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AMCPM-PE-E, Redstone
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EMP GENERATED WAVEFORMS
SPECIFICATIONS FOR
SUBSYSTEMS AND CIRCUITS

Contract No. DAAH01-71-C-1366

OR 11,930  15 May 1972

Prepared by:
Special Programs and
Analytical Support Group

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Material Command
attr. AMC PM-PE-E,
Redstone Arsenal Ala 35809

Martin Marietta Corporation
Orlando, Florida
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I. INTRODUCTION

This document summarizes the results of efforts performed on contract number DAAH01-71-C-1366 for the U.S. Army Missile Command.

Many military systems are required to perform in an Electromagnetic Pulse (EMP) environment caused by nuclear explosions. The environment is normally specified in terms of EMP field amplitude and duration. The designer of an individual subsystem, however, needs a technique for predicting the impact which the EMP field may have on the subsystem. Prediction capability is required for susceptibility studies, conceptual design, hardware design, and test criteria.

The predominant impact is caused by energy from the external field being coupled onto interconnecting cables of the system and presenting electrical stress to subsystem components. Adequate prediction of the coupled energy is a difficult capability to achieve. The problem is complicated by many variables encountered in system architecture and their related effect on coupling to a specified field. The cable types, lengths, shielding, enclosures, terminations, geometry and proximity to ground planes all have a major effect on the coupling mode.
II. PURPOSE

The primary objective of contract DAAH01-71-C-1366 was to develop a technique for predicting the characteristics of coupled energy on system cables having a wide range of parameters. It was also desired to present the results of the prediction technique in the form of nomographs or charts to be used by system designers. Energy coupled into complex systems generally contains many frequency components and amplitude variations. In order to facilitate design and test specification requirements, a simplified mathematical model of the major frequency and amplitude components was desirable.

The Pershing Weapon System has been tested in a simulated EMP environment during the Special Test Program (STP). A comprehensive review of the STP test data indicates that the major characteristics of coupled energy on a complex system can be described generally by the following equation of a damped oscillatory waveform.

\[ Y(t) = A e^{-Bt} \sin 2\pi Ft \]

where

- \( Y(t) \) = Current or voltage time history
- \( A \) = Amplitude factor
- \( B \) = Damping factor
- \( t \) = Time from beginning of field
- \( F \) = Frequency

The three variables \( A, B, \) and \( F \) are dependent on cable parameters.
III. TECHNICAL APPROACH

A. TEST EQUIPMENT AND CONDITIONS

Many combinations of parameters or variables must be considered in determining a prediction capability for EMP induced signal waveforms on system cables. Equations, charts, or nomographs would be desirable to assist the designer in performing rapid analysis of a specific system configuration. Equations, however, are entirely too complex to use without the aid of a computer. It was therefore decided to provide either nomographs or charts.

Either prediction aid (nomographs or charts) requires analysis of several values of each parameter so that the effect of intermediate values can be extrapolated. The Martin Marietta Corporation has developed an analytical capability for establishing effect of key parameter values by refining a previously developed computer program, CTRAN. This program employs a transmission line model that required modification. Consequently, a means for verifying the effectiveness of CTRAN is required.

Time domain data, illustrating the effect that several variations of cable parameters has on coupling characteristics, was required for verification purposes. This data may be compared directly with CTRAN predictions which are plotted in the time domain. In order to obtain the necessary empirical data, cable coupling tests were performed in a simulated EMP environment provided by a radiating long wire antenna facility.

All tests were conducted at a location that was on geometric center line with the field and at a distance of 650 feet. The long wire antenna is approximately 1000 feet long and 46 feet above the earth. Due to time limitations, all tests were performed with cables oriented parallel to the antenna only.

Two shielded terminal boxes were employed to house the boxes containing the terminating impedance. One terminal box was located on center line (west box), the other located 50 feet east of center line (east box). Transmission lines longer than 50 feet were not terminated at the east end. All measurements were taken at the west end.

Shielded twisted pair and coax (type RG-58/U) were used in the tests. In cases where single conductor is indicated, all shields and conductors are shorted together at both ends.

The ground plane used consisted of two 100-foot strips of 48-inch wide hardware cloth (1/4 in. mesh). The two 100-foot strips were overlapped approximately 4 inches for their entire length. Both strips were joined together at 2-foot intervals and grounded to a 1-inch wide copper braid strap at each end. The copper braid straps were buried 4 feet deep in the earth. The ground plane overlapped both terminal boxes by approximately 6 feet. In cases where 100-foot cables were used, the ground plane was extended to the full 100-foot length.
The following cables were used in the test:

- 50-foot single conductor
- 100-foot single conductor
- 50-foot coax RG-58/U
- 100-foot coax RG-58/U
- 50-foot shielded twisted pair
- 50-foot parallel transmission line - horizontal to earth.

Tests were conducted with the above cables in several configurations. The conditions for tests using cables parallel to the antenna were as follows:

- on earth
- on a ground plane
- 3 inches above earth
- 20 inches above earth
- 20 inches above ground plane
- 70 inches above ground plane.

The source of the pulsed electromagnetic field is a horizontal 1000-foot tapered long wire antenna resistively loaded so that an electromagnetic field pulse is propagated from the antenna when spark gap switching occurs. The long wire antenna consists of two halves of 24 resistance loaded sections tapering from a 12-inch diameter to a 4-inch diameter. The antenna, oriented axially east and west, is suspended 46 feet above the earth's surface by high voltage insulators between telephone poles. The aluminum discs are corona rings placed between the antenna sections on each side of the resistor blocks. Two adjustable voltage power supplies with a rating of 250 kV maximum voltage of opposite polarity with respect to the ground are used to feed the antenna through the end resistor feeder lines. The pressurized spherical gap assembly is continuously purged with an industrial nitrogen gas mixture and can be adjusted from 1 to 20 atmospheres with the voltage range across the gap from 30 to 330 kV. The pulsed electromagnetic field from the antenna has the E field horizontally polarized, an adjustable rise time, and an adjustable pulse repetition rate. The east and west control stations contain the current and pulse repetition rate monitors and the power supply controls.

The parameter variation effects on coupling the electromagnetic energy with the specimen transmission lines were determined while varying the following:

1. Geometric parameters of the transmission line (wire size, separation between conductors, length, etc.)

2. Terminal impedances $Z_1$ and $Z_2$ (Figure 1)
3. Orientation of transmission line with respect to the ground plane
4. Height of transmission line above the ground plane.

![Diagram of transmission line configuration](image)

Figure 1. Two-Wire Configuration Used In the Model
(Assumes Ground Plane = Earth Plane)

B. TRANSMISSION LINE MODEL

The transmission line model used has been previously derived (Reference 1). The model, however, was developed for use on a two-wire uniform transmission line pair. Since our problem involves a single uniform transmission line, it was necessary to employ the method of images to complete the line pair (Figure 1).

The dipole mode current, \( I_D \), induced on the cable has an equal image dipole current, \( I_{Di} \), with orientation as shown in Figure 1. These currents can be modeled as the transmission line mode current in the lossy transmission line model previously derived (Reference 1). The cable and cable image are the conductor pair in the lossy transmission line model. The transmission line characteristic impedance, \( Z_0 \), is that of the cable and its image or, equivalently, twice the characteristic impedance of the cable with respect to the ground plane.

The characteristic impedance, \( Z_0 \), is computed directly from the distributed line parameters using the equation:

\[
Z_0 = \sqrt{\frac{\text{Distributed Series Impedance}}{\text{Distributed Shunt Admittance}}} = \sqrt{\frac{R+jwL}{G+jwC}}
\]

The distributed parameters of the transmission line above the ground plane or sufficiently far above the earth are obtained from the following series R-L and shunt G-C expressions:

\[
\text{Resistance} = R = \frac{1}{\pi A^2 \sigma_c} \quad \text{(ohms/meter) for } f \leq 60 \text{ Hz} \tag{1A}
\]

\[
= \frac{3.31 \times 10^{-8} \sqrt{\frac{\omega}{A}}}{\frac{\omega}{A}} \quad \text{(ohms/meter) for } f > 60 \text{ Hz} \tag{1B}
\]

\[
\text{Inductance} = L = \frac{\mu}{\pi} \ln \left( \frac{2H}{A} \right) \quad \text{(henry/meter)} \tag{2}
\]

\[
\text{Conductance} = G = \frac{\pi \sigma_A}{\ln \left( \frac{2H}{A} \right)} \quad \text{(mhos/meter)} \tag{3}
\]

\[
\text{Capacitance} = C = \frac{\pi \epsilon}{\ln \left( \frac{2H}{A} \right)} \quad \text{(farads/meter)} \tag{4}
\]

where:  
- A is conductor radius
- H is height of conductor above ground plane
- \(\sigma_c\) is conductivity of conductor
- \(\sigma_A\) is conductivity of air
- \(\mu\) is permeability of air
- \(\epsilon\) is permittivity (dielectric constant)

The following current expressions from Reference 1 are used as the basic coupling equations:

\[
I_B^1(\omega) = \frac{j \omega b B(\omega)}{2 \gamma Z_0} T_2 \left[ \frac{1 - \rho_1 e^{-2\gamma s} - \rho_2 e^{-2\gamma s}}{1 - \rho_1 \rho_2 e^{-2\gamma s}} \right] \tag{5}
\]

\[
I_E^1(\omega) = \frac{-b E(\omega)}{2 \gamma Z_0} T_2 \left[ \frac{1 + \rho_1 e^{-2\gamma s} - \rho_2 e^{-2\gamma s}}{1 - \rho_1 \rho_2 e^{-2\gamma s}} \right] \tag{6}
\]

where:
- \(Z_0\) characteristic impedance of transmission line
- \(\gamma\) propagation constant
- \(b\) conductor pair separation
The transmission line model and method of images employed to predict the current induced on a cable above the ground plane have been experimentally verified. The assumptions made for the transmission line model are violated near the earth. Therefore, additional effort needs to be expended to determine the correct model to use for the cable above the earth plane. The damped sinusoid prediction and design aids given in this report are to be used as bounds for design and tests. Use of these design aids should lessen considerably the probability of failing a cabled system test under a threat environment.

C. PROGRAM CTRAN

Program CTRAN was developed specifically for the model used. It utilizes complex arithmetic and operates in both the time and frequency domains. Program CTRAN requires transmission line distributed characteristics, terminal parameters and a time domain field definition as inputs. These inputs are transformed into the frequency domain where the model of Dr. Whitescarver is applied. All key parameters, calculated by the model, are stored on magnetic tape and are available for output. The induced current is transformed back into the time domain and is printed and plotted. Figure 2 is a block diagram of Program CTRAN. Resultant plots of representative cases are presented in Appendix B.

Line characteristics required are: type of conductor, radius of conductor, height above ground plane and length of the line. Internally, the program determines the distributed line parameters and the characteristic line impedance based on these inputs.

Terminal impedance definition is accomplished for each terminal by the input of "terminal structure" (integer corresponding to one of seventeen possible configurations presently available: e.g., L, R, C in parallel, LRC in series, etc.), and the values of the inductance (henrys), the resistance (ohms) and the capacitance (farads). Internally, the program determines the respective terminal impedances based on these inputs.

The field is defined by a "ROSCHAR DECK" which is a list of X-Y coordinates of points on a time plot of the field. This list of points must include all major peaks of the field in order that it can be properly described. Internally, the program interpolates the data and transforms it into the frequency domain for use by the model.
Figure 2. Frequency Domain Computer Program, CTRAN
The model section of the program defines and computes all the key variables exactly as they are defined in Dr. Whitescarver's model. The propagation constant, reflection coefficients, transmission coefficients and transfer functions are all computed for each of 4096 values of frequency. The capability exists to print out any or all of these parameters. The induced current is transformed to the time domain and is both printed and plotted.

The model has been checked out and is presently being used to predict the effects of conditions not empirically defined. It utilizes either the IBM 360/65 or IBM 370/155 computer and the CALCOMP Model 763 Plotter. Three scratch discs or tapes and 320K bytes of memory are required. Each individual run requires approximately 1 minute of CPU time.
IV. PREDICTION AND DESIGN AIDS

Program CTRAN was used to determine the coupling waveforms for variations in transmission line parameters. The computer plot results were analyzed for damped sinusoid parameters. The parameter variations were analyzed manually since a technique for automated analysis has not yet been incorporated in the CTRAN program.

CTRAN requires a description of the EMP field as an input. A typical field was assumed to have a peak amplitude of 1 ampere/meter, rise time of 10 nanoseconds, decay time of 300 nanoseconds, and a duration of 550 nanoseconds at zero level (Figure 3).

The variations of peak current amplitude coupled into single transmission lines greater than 10 feet long as a function of terminating impedance and height above ground is summarized in Figure 4. The length of a cable affects frequency and damping factor of the induced signal. However, cable lengths in excess of 10 feet have no significant effect on peak amplitude. Cable lengths shorter than 10 feet for the assumed field parameters modify the peak amplitudes given in Figure 4 by a ratio of the effective length to 10 feet (e.g., a 5-foot transmission line having a unity propagation factor will have a signal induced with a peak amplitude of approximately 5/10 or 50 percent of the peak amplitude indicated in Figure 4). This effect of short cables is due to the rise time of 10 nanoseconds for the assumed field as well as the fine structure of the field and a propagation factor for the sample transmission line of approximately 1. Faster rise times for the field would have a greater effect on shorter cables.

Since CTRAN does not completely model all of the coupling effects and since there are some uncertainties in the results, it is recommended for conservative designs to add 50 percent to the peak amplitude values given in Figure 4.

The frequency of the induced current waveform is directly related to the length of the transmission line. For a propagation factor of unity, the time duration of each cycle is approximately 2 or 1 nanoseconds for each foot of effective length, depending on the far end terminating impedance. The effective length is twice the actual length since the induced waveform is the resultant primarily of the reflection and transmission coefficients operating upon the incident transient field.

For the conditions $Z_2 > Z_0$ at the end of the cable opposite the measurement, the frequency, $F$, of the damped sinusoid may be determined by the following equations:

$$F (\text{Hz}) = \frac{1}{2\tau} \text{ for } Z_2 > Z_0$$  \hspace{1cm} (7)

$$F (\text{Hz}) = \frac{1}{4\tau} \text{ for } Z_2 < Z_0$$  \hspace{1cm} (8)

where $\tau$ is the propagation time from end to end of cable in seconds.
For the condition $0 < Z_1 < Z_0$ at the end of the cable opposite the measurement, the frequency, $F$, of the damped sinusoid may be determined by the following equation:

$$F (Hz) = \frac{1}{4\tau} \text{ for all } Z_2 \neq Z_0.$$  \hspace{1cm} (9)

For the conditions $Z_1 = Z_0$ and/or $Z_2 = Z_0$ the output is not periodic and the damped sinusoid does not apply. This possibly explains why some Pershing Weapon System data cannot be modeled by a damped sinusoid.

For the conditions $Z_1 = Z_2 = 0$ the current is the driving field multiplied by a constant; consequently, the damped sinusoid does not apply.

The determination of the frequency is summarized in the frequency matrix in Table I. Table I is reprinted in Appendix A to be used as a design aid.

### TABLE I

**Frequency Matrix**

<table>
<thead>
<tr>
<th>$Z_1$ (Far-End Termination)</th>
<th>0</th>
<th>$0 &lt; Z_1 &lt; Z_0$</th>
<th>$Z_0$</th>
<th>$&gt; Z_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_2$ (Near-End Termination)</td>
<td>0</td>
<td>NA (1)</td>
<td>$F = \frac{1}{4\tau}$</td>
<td>NA (3)</td>
</tr>
<tr>
<td>$0 &lt; Z_2 &lt; Z_0$</td>
<td>NA (1)</td>
<td>$F = \frac{1}{4\tau}$</td>
<td>NA (2)</td>
<td>$F = \frac{1}{4\tau}$</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>NA (2)</td>
<td>NA (4)</td>
<td>NA (2)</td>
<td>NA (4)</td>
</tr>
<tr>
<td>$&gt; Z_0$</td>
<td>NA (1)</td>
<td>$F = \frac{1}{4\tau}$</td>
<td>NA (2)</td>
<td>$F = \frac{1}{2\tau}$</td>
</tr>
</tbody>
</table>

NA - Damped sinusoid does not apply

(1) - Output = driving field times a constant

(2) - Output = driving field convolved with $B(t-2\tau)$

(3) - Output = driving field convolved with $B(t-\tau)$

(4) - Output = driving field convolved with $B(t-\tau)$ and $B(t-2\tau)$

The damping factor of the induced current waveform is related to termination impedance, length of transmission line and height above ground. Figure 5 illustrates the damping factor for several lengths of cables having zero ohms terminating impedance. The values, however, do not have a consistent pattern with respect to cable length. This is probably due to the effects of the fine structure of the field time domain waveform. Therefore the dashed line at the bottom of Figure 5 is considered worst case and recommended for application to all cable lengths terminated in zero ohms.
Figure 6 illustrates the damping factors for several lengths of cables terminated in zero ohms. Cables shorter than 20 feet have very high damping factors; therefore, the factors for the 20-foot cable may be used as worst case values for the shorter cables. Also, as the terminating impedance approaches the characteristic impedance of the cable (300-1000 ohms), the damping factor approaches very high values (i.e., the incident pulse and the first reflected pulse are the only two pulse components of significant amplitude). Effects of intermediate impedance values between 0 and 100 ohms may be interpolated.

Example of Using Plots

Figures 4, 5, and 6 are reprinted in Appendix A to be used as design aids. Determine the following parameters for the cable:

1. Length (e.g. 40 feet)
2. Diameter (e.g. 1/32 in.)
3. Height above ground plane (e.g. 30 in.)
4 Field strength (e.g. 10 amps/meter)

5 Termination resistance (e.g. 50 ohms)

Use Figure 4 to determine the peak amplitude basis for a 1 amp/meter field. Interpolate for impedance and height. The peak amplitude in this case is found to be approximately 0.75 amps for the 1 amp/meter field. Therefore, for a 10 amp/meter field the peak amplitude will be 7.5 amps. The value of peak amplitude to be used for this example cable is 7.5 amps.

Damping Factor

Referring to Figure 6 the damping factor is interpolated between the 30-foot and 50-foot curves. The value of damping factor for this cable is approximately $6 \times 10^6$ if it were terminated in 100 ohms. Since the terminating impedance is actually 50 ohms, the damping factor should be about $3 \times 10^6$ (i.e., about half of the difference between the 0 ohm and 100 ohm value).
Assuming the propagation factor for the cable is unity, the propagation time per cycle of excitation is 160 nanoseconds \((4\tau)\) for a 40-foot cable that is unterminated at the far end. Using Table I in Appendix A, this represents a frequency of approximately 6 MHz \((1/4\tau)\).
V. LIMITATIONS

A. PREDICTION MODEL LIMITATIONS

Present limitations to the transmission line model involve minimum line length, minimum height above the ground plane and the assumption that the earth and electrical ground planes coincide.

The first limitation is not truly a limitation of the model but rather a limitation of the output analysis capability. Since the duration of the driving field (\( \approx 600 \text{ ns} \)) is longer than the time required for the induced current to travel the length of the wire (\( T = S \text{ ns} \) where \( S \) is line length in feet), the model must convolve the effects of the field. The model, as well as the computer program, will accurately handle all line lengths. However, for short line lengths (\( S \leq 20 \delta T \)), interpolation between successive computer runs is not sufficiently accurate. This is caused by the self-interfering nature of the fine structure of the driving field (see Figure 3). For greater line lengths fine structure does not significantly affect output and interpolation can be made between different run parameters (e.g., height, terminal impedances, length, etc.)

The second limitation is inherent in both the computation of line parameters and the derivation of the model itself. The restriction is that height must be much greater than the radius of the conductor. For purpose of this contract, a conductor radius of \( \frac{1}{16} \) inch was assumed. All computed measurements were made at heights above 5 inches to ensure that this restriction was not violated. It is felt, however, that heights between 1 and 5 inches can be tested with acceptable accuracy.

Presently, the computation of input line parameters is restricted to problems where the electrical ground plane is located on the earth's surface. The more general case of a "buried" ground plane does not effect the coupling equations (equations 5, 6) or the distributed resistance equation (equation 1). However, there is a significant effect on distributed inductance, conductance and capacitance (equations 2, 3, 4). A possible modified configuration is shown in Figure 7.

Provisions must be made to account for differences in conductivity and permittivity between earth and air. Also the equivalent separation distance (between the wire and its image) must be determined. At present, there is no mathematical model to handle these alterations.

Additional effort is required to thoroughly investigate and eliminate the above limitations. The present ambiguities concerning very short wires can be eliminated either by extensive investigation into the analytic effects that field fine structure has on induced current or by establishing best and worst case limits on the problem by repeated runs of the computer program for very short line lengths. The most meaningful solution would be a thorough analytic investigation verified by computer runs of selected cases. The minimum height restraint is not believed to be a significant practical
limitation. Empirical data indicates that the induced current approaches zero as the height above ground plane approaches zero. Therefore, a 1 inch limitation on minimum height should not present any problems. This limitation does not exist for any separation between the earth and true ground planes greater than 1 inch. Non-coincident ground and earth planes will require additional modeling and investigation of either the effects on or the changes to the model parameters. This may necessitate revision of the present method of computing characteristic impedance and the development of an entirely new model. As a minimum it will be necessary to thoroughly investigate the effects of non-uniform or multi-layer dielectric and the development of a means of precisely locating the electrical ground plane.

B. AMPLITUDE LIMITATIONS

The resultant peak amplitude for fields having intensities other than 1 amp/meter may be found by linearly extrapolating to the field. Peak amplitudes are given in Figure 4 for cable lengths greater than 10 feet. Cables shorter than 10 feet have peak amplitudes that may be determined by linearly extrapolating the ratio of lengths to the 10-foot value. Values of peak amplitude for cables having terminating impedances other than those given may be determined by extrapolating. Uncertainties due to non-linear characteristics of response should be biased in favor of the maximum value. Peak amplitudes given in Figure 4 are not effected significantly by cable diameter. The values given in Figure 4 are for cables unterminated at the far end which is worst case. The same values may be applied to cables that are terminated at the far end.
C. DAMPING FACTOR LIMITATIONS

Damping factors are given in Figure 5 for cable lengths of 20, 50, and 100 feet and terminated with 0 ohms impedance. The damping factor may also be applied to cables shorter than 20 feet as a worst case value. Damping factors are given in Figure 6 for cable lengths of 20, 30, 50, and 100 feet and terminated with 100 ohms impedance. Damping factors for intermediate lengths may be determined by linear interpolation without significant error. However, the smallest estimates should be used for worst case values. Similarly, estimates may be made to determine damping factors for cables terminated in impedances other than 0 or 100 ohms. The damping factor becomes very large as the value of terminating impedance approaches the characteristic impedance of the cable. Therefore, a practical limit of 10 should be used for terminations greater than 100 ohms.

The damping factors, however, cannot be determined precisely due to the following reasons:

1. Waveform damping does not behave exactly exponentially

2. Selection of peak amplitudes is performed manually to determine the damping factor. (Errors may be caused by the particular selection of peaks.)

3. Certain combinations of transmission line parameters cause sharp peaks in the early time waveforms due to the fine structure in the field time domain waveform. (The sharp peaks are frequently rounded off in later time data in some cases.)

The variations obtained in determining damping factors for specific cases is not regarded as a significant problem however. In cases where terminating impedances are 10 ohms or more, the damping factor is such that the energy in successive peaks decreases rapidly. The energy in the first few peaks which have the most significant amount of energy are least affected by the errors in the damping factor value. In cases where interpolation is required, the least damping factor should be selected for application since the maximum energy in the waveform would be represented.
VI. CONCLUSIONS

The prediction procedure developed provides the designer with worst case limits for the parameters of a damped sine wave representing the current induced on a resistively terminated cable near a ground plane.

Mathematical expressions directly relating the induced current to the various line parameters were investigated but were found to be very complex and therefore, of questionable value as prediction tools. Consequently a parametric solution technique was developed. A computer program (CTTRAN) was developed and verified by empirical measurements. The program was used to compute induced current for a large number of transmission line conditions. Since computations for all combinations of conditions are impractical, interpolation for intermediate conditions is considered permissible. This was verified within reasonable limits by several test cases.

Program CTRAN results conformed closely to empirical test data in most cases where test data is available. The CTRAN model simulates adequately the effects of transmission lines near ground planes. Modifications to the program are required to simulate effects of cables near earth without intervening ground plane.
VII. RECOMMENDATIONS

The prediction capability provides parameter bounds for relatively simple cables and does not consider effects of earth plane, cable bundles, coax, shielding, flat ribbon conductors or conducting slabs. Also, the cable orientation considered in analysis for this task is horizontal to the earth and parallel to the field's plane wave. This is considered to be worst case orientation. Vertical or perpendicular segments of a cable may affect the overall response significantly but not severely. Additional modification to the CTRAN program is required to accommodate variations in orientation and earth effects.

It is recommended that further effort be expended to expand the transmission line model and to investigate the effects of other factors not presently modeled. These other factors include the effects of:

1. Earth and ground plane location with respect to the earth plane
2. Cable orientation with respect to the ground plane and the field's plane wave
3. Multi-cable configurations (e.g., bundled cables, coax, ribbons, etc.)
4. Field fine structure in relation to short cables
5. Cross coupling between adjacent cables and an artificial ground plane.

Of eventual concern are the problems of non-uniform configurations (such as tapered cables, non-level heights, and bent cables) and non-uniform driving fields.
<table>
<thead>
<tr>
<th>$Z_2$ (Near-End Termination)</th>
<th>$Z_1$ (Far-End Termination)</th>
<th>$0$</th>
<th>$0&lt;Z_1&lt;Z_0$</th>
<th>$Z_0$</th>
<th>$&gt;Z_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>NA (1)</td>
<td>$F = \frac{1}{4\tau}$</td>
<td>NA (3)</td>
<td>$F = \frac{1}{4\tau}$</td>
<td></td>
</tr>
<tr>
<td>$0&lt;Z_2&lt;Z_0$</td>
<td>NA (1)</td>
<td>$F = \frac{1}{4\tau}$</td>
<td>NA (2)</td>
<td>$F = \frac{1}{4\tau}$</td>
<td></td>
</tr>
<tr>
<td>$Z_0$</td>
<td>NA (2)</td>
<td>NA (4)</td>
<td>NA (2)</td>
<td>NA (4)</td>
<td></td>
</tr>
<tr>
<td>$&gt;Z_0$</td>
<td>NA (1)</td>
<td>$F = \frac{1}{4\tau}$</td>
<td>NA (2)</td>
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</tr>
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</table>

NA - Damped sinusoid does not apply

(1) - Output = driving field times a constant
(2) - Output = driving field convolved with $B(t-2\tau)$
(3) - Output = driving field convolved with $B(t-\tau)$
(4) - Output = driving field convolved with $B(t-\tau)$ and $B(t-2\tau)$
Figure 4. Peak Currents for Cables Longer Than 10 Feet In Normalized Field of 1.0 AMP/Meter
Figure 5. Ω Termination
Figure 6. 100Ω Termination
CTRAN CASE 6  (L = 100 F  H = 20 JN  R2 = 100 OHMS)

CURRENT - AMPERES

3.0 x 10^{-1}
1.0 x 10^{-1}
0.0
-1.0 x 10^{-1}
-2.0 x 10^{-1}
-3.0 x 10^{-1}
-4.0 x 10^{-1}
-5.0 x 10^{-1}
0.0 2.0 4.0 6.0 8.0 1.0 1.2 1.4 1.6 1.8 2.0
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1}

TIME - SECONDS
CTran Case 9  \( L = 100 \, \text{ft} \quad M = 5 \, \text{JN} \quad R_2 = 0 \, \Omega \)
CTRAN CASE 12  (L = 50 FT   H = 70 IN   R2 = 100 OHMS)

CURRENT - AMPERES

TIME - SECONDS
CTRAN CASE 13 (L = 50 FT  H = 20 IN  R2 = 0 OHMS)

Current - Amperes

Time - Seconds
CTRAN CASE 17 ( L = 50 FT   H = 5 IN   R2 = 0 OHMS )

CURRENT - AMPERES

-2.00 x 10^-1
-1.50 x 10^-1
-1.00 x 10^-1
-5.00 x 10^-2
0.00
5.00 x 10^-2
1.00 x 10^-1
1.50 x 10^-1
2.00 x 10^-1

TIME - SECONDS

0.00 2.00 4.00 6.00 8.00 1.00 1.20 1.40 1.60 1.80 2.00
10^-7 2.00 4.00 6.00 8.00 1.00 1.20 1.40 1.60 1.80 2.00
10^-7 10^-7 10^-7 10^-7 10^-6 10^-6 10^-6 10^-6 10^-6 10^-6 10^-6
CTRAIN CASE 23  ( L = 20 FT  H = 10 IN  R2 = 0 OHMS )

CURRENT - AMPERES

-4.00 x 10^-1
-3.00 x 10^-1
-2.00 x 10^-1
-1.00 x 10^-1
 0.00
+1.00 x 10^-1
+2.00 x 10^-1
+3.00 x 10^-1
+4.00 x 10^-1

TIME - SECONDS

0.00  2.00  4.00  6.00  8.00  1.00  1.20  1.40  1.60  1.80  2.00
10^-9 10^-7 10^-5 10^-3 10^-1 10^-9 10^-7 10^-5 10^-3 10^-1 10^-9
CTRAN CASE 28 (L = 10 FT, H = 70 IN, R2 = 100 OHMS)
CTARAN CASE 30  ( L = 10 FT  H = 20 IN  R2 = 100 OHMS )

CURRENT - AMPERES

TIME - SECONDS
CTRAN CASE 31  ( L = 10 FT  H = 10 IN  R2 = 0 OHMS )

CURRENT - AMPERES

4.00 x 10^-1
3.00 x 10^-1
2.00 x 10^-1
1.00 x 10^-1
0.00
-1.00 x 10^-1
-2.00 x 10^-1
-3.00 x 10^-1
-4.00 x 10^-1

0.00  2.00  4.00  6.00  8.00  1.00  1.20  1.40  1.60  1.80  2.00
10^-7  10^-7  10^-7  10^-7  10^-6  10^-6  10^-6  10^-6  10^-6  10^-6  10^-6

TIME - SECONDS
CTRAN CASE 32  ( L = 10 FT  H = 10 IN  R2 = 100 OHMS )

CURRENT - AMPERES

-2.50 x 10^{-1}  -2.00 x 10^{-1}  -1.50 x 10^{-1}  -1.00 x 10^{-1}  -5.00 x 10^{-2}  0.00  5.00 x 10^{-8}  1.00 x 10^{-7}  1.50 x 10^{-7}  2.00 x 10^{-7}  2.50 x 10^{-7}  3.00 x 10^{-7}  3.50 x 10^{-7}  4.00 x 10^{-7}  4.50 x 10^{-7}  5.00 x 10^{-7}  5.50 x 10^{-7}  6.00 x 10^{-7}  6.50 x 10^{-7}  7.00 x 10^{-7}  7.50 x 10^{-7}  8.00 x 10^{-7}  8.50 x 10^{-7}  9.00 x 10^{-7}  9.50 x 10^{-7}  1.00 x 10^{-6}  1.05 x 10^{-6}  1.10 x 10^{-6}  1.15 x 10^{-6}  1.20 x 10^{-6}  1.25 x 10^{-6}  1.30 x 10^{-6}  1.35 x 10^{-6}  1.40 x 10^{-6}  1.45 x 10^{-6}  1.50 x 10^{-6}  1.55 x 10^{-6}  1.60 x 10^{-6}  1.65 x 10^{-6}  1.70 x 10^{-6}  1.75 x 10^{-6}  1.80 x 10^{-6}  1.85 x 10^{-6}  1.90 x 10^{-6}  1.95 x 10^{-6}  2.00 x 10^{-6}

TIME - SECONDS
CTRAN CASE 40  ( L = 40 FT  H = 70 IN  R2 = 100 OHMS )

CURRENT - AMPERES

TIME - SECONDS
CTRAN CASE 41

\( L = 40 \text{ ft} \quad \eta = 70 \text{ in} \quad R_2 = 50 \text{ ohms} \)

- \( \text{TIME - SECONDS} \)
- \( \text{CURRENT - AMPERES} \)

\( \text{AMPERES} \):
- \( 1.50 \cdot 10^{-6} \)
- \( 1.00 \cdot 10^{-6} \)
- \( 5.00 \cdot 10^{-7} \)
- \( 0.00 \)
- \( -5.00 \cdot 10^{-7} \)
- \( -1.00 \cdot 10^{-6} \)
- \( -1.50 \cdot 10^{-6} \)
- \( -2.00 \cdot 10^{-6} \)

\( \text{SECONDS} \):
- \( 10^{-10} \)
- \( 10^{-9} \)
- \( 10^{-8} \)
- \( 10^{-7} \)
- \( 10^{-6} \)
- \( 10^{-5} \)
- \( 10^{-4} \)
- \( 10^{-3} \)
- \( 10^{-2} \)
- \( 10^{-1} \)
- \( 10^{0} \)
- \( 10^{1} \)
- \( 10^{2} \)
- \( 10^{3} \)
- \( 10^{4} \)
- \( 10^{5} \)
- \( 10^{6} \)
CTRAH CASE 48 \( (L = 30 \text{ FT} \quad H = 20 \text{ IN} \quad R_2 = 200 \text{ OHMS}) \)
CTRAN CASE 50  ( L = 30 FT  H = 20 IN  R2 = 50 OHMS )

CURRENT - AMPERES

TIME - SECONDS