraction of the <u>long wave speed</u>, so that the <u>modulations</u> in surface speed at the different long wave crests are a (fairly small) fraction of the long wave speed. However, they are a substantial fraction of the <u>short wave speed</u>. Short waves at different phases of the internal wave experience surface drifts at the long wave crests that vary by a substantial fraction of their own speed; in the previous section it was shown that the maximum amplitude the short waves can have against breaking and in the presence of a regenerating wind, depends <u>sensitively</u> on the value of the drift q as a fraction of <u>their</u> phase speed c. Consequently, there will be modulations in the short wave energy density, with maxima where the long waves are smallest, that is, when the surface current in the direction of wave propagation is greatest. Minima will occur when the current is least or in the reverse direction.

These statements will now be put into more quantitative form. The variation in (root mean square) amplitude of the long waves (no resonance) is given by [5.17] as

$$\frac{\delta a}{a} = -\left(\frac{\delta U_x}{C_\ell}\right)\frac{1 + \cos 2\theta - y(2 + \cos\theta)}{(\cos\theta - 2y)(1 - 2y)} \qquad [6.8]$$

and the variation in wave-number by [5.14] as

$$\frac{\delta k}{k} = -\left(\frac{\delta U_x}{C_\ell}\right)\frac{2\cos\theta}{1 - 2y} \qquad [6.9]$$

where $y = -(\overline{U}_s - C)/C_\ell$, $\overline{U}_s$ being the component of the mean surface current (in the top few meters) in the direction of the long waves. The slope variation is accordingly

$$\frac{\delta(ak)}{ak} = -\left(\frac{\delta U_x}{C_\ell}\right)\frac{4\cos^2\theta - y(2 + 5\cos\theta)}{(\cos\theta - 2y)(1 - 2y)} \qquad [6.10]$$

Modulations in long wave slope when $\delta U_x/C = 0.2$ are shown in Figure 18 as a function of C_ℓ/C. The resonance point is when $C_\ell/C = 2$; we are concerned with considerably longer waves such that $C_\ell/C > 2$. The slope modulations, opposed in phase to the surface current, decrease as C_ℓ/C increases; a characteristic magnitude is 0.2 or 20%.

Now from [6.5], taking differential increments, the modulations in surface wind drift at the crests of the long waves are given as a fraction of the long wave speed by

$$\frac{\delta q}{C_\ell} = \delta(ak)\left\{\left[(1 - ak)^2 - \frac{q_o}{C_\ell}\left(2 - \frac{q_o}{C_\ell}\right)\right]^{-\frac{1}{2}}(1 - ak) - 1\right\} \qquad [6.11]$$

Note that q_o, the average surface wind drift is essentially constant with respect to phase of the internal wave on account of the short response time of the wind drift layer — see Section 2. Note also that in the field, $q_o/C_\ell << 1$, the average surface wind drift is small compared with the long wave speed. The expression [6.11] is illustrated in Figure 19 for various values of q_o/C_ℓ.

Consider now the short wavelets, constantly regenerated by the wind, riding on these long ones. The maximum amplitude they can have against breaking is given by [6.4], i.e.

$$\zeta_{max} = (2g')^{-1}\ (c - q)^2$$

where $g' = g(1 - ak)$ is the apparent gravity acting on the short waves as they ride over the long wave crest, $c = c_o\ (1 - ak)$ is the local wave speed of the short waves and q is the local surface wind drift at the long wave crest. Again taking differential increments, the variations are given by

$$\frac{\delta\zeta_{max}}{\zeta_{max}} = -\frac{\delta g'}{g'} + 2\,\frac{\delta c - \delta q}{c - q} = \frac{\delta(ak)}{1-ak} - \frac{2\ \delta(ak)\ c_o}{(c-q)(1-ak)} - \frac{2\ C_\ell}{c_o - q}\,\frac{\delta q}{C_\ell}$$

$$= \frac{\delta(ak)}{1-ak}\left\{1 - \frac{2\ c_o}{c_o - q}\right\} - \frac{2\ C_\ell}{c_o - q}\,\frac{\delta q}{C_\ell} \qquad [6.12]$$

In this expression, $\delta(ak)$ and δq are both directly proportional to δU_s according to [6.11] and [6.10]; in the calculations below, δU_s is taken as 0.2C (a rather low value: see Figure 5) so that for other values of the ratio, the modulations can be found by simple multiplication.

Since the modulations in short wave energy density $\delta E/E \approx 2(\delta\zeta_{max})/\zeta_{max}$ they are given by

$$\frac{\Delta E}{E} = -2ak\ \{\text{Fig. 18}\} \left[\frac{c_o + q}{(1-ak)\ (c_o-q)} + \frac{2\ C_\ell}{c_o-q}\ \{\text{Fig. 19}\}\right] \qquad [6.13]$$

where {Fig. 18}, {Fig. 19} represent the ordinates for appropriate conditions from these curves. Note that when $q \to c_o$, the amplitude of the proportional modulations becomes indefinitely large, the short waves being erased from at least part of the longer wave cycle.

The results of some detailed calculations based on these expressions are shown in Figures 20 through 27. In interpreting these figures, several points should be kept in mind.

1. In each case, the ordinate represents the amplitude of the modulations in short wave energy density over a period of the internal wave.

2. The short waves considered here are freely travelling waves under the influence of wind, not the Fourier harmonics of longer waves whose basic wave length is a multiple of the wave length given. For freely travelling short waves of a given wave length, there is a cut-off produced by the wind drift, while longer waves, with harmonics of the same wave-number can still propagate.

3. Calculated variations of $\Delta E/E$ larger than some limit of order 1 can no longer be regarded as perturbations and the numerical calculations are inaccurate. Indicated modulations greater than this simply indicate a very large response. Values of $\Delta E/E$ less than 10% may possibly be regarded as insignificant and are not plotted.

4. All calculations are for surface current variations whose amplitude is 0.2 times the internal wave speed. For other values, the curves should be increased or decreased in linear proportion.

5. Under natural conditions, the wind wave spectrum is generally narrow and sharply peaked. The complex surface wave field is here simplified to a long wave, together with superimposed local short waves limited by breaking.

Figure 20, 21 and 22 give $\Delta E/E$ for 2.5, 10 and 20 centimeter waves as a function of wind speed for the case of no mean surface current, long wave slope 0.2, for various values of the wave length of the longest waves. The bracketed pairs of curves show the influence of variations in internal wave speed from 30 to 40 cm/sec. Note the cut-off in these curves when the surface wind drift is sufficient to suppress freely travelling waves ($q_o = c$).

Figures 23, 24, 25 give $\Delta E/E$ again for 2.5, 10 and 20 centimeter waves as a function of the slope of the longest waves, again with no mean surface current and with the wind speed of 7.5 m/sec (15 knots). Note the much greater response of shorter waves.

Figure 26 gives $\Delta E/E$ for the three short wave lengths as a fuction of mean surface current referred to the internal wave speed (though with a constant value of 0.2 for the variations in surface current to internal wave speed, that are produced by the internal waves). The longer wave has 1 m wavelength, internal wave speed 30 cm/sec and again a 7.5 m/sec (15 knot) wind.

Figure 27 gives the directional dependence of the response as a function of angle θ between the long surface waves and the internal wave. There is some directional weakening, but the response is surprisingly broad band.

The general trends of the calculations are indicated below.

6.4 Summary

1. The response of 2.5 cm waves is larger than that of 10 or 20 cm waves by a factor of order 3 (at low wind speeds, $U \sim 2$ m/sec, or 4 knots) to 10 or much more at higher wind speeds (greater than 7.5 m/sec, 15 knots).

2. The phase of the short wave minimum corresponds to the internal wave phase where the current has its maximum adverse value with respect to the longer waves in the surface wave field (above the crests if the two are concurrent). The maximum occurs between this point and the following internal wave trough.

3. Orders of magnitude of variations with the various parameters are as follows:

With Increasing:	The Modulation Amplitude for Freely Travelling Short Waves	By Factor of Order	Over the Range
Wavelength of Short Waves	Decreases	At Least 3	2.5 to 20 cm
Wind Speed	Increases	10	4 - 8 m/sec
Internal Wave Speed	Increases	2	30 - 40 cm/sec
Surface Current Modulations	Increases Linearly	Always	
Longest Surface Wavelength	Decreases	10	3 - 30 m
Longest Wave Slope	Increases	3	0.2 to 0.4
Mean Surface Current in Wind Direction	Decreases for Given Values of Modulations $\frac{\text{Given}}{\delta U/C}$	4	-0.5 to + 0.5
Angle Between Surface and Internal Waves	Decreases	3	0 - 60°

7. EPILOGUE

This work has sought to evaluate the predominant influences of internal waves on short surface waves under natural conditions where a variety of physical processes occur in combination. The strongest and most widespread effect appears to arise from a relatively small modulation to long waves that run through the internal wave-induced motion, these modulations having a sometimes very strong influence on the maximum height that short waves can maintain against energy loss by breaking while under the continued influence of the wind. The results are summarized in Section 6.4. Modulations in short wave energy density and in density of short wave breaking and surface stirring are important. Since the present report is already long enough, these questions will be considered in a sequel.

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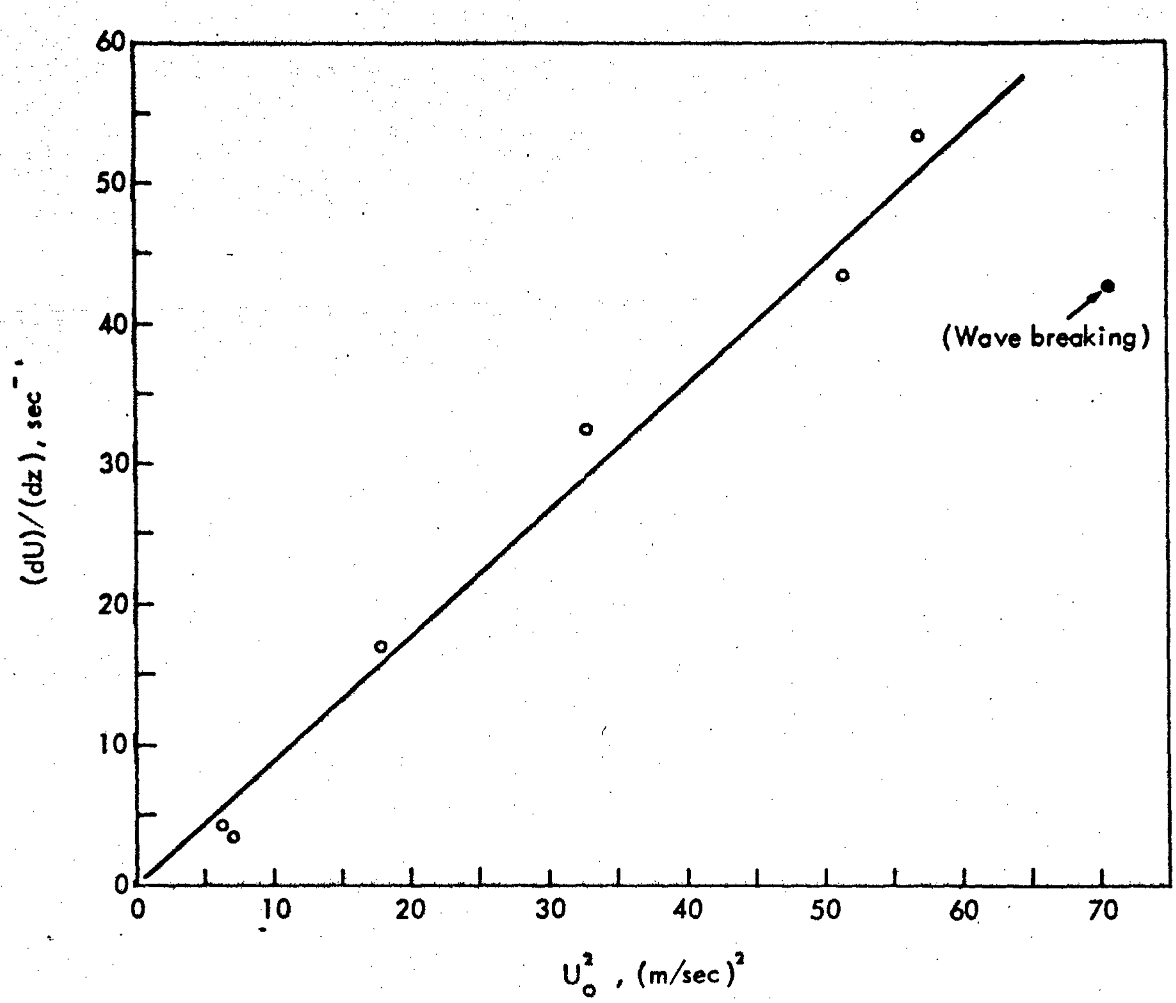


FIGURE 1 - THE VELOCITY GRADIENT AT THE WATER SURFACE AS A FUNCTION OF THE SQUARE OF THE CENTERLINE SPEED. The linearity confirms that the shear stress is supported predominantly by molecular viscosity. (After Wu, 1968)

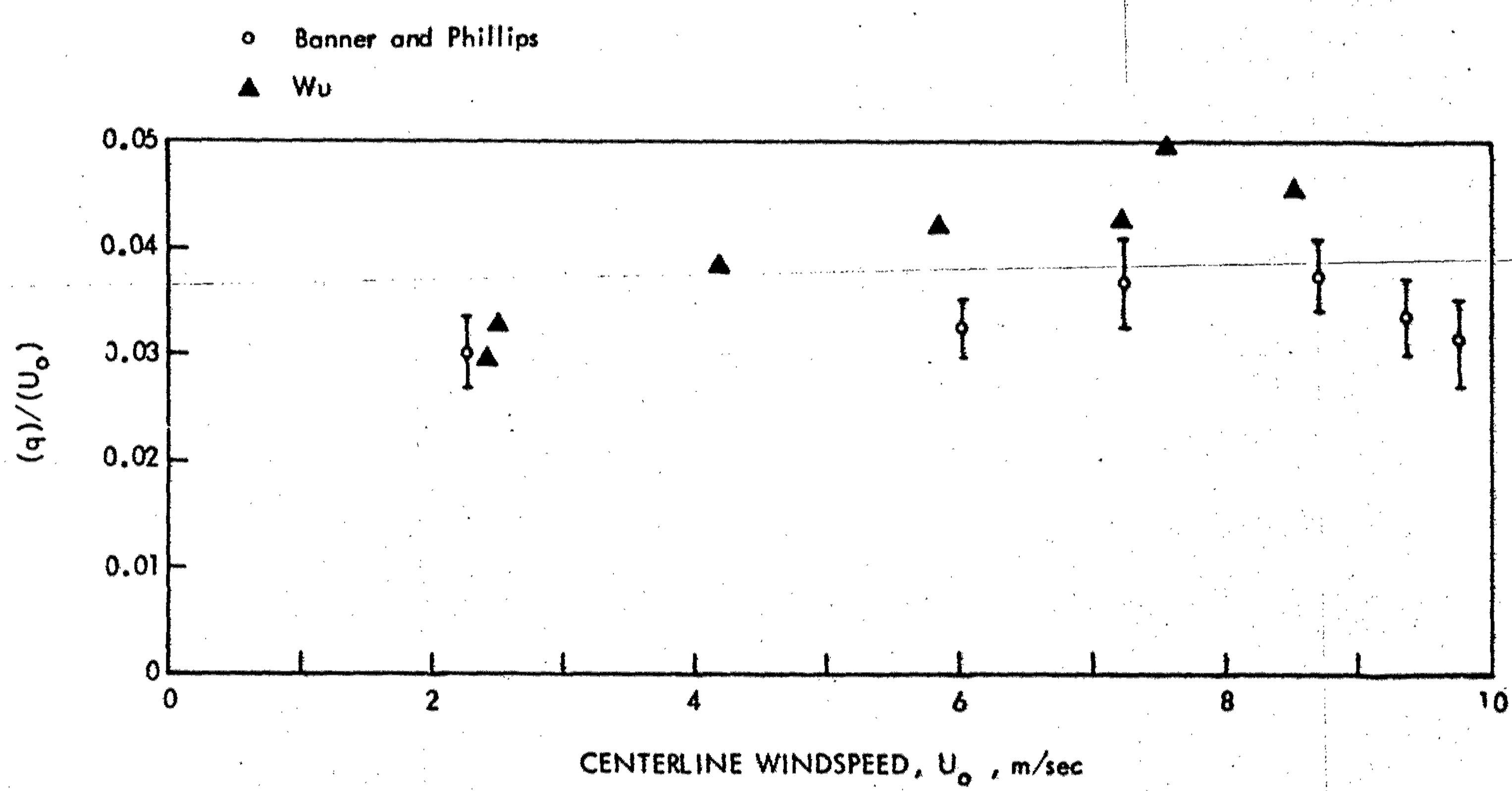


FIGURE 2 - SURFACE DRIFT CURRENT AS A FUNCTION OF CENTERLINE WIND SPEED. Measurements by Wu (1968) and Banner and Phillips (1974) in two different wind-wave facilities.

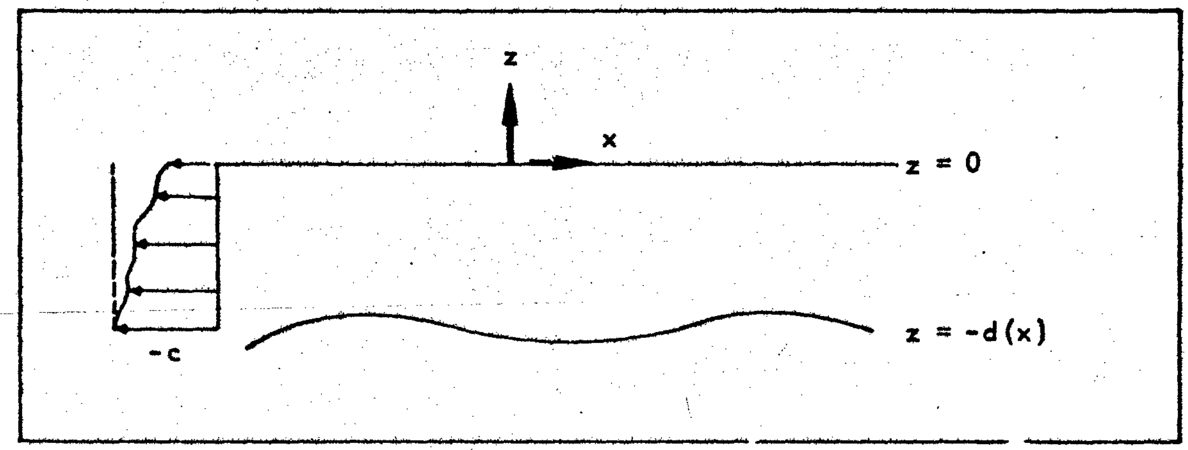


FIGURE 3 - A DEFINITION SKETCH. The frame of reference is moving with the internal wave train.

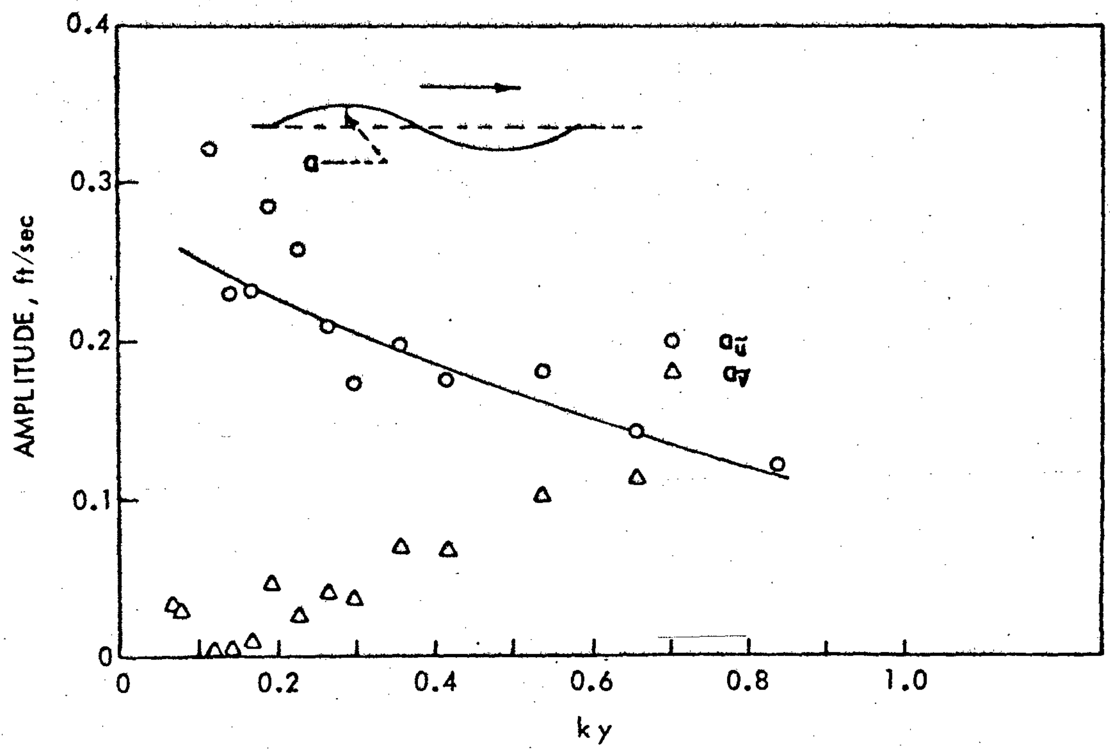


FIGURE 4 - THE AMPLITUDE OF WAVE-INDUCED FLUCTUATIONS AS A FUNCTION OF HEIGHT. The line represents amplitudes calculated from the Lighthill-Miles approximation. (After Karaki and Hsu, 1968)

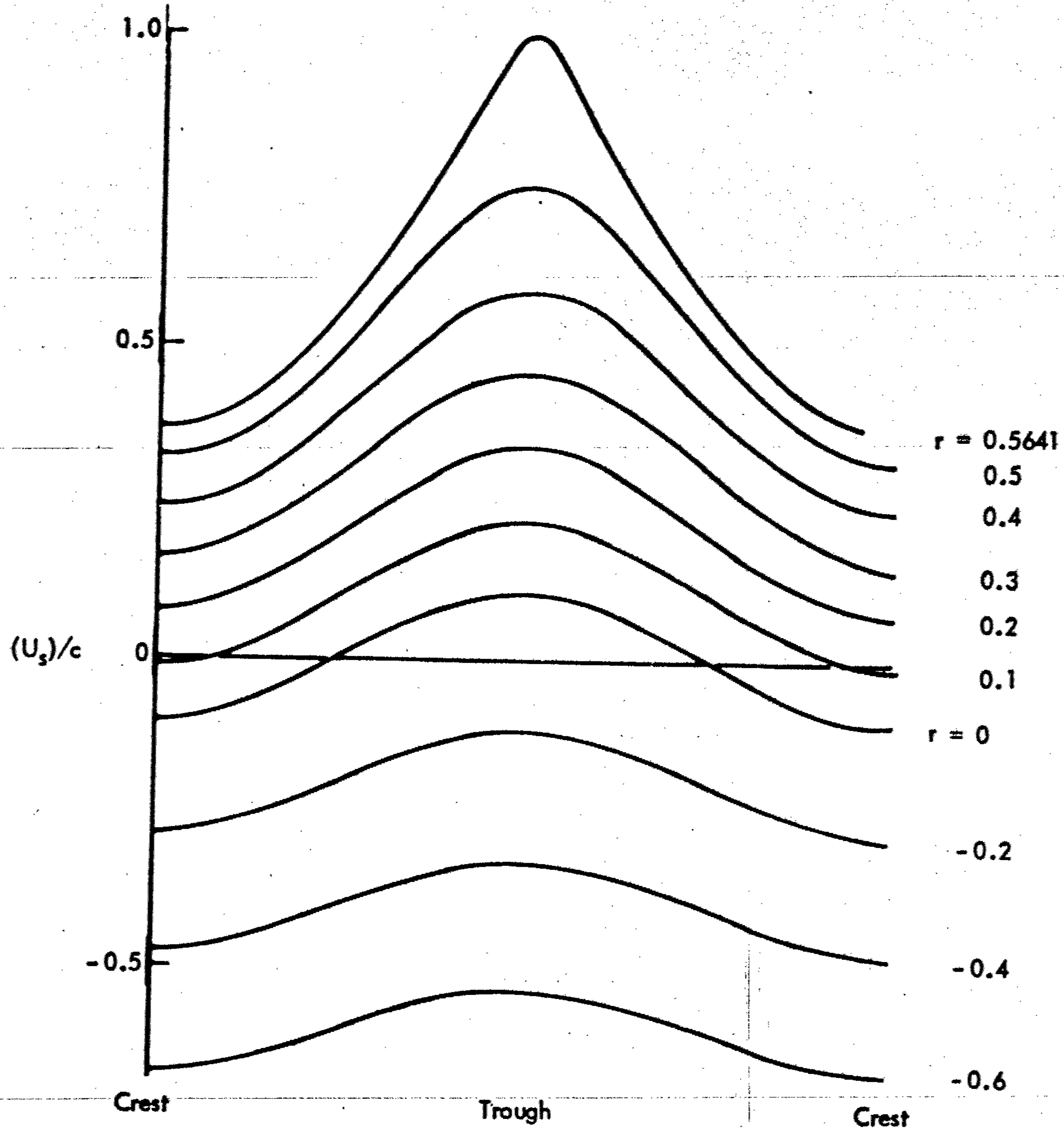


FIGURE 5 - DISTRIBUTIONS OF SURFACE CURRENT, WITH RESPECT TO PHASE OF AN INTERNAL WAVE, WITH $a/d = 0.1$. $r = \bar{U}/C$, the ratio of mean current to internal wave speed. Note that the magnitude of the modulations increases with r; above r = 0.5641 in this case, fluid above the wave trough moves faster than the wave profile and a region of closed mean circulation develops. r = 0 corresponds to the classic irrotational case.

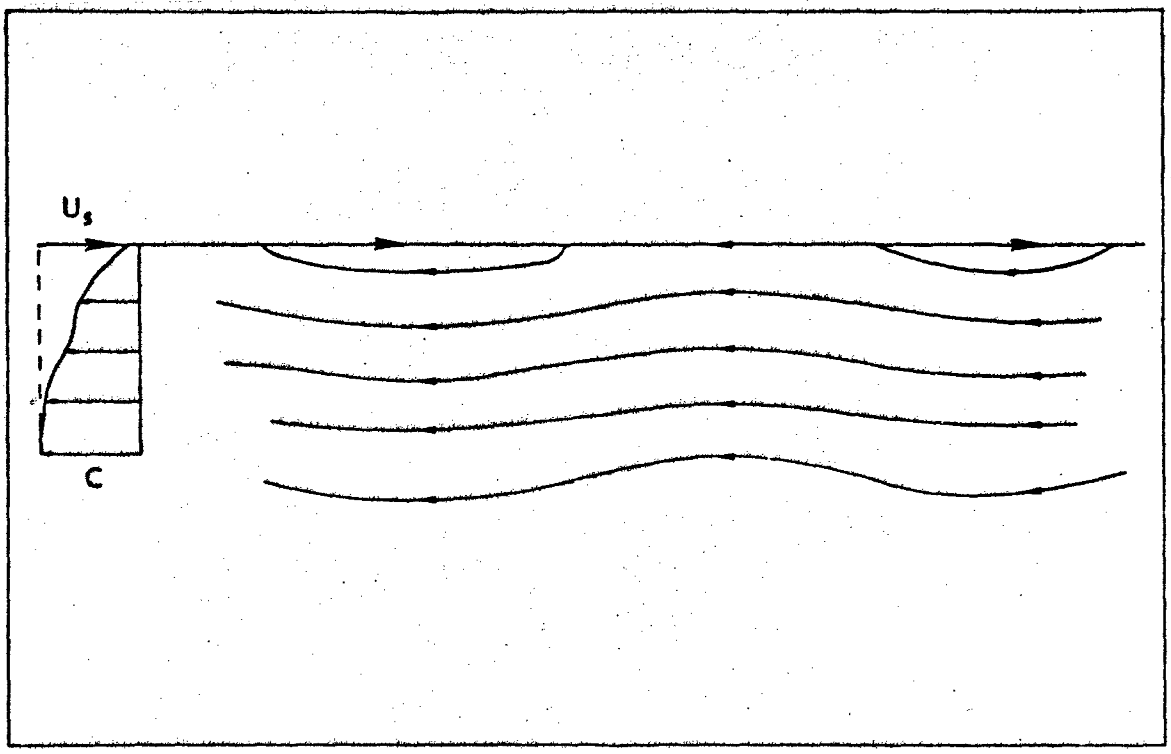


FIGURE 6 - QUALITATIVE PATTERN OF MEAN STREAMLINE WHEN $\overline{U}_s/C \sim 1$

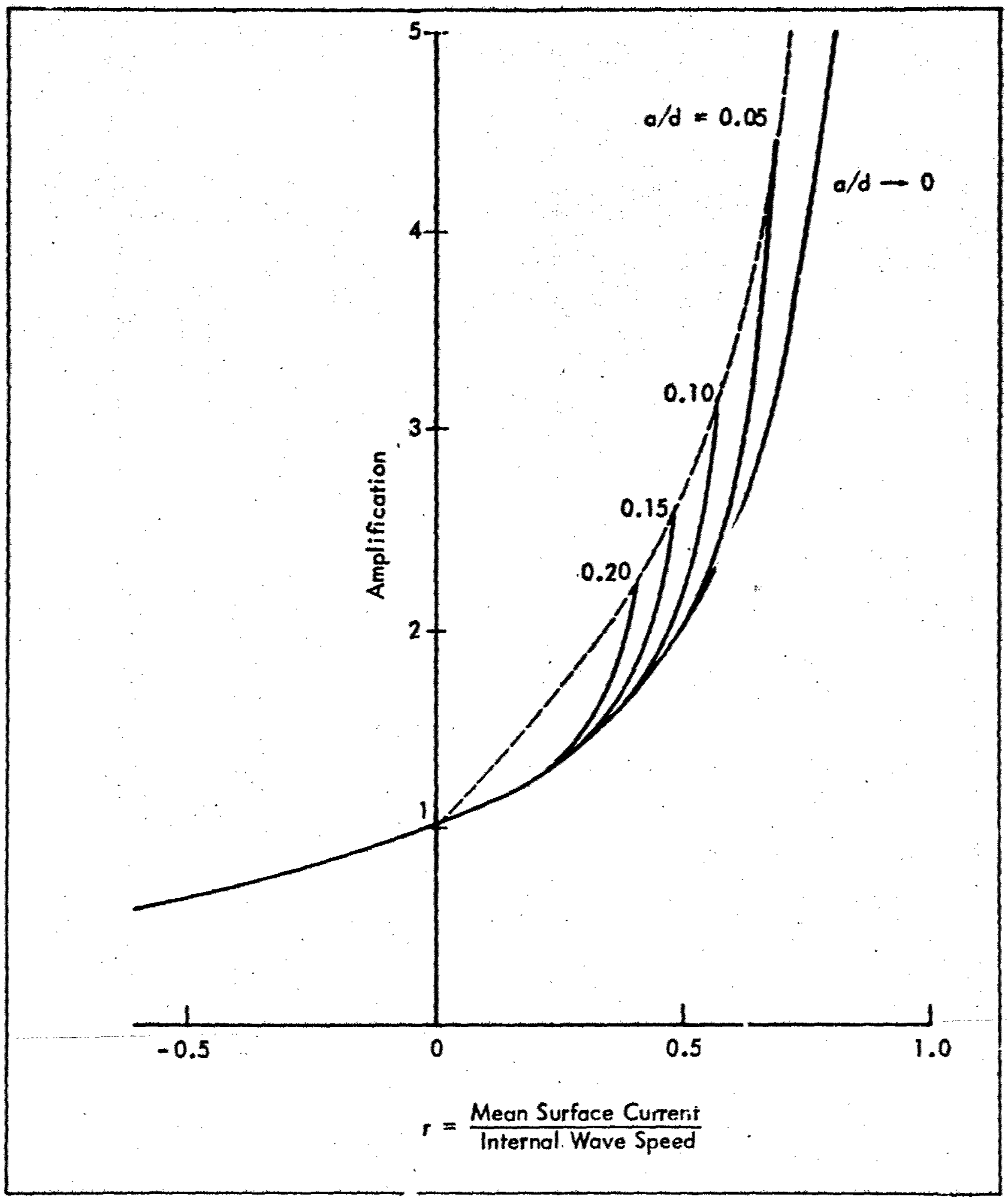


FIGURE 7 - THE AMPLIFICATION IN SURFACE CURRENT MODULATIONS RESULTING FROM THE EXISTENCE OF A MEAN CURRENT. Above the broken line, fluid at the surface above the internal wave troughs moves forward faster than the wave itself and a closed eddy forms.

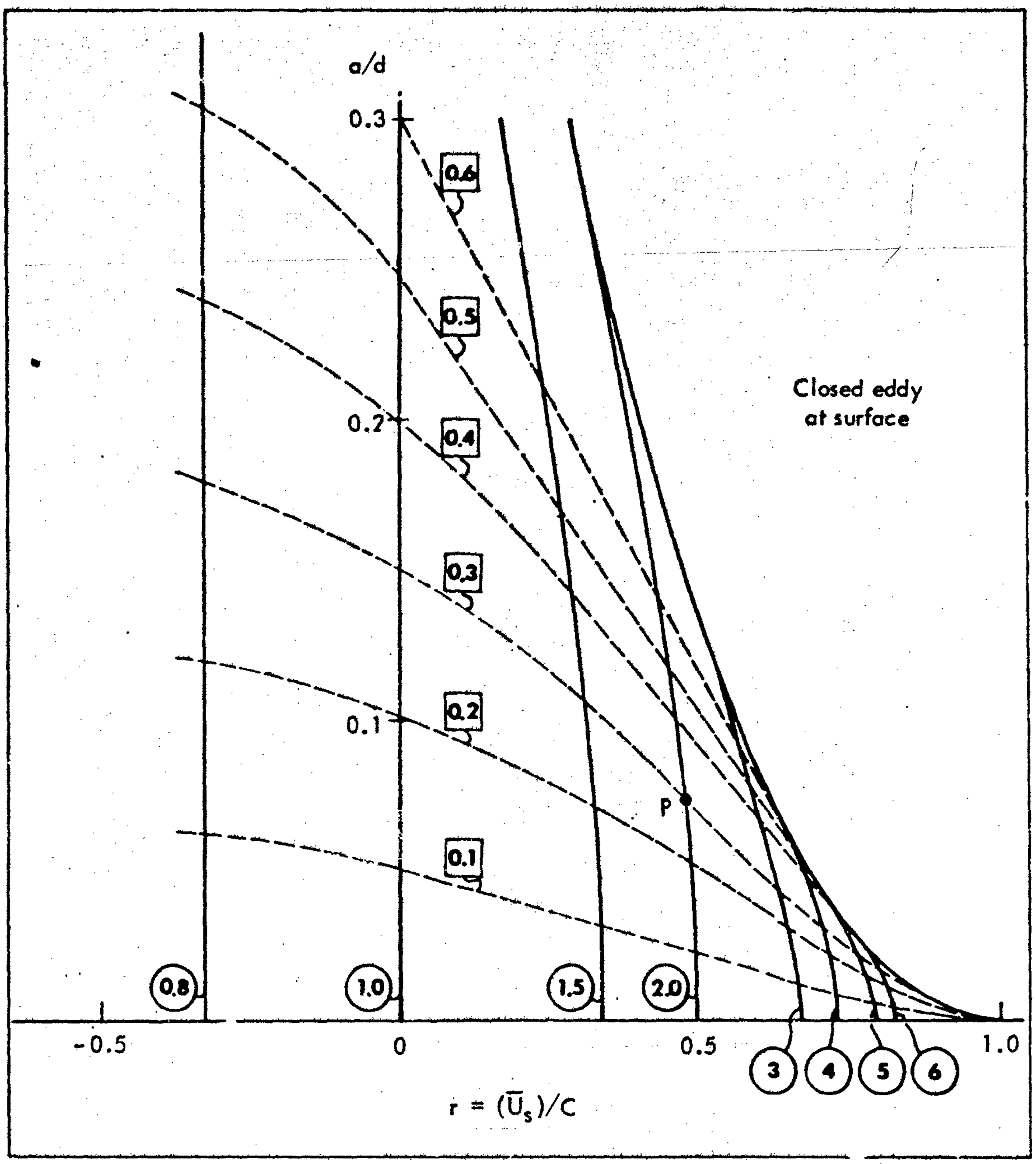
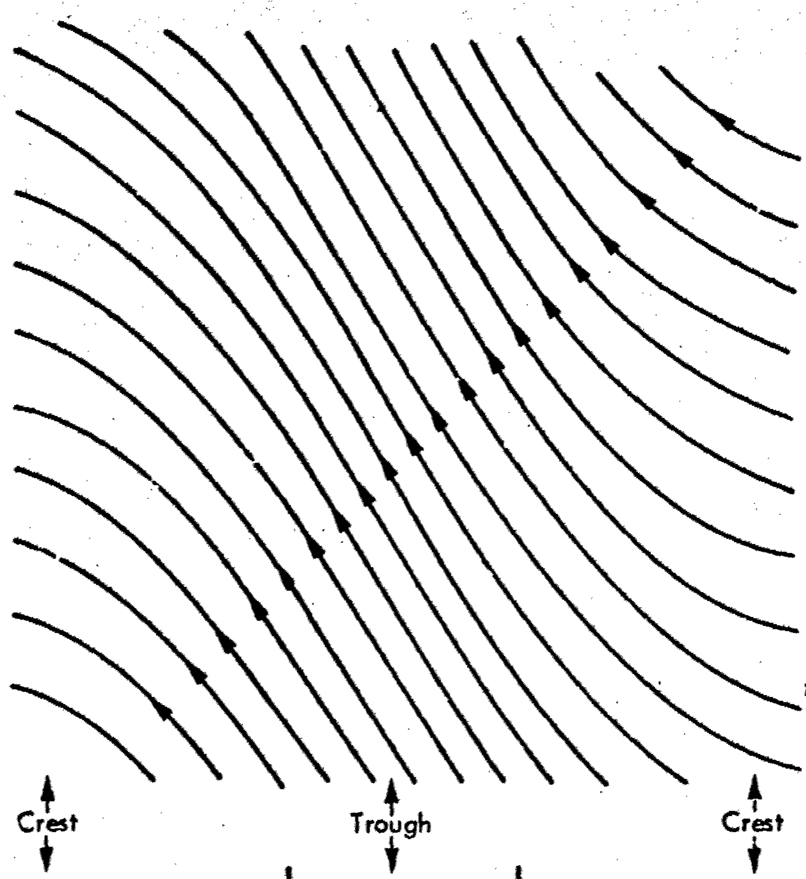


FIGURE 8 - CONTOURS OF THE AMPLIFICATION FACTOR OF SURFACE CURRENT MODULATIONS (solid curves, identified by numbers in circles) AS A FUNCTION OF a/d AND $\bar{U}_s$/C AND OF THE TOTAL AMPLITUDE OF CURRENT MODULATION DIVIDED BY INTERNAL WAVE SPEED (broken curves, identified by numbers in boxes).

(a)



(b)

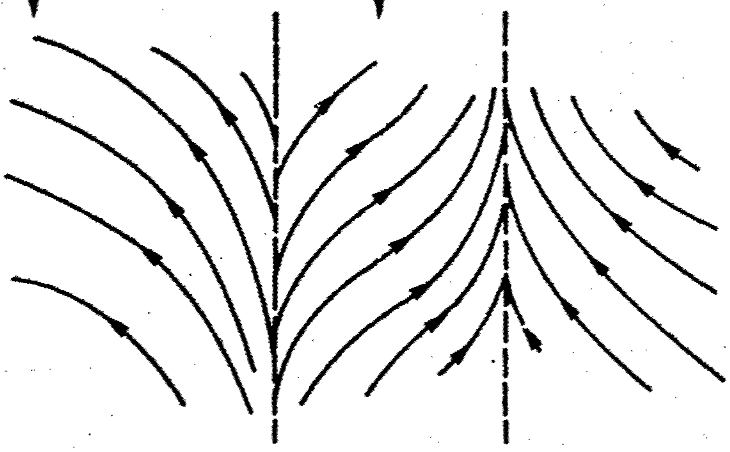


FIGURE 9 - SURFACE STREAMLINES OBSERVED MOVING WITH THE INTERNAL WAVE. As $\overline{U}_s/C$ becomes of order of unity, a detached eddy forms, with both a divergence and a convergence line, as in the bottom diagram.

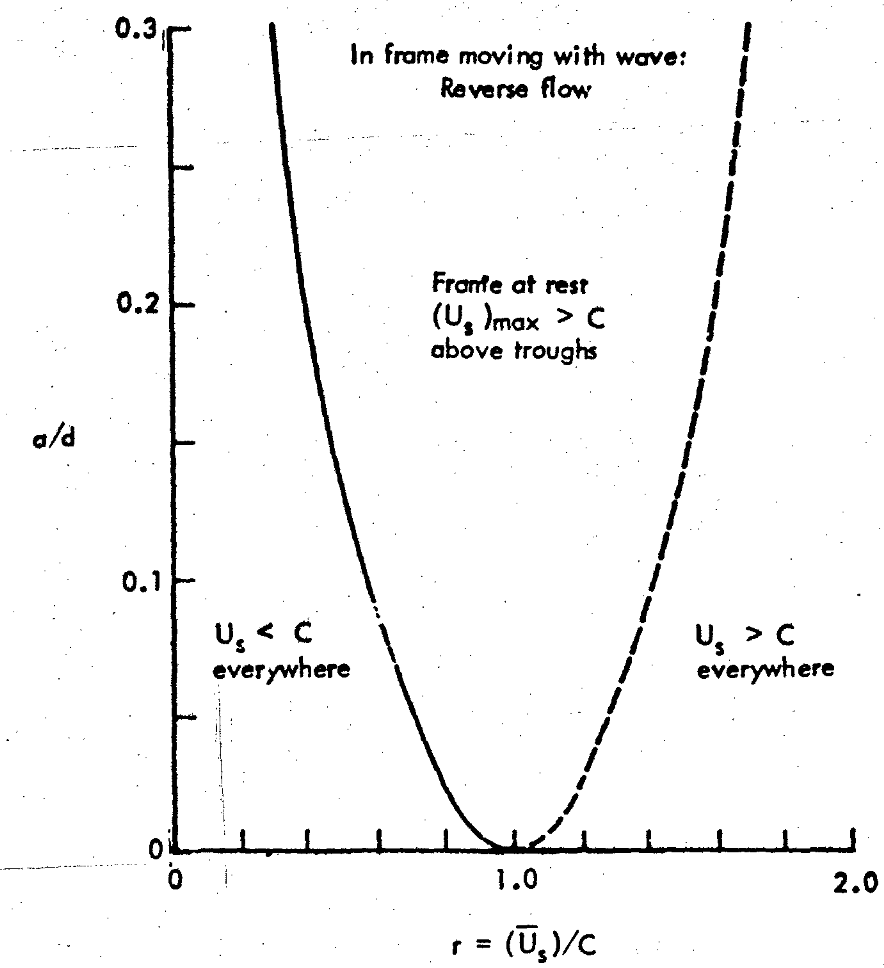


FIGURE 10 - PARAMETER RANGES WHERE REVERSE FLOW OCCURS.
The broken line is of low accuracy.

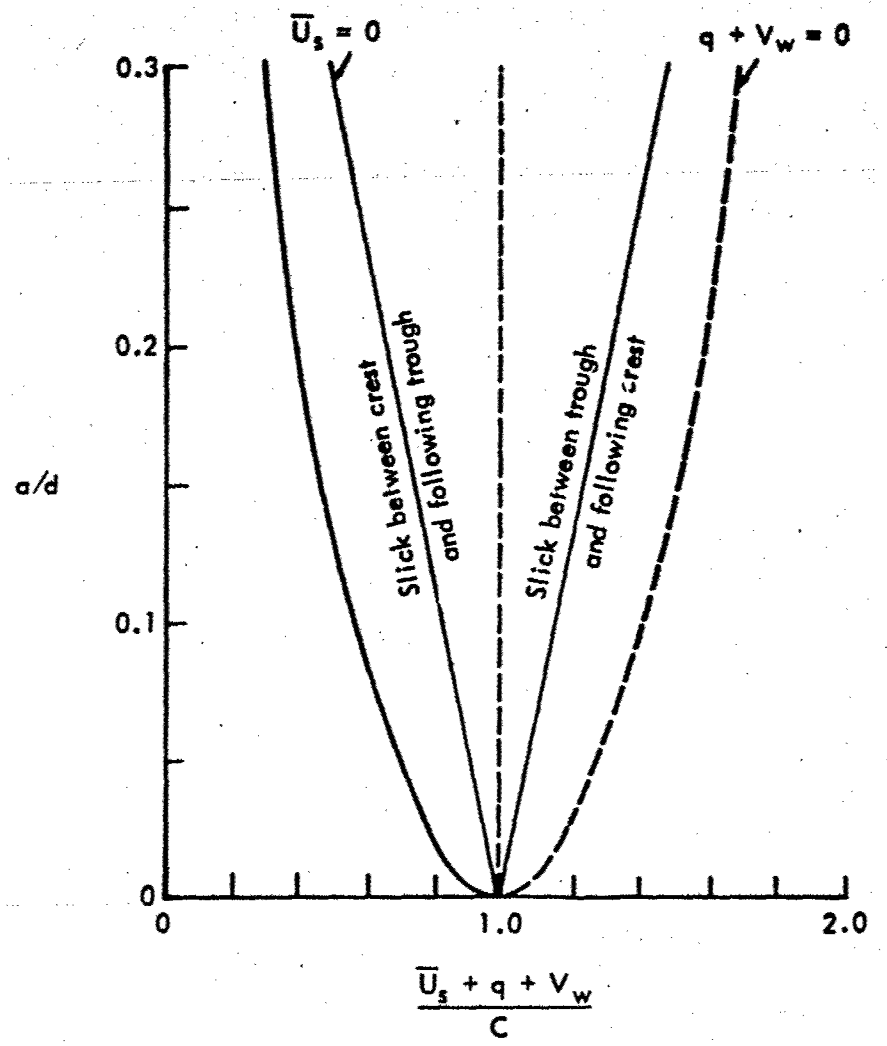


FIGURE 11 - THE CONDITIONS FOR FORMATION OF A SATURATED SLICK IN GENERALLY DISPERSED SURFACE MATERIAL. Slicks will be formed under conditions that correspond to points inside the bowl if $V_w + q = 0$, or inside the V-shaped region if the surface current vanishes. a = internal wave amplitude, d = thermocline depth.

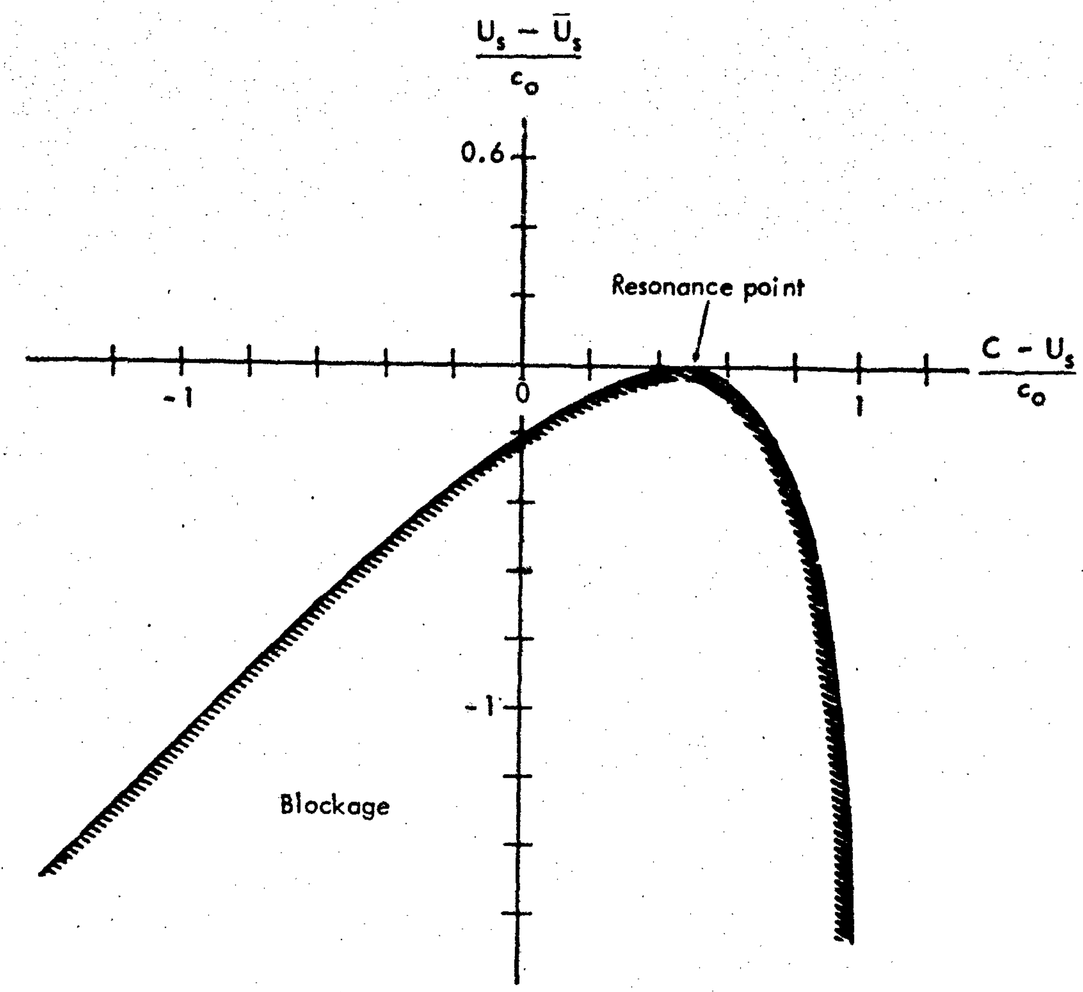


FIGURE 12 - BLOCKAGE CONDITIONS FOR SURFACE WAVES OF SPEED c_o WITH INTERNAL WAVES CF SPEED C AND A SURFACE DRIFT, WHOSE MEAN VALUE IS $\overline{U}_s$ AND WHOSE MINIMUM IN THE PRESENCE OF OF INTERNAL WAVES IS U_s. Note that when the variations in surface current $U_s - \overline{U}_s \rightarrow 0$, blockage is reduced to a single point: $C - \overline{U}_s = (\frac{1}{2})\, c_o$. the resonance condition (5.8).

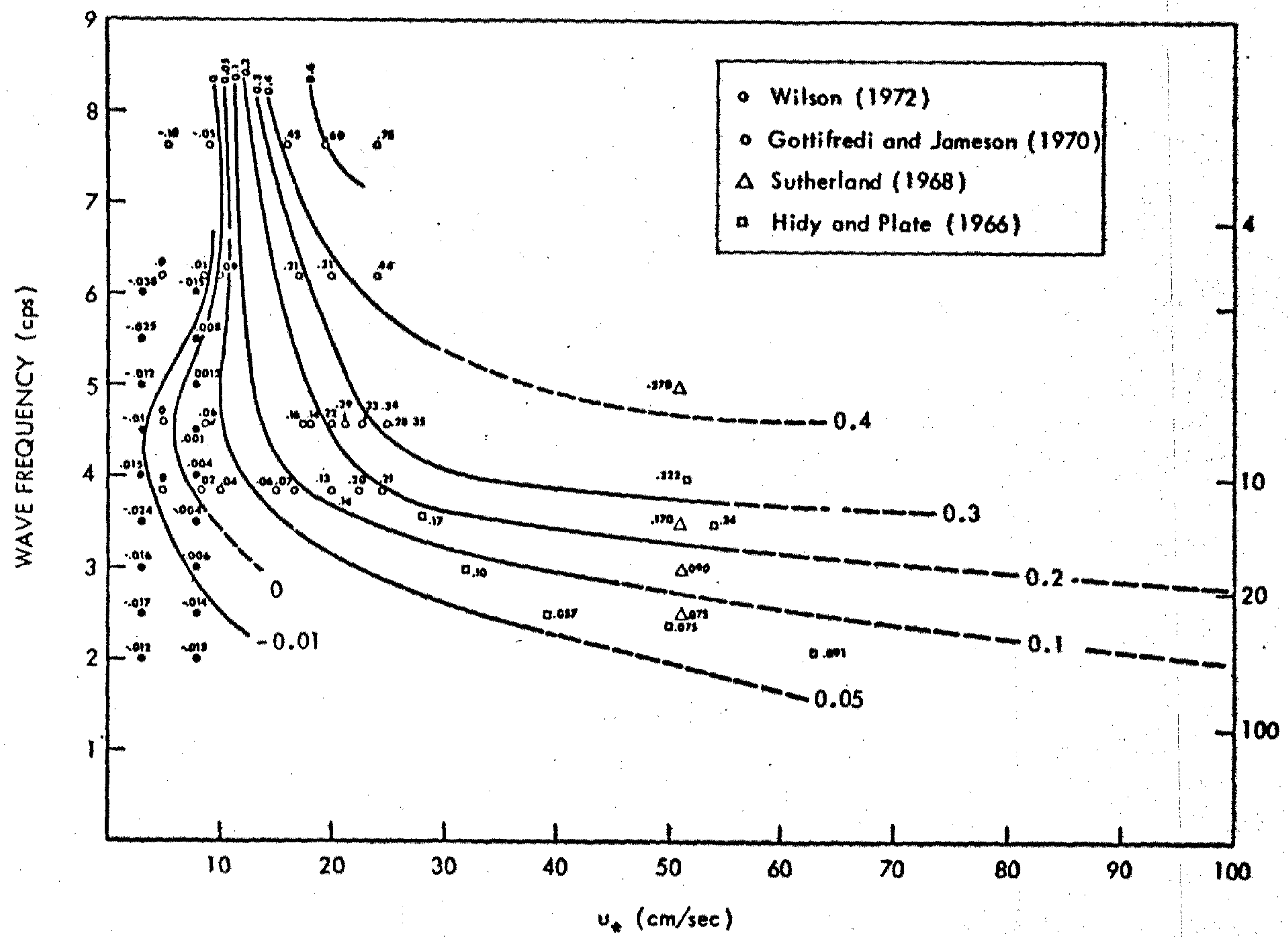


FIGURE 13 - EXPLORATORY RESULTS: CONTOURS OF THE GROWTH RATES OF WIND-GENERATED WAVES (sec^{-1}) AS A FUNCTION OF WAVE FREQUENCY AND FRICTION VELOCITY.

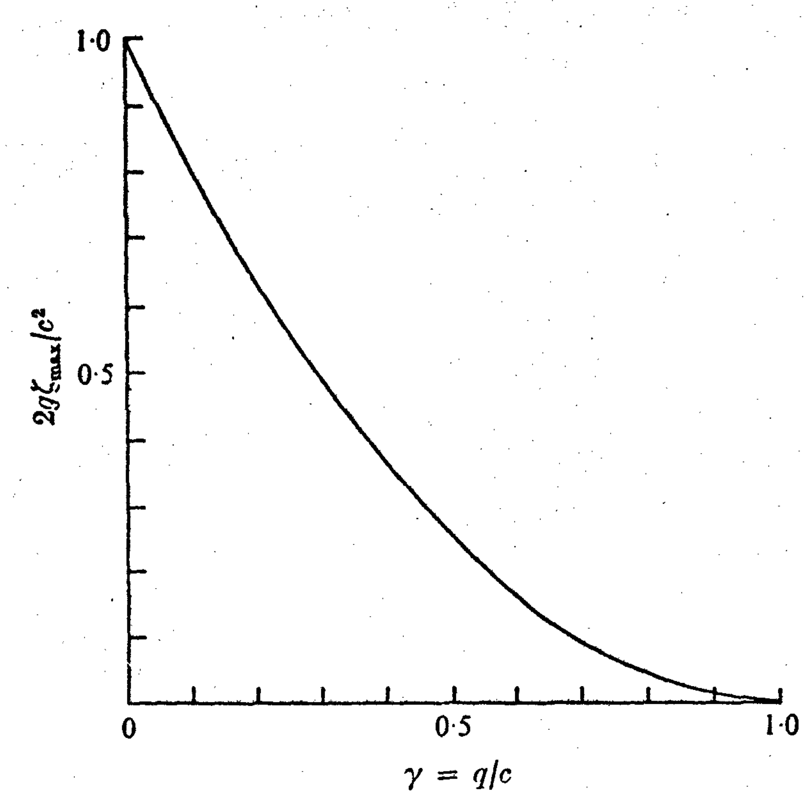


FIGURE 14 - THE MAXIMUM ELEVATION ζ_{max} ABOVE MEAN WATER LEVEL THAT CAN BE ATTAINED WITHOUT WAVE BREAKING, AS A FUNCTION OF THE SURFACE DRIFT q AT $\zeta = 0$.

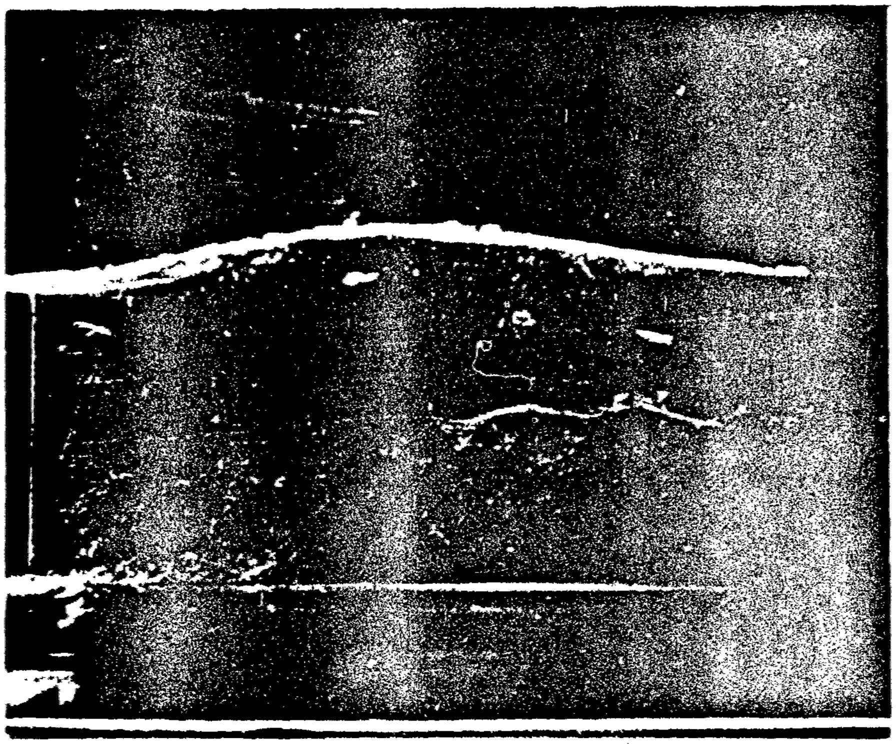


FIGURE 15 - STREAK LINES IN A SMALL SCALE BREAKING WAVE.
For identification, see Figure 16.

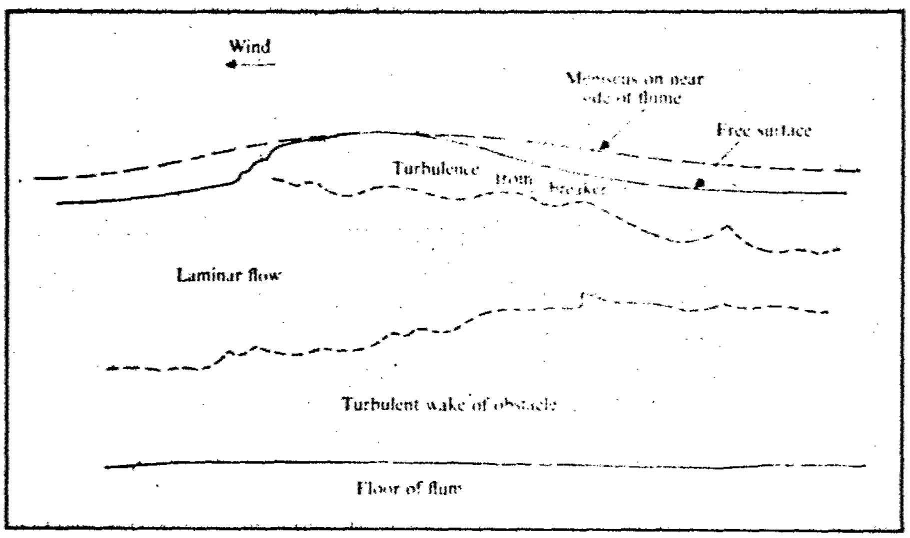


FIGURE 16 - AN IDENTIFICATION OF FIGURE 15.
The undisturbed water depth is 15 cm.

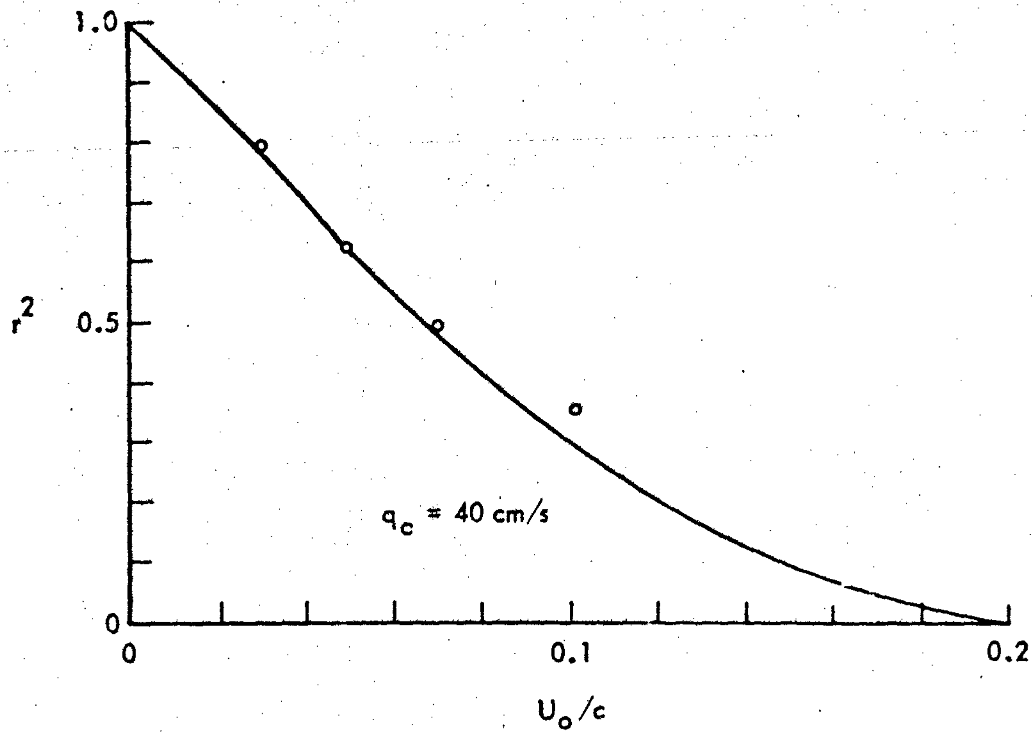


FIGURE 17 - THE REDUCTION IN ENERGY DENSITY OF SMALL-SCALE BREAKING WAVES PRODUCED BY SUPERIMPOSED LONGER WAVES AFTER PHILLIPS AND BANNER (1974). The measurements are by Mitsuyasu (1966); the line represents the theoretical calculation, with no adjustable constants.

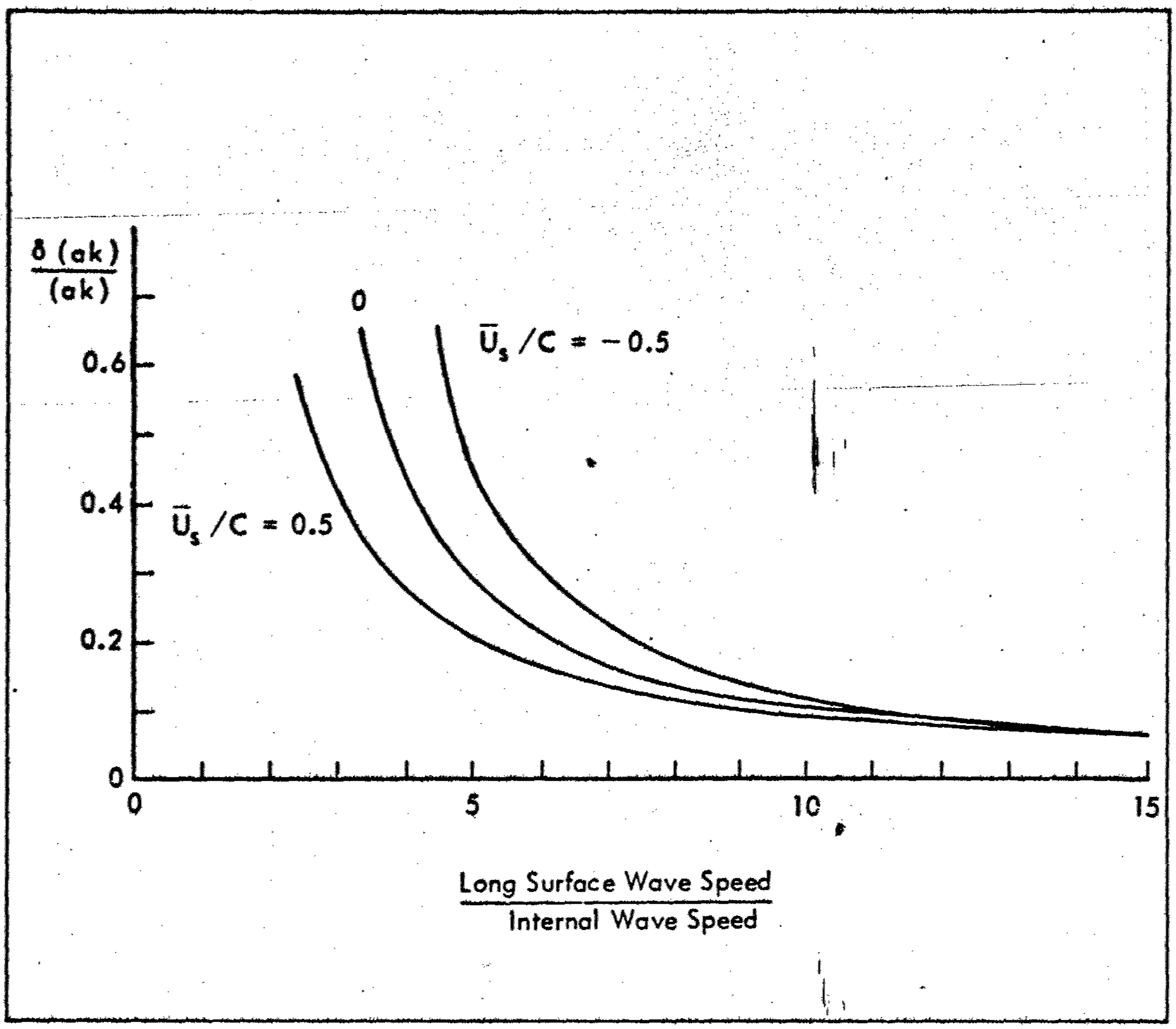


FIGURE 18 - RELATIVE MODULATIONS IN LONG WAVE SLOPE PRODUCED BY INTERNAL WAVES WITH $\delta U_x/C = 0.2$. The modulation amplitude is linear in this factor. The effect of mean surface current is shown; it is significant here (for given modulations in current) only for small values of C/C_ℓ .

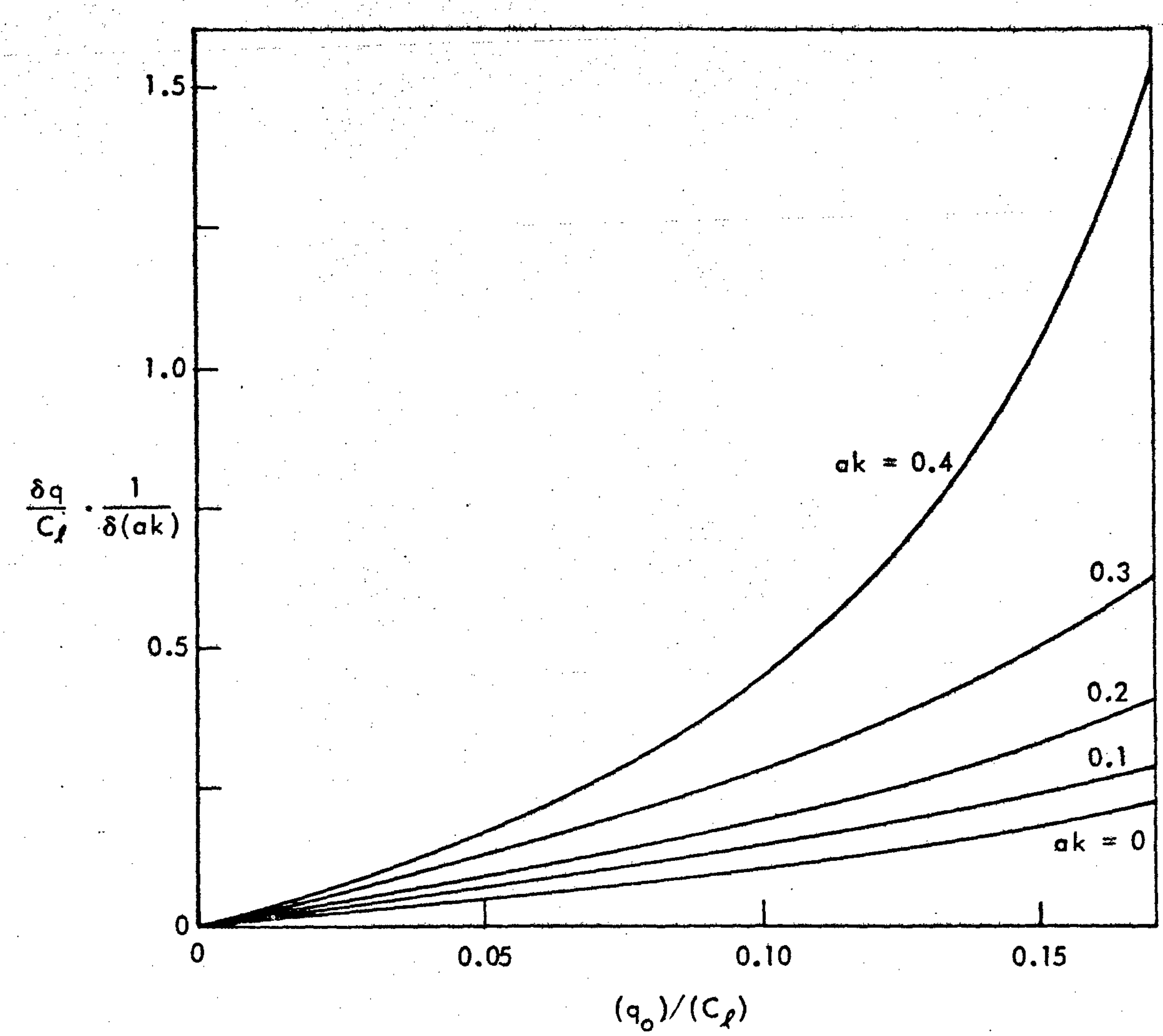


FIGURE 19 - VARIATIONS IN SURFACE DRIFT AS A FRACTION OF INTERNAL WAVE SPEED, DIVIDED BY MODULATIONS IN LONG WAVE SLOPE, FOR VARIOUS VALUES OF THE LONG WAVE SLOPE AS A FUNCTION OF SURFACE DRIFT. See Eq. (6.11).

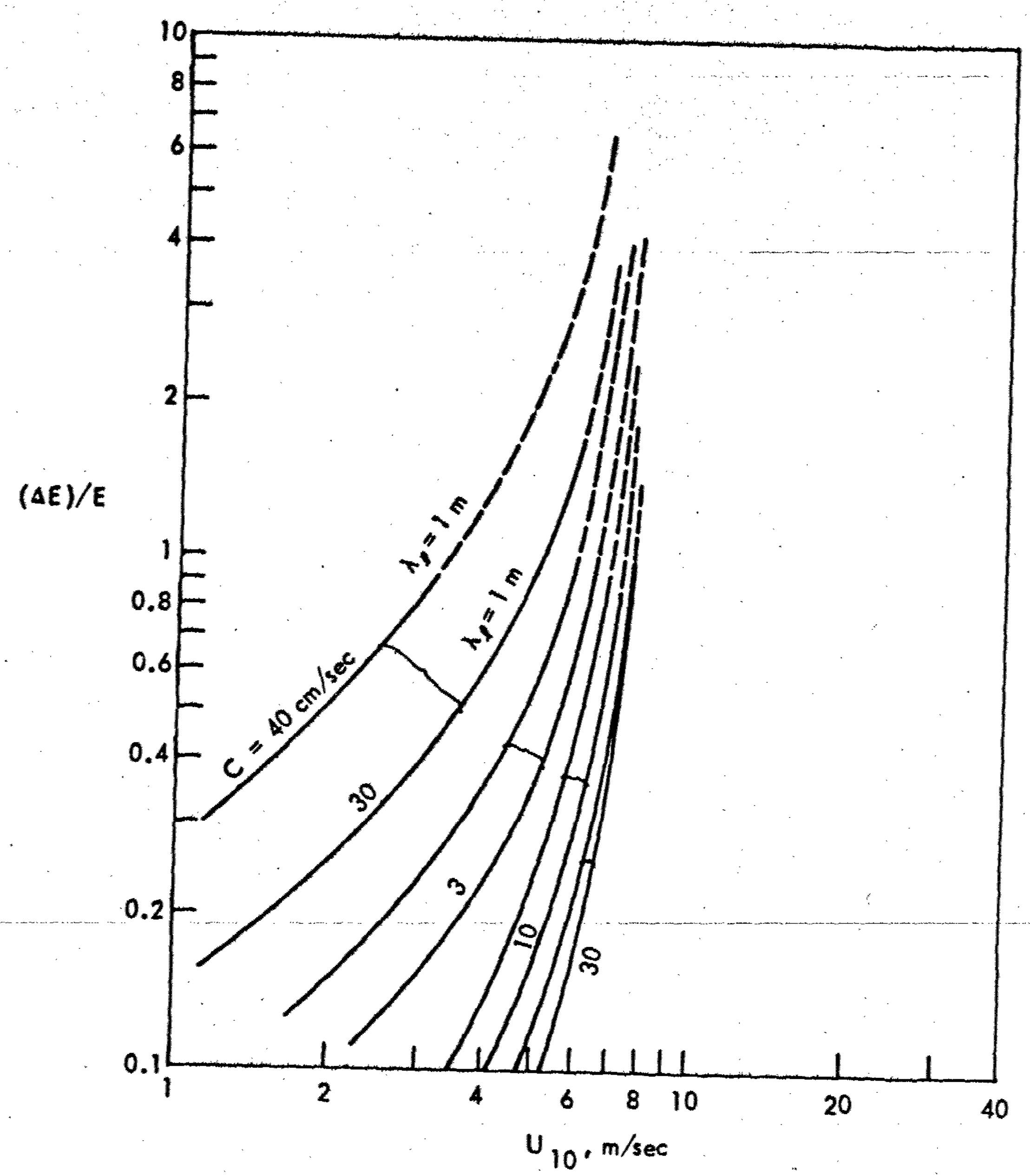


FIGURE 20 - PROPORTIONAL MODULATIONS IN ENERGY DENSITY OF SHORT BREAKING WAVES AS A FUNCTION OF WIND SPEED. Short wavelength ≈ 2.5 cm, surface current = 0, long wave slope = 0.2. The four pairs of curves, bracketed, are for long wavelengths of 1 meter, 3 meters, 10 meters, and 30 meters, the left-hand member of each pair being for an internal wave speed of 40 cm/sec, the other for 30 cm/sec.

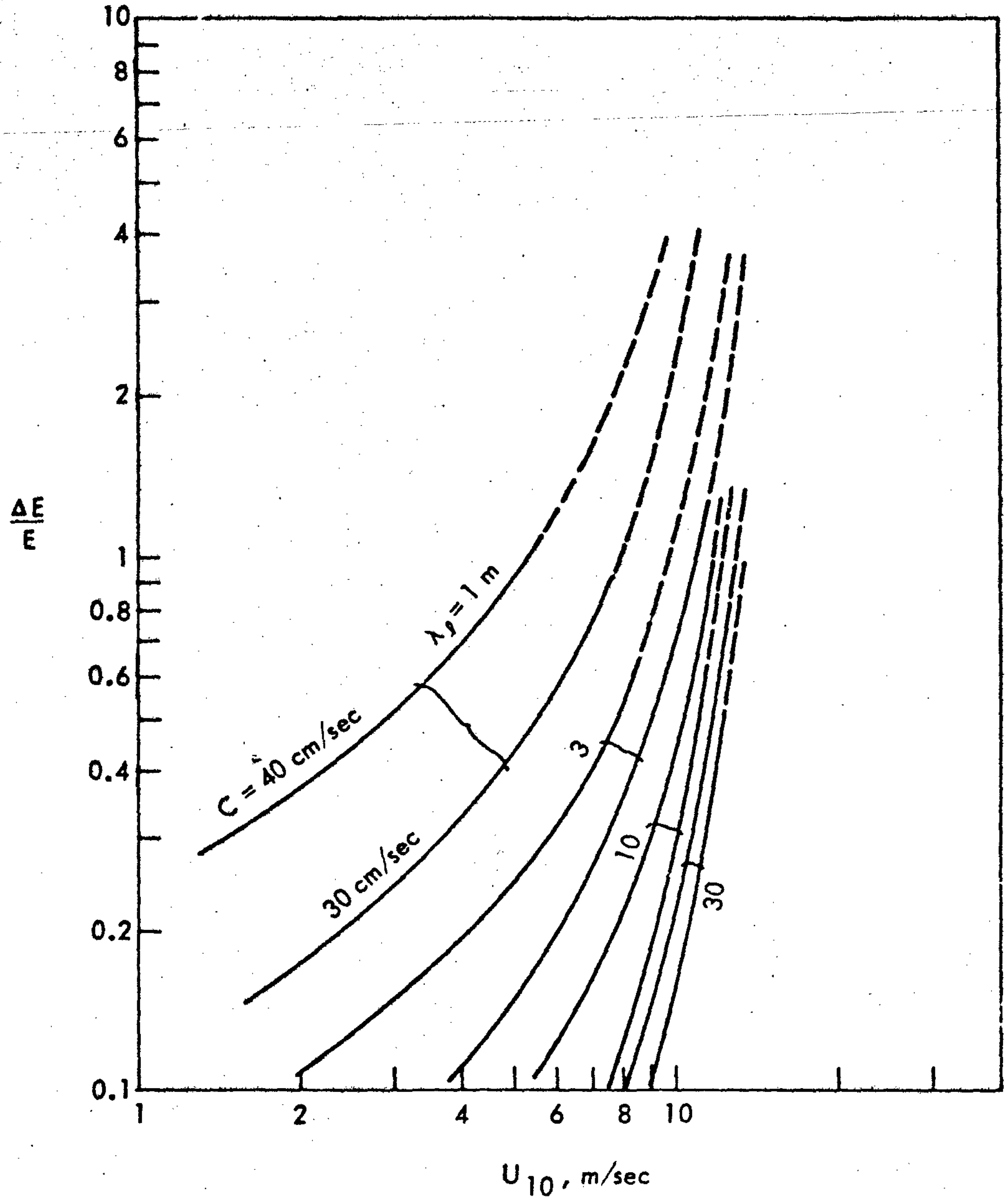


FIGURE 21 - PROPORTIONAL MODULATIONS IN ENERGY DENSITY OF SHORT BREAKING WAVES AS A FUNCTION OF WIND SPEED. Short wavelength = 10 cm, surface current = 0, long wave slope = 0.2. The four pairs of curves, bracketed, are for long wavelengths of 1 meter, 3 meters, 10 meters, and 30 meters, the left-hand member of each pair being for an internal wave speed of 40 cm/sec, the other for 30 cm/sec.

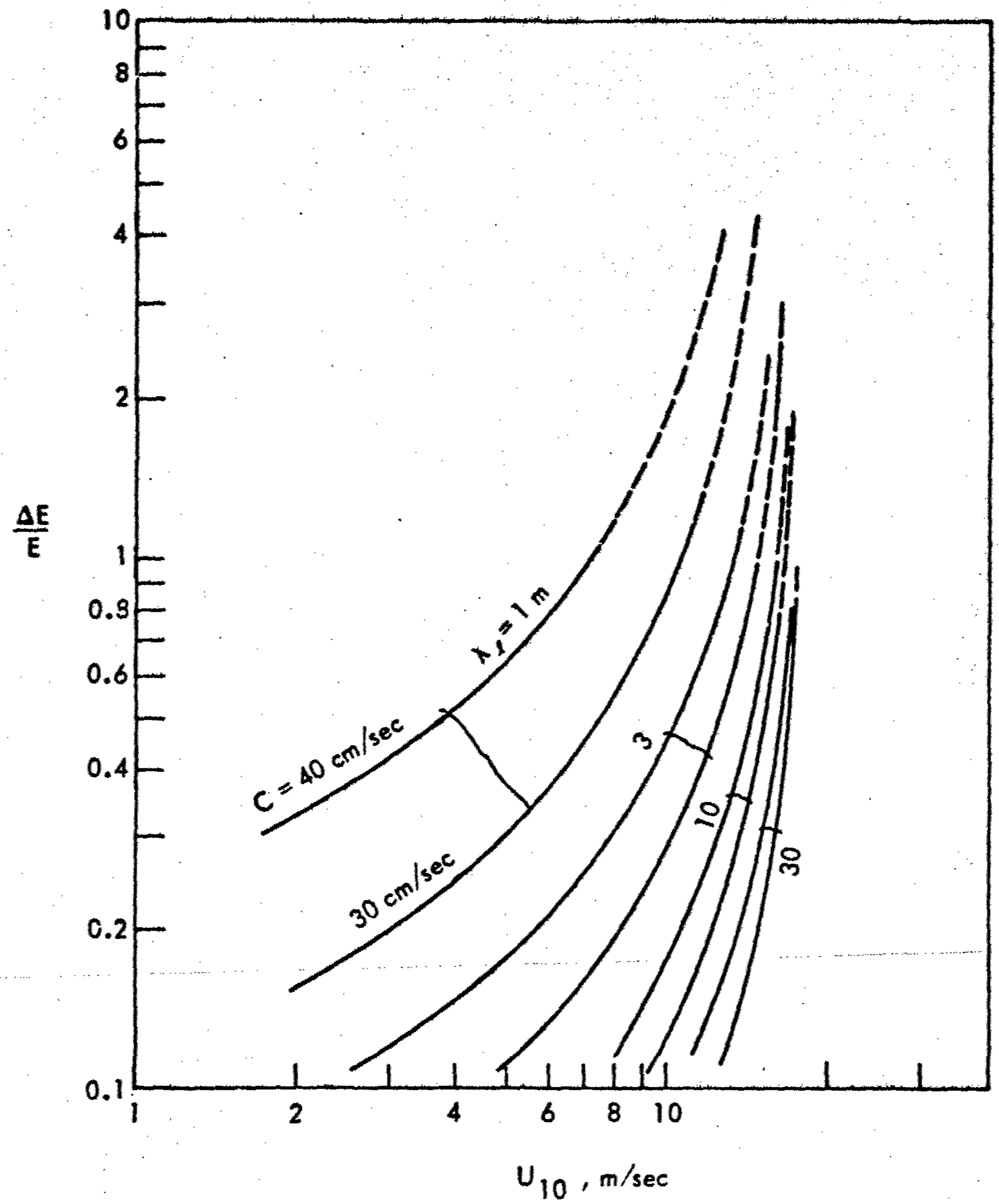


FIGURE 22 - PROPORTIONAL MODULATIONS IN ENERGY DENSITY OF SHORT BREAKING WAVES AS A FUNCTION OF WIND SPEED. Short wavelength = 20 cm , surface current = 0, long wave slope = 0.2. The four pairs of curves, bracketed, are for long wavelengths of 1 meter, 3 meters, 10 meters, and 30 meters, the left-hand member of each pair being for an internal wave speed of 40 cm/sec, the other for 30 cm/sec.

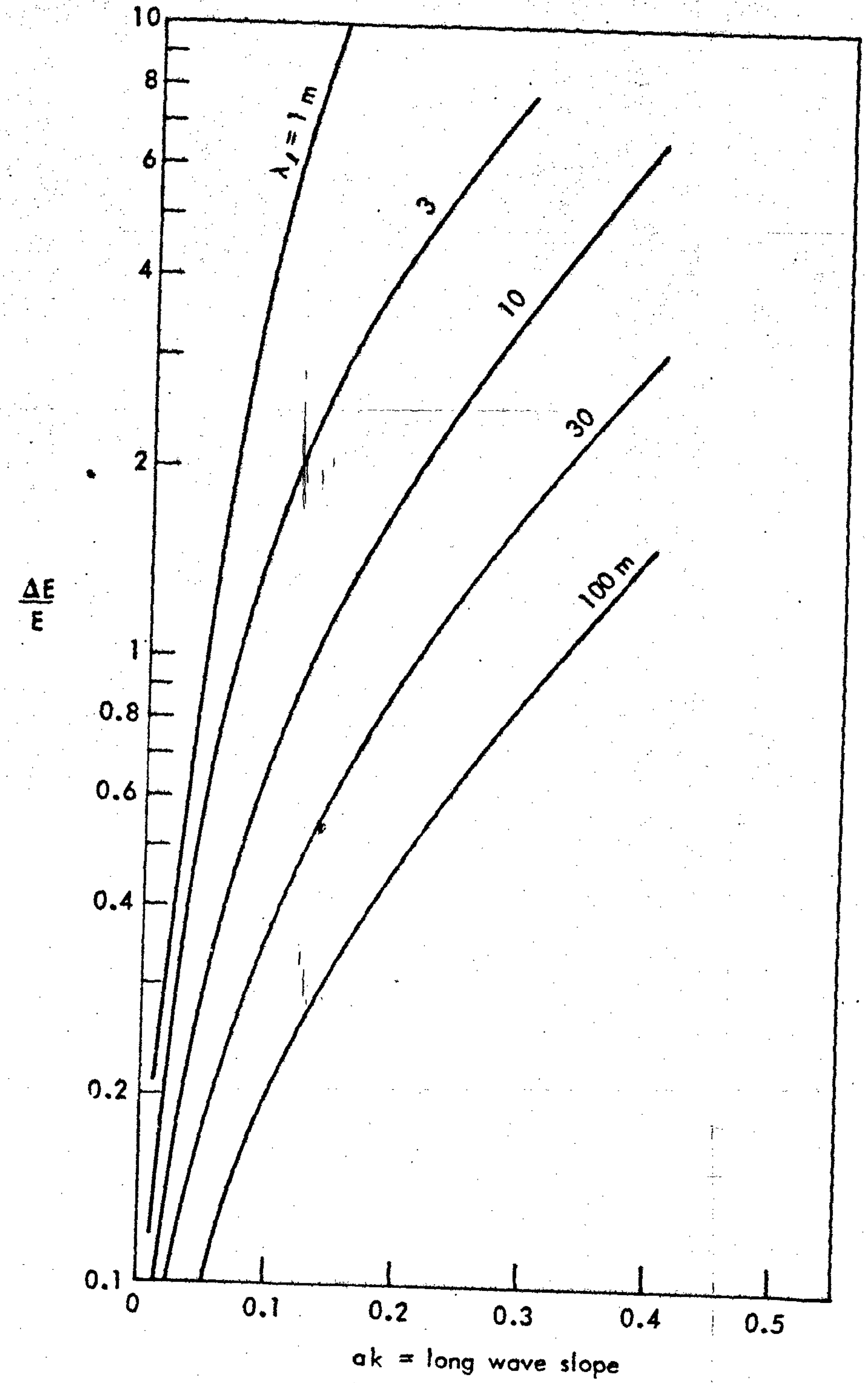


FIGURE 23 - FRACTIONAL MODULATIONS IN 2.5 cm WAVE ENERGY DENSITY, PRODUCED BY LOCAL BREAKING, AS A FUNCTION OF LONG WAVE SLOPE. No surface current, wind speed = 7.5 m/sec. Wavelengths of the long waves, 1 meter, 3 meters, 30 meters, and 100 meters.

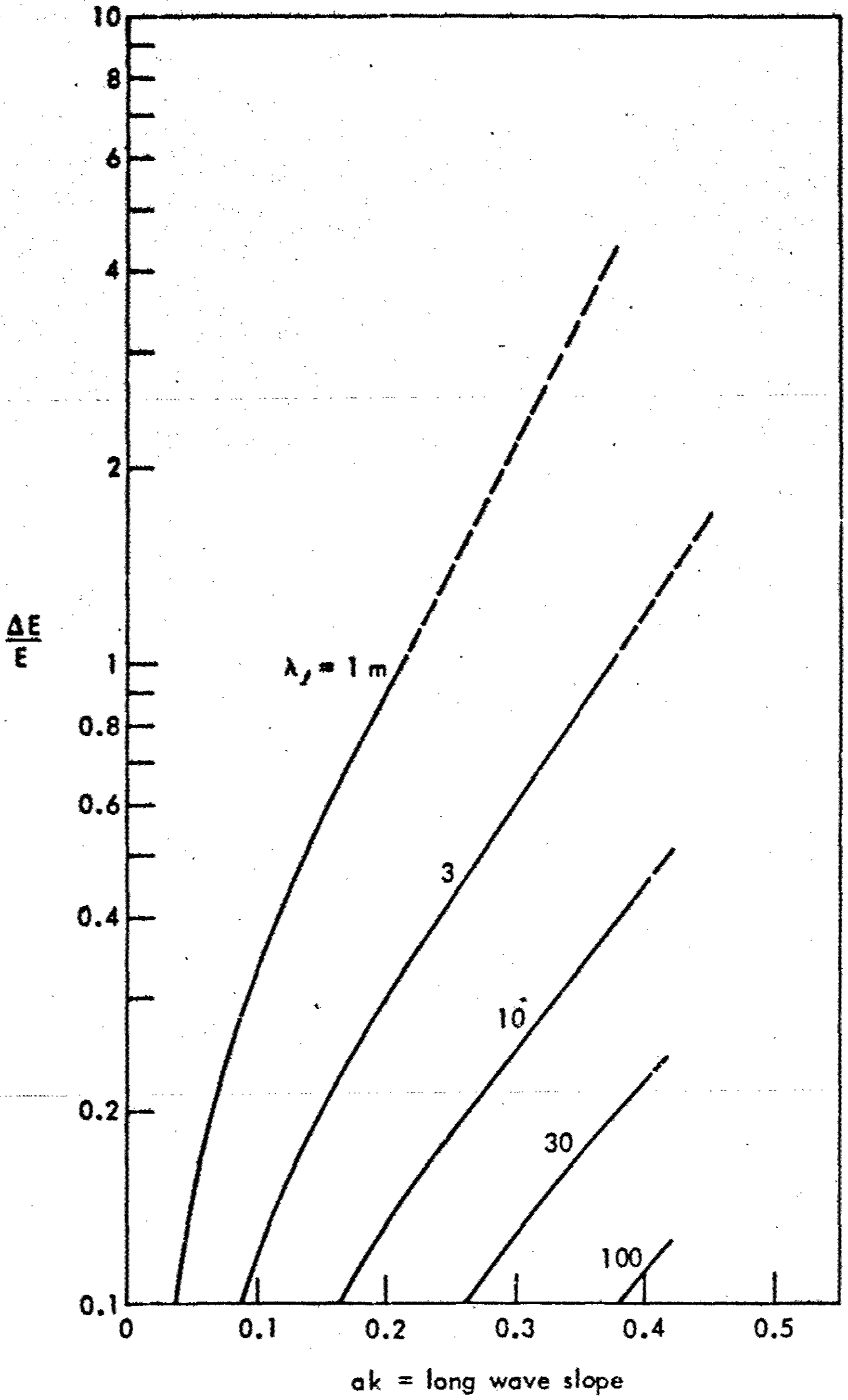


FIGURE 24 - FRACTIONAL MODULATIONS IN 10 cm WAVE ENERGY DENSITY, PRODUCED BY LOCAL BREAKING, AS A FUNCTION OF LONG WAVE SLOPE. No surface current, wind speed = 7.5 m/sec. Wavelengths of the long waves, 1 meter, 3 meters, 30 meters, and 100 meters.

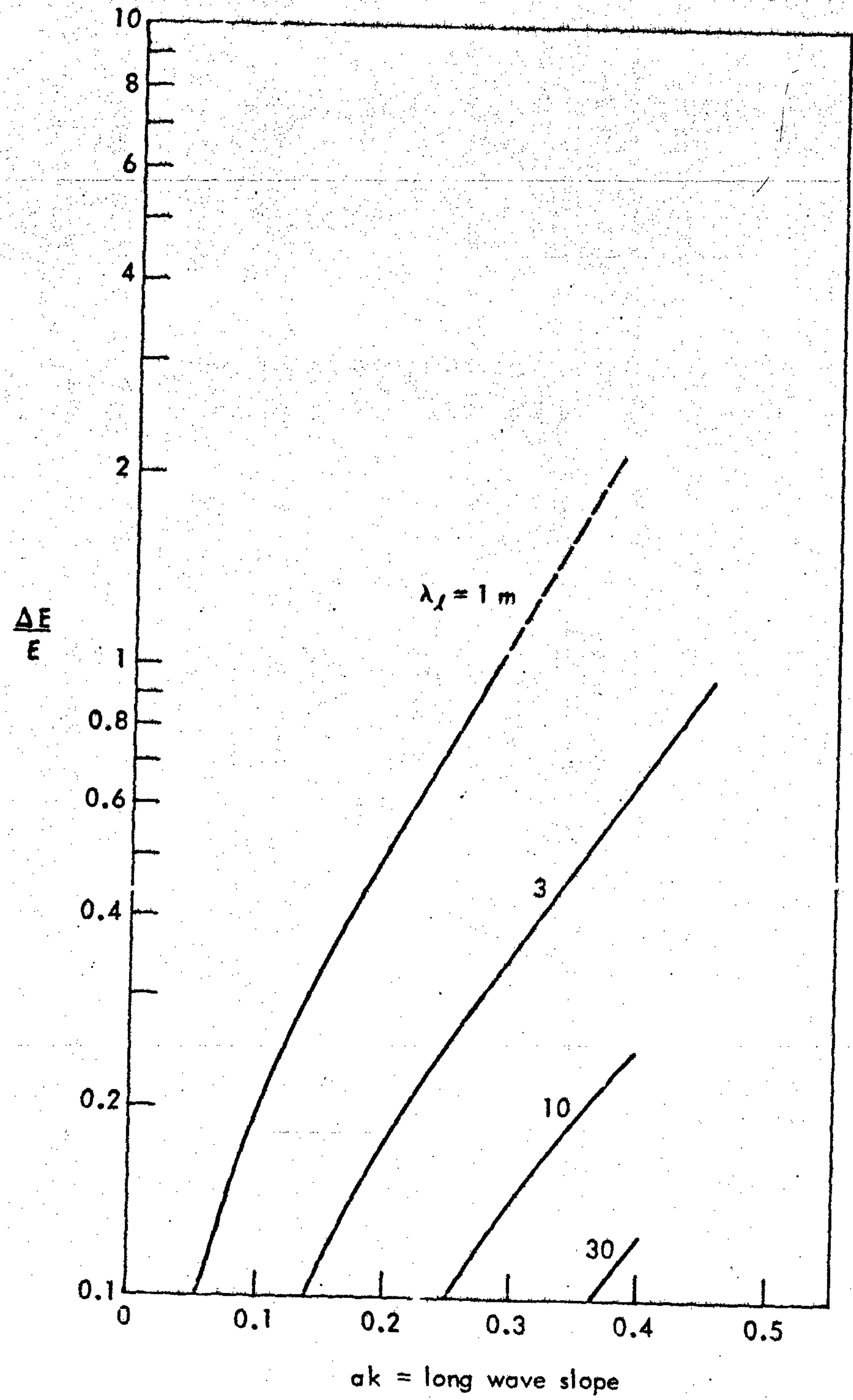


FIGURE 25 - FRACTIONAL MODULATIONS IN 20 cm WAVE ENERGY DENSITY, PRODUCED BY LOCAL BREAKING, AS A FUNCTION OF LONG WAVE SLOPE. No surface current, wind speed = 7.5 m/sec. Wavelengths of the long waves, 1 meter, 3 meters, 30 meters, and 100 meters.

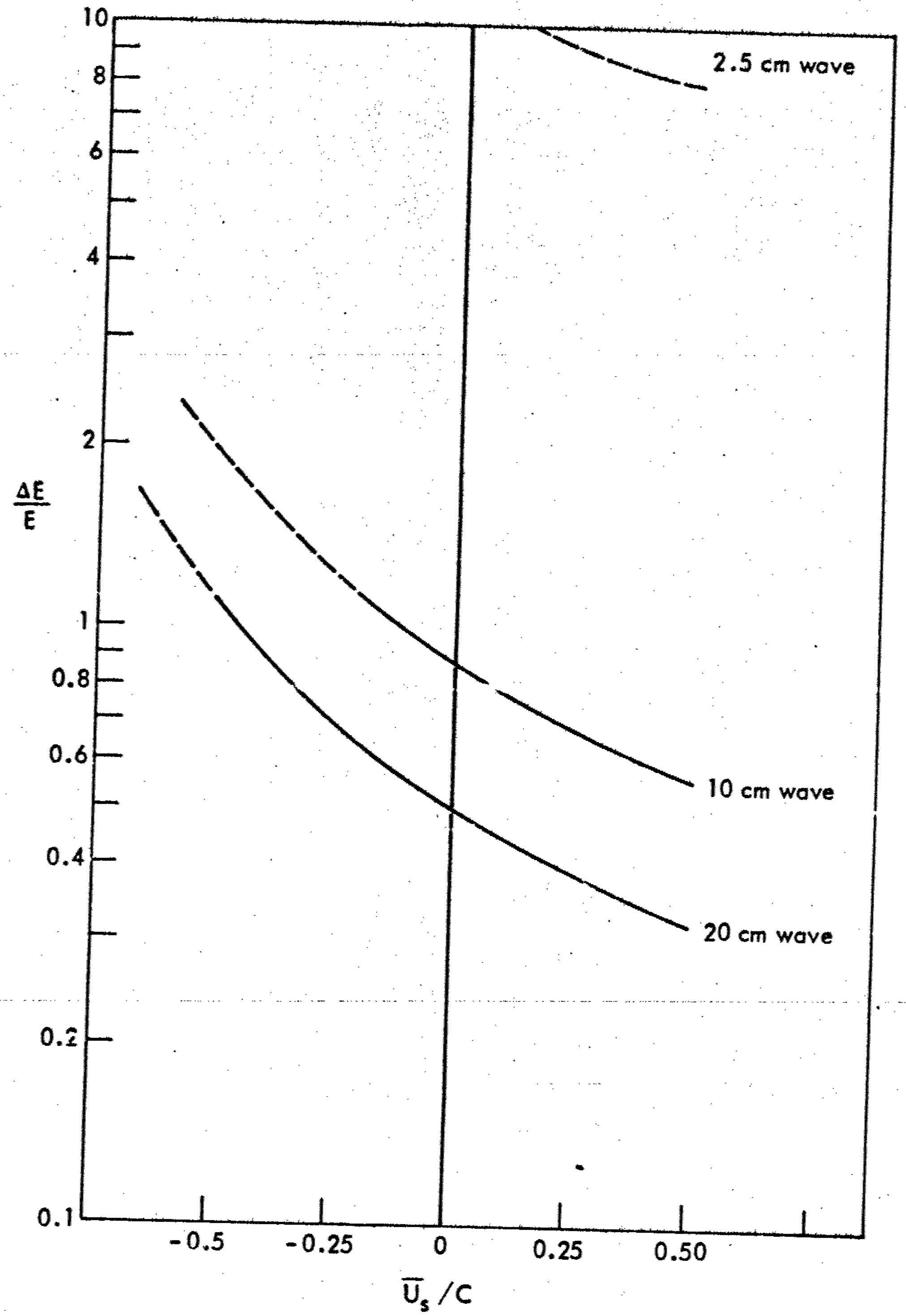


FIGURE 26 - THE FRACTIONAL MODULATIONS IN ENERGY DENSITY FOR SHORT WAVELENGTHS OF 2.5, 10, and 20 cm AS A FUNCTION OF MEAN SURFACE CURRENT, BUT WITH A CONSTANT VALUE OF 0.2 FOR THE VARIATIONS IN SURFACE CURRENT. Longer wavelengths = 1 m, internal wave speed = 30 cm/sec, wind speed= 7.5 m/sec.

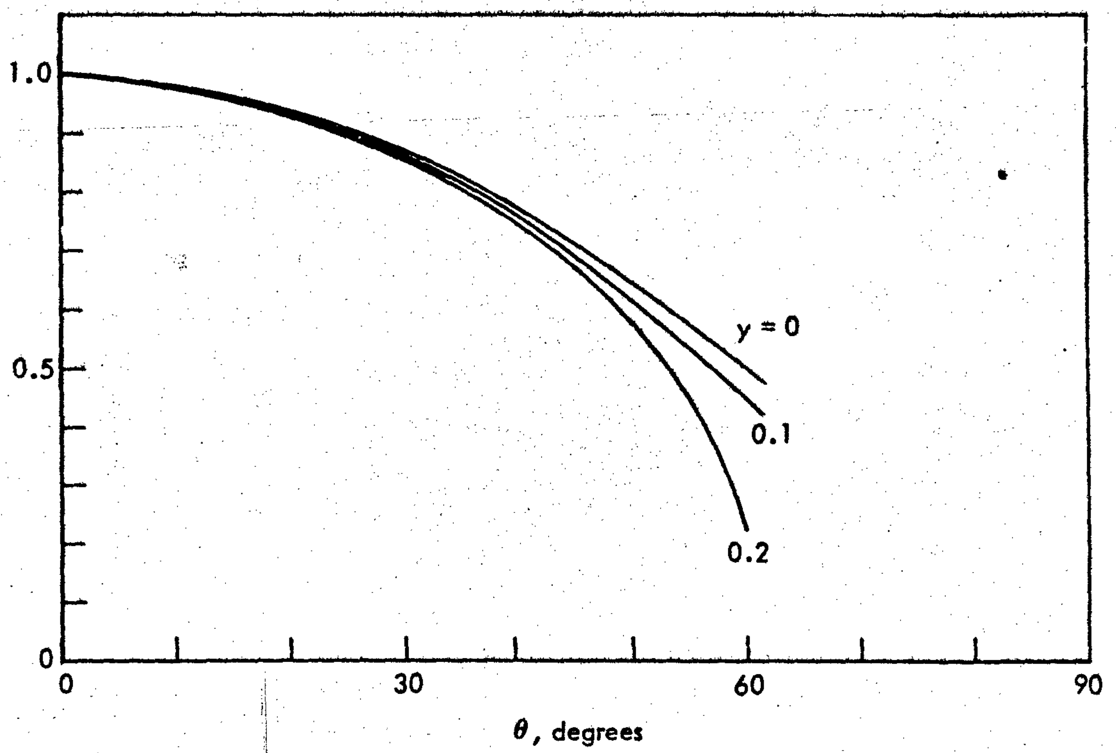


FIGURE 27 - THE DIRECTIONAL RESPONSE IN SHORT WAVE ENERGY MODULATIONS AS A FUNCTION OF ANGLE BETWEEN LONG SURFACE WAVES AND THE INTERNAL WAVE. When this angle is as large as 60°, the magnitude of the modulations is reduced only by about one-half.