A TWO DIMENSIONAL GLOBAL CLIMATIC MODEL

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**Title:** A Two Dimensional Global Climatic Model

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**Abstract:**
A two-dimensional horizontal box model of the global climate is being expanded to include a hydrologic cycle. Output variables would include cloud cover, relative humidity, and precipitation and evaporation rates. The proposed hydrologic cycle is outlined and a flow diagram for the complete model is presented.
A Two Dimensional Global Climatic Model

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1. Summary

The purpose of this project is to develop a simple global climatic model which may be used to study the distribution of climates on the earth and their causes, the factors that influence climatic change, and the possible influence that man might have on climate. The model structure was described in the previous Technical Report for this project. Briefly, it utilizes a global box grid, with each box having dimensions of 10° of latitude by 10° of longitude. The rates at which humidity in the atmosphere and temperature and the north-south wind or current speed in the atmosphere and oceans vary vertically are specified. The vertically-averaged thermodynamic energy equation, the equation of motion for the surface boundary layer, and the hydrostatic equation are then solved jointly for each box at one-month time steps to determine the global distribution of sea-level pressure, temperature, and wind speed and direction.

A few small changes have been made in the parameterization of the model. These involve, principally, the sea-level pressure field, the divergence of the zonal transport of heat, water vapor, and potential energy in the atmosphere, and the transport of heat by ocean currents. Each change is discussed briefly in the following section.

Also described in the following section is a proposed hydrologic cycle to be used with the model. As it stands now, the model already yields values for the storage rate of water vapor in the atmosphere and the divergence of the water vapor flux by the atmospheric circulation.
These may be combined to give the difference between monthly evaporation and precipitation rates for each grid box. Over the oceans the evaporation rate may be estimated from established formulae relating it to the near-surface wind speed, temperature, and relative humidity. Over land evaporation rates depend mainly on the amount of water in the soil and the potential evapotranspiration rate and may be determined from the water balance equation for a soil column using a simple bookkeeping approach. As by-products here, the soil moisture content and runoff rates are obtained.

Knowing the evaporation rate, the monthly precipitation may be obtained directly (with the necessary constraint that it may never be less than zero). Depending on the surface temperature, a prescribed fraction of this precipitation will fall as snow, adding to that which may be already present on the ground or ice. Sea ice forms if the air temperature is at or below freezing and decreasing. It melts from below when the temperature is rising. Both snow and ice may melt at the surface when the temperature is above freezing. Part of the snow melt over land can infiltrate into the soil, increasing the soil moisture content. The fraction of each box covered by snow and/or ice is a simple function of their respective depths.

The relative humidity over water, snow, or ice may be expressed as the mid-ocean value (prescribed as a function of latitude) modified by the zonal advection of wetter or drier air from upwind regions. Over land the surface relative humidity depends mainly on the soil moisture content. To complete the cycle, cloud cover can be empirically estimated from the precipitation and evaporation rates and the surface relative humidity.
With the incorporation of a hydrologic cycle, the only prescribed variable in the model (other than the mid-ocean relative humidity) is the extraterrestrial solar radiation. Everything else can vary. With such a system, incorporating numerous feedbacks, both positive and negative, it is possible that, without proper coupling, one process may dominate and, after a sufficiently long run of the model, produce nonsense results. Therefore, it will be necessary to make one or more runs of about 100 model years to check the stability of the model. Any further modifications or applications will depend largely on the results of this experiment. Assuming that it is successful, there are several useful projects that can be carried out. These include comparing results of a particular experiment, for example that involving a change in the amount of carbon dioxide in the atmosphere, with those obtained using one of the three-dimensional general circulation models. Tests on the sensitivity of the model to changes in the prescribed initial conditions and studies of the stability of the Arctic and Antarctic continental and sea ice would also be useful.

2. Model revisions
   a. sea-level pressure

   Instead of specifying the meridional pressure gradient averaged over all longitudes, it is now expressed as a function of the corresponding sea-level temperature gradient. This is similar to the procedure used in Sellers (1973). However, the constant of proportionality is now assumed to be inversely proportional to the zonally-averaged surface friction coefficient, which equals $4 \times 10^{-7}$ cm$^{-1}$ over land and ice
and $2 \times 10^{-8}$ cm$^{-1}$ over water. This permits asymmetry in the wind field between the two hemispheres, with much stronger surface westerlies in the southern hemisphere.

b. divergence of the zonal transport

Because the model contains no stratosphere, the zonal wind increases without limit to the top of the atmosphere. This yields unrealistically large values for the divergences of the zonal transport of water vapor, heat, and potential energy. To get around this, the magnitude of each divergence is constrained and not permitted to exceed about 10 kly mo$^{-1}$. This is done by not allowing the magnitudes of $a_{xV}u_o$, $a_{xH}u_o$, and $a_{xP}u_o$ (see Sellers, 1973) to exceed 20, 1.4A, and 4.9A cm sec$^{-1}$, respectively, where A is the box area in units of $10^{14}$ cm$^2$ and $u_o$ is the surface zonal wind speed. The restriction on the magnitude of the zonal vapor flux divergence should be kept in mind, since it will probably have an influence on the precipitation field obtained.

c. ocean-current heat transport

In determining the meridional transport of heat associated with wind-driven ocean currents what is needed in the model is the meridional component $v_{wo}$ of the component of the surface wind-driven current (directed 45° to the right of the wind $V$ in the northern hemisphere) in the direction of the net mass transport (45° to the right of the surface current and 90° to the right of the wind). Following Adem (1970, pp. 415, 416), this implies that

$$v_{wo} = \mp c_1 u_o,$$  

(1)
where the minus sign applies to the northern hemisphere and the plus sign to the southern hemisphere.

\[
C_1 = 0.0001522 (|f|)^{-2} \quad \text{when } |V| \geq 600 \text{ cm sec}^{-1}
\]

and

\[
C_1 = 0.003278 (|fV|)^{-2} \quad \text{when } |V| < 600 \text{ cm sec}^{-1}
\]

Initial runs of the model with this formulation indicate instabilities arising near the equator because of the small values of the coriolis parameter \(f\) there. To get around this, it may be necessary to let \(f = 10^{-4} \text{ sec}^{-1}\) at all latitudes.

d. the hydrologic cycle

The hydrologic cycle subroutine will be embedded within the main program, so that most quantities will be determined more than once during a given month, the final values used being those that give satisfactory convergence. This will be explained more clearly in the next section. At any rate, if it seems in the following discussion that some quantities are being used that have not been evaluated yet, this is really not the case. They have been, but they are subject to change before the calculations are completed.

The first step is to estimate the difference between the rates of evaporation \(E\) and precipitation \(r\) from estimated or previously-determined values for the vapor flux divergence and the change in atmospheric moisture storage. This is done for each grid box. Over the oceans and snow-covered land the surface relative humidity \(RH\) of the \(j\)-th box is estimated from

\[
RH = RH' = 0.5(RH_w + e'/e_s), \quad (2)
\]
where $RH_w$ is the (specified) mid-ocean relative humidity, $e'$ is the surface vapor pressure of the upwind box and $e_s$ is the saturation vapor pressure of the $j$-th box. The evaporation rate in cm mo$^{-1}$ may then be determined from

$$E = E_p = E' = 0.004|V|e_s(1 - RH).$$

(3)

Over snow-free land at temperatures above 271K the potential evapotranspiration rate $E_p$ in cm mo$^{-1}$ may be estimated from

$$E_p = 0.0016(T_s - 271)^3,$$

(4)

where $T_s$ is the surface temperature. This relationship was derived from data given by McIlroy and Angus (1964) for Aspendale, Australia. Actual evapotranspiration from land, which may be partially snow-covered, is given by

$$E = w_1 E_p (1-A_s)/w_k + E'A_s,$$

(5)

where $w_1$ is the soil moisture content at the start of the given month, $w_k$ is the critical soil moisture content, below which evaporation proceeds at less than the potential rate. It is assumed to equal 0.75 of the moisture content $w_m$ at field capacity. $A_s$ is the fraction of the box covered by snow. This is obtained from

$$A_s = 0.1 s_d \quad (s_d < 10 \text{ cm})$$

(6)

where $s_d$ is the existing value for the average snow depth in cm (water equivalent). In (5) when $w_1 > w_k$, the ratio of the two is
set equal to 1.0. Knowing the evaporation rate from (3) and/or (5) and the difference E-r, the precipitation rate may be obtained directly, with the condition that it cannot be less than zero.

In order to estimate cloud cover and the surface albedo, both needed in the radiation calculations, it is now necessary to redetermine the snow and ice coverage and the soil moisture content. To do this, it may first be assumed that the snowfall s equals 0 when \( T_s \) is greater than 275K and equals r when \( T_s \) is less than 265K. In between

\[
s = (27.5 - 0.1 T_s)r .
\]  
(7)

It is further assumed that the potential snow melt equals 7.5 \( E' \). The actual snow melt \( s_m \), of course, cannot exceed the amount of snow present initially plus the snowfall. If \( s_d' \) is the average snow depth over the box (or over ice on the oceans) at the beginning of a given month, then its value at the end of the month is

\[
s_d = s_d' + s - s_m A_s \quad \text{over land}
\]  
(8)

and

\[
s_d = s_d' + (s - s_m (A_s))A_i \quad \text{over water} .
\]  
(9)

An exception to (8) is made in Antarctica and Greenland. The depths of these two ice caps are held constant.

\( A_i \) in (9) is the fraction of the box covered by ice. It is related to the existing value for the average ice thickness \( i_d \) in cm by

\[
A_i = 0.003 i_d \quad \text{for} \quad (i_d < 333 \text{ cm})
\]  
(10)
The ice thickness can change by melting $i_m$ or freezing $i_f$ from below or by melting $i'_m$ from above in snow-free regions at a rate equal to or less than the potential rate $7.5E'$. It is assumed that

$$i_m = 6(T_s - T'_s) , \quad i_f = 0 \quad \text{when } T_s > T'_s \quad (11)$$

and

$$i_f = 6(T'_s - T_s) , \quad i_m = 0 \quad \text{when } T_s < T'_s < 275K , \quad (12)$$

where $T'_s$ is the surface temperature at the start of the given month. The numerical value of the constant was selected on the basis of a statement made by Wittman and Schule (Fletcher, 1966, p. 219) to the effect that one season's growth of ice varies from 157 to 183 cm. In (11) $i_m$ cannot exceed the total amount of ice that can be melted. The ice thickness $i_d$ at the end of the month is given by

$$i_d = i'_d + i_f - (i_m + i'_m - s_m A_s)A_i . \quad (13)$$

The heat required to melt the ice from below and that released by the formation of snow and ice should be included in the energy conservation equation. The energy used to melt surface snow and ice is already included in the evaporation term.

With $s_d$ and $i_d$ obtained from (8) or (9) and (13), $A_s$ and $A_i$ may be redetermined from (6) and (10). The surface albedo $\alpha_s$ over water may then be estimated from

$$\alpha_s = (0.8 A_s + 0.6(1 - A_s))A_i + (1 - A_i)\alpha_w . \quad (14)$$
where \( a_w \) is the albedo of open water (prescribed as a function of latitude and time of year) and values of 0.8 and 0.6, respectively, are used for the albedos of snow and ice.

Over land the surface albedo depends to some extent on the soil moisture content. This may be estimated from the water balance equation for a soil column. Following Sellers (1965) and assuming that runoff \( r_0 \) is given by

\[
    r_0 = 0.8 \frac{w \cdot r'^2}{w \cdot (E_p + r')} = b \bar{w}
\]

where \( r' \) is the effective precipitation,

\[
    r' = r + s_m a_s - s
\]

and that

\[
    E = E_p' = E_p (1 - A_s) \text{ when } \bar{w} \geq w_k
\]

and

\[
    E = \bar{w} E_p (1 - A_s)/w_k \text{ when } \bar{w} < w_k,
\]

where \( \bar{w} = 0.5(w_1 + w_2) \) is the average soil moisture content for the month, it follows that

\[
    \bar{w} = \frac{2w_1 - E_p + r'}{2 + b} \text{ when } \bar{w} \geq w_k
\]

and

\[
    \bar{w} = \frac{2w_1 + r'}{2 + b + E_p/\bar{w}_k} \text{ when } \bar{w} < w_k.
\]
Having determined \( \bar{w} \) and, hence \( w_2 \), which becomes \( w_1 \) for the following month, the surface albedo for land may be obtained from

\[
\alpha_s = 0.8A_s + (1 - A_s) (0.3 - 0.25 \bar{w}/w_m) .
\] (21)

Further, the surface relative humidity and total potential evapotranspiration are given by

\[
RH = 0.9(1 - A_s)\bar{w}/w_m + A_s RH' 
\] (22)

and

\[
E_{pt} = E_p (1 - A_s) + A_s E' 
\] (23)

To conclude the cycle, it is assumed that cloud cover \( n \) is related to the relative humidity and the precipitation and total potential evapotranspiration rates by

\[
n = (0.75 + 0.25 \frac{r - E_{pt}}{r + E_{pt}})RH ,
\] (24)

where the coefficients have been determined so as to give and average cloud cover of slightly greater than 0.5 under present conditions. In the model, then, cloud cover increases with increasing precipitation, increasing relative humidity, and decreasing potential evapotranspiration (or decreasing temperature).

3. Model flow diagram

A simple flow diagram will be used to show how the hydrologic cycle is incorporated into the model. This is presented in Figure 1 and explained in the following paragraphs.
The first step is to estimate or specify the sea-level temperatures for each grid box at the beginning and middle of a given month (block 0). The calculations are started with January and the two temperatures are assumed equal. The sea-level pressure, wind, and ocean current fields corresponding to the specified temperature field are computed next for the whole grid (block 1). Using estimated relative humidities (either specified or those for the previous month), the sea-level vapor pressure is then determined for each box (block 2).

The calculations in the next three blocks are carried out at one longitude i at a time. First, the temperature and moisture dependent parameters are evaluated (block 3). These are principally the a coefficients in Sellers (1973), relating vertical integrals in the thermodynamic energy equation to their sea-level values. Next (block 4) the components of the hydrologic cycle, including cloud cover and surface albedo are computed. This requires specified or predetermined estimates for ice and snow thickness and the soil moisture content, as outlined in the previous section. Then, in block 5, the sea-level temperature in each 10° latitude box is determined from the energy equation.

When this cycle is completed for all longitudes, the resulting temperatures for each box are compared with those used originally (block 6). If each pair of temperatures differ by no more than a specified amount (which varies with latitude, ranging from 1.0K near the poles to 0.2K near the equator) the calculations proceed directly
to block 7. Otherwise, the initial temperatures are adjusted and
the sequence of blocks 2, 3, and 5 repeated (but never more than
six times). Notice that the components of the hydrologic cycle
are not recomputed here.

When the temperatures have converged or the above cycle
has been completed six times, the temperatures at one longitude
(arbitrarily, at 5°W) are compared with those used originally to
find the pressure and wind fields (block 7). Whether they agree
or disagree to a specified amount (1.0K), the program returns to
block 1. With non-convergence, the complete cycle, including the
calculation of the components of the hydrologic cycle, is repeated,
but not more than ten times. Satisfactory convergence is usually
achieved after no more than four cycles, in which case a final pass
is made through block 5, with final values for snow and ice depths
and the soil moisture content made along the way (block 8). After
the values of desired variables are printed out (block 9) the program
proceeds to the next month (block 0). At the end of each year
of calculations the average annual temperature in each box and the
temperature of the bottom water of the oceans are recomputed.

4. References

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