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POLARIZATION EFFECTS ON DIRECTION FINDING

Lee A. Morgan
Philip A. Hicks

TELEDYNE MICRONETICS

TECHNICAL REPORT AFAL-TR-73-262
July 1973

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FOREWORD

The research described in this Final Technical Report was performed by Teledyne Micronetics, San Diego, California, under Air Force Contract F33615-71-C-1579, Modification No. P00002, for the Air Force Avionics Laboratory, Air Force Systems Command, United States Air Force.

Mr. Lee A. Morgan was the Principal Investigator, under the direction of Dr. Steven Weisbrod.

Mr. John Tehan (AFAL/WRP) and later Major Christo Christodoulou, USAF, were the designated Project Engineers for the Air Force Avionics Laboratory.

This work was performed under AFAL Project No. 7633-02-11, from 1 April 1972 to 1 June 1973.

This report, Teledyne Micronetics No. R14-73, was submitted on 1 July 1973.

TECHNICAL APPROVAL

This Technical Report has been reviewed and is approved.

W. J. FORTUNE, JR.
Assistant Chief
Electronic Warfare Division

ABSTRACT

The results of an investigation of techniques to minimize or correct polarization errors in Direction Finding Systems are presented. Four techniques are considered. These are "Flagging," "Switching," "Tracking," and "Polarimeter." The flagging technique senses situations when the polarization error would be unacceptably large and notifies the DF system of this fact. The switching technique senses the field polarization and chooses the one of two receiving polarizations which is closest. The tracking technique senses the field polarization and adjusts the receiving antenna polarization to match. The polarimeter technique measures the field polarization and computes a correction to the DF output.

The residual error after correction by these techniques is analyzed and numerical results for a phase interferometer presented. For the purposes of evaluating these residual errors, a randomly polarized field (in the sense that there is no apriori knowledge of its polarization), is assumed and the cumulative probability distribution of the errors computed. It is shown that all four techniques are capable of reducing the 95% confidence level on the errors to a value consistent with other errors in a very good DF system. The flagging technique is accompanied by a potentially serious amount of data loss and the polarimeter technique requires too much equipment complexity and cost to be acceptable for all but the most demanding applications. Both the switching and tracking techniques have sufficient error improvement and equipment simplicity to make their inclusion in high accuracy DF systems desirable. Switching

is the simpler of the two and is ideally suited to a retrofit application. Tracking is the more accurate of the two and is well suited to incorporation in new designs. The ultimate in correction would be provided by a combination of the switching and polarimeter techniques.

Also included are discussions of time of arrival polarization errors, effects of multipath on the corrective techniques, and methods of measuring polarization. An annotated bibliography is included.

TABLE OF CONTENTS

1. INTRODUCTION 1
   1.1 General Discussion 1
   1.2 Scope and Objectives 2
   1.3 Summary of Major Results 4
2. SUMMARY OF FIRST PHASE RESULTS 21
   2.1 General Considerations 21
   2.2 Phase Interferometer 22
   2.3 Amplitude Monopulse. 26
   2.4 TOA System 28
   2.5 Measurement Program. 30
   2.6 DF Evaluation Techniques 30
3. POLARIZATION ERROR CORRECTIONS 31
   3.1 General Discussion 31
   3.2 Polarization Flagging. 33
   3.3 Polarization Switching 35
   3.4 Polarization Tracking. 38
   3.5 Polarimeter Correction 47
   3.6 Effects of Randomizing Antenna Parameters. . 50
4. POLARIZATION ERRORS IN TOA SYSTEMS 55
   4.1 General Discussion 55
   4.2 Polarization Effects on TOA Systems. 56
   4.3 Examples 58
5. MULTIPATH EFFECTS ON CORRECTIVE TECHNIQUES . 67
6. CONCLUSIONS AND RECOMMENDATIONS. 83
APPENDIX A - DETAILED MATHEMATICAL ANALYSIS. 85
   A-1 Polarization Switching Analysis. 85
   A-2 Polarization Tracking Analysis 88
   A-3 DF Error Correction by Polarimeter Measurements 95

TABLE OF CONTENTS (Cont.)

A-4 Antenna Parameter Randomization Analysis . . 105

A-5 Grating Lobe Errors in Multilevel Interferometers. 109

A-6 Discussion of Interferometer Normalized Arrival Angle. 115

APPENDIX B - POLARIZATION DESCRIPTION. 117

B-1 Polarization Representation of Antennas. . . 117

B-2 Randomly Polarized Fields. 121

APPENDIX C - POLARIZATION MEASUREMENT. 125

C-1 General Discussion 125

C-2 Rotating Antenna Polarimeter 127

C-3 X-Y Lissajous Display Techniques 128

C-4 Polarization Parameter Computation from Circularly Polarized Components. 129

C-5 Computation From Amplitudes Only 131

C-6 Relative Performance and Cost. 133

APPENDIX D - ANNOTATED BIBLIOGRAPHY. 135

REFERENCES 143

LIST OF FIGURES

1. Polarization Flagging Technique. 10
2. Polarization Switching Technique 11
3. Polarization Tracking Technique. 13
4. Polarimeter Correction Technique 15
5. Comparison of Interferometer Uncertainty With and Without Correction. 17
6. Cumulative Probability Distribution of Interferometer Error. 24
7. Cumulative Probability Distribution of Interferometer Error. 25
8. Monopulse Error Probability Distribution ψ = 45 Degrees 27
9. Error Probability Distributions (r = 0.1). 40
10. Interferometer Error Probability Distribution (r = 0.5) 41
11. Vector Representation for Polarimeter Correction of a Phase Interferometer System 49
12. Interferometer Error Probability Distributions. 52
13. Interferometer Error Probability Distributions. 53
14. Multipath Geometry 69
15. Multipath Polarization Geometry. 69
16. Probability of Selecting Cross Polarized Component. 77
17. Probability of Slecting Cross Polarized Component. 78
18. Probability of Selecting Cross Polarized Component. 79
19. Probability of Selecting Cross Polarized Component. 80

A-1 Interferometer Error Probability Density Function with Polarimeter Correction 102

LIST OF TABLES

I. Contributions to Interferometer Phase Error. 5

II. Correspondance Between Antenna Cross Polarization Ratio and Other Antenna Specifications 6

III. Maximum Interferometer Error With Polarization Switching 39

A-1 Probability of Ambiguity Error in a Multilevel Interferometer. 113

A-2 Conversion from Interferometer Phase (Normalized Arrival Angle) to Space Angle. 115

C-1 Comparison of Polarimeter Methods. 134

1. INTRODUCTION

1.1 General Discussion

This report contains the results obtained in the second phase of a program entitled "Polarization Effects on DF."

Alert workers in the field of radio-direction finding (DF) have long recognized that the interaction between the polarization of the incident field and the polarization of the receiving antenna system could substantially affect the accuracy of the DF system. Although well understood for certain types of direction finding, such as the classical rotating loop where sophisticated approaches are used to minimize the cross polarized response, this problem has been ignored in all too many cases. The result of this neglect has often been an overoptimistic estimate of the accuracy of DF systems and, frequently, the expenditure of considerable effort to correct or compensate for other, less serious, sources of error. Ironically, although ignored by those who design and analyze DF systems, the effects of polarization are sufficiently well recognized by those who operate antenna evaluation ranges to cause them to take considerable care to ensure that the test transmitter is matched in polarization to the receiving antenna system. The result of course is an optimistic assessment of the capabilities of the DF system under test.

In view of these factors, and the need of the Air Force to obtain the best possible results from its DF systems, the current program was initiated.

1.2 Scope and Objectives

The first phase of this program, whose results are completely contained in the Final Technical Report for that phase (AFAL-TR-72-165, May 1972) and summarized in Section 2 of this report, was designed to bring together all of the knowledge gained through research studies on polarization and to add to this knowledge where necessary. The effort, though oriented towards the airborne DF problem, led to results applicable to almost any DF application. Four specific tasks were defined and carried out:

1) Survey the literature and prepare a bibliography.

2) Theoretically study the relationships between polarization, antenna responses, and angle of arrival measurements.

3) Design a program to collect data to verify the theoretical results, collect such data as can be done within the scope of the program, and compare the results with the theory.

4) Examine the methods used to evaluate DF systems and prepare standard definitions of terminology and measurements. Define a "Figure of Merit" that can be used to compare different DF systems with respect to their sensitivity to polarization errors.

Specific areas of interest included the following:

1) Polarization definitions and polarization properties of general electromagnetic fields.

2) Polarization properties of antennas and methods of describing them.

3) Relationships between polarization and measured arrival angle for general types of DF systems.

The DF systems of interest were those proposed or currently in use by the Air Force, and were primarily limited to azimuth-only measurements.

The second phase, whose results form the major portion of this report, had as its major objective the delineation of corrective techniques to minimize or eliminate polarization error. Two major tasks were defined as follows:

1) Maintain currency of the literature sources,

2) Investigate corrective techniques for representative types of systems.

Of these, the second task was considered to be, by far, the most important. Under task two, the primary emphasis was to be on phase comparison and TOA systems.

The analysis was to include the cost and complexity tradeoffs for utilizing various approaches to the problem and error analyses in sufficient detail to permit the Air Force to make a realistic evaluation of the practicality of reducing this source of error in direction finding systems.

The results are presented in summary form in Section 1.3 and in detail in Sections 3, 4, and 5 of this report. Conclusions and recommendations are contained in Section 6 and mathematical details, the new results of the literature survey, and a discussion of the state-of-the-art of polarization measuring equipment are contained in the Appendices. Section 3 is the section of primary importance, being devoted to the evaluation of corrective techniques, with Sections 4 and 5 containing the results of the study in related areas.

1.3 Summary of Major Results

1.3.1 Corrective Techniques - As stated above, the major area of emphasis was the evaluation of corrective techniques to minimize or eliminate polarization errors from DF systems. The need for corrective techniques is apparent when one compares the magnitude of polarization errors to the magnitudes of errors from other sources. Table I contains such a comparison for a phase interferometer. In considering the polarization errors, the crucial parameter is the antenna cross polarization ratio, r, which is a measure of the polarization mismatch between the two antennas in the interferometer system. Table II shows the correspondence between this parameter and other antenna specifications. It is apparent that unless a very good antenna system (r = 0.02) is used, polarization will be the dominant error source in many applications. In particular, without correction polarization will probably be the dominant source of errors associated with the system itself even for the very good antenna system.

TABLE I

CONTRIBUTIONS TO INTERFEROMETER PHASE ERROR(1)

NOISE:	95% CONFIDENCE LEVEL	
	SNR = 20 dB	11.2°
	SNR = 30 dB	3.6°
MULTIPATH:	95% CONFIDENCE LEVEL(2)	
	RATIO = 20 dB	8.1°
	RATIO = 30 dB	2.6°
POLARIZATION ERROR:	95% CONFIDENCE LEVEL	
	r = .1	16.6°
	r = .05	9.0°
	r = .02	3.6°
	r = .01	1.8°
EQUIPMENT PHASE ERRORS:		
	MAXIMUM(3)	7°
	TYPICAL(4)	2°

NOTES:

1) PHASE ERROR IS ALSO ERROR IN NORMALIZED ARRIVAL ANGLE.

2) MULTIPATH COMPONENT REMOVED ONE-FOURTH OF AMBIGUITY INTERVAL FROM DESIRED COMPONENT. RATIO = MULTIPATH AMPLITUDE/DIRECT PATH AMPLITUDE.

3) SPECIFICATIONS FOR BEST OFF-THE-SHELF DOWNCONVERTER-IF AMPLIFIER COMBINATION OVER FULL MIL-SPEC TEMPERATURE RANGE AND FULL OPERATING DYNAMIC SIGNAL STRENGTH RANGE.

4) MAXIMUM MEASURED ON SAMPLE OF ABOVE UNITS OVER LIMITED TEMPERATURE AND SIGNAL STRENGTH RANGE.

TABLE II

CORRESPONDENCE BETWEEN ANTENNA CROSS POLARIZATION RATIO AND OTHER ANTENNA SPECIFICATIONS

CIRCULAR POLARIZATION:

ANTENNA 1 IS PERFECT: AXIAL RATIO = 1
ANTENNA 2 IS IMPERFECT: AXIAL RATIO = AR

LINEAR POLARIZATION:

ANTENNA 1 IS VERTICAL
ANTENNA 2 IS TILTED BY ANGLE θ

ANTENNA CROSS POLARIZATION RATIO MAGNITUDE = r

r	AR	θ
0.01	0.2 dB	0.6°
0.02	0.35 dB	1.2°
0.05	0.9 dB	2.9°
0.1	1.7 dB	5.7°
0.5	9.5 dB	26.6°

r = 0.02 IS VERY GOOD
r = 0.1 IS AVERAGE
r = 0.5 IS POOR

A brief discussion of the meaning of the polarization errors is in order. Almost all of the numerical results are presented in terms of the probability of an error when the incident field is randomly polarized. The statistical properties of randomly polarized field are discussed in Section B-2 of Appendix B. For most manmade sources, the field can be considered randomly polarized only in the sense that there is no apriori knowledge of its polarization. The field from a specific source will have a definite polarization, in the absence of propagation effects, and is a random variable only by reference to the ensemble of all possible sources. Thus, an error probability of one percent implies that for DF measurements on 100 sources, one will be in error. If the set of DF measurements is repeated, the same one will again be in error. Thus, while a one-percent error probability may be quite acceptable in terms of a single DF measurement, it may well be non-acceptable if it means that one out of 100 sources cannot be located with sufficient accuracy.

Four basic techniques were investigated ranging from very simple to quite complex. These four techniques were:

1) Polarization Flagging
2) Polarization Switching
3) Polarization Tracking
4) Polarimeter.

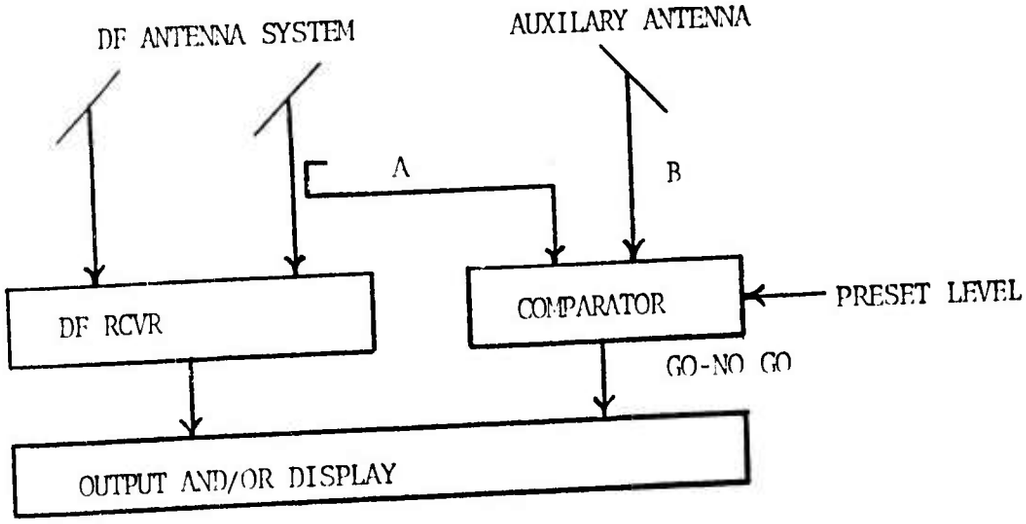
Block diagrams of these approaches are shown in Figures 1 through 4.

Before discussing these techniques it is necessary to explain the underlying philosophy and rationale.

A major result of the first phase, in which the polarization errors were evaluated, is that the significant errors occur when the incident field is nearly cross polarized to the antenna system. Qualitatively, this is explained in the following manner. Every real antenna is really two "antennas." The first of these has the designed polarization and possesses known properties. The second "antenna" is cross polarized and, since it exists because the real antenna does not precisely correspond to the ideal antenna assumed in the design, has properties which are usually unknown. The DF system assumes that the signal is being received by the design antenna and determines angle of arrival by utilizing the known properties of this antenna to interpret the voltages seen at the antenna terminals. Since the gain of the design antenna is usually much greater than that of the cross polarized one, its contribution to the output voltage is usually dominant and the assumptions implicit in the DF system are valid. However, when the incident field is nearly cross polarized, the contribution of the second "antenna" becomes dominant and the assumptions as to the antenna properties become invalid. Substantial errors then occur. The obvious solutions then are: 1) avoid situations where the incident field is cross polarized or 2) measure the properties of the antennas and the polarization of the incident field and obtain the direction of arrival by using a correct set of assumptions. The first three techniques follow the first approach while the fourth technique attempts to implement the second approach.

The first technique is "Polarization Flagging" and has the block diagram shown in Figure 1. In this approach a third antenna is added to the system which is nominally cross polarized to those used to DF. The signal received by this antenna is compared with that received by one of the DF system antennas and, if the ratio of the two signals exceeds a preset level a "No-Go" signal is sent to the DF output. When the comparison yields a "Go" condition, the errors will be the same as for polarization switching (described next) if the decision level for flagging is the same as the switching level. The principal advantages of this technique are its extreme simplicity, its inherently wide bandwidth, and its effectiveness under conditions where the field polarization is rapidly varying. The major disadvantage is the substantial loss of data which will occur when the incident field is fixed, or slowly varying, and nearly cross polarized.

The second technique, "Polarization Switching," has the block diagram shown in Figure 2. This approach is designed to overcome the principal disadvantage of the flagging technique. To implement this approach, the DF antenna system is replaced by cross polarized pairs. One of these pairs also provides inputs to a comparator which determines which of the two orthogonal antenna polarizations is closest to that of the field. The information is used to control a switch which selects the signal from the antenna with the best polarization to become the input to the DF receiver. In order to avoid excessive switching back and



POLARIZATION FLAGGING

NO GO IF (SIGNAL B)/(SIGNAL A) > PRESET LEVEL

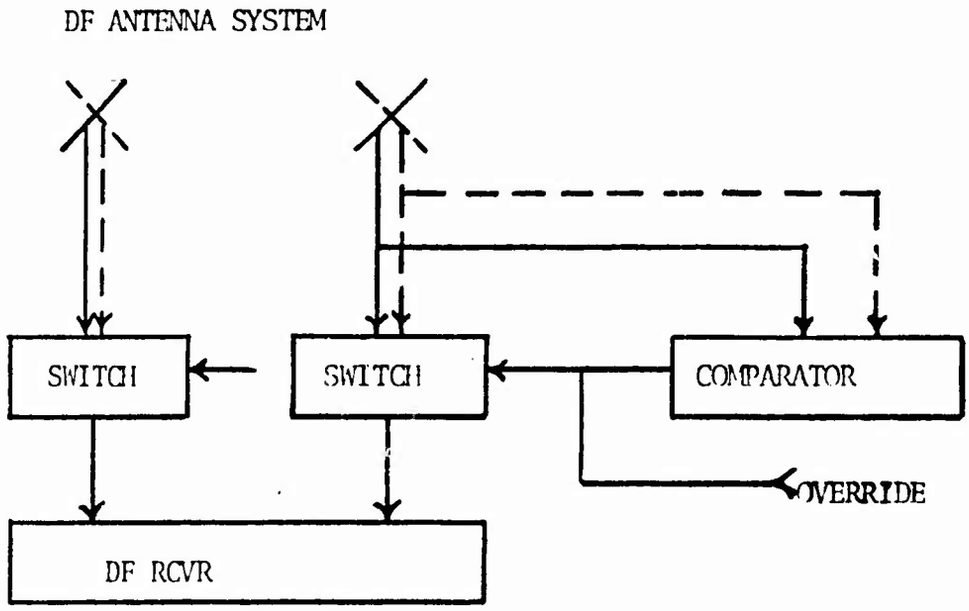
ADVANTAGES:

1. SIMPLICITY
2. EFFECTIVE WHEN FIELD POLARIZATION IS RAPIDLY VARYING

DISADVANTAGES:

1. SUBSTANTIAL LOSS OF DATA WHEN FIELD IS SLOWLY VARYING AND NEARLY CROSS POLARIZED.

Figure 1 Polarization Flagging Technique



POLARIZATION SWITCHING

ADVANTAGES:

1. NEARLY AS SIMPLE AS SENSING.
2. EFFECTIVE IN ELIMINATING OR REDUCING MAJOR ERRORS.
3. NO LOST DATA

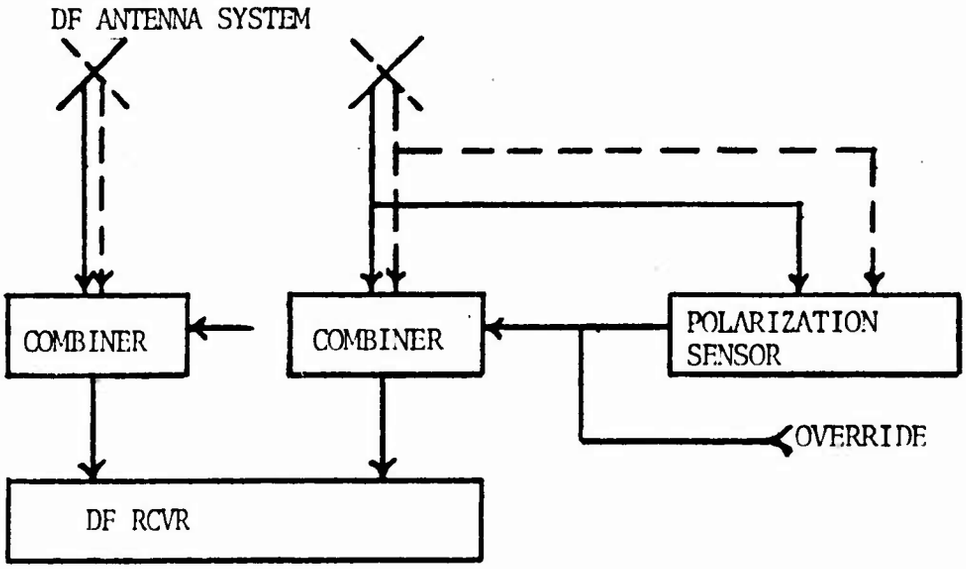
DISADVANTAGES:

1. POSSIBLE SWITCHING TRANSIENTS WHEN FIELD POLARIZATION VARRIES VERY RAPIDLY.

Figure 2 Polarization Switching Technique

forth when the field polarization is midway between those of the two orthogonal antennas it is necessary to incorporate a form of hysteresis. This is accomplished by switching when the ratio of the signal from the (currently) unused antenna to the signal from the (currently) used antenna exceeds a preset value. It is convenient to denote this ratio by tan θ_o. Thus, a value for θ_o of 45 degrees implies switching at equality and provides no hysteresis while a value of 60 degrees implies switching when the ratio reaches 1.73 and provides considerable hysteresis and therefore much less susceptibility to frequent switching transients. The principal advantages of this technique are its relative simplicity, inherently wide bandwidth, freedom from loss of data, and overall effectiveness in reducing errors. The major disadvantage appears to be the switching transients which may distress some DF receivers. A possible disadvantage is the fact that a wrong selection can be made under certain multipath conditions (see Section 5).

The third technique is "Polarization Tracking." The block diagram for this system is shown in Figure 3. This technique is an extension of the second technique to provide continuous "switching," always to the optimum polarization. Again, a cross polarized receiving antenna system is required. The signals from one pair of cross polarized antennas are fed to a polarization sensor where the polarization of the incoming field is, perhaps implicitly, measured. The polarization parameters are used to control combiners which form a new DF antenna system which is



POLARIZATION TRACKING

ADVANTAGES: INHERENTLY SMALL RESIDUAL ERRORS.

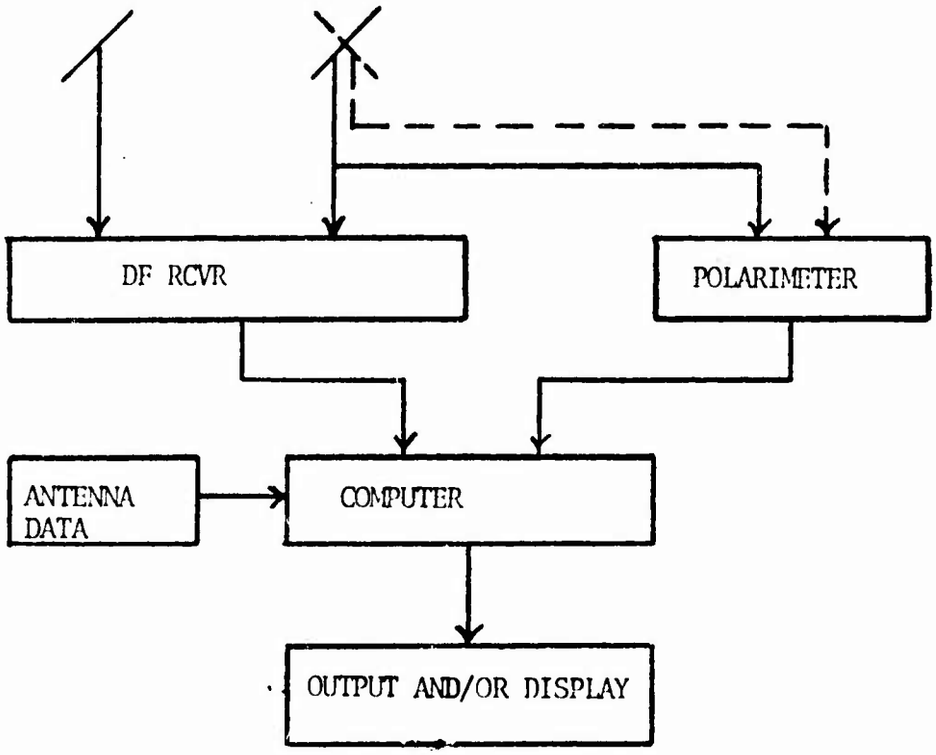
DISADVANTAGES:

1. MODERATELY HIGH COMPLEXITY.
2. SOME BANDWIDTH PROBLEMS.

Figure 3 · Polarization Tracking Technique

matched in polarization to the incoming field. The major advantage of this system is good effectiveness. The complexity might be considered moderate and is a disadvantage when compared with the first two techniques but an advantage when compared with the fourth one. Bandwidth problems in the combiners are likely to constitute the major disadvantage.

The fourth technique is loosely designated the "Polarimeter" method. The block diagram is shown in Figure 4. In this technique a precision polarimeter is used to determine the polarization of the incoming signal. This information is used, together with premeasured antenna data and the raw information from the DF system, to compute a correction for the polarization error. The error correction is extremely good for small errors, say those which occur more than 80 percent of the time, but degrades rapidly as the errors become larger and, for the very largest errors, the error after correction may be worse than that before correction. The point at which this approach becomes useless depends upon the accuracy of the premeasured antenna data and the polarimeter measurement. The effect of uncertainties in the antenna data, errors in the polarimeter outputs, and deviations of the operation of the DF system from the model used in the computations is to introduce errors into the correction. The sum of all of these factors can be approximated by introducing an equivalent signal to noise ratio. This ratio, which will probably be controlled by the uncertainties in the antenna data in an operational system, could be expected to be not better than



POLARIMETER CORRECTION

ADVANTAGES:

1. EXTREMELY GOOD FOR SMALL ERRORS.

DISADVANTAGES:

1. COMPLEX SYSTEM.
2. REQUIRES EXTENSIVE ANTENNA CALIB. DATA.
3. VERY LARGE ERRORS ARE NOT CORRECTED WELL.

Figure 4 Polarimeter Correction Technique

20 dB and might be as low as 10 dB. The major advantage of this technique is the very good correction of small errors. Its major disadvantages are its complexity, computational requirements, and the requirement for precise and extensive antenna data. As a consequence of these factors, it is apparent that this technique is primarily suitable for research applications and is not feasible for operational systems.

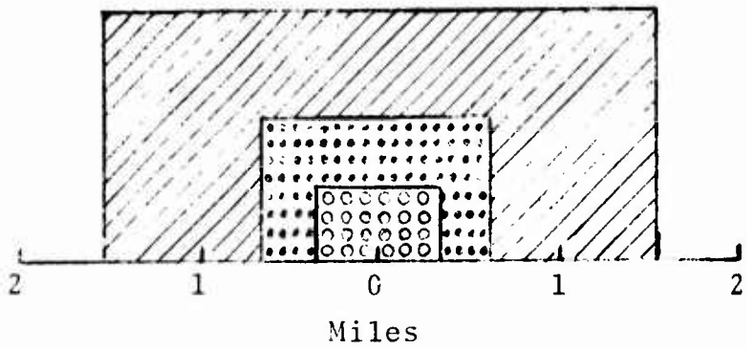
Since the first technique has errors comparable to the second, and the fourth technique does not appear to be practical in an operational system, we are left with the second and third systems to be compared for accuracy. A graphic comparison is provided by Figure 5 which shows the width of the 95 percent confidence limits for an eight wavelength interferometer at 100 miles range. The uncertainty area is shown for a poorly matched antenna pair and a well matched pair* and for the uncorrected and corrected by switching ($\theta_o = 60°$) and tracking. Two things are evident. First, the well matched antenna pair even without correction is almost as good as the poorly matched one with correction. Second, the correction by either method is effective but the additional complexity of the tracking technique as opposed to the switching technique does result in improved accuracy.

The general conclusions that can be drawn are as follows.

---

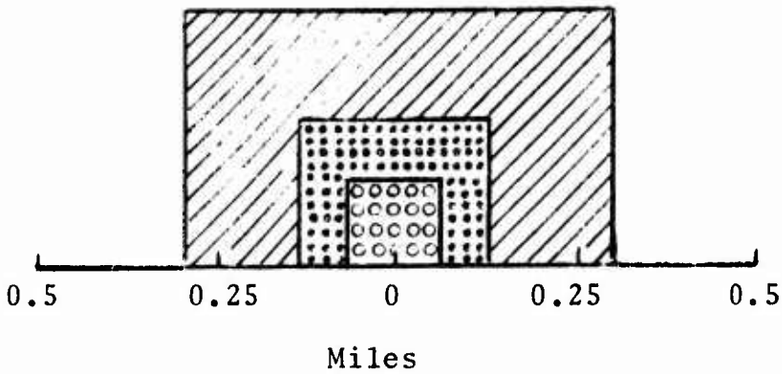
*This refers to the degree to which they are matched to each other in polarization. For example, cross polarization ratio of 0.1 applies to two linear dipoles misaligned by 5.7 degrees or to a perfect circularly polarized antenna and one with an axial ratio of 1.75 dB.

DF SYSTEM AREA OF UNCERTAINTY DUE TO POLARIZATION ERRORS



POOR ANTENNA CROSS POLARIZATION RATIO (0.5)

NO CORRECTION
SWITCHING
TRACKING



GOOD ANTENNA CROSS POLARIZATION RATIO (0.1)

95% CONFIDENCE LIMITS AT 100 MILES
8 WAVE LENGTHS INTERFEROMETER

Figure 5 Comparison of Interferometer Uncertainty Area With and Without Corrections

Polarization tracking is the system of choice for future DF systems. If this technique is adopted, however, considerable care in the design and manufacture of the combiner networks must be exercised in order to prevent mistracking between channels, especially with regard to phase shift, in excess of that contributed by other portions of the DF system. This can probably be accomplished without excessive difficulty in the design of new systems but may make retrofitting to existing systems difficult. Polarization switching is well suited, by its lack of complexity and its loose component requirements, to the retrofitting of present systems. The only component requirement is matching of the insertion loss and phase in the two switches, a relatively easy requirement to meet. Polarization flagging might be the technique of choice for immediate retrofit in applications where possible, but relatively infrequent, loss of data can be accepted. It is by far the simplest technique and requires no modifications, other than the addition of a directional coupler to one antenna line, to the path of signal flow for the DF system.

1.3.2 Other Areas of Consideration - In addition to the analysis of corrective techniques, two other areas received some attention during the course of the study. These were 1) further analysis of TOA systems and 2) polarization changes due to multipath.

In the first area, a further analysis of TOA systems, it was concluded the effects of polarization are generally to distort an incoming pulse but that such distortion is

equivalent to echoes whose delays are of the order of transit times between various parts of the antenna system. Such echoes should not be a significant source of errors when pulsed signals are involved. They could be serious however in the case of communications or other continuous signals if the signal bandwidth is comparable with the reciprocal of the echo delay. A more serious error can occur however if multipath propagation conditions are involved. Since there is usually a change of polarization upon reflection, the multipath signal, although weaker in general than the direct signal, may be matched to the receiving antenna polarization better than the direct signal and thus appear to be the stronger of the two signals. If this occurs, the measured time of arrival will correspond to the reflected path rather then the direct path and very substantial errors will occur. Since the direct path signal is usually the stronger of the two, either polarization switching or tracking will generally tend to match the receiving system polarization to that of the direct path signal and thus significantly reduce this type of error. An exception to this statement will occur if the polarization conversion upon reflection is only partial. Then there exist phase relationships between the signals arriving over the two paths such that destructive interference between the direct path signal and the unconverted part of the reflected signal cause the received signal to be dominantly cross polarized to the transmitted signal. When this occurs, the corrective techniques will accentuate the multipath problem. These considerations led to the inclusion of the second area mentioned at the beginning of this section.

The question of polarization changes due to multipath, in particular the conditions under which the corrective techniques would lead to worse results than if no correction were applied, was studied in some detail. The results generally are as follows. When the transmitted polarization is circular and the combination of transmitting and receiving antenna gains over the direct path is equal to or greater than that over the reflected path, the transmitted polarization will always be dominant except for reflections at low angles from seawater at frequencies below one or two GHz. When the combined antenna gains are greatest over the reflected path, there will be a fraction of time when the cross polarization will predominate and, if the difference is sufficiently great, the cross polarization will always predominate at incidence angles above the Brewster angle. If the transmitted polarization is horizontal, and the reflection takes place from terrain with a tilt, there is a small probability that the cross polarized signal will predominate even when the antenna gains are equal over the two paths.

Situations where the cross polarized component is dominant could cause catastrophic failures in the polarization switching technique if the transmitted polarization is one of those switched between. Since such situations generally depend upon the relative phase between two paths which is time variable in most airborne situations, there would be many circumstances in which operator override of the switching operation would be desirable in order to avoid this possibility. A similar but less catastrophic failure would occur in the polarization tracking technique. For this reason, an override provision has been indicated in Figures 2 and 3.

2. SUMMARY OF FIRST PHASE RESULTS

2.1 General Considerations

A general approach to the problem of evaluating DF errors due to polarization was developed and applied to three general classes of DF systems. This approach is based upon the concept of a complex effective vector length and its unitized form which is called the polarization vector. This concept enables one to concisely define the polarization properties of an arbitrary antenna and the incident field as well as the interaction between these two polarizations.

The analytical approach followed was directed towards the definition of a minimal number of antenna parameters applicable to a specific class of DF systems, which quantitatively characterize the sensitivity of the system to errors due to polarization. A conscious effort was made to have these theoretically derived quantities correspond to physically measurable parameters of the antenna system. Broadly speaking, these goals were achieved.

A major conclusion is that there will be no polarization error unless the antenna system has some response to a field which is cross polarized to the design polarization. This is small comfort since all real antennas will have such a response but it does suggest one goal of the antenna design.

Appendix B contains a general discussion of polarization and polarization descriptions. Included in this section is a discussion of antenna polarization and what is meant by cross polarization and the properties of randomly polarized fields. In this study, random polarization usually implies statistical randomness in the sense that the polarization of the field in a particular situation is not known a priori. It does not usually imply that the polarization of the field received over a specific path is a random function of time although this may also be true.

The three specific DF system classes considered were: (1) phase interferometer, (2) amplitude monopulse, and (3) time of arrival (TOA). The results for each class are briefly summarized in the following paragraphs.

2.2 Phase Interferometer

The two element phase interferometer was found to be characterized by a single parameter of the antenna system. This parameter, denoted by r, is the antenna cross polarization ratio defined as the output of one antenna when illuminated by a completely "cross polarized" field to the output of the same antenna when illuminated by an equal amplitude field which has the "correct" polarization. The "correct" polarization is the polarization of the other antenna. This antenna cross polarization ratio, which is a function of the angle of arrival of the incoming field, completely describes the sensitivity of the system to polarization errors.

The complex product of the antenna and field cross polarization ratios (amplitude and phase) uniquely specifies the error in electrical angle of arrival due to polarization leading to universal error curves. The probability distribution of error, assuming a randomly polarized incident field, was computed and is presented* in Figures 6 and 7. As an example, there is a 15 percent probability that the absolute value of the error will exceed one-twentieth of the interferometer ambiguity interval for an antenna cross polarization ratio (r) equal to 0.2. This value of r would be obtained by two linear antennas misaligned by 11 degrees, for instance.

For an interferometer of the type studied, there exists a field polarization which will cause an error equal to half the interferometer ambiguity range. This implies that the proper sort of field polarization variation would cause a tracking interferometer to move from one grating lobe to another, thus causing an unbounded error. This possibility was not examined explicitly since such large errors depend upon the specific operation of a given system, including whether or not a shorter baseline interferometer is available for ambiguity resolution. The results presented are therefore optimistic in that it was assumed that the correct grating lobe was always selected. Grating lobe resolution errors were considered during the second phase and are discussed in Section A-5 of Appendix A.

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*Refer to Section A-5 of Appendix for an explanation of the relationship between normalized arrival angle and space arrival angle for phase interferometers.

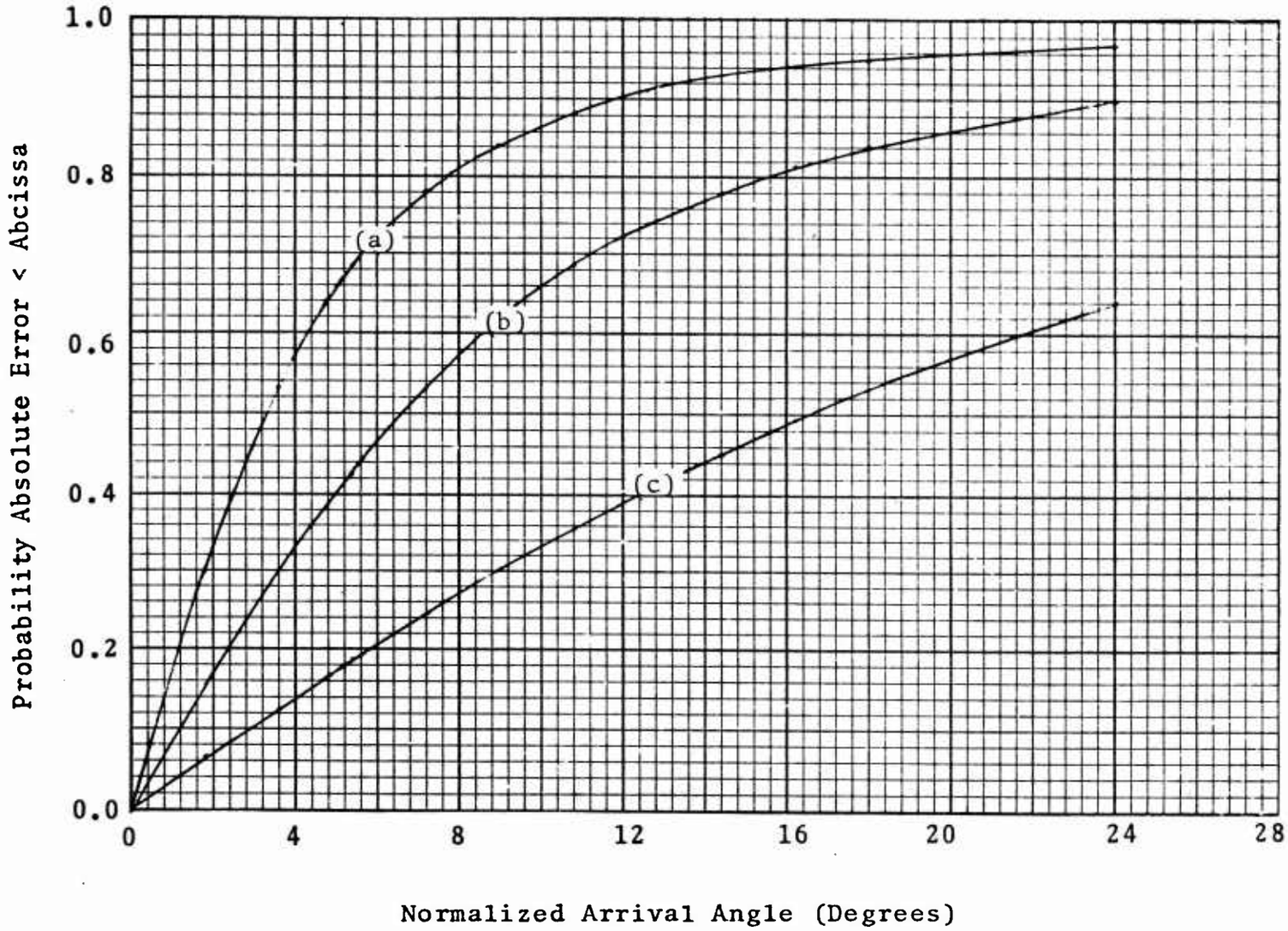

Figure 6 Cumulative Probability Distribution of Interferometer Error.
a) r = 0.1, b) r = 0.2, c) r = 0.5

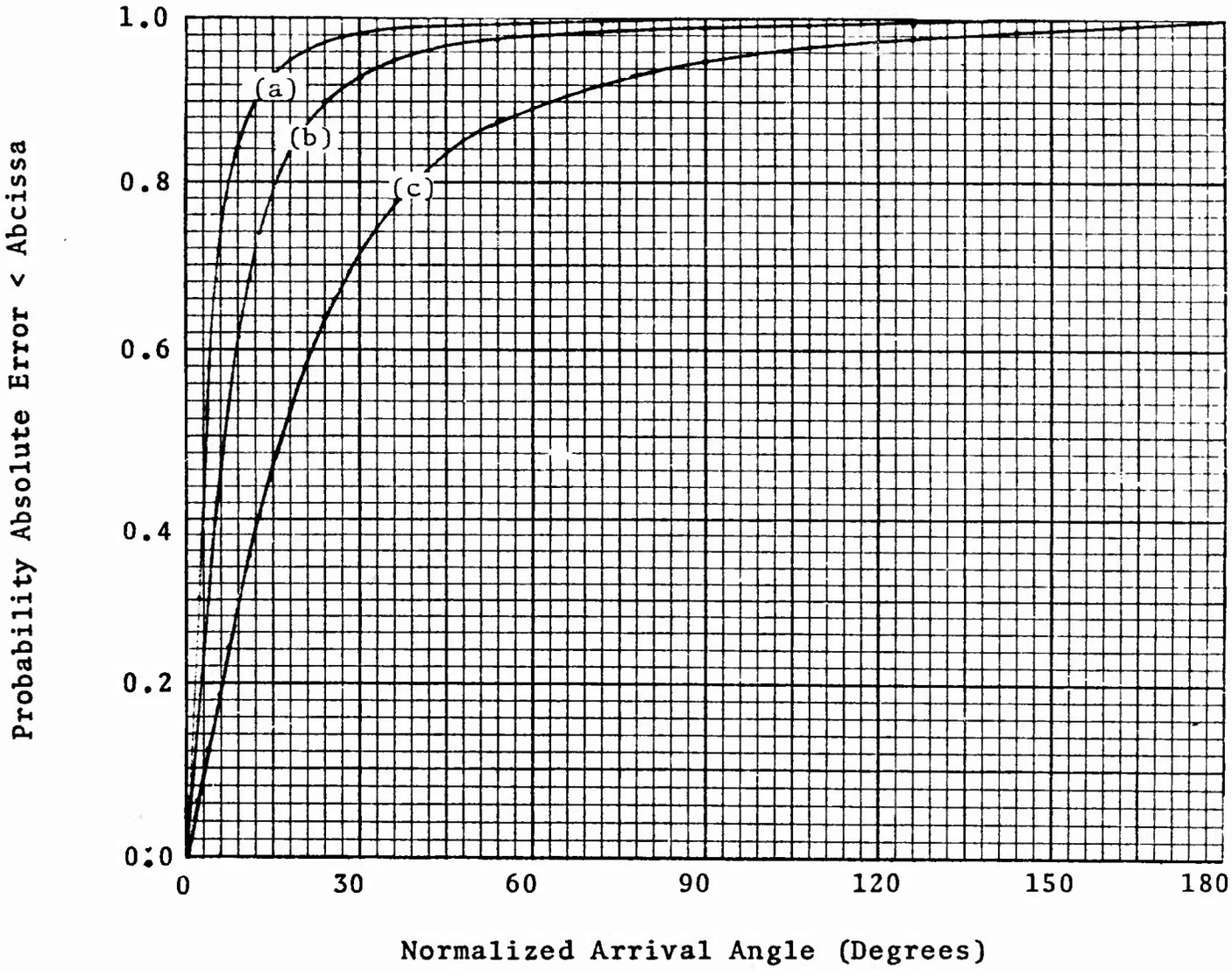

Figure 7 Cumulative Probability Distribution of Interferometer Error.

a) r = 0.1, b) r = 0.2, c) r = 0.5

In a randomly polarized field, the probability distribution of the error due to polarization is similar to the probability distribution of the error due to noise. The equivalent signal-to-noise ratio is approximately 2 dB less than the reciprocal of the antenna cross polarization ratio. That is, $r = 0.1$ corresponds to a signal-to-noise ratio of 18 dB.

2.3 Amplitude Monopulse

This class of DF systems was found to require at least three parameters to characterize it. In most real situations, however, it was found that only one of these, denoted by k_1, was of any real importance. This parameter, which has the dimensions of an angle, is defined as the magnitude of the ratio of the difference pattern output measured on boresight in a cross polarized field to the slope of the difference pattern measured on boresight in a correctly polarized field of the same amplitude. The "Correct" polarization for this system is defined as the boresight polarization of the sum pattern. For a symmetric monopulse antenna in free space, this parameter would be independent of direction of arrival. This is not generally true of an airborne monopulse antenna due to radome and aircraft structure effects.

A typical probability distribution for the polarization error in a randomly polarized field is presented in Figure 8. For most antennas, the curve for $k_2 = 0$ in this figure is applicable. Under this assumption, we find from the

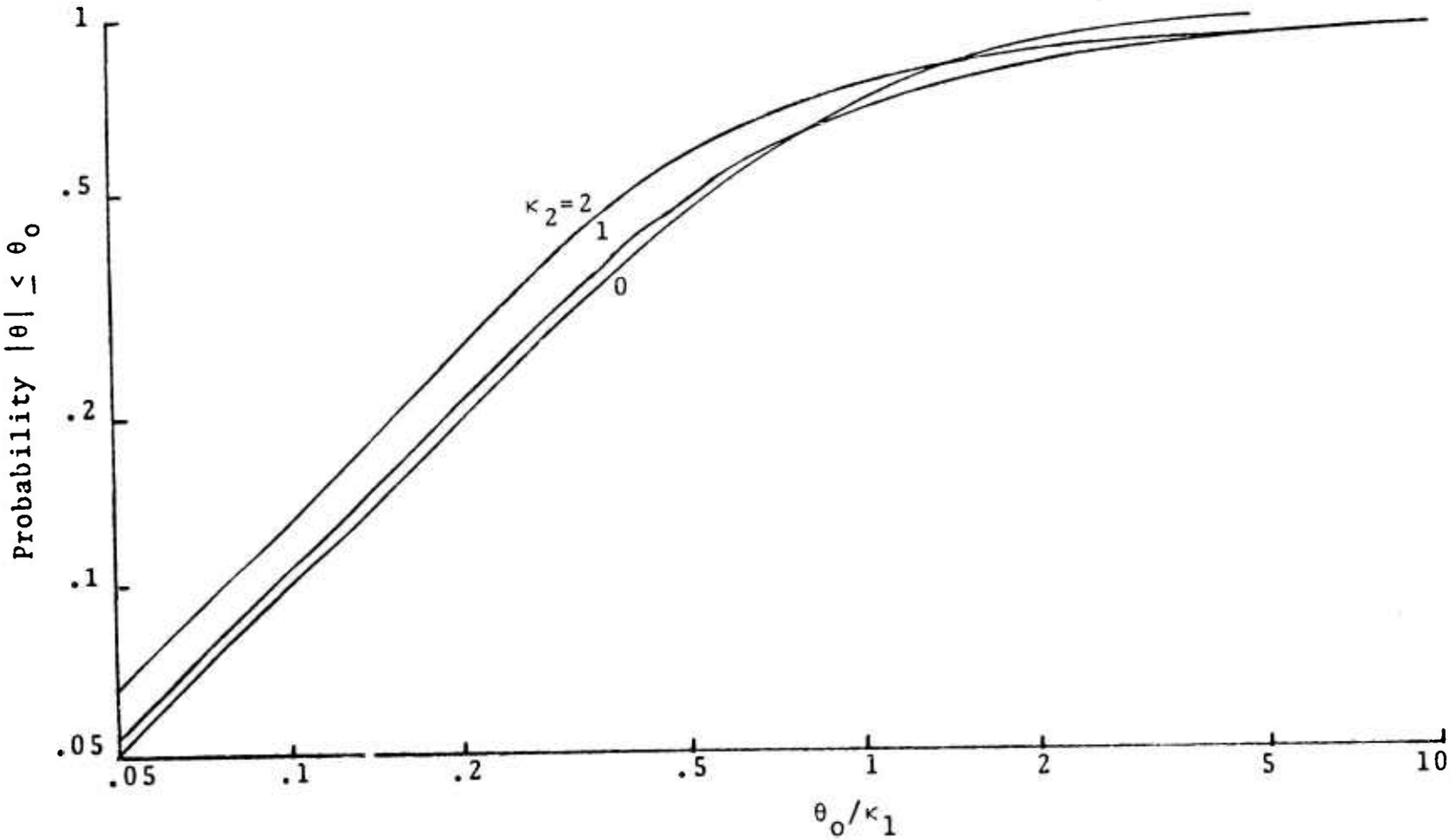

Figure 8. Monopulse Error Probability Distribution ψ = 45 Degrees

figure that there is a 25 percent probability that the error exceeds k_1. For a good antenna system, the parametric angle k_1 might be of the order of one-twentieth of the sum pattern 3 dB beamwidth.

The simple first order model of a one-dimensional monopulse used in the computations does not permit the prediction of errors larger than the antenna beamwidth. A more complete analysis would show that such large errors are theoretically possible, especially when a two-dimensional monopulse is considered, if the incident field polarization has the correct behavior. The results presented are therefore to this extent optimistic. Such large errors would, however, be highly improbable unless a very intelligent effort was made to confuse the system.

As in the case of the interferometer, an equivalent signal-to-noise ratio can be assigned to a monopulse antenna with a given set of parameters. As a rule of thumb, the equivalent signal-to-noise ratio is approximately equal to the ratio of the beamwidth to the parametric angle k_1.

2.4 TOA System

Three models of a TOA system were considered. The three models are: (1) Split-gate or center-of-gravity detector; (2) Maximal-slope detector; and (3) 3-dB-below-peak detector. Each of these models determines time of arrival by measuring the time of occurrence of a specific feature of the pulse, the first utilizing the pulse center

and the second two utilizing points on the leading edge. It was not possible to derive a general error expression for the TOA class of systems because of the extremely large number of system parameters involved. For this reason, some specific examples were used and the errors computed.

The mechanism by which polarization affects a TOA system is essentially one of dispersion. If the antenna polarization and/or the field polarization varies with frequency, the time signal appearing at the antenna terminals will be different from that which would appear if polarization were not a factor.

Although no general error equations could be found, one result that is widely applicable was derived. This occurs when the bandwidth of the signal is relatively small and a simple antenna is used so that the antenna-field polarization coupling may be represented as a linear function of frequency across the signal passband. In this case, the first order effect of the dispersion due to polarization is to shift the pulse in time. This shift, which may be either positive or negative, is bounded by the coefficient of $2\pi(f-f_o)$ in the linear expansion of the antenna-field polarization coupling, where f_o is the center frequency of the undistorted signal. A typical result is that there is a 94 percent probability that the time shift is bounded by 0.1 of the pulsewidth for a 20 percent signal bandwidth, a simple antenna, and a randomly polarized incident field.

2.5 Measurement Program

A series of measurements, utilizing available components, was made for comparison with the theoretical results. A narrow band variable polarization source was used and the transmitted polarizations were varied through all polarizations in such a fashion that the results could be easily interpreted in terms of the response that would be observed in a randomly polarized field. An 8λ baseline interferometer and a one-dimensional monopulse were evaluated. In general, the agreement between theory and experiment was excellent.

2.6 DF Evaluation Techniques

Presently used techniques to evaluate DF system accuracies were examined and found to be faulty in that they are consistently designed to minimize errors due to polarization.

Based upon the results of the analytical study, it was recommended that a polarization "Figure of Merit" be assigned to all DF systems. This figure of merit can be consistently defined by specifying a point on the probability distribution for errors in a randomly polarized field. Such a definition is independent of the specific DF technique used.

The detailed results of this first phase are contained in the Final Technical Report, Reference 1, along with an annotated bibliography and a dictionary of polarization terms.

3. POLARIZATION ERROR CORRECTIONS

3.1 General Discussion

An overall summary of the four basic corrective techniques investigated has already been presented in Section 1.3.1. In this section we present more detailed discussion of these techniques, their limitations, and the results obtainable. Before proceeding with these discussions of the individual techniques, it is proper to restate the aspects of polarization errors which make correction possible. These factors are:

1) Polarization errors result from polarization mismatch between the antenna elements of the DF system.

2) Polarization errors are directly related to the degree to which a polarization mismatch between the incident field and the antenna system exists.

3) Given sufficient knowledge of the antenna system polarization and the field polarization, the errors are computable.

The first factor suggests that the first step in minimizing polarization errors is to exercise due care in polarization matching the antenna elements of the DF system. This is an obvious preventive measure which will negate a requirement for further corrective steps in many applications. It is, however, not possible to completely match antennas, especially when mounted on an aircraft or other

vehicle where coupling to the external structure will occur. It is also not feasible to redesign existing antenna systems in order to obtain an improved polarization match. Thus, there still exists a need for corrective techniques, especially in high accuracy applications or those requiring a high degree of confidence. The corrective techniques studied are suggested by the second and third factors cited above.

The second factor suggests that polarization errors may be minimized by insuring that the incident field polarization is always more or less matched to that of the antenna system. This concept leads to the first three techniques to be described below.

The third factor, that the errors are computable, suggests that a corrective technique based upon the acquisition of detailed knowledge may be effective. This is the fourth technique to be described below.

In the following discussions we present for each technique the principles of operation, the limitations on accuracy, the general results obtainable and, specifically, the resultant errors in a phase interferometer DF system. The results of the mathematical analysis are given here with the details of the derivations contained in Appendix A for those interested.

In the following discussions we shall refer to the two antennas of a DF system. It is recognized that many DF systems have more than two antenna elements and some DF

systems do not explicitly have more than one element. However, a small amount of reflection will convince one that all DF systems, in reality, must consist of antenna pairs. Fundamentally, there must be at least two antennas because a DF measurement cannot be made using only the output voltage from a single, fixed antenna; there is simply not enough information. Even the simple case of a scanned beam DF system in which the direction of arrival is defined as the pointing direction which yields maximum signal strength is no exception. In this case, there are actually an infinite number of antennas, one for each pointing direction and the final angle of arrival determination is made by comparing, pairwise, the output of the three "antennas" most nearly pointed in the direction from which the signal is coming. In the case of a monopulse system, the two antennas are clearly the sum and difference antennas. A single level phase interferometer clearly has two antennas and a multilevel interferometer is merely a set of parallel single level interferometers each of which is essentially an independent DF system. Thus, the two antenna model of a general DF system used in the following discussion is well justified.

3.2 Polarization Flagging

This extremely simple technique is well suited for immediate retrofit to many DF systems. The system block diagram is shown in Figure 1. Implementation of this approach requires the addition of a third antenna which is nominally cross polarized to those used by the

DF system, a coupler to tap off some of the signal from one of the DF antennas, and a power comparator. The signals received in the two orthogonal polarizations are compared and, based upon the value of their ratio, a decision is made as to whether or not the probable error due to polarization will be acceptable. If it is not, a message indicating this fact is sent to the DF system output device. The unflagged data will of course contain the same errors that would occur without the incorporation of this technique. These errors can be computed for the system in question using the same formulas previously derived except for the distribution of field cross polarization ratio which is changed. The modified distribution of the field cross polarization ratio is presented in the next section and is a function of the decision level chosen. The error distribution after correction by this technique is the same as that which exists after correction by polarization switching.

The principal disadvantage of this technique is clearly the fact that, when the flag is up, there is no DF data. This loss of data could potentially be serious, although not as serious as the erroneous data that would be obtained if flagging were not used, in situations where the incident field is substantially cross polarized to the DF antenna system and does not change. If the field polarization is rapidly changing, because of intentional action by the transmitter or unintentially as a result of antenna scanning etc., this would not be a significant problem. An exception to this statement occurs if the polarization changes are due to multipath (see the discussion in Section 5).

This technique is especially attractive from the hardware standpoint since none of the components have critical requirements. Small errors in the comparator or the decision level are not important, for example. Nor is it necessary for the cross polarized antenna pair to be truly orthogonal. The only real hardware problems would be those which arise from the need to not affect the DF system's performance in the process of interconnecting with the flagging system. Of principal concern here would be the method used to couple energy from one of the DF system antennas and the extraction of an LO source if the comparator utilizes heterodyne techniques. These should not present problems as long as they are recognized to be potential sources of errors and care is exercised.

There are no apparent bandwidth problems with this technique.

3.3 Polarization Switching

The polarization switching technique is specifically designed to overcome the major disadvantage (i.e., loss of data) of the polarization flagging technique. This technique, whose block diagram is shown in Figure 2, involves replacement of the usual DF antenna system by cross polarized pairs, the addition of a comparator and the insertion of switches into the signal lines from antenna to DF receiver. A portion of the signals from one of the cross polarized pairs is sent to the comparator where the relative signal strengths are compared. If the ratio of the signal received in the polarization not currently

being used by the DF system to the signal received in the currently used polarization exceeds a preset value, the DF system is switched to the other polarization. This results in limiting the maximum possible value of the field cross polarization ratio to the reference value used by the comparator just as in the case of the flagging technique but without the loss of data. Ideally, the reference value would be unity. This could, however, result in continuous switching if the polarization of the incident field were midway between the two orthogonal polarizations of the antenna system. This would occur, for example, if the antenna pairs consisted of right and left circular and a linear polarization was received. Continuous switching of this sort would introduce frequent transients into the signal as seen by the DF system which would degrade the performance of some systems. This situation can be avoided by choosing a reference ratio greater than unity. If we denote the signals from the two antennas of a cross polarized pair by A and B respectively and choose as an example a reference ratio of 1.7, we find the following. If signal A is presently being received, switching will not occur until B/A = 1.7. Once the signal has been switched, another switching command will not occur until A/B = 1.7 or B/A = 0.6. Thus, the relative values of the two signals must change by 1.7/.6 (9 dB) before the second switching will occur. This would result in infrequent switching even under worst case conditions and, thus, relative freedom from transient effects. The larger the value of the reference ratio used, the larger the possible value of the field cross polarization ratio and, therefore, the larger the possible errors.

As noted above, the process of switching has the effect of limiting the maximum possible value of the amplitude (s) of the field cross polarization ratio to the value of the reference ratio for switching. Under these conditions the probability density function of s in a randomly polarized field becomes

$$p(s) = \csc^2\theta_o \, [2s/(1+s^2)^2] \quad \text{for } 0 \leq s \leq \tan\theta_o \qquad (1)$$
$$= 0 \qquad \text{otherwise}$$

where $\tan\theta_o$ is the reference ratio for switching. The probability distribution of the errors after correction by polarization switching can be found by using equation (1) in the appropriate formula of reference 1.

In order to evaluate the improvement obtained by this technique, let us examine the single level interferometer. For this case the cumulative error distribution in a randomly polarized field becomes

$$P[|\delta|<\delta_o] = (2/\pi)\,\{\theta_1 + \theta_2\} \qquad \text{for } x < \tan\theta_o \qquad (2)$$
$$= 1 \qquad \text{for } x > \tan\theta_o$$

where $x = (1/r)\sin\delta_o$,

$$\theta_1 = \sin^{-1}(x\cot\theta_o),$$

$$\theta_2 = (x\csc^2\theta_o/\sqrt{1+x^2})\cos^{-1}(\sqrt{1+x^2}\cos\theta_o)$$

and r is the antenna cross polarization ratio*. This equation assumes that $r \tan \theta_o < 1$ which includes almost all situations of practical interest. We note that a major effect of switching is to limit the maximum error that can occur. This is important in multilevel interferometers since ambiguity errors occur only if the error in normalized arrival angle measured on one of the short base line interferometers exceeds a certain amount. This amount, which depends upon the number of interferometer levels used for ambiguity resolution, is about 45 degrees for a three-level interferometer. The maximum possible error is a function of the antenna cross polarization ratio, r, and the switching reference ratio ($\tan \theta_o$) and is tabulated in Table 3.

The probability distribution of the error after correction is plotted in Figures 9 (r=0.1) and 10 (r=0.5) along with the distributions for no corrections and correction by tracking and polarimeter. Switching levels corresponding to 45° (ratio = 1) and 60° (ratio = 1.7) are plotted.

3.4 Polarization Tracking

The accuracy of correction in the polarization switching is limited by the fact that only two receiving polarizations are available. This limits the degree to which the antenna polarization can be matched to that of the incident field. A logical extension then would be

*The symbol ε is used for the error without correction and δ for the error after correction. For the interferometer, these refer to the error in normalized arrival angle.

TABLE III

Maximum Interferometer Error

With Polarization Switching

r	Maximum Error (Degrees) $\theta_o=45°$	$\theta_o=60°$
.1	5.7	10
.2	11.5	20.3
.3	17.5	31.3
.4	23.6	43.9
.40825	-	45
.5	30.0	60
.57735	-	90
.6	36.9	-
.7	44.4	-
.7071	45	-

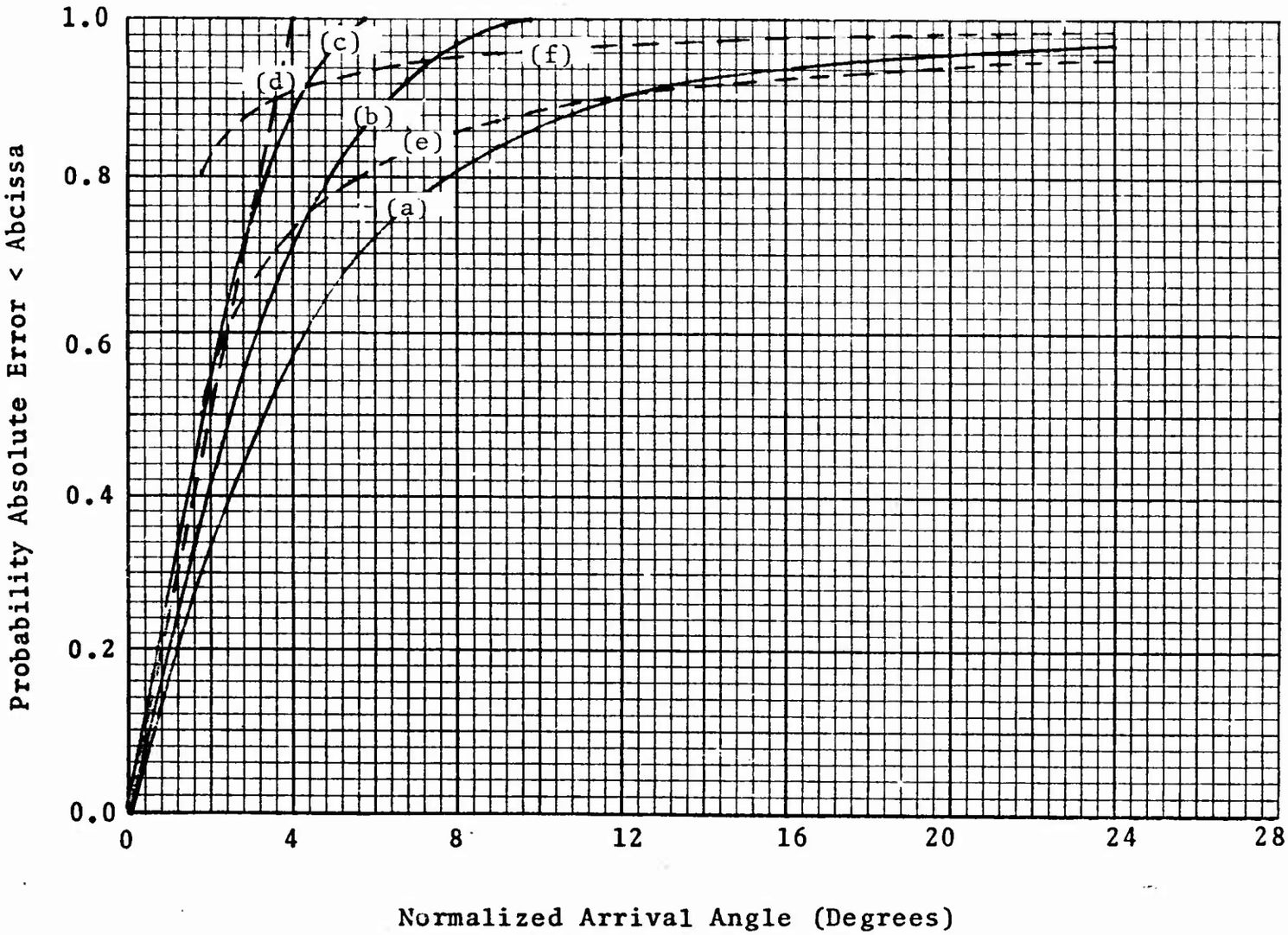

Figure 9 Error Probability Distributions (r = 0.1)
a) No correction, b) Switching θ_o = 60°, c) Switching θ_o = 45°
d) Tracking, e) Polarimeter SNR=10 dB, f) Polarimeter SNR=20 dB.

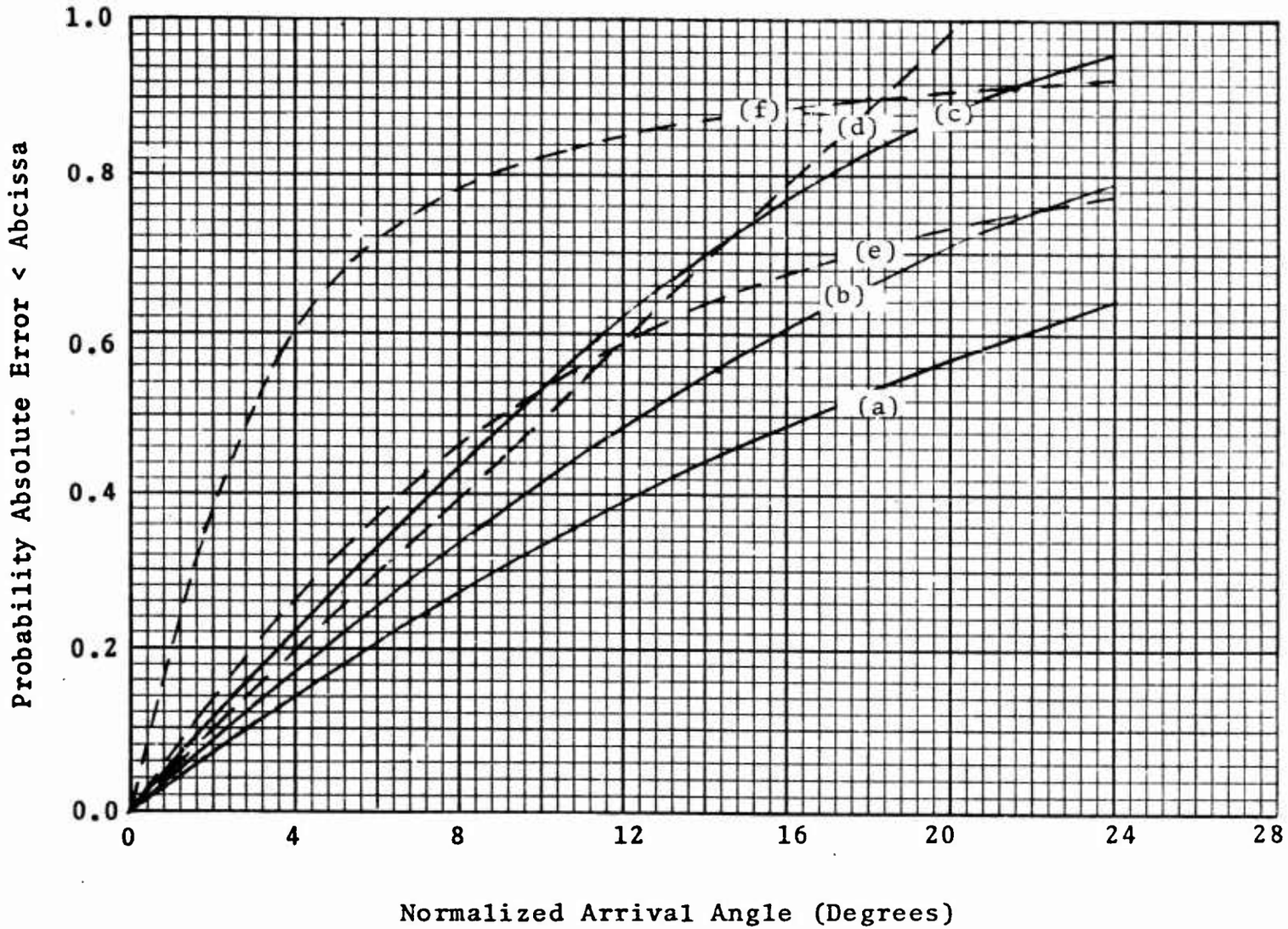

Figure 10 Interferometer Error Probability Distribution (r = 0.5)
a) No correction, b) Switching θ_o=60°, c) Switching θ_o=45°
d) Tracking, e) Polarimeter SNR=10 dB, f) Polarimeter SNR=20 dB

the incorporation of additional antenna polarizations. When carried to the limit of a large number of antenna polarizations, this suggests the concept of polarization tracking. A block diagram of a system to implement this concept is shown in Figure 3. In this system, each DF antenna system element consists of a cross polarized pair. The signals from one of these pairs are sent to a polarization sensor where the polarization of the received field is determined. This information is used to control combiners which take the signals from each pair and form a virtual set of DF system antennas which are matched in polarization to the incident field. The fundamental fact that any polarization antenna can be represented as a linear combination of an arbitrary pair of cross polarized antenna elements is utilized. In principle, polarization tracking would completely eliminate polarization errors. In practice, however, this is not quite true for the following reasons. First, the polarization sensor and the combiners are never perfect which means that the receiving antenna polarization will not be precisely matched to that of the incident field nor will the antenna polarizations be matched precisely to each other. Secondly, and of greater importance, the combination process will introduce an additional source of error into the system that is a result of the fact that the elements of the "cross polarized" pairs are not strictly orthogonal. This results in a gain and phase mismatch between the DF antenna system elements under some circumstances. Since this effect is rather subtle and is the major source of error after correction by the technique an explanation is appropriate at this point.

The details of the mathematics are contained in Section A-2 of Appendix A, only the results are presented here. It is first necessary to define some terms however. Let the two reference polarizations for the analysis be denoted by $\hat{h}_o$ and $\hat{h}_x$. Then we can express the polarization of the incident field, $\hat{h}_e$, as

$$\hat{h}_e = c_o^* \hat{h}_o^* + c_x^* \hat{h}_x^*. \tag{3}$$

Let the four antenna elements be denoted by

$\hat{h}_{ij}$ where $i = 1,2$ and $j = 1,2$.

The first subscript denotes the DF antenna pair and the second subscript denotes the member of a pair. Nominally

$$\hat{h}_{ij} = \hat{h}_o$$

and

$$\hat{h}_{2j} = \hat{h}_x.$$

We now define complex antenna cross polarization ratios (ρ) which specify the polarization mismatch between corresponding elements of the different antenna pairs. That is

$$\rho_1 / \sqrt{1 + r_1^2} \equiv \hat{h}_{11} \cdot \hat{h}_{21x}^*, \tag{4}$$

and

$$\rho_2 / \sqrt{1 + r_2^2} \equiv \hat{h}_{12x} \cdot \hat{h}_{22}^*$$

where the third subscript, x, denotes polarization orthogonal to the antenna defined by the first two subscripts. The voltages presented to the DF receiver are

$$V_1 = \hat{h}_1 \cdot \hat{h}_e$$

and (5)

$$V_2 = \hat{h}_2 \cdot \hat{h}_e$$

where $\hat{h}_1$ and $\hat{h}_2$ are the resultant antenna polarizations formed by the combiners. The phase error due to tracking, δ, is given by

$$\delta = \text{phase } (V_1) - \text{phase } (V_2) \tag{6}$$

and the amplitude error, a, is given by

$$a = \text{amplitude } (V_1) - \text{amplitude } (V_2). \tag{7}$$

It is assumed that the system has been properly aligned by transmitting the two reference polarizations and performing the required amplitude and phase trimming. If this is done, the first order phase and amplitude errors due to tracking are

$$\delta = \text{Im } \{c_o c_x^* (\rho_2 - \rho_1)\} \tag{8}$$

and

$$a = -\text{Re } \{c_o c_x^* (\rho_2 + \rho_1)\} \tag{9}$$

It will be noted that these errors maximize when the incident field polarization is midway between the two orthogonal polarizations of the antenna system. As an example, if the two orthogonal polarizations are left and right circular, the amplitude and phase tracking errors maximize for a linear field polarization. It will be further noted that, if the two antenna cross polarization ratios are independent random variables, the amplitude and phase errors are independent.

The phase tracking error of course represents, directly, an error in an interferometer system. The importance of these errors in an amplitude monopulse system depends upon where the combining takes place. If the two antenna pairs referred to above are the primary antennas from which the monopulse sum and difference patterns are formed, then the phase tracking error will result in a decrease in the null depth of the difference pattern and the amplitude error will cause a shift in the null position. On the other hand, if the two antenna pairs already form the sum and difference patterns, then the amplitude and phase tracking errors will not directly lead to a DF error. Consequently, at least for the amplitude monopulse and related systems, the DF error due to the combiner action can be minimized by properly positioning the combiners in the path of signal flow. In particular, for the amplitude monopulse system, it is better to form cross polarized sum and difference patterns and combine the outputs of these to obtain polarization tracking than it is to use cross polarized element patterns which are first combined to achieve the polarization tracking function and then combined to form the monopulse sum and difference patterns.

In order to evaluate the improvement obtainable through the use of polarization tracking, let us again look at the single level interferometer. The cumulative probability distributions of the error in normalized arrival angle is given by

$$P[|\delta|<\delta_o] = 2\delta_o/R \qquad \text{for } 2\delta_o/R < 1$$
$$= 1 \qquad \text{for } 2\delta_o/R > 1 \tag{10}$$

where $R = |\rho_2 - \rho_1|$.

The value of the resultant antenna cross polarization ratio, R, depends upon, among other things, the relative phases of the two individual ratios. If the two individual ratios have equal magnitudes (r) but a variable, or unknown, phase difference, the rms value of R is 1.414r. Under this assumption, the probability distribution of the errors after correction by polarization tracking are as shown in Figures 9 (r = 0.1) and 10 (r = 0.5). Comparison with the results for polarization switching shows a considerable reduction in the size of the 95% confidence region, especially for a switching ratio of 1.7. The advantage of tracking over switching is nonexistent if a switching ratio of 1 and confidence levels of 80% or less are considered. At confidence levels of 50% or less, there is no advantage of tracking over switching even for a switching ratio of 1.7. A result of these considerations is that the additional complexity of polarization tracking, as compared with switching, is justified primarily in applications requiring a high confidence in the DF information.

3.5 Polarimeter Correction

The principles of the polarimeter correction technique are quite straightforward. Referring to the block diagram shown in Figure 4, this corrective technique involves three basic components. These are a polarimeter to measure the polarization of the incident field, a data bank containing a complete polarization description of the antenna system, and a computer. The basic concept is that the DF system error due to polarization is computable if the polarization of the incident field and the polarization properties of the antenna system are known. If the error is computed, it can be subtracted from the measured DF value to produce the true angle of arrival. This ideal result cannot be achieved in practice, however, because the computed correction will itself contain errors. These errors arise from a number of sources including:

1) Errors in the polarimeter measurement of the incident field polarization.

2) Errors in the antenna polarization description.

3) Errors in the model of the DF system operation.

The combined effect of these factors will be a more or less random error in the computed correction. This random error can be considered to be a noise whose effect can be described by an equivalent voltage signal-to-noise ratio (SNR). While it is difficult to assign a value to this signal-to-noise ratio, a value of 20 dB might be expected to be typical and a value of 10 dB would probably represent a pessimistic estimate.

The analysis for this system (Section A-3 of Appendix A) is quite difficult and an exact result was not obtained. A good approximation for the cumulative distribution function of the residual errors in the phase interferometer was obtained and is given by

$$P[|\delta|<\delta_o] = (1/90) \text{ arc tan } [2(SNR/r) \tan (\delta_o/2)] \quad (11)$$

where the arc-tangent is in degrees and the other symbols have their usual meanings. Plots of equation 11 for 10 and 20 dB signal-to-noise ratios are shown in Figures 9 and 10.

Of all of the techniques considered, this one produces the best results most of the time. The very largest DF errors (those occurring with a very small probability) are not, however, well corrected. In fact, there is some error value above which the probability of such an error after correction is greater than the probability before correction. This effect can be qualitatively explained by reference to Figure 11. This corrective technique can be interpreted physically as the subtraction of the vector $\rho\sigma^m$ (the computed error vector) from the vector $\rho\sigma$ (the true error vector). When the true error vector is large, even a small relative error in the computed error vector can cause an increase in the interferometer error. This result would be intuitively expected, the only surprising fact is the point at which this correction technique degrades the DF data. For a relatively poor system, this occurs at approximately the 90 or 95% confidence level (c.f. Figure 9) which means that in 5 to 10% of the cases, degradation would occur. For the typical system (SNR = 20 dB) only about 1% of the cases are degraded.

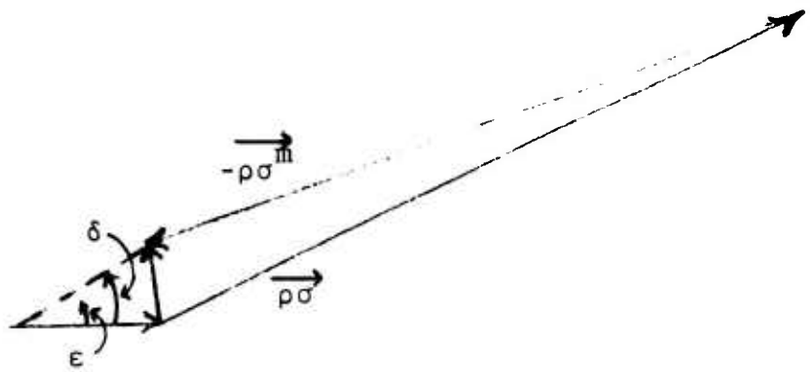

Figure 11 Vector Representation for Polarimeter Correction of a Phase Interferometer System

The conclusion then is that this technique is called for when extremely good correction of most of the values is important and poor or no correction of infrequent large errors can be tolerated. On the other hand, if it is most important to place a limit upon the maximum possible error, then one of the other techniques is preferable. In comparing techniques, it is also important to remember that this approach involves considerably more equipment complexity than the others and this complexity is probably justifiable in laboratory rather than operational applications.

3.6 Effects of Randomizing Antenna Parameters

Throughout the preceding discussions the various antenna parameters have been treated as though they were constants. This is certainly true at any instant of time for a specific antenna pair receiving a signal from a specific direction. It is not true however when sample-to-sample variation between antennas and, more importantly, variable arrival angles are taken into account. Some parameters, such as gain or beamwidth, may be assumed the same for all antennas of a given type and to have a known variation with arrival angles. Other parameters, in particular the complex antenna cross polarization ratio, will differ significantly between antennas of a given type, will vary in a rather unpredictable manner with arrival angle and frequency and can even be expected to change considerably with time. These latter parameters are those that are of primary importance in assessing polarization errors. They are so variable because the fact that they are non-zero is itself due to the non-ideal nature of real antennas in a real environment.

The ideal way to assess the effects of this sort of parameter variation would be to make extensive measurements of their values using the real antenna system in its final configuration (e.g., mounted on the aircraft for an airborne application), over the entire frequency and angle of arrival ranges for which they are to be used. Several samples of a given antenna type should be utilized in order to include the effects of manufacturing tolerances. The various error expressions and probability distributions presented in this report could then be treated as conditional expressions which are to be weighted by the probability distributions of the antenna parameters and the product integrated to give the statistical properties of the errors in the real world.

The procedure outlined above would be expensive and time consuming at best and would probably be unnecessary. Certainly, one would like to think that he could use the mean (or median or most probable or some other measure) value of the antenna parameters in the conditional error expressions and obtain reasonably accurate results.

In order to test this hypothesis, the effects of a noise-like distribution of antenna cross polarization ratio on the interferometer error probability distributions, with and without polarization tracking, were evaluated. The theoretical analysis is presented in Section A-4 of Appendix A. Typical results are shown in Figures 12 and 13. Figure 12 shows the cumulative interferometer error probability distribution for two fixed values of antenna

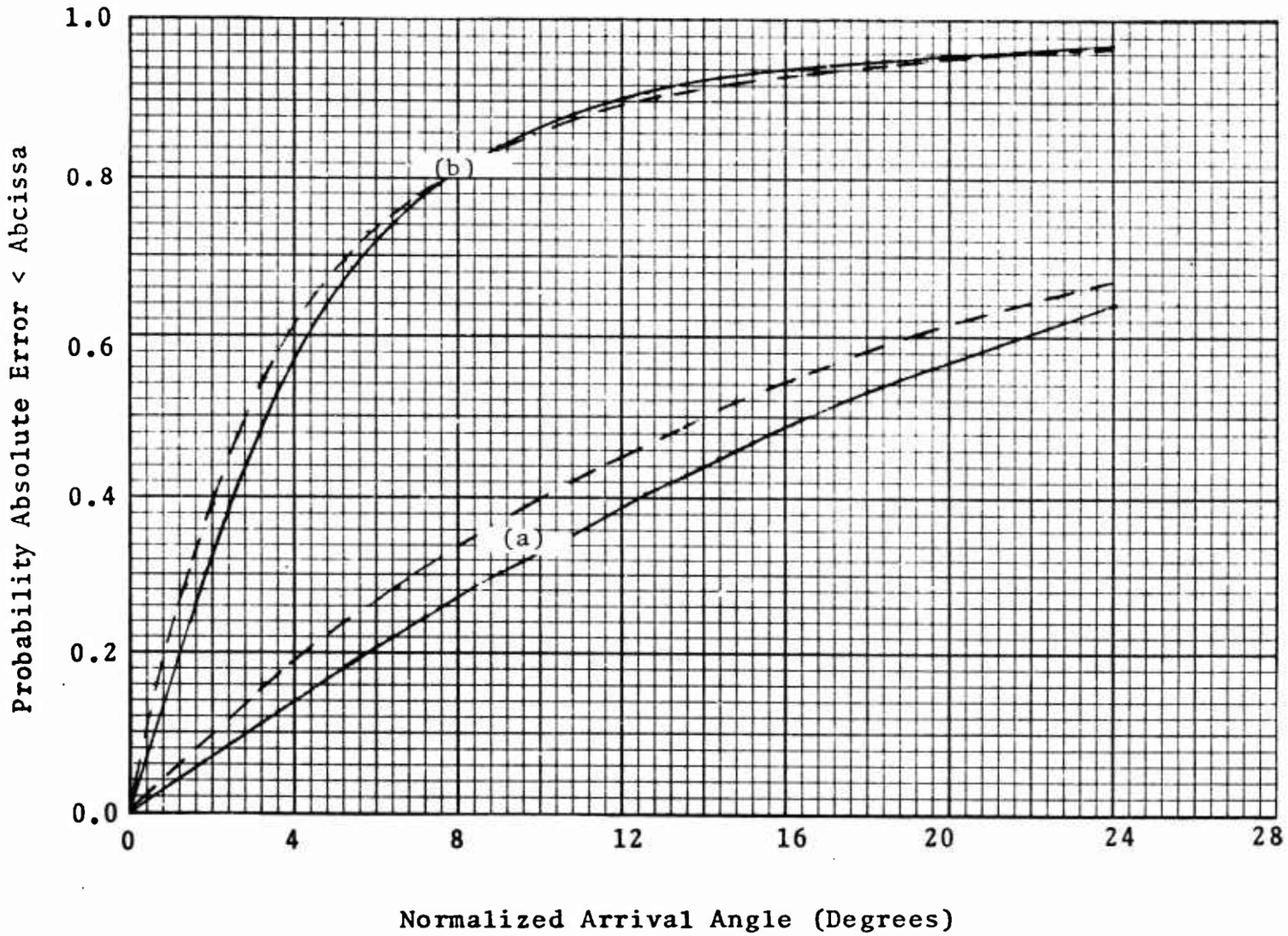

Figure 12 Interferometer Error Probability Distribution

——— Fixed r a) r=0.1, b) r=0.5

– – – Random r a) Mean=0.1, b) Mean=0.5

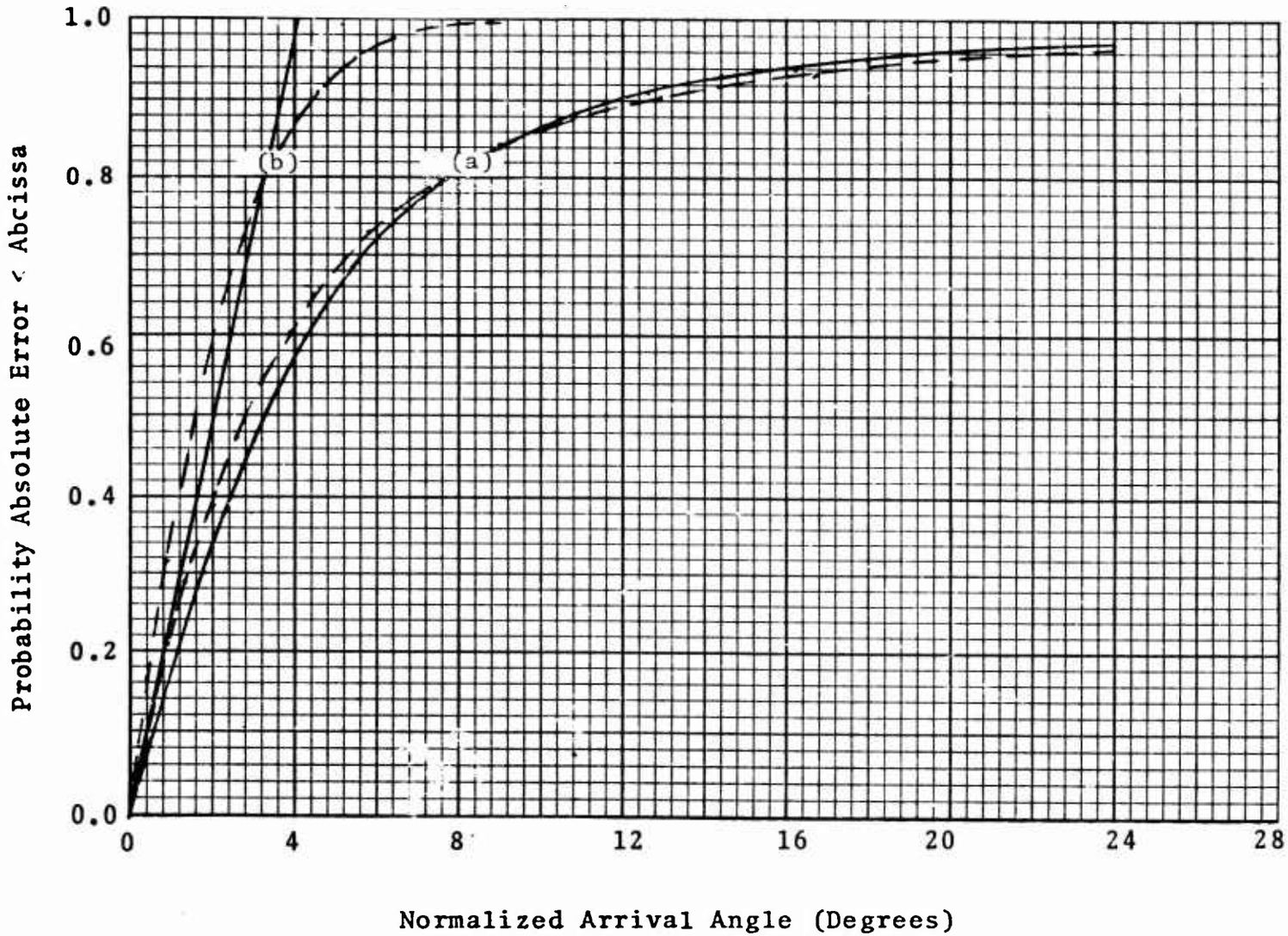

Figure 13 Interferometer Error Probability Distributions

——— Fixed r, a) No correction, b) Tracking

— — — Random r, a) No correction, b) Tracking

cross polarization ratio (r = 0.1 and 0.5) and two random antenna cross polarization ratios whose mean values are also 0.1 and 0.5. Clearly, not much error is generated by using the mean value. Even less error in the low probability portions of the curve would result if the median value had been used. Figure 13 shows the cumulative interferometer probability distribution with and without correction by polarization tracking for a fixed antenna cross polarization ratio of 0.1 and for a random antenna cross polarization ratio whose mean value is 0.1. Again, there is no qualitative, and very little quantitative, error introduced by simply using the mean value.

In summary then, this rather brief assessment indicates that the mean values of antenna parameters can be used in the conditional error expressions presented in the report to arrive at reasonably accurate values for polarization errors. Errors occurring 50 percent of the time or less might possibly be better evaluated by using the median value of antenna parameters when this is significantly different from the mean value.

4. POLARIZATION ERRORS IN TOA SYSTEMS

4.1 General Discussion

Polarization errors in TOA systems were considered in the first phase of the program. The results were not however as instructive as might be desired. Consequently, a portion of the effort during the second phase was devoted to a reconsideration of the problem. Some alternative general formulations of the problem were obtained. These general formulations were not, however, very useful either in quantitatively describing the problems or in suggesting solutions. For this reason, a number of special cases were examined in detail. By specializing, it is possible to concentrate one's attention on one aspect of the problem without being distracted by other aspects. What makes the TOA system hard to evaluate is the fact that polarization can lead to errors in many ways, each depending upon the details of the situation. Three of these special cases are discussed below in order to illustrate these effects.

In the first example, corresponding to a classical radio astronomy TOA system, the polarization error is equivalent to a signal-to-noise degradation. In the second example, which manifests itself as intersymbol interference in a communication type of signal, polarization errors are related to multipath with short delays (of the order of reciprocal of the signal bandwidth). The third example, which considers a true multipath situation illustrates a case where polarization effects, while not directly causing errors, can be very serious by accentuating problems due to other causes. In this case, by possibly causing one of the multipath signals to be the dominant one.

The third type of error is probably the only one which is of significant importance in most applications and therefore requires corrective action. Either polarization switching or tracking will usually provide a good corrective technique; however, the factors considered in Section 5 need to be taken into account.

4.2 Polarization Effects on TOA Systems

Let us consider a very general sort of TOA system which consists of I receiving locations. The i'th location has an antenna whose polarization vector is given by:

$$\hat{h}_i = A_i \, (\hat{h}_o + \rho_i \, \hat{h}_x) \tag{12}$$

where

$\hat{h}_o$ = reference polarization vector,

$\hat{h}_x$ = polarization vector cross polarized to $\hat{h}_o$,

$\rho_i = r_i \exp(j \, \zeta_i)$ = complex antenna cross polarization ratio,

and

A_i = normalizing factor = $1/\sqrt{1 + |\rho_i|^2}$.

In general, A and ρ are functions of frequency. In the most general case of course, there may be a phase associated with A but we will not explicitly include this possibility in the present analysis. The field incident at the i'th receiving antenna is given by:

$$\vec{E}_i = F_i \hat{h}_o^* + G_i \hat{h}_x^* \tag{13}$$

where F and G are the complex amplitude spectra of the signals in the reference and cross polarizations respectively. The received voltage at the i'th location is:

$$V_i = \vec{E}_i \cdot \hat{h}_i = A_i [F_i + \rho_i G_i]. \tag{14}$$

The complex time signal at the i'th location is the Fourier Transform or:

$$v_i(t) = (1/2\pi) \int V_i (\omega) \exp (j\omega t) d\omega. \tag{15}$$

Most TOA systems do not work directly with these complex amplitudes but rather with their envelopes.

In this form, the expressions are too general to provide any insight into either the errors or the appropriate remedial steps. It is much more instructive to consider some specific examples. This is done in the following examples.

4.3 Examples

4.3.1 Example 1 - In this example we make the following assumptions:

1) The receiving antennas have different, but frequency independent, polarizations.

2) The fields at the different sites are all identical except for a time shift, and consist of a signal which is randomly polarized as a function of time.

Assumption 1 requires that the A and ρ be independent of frequency but different at the different receiving sites. Assumption 2 requires that the F and C be the same except for a time shift factor at all sites but that they correspond to two time functions (f and g) which are uncorrelated in time. The complex time signal at the i'th receiving site is then given by:

$$v_i(t) = A_i \, [f(t - \tau_i) + \rho_i \, g(t - \tau_i)], \qquad (16)$$

where τ_i is the relative delay of the signal received at the i'th site. Clearly, the effect of polarization is to cause the signals received at the different receiving sites to deviate from the TOA requirement that all signals be time shifted replicas. If the antennas at the different sites have nearly the same polarization, then the antenna cross polarization ratios may be assumed to have a magnitude less than unity. The second term in equation 16 then represents a form of noise which causes the observed signal (v) to differ from a time shifted replica of the "correct"

signal (f). If the reference polarization is properly defined, the average of the quadrature components of ρ, averaged over the ensemble of receiving sites, will be zero. The consequences of these assumptions and definitions will be to cause the polarization noise, represented by the second term in (16), to tend to be uncorrelated between the various sites. When several sites are involved, however, it will be found that some site combinations will have partially correlated polarization noise. To the extent that the polarization noises are uncorrelated, the effect of polarization errors is to decrease the overall TOA system signal-to-noise ratio. In fact, the best achievable signal-to-noise ratio will be the average of the magnitude of the antenna cross polarization ratios.

This example is perhaps not very representative of those applications which are of primary interest in the present study. There are however some real life applications where these assumptions are valid. One of these is in radio astronomy. Another application would be that of direction finding on a noise-like source.

4.3.2 Example 2 - In this example, we consider the case where the fields at the various sites are identical, except for time delay, and consist of a signal with a well-defined polarization which we take to be the reference polarization. The antenna polarizations are assumed to be different at the different sites and are further assumed to be frequency dependent. In this case then, the voltage of the i'th site is

$$v_i(t) = \int f(t' - \tau_i)\, a_i(t' - t)\, dt'. \qquad (17)$$

where a (The Fourier Transform of A) is the impulse response of the i'th antenna in the reference polarization. Generally, the antenna function, A, will be of the form

$$A(\omega) = C + D(\omega)$$

where C is the value of A at the center of the signal bandwidth. Then, the voltage can be written as

$$V_i(t) = C_i\, f(t - \tau_i) + \int f(t' - \tau_i)\, d_i(t' - t)\, dt'. \qquad (18)$$

The second term in (18) is of the same form as an error due to multipath. In fact, if the antenna cross polarization is frequency dependent because of reflections in the antenna, its environment, and associated circuitry, the second term in (18) can be physically associated with these reflections and is truly a type of multipath.

Qualitatively, the effect of polarization errors for this example is to spread the received signal in time by an amount equal to the width of the impulse response of the antenna in the reference polarization and add the smeared signal to a fraction of the correct signal. The width of the antenna impulse response will generally be of the order of delays in the antenna itself and will therefore be related to physical path lengths in the antenna. In some cases, this physical path length may be surprizingly large. In a planar spiral antenna for example, the cross polarized response is often associated with the fact that the cross polarized signal component

induces currents which travel from the active region to the outside of the spiral, undergo partial reflection, travel back through the active region, where they are attenuated by radiation, to the antenna terminals. The excess path length for the cross polarized component then is twice the distance from the active region to the outer edge of the sprial measured along the spiral. For a tightly wound spiral, this length may be many times the outer circumference.

The seriousness of this type of polarization error depends substantially upon the type of signal modulation. When the signal consists of discrete events well separated in time (e.g., radar pulses) the effect of the second term in (18) is to smear the discrete events out in time. However, the maximum displacement of a demodulated pulse characteristic, such as the leading edge, will be of the order of an antenna dimension which would certainly not be a significant error. If the signal is continuous however (e.g., a communications signal), the effect of the second term will be to introduce intersymbol interference. When this intersymbol interference is large, which will occur if C is relatively small and the data rate is of the same order as the duration of the antenna impulse response, the detected signal will have little similarity to the correct signal and determining the relative time of arrival at various sites will not be possible. A quantitative measure of the intersymbol interference is provided by the ratio of the power in the delayed signals relative to the power in the undelayed signal. Mathematically, this ratio is given by R where

$$R_i = \int |d_i(t)|^2 \, dt / |C|^2. \tag{19}$$

The errors are minimized by making R small which can be done either by decreasing the numerator (making the antenna polarization frequency independent) or maximizing the denominator (matching the antenna polarization to that of the incident field). Careful antenna design can minimize, but not eliminate, the frequency dependence of the antenna polarization. Given that this has been done, significant errors due to intersymbol interference will occur only for small values of C (i.e., when the field is nearly cross polarized to the receiving antenna). This situation can be avoided in practice by using two (cross polarized) antennas at each receiving site and selecting the signal from the antenna which has the greatest output. This polarization selection will guarantee that $|C|^2$ is never much below 0.5 and thus provide an output that is reasonably free of intersymbol interference. An examination of the data sheets for one commercially available cavity backed spiral antenna designed for the 2 - 10 GHz frequency range indicates a value for the numerator in equation (19) of the order of 0.01 over most of a hemisphere. Polarization selection using this antenna then would result in a value of R of between -17 dB and -20 dB for all polarizations of the incident field. While intersymbol interference of this order will degrade system performance by decreasing the available signal-to-noise ratio, it will not make TOA measurements impossible.

4.3.3 Example 3 - For this example, we assume that the antenna polarizations are frequency independent but not necessarily the same at all sites. The incident fields at each site are assumed to consist of a combination of a direct signal with the reference polarization and a number of multipath signals with different delays and polarizations. For this case, a representation of the incident field differing somewhat from equation (13) is desirable. An appropriate form is

$$\vec{E}_i = F(\omega) \quad \Sigma \exp(-j\omega\tau_{i,n}) \, B_{i,n} \, (\hat{h}_o^* + \sigma_{i,n}\hat{h}_x^*) \tag{20}$$

where

$\tau_{i,n}$ = delay of the n'th field component at site i.

$\sigma_{i,n}$ = complex field cross polarization ratio for the n'th component at site i,

$B_{i,n}$ = field polarization normalization factor,

and the sum is over the n multipath components. The coefficients, B, also contain the relative amplitudes of the multipath components. The complex received signal at the i'th site is then

$$v_i(t) = \Sigma \, B_{i,n} \, A_i \, (1 + \sigma_{i,n}\rho_i) \, f(t - \tau_{i,n}). \tag{21}$$

Physically, equation (21) expresses the fact that the voltage at the i'th site is the sum of the multipath signals received at that site with each multipath signal weighted by the degree to which its polarization matches that of the receiving antenna. In many direction finding applications, the geometry will be such that the largest B and/or $B\sigma$ product may be associated with one of the reflected signals. Furthermore, the delays of the multipath components can be large enough to cause very substantial position location errors. Consequently, the fact that the receiving antenna will modify the distribution of amplitudes of the multipath components by weighting some polarizations more heavily than others can cause substantial time of arrival errors. The most serious errors will clearly occur when the direct signal is nearly cross polarized to the receiving antenna while one of the multipath components has the same polarization as the receiving antenna. The technique to reduce this type of error must obviously involve matching the polarization of the receiving antenna to that of the direct signal. In this case, however, simply switching between two cross polarized antennas on the basis of which has the strongest signal will not necessarily suffice. This is due to the fact that under some circumstances this technique will result in the wrong choice as shown in Section 5. For a radar type signal, however, one can postulate a polarization matching technique. Conceptually, the idea is to vary the polarization of the receiving antenna at each site through all possible states. Since the direct field component is assumed to arrive first and it is the time of

arrival of this component which we seek to determine, we would choose that polarization yielding the earliest time of arrival as the correct polarization and time. Although this technique will work conceptually, it is not now clear whether or not its implementation, especially in a real time operational sense, is feasible.

5. MULTIPATH EFFECTS ON CORRECTIVE TECHNIQUES

Multipath will confuse not only the DF system, a fact that is well known, but also the polarization error correction techniques considered in this study. While all of the techniques will be affected to some extent, the effect on polarization switching can be particularly disastrous. For this reason, the following discussion is primarily aimed at that technique.

In a multipath environment, there exist situations where the simple polarization switching system will choose the polarization that is cross polarized to the desired signal. When this is done, the DF system will be looking solely at the multipath component and, in essence, be locating a phantom emitter. The probability of these situations occuring is obviously of considerable importance in determining the practicability of utilizing polarization selection.

In order to assess this probability, let us begin by redefining the polarization switching system. The polarization vectors of the two cross-polarized antennas are denoted by $\hat{h}_1$ and $\hat{h}_2$ where

$$\hat{h}_1 \cdot \hat{h}_1^* = \hat{h}_2 \cdot \hat{h}_2^* = 1$$

and (22)

$$\hat{h}_1 \cdot \hat{h}_2^* = 0.$$

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Let the polarization of the total electric field incident at the receiving antenna system be $\hat{h}_t$ and its amplitude be E_t. Then the antenna voltages at antennas 1 and 2 are

$$V_1 = E_t \, \hat{h}_t \cdot \hat{h}_1^*$$

and (23)

$$V_2 = E_t \, \hat{h}_t \cdot \hat{h}_2^*$$

respectively. We define a selection parameter Δ by

$$\Delta = |V_1|^2 - |V_2|^2. \tag{24}$$

The switching strategy is to choose the signal from antenna 1 if Δ is positive and the signal from antenna 2 if Δ is negative.*

Before proceeding further, we must investigate the nature of the total incident field in some detail. Consider the propagation geometry shown in Figure 14. In this figure, the geometry of reflection from a tilted plane is shown. The Z-axis is vertical and the Y-axis is defined such that reflection occurs at X = Z = 0. The orientation of the tilted reflecting plane, defined by the unit normal to the plane, $\hat{n}$, is given by

*This corresponds to a switching ratio of unity. Another value will modify the results somewhat.

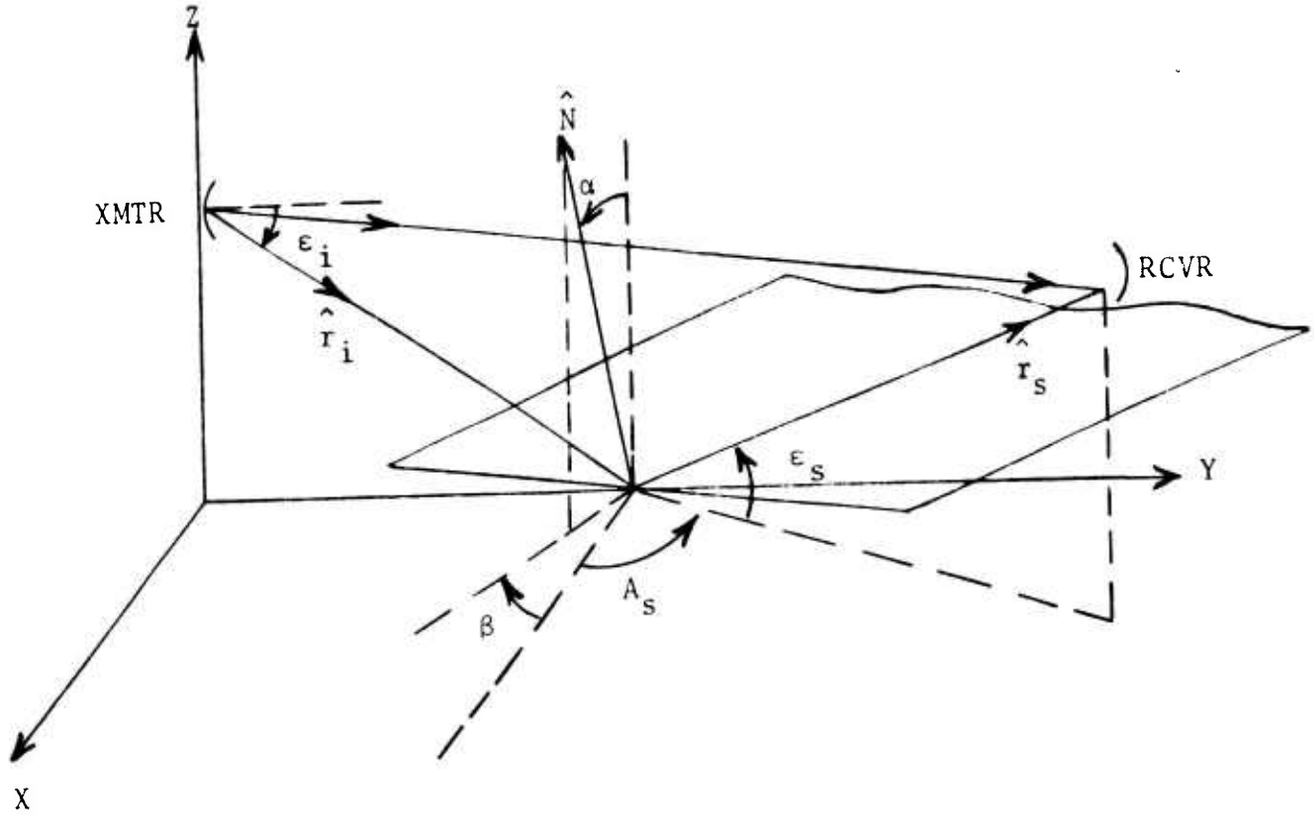

Figure 14 Multipath Geometry

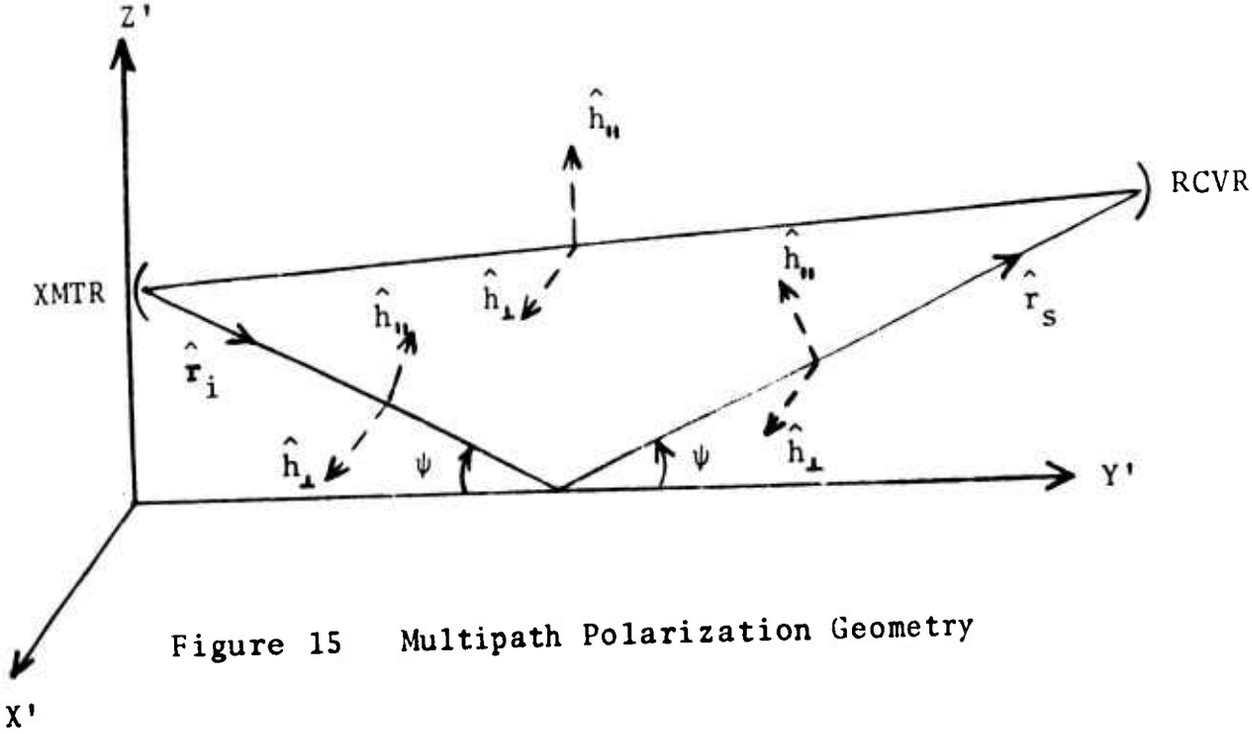

Figure 15 Multipath Polarization Geometry

$$\hat{n} = \sin\alpha \cos\beta\, \hat{e}_x - \sin\alpha \sin\beta\, \hat{e}_y + \cos\alpha\, \hat{e}_z. \qquad (25)$$

The unit vectors from the transmitter to the reflection point and from the reflection point to the receiver are $\hat{r}_i$ and $\hat{r}_s$, respectively. The conditions for specular reflection are

$$\hat{n} \cdot (\hat{r}_i + \hat{r}_s) = 0 \qquad (26)$$

and

$$\hat{n} \times (\hat{r}_i - \hat{r}_s) = 0. \qquad (27)$$

The first of these specifies that angle of incidence equal the angle of reflection and the second specifies that the incident, reflected, and normal unit vectors be coplaner.

Let us express the incident and reflected unit vectors in terms of depression angle (ε) and "azimuth" (A).

$$\hat{r}_i = \cos\varepsilon_i\, \hat{e}_y - \sin\varepsilon_i\, \hat{e}_z, \qquad (28)$$

and

$$\hat{r}_s = \cos\varepsilon_s \cos A_s\, \hat{e}_x + \cos\varepsilon_s \sin A_s\, \hat{e}_y + \sin\varepsilon_s\, \hat{e}_z. \qquad (29)$$

If we specify the incident depression angle, ε_i, and the orientation of the reflecting plane, α and β, equations (26) - (29) define the reflected unit vector, (i.e., ε_s and A_s), as well as the angle of incidence ψ (used here as a grazing angle). Specifically,

$$\sin \psi = -\hat{n} \cdot \hat{r}_i = \cos \alpha \sin \varepsilon_i + \sin \alpha \cos \varepsilon_i \sin \beta, \qquad (30)$$

$$\sin \varepsilon_s = 2 \cos \alpha \sin \psi - \sin \varepsilon_i, \qquad (31)$$

and

$$\sin \alpha \cos \varepsilon_s \cos (\beta + A_s) = \sin \psi - \cos \alpha \sin \varepsilon_s. \qquad (32)$$

Let us now examine the polarization of the reflected signal. To do this we consider the geometry shown in Figure 15. In this coordinate surface the reflecting plane defines the X' - Y' plane (i.e., Z' is along the normal to the plane) and the Y' - Z' plane contains the incident and reflected unit vectors. The incident and reflected electric vectors may be defined in terms of components in the plane of incidence (parallel polarized) and normal to the plane of incidence (perpendicular polarized). Thus

$$\vec{E}_i = E_i \, (a_i \hat{h}_{\parallel} + b_i \, \hat{h}_{\perp}) \qquad (33)$$

and

$$\vec{E}_s = E_s \, (a_s \, \hat{h}_{\parallel} + b_s \, \hat{h}_{\perp}), \qquad (34)$$

where E_i and E_s are the magnitudes of the incident and reflected fields respectively. We note that the perpendicular unit vector, $\hat{h}_{\perp}$, is the same for incident, reflected, and direct components and is directed along the X' axis. The parallel unit vector, $\hat{h}_{\parallel}$, is however different

for each component and is defined by the condition that it be normal to the direction of propagation and lie in the plane of incidence (Y' - Z' plane). The reflected (scattered) electric field components are related to the incident components by the familiar Fresnel reflection coefficients ($R_{\parallel}$ and $R_{\perp}$) for parallel and perpendicular polarization. Thus,

$$E_s \, a_s = E_i \, a_i \, R_{\parallel} \tag{35}$$

and

$$E_s \, b_s = E_i \, b_i \, R_{\perp}.$$

In order to determine the polarization and magnitude of the reflected field, the following procedure is adopted. We first express the incident field (assumed to have unity amplitude) as a combination of "vertically" and "horizontally" polarized components. Horizontal polarization is parallel to the X - Y plane of Figure 15 and vertical polarization is normal to the direction of propagation and in the plane of incidence. Thus,

$$\vec{E}_i = c_i \, \hat{h}_V + d_i \, \hat{h}_H \tag{36}$$

where $\hat{h}_V$ and $\hat{h}_H$ are vertical and horizontal polarization unit vectors, respectively. Note again that while the horizontal polarization unit vector is the same for all three field components, the vertical polarization unit vector is not. We must now express this field in terms of parallel and perpendicular polarization. The required coordinate transformation equations are

$$\hat{h}_{Hi} = X_i \hat{h}_{\perp i} + Y_i \hat{h}_{\parallel i},$$

$$\hat{h}_{vi} = - Y_i \hat{h}_{\perp i} + X_i \hat{h}_{\parallel i},$$

$$\hat{h}_{\perp s} = X_s \hat{h}_{Hs} - Y_s \hat{h}_{vs}, \tag{37}$$

$$\hat{h}_{\parallel s} = Y_s \hat{h}_{Hs} + X_s \hat{h}_{vs},$$

where

$$X_i = \sec \psi \, (\cos \alpha \cos \varepsilon_i - \sin \alpha \sin \beta \sin \varepsilon_i),$$

$$Y_i = \sec \psi \sin \alpha \cos \beta,$$

$$X_s = \sec \psi \sec \varepsilon_s \, (\cos \alpha - \sin \varepsilon_s \sin \psi),$$

and

$$Y_s = \sec \psi \sec \varepsilon_s \sin \alpha \cos \beta \cos \varepsilon_i.$$

Thus the incident field becomes

$$\vec{E}_i = \hat{h}_{\perp i} \, (c_i X_i - d_i Y_i) + \hat{h}_{\parallel i} (c_i Y_i + d_i X_i) \tag{38}$$

and the scattered field is

$$\vec{E}_s = R_\perp \hat{h}_{\perp s} (c_i X_i - d_i Y_i) + R_\parallel \hat{h}_{\parallel s} (c_i Y_i + d_i X_i) \tag{39}$$

$$= \hat{h}_{Hs} \, [R_\perp X_s \, (c_i X_i - d_i Y_i) + R_\parallel Y_s \, (c_i Y_i + d_i X_i)]$$

$$+ \hat{h}_{vs} \, [- R_\perp Y_s \, (c_i X_i - d_i Y_i) + R_\parallel X_s \, (c_i Y_i + d_i X_i)].$$

The total field at the receiving system is the sum of the direct signal and the field given by (39).

In order to be general, let the ratio of the combined antenna gains along the direct path to the combined antenna gains along the reflected path be C and the relative phase between the two paths (exclusive of the phase change on reflection) be ϕ. Then the total field $\vec{E}_t$ is given by

$$\vec{E}_t = C\ [c_d \hat{h}_{vd} + d_d\ \hat{h}_{Hd}] + \vec{E}_s \exp\ (-j\phi). \tag{40}$$

We shall assume that the polarization of the transmitting and receiving antennas, when expressed in terms of vertical and horizontal polarization as defined relative to the direction of propagation, is independent of look direction. This is equivalent to assuming that $c_d = c_i$ and $d_d = d_i$. It further implies that we may use the following expressions.

If

$$\hat{h}_1 \equiv \gamma_1\ \hat{h}_H + \delta_1\ \hat{h}_v$$

and

$$\hat{h}_2 \equiv \delta_1^*\ \hat{h} - \gamma_1^*\ \hat{h}_v$$

then

$$\hat{h}_1 \cdot \hat{h}_{Hs} = \hat{h}_1 \cdot \hat{h}_{Hd} = \gamma_1,$$

$$\hat{h}_1 \cdot \hat{h}_{vs} = \hat{h}_1 \cdot \hat{h}_{vd} = \delta_1,$$

and similarly for $\hat{h}_2$.

Using these assumptions and the equations given above we may compute the voltage due to the unit direct path signal (V_d), and the voltage due to the reflected signal (V_s) if the transmitting and receiving polarizations are defined and the geometry is specified. For a given receiving polarization, the total output power (P) is then given by

$$P = C^2 \, |V_d|^2 + |V_s|^2 + 2C \, \mathrm{Re}[V_d \, V_s^* \exp(j\phi)]. \tag{41}$$

The selection parameter Δ is then given by

$$\begin{aligned} \Delta &= C^2 \, [|V_{d1}|^2 - |V_{d2}|^2] + [|V_{s1}|^2 - |V_{s2}|^2] \\ &\quad + 2 \, C \, \mathrm{Re}[(V_{d1} \, V_{s1}^* - V_{d2} \, V_{s2}^*) \exp(j\phi)] \qquad (42) \\ &= C^2 \, K_1 + K_2 + CK_3 \cos\theta. \end{aligned}$$

The parameters K_2 and K_3 are functions of the tilted plane orientation angle β (which should be considered a uniformly distributed random variable), the plane tilt angle α, the

ground constants, the depression angle ε_i, and the antenna polarization. For an aircraft application, the angle θ should be assumed to be a uniformly di ·ibuted random variable because it changes rapidly as the aircraft moves. The probability that antenna 1 will be selected (i.e., that Δ is greater than zero) with everything fixed except θ is

$$P\ [\Delta > 0 \mid \beta] = (1/\pi) \text{ arc cos } [(C^2K_1 + K_2)/(CK_3)].$$

Integrating over β yields the probability that antenna 1 will be selected for a given tilt angle, depression angle, ground constants, and antenna combinations. Of particular interest are those antenna combinations where antenna 1 has the transmitted polarization and antenna 2 is cross polarized to the transmitter. In this case, the probability that antenna 2 will be selected is the probability that a serious error will occur. This probability has been computed for seawater at several frequencies above 500 MHz with no tilt angle and for typical soil* with tilt angles of 5 and 10 degrees. These computations were performed for several different depression angles. Typical probability curves** are shown in Figures 16, 17, 18, and 19. The depression angle of the incident field is labeled on the individual curves. When the antenna gain

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* For soil, the results are independent of frequency above 500 MHz because of the low conductivity. This is not true for seawater where the reflection coefficients are frequency dependent.

**The 90 degree incidence angle curves on Figures 18 and 19 represent mathematical limit curves and should be interpreted with care. Physically the distinction between horizontal and vertical transmitted polarization disappears at this incidence angle.

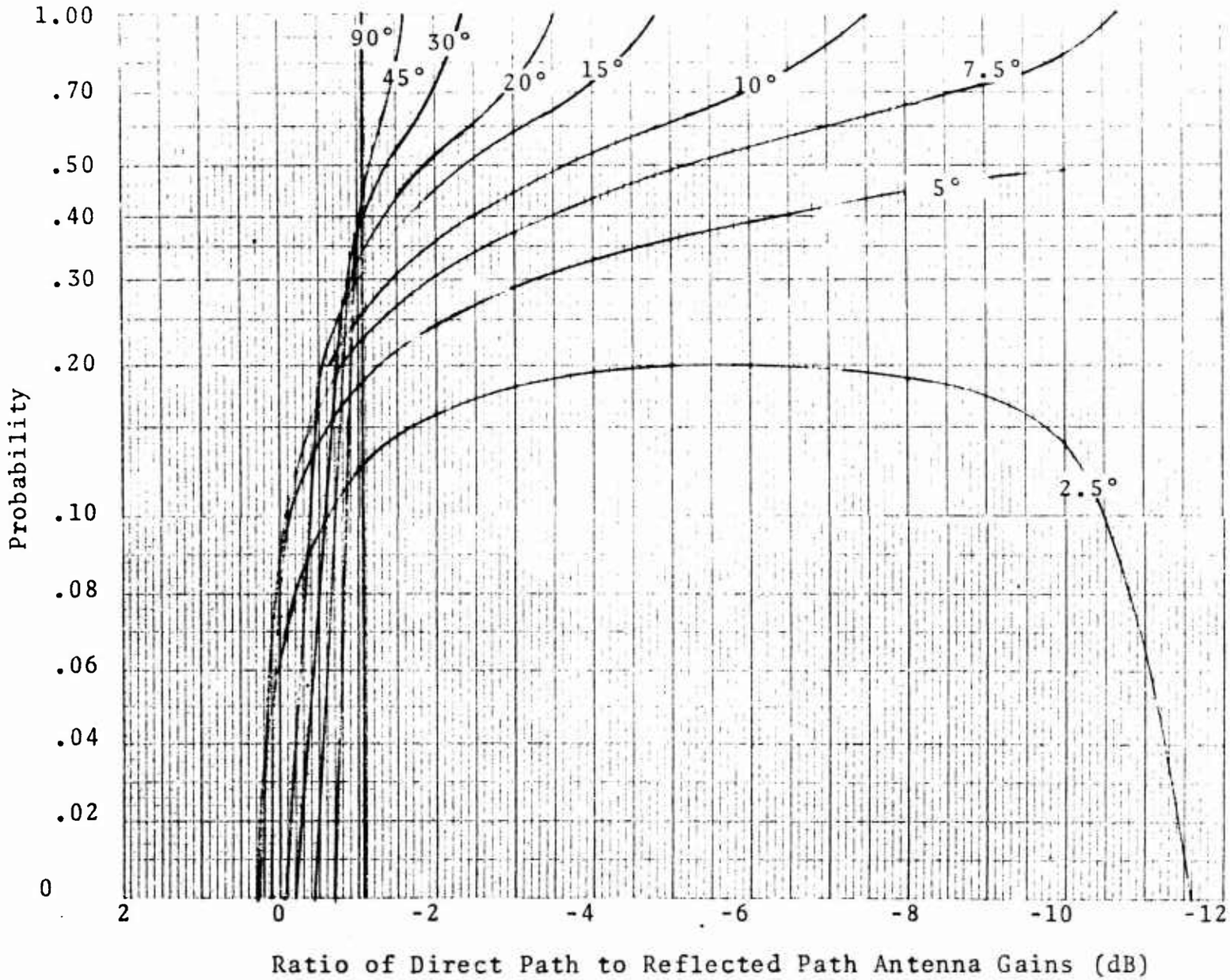


FIGURE 16 Probability of Selecting Cross Polarized Component.
Sea Water, Freq. = 0.5 GHz, RCV Ant. CP & CP, XMT Ant. CP.

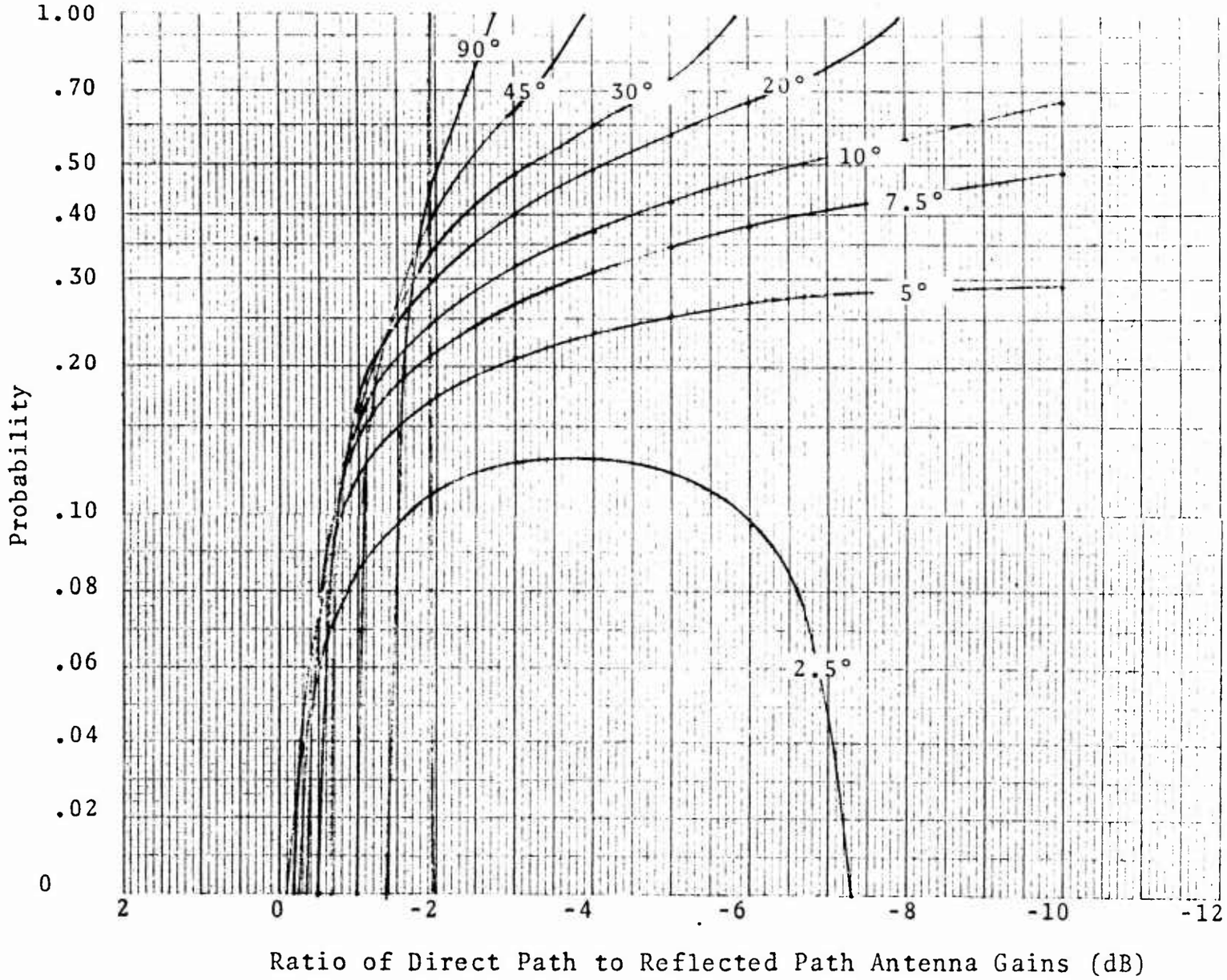


FIGURE 17 Probability of Selecting Cross Polarized Component.
Sea Water, Freq. = 10.0 GHz, RCV Ant. CP & CP, XMT Ant. CP.

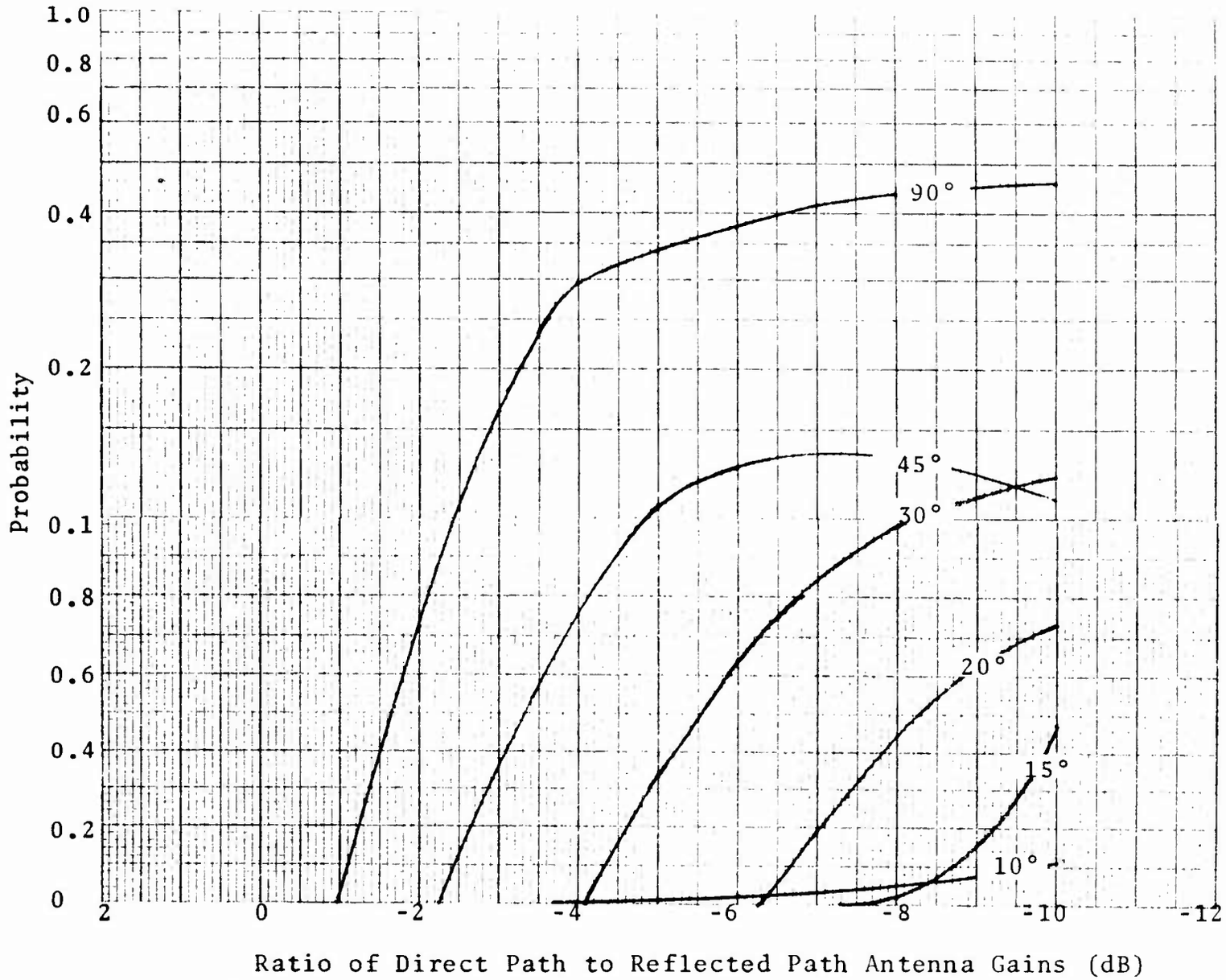


FIGURE 18 Probability of Selecting Cross Polarized Component.
Good Soil, Tilt Angle = 10°, RCV Ant. V & H, XMT Ant. V

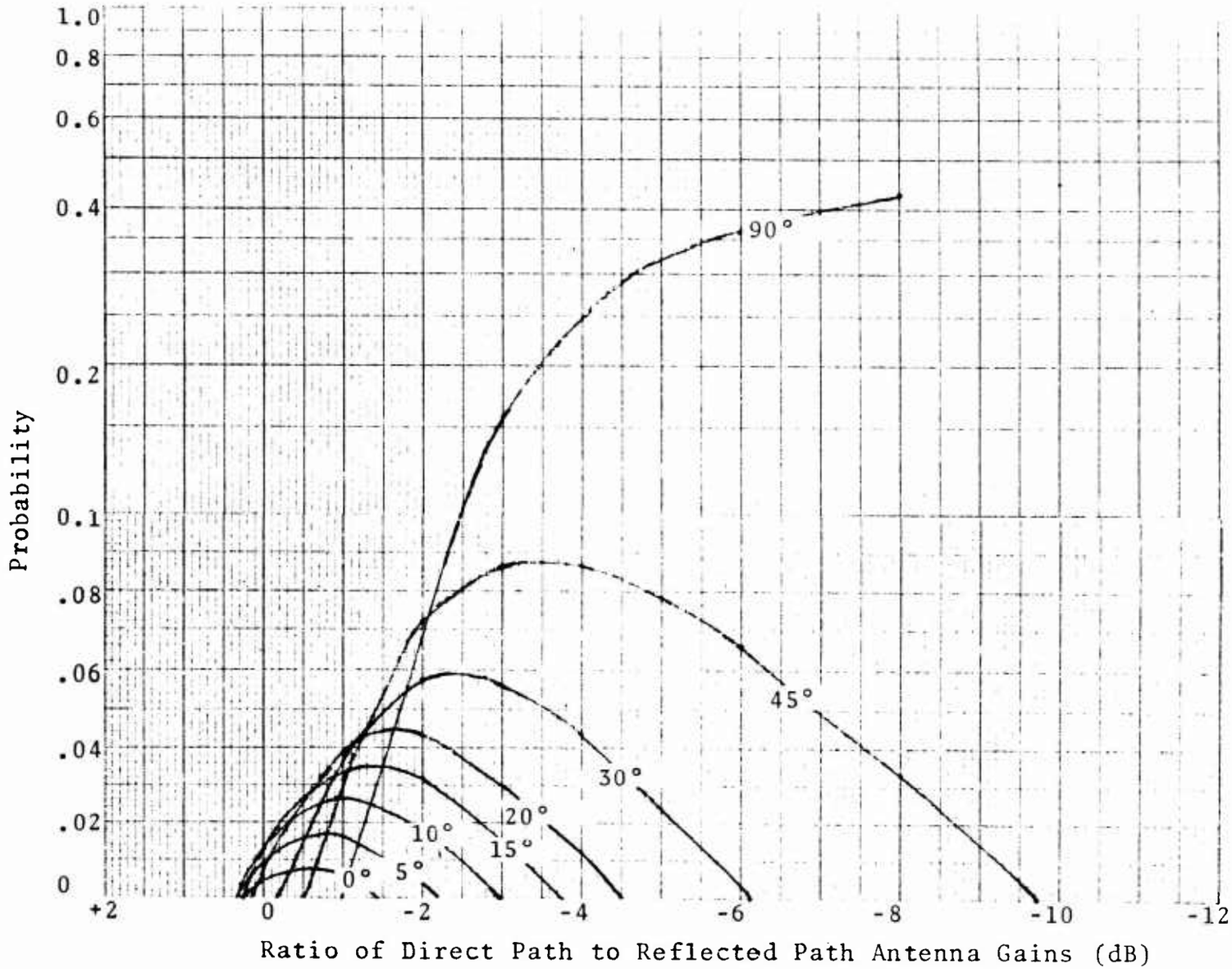


FIGURE 19 Probability of Selecting Cross Polarized Component.
Good Soil, Tilt Angle = 10°, RCV Ant. V & H, XMT Ant. H

factor, C, is greater than or equal to unity, the following results are obtained. For seawater, a wrong choice can occur only for circularly polarized transmitting and receiving antennas at frequencies below 2 GHz and low incidence angles. For good earth, a wrong choice will result only if the receiving antenna pair consists of horizontally and vertically polarized elements and horizontal polarization is transmitted, at least for ground tilts of 10 degrees or less. The smaller the antenna gain factor, the more probable is a wrong selection as should be expected.

It is comforting to note that in the important class of situations where the antenna gain ratio is substantially greater than unity (say +1 dB), there is zero probability of making a wrong selection.

When the two receiving polarizations do not include the transmitted polarization (e.g., vertical and horizontal receiving antennas and a circularly polarized transmitting antenna) the selection does not appear to have much effect. The ratio of multipath to direct signal strength is nearly the same with and without switching.

There are two alternative methods of reducing the possibility of a wrong selection, both based upon the fact that the fields change relatively rapidly in a multipath environment. The first of these is to incorporate a manual override to prevent switching. Since the probability of making a wrong selection is small, the correct choice will

be made most of the time. Consequently, a short trial period would often suffice to determine the correct choice, by observing which polarization was predominantly selected, after which the switching process could be locked out. The second possibility is to increase the reference switching ratio to a value greater than unity. This would tend to delay switching to the wrong polarization once the correct polarization has been chosen and, if set high enough, might prevent switching altogether.

To summarize the above discussion, the following points should be noted. First, only certain combinations of transmitting and receiving antenna polarization are susceptible to important switching errors in a multipath environment. Second, the probability of making a wrong selection is small unless the multipath environment is extremely bad due to either the transmitting or receiving (or both) antenna patterns pointing primarily at the reflection point. Third, as long as the probability of a wrong selection is reasonably small, it may be reduced still further by taking appropriate action as outlined in the preceding paragraph.

6. CONCLUSIONS AND RECOMMENDATIONS

The main conclusions to be drawn from the two phases of this study are contained in the following paragraphs. These conclusions lead to the recommendations presented at the end of this section.

The primary conclusion drawn from the first phase of the study is that polarization effects can cause DF errors which are quite significant when compared with the errors due to other sources. This result is apparent from the information contained in Table I.

As a result of the second phase of the study, which is the primary subject of this report, several conclusions may be drawn. First, polarization errors may be reduced to a level consistent with the total error in a very good DF system. Second, there are several techniques, ranging from simple to complex, for achieving this reduction. Third, the degree to which polarization errors can be reduced is limited by the amount of added equipment complexity and cost that is acceptable. Fourth, for most applications either the polarization switching or polarization tracking technique will reduce the polarization error to a level consistent with the overall system errors. These two techniques would involve relatively insignificant cost and complexity increments to most modern DF systems.

Both of these techniques are suitable for either inclusion in future systems or retrofitting to existing systems. Fifth, the ultimate in polarization error reduction is obtainable by combining the polarization switching and polarimeter correction techniques. This combined approach, while reducing polarization errors to an absolute minimum, would require a substantial investment in terms of cost and system complexity. It is therefore probably justified only for applications wherein extreme accuracy is required and equivalent effort to reduce the errors due to other sources is also undertaken.

Based upon these conclusions, the following recommendations are made.

1. Existing high accuracy systems should undergo an assessment of the importance of polarization errors relative to other system errors. If necessary, this assessment should include range evaluations. A method of performing such evaluations was presented in the Final Report for the first phase (Reference 1).

2. Where the polarization error is significant, retrofitting of one of the corrective techniques described in this report should be undertaken.

3. Future DF system designs should include polarization errors as a design factor. High accuracy systems should incorporate one of the corrective techniques described in this report.

APPENDIX A DETAILED MATHEMATICAL ANALYSIS

A-1 POLARIZATION SWITCHING ANALYSIS

Polarization switching merely limits the maximum magnitude of the field cross polarization ratio that can exist. Thus, the only change in the analysis, as compared with those previously given (Reference 1) is the use of a modified probability distribution for the magnitude (s) of the field cross polarization ratio. If switching occurs at a reference ratio denoted by $\tan \theta_o$, the modified density function is

$$p(s) = [2s/(1 + s^2)^2] \csc^2\theta_o \quad \text{for } 0 \leq s \leq \tan \theta_o \qquad \text{(A-1)}$$
$$= 0 \quad \text{otherwise.}$$

In order to obtain a quantitative measure of the improvement obtainable by this technique, we consider the phase interferometer. It was shown in reference 1 that the cumulative probability distribution of the error in normalized arrival angle is given by

$$P[|\delta|<\delta_o]=P[s<x]+(1/\pi)\{P[s>y]+2I(x,x)-I(y,x)\} \text{ for } \varepsilon<\pi/2 \qquad \text{(A-2)}$$
$$= P[s<y]+(1/\pi)\{P[s>y)+I(y,x)\} \text{ for } \varepsilon>\pi/2$$

where $x = \sin \delta_o/r$,
$y = 1/r$,

and

$$I(a,x) = \int_a^\infty \text{arc sin } (x/s)\ p(s)ds.$$

The modified cumulative distribution function of s is given by

$$P[s>u] = 1-(u \csc \theta_o)^2/(1 + u^2) \quad \text{for } u < \tan \theta_o$$
$$= 0 \quad \text{for } u > \tan \theta_o. \tag{A-3}$$

The auxiliary function, I, can be evaluated in the following manner. Integration by parts and manipulating yields

$$I(a,x) = 0 \quad \text{for } a > \tan \theta_o \tag{A-4}$$
$$= \theta_1+\csc^2\theta_o\{-[a^2/(1+a^2)]\theta_3+x^2\theta_4\} \text{ for } a<\tan \theta_o$$

where $\theta_1 = \sin^{-1} (x \cot \theta_o)$,

$\theta_3 = \sin^{-1} (x/a)$,

$\theta_4 = \int_b^c (2x^2+1+\cos \theta)^{-1} d\theta$,

$b = \pi - 2\theta_3$,

$c = \pi - 2\theta_1$,

and the change of variables $s = x \sec (\theta/2)$ has been made. The integral for θ_4 is in a standard form found in tables yielding

$$x^2\theta_4 = (x/\sqrt{1 + x^2})(\theta_5 - \theta_6) \tag{A-5}$$

where

$$\theta_5 = \cos^{-1}(\sqrt{1 + x^2}\cos\theta_o),$$

and

$$\theta_6 = \cos^{-1}(\sqrt{1 + x^2} / \sqrt{1 + a^2}).$$

All cases of practical interest satisfy the conditions that $\delta_o < \pi/2$ and $r \tan\theta_o < 1$. For these conditions, equation (A-2) becomes

$$\begin{aligned} P[|\delta|<\delta_o] &= (2/\pi)\{\theta_1 + \theta_2\} \qquad \text{for } x < \tan\theta_o \\ &= 1 \qquad\qquad\qquad\qquad\quad \text{for } x > \tan\theta_o \end{aligned} \tag{A-6}$$

where

$$\theta_1 = \sin^{-1}(x \cot\theta_o),$$

and

$$\theta_2 = (x \csc^2\theta_o / \sqrt{1+x^2}) \cos^{-1}(\sqrt{1+x^2} \cos\theta_o).$$

A-2 POLARIZATION TRACKING ANALYSIS

This is the third error correction technique considered. The generalized block diagram is shown in Figure 3.

Each original antenna element is replaced by a nominally cross polarized pair. The complex unit polarization vectors of the four real antennas are denoted by

$$\hat{h}_{11} = (1-\varepsilon_{11})\,\hat{h}_o + \delta_{11}\,\hat{h}_x,$$

$$\hat{h}_{12} = \delta_{12}\,\hat{h}_o + (1-\varepsilon_{12})\,\hat{h}_x,$$

$$\hat{h}_{21} = (1-\varepsilon_{21})\,\hat{h}_o + \delta_{21}\,\hat{h}_x, \qquad \text{(A-7)}$$

and

$$\hat{h}_{22} = \delta_{22}\,\hat{h}_o + (1-\varepsilon_{22})\,\hat{h}_x$$

where $\hat{h}_o$ and $\hat{h}_x$ are the complex unit polarization vectors of the designed cross polarized pairs. The epsilons and deltas represent the departures of the true antenna polarizations from the desired ones. In the combiners, two new antennas are synthesized whose complex unit polarization vectors are given by

$$\hat{h}_1 = \gamma_{11}\,\hat{h}_{11} + \gamma_{12}\,\hat{h}_{12}, \qquad \text{(A-8)}$$

and

$$\hat{h}_2 = \gamma_{21}\,\hat{h}_{21} + \gamma_{22}\,\hat{h}_{22}.$$

The polarization of the incoming field ($\hat{h}_e$) is determined in the polarization sensor and the results used to control the combiner coefficients. Let the incoming field polarization be given by

$$\hat{h}_e = c_o^* \hat{h}_o^* + c_x^* \hat{h}_x^*. \tag{A-9}$$

Then the combining coefficients are given by

$$\begin{aligned} \gamma_{11} &= (c_o + \varepsilon_{s1})(1 + \varepsilon_{M1}) + \varepsilon_{z1}, \\ \gamma_{12} &= (c_x + \varepsilon_{s2})(1 + \varepsilon_{M2}) + \varepsilon_{z2}, \\ \gamma_{21} &= (c_o + \varepsilon_{s1})(1 + \varepsilon_{M3}) + \varepsilon_{z3}, \end{aligned} \tag{A-10}$$

and

$$\gamma_{22} = (c_x + \varepsilon_{s2})(1 + \varepsilon_{M4}) + \varepsilon_{z4}$$

where

ε_{si} = polarization sensing error,

ε_{Mi} = multiplicative combiner error,

and

ε_{zi} = additive combiner error.

If the combiner is formed of a gain controlled amplifier and phase shifter, the additive combiner errors will be negligible.

The antenna cross polarization ratio, ρ, and field cross polarization ratio, σ, can be formally found by using $\hat{h}_2$ as the reference polarization. Thus

$$\rho/\sqrt{1 + r^2} = \hat{h}_1 \cdot \hat{h}_{2x}^*$$

and (A-11)

$$\sigma/\sqrt{1 + s^2} = \hat{h}_e \cdot \hat{h}_{2x},$$

where

$$\hat{h}_1 = [\gamma_{11}(1-\varepsilon_{11}) + \gamma_{12}\delta_{12}]\hat{h}_o + [\gamma_{11}\delta_{11} + \gamma_{12}(1-\varepsilon_{12})]\hat{h}_x$$

and

$$\hat{h}_{2x} = [\gamma_{21}\delta_{21} + \gamma_{22}(1-\varepsilon_{22})]^* \hat{h}_o - [\gamma_{21}(1-\varepsilon_{21}) + \gamma_{22}\delta_{22}]^* \hat{h}_x.$$

After some algebra we obtain

$$\rho/\sqrt{1 + r^2} = c_o^2(\delta_{21} - \delta_{11}) + c_x^2(\delta_{12} - \delta_{22})$$
$$+ c_o c_x(\varepsilon_{12} + \varepsilon_{21} - \varepsilon_{11} - \varepsilon_{22} + \varepsilon_{M1} - \varepsilon_{M2} - \varepsilon_{M3} + \varepsilon_{M4}) \quad \text{(A-12)}$$

and

$$\sigma^*/\sqrt{1+s^2} = c_o^2\delta_{21} - c_x^2\delta_{22} + c_o\varepsilon_{s2} - c_x\varepsilon_{s1}$$
$$+ c_o c_x(\varepsilon_{21} - \varepsilon_{22} + \varepsilon_{M4} - \varepsilon_{M3}) \quad \text{(A-13)}$$

where only first order error terms have been kept and the additive combiner errors have been assumed to be negligible. These expressions for the antenna and field polarization ratios in terms of combiner and antenna element errors do not however define completely the DF system error as might at first be supposed. This becomes quite clear when the interferometer system is considered.

Application to a Phase Interferometer

The error in normalized arrival angle due to polarization (after correction by polarization tracking) in a two-element interferometer is δ and is given by

$$\delta = \text{phase}\ [(\hat{h}_1 \cdot \hat{h}_e)(\hat{h}_2 \cdot \hat{h}_e)^*]. \tag{A-14}$$

Again retaining only first order error terms this becomes

$$\begin{aligned}\delta = \text{phase}\ \{&(1 + c_o\varepsilon_{s1}^* + c_o^*\varepsilon_{s1} + c_x\varepsilon_{s2}^* + c_x^*\varepsilon_{s2}) \\ &+ |c_o|^2\ (\varepsilon_{M3}^* + \varepsilon_{M1} - \varepsilon_{11} - \varepsilon_{21}^*) \\ &+ |c_x|^2\ (\varepsilon_{M4}^* + \varepsilon_{M2} - \varepsilon_{12} - \varepsilon_{22}^*) \\ &+ c_o c_x^*\ (\delta_{11} + \delta_{22}^*) + c_o^* c_x(\delta_{12} + \delta_{21}^*)\}.\end{aligned} \tag{A-15}$$

The first term in this expression is of course real. The imaginary parts of the second and third terms represent phase errors which exist even if the received polarization is one of the two design polarizations. In order to be consistent with the previous error analysis, it

must be assumed that the system has been properly aligned so that these errors are zero. This would be done, for example, by transmitting $\hat{h}_o$ $(c_x = 0)$ and adjusting the relative phases of the signals from $\hat{h}_{11}$ and $\hat{h}_{21}$ to yield zero error and then by transmitting $\hat{h}_x(c_o = 0)$ and similarly adjusting the phases of the signals from $\hat{h}_{12}$ and $\hat{h}_{22}$. Subject to the assumption that the interferometer has been properly aligned, the first order polarization error becomes

$$\delta = \mathrm{Im}\,\{c_o c_x^*(\delta_{11}+\delta_{22}^*)+c_o^* c_x\,(\delta_{12}+\delta_{21}^*)\}. \tag{A-16}$$

This can be expressed in terms of the antenna cross polarization ratio between corresponding elements of the two antenna pairs. Thus

$$\rho_1/\sqrt{1 + r_1^2} \equiv \hat{h}_{11}\cdot\hat{h}_{21x}^* = \delta_{21} - \delta_{11} \tag{A-17}$$

and

$$\rho_2/\sqrt{1 + r_2^2} \equiv \hat{h}_{12x}\cdot\hat{h}_{22}^* = \delta_{22}^* - \delta_{12}^*, \tag{A-18}$$

where again we have kept only the first order terms. The first order polarization error becomes

$$\delta = \mathrm{Im}\,\{c_o c_x^*\,(\rho_2 - \rho_1)\} \tag{A-19}$$

to the first order. After some final manipulations this becomes

$$\delta = |c_o|\;|c_x|\;R \sin \eta \tag{A-20}$$

where

$$R^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos\,(\zeta_1-\zeta_2).$$

The cumulative probability function of the polarization error is

$$P[|\delta|<\delta_o]=\int_o^1 P_{|c_o|}(x)P[|\sin\eta|<\delta_o/(Rx\ \ 1-x^2)]\,dx, \qquad \text{(A-21)}$$

where

$$P[|\sin\eta|<y]=\begin{matrix}(2\pi)\text{arc sin }(y) & \text{for } y<1\\ 1 & \text{for } y>1.\end{matrix} \qquad \text{(A-22)}$$

Hence

$$P[|\delta|<\delta_o]=\begin{matrix}1 & \text{for } 2\delta_o/R>1\\ P[|c_o|>\cos\theta]+P[|c_o|<\sin\theta]+I & \text{for } 2\delta_o/R<1\end{matrix} \qquad \text{(A-23)}$$

where

$$I=(2/\pi)\int_{\sin\theta}^{\cos\theta} P_{|c_o|}(x)\text{arc sin}\left(\frac{\sin 2\theta}{2x\sqrt{1-x^2}}\right)\,dx$$

and $\sin 2\theta = 2\delta_o/R$. Using the probability density function of $|c_o|$, (i.e., $p(|c_o|) = 2|c_o|$), this becomes for $2\delta_o/R<1$

$$P[|\delta|<\delta_o]=1+(1/2\pi)\sin 2\theta\int_{\sin^2\theta}^{\cos^2\theta}(2-1/t)(a+t-t^2)^{-1/2}\,dt \qquad \text{(A-24)}$$

where

$$a = -\sin^2\theta\cos^2\theta.$$

and we have integrated by parts and then made the substitution $x^2 = 1-t$. The integral is in a standard form given in tables and the final result is

$$P[|\delta|<\delta_o] = \sin 2\theta = 2\delta_o/R \quad \text{for } 2\delta_o/R < 1 \qquad \text{(A-25)}$$
$$= 1 \quad \text{for } 2\delta_o/R > 1.$$

A-3 DF ERROR CORRECTION BY POLARIMETER MEASUREMENTS

The theoretical concepts involved in the assessment of the residual errors which result from using polarimeter measurements to correct DF data for the effects of polarization are relatively simple. In any DF system, the polarization error can be expressed as a function of the complex cross polarization ratio of the incident field (σ) and the complex antenna cross polarization ratio (ρ). Ideally then, one could measure the incident field polarization using a polarimeter and thus determine σ. This, together with the known value of ρ (assumed known either by computation or range measurements of the antenna system) would be used to compute the polarization error. This computed error would then be subtracted from the measured direction of arrival to produce a result free of polarization error. This ideal result can never be achieved in practice because of inaccuracies in the polarimeter measurements, uncertainties in the true antenna cross polarization ratio, and deviations in the actual DF system processing method from that assumed (e.g., nonlinearities in a phase detector). In order to establish the practicability of utilizing polarimeter measurements, it is therefore necessary to consider the residual errors due to these effects. The difficulties which arise in such a consideration are well illustrated by the case of the phase interferometer.

The phase interferometer error* due to polarization (ε_p) has been shown to be given by

$$\varepsilon_p = \text{phase}\,\{1 + r\,s\,\exp[j(\zeta+\eta)]\} \qquad \text{(A-24)}$$

where $r\exp(j\zeta) = \rho$ is the complex antenna cross polarization ratio and $s\exp(j\eta) = \sigma$ is the complex incident field cross polarization ratio. This latter quantity is defined by $\sigma = E_x/E_o$ where $E_x = \vec{E}\cdot\hat{h}_x$ is the "cross" polarized part of the incident field (E) and $E_o = \vec{E}\cdot\hat{h}_o$ is the "parallel" polarized part of the incident field. The function of the polarimeter is to measure σ. In its basic form a polarimeter consists of two orthogonally polarized antennas. Let us denote the polarization vectors of these two antennas by $\hat{h}_\alpha$ and $\hat{h}_\beta$, respectively. The reference polarization vectors can be expressed as a linear combination of the polarimeter polarization vectors. That is we can write

$$\hat{h}_o = \alpha_o\hat{h}_\alpha + \beta_o\hat{h}_\beta$$

and (A-25)

$$\hat{h}_x + \alpha_x\hat{h}_\alpha + \beta_x\hat{h}_\beta,$$

where the α's and β's are complex numbers. It then follows that the incident field polarization components are linear functions of the field components measured by the polarimeter. That is,

---

*In this discussion, ε_p and δ are errors in the equivalent arrival angle $kd\sin\theta$ where θ is the spatial arrival angle.

$$E_o = \alpha_o E_\alpha + \beta_o E_\beta$$

and (A-26)

$$E_x = \alpha_x E_\alpha + \beta_x E_\beta$$

where $E_\alpha (= \vec{E} \cdot \hat{h}_\alpha)$ and $E_\beta (= \vec{E} \cdot \hat{h}_\beta)$ are the field components measured by the polarimeter. Errors enter into these determinations of E_o and E_x because of errors in the measurements of E_α and E_β (due to noise, system inaccuracies, etc.) and errors in the assumed values of the α's and β s (due to imperfect knowledge of h_α, h_β, h_o, and h_x). Let the superscript m denote a quantity derived from the polarimeter measurement including these errors. Then we may write

$$E_x^m = E_x + \delta_x,$$

$$E_o^m = E_o + \delta_o, \qquad \text{(A-27)}$$

and

$$\sigma^m = E_x^m / E_o^m$$

where δ_x and δ_o are the errors in the determination of E_x and E_o, respectively. Considering the sources of these errors it is reasonable to assume that δ_x and δ_o consist of independent quadrature normally distributed random variables.

An expression for the residual interferometer error (δ) can be found by considering the geometry shown in Figure 11. Clearly,

$$\begin{aligned} \delta &= \text{phase } (1 + \rho\sigma^m) - \text{phase } (1 + \rho\sigma) \\ &= \text{phase } [(1 + \rho\sigma^m)/(1 + \rho\sigma)]. \end{aligned} \tag{A-28}$$

Without loss of generality, we may combine the phase of the antenna cross polarization ratio with that of the incident field resulting in

$$\rho\sigma = rs \exp[j\eta]$$

where r (the magnitude of the antenna cross polarization ratio) is a real number characterizing the antenna system and s and η are the magnitude and phase, respectively, of the incident field cross polarization ratio. If we introduce a constant power constraint on the incident field, we have

$$1 = |E_o|^2 + |E_x|^2 = |E_o|^2(1 + s^2) \tag{A-29}$$

and thus define s as a function of the magnitude of E_o. Returning now to the measured field cross polarization ratio, we have

$$\sigma^m = (E_x + \delta_x)/(E_o + \delta_o) = (\sigma + \delta_x/E_o)/(1 + \delta_o/E_o). \tag{A-30}$$

Since, by assumption, the phases of δ_x and δ_o are uniformly distributed over all possible values, it is possible to replace E_o in the above expression by its magnitude without changing any of the statistical properties of σ^m. Thus, we have expressed the measured cross polarization ratio as a function of two incident field parameters ($|E_o|$ and η) and the two complex random variables δ_x and δ_y. Equation (A-28) therefore expresses the residual error δ as a function of these parameters and the antenna cross polarization ratio amplitude r. Since the statistical properties of δ_x and δ_o are known (by assumption) it is possible, in principle at least, to compute the statistical porperties of δ for specified values of r, $|E_o|$, and η. We are, however, more interested in the statistical properties of δ as a function of r when $|E_o|$ and η are themselves random variables corresponding to a randomly polarized incident field. These properties are found in the following fashion. The statistical properties of the residual error, δ, are known if the probability density function of δ is known. Let us denote this density function, for a given value of antenna cross polarization ratio r, as $p_\delta(\delta|r)$ (read as density function of δ given r). We further introduce the conditional probability density function of δ given $|E_o|$ and η denoted by $p_\delta(\delta|r,\eta,|E_o|)$. Then the following relationship applys

$$p_\delta(\delta|r) = \int p_\eta(y)dy \int p_{|E_o|}(x)dx \; p_\delta(\delta|r,y,x) \qquad \text{(A-31)}$$

where $p_\eta(y)$ and $P_{|E_o|}(x)$ are the probability density functions of η and $|E_o|$, respectively, and the integrations are over all possible values of these variables. For a randomly polarized field

$$P_\eta(y) = y/2\pi,$$

$$P_{|E_o|}(x) = 2x,$$

the y range is any 2π interval, and the x interval is zero to one. Analytically, it was possible only to reduce the computation of the probability density function for the residual error (δ) in normalized arrival angle to a triple integral. Numerical integration was performed for several selected values of antenna cross polarization ratio (r) and polarimeter measurement signal-to-noise ratio* (SNR). These numerical integrations were very time consuming, requiring a great many points in order to achieve a reasonable accuracy. An examination of the resulting probability distribution functions, however, revealed that they are well approximated by the following simple formula:

$$p(\delta) = 1/(a + b \cos \delta) \qquad \text{(A-32)}$$

where

$1/(a+b) = .006\ (SNR/r)$ per degree,

SNR = voltage signal-to-noise ratio,

and

$$a-b = (360)^2/(a+b).$$

---

*As defined here, the value of SNR includes uncertainties in the proper values to be used in computing the interferometer error from the polarimeter data.

In order to demonstrate how well Equation (A-32) approximates the exact results, Figure A-1 is included. In Figure A-1, the dots and crosses are the results found by numerical integration and the solid curves were computed from Equation (A-32). The approximation is within a few percent, while the function varies over many orders of magnitude. Comparisons of Equation (A-32) with the values found by numerical integration were made for values of r between 0.1 and 0.5 and values of SNR of 10 and 20 dB. The approximation is excellent for this entire range of parameters which includes most cases of practical interest. One advantage of the approximation is that the cumulative distribution is also a simple function and is given by

$$P[|\delta|<\delta_o] = \int_{-\delta_o}^{\delta_o} p(\delta)\, d\delta$$

$$= (1/90)\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\delta_o/2\right) \qquad \text{(A-33)}$$

$$= (1/90)\tan^{-1}[2(SNR/r)\tan(\delta_o/2)]$$

where the arc-tangent is in degrees.

Cumulative probability curves, computed from Equation (A-33), are presented in Figures 9 and 10.

Two features of these curves are immediately apparent. First, the mean error is significantly reduced even when the polarimeter data is quite crude, a reduction by about a factor of two resulting when the effective polarimeter signal-to-noise ratio is only 10 dB. The improvement in the mean error is approximately a factor of five for a signal-to-noise ratio of 20 dB.

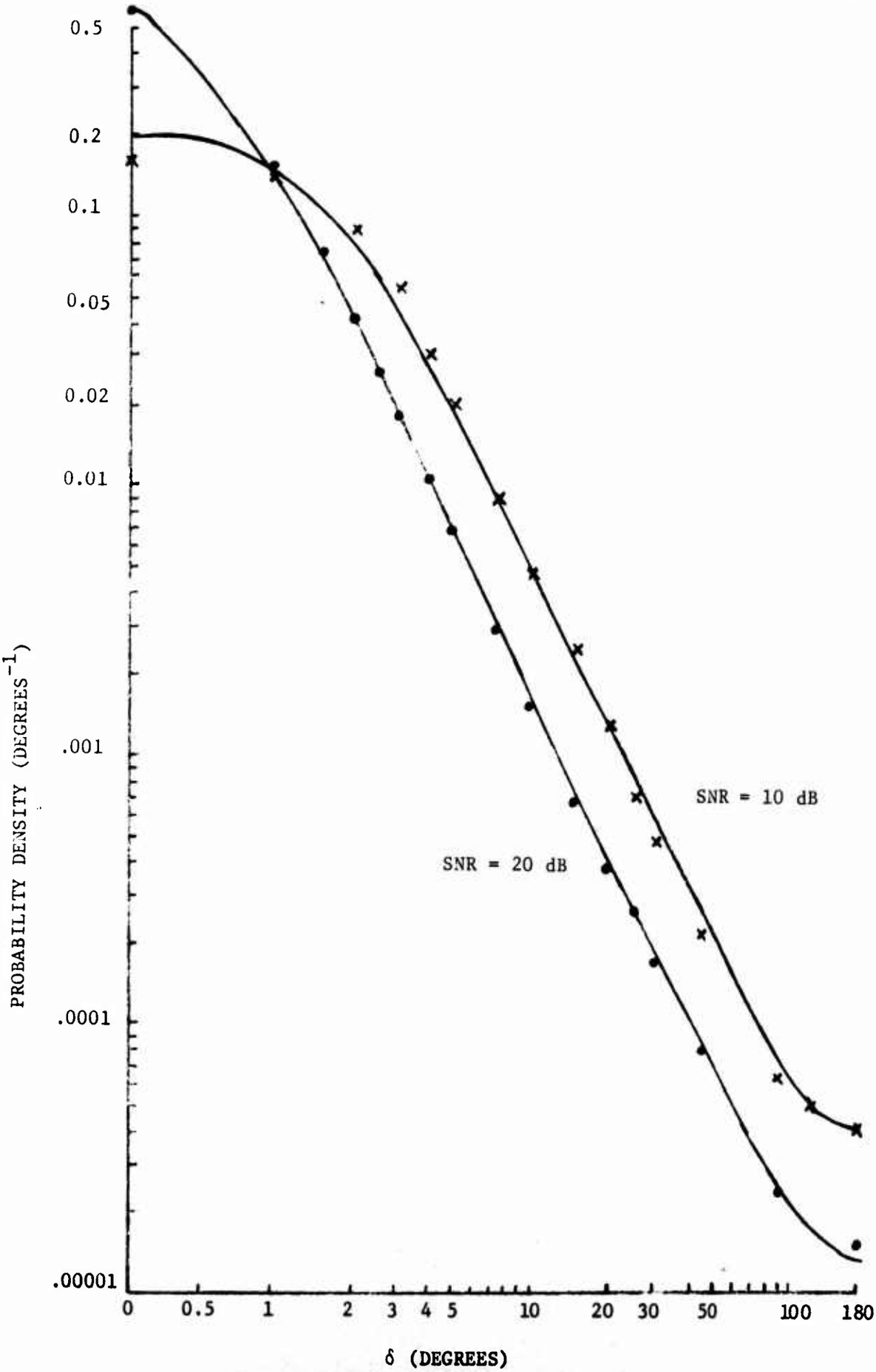


FIGURE A-1 INTERFEROMETER ERROR PROBABILITY DENSITY FUNCTION WITH POLARIMETER CORRECTION. ANTENNA CROSS POLARIZATION RATIO = 0.1.

Second, the probability of really large errors is increased when poor polarimeter data is used. This is indicated by the crossing of the before and after correction curves. That such an effect must occur is clear when one considers the fact that for a negative signal-to-noise ratio the correction would become essentially a random guess obviously resulting in larger errors after correction. What is somewhat surprising, however, is the fact that, for SNR = 10 dB, this degradation occurs at a significant probability level (greater than five percent) and therefore becomes an important consideration for applications where large errors with low probabilities are important.

Upon a closer examination of the numerical integration results, it appears that the degradation occurs when the corrections are applied under conditions of a large incident field cross polarization ratio and that further improvement in the mean square error would be obtained if no correction were made under these conditions. This fact strongly suggests that a sophisticated strategy combining corrected and uncorrected DF measurements in a weighted fashion might be beneficial in a critical application. Even better would be combining this technique with the polarization switching technique. This latter approach would of course double the amount of antenna data that must be made available to the computer. Combining polarization tracking with the polarimeter correction does not appear to be feasible as the required amount of antenna data would be untenable.

A further comparison of the data presented in Figures 9 and 10 graphically illustrates how either improving the interferometer antenna system (compare curves f from Figures 9 and 10, for example) or using good polarimeter measurements (compare curves e and f in Figures 9 and 10) will result in the best DF measurements. For example, an antenna cross polarization ratio of 0.1 uncorrected gives essentially the same results as an antenna cross polarization ratio of 0.5 corrected by polarimeter measurements with a 10 dB SNR.

A-4 ANTENNA PARAMETER RANDOMIZATION ANALYSIS

The problem is to evaluate the effects of considering the antenna parameters to be random variables rather than constants. We shall consider the phase interferometer with and without correction by polarization tracking. For this system, the antenna parameter of interest is the magnitude of the antenna cross polarization ratio (r) which we shall take to be Rayleigh distributed. It therefore has a probability density function given by

$$p(r) = (r/\sigma_r^2) \exp(-r^2/2\sigma_r^2) \quad \text{for } r > 0$$
$$= 0 \quad \text{for} < 0. \tag{A-34}$$

The conditional cumulative probability density function of the polarization error given r is (from Reference 1)

$$P[|\varepsilon_p| < \varepsilon | r] = (1/\pi)\{\varepsilon + u \sin \varepsilon [\pi - \tan^{-1}(u/\cos \varepsilon)]\} \text{for } \varepsilon < \pi/2 \tag{A-35}$$

where

$$u = \sqrt{r^2 + \sin^2\varepsilon}.$$

The cumulative probability density function of the polarization error is found by integration of equation (A-35) weighted by equation (A-34) with respect to r. If we make the change of variables $x = u/\sqrt{2}\sigma_r$, we can rather easily find

$$P[|\varepsilon_p|<\varepsilon] = \sqrt{\pi}\, v \exp(v^2)\, \mathrm{erfc}(v) + (2/\pi)[\varepsilon/2) - v\exp(v^2) I] \tag{A-36}$$

where

$$v = \sin \varepsilon / \sqrt{2}\sigma_r$$

$$I = \int_v^\infty \tan^{-1}(vx \tan \varepsilon)\exp(-x^2)dx,$$

and

$$\mathrm{erfc}(v) = 2/\sqrt{\pi} \int_v^\infty \exp(-x^2)\, dx = \text{complementary error function.}$$

The remaining integral may be evaluated asymptotically by using the series expansion for the arc tangent. When this is done the result is

$$P[|\varepsilon_p|<\varepsilon] \simeq \sqrt{\pi}\, v \exp(v^2)\, \mathrm{erfc}(v) + O\,(\sigma_r^{\,2} \sin \varepsilon) \tag{A-37}$$

where the approximation is valid for $\varepsilon < \pi/4$ which is the region of primary interest.

Let us now consider the case when polarization tracking has been used.

We assume that the quadrature components of the complex antenna cross polarization ratio are independent, normally distributed, random variables with zero means and standard deviations σ_r. Furthermore, the quadrature components of the sum (or difference) of N independent antenna cross polarization ratios are also independent, normally distributed, random variables with zero mean and standard deviations $\sqrt{N}\,\sigma_r$. The probability density function of the parameter R is therefore

$$p(R) = (R/2\sigma_r^2)\exp(-R^2/4\sigma_r^2) \quad \text{for } R \geq 0 \qquad \text{(A-30)}$$
$$= 0 \quad \text{for } R < 0$$

In this context, equation (A-25) is interpreted as the conditional polarization error probability given R. The final polarization error probability distribution is found by integrating equation (A-29) weighted by the probability density function of R. Thus

$$P[|\delta| < \delta_o] = P[R < 2\delta_o] + 2\delta_o \int_{2\delta_o}^{\infty} p(R)/R \, dR. \qquad \text{(A-39)}$$

This is easily evaluated to give

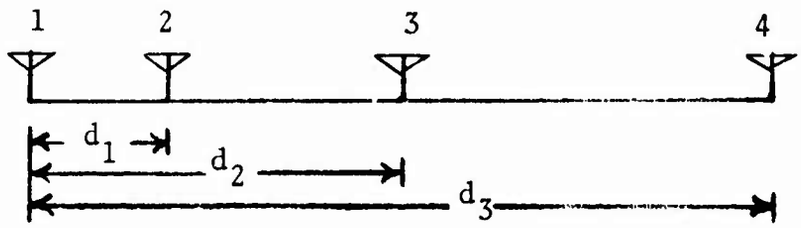
$$P[|\delta| < \delta_o] = 1 - \exp(-x^2) + \sqrt{\pi}\, x \operatorname{erfc}(x) \qquad \text{(A-40)}$$

where $x = \delta_o/\sigma_r$. We note that this expression is also proportional to δ for small x just as equation (A-25) is. The initial slope is however different. In general, equation (A-25) gives a higher probability of small errors and a lower probability of large errors than equation (A-40). If the most probable value of $R(= 1.414\ \sigma_r)$ is used in equation (A-25), the two equations match at the 50 percent probability level. If the median $(R = 1.665\ \sigma_r)$ or mean $(R = 1.772\ \sigma_r)$ is used, the match is between the 70 and 80 percent level.

The results given in the preceding paragraphs imply that when the antenna cross polarization ratio is a random variable but its mean (or median or most probable or some other statistical measure) is used in the previously derived error expressions as though the ratios were a constant, then the probability of small DF errors will be overestimated and that of large DF errors underestimated. This is hardly surprising but is included here as a cautionary note in interpreting specific error curves.

A-5 GRATING LOBE ERRORS IN MULTILEVEL INTERFEROMETERS

A three-level interferometer is shown in the figure below.



In this interferometer there are three unique phase differences that can be measured. These are given by:

$$\phi_i = 360\ (d_i/\lambda)\ \sin\alpha \text{ degrees } i = 1, 2, 3 \qquad \text{(A-41)}$$

where α is the arrival angle measured from the vertical. A configuration which is optimum in several respects has $d_1/\lambda = 0.5$ and $d_2/\lambda = \sqrt{d_3/2\lambda}$. This configuration permits the phase measurement accuracy requirements for all phases to be equal and provides complete resolution of ambiguities. We shall consider the specific case where $d_3/\lambda = 8$.

The long baseline interferometer is used to make the precision angle of arrival measurement, the intermediate baseline interferometer is used to resolve the ambiguities in the long baseline measurement, and the short baseline interferometer is used to resolve the ambiguities in the intermediate baseline measurement. As an example, suppose the true arrival angle is 20 degrees and there are no measurement errors. Then $\phi_1 = 62°$, $\phi_2 = -114°$, and $\phi_3 = -95°$. For a measured $\phi_3 = -95°$, possible arrival

angles are -65, -52, -41, -32, -24, -16, -9, -2, +5, +13, +20, +28, +36, +46, +57, and +75 degrees. For a measured $\phi_2 = -114°$, possible arrival angles are -41, -9, +20, and +57 degrees. For a measured $\phi_1 = 62°$ the only possible arrival angle is 20 degrees. Now let us look at the effects of phase measurement errors. In order to examine this we must first formulate the rules for resolving the ambiguities. These rules can be expressed mathematically by:

$$\alpha = \arcsin (\phi_3/2880 + N/8), \tag{A-42}$$

where

$$N = [\phi_2/90 - \phi_3/360 + 4M + .5], \tag{A-43}$$

$$M = [\phi_1/90 - \phi_2/360 + .5], \tag{A-44}$$

and the square brackets indicate greatest integer less than the quantity inside the brackets. For the given example, M = 1 and N = 3 yielding the correct answer α = 20 degrees. Equation (A-44) is the mathematical formulation for the process of finding how many multiples of 360 degrees must be added to the phase for the intermediate baseline to deduce the arrival angle closest to the value given by the short baseline. Equation (A-43) similarly defines the long baseline phase measurement in order to achieve closest agreement with the intermediate baseline result. There are two types of ambiguity errors. An ambiguity error of the first type results from an error in N with M correct and an ambiguity error of the second type results if M is in error. Ambiguity errors of both types result from errors in the phase measurement.

If we denote the error in determining ϕ_i by ε_i, the probability of an error of the second type is given by P_{II} where

$$P_{II} = \text{Probability } (|\varepsilon_1 - \varepsilon_2/4| > 45 \text{ degrees}). \qquad \text{(A-45)}$$

and the probability of an error of the first type is given by P_I where

$$P_I = \text{Probability } (|\varepsilon_2 - \varepsilon_3/4| > 45 \text{ degrees}) \qquad \text{(A-46)}$$

The error in the i'th phase measurement is given by

$$\tan \varepsilon_i = r_i \, s \sin(\delta_i + \eta)/[1 + r_i \, s \cos(\delta_i + \eta)] \qquad \text{(A-47)}$$

if we assume that the complex field cross polarization ratio $s \exp(j\eta)$ is the same for all antenna pairs. Since equations (A-45) and (A-46) have the same form, we shall evaluate only P_{II} and recognize that P_I will be the same with subscript 2 replacing 1 and subscript 3 replacing 2. Since the phase of the field cross-polarization ratio may be assumed to be uniformly distributed over a 360 degree range, we can make the following replacements; $\eta \to \eta + \delta_1$ and $\eta + \delta \to \eta + \delta_2$ where $\delta = \delta_2 + \delta_1$. Now, since without referring to a specific set of antennas we can say nothing about δ, we shall assume it to be a uniformly distributed random variable. This implies that the random functions ε_1 and ε_2, given the magnitude of the field cross-polarization ratio (s), are independent with conditional probability density functions $p_1(\varepsilon_1|s)$ and $p_2(\varepsilon_2|s)$, respectively. The conditional probability density function of $\delta = \varepsilon_1 - \varepsilon_2/4$ given s is then

$$p(\delta|s) = \int_{-\pi}^{\pi} p_2(\varepsilon_2|s)\, p_1\, (\varepsilon_1 = \delta + \varepsilon_2/4|s)\, d\varepsilon_2. \qquad \text{(A-48)}$$

After considerable manipulation, the probability of an ambiguity error of the second type, P_{II}, can be put in the form

$$P_{II} = P(|\delta|>\pi/4) = (1/\pi) \int_0^{\infty} p_s(s)ds \int_0^{\pi/2} d\alpha\, R(s,\gamma) \qquad \text{(A-49)}$$

where

$$R(s,\gamma) = 2\, P_2[-\pi<\varepsilon^2<\chi] \text{ for } s<1/r_1$$

$$= P_2[-\pi<\varepsilon_2<\chi] + (1/2) \text{ for } s>1/r_1$$

and

$$\chi = 4 \text{ arc sin } (r_1 s \sin\alpha)-\pi \text{ for } |r_1\, s \sin\alpha|<1$$

$$= \pi \qquad \text{for } |r_1\, s \sin\alpha|>1.$$

The function p_s (s) is the probability density function of s and the function $P_2[-\pi<\varepsilon_2<\chi]$ is the probability that ε_2 lies between $-\pi$ and χ. The error probability P_{II} is a function of the magnitude of the antenna cross-polarization ratios r_1 and r_2 which apply to the antenna pairs (1,2) and (1,3) respectively. The results of numerical integration of Equation (A-49) for realistic values of r_1 and r_2 are shown in Table A-1.

TABLE A-1

Probability of Ambiguity Error In A Multilevel Interferometer

r_2/r_1	.05	.1	.2
0	.00226	.00895	.0342
.05	.00240	.00913	.03411
.1	.00305	.00953	.0344
.2	.00473	.0120	.0360

As noted above, the probability of an ambiguity error of the first type is the same with subscript 2 replacing subscript 1 and subscript 3 replacing subscript 2.

It will be noted from the entries in this table and the results shown in Figure 7 that the probability of an ambiguity error of type II is very nearly the same as the probability that the phase error on the shortest baseline interferometer exceeds 45 degrees. The result is to be expected since, for antenna cross-polarization ratios of this order, the probability density function of the phase error is a rapidly decreasing function of the error. The implication of this result is that an ambiguity error of type II is primarily a result of a phase measurement error in the short baseline interferometer while an ambiguity

error of type I is primarily a result of a phase measurement error in the intermediate baseline interferometer. Thus, the two types of ambiguity errors should be nearly independent and the probability of at least one type of ambiguity error occuring will be approximately the sum of the individual probabilities for the antenna cross-polarization ratios considered.

A-6 DISCUSSION OF INTERFEROMETER NORMALIZED ARRIVAL ANGLE

All of the interferometer results, with the exception of those in Figure 5, are presented in items of the normalized arrival angle. This normalized arrival angle is the one which is directly measured by all interferometer systems and is merely the phase difference between the signal received at the two antennas. The space angle, α, is related to the normalized arrival angle, ϕ, by equation (A-41). This relationship is summarized for the interferometer baseline length in Table A-2.

TABLE A-2

Conversion From Interferometer Phase to Space Angle

Interferometer Phase (Degrees)	Space Angle (Degrees)		
	$d=\lambda/2$	$d=2\lambda$	$d=8\lambda$
0	0	0	0
30	9.6	2.4	0.6
60	19.5	4.8	1.2
90	30.0	7.2	1.8
120	41.8	9.6	2.4
150	56.4	12.0	3.0
180	90.0	14.5	3.6

All interferometer space angle results are ambiguous. If the baseline is more than one-half wavelength, there are multiple ambiguities. In interpreting Table A-2 it should be noted that only the space angle corresponding to the first grating lobe has been tabulated.

APPENDIX B

POLARIZATION DESCRIPTION

B-1 POLARIZATION REPRESENTATION OF ANTENNAS

Consider an antenna at the origin (transmitting) and an observation point at P. The electric field at P is given by:

$$\vec{E}(r,\theta,\phi,t) = (E_r\hat{e}_r + E_\phi\hat{e}_\phi + E_\theta\hat{e}_\theta)\ \{\exp[j(\omega t-kr)]/r\} \qquad \text{(B-1)}$$

where E_r, E_ϕ, E_θ are functions of r, ϕ, ϵ, and $\hat{e}_r$, $\hat{e}_\phi$, $\hat{e}_\theta$ are unit vectors in the r, ϕ, θ directions. We have assumed a sinusoidal time dependence and a spherical wave expansion. Now rewrite Equation (B-1) in the following form,

$$E(r,\theta,\phi,t) = E_o G(r,\theta,\phi)\{\exp[j(\omega t-kr)]/r\}\hat{h}(r,\phi,\theta) \qquad \text{(B-2)}$$

where

$$E_o^{\,2} = (r^2\ \vec{E}\cdot\vec{E}^*)\Big|_{\substack{r\to\infty\\ \theta=\theta_o\\ \phi=\phi_o}}$$

and

$$G(r,\theta,\phi) = (r/E_o)\sqrt{\vec{E}\cdot\vec{E}^*}\ .$$

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Physically, E_o is a normalizing constant containing the power transmitted and constants of the propagation medium. The direction (θ_o, ϕ_o) would normally be the direction of the beam maximum. The function G is a pattern function which, as the distance approaches infinity, becomes the conventional antenna pattern. The polarization properties are contained in the unit complex vector $\hat{h}$ which satisfies the property $\hat{h} \cdot \hat{h}^* \equiv 1$. By the definition of radiation or far field, in the far field the functions G and h are independent of the distance r. The composite complex vector $\vec{h} = G\,\hat{h}$ is sometimes called the complex effective vector length, and has the property that the open circuit voltage (V_{oc}) appearing at the antenna terminals when used as a receiving antenna is given by

$$V_{oc} = C\,(\vec{h} \cdot \vec{E}_i) \tag{B-3}$$

where $\vec{E}_i$ is the incident field due to a source in the direction (θ, ϕ) and C is the numerical constant. In all further work, we shall assume only plane wave incident fields and use the far field limit of the effective vector length $\vec{h}$. This assumption is permissible since <u>any</u> incident field can be expressed as a linear combination of plane wave fields each of which may be considered separately. The total antenna voltage will of course be a linear combination of the responses to each of the plane wave components.

In the far field, the electric field given by Equation (B-1) becomes a plane wave. In isotropic media it can be shown that the electric and magnetic components of a plane wave are normal to the propagation vector $\vec{k}$.

Two antennas (a and b) are orthogonally polarized (in a specified direction) if

$$\frac{\vec{E}_a \cdot \vec{E}_b^*}{G_a \cdot G_b} = 0$$

which implies $\hat{h}_a \cdot \hat{h}_b^* = 0$. Since $\hat{h}_a$ is normal to the unit propagation vector $\hat{k}$, it is clear that the orthogonally polarized $\hat{h}_b$ can be found from the vector cross product,

$$\hat{h}_b = \hat{h}_a^* \times \hat{k}. \tag{B-4}$$

To illustrate this, let us consider a general polarization vector

$$\hat{h}_a = \frac{\hat{e}_\mu + re^{j\zeta}\hat{e}_\nu}{\sqrt{1+r^2}} \tag{B-5}$$

where $\hat{e}_\mu$ and $\hat{e}_\nu$ are orthogonal unit vectors perpendicular to the direction of propagation $\vec{k}$ and r and ζ define the relative complex amplitude of the linearly polarized components. By definition

$$\hat{e}_\mu \times \hat{e}_\nu = \hat{k}.$$

Then,

$$\hat{h}_b = \frac{re^{-j\zeta}\hat{e}_\mu - \hat{e}_\nu}{\sqrt{1+r^2}} \quad . \tag{B-6}$$

B-2 RANDOMLY POLARIZED FIELDS

Let $\hat{h}_R$ be the random polarization given by

$$\hat{h}_R = a\,\hat{h}_o + b\,\hat{h}_x \tag{B-7}$$

where $\hat{h}_o$ and $\hat{h}_x$ are two fixed orthogonal polarization vectors and a and b are complex random variables. We are interested in the distribution of the complex random variable σ, defined by

$$\sigma = b/a = s\,\exp(j\eta). \tag{B-8}$$

Let the real and imaginary parts of a and b be normally distributed random variables. It can then be shown that the phase η is uniformly distributed in the range 0 to 2π radians and the probability density function of s is

$$\begin{aligned} p(s) &= 2\,s/(1+s^2)^2 \qquad && \text{for } s > 0 \\ &= 0 && \text{for } s < 0. \end{aligned} \tag{B-9}$$

The cumulative probability that s is greater than any constant s_o is

$$p(s_o) = 1/(1+s_o^2). \tag{B-10}$$

This has the essential property that the cumulative distribution functions for s and 1/s are identical.

One property which a uniformly distributed random polarization, $\hat{h}_R$, should possess is that the probability that the dot product with any fixed polarization, $\hat{h}$, is less than a constant ε should be the same for all fixed polarizations. This means that the probability

$$p(|\hat{h} \cdot \hat{h}_R^*| \leq \varepsilon)$$

for $\hat{h} = \alpha \hat{h}_o + \beta \hat{h}_x$ must be independent of α and β. It can be verified that this is true and, in fact, the probability is just ε^2. The proof of this statement is as follows. We require the probability, P_R, that $|h \cdot h_R^*|^2$ is less than or equal to ε^2. This can be rewritten as

$$P_R = P\,[|a|^2\,|\alpha|^2 + (1-|a|^2)|\beta|^2 + 2|a|\,|\alpha|\,|\beta|\sqrt{1-|a|^2}\,\cos(\eta-\zeta) \leq \varepsilon^2]. \tag{B-11}$$

where the angle ζ is the phase of β/α.

Before proceeding with the problem at hand, we must first determine the probability density function $p_a(s)$, which is the probability density function for the magnitude of a. This is done in the following fashion. We first note that the following relationships are satisfied:

$$P[s = \sqrt{1-|a|^2}/|a| > S_o] = 1/(1+S_o^2) = P[|a|^2 < 1/(1+S_o^2)]. \tag{B-12}$$

It therefore follows that

$$P\,[|a|<a_o]=P[|a|^2<a_o^2] = \int_o^{a_o} p_a(x)dx = a_o^2 \qquad \text{(B-13)}$$

and that

$$p_a(a_o) = \frac{d}{da_o} P[|a|<a_o] = 2\,a_o. \qquad \text{(B-14)}$$

It should be noted that this implies that $|a|^2$ is uniformly distributed between zero and one.

The evaluation of P_R can be made by direct methods. However, a more meaningful, and at the same time computationally simple approach results if the following substitutions are made. We let

$$|a| = \sin \theta/2,$$

and

$$|\alpha| = \sin \phi/2,$$

where the angle θ is a random variable and the angle ϕ is a constant. The distribution of θ is found from

$$P[|a|<a_o] = P[\theta < \theta_o]=a_o^2=\sin^2(\theta_o/2) = \int_o^{\theta_o} p_{\theta_o}(\theta)\, d\theta \qquad \text{(B-15)}$$

and hence $p_\theta(\theta) = (\sin \theta)/2$.

The random polarization vector, $\hat{h}_R$, is defined by the two two random angles η and θ. The probability that η lies between η and $\eta+d\eta$, and θ lies between θ and $\theta+d\theta$ is $(1/4\pi)$ $\sin\theta\, d\theta\, d\eta$, which is equal to the differential element of area on a unit sphere divided by the total area of the sphere. Thus, if θ is interpreted as a polar angle and η as an azimuthal angle, the points defining the random polarization vector are uniformly distributed over the surface of the unit sphere. This unit sphere, with the north pole shifted to $\hat{h}_x$, is, in fact, the Poincaré sphere. The test polarization, $\hat{h}$, is similarly defined by the constant angles ζ and ϕ.

Upon making the above substitutions, and performing some simplification,

$$P_R = P[1 + \cos\theta\cos\phi + \sin\theta \sin\phi \cos(\eta-\zeta) \leq 2\varepsilon^2]$$

$$= P[1 + \cos U \leq 2\varepsilon^2] = P[|\cos(U/2)| \leq \varepsilon] \qquad \text{(B-16)}$$

where U is the angular great circle distance between the point on the unit sphere representing $\hat{h}_R$ and the point representing $\hat{h}$. Since the random points are uniformly distributed over the sphere, any axis may be taken as the polar axis without changing the probability density function of the polar angle. Consequently, if the point representing $\hat{h}$ is taken as the pole, the probability density function of the angle U is just $(\sin U)/2$ and we easily find

$$P_R = P[U \geq 2 \arccos\varepsilon] = (1/2) \int_{2\cos^{-1}\varepsilon}^{\pi} \sin U\, dU = \varepsilon^2 \qquad \text{(B-17)}$$

which completes the proof.

APPENDIX C

POLARIZATION MEASUREMENT

C-1 GENERAL DISCUSSION

The polarization of an electromagnetic plane wave describes the locus of end points of the electric field vector in a plane normal to the direction of propagation. In the case of a linearly polarized wave, the amplitude varies sinusoidally at the carrier frequency with a constant orientation in the normal plane. A circularly polarized wave has a constant amplitude which rotates in the normal plane at the carrier frequency. In the more general case the wave polarization is elliptical with linear and circular polarization as limiting cases.

Three parameters suffice to completely specify the instantaneous polarization of an elliptically polarized wave. They are: (1) axial ratio ρ, the ratio of the major axis of the polarization ellipse to its minor axis, (2) tilt angle τ, the angle which the major axis of the polarization ellipse makes with the horizontal of the measurement system ($0 \leq \tau < 180°$), (3) sense of rotation, the direction of rotation of the electric field vector. When viewed in the direction of propagation, the sense of rotation is right-handed or left-handed as the electric field vector is rotating clockwise or counterclockwise.

The polarization parameters can be time varying quantities and in that case, to completely define the dynamic polarization, it is necessary to also specify the time variation of the parameters.

There are four basic methods of measuring the polarization parameters. These vary considerably in complexity, cost, and ability to define the instantaneous polarization of the received signal. These approaches are the rotating linear antenna, the use of orthogonal linear antennas in conjunction with an X-Y display to trace out the polarization ellipse, the computation of the polarization parameters from the amplitude and phase of the circularly polarized components, and the computation of the polarization parameters from amplitudes only of circularly polarized, vertically and horizontally polarized linear, and $\pm$ 45° polarized linear components. Each of these measurement techniques will be described in the following paragraphs.

C-2 ROTATING ANTENNA POLARIMETER

The simplest, least expensive, and possibly the most accurate system for measurement of polarization is that of the rotating linear antenna. This system consists of a linearly polarized antenna followed by a receiver channel and an amplitude detector. The antenna is rotated in a plane containing its aperture and the plane normal to the direction of propagation of wave being measured. By recording the maximum and minimum amplitudes and taking their ratio, one obtains the axial ratio. The tilt angle is obtained by measuring the space angle which the direction of the maximum makes with the horizontal. Sense of rotation, however, cannot be obtained from these measurements. Kraus (3) and Bohnert (4) have described methods for using this technique.

The rotating antenna approach has the advantage of using a single receiver channel. Because of the simplicity of the system and lack of requirements for specialized components, it can easily be assembled from ordinary, off-the-shelf components, making this system relatively inexpensive. This approach has the disadvantage that it cannot be used for instantaneous polarization measurements. The frequency response of the system is limited by the rate at which the antenna is physically rotated. While electronic techniques for producing the effect of rotating the antenna are feasible, they are in general narrow band and unless performed at intermediate frequency, severely limit the tuning range of the equipment. If this function is performed at intermediate frequency, the portion of the receiver prior to the processing must be comprised of carefully phase and amplitude matched channels.

C-3 X-Y LISSAJOUS DISPLAY TECHNIQUES

A commonly used method of instantaneous polarization measurement consists of using horizontally and vertically polarized linear antennas. The outputs of these two antennas are downconverted by a common local oscillator and are fed through identical intermediate frequency amplifiers to provide outputs of sufficient amplitude for direct application to the inputs of an X-Y oscilloscope. The resulting Lissajous pattern traced out on the oscilloscope face is a direct representation of the polarization ellipse and the tilt angle and axial ratio may be measured directly from the ellipse. The direction of rotation of the ellipse cannot be obtained directly, but must be determined by further resolving the vertical and horizontal components into right and left-hand circular components and comparing the amplitudes to determine the sense of rotation.

While the approach provides a display which is most simply and directly related to the parameters normally measured, it has the disadvantage that instantaneous polarization can only be measured by taking a sequence of photographs of the oscilloscope face and measuring from this output. It has essentially the same requirements for amplitude and phase matching of the intermediate frequency amplifier channels as other approaches which are capable of providing the desired parameters in analog form as an output.

C-4 POLARIZATION PARAMETER COMPUTATION FROM CIRCULARLY POLARIZED COMPONENTS

The outputs of two opposite sense circularly polarized antennas are rather simply related to the polarization parameters. For this reason it is attractive to use these parameters to directly compute the polarization parameters. The axial ratio ρ is

$$\rho = \frac{|E_R| + |E_L|}{|E_R| - |E_L|} .$$

The tilt angle of the ellipse, τ, is $\tau = -\phi/2$. ϕ is the phase of the left-hand circularly polarized component E_L with reference to the phase of the right-hand circularly polarized component E_R, and the third parameter, the sense of rotation, is right if E_R is greater than E_L and left if E_R is less than E_L. A polarimeter using this method of measurement uses either two circularly polarized antennas of opposite sense or two linearly polarized antennas in conjunction with a quadrature hybrid to resolve the linearly polarized antenna outputs into right and left-hand circular components. These right and left-hand circular outputs are then downconverted using a common local oscillator and are amplified in carefully phase- and amplitude-matched receiver channels. The outputs of these channels are detected to provide the necessary amplitude components for computation of the axial ratio. They are also phase compared in order to determine the tilt angle τ.

As previously mentioned, this approach requires carefully matched receiver channels, both in amplitude and phase. The error in tilt angle due to phase tracking errors in the channels is equal to half the tracking error. Errors in amplitude tracking contribute to axial ratio errors, with the largest error resulting when the right and left-hand circularly polarized components are nearly equal since the difference of the right and left-hand components approaches zero. This error rapidly decreases as the axial ratio decreases and goes to zero at unity axial ratio. Amplitude tracking errors also result in inability to determine the sense of rotation for very high axial ratios.

C-5 COMPUTATION FROM AMPLITUDES ONLY

Another attractive approach to the computation of polarization parameters utilizes both senses of circular polarization, vertical and horizontal polarization, and two additional linear components of polarization which are displaced from the vertical and horizontal by 45°. Using these quantities, it is possible by manipulation to determine all of the parameters using amplitudes only. While the requirement for amplitude tracking still exists, the requirement for phase tracking is completely eliminated in the system. Such an approach was utilized by Allen, Olin, and Queen (5,6) of NRL using a unique antenna system for polarization resolution. A similar approach using linearly polarized antennas in conjunction with external processors to resolve the desired polarization outputs has recently been reported by Shnitkin (7) of Maxson Electronics. This latter approach differs slightly from the previously reported one in that it uses log IF's for the various channels, thus making possible the computation of the various parameters by addition and subtraction only of varying DC voltages.

While these approaches using amplitude only have the advantage that phase shift through the channels is unimportant, they require six receiver channels which are matched in amplitude response over the dynamic range in question. Also, the computation of the desired parameters is much more complex than that involved using the circularly polarized components only, which are more simply and directly related to the desired parameters. The receiver

expense of the amplitude-only approach is probably comparable to that of the approach using circular polarization only. However, the processing expense using amplitudes only is considerably greater and it is more difficult to perform an error analysis because of the more complex computations, and the additional channels involved.

C-6 RELATIVE PERFORMANCE AND COST

It is of primary interest to compare the performance and cost of the several approaches to polarization parameter measurements. In making this comparison, it should be emphasized that the rotating antenna approach is not suitable for making instantaneous measurements.

Table C-1 is a comparison of the four approaches described. The errors are estimated using limit specification of commercially available hardware. Typical performance could be expected to be better by at least a factor of two except for the rotating antenna case, where it is assumed that errors are calibrated out.

The cost estimates are based on the assumption of an existing design using standard commercially available conponents and requiring only the ordinary alignment and checkout procedures. No non-recurring engineering costs are included.

TABLE C-1

Comparison of Polarimeter Methods

Approach	ρ = 10 dB		ρ = 5 dB		No. of Channels	Approx. Cost
	$\Delta\tau$	$\Delta\rho$ dB	$\Delta\tau$	$\Delta\rho$ dB		
Rotating Antenna	$\pm 2°$	± 0.2	$\pm 5°$	± 0.2	1	$10 k
X-Y Lissajous	$\pm 3°$	± 1	$\pm 5°$	± 1	2	20 k
CP Amplitude-Phase	$\pm 5°$	± 1.5	$\pm 5°$	± 0.7	2	25 k
6-component amplitude only	$\pm 5°$	± 1.5	$\pm 8°$	± 0.7	6	30 k

APPENDIX D

ANNOTATED BIBLIOGRAPHY

D-1 Introduction to the Bibliography

This bibliography is an addition to that provided in AFAL-TR-72-165 and is similarly arranged. Section D-2 is an annotated bibliography arranged alphabetically by the last name of the principal author. Section D-3 is a subject index cross-referenced to the entries in Section D-2.

D-2 Bibliography

1. Adams, A. T. and Lumjiak, Chaikomal, "Optimization of the Quadrifilar Helix Antenna," IEEE Transactions on Antennas and Propagation, AP-19, No. 4, pp. 547-548 (July 1971).
   Data and descriptions. Axial ratio vs. frequency on axis.

2. Adams, A.T., and Mendelovicz, E., "The Near Field Polarization Ellipse," IEEE Transactions on Antennas and Propagation, AP-21, No. 1, pp. 124-126 (Jan. 1973).
   Polarization in near field of antenna analysis. Shown to be a planar ellipse.

3. Allen, P. J. and Olin, I. D., "A Four-Component Polarization Resolver for Microwaves," N.R.L. Memorandum Report 1086, April 1960.

4. Atia, A. E. and Mei, K. K., "Analysis of Multiple-Arm Conical Log-Spiral Antennas," IEEE Transactions on Antennas and Propagation, AP-19, No. 3, pp. 320-331 (May 1971)
   Analysis and computations. Curves from which axial ratio off-axis may be found.

5. Bohnert, J. I., "Measurements on Elliptically Polarized Antennas," Proc. of the IRE, 39, No. 5, pp. 549-552 (May 1951).

6. Bolster, M. F., "A New Type of Circular Polarizer Using Crossed Dipoles," IRE Transactions on Microwave Theory and Techniques, MTT-9, No. 5, pp. 385-388 (Sept. 1961).
Describes use of unequal length to get circular polarization. Curves given.

7. Breithaupt, R. W., "Cross Polarization In a General Two Plate Polarizer," IEEE Transactions on Microwave Theory and Techniques, MTT-15, No. 1, pp. 55-56 (Jan. 1967).
Analysis and curves.

8. Bridge, W. M., "Cross Coupling in a Five Horn Monopulse Tracking System," IEEE Transactions on Antennas and Propagation, AP-20, No. 4, pp. 436-442 (July 1972).
Analysis of polarization error. Measurements and computer results. Serious polarization errors and polarization induced instabilities shown.

9. Buizert, H., "Circular Polarization at Millimeter Waves by Total Internal Reflection," IRE Transactions on Microwave Theory and Techniques, MTT-12, No. 4, pp. 477-478 (July 1964).
Technique description.

10. Bullock, L. G.; Oeh, G. R.; and Sparagna, J. J., "An Analysis of Wideband Microwave Monopulse Direction Finding Techniques," IEEE Transactions on Aerospace and Electronic Systems, AES-7, No. 1, pp. 188-203 (Jan. 1971).
See also Report No. PRE-748, GTE Sylvania; and Electronic Warfare, (Winter/Spring 1972) for additional materi l by these authors on DF systems in general.

11. Burrows, M. L., "On the Composite Model for Rough-Surface Scattering," IEEE Transactions on Antennas and Propagation, AP-21, No. 2, pp. 241-243 (Mar. 1973)
Includes some discussion of polarization effects.

12. Chen, C. C., "Turnstile Antenna for Space Communications," 1966 IEEE Intl. Conv. Record, Part 4, pp. 120-130.
Discusses polarization properties, includes measured data.

13. Chen, M. H. and Tsandoulas, G. N., "A Wide-Band Square-Waveguide Array Polarizer," IEEE Transactions on Antennas and Propagation, AP-21, No. 3, pp. 389-391 (May 1973).
Description, data.

14. Chu, T. S. and Kuoyoumjian, R. G., "An Analysis of Polarization Variation and its Application to Circularly-Polarized Radiators," IRE Transactions on Antennas and Propagation, AP-10, No. 2, pp. 188-192 (Mar. 1962).
General polarization vs. direction theorems for antennas.

15. Chu, T. S. and Turrin, R. H., "Depolarization Properties of Offset Reflector Antennas," IEEE Transactions on Antennas and Propagation, AP-21, No. 3, pp. 339-345 (May 1973).
Analysis and measurements. Beam offset due to polarization found along with relatively large cross polarized lobes.

16. Cox, R. M. and Rupp, W. E., "Circularly Polarized Phased Array Antenna Element," IEEE Transactions on Antennas and Propagation, AP-18, No. 6, pp. 804-807 (Nov. 1970).
Dipole-slot combination to provide circular polarization. Patterns and axial ratio.

17. Deschamps, G. A. and Dyson, J. D., "The Logarithmic Spiral in Single-Aperture Multimode Antenna System," IEEE Transactions on Antennas and Propagation, AP-19, No. 1, pp. 90-96 (Jan. 1971).
Multiarm spiral analysis and measured data, patterns, and axial ratio. Includes higher order modes.

18. Dyson, J. D., "Investigation of Some Electrically Small Conical Log-Spiral Antennas," Univ. of Illinois, Antenna Laboratory, Report No. ESD-TR 71-297 (Sept. 1971) AD 736 500.
Polarization vs. angle measurements by the rotating linear dipole method at several frequencies.

19. Fessenden, D. E., "Space Wave Field Produced by a Vertical Electric Dipole above a Perfectly Conducting Sinusoidal Ocean Surface," IEEE Transactions on Antennas and Propagation, AP-21, No. 1, pp. 32-37 (Jan. 1973).
Analysis. Predicts a depolarization term.

20. Haas, H. W., et. al., "Circularly Polarized Dual Frequency Fan Beam Array and Hemispherical Coverage Antennas for Satellite Tracking," New Mexico State Univ., Report No. CR-14 (Aug. 1964). AD 607 988. Description, polarization patterns.

21. Hass, Finn A., "The Modified Short Backfire Antenna-Final Technical Report," Technical Univ. of Denmark, Report No. S204,R92 (Aug. 1971) AD 732 506. Description, comparison with other antenna types. Brief discussion of polarization. Theory and measurements.

22. Kantor, I. J.; Rai, D. B.; and de Mendonca, F., "Behavior of Polarization of Downcoming Radio Waves Including Transverse Magnetic Field," IEEE Transactions on Antennas and Propagation, AP-19, No. 2, pp. 246-254 (Mar. 1971).
Analysis and computations for satellite to ground path.

23. Knott, E. F. and Senior, T. B. A., "Cross Polarization Diagnostics," IEEE Transactions on Antennas and Propagation, AP-20, No. 2, pp. 223-224 (Mar. 1972).
Analysis of returns from axially symmetric objects. Data on cross polarized RCS of several objects.

24. Kraus, J. D., "Antennas," McGraw-Hill Book Co., Inc., New York (1950).

25. Leader, J. C., "The Relationship Between the Kirchoff Approach and Small Perturbation Analysis in Rough Surface Scattering Theory," IEEE Transactions on Antennas and Propagation, AP-19, No. 6, pp. 786-787 (Nov. 1971).
Theory, depolarization is crucial in the arguement. Comments on above paper contained in AP-20, No. 4 (July 1972) by G. R. Valenzuela (pp. 536-537) and J. W. Wright (p. 537) Reply by author (pp. 537-539).

26. Lee, Shung W., "Aperture Matching for an Infinite Circular Polarized Array of Rectangular Waveguides," IEEE Transactions on Antennas and Propagation, AP-19, No. 3, pp 332-342. (May 1971).
Analysis and computations for iris matching techniques. Best polarization not necessarily occurring with minimum reflection.

27. Ludwig, Arthur, C., "The Definition of Cross Polarization," IEEE Transactions on Antennas and Propagation, AP-21, No. 1, pp. 116-119 (Jan. 1973). Three definitions given, none identical with that used in this report. Discussion of reflector cross polarized patterns.

28. Olin, I. D. and Queen, F. D., "Measurements Using a Polarization Instrumentation Radar on Navigational Buoys," NRL Report 5701 (1961).

29. Orejuela, H. and Myers, G. A., "Theory and Performance of a Matched-Filter Doppler Direction-Finding System," U.S. Naval Postgraduate School, Report No. NPS-52MV70091A (Sept. 1970) AD 720 699. Description, measurements.

30. Pfendtner, Felix, "Adaptive Angle Tracking and Correlation for Airborne Direction Finding," U.S. Naval Postgraduate School, Thesis, (Mar. 1971) AD 722 564.
Multiple emitter system is described.

31. Raburn, L. E., "The Calculation of Reflector Antenna Polarized Radiation," IRE Transactions on Antennas and Propagation, AP-8, No. 1, pp. 43-49 (Jan. 1960). Procedure for computing cross-polarized patterns for focused but not necessarily symmetrical paraboloid is given.

32. Reuss, M. L., Jr., "A Nonreciprocal Circular Polarizer," IEEE Transactions on Microwave Theory and Techniques, MTT-15, No. 1, pp. 37-41 (Jan. 1967). Device description and data.

33. Saunders, M. J., "Cross Polarization at 18 and 30 GHz Due to Rain," IEEE Transactions on Antennas and Propagation, AP-19, No. 2, pp. 273-277 (Mar. 1971).
Theory of conversion from horizontal to vertical polarization. Computations.

34. Silberberg, R. W., "The 'Paradisc' Antenna-A Novel Technique to Improve the Axial Ratio of a Circularly Polarized High Gain Antenna System," IEEE Transactions on Antennas and Propagation, AP-21, No. 1, pp. 108-110 (Jan. 1973).
Description and measurements.

35. Shnitkin, Harold, "Instantaneous Polarization Measuring Antenna System," 21st Annual Symposium on USAF Antenna Research and Development (12-14, Oct. 1971).

36. Turley, D. G., "Electromagnetic Polarization: Representation and Measurement-Special Report," Smyth Research Associates, Report No. SRA-343 (June 1963).
Overall discussion.

37. Vartanesyan, V. A.; Goikhman, E. Sh.; Rogatkin, M. I., "Radio Direction Finding," Army Science and Tech. Center, FSTC HT 23828 70 (Feb. 1970) Translation. AD 714 509.
Complete section on propagation. Polarization errors discussed. Primarily ground based at VHF and below.

38. Watson, P. A. and Ghobrial, S. I., "Cross Polarizing Effects of a Water Film on a Parabolic Reflector at Microwave Frequencies," pp. 668-671 (Sept. 1972).
Analysis, very small effect predicted.

39. Watson, P. A., and Ghobrial, S. I., "Off-Axis Polarization Characteristics of Cassegrainian and Front-Fed Paraboloidal Antennas," IEEE Transactions on Antennas and Propagation, AP-20, No. 6, pp. 691-698 (Nov. 1972).
Analysis predicts Cassegrain superior at least when symmetrical.

40. Young, Leo, et. al., "Meander-Line Polarizer," IEEE Transactions on Antennas and Propagation, AP-21, No. 3, pp. 376-378 (May 1973).
Description and data.

D-3 Subject Cross References

Depolarization: 11, 19, 22, 23, 25, 34, 38

DF Systems, General: 10, 29, 30, 37

Polarization, Effect on DF: 8, 15, 37

Polarization, General: 27, 36

Polarization, Generation: 6, 7, 9, 13, 16, 32, 40

Polarization, Measurement: 3, 5, 24, 28, 35, 36

Polarization Properties of Antennas, Theory: 1, 2, 4, 6, 8, 12, 14, 15, 16, 17, 19, 20, 21, 26, 31, 39

Polarization Properties of Antennas, Measurements: 1, 6, 8, 12, 15, 16, 17, 18, 20, 21, 26, 35

REFERENCES

1. Morgan, Lee A. and Blackwell, Harry L., "Polarization Effects on Direction Finding Accuracy," AFAL-TR-72-165 (May 1972).

2. Collin, R. E. and Zucker, F. J. "Antenna Theory - Part I," McGraw-Hill Book Co., Inc., New York (1967).

3. Kraus, J. D., "Antennas," McGraw-Hill Book Co., Inc., New York (1950).

4. Bohnert, J. I., "Measurements on Elliptically Polarized Antennas," Proc. of the IRE 39, No. 5, pp. 549-552, (May 1951).

5. Olin, I. D., and Queen, F. D., "Measurements Using a Polarization Instrumentation Radar on Navigational Buoys," NRL Report 5701 (1961).

6. Allen, P. J. and Olin, I. D., "A Four-Component Polarization Resolver for Microwaves," NRL Memorandum Report 1086, April 1960.

7. Shnitkin, Harold, "Instantaneous Polarization Measuring Antenna System," 21st Annual Symposium on USAF Antenna Research and Development (12-14, Oct. 1971).

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The results of an investigation of techniques to minimize or correct polarization errors in Direction Finding (DF) Systems are presented. Four techniques are considered. These are "Flagging," "Switching," "Tracking," and "Polarimeter." The flagging technique senses situations when the polarization error would be unacceptably large and notifies the DF system of this fact. The switching technique senses

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the field polarization and chooses the one of two receiving polarizations which is closest. The tracking technique senses the field polarization and adjusts the receiving antenna polarization to match. The polarimeter technique measures the field polarization and computes a correction to the DF output.

The residual error after correction by these techniques is analyzed and numerical results for a phase interferometer presented. For the purposes of evaluating these residual errors, a randomly polarized field (in the sense that there is no a priori knowledge of its polarization), is assumed and the cumulative probability distribution of the errors computed. It is shown that all four techniques are capable of reducing the 95% confidence level on the errors to a value consistent with other errors in a very good DF system. The flagging technique is accompanied by a potentially serious amount of data loss and the polarimeter technique requires too much equipment complexity and cost to be acceptable for all but the most demanding applications. Both the switching and tracking techniques have sufficient error improvement and equipment simplicity to make their inclusion in high accuracy DF systems desirable. Switching is the simpler of the two and is ideally suited to a retrofit application. Tracking is the more accurate of the two and is well suited to incorporation in new designs. The ultimate in correction would be provided by a combination of the switching and polarimeter techniques.

Also included are discussions of time-of-arrival (TOA) polarization errors, effects of multipath on the corrective techniques, and methods of measuring polarization. An annotated bibliography is included.