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AN ANALYTICAL STUDY OF THE EFFECTS OF SURFACE ROUGHNESS ON BOUNDARY-LAYER TRANSITION

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stability theory, and is related to transition by an empirical transition criterion. Comparisons between the predictions of the model and available experimental results for incompressible boundary layers are in reasonable agreement. In addition, the model indicates that for large roughnesses, the roughness completely controls the transition location, regardless of other parameters in the problem. Similar conclusions have also been suggested by experimental results.

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by

C. L. Merkle, T. Kubota, and D. R. S. Ko

ABSTRACT

An analytical model has been developed to describe the manner in which distributed surface roughness affects transition. The model pictures the roughness as having two distinct effects: one, it introduces higher disturbance levels in the boundary layer; and two, it alters the mean velocity profile and, hence, the growth rate of the disturbances. The alteration of the mean velocity profile is described by means of a "turbulent sublayer", which visualizes an enhanced momentum transfer in a narrow layer next to the surface. The corresponding change in the amplification of disturbances is then determined by means of linear stability theory, and is related to transition by an empirical transition criterion. Comparisons between the predictions of the model and available experimental results for incompressible boundary layers are in reasonable agreement. In addition, the model indicates that for large roughnesses, the roughness completely controls the transition location, regardless of other parameters in the problem. Similar conclusions have also been suggested by experimental results.

1. INTRODUCTION

As a re-entry vehicle descends, the boundary layer on its nosetip may be either laminar, transitional or turbulent, depending on the altitude and the trajectory of the body. These variations in the local structure of the boundary layer affect both the drag of the vehicle and the rate of heat transfer to the body. The changes in the heat transfer characteristics are particularly important because of the coupling that exists between the heating rate and the shape of the surface of the ablating nosecone. This coupling causes an initially sphere-cone configuration to have different "equilibrium" shapes, depending on whether the boundary layer is laminar or turbulent. Thus at high altitudes, where the boundary layer is completely laminar, the nosetip is somewhat more blunt than at low altitudes, where the boundary layer is almost completely turbulent. However, between these two regimes, transition from laminar to turbulent flow occurs on the nosetip. In general, the transition point moves forward with time at a rate determined by the flight trajectory, but if transition remains fixed at a given location on the nose for an extended period, "irregular" shapes may form. The formation of such irregular shapes can result in the deterioration of the vehicle performance and may even lead to catastrophic failure because of unpredictable aerodynamic forces or ablation rates. Recent test data have also shown that these irregular shapes can give rise to violent oscillations of the shock layer near the nosetip, hence, leading to conditions which can exceed the designed margins of safety.

As indicated above, the appearance of abnormal nosetip shapes is intimately tied to the transition location of the nosetip boundary layer; however, in spite of a good deal of research, the factors which influence boundary-layer transition are not completely understood. These factors include surface roughness, pressure gradient, wall temperature, local Mach number, and even the free-stream noise level. Of these various factors, experimental evidence has indicated that surface roughness is frequently the most important variable in controlling the location of transition on the nosetip. Although a considerable experimental data base is available to describe the dependence of the transition location on most of these parameters, very little of this data is capable of explaining why or how these factors influence transition. Consequently, it is difficult for the designer to use these data to obtain an accurate estimate of the locus of the transition point on the nosetip during re-entry, or to choose the trajectory in a reliable manner so as to preclude the appearance of an abnormal nosetip shape.

This Report presents the results of one phase of a research program which is aimed at understanding the mechanisms which lead to transition in a re-entry environment, so that improved transition prediction techniques can be developed. In particular, this Report is concerned with describing and predicting the effects of distributed surface roughness on transition by means of an analytical model. The approach is based on applying the results of linear stability theory to the transition problem. One advantage of this approach is that one can consider the various parameters separately and, thereby, determine their individual effects. Once these individual effects have been assessed, the inclusion of multiple effects into a single problem can be easily accomplished. All the work which is discussed herein is limited to incompressible flow. The justification for using incompressible flow as the first step in predicting transition on a re-entry body are several. First, we are interested in investigating the effects of roughness in the simplest possible flow field so as to more easily understand the basic phenomena. Second, the only relevant experimental data which describes the mechanism whereby roughness affects transition is incompressible. Third, a good base of linear stability calculations is available for incompressible flow (due, in part, to the fact that there are fewer parameters governing boundary-layer stability in incompressible flow, as compared to compressible flow), and a good deal of work has been done previously to relate these linear stability results to transition. Thus, the approach is to first prove the model in incompressible flow and then include the effects of other variables later.

The phenomena which occur during transition have been determined from a number of basic experiments (see the review papers by Tani, 1969 and Morkovin, 1969). Based on these results, we can describe transition by means of several steps, some of which may be absent in certain situations. Transition may be viewed as starting from a laminar boundary layer within which small disturbances are present. As the boundary layer grows thicker, a narrow band of the disturbance spectrum starts to grow in a linear fashion (Tollmien-Schlichting waves). Eventually these amplified disturbances become so large that they extract a finite amount of energy from the mean flow and cause an alteration in the mean flow profile. This altered mean flow profile possesses a local scale which is considerably smaller than the boundary-layer thickness and which causes a second band of frequencies (much higher in frequency than the Tollmien-Schlichting waves) to become amplified. The rapid amplification of these higher frequencies generate the beginnings of a turbulent spot which spreads as the flow moves downstream until the entire stream becomes turbulent.

Of these processes, only the linear amplification of waves can be described by stability theory. However, as long as the waves remain linear, the stability results represent a solution of the complete Navier-Stokes equations and, hence, describe the boundary-layer phenomena accurately. However, when the waves begin to interact with the mean flow profiles, linear stability theory ceases to be appropriate. Consequently, at this point empiricism must enter, although the formal extensions of stability theory to non-linear regimes can be used as a guide for this empiricism. Because of this limitation to linearity, stability theory can only be used to predict the beginning of transition. It cannot be extended to determine the end (and, hence, the length) of transition. Nevertheless, since pre-transition phenomena are exactly described by the stability equations, a basic understanding of these mechanisms can be obtained from the linear stability approach.

2. BACKGROUND, EXPERIMENTAL EVIDENCE

Much of the initial work in the field of linear stability theory was performed by Tollmien and Schlichting and has been reviewed in Schlichting's text (1960). Their results showed that for a laminar boundary layer on a flat plate there was a range of frequencies and Reynolds numbers over which small disturbances inside the boundary layer would be amplified. The existence of these theoretically predicted "Tollmien-Schlichting waves" was first detected experimentally in the classic experiments by Schubauer and Skramstad (1948). In these experiments, Schubauer and Skramstad introduced an artificial single-frequency disturbance into the boundary layer by means of a vibrating ribbon. By measuring the amplitude of the wave as it was swept downstream, they were able to ascertain the rate of growth or decay of the wave for a series of Reynolds numbers and frequencies. Comparisons between their measured neutral stability lines (for which the amplitude of the artificially induced waves remained constant) and the predictions of linear stability theory were in good agreement. Later, more accurate numerical calculations of the Tollmien-Schlichting waves resulted in excellent predictions of both the stream-wise growth rate and the cross-stream variation of the amplitude of the fluctuations. Additional experiments by Schubauer and Klebanoff (1955), Klebanoff, Tidstrom and Sargent (1962), and others gave further substantiation of the linear stability theory and also established that the existence of growing Tollmien-Schlichting waves served as a precursor to transition in many boundary layers of practical interest.

More recently, the measurements by Wells (1967) and Spangler and Wells (1968) have shown that transition is strongly dependent upon both the amplitude and the spectrum of the disturbance environment. In these experiments, the transition Reynolds number was delayed to values which were nearly twice as high as the ones observed by Schubauer and Skramstad, even though the magnitude of the free-stream disturbances was about the same for both experiments. Further, by changing only the spectral content of the external disturbance environment, while maintaining its amplitude constant, they were able to change the transition Reynolds number by more than a factor of five. Although exact comparisons of these experiments with the predictions of linear stability theory have not been reported, Spangler and Wells did note that when most of the energy in the disturbance spectrum was in the region corresponding to the unstable portion of the boundary layer (as predicted by linear stability theory), the location of transition was strongly influenced by changes in the magnitude of the external disturbances. Conversely, when most of the energy in the external disturbance was in the stable portion of the boundary-layer spectrum, the location of transition was nearly independent of the free-stream disturbances. This experimental verification that transition is dependent on the spectrum as well as the amplitude of the disturbance environment is, again, evidence of the relationship between stability and transition.

Similar verifications of the presence of linearly amplified waves in supersonic boundary layers have been given by Laufer and Vrebalovich (1960) and Kendall (1971). Their results show that the growth of artificially induced waves is in close agreement with the linear stability calculations of Mack (1971).

Thus, it is quite well substantiated that transition often begins as a linear amplification of Tollmien-Schlichting waves, whose growth continues until non-linear effects set in. These non-linear effects then extract finite amounts of energy from the mean flow, causing the mean flow profiles to depart from their laminar shape. This alteration of the mean flow profile in turn affects the amplification of disturbances, and marks the "start of transition", as measured experimentally.

All the experiments discussed above were concerned with investigating and documenting the phenomena of transition to turbulence in boundary layers on smooth walls. As a result of these investigations, a reasonably clear understanding of the mechanisms which are important during, and preceding, smooth-wall transition has been developed. However, for the corresponding case of transition in boundary layers over rough walls, much less information is available.

A number of experimenters have shown that surface roughness reduced the transition Reynolds number. The magnitude of the reduction depends on the shape and the location of the roughness elements. For example, a single two-dimensional roughness element has a different effect on the transition location than does a single three-dimensional element; a summary of these effects is given in the review paper by Tani (1969). The type of surface roughness which is expected on the nosetip of a re-entry vehicle is not like that of either of these two special cases. Instead, the nosetip has a large number of roughness elements distributed over the surface. The characterization of these numerous elements requires the definition of a length scale (or scales). One commonly chosen scale is the height of the roughness elements or the "effective" height of the elements when there is a statistical distribution of roughness heights. However, a second important length scale is the distance between the elements or alternatively, their number per unit surface area. In addition, it is noted that experimental evidence has indicated (see references cited by Tani) that the effects of roughness on transition are strongly dependent on the height of the element. Thus on a surface with distributed roughness, the height and number per unit area of the very largest elements may completely dominate the effect of the roughness on transition. Consequently, a description of the roughness characteristics is important in a model of the effect of distributed roughness on transition. In the model to be discussed below, this description has been bypassed, and it has been assumed that an effective roughness height is known. Some efforts to determine the roughness characteristics of a typical nosetip surface have been reported by Powars (1973).

Most of the roughness experiments reviewed by Tani (1969) have been concerned with determining the magnitude of the effect of roughness on transition, rather than with the mechanisms which are involved. In the absence of explicit experimental evidence, several potential mechanisms have been formulated to explain the observed experimental results. For example, there has been a general class of transition which is traditionally tied to a "by-pass" mechanism (see, for example, Morkovin, 1969) in which transition is believed to be independent of stability. Among the variables included in this class are the effects of roughness. The foundation for the bypass theory is based on experimental evidence such as that reported by Dryden (1959) in which transition on rough surfaces occurred at Reynolds numbers at or below the critical Reynolds number of linear stability theory. While such bypass mechanisms undoubtedly do exist, the effects of roughness on the mean profile (as described below) can significantly lower the critical Reynolds numbers so that results, such as those cited by Dryden, can still be dependent on stability mechanisms.

Another suggested mechanism for the effect of roughness on transition is the wavy-wall analogy. However, in order for a wavy wall to affect transition, the characteristic length of the waviness must be on the order of a boundary-layer thickness (i.e., a wavelength similar to the Tollmien-Schlichting wavelength). Although this effect cannot be completely ruled out, we believe that because of the small characteristic length associated with the roughness on a re-entry vehicle nosetip (as compared to the boundary-layer thickness), this effect is of minor importance.

Finally, a third potential mechanism for the effects of roughness on transition is one which views an interaction between the roughness and the mean velocity profile. This effect has been observed by Klebanoff and Tidstrom (1972). This experiment represents the only experiment known to the authors which is concerned with determining the mechanism whereby roughness affects transition. In this experiment, a cylindrical, two-dimensional roughness element was placed on the surface of a flat plate at a stream-wise location at which the boundary layer was still laminar, but was sufficiently thick to completely submerge the cylinder. Measurements taken inside the laminar boundary layer downstream of the cylinder indicated that the fluctuations were amplified or damped as they were swept downstream, depending on their frequency. Comparisons with computations from flat plate linear stability theory showed that the experimentally measured amplification rates were much higher than the predicted values. However, when the predicted amplification rates were computed from the measured mean velocity profiles (which were distorted from the Blasius shape by the presence of the wire), the results were in good agreement with the measured growth rates.

In addition to this increased amplification rate, which indirectly increased the fluctuation level, Klebanoff and Tidstrom found that the roughness element also generated additional fluctuations in the laminar boundary layer by a direct method. Consequently, Klebanoff and Tidstrom's findings can be summarized by noting that roughness affects transition in the following two ways:

1. The presence of surface roughness alters the mean velocity profiles in such a manner that disturbances in the laminar boundary layer are amplified at a faster rate.
2. The presence of surface roughness generates additional disturbances in the boundary layer and, hence, changes the initial disturbance level before amplification begins.

This experimental observation not only further justifies the link between instability and transition, but also forms the foundation of the approach reported herein.

In the following sections of this Report, an analytical model which describes the effects of distributed surface roughness on the mean velocity profile is developed. This model is then applied to the prediction of transition in incompressible boundary layers over walls with distributed roughness. It is noted that the above physical observations were made for flow over a single, two-dimensional roughness element, not for flows over walls having distributed roughness. In the absence of definitive experimental evidence linking distributed roughness to transition, we have applied this information on the physics of transition behind a two-dimensional roughness element to the analysis of transition on a surface with distributed roughness.

It should be noted that the present approach bypasses some of the phenomena which are present in the transitional boundary layer on the nosetip of a re-entry vehicle. Because of the ablative surface of the nosecone, the roughness is, itself, coupled to the flow processes. A schematic of the complete, coupled process is shown as Fig. 1. Referring to this figure, nosetip transition is initially affected by the surface roughness which, as indicated above, causes a two-fold change in the boundary layer: it increases the local disturbance level, and it also causes local changes in the mean flow profile. The mean flow changes alter the stability characteristics of the boundary layer and these altered stability characteristics, plus the increased disturbance levels, cause a change in the energy transfer between the mean flow and the disturbances, which results in a further alteration of the mean profile shapes. These adjustments in the mean profile cause changes in the ablation characteristics which, in turn, affect the roughness of the surface and, hence, complete the coupling between the flow and the surface characteristics. Finally, if these mean flow adjustments are computed by a non-linear stability theory, we could then deduce the onset of transition directly by observation of the mean profile shapes. In order to simplify the analysis, we have removed the coupling between the disturbances and the mean flow (the non-linear stability effects) by means of an empirical transition criterion so that we can deduce transition directly from the linear stability results. In addition, the results in this Report do not include a computation of the material ablation characteristics but rather, assume that the roughness characteristics of the surface are specified.

3. TURBULENT SUBLAYER MODEL FOR THE EFFECT OF ROUGHNESS ON TRANSITION

The analytical model for the effect of distributed roughness on transition pictures the flow over the many individual roughness elements as having an unsteady nature which is similar to that commonly observed behind isolated bodies at intermediate Reynolds numbers. This postulated unsteadiness could occur in the form of either a vortex street or wake turbulence. (Order-of-magnitude estimates indicate that the local Reynolds number of the flow over a roughness element, based on the roughness height and the velocity at this height, which is just large enough to begin to affect the location of transition, is near the Reynolds number at which vortex shedding would begin in a uniform flow.) The model views these unsteady velocity fluctuations near the wall as a source of augmented momentum transfer. This increased momentum transfer near the surface is modelled by means of a local eddy viscosity. Consequently, in contrast to the familiar laminar sublayer which exists near the wall in a fully turbulent boundary layer, the surface roughness model pictures a "turbulent sublayer" beneath the laminar boundary layer on a rough wall. Because of this high effective viscosity near the wall, a small velocity gradient is expected there. By "pulling down" the velocity profile near the wall, the overall mean velocity profile for the zero pressure gradient case develops an inflection point which is dynamically more unstable, according to the linear stability theory. Thus, this modified mean profile results in substantially increased linear amplification rates and, hence, earlier boundary-layer transition.

A schematic comparison between the above-described "turbulent sublayer" model and the effective viscosity distribution in a typical turbulent boundary is shown in Fig. 2. This figure also shows that we expect the effective viscosity, which approximates the momentum transfer by the unsteady velocities near the wall, to be a maximum there and to vanish away from the wall. Again, by analogy with isolated bodies in a free stream, we expect the "width" of the turbulent sublayer to be of the same order of magnitude as the local roughness height. Thus, mathematically we represent the effective viscosity in the "turbulent sublayer" as

$$\frac{\varepsilon_{eff}}{\nu} = \frac{\varepsilon_{max}}{\nu} F(y/k) \tag{1}$$

where the function, F(y/k), is a function which is a maximum near the wall, decreases as we move away from the wall, and vanishes at distances which are large compared to the roughness height, k. In analogy to wake flow, we assume

$$\frac{\varepsilon}{\nu} = \frac{\varepsilon_{max}}{\nu} e^{-\beta(y/k)^2} \qquad \beta = \mathcal{O}(1) \tag{2}$$

where β is a constant which is required to be of the order of unity so that width of the region of amplified momentum transfer is similar to the roughness height.

In order to obtain a reasonable estimate for the numerical values of the constants in the model, we turn to analogous mechanisms for which experimental data are available. For a turbulent wake, the eddy viscosity can be expressed as (Schlichting, 1960)

$$\varepsilon = K_\varepsilon' \; B_{1/2}(U_e - U_{\mathbb{C}}) \;, \tag{3}$$

where K_ε' is an empirically determined constant, and $B_{1/2}$ is the half-width of the jet. If we re-write Eq. (3) in terms of the characteristic dimensions of our problem, we have

$$\left(\frac{\varepsilon}{\nu}\right)_{MAX} = K_\varepsilon'' \frac{U_k k}{\nu} = K_\varepsilon'' \, Re_k \;, \tag{4}$$

where k refers to the roughness height which characterizes the surface, and U_k is the velocity at the top of the roughness elements. The Reynolds number, Re_k, which is defined by Eq. (4), will be referred to as the roughness Reynolds number. To accout for differences between the momentum transfer in the wake of an isolated body and the wake behind a roughness element on a surface, we have shown the empirical coefficient, K_ε'', in Eq. (4) as different from the analogous proportionality constant in Eq. (3). From Schlichting (1960), we find $K_\varepsilon' = \mathcal{O}(0.1)$ and we, likewise, require $K_\varepsilon'' = \mathcal{O}(0.1)$.

Finally, we expect the momentum transfer by the unsteady, roughness-induced velocities to have some threshold Reynolds number below which the flow is completely stable so that no "turbulent" momentum transfer takes place in the region near the wall. Again, appealing to the case of low Reynolds number flow over an isolated body, experimental results show that the flow field is completely stable up to some particular Reynolds number, after which unsteadiness begins. This threshold Reynolds number has been introduced into the model by allowing the coefficient, K , to have a van Driest-type dependence on the local Reynolds number. Thus, we represent

$$K_\varepsilon'' = K_\varepsilon \left[1 - \exp(-Re_k/A^+)\right] \ . \tag{5}$$

Based on experimental results taken from bodies in an undisturbed flow, we expect this threshold Reynolds number to be of the order of 40 (corresponding to the Reynolds number at which the vortex street behind a cylinder begins).

In summary, the model of the effect of roughness on transition pictures an additional momentum transport near the wall which augments the transfer of momentum by molecular viscosity. This additional momentum transfer is included by means of an effective viscosity which is large near the wall ("turbulent sublayer") and diminishes to the molecular viscosity far from the wall. The complete mathematical formulation can be obtained by grouping Eqs. (2), (4), and (5):

$$\frac{\varepsilon}{\nu} = 1 + K_\varepsilon Re_k \left\{1 - \exp(-Re_k/A^+)\right\} \exp\left[-\beta(y/k)^2\right] \tag{6}$$

where, as indicated above, we expect

$$\begin{aligned} K_\varepsilon &= \mathcal{O}(0.1) \\ A^+ &= \mathcal{O}(40) \\ \beta &= \mathcal{O}(1) \end{aligned} \tag{7}$$

A qualitative picture of the effects of a viscosity distribution such as this is shown in Fig. 3. Here, the relative shapes of the fully developed turbulent boundary layer with its steep gradient at the wall is compared with the less steep Blasius profile. The rough wall mean velocity profile, in which the viscosity varies in the opposite manner from that for the turbulent boundary layer (see Fig. 2), is less steep near the wall than the Blasius profile and contains an inflection point. It is emphasized that Fig. 3 is intended to be exemplary in nature, and that the relative differences between the boundary-layer profiles have been exaggerated in order to more clearly discuss their qualitative differences.

Having obtained a tentative model of the effects of distributed surface roughness on the mean velocity profile, it remains to determine the effects these alterations will have on the stability (and hence, transition) characteristics of a laminar boundary layer. Two separate procedures for computing the effects of these profile changes on the stability characteristics of the boundary layer are given below. The two procedures are, respectively, an "exact" procedure and an "approximate" procedure. Although there is no difficulty in applying the exact procedure, the results presented in this Report are based solely on the approximate technique. The reason for this choice of the approximate technique over the exact technique was to give us a rapid, efficient evaluation of the potential of the turbulent sublayer model for predicting changes in transition Reynolds numbers caused by the presence of surface roughness. The approximate technique made use of currently available tabulations of linear stability calculations, so that no new stability calculations were required. In view of the favorable comparisons between the turbulent sublayer predictions and experimental results, additional calculations based on the exact analysis are planned. Although the discussions of the two procedures of predicting transition from linear stability results are presented in terms of the rough wall analysis, they can be made to apply to the prediction of transition on smooth walls by simply taking the effective viscosity to be everywhere constant.

3.1 "Exact" Analysis

In order to calculate the stability properties of a laminar boundary layer, the mean flow profiles must first be known, as discussed in standard textbooks (Betchov and Criminale, 1967). Thus, to predict transition, one must start from a knowledge of the stream-wise variation of the pressure field which is impressed on the boundary layer by the outer, inviscid flow field. Then, by means of a numerical solution of the laminary boundary-layer equations, the mean velocity profiles can be found along the expected laminar length of the body. Having obtained the cross-stream boundary-layer profiles, the linear stability equations can be integrated at each stream-wise location (or Reynolds number) to obtain the amplification rate of each individual frequency component as a function of the boundary-layer Reynolds number. This amplification rate describes the rate at which a disturbance grows as it is convected downstream. Thus, if the spectral portion of the disturbance in the boundary layer, which has the frequency, ω, is given by

$$\phi'(x,y,t) = \hat{\phi}(y) \exp\left[i(\alpha x - \omega t)\right] , \tag{8}$$

where ω is real, and $\alpha = \alpha_r + i\alpha_i$ is complex, then the spatial amplification rate is $(-\alpha_i)$. By integration, it can be shown that the amplitude, A, of this spectral component of the disturbance is related to its initial amplitude by

$$A/A_o = \exp\left\{-\int_{x_0}^{x} \alpha_i dx\right\}. \tag{9}$$

However, even though a large body of experimental evidence has shown that these linearly growing waves are a precursor to transition, transition cannot begin until non-linearities enter. That is to say, transition is an inherently non-linear phenomenon. Consequently, some additional information must be added to the linear analysis before we can proceed to the transition problem. Since the linear stability equations represent a first-order approximation to the Navier-Stokes equations, the non-linearities can formally be included into the analysis by solving additional sets of equations corresponding to higher-order approximations. An example of such a calculation has been given by Benney (1964). Even though such an approach seems promising and within the capability of present-day high-speed computers, we have chosen to include the non-linear effects by means of empirical information rather than by this purely mathematical procedure. Jaffe, Okamura and Smith (1968) have shown that if the linear stability theory is used to predict the growth of each individual spectral component of the disturbances in the boundary layer, a reasonable prediction of transition can be obtained by observing when the local amplification of the disturbance has first exceeded some critical value, $(A/A_o)_{CRIT}$. By comparison with experimental data, including both favorable and unfavorable pressure gradient cases, Jaffe, Okamura and Smith (1970) have shown that this critical value is

$$(A/A_o)_{CRIT} = e^n, \tag{10}$$

where the exponent, n, is of the order of 9 or 10.

One might object that the actual magnitude of the disturbance, A, rather than the total amount by which it has been amplified, A/A_o, would be a more plausible transition criteria. However, since initial disturbance levels, A_o, are seldom known (and their spectral content, known even less frequently), we have been forced to use the amplification ratio rather than the magnitude of the disturbance. The relative success of the "e^9" criterion is, in part, tied to the high amplification rates which occur in incompressible boundary layers, as well as to a probable reflection of the fact that most low-speed wind tunnels have similar disturbance environments. The high amplification rates ensure that the predicted transition Reynolds number is relatively insensitive to the magnitude of the transition criterion. Thus, the transition Reynolds number which would be obtained from an "e^{10}" criterion is only slightly higher than that Reynolds number corresponding to "e^9". Similarly, if most wind tunnels have similar background disturbances, the amplification criterion, $(A/A_o)_{CRIT}$, is equivalent to an amplitude criterion, A_{CRIT}.

Thus, once the mean flow field and the linear stability map for a given boundary layer has been computed, the transition Reynolds number can be deduced directly from the stability results by means of the empirical "e^9" criterion. Consequently, the effects of variables such as the local pressure gradient can be taken into account in a direct fashion, without need for additional experimental results. Likewise, the effects of surface roughness can be taken into account in the "e^9" method by means of the "turbulent sublayer" method.

3.2 "Approximate" Procedure

By contrast to the above exact procedure, an approximate procedure has been developed for the purpose of obtaining rapid assessments of the location of transition in the presence of coupled effects such as pressure gradients and surface roughness. This approximate method has been used to test the validity of the proposed turbulent sublayer model for the effects of distributed surface roughness. As yet, no comparisons between the exact and the approximate methods have been made, although such comparisons are planned.

The approximate method short-cuts the prediction of transition in the presence of smooth walls by first approximating the mean velocity profiles in the boundary layer as being composed of a series of similar (i.e., Falkner-Skan) profiles. This saves only a modest amount of computational time as compared to a completely numerical solution of the boundary-layer equations, but it allows the stability equations to be parameterized in terms of the pressure gradient parameter for similar flows. Thus, rather than re-computing the linear stability results for each arbitrary velocity profile of interest, the stability results can be tabulated once and for all for various values of the Falkner-Skan pressure gradient parameter, β. Further, an extensive tabulation of linear stability results has been published by Wazzan, Okamura and Smith (1968) so that the complete set of stability results which are necessary for the smooth wall problem is already available.

For the rough wall case, similar velocity profiles can again be obtained. An outline of their derivation is given in the Appendix. However, in the presence of surface roughness, the similar solutions contain two parametric variables: the pressure gradient, β (which was the only parameter in the smooth wall solutions), plus the roughness parameter, Re_k. Thus, in order to reduce the rough wall case to a single parameter dependence, we have defined a one-to-one approximation between the rough wall similar solutions and the Falkner-Skan solutions. The approximation assumes that the shape factor, $H = \delta^*/\theta$, completely characterizes the mean flow profile for either the rough or the smooth wall case. Thus, in the analysis, we first compute the rough wall mean velocity profiles (from the turbulent sublayer model). Then, we compute the local shape factor, H, and assume that the stability characteristics of the rough wall profile are identical to those of the Falkner-Skan profile, having the same shape factor. Finally, the transition Reynolds number is determined by the semi-empirical "e^9" criterion in a manner identical to that used for the exact analysis.

As indicated, the link between the rough wall profiles and the Falkner-Skan profiles is based on the shape factor. Thus, starting from the stability results for smooth wall Falkner-Skan flows, the transition Reynolds number, $Re_{x_{TR}}$, has been computed as a function of the pressure gradient parameter, β(using an "e^9" transition criterion). However, to each value of β, there corresponds a unique shape factor, H, so that we can express $Re_{x_{TR}}$ as a function of H, as shown in Fig. 4. Note that these results are based on an approximate mean velocity profile as well as an approximate viscosity distribution. That is to say, the effective viscosity of the turbulent sublayer has been ignored in the stability calculations. However, we believe that the errors introduced by this approximation are small, and that we can obtain a reliable assessment of the capabilities of the roughness model without the need for making additional stability calculations. An exact assessment of the errors introduced by this approximation is planned.

4. DISCUSSION OF RESULTS

Some typical rough wall, mean velocity profiles, which have been obtained from the "turbulent sublayer" model, are shown in Fig. 5. This figure shows the non-dimensional velocity, u/u_∞, as a function of the non-dimensional distance from the wall, η. The solid lines represent two rough wall profiles in zero pressure gradient flows. The value of the shape factor, H, for these two cases is H = 3.02 and H = 3.33. The smooth wall Falkner-Skan solutions having these same shape factors are also shown so that a quantitative comparison can be made between the velocity profiles when the shape factors are identical. Of course, this comparison does not give a direct indication of the differences in the stability characteristics of two profiles with the same value of H, but some inferences can be drawn. For instance, since the rough wall profile for the H = 3.33 case lies between the Falkner-Skan profiles for H = 3.33 and H = 3.02, we can infer that a complete stability calculation, based on the rough wall profile, would lead to a somewhat smaller predicted effect of roughness on transition than is obtained from the shape factor approximation. Finally, note that the Blasius profile (H = 2.59, β = 0) and the profile of incipient separation (H = 4.03, β = -.1988) are also shown for reference in Fig. 5.

Figs. 6 through 11 present some transition predictions which have been obtained from the turbulent sublayer model using the "approximate" technique described above. Unless otherwise noted, the values of the empirical coefficients in the model, which have been used for these predictions, are those which are obtained by replacing the order of magnitude symbols in Eq. (7) by exact equalities. Specifically, the "baseline" values of the constants were taken as $K_\varepsilon = 0.1$, $A^+ = 40$ [illegible] 1.0. However, in order to determine the sensitivity of the predic[illegible] he values selected for these coefficients, a parametric study has be[illegible] and the results are given in Figs. 6, 7, and 8. Fig. 6 shows th[illegible] ed transition Reynolds number as a function of the roughness height, k/[illegible] three values threshold Reynolds number; a large value ($A^+ = 80$), the ba[illegible] ($A^+ = 40$), and a small value ($A^+ = 4$). Note that small values of A^+ imp[illegible] hat the unsteadiness in the flow over the roughness elements begins at ve[illegible] Reynolds numbers. Similarly, large values of A^+ imply the unsteadiness i[illegible] to higher Reynolds numbers. A corresponding study of the sublaye[illegible] ess parameter, β, is shown in Fig. 7. Note from Eq. (6) that a la[illegible] corresponds to a thin sublayer and that the thickness varies as the squ[illegible] of β, rather than as β itself. The effect of varying the last of [illegible] meters, K_ε, is shown in Fig. 8. (Note that Fig. 8 also contains exp[illegible] ults, but these are discussed in the following paragraph.) A re[illegible] three figures shows that the predicted transition Reynolds numb[illegible] moderately affected by the choice of the constants; nevertheless, a [illegible] experiment of a transitional and pre-transitional boundary layer in [illegible] sence of distributed surface roughness is needed to obtain improved va[illegible] the constants and to completely validate the model.

We now consider the experimental results which are included in Fig. 8 and compare them with the predictions. These experimental results are taken from Feindt (1957) and show the variation of the transition Reynolds number as the wall roughness is varied. As can be seen, the prediction is good for relatively high roughness Reynolds numbers, but at low roughnesses, the predictions are nearly an order of magnitude above the measurements of Feindt. The reason for this discrepancy is that Feindt's experiments were made with a free-stream turbulence level of some 1.2%, whereas, the predictions did not take into account this high free-stream turbulence level. Because of this shortcoming of Feindt's experiment, the general level of some other experiments for the smooth wall case, including those of Schubauer and Skramstad (1948) and Wells (1967), are shown at the smooth wall limit ($Re_k = 0$). Thus, it is seen that the prediction agrees quite well with these smooth wall experiments which were taken in low disturbance wind tunnels. (Note that agreement with these tests is to be expected since the "e^9" factor was, in part, determined by comparing stability predictions with these same experiments.) However, even despite the high turbulence levels in Feindt's experiments, we believe the agreement between the predictions and the experiments at large roughness Reynolds numbers is meaningful for the following two reasons. First, the theory predicts that for sufficiently large roughness, the location of transition will be controlled by the roughness regardless of the other parameters in the problem. (Some numerical results which indicate this are given later.) Second, some further experimental results by Feindt (1957) compared the effects of two different free-stream turbulence levels. The results of these additional experiments are given in Fig. 9. This figure shows that for small roughnesses, increasing the free-stream turbulence level decreases the transition Reynolds number, but as the roughness Reynolds number is increased, the curves for the two different turbulence levels approach each other so that, within the scatter of the data, there is no effect of free-stream turbulence on transition above a certain roughness Reynolds number.

Some further predictions of the linear stability theory, which show the effect of pressure gradient on transition, are shown in Fig. 10, where they are compared with some experimental results which have been collected by Granville and reported in Schlichting (1960). The experimental results are for a variety of transition measurements which were made in the presence of both favorable and adverse pressure gradients. As can be seen from Fig. 10, the transition predictions are in reasonably good agreement with the data, indicating that a constant amplification criterion can be used for either a zero or a non-zero pressure gradient situation. Further testing of the "e^9" criterion in the presence of pressure gradient has been reported by Jaffe, Okamura and Smith (1970).

Finally, some predictions of the combined effects of pressure gradient and roughness on the transition location have been made and are shown in Fig. 11. The predictions are for a number of similar flow (Falkner-Skan) profiles. The results show that the effect of pressure gradient on the transition Reynolds number is very important at the low roughness levels, but that at the larger roughness levels, the location of transition is nearly independent of the free-stream pressure gradient. This is in agreement with the observations made with respect to Figs. 6 and 7. Fig. 11 also includes (in the insert) the experimental results by Feindt (1957) showing the coupled effects of pressure gradient and roughness. Because of the high free-stream turbulence levels which are present in Feindt's experiments (as discussed earlier), a quantitative comparison cannot be made; however, it is noted that the qualitative agreement is good.

5. SUMMARY AND CONCLUSIONS

In summary, a model for the effect of distributed surface roughness on the location of transition has been developed. The model is based on the experimental evidence of Klebanoff and Tidstrom (1972) which showed that roughness not only generates additional fluctuations in the boundary layer, but that it also modifies the mean flow profile in such a manner as to make the fluctuations grow more rapidly. In the distributed roughness case, this modification in the mean profile was taken into account by the "turbulent sublayer" model. Transition predictions, which have been based on this turbulent sublayer model, linear stability theory and the "e^9" transition criterion, have shown promising agreement with the available data for both zero pressure gradient and non-zero pressure gradient situations. However, the model does have three empirical constants which have been evaluated from order-of-magnitude arguments. Although parametric studies have shown that the predicted transition locations are relatively insensitive to the values of these constants, some direct experimental evidence is required to completely specify their magnitudes. Finally, it is noted that experimental evidence is also necessary to verify the turbulent sublayer concept, as well as to verify the mechanisms by which distributed surface roughness affects transition. The verification of the turbulent sublayer concept, as well as the evaluation of the empirical constants, could be obtained from a carefully performed flat plate experiment in which both the mean velocity profiles and the growth rate of the disturbances in the laminar boundary layer were measured.

APPENDIX: <u>Similar Solutions of the Rough Wall Boundary Layer Equations</u>

When the "turbulent sublayer" model is incorporated into the incompressible boundary layer equations, they become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{A.1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = v \frac{dU}{dx} + \frac{\partial}{\partial y} \left[(\nu + \varepsilon) \frac{\partial u}{\partial y} \right] . \tag{A.2}$$

In these equations, x and y represent the coordinates along and normal to the wall, u and v represent the corresponding velocity components, and U represents the free-stream velocity. The kinematic viscosity is given by ν, while ε represents the effective viscosity in the turbulent sublayer.

By standard methods, Eqs. (A.1) and (A.2) can be reduced to the similar form,

$$\left[C f'' \right]' + ff'' + \beta(1 - f'^2) = 0 , \tag{A.3}$$

where primes refer to differentiation with respect to the similarity variable, η, which is given by

$$\eta = \sqrt{\frac{n+1}{2}}\; y \sqrt{\frac{U}{\nu x}} . \tag{A.4}$$

Other quantities in Eq. (A.3) are

$$f' = u/U \tag{A.5}$$

and

$$C = 1 + \varepsilon/\nu . \tag{A.6}$$

The quantities β and n (in Eq. A.4) represent pressure gradient parameters. They are related to each other by

$$n = \beta/2 - \beta , \tag{A.7}$$

and n is defined from the similarity requirement for the free-stream flow, namely

$$U = U_0 \, (x/L)^n . \tag{A.8}$$

As can be seen, Eq. (A.3) becomes the classical Falkner-Skan equation in the smooth wall case, where $C = 1$ ($\varepsilon = 0$).

In addition to the restriction on the free-stream velocity, Eq. (A.8), the effective viscosity, ε, must also be restricted in order to obtain similar solutions. Thus, the ratio, ε/ν, must be a function of η only. As shown below, this imposes a restriction on the stream-wise variation of the surface roughness height, k. From Eq. (6) it can be seen that for the effective viscosity to depend on η only, we must require that the roughness Reynolds number, Re_k, depend on η only. Now, for small roughnesses, we can express the velocity at the roughness height, U_k, by a Taylor series,

$$u_k \approx \left(\frac{\partial u}{\partial y}\right)_{y=0} k \,, \tag{A.9}$$

so our requirement for similarity becomes,

$$Re_k = \frac{u_k k}{\nu} \approx \left(\frac{\partial u}{\partial y}\right) \frac{k^2}{\nu} = \text{const.} \tag{A.10}$$

But for a laminar boundary layer, the velocity gradient at the wall is

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \sqrt{\frac{n+1}{2}}\sqrt{\frac{U^3}{\nu x}}\, f''(0) \,. \tag{A.11}$$

Thus, by combining Eqs. (A.10) and (A.11), we see that if the roughness Reynolds number is to be independent of x, the roughness height, k, must vary as

$$k \sim x^{1-3n/4} \,. \tag{A.12}$$

Thus, for a flat plate ($n = 0$), the roughness height is required to grow slowly with the stream-wise coordinate (as the one-fourth power).

However, if the viscosity in the "turbulent sublayer" is to depend on η only, we must also require that the ratio, y/k, depend on η only (again, see Eq. 6). If the roughness height is constrained to vary with x according to Eq. (A.12), it is seen that y/k has a weak dependence on x. Thus, a local similarity approximation is required to obtain the similar solution in the rough wall case, although the departure from "exact" similarity is small.

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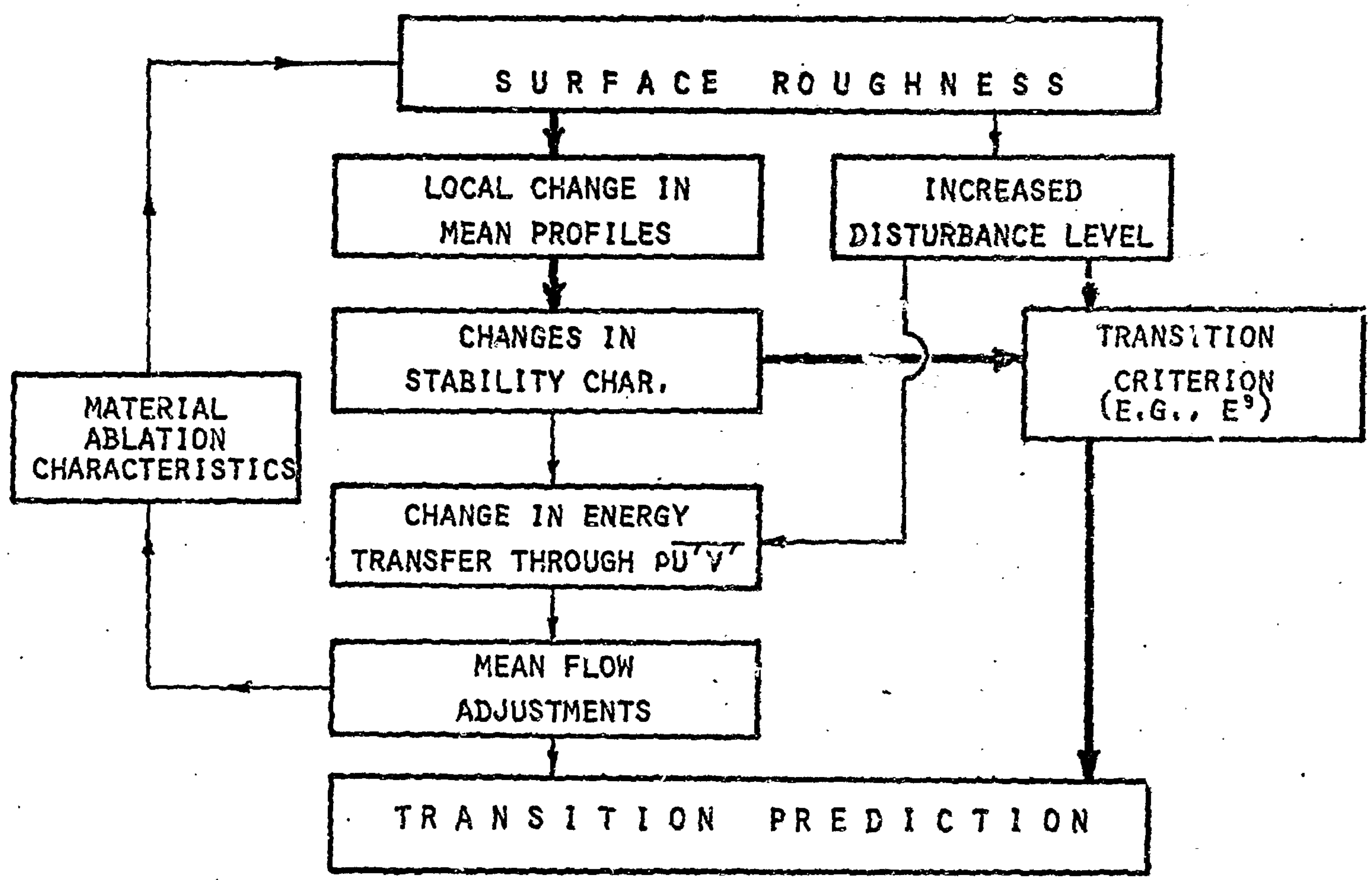

Fig. 1 Roughness/Transition Block Diagram for Phenomena Leading to Transition on a Re-Entry Vehicle Nosetip.

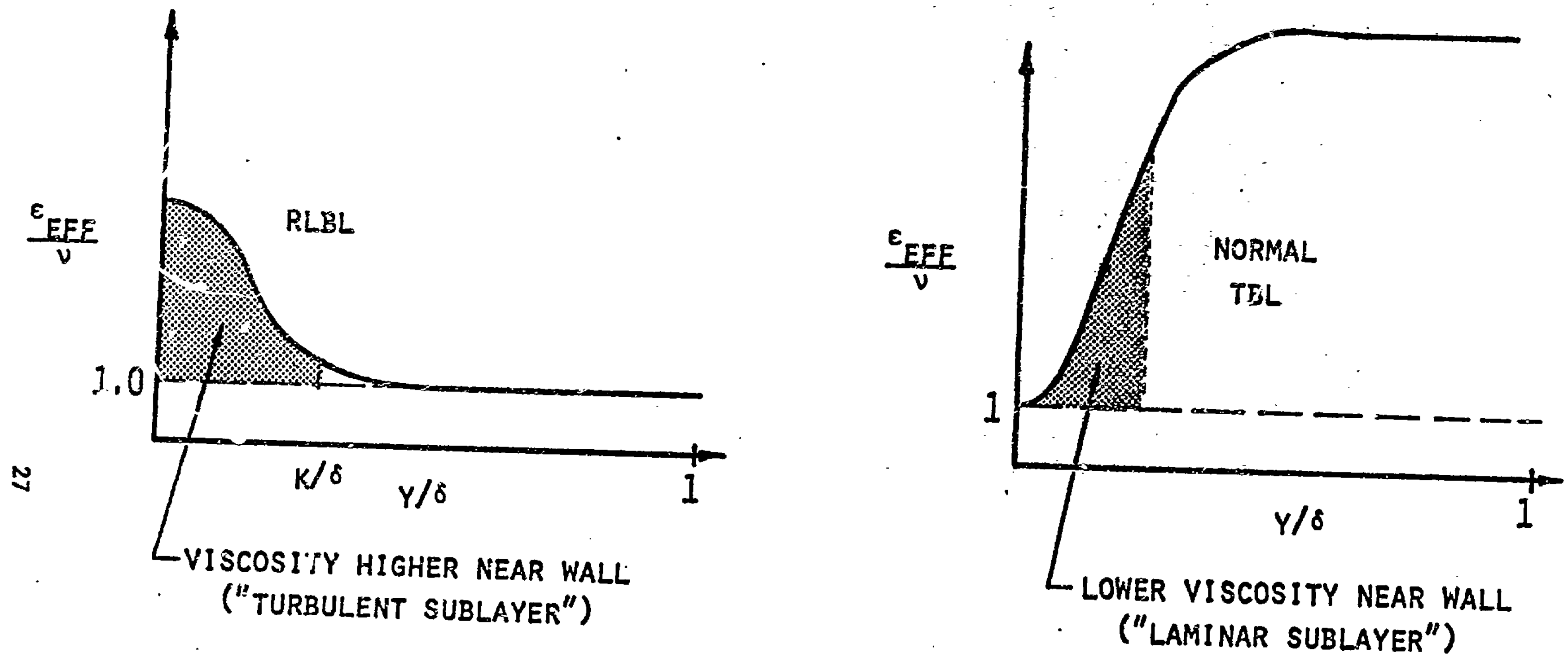

Fig. 2 Momentum Transport by Unsteadiness Near Wall
(Turbulent Sublayer Model)

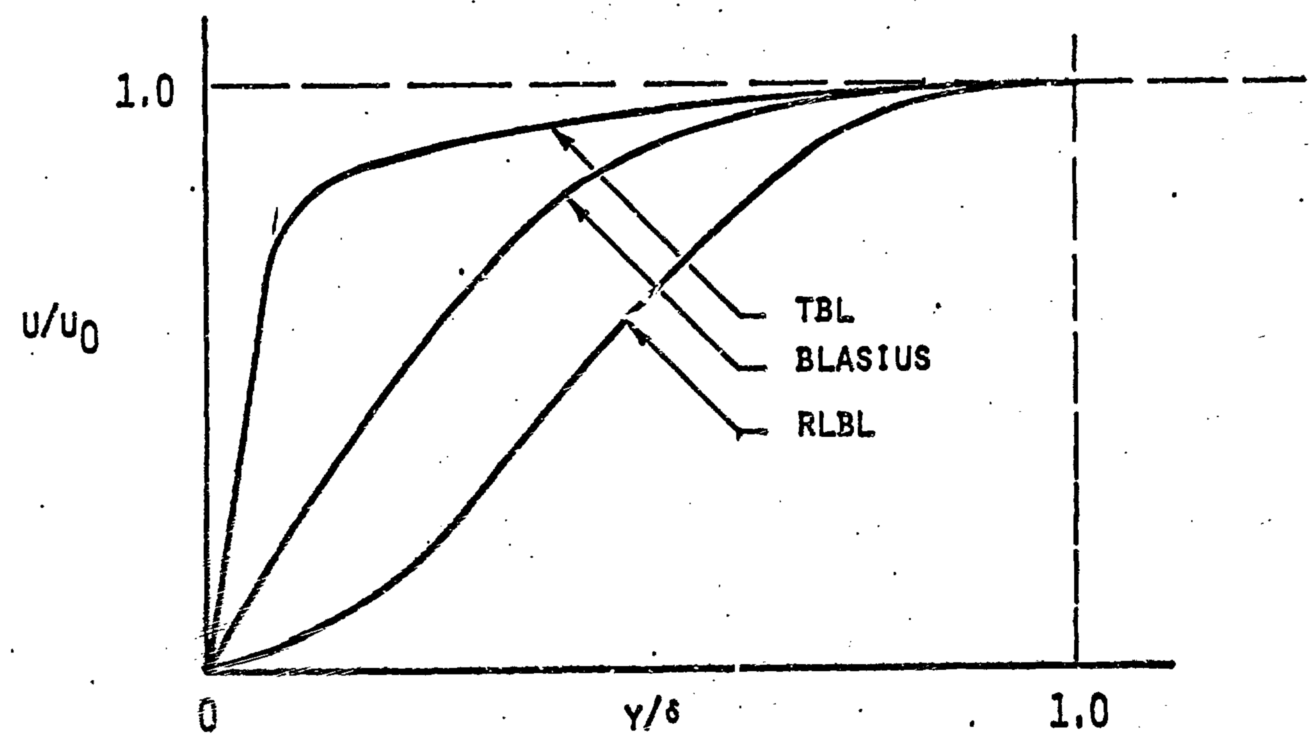

Fig. 3 Qualitative Comparison between Mean Velocity Profiles for Turbulent Boundary Layer, Blasius Boundary Layer, and Rough-Wall Laminar Boundary Layer.

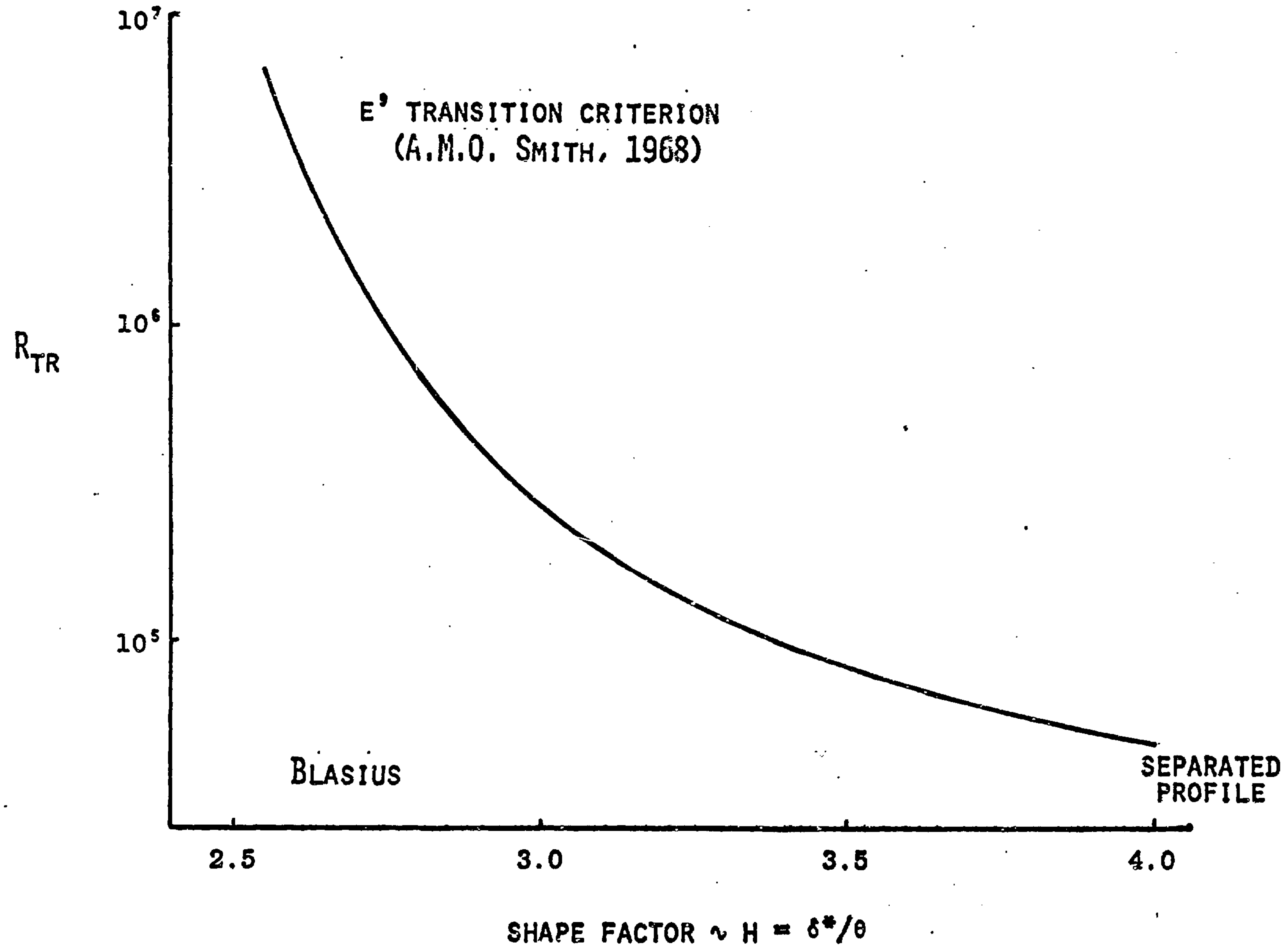

Fig. 4 Transition Reynolds Number vs Shape Factor.

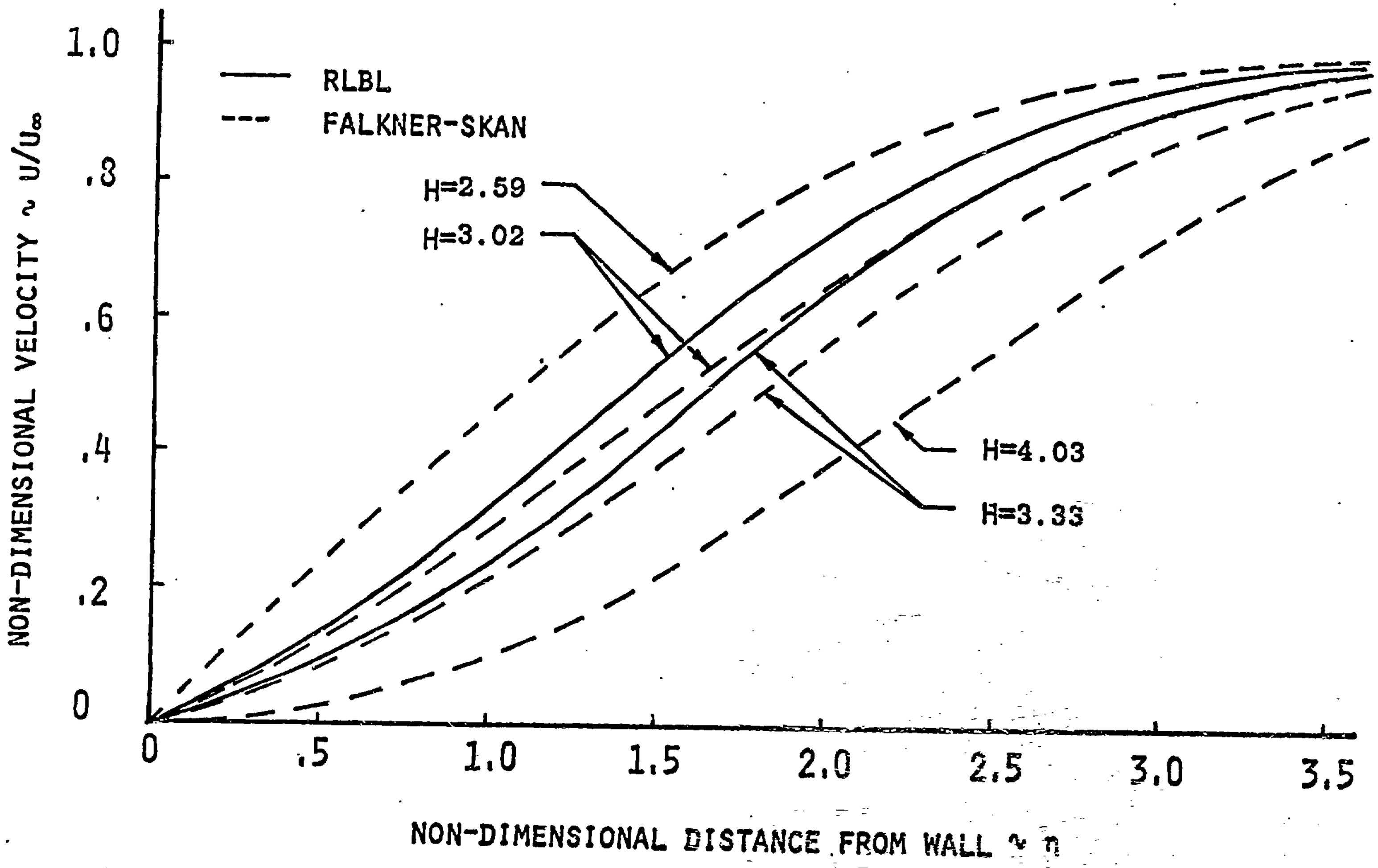

Fig. 5 Comparison between Mean Velocity Profiles on Rough and Smooth Walls.

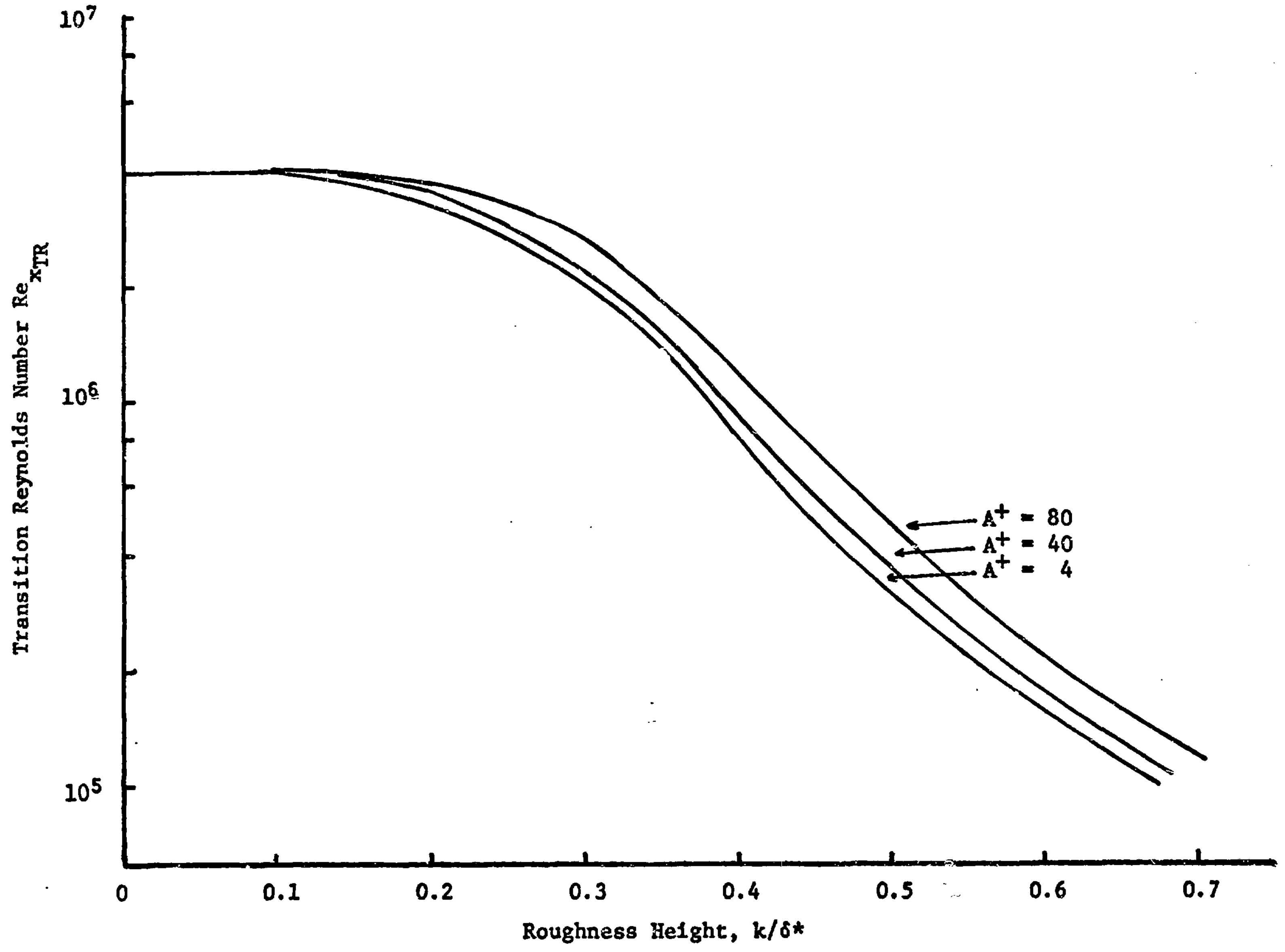

Fig. 6 Effect of Variations in Threshold Reynolds Number on Predicted Transition Location.

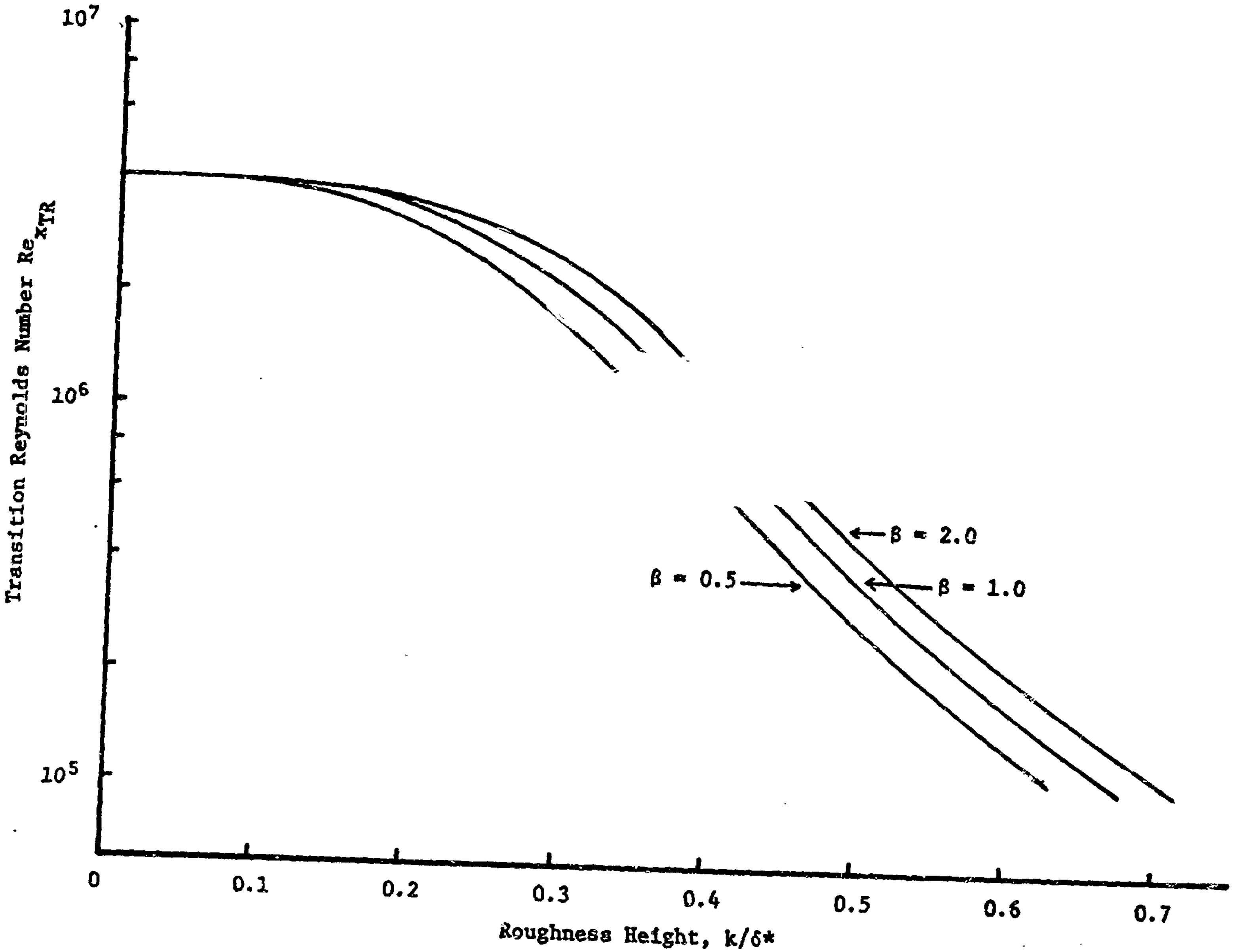

Fig. 7 Effect of Turbulent Sublayer Thickness on Predicted Transition Location.

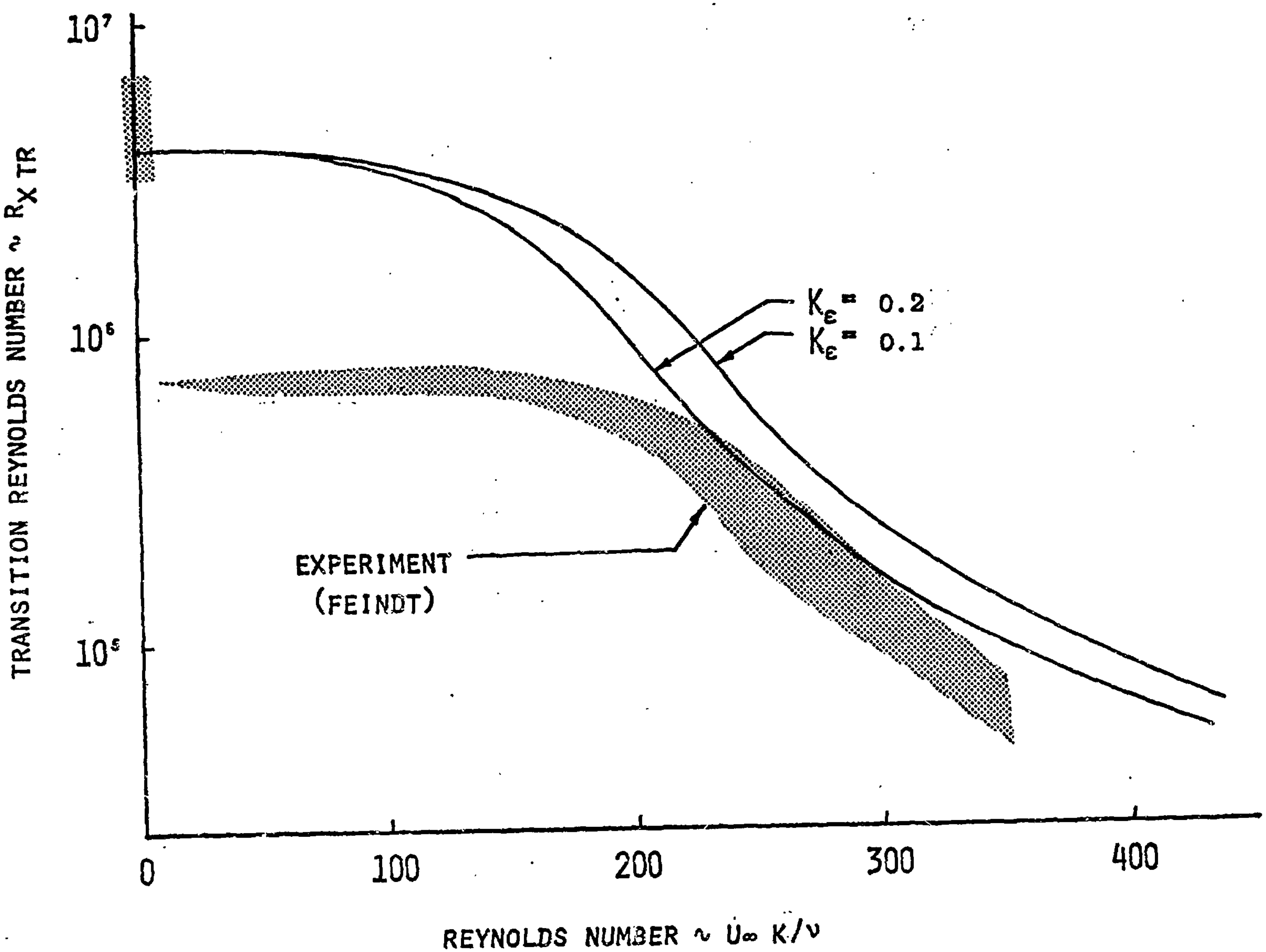

Fig. 8 Effect of Distributed Roughness on Transition Comparison between Theory and Experiment.

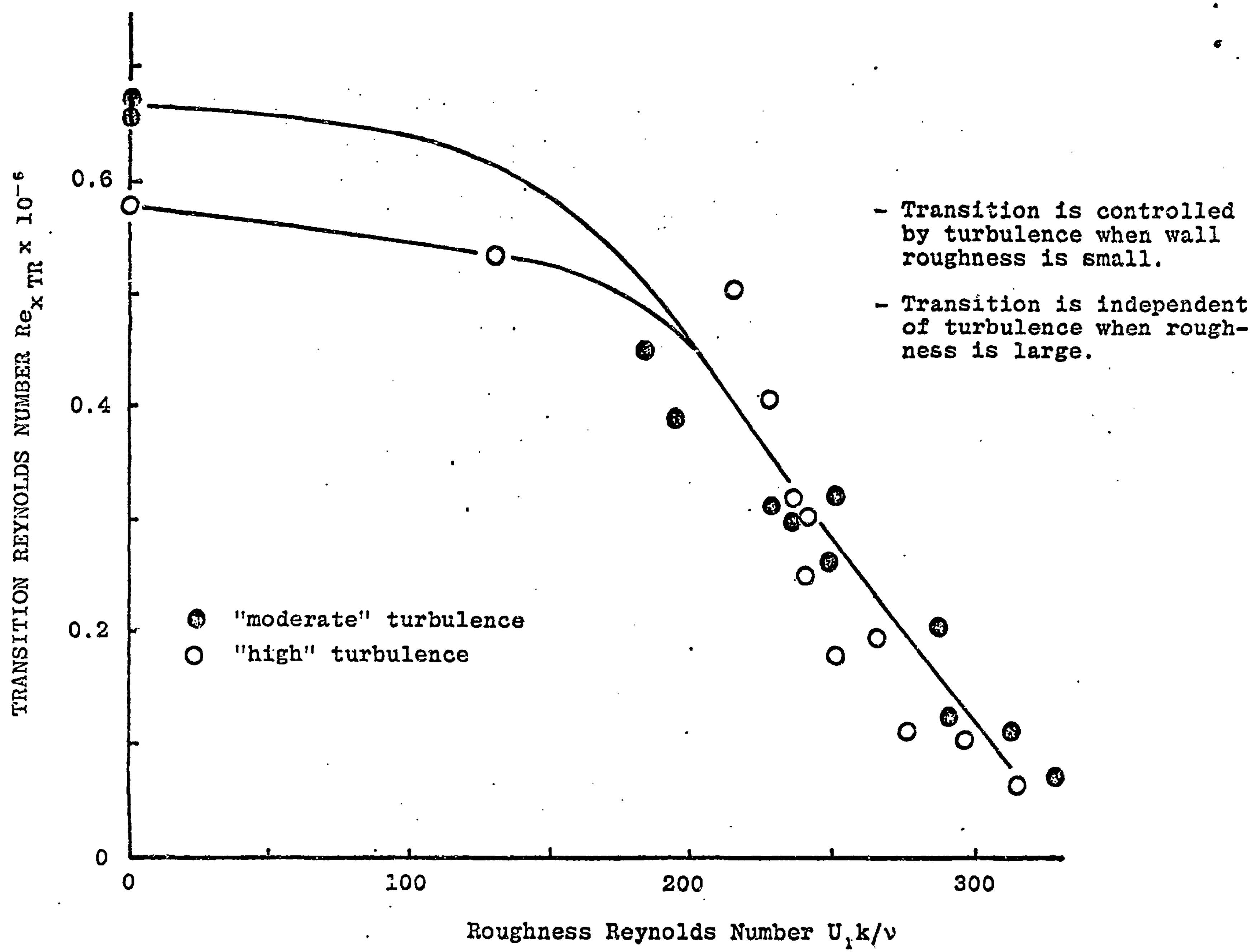

Fig. 9 Combined Effects of Free-Stream Turbulence and Roughness (Feindt).

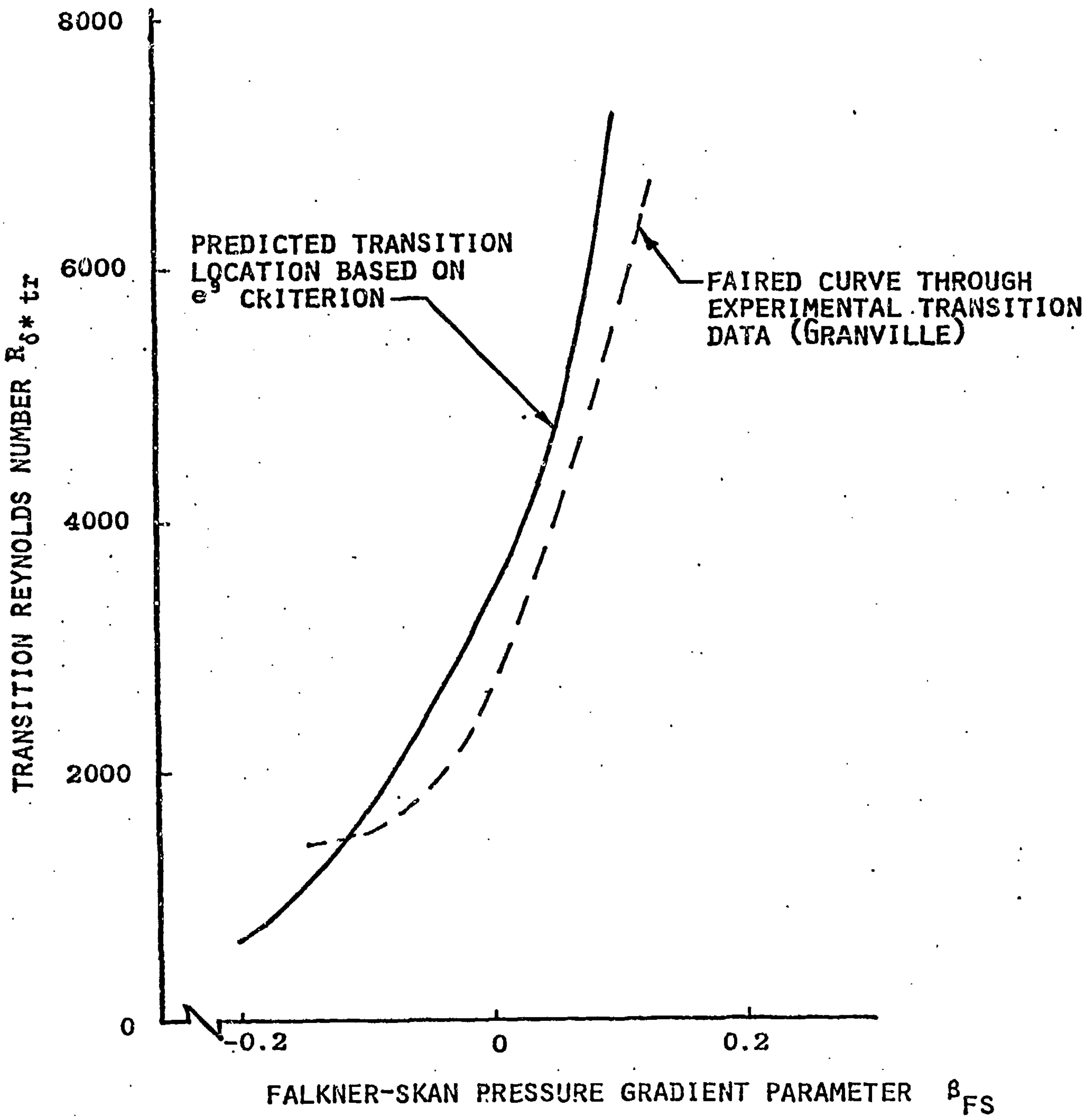

Fig. 10 Effect of Pressure Gradient on Transition Comparison between Theory and Experiment.

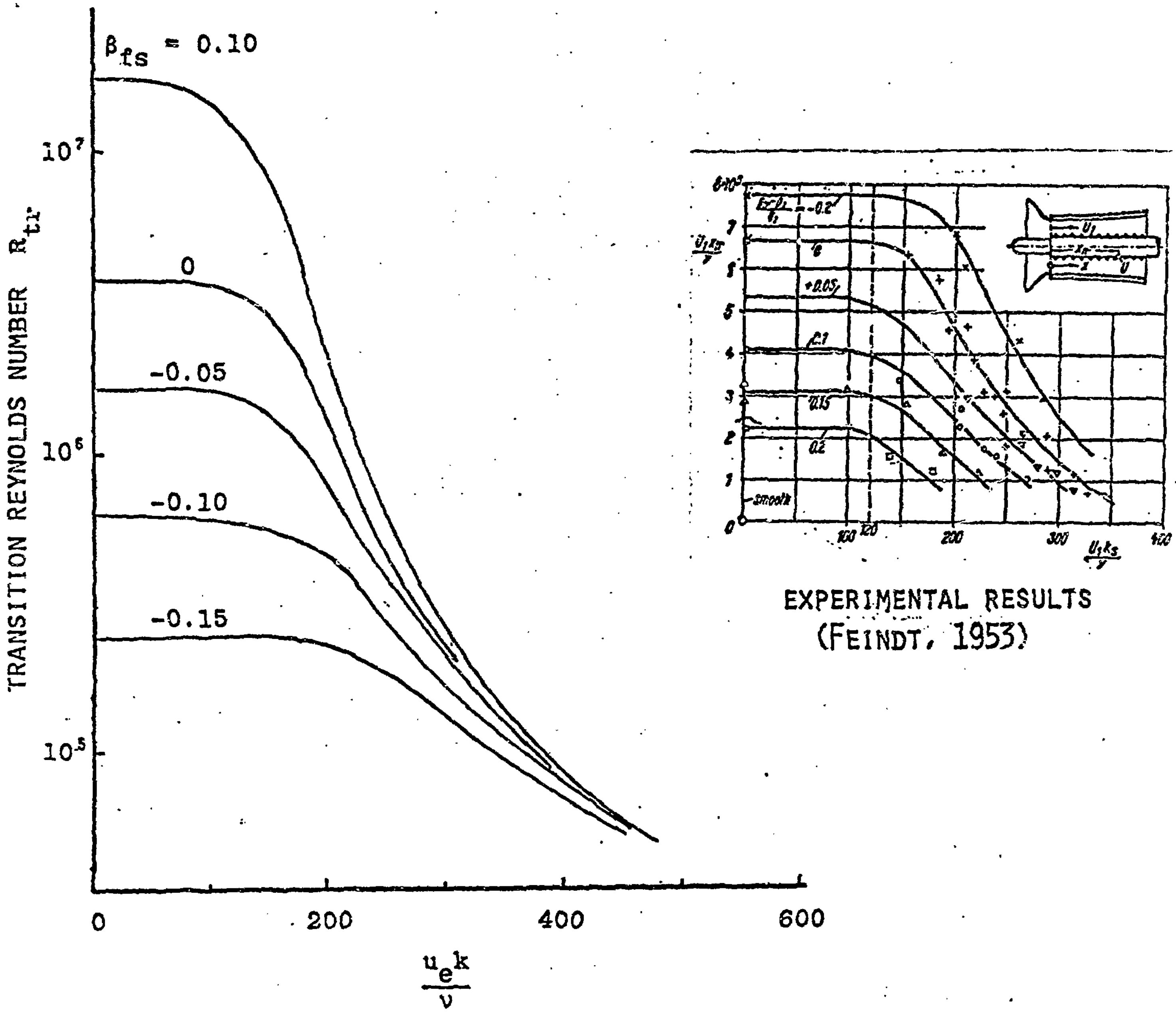

Fig. 11 Combined Effects of Surface Roughness and Pressure Gradient on Transition.