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A THEORETICAL APPROACH TO MULTIOBJECTIVE DECISION PROBLEMS

Gordon B. Hatfield
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The basis for a "synthetic" decision theory has been developed and operationalized through decision programming. The theory is addressed to higher order decision problems characterized by multiple objectives, conflicting goals, and an uncertain decision environment. A novel canonical form embracing three "primitive" concepts--goals, preferences, and constraints--provides a framework which accommodates a wide diversity of decision behavior. In this form, although constraints are treated in the conventional manner, preference ordering may be incomplete (or even intransitive) and goals may be conflicting.

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FOREWORD

This research was performed under Exploratory Development Task Area PF55.521.010 (Manpower Management Decision Technology). This is one of a series of reports concerning the development of a decision technology for use by upper level managers involved in manpower planning and policy formulation. The results are intended to provide the theoretical underpinning necessary for the development of operational algorithms to handle multiobjective decision problems. In addition, advances in "synthetic decision theory" are expected to reveal useful directions for further research and development in Navy management systems.

J. J. CLARKIN
Commanding Officer

SUMMARY

Problem

With few exceptions, manpower and personnel management decisions considered at the policy and planning levels of the Navy are concerned with multiobjective problems. Such problems are characterized by the number and variety of objectives to be served by a decision, the frequent occurrence of conflicting objectives, and the uncertainty of the decision environment. In practice, problems of this type often involve "tradeoff" or "hedging" solutions. These decision problems are not only widespread but also have a considerable impact on the Navy's ability to achieve its manpower objectives. As a result, both the research community and manpower managers have become increasingly sensitive to the need for new or more realistic approaches to the solution of large, complex decision problems. This new interest has shifted the focus of attention from well-defined, single-objective operational problems at the lower levels of organizations to the more uncertain problems typical of planning and policy decisions at the upper level. The shift has not yet been rewarded with the same kind of dramatic success associated with highly-structured operational problems (e.g., applications of linear programming, inventory theory, queueing, etc.). Consequently, the research described in this report is an intitial attempt to develop an adequate theoretical basis for the treatment of multiobjective decision problems.

Background

As a part of this Center's program of research in manpower systems, special emphasis has been placed on the development of computer-based systems for more effective manpower and personnel planning. In the course of this research it became apparent that the ability of the Navy to achieve its manpower objectives is heavily dependent on management decisions concerning recruitment and training input. An investigation of the flow of enlisted personnel from recruitment to petty officer revealed a number of decision variables relevant to the recruit planning problem. Among them, the manpower manager must account for future vacancies, experience requirements for promotion, school capacity, level loading of recruit input, as well as cost considerations. Like other problems of manpower/personnel planning, the determination of rating input is a multiple objective decision problem that often involves drastic tradeoffs. While attempting to model this problem quantitatively, a review of the theory and applications relevant to complex decision processes was made. This review indicated some technical and logical difficulties in both theoretical development and the behavioral assumptions underlying that theory.

Approach

Because the ultimate effectiveness of much manpower planning at the higher echelons is dependent on the accuracy and timeliness of policy

decisions, the development of a realistic decision technology becomes a fundamental research task. The purpose of such research is to provide a theoretical framework for decisionmaking which (a) accomodates the most useful features of existing decision technology from game theory to statistical decision theory to goal programming, (b) recognizes some of the effects that organizational behavior has on decision-making, (c) accounts for the "limits of rationality" in the individual decision behavior, (d) explicitly treats the malleability of a decision problem structure over time and among situations, and (e) directly encompasses problems of multiple constraints and multiple conflicting goals, when transfer occurs between the goal set and the constraint set.

Conclusions

A paradigm has been developed for higher order decision-making characterized by multiple objectives. In an organizational context, the operational surrogates for these objectives are termed goals. Thus, for complex decision problems we posit a set of multiple goals, a set of preferences among those goals, and the desire of the decision-maker to satisfy those preferences and goals subject to constraints. By operationalizing these concepts, a decision programming methodology has been developed which provides a computational vehicle for a viewpoint described as a "synthetic" decision theory.

A THEORETICAL APPROACH TO MULTIOBJECTIVE PROBLEMS*

The intent of this report is to develop a mathematical and conceptual framework for a normative theory of "higher order" decisionmaking. The term "higher order" is meant to connote several ideas: (1) the organizational level at which such decisions predominate--namely, the executive strata; (2) a process whereby decisions are the end result of complex behaviors rather than an isolated act of choice; and (3) the mental operations associated with those psychological processes involved in "thinking" and "problem solving." Given the hierarchical placement of these decisions and their consequences for organizational survival, it is difficult to conceive of the decision environment as being anything other than uncertain. As a result, the essential features of higher order processes are usually described in terms such as "novel, unstructured, and consequential" [16], and take on some of the attributes of what March and Simon [8] refer to as "nonprogrammed" decisions. While there are many sources for the uncertainty which infuses policy decisions, much of the ambiguity derives from the decision process itself. In this regard, it is worth noting that the search for decision alternatives is rarely exhaustive. Of those alternatives that can be specified, the consequences are difficult to determine with either precision or accuracy. In addition, the criteria for evaluating such alternatives and their estimated outcomes are frequently vague or subjective.

The framework developed in this paper is intended to be consistent

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*This report is an extended and modified version of a paper presented at the TIMS Twentieth International Meeting, Tel Aviv, Israel, June 1973.

with the decision behavior observed in "real-world" organizations. Regarding the behavioral properties of decision actors, Newell and Simon [10], in their landmark study of human problemsolving, concluded there are few expressions of "higher order" behavior that are invariant among individuals. The wide variation in perceptual responses and cognitive capabilities among humans implies the need for a decision theory which accommodates a great variety of individual abilities and preferences, considerable differences in the "internal" models for exercising those preferences, and a sizeable variance in the outward expressions of behavior.

While the observation of human variability is well established, the reasons for that variability are a matter of considerable speculation. However, foregoing the metaphysical thicket, it should simply be noted that a normative theory of decision carries with it an assumption of *purposeful* behavior. That is, a "first principle" in any prescriptive theory concerning systems of human interaction is the stipulation of some degree of autonomy on the part of those who would apply the theory in practice. Assuming a decision actor of some autonomy and a behavioral dynamic which appears to be teleological, we can then concentrate on the objects of purposeful behavior.

There is considerable confusion concerning the "objectives" of human behavior which, unfortunately, is not clarified by locating that behavior in an organizational setting. Consider the variety of terms used to express the teleological object: goals, needs, purposes, motives, objectives, values, aspirations, ends, criteria, preferences, norms, etc. While not clearly synonymous in their application, the terms are usually employed with a sense of some form of purposeful behavior. In a decision-making context, Feldman and Kanter [4, p. 628] provide some clarification, indicating that such terms frequently "denote one or both of two ideas: (a) a function which is to be used to evaluate consequences of alternatives and (b) some end point which is to be achieved by means which are to be discovered." Following this lead, we will define the latter as an objective. By way of elaboration, an objective will be considered as some desired but unquantified state of being to be attained at some future but possibly unspecified time. In contrast, goals are defined as the operational surrogates for objectives. As such, they can be used to evaluate alternatives presumably leading to the achievement of desired states. Goals defined in this way become the means for implementing a particular condition at some specific point in time, that support one or more objectives. Because goals are essentially instrumental, they are modified to fit changing situations and, of course, vary considerably among individuals ostensibly pursuing the same objectives.

To account for higher order processes, a theory of decision must provide for the case in which an individual's goal set contains more than one element. There is good reason for considering this the general condition--especially in the case of policy and planning decisions--since the observation of multiple goals is virtually universal.

In an individual's set of operational goals, it is possible to identify at least two classes: (a) those goals intended to implement personal objectives (e.g., such objectives might include power, advancement, aggrandizement, social approbation, or respect), and (b) those more directly related to organizational objectives (e.g., efficiency, effectiveness, productivity, or profit). It is important to recognize that in the case of personal objectives, all surrogate goals must be dressed in organizational garb--whatever their motivation. As a result, personal objectives are reflected in the decision process in various guises; including (1) the selection of goals to be pursued and the constraints under which that pursuit will take place, and (2) the ordering of preferences among the various goals (in organizational terms, the priorities).

While those goals prescribed by an individual's role in the organization remain relatively constant (unless of course the role changes), the goals which are instruments of personal motivation are likely to change as the individual responds and adapts to a fluid personal environment. Consequently, the set of multiple goals bearing on a particular decision problem is subject to considerable change through the introduction of new kinds of goals, discharge of "obsolete" goals, and the modification of "aspiration" levels reflected in continuing goals. Not only are there multiple goals which are subject to great change, but many of these goals are incompatible--and intendedly so. Along these lines, an interesting hypothesis might assert that the pursuit of a wide variety of goals reflects the need to offset the penalties possible in pursuing the "wrong" goal--optimally or otherwise. Goal conflict has generated a considerable literature in the areas of interpersonal relations, economics and game theory, international relations, political behavior, and organization management. In the latter area, case studies abound in which an organizational unit (e.g., production) will seek its goals at the expense of other units (e.g., sales). At the individual level, goal conflict has been most often described in terms of a hypothesized incompatibility between personal needs and organizational demands. In any event, it is not difficult to discover measureable goals of "efficiency" and "effectiveness" operating in an inverse relationship within the same goal set. The classic production tradeoff between time, cost, and quality represents a common organizational example of goal conflict.

In summary, the goals relevant to higher order decision processes reflect personal as well as organizational objectives, and are the operational instruments for the achievement of desired states or objectives. The goal set is subject to considerable change, reflecting the use of new or modified surrogates for the attainment of objectives. While it may not be the general case, given the variety of objectives some conflict among the various goals may occur. For these and other reasons, the set of goals operationally employed in higher order decisions are characterized as being "flexible." That a decisionmaker may express preferences among some of the goals is entirely reasonable. If nothing else, consider the preferences possible between a single personally-derived goal and one organizationally-derived goal.

It has been observed that in organizations, "...decisions are constrained by the actions of the organization itself, by the physical and mental characteristics and previous experience of its members, and by the social, political, and economic environment of the organization and its members " [4, p. 619]. To avoid the possible confusion between goals and constraints, it is useful to follow Dorfman [2, p. 609], who defines a requirement as a constraint if, "...it must not be violated at any cost however high or with any probability however low...on the other hand, a requirement is one of the objectives (i.e., goals) of the firm if it can be violated, though at a cost or penalty, or if there is an advantage in overfulfilling it." Consequently, constraints may be conceived as being "rigid"--and any constraint that is not rigid is, *ipso facto*, a goal. This view of goals and constraints is in sharp contrast to some past approaches to multiple objective problems. For instance, Thompson [17, p. 301] observes that "...(the) procedure in situations like these is to maximize one goal in the analysis and then treat the remaining goals as constraints by specifying some minimum or maximum level with respect to each."

The treatment of constraints as "rigid" refers to their use as fixed boundaries rather than any permanency of their membership in a given set of constraints. If, for a given problem, the set of constraints is unduly restrictive (i.e., the solution space does not contain at least one feasible alternative), the decision-maker may convert the overly-restrictive constraint to a goal in order to derive a feasible solution. From this, it can be seen that there may be some interchange between the "flexible" goal set and the "rigid" constraint set in the course of the decision process. This is particularly the case in the higher order decisions involved in iterative planning over time. For example, the minimization of "cost" may be stated as a goal in the early stages of a planning cycle. However, as the cycle moves toward closure, cost can become a fixed constraint on the achievement of other kinds of goals by its transformation into "budget."

Based on the preceding discussion, we can now posit an individual decision actor faced with a problem in which some of the "rigid" constraints and some of the "flexible" conflicting goals have been specified. In addition, we can hypothesize at least ordinal preferences among some of these goals. That both the constraint set and goal set are "incomplete" is a necessary result of human limitations in perception and cognition [15], considerations relating to cost and quality of information [5], and some implications of the managerial work environment [9]. Without elaborating on the reasons for the existence of incomplete sets, it should suffice to note that the condition is one that not only characterizes "uncertainty" but also encourages certain decision behaviors in the face of that uncertainty.

Following the above, it is useful to think of decisionmakers as "goal achievers" rather than "optimizers." Perhaps the most thorough critique of the decision-maker as "optimizer" has been delivered by Simon in a body of work revealing the psychological and organizational limits of human decision behavior [14], [15], [16]. As Eilon [3] recently observed, "...(while) the optimizing philosophy is the one that prevails in the literature,...experience and observation suggest that satisficing is the approach that prevails in practice." Because all higher order decision problems are necessarily incomplete--or at best, of uncertain structure--there may be heavy latent penalties associated with "optimal decisions" (in the traditional sense) whose implementation may be organizationally or behaviorally dysfunctional. That is, the maximization of a utility function synthesized from a decision problem whose constraints, goals, and preferences are easily subject to change would appear to be a rash strategy as an absolute guide to action. In this regard, the personal exposure involved in gambling on the outcome of "extreme point" solutions may help to explain managerial behavior tending toward the avoidance of risk and the limitation of decision liability. It might be hypothesized that those who reach the upper levels of large organizations exhibit a relatively conservative style of decision-making. If organizations do tend to reward "moderate" decision behavior and penalize "gamblers," few "optimizers" are likely to persist or advance in the same organization long enough to attain executive status. Along these same lines, Wilcox [18] presents an illuminating discussion of risk aversion easily justified "in a world of limited cognitive capability" and on problems with strong "externalties."

Based on the preceding discussion, some of the elements of a distinct viewpoint begin to emerge. In order to give expression to this viewpoint, a methodology has been devised and termed "decision programming." While only a brief introduction to the fundamental ideas of decision programming can be given here, a more complete mathematical treatment is provided by Hatfield [6]. In decision programming, the abstract characterization of decision-making problems is given by the canonical form;

$$\begin{aligned} \text{Satisfy:} \quad & \{\text{goals}\} \\ & \{\text{preferences}\} \\ \text{subject to:} \quad & \{\text{constraints}\} \quad . \end{aligned} \tag{1}$$

This is read "satisfy a set of goals and a set of preferences among the goals subject to a set of constraints." As stated, the canonical form is not yet operational because the term "satisfy" refers to the selection of a norm for goal achievement. Consequently, satisfy replaces maximize or minimize, multiple objectives are cast in the form of "flexible" goals, and constraints are viewed in the conventional manner, i.e., rigid or inviolable. In addition, we permit a preference structure among the goals, in which the ordering may be complete or incomplete. In this sense, it is useful to think of the preference sets as implicit "flexible" goals.

In considering preferences, we adopt the notation,

$$P_i = \{j,k,\ldots\}, \qquad\qquad i = 1,\ldots,p$$

indicating that goal i is preferred to goals j,k,... but saying nothing about the preferences between goals j,k,.... The completely degenerate case of no preferences is indicated by $P_i = 0$, $i = 1,\ldots,p$. Since some goals are usually perceived as more important than others, this case is the exception rather than the rule. Generally, it is difficult to state *precisely* how much more important one goal is than another. In addition, we may not be able to order *all* of the goals in a complex, multigoal problem. Nevertheless, it should be possible to specify preferences among some of the goals and, by using preference sets, a complete ordering of the goals is obviated. For example, in decision programming the following partial ordering is permissible: $P_1 = \{2,3\}$, $P_2 = 0$, and $P_3 = 0$. This indicates that goal one is preferred to goals two and three but that no ordering among goals two and three is given.

The construction of the preference sets depends upon preference relations and "indifference" relations. That is, if P is a preference relation and I is an indifference relation, then we denote iPj when goal i is preferred to goal j and iIj when goal i is "indifferent" to goal j. In practice, we generally construct the preference sets so that the preference relation and the indifference relation are transitive.

In this case the preference sets satisfy the definition of a "rational preference ranking" (see Davidson, McKinsey, and Suppes [1, p. 143]). However, a transitive ordering may not be desired, and it is not a "necessary condition" of decision programming.

With the canonical form (1), it is possible to interpret "satisfying" behavior in the context of optimization. In complex decision problems it is quite common to find goals that are unattainable and/or conflicting. For simplicity, suppose we assume that all of the goals and constraints are linear. Suppose further that we take "satisfy" to mean, *minimize the sum of the ordinary Euclidean distances to all the goals (explicit and implicit)*. Then, the solution of (1) is the feasible point closest to all the goals. Since some of the goals may be conflicting, the solution may be an interior point or a boundary point of the convex set. Thus, what may appear to be a satisfying solution can in fact be obtained by optimizing. While these arguments are obviously not offered in proof of anything, they are intended to provide some rationale for the idea that higher order decision-making may be treated as an optimization problem--although not in the traditional sense.

In this report we have restricted our attention to *linear* decision programming problems, that is, canonical form (1) when the constraints and goals are taken as linear equations or inequalities. We will not consider nonlinearities in either the constraints or goals, nor will we consider discontinuities in the feasible set by restricting the variables to be integer-valued. This is not to imply that these problems are not important. In fact, they may account for a large class of "real-world" decision problems. Nevertheless, a convenient point to begin our mathematical development is with a completely linear structure.

Let F be the feasible set of all $\underline{y}$ such that*

$$\underline{a}_i\underline{y} = r_i, \qquad i = 1,\ldots,m_1$$

$$\underline{a}_i\underline{y} \leq r_i, \qquad i = m_1+1,\ldots,m$$

$$y_j \geq 0, \qquad j = n_1+1,\ldots,n$$

where m_1 is the number of equations and n_1 is the number of free variables. We refer to the following problem statement as a linear decision programming problem:

Satisfy: $\underline{c}^i_j\underline{y} = g_i, \qquad i = 1,\ldots,p_1;\ j = 1,\ldots,t_i$

$$\underline{c}^i_j\underline{y} \leq g_i, \qquad i = p_1+1,\ldots,p;\ j = 1,\ldots,t_i \qquad (2)$$

$$P_i = \{j,k,\ldots\}, \qquad i = 1,\ldots,p$$

subject to: $\underline{y} \in F$

where p_1 is the number of goal equations, t_i is the number of possible outcomes for goal i, $i = 1,\ldots,p$, and P_i, $i = 1,\ldots,p$, are the preference sets of the goals. For given i and j the outcome, $\underline{c}^i_j\underline{y}$, for goal i has the possibility of occurrence q_{ij},

where $$\sum_{j=1}^{t_i} q_{ij} = 1, \qquad i = 1,\ldots,p$$

$$q_{ij} \geq 0, \qquad i = 1,\ldots,p;\ j = 1,\ldots,t_i.$$

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*Vectors are denoted by underlined lower case letters.

In (2), when for each goal there is only one possible outcome, i.e.,

$$q_{ik} = 1 \text{ and } q_{ij} = 0 \text{ for all } j \neq k, \qquad i = 1,\ldots,p,$$

we have:

$$\text{Satisfy:} \quad \underline{c}_i\underline{y} = g_i, \qquad i = 1,\ldots,p_1$$

$$\underline{c}_i\underline{y} \leq g_i, \qquad i = p_1+1,\ldots,p \qquad (3)$$

$$P_i = \{j,k,\ldots\}, \qquad i = 1,\ldots,p$$

$$\text{subject to:} \quad \underline{y} \in F$$

Having introduced the canonical form and the preference structure permitted in decision programming, we can now turn to the problem of goals. Two questions arise in this regard: (1) what are "flexible" goals? and (2) what is meant by satisfying "flexible" goals? As to the first question, observe that in the feasible set F the first m equations or inequalities plus the non-negativity restrictions are the constraints of ordinary linear programming. Moreover, they are rigid in the sense that if there does not exist a point $\underline{y}^o$ satisfying all of the constraints then the problem is infeasible. In contrast, the p goal equations or inequalities are "flexible" since it is not required that they be satisfied exactly. For example, consider problem (3). If $\underline{y}^o$ is feasible, then we permit

$$\underline{c}_i\underline{y}^o \neq g_i, \qquad i = 1,\ldots,p_1$$

and

$$\underline{c}_i\underline{y}^o > g_i, \qquad i = p_1+1,\ldots,p.$$

In fact, this is the only case where this class of problems is of any interest. The reason for this is that if there are $\underline{y}^0$ which satisfy the constraints *and* the goals then we are simply finding feasible solutions to ordinary linear programming problems.

In order to answer the second question, we must choose a measure of satisfaction. To this end, let

$$s_i = \underline{c}_i\underline{y} - g_i, \qquad i = 1,\ldots,p_1$$

and

$$s_i = \begin{cases} 0 & \text{for } g_i - \underline{c}_i\underline{y} \geq 0 \\ \underline{c}_i\underline{y} - g_i & \text{for } g_i - \underline{c}_i\underline{y} < 0 \end{cases} \qquad i = p_1+1,\ldots,p$$

be the slacks in goal i at $\underline{y}$. Assume for the moment that $P_i = 0$, $i = 1,\ldots,p$. Then a convenient first choice of a measure of satisfaction is to minimize a weighted sum of the absolute values of the slacks in the goals, i.e.,

$$\text{Min} \sum_{i=1}^{p} w_i \, |s_i| \quad . \tag{4}$$

The weights are introduced in order to establish certain theoretical connections between goal programming, vector maximization, and decision programming. In addition they facilitate the development of a minimum distance method for solving (3).

It can be shown that with *satisfy* defined by (4), a linear goal programming problem is a special case of (3), solving a specific linear decision programming problem (3) is equivalent to solving a linear vector minimization problem, and the satificing method of Simon [8] can be made operational as a linear decision programming problem. Concerning the latter, Odhnoff [11] has noted that the aspiration levels of Simon's satisficing method can be operationalized as inequality goals. In addition, a minimum distance method follows directly from the canonical form. Consider the linear decision programming problem (3) with *satisfy* defined by (4). If we let

$$w_i = \frac{1}{\left(\sum_{j=1}^{n} c_{ij}^2\right)^{\frac{1}{2}}}, \qquad i = 1,\ldots,p \tag{5}$$

then $w_i\,|s_i|$, $i = 1,\ldots,p$ is the Euclidean distance from $\underline{y}$ to the hyperplane $\{\underline{y}|\underline{c}_i\underline{y} = g_i\}$. With the weights chosen by (5) the minimum distance problem is

$$\text{Minimize:} \quad f(\underline{y}) = \sum_{j=1}^{p_1} w_i|s_i(\underline{y})| + \sum_{i=p_1+1}^{p} w_i s_i(\underline{y}) + \sum_{i=1}^{p}\sum_{j=1}^{p} p_{ij}(\underline{y}) \tag{6}$$

where $p_{ij}(\underline{y})$ is the preference function of goals i and j.

The preference functions are constructed from the information contained in the preference sets. To understand the rationale for this construction, we must consider what is meant by the statement "goal i is preferred to goal j," i.e., $P_i\ \{j\}$. Suppose that i and j refer to goal equations and we examine those points $\underline{y}^0$ that are closer (in the sense of the Euclidean norm) to the hyperplane corresponding to goal i. We can then define those points by observing that they lie on one side of an "indifference" hyperplane. Accordingly, it seems reasonable to associate preference with distance and to assert "goal i is preferred to goal j" for all $\underline{y}^o$ satisfying the "indifference" hyperplane. When "indifference" is indicated in the preference sets no construction is made and we assert "goal i is 'apreferred' to goal j" for all $\underline{y}^o$. Parenthetically, since a complete ordering of the goals is not required, we cannot include information about preferences by simply weighting each term of

$$\sum_{i=1}^{p_1} w_i|s_i| + \sum_{i=p_1+1}^{p} w_i s_i \quad .$$

To illustrate, suppose $P_1 = \{3,4\}$, $P_3 = \{4\}$, $P_2 = \{4\}$ and consider

$$\text{Min} \sum_i M_i w_i |s_i| \;, \qquad \text{for } i \leq P_1 \quad .$$

To achieve P_1 and P_3 we might select $M_1 = 100$, $M_3 = 10$ and $M_4 = 1$. However, any choice for M_2 requires that we choose a preference for goal 2 relative to goals 1 and 3. Thus, the inclusion of preference information is a nontrivial problem.

As an illustration of a linear decision programming problem which can be solved by the minimum distance method, consider the following:

Satisfy: (1) $3y_1 + 2y_2 = 16$

(2) $-2y_1 + y_2 = 3.2$

(3) $3y_2 - y_2 = 3$

(4) $y_2 \leq 4$

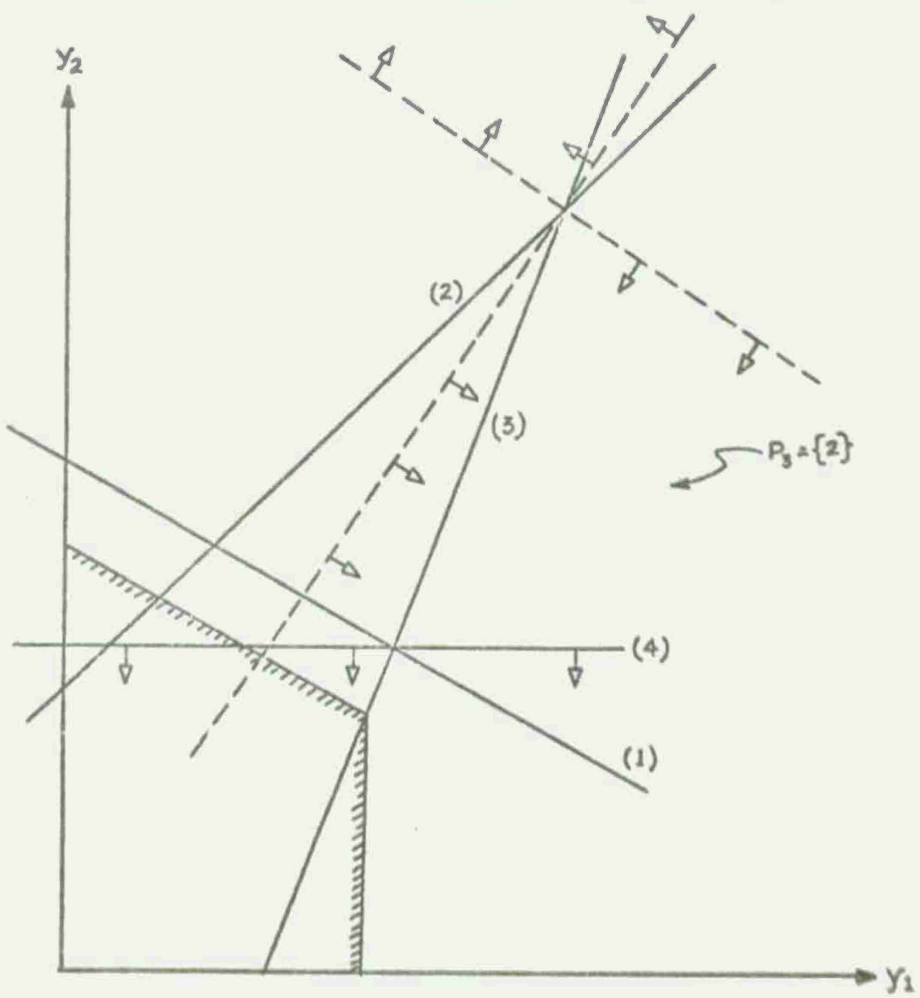
$P_1=0, \quad P_2=0, \quad P_3=\{2\}, \quad P_4=0$

subject to: $3y_1 + 2y_2 \leq 12$

$y_1 \leq 2$

$y_1, y_2 \geq 0$.

Graphically, this problem is shown below.



The objective is to find a feasible point that minimizes the distance to all the goals and satisfies the goal preferences (indifference hyperplanes). Using this approach, goal preferences become additional goals rather than rigid constraints and the objective of minimizing the distance to all the goals is maintained.

Although we cannot go into details here, it is sufficient to note that solutions of the minimum distance problem (6) can be obtained [6]. In addition, a "real-world" application employing the methodology of linear decision programming and the minimum distance method of solution has been undertaken. This application concerns a problem in planning military personnel promotions involving multiple, conflicting goals and incomplete ordering in the preference structure [13].

In returning to the question of satisfying goals, we note that, while the operational definition of satisfy given by (4) has a number of important implications, there are other choices of interest. For instance, consider the possibility of giving satisfy a minimax interpretation. To this end consider (2) with $P_i = 0$, $i = 1,\ldots,p$. As a measure of satisfaction suppose we choose to minimize the maximum of the $w_{ij}\ |s_{ij}|$, i.e.,

$$\min_{\underline{y}\in F}\ \max_{\substack{i=1,\ldots,p\\ j=1,\ldots,t_i}}\ \{w_{ij}|s_{ij}(\underline{y})|\} \tag{7}$$

where

$$s_{ij} = \underline{c}^i_j\underline{y} - g_i\ , \qquad i = 1,\ldots,p_1$$

$$s_{ij} = \begin{cases} 0 & \text{for } g_i - \underline{c}^i_j\underline{y} \geq 0 \\ \underline{c}^i_j\underline{y} - g_i & \text{for } g_i - \underline{c}^i_j\underline{y} < 0 \end{cases} \qquad i = p_1+1,\ldots,p\ .$$

Then (2), with satisfy defined by (7), is

$$\underset{\underline{y}}{\text{Min}} \quad \underset{\substack{i=1,\dots,p \\ j=1,\dots,t_i}}{\text{Max}} \quad \{w_{ij}|s_{ij}(\underline{y})|\} \tag{8}$$

subject to: $\underline{y}\varepsilon F$.

With satisfy defined by (7), it can be shown that the Chebyshev approximation problem is a special case of (8), and that a two person, zero-sum game has an equivalent linear decision programming problem (2).

Another interesting choice of measure of satisfaction is to maximize the number of goals having zero slack. To this end consider (3) with $P_i = 0$, $i = 1,\dots,p$. Let

$$\delta_i = \begin{cases} 1 \text{ if } s_i = 0 \\ 0 \text{ otherwise} \end{cases} \qquad i = 1,\dots,p \quad .$$

As a measure of satisfaction take

$$\text{Max} \sum_{i=1}^{p} \delta_i \quad . \tag{9}$$

The combinatorial nature of this problem is apparent. In selecting a measure of satisfaction, the choices suggested above hardly exhaust the range of possibilities. They do, however, pose some interesting problems for future development.

By way of summary, the preceding discussion (both verbal and mathematical) may be thought to provide the initial framework for what might be termed a "synthetic" decision theory. It should be noted at this stage of development that the formal designation "theory" is much too ambitious a term for a hypothetical array of concepts--even as operationalized through decision programming. Nevertheless, we believe the ideas introduced in this paper provide a reasonably precise framework for further theoretical development.

We have described decision behavior as being intentionally purposeful, and directed toward a set of rather ambiguous objectives--some of which may be personal in origin. In an organizational context, the operational surrogates for these objectives are labelled goals. Note that no attempt is made to define a "mapping" from the region of objectives to the domain of goals. And certainly no assumptions are made concerning a one-to-one relationship between goals and objectives. For higher order decisions we posit a multiplicity of "flexible" goals with a relationship that may be conflicting. Further, a set of preferences among these goals describes the decision actor's priorities or desires, however indifferent that preference structure may be. Finally, we define the level of aspiration which triggers the decision as being one of "satisfaction"--or relative contentment with the solution emerging from the decision process. By operationalizing these concepts through the methodology of decision programming, it may be possible to undertake the solution of more complex decision problems within a more parsimonious framework--and, hopefully, in a way more compatible with human decision behavior.

In the narrow sense, a decision problem is manifested one form at a time, producing a single class of solutions bounded by the feasible set. However, in the larger view, and in terms of human behavior, the definition of a complex decision problem begins with its solution. In other words, a "satisfactory" solution dictates the form of the problem found to be acceptable. Since prior to a satisfactory solution a decision actor may manipulate the constraints, goals, and preferences to yield a different class of problems, the process stabilizes only when the decision is finally made. As Rivett [12] observed,

> *"...in most problems we do not, at the beginning of the research, have any precise understanding of the objectives involved. Nor can we formulate a set of objective functions. These only become clear toward the end of the study, that is, at the stage where we have been largely committed to the form of the model."*

The theoretical approach outlined here is intended to avoid some of the difficulties posed above by introducing a more general and more flexible model of the problem solving process.

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