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TECHNICAL REPORT M-71-4

AN ANALYSIS OF GROUND MOBILITY MODELS (ANAMOB)

by

A. A. Rula, C. J. Nuttall, Jr.

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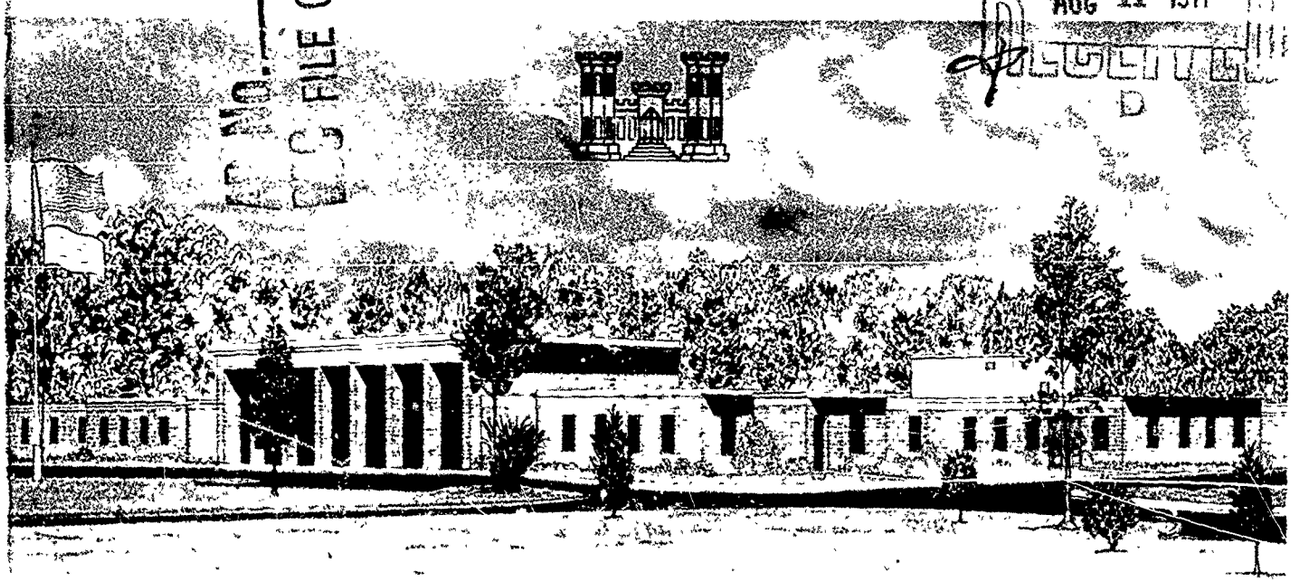

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13. ABSTRACT

Practically every research effort and subsequent procurement of materiel for Army use is backed up with a system effectiveness-cost effectiveness study. The development of good ground mobility models in support of these studies was of particular interest to the Office of the Assistant Vice Chief of Staff, U. S. Army, to assist in the construction of new models or in the evaluation of existing models. As a first step, it was highly desirable to survey the existing models. With the need established, and with U. S. Army Engineer Waterways Experiment Station (WES) interest and technical capabilities in the field, a study aimed at the analysis of existing ground mobility models was initiated by WES and WNRE, Inc., under contract to WES. The objectives of the study were to analyze existing ground mobility models in order to: (a) determine their general level of usefulness and applicability to predicting cross-country performance of ground vehicles in the real world; (b) select the models that appear to be the more promising for this purpose and determine their usefulness and applicability in more definitive terms; (c) point out areas of the latter models in which additional research is needed; and (d) develop guidelines for future development of ground mobility models. Various Army sources and unclassified literature were canvassed for cross-country performance models that have been used or seriously proposed. Examination of these available simulations revealed a considerable degree of fundamental commonality, despite initial differences in appearance. In light of the basic commonality, the available cross-country models were next examined for the single-feature vehicle-terrain interaction models that they utilized. Those single-feature models found in the existing cross-country models were identified. Each cross-country and single-feature model was then examined by a study-team member familiar with the type of problem it dealt with. Each model was briefed, classified, and evaluated on the basis of the degree of objective validation available and of subjective considerations as to adequacy, real-world verisimilitude, probable accuracy, etc. Models covering a given single-feature/vehicle interaction were grouped and the characteristics, assumptions, and limitations that they shared were outlined. When data were available, checks of the prediction accuracy of the single-feature model were made. Details for several classes of single-feature models are presented in two appendixes to the report. Points of divergency were then examined for their significance, and (a) an overall judgment was made as to the most advanced existing model of the class, and its principal assumptions and limitations were evaluated; (b) modest suggestions were made for immediate improvements, as possible; (c) from (a) and (b), recommendations were formulated for the NOW model; and (d) specific further work to improve the model was suggested. In examining each type of model, modeling strategies available were (Continued)

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Ground mobility models						
Mobility						
Off-road vehicles						
Soil-vehicle interaction						

13. ABSTRACT (Continued)

outlined in the broadest sense. This procedure facilitated classification of existing models and indicated the existence of alternatives that might not yet have been explored. In general, the strategies consisted of a number of approaches to each of several segments of the problem and a model was classed by the serial path that it represented through a matrix. It was found that, in general, the sequence of a rational path did not include successive steps that proceed from the more specific to the less. The study was conducted within several constraints. Only those current models for cross-country operation that are functioning and that offer the potential to do a simulation job now were considered. Ten cross-country models meeting most of these criteria were selected for detailed examination. The study produced a compendium of existing ground mobility submodels and comprehensive cross-country vehicle performance models that have been used or have a potential for future use. A structure for a NOW cross-country ground mobility model is suggested, along with some minor additions that do not require a great amount of effort. The study also presents a list of guidelines for the future development of ground mobility models along with plans for a future research program. This report consists of: a main text containing an introduction, a presentation and analysis of single-feature models, a description and analysis of cross-country performance models, structure for a NOW comprehensive cross-country model, and guidelines and plans for future development of ground mobility models; and two appendixes that present in detail the soil-vehicle models (Appendix A) and stream-crossing models (Appendix B) examined for this study.

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FOREWORD

The analysis of ground mobility models reported herein was performed by the U. S. Army Engineer Waterways Experiment Station (WES) for the Weapon Systems Analysis Directorate, Office of the Assistant Vice Chief of Staff, Department of the Army. The study was authorized by the Office, Chief of Engineers, by 1st Ind dated 18 April 1969 to letter from the Office, Chief of Staff, DA (CSAVCS-W-CS), dated 10 April 1969, subject: Analysis of Ground Mobility Models (ANAMOB).

During the preparation of plans for executing the study, it became apparent that to ensure a comprehensive and unbiased study, it would be advantageous to utilize the services of a consulting firm that has been closely associated with the experimental and theoretical programs related to developing terrain-vehicle models of Government laboratories, universities, and industry. Accordingly, a contract was negotiated with WNRE, Inc., of Chestertown, Maryland, for participation in the study.

Acknowledgments are made to Mr. Hunter M. Woodall, Jr., Chief, Combat Support Systems Group, and LTC Gerald E. Galloway, Weapon Systems Analysis Directorate, Office of the Chief of Staff, U. S. Army, for establishing the need for the study and for preparing the outline for proposed study, and to LTC A. J. Gow, Weapon Systems Analysis Directorate, Office of the Assistant Vice Chief of Staff, Department of the Army, who monitored the study and provided helpful suggestions.

Acknowledgment is also made to personnel of the U. S. Army Tank-Automotive Command, Aberdeen Proving Ground, and Cornell Aeronautical Laboratory for providing information and reports upon request, and to the following personnel and organizations for their cooperation in discussions, correspondence, and reports furnished on modeling lunar vehicle performance:

Messrs. R. W. Wong and C. Kern, The Bendix Corporation; J. P. Finelli, General Motors Corporation; Mr. E. Markow, Grumman Aircraft Engineering Corporation; and Mr. R. Love, National Aeronautical and Space Administration. Special recognition is given to Mr. B. D. Van Deusen, Chrysler Corporation, for providing the results of Chrysler's vehicle dynamic response prediction for a selected terrain.

This study consisted essentially of a review of pertinent unclassified literature that was readily available and information gathered from discussions with personnel actively engaged in past and present development of ground mobility knowledge.

The information used in this report was prepared by the following personnel:

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A. S. Lessem	Mobility Research Branch
N. R. Murphy, Jr.	Mobility Research Branch
C. J. Nuttall, Jr.	WNRE, Inc.

This study was performed during 1969 and 1970 by the Vehicle Studies Branch under the direction of Mr. Rula, Chief, and under the general supervision of Mr. S. J. Knight, Assistant Chief, Mobility and Environmental (M&E) Division, and Mr. W. G. Shockley, Chief, M&E Division. The material for this report was assembled and analyzed by the various team members; the report was written by Messrs. Rula and Nuttall.

Directors of the WES during this study and preparation of the report were COL Levi A. Brown, CE, and COL Ernest D. Peixotto, CE. Technical Director was Mr. Frederick R. Brown.

---

* Mobility and Environmental (M&E) Division, WES.

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CONVERSION FACTORS, BRITISH TO METRIC UNITS OF MEASUREMENT

British units of measurement used in this report can be converted to metric units as follows:

Multiply	By	To Obtain
inches	25.4	millimeters
feet	30.48	centimeters
square inches	6.4516	square centimeters
pounds	0.45359237	kilograms
tons (2000 lb)	907.185	kilograms
pounds per square inch	0.070307	kilograms per square centimeter
pounds per cubic inch	276.80	kilograms per cubic centimeter
foot-pounds	0.138255	meter-kilograms
feet per second	30.48	centimeters per second
miles per hour	1.609344	kilometers per hour

SUMMARY

Practically every research effort and subsequent procurement of materiel for Army use is backed up with a system effectiveness-cost effectiveness study. The development of good ground mobility models in support of these studies was of particular interest to the Office of the Assistant Vice Chief of Staff, U. S. Army, to assist in the construction of new models or in the evaluation of existing models. As a first step, it was highly desirable to survey the existing models. With the need established, and with U. S. Army Engineer Waterways Experiment Station (WES) interest and technical capabilities in the field, a study aimed at the analysis of existing ground mobility models was initiated by WES and WNRE, Inc., under contract to WES.

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Various Army sources and unclassified literature were canvassed for cross-country performance models that have been used or seriously proposed. Examination of these available simulations revealed a considerable degree of fundamental commonality, despite initial differences in appearance. In light of the basic commonality, the available cross-country models were next examined for the single-feature vehicle-terrain interaction models that they utilized. Those single-feature models found in the existing cross-country models were identified. Each cross-country and single-feature model was then examined by a study-team member familiar with the type of problem it dealt with. Each model was briefed, classified, and evaluated on the basis of the degree of objective validation available and of subjective considerations as to adequacy, real-world verisimilitude, probable accuracy, etc.

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report. Points of divergency were then examined for their significance, and

a. An overall judgment was made as to the most advanced existing model of the class, and its principal assumptions and limitations were evaluated.

b. Modest suggestions were made for immediate improvements, as possible.

c. From a and b, recommendations were formulated for the NOW model.

d. Specific further work to improve the model was suggested.

In examining each type of model, modeling strategies available were outlined in the broadest sense. This procedure facilitated classification of existing models and indicated the existence of alternatives that might not yet have been explored. In general, the strategies consisted of a number of approaches to each of several segments of the problem and a model was classed by the serial path that it represented through a matrix. It was found that, in general, the sequence of a rational path did not include successive steps that proceed from the more specific to the less.

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This report consists of: a main text containing an introduction, a presentation and analysis of single-feature models, a description and analysis of cross-country performance models, structure for a NOW comprehensive cross-country model, and guidelines and plans for future development of ground mobility models; and two appendixes that present in detail the soil-vehicle models (Appendix A) and stream-crossing models (Appendix B) examined for this study.

AN ANALYSIS OF GROUND MOBILITY MODELS (ANAMOB)

PART I: INTRODUCTION

Objective

1. The objectives of the study, "An Analysis of Ground Mobility Models (ANAMOB)," were to analyze existing ground mobility models in order to:

a. Determine their general level of usefulness and applicability to predicting cross-country performance of ground vehicles in the real world

b. Select the models that appear to be the more promising for the purpose and determine their usefulness and applicability in more definitive terms

c. Point out areas of the latter models in which additional research is needed

d. Develop guidelines for future development of ground mobility models.

Background

2. Recent growth in the sophistication and cost of weapons systems has made it desirable to formalize procedures for making objective selections among them. The principal procedure developed has been cost-effectiveness comparisons using increasingly complex or comprehensive computer programs.

3. To satisfy the requirements of a computer, all input information must be quantified, whether or not rational means exist for doing so. Moreover, despite the most painstaking quantification, the basic concept of large-scale cost-effectiveness computations ultimately involves comparison among things that are so unlike that comparison seems irrational. These fundamental difficulties may be greatly mitigated, but seldom eliminated, when cost-effectiveness comparisons are made between smaller systems that

have, to the naked eye at least, a large measure of similarity, such as between competing forms of ground vehicles in battlefield logistic support, for example.

4. Notwithstanding, to compare systems of whatever size, both cost and effectiveness must be computed. Despite the usual gross simplifications (use of peacetime dollar values for wartime studies, assignment of no intrinsic value to human life, etc.), even the cost element in such trade-off relations has proven most elusive. From experience with the F-111, the C-5A, and the MBT70, all of which received the benediction of extensive analyses, it appears increasingly difficult to estimate even the first cost of a new machine with an error of much less than plus 100 percent.

5. Quantitative assessment of the effectiveness of a weapons system presents even larger practical and philosophical problems, leading to the suspicion that computed values for effectiveness indexes can hardly be more reliable than their cost partners. Consider the case of a tank engagement. Predicting behavior of each vehicle involved requires knowledge of:

a. Weapon reliability and accuracy.

b. Personnel behavior in acquiring targets, serving the weapon, and operating the vehicle, all in an environment conditioned by the battle, by the vehicle, and by the terrain.

c. Vehicle performance under the constraints of terrain, of mission, and of driver judgment.

In order to simulate* more than a simplistic one-on-one situation, projections must also be made of the ergonomic and technical performances of communications systems, and of the interrelated responses of other man-weapon-vehicle units, friendly and hostile, like and unlike. Finally, when, once all the input parameters are given, the ensuing action can be predicted for a reasonable time ahead, there remains the crucial task of specifying realistic terrain inputs and initial conditions.

6. The ANAMOB study addresses itself to but a small part of this broad picture and eschews entirely such formidable tasks as supplying

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* In this report, simulation is defined as the act of representing some aspect of the real world by numbers or symbols that can be easily manipulated to facilitate their study.

reasonable scenarios. ANAMOB examines only methods to predict the mobility performance* of a single ground vehicle in a sequence of terrain conditions. The outputs can be any measures of vehicle behavior that are desired. Even this small part of the overall problem of simulating behavior of a vehicle system in a given operational situation is extremely complex.

7. Despite the great utility of the helicopter in Southeast Asia, requirements for increased ground mobility in our military vehicles continue. High-mobility ground vehicles support the general need in limited warfare to amplify our manpower through the use of efficient machines, to widen limited options, and to avoid predictable channeling of traffic. When they can be used, ground vehicles will usually prove less costly, and sometimes less vulnerable, than helicopters. Helicopters and truly mobile ground vehicles in combination offer a useful increase in operational flexibility. Accordingly, between the continuing needs for ground vehicles and for systems studies, it appears that cross-country traverse simulations will be in use for some years to come. Such simulations must be as good as possible if the entire systems study effort is to give meaningful results. The ANAMOB study is addressed to the problem of improving cross-country traverse simulation.

8. Current cross-country performance simulations are mostly ad hoc models. Largely for this reason, a particular cross-country model will sometimes, usually deliberately, be limited to the simulation of a part of the overall problem only, that part specifically needed in the particular larger simulation of which it is a subassembly. Limitations of this sort are usually manifested in arbitrary restrictions on the terrain types considered.

9. The sponsor has recognized that further proliferation of ad hoc model development not only leads to chaos (or at least the appearance thereof), but also reduces the value of individual systems studies by

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* Mobility performance in this context is defined as the ability of a ground vehicle to move freely in a given terrain context. If predictions are made by use of a comprehensive model, speed is usually used as the parameter to describe performance, whereas if the predictions are made by use of a submodel such as a soil-vehicle submodel, drawbar pull and motion resistance are usually the important performance parameters.

making them to a greater or lesser degree not comparable with others. The sponsor desires the promulgation of a complete, standard terrain-vehicle interaction model of the best accuracy and highest degree of validation currently possible, for regular and consistent use in more complete systems simulations. This will be termed the "NOW" model. The sponsor recognizes that despite some 25 years of mobility research, the NOW model will still leave much to be desired. Accordingly, development of a research and development program to produce a significantly more accurate and more realistic model appears necessary.*

10. While one overall objective of ANAMOB is to propose a NOW model for uniform use in more complete system evaluation simulations, modeling for less ambitious use (as part of engineering design optimization, for mixed-mode comparisons in which the performance of a paper vehicle is compared with that of existing machines, etc.) has also been kept in mind. As a result, the NOW model recommended in Part III of this report is outlined in a modular form, so that different relations may be used in some modules according to the particular object of the model's use. A specific composition of each module is recommended for each specific use of the NOW model. In the design process it is not intended to restrict use of any method that appeals to the designer, but alternate methods must be used at his risk, because the controlling concept of ANAMOB is that all final evaluations will be made using the NOW model in its evaluation configuration. The NOW model proposed is recommended for operational use in evaluation simulations for the immediate future (five years), irrespective of any minor improvements that might be possible in the near future, so as to preserve internal consistency among all studies in that time frame. Past experience indicates that the rate of technical progress will be such that five years will be a

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* This is not the first time in recent years that this general state of affairs has been recognized, and steps begun to rectify it. Two previous efforts in this direction were initiated by Advanced Research Projects Agency (ARPA) in 1964-66 (Mobility Environmental Research Study (MERS) at the U. S. Army Engineer Waterways Experiment Station (WES)), and in 1967-68 (the Off-Road Mobility Research (ORMR) program at Cornell Aeronautical Laboratory (CAL)). For reasons that have never appeared technically justifiable, both programs were aborted just as they approached their payoff period.

minimum time increment for significant improvement. This is not to say, however, that new elements should not be added to the model where it can be demonstrated that they are necessary to give a radically different machine a fair evaluation.

Perspective

11. Formal study of military mobility problems began during World War II and was concerned initially with the failure of vehicles to negotiate the muds and weak soils found in Italy, Germany, Okinawa, and other battle areas. Research on the vehicle-soil relations that might explain the failures and indicate ways to design vehicles less prone to soft-soil immobilizations continued on a low-budget, essentially ad hoc basis into the early 1950's. At that time, under the impetus of renewed tank immobilizations, this time in Korea, the problem was more formally recognized, and there has been a modest but continuous program of research in this general area under U. S. Army sponsorship ever since. Ground vehicle immobilizations are still with us, however, now in Southeast Asia. And despite the fact that available vehicle-soft ground relations have generally clarified some broad design and operational problems, and that they are fundamental to the prediction of all ground vehicle operations, they are not yet entirely satisfactory for many purposes.

12. In the early 1950's, Bekker expanded the purview of ground mobility research from examination of vehicle-soil relations to study of more general vehicle-terrain relations. Trees and hills and lumps and bumps were recognized as elements contributing to the total impedance that a ground vehicle encounters in a normal cross-country traverse. Other researchers--partially despairing of advancing the solution of the soft-soil problem, partially in recognition of the complexity of real terrain--followed suit by the end of the decade. Further impetus to treat the complete terrain complex came from pressures at DOD level for cost-effectiveness and/or systems studies in justification of new hardware proposals. By the mid-1960's, a similar systems approach was adopted in then-beginning studies of possible moon vehicles.

13. In the mid-1950's, consideration was given to the interplay of vehicle vibrations, or "ride," and speed, through the reaction of the driver. A decade later, modest efforts were started to model the driver and his vast influence upon actual performance more completely. Thus far, progress has been slow, and no cross-country performance model in use today incorporates the driver except as a vibration-visibility speed governor.

14. In a total systems approach, estimation of vehicle performance in a particular situation is only one of several complex elements that require simulation. Under-the-gun for decision-fodder, the approach taken over the past ten years to simulate cross-country performance has generally been to consult a limited number of research reports (of various vintage) and Bekker's latest book, make a brief brainpicking tour of a few handy experts, add a lot of imagination (of various quality), and finally, erect still another ad hoc model. And then the analyst moves on to the simulation of the next element with a prayer that it will prove more tractable and offer a more elegant solution. To some extent such almost casual inclusion of terrain-vehicle interrelations may have been justified in the past by the broad scope of the total simulations attempted. Individual vehicle performance was but one of many elements affecting the final result. On the other hand, the function of an army is to bring to bear on an enemy a winning mix of firepower, armor, and mobility. In relation to ground vehicles, then, one of the Army's major responsibilities is to understand vehicle tactical mobility in order to optimize its fundamental three-way mix. Any slighting of this critical element in a simulation upon which equipment and doctrine decisions might be made is no longer defensible. Even simulations that project reasonable, average performance in a "typical" terrain only short-circuit the intention of those comparisons of new and/or existing equipment which include candidates having nonaverage performance profiles.

Basic Concepts

15. Modeling of cross-country performance ideally involves four major concepts: the vehicle, the driver, the terrain, and the model. While it would seem that each is self-explanatory, some brief discussion at this

point of the particular interpretations that are reflected in this study may save some misunderstanding later on.

The vehicle

16. Of the four, the concept of the vehicle is essentially the most straightforward. As noted earlier, the study is aimed primarily at ground vehicles and at single vehicles operating without appreciable engineer support to mitigate terrain severity. In terms of army operations, this treatment is to some degree unrealistic. Even reconnaissance operations will usually involve a group of vehicles rather than only one, while most other operations will involve many. In most cases, the single vehicle assumption will be conservative, but not always. In general, the larger the group of vehicles in a given movement, the more likelihood of some organic engineer support for bridging, bulldozing of trees and banks, etc. The operation of a group of vehicles usually permits more rapid recovery of individual vehicles temporarily immobilized by terrain impedances, through towing rather than winching, for example. As the number of vehicles involved increases, the necessity for more than one vehicle to operate essentially in the tracks left by the preceding vehicle increases. In some situations the impedances of following vehicles operating in this manner will be reduced; but where weak remolding soils are involved, the likelihood of a later vehicle miring in the ruts of the preceding ones increases. If the vehicles are such that they are not homogeneous in their general off-road performance and/or are not compatible in scale and destructiveness to the terrain, the problems of following vehicles can be increased rather than decreased.

17. The degree of traffic channeling, and its probable effects upon movement of a group of vehicles, is of course a function of the number of vehicles, of the terrain, and of operational constraints. Either of the latter two may force traffic channeling even where only a small number of vehicles is involved and may, moreover, force such channeling where least desired from a mobility viewpoint. Accordingly, the present treatment of the vehicle proceeding solo must someday be expanded to properly reflect terrain influences upon multiple vehicle operations.

The driver

18. The driver enters existing simulations, when at all, only as a

speed limiter in response to excessive ride vibrations or to reduced visibility. His actual influence is often far greater. The general area of driver decision (or commander-and-driver decision, in the case of a tank) in selecting a detailed path through a given stretch of off-road terrain, for example, is at the moment still unmodeled in practice. This, despite the well-known fact that an experienced (and lucky?) driver can often get through a given area by intelligent, on-the-spot, close-range route selection where an inexperienced driver would be constantly riding the winch line. A complete model must eventually reflect the full impact of all driver influences.

The terrain

19. The terrain component of the vehicle-driver relation is also a familiar concept. Terrain is an area and in the military sense denotes the physical features of that area. In context of the present study, the features of interest are those that specifically interact with a vehicle attempting to traverse the area. These features are for the most part areally distributed, even though a vehicle ultimately experiences terrain more simply as a path. It is desirable in modeling cross-country operations to incorporate terrain in its full areal scope so that vehicle paths may be generated through realistic interaction of the capabilities of the vehicle, the requirements of the operation, and the configuration of the terrain. Simulation that does not permit this degree of flexibility cannot be expected to reflect the true differences in effectiveness among vehicles whose overall mobility profiles are significantly different.

20. There is an overwhelming consensus that the primary terrain attributes of interest to a cross-country vehicle operation are those that describe in engineering terms the geometry of the surface and the strength of the materials in it from place to place (in both cases including vegetation, streams, etc.). In this report, a single measurement of a single attribute of the terrain will be called a factor. Gross slope, mean spacing of trees of specified diameters, soil rating cone index, etc., are examples of terrain factors.

21. Factors that describe the terrain segment of the environment are of two kinds: those reflecting relatively stable, long-term attributes,

and those reflecting the immediate state of elements subject to seasonal and/or daily variation. A vehicle's performance is influenced by the geometric and mechanical features and properties it finds point-by-point and moment-by-moment in the terrain. These, in turn, reflect the combined effects of both kinds of factors.

22. Consider a slope of reasonable length and uniformity, extreme but not impossible--say one having a 30 percent grade. Whether or not a given, relatively mobile vehicle can negotiate it depends on many factors acting in concert with the slope. All may be considered, for the moment, to be independent. It is readily apparent, for example, that the types of soils involved, their stratification and moisture contents (and hence strengths), the extent of superimposed minor relief (microrelief), the surface roughness (minirelief), and the kind and condition of vegetative cover may each have some influence.

23. Slope (macrorelief), soil type and stratification, general vegetative cover, microrelief, and perhaps minirelief are relatively stable. Soil moisture content and its stratification will usually vary from day to day, the state of vegetation, from week to week. All will vary, more or less, to some degree at different points in any given vehicle's path even in a nominally homogeneous stretch of terrain, and most will be altered to some extent by the passage of a single vehicle.

24. The long-term factors--those describing topography (macro-, micro-, and mini-relief), the general vegetative picture, soil types and distributions, overall seasonal groundwater regime, semipermanent cultural features, etc.--may for the most part be considered on the basis of relatively large areal units. They may usefully be classified and analyzed essentially within the framework of classical naturalistic studies. The variable attributes must be treated on a time-dependent basis, reflecting temporal variations in weather, cyclic influences of climate, the mechanics of soil moisture and plant growth, and the onslaught of mankind.

25. In the aggregate, proper, long-term, broad classifications--in terms of landform, geology, ecology, climate, etc.--with their interrelations appear to constitute a sound base for predicting conditions to be found in unsampled (and sometimes unsampleable) regional areas of the world,

and hence for predicting equipment performance (or, conversely, requirements) in such areas. The considerable environmental research that must be done to develop this potential, via the correlation of the naturalistic classification systems with the occurrence of geometric and mechanical terrain features that directly affect the performance of military equipment, appears to be one of the steps essential to the development of more rational terrain information.

26. Terrain features will, in general, be such things as a streambank or a forest, described in terms of a number of factors. A streambank, for example, might be described in terms of a series of measurements defining its geometry in a single plane normal to the direction of the streamflow plus the location of the free waterline in relation to it; a forest, in terms of frequency distributions of size and spacing of trees and indexes of straight-line visibility and perhaps of effective strength of the trees (functions of species). Although groupings of factors into features are essentially arbitrary, there is a basic consensus, even here, following lines of division laid down essentially by the prediction scheme employed.

27. To describe an actual section of terrain as a vehicle sees it, it will normally be necessary to specify a combination of features. For example, data on vegetation plus surface geometry plus surface composition are needed to define a small piece of real estate that is significantly forested. Vegetation occurs in a specific topographic situation and soil body, and is often intermixed with deadfalls, stumps, and minor drainage features. All in concert influence a vehicle's performance. A distinct combination of factors that provide a relatively complete quantitative description of terrain over an area (which may be quite small or very large) will be termed a unit terrain.

28. In making vehicle performance predictions, unit terrains are generally assumed for practical purposes to be homogeneous; i.e., values for each single factor measurement are considered to be constant, or to lie within the same class range, or to be described by the same probability distribution. The number of discrete unit terrains necessary to describe an area is obviously a function both of the number of factors used and of the resolution with which each is defined. The problem is inescapably one of

statistics. Actual terrain is endlessly variable and its "full" description for vehicular purposes alone might conceivably run to the complete mapping of 30 or more measurements at a resolution of approximately 5 ft.*

29. Clearly the cost of a truly precise set of maps in such terms, even for a small area, would be prohibitive. Moreover, initial precision would disappear with the passage of a few vehicles or a few rainstorms or a few months. Accordingly, the practical description of terrain must accept both considerably reduced resolution and extensive idealization of the terrain geometry in order to keep the total number of factors manageable. The tradeoff is not hopeless, but it does lead to uncertainty in any individual result. The effects of this uncertainty upon most practical decision making may average out in areas characterized by a large number of unit terrains, but this remains to be objectively checked.

30. Mapping terrain directly in terms of its factors, as in the current generation of WES factor family maps, represents a straightforward interpretation of the engineering numbers required to describe terrain. Some experience has shown, however, that this approach may lead to the problems just mentioned; i.e., reasonable, feasible, economical resolution results in potential errors at any given point in the terrain. This is not unexpected, of course. Mitigating this by squinting at the results through a statistical filter is a suitable fix for some uses, as in design study, but even here some desired sensitivity must inevitably be lost.

The model(s)

31. A model for simulating cross-country operation is one or more expressions and analytical processes that relate desired measures of vehicle performance to quantitatively expressed vehicle, driver, terrain, and operational factors.** A complete cross-country performance model comprises the following basic elements:

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* A table of factors for converting British units of measurement to metric units is presented on page xv.

** While ultimately any such model must be set up for computer use, not all current examples are fully computerized at this time. In some cases, possible geometric interference between the vehicle underside and the terrain is examined through the use of two-dimensional static scale models. This permits somewhat more complete representation of each, but at considerable obvious expense in computation time.

a. Means to specify the vehicle in meaningful parametric form.

b. Means to depict the terrain as a quantitative areal phenomenon.

c. Criteria and procedures for generating and/or selecting paths through the terrain (including, as an almost trivial end point, fully specifying a path a priori).

d. Means to quantify terrain factors for all segments of any selected path.

e. Means to predict desired vehicle performance measures along each segment of any selected path.

f. Rules for joining performances along individual segments into a continuous traverse.

g. Means to aggregate predictions for individual route segments into maps, totals, statistical measures, etc., according to the needs of the model user or of the larger simulation of which the cross-country performance model is a part.

Elements a through d, which precede prediction of vehicle performance in a given unit terrain, are considered to be preprocessing of the data; steps f and g, which follow, are output processing. The logic and functioning of both will be determined largely by the intended use of the complete model.

32. The heart of the model, however, is the calculation of vehicle performance on a homogeneous path segment once all terrain factors pertinent to that segment are specified (element e). The resolution of terrain data available, practical computing considerations, and terrain diversity will together dictate the number of distinguishable homogeneous unit terrains by which a given area will be represented. Thereafter, the validity and resolution of the complete simulation is directly dependent upon the adequacy and accuracy of the performance model per se.

33. As a result of the large number of vehicle and terrain parameters involved, and of the true complexity of detailed relations among them, most predictions of vehicle performance are presently done by computing interactions of single terrain features with the vehicle, and subsequently superimposing the results in various reasonable combinations and according

to several exclusivity rules. The currently accepted breakdown of real vehicle-terrain interactions into manageable submodels--termed single-feature models--is as follows.

a. A soil-vehicle model which, as a function of quantitative soil values, estimates (1) vehicle motion resistance due to soil plastic and/or visco-plastic deformation and (2) net traction available to overcome other impedances.

b. A soil-slope model which adds or subtracts gravity effects to the soil-vehicle system. (These may include load transfer effects, reduced initial soil stability due to its sloped surface, etc., as well as simple changes in net traction requirements.)

c. An obstacle interference model which searches for possible geometric interferences during vehicle passage over an obstacle, which may be deformable but generally is treated as rigid.

d. An obstacle-surmounting traction model which projects torque and traction required to override a vertical obstacle.

e. An obstacle-avoidance model in which is examined the possibility of threading the vehicle through a planar array of insurmountable or especially troublesome obstacles, such as large trees, stumps, and/or large boulders.

f. A vegetation override model which estimates traction and torque required to maintain headway while overriding vegetation which cannot or may not be avoided.

g. A ride vibrations model which computes, as a function of vehicle speed, the dynamic response of a vehicle to surface roughness and compares this with personnel comfort, safety, and performance tolerances, cargo tolerances, and/or vehicle structural tolerances to establish a probable maximum speed and corresponding average power expenditure.

h. A visibility model which forecasts the speed reduction which will result, through driver election, from terrain-induced reductions in the driver's view of the prospect before him.

i. A stream-crossing model which examines vehicle ingress, swimming, and egress as a static or dynamic mechanics problem.

In all except obstacle avoidance, the vehicle is tacitly assumed to be moving in essentially a straight line. Incremental effects of vehicle maneuvering on performance in the other terrain situations are sometimes estimated and combined by further superposition.

34. Forces and torques calculated from several of the single-feature models, in appropriate combinations, are transformed to estimates of speed, fuel consumption, etc., using straightforward automotive engineering models relating speed to vehicle engine and power train torque characteristics.

Approach to the ANAMOB Study

35. Various Army sources and unclassified literature* were canvassed for cross-country performance models that have been used or seriously proposed. Examination of these available simulations revealed a considerable degree of fundamental commonality, despite initial differences in appearance. All were conceived within the common framework of Newtonian mechanics, and all consider at least a part of the overall vehicle-terrain interrelation. Differences and discrepancies arise (apart from degrees of completeness) largely in the idealizations or simplifications adopted in describing the vehicle, in describing the terrain, and in formulating the equations relating the two.

36. In light of the basic commonality, the available cross-country models were next examined for the single-feature vehicle-terrain interactions models that they utilized. Those single-feature models found in the existing cross-country models and those in practical use in less ambitious computations were identified. Each cross-country and single-feature model was then examined by a study team member familiar with the type of problem it dealt with. Each model was briefed, classified, and evaluated on the basis of the degree of objective validation available and of subjective considerations as to adequacy, real world verisimilitude, probable accuracy, etc.

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* See Selected Bibliography.

37. Models covering a given single-feature/vehicle interaction were grouped and those characteristics, assumptions, and limitations that they shared were outlined. Points of divergency were then examined for their significance, and

a. An overall judgment was made as to the most advanced existing model of the class, and its principal assumptions and limitations were evaluated.

b. Modest suggestions were made for immediate improvements, as possible.

c. From a and b, recommendations were formulated for the NOW model.

d. Specific further work to improve the model in an approximate five-year time frame was suggested.

38. In examining each type of model, modeling strategies available were outlined in the broadest sense. This procedure facilitated classification of existing models and indicated the existence of alternatives that might not yet have been explored. In general, the strategies consisted of a number of approaches to each of several segments of the problem and a model was classed by the serial path that it represented through a matrix. As will be seen later, some paths were not truly feasible and others would simply be nonsense. It was found that, in general, the sequence of a rational path did not include successive steps that proceed from the more specific to the less. This will become clearer once the strategy diagrams are presented.

39. The study was conducted within several constraints. As noted earlier, only those current models for cross-country operation were considered that are functioning and that offer the potential to do a simulation job now. Cross-country models meeting these criteria are listed in table 1.

40. Single-feature models considered are also essentially only those presently in use, although a few promising paper candidates will be mentioned. The study is aimed at models dealing with ground vehicles only. Extension of any model to cover surface effects vehicles (SEV's), however, would not be difficult. Because SEV operation would not involve soil

Table 1

Models Used in ANAMOB Study

Title of Report or Reports in Which Model is Discussed	Model Abbreviation or Acronym	Developer	Sponsor
1. Analytical Model for Predicting Cross-Country Vehicle Performance	WES	U. S. Army Engineer Waterways Experiment Station (WES)	Office, Secretary of Defense; Advanced Research Projects Agency (OSD/ARPA) and U. S. Army Materiel Command (AMC)
2. Tank-Antitank and Assault Weapons Requirements Study	TATAWS	Lockheed Missiles and Space Co.	U. S. Army Combat Development Command
3. Army Research and Development Newsmagazine, Jan 1969	OMEGA	U. S. Army Tank-Automotive Command	U. S. Army Materiel Command
Status Report: Systems Analysis Models at U. S. Army Tank-Automotive Command		U. S. Army Tank-Automotive Command	U. S. Army Materiel Command
4. Analysis and Simulation of Dynamical Vehicle-Terrain Interaction	CAL	Cornell Aeronautical Laboratory, Inc. (CAL)	Office, Secretary of Defense; Advanced Research Projects Agency
5. Armored Reconnaissance Scout Vehicle (XM800); Parametric Design/Cost Effectiveness Study	ARSV-BA	Booz-Allen Applied Research, Inc. (BA)	U. S. Army Tank-Automotive Command
6. Evaluation of Counterinsurgency Visibility Relation to Environment	GM	General Motors Corp. Defense Research Laboratories	Office, Secretary of Defense; Advanced Research Projects Agency

(Continued)

Table 1 (Continued)

Title of Report or Reports in Which Model is Discussed	Model Abbreviation or Acronym	Developer	Sponsor
7. The Tank Weapons System	OSU	Ohio State University Research Group	U. S. Army Combat Development Armor Agency
8. Guide Report for Terrain-Vehicle System Evaluation	Bekker	General Motors Corp. Defense Laboratories	U. S. Army Office of the Chief of Research and Development
Engineering Optimization of Terrain-Vehicle Systems; A Case Study of a Concept Analysis of a New Amphibian		General Motors Corp. Defense Laboratories	U. S. Army Tank-Automotive Command
9. A Study of the Vehicle Ride Dynamics Aspect of Ground Mobility	Chrysler	Chrysler Defense Engineering	Office, Secretary of Defense; Advanced Research Projects Agency
A Statistical Technique for the Dynamic Analysis of Vehicles Traversing Rough Yielding and Nonyielding Surfaces		Chrysler Defense Engineering	National Aeronautics and Space Administration
Experimental Verification of Surface Vehicle Dynamics		Chrysler Defense Engineering	National Aeronautics and Space Administration
10. A First-Cut at Developing Testable Off-Road Performance Specifications for Vehicles Adapted to Specific Terrain	WNRE	WNRE, Inc.	Office, Secretary of Defense; Advanced Research Projects Agency

strength relations* (unless the vehicle was a hybrid), the terrain and terrain-vehicle simulation would be simpler in many respects. Soil and soil-vehicle models for inorganic soils only are examined. Again, an extension to organic soils (muskeg, etc.) and to snows and snow-covered terrains is entirely feasible, although the problem of validated "soil"-vehicle models in these cases is more severe; i.e., there is considerably less in the way of available models and reliable supporting laboratory and/or field tests.

41. The present study considers only the problem of a single vehicle operating in unprepared terrain. Elaboration to include multiple vehicle operation (the traffic problem) and various degrees of engineer support in preparing the terrain for the vehicle remains largely for the future. Finally, while a complete model must, by general agreement, ultimately include a number of driver inputs, the present study considers only those that have actually been modeled to date. There are but two. One is driver reaction to more-or-less continuous vehicle vibrations (the ride problem), in which the driver is presumed to alter the general level of his vibrational environment to some tolerable level through speed adjustment. The second is the strong interrelation between the vehicle and the terrain through the mechanism of the driver, who adjusts his operating speed to match the terrain information that he can identify (the visibility problem). In many situations this will be the only impediment. The visibility problem, which is as yet fuzzy at best, is a good instance where poor simulation of the vehicle-driver-terrain interactions could lead to seriously erroneous results. For example, without a good model for this important part of the problem, the value of supplementary on-board sensing equipment, new approaches to control, or even of advances in vehicle suspension capabilities, obviously cannot be estimated intelligently.

Realism and Validation

42. Two important guiding criteria in the ANAMOB study are realism

* Such factors as dust, which are ignored as secondary in present ground vehicle modeling, would become primary, however.

and validation. The two are, of course, related but they are far from identical. To make a working distinction, validation of a model or submodel is the extent to which its predictive accuracy has been demonstrated in reliable, relevant tests.

43. Realism in a model is the degree to which:

a. Parameters used to describe the vehicle do in fact describe all of the functionally important features of real vehicles.

b. Parameters used to describe the terrain do in fact define real terrain adequately as a vehicle sees it.

c. Vehicle behavior predicted on the basis of a and b agrees with actual behavior of actual vehicles in actual terrains.

Note that a submodel may, by these special definitions, be valid, even though it lacks realism. Ideally, of course, an overall model and all submodels within it should be both realistic and validated. The nature of the problem makes either requirement difficult to achieve.

Realism

44. Realism is a function both of the inputs to the model and of the process by which the model predicts performance from them. The inputs include both vehicle and terrain parameters, but the latter are by far the more troublesome. Realism of the inputs involves such questions as: "Are the descriptors valid measures of relevant features? Do they, in the aggregate, adequately describe the total situation? Is their combined resolution sufficient? How reliable are available values?" And finally, "What is the true availability of reliable terrain data in terms of these measures at the present time and what are the prospects for the future?"

45. In describing simple terrains, the questions of data precision, resolution, and reliability are paramount. For more complicated terrains, the way in which complex situations are idealized and digested for presentation also becomes important. In any system, there will be inherent limits to realism at each level.

46. A large measure of idealization is necessary to describe both the vehicle and the terrain quantitatively if an impossibly long list of descriptors is to be avoided. Usual idealizations in vehicle specification are to use simple straight-line and circle envelopes to describe the

vehicle geometry and to ignore (except in specifically dynamic situations) changes of loading and geometry due to running gear and frame compliance. In tracked vehicle representation, track and suspension details are frequently omitted entirely and the track replaced by a simple flat plate with idealized ground-gripping elements. Tire tread and deformation characteristics, except in their crudest aspects, are normally ignored. By and large, such simplifications presently have some justification, not only from a practical computational viewpoint but also because the effects of these simplifications appear to be generally well within the uncertainty band associated with the assignment of associated terrain factor values.

47. Idealizations used to describe the terrain in manageable form are potentially far more troublesome. Needless to say, real terrain is endlessly complex. There is no hope of dealing with it in any practical way, as there might be for the vehicle itself, by simply increasing the number of measurements used. While there is a barely tangible limit when such a process is applied to any single vehicle, when applied to the terrain, it implies rapidly increasing resolution whose end point is description in terms of geometric and mechanical properties inch by inch in all directions. Adequate terrain representation is clearly hopeless without a relatively high degree of sophisticated simplification and classification.

48. The 1966 MERS exercise represents the most ambitious terrain mapping exercise thus far attempted in relation to vehicle operations. The general scheme for classifying terrain factors and the resolution attempted are shown in table 2. Surface composition is described in terms of rating cone indexes and the direct shear parameters, cohesion and angle of internal friction. Surface geometry is given in terms of gross slope, and obstacle spacing, approach angle, and step height. Obstacle shaping is described by a small number of straight-line segments and their intersection angles. Vegetation is described by accumulation curves of stem density as functions of increasing and decreasing stem diameters. These three classes of factor families--surface composition, surface geometry, and vegetation--describe the terrain on an areal basis. Values for some 17 factors (table 3) are assigned for every point in a given land area to the resolution indicated in table 2.

Table 2

Terrain Factors and Class Ranges Used in Preparing MERS Factor Family Maps

Terrain Factor Family	Terrain Factor	Unit of Measure	Class Ranges 1	2	3	4	5	6	7	8	9
Surface composition*	Soil mass strength	RCI	<10	10-25	>25-60	>60-100	>100				
	Soil surface strength: a_{ur}	kg/cm^2	0-0.07	>0.07-0.14	>0.14-0.28	0-0.07	>0.07-0.14	>0.14-0.28	0-0.07	>0.07-0.14	>0.14-0.28
		psi	0-1	>1-2	>2-4	0-1	>1-2	>2-4	0-1	>1-2	>2-4
	α_{ur}	deg	0-10	0-10	0-10	>10-20	>10-20	>10-20	>20-40	>20-40	>20-40
Surface geometry	Slope	deg	0-1.5	>1.5-4.5	>4.5-9	>9-18	>18-30	>30-45	>45		
	Vertical obstacle spacing	m	0-2.1	>2.1-3.7	>3.7-15.2	>15.2-45.7	>45.7				
		ft	0-7	>7-12	>12-50	>50-150	>150				
	Terrain approach angle	deg	<100	100-<125	125-<150	150-<165	165-<180	180-<200	200-<210	210-<220	≥220
	Step height	cm	0-10	>10.2-25	>25-46	>46-76	>76-122	>122-168	>168-213	>213	
		in.	0-4	>4-10	>10-18	>18-30	>30-48	>48-66	>66-84	>84	
Vegetation**	Spacing of stems ≤2, 5, 9, and 50 in. in diameter	m	>9.1	>3.0-9.1	>1.5-3.0	0-1.5					
		ft	>30	>10-30	>5-10	0-5					
	Spacing of stems ≥1, 3, 6, and 10 in. in diameter	m	>9.1	>3.0-9.1	>1.5-3.0	0-1.5					
		ft	>30	>10-30	>5-10	0-5					
Hydrologic geometry†	Contact approach angle	deg	<145	145-155	>155-165	>165-180					
	Step height††	cm	<30	30-<61	61.0-<91	91.0-122	>122				
		in.	<12	12-<24	24-<36	36-48	>48				
	Position of step base‡	cm	>122	>91-122	>46-91	3-46	At water level	3-30	>30-76	>76-122	>122
		in.	>48	>36-48 bwl‡‡	>18-36 bwl	1-18 bwl		1-12 awl§	>12-30 awl	>30-48 awl	>48 awl
	Water depth	m	0.9-1.4	>1.4							
		ft	3-4.5	>4.5							

* Maximum and minimum moisture conditions mapped.
** Class ranges for each diameter are mapped.
† Considered a surface geometry feature when water depth is <3 ft (<0.9144 m). Mean maximum and minimum water depths mapped.
†† Step is a slope change >35 deg.
‡ Referenced to water level.
‡‡ Below water level.
§ Above water level.

Table 3

MERS Factors Used to Describe Terrain for Ground Mobility

Areal Terrain

	Factor Family	Factor	Dim.	MERS Symbol
1.	Surface Composition	Rating Cone Index	-	RCI
2.		Soil Cohesion	psi	a_{MP}
3.		Soil Angle of Internal Friction	deg	a_{MP}
4.	Surface Geometry	Slope	deg	SL
5.		Vertical Obstacle Specing	ft	OS
6.		Obstacle Approach Angle, East	deg	AA(E)
7.		Obstacle Approach Angle, West	deg	AA(W)
8.		Step Height, East	in.	SH(E)
9.		Step Height, West	in.	SH(W)
10.	Vegetation	Mean Stem Spacing for Stems with Diameters ≤2 in.	ft	DO(1)
11.		≤5 in.	ft	DO(2)
12.		≤9 in.	ft	DO(3)
13.		≤50 in.	ft	DO(4)
14.		≥1 in	ft	DA(1)
15.		≥3 in.	ft	DA(2)
16.		≥6 in.	ft	DA(3)
17.		≥10 in.	ft	DA(4)

Linear Terrain (Hydrologic)

	Factor Family	Factor	Dim.	MERS Symbol
1.	Surface Composition	Rating Cone Index	-	RCI
2.		Soil Cohesion	psi	a_{MP}
3.		Soil Angle of Internal Friction	deg	a_{MP}
4.	Hydrologic Geometry (Wet)	Lower Approach Angle, East Bank	deg	AA(E)
5.		Lower Approach Angle, West Bank	deg	AA(W)
6.		Step Height, East Bank	in.	SH(E)
7.		Step Height, West Bank	in.	SH(W)
8.		Step Base vs Water Line, East	in.	PS(E)
9.		Step Base vs Water Line, West	in.	PS(W)
10.		Water Depth	ft	WD
	-OR-			
4.	Hydrologic Geometry (Dry)	Upper Approach Angle, East Bank	deg	EA(E)
5.		Upper Approach Angle, West Bank	deg	EA(W)
6.		Lower Approach Angle, East Bank	deg	IA(E)
7.		Lower Approach Angle, West Bank	deg	IA(W)
8.		Step Height, East Bank	in.	SH(E)
9.		Step Height, West Bank	in.	SH(W)

49. Water courses, drainage features, and other major linear terrain features are mapped separately and treated as potential linear barriers. Nine terrain factors* (table 3) are used to describe the geometry of typical cross sections of these obstacles, and the water depth, if any, accounts for one more factor.

50. The MERS terrain description gives no precise positional information about specific surface geometry and vegetational obstacles, no locations for soil hard and soft spots within an area of a given class, and no slope orientation information. Moreover, hydrologic feature description does not directly indicate the soils to be found on the banks and in the beds of streams, stream velocity, stream width, or the frequency with which potential vehicle entrance and egress windows occur along the banks, which is a critical feature of real life streams. These omissions were, for the most part, made deliberately in order to make possible description of the terrain in what was considered a reasonable number of dimensions. Their omission, however, and the idealizations involved in creating the measures actually used both illustrate the terrain mapping dilemma.

51. Terrain quantification is further complicated by the question of resolution in the classing of individual factors. The class intervals shown in table 2 do not appear unreasonably large, especially when it is realized that in practice they must be inferred largely from aerial photographs. Nonetheless, and even presuming that the actual values on the ground in fact lie in all cases within the stated class intervals, it has been found that <u>in combination</u> these intervals can create an unacceptably broad uncertainty band. For example, in a go-no go study of the Khon Kaen map sheet No. KK1, the performance of the M29C Weasel was computed to be 100 percent go across all areal features over the entire map if each factor was assumed to be at the lower severity bound within its assigned class, and was 99 percent no go when each factor was taken at its highest severity bound.

52. Obviously, the probability that either of these extremes would

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* After selection among multiple values for some factors according to season.

prevail over the entire area is negligibly small. However, such wide bounds are disappointing, to say the least. This result indicates that the question of go or no go or of any more elaborate projection such as speeds, can in fact only be realistically approached on a statistical basis. That is, the proper description of go or no go or of speed in a given unit terrain is a statistical distribution. This immediately suggests that terrain factors would better be expressed in terms of joint distributions (JPD) among themselves. Such distributions would lend themselves to transformation to vehicle performance probabilities more-or-less directly by mathematical methods. In turn, description of the terrain in terms of JPD's suggests that statistically homogeneous terrain units might prove to be larger and far less numerous than would appear from the many different unit terrains that arise in the discrete class-interval approach. The consequences of this from an air-photo interpretation viewpoint, or from the viewpoint of probabilities for automatized analysis of data from other types of remote sensors, clearly would be beneficial.

53. Realism of terrain representation for vehicular mobility purposes is fundamentally the most difficult problem in cross-country performance simulation. Indeed it is the keystone of realism for the entire operation. Accordingly, an unromantic estimate of the feasibility of improving upon present capabilities to specify terrain should, to some degree, govern objectives elsewhere in the total model. A high degree of accuracy in predicting vehicle behavior in a seriously unrealistic terrain, or in one that is merely an imaginative abstraction of what is actually on the ground, will for many purposes prove nearly useless.

54. The third leg of the stool supporting realism is the accuracy of predictions when the inputs are accurately specified. Good representation of the vehicle and of the terrain obviously does not of itself guarantee realistic vehicle behavior predictions. Not only must the vehicle and terrain inputs be correct, but the prediction model must also do the right things with the input data. In further discussions of terrain-vehicle models, attention is directed largely to the latter problem. In this reduced context, realism becomes more nearly akin to accuracy of prediction.

Validation

55. Validation is defined in this report as the demonstration of predictive accuracy of a model or submodel by reliable, relevant tests. Validation also implies correct reasoning and solidity in the grounds upon which a model is based. Note that since simulation involves the representation of reality by more convenient symbols, a model may be valid but lack realism.

56. Review of the literature shows that information pertinent to vehicle performance prediction and testing is abundant, but that each set is essentially relevant only to a specific single-feature model. Data to verify comprehensive cross-country models or to compare the prediction accuracy of two or more single-feature models intended to predict the same performance parameters are almost nonexistent. Soil-vehicle model data predominate. They include soil performance tests conducted in the field with full-scale vehicles and in the laboratory primarily with single wheels in prepared soil bins. Both field and laboratory scale-model vehicle test data are available. Performance parameters for which soil strength relations have been developed using full-size vehicles include drawbar pull, motion resistance, slope climbing, speed, and minimum soil strength required to permit completion of a prescribed number of passes (usually one pass and 50 passes). Laboratory tests of soil strength relations usually include drawbar pull, motion resistance, and torque measurements. Most of the testing reported in the literature on soil-vehicle models is based on the WES and Land Locomotion Division (LLD), TACOM, soil measurement systems.

57. Although there are considerable full-scale vehicle test data in a variety of soil conditions, most of these are not usable in model validation. Most of the field programs were planned for purposes other than validation of any kind of model, with the result that their test results are of little value due to omission of data required by one or another (or all) model(s). In some field exercises equipment used to obtain soil data was inadequately specified. Published reports frequently do not include sufficient of the basic data to perform independent analysis of measured vehicle performance. Some that include both measured and predicted vehicle performance fail to mention the specific version of the prediction system used,

or to cite exact procedures and quantities used to make the predictions. In still other cases, quoted predictions are made only after a prediction system has been modified to reflect the test results to which they are subsequently compared. Testing vehicles in the off-road environment presents many problems, and the state-of-the-art in standardized testing and data collection must be improved before consistent, reliable, and relevant validation data will be available for ground mobility purposes. This does not excuse sloppy reporting, however.

58. By comparison, reported laboratory test results are generally complete. Differences between laboratory and field procedures preclude their use interchangeably, however. Laboratory tests are usually run in homogeneous soils under closely controlled procedures with sophisticated instrumentation. Field tests are usually run on comparatively nonhomogeneous surfaces containing variations in soil strength with depth, or along the length of a test lane, or both, and in surface profile and vegetation. The accuracy and repeatability to which vehicle response measurements can be made in the field are much less than are generally obtained under laboratory conditions. Laboratory results provide a useful body of knowledge to guide the interpretation of field data, but considerable effort is yet required before laboratory results can duplicate field test results. Relating laboratory soils to field soils and developing methods to translate the performance of a single traction element to that of a full-scale multielement vehicle still remain largely to be done.

PART II: SINGLE-FEATURE MODELS

59. Existing cross-country performance models all treat the terrain as a series of homogeneous, multifactored units, and treat the terrain-vehicle relations in terms of combinations of single-feature interactions. Differences arise in:

a. The parameters used to describe the terrain.

b. The details of the single-feature interaction modeling.

c. The manner in which the effects on the vehicle of several features in a single-unit terrain are combined.

d. The way in which predictions for a series of unit terrains are joined to approximate the continuous motion of a vehicle.

Despite differences of these several types, there is still much in common as to the specific single-feature models and rationale that are used in formulating cross-country models. For this reason and because some results of single-feature model testing are available for verification use, special attention is given to single-feature models and they are discussed and evaluated before the comprehensive models are treated.

60. The terrain feature-vehicle models that were selected for detailed comment include soil, obstacle, vegetation, and water crossing. The obstacle and vegetation considerations are both static and dynamic. These terrain-vehicle interactions are incorporated in most comprehensive models.

Soil-Vehicle Models

61. Organized research on the off-road performance of ground vehicles began, under the impetus of World War II experience with vehicle immobilizations in the muds of the world, with study of soil-vehicle interrelations. The soil-vehicle problem was and still is a first-order one. Vehicles still can and too often do become immobilized on level surfaces simply through excessive sinkage coupled with inadequate thrust. Not only are obvious weak-soil immobilizations not yet behind us, but success or failure in many more complex terrain situations, such as those characterized by soil, slopes, and discrete obstacles, also depends upon

the soil-limited net traction that a vehicle can develop. Even in subcritical situations, the external motion resistance generated as a result of progressive soil failure under the moving vehicle is often a substantial element in the energy absorption that ultimately determines power-limited speed.*

62. All cross-country traverse models in current or recent-past use are based largely upon applicable engineering mechanics; i.e., the behavior of the vehicle is calculated primarily from the external forces acting upon it. By and large, the principal propelling forces and many of the impedances experienced by a ground vehicle are developed at points and areas where the vehicle contacts the terrain. Generally, the off-road terrain surface is soil (or snow, or muskeg, etc.) and the pertinent forces at any time and place are accordingly dependent upon the stress-strain behavior of that material, at that time and place. Thus, means to represent the soil strength from point to point realistically, and reliable methods to calculate interactions between any soil so depicted and the vehicle of interest are central to the prediction of the vehicle's performance. Selection of a suitable system for quantifying pertinent soil properties, and of related means to predict soil forces on a vehicle, is clearly a most critical problem that must be resolved in constructing a meaningful cross-country performance simulation. It will also be the most controversial. Soil-vehicle interactions have occupied the energies and emotions of vehicle mobility researchers for the 27 years there have been such workers.

Perspective

63. Study of the vehicle-soil relation began on an ad hoc basis in Great Britain, the United States, and Canada in the early and mid-1940's. The British work looked initially and primarily at vehicle flotation--at

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* Power consumption is of special interest in the design of lunar vehicles because of stringent, nonnegotiable weight limitations for such vehicles. Electric motors and supporting energy systems must be selected with an absolute minimum of overdesign. In this circumstance, it is embarrassing to acknowledge the uncertainty with which soil-related power consumption can in fact be calculated at present, even assuming that values for the parameters used to describe lunar soil strengths are essentially correct. Serious mistakes on the moon cannot be buried in a classified file. They will be broadcast immediately to a worldwide, prime-time TV audience.

the prediction of vehicle sinkage and motion resistance when proceeding in weak soils. The early work of Evans and others, simplified and extended in the 1950's by Uffelmann, is the foundation for soil-vehicle interaction expressions currently used by the Fighting Vehicle Research and Development Establishment (FVRDE)*. Soil-vehicle work in the United States began during World War II with the problem of selecting optimal tires for off-road military vehicles (Eklund, 1945), and with a study of tracked vehicle traction (Gross and Elliott, 1946). In 1945, development of means to predict soil-vehicle interaction under traffic was undertaken by the U. S. Army Corps of Engineers. Only the latter United States effort led directly to a system of vehicle-soil relations that need enter present considerations, but it generated two systems that will be discussed--the WES VCI and the WES mobility number systems. Canadian work began by treating the vehicle-soil relation in terms of the combined loading that vehicle support and traction placed on soil (Leggett and Bekker, 1948). This viewpoint was shortly eclipsed by the beginnings of the system of soil-vehicle interaction relations eclectically fathered by Bekker (1947 to 1953).** In the years following, Bekker's proposals, elaborated and widely published, were espoused by the U. S. Army Ordnance Corps and its successors, and are today generally referred to almost interchangeably as the Bekker or the LLD system. The Bekker/LLD system has seen use in recent design efforts (ATAC, NASA) and has invited a number of modifications (Firth, Janosi, Reece, McRae, Perloff, etc.) aimed at improving its realism, accuracy, and/or mathematical elegance. The most important of these has been the rational reintroduction by Reece of the essential and fundamental concept of combined soil loading under vehicle weight and traction; i.e., rejection of the convenient simplifying assumption that these may be treated independently and subsequently simply superimposed. Reece's proposals will be briefly treated, even though they currently lack the degree of development, validation, or pragmatic acceptance postulated as part of the general

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* Name recently changed to Military Vehicles and Engineering Establishment (MVEE).

** More recent Canadian work by Dickson, Leger, Nicholson, et al. (1960 to 1965) did not proceed far enough to warrant inclusion.

criterion for direct inclusion in the present survey of single-feature models.

General status

64. Despite the efforts cited, there is today still no single agreed-upon, reliable system of relations to predict the performance even of seemingly simple soil-vehicle systems. Instead, there are several, largely empirical procedures, each of which, when correctly applied to a problem within its proper scope, will provide results generally within about ±20 percent of the truth.

65. The reason the soil-vehicle interaction problem remains somewhat controversial at this late date is essentially that it is exceedingly complex, and has not yet in practice yielded to any manageable, fully theoretical approach.

66. Consider a single tire- or track-suspension assembly simply propelling itself at a constant speed, in a straight horizontal line, over and through a smooth, level, uniform homogeneous laboratory soil. Consider further that after its passage, there remains an appreciable rut. Prediction of such a seemingly elementary performance measure in this system as the depth of that rut is a problem in three-dimensional, dynamic, failure mechanics. The failing material, after deforming elastically, will compact and/or flow, and its shear strength will in general, but in markedly variable degree from soil to soil, be related (perhaps nonlinearly) to normal stresses, to strains, and to rates of strain. The dynamically applied load will be nonuniform and nonvertical. It will be applied to the soil through a readily deformable, three-dimensional interface whose shape is complex, and whose flexural responses to soil reactions are still more complex.

67. That is the laboratory situation. In the field, the soil will often be neither entirely level nor have a smooth plane surface. It will rarely be even nominally uniform with depth or for significant distances along a traverse. Soil in the zone affected by a vehicle will often exhibit mechanical responses after a vehicle running gear unit passes which differ from those it displayed as the unit first approached. The vehicle will, of course, have more than one wheel and/or track, each probably carrying different loads. Some vehicles will usually run in varying degrees in

the ruts left by others. The vehicle will generally be accelerating, braking, and/or turning in some degree; sometimes bouncing; often proceeding under the influence of other loads generated by slopes, by pushed or trailed devices, by the overriding of vegetation, and/or by the surmounting of minor obstacles.

68. The situation is not hopeless, only nearly so. Needless to say, however, considerable simplification and idealization of the total real world picture are required. And, as it has thus far resulted, even after drastic pruning, the problem still requires an almost wholly empirical approach.

Strategies available

69. Possible approaches to computing soil-vehicle interactions are to:

a. Describe the overall situation in fundamental theoretical terms, using equations of motion and a complete set of constitutive equations for the soil material (along, of course, with suitably precise boundary definitions).

b. Generalize sound, systematic test data from the laboratory and the field in terms of proper nondimensional coefficients (numerics).

c. Postulate a grossly simplified soil loading system whose response is assumed to be analogous to that of a wheel and/or track in the same soil, and to develop reasonable analytical expressions relating the response of a real system to that of the simplified system or analog.

d. Seek statistically reliable empiric correlations among various arrays of quantitative vehicle and soil parameters without concern for underlying theoretical causes.

Possible variations among these strategies are illustrated in fig. 1. All four strategies, sometimes mislabeled, have been attempted in some degree at various times and places. Attempts at a respectably theoretical treatment continue at several levels, on several computers, in several places, but there is as yet no operational soil-vehicle interaction model that may seriously be classed as theoretically founded. The development of such a

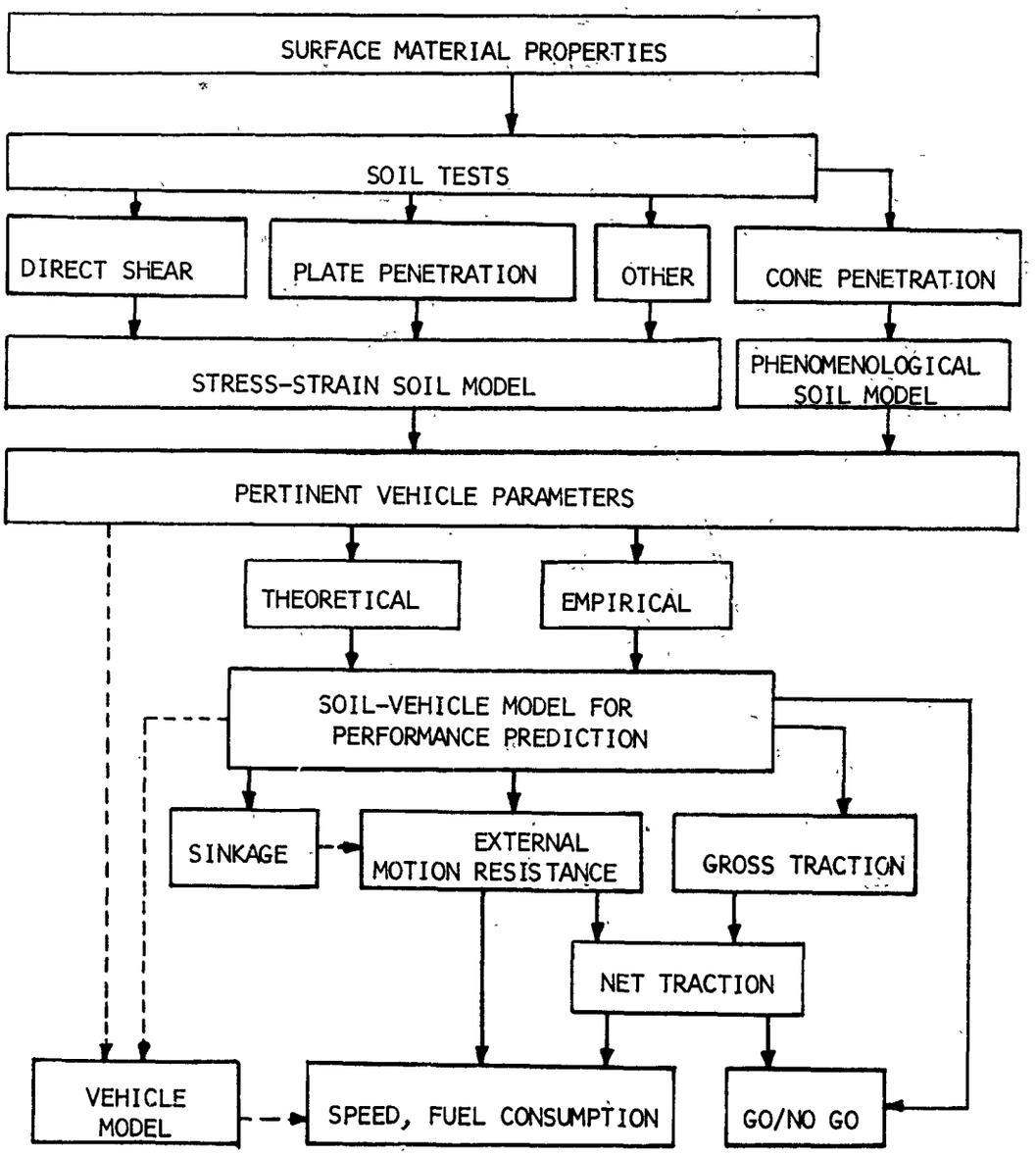


Fig. 1. Flow chart illustrating possible strategies for coupling soil and vehicle models to predict vehicle performance

model, finite-element and other powerful computer techniques notwithstanding, awaits the statement and validation of suitable constitutive equations for soils. Such wisdom will probably come, when it does, from more fundamental soils and mechanics studies than are now embraced by ongoing soil-vehicle research.

70. In further discussions it is useful first to consider soil-vehicle models in terms of the soil models and the vehicle models employed. A soil model details a quantitative working concept of soil behavior under load, defines soil parameters by which to quantify soil strength within this concept, and, if it is indeed a working model, it specifies procedures to conduct and interpret tests on a given soil body by means of which values may be assigned to the soils-system parameters used. Idealizations and simplifications made in describing the vehicle, or its contact areas with the soil and their loadings, constitute a vehicle model. Quantifications of vehicle characteristics that enter this model are the vehicle parameters. A set of relations between measures of vehicle performance and some combination of the soil and the vehicle parameters comprises a soil-vehicle model.

Soil models

71. The common assumption of all soil models, whether explicit or not, is that the controlling mass soil reaction in vehicle-related behavior is the soil's shearing resistance, and that this in turn is related to soil displacements. Freitag et al. summarized the current status of soil modeling from the vehicle viewpoint in a paper for the Fourth ASAE Symposium on Similitude (1969). Tables 4 and 5 are reproduced from this paper. Table 4 lists soil properties and table 5 presents a compilation and classification of soil-measuring devices. Table 4 includes a number of soil dynamic properties that are in fact not yet used in any soil-vehicle model, nor even widely measured in conjunction with related laboratory research. Principal among the latter are coefficients reflecting rate dependent shear strength or viscous behavior, which in most soils will clearly be non-Newtonian, and probably nonlinear. In addition, viscous reactions may be expected to differ, sometimes radically, when strains are small and shear rates low, and when large strains and high shear rates are involved. The dramatic example

Table 4

Soil and Soil-Material Properties

Soil Property	Symbol	Dimension
Cohesion	c	FL^{-2}
Friction angle	ϕ	-
Viscosity	η	$FL^{-2}T$
Wet unit weight	γ	FL^{-3}
Density	ρ	$FL^{-4}T^{2}$
Coefficient of compressibility	m, n_c, b	$F^{-1}L^{3}$
Tensile strength	T	FL^{-2}
Modulus of elasticity	E	FL^{-2}
Modulus of rupture	R	FL^{-2}
Grain size	r	L
Soil-Material Property		
Adhesion	A	FL^{-2}
Soil-material friction angle	δ	-

Table 5

Compilation and Classification of Soil-Measuring Devices

Test or Device	Measurement	Derived Outputs	Class of Output	Soil Condition*	Remarks
Triaxial	Stress, strain, time	c, ϕ, η, m, n_c, b, E	Property	1, 2, 3	Laboratory, complex
Unconfined compression	Stress, strain, time	q_u ----------	Parameter ------	2, 3	Special case of triaxial
		c ----------	Property -------	2	$\sigma_2 = \sigma_3 = 0$
Direct shear	Stress, strain, time	c, ϕ, η	Property	1, 2, 3	Drained tests, laboratory special equipment required
Ring shear	Torque, angle, load, rate of rotation, sinkage	c, ϕ, η -------	Parameter ------	1, 2, 3	Boundary conditions not adequately defined. Usable as analog of traction
		K ----------	Analog ---------	1, 2, 3	
Sheargraph	Torque, load	c, ϕ	Parameter	1, 2, 3	Boundary conditions not adequately defined
Shear vane	Torque (angle)	c ----------	Property -------	2	
		s ----------	Parameter ------	1, 2, 3	Stress and boundary conditions unknown
Plate penetration	Pressure, sinkage, time	k_c, k_ϕ, n ----	Parameter ------	1, 2, 3	Usable as an analog of vertical load application
		k'_c, k'_ϕ, n' ----	Parameter ------	1, 2, 3	
		c, η ----------	Property -------	2	
		C_r ----------	Analog ---------	1, 2, 3	

* 1 designates that $c = 0$ soil; 2, $\phi = 0$ soil; and 3, c and $\phi \neq 0$ soil.

Table 5 (Continued)

Test or Device	Measurement	Derived Outputs	Class of Output	Soil Condition*	Remarks
Tilting plate penetrometer	Horizontal and vertical forces; horizontal and vertical displacements	H/Q, $\dot{x}/\ell$	Analog	1, 2, 3	Analog of combined traction and flotation
Cone penetrometer	Pressure, sinkage, time	CI, PR --------- c, η ---------- G ----------	Parameter ------ Property ------- Parameter ------	1, 2, 3 2 1	
Vibratory test	Frequency, amplitude, phase	η, E	Property	1, 2, 3	Viscoelastic response at small strains
Tension test	Axial load	T	Property	2, 3	
Beam loading	Load	R	Property	2, 3	
Nuclear devices	Counts	γ, ρ	Property	1, 2, 3	Sampled volume not always known
Samplers	Volume, weight	γ, ρ	Property	1, 2, 3	
Particle-size test	Percent passing	r	Property	1, 2, 3	
Exact models, two sizes	Performance parameters of interest	Prediction relations	Model	1, 2, 3	Validity limited to soil conditions tested, and responses are in regime of same mode
Prototype	Performance parameters of interest	Performance parameters	Model	1, 2, 3	Valid only in soil conditions tested

of silicone putty, which flows under long-term gravity loads but shatters on impact, may be only an extreme illustration of the behavior of many soils.

72. Pedologically, soils are bodies of material, one of whose most striking features is stratification in essentially horizontal layers: the A horizon, the B horizon, etc. In civil engineering work, the upper horizons are normally removed and the assumption made, barring gross anomalies, that the material is essentially uniform throughout the remainder of the body. This assumption is not appropriate when considering vehicle performance. A vehicle of anything like normal size will deal most of its life with soils that are distinctly and noticeably stratified. This stratification will not necessarily follow the pedologic definitions and will in most cases be potentially more complex. Instead of two or three visibly discernible horizons, there will usually be considerable variation in strength with depth within the soil layers that the vehicle involves. Strength profiles will range from thin layers of weak, slippery materials over firm, strong ones to relatively strong crusts over weak layers, with endless possible variations between. Some typical cone index profiles for different types of marginal surface materials from the ground mobility standpoint are shown in fig. 2. Accordingly, field descriptions of soils for vehicle purposes must in some way reflect the soil strength profile to a suitable depth. The "suitable depth" will in general vary with the vehicle size and weight, its running gear type, the mean level of soil strength, and the shape of the soil strength profile. The profile effect may be incorporated by presenting complete or idealized profiles or by using various averaging rules. Field measurements must include sampling in some way over the depth of interest.

73. The strength of a given soil material will in general vary with its water content and its state of compaction or density. It may be largely affected also by its in situ microstructure. The reactions of a body of this same soil material in nature or in a system simulation in the laboratory will, in addition, be affected by its geometry (its boundary conditions or stratification). When the soil body is greatly disturbed, as by the passage of a wheel leaving a rut, the boundary conditions are

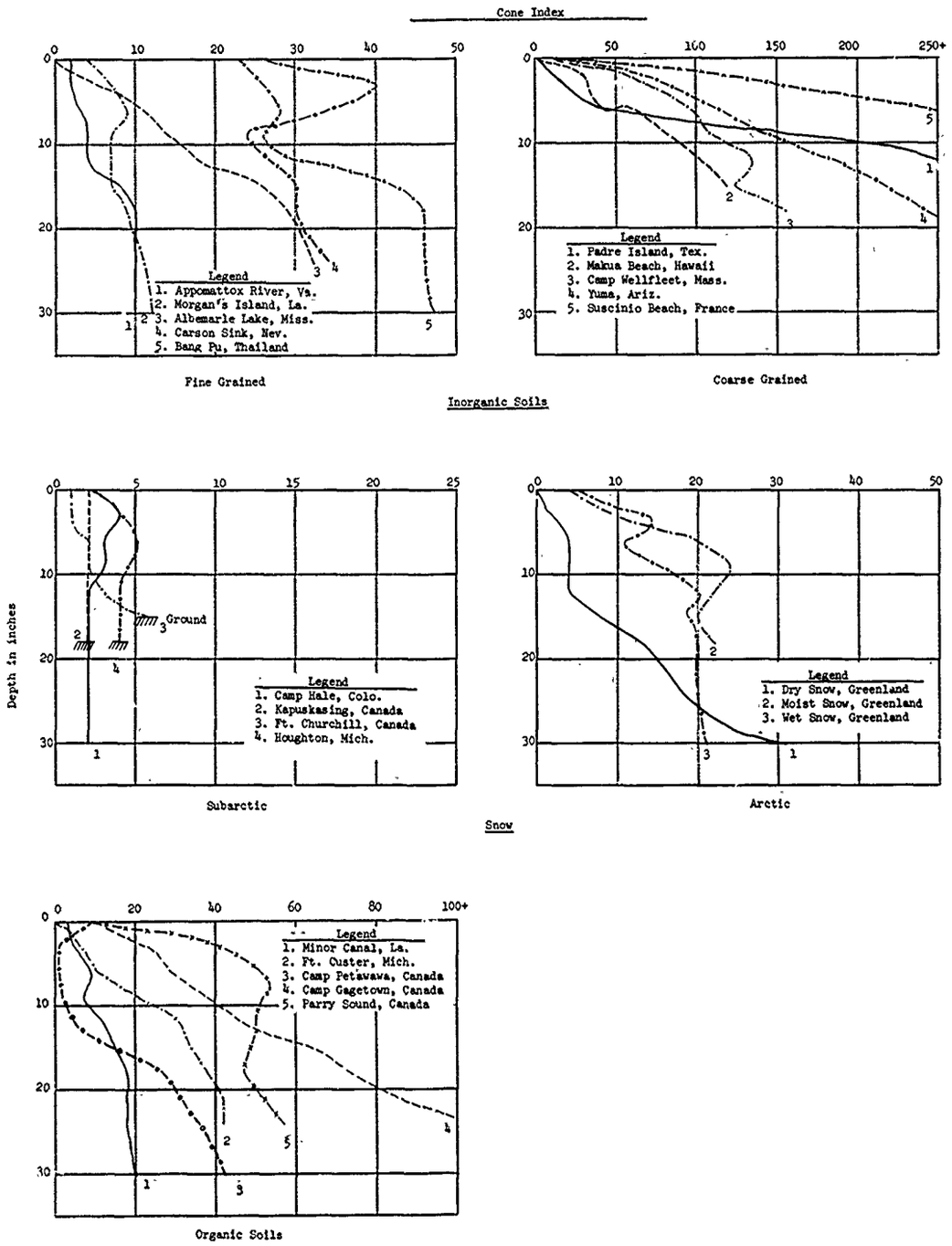


Fig. 2. Cone index profiles for different types of marginal surface materials

changed for the next wheel, and the in situ microstructure of the soil may be disturbed to the degree that its strength is also changed. The importance of the latter effect, soil "remolding," to vehicle performance, especially in marginally weak soils, was demonstrated early in the WES trafficability research. Realism in modeling soils dictates that this phenomenon be reflected in some way. This is particularly important for the real world case of a group of vehicles moving off-road where the tendency is in fact to track one another even where terrain or other real constraints do not force such channeling.

74. The ability of a soil to develop adequate bearing and traction capacity under the traction elements of ground vehicles is primarily a function of the shearing resistance of the soil. There are essentially two methods of describing soil strength for ground mobility purposes and three soil models that are currently in use in the United States. The soil models are the WES vehicle cone index (VCI) method, the WES mobility number method, and the Bekker/LLD method. Both WES methods use a cone penetrometer (fig. 3) to obtain an index of the shear strength of the soil in terms of a single number, and the Bekker/LLD method uses a two-part bevameter (fig. 4) consisting of a plate penetration and an annular ring shear device for obtaining pressure-sinkage and surface shear strength data, respectively, from which five soil parameters are currently derived.

75. WES VCI soil model. The WES VCI model is wholly empirical. The surface body of material is described in terms of its general nature (fine-grained or coarse-grained inorganic soil, organic soil, or snow) and its profile of resistance to penetration by a small standardized cone (cone index profile), corrected in soft, fine-grained inorganic soils for decreases in strength due to remolding as determined by a remolding test (equipment is shown in fig. 3). When the remolding test is used to obtain a remolding index, the soil is described in rating cone index (RCI),* which

* The rating cone index is given by

$$RCI_i = CI_i \times RI_i$$

where CI = cone index for soil layer "i" and $RI_i \leq 1.0$ = remolding index for that layer.

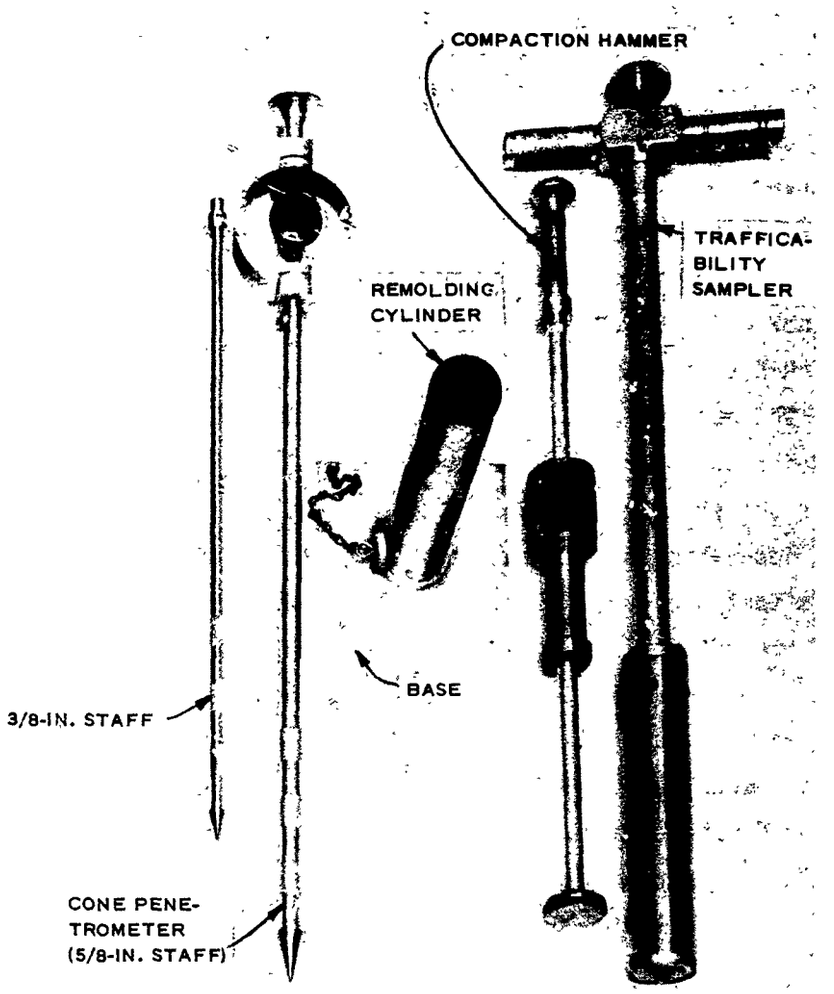
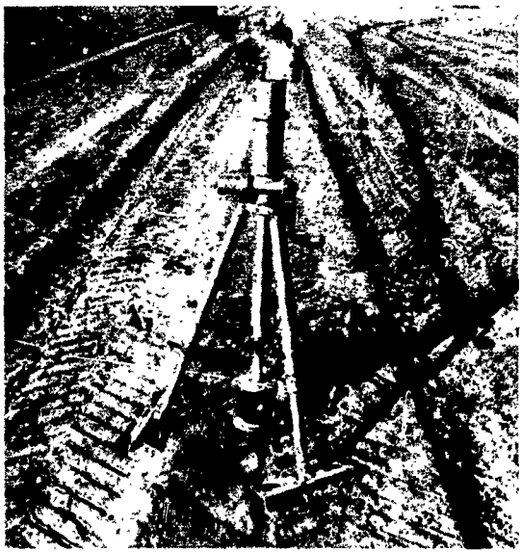


Fig. 3. WES soil trafficability equipment

is the product of the cone index and remolding index for the same soil layer. While cone penetration resistance is assumed to be a function of shear strength, no attempt is made to define the specific relation which is different for the several soil classes. An average cone index (CI) over a "critical layer" appropriate to the vehicle type and weight and the strength profile of the soil and corrected for remolding as necessary is usually used in place of the full profile. If the strength of the soil increases with depth in the critical layer and the 6-in. layer below the critical layer, the standard procedure for determining the critical layer is

a. Plate penetration device



b. Annular ring shear device

Fig. 4. LLD portable bevameter

used. However, if the 6-in. layer below the critical layer is weaker than the normal critical layer, the deeper layer is considered to be the critical layer. In this way and with obvious, considerable loss of information, the soil strength at a given point may be expressed, and therefore easily mapped, in terms of a single quantitative index, plus, as necessary, an indication of the class of materials involved.

76. Considerable terrain mapping has been done on a worldwide basis using the VCI method. Simplifying procedures have been developed for crudely estimating changes in cone indexes due to various changes in soil moisture or, more directly, due to rainfall, drainage, and drying. Other procedures have been developed for making reasonable estimates of on-the-ground cone indexes through air-photo interpretation. An example of the resolution that can be obtained from good quality, large-scale aerial photography is shown in table 2, which shows the map units used in the MERS study. The indicated level of resolution, which may in fact be even less in practice, is still really only sufficient to distinguish three levels of soil strength for a given vehicle: strong enough, maybe, and too weak. This is probably as much resolution as can be hoped for for some time to come unless considerable advances are made in meaningful remote sensing, and then exploited. More numbers might be added in order to play more elaborate games, but improvement in the results would be questionable at best.

77. Over the years considerable field data have been collected from various parts of the world using the WES VCI method. These data have been analyzed for the purpose of establishing a soil classification system and procedures for estimating soil moisture content and strength in terms of CI and RCI for the wet season only. The dry season was not considered because soils that undergo seasonal wetting and drying are normally trafficable in the dry season. The classification system has been applied to soils in temperate and tropical climates and separate reports have been published. The classification system and the data available in the reports have proved useful in preparing tactical and strategic soil trafficability maps of remote areas of the world.

78. <u>WES mobility numeric soil model.</u> The WES mobility numeric model is also essentially empirical and, at this time, is applicable to

pneumatic-tired vehicles. The mobility numeric model is based upon generalization of systematic single wheel tests on prepared homogeneous (uniform strength profiles) laboratory soils through dimensional reasoning. Further work is required to improve procedures for predicting multi-wheeled vehicle performance from the single wheel results and in defining soil descriptors for natural soil conditions equivalent to those used to reduce laboratory results.

79. In sands, both in the laboratory and in the field, penetration-resistance gradient* is used to characterize soil strength. In laboratory clay soil, CI** is used as an index of soil strength. In natural clay soil, field test data indicate that prediction accuracy was slightly better when RCI was used rather than CI.

80. Bekker/LLD soil model. This model describes soil responses to loading in terms of three parameters that reflect soil deformation under vertical loads, and three more (one of which is currently assumed to be constant) that express the horizontal shearing resistance of the soil as a function of mean normal pressure and horizontal deformation. The two sets are treated independently. Horizontal shear parameters (c, ϕ, and K) are extracted from tests with a small grousered annular ring-shear device by best-fitting the results to a modified version of the Coulomb-Micklethwait expression whose current form is:

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* Penetration resistance gradient (G) is also obtained from tests with the standard cone penetrometer. It is given by

$$G = \frac{\overline{CI}_z - CI_0}{z/2}$$

where

$\overline{CI}_z$ = average cone index in the 0- to z-in. soil layer

CI_0 = cone index when $z = 0$

z = depth of penetration of the cone base into the soil surface

** Cone index is obtained from tests with the same standard cone penetrometer used in the VCI method and is analogous to the ultimate bearing capacity of clay. In a clay homogeneous with depth, there is little change in cone penetration resistance after the base of the cone passes below the soil surface.

$$\tau = (c + p \tan \phi)\ (1 - e^{-j/K}) \qquad (1)$$

where

τ = mean shear stress

p = mean ground pressure on shear area

j = horizontal travel of shear elements from initial position

The parameters c and ϕ are clearly analogous to soil cohesion and angle of internal friction as used in engineering soil mechanics, but values measured by this method in a given soil do not duplicate those obtained by such more familiar soil mechanics procedures as the triaxial test. The parameter K tends to give calculated curves of vehicle pull versus slip a reasonable shape, but is erratic in practice, and is highly instrument-dependent. In the most recent field correlation study, values of K derived from actual ring-shear tests were not used, and K was instead assumed constant and equal to one (inch).

81. Vertical deformation parameters for a soil are derived by force-fitting results from pressure-penetration tests with small flat plates of two or more diameters to the equation, traceable to Bernstein via Goriatchkin:

$$p = kz^n \qquad (2)$$

where

p = average pressure

z = depth of plate penetration from the surface

$k = \frac{k_c}{r} + k_\phi$

r = plate radius

and k_c, k_ϕ, and n are the soil response parameters. The plate tests appear to give results that correlate with vehicle tests in the laboratory but not in the field. This is probably due to variations in soil strength profile with depth, which are always present in the field (and seldom in well-run laboratories), whose influence upon a vehicle cannot properly be sensed by small plates penetrating only a few inches below the soil surface.

82. Extensive publication of the Bekker/LLD soil value system has

attracted several proposals for modifications to its plate pressure-sinkage treatment (Reece, Firth, McRae, Perloff, Assur, etc.). The most substantial are those of Reece and of Firth. Reece's preliminary revision substitutes for Bekker's version of the Bernstein equation the dimensionally more digestible expression:

$$p = (ck'_c + \gamma \times \frac{b}{2} \times k'_\phi)(z/b)^n \qquad (3)$$

where

c = soil cohesion

γ = soil density

b = the least linear dimension of the test plate

The new constants, k'_c and k'_ϕ, are, like n, now dimensionless, which makes the method more civilized, at least. Firth's modification incorporates a plate shape or aspect-ratio term. According to both investigators, their respective alterations improve the fit of the basic power equation to laboratory data, as might reasonably be expected. Neither, however, increases the ability of small, shallow plate tests to quantify real natural soil bodies for mobility purposes. The order of improvement in predictive precision that either offers in real terrain is thus minor, and it is not surprising that neither has as yet been incorporated into any operating cross-country model.

83. It is to be noted that there are a number of forms of three-parameter equations other than the power equation that may be fitted satisfactorily to the normal run of plate pressure-sinkage test data. Assur has proposed one alternative, Murphy another, and several more were examined by M. E. Smith during the ANAMOB study for demonstration purposes. In the present absence of a sound basic theory to explain (rather than merely to represent) the phenomenon, selection of one or another is largely a matter of taste, intuition, and/or mathematical tractability. At this juncture, the broad publication by Bekker of the Bernstein power equation has effectively preempted the curve-fitting approach.

Vehicle models

84. The vehicle models consist essentially of a description of the

vehicle design and potential contact areas with the soil, and their loadings, in an idealized and simplified form compatible with a particular soil-vehicle model. Thus, the vehicle characteristics used are dictated from a soil point of view. Most laboratory work has been with scale vehicle models or with single traction elements, either scaled or prototype. The Bekker/LLD method has evolved primarily from small-scale laboratory tests, the WES VCI method from prototype natural soil tests, and the WES mobility number from laboratory single-pneumatic-wheel tests.

85. In a vehicle sense, a vehicle model also includes a straightforward automotive engineering model relating speed to vehicle engine and power train-torque characteristics or to total traction available for propelling the vehicle. All comprehensive models incorporate such a submodel as a base line from which terrain demands for traction force required to overcome resisting forces are subtracted.

86. Data for speed-traction force models are usually available for all vehicles that have undergone engineering tests at Aberdeen Proving Ground (APG). This test is run on a level, paved surface in all forward gears.

87. If performance test data are not available, a speed-traction force model can be computed from engineering data in the form of a curve relating transmission torque output to transmission rpm output provided by the manufacturer. Traction coefficients and speeds can be computed for selected transmission torque-speed output pairs using the following simple equations:

$$T_c = \frac{\eta \times Q_t \times D_g}{R_w \times W} \tag{4}$$

where

T_c = traction force coefficient

η = overall power train efficiency

Q_t = transmission torque output, ft-lb

D_g = final drive gear reduction ratio

R_w = pitch radius of drive wheel or sprocket, ft

W = gross vehicle weight, lb

and

$$S_t = \frac{0.6818 \times rps \times C_w}{D_g} \tag{5}$$

where

S_t = track or wheel speed, mph

rps = transmission output

C_w = circumference of drive wheel or sprocket, ft

Maximum traction coefficient on a paved surface is usually limited to about 0.80, at which level tire or track slip will approach 15 percent. For traction levels below about 0.60, slip is small, and track or wheel and road speeds are considered to be equal.

Soil-vehicle models

88. The mechanics involved in coupling soil and vehicle models to predict vehicle performance on a smooth, level soil surface are outlined in fig. 1. All soil-vehicle models specify a set of surface material properties for which soil tests are conducted to obtain the pertinent data, and implicitly or otherwise, involve a soil model to arrive at the necessary soil descriptors. The soil model is coupled with pertinent vehicle parameters to form, theoretically or empirically, a soil-vehicle model from which predictions are made. Go-no go performance may be predicted directly from a soil-vehicle model or handled by comparing available traction to total impedance. Vehicle speed is more usually computed by first determining external motion resistance and gross traction and subsequently adjusting the traction to the level required to propel the vehicle. Details of the soil-vehicle models discussed in the following paragraphs are presented in Appendix A.

89. WES VCI soil-vehicle model. The WES VCI soil-vehicle model predicts the performance of a vehicle in terms of go or no go for a prescribed number of passes (usually 1 and 50) over the same soil traveling in a straight-line path. The model also predicts the maximum drawbar pull and maximum slope negotiable, and motion resistance, for soils with strengths above the minimum needed to just go. To be adequate for passage of a vehicle, a soil must have sufficient bearing capacity to prevent the vehicle

from sinking too deeply and sufficient traction capacity to provide the thrust needed to overcome the motion resistance generated in rut formation, slope climbing, obstacle negotiation, etc. Bearing and traction capacities are both functions of the shear strength of soil. Traction failure can occur on a soil having high bearing strength and with little vehicle sinkage. Sinkage failure to the point of vehicle immobilization is always accompanied by traction failure.

90. Soil strength is measured empirically in terms of cone index (CI) and remolding index (RI), and when remolding index is used, soil strength is expressed in terms of rating cone index (RCI). For coarse-grained soils, organic soils, and snow, vehicle performance is related to CI; and for fine-grained soils, it is related to RCI. Empirically derived equations and graphs and their use to relate strength of inorganic soils to vehicle performance are given in Appendix A. The system has been used extensively by WES in conduct of mobility research programs. It has been used in a number of studies including Swamp Fox I and II, REVAL WHEELS, Wheeltrack, RAMS, 1/2-ton carrier development program, and RAC analysis of Surface Cross-Country Vehicles in Vietnam. The system has also been used to describe terrain conditions for QMR's and SDR's.

91. WES mobility numeric soil-vehicle model. The basic WES numeric soil-vehicle model was developed from a dimensional analysis of results obtained in single-wheel laboratory tests in homogeneous sand and clay soil. The WES sand mobility numeric is

$$\pi_s = \frac{G(bd)^{3/2}}{W} \times \frac{\delta}{h} \tag{6}$$

where

G = average rate of increase of cone index with depth in sand

b = tire section width

d = tire diameter

W = load on tire

δ = tire deflection

h = tire section height

The original WES clay numeric was developed from test data with toroidal tires only and took the form

$$\pi_c = \frac{Cbd}{W}\left(\frac{\delta}{h}\right)^{1/2} \tag{7}$$

where C = average CI in the 0- to 6-in. layer. Analysis of later test data showed that the performance of tires of a wide range of b/d values was predicted best by

$$\pi_c = \frac{Cbd}{W}\left(\frac{\delta}{h}\right)^{1/2}\left(\frac{1}{1 + \frac{b}{2d}}\right) \tag{7a}$$

Performance* coefficients (pull/load and towed force/load) are each related to the mobility numeric by curves derived from extensive laboratory tests.

92. An anlysis of data from prototype vehicle tests in natural clay soil showed that slightly better predictions were obtained using RCI rather than CI. Therefore, the revised clay numeric used for prototype vehicles on natural clay soil is

$$\pi_c = \frac{RCI\ bd}{W}\left(\frac{\delta}{h}\right)^{1/2}\left(\frac{1}{1 + \frac{b}{2d}}\right) \tag{7b}$$

The general curves and equations for vehicle performance in the field are shown in fig. A8, Appendix A. The curves used in the design** of the MEXA vehicles shown in fig. A9, Appendix A, were based on the original clay numeric (the original numeric was the only one that existed at the time the MEXA vehicles were designed). The abscissa of these curves is the reciprocal of the original clay numeric.

93. Several types of equations have been written to express the empirical relation of performance to the clay mobility numeric. Two equations for rectangular hyperbola that reasonably approximate the experimental relation of the pull coefficient and the towed coefficient to the sand mobility numeric are given in fig. A10, Appendix A.

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* In this model the optimum vehicle performance is considered to be in the vicinity of 20 percent slip. While the approach and supporting data permit predictions at other slips, predictions of pull, torque, and sinkage are usually made for the 20 percent slip point only.

** The obvious reason for offsetting the curves was to have a "built-in" safety factor.

94. Bekker/LLD soil-vehicle model(s). The Bekker/LLD soil-vehicle model assumes implicitly that the plates used in penetration tests are simple analogs of the running gear of tracked vehicles and wheeled vehicles equipped with relatively large, deflected tires. The mathematical model, $p = kz^n$, used to represent the results of plate penetration tests, becomes the basic mathematical model for tracks and large tires as well. Solving the p-z relation for z, the uniform sinkage relation for tracks and large tires is

$$z = (p/k)^{1/n} = (W/Ak)^{1/n} \tag{8}$$

where

z = sinkage

p = pressure

$k = k_c/b + k_\phi$

n = exponent of sinkage

W = load per tractive element

$A = b\ell$ for track: b = width, ℓ = length;

for wheel: b^* = maximum width of unloaded tire, $\ell = 2\sqrt{\delta(D - \delta)}$ where D = diameter and δ = deflection.

Motion resistance (R) per tractive element in all cases is the same and is the sum of compaction resistance (R_c), gravitational resistance (R_θ), and mechanical rolling resistance of suspension components (R_γ).** A bulldozing term, used at one time, is no longer applicable to R.

95. The Bekker/LLD soil-vehicle model assumes that the action recorded in ring-shear tests simulates the shearing action between the running gear of vehicles and the soil. The mathematical expression (eq 3) used to represent the results of a ring-shear test is accordingly considered to be the basic model for vehicle response. Equations for vehicle traction under various conditions that are derived from this expression are listed in Appendix A. In its present form, the Bekker/LLD soil-vehicle

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* Sometimes b = minor axis of the ellipse representing the print of a tire.

** R_γ for an existing vehicle is usually obtained from the vehicle manufacturer or APG engineering tests.

model relates gross traction and slip in the equation

$$H = (Ac + W \tan \phi) \left[1 - \frac{K}{s\ell} \left(1 - e^{\frac{-s\ell}{K}} \right) \right] \tag{9}$$

where

K = tangent modulus of deformation

s = slip ratio

c and ϕ are soil cohesion and friction, respectively, as measured by the ring-shear apparatus

96. Probably the greatest difficulty in using the Bekker/LLD model arises in deciding what equations should be applied to a specific set of conditions. For example, the equations used for predicting the performance of rigid wheels differ from those used for predicting the performance of deflected wheels. Both are used on occasion for pneumatic tires. The difficulty arises because a tire on a yielding surface may assume any shape between that of a rigid wheel and that of a deflecting wheel on a hard surface, depending on the relative stiffness of the tire and the surface.* Presumably the true condition will be bounded by the two extreme assumptions. However, in the absence of quantitative guidelines for making this selection, one investigator may select the rigid-wheel equations and another the deflected-wheel equations for the same set of conditions and obtain significantly different prediction values from the same set of input data. This may or may not be important when the model is used only to compare the performance of a group of vehicles for a given soil condition, but becomes of clear importance when actual performance of one vehicle must be predicted.

97. Reece modified the Bekker/LLD equation for total traction as related to slip by including an expression for frictional thrust from the sides of a grousered track or wheel. Appendix A lists both equations.

98. WES-WNRE soil-vehicle model. The WES-WNRE soil-vehicle model was developed basically along the lines of the WES VCI system, but solely for one-pass performance predictions. In addition to computing go or no go,

* On an unyielding surface, only the tire yields to the pressure of the load carried by the tire, whereas on a soil surface both the tire and the soil yield. When a tire is operating in a soft soil, the yielding of the soil may be sufficient to allow very little yielding of the tire. In this case, the shape of the tire is similar to that of a rigid tire. (See WES Technical Report No. 3-516, Report 3.)

the model includes an equation for predicting soil strength, in cone index terms, required to permit one pass of a vehicle in unaccelerated, straight-line motion over smooth, level soil, and the soil-generated component of its motion resistance. These relations were developed empirically from tests of a variety of medium and lightweight military vehicles in fine-grained soils. They have served as an interim basis for making first-pass performance predictions and were adapted by WNRE to the needs of two subsequent vehicle go-no go mapping studies.

99. WNRE-CAL soil-vehicle model. This model was designed to utilize soil strength data in the form given in the WES/ARPA MERS surface composition factor maps of selected areas in Thailand. In these maps soil mass strength is presented in cone index terms, and surface tractive capacity by values for the direct shear parameters c and ϕ. The WNRE-CAL model joins the now-conventional Micklethwait-Coulomb assumptions for gross traction with the WNRE-WES projection of motion resistance (paragraph 98) to predict vehicle net traction. It was first used in a computer program to prepare go-no go maps for specific vehicles operating in terrain prescribed according to the quantitative descriptive system used in the MERS factor family maps. It was more recently adapted for use in a conceptually similar go-no go study of lunar vehicles in lunar landscapes. There has been no experimental verification of the reliability of the WNRE-CAL model.

100. Perloff soil-vehicle model. This model was developed to provide soft-soil mobility input to a digital computer simulation of tank-to-tank combat intended for tank design evaluations. When this study began in 1960, Perloff concluded that neither the Bekker/LLD formulations for tracked vehicle performance in soft soils nor then-extant procedures for expanding scale-model results to full-size predictions were acceptable. The new model he proposed as a result accepts one-dimensional pressure-sinkage relations generalized from observations on small-scale models (or from any other source) and by computer iteration determines a vehicle sinkage and trim that satisfy its requirements plus those of static equilibrium, including a slope component. Traction and slip are computed for the resulting equilibrium track sinkage and attitude on the basis of an approximate shear stress/strain relation derived from a traction oriented analysis of

soil direct shear test results. Despite its considerable complexity, the Perloff model treats the effects of tractive and normal forces as independent within a two-dimensional static framework, which makes it fundamentally as suspect as the less pretentious models it was intended to supplant. Since there have been no demonstrations of the predictive accuracy of the Perloff model, of the usefulness of the several elaborate new assumptions it employs, or even of practical means to measure in the field the soil parameters involved, this model is not considered a serious candidate for use in a complete vehicle cross-country performance simulation at this time. Equations pertinent to the explanation of the system are listed in Appendix A.

101. Lunar models. Several agencies and/or individuals have developed computer programs or models for NASA lunar roving vehicle studies. The soil-vehicle interaction used in these models in most instances is a simplified version of one or another of the soil-vehicle models discussed. For example, one study cites Assur's pressure-sinkage relations but a linear relation is actually used. In the other cases, the Bekker system is frequently cited but not strictly adhered to. In no case prior to a modest program begun by NASA at WES early in 1970 was any attempt made to validate the soil-vehicle interaction part of the analytical model objectively. The work of agencies that fall into this category are Chrysler, Brown Engineering, Bendix, Grumman, Cornell Aeronautical Laboratory, and Booz-Allen.

102. British soil-vehicle model. Using essentially the classic soil mechanics soil model, Uffelmann derived equations for maximum drawbar pull for tracked vehicles and rigid wheels. These employed Micklethwait's equation for maximum tractive effort and include a correction for slope angle of the terrain. Uffelmann's equation for motion resistance for tracked vehicles and rigid wheels in sand is derived by using Bekker's equation for motion resistance to estimate an unknown power function which he postulated. Equations for motion resistance and sinkage in clay of tracked vehicles assume Evans' theory (1950). These and other equations are listed in Appendix A. Laboratory verification of the wheeled vehicle relations on uniform remolded clays is reported in the literature. Confirmatory field trials of the Evans' tracked vehicle equations were reported.

103. Other soil-vehicle models. The works of CARDE/Dickerson, Eklund, and Kraft-Luming/Dayton were reviewed but are not included because they are not pertinent to the discussions of the analytical models nor are they complete models.

Verification

104. In conducting the literature review for this study, the literature that contained measured and predicted off-road prototype vehicle performance and soil data that could be used in an objective evaluation of the prediction accuracy of soil-vehicle models was sorted out for study. The specific vehicle performance parameters of interest in the sorting process were minimum soil strength requirements in terms of VCI, drawbar pull, gradeability, motion resistance, and sinkage. Gradeability was included because of its association with drawbar pull performance prediction; i.e., the tangent of the maximum slope angle negotiable can be closely estimated from the drawbar pull coefficient. In addition to the requirements set forth for vehicle performance and accompanying soil data, the data were also assembled as to the applicability of the model to vehicle and soil types.

105. Selection of field test programs. The vehicle field test exercises selected for preliminary study included Project Wheeltrack (1963), Swamp Fox II (1964), MERS One-Pass Program (1965), Keeweenaw Tracked Vehicle Speed Tests (1969), and Mobility Exercise A (1965-1970). A preliminary examination of the exercises revealed that except for the MEXA program, only the Bekker/LLD and the WES VCI soil-vehicle models were employed and that several versions of the models were used in evaluating measured versus selected prediction performance parameters. With the advent of the MEXA program, the WES mobility number models were used in the design phase and later exposed to the prediction evaluation phase of the program.

106. At first glance, it appeared that the field exercises reviewed contained considerable soil-vehicle test data to perform an appraisal of soil-vehicle models cited, but when the data were organized in a manner required to produce a meaningful evaluation, the resultant data were minimal for reasons stated in paragraphs 56 and 57. Similar difficulties were also expressed by Pleuthner (1969) in his recent report on a critique on the

performance of off-road vehicles, full-scale test results, and prediction method evaluation. In the final analysis, it was decided to use only the MEXA data to conduct an evaluation of prediction accuracy of soil models because the data met most of the evaluation objectives set forth. Of equal importance was the fact that the MEXA data permitted checking the most recent versions of three soil-vehicle performance prediction models (WES VCI, WES mobility number, and Bekker/LLD) used in current vehicle design and evaluation studies.

107. Before discussing the MEXA program, some general comments are presented regarding the limitations of past off-road vehicle testing and reporting with the hope that these comments will help to improve future vehicle testing programs, at least to a point that the data produced will be more reliable and useful for model verification. The test programs and reports reviewed were not planned to permit an orderly assessment of the validity of the model or models used. Usually all the performance parameters of a given model were not tested. Frequently, a field judgment was made by the project leader to collect only minimal data, thus allowing only a check of a simplified or old version of the model. VCI and drawbar pull tests are the most common performance parameters found in the literature. Motion resistance and sinkage seldom accompany drawbar pull data for the same soil condition. Some slope climbing and speed tests with accompanying soil data are reported. Between and within exercises, testing procedures varied in the number, type, and spacing of soil strength measurements made; in conducting the vehicle test; and in recording and reducing vehicle response measurements. Most field tests are reported in summary type reports and are incomplete in detail of testing procedures, presentation of actual data measurements, model used and any modification made, sample computations, and a listing of all measured and predicted performance for comparative purposes.

108. MEXA program. In 1964 the MEXA program was initiated by the late R. R. Philippe and conducted at Vicksburg, Miss., to consider the various quantitative elements of the off-road ground mobility problem by a group of experts in the field. The specific purpose of the program was to design three vehicles capable of carrying a 2-1/2-ton payload with

approximately a 50 percent improvement in soft-soil performance over conventional military vehicles. The design soil strength selected was an RCI of 7, which is just adequate for a man to walk on with some difficulty. Methods of design based on research studies at WES and LLD, TACOM, were employed to specify vehicle traction components that would yield the desired soft-soil performance. The final configurations were two wheeled vehicle concepts and one tracked vehicle concept that reflected chiefly the soft-soil requirements, but other requirements such as obstacle performance were also considered in the design. One requirement was the development of a design that would utilize a minimum of new components.

109. After the test beds proposed in the MEXA study were fabricated and delivered, a second meeting of the expert group was held to design and implement a field program for testing the three vehicle test beds. The primary purpose of the test program was to evaluate the methods used in the design, compare the performance of the MEXA test beds with that of approximately comparable conventional military vehicles on smooth, level surfaces, and check or develop pertinent terrain-vehicle relations as required in analytical modeling of vehicle performance in terms of speed. The MEXA and military vehicles used in the test program are shown in figs. 5 and 6, respectively. Pertinent characteristics of these vehicles are given in table 6.

110. Most of the field tests in soft soils were conducted at Carson Sink and Four Mile Flat Playa areas near Fallon, Nev. Tests were run in smooth, bare, level, soil--near ideal field soil conditions. The soil was a heavy clay, and the strength range was sufficient to determine no go and a degree of go for the go conditions for all vehicles tested.

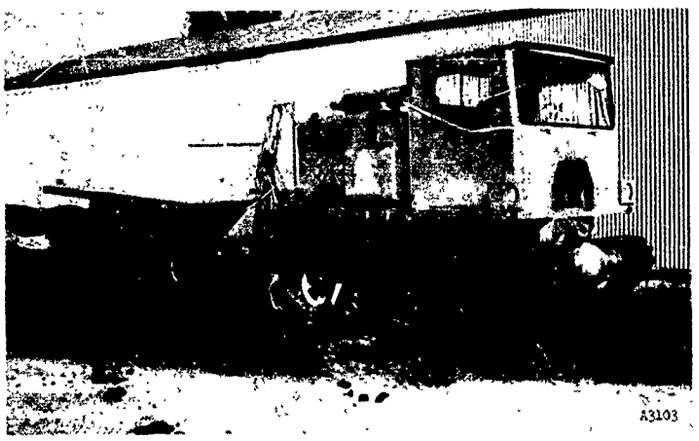
111. One object in the program was to examine the accuracy of the several soil-vehicle performance calculation procedures originally used in the preliminary design, and, if sufficient data were available, to modify or refine the prediction procedures to achieve improvement in prediction accuracy. Where revisions were made, they are identified as such in the comparative analysis. In order to ensure that supporting soil measurements and predictions were made in the most acceptable manner, personnel familiar with each model were involved in the vehicle tests, in making the in situ

soil measurements, and in making comparative analysis of prediction accuracy. The MEXA results are both instructive and timely; while they must not be considered as a full and final evaluation of the vehicle-soil models examined, they are the best data of the sort available. The MEXA program is representative of the kind of exercises needed in the future to clarify the vehicle-soil model problem further and to examine critically each of the other single-feature models that form the building blocks of comprehensive analytical models.

112. Sufficient data were extracted from the MEXA program to make a meaningful comparative evaluation of measured and predicted performance parameters common to the WES VCI, WES numeric, and LLD soil-vehicle models. Results of comparisons of original or design and revised methods and

a. MEXA 8x8

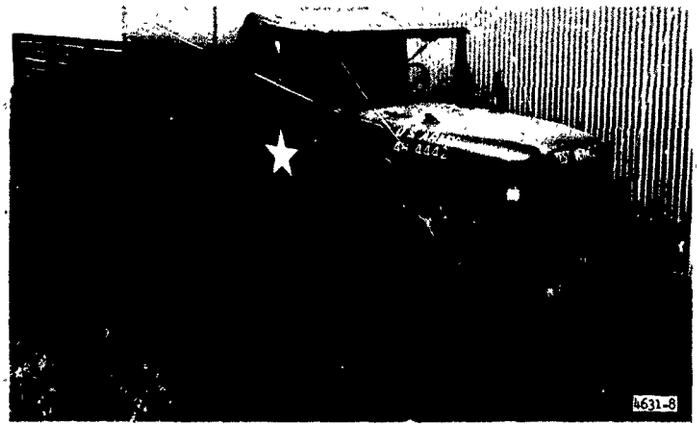
b. MEXA 10x10



c. MEXA track

Fig. 5. MEXA test beds

a. XM410E1



b. M35A2

c. M113

Fig. 6. Military vehicles used in MEXA test program

comparisons of methods are presented along with pertinent comments. In all cases computations were made using the final vehicle characteristics and (where appropriate) measured soil values in essentially the same equations utilized in developing the preliminary designs as outlined in the first MEXA report. Data from which the graphical comparisons of measured and predicted results were prepared are not presented in the report; they are available at the WES. The MEXA results have suggested some refinements to each of the models considered, both to those who have been working with each particular system and to those who have been looking over their shoulders.

113. <u>Evaluation of soil-vehicle model performance parameters (MEXA data).</u> First-pass performance parameters used in the soil-vehicle models examined that were

Table 6

Pertinent Characteristics of Vehicles Used in the MEXA Field Test Program

Vehicle	Gross Weight, lb: Empty	Gross Weight, lb: Payload	Gross Weight, lb: Test Weight	Wheel and Tire Data: Size*	Nominal Width in.	Rim Diam in.	Wheel Diam in.	No. of Wheels	No. of Axles	Average Inflation Pressure psi	Average Deflection %	Total Contact Area sq in.	Average Contact Pressure psi	Ply Rating	Ground Clearance in.	Brake Horsepower	Transmission Type
Wheeled Vehicles																	
2-1/2 ton XM410E1, 8x8 truck	11,504	5000	16,504	14x18	14	18	40	8	4	12.2	25.0	975	16.9	6	15.0	154	Automatic
2-1/2 ton M35A2, 6x6 truck (modified)	13,225	5000	18,225	11x20	11	20	42	6	3	20.1	25.0	664	27.4	12	12.5	140	Mechanical
2-1/2 ton MEXA 10x10	13,030	5000	18,030	42x40-16A	40	16	42	10	5	7.3	20.0	2654	6.8	4	11.5	214	Automatic
2-1/2 ton MEXA 8x8	14,013	5000	19,013	48x31-16A	31	16	48	8	4	9.0	20.0	2201	8.6	4	12.0	214	Automatic

Vehicle	Gross Weight, lb: Empty	Gross Weight, lb: Payload	Gross Weight, lb: Test Weight	Tracked Vehicles: Contact Length in.	Contact Width in.	Shoe Length in.	Total Contact Area (All Tracks) sq in.	Average Contact Pressure psi	No. of Bogies Per Side	Ground Clearance in.	Brake Horsepower	Transmission Type
M113 armored personnel carrier (APC)	18,600	4000	22,600	105	15	6	3150	7.2	5	16.0	202	Automatic
MEXA track	14,680	5000	19,680	85 and 104	20	4	7560	2.6	9	12.0	214	Automatic

* From manufacturer's specifications.

evaluated by using pertinent MEXA data include: go-no go for all models, optimum drawbar pull for all models, towed motion resistance for all models, and sinkage and optimum slip only for the Bekker/LLD model. The prediction accuracy of all performance parameters except go-no go was evaluated on the basis of simple statistical parameters of algebraic deviation (predicted minus measured), percent error* of algebraic deviation, and root mean square.** Algebraic deviation is used to indicate the tendency for a model to predict higher or lower than the measured values.

114. The Bekker/LLD soil models and pertinent notations used in the evaluation are given in table 7. It is to be noted in table 7 that many of the elements that make up a Bekker/LLD soil-vehicle model are used in more than one model. The commonality of elements and models are shown in the following tabulation. Each letter represents a particular equation used in the various soil-vehicle models starting with the letter "a" being common to soil-vehicle model A_T.

Soil-Vehicle Model	Sinkage	Compaction Resistance	Bulldozing Resistance	Motion Resistance	Gross Traction
		Tracked Vehicles			
A_T	a	a	a	a	a
B_T	b	a	Not used	b	b
C_T	a	a	Not used	b	c
		Wheeled Vehicles			
A_W	c	a	a	a	a
B_W	c	a	Not used	b	d
C_W	a	a	Not used	b	c

* $\% \text{ error} = \frac{\text{predicted} - \text{measured}}{\text{measured}} \times 100$

** Root mean square (RMS) is a measure of the quality of a relation in terms of the RMS of the deviation:

$$\text{RMS} = \sqrt{\frac{\Sigma(d^2)}{n}}$$

where

Σ = the sum of
d = deviation
n = number of deviations

A

Soil-Vehicle Model	Sinkage	Compaction Resistance	Bulldozing Resistance	
A_T	$z = \left(\frac{W}{Ak}\right)^{\frac{1}{n}}$	$R_c = \left(\frac{bk}{n+1}\right) z^{n+1}$	$R_b = b\left(2zcK_b + \gamma z^2 K_\theta\right)$	R
B_T	$z_m = \left[\frac{(n+1)W}{Ak}\right]^{\frac{1}{n}}$	$R_c = \left(\frac{bk}{n+1}\right) z_m^{n+1}$	Not used	R
C_T	$z = \left(\frac{W}{Ak}\right)^{\frac{1}{n}}$	$R_c = \left(\frac{bk}{n+1}\right) z^{n+1}$	Not used	R

Method	Sinkage	Compaction Resistance	Bulldozing Resistance	
A_W	$z = \left[\frac{3W}{bk\sqrt{D}\,(3-n)}\right]^{\frac{2}{2n+1}}$	$R_c = \left(\frac{bk}{n+1}\right) z^{n+1}$	$R_b = b\left(2zcK_b + \gamma z^2 K_\theta\right)$	
B_W	$z = \left[\frac{3W}{bk\sqrt{D}\,(3-n)}\right]^{\frac{2}{2n+1}}$	$R_c = \left(\frac{bk}{n+1}\right) z^{n+1}$	Not used	
C_W	$z = \left(\frac{W}{Ak}\right)^{\frac{1}{n}}$	$R_c = \left(\frac{bk}{n+1}\right) z^{n+1}$	Not Used	

List of Symbols

W = weight on the tractive element, lb
b = nominal tire width or track width, in.
ℓ = chord length $2\sqrt{(D-\delta)\delta}$ for wheels or track length ground contact
n = sinkage exponent, dimensionless
k = soil consistency, $k = \frac{k_c}{b} + k_\phi$, lb/in.$^{n+2}$
c = cohesion, lb/in.^{2}
ϕ = angle of integral friction, deg
K_θ and K_b = coefficient of bulldozing resistance, dimensionless
K = tangent modulus of deformation, in.
s = slip, dimensionless
e = Naperian logarithm (base 2.718), dimensionless

δ = tire deflec
γ = bulk densit
A = computed co
p = computed gr
z = sinkage, in
z_m = maximum sin
R_c = compaction
R_b = bulldozing
R_T = total motio
H = gross tract
DP = drawbar pul
D = tire diame
θ = angle that

B

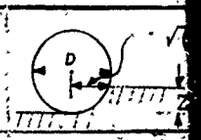
Table 7

Bekker/LLD Soil-Vehicle Models Evaluated in MEXA Program

Tracked Vehicles

Bulldozing Resistance	Motion Resistance	Gross Traction	Con
$R_b = b\left(2zcK_b + \gamma z^2 K_\theta\right)$	$R_T = R_c + R_b$	$H = A_c + W \tan \phi$	$A = b \cdot \ell$
Not used	$R_T = R_c$	$H = A\left\{c\left[1 - \frac{K}{s\ell}\left(1 - e^{\frac{-s\ell}{K}}\right)\right] + kz_m^n \tan \phi \left(\frac{1}{n+1} - \frac{1}{\ell^{n+1}}\int_0^\ell x^n e^{\frac{-sx}{K}} dx\right)\right\}$	$A = b \cdot \ell$
Not used	$R_T = R_c$	$H = (Ac + W \tan \phi)\left[1 - \frac{K}{s\ell}\left(1 - e^{\frac{-s\ell}{K}}\right)\right]$	$A = b \cdot \ell$

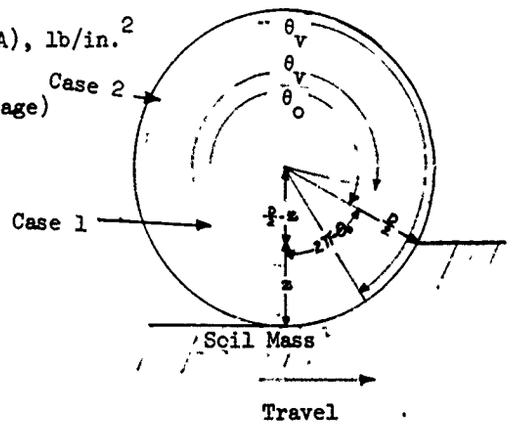
Wheeled Vehicles

Bulldozing Resistance	Motion Resistance	Traction	Co
$R_b = b\left(2zcK_b + \gamma z^2 K_\theta\right)$	$R_T = R_c + R_b$	$H = Ac + W \tan \phi$	
Not used	$R_T = R_c$	$*H = H_1 - H_2$ (cycloid method)	
Not Used	$R_T = R_c$	$H = (Ac + W \tan \phi)\left[1 - \frac{K}{s\ell}\left(1 - e^{\frac{-s\ell}{K}}\right)\right]$	

abols

δ = tire deflection, in.
γ = bulk density of soil, lb/in.^{3}
A = computed contact area, in.^{2}
p = computed ground contact pressure (W/A), lb/in.^{2}
z = sinkage, in.
z_m = maximum sinkage, in. (nonuniform sinkage)
R_c = compaction resistance, lb
R_b = bulldozing resistance, lb
R_T = total motion resistance, lb
H = gross tractive effort, lb
DP = drawbar pull, DP = H - R_T, lb
D = tire diameter (undeflected), in.
θ = angle that varies between θ_v and 2π

less



* $\theta_v = 2\pi - \cos^{-1}(1 - \frac{2z}{D})$, angle of
$\theta_o = 2\pi - \cos^{-1}(1 - s)$, angle of

$\theta_o \leq \theta_v$ Case 1
$\theta_o > \theta_v$ Case 2

$p = k[\frac{D}{2}(\cos \theta - \cos \theta_v)]^n$

Case 1

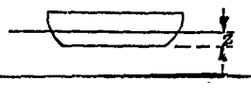
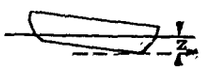
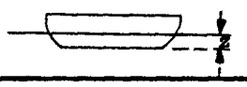
$H_1 = \frac{bD}{2}\int_{\theta_v}^{2\pi} (c + p \tan \phi)(1 -$

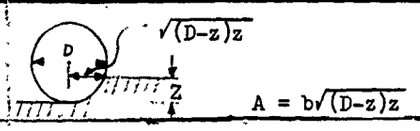
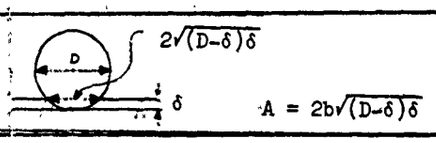
$H_2 = 0$

$j_1 = \frac{D}{2}[(1 - s)(\theta_v - \theta) + \sin$

$j_2 = 0$

MEXA Program

	Contact Area	Remarks
	$A = b \cdot \ell$	Type of sinkage 
$\frac{1}{n+1} - \frac{1}{\ell^{n+1}} \int_0^{\ell} x^n e^{\frac{-sx}{K}} dx \Big)\Big\}$	$A = b \cdot \ell$	Type of sinkage 
	$A = b \cdot \ell$	Type of sinkage 

	Contact Area	Remarks
		Rigid wheels; H computed at high slip only
		Rigid wheel; H computed at selected slip values
		Deflected tire; H computed at selected slip values

* $\theta_v = 2\pi - \cos^{-1}(1 - \frac{2z}{D})$, angle of contact with original soil surface

$\theta_o = 2\pi - \cos^{-1}(1 - s)$, angle of slip

$\theta_o \leq \theta_v$ Case 1

$\theta_o > \theta_v$ Case 2

$$p = k[\frac{D}{2}(\cos\theta - \cos\theta_v)]^n$$

Case 1

$$H_1 = \frac{bD}{2}\int_{\theta_v}^{2\pi} (c + p\tan\phi)(1 - e^{\frac{-j_1}{K}})\cos\theta\, d\theta$$

$$H_2 = 0$$

$$j_1 = \frac{D}{2}[(1 - s)(\theta_v - \theta) + \sin\theta - \sin\theta_v]$$

$$j_2 = 0$$

Case 2

$$H_1 = \frac{bD}{2}\int_{\theta_o}^{2\pi} (c + p\tan\phi)(1 - e^{\frac{-j_1}{K}})\cos\theta\, d\theta$$

$$H_2 = \frac{bD}{2}\int_{\theta_v}^{\theta_o} (c + p\tan\phi)(1 - e^{\frac{+j_2}{K}})\cos\theta\, d\theta$$

$$j_1 = \frac{D}{2}[(1 - s)(\theta_o - \theta) + \sin\theta - \sin\theta_o]$$

$$j_2 = \frac{D}{2}[(1 - s)(\theta_v - \theta) + \sin\theta - \sin\theta_v]$$

From the tabulation on page 60 it can be seen that three methods are used to determine sinkage, one method is used to determine compaction resistance, one method to determine bulldozing resistance in two models, two methods to determine motion resistance, and four methods to determine gross traction. Soil-vehicle models C_T for tracked vehicles and C_W for wheeled vehicles are the same.

115. For each comparative analysis the data were selected on the basis of the soil-vehicle model that had the least amount of complete data so that the same measured performance data could be used to compare the prediction accuracy of the models. The soil data used in the analysis are shown in table 8.

a. Go-no go. The MEXA test items selected for making a comparison of measured and predicted first-pass go-no go performance are given in table 9. Item numbers less than 100 designate VCI tests, and items 100 and greater designate drawbar pull tests conducted in a soil strength range that was near critical from the go-no go standpoint. Go-no go performance for the Bekker/LLD model was made on the basis of drawbar pull prediction. A go condition was assigned for positive drawbar pull predictions and a no go for negative drawbar pull predictions. Data from 19 tests were used for the wheeled vehicles; and the number of tests ranged from 1 to 10 for the vehicles for which data were available. Only five tests were available for the tracked vehicles: two for the MEXA track and three for the M113. Information presented in table 9 is summarized below.

Soil-Vehicle Model	% Prediction Correct			Remarks
	Wheeled	Tracked	All Vehicles	
WES-VCI				
Original, VCI_1 (50)	79	100	83	
Revised, VCI_1	95	80	92	

(Continued)

Table 8

Summary of Soil Data

Item No.	Avg CI 0- to 6-in. Layer	RCI 0- to 6-in. Layer	RCI 6- to 12- in. Layer	RCI 12- to 18-in. Layer	Sinkage Parameters k_c	Sinkage Parameters k_ϕ	Sinkage Parameters n	Shear Parameters c	Shear Parameters ϕ	Shear Parameters k
					XM410E1					
89	48	37	35	19	19.7	9.0	0.55	0.28	30.9	1.0
90	48	37	35	19	19.7	9.0	0.55	0.28	30.9	1.0
91	34	24	21	18	10.4	12.4	0.37	0.23	32.3	1.0
92	34	24	21	18	10.4	12.4	0.37	0.23	32.3	1.0
					M35A2 (Mod)					
22	36	27	22	21	10.4	12.4	0.37	0.23	32.3	1.0
96	48	37	35	19	19.7	9.0	0.55	0.28	30.9	1.0
97	48	37	35	19	19.7	9.0	0.55	0.28	30.9	1.0
					M113					
40	19	11	14	14	0	3.8	0.17	0	11.2	1.0
41	15	8	13	17	0	3.8	0.17	0	11.2	1.0
99	71	53	40	31	20	10.1	0.58	0.28	30.9	1.0
100	71	53	40	31	20	10.1	0.58	0.28	30.9	1.0
101	48	37	35	19	19.7	9.0	0.55	0.28	30.9	1.0
102	48	37	35	19	19.7	9.0	0.55	0.28	30.9	1.0
103	34	24	21	18	10.4	12.4	0.37	0.23	32.3	1.0
104	34	24	21	18	10.4	12.4	0.37	0.23	32.3	1.0
105	29	18	21	26	0.8	5.3	0.14	0	9.3	1.0
106	27	18	18	19	0.5	6.8	0.155	0.17	13.8	1.0
					MEXA 10x10					
53	44	35	32	22	19.7	9.0	0.55	0.28	30.9	1.0
54	35	24	19	19	10.4	12.4	0.37	0.23	32.3	1.0
55	14	6	10	13	0.8	5.3	0.14	0	9.3	1.0
57	10	5	11	17	0.8	5.3	0.14	0	9.3	1.0
58	11	6	11	17	0.8	5.3	0.14	0	9.3	1.0
59	15	8	14	20	0.8	5.3	0.14	0	9.3	1.0
108	46	35	30	17	14	12.7	0.53	0.28	30.9	1.0
109	46	35	30	17	10	18.0	0.53	0.28	30.9	1.0
111	33	23	20	20	10.4	12.4	0.37	0.23	32.3	1.0
112	33	23	20	20	10.4	12.4	0.37	0.23	32.3	1.0
115	20	13	13	17	0	3.8	0.17	0	11.2	1.0
116	23	15	15	15	0	3.8	0.17	0	11.2	1.0
118	23	17	18	23	0.5	6.8	0.155	0.17	13.8	1.0
119	22	16	20	23	0.5	6.8	0.155	0.17	13.8	1.0
120	16	10	16	20	0	3.8	0.17	0	11.2	1.0
121	18	11	14	18	0	3.8	0.17	0	11.2	1.0
122	19	12	17	17	0	3.8	0.17	0	11.2	1.0
					MEXA 8x8					
69	12	6	8	16	0.8	5.3	0.14	0	9.3	1.0
70	18	10	10	14	0.8	5.3	0.14	0	9.3	1.0
72	34	7	13	16	0.8	5.3	0.14	0	9.3	1.0
73	19	12	15	19	0.8	5.3	0.14	0	9.3	1.0
124	35	25	21	18	10.4	12.4	0.37	0.23	32.3	1.0
125	35	25	21	18	10.4	12.4	0.37	0.23	32.3	1.0
126	40	29	32	17	19.7	9.0	0.55	0.28	30.9	1.0
127	40	29	32	17	19.7	9.0	0.55	0.28	30.9	1.0
129	21	13	15	17	0	3.8	0.17	0	11.2	1.0
130	21	13	15	17	0	3.8	0.17	0	11.2	1.0
131	23	14	22	21	0.5	6.8	0.155	0.17	13.8	1.0
					MEXA Track					
132	60	52	38	34	20	10.1	0.58	0.29	35.3	1.0
133	60	52	38	34	20	10.1	0.58	0.29	35.3	1.0
134	41	32	28	21	19.7	9.0	0.55	0.28	30.9	1.0
135	41	32	28	21	19.7	9.0	0.55	0.28	30.9	1.0
136	33	23	20	19	10.4	12.4	0.37	0.23	32.3	1.0
137	33	23	20	19	10.4	12.4	0.37	0.23	32.3	1.0
140	69	49	42	31	20	10.1	0.58	0.29	35.3	1.0
141	69	49	42	31	20	10.1	0.58	0.29	35.3	1.0
142	20	16	15	19	0.5	6.8	0.155	0.17	13.8	1.0
143	18	9	14	17	0.5	6.8	0.155	0.17	13.8	1.0
144	10	6	12	15	0.8	5.3	0.14	0	9.3	1.0

Table 9

MEXA Data Used in Comparing Measured and Predicted First-Pass Go-No-Go Performance

Item	Measured		Predicted WES VCI				Predicted WES Numeric				Predicted Bekker/LLD (See footnote for combination of methods)*											
			VCI_1 (50)		VCI_1						Wheeled Vehicles								Tracked Vehicles			
			Original		Revised		Original		Revised		A_W		B_W		C_W		A_T		B_T		C_T	
	Go	No Go	Go	No Go	Go	No Go	Go	No Go	Go	No Go	Go	No Go	Go	No Go	Go	No Go	Go	No Go	Go	No Go	Go	No Go
Wheeled Vehicles																						
M35A2 (Modified):																						
22		x		x		x		x		x	x		x		x							
MEXA 10x10																						
53	x		x		x		x		x		x		x		x							
55		x	x			x	x		x			x		x	x							
57		x	x			x		x		x		x		x	x							
58		x	x			x	x			x		x		x	x							
59	x		x		x		x			x		x		x	x							
115	x		x		x		x		x			x		x	x							
116	x		x		x		x		x			x		x	x							
120	x		x		x		x		x			x		x	x							
121	x		x		x		x		x			x		x	x							
122	x		x		x		x		x			x		x	x							
MEXA 8x8:																						
69		x		x		x		x		x		x		x	x							
70		x		x		x	x			x		x		x	x							
71	x		x			x	x			x		x		x	x							
72		x	x			x		x		x		x		x	x							
73	x		x		x		x		x			x		x	x							
129	x		x		x		x		x			x		x	x							
130	x		x		x		x		x			x		x	x							
131	x		x		x		x		x		x		x		x							
All wheeled vehicles % correct			79		95		84		84		42		42		63							
Tracked Vehicles																						
M113:																						
40		x		x		x												x		x		x
41		x		x		x												x		x		x
105	x		x		x												x			x	x	
MEXA Track:																						
143	x		x		x												x		x		x	
144	x		x			x											x		x		x	
All tracked vehicles % correct			100		80												100		80		100	
All Wheeled and Tracked Vehicles																						
% correct			83		92						54**				71†							

* See table 7 for specific equations used.
** Denotes combination of methods A_W and A_T (most representative of the original method).
† Denotes combination of methods C_W and C_T (revised method).

Soil-Vehicle Model	% Prediction Correct Wheeled	Tracked	All Vehicles	Remarks
WES numeric				
Original	84	--	--	Tracked vehicle numeric not available
Revised	84	--	--	No revisions made
Bekker/LLD				
A_W	42			
B_W	42			
C_W	63			
A_T		100		
B_T		80		
C_T		100		
A_W and A_T			54	A_W and A_T combined (most representative of design method)
C_W and C_T			71	C_W and C_T combined (revised method)

The tabulation above shows that for wheeled vehicles the WES VCI models gave the highest prediction accuracy, with the VCI_1 model being best, the WES numeric method second, and the Bekker/LLD models gave the lowest prediction accuracy. Of the three Bekker/LLD models, A_W and B_W gave the same results, and C_W was the most accurate. For tracked vehicles, the Bekker/LLD A_T and C_T and the WES $VCI_1(50)$ models gave the same and best prediction accuracy, the WES VCI_1 and Bekker/LLD B_T models gave the same results and were next best. On the basis of all vehicles, the prediction accuracy for the WES VCI_1 model was best, the WES $VCI_1(50)$ was second, the Bekker/LLD C_W and C_T model combination third, and the Bekker/LLD A_W and A_T combination model last.

b. VCI. A comparison of measured and predicted 50- and first-

pass vehicle cone indexes for the WES VCI and first pass for the WES clay numeric soil-vehicle models is given in table 10. Plots of measured versus predicted VCI_{50} original and revised and $VCI_1(50)$ and VCI_1 for the WES VCI methods are shown in fig. 7 (a, b, c, and d). Fig. 7e and f are similar plots for the WES clay numeric method. The data given in table 10 are summarized below. Deviations are in terms of RCI points.

Soil-Vehicle Model	Wheeled		Tracked		All Vehicles	
	Dev	% Error	Dev	% Error	Dev	% Error
WES VCI						
VCI_{50} (original)	6	19	13	50	8	29
VCI_{50} (revised)	6	18	3.0	10	5	16
$VCI_1(50)$ (original)*	-2.5	-17	4.5	49	0.17	5
VCI_1 (revised)	0.5	4	1.0	10	1	6
WES numeric						
VCI_1 (original)	0.5	-3	--	--		
VCI_1 (revised)	4	25	--	--		

* 0.40 of predicted VCI_{50}.

From the tabulation above it can be seen that the WES VCI prediction for VCI_{50} original and revised is higher than measured VCI_{50} and thus conservative for both wheeled and tracked vehicles. In the development of these relations, they were deliberately designed to give conservative results. When 40 percent of VCI_{50} was used as $VCI_1(50)$, predictions were lower than measured values for wheeled vehicles and higher than measured for tracked vehicles. The overall prediction for all vehicles was conservative. WES VCI_1 predictions were slighly conservative and they gave the highest prediction accuracy of the WES VCI methods. VCI_1 predictions made by the WES clay numeric (original) for wheeled vehicles was slightly low. VCI_1 predictions made by the WES numeric (revised) for wheeled vehicles were higher, and when compared with the WES VCI_1 method for

Table 10

MEXA Data Used in Comparing Measured and Predicted Vehicle Cone Indexes

Vehicle	Predicted	Measured*	Deviation Predicted-Measured	Percent Error
		WES - VCI_{50} (Original)		
XM410E1	42	32	10	31
M35A2 (mod)	62	56	6	11
M113	47	29	18	62
Std mil vehicles			Avg 11	35
MEXA 10x10	21	18	3	17
MEXA 8x8	27	23	4	17
MEXA track	29	21	8	38
MEXA vehicles			Avg 5	24
All vehicles			Avg 8	29
		WES - VCI_1(50) (Original)		
XM410E1	17	18	-1	-6
M35A2 (mod)	25	27	-2	-7
M113	19	15	4	27
Std mil vehicles			Avg 0.33	5
MEXA 10x10	8	12	-4	-33
MEXA 8x8	11	14	-3	-21
MEXA track	12	7	5	71
MEXA vehicles			Avg -1	6
All vehicles			Avg -0.17	5
		WES Numeric VCI_1 (Original)		
XM410E1	18	18	0	0
M35A2 (mod)	32	27	5	19
Std mil vehicles			Avg 2	10
MEXA 10x10	6	7	-1	-14
MEXA 8x8	9	11	-2	-18
MEXA vehicles			Avg -2	-16
All vehicles			Avg 0.5	- 3

Vehicle	Predicted	Measured*	Deviation Predicted-Measured	Percent Error
		WES - VCI_{50} (Revised)		
XM410E1	42	32	10	31
M35A2 (mod)	63	56	7	12
M113	38	29	9	31
Std mil vehicles			Avg 9	25
MEXA 10x10	21	18	3	17
MEXA 8x8	26	23	3	13
MEXA track	19	21	-2	-10
MEXA vehicles			Avg 1	7
All vehicles			Avg 5	16
		WES VCI_1		
XM410E1	18	18	0	0
M35A2 (mod)	28	27	1	4
M113	16	15	1	7
Std mil vehicles			Avg 1	4
MEXA 10x10	8	7	1	14
MEXA 8x8	11	11	0	0
MEXA track	8	7	1	14
MEXA vehicles			Avg 1	9
All vehicles			Avg 1	6
		WES Numeric VCI_1 (Revised)		
XM410E1	22	18	4	22
M35A2 (mod)	38	27	11	41
Std mil vehicles			Avg 8	31
MEXA 10x10	9	7	2	29
MEXA 8x8	12	11	1	09
MEXA vehicles			Avg 2	19
All vehicles			Avg 4	25

* The 6- to 12-in. layer is the critical layer for the WES VCI_1(50) and VCI_{50} methods, and the 0- to 6-in. layer is the critical layer for the WES VCI_1 and numeric VCI_1 methods.

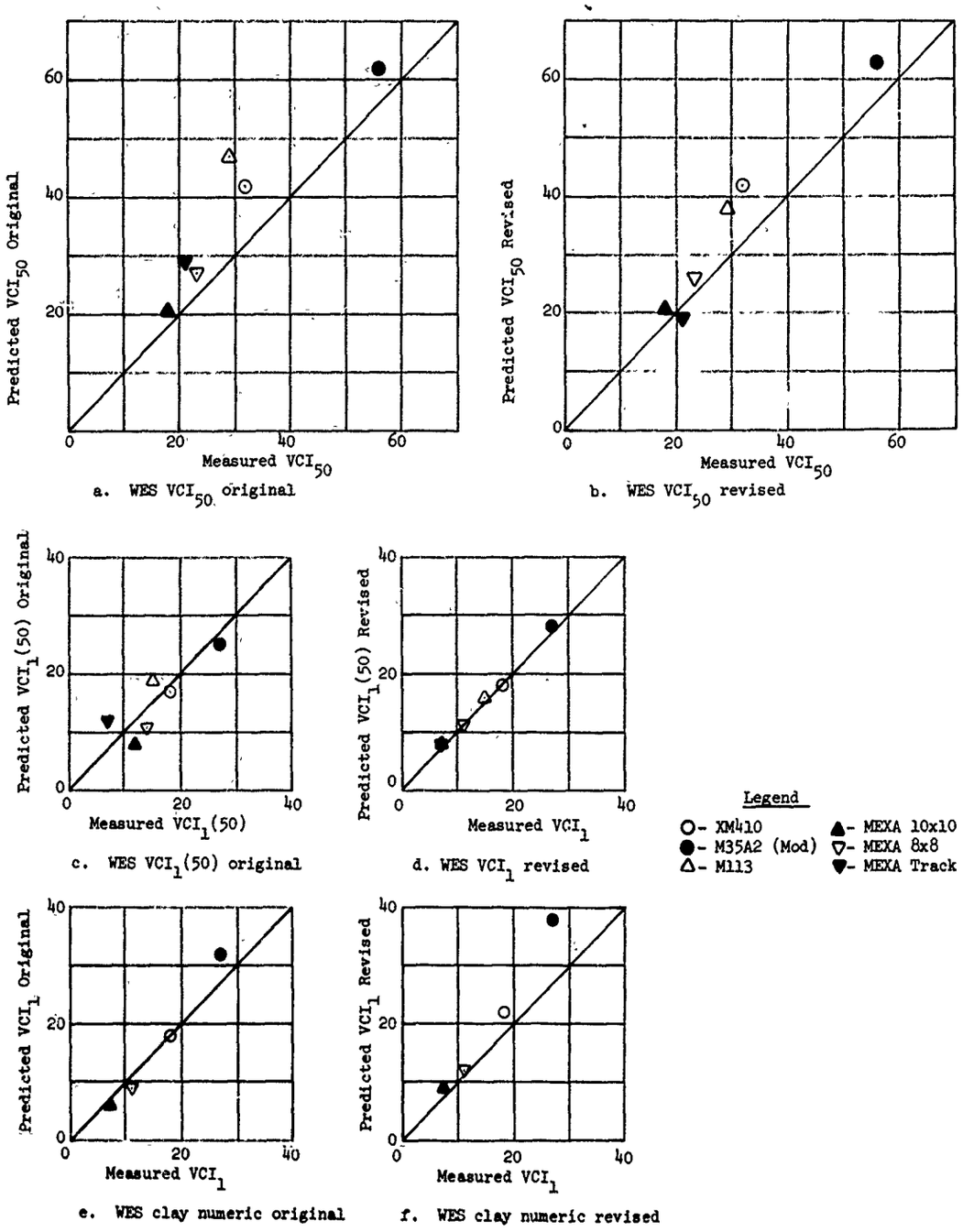


Fig. 7. Comparison of measured and predicted vehicle cone indexes

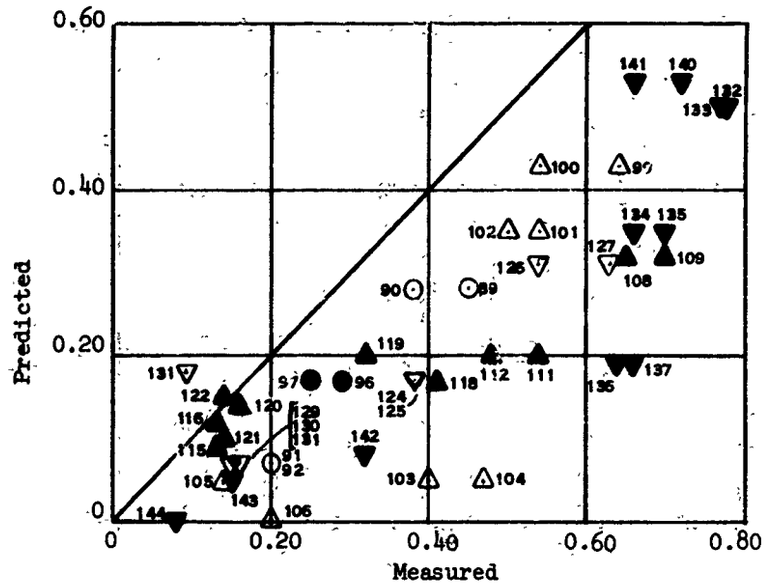
wheeled vehicles, its accuracy was somewhat lower.

c. Optimum drawbar pull.* The MEXA data used in comparing first-pass measured and predicted optimum drawbar pull coefficients for the WES VCI, WES clay numeric, and Bekker/LLD soil-vehicle models are given in table 11. Results from 24 wheeled and 19 tracked vehicle tests were used in the comparison. Plots of measured versus predicted optimum drawbar pull for the WES VCI model for $VCI_1(50)$ and VCI_1 are shown in fig. 8, for the WES clay numeric model for the original and revised methods in fig. 9, and for the Bekker/LLD model for the design and revised methods in fig. 10. The data in table 11 are summarized below. The deviations shown are in terms of drawbar pull coefficients.

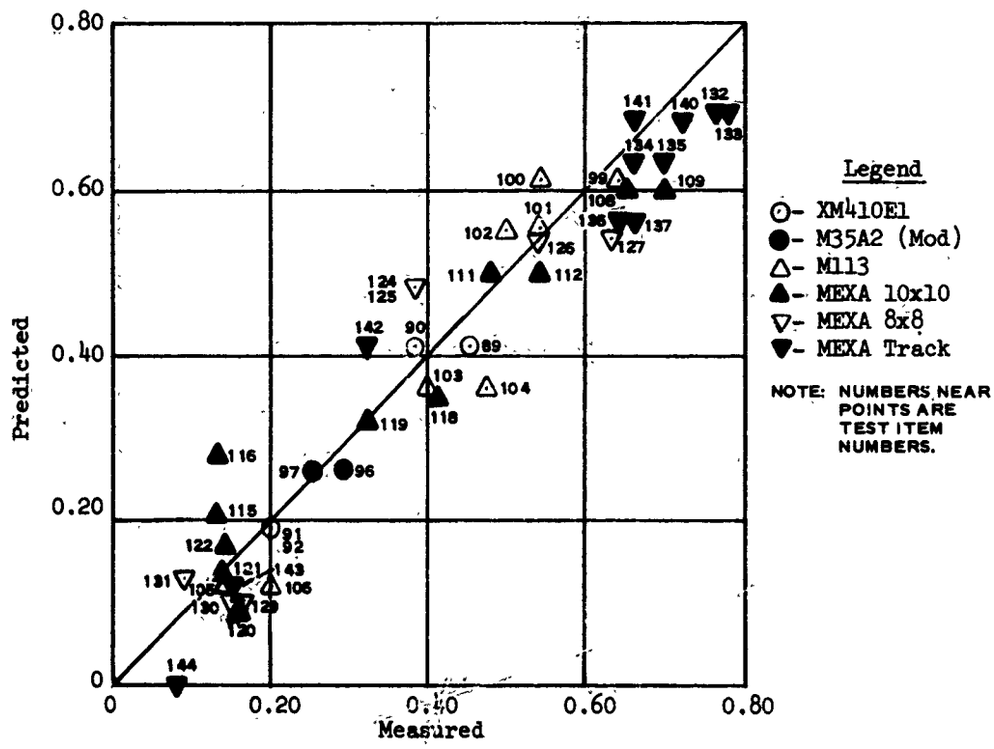
	Wheeled			Tracked			All Vehicles			
Soil-Vehicle Model	Dev	% Error	RMS	Dev	% Error	RMS	Dev	% Error	RMS	Remarks
WES VCI										
Original, $VCI_1(50)$	-0.15	-36	0.19	-0.24	55	0.25	-0.19	-45	0.22	
Revised, VCI_1	0.002	4	0.06	-0.034	-12	0.07	-0.016	-3	0.06	
WES clay numeric										
Original	0.13	46	0.20	--	--	--	--	--	--	Tracked vehicle numeric not available
Revised	0.01	12	0.13	--	--	--	--	--	--	
Bekker/LLD										
A_W*	-0.10	-35	0.18							
B_W	-0.17	-59	0.21							
C_W	0.04	25	0.13							
A_T*				0.01	5	0.12				
B_T				0.06	14	0.10				
C_T				0.03	11	0.08				
A_W and A_T							-0.06	-18	0.15	A_W and A_T combined as most representative of design method
C_W and C_T							0.04	19	0.11	C_W and C_T combined as revised method

* These methods predict for maximum drawbar pull only.

* Drawbar pull at optimum slip. Optimum slip is that slip at which drawbar horsepower is a maximum; i.e. the product $\frac{\text{drawbar pull}}{\text{weight}}$ times (1 - slip ratio) is greatest. In fine-grained soils, optimum slip is about 20 percent.

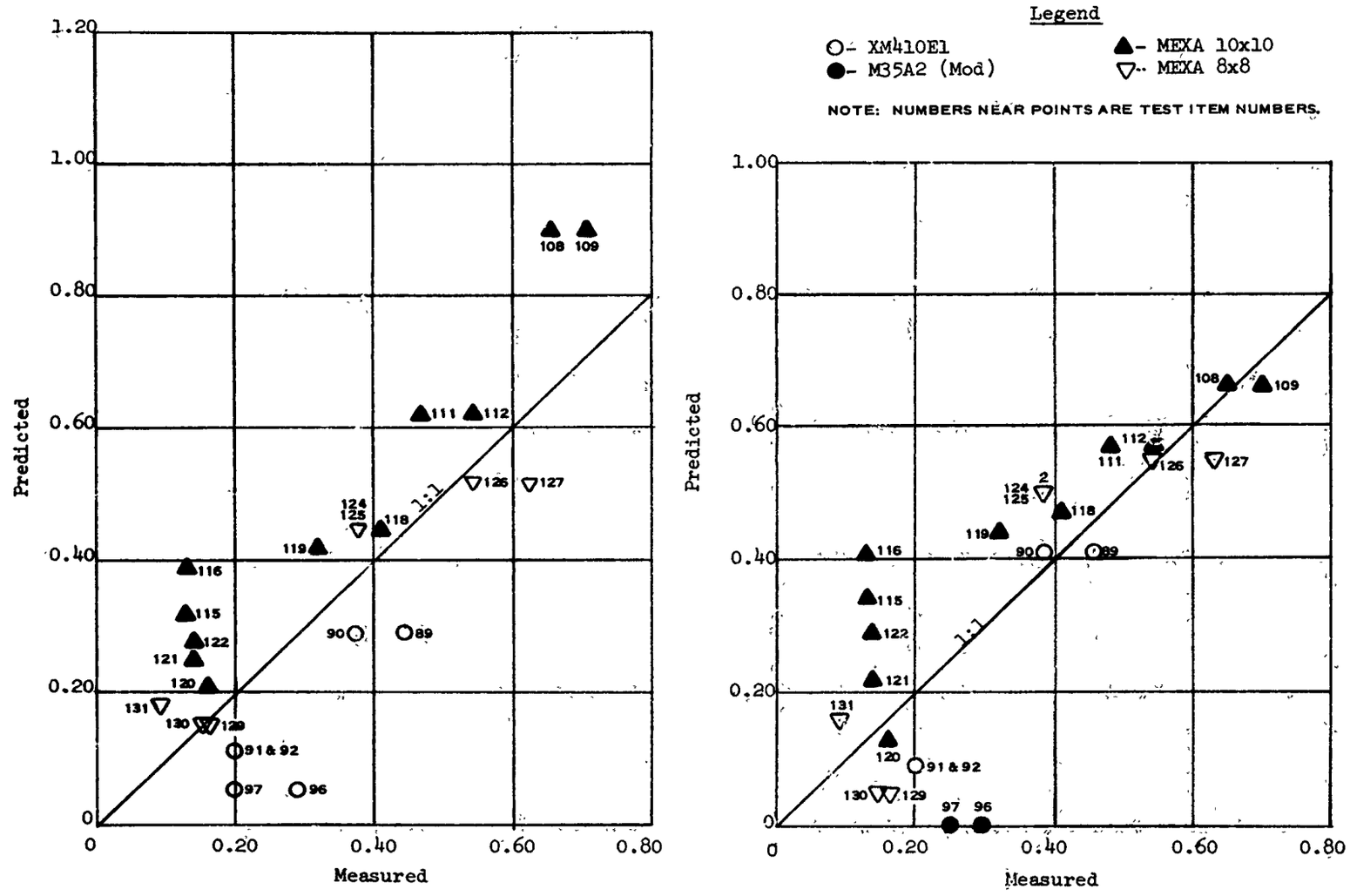


a. WES-VCI$_1$(50) model



b. WES-VCI$_1$ model

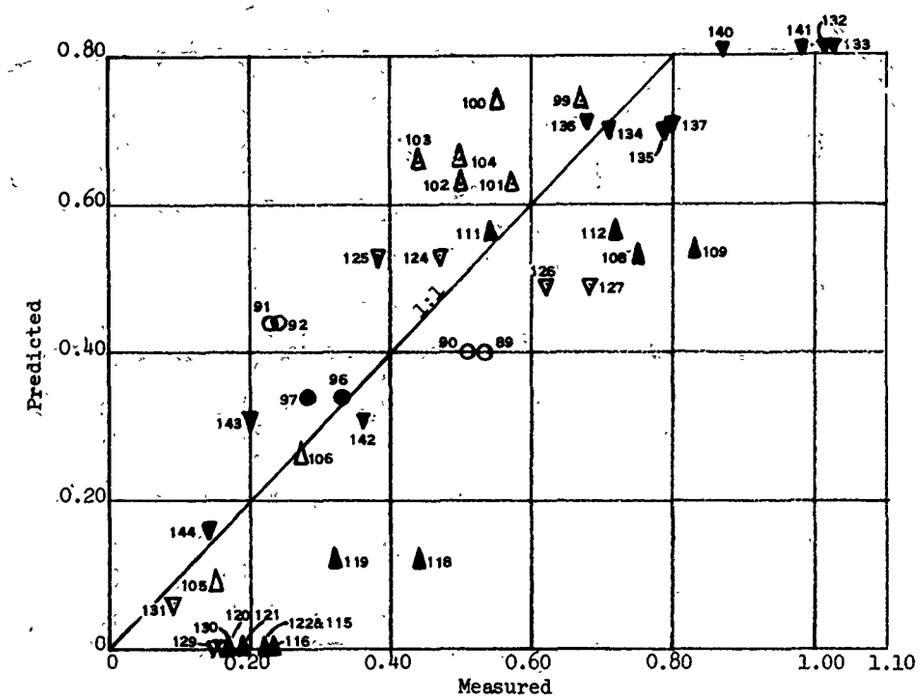
Fig. 8. Comparison of measured and predicted first-pass optimum drawbar pulls for WES VCI models



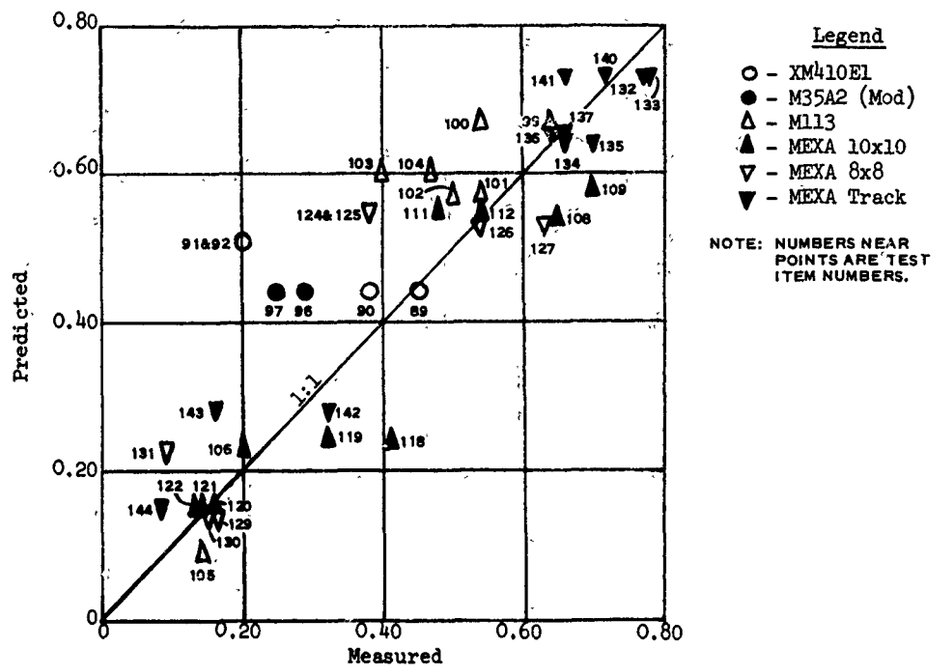
a. WES clay numeric original

b. WES clay numeric revised

Fig. 9. Comparison of measured and predicted first-pass optimum drawbar pulls for WES clay numeric models



a. Bekker/LLD Model - A_W and A_T
(These methods predict for maximum drawbar pull only)



b. Bekker/LLD model - C_W and C_T

Fig. 10. Comparison of measured and predicted first-pass optimum drawbar pulls for Bekker/LLD models

Table 11

MEXA Data Used in Comparing Measured and Predicted First-Pass Optimum Drawbar Pull Coefficients

Number of Tests	WES VCI: $VCI_1(50)$ Original† Algebraic Dev	WES VCI: $VCI_1(50)$ Original† Algebraic %Error	WES VCI: $VCI_1(50)$ Original† Algebraic RMS	WES VCI: VCI_1 Revised Algebraic Dev	WES VCI: VCI_1 Revised Algebraic %Error	WES VCI: VCI_1 Revised Algebraic RMS	WES Numeric: Design Algebraic Dev	WES Numeric: Design Algebraic %Error	WES Numeric: Design Algebraic RMS	WES Numeric: Revised Algebraic Dev	WES Numeric: Revised Algebraic %Error	WES Numeric: Revised Algebraic RMS	Bekker/LLD (See footnote for design and revised methods): A_T or A_W Algebraic Dev	A_T or A_W Algebraic %Error	A_T or A_W Algebraic RMS	B_T or B_W Algebraic Dev	B_T or B_W Algebraic %Error	B_T or B_W Algebraic RMS	C_W or C_T Algebraic Dev	C_W or C_T Algebraic %Error	C_W or C_T Algebraic RMS
										Wheeled Vehicles											
										XM410E1											
4	-.13	-48	.14	-.008	-3	.03	-.11	-38	.11	-.06	-28	.08	.04	32	.17	-.08	-13	.15	.15	76	.22
										M35A2 (Mod)											
2	-.10	-36	.10	-.010	-3	.02	-.20	-79	.20	-.27	-100	.27	.03	0	.04	-.09	-31	.09	.15	51	.15
										MEXA 10x10											
11	-.16	-36	.22	.005	10	.07	.15	65	.16	.09	.54	.13	-.20	-66	.22	-.23	-80	.26	-.03	-3	.08
										MEXA 8x8											
7	-.15	-30	.20	.006	2	.07	.02	17	.05	.004	-0.8	.09	-.08	-37	.12	-.14	-60	.16	.06	33	.11
										All Wheeled Vehicles											
24	-.15	-36	.19	.002	4	.06	.04	-22	.13	.01	12	.13	-.10	-35	.18	-.17	-59	.21	.04	25	.13
										Tracked Vehicles											
										M113											
8	-.22	-57	.24	-.018	-8	.06	-	-	-	-	-	-	.10	15	.13	.08	10	.13	.07	13	.10
										MEXA Track											
11	-.26	-54	.26	-.045	-14	.07	-	-	-	-	-	-	-.06	-2	.11	.04	19	.06	.01	10	.06
										All Tracked Vehicles											
19	-.24	-55	.25	-.034	-12	.07	-	-	-	-	-	-	.0	5	.12	.06	14	.10	.03	11	.08
										All Wheeled and Tracked Vehicles											
43	-.19	-45	.22	-.016	-3	.06	-	-	-	-	-	-	-.06*	-18*	.15*	-	-	-	.04**	19**	.11**

* A_W and A_T denote maximum drawbar pull coefficients for wheeled and tracked vehicles, respectively (original method).

** Denotes combination of methods C_W and C_T (revised method).

† The WES VCI method was not included in the design, but it was included in this analysis for comparative purposes.

For wheeled vehicles, all soil-vehicle model prediction accuracies were greatly improved by the revised methods. On the basis of the revised methods, the WES clay numeric, WES VCI, and the Bekker/LLD C_W methods, the predicted performance was higher than the measured performance. In terms of RMS, the WES VCI_1 prediction method was best, the Bekker/LLD C_W method next best, and the WES clay numeric last. As with wheeled vehicles, the revised tracked vehicle methods improved prediction accuracy. The WES VCI_1 prediction method was conservative (predictions lower than measured) whereas the Bekker/LLD C_T predictions were higher than measured results. For the WES VCI_1 and Bekker/LLD tracked vehicle performance prediction methods, the prediction accuracy was about the same (RMS of 0.07 versus 0.08). For all types of vehicles, the RMS for the WES VCI_1 method was lower than that for the Bekker/LLD C_W and C_T combination method.

d. Towed motion resistance. The MEXA data used in the comparative analysis of towed motion resistance coefficient for the four WES and six* Bekker/LLD models are given in table 12. For the comparative analysis, data from 11 wheeled and seven tracked vehicle tests were available. Plots of measured versus predicted towed motion resistance coefficients are shown for the WES VCI model for $VCI_1(50)$ and VCI_1 in fig. 11, for the WES clay numeric and revised models in fig. 12, and for the Bekker/LLD models combining A_W and A_T, and C_W and C_T in fig. 13. The data in table 12 are summarized on page 80. The deviations shown are in terms of motion resistance coefficients. The WES $VCI_1(50)$ prediction results were very conservative and very poor. The particular soil strength-towed motion resistance

* Although two motion resistance equations are given in table 7, sinkage used as an input is determined by three equations, giving six different methods for calculating motion resistance.

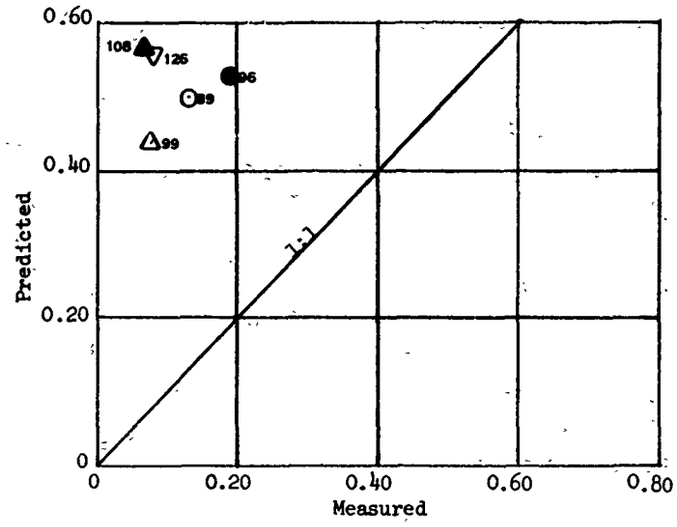
Table 12

MEXA Data Used in Comparing Measured and Predicted First-Pass Towed Motion Resistance Coefficients

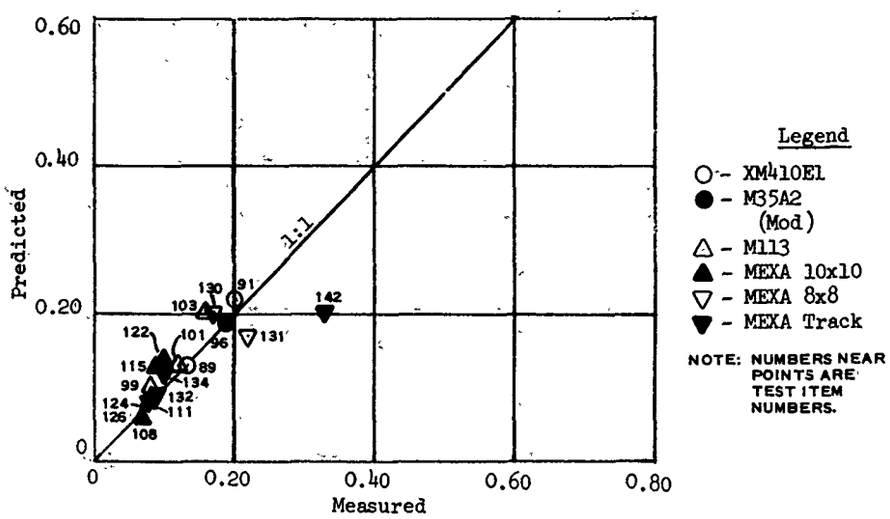
Total Number of Tests	WES VCI: VCI_1(50) Original Algebraic Dev	%Error	RMS	WES VCI: VCI_1 Revised Algebraic Dev	%Error	RMS	WES Numeric: Original Algebraic Dev	%Error	RMS	WES Numeric: Revised Algebraic Dev	%Error	RMS	Bekker/LLD (See footnote for combination of methods): A_T or A_W Algebraic Dev	%Error	RMS	B_T or B_W Algebraic Dev.	%Error	RMS	C_W or C_T Algebraic Dev	%Error	RMS
Wheeled Vehicles																					
XM410E1																					
2	.37	285	.37	.010	5	.01	-0.5	-32	.05	-.02	-13	.02	.07	69	.09	.07	69	.09	-.13	-86	.14
M35A2 (Mod)																					
1	.34	179	.34	0	0	.00	.01	5	.01	.02	10	.02	.11	73	.09	.11	73	.09	-.09	-60	.11
MEXA 10x10																					
4	.50	714	.50	.018	18	.03	-.02	-24	.03	.01	12	.02	.11	210	.13	.11	210	.13	-.05	-100	.05
MEXA 8x8																					
4	.48	600	.48	-.005	-1	.03	-.04	-27	.04	-.01	-4	.03	.08	112	.10	.08	112	.10	-.10	-97	.12
All Wheeled Vehicles																					
11	.42	444	.43	.006	7	.02	-.03	-24	.04	-.0009	+2	.02	.09	136	.11	.09	136	.11	-.09	-93	.10
Tracked Vehicles																					
M113																					
3	.36	450	.36	.02	19	.03	-	-	-	-	-	-	-.08	-100	.08	-.07	-88	.08	-.08	-100	.08
MEXA Track																					
4	-	-	-	.037	-6	.08	-	-	-	-	-	-	-.18	-100	.24	-.18	-100	.24	-.18	-100	.24
All Tracked Vehicles																					
7	.36	450	.36	-.007	6	.06	-	-	-	-	-	-	-.14	-100	.19	-.14	-95	.19	-.14	-100	.19
All Wheeled and Tracked Vehicles																					
18	.34	371	.38	.002	7	.04	-	-	-	-	-	-	.003	44*	.15*	-	-	-	-.11**	-96**	.14**

* Denotes combination of methods A_W and A_T (original methods).

** Denotes combination of methods C_W and C_T (revised methods).

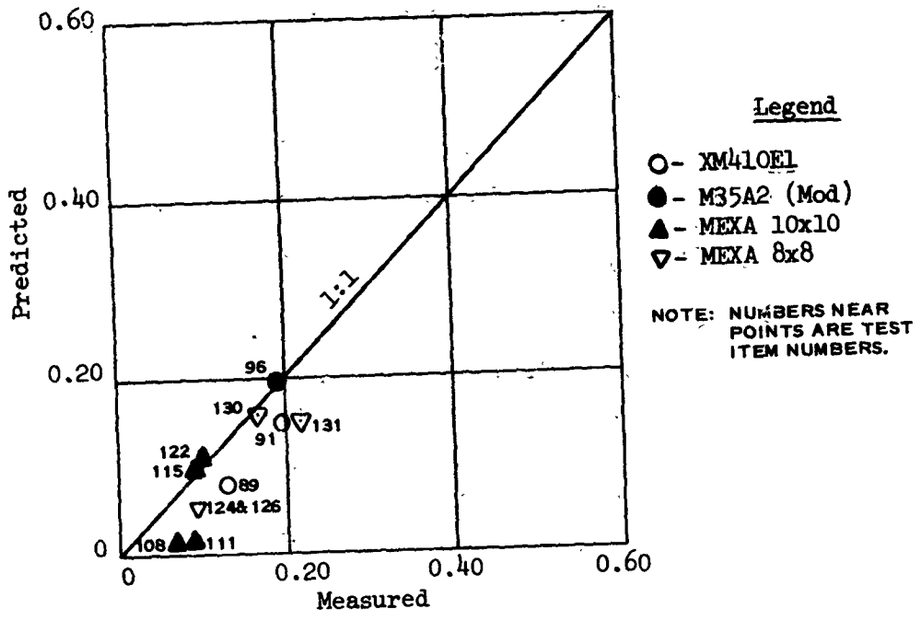


a. WES-VCI$_1$(50) model

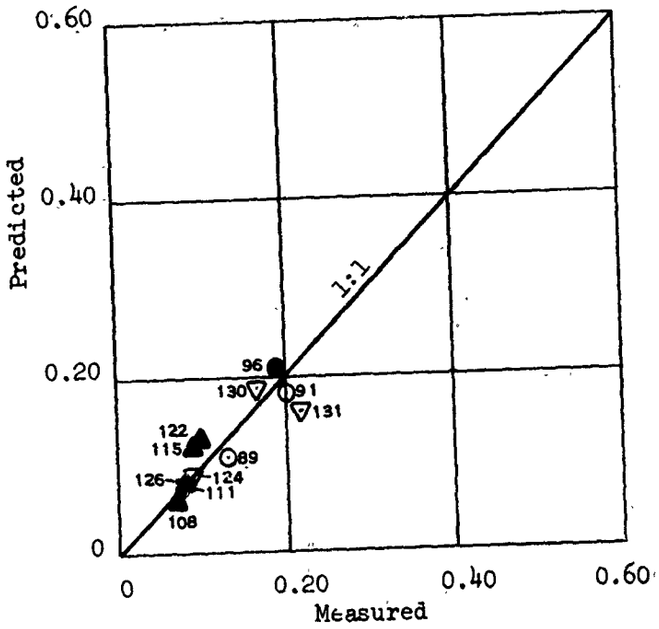


b. WES-VCI$_1$ model

Fig. 11. Comparison of measured and predicted first-pass towed motion resistance coefficient for WES VCI models

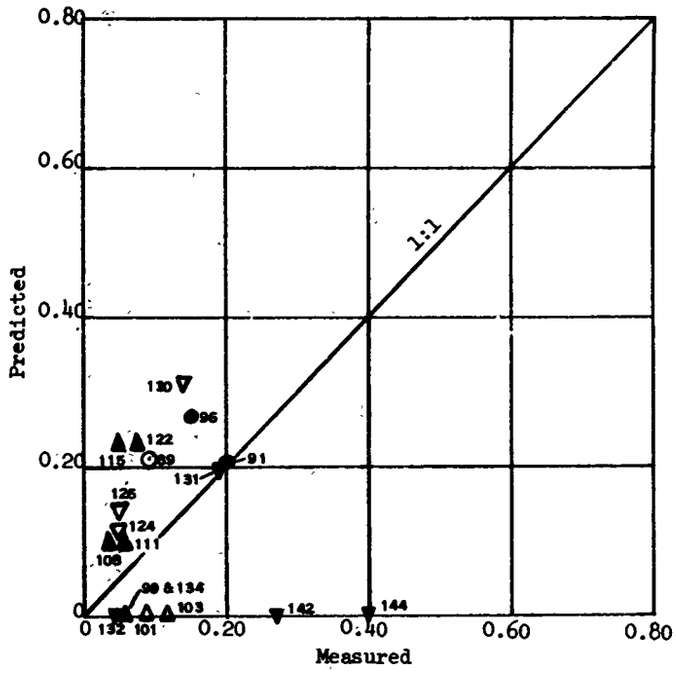


a. WES clay numeric original

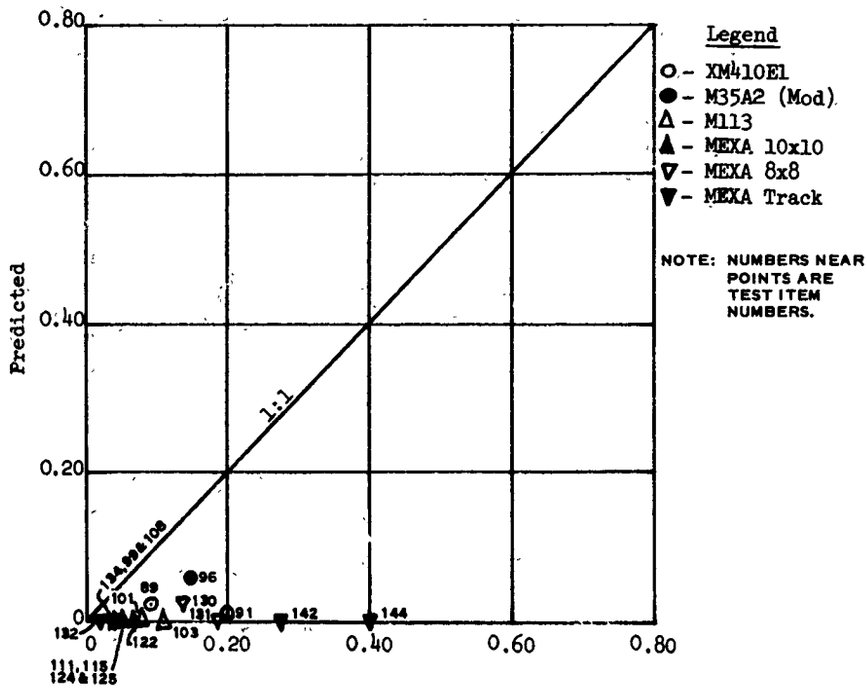


b. WES clay numeric revised

Fig. 12. Comparison of measured and predicted first-pass towed motion resistance coefficients for WES clay numeric model



a. Bekker/LLD Model - A_W and A_T



b. Bekker/LLD Model - C_W and C_T

Fig. 13. Comparison of measured and predicted first-pass towed motion resistance coefficients for Bekker/LLD models

Soil-Vehicle Model	Wheeled Dev	Wheeled % Error	Wheeled RMS	Tracked Dev	Tracked % Error	Tracked RMS	All Vehicles Dev	All Vehicles % Error	All Vehicles RMS	Remarks
WES VCI										
$VCI_1(50)$ original	0.42	444	0.43	0.36	450	0.36	0.34	371	0.38	Data available for only one tracked vehicle
VCI_1 revised	0.006	7	0.02	-0.007	6	0.06	0.002	7	0.04	
WES clay numeric										
Original	-0.02	-12	0.03	--	--	--	--	--	--	
Revised	-0.009	+ 2	0.02	--	--	--	--	--	--	
Bekker/LLD										
A_W	0.09	136	0.11							
B_W	0.09	136	0.11							
C_W	-0.09	-93	0.10							
A_T				-0.14	-100	0.19				
B_T				-0.14	-95	0.19				
C_T				-0.14	-100	0.19				
A_W and A_T							0.003	44	0.15	A_W and A_T combined as most representative of design method
C_W and C_T							-0.11	-96	0.14	C_W and C_T combined as revised method

relations used to make these predictions were developed for multiple-pass performance (25 passes) many years ago and because of lack of better relations, they have been used in first-pass predictions. In recent years, first-pass motion resistance tests have been run in addition to the MEXA program with a variety of wheeled and tracked vehicles, and these data have been used to develop the VCI_1 method. The RMS in the tabulation above indicates that the VCI_1 predictions in clay were good for both vehicle types. The prediction accuracy for the WES numeric for wheeled vehicles was considered good. For the Bekker/LLD models, prediction accuracies for wheeled vehicle methods A_W and B_W were the same; the predictions were higher than the measured results. Wheeled vehicle method C_W predictions were somewhat better than methods A_W and B_W, and the predictions were lower than measured results. The RMS of the tracked vehicle model (A_T, B_T, and C_T) predictions shows that the same level of accuracy was achieved, and the predictions were lower than the measured results. Prediction of motion resistance by the Bekker/LLD models was considered generally poor. Considering the prediction accuracy in terms of RMS for both vehicle types, the combination of

C_W and C_T models gave slightly better results than the combination of A_W and A_T models. The predictions for the combination of A_W and A_T were higher than measured results, while the reverse was true for combinations of C_W and C_T. Of the WES and Bekker/LLD models examined, the WES VCI_1 model was most accurate.

e. Sinkage. Of the soil-vehicle models examined in detail, only the Bekker/LLD models are concerned with predicting sinkage which is used as an input to the motion resistance equations. The MEXA data used in the comparative analysis of measured and predicted results are given in table 13. Plots of measured versus predicted sinkage as used in the various models are given in fig. 14. Results from 20 wheeled and 11 tracked vehicle tests were used in the comparative analysis. The sinkage equations (table 7) were used to predict sinkage for wheeled vehicles. The sinkage equations for methods A_W and B_W are the same (table 7); therefore, the values shown for prediction accuracy in table 13 are the same. In terms of RMS, the method for predicting sinkage for the A_W and B_W models gave more accurate overall sinkage predictions than that for the C_W model. The sinkage predictions made for the C_W model were lower than the measured results, and the reverse was true for sinkage predictions made for the A_W and B_W models. The sinkage predictions for the tracked vehicles were much poorer than those for the wheeled vehicles. The sinkage equations for methods A_T and C_T are the same. Sinkages predicted for the A_T and B_T models are much less than the measured sinkages. Apparently the two methods used to predict sinkage of tracked vehicles are not sensitive to predicting sinkage of low-ground-pressure vehicles. In fig. 14 it can be seen that little or no sinkage was predicted for the MEXA tracked vehicle, yet measured sinkage ranged from about 2 to 10 in. For all types of vehicles,

Table 13

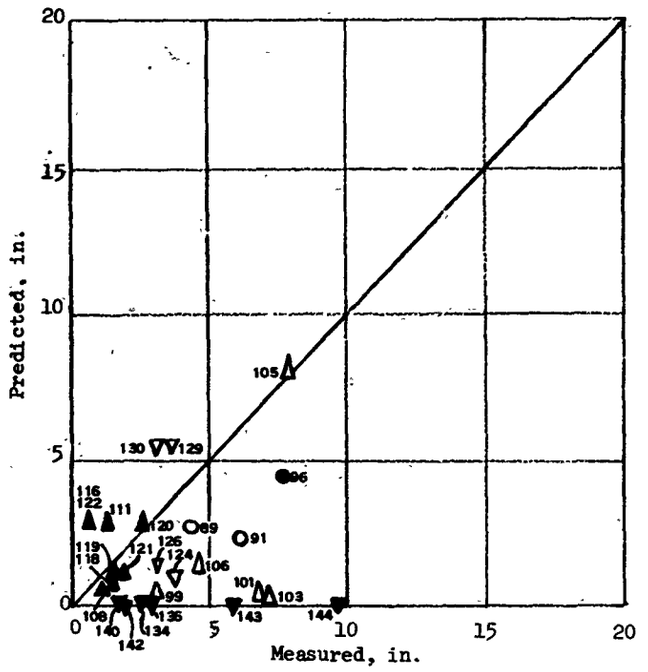
MEXA Data Used in Comparing Measured and Predicted First-Pass Sinkage

Bekker/LLD Models (See footnote for design and revised methods)

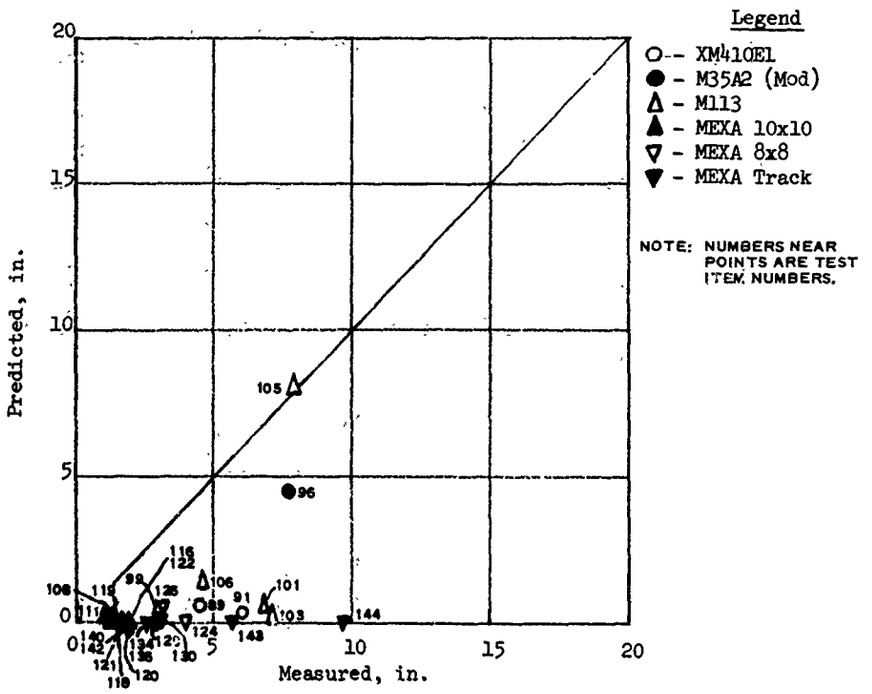
Total Number of Tests	A_T or A_W Algebraic Dev	A_T or A_W Algebraic % Error	A_T or A_W RMS	B_T or B_W Algebraic Dev	B_T or B_W Algebraic % Error	B_T or B_W RMS	C_W or C_T Algebraic Dev	C_W or C_T Algebraic % Error	C_W or C_T RMS
Wheeled Vehicles									
XM410E1									
3	-1.8	-34	2.4	-1.8	-34	2.4	-3.8	-87	4.1
M35A2 (Mod)									
2	-1.4	-14	2.3	-1.4	-14	2.3	-4.1	-70	4.5
MEXA 10x10									
10	0.3	64	1.2	0.3	64	1.2	-1.3	-105	1.4
MEXA 8x8									
5	-0.4	-11	2.1	-0.4	-11	2.1	-3.1	-92	3.1
All Wheeled Vehicles									
20	-0.4	23	1.8	-0.4	23	1.8	-2.4	-96	2.9
Tracked Vehicles									
M113									
5	-3.8	-68	4.6	-0.5	-21	7.0	-3.8	-68	4.6
MEXA Track									
6	-4.0	-98	5.0	-4.0	-97	5.0	-4.0	-98	5.0
All Tracked Vehicles									
11	-3.9	-85	4.8	2.4	-62	6.0	-3.9	-85	4.8
All Wheeled and Tracked Vehicles									
31	-1.6*	-15*	3.2*	-	-	-	-3.0**	-92**	3.7**

* Denotes combination of methods A_W and A_T (original methods).

** Denotes combination of methods C_W and C_T (revised methods).



a. Bekker/LLD - A_W and A_T



b. Bekker/LLD - C_W and C_T

Fig. 14. Comparison of measured and predicted first-pass sinkage for Bekker/LLD models

the overall prediction accuracy obtained for the combination of A_W and A_T models was better than that obtained for the combination of C_W and C_T models.

f. Slip. All Bekker/ILD models except A_W and A_T use selected slip values to compute gross traction. In this comparative analysis of measured and predicted slip, the evaluation is made only for slip at optimum drawbar pull. The MEXA data used for this comparison are given in table 14. The deviations are in terms of percent slip. The wheeled vehicle slip predictions for both methods were higher than measured values. The predictions obtained with method B_W were poor, but the predictions obtained with method C_W were very good. In terms of RMS, the prediction accuracy for the tracked vehicle was about the same for both methods. The predictions were lower than measured results.

116. Summary discussion. Because of the importance that a soil-vehicle model plays in the overall prediction accuracy of a comprehensive ground mobility model for predicting speed performance, a few summary statements are made to place the results of the soil-vehicle model verification analysis in the proper perspective.

117. All comprehensive ground mobility models include a soil-vehicle model to obtain net traction (DBP). Net traction is defined universally as the tractive force developed at the vehicle drawbar when the vehicle is traveling in a straight line on a level surface at a slow constant speed. It is normally obtained as the difference between total or gross traction (H) available and external motion resistance (R). The general equation is $H = DBP + R$. Net traction is a function primarily of the soil strength, the vertical load imposed by the vehicle on the soil, slippage between the traction elements and the soil, and the vehicle speed. All of the soil-vehicle models verified using the MEXA data accounted for all the factors that affect net traction or drawbar pull in some form except vehicle speed, which in all tests the speed of the traction elements was constant or nearly so. Drawbar pull prediction by the WES VCI model is at the optimum or maximum pull, and it is obtained from a set of curves relating excess

Table 14

MEXA Data Used in Comparing
Measured and Predicted Optimum Slip

Bekker/LLD Models (See footnote for design and revised methods)

Total Number of Tests	B_T or B_W Algebraic Dev	B_T or B_W Algebraic % Error	B_T or B_W RMS	C_W or C_T Algebraic Dev	C_W or C_T Algebraic % Error	C_W or C_T RMS
Wheeled Vehicles						
XM410E1						
6	21	110	21	0.33	2	2
M35A2 (Mod)						
4	16	66	16	1	4	1
MEXA 10x10						
13	7	39	25	-0.08	-0.08	2
MEXA 8x8						
9	7	46	20	1	6	3
All Wheeled Vehicles						
32	11	58	22	0.38	3	2
Tracked Vehicles						
M113						
8	-8	-44	8	-6	-36	6
MEXA Track						
11	-10	-37	16	-10	-36	17
All Tracked Vehicles						
19	-9	-40	13	-9	-36	14
All Wheeled and Tracked Vehicles						
				-3*	-12*	8*

* Denotes combination of methods C_W and C_T (revised methods).

soil strength to drawbar coefficient for different types of vehicles or category of vehicles within a given type. A simple equation of dimensionless form is used by the WES numeric (wheeled vehicles only) to predict drawbar pull at 20 percent slip. For most of the Bekker/LLD models, complex equations are used to calculate drawbar pull at selected slip values from which a drawbar pull-slip curve is constructed. The discussion on drawbar pull is also applicable to the prediction of motion resistance by the various models.

118. An examination of the elements of each of the models verified and the results obtained shows that the calculations required in predicting drawbar and motion resistance by the WES methods are simple as compared to the Bekker/LLD models. Furthermore, the WES models produced more accurate predictions. The WES VCI and WES numeric (wheeled vehicles only) models for predicting optimum drawbar pull coefficient have an RMS of 0.06 and 0.13, respectively. The RMS for motion resistance coefficient (wheeled vehicles only) for the WES VCI_1 was 0.02 and for the WES numeric 0.02. The combination of C_W and C_T considered as the Bekker/LLD revised model uses the same equations for predicting gross traction and motion resistance. For these models, sinkage is presumed to be uniform along the track length and when applied to wheeled vehicles, the contact area is presumed to be defined by the product of hard-surface contact length of the deflected tire by the section width of the tire times the number of tires. An examination of the Bekker/LLD revised method (C_W and C_T combination) prediction data indicates that the prediction accuracy for all vehicles in terms of RMS was 0.11 for optimum drawbar pull coefficient, 0.14 for motion resistance coefficient, 3.7 in. for sinkage, and 8 percent for slip. The predictions for tracked vehicles were worse than those for wheeled vehicles.

119. The large deviations in sinkage are reflected in the prediction of motion resistance. The relatively low RMS obtained for the optimum drawbar pull suggests that the error in computing sinkage and motion resistance must have been matched by compensating errors in computing traction. With the extremely low sinkage predicted (see fig. 14b) as compared with the measured results, approximately the same prediction accuracy for drawbar pull would have resulted, with perhaps some modification to the traction

equation, had there been no allowance made for soil motion resistance at all.

120. In light of the overall results achieved by the Bekker/LLD C_W and C_T soil-vehicle models, it appears reasonable to consider using the Bekker/LLD shear measurements and tractive area assumptions to compute available vehicle traction without bothering with the controversial and worrisome problems of predicting sinkage and motion resistance from small plate tests. This suggestion is further reinforced by examination of fig. 15, which compares measured cone index with cone index predicted from bevameter parameters using equations proposed by Janosi for MEXA tests conducted at the Nevada test sites. The data used in the comparison and the equation used are given in table 15. The tendency to underestimate the strength of weaker materials and to overestimate the strength of stronger materials, seen in the vehicle test results, is repeated. If the penetration resistance of a simple, rigid, asymmetric cone of the same general scale as the test plates cannot be reliably predicted from the plate penetration results, the chance for achieving acceptable estimates of the sinkage of the larger and more complex system

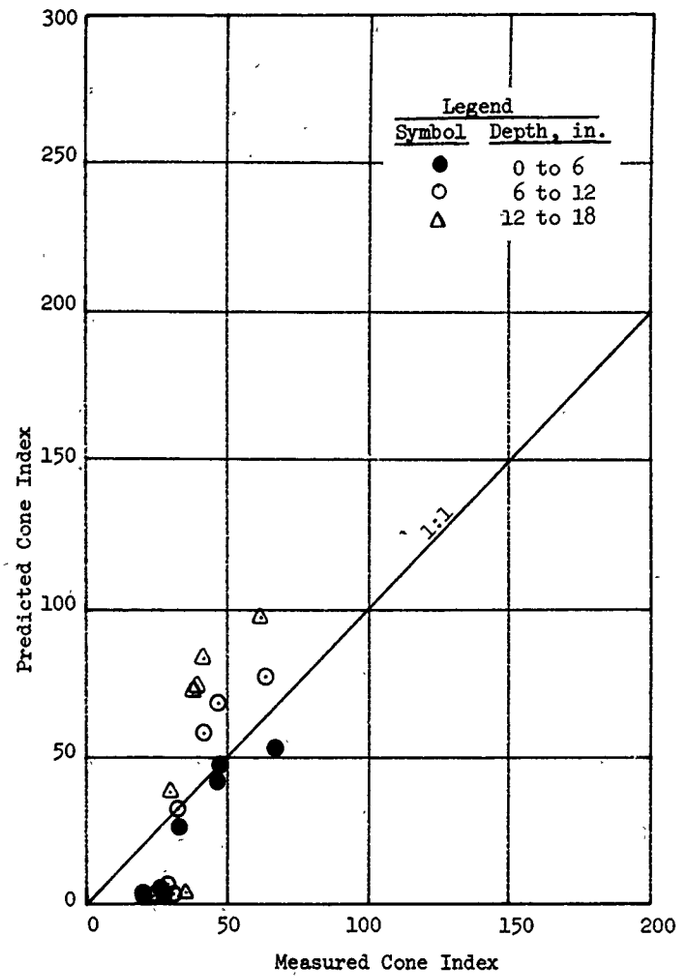


Fig. 15. Comparison of measured cone index and cone index predicted from bevameter parameters using equation prepared by Janosi

Table 15

MEXA Soil Data Used to Compare Measured Cone Index with Cone Index Predicted from Bevameter Parameters (k_c, k_ϕ, and n) Using Equations Prepared by Janosi

Item No.	Site No.	Soil No.	Cone Index of Layers, in. Measured 0-6	Measured 0-12	Measured 0-18	Predicted 0-6	Predicted 0-12	Predicted 0-18	Measured or Computed Bevameter Values k_c	k_ϕ	n	c	ϕ	K
87	4	4	67	64	62	53	76	96	20	10.1	0.58	0.29	35.3	1.0
89	2	2	48	47	42	48	68	84	19.7	9.0	0.55	0.28	30.9	1.0
91	3	3	34	32	30	26	32	37	10.4	12.4	0.37	0.23	32.3	1.0
105	6	10	29	31	35	4	4	5	0.8	5.3	0.14	0	9.3	1.0
106	6	6	27	28	29	5	6	6	0.5	6.8	0.16	0.17	13.8	1.0
108	2	11	46	43	39	41	58	71	14	12.7	0.53	0.28	30.9	1.0
109	2	12	46	43	39	41	58	72	10	18.0	0.53	0.28	30.9	1.0
115	16	6	20	20	22	3	3	3	0	3.8	0.17	0	11.2	1.0

The rearranged form of Janosi's equations used to obtain predicted cone index for the comparison is as follows:

$$CI = \frac{1}{K\lambda_1}\left(k_\phi + \frac{K\lambda_2}{d} k_c\right) \qquad (10)$$

where

$$K\lambda_1 = \frac{(n+1)(n+2)}{2H^n} \times \frac{(\lambda - 1)^{n+2}}{\lambda^{n+2} - (n+2)\lambda + (n+1)}$$

$$K\lambda_2 = \frac{(n+2)}{2} \times \frac{(\lambda - 1)(\lambda^{n+1} - 1)}{\lambda^{n+2} - (n+2)\lambda + (n+1)}$$

$$\lambda = \frac{H}{z} + 1$$

and where H = cone length, from tip to base = 1.5 in.
z = cone sinkage measured at the cone base, in.
d = 0.8 in.

represented by a number of wheels or tracks is small. An extensive survey of plate sinkage relations by Hvorslev (1969), including the bevameter equations, concludes with

> ...the results of routine sinkage tests with small plates cannot be adequately represented by the sinkage equations prepared by Bekker and others...

which

> means that the sinkage of large plates cannot always be predicted with acceptable accuracy by means of results of sinkage tests with small plates and currently available theories. Ability to do so is required before the tests and theories can be applied with confidence to the much more complicated conditions encountered in the mobility of vehicles [underline added].

Obstacle-Vehicle Models

121. Surface configuration produces a profound effect on ground contact vehicles whether they are operating on a prepared surface or natural terrain. Exclusive of vehicle characteristics, vehicle road speed is dependent on road alignment, grade, surface, roughness, visibility, and traffic. However, with current technology in roadway design and construction, roadways can be constructed and maintained to yield a specified level of performance in any type of terrain. On the other hand, military vehicles are often required to perform missions on unimproved terrains in various parts of the world. Terrain features occur in many sizes, shapes, and spacings, producing an almost infinite number of combinations of surface configurations; and their effects on vehicle performance range from immobilizations caused by interference to reduction in speed caused by vehicle vibration or obscuration of the surface by vegetal cover. Means to characterize surface configuration (including discrete obstacles) and cover adequately, and methods and techniques for predicting the interactions between any surface configuration, visibility, and the vehicle and driver are thus required as necessary inputs to any comprehensive model for predicting cross-country vehicle performance with some degree of realism. Compared with soil-vehicle models, obstacle-vehicle models, including vehicle

dynamics, are in the early stages of development.

122. Cross-country operation differs from on-road operation in essentially four ways:

a. In off-road travel the path is somewhat more freely selected by the driver.

b. The surface bearing capacity and traction characteristics are more variable off-road than on, and frequently are very low.

c. The surface will generally be much rougher in terms of minor geometric anomalies and major slopes.

d. The selected path will usually be strewn with obstacles of various sizes and shapes, ranging from linear features (streams, ditches, etc.), discrete three-dimensional hazards (rocks, deadfalls, etc.), to systems of clearly insurmountable, small area obstacles in planar array (large trees).

123. The way in which a driver exploits the greater freedom available to him in selecting a path off-road has not yet been modeled. The effects of variations in strength of the operating surface are the subject of soil-vehicle modeling; minor surface anomalies are dealt with as a part of the "ride" dynamic problem, and slopes in soil-slope modeling. All of these have some analogs, however tenuous, in on-road vehicle motion mechanics. The obstacle problem, however, is essentially peculiar to off-road operations.

124. As previously stated, obstacles affect vehicle performance on two levels. They may either reduce the potential operating speed of a vehicle severely or totally immobilize it. While the latter, of course, is merely the end point of the former, the distinction is useful because study at the go-no go level can be adequately handled as a problem in static mechanics, whereas the speed problem should be treated as one in dynamics.

125. In this part of the report, obstacle-vehicle models are discussed separately according to the kind of feature and kind of terrain, vehicle, and driver interaction exhibited. The models considered include those that account for the effects of continuous, irregular terrain where

the profile includes small, irregular terrain features that produce vehicle vibration or occasional shock; slope models that account for gravity effects; discrete obstacle models that account for geometric obstacle-vehicle interference and if they can be overridden, the force required to override them or when randomly spaced, the demands imposed by maneuvering; visibility models that account for the effects of recognition distance on performance; and stream-crossing models that account for the interaction a vehicle encounters when entering, crossing, and exiting a stream.

Perspective

126. Since the advent of the automobile (1920's), research by government and industry has been conducted on a continuing basis in various countries of the world in highway design and construction, and vehicle design, particularly in the areas of steering control, power train, and suspension. These areas are the principal contributors to the safety, efficiency, and riding comfort of on-road vehicles.

127. The speed at which a driver of a vehicle will traverse obstacles or continuous irregular terrain is controlled primarily by the level of vibration activity that does not exceed his particular ride comfort level. Vehicle vibration or ride is sensed by a driver or passenger through sight, touch, and hearing in response to external stimuli, such as motions, forces, and sounds. Whenever this sensation becomes too severe, the driver will alter the vehicle's speed until the sensation reaches an acceptable level. This sensation, therefore, is a significant factor in determining the speed of a vehicle over a given terrain. The maximum vibration activity may be limited by the type of cargo or the structural elements of a vehicle. Generally speaking, the driver or the cargo establishes the ride limits. The irregular terrain-vehicle problem is essentially one of dynamics, and its solution must include the combined effects of the surface being traversed, the vehicle, and the driver. Because of the complexity of the problem and the desire to produce better riding vehicles, considerable effort has been expended on modeling dynamic vehicle response.

128. Because of the lack of mathematical techniques required in modeling suspension systems, much of the early work consisted of

cut-and-try methods. The first significant contributions to an analytical treatment of vehicle dynamics were made by Rowell, Guest, and Olley in the early 1920's and 30's. In 1922 Rowell made one of the first attempts to analytically define the problem. This work was extended in 1926 by Professor Guest and again in 1934 by Olley, who introduced an analytical treatment of independent wheel suspension.

129. In 1941 a mechanical differential analyzer was built by Schilling and Fuchs specifically for suspension analysis, and although it was suited to only a single-degree-of-freedom system, it did permit the inclusion of a nonlinear characteristic of shock absorbers. An important use of this analyzer was the continuous determination of transient motions and portrayal of the effect on motion by changes in the characteristics of the shock absorber. This differential analyzer was the forerunner of today's analog computer, and its capabilities led to rapid advances in suspension analysis and design. By the 1950's, it was widely exploited by the automotive industry.

130. In 1953, Jeska developed a 4-degree-of-freedom model that included pitch and bounce of the body and vertical motions of the front and rear wheels. The forcing function was an actual road wave measured by a photographic technique. In 1955, Bodeau, Bollinger, and Lipkin of Ford Motor Company developed a detailed ride analysis in which a 9-degree-of-freedom model was used to describe a passenger car. In 1960, Kohr of General Motors Corporation developed a mathematical simulation of automobile ride. In his simulation, a measured road profile was recorded on magnetic tape, and the tape was fed through an analog computer model of the vehicle to predict the vehicle motions, i.e. pitch, bounce, and roll. The resulting motions were used to drive a vibration simulator, which was used as a laboratory means of assessing the effect of the vibration on humans.

131. Until about 1960, the analysis of ride had been concerned primarily with the suspension system and means of improving the ride quality. Although considerable work was done in the area of human tolerance to vibration, a means for quantifying human tolerance to vibrations had not been developed. Van Deusen has shown that very little of the research done actually pertains to the surface vehicle problem. Most experiments have been

devised to assess human response to sinusoidal motion in only one direction, while the more complex ride comfort problem involves random vibrations in various directions. The most frequently used criteria have been those of Dieckman and Janeway, who developed simple formulas for relating comfort limits to amplitude and frequency of vibration. There have been several studies of "on-the-road" measurements of ride comfort. For example, von Eldik Thieme examined the Dieckman-Janeway criteria in the actual vehicle environment, but he met with little success. Van Deusen and Versace used a technique, referred to as cross modality, in which subjects received noise signals through earphones and adjusted and matched their level to the sensation level of ride vibration. A statistical analysis showed favorable correlations of the measured accelerations with ride sensation, and at least indicated that correlations between ride sensation and vibration were possible.

132. In the late 1950's the military began to recognize the significance of vehicle vibration on off-road mobility and fire effectiveness. This led to an extensive effort to quantify the vehicle vibration problem and to correlate it with human response and terrain characteristics. Interest was shifted from deterministic to stochastic techniques. The latter consists of classifying the terrain profiles by certain pertinent statistics and analyzing the response statistically. The groundwork for this type of analysis was begun in 1959 by Bogdanoff and Kozin, who described in detail the statistical analysis of the responses of simple linear systems to random terrain inputs. Although the vehicle models were simple and idealized, the analyses provided a starting point and yielded much useful information regarding fundamental relations between pertinent vehicle parameters and statistical terrain quantities. This study preceded those of Bieniek (1960), Van Deusen (1962), and Bussman (1964), who followed essentially the same approach as that described by Bogdanoff and Kozin. The one notable exception was Van Deusen's introduction of a nonlinear vehicle system into his statistical analysis.

133. In 1963, the U. S. Army Tank-Automotive Command began basic research on the effects of vehicle vibration on human response. This work was based on the results of past studies and so was oriented towards

quantifying the effects of random vibration on vehicle-driver performance. Two performance parameters were developed to describe human response--"acceleration density" and "absorbed power." Of the two performance parameters, absorbed power is preferred in quantifying human response to vibration since it is a descriptor of the flow of energy from the vibrating vehicle to the driver. During the 1960's the military sponsored several studies in the development and application of ad hoc comprehensive cross-country models in which V-ride was incorporated as a submodel.

134. In the early 1960's the scope of WES mobility research was expanded, and static and dynamic surface configuration-vehicle-driver interaction studies were initiated. In 1965, FMC Corporation conducted a study for WES to determine the feasibility of using a digital computer to simulate the dynamic response of ground vehicles traveling over unyielding irregular terrain segments. This study resulted in the development of a generalized mathematical model of an n-axle vehicle which, within limits, is suitable for both wheeled and tracked vehicles. Recent efforts by WES have produced improvements in vehicle dynamics modeling.

135. In 1966, CAL, under contract to ARPA,* began a comprehensive study designed to augment knowledge of ground mobility and to refine methods available to planning, engineering, and field personnel for improving it. One result of this study was the development of a comprehensive vehicle dynamics model that included both viscous and Coulomb damping properties of the suspensions. The model, although virtually untried, shows promise for achieving a greater degree of realism than other models.

136. Recent events in the United States space program have placed even more emphasis on the dynamic behavior of ground vehicles. Several studies of ride quality of extraterrestrial vehicles have already been conducted by contractors for NASA. By their very nature, such studies require a statistical analytical approach since cost and complexity preclude experimental testing in extraterrestrial environment.

137. Organized discrete obstacle-vehicle research in the Western

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* This program was terminated prior to the completion of the phases of the program initially agreed to by ARPA.

world was given special attention as the result of World War II experiences. Early United States military efforts were concerned with designing military vehicles that would reduce immobilizations by obstacle interference. Obstacle test courses were constructed (APG) and tests on these courses have become part of the overall vehicle engineering evaluation test program. The results of these studies have led to the recent development of articulated vehicles. United States discrete obstacle-vehicle research studies gained more emphasis about the mid-1950's when terrain factors other than soils were introduced as deterrents to off-road vehicle travel and more attention was given to obstacle geometry interference and the effects of dynamic response on vehicle performance. By the early 1960's these studies produced several static and quasi-dynamic models which related, by simple two-dimensional static mechanics, slope and obstacle geometry to go-no go performance. A number of terrain-vehicle interactions not previously investigated were included in the MERS program. For example, the forces involved in overriding trees and obstacles, and the effects of obscuration or recognition distance on vehicle performance were investigated. Special attention has also been given by the British and Canadians to riverine crossings, and several devices for aiding vehicles in crossing streams and rivers have been developed. In addition to vehicle studies, research in the United States and Britain has continued since World War II to develop expedients for bridging gaps and stabilizing weak soils.

General status

138. Today's practice in modeling the effects of surface configuration and vegetation on vehicle performance consists essentially of two types of analysis which are separated on the basis of the kind of vehicle and/or driver interaction anticipated. Regardless of the analysis performed, the terrain profile and associated discrete obstacles except vegetation are considered rigid. This consideration represents the worst conditions from the standpoint of the vehicle's behavior. If a terrain unit contains an irregular surface that can be easily overridden by a vehicle without inducing frequent shock, vehicle performance is predicted by a dynamics model. In this case, the problem is commonly identified as surface roughness, and the profile used is a statistically uniform surface profile.

If a terrain unit contains discrete obstacles larger than those included in the rough terrain analysis and which are likely to produce immobilizations, it is assumed that traversing or circumventing the obstacles will be accomplished at a creep speed and the interactions are treated as static phenomena. The models used in such terrain situations are thus static models. The controlling factors in the relation of a static model are the geometry of the obstacle and vehicle configuration. If the discrete obstacles that the vehicle must pass over (dikes) occur at wide spacings, and they can be overridden at greater than creep speeds, a dynamic model is used to determine the maximum override speed. If a vehicle can override some obstacles and avoid some such as trees, override and avoidance models are used to select the optimum speed. For terrain units whose surfaces are obscured by vegetation, namely grasses, a separate model is used to predict the effects of recognition distance on vehicle speed.

139. Vehicle dynamic models simulate mathematically the dynamic response of selected points within a vehicle (usually driver's seat or cargo compartment) in traversing discrete obstacles or rough terrain. Performance is generally expressed in terms of relations between speed and such quantities as absorbed power, RMS, or peak acceleration, and referenced to established horizontal and vertical acceleration and power limits. Most dynamic models available are designed so that the motion of any generalized rigid-body vehicle can be analyzed by one basic computer program. Most dynamic models predict vertical acceleration; however, they can easily be modified to include horizontal acceleration. The mathematical techniques and physical laws involved in the formulation of dynamic models are common to all models but differences occur in the details of representing the terrain, vehicle, and driver limits, and the size of the computer required.

140. Mathematical descriptions of vehicle behavior in surmounting or maneuvering around obstacles are essentially static models. Discrete obstacles such as rocks, boulders, mounds, scarps, ditches, stumps, etc., are usually first examined to determine whether or not there will be spatial interference between the obstacles and the nonpropelling vehicle structure. This examination may proceed in either two or three dimensions: with or without compliance of the vehicle running gear, suspension, or structure;

with or without compliance of the obstacle itself; by spatial matching of the vehicle and the obstacle, either of which may be described more or less completely. In some instances, the vehicle underside is scale-modeled, usually in two dimensions. In others, the vehicle shape is idealized to the quantitative description and location of salient features and matching is done through the application of complex but ordinary trigonometry and geometry. The latter procedure is, of course, more suitable for computer use. Where the number of obstacle configurations assumed in an area is relatively small, however, the scale-model experiments can be conducted once and for all and the results stored for subsequent computer consultation as needed. Most comprehensive models include the effects of vegetation but only to the extent of assigning a constant speed when operating in forested areas. A tentative submodel has been developed by WES for determining the trees to be overridden and avoided to obtain optimum vehicle speed. A first-generation submodel for determining the effects of obscuration on vehicle speed has also been developed by WES. Most available stream-crossing submodels are primarily concerned with exiting a stream only and a simple slope model is generally used.

141. Although it is recognized that no vehicle (current or future) can negotiate all terrain features that it will encounter in performing some of its assigned operation mission, no known research is being conducted to assess the kind and amount of engineer effort that may be necessary to render the obstacle or stretch of rough terrain passable. Methods of assessing the engineer effort required to modify a vehicle path to a condition that is passable are required for performance prediction purposes as well as for planning and operational purposes. Those comprehensive models that assess a time penalty to employ engineer effort to modify or remove an obstacle or segment of terrain to a passable condition use crude methods for estimating time required.

142. In spite of the information available in various obstacle-vehicle-driver studies performed to date, the rough terrain and obstacle problems still require considerable research effort to reap the benefits of available technology. Considerable work still remains to be done to resolve the selection of a suitable system for quantifying and portraying

surface configuration for mobility purposes, develop means to predict forces resulting from obstacle-vehicle interactions, and structure the interaction modules or submodels into manageable components so that a meaningful cross-country performance prediction can be simulated by a comprehensive model. When this is accomplished, verification of the submodels and the comprehensive model must be accomplished to obtain the level of prediction accuracy that can be achieved in real terrains.

Strategies available

143. Possible approaches to computing terrain roughness and obstacle-vehicle interactions are to:

a. Separate the overall problem into component parts on the basis of dynamic or static interactions involved, using equations of motion and mechanics and geometric and trigonometric relations in matching obstacle-vehicle interference.

b. Treat continuous irregular terrain surfaces and obstacles as unyielding or yielding under a vehicle load, with unyielding representing the worst case of vehicle behavior.

c. Include a force or traction required in overriding or maneuvering around individual obstacles.

d. Treat the terrain profile either as deterministic or statistical with probability statements.

e. Describe vegetation as a separate kind of obstacle, accounting for the forces involved in override and maneuver or a combination of both, or assign a constant speed when a vehicle operates in a forested area.

f. Assign a constant speed to areas where the surface is obscured by vegetation or model the effects of obscuration or visibility on vehicle performance.

144. A possible application of most of the above-listed strategies is illustrated in fig. 16. Currently operating cross-country simulations include all the strategies outlined above, in one form or another. The path selected through the strategy diagram to deal with terrain roughness or particular types of obstacles can be, and frequently is, difficult even

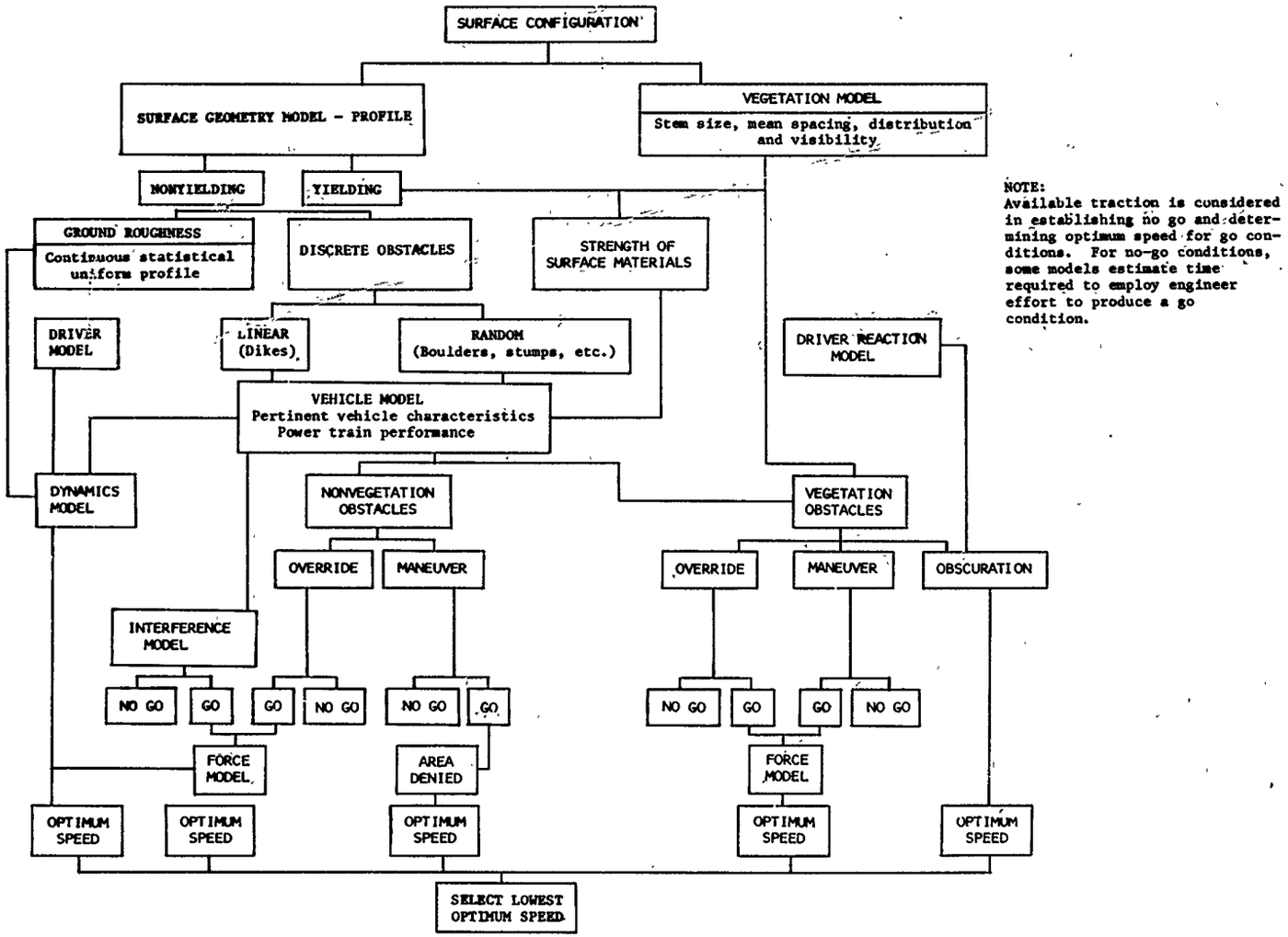


Fig. 16. Flow chart illustrating possible strategies for coupling surface geometry, vegetation, vehicle, and driver models to predict vehicle performance

in a given overall simulation. This comes about, in part at least, because of differences in form and availability of relevant terrain and vehicle data. In its ultimate treatment, the surface configuration (surface geometry and vegetation) problem is a dynamic one in three dimensions involving obstacles that may yield. Traction, inertia, and impedance forces in equilibrium at all times will replace simple considerations of potential geometric interferences. In many instances driver decision will determine (both correctly and incorrectly) whether the problem is one of obstacle avoidance or obstacle surmounting and, where terrain roughness is involved, driver influence will enter again in selecting an acceptable speed on the basis of subjective ride and vibration criteria.

145. In the next sections of this report, discussions are presented on ride dynamic modeling of vehicle vibrations or the surface roughness problem, negotiation or avoidance modeling of discrete obstacles (including trees) on the basis of static relations, visibility modeling, and stream-crossing modeling.

Vehicle ride dynamics model

146. Eight vehicle ride dynamics models were examined in this study. They are: (a) U. S. Army Tank-Automotive Command, (b) Chrysler Corporation (Van Deusen), (c) General Motors Corporation (McKenzie), (d) Ohio State University (Bussman), (e) WES-FMC Corporation (Smith), (f) NASA-GMC, (g) National Aeronautical Space Administration-Brown Engineering, and (h) Cornell Aeronautical Laboratory. Models (a), (b), and (e) have been partially validated. No validation data are available for the others.

147. In reviewing the literature to select models for detailed study, considerable commonality became apparent in the elements of building blocks from which the models were constructed, whether intended for a specific or a generalized vehicle. Because of this commonality in approaches to ride dynamics modeling, and to avoid considerable repetition of information, a general discussion on ride dynamics is first presented, and later differences in models are singled out for discussion.

148. The building blocks common to all of the ride dynamics models examined in detail are shown in fig. 17. This figure shows the commonly used choices in representing terrain, the vehicle, the terrain-vehicle

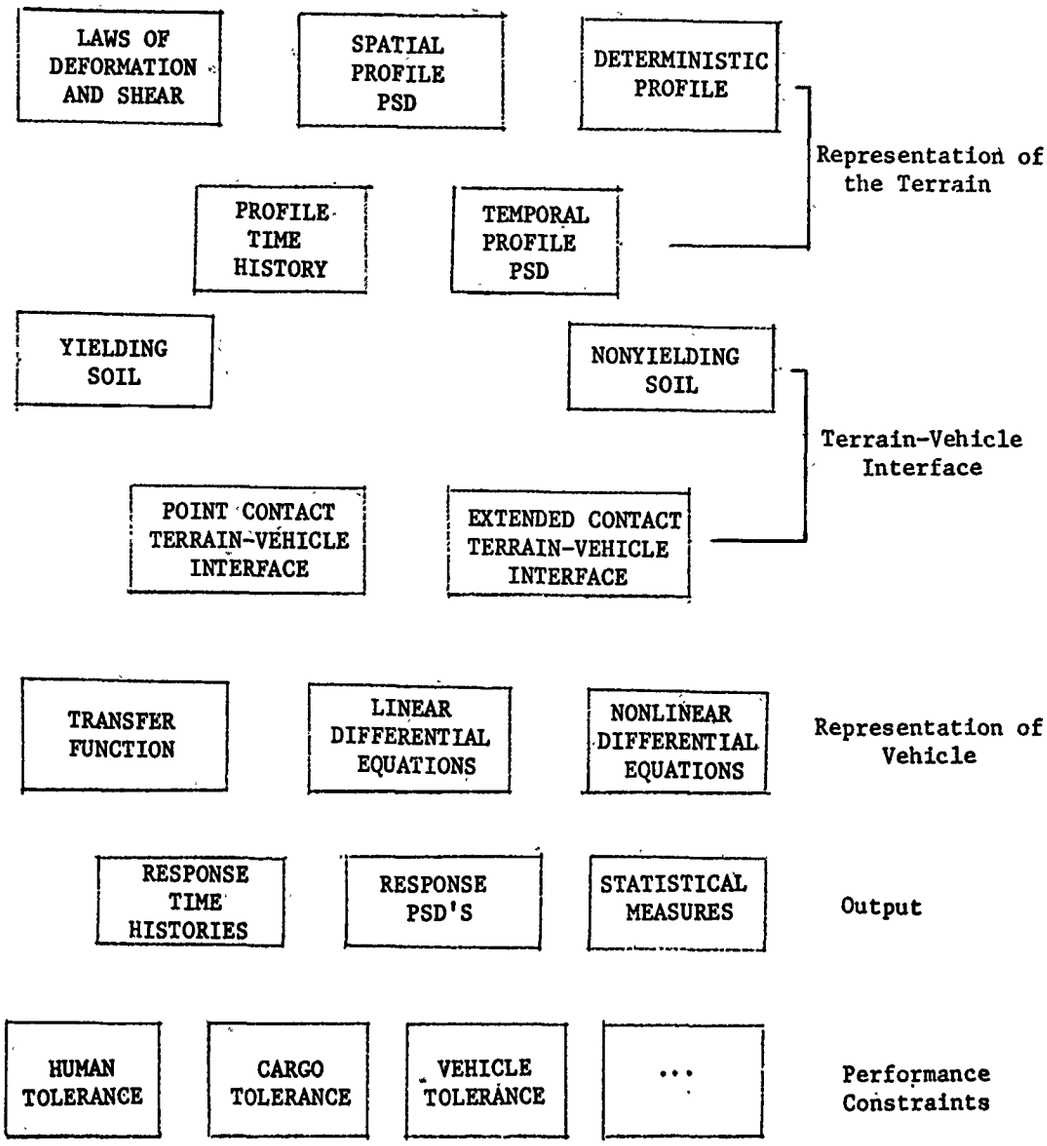


Fig. 17. Principal building blocks used in constructing vehicle ride dynamics models

interface, performance constraints, and the output. In figs. 18-25, the principal building blocks used in the construction of the several models examined are indicated, and the manner in which the building blocks are coupled is shown by the solid line connecting them. When a particular model has more than one coupling approach, alternative couplings are shown as dashed lines.

149. Representation of terrain. A vehicle operating off-road encounters two types of hard-ground roughness. One type consists of discrete obstacles such as boulders, tree stumps, logs, rice-field dikes, etc., to which the driver tends to give individual attention. The other type consists of continual more-or-less gradual variations in elevation that are uniform over reasonably large areas and which the driver treats as an element of the environment. The latter type, often referred to as stable ground roughness, is characteristic of many virgin terrains and is of principal interest in ride analysis. A distinction is made between these two types, and they are generally treated separately in vehicle dynamics studies.

150. The simulation of rough stable ground is normally accomplished by first measuring an actual terrain profile. The measurements, which constitute an elevation-distance sequence for the area of interest, are transferred to magnetic tape, punched cards, or some other means suitable for computer analysis.

151. Terrain profiles used in vehicle ride dynamics analyses may be in either deterministic or statistical form. Selection of the form to be used is dictated by the desired accuracy of the output and by the details of available terrain data. A deterministic profile can be used as-recorded in the form of a deterministic displacement-distance function to excite a vehicle model passing over it. Vehicle response is then outputted in the form of motion-time histories of various parts of the vehicle and generally is not easy to interpret. As an alternate to the deterministic approach, a terrain profile in statistical form can be used. In this form, the terrain profile is expressed in terms of its frequency components.

152. A terrain profile can be converted to a statistical statement of its frequency components. This is accomplished for a random stationary segment of terrain by first computing the autocorrelation function. The

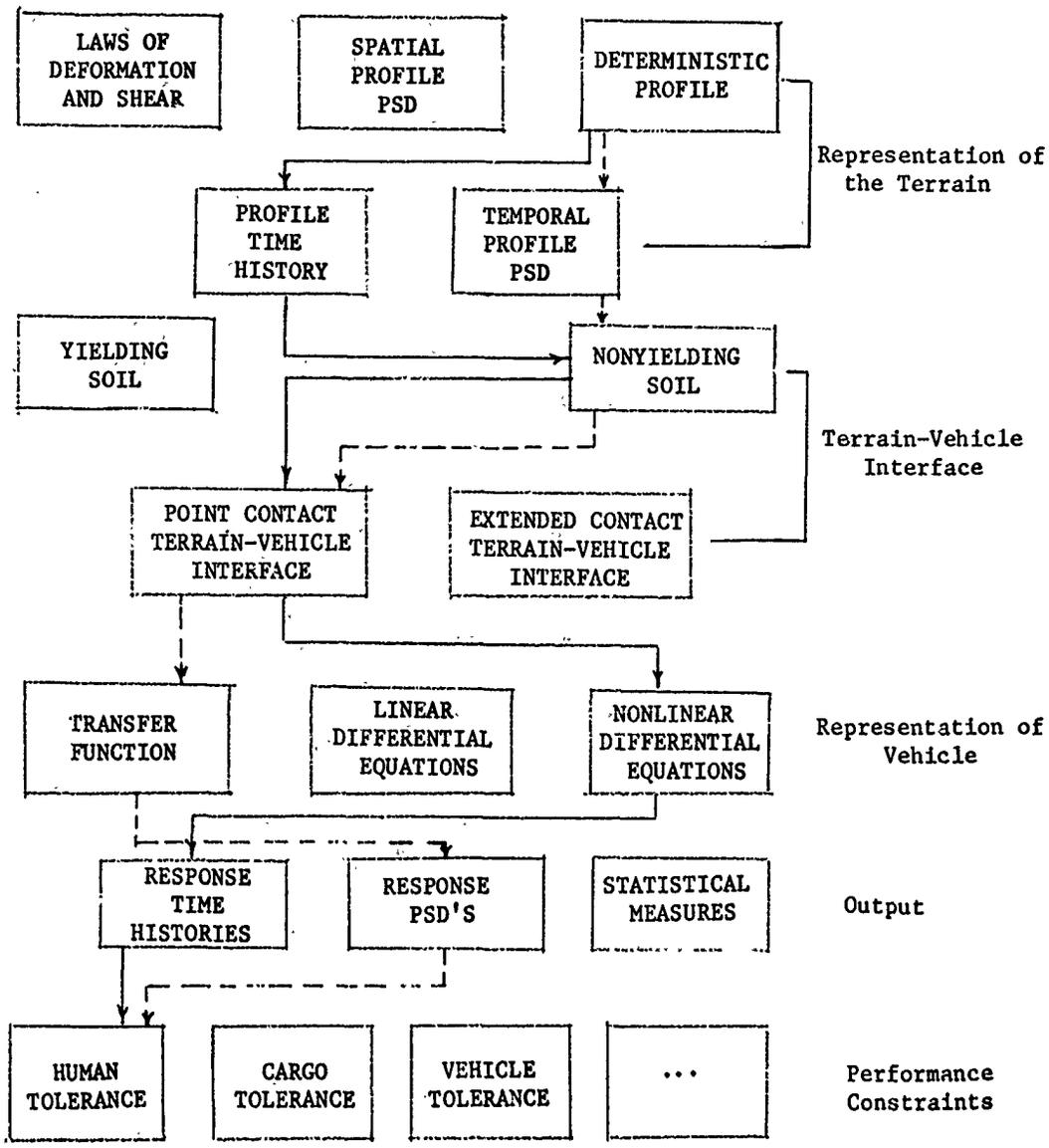


Fig. 18. U. S. Army Tank-Automotive Command ride dynamics model

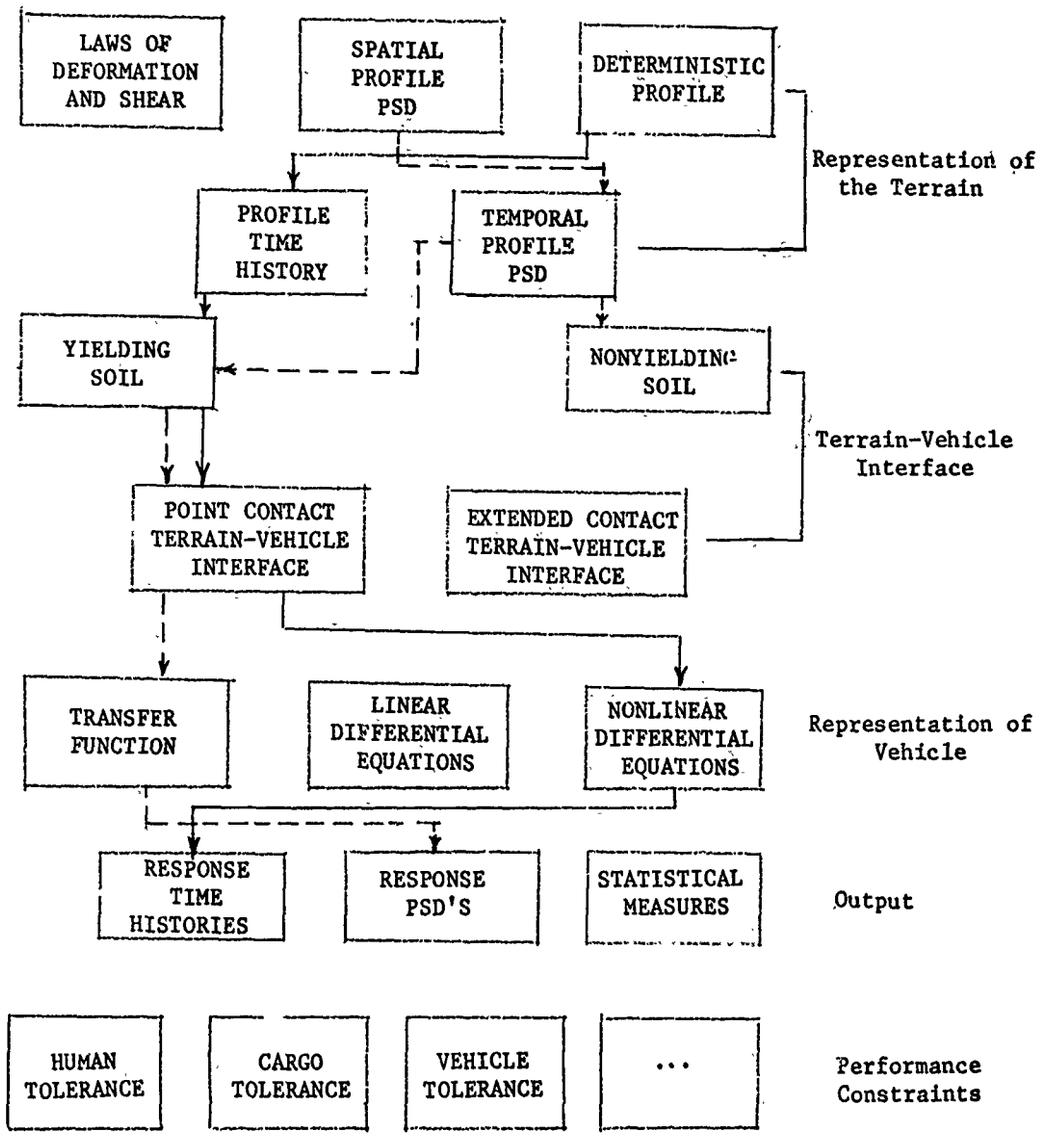


Fig. 19. Chrysler Corporation (Van Deusen) ride dynamics model

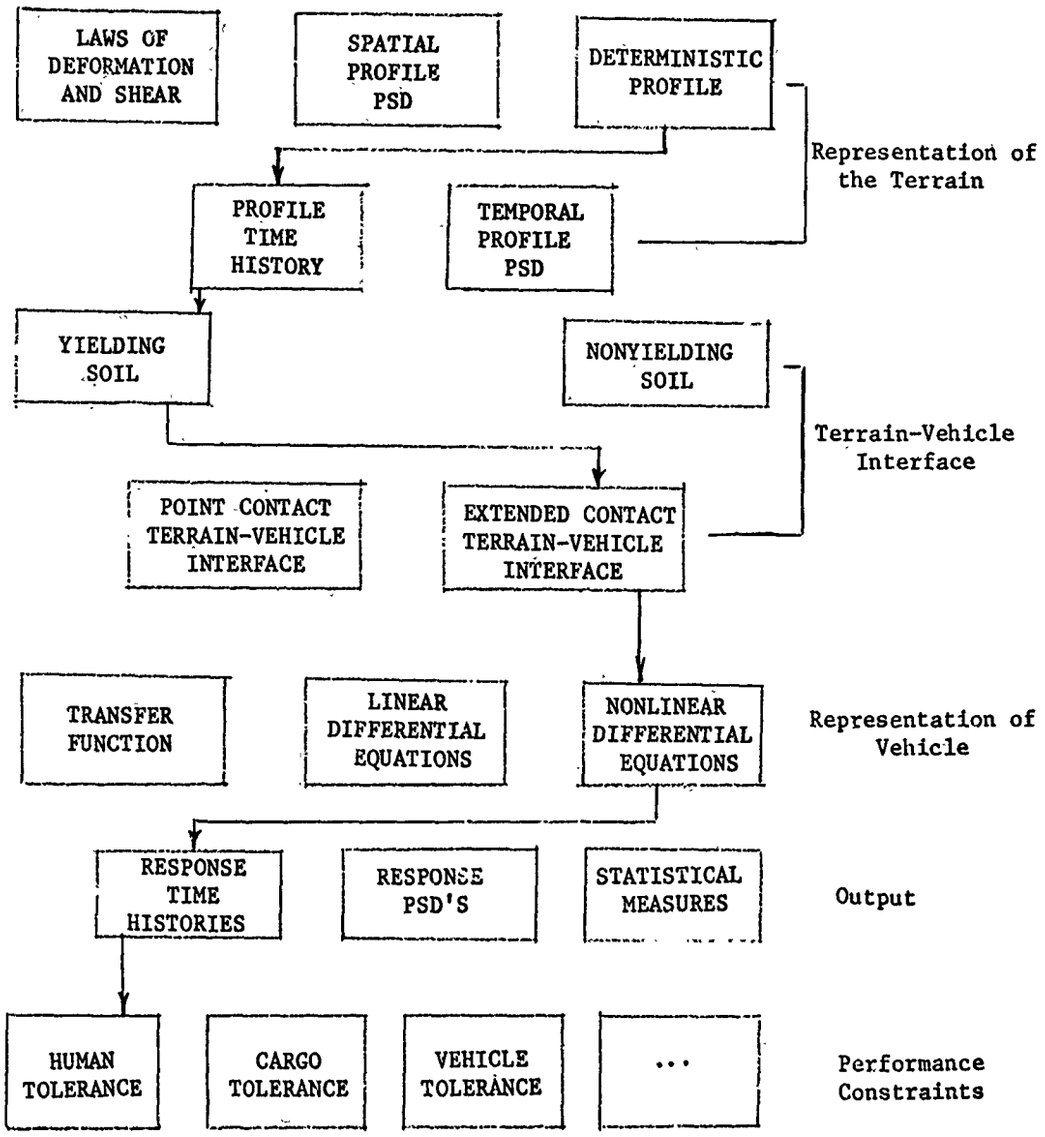


Fig. 20. General Motors Corporation (McKenzie) ride dynamics model

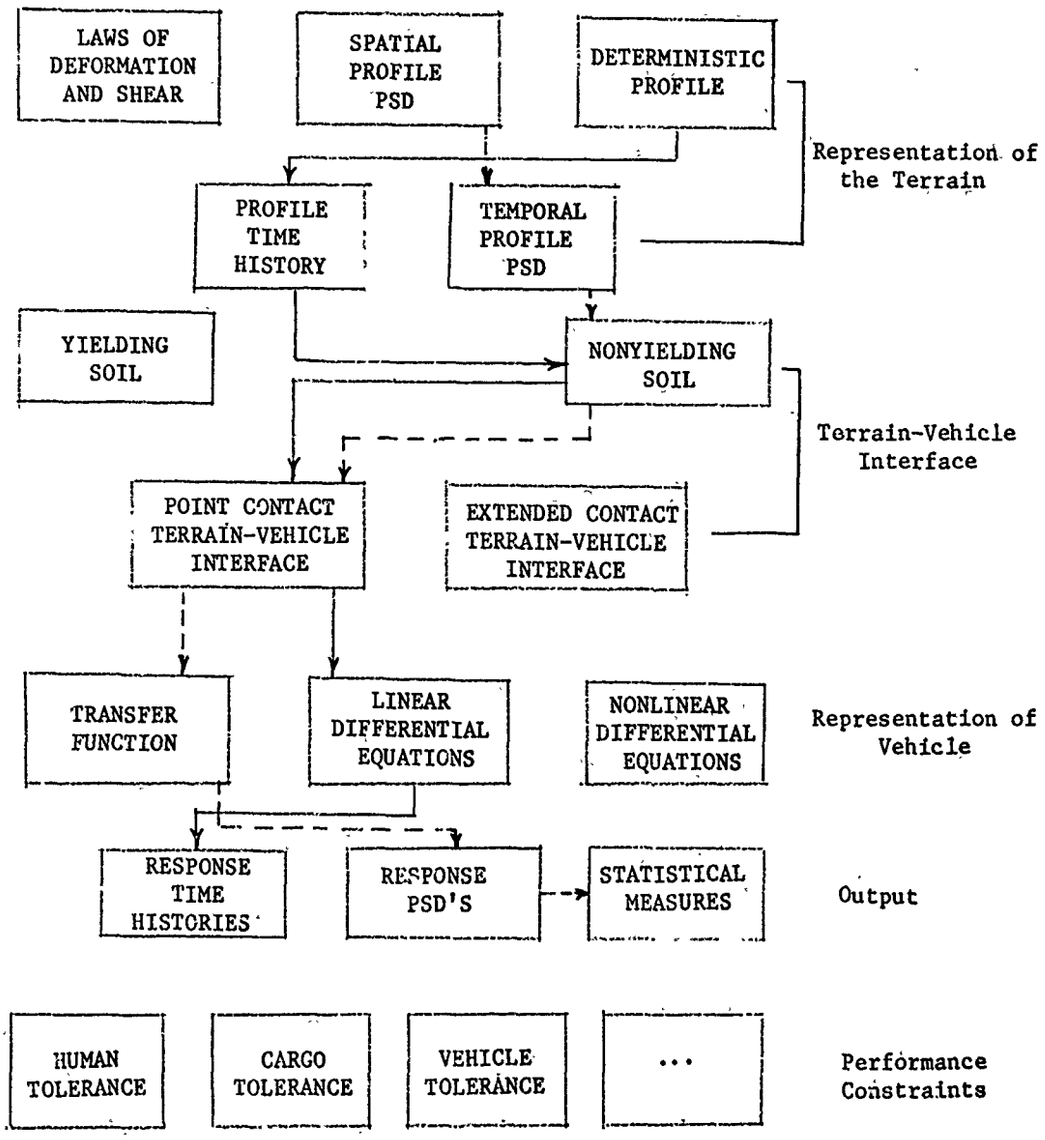


Fig. 21. Ohio State University (Bussman) ride dynamics model

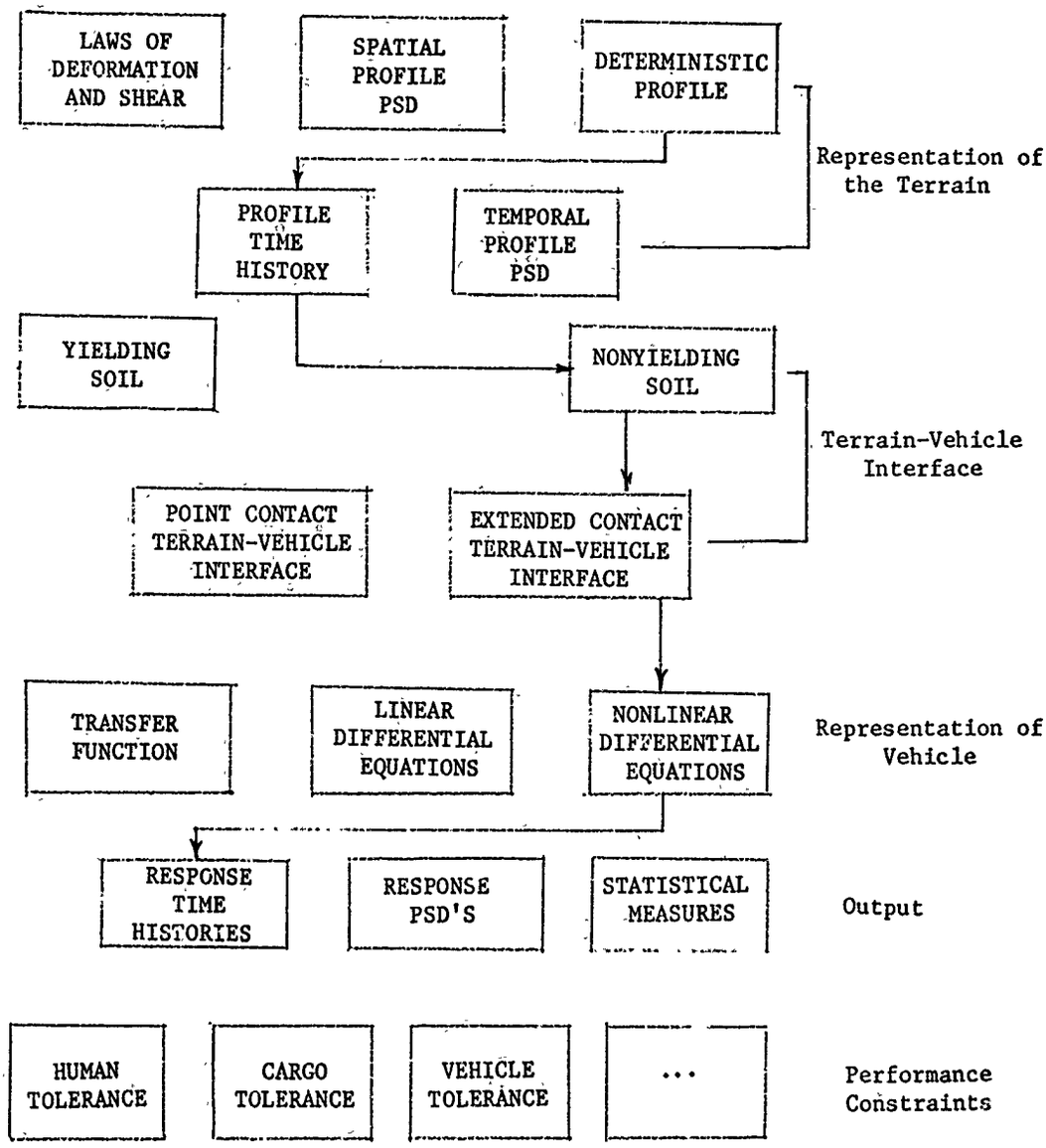


Fig. 22. WES-FMC Corporation (Smith) ride dynamics model

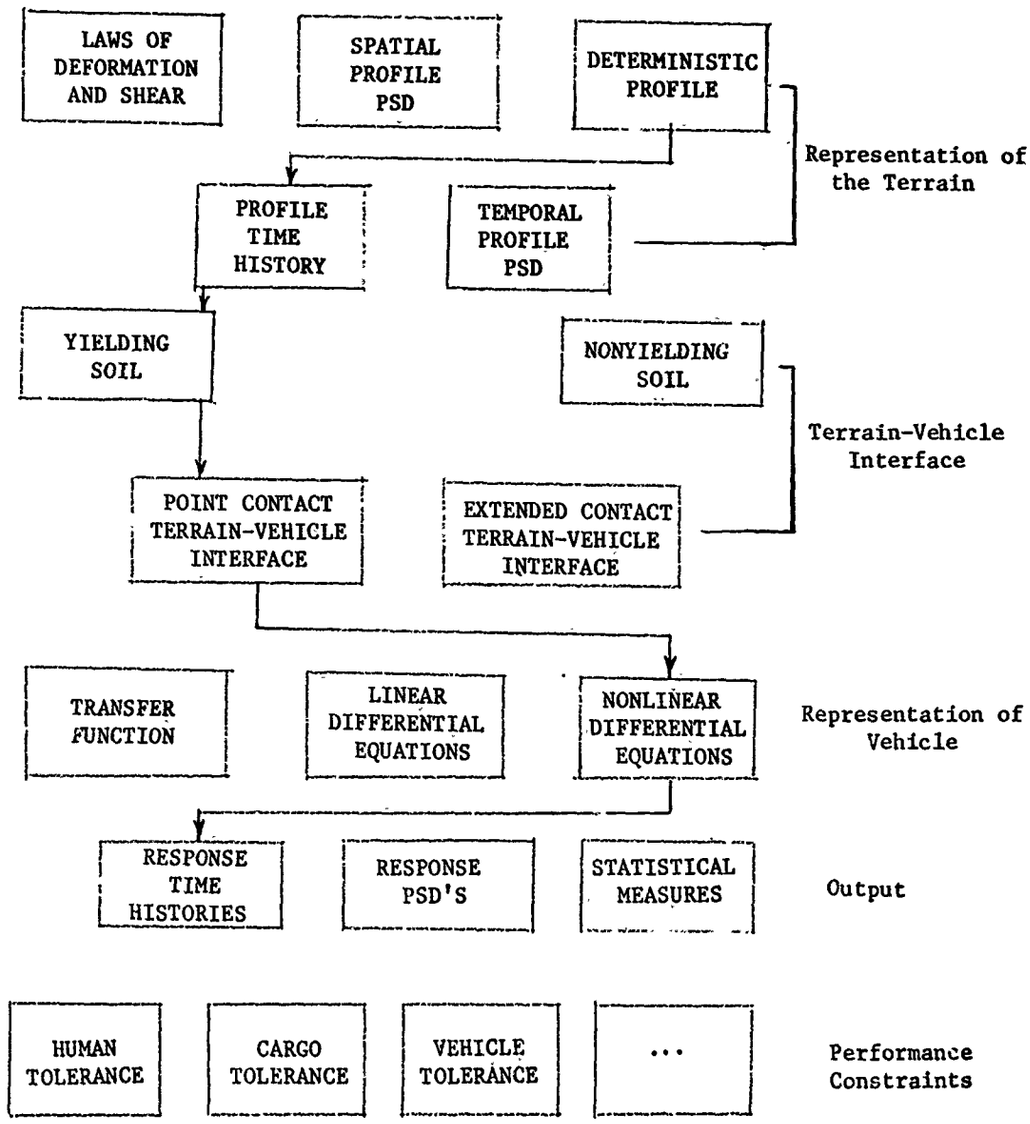


Fig. 23. National Aeronautics Space Administration-General Motors Corporation ride dynamics model

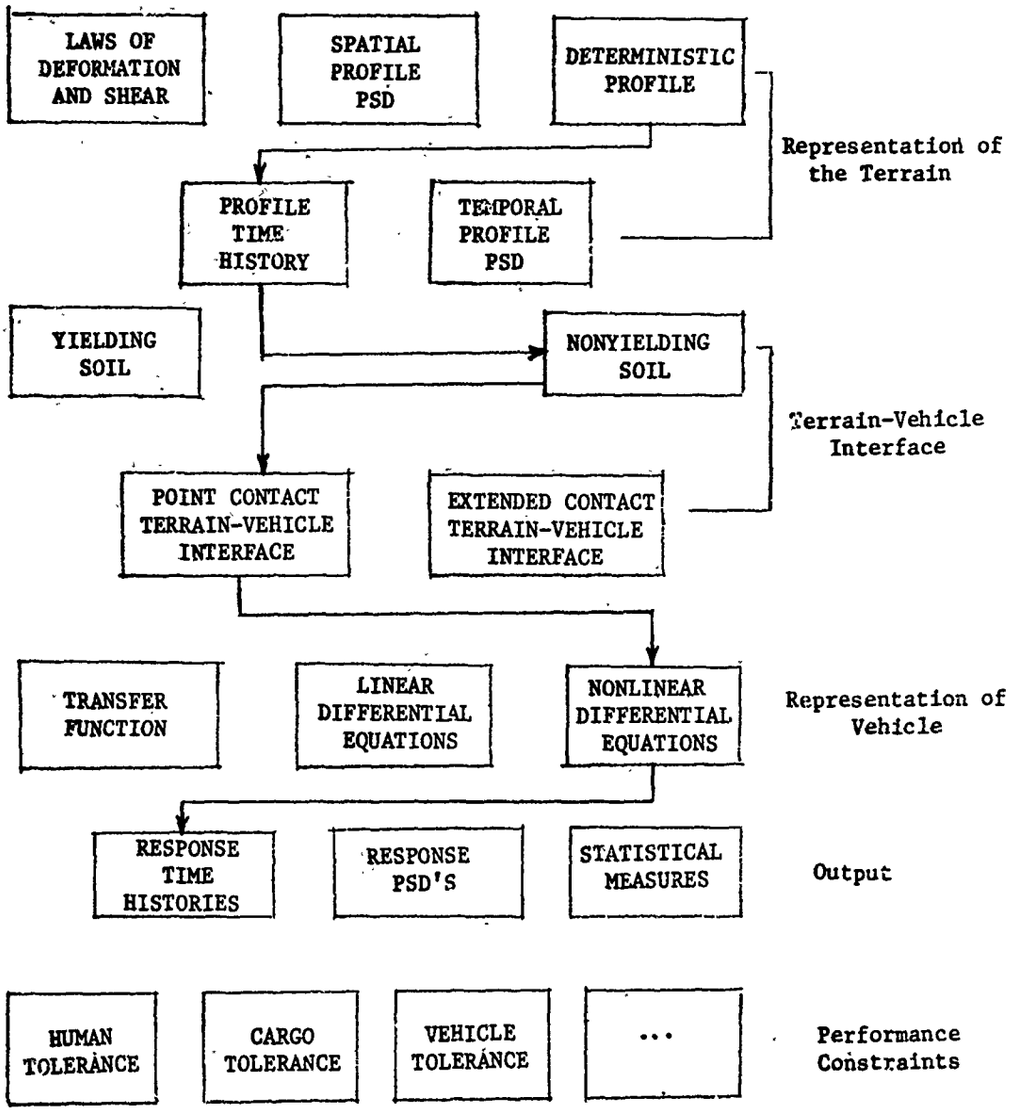


Fig. 24. National Aeronautics Space Administration-Brown Engineering ride dynamics model

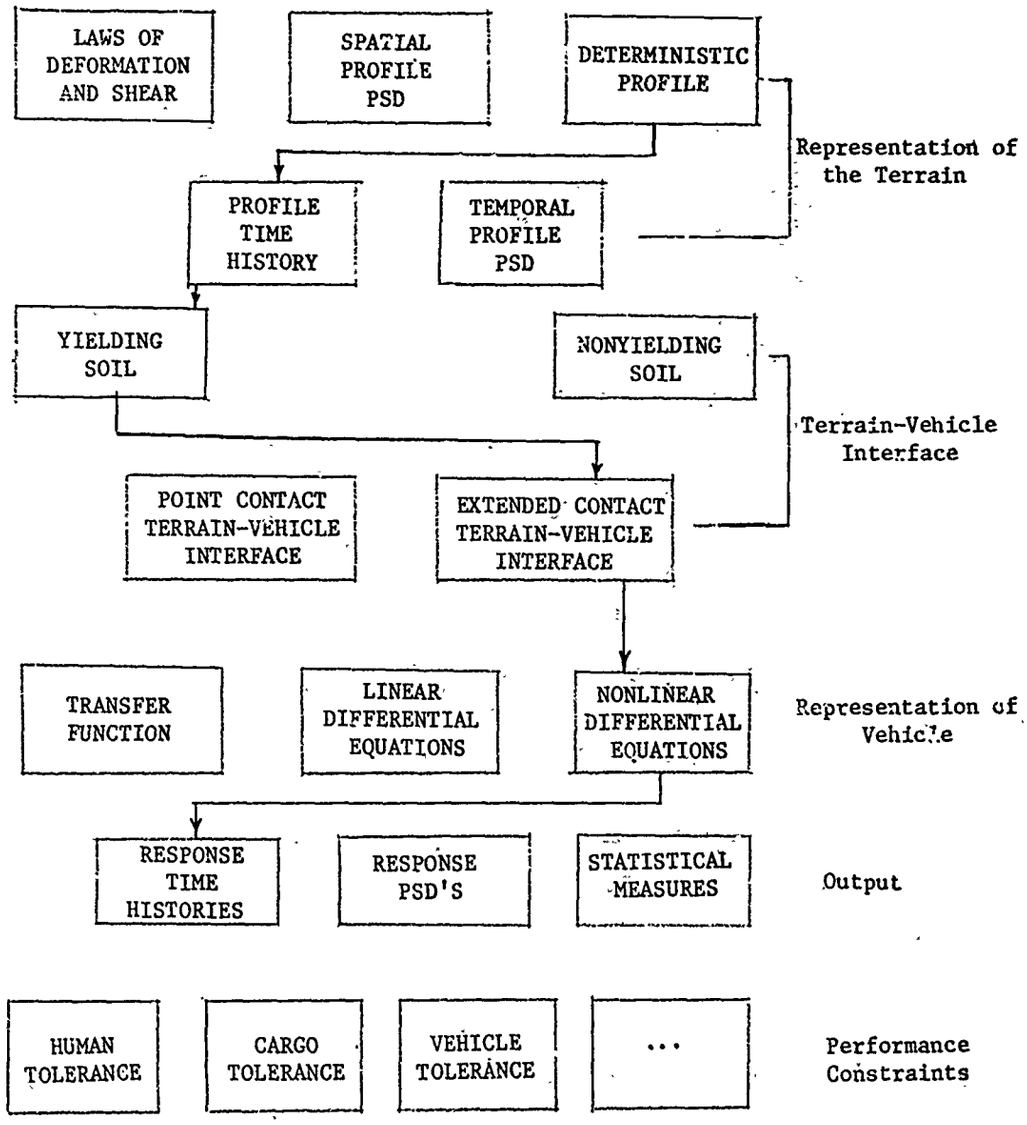


Fig. 25. Cornell Aeronautical Laboratory ride dynamics model

autocorrelation function is computed by taking two identical waveforms (profiles), repeatedly shifting one with respect to the other, and then calculating the correlation coefficient for each shift. The correlation coefficient for a given shift is obtained by multiplying the elevations of the two profiles at corresponding points on the horizontal scale and averaging the products over a sufficiently long range, usually several periods of the lowest frequency present, to provide a steady average. An example is illustrated in fig. 26. The products a_1b_1 , a_2b_2 , a_3b_3 , etc., are computed, added together, and then divided by the total number of products in the sample. The correlation function is obtained from a plot of the correlation coefficient as a function of the shift or lagged values (fig. 27). The shift values may be in terms of time or distance, depending

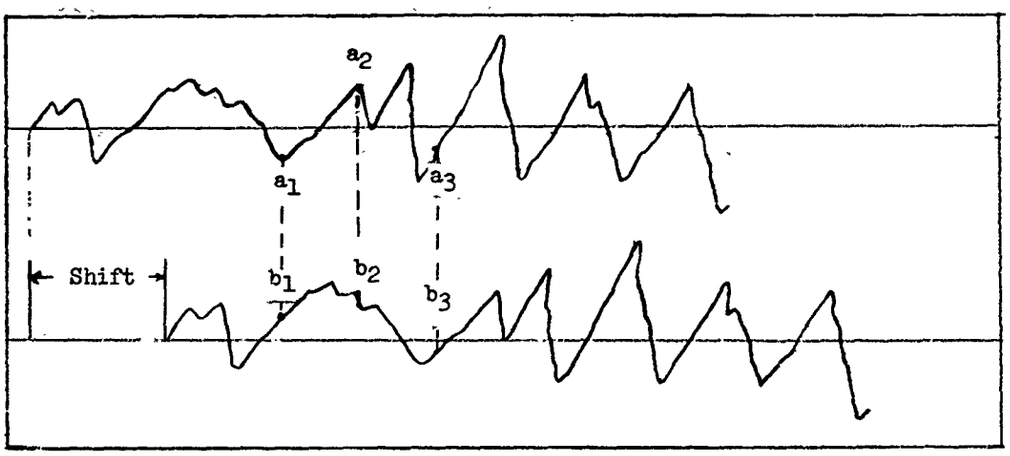


Fig. 26. Calculation of correlation coefficient. At corresponding points on horizontal scale, the ordinates of the two waveforms are multiplied. The correlation coefficient is the average of the a_ib_i products

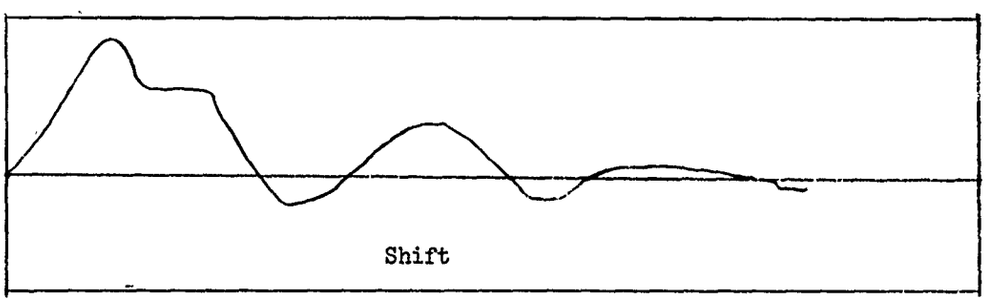


Fig. 27. The correlation function

upon whether the original waveforms are functions of time or distance.

153. The autocorrelation function has a maximum value for zero shift, which is equal to the mean square value of the waveform, and decreases to zero if the mean value of the waveform is assumed to be zero for infinite value of the shift unless the waveform contains periodic components. If the waveform has periodic components, the autocorrelation function also has periodic components of the same period. The autocorrelation functions for several common waveforms are shown in fig. 28. The autocorrelation function of a discrete sequence is defined by

$$R_x(t_1, t_1 + \tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} X_k(t_1)\, X_k(t_1 + \tau) \tag{11}$$

where $R_x(t_1, t_1 + \tau)$ is the value of the autocorrelation function when the lagged distance is τ and $X_k(t)$ is the original sequence. The discrete form of equation 11 is a direct translation of the autocorrelation

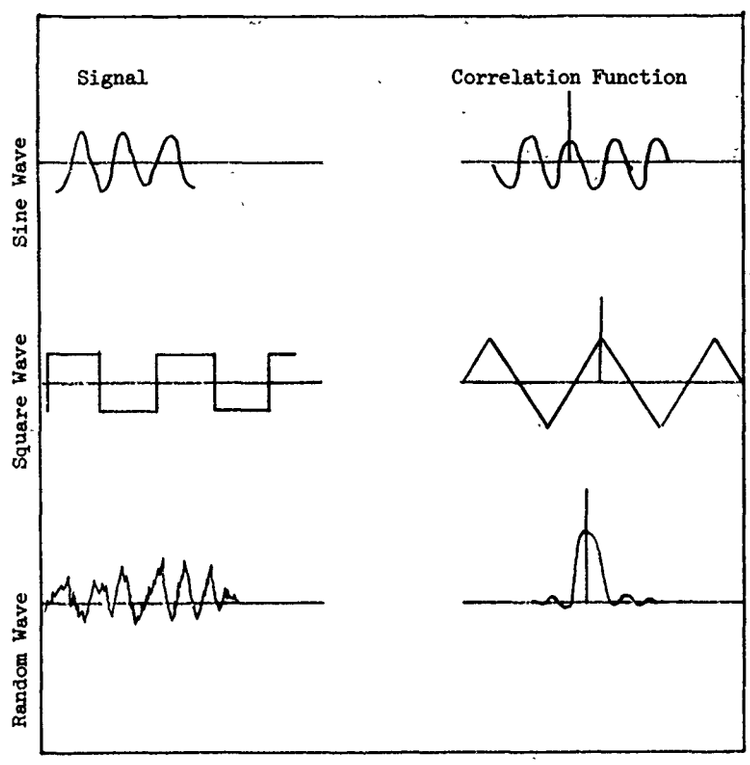


Fig. 28. Autocorrelation functions of several common waveforms

function for a continuous and infinite sequence given by equation 12.

$$R_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T X_k(t) X_k(t + \tau) dt \tag{12}$$

This equation is true for a sequence that has an infinite length. However, for sequences with finite length, consistent results require that the autocorrelation function (equation 11) be modified to the form

$$R_x(t_1, t_1 + \tau) = \frac{1}{N - \tau} \sum_{k=1}^{N-\tau} X_k(t_1) X_k(t_1 + \tau) \tag{13}$$

This equation differs from equation 11 in that the quantity $X_k(t_1)X_k(t_1 + \tau)$ is averaged by dividing by the actual length over which $X_k(t_1)$ and $X_k(t_1 + \tau)$ are multiplied. For a random sample with a zero mean, the autocorrelation function should approach zero and remain at zero as the lag values increase, and the function should not be periodic. (See fig. 28 for random wave.) The extent to which the function fulfills these conditions determines whether the waveform (terrain profile) is sufficiently random to be described by this technique. However, an unsatisfactory autocorrelation function can frequently be improved by performing a filtering operation (detrending) on the original data to remove the influence of longer wavelengths. Another useful and very significant property of the autocorrelation function is that it happens to be the Fourier transform of the power spectral density (PSD) of a random, stationary waveform. Having obtained the autocorrelation function, it is then possible to compute the power spectral density function.

154. The PSD is a second moment or variance density spectrum. PSD of a random, stationary function is defined as the Fourier transform of its autocorrelation function as follows:

$$P(\Omega) = 2 \int_{-\infty}^{\infty} R_x(\tau) e^{i2\pi\Omega\tau} d\tau \tag{14}$$

where $R_x(\tau)$ is the value of the correlation function for shift τ, and

Ω is the frequency in cycles per inch if τ is measured in inches and cycles per second if τ is measured in seconds. The integral over the frequency range $0 \rightarrow \infty$ of the PSD is the variance.*

155. In the application of equation 14, a raw estimate of the power spectrum can be obtained by the following numerical method:

$$p_x^{(h)} = \frac{2\Delta t}{\pi} \sum_{\tau=0}^{m} \epsilon_\tau R_x^{(\tau)} \cos \frac{h\tau\pi}{m} \qquad h = 0 , 1 ,\ldots m \tag{15}$$

where

$p_x^{(h)}$ = raw power spectral estimate of series x at frequency $\frac{h\pi}{m\Delta t}$

Note: $p_x^{(h)} = P_x\left(\frac{h\pi}{m\Delta t}\right)$

Δt = constant sampling interval (may be time or distance)

m = maximum lag

$$\epsilon_\tau = \begin{cases} 1 & 0 < \tau < m \\ 1/2 & \tau = 0 , m \end{cases}$$

$R_x^{(\tau)}$ = autocorrelation value of series x at lag τ

The raw estimates are then customarily smoothed by using "smoothing" coefficients. An accepted method of smoothing using "Hamming" coefficients is as follows:

$$SP_x^{(0)} = 0.54\ P_x^{(0)} + 0.46\ P_x^{(1)}$$

$$SP_x^{(h)} = 0.23\ P_x^{(h-1)} + 0.54\ P_x^{(h)} + 0.23\ P_x^{(h+1)},\ 0 < h < m$$

$$SP_x^{(m)} = 0.54\ P_x^{(m)} + 0.46\ P_x^{(m-1)}$$

where $SP_x^{(h)}$ = smoothed power spectral estimate of series x at frequency

$$\frac{h\pi}{m\Delta t}$$

Smoothing is necessary because the raw estimate is an inefficient estimate of the true spectral density since a finite record length precludes

* This is true if the function has a zero mean. For functions with a zero mean, the mean square and variance are the same.

identifying frequencies exactly and a more reliable statistical estimate can be obtained by averaging over neighboring frequencies. Trial-and-error methods such as the method developed by Hamming have proved more useful in obtaining sets of averaging coefficients that produce reliable spectral estimates than has any particular theory. There are several well-established sets of coefficients available, and in general, the Hamming method is quite satisfactory.

156. A logarithmic plot of two terrain power spectra is shown in fig. 29. The ordinate is in terms of inch^2 per cycles per inch, and the

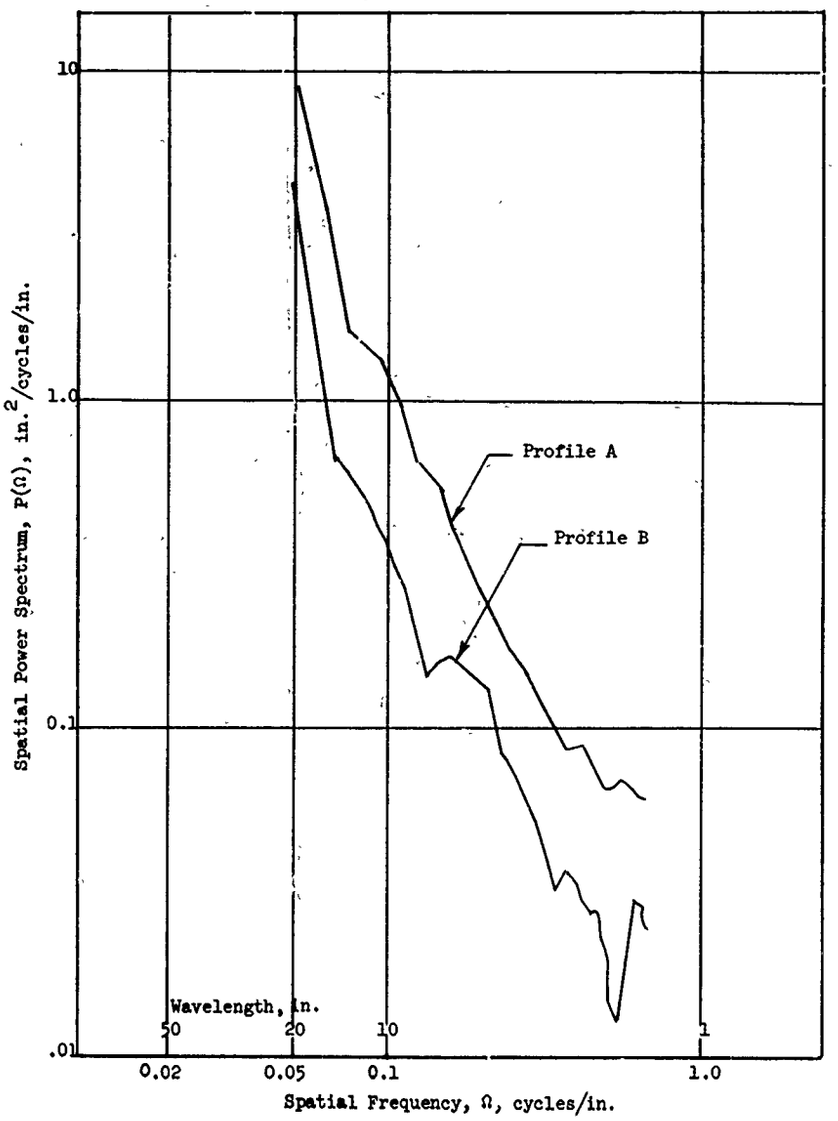


Fig. 29. Examples of terrain power spectra

abscissa is in units of cycles per inch, the reciprocal of wavelength. These spectra measure the rate of change of the average elevation with frequency and are primarily geometric quantities that do not contain any units of time. The area under each power spectrum represents the mean square value of the total roughness for the respective profile. The area under the curve between any two selected wavelengths will indicate the contribution to the total roughness that this range of wavelengths produces. Thus, from the terrain power spectrum curve, the wavelengths that make the greatest contribution to the variation in the elevation measurements can be determined.

157. The terrain elevation power spectrum can be used also with certain vehicle characteristics to determine wavelengths and frequencies that induce significant forces and motions if the velocity of the vehicle is introduced. This is done by multiplying the abscissa of the terrain power spectrum by the vehicle speed (inches per second) to obtain the frequency cycles per second (cps), and by dividing the ordinate of the power spectrum by the vehicle velocity to obtain the units of $inch^2$ per cycles per second. A different spectral curve obviously results for each velocity. This calculation was performed on the terrain spectrum of profile A (fig. 30) to represent vehicular speeds of 44 and 88 in./sec (2.5 and 5 mph, respectively). It is seen that increasing the vehicle's velocity tends to shift the spectrum to the right along the frequency axis and expand the frequency range. For example, if a vehicle is excited by a sinusoidal exciter platform with a controlled frequency, adjustable amplitude, and electronic transducers to measure the resulting reaction forces and motions, a response of the force ratio versus frequency (fig. 31) can be obtained by systematically varying only the frequency. Similar relations can be obtained for accelerations. For a linear system, this spectral relation is referred to as the systems transfer function. Note in fig. 31 the two predominant frequencies at 2 and 15 cps. These are the basic natural frequencies of the center of gravity and the axles, respectively. Variations in the terrain profile that will excite frequencies of 2 and 15 cps will thus generate large forces and motions between the vehicle and the terrain. These frequencies are, therefore, of interest in determining the influence

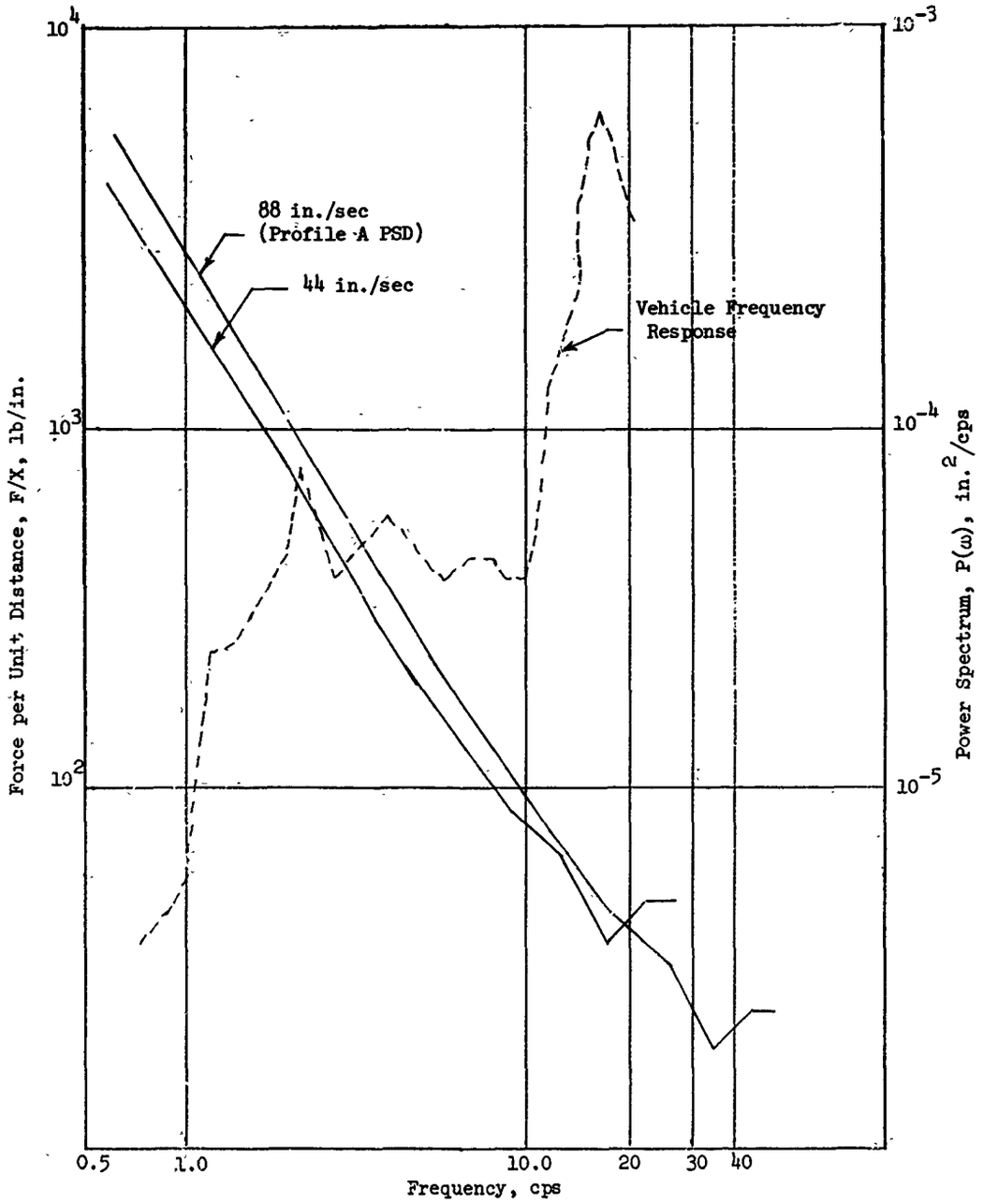


Fig. 30. Terrain and vehicle characteristics as a function of frequency

Fig. 31. Force-frequency vehicle response

of the terrains on the reaction of the vehicle. This relation can now be combined with the terrain power spectrum to obtain an estimate of the dynamic force or motion that the vehicle will exert on the terrain, and vice versa. To do this, the terrain spectrum must first be modified by introducing the vehicle velocity, as previously described. If this is done, then to determine a power spectrum of the dynamic force that the vehicle will exert on the terrain is simply a matter of multiplying the two spectra. It is worth mentioning that the vehicle characteristics (fig. 31) must be squared when this mathematical operation is performed to give the approximate units of pounds^2 per cycles per second (lb^2/cps) on the ordinate of the response spectrum. A typical result of these computations is shown graphically in figs. 30 and 32 for the vehicle traversing profile A at a speed of 44 in./sec. The influence of the vehicle's natural frequencies (2 and 15 cps) is vividly portrayed by the dynamic response spectrum (fig. 32). This response spectrum also has the same characteristics as previous spectra, i.e. the area under the curve represents the mean square value of the response. With this value, it is easy to determine the root mean square (effective) value. This RMS value is the effective force that the vehicle exerts on the terrain for that velocity. Such a spectral analysis provides a great deal of information on and insight into just how the frequencies of the terrain influence a vehicle's response and further pinpoints critical speeds and terrain wavelengths. However, this analysis is based on the fact that the vehicle is a linear system; that is, if the amplitude of the sinusoidal shaker is doubled, the only effect on the vehicle's response is the doubling of the output for all frequencies. This, however, is not the case for actual vehicles, which, unfortunately, are nonlinear.

158. Most terrain models employ a nonyielding soil-vehicle interface. No additional equations are required, and it is also reasonable since it represents the worst case from a vehicle ride dynamics standpoint. Some models, however, are more realistic and include yielding interfaces (Van Deusen). A consideration of yielding surfaces increases the complexity of the model, but permits a more realistic simulation of the real world terrain situation.

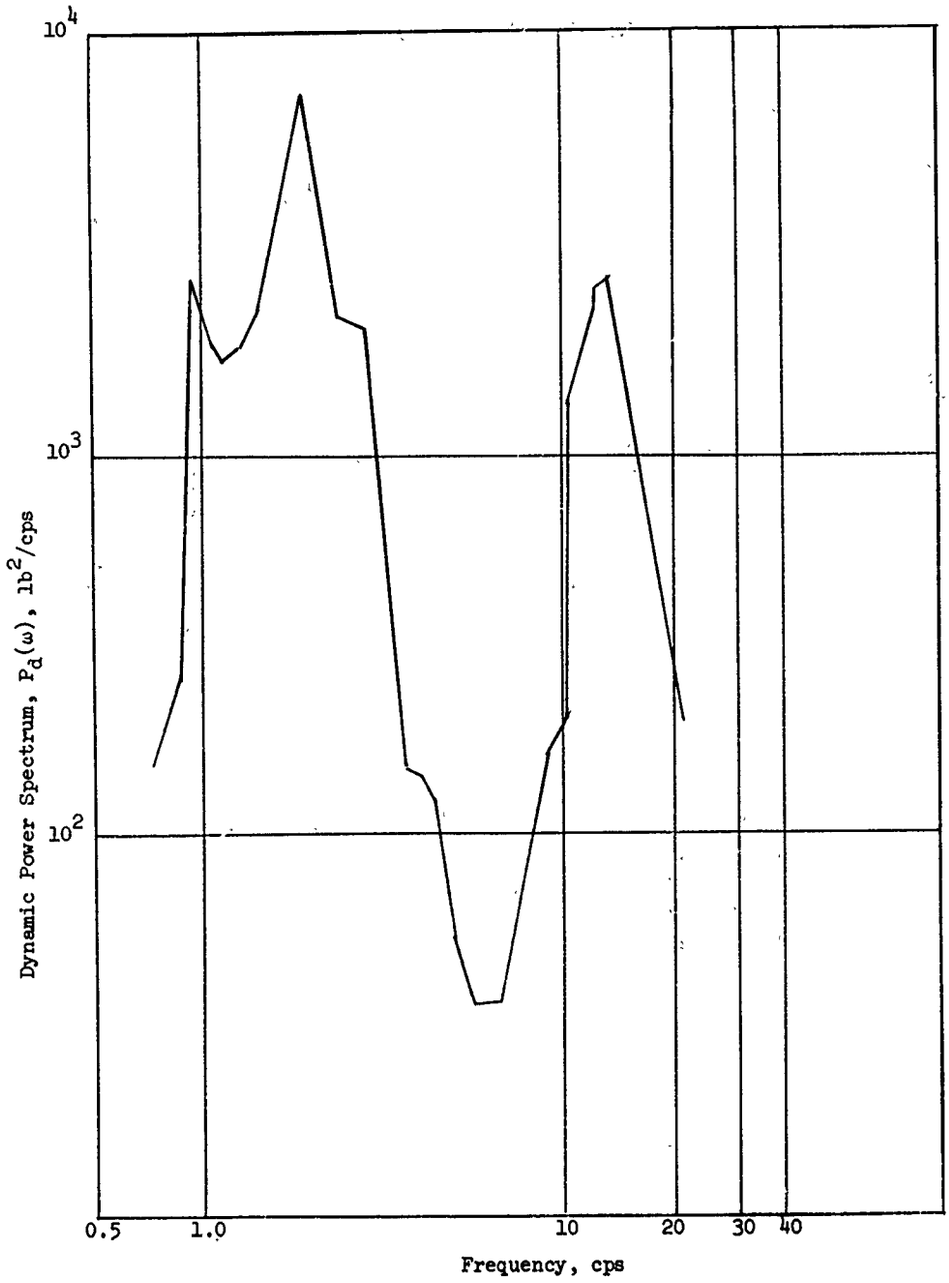


Fig. 32. Vehicle response-frequency relation

159. Representation of the vehicle. The central problem of ride analysis is the representation of the vehicle. An accurate mathematical model that accounts for structural flexing is too unwieldy for practical applications, so the use of rigid-body mechanics has become the standard approach. Vehicles are represented as collections of masses, springs, and damping elements suitably configured for the job at hand (see schematics in figs. 33 and 34). These give rise to sets of differential equations that describe motions throughout the vehicle model. Nonlinearities, inevitable in the vehicle structure, arise from the physics of suspension components, limitations on suspension travel, large pitch and roll motions, etc. A trade-off is therefore required between the inclusion of these important nonlinearities in the model and the complexity of analysis.

160. As previously stated, methods of analysis have developed along two complementary lines: time history and frequency response. In the first, differential equations are "driven" by time-dependent forcing functions and produce time-dependent responses; this corresponds to real life usage of the vehicle. In the second, properties of interest (typically

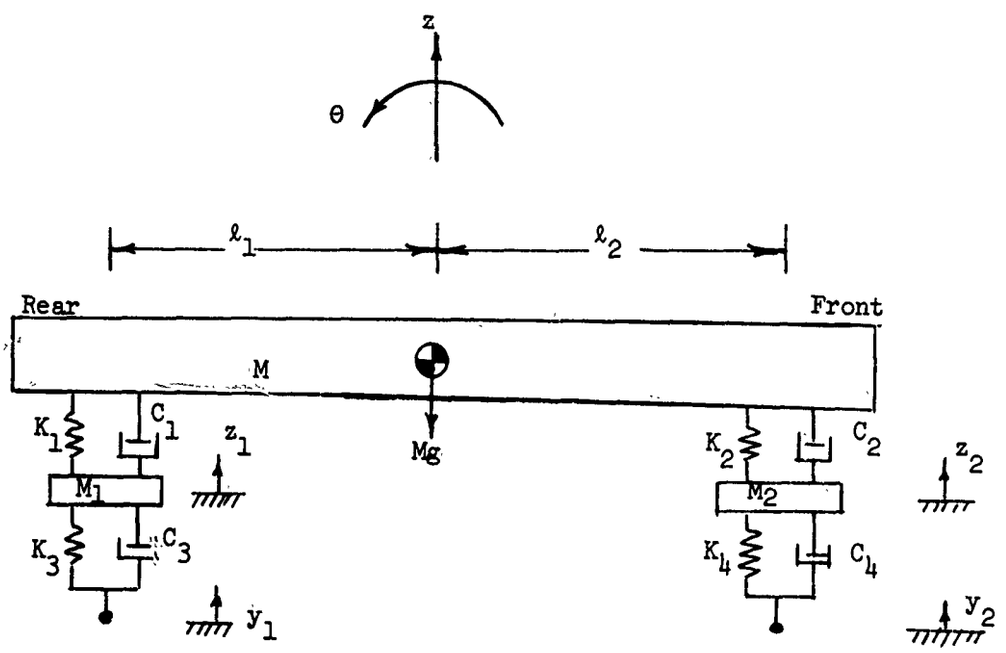


Fig. 33. Idealized 4-degree-of-freedom model of vehicle

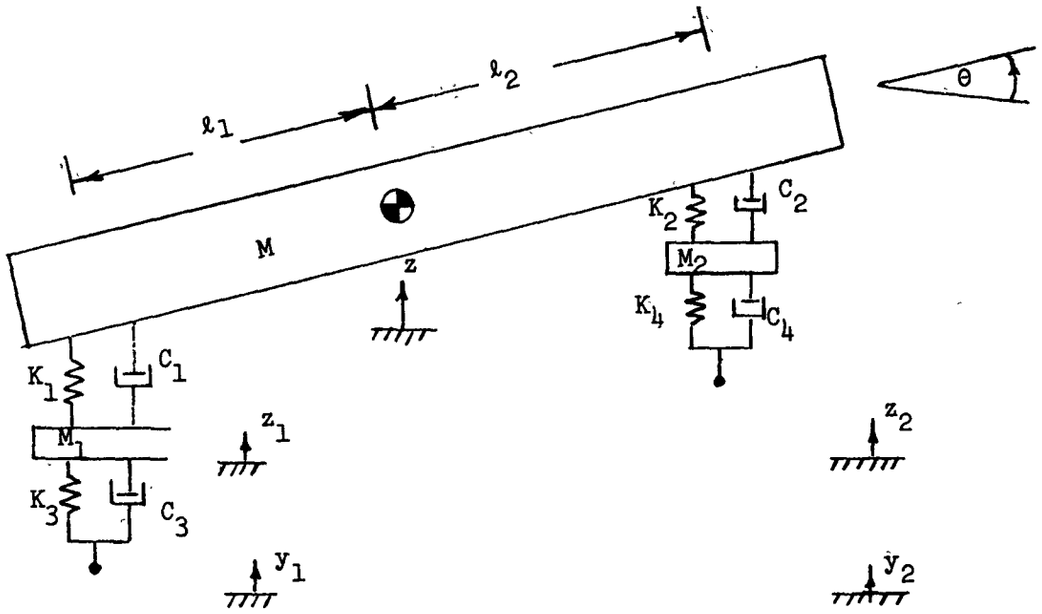


Fig. 34. Vehicle model in displaced configuration for determining forces

statistical properties, such as root mean square elevation or acceleration), are displayed against a frequency parameter; this compacts a great deal of information into a few graphs. An analysis predicated on time histories can also be used for frequency response at little extra cost. Most vehicle dynamics models reflect this fact by being constructed to allow either method of analysis. Frequency response analysis has great appeal, because, as shown in previous paragraphs, under conditions where linearity is a reasonable assumption, the use of differential equations (and thus the detailed workings of the vehicle) can be bypassed entirely and input-output correspondences can be dealt with directly by means of transfer functions.

161. The schematic in fig. 33 represents a simple, two-dimensional, 4-degree-of-freedom model of a vehicle. It is composed of a sprung mass, M , representing the vehicle body and two unsprung masses, M_1 and M_2 , representing the axle or individual wheel or bogie assemblies. The spring and damper assemblies connecting the unsprung masses to the main frame decribe the suspension compliance, while the spring and damper assemblies below the unsprung masses serve to describe the tire compliance. In the usual sense of model development, the masses, springs, and dampers serve to

represent the body, elasticity, and friction of the system, respectively. The mass elements are assumed to be rigid bodies. The spring elements possess elasticity and are assumed to be of negligible mass. A spring force exists only if the spring is deformed, such as the extension or compression of a coil spring. Therefore, the spring force exists if there is a relative displacement between the two ends of the spring. The spring force-deformation relation may be either linear or nonlinear. The damping elements have neither mass nor elasticity. Damping force exists only if there is relative motion between the two ends of the damper. Many types of damping may be encountered in engineering systems, most of which are nonlinear. However, there are but two common types of damping occurring in vehicle suspensions. One type is Coulomb damping, sometimes called frictional damping, in which the damping force is a function of the normal force between the sliding bodies as well as the materials involved. This type of damping force occurs in suspension leaf springs and is assumed to be independent of the relative velocity between the sliding bodies and dependent on the direction of the motion. The other type, which is used most often in modeling, is that which is characterized by dashpot action in which the damping force depends on the relative velocity of the deformations. This type of damping occurs in shock absorbers. Some recent vehicle models have included both types of damping in the suspension elements, but generally the gross damping in suspensions is lumped in the latter type. All the damping forces in the vehicle model in fig. 33 are assumed to be solely dependent on relative velocity.

162. The number of degrees of freedom of any system is determined by the number of independent coordinates required to describe the configuration of each mass of the system. For the model shown in fig. 33, two coordinates are required to describe the configuration of the sprung mass--a vertical motion of the center of gravity and a rotation about the center of gravity. The axles are permitted to have motion in only the vertical direction, thus requiring but a single coordinate to describe each axle's configuration.

163. The differential equations describing the motion of the system are arrived at through Newton's second law, $F = ma$. Determining the

equations of motion of any system first requires the establishment of a coordinate system and appropriate sign convention for each degree of freedom. This coordinate system and sign convention are completely arbitrary, but once chosen, they must then be adhered to throughout the development of the equations. Regarding the model in fig. 33, it is seen that all upward translational motions and counterclockwise rotations are considered positive. Once the coordinate system and sign convention have been established, the next step is to displace the model in a configuration such that all the coordinates are affected, i.e. greater than or less than zero. For example, assume that some arbitrarily positive displacements are applied to points of contact of the front and rear tires displacing the system such that the following conditions exist:

$$y_1 > 0 \quad y_2 > 0 \quad y_2 > y_1$$

$$y_1 > z_1 > z \qquad \dot{y}_1 > \dot{z}_1 > \dot{z}$$

$$y_2 > z_2 > z \qquad \dot{y}_2 > \dot{z}_2 > \dot{z}$$

$$\theta > 0$$

where

y_1, y_2 = vertical displacements of rear and front tires, respectively

z_1, z_2 = vertical displacements of rear and front axles, respectively

z = vertical displacement of center of gravity of vehicle body

θ = rotation, in radians, of vehicle body about the center of gravity

The terms with the dot notation denote time derivatives (velocities) of the appropriate coordinates.

This is a possible configuration for the system and is shown schematically in fig. 34. In such a configuration, it is obvious that all the spring elements are in compression. The amount of deformation for the front suspension spring, Δ_2, is given by the relative vertical displacements of the front axle and the point of attachment of the front spring to the

vehicle body. This deformation is denoted symbolically with reference to the appropriate coordinates as:

$$\Delta_2 = \left[z_2 - (z + \ell_2 \sin \theta)\right]$$

Since $z_2 > z$, the quantity in brackets is positive. Thus, positive deformations denote spring compressions and negative deformations denote extensions of the springs.

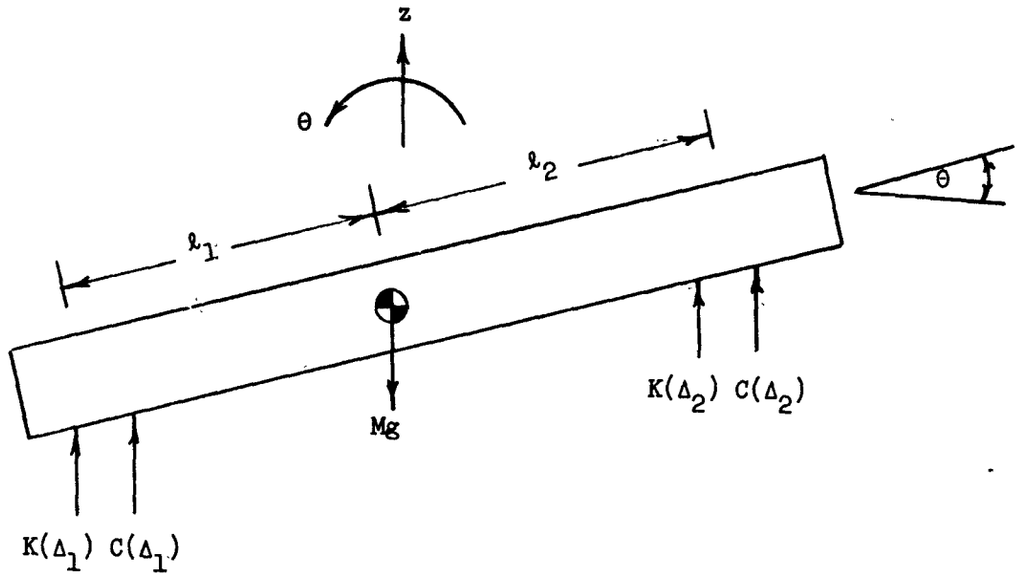
164. Similarly, the deformation of the rear suspension spring is:

$$\Delta_1 = \left[z_1 - (z - \ell_1 \sin \theta)\right]$$

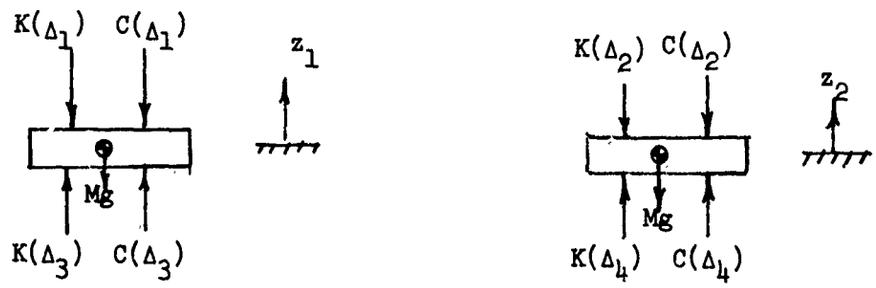
The quantities ℓ_1 and ℓ_2 are the distances from the center of gravity to the rear and front suspensions, respectively. The relative velocities are obtained from the time derivatives of the deformations, e.g. for the rear suspension

$$\frac{d}{dt}(\Delta_1) = \dot{\Delta}_1 = \left[\dot{z}_1 - (\dot{z} - \ell_1 \dot{\theta} \cos \theta)\right] \tag{16}$$

Assuming that the spring force-deformation and damper force-velocity relations are obtained for the suspension and tires through some means of measurement, it is then possible to formulate functional relations, $K(\Delta_i)$ and $C(\Delta_i)$, that will allow for the calculation of spring and damper forces due to variations in the tire and suspension deformations and relative velocities. These relations may be in the form of tables, polynomials, or piecewise linear segments. The suspension forces created by the configuration in fig. 33 act upward on the vehicle body and downward with an equal but opposite effect on the axles. The forces generated by tire deflection and damping act upward on the axles. The three masses, isolated into separate free bodies with the appropriate forces applied to each mass, are shown in fig. 35. The equations of motion, one equation for each degree of freedom, are readily determined by summing the forces and moments acting on each mass. They are as follows:



a. Forces acting on sprung mass



b. Forces on rear axle

c. Forces on front axle

Fig. 35. Forces acting on a vehicle at selected locations

<u>Forces and moments on vehicle body:</u>

$$M\ddot{z} = K(\Delta_1) + C(\dot{\Delta}_1) + K(\Delta_2) + C(\dot{\Delta}_2) - Mg \tag{17}$$

$$I\ddot{\theta} = \left[-K(\Delta_1)\ell_1 - C(\dot{\Delta}_1)\ell_1 + K(\Delta_2)\ell_2 + C(\dot{\Delta}_2)\ell_2\right]\cos\theta \tag{18}$$

<u>Forces on rear axle:</u>

$$M_1\ddot{z}_1 = -K(\Delta_1) - C(\dot{\Delta}_1) + K(\Delta_3) + C(\dot{\Delta}_3) - M_1 g \tag{19}$$

Forces on front axle:

$$M_2\ddot{z}_2 = -K(\Delta_2) - C(\dot{\Delta}_2) + K(\Delta_4) + C(\dot{\Delta}_4) - M_2 g \tag{20}$$

where

Δ_1 and Δ_2 = the deflections of the rear and front suspensions, respectively

Δ_3 and Δ_4 = the deflections of the rear and front tires, respectively

$\dot{\Delta}_1$ and $\dot{\Delta}_2$ = the velocities of the rear and front suspensions, respectively

$\dot{\Delta}_3$ and $\dot{\Delta}_4$ = the velocities of the rear and front tires, respectively

165. The response to arbitrary inputs, y_1 and y_2, may be solved using conventional methods for solving differential equations on either digital or analog computers.

166. A two-dimensional model, although incorporating many nonlinear features, is a simple representation of a vehicle. For example, the model is restricted to motions in a plane. Expansion to a more general three-dimensional model greatly increases the cost and complexity of analysis. More degrees of freedom are introduced, thus increasing the number of equations, and the effects of independent and solid-axle suspensions become more significant than with the two-dimensional model; and for travel other than along straight paths, the concepts of space-fixed and body-fixed axes and Euler angle references are required.

167. Representation of terrain-vehicle interface. Much of the work involving mathematical modeling of vehicle dynamic performance uses a common representation of the vehicle traction element. When this element is a pneumatic tire, a spring and viscous damper are used to represent tire compliance; when the element is a track, the track pads are neglected and the spring-damper representation is applied to the road wheels. The terrain is represented as a vertical displacement input to each traction element, which, in turn, transmits vertical forces to the vehicle. The interface of terrain and traction element is represented as a single point contact. Such representations should yield generally satisfactory results where the terrain is not too severe, i.e. where the radius of curvature of an

obstacle or terrain undulation is larger than the rolling radius of the tire. Inclusion of more realistic conditions of variable area contact and envelopment have been employed by several investigators (see figs. 20, 22, and 23). This extended contact feature provides a more accurate simulation of tire compliance and enables the inclusion of horizontal force inputs to the model. Here again, realism versus complexity influences the choice between a point-contact interface--the easiest scheme to implement--and an extended contact interface.

168. Response of model. The generation of outputs from vehicle models is a relatively simple task. The outputs consist of time histories of various responses and their counterparts, power spectral densities, and statistical measures. These outputs represent dynamic inputs to drivers, passengers, and cargo, responses of which govern the contribution of ride dynamics to vehicle mobility. Because human tolerance to vibration is subjective, its determination is obscure. But regardless of how future research may improve present knowledge of human vibration tolerance, vehicle models should be capable of providing the raw data inputs required.

169. Comparison of measured and predicted vehicle dynamics behavior. To compare model and prototype vehicle performances, pertinent vehicle response data were measured while an M37, 4x4, 3/4-ton cargo truck traveled over a section of firm natural terrain and over rigid obstacles. Vehicle vibration was the principal factor controlling vehicle performance. The cross-country test was conducted at a test site near Carson Sink, Nev. Fig. 36 is an overview of the site. In the test the vehicle traveled

Fig. 36. Overview of terrain roughness test site near Carson Sink, Nev.

over a 1000-ft-long, mildly undulating course at an almost constant speed of 8 mph. The obstacle tests were conducted on the WES obstacle test course. In these tests the vehicle traveled at an average speed of 15 mph in a path that was at a right angle to the obstacle. The obstacle was a nonyielding, triangular-shaped object, 6 in. high, with an approach angle of 30 deg (see fig. 37). The response data from the cross-country and obstacles tests were recorded on analog magnetic tape and processed on an analog computer. In addition, results from a previous study with the M37 were used to augment the analysis. Two mathematical models of the M37 were used to simulate the actual tests, one using a point-contact interface for the tire and terrain and linearized rotational motions, and the other using an extended contact interface and nonlinear rotational motions.

a. Single obstacle test.

(1) The acceleration-time history of the response of the vehicle's front axle to a single obstacle is shown in fig. 38. The extended contact model yields the more realistic motions, which very closely approximate the measured motions in the initial portion. However, this correspondence is of short duration, and the deviations quickly increase to a significant level. The nature of these deviations suggests that modeling of the suspension damping may not be sufficiently accurate. The shortcomings of the point-contact model are revealed by the extremely large accelerations occurring during and just after the period of tire-obstacle contact. The adverse effects of the inability of the point-contact model to simulate the envelopment characteristics of the tire (see fig. 37) are accentuated for cases such as this when the input forcing function contains significant discontinuities. An almost infinite negative acceleration occurred when the point of contact became abruptly separated from the obstacle, so that for a short time only negative downward forces were applied to the axle.

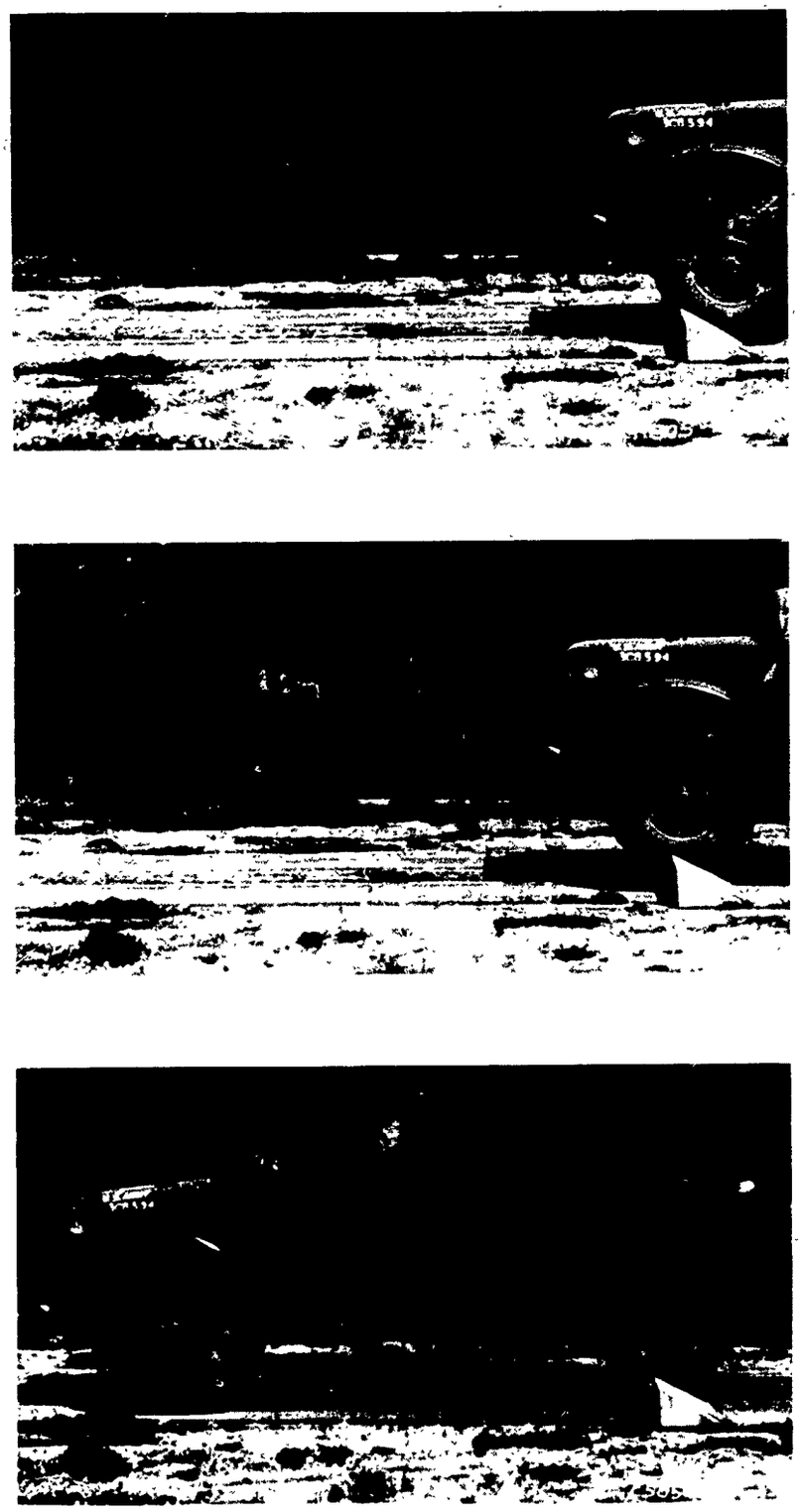


Fig. 37. M37, 4x4, 3/4-ton cargo truck negotiating a rigid obstacle at 22 fps

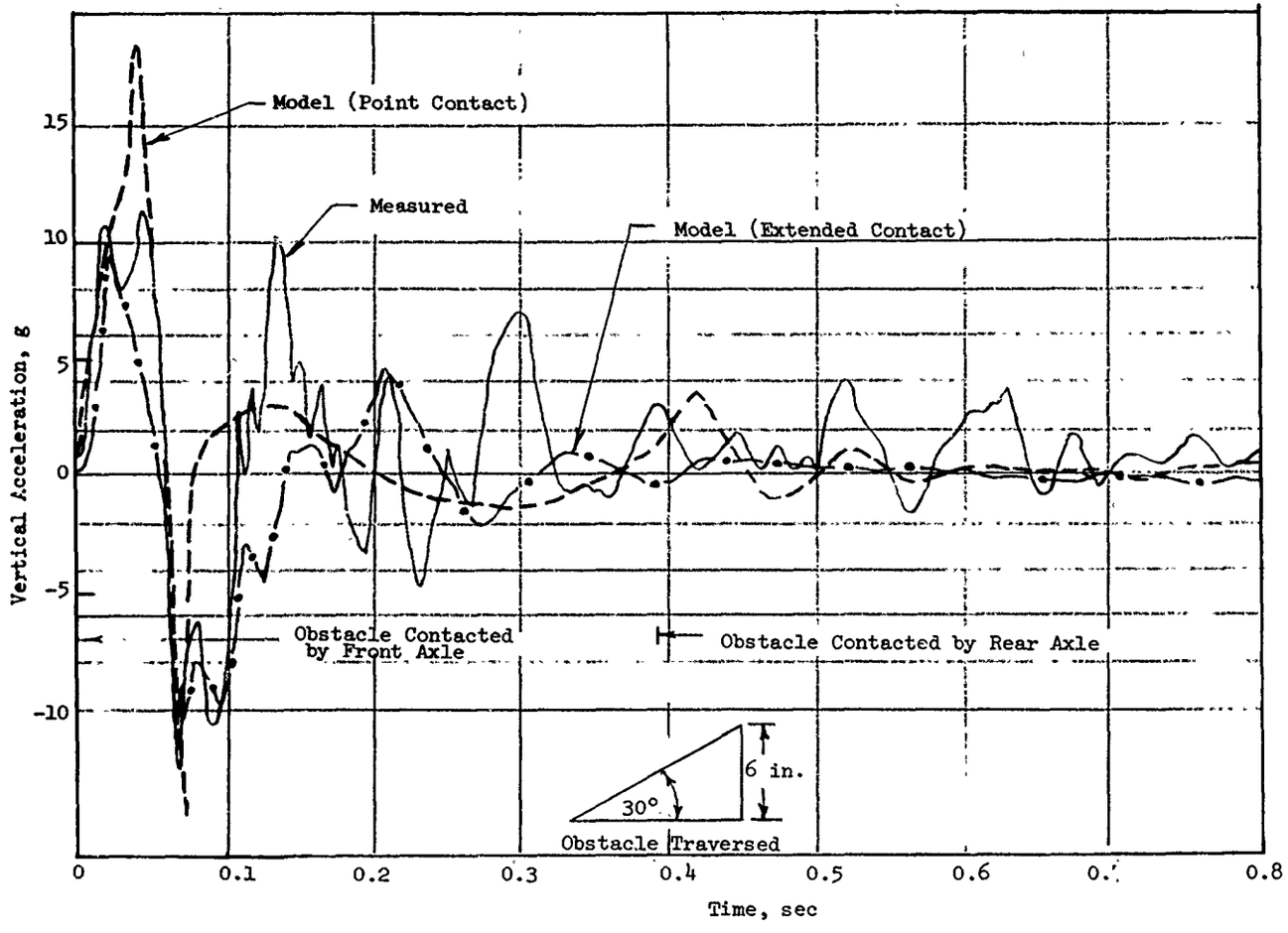


Fig. 38. Acceleration-time history; M37's front-axle response to single obstacle at 15 mph

(2) The tire envelopment characteristics can be included to a certain degree in the point-contact model by modifying the obstacle shape in terms of the tire geometry. This is done by determining the vertical movement of the axle as a tire (considered rigid) passes over the obstacle. This movement then serves as the modified displacement input to the point of contact. Such a modification was made for the obstacle run, and the response is compared with the original response in fig. 39. In this case, the responses are not very different.

(3) Little difference between the two models is noted when the responses of the vehicle's center of gravity are compared (fig. 40). The two distinct peaks, representing the contacts of the front and rear axles with the obstacle, occur at about the same time as the measured peaks, but the magnitudes for both models are considerably smaller than the measured values.

(4) Generally, the essential features of the simulated acceleration-time histories do not compare well with the measured waveforms.

(5) A noticeable difference occurs in the models in the simulation of the vehicle's pitch motion. This difference is due largely to the different restrictions placed on the rotational motions. The point-contact model with the linearized rotational motions overpredicts the pitch motions, and the extended contact model underpredicts them (fig. 41).

b. Cross-country test.

(1) The results of the cross-country test are compared in fig. 42 in the form of an RMS acceleration-time history at the vehicle's center of gravity and in fig. 43 in the form of an absorbed power-time history computed from the accelerations at the driver's seat. The

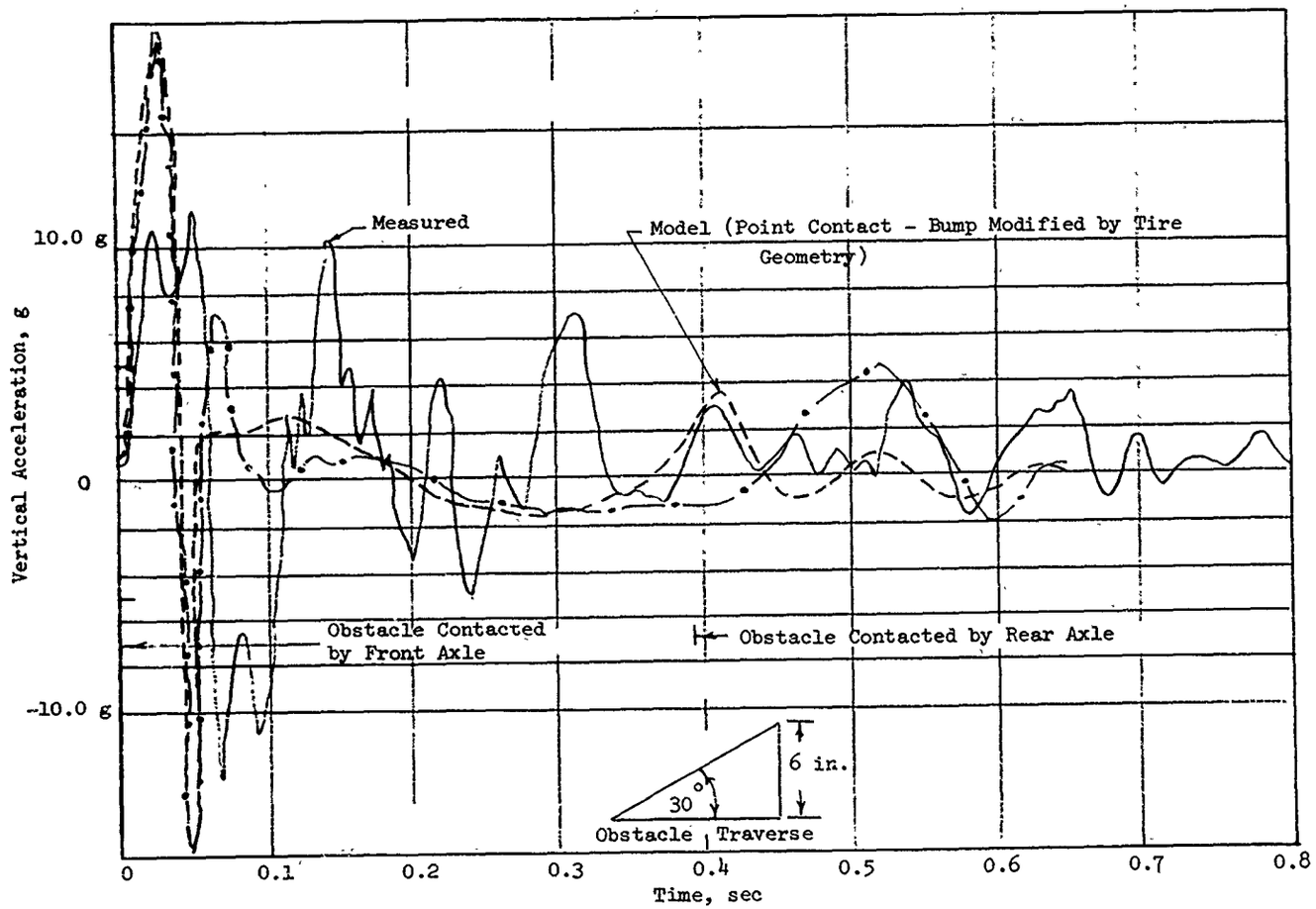


Fig. 39. Acceleration-time history; M37's front-axle response to modified single obstacle shape at 15 mph

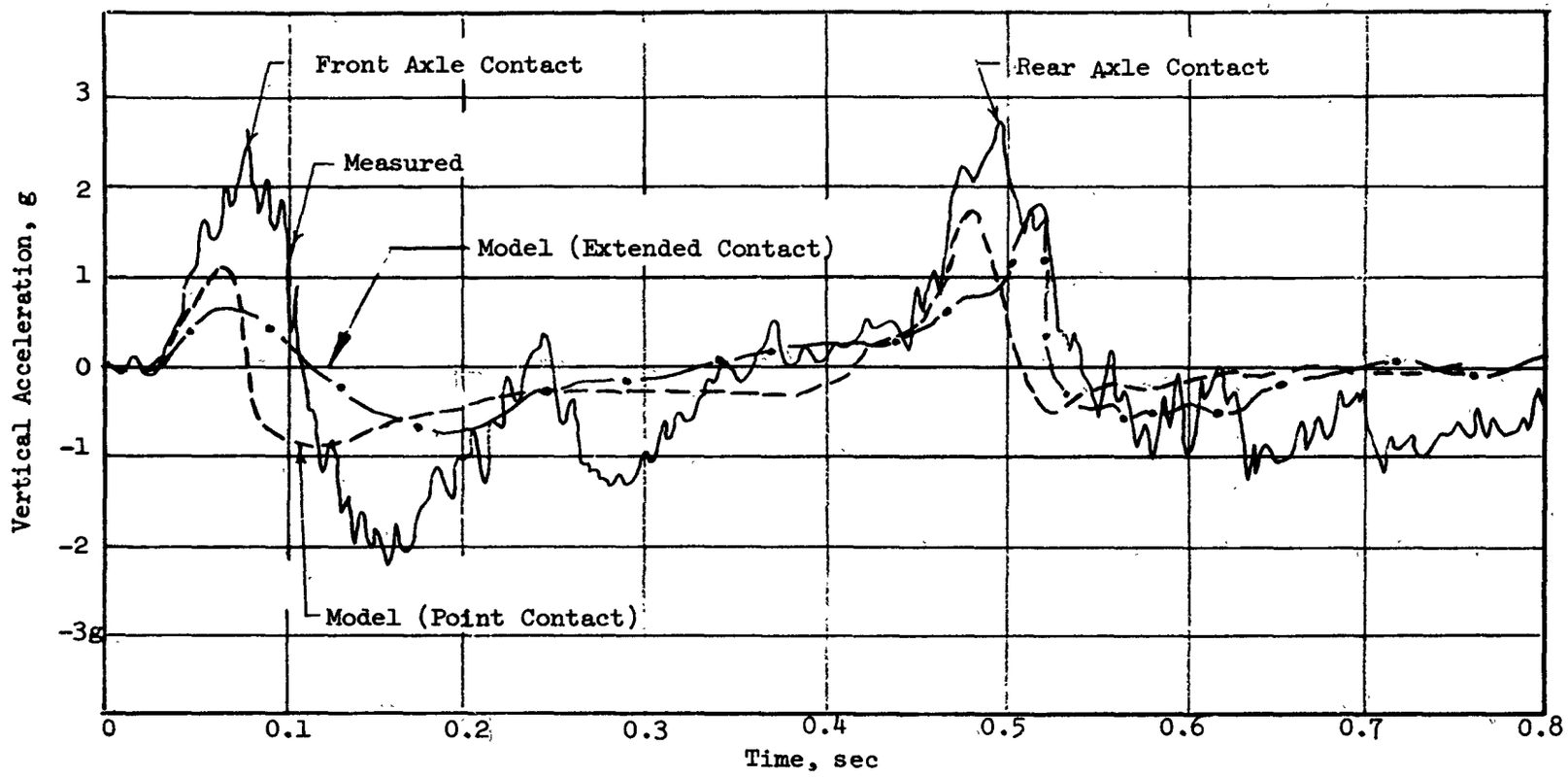


Fig. 40. Acceleration-time history; M37's center-of-gravity response on single obstacle at 15 mph

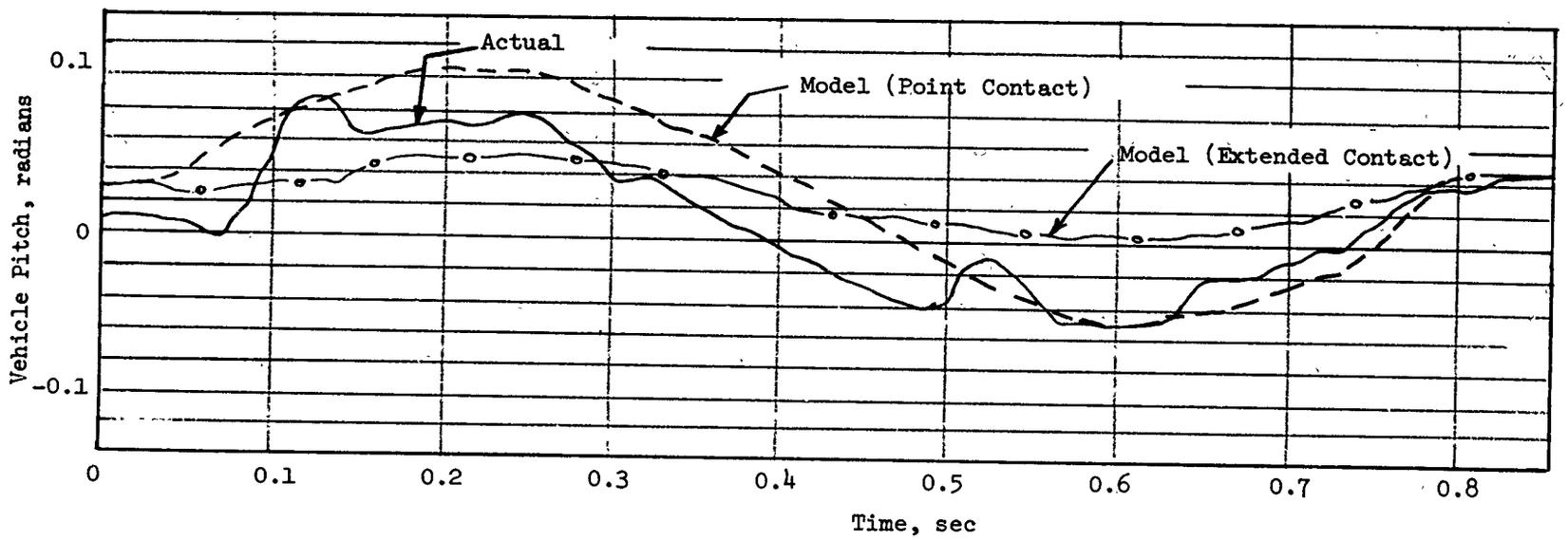


Fig. 41. Pitch-time history; M37's center-of-gravity response on single obstacle at 15 mph

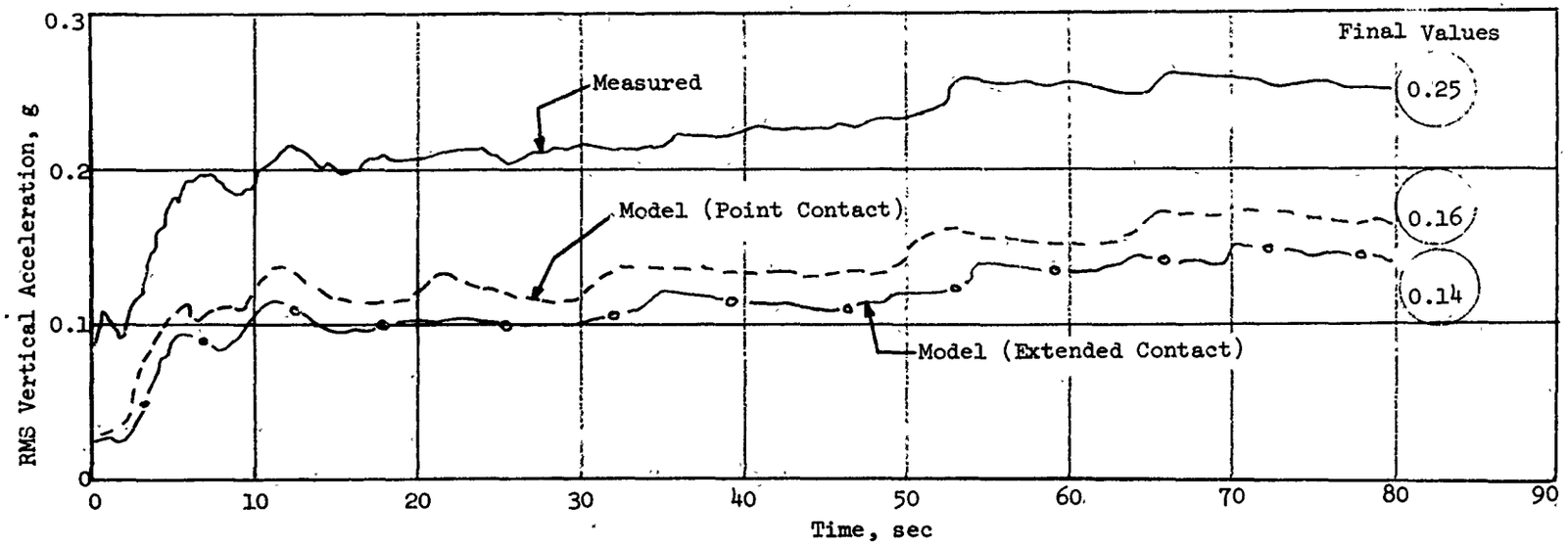


Fig. 42. Acceleration-time history; M37's center-of-gravity response on cross-country course at 8 mph

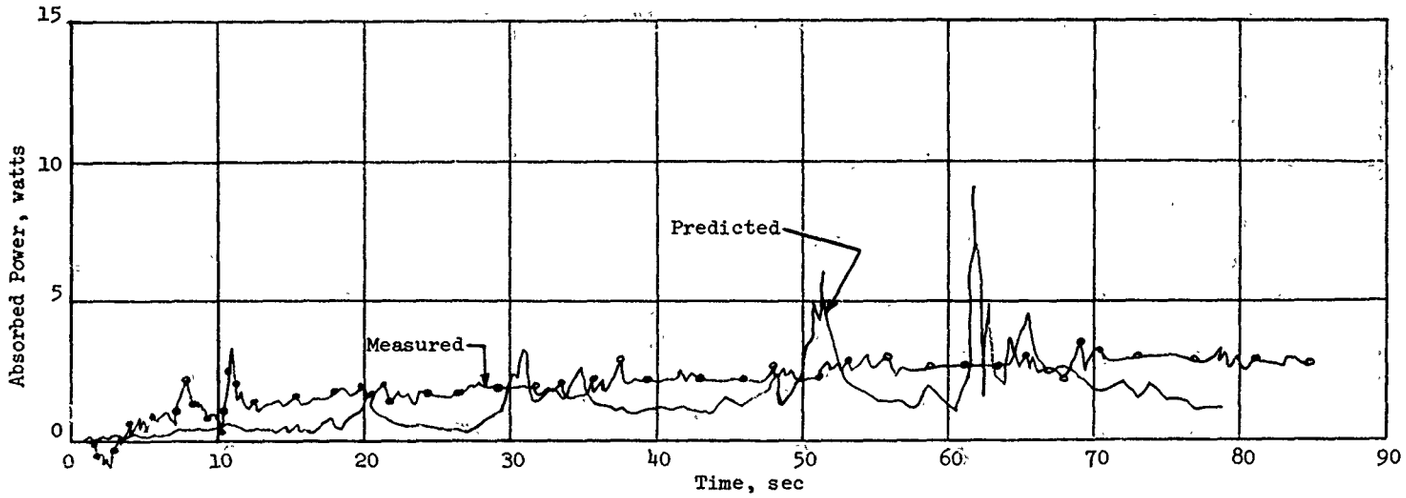


Fig. 43. Absorbed power-time history; M37's response on cross-country course at 8 mph

predicted and measured RMS acceleration-time histories are similar in form (fig. 42), but the model predictions are both low by a factor of nearly 2.

(2) These results are the opposite of those from a similar program reported by Van Deusen, in which the predicted RMS vertical accelerations of the M37 were consistently higher (by a factor of about 2) than measured values. This difference probably is due to the different methods for obtaining values of the vehicle parameters. Van Deusen obtained his values from shaketable tests in which the entire vehicle was vibrated over a selected range of frequencies, while those used in the two models in this study were obtained by measuring individual components while they were isolated from the vehicle.

170. Problems associated with vehicle modeling. Determining accurate and consistent values for the appropriate vehicle parameters is a problem of tantamount importance in today's vehicle modeling. There are others, however, that must be recognized, particularly when model and prototype responses are compared. An example of a pertinent problem is shown in fig. 44, where RMS vertical acceleration-time histories for differing degrees of filtering are plotted. The unfiltered acceleration signal produces an RMS time history that is approximately twice the histories of the three filtered signals. This is due to high-frequency vibrations transmitted by the engine, flexural modes of the vehicle frame, and transducer peculiarities, among others. When the acceleration is filtered with a 30-cps filter, the RMS time history is reduced by nearly 50 percent of the unfiltered value. Filtering with 10- and 3-cps filters has only slight effects on further reducing the magnitude. In fact, the 3-cps filter reduces the RMS time history only 20 percent less than that obtained with the 30-cps filter. These filtered results, however, are still higher than those produced by the models. The models employing the concept of rigid-body mechanics are an assemblage of lumped masses, springs, and dashpots that have discrete principal modes of vibration, usually not exceeding 20 to 25 cps.

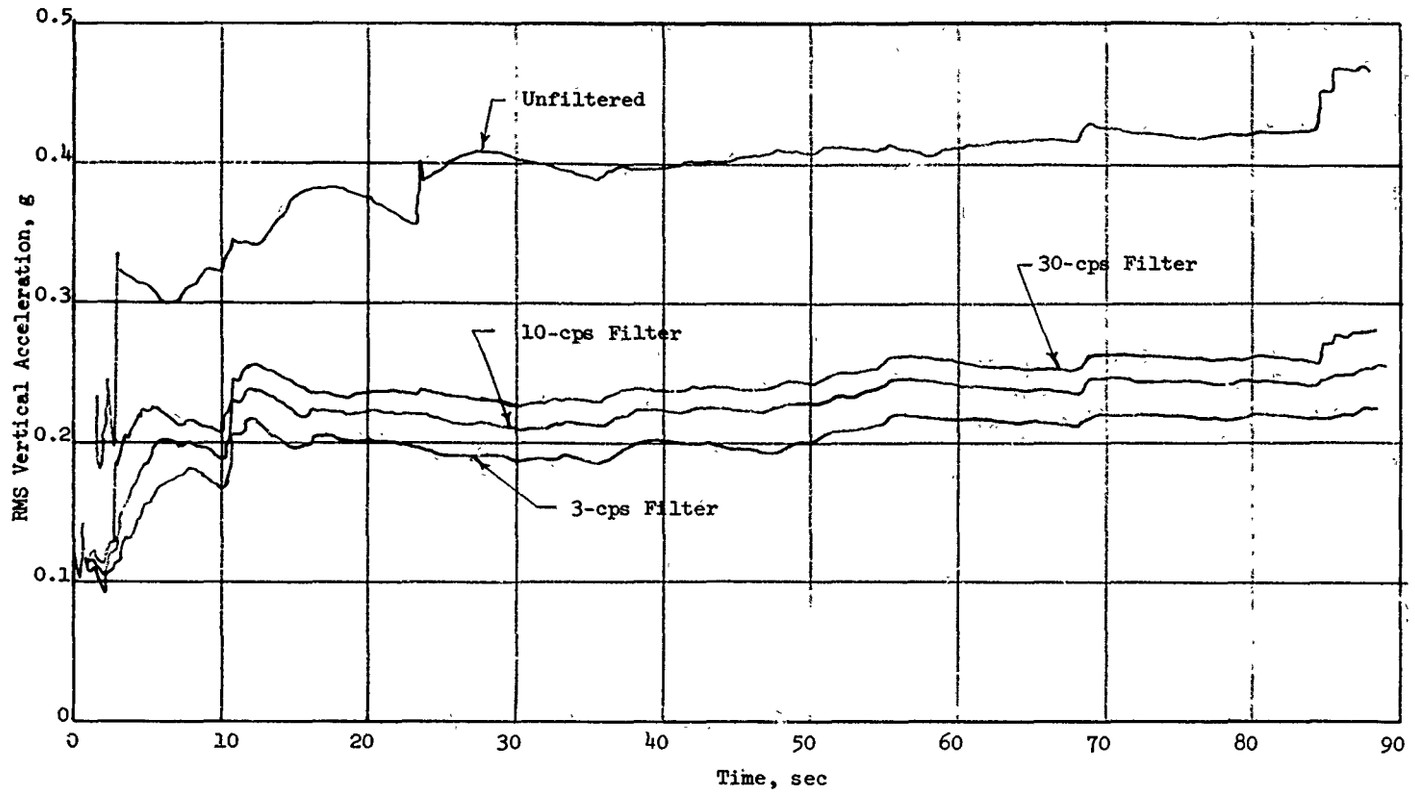
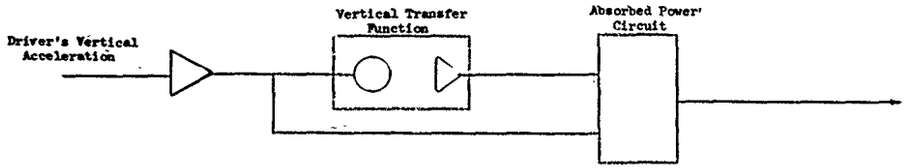


Fig. 44. RMS vertical acceleration at driver's seat; M37's response on cross-country course at 8 mph

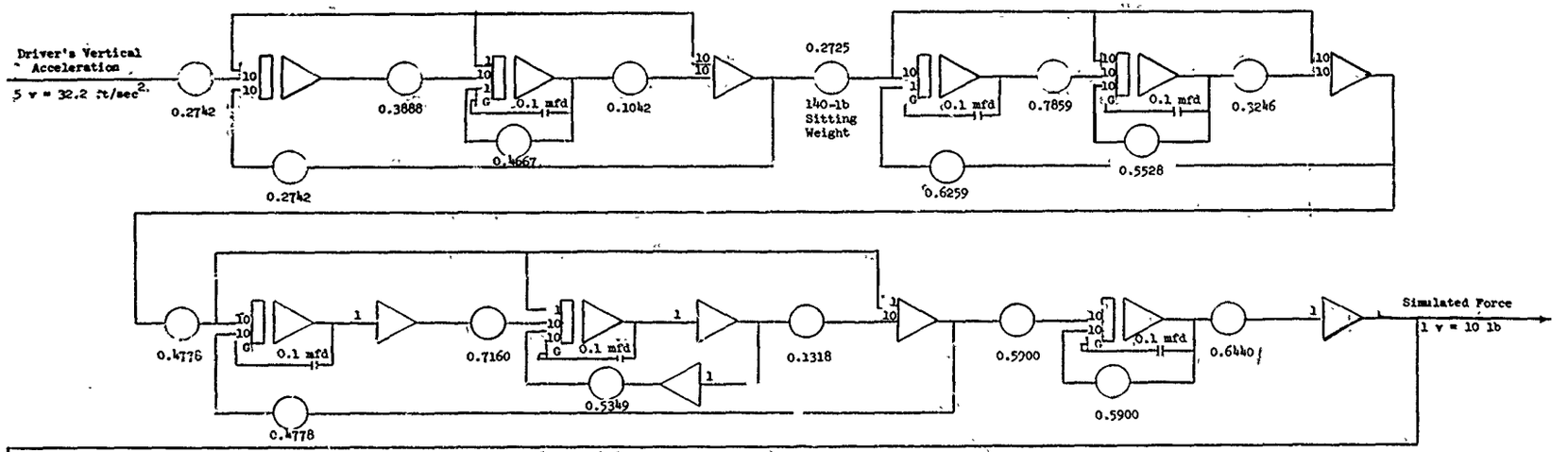
Therefore, the models do not simulate the influence of engine vibrations, bending modes of the vehicle frame, and other high-frequency components that are present in the prototype response. Since these high-frequency motions are not bothersome to passengers and cargo, it is reasonable to filter out these effects when comparisons are made between model and prototype responses. The measured RMS time history shown previously in fig. 42 was processed with a 10-cps filter.

171. The modeled values of absorbed power (fig. 43) were generally lower than the values that were computed from the vertical accelerations recorded at the driver's seat. The computation was implemented on the analog computer following the circuit diagrams (fig. 45) furnished by the Scientific Computer Branch of the U. S. Army Tank-Automotive Command. A problem lies in the inability to reproduce the absorbed power-time history. Each time the tape was processed a different time history was obtained. This inconsistency is believed to be due to an open-loop, double integration in the absorbed power circuit.

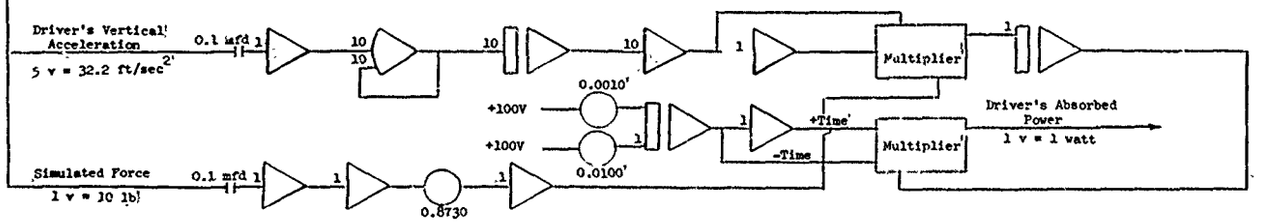
172. Normally, models are validated by exciting them with simple, well-defined inputs, such as a step function or sine wave, and comparing the model response with that of a prototype vehicle driven over a course with the same input configurations. Some criterion is chosen to determine what is deemed a tolerable correspondence of the responses, and the model is then adjusted until this degree of correspondence is met. The model is then considered validated and, hence, representative of the vehicle. However, models validated in this manner to certain well-defined inputs are generally valid only for those particular inputs. A vivid example of this is illustrated in figs. 46, 47, and 48, which are plots of RMS vertical acceleration at the driver's seat of the M37 versus speed for both measured and predicted or modeled results for different sizes and shapes of single obstacles. The model was "tuned" for the 8-in., half-round obstacle shape. A close agreement exists between measured and modeled responses of the vehicle to an 8-in. half-round obstacle (fig. 46). On the basis of these results, the model is considered truly representative of the vehicle. However, measured and modeled responses to an 8-in.-high obstacle with a 30-deg face (fig. 47) deviate considerably. Furthermore, measured responses



a. Schematic of analog program for absorbed power calculation



b. Analog circuit for vertical transfer function



c. Analog circuit for absorbed power

Fig. 45. Schematic and circuit diagrams for calculation of absorbed power

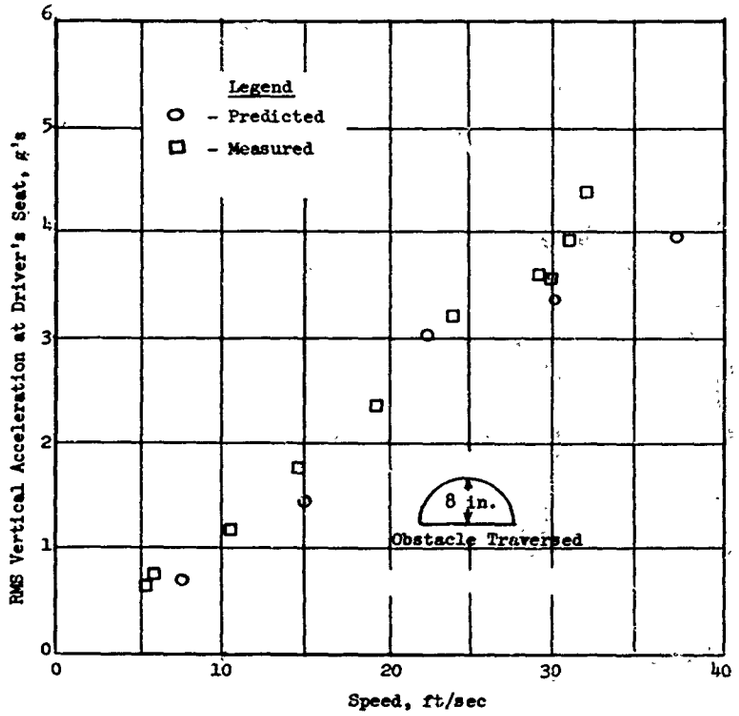


Fig. 46. M37 driver's seat response to single 8-in.-high, half-round obstacle at various speeds

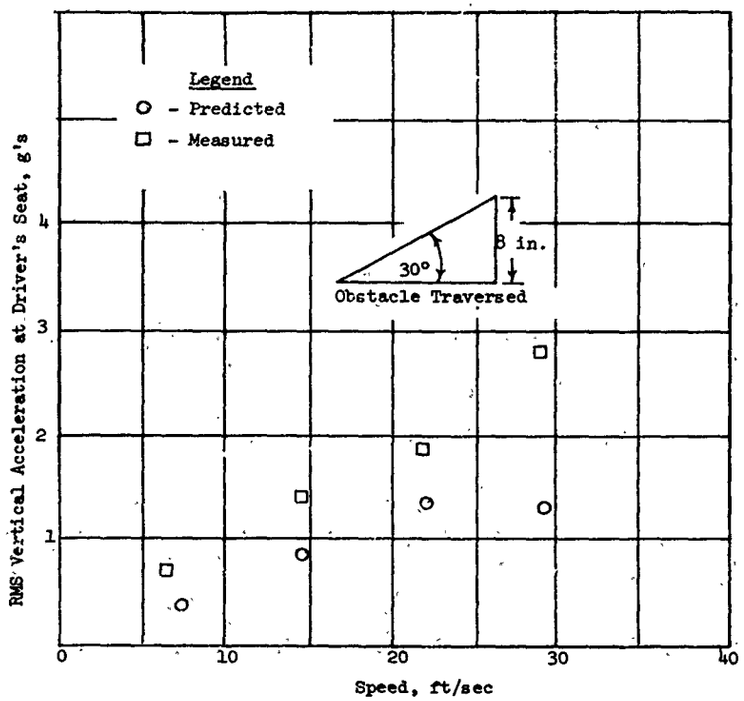


Fig. 47. M37 driver's seat response to single 8-in.-high obstacle with 30-deg face at various speeds

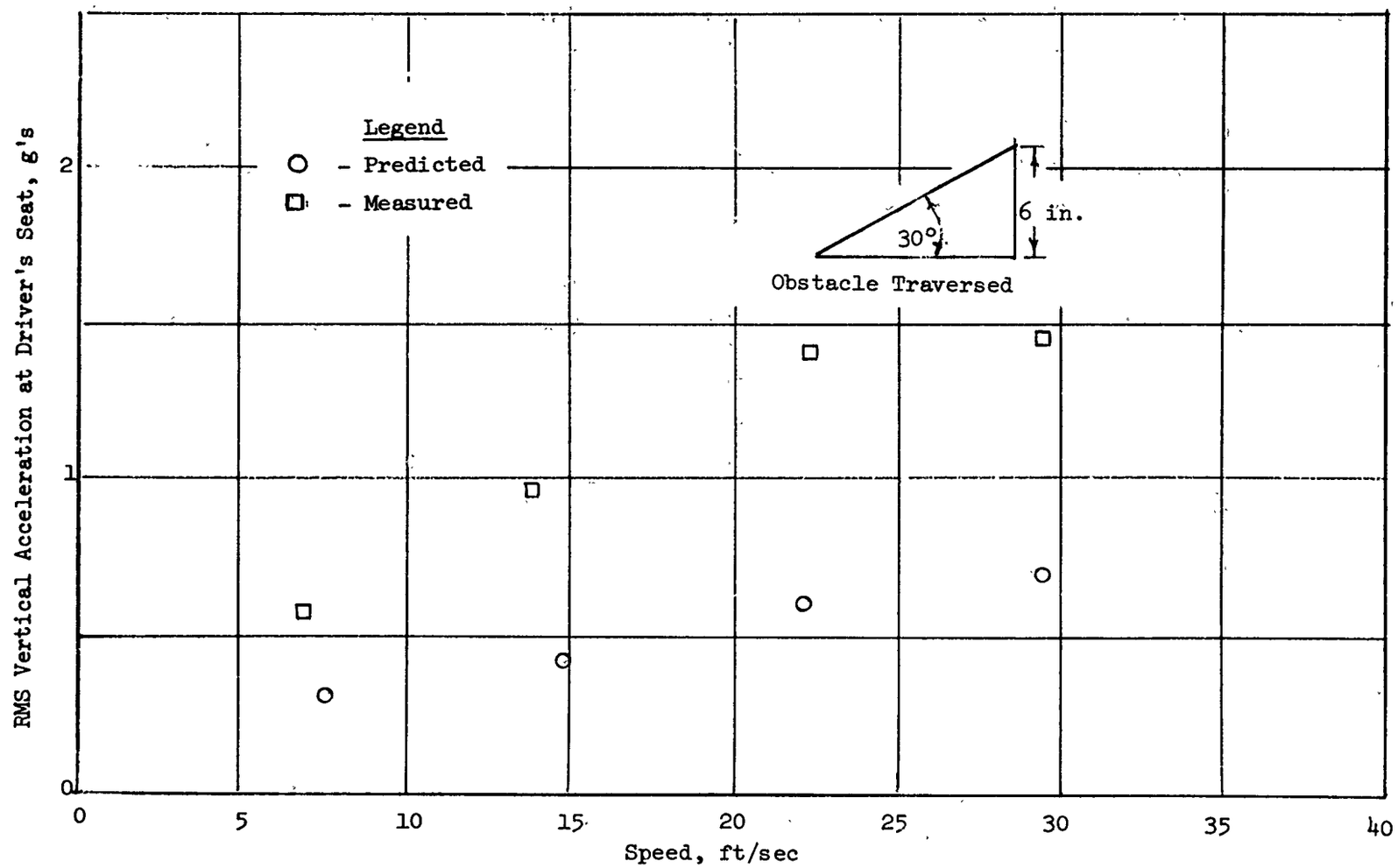


Fig. 48. M37 driver's seat response to single 6-in.-high obstacle with 30-deg face at various speeds

to a 6-in.-high obstacle with a 30-deg face (fig. 48) are considerably higher than the modeled values and the deviation increases with increases in vehicle speed.

173. Summary. The basic structure of all vehicle models follows essentially the same general framework in that each is composed of an assemblage of lumped masses, springs, and damper elements suitably arranged to represent a specific vehicle or vehicle type. Differential equations, formed from the assemblage via Newton's second law, describe the motions of the masses which may be analyzed as time histories or statistical quantities.

174. For tracked vehicles, the effects of the tracks are ignored. The basic differences in models lie generally in the degrees of freedom employed and the amount of emphasis placed on representing the unsprung masses such as the suspension assemblies and tire compliance, and in the trade-offs between inclusion of linear and nonlinear interactions. The majority of vehicle dynamics simulations are restricted to travel along a straight path over a rigid surface at constant speed. However, a few models have been developed which do include the features of variable speeds and yielding surfaces.

175. The most significant deficiency of vehicle modeling to date, however, is the lack of verification. Although very little information exists in published form in which predicted and measured responses are compared, it is quite evident from the foregoing discussion that considerable improvement in vehicle modeling is needed before reliable predictions can be attained. To effect these improvements will require first the identification of the pertinent problem areas and systematic studies in these areas to improve understanding between the model and the physical system.

176. One of the most notable of these problem areas is lack of understanding concerning the measurement of appropriate vehicle parameters to use in these lumped mass models. Another problem of equal importance concerns model verification--that is, once a model is implemented, how to validate it to ensure that it is representative of the desired vehicle. Even "tuning" a model to match the actual vehicle's response to a given obstacle configuration only produces a model verification for that particular

obstacle configuration. Since both the vehicle and the model responses are frequency dependent, it seems more logical to validate a model by using as input some general form of random profile composed of many frequencies and comparing the outputs in terms of their frequency response. Actual terrain profiles of suitable length should serve this purpose.

177. Another problem that frequently causes significant differences between the model prediction and vehicle measured responses is due to the limited frequency range of the model as compared with the actual vehicle. The contribution of the higher frequencies transmitted by the actual vehicle can significantly affect the response, particularly so when evaluated in terms of such statistical quantities as RMS, standard deviation, etc. These high-frequency oscillations are negligible, however, with regard to the ride problem and should be filtered from the measurements when making comparison with model responses.

178. With these major problems in mind, it is apparent that considerable study is needed in the area of vehicle dynamics modeling. However, an adequate correspondence between predicted and measured responses can most likely be achieved with present modeling techniques through proper filtering of the measured data and inclusion of accurate vehicle parameters.

Slope models

179. Many terrains contain slopes of sufficient magnitude that their effects must be accounted for in the prediction of off-road vehicle performance. Even when traction required to negotiate a slope does not exceed the propelling force available, it will increase power required and fuel consumption, reduce the margin of traction available to overcome further impedances, and will usually decrease speed. Modeling vehicle and slope interaction is thus a necessary element in any comprehensive model for predicting off-road vehicle performance. When a vehicle is operating on a slope, there is an accompanying weight transfer and need for a margin of excess traction to maintain steering control of the vehicle. In addition, especially when side-sloping, a vehicle may overturn. Multipass operation on slopes degrades or modifies the shape of the surface. In rough order of magnitude of its effects on off-road vehicle operations generally, the slope problem is probably second only to the soft-soil problem.

180. Despite this, there are essentially only three models in use that deal specifically with the effect of gravitational forces acting on a vehicle when operating on a slope. The most common model treats the problem of moving straight up or down a slope on the basis of traction available when the vehicle is on the level. A second model attempts to account for the effects of weight transfer upon the available net traction. The third deals with the static roll stability of a vehicle when traversing a slope at 90 deg to the maximum slope line. In all three, the problem is handled as a simple static phenomenon, and it is assumed that the surface is smooth and that the vehicle is rigid with no suspension compliance.

181. Common slope-climbing model. The most commonly used slope-climbing model treats a vehicle on a soil slope as a problem in simple static friction, using the drawbar pull-to-weight ratio that the vehicle can develop in the same soil when on a level surface as though it were a coefficient of friction. By these assumptions, the drawbar pull that a vehicle can develop on a slope (DBP_s) is given by

$$DBP_s = \left(\frac{DBP_L}{W}\right) \times W \cos \theta - W \sin \theta \tag{21}$$

where DBP_L = drawbar pull measured on the level, and the maximum negotiable slope angle (θ_m), which (neglecting requirements for steering control) corresponds to zero DBP_s, is defined by

$$\tan \theta = \frac{DBP_L}{W} \tag{22}$$

182. In tests conducted by WES (1951) with wheeled and tracked vehicles on natural, fine-grained soil slopes of up to 20 percent (i.e. distinctly less than the maximum slope negotiable), the difference between measured and predicted drawbar pulls (using equation 21) was less than 1 percent. A plot of measured versus computed drawbar pull for these tests is shown in fig. 49. Empirical relations developed by WES for predicting the maximum drawbar pull on a level-fine-grained soil surface and the corresponding maximum slope negotiable are shown as figs. A2 and A4, Appendix A.

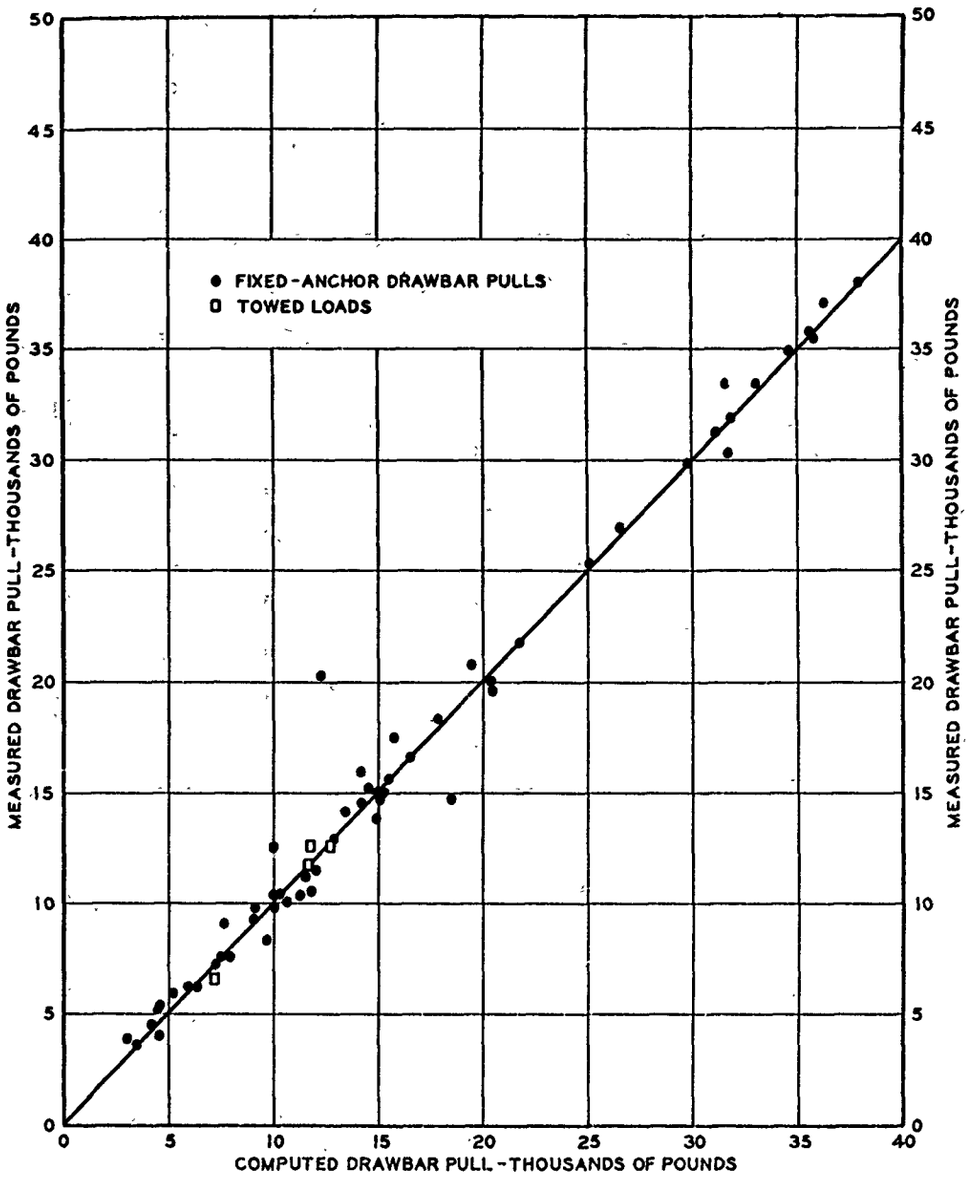


Fig. 49. Comparison of measured and computed drawbar pulls on sloping surfaces

183. In WES tests (1963) with wheeled vehicles on clean sand slopes, the maximum slope that a given wheeled vehicle could climb was generally 2 percent less than its maximum drawbar pull coefficient in level sand of the same strength. An empirical equation developed by WES from these data for predicting the maximum sand slope negotiable is given in table A4,

Appendix A. Measured maximum slope negotiable compared with values predicted from this regression equation yield a multiple correlation coefficient (R^2) of 0.96.

184. Laboratory wheel studies (cf. Smith 1969) show that the ratio of drawbar pull to wheel load (DBP_I/W) decreases monotonically with increasing load (fig. 50). This implies that predictions made with equation 21 should be slightly conservative insofar as the basic assumptions are adequate. A recent lunar study by WES for NASA, reported by Freitag, Green, and Melzer (1970), shows that for very light wheel loads, the relation given by equation 21 is applicable, right up to maximum negotiable slopes.* However, laboratory findings reported by Jones and Dewhirst (1964) with model vehicles on sand and gravel slopes indicate that at somewhat higher wheel loadings and high wheel slips the maximum grade predicted by equation 22 was optimistic by as much as 20 percent.

185. LLD weight transfer model. When a vehicle is climbing a slope, there is an effective transfer of weight from front to rear. Since ground pressure is a primary determinant of tractive effort in soils, this weight transfer should be accounted for in predicting vehicle slope operation. LLD has recently published a model applicable to tracked vehicles which makes some such allowances. This model assumes a trapezoidal ground pressure distribution under a tracked vehicle when on a slope, as shown in fig. 51. Values for contact pressure at the front (a) and at the rear (b), reflecting both the slope angle and the location of the vehicle center of gravity, are then computed for two-dimensional static equilibrium. These values are used somewhat inconsistently, with the basic Bernstein-Bekker relation

$$z = (p/k)^{1/n} \tag{23}$$

to calculate front and rear sinkage, and hence resistance generated by soil compaction, and a slope resistance due to the vehicle trim in relation to

* Kloc, in "Vehicle Mobility Tests, Soft Soil Slopes," June 1970, purports to show that this relation is not true at low loadings, but close analysis of his data does not support this conclusion.

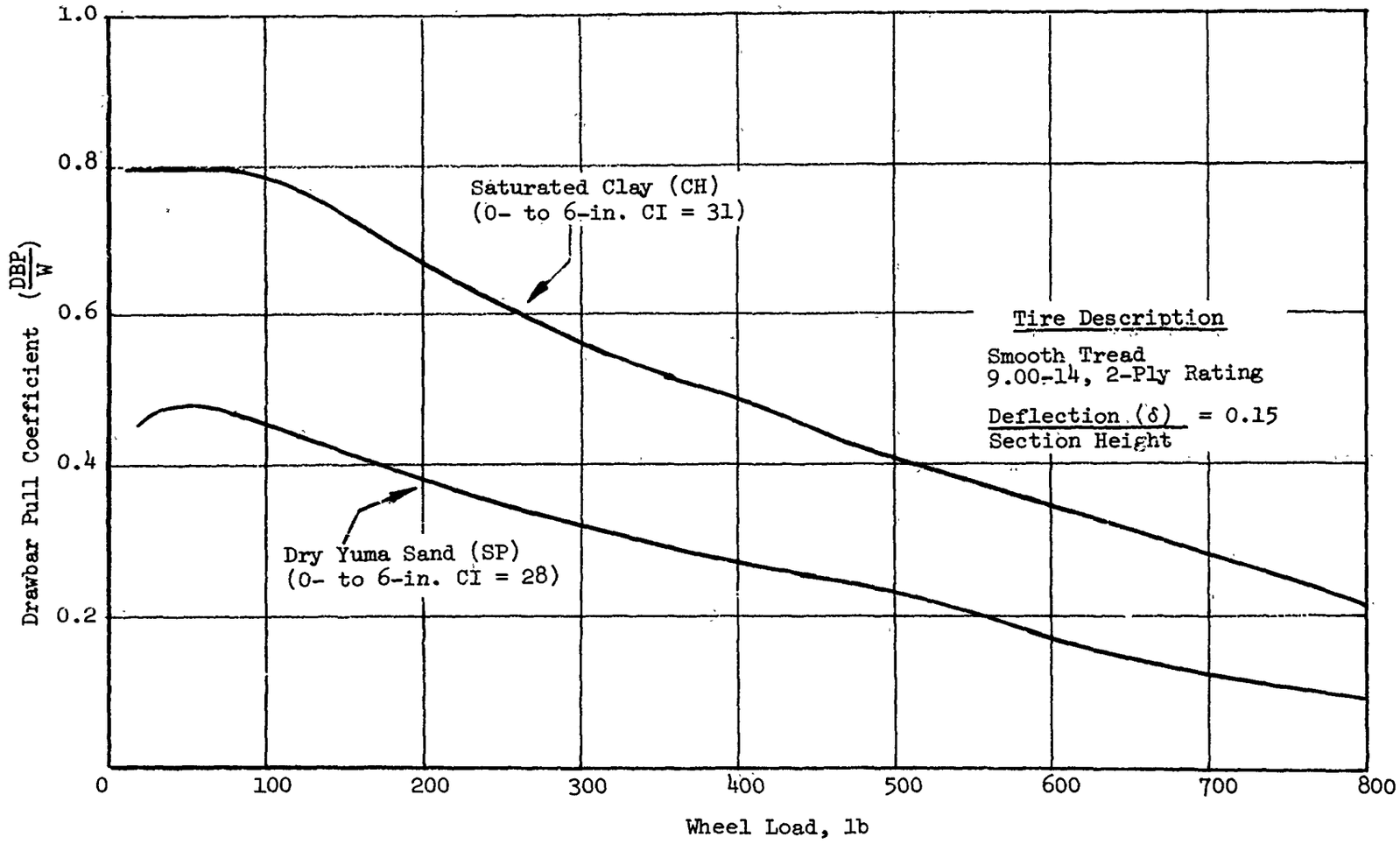


Fig. 50. Effect of wheel load on drawbar pull performance on level surfaces

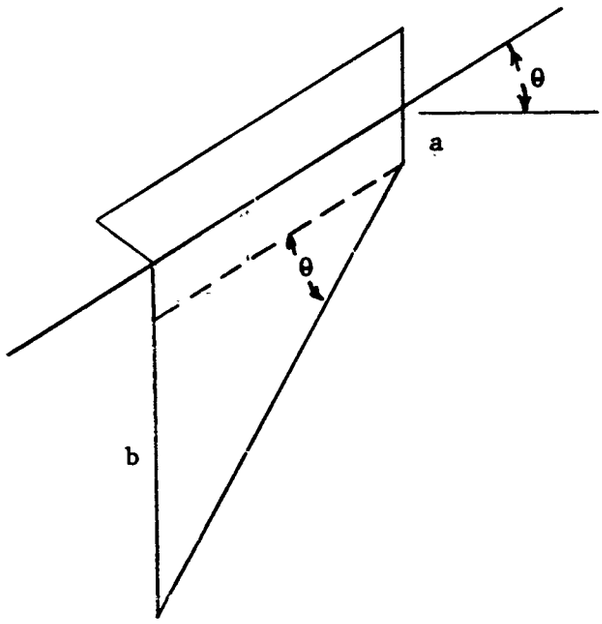


Fig. 51. Ground pressure distribution under tracked vehicle

the undisturbed sloped soil surface. Note that the basic LLD traction model is insensitive to changes in pressure distribution, so that only these two added resistance components are involved in this elaboration. The model has thus far been only briefly spot-checked, and has not been incorporated in any comprehensive cross-country model.

186. Side-slope models. The only side-slope model regularly used checks the static roll equilibrium of a vehicle when placed on a given side slope, based upon the height of the vehicle center of gravity and its center-to-center tire or track tread. Effects of suspension compliance are usually ignored.

Obstacle-vehicle
geometry interference models

187. Obstacle-vehicle geometry interference models are essentially the same in the manner in which the interactions are accounted for. Differences occur in the application of the model and in the types of vehicles for which they are applicable. Two approaches are generally followed. One approach is to manually pass a two-dimensional model over a two-dimensional obstacle to determine interference; the other approach uses geometric and trigonometric obstacle-vehicle relations. Little verification exists for

prototype vehicles except for standard rigid obstacles used in APG engineering tests. Some scale-model verification results are available. Those obstacle-vehicle geometry interference models that are operational are discussed in the following paragraphs.

188. WES obstacle-vehicle geometry model (GM or Chrysler or others). These models manually test a two-dimensional vehicle scale model against two-dimensional obstacles. Neither vehicle nor obstacle is considered to have any compliance. The obstacle, but not the vehicle, may be stylized. The vehicle is presumed to approach the obstacle at 90 deg. Immobilization is usually assumed if there is any interference at any time during the complete passage of the vehicle over an obstacle. If the vehicle can cross the obstacle or a section of irregular terrain, the WES model also determines the maximum attitude angle of tracked vehicles and the effective obstacle angle for wheeled vehicles when crossing the obstacle. The effective obstacle angle for wheeled vehicles is the angle formed by a tangent to the point of tire-obstacle contact and a horizontal plane.

189. LLD obstacle-vehicle geometry model. The LLD model is a series of computerized, two-dimensional comparisons between the vehicle and the obstacle, both considerably idealized. This modeling is based upon equations published by Janosi in September 1968, which in turn, by way of manuscript exchange, is due somewhat to suggestions subsequently published by Bekker in 1969. The LLD model will handle four-wheeled vehicles and normal skid-steered tracked vehicles. Obstacle shapes that can be tested include steps and vertical or V-walled ditches and crests, large and small. Possibilities for interference are checked in a series of critical vehicle/terrain orientations and any interferences found are signaled. The assumption is made that an interference equals an immobilization. These equations were checked in a brief series of very slow-speed tests using three 4x4 vehicles and a few specially constructed concrete obstacles. Agreement was good, as would be expected.

190. WNRE obstacle-vehicle geometry model. The WNRE model is philosophically similar to the LLD model except that the range of obstacle shapes treated is more general and means are included for handling wheeled vehicles with more than two axles. The WNRE model is a small part

integrated into a more elaborate go-no go program published in July 1968. It has since been further elaborated to handle articulated 6x6 vehicles and still more complex shapes, as described in a July 1970 report of methods to quantify lunar roving vehicle performance. In both cases, obstacle geometry is stylized in the general manner of the two-dimensional MERS terrain map units and interference is considered tantamount to immobilization.

Obstacle-traction models

191. When no geometric interferences are found, obstacle-traction models determine whether or not sufficient traction can be developed to surmount the obstacle. The soil-slope model may be considered as one possible member of this class but is usually treated separately. There are two obstacle-traction models presently in the literature of complete cross-country simulations; however, there has been no experimental verification of these models.

192. WES obstacle-traction model. In the WES obstacle-traction model, determinations of the maximum force required and the maximum force that a vehicle can develop when at the maximum attitude angle while crossing an obstacle are made using the following equations. For tracked vehicles

$$F_{mr} = W \sin \alpha_{mv} \tag{24}$$

where

F_{mr} = maximum force required, lb

W = gross vehicle weight, lb

α_{mv} = maximum attitude angle vehicle will attain in crossing obstacles, deg

and

$$F_{md} = DBP \cos \alpha_{mv} \tag{25}$$

where

F_{md} = maximum force that can be developed, lb

DBP = maximum drawbar pull on level surface, lb

For wheeled vehicles, it is considered one axle at a time crosses the

obstacle, and W in equation 24 is replaced by $\frac{W}{n_a}$, where n_a is the number of axles and α_{mv} is replaced by α_e as the effective obstacle angle. If F_{md} is greater than F_{mr}, sufficient traction can be developed for the vehicle to negotiate the obstacle.

193. WNRE obstacle-traction model. The WNRE model computes, for various critical positions of the vehicle on an obstacle, a single coefficient of friction between the vehicle and the surface of the obstacle needed for static equilibrium. This value is then compared with an effective coefficient of friction, taken as the vehicle net traction on level ground divided by its weight, computed for soil having the given strength characteristics. When effective coefficient of friction based on the appropriate soil values is less than that required for equilibrium, the condition is considered no go. There is obvious crudity in ignoring the redistribution of weight on the contact surfaces as it affects motion resistance, tire deflection, actual single wheel traction, etc., for these may seriously alter the magnitude of unit traction at each contact point. The WNRE obstacle-traction model treats the problem as a quasi-static two-dimensional one, with the vehicle approaching each discrete, linearized obstacle at right angles. No vehicle or obstacle compliance is allowed, except (in the 1970 version) at the pitch joint of an articulated 6x6. However, the model will handle stylized wheeled and tracked vehicles of normal configuration and a wide variety of obstacle sizes and configurations. The basic model, incorporated in a larger simulation published in July 1968, has recently been extended to cover two-dimensional, craterlike obstacles and articulated, three-axle wheeled vehicles.

194. WES obstacle override speed model. In determining the speed at which a vehicle can override an obstacle, the resisting force due to override is added to the other resisting forces acting on the vehicle. The override force is determined by dividing the work required to override the obstacle by the average distance between obstacles, or

$$F_{om} = \frac{Wh_o}{12D_o} \tag{26}$$

where

F_{om} = average force required to override obstacle, lb

W = weight of vehicle, lb

h_o = height of obstacle, in.

D_o = average distance traveled between obstacles, ft

The average distance traveled between two obstacles is computed by converting the area of the sample cell* to a rectangular area whose width is equal to the vehicle width and dividing the length of the rectangular area by the number of obstacles in the sample. The equation is

$$D_o = \frac{\pi D^2}{4wn} \tag{27}$$

where

D = sample cell diameter, ft

w = vehicle width, ft

n = number of obstacles in sample

WES maneuver model

195. Abrupt nondeformable surface irregularities such as rock outcrops, stumps, boulders, termite mounds, logs, etc., are deterrents to vehicle performance and cause the vehicle to slow down to maneuver around some and perhaps override others. To account for the maneuvering effects on speed, the WES maneuver model uses semiempirical relations of percent area denied and speed. By definition, "area denied" by a single obstacle is the area encompassed by the obstacle itself plus one-half the vehicle width around the obstacle.

196. The equation for determining percent area denied is:

$$\%A_d = \frac{\left[\ell_o w_o + (\ell_o + w_o)w + \frac{\pi w^2}{4}\right] n}{A} \times 100 \tag{28}$$

---

* The structural cell concept with its derivatives, mean tree spacing, nearest neighbor distance, etc., has been explored with some intensity by WES. The concept is described in "Quantitative Physiognomic Analysis of the Vegetation of the Florida Everglades," by H. L. Mills, Contract Report No. 3-72, 1963, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.; prepared by Marshall University, Huntington, W. Va.

where

$\%A_d$ = area denied, percent

ℓ_o = length of obstacle, ft

w_o = width of obstacle, ft

w = width of vehicle, ft

n = number of obstacles in sampled area

A = area of sample, ft^2

When the obstacle configuration is of such dimensions that the vehicle in question can pass over the obstacle without contacting it, twice the width of the traction element is substituted for the width of the vehicle in the equation above. Area denied computations are illustrated for two single obstacles in fig. 52.

197. When trees are present, percent area denied by the trees is computed by the following equation

$$\%A_d = \frac{n(d_s + w)^2}{D^2} \times 100 \qquad (29)$$

where

n = number of tree stems

d_s = stem diameter, ft

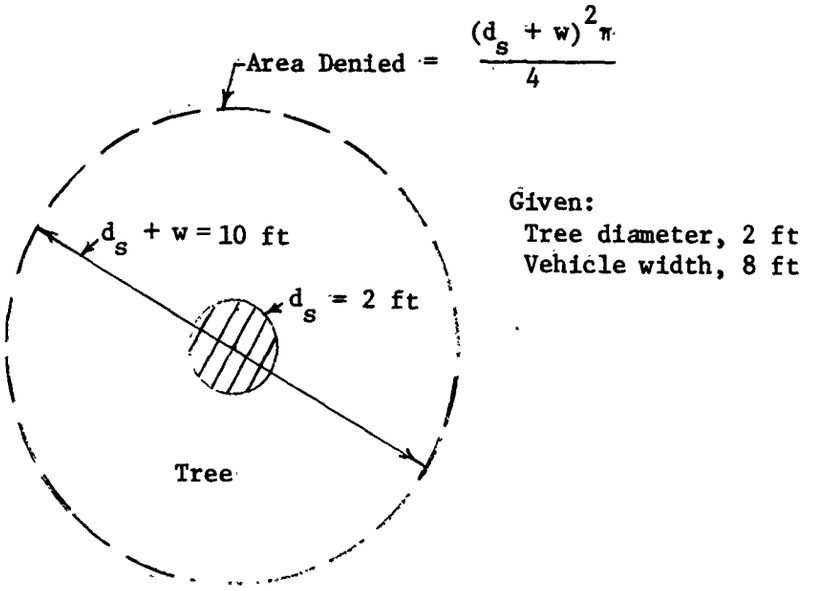
w = vehicle width, ft

D = sample cell diameter, ft

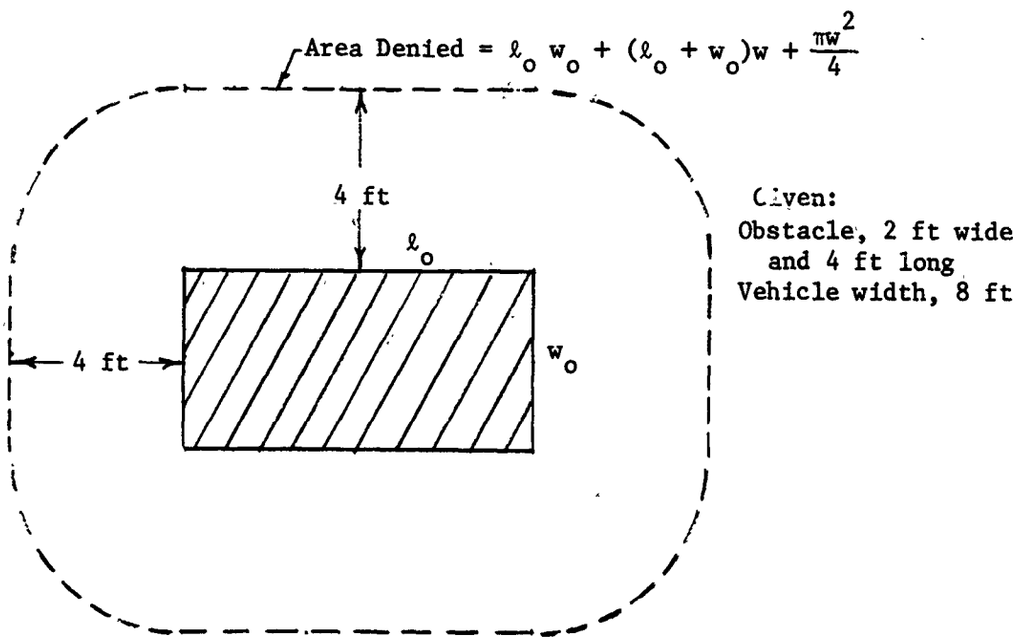
If an area contains a combination of discrete obstacles and trees, the trees that occur in the area denied by the discrete obstacles are not included in the calculation. This is accomplished by reducing n in the equation above by percent of area denied by the discrete obstacles. For example, if 20 trees occur in a sample in which 10 percent of the area is denied by discrete obstacles, n in the equation above is reduced 10 percent. The percent area denied by trees is added to the percent area denied by other types of discrete obstacles present.

198. The equation used to determine speed as a function of area denied is:

$$S_m = \frac{S_o}{40}(50 - \%A_d) \qquad (30)$$



a. Tree



b. Obstacle

Fig. 52. Example of area denied computations for single obstacles

where

S_m = maneuver speed, mph

S_o = maximum speed, mph, as controlled by soil, slope, dynamic response, and visibility

199. It has been established by field tests that when the area denied is less than 10 percent, the effect on vehicle speed is insignificant; but when the area denied is greater than 50 percent, the vehicle must override some trees or obstacles in order to traverse the area.

WES vegetation models

200. Vegetation such as trees, bamboo clumps, brush, and grasses are deterrents to vehicle performance since woody plants may cause the vehicle to slow down to maneuver around or override single or multiple stems. In addition to the maneuvering and overriding problem, visibility restrictions caused by vegetation alone may limit the vehicle's speed.

201. The WES vegetation model considers the effects of vegetation in terms of the maximum and average forces required to override trees and the limits imposed by obscuration. Methods used in analyzing the separate effects of vegetation on vehicular movement are discussed in the following paragraphs.

202. Vegetation impact model. A determination of the maximum forces required to fail a single tree is used to establish whether or not the strength of the leading edge of the vehicle and/or the driver's tolerance to horizontal acceleration will be exceeded.

203. In deriving the equation for maximum force required to fail a single tree, a rational and an empirical approach were used. In the rational approach, it was assumed that a tree failed as a cantilever beam. The fiber-stress equation for the resisting moment at any cross section of a beam is equal to the unit stress (S_m) times the moment of inertia of the cross section with respect to the neutral axis divided by the distance from the neutral axis to the outermost fiber. In applying this equation to a tree, the resisting moment is the product of the horizontal pushbar force and pushbar height ($F_h \times h_p$), the moment of inertia of a cross section of a tree stem is $\pi d_s^4/64$, and the distance from the neutral axis to the outermost fiber is $d_s/2$; the equation is:

$$F_h \times h_p = \frac{S_m \frac{\pi d_s^4}{64}}{\frac{d_s}{2}} \tag{31}$$

where

d_s = stem diameter, in.

204. The ultimate unit stress for most trees tested was approximately 7500 psi. Using this value for unit stress and pushbar height as an approximation of the length of the moment arm gave reasonable results when compared with test results. Since unit stress and length of moment arm are approximations, an empirical equation derived from test results is used to determine maximum force required to override a single tree, as follows:

$$F_h = \left(40 - \frac{H_p}{2}\right) d_s^3 \tag{32}$$

where

H_p = pushbar height, in.

An example illustrating the quality of the relations established between maximum horizontal pushbar force and stem diameter by equation 32 and line of best visual fit is given in fig. 53.

205. To determine if the impact force will exceed the force that the leading edge of the vehicle can withstand, the maximum force required to override a single stem is compared with the strength of the leading edge, as provided by the vehicle manufacturer. If the information is not available, the bending strength of the leading edge is computed.

206. To determine whether or not the force of override will exceed the driver's tolerance of 2-g horizontal acceleration, the maximum override force is divided by twice the weight of the vehicle.

207. Vegetation override speed model. In order to determine the speed at which a vehicle can override vegetation, the resisting force due to override is added to the other resisting forces acting on the vehicle. The override force is determined by dividing the work required to override a single tree by the average distance between trees, or

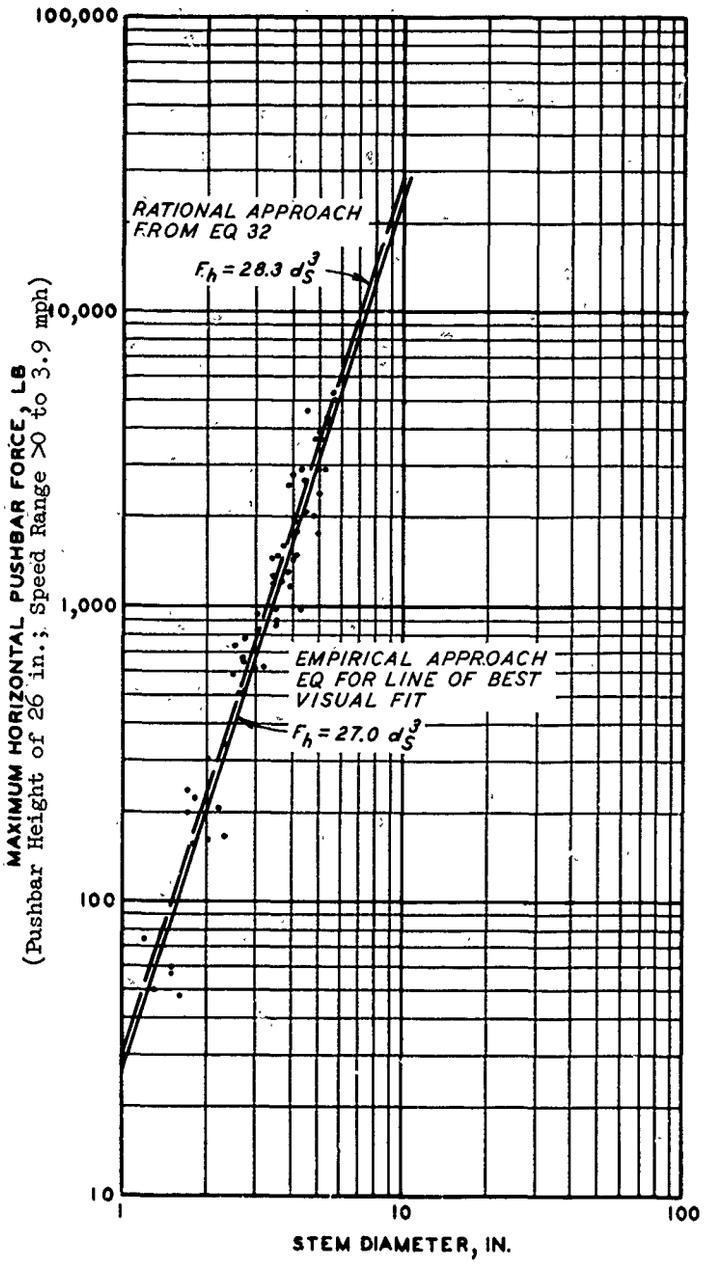


Fig. 53. Force required to fail tree

$$F_o = \frac{W_t}{D_x} \tag{33}$$

where

F_o = average force required to override a single tree, lb

W_t = work required to override single tree, ft-lb

D_x = average distance traveled between trees, ft

The work required to override a single tree (W_t) is determined by an empirical equation derived from field work. The relation is shown in fig. 54 where work required per tree is plotted against stem diameter. The equation is

$$W_t = 100\ d_s^3$$

The average distance traveled between trees is computed by converting the area of the sample cell to a rectangular area whose width is equal to the vehicle width and dividing the length of the rectangular area by the number of trees in the sample. The equation is the same as equation 27. When the resisting force due to override is obtained, it is added to the other resisting forces acting on the vehicle, and a vehicle speed is determined from the tractive force-speed curve.

208. Vegetation traction model. When a vegetation override speed has been determined, a determination of the maximum tractive force required to override a single tree is made. For a go condition the maximum tractive force available must be greater than the force required to fail a tree minus the kinetic energy at the override speed. The tractive force required is determined by the following equation

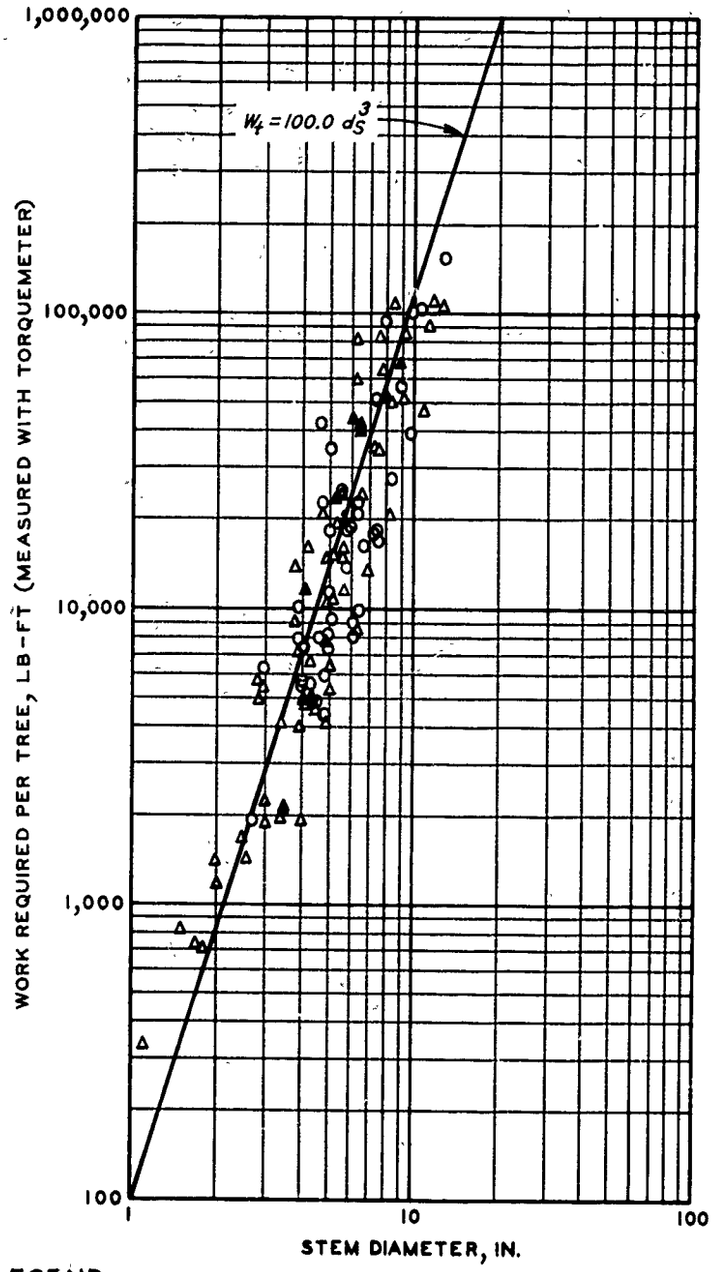
$$T_o = \frac{W_p}{5.8} - \frac{\frac{W}{g} \times V^2}{2 \times 5.8} \tag{34}$$

where

T_o = maximum tractive force required to override single tree, lb

W_p = work required to fail a tree, ft-lb

W = weight of vehicle, lb



LEGEND

Δ EGLIN AFB, FLA., SPEED RANGE >0-3.9 MPH

▲ EGLIN AFB, FLA., SPEED RANGE 4-12 MPH

O THAILAND, SPEED RANGE >0-3.9 MPH

Fig. 54. Work required to override tree

g = gravitational constant, fps^2

V = vehicle velocity, fps

The work required to fail a tree is obtained from the empirical equation illustrated in fig. 55. The force required to fail a tree is determined by dividing the work required to fail the tree by the average distance the vehicle travels to fail the tree, i.e. 5.8 ft or $W_p/5.8$.

209. Visibility model. In some terrain situations visibility alone can control off-road vehicle speed. The surface may be covered with grasses or ferns of such height that surface features are completely obscured, requiring the driver of a vehicle to reduce his speed to that of a man walking ahead of the vehicle. In other situations, surface features may be partially obscured by leaves or brush, or even by the arrangement of stems and branches, with an accompanying, although lesser, effect on vehicle speed. In any terrain situation, there is a very practical limit imposed upon the speed a vehicle may safely achieve--the vehicle should at no time exceed that speed at which the driver can stop his vehicle in time to avoid striking a damaging obstacle. This is, of course, equally true whether the obscuration is occasioned by vegetation in a wooded area or by fading light on a trail, or by fog on a superhighway. In each case, the driver chooses a speed by integrating two factors: (a) the distance required to stop the vehicle, and (b) the distance that he can see and recognize a significant obstacle. From the reaction time, braking force, and recognition distance established for a specific set of conditions, the vehicle speed as controlled by visibility is computed by the following equation:

$$V = -at_r + \sqrt{a^2 t_r^2 + 2as} \tag{35}$$

where

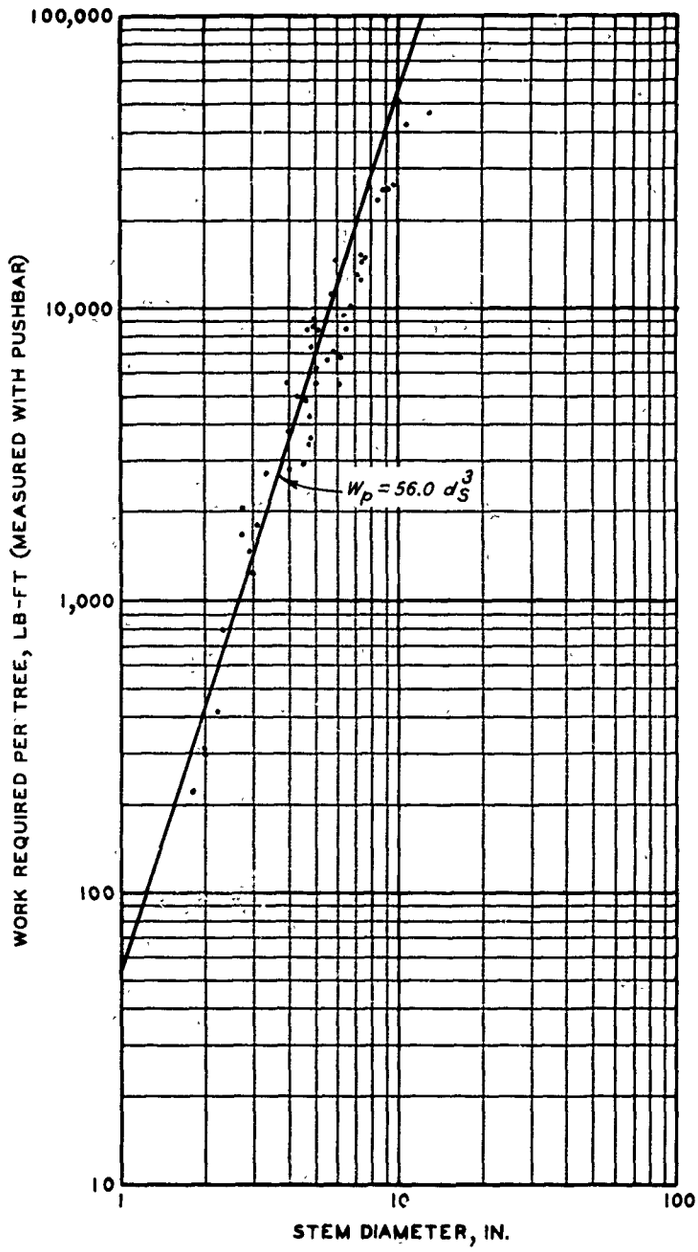
V = speed controlled by visibility, fps

a = deceleration $\left(\frac{\text{braking force}}{\text{vehicle weight}} \times 32.2\right)$, fps^2

t_r = reaction time, sec

s = recognition distance, ft

Reaction time is a function of the stimulus, the driver, and the vehicle



LEGEND

△ EGLIN AFB, FLA., MAY 1965, SPEED RANGE >0-3.9 MPH

▲ EGLIN AFB, FLA., MAY 1965, SPEED RANGE 4-12 MPH

□ NASA MARSHALL SPACE FLIGHT CENTER, MISS., AUG 1964, SPEED RANGE >0-3.9 MPH

Fig. 55. Work required to fail tree

characteristics. Tests have been conducted with a driver reaction simulator; the results indicate that 0.5 sec is a reasonable estimate of reaction time. Recognition distance is the maximum distance at which an obstacle of mobility significance can be recognized. Values of recognition distance are obtained from field measurements. The braking force used to obtain deceleration is assumed to be equal to the maximum tractive force that can be developed on the soil condition under consideration.

Acceleration and deceleration models

210. In negotiating terrain conditions that contain widely spaced discrete obstacles that must be overridden (e.g. rice-field dikes), the driver will accelerate after crossing an obstacle and decelerate before striking the next obstacle. To account for the speedup and slowdown processes, acceleration and deceleration models are used. The WES method for predicting average speed in terrains containing widely spaced obstacles that are to be overridden is illustrated in fig. 56. The acceleration and deceleration models used by WES are discussed in the following paragraphs.

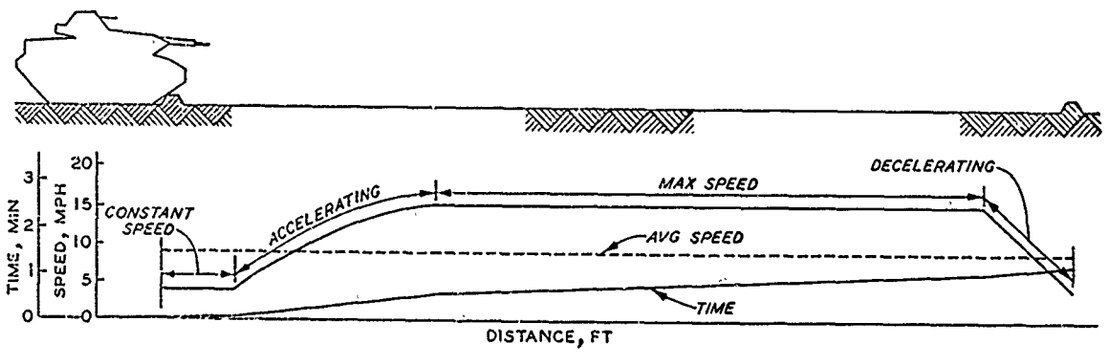


Fig. 56. Graphical illustration of the WES method for predicting the average speed in terrains containing widely spaced discrete obstacles that are to be overridden

211. Acceleration model. The effect of acceleration on average vehicle speed is determined by the use of speed-distance relations and time. Specific values of vehicle speed and drawbar pull are obtained from a drawbar pull-speed relation to develop the velocity-distance relation. Time is computed from values obtained for acceleration. The procedures used are discussed below.

a. An increment for a velocity change is chosen on the drawbar pull-speed curve* and the average DBP and average velocity for that increment are determined by the following equations

$$\overline{DBP} = \frac{DBP_1 + DBP_2}{2}$$

$$\overline{V} = \frac{V_1 + V_2}{2}$$

where

V = velocity, fps

b. The average acceleration $\overline{a}$ between V_1 and V_2 is computed as follows

$$\overline{a} = \frac{\overline{DBP}}{W} g \tag{36}$$

where

W = vehicle weight, lb

g = acceleration of gravity, ft/sec^2

c. The time t required to accelerate from V_1 to V_2 is computed as follows

$$t = \frac{V_2 - V_1}{\overline{a}} \tag{37}$$

d. The distance D_n traveled in time t is computed as follows

$$D_n = \overline{V} \times t \tag{38}$$

This procedure is repeated for all increments between initial and final velocities. If the vehicle is operating on a slope, the drawbar pull is

---

* Obtained by converting tractive force coefficient into tractive force and subtracting motion resistance.

corrected for the effect of the slope

212. Deceleration model. The rationale and the method of predicting the speed and distance traveled for a decelerating vehicle are as follows:

a. Rearranging equation 37, $\frac{V_2 - V_1}{t} = -a$, the change in velocity from V_1 to V_2 divided by the time required to effect that change is the deceleration when $V_1 > V_2$.

b. The force required to produce this deceleration is equal to the mass W/g of the vehicle times the deceleration:

$$F = \frac{W}{g}\left(\frac{V_2 - V_1}{t}\right) \tag{39}$$

c. The force available to produce this deceleration (braking force is discussed in paragraph 213) is the motion resistance, MR . Substituting into equation 39

$$MR = \frac{W}{g}\left(\frac{V_2 - V_1}{t}\right)$$

d. Again, from the basic equation $\frac{V_2 - V_1}{t} = -a$, $-a$ may be substituted into the equation above and the equation rearranged

$$\frac{MR}{W} \times g = -a$$

e. Or

$$\frac{MR}{W} = \frac{-a}{g}$$

f. Speed at the end of any increment of velocity change may be determined by the equation

$$V_2 = V_1 + at \tag{40}$$

(a may be either positive or negative)

g. When V_1 is equal to zero, equation 40 becomes:

$$V_2 = at = \left(\frac{MR}{W} \times g\right) t$$

Or for any time increment from $V_1 = 0$

$$V = at = \left(\frac{MR}{W} \times g\right) t$$

h. The distance d traveled during deceleration is predicted by the general equation for distance while accelerating from zero velocity:

$$d = \frac{1}{2} at^2 = \frac{1}{2}\left(\frac{MR}{W} \times g\right) t^2 \qquad (41)$$

213. The computations made to determine the speed and distance traveled when the vehicle is decelerating by braking are identical to those for when the vehicle is rolling except that a braking force is used in lieu of motion resistance as the force available to decelerate the vehicle. Braking force is assumed to be equal to the maximum tractive force the vehicle could develop on the soil strength on which the vehicle is operating.

214. Average speed predictions. When a section of terrain is of sufficient length for the vehicle under consideration to accelerate to its maximum speed, the average speed is predicted by dividing the length of the terrain section by the sum of the time required for acceleration to maximum speed, the time traveled at the maximum speed, and the time required to decelerate from the maximum speed to the desired speed at the end of the terrain section.

Stream crossing models

215. Crossing streams and rivers constitutes one of the most troublesome and hazardous operations in cross-country movement. Moreover, throughout the humid climates of the world, it is a hazard that must be faced over and again in any realistic traverse. Accordingly, full simulation requires incorporation of a reasonable realistic river/stream crossing model, plus proper relevant data on the configuration of streams in nature. Description of a section of streambank for vehicle performance prediction requires quantification not only of bank geometry, but also of bank soils--which

will often vary rapidly from the stream bottom up to the highest point of the bank--and bank vegetation. Vegetation on the bank will frequently be quite different from that just a few yards back due to levee formation, local erosion, edge exposure of the forest, etc.

216. A typical stream crossing poses three separate problems. The first is entrance to the stream or river over a bank, the ingress problem; the second, swimming and/or fording the stream (or lake or river); the third is exiting up the far bank (egress). Ingress and egress are essentially obstacle surmounting problems with occasionally the added influence of buoyancy, and perhaps hydrodynamic forces due to vehicle wave-making, stream velocity, etc.

217. Negotiation of even a relatively minor drainage feature is almost invariably a critical maneuver. A principal difficulty in modeling this realistically is to achieve realism in the depiction of the stream itself. Even a small amount of conservatism can make a stream appear to be a complete barrier to a vehicle where, in fact, it might be negotiable in many, many places. A feature of most streambanks, except where highly channelized through the efforts of man, is that they are highly variable as to their severity from the point of view of vehicle negotiation. Short stretches of a streambank can usually be found with some usable frequency which present considerably less impedance to vehicle passage than do adjacent sections that more nearly characterize the bank on an average basis. These may be termed entrance/exit "windows." (Note that whether or not these windows are in fact negotiable to a given vehicle is not part of the present definition.) Proper description of rivers and streams for vehicle crossing purposes must include statistics describing the configuration and frequency of such windows.

Water crossing model

218. The effects of the water per se upon a vehicle transiting a stream or river may be of several kinds. If the water is shallow enough, and it usually is, the vehicle will cross between its ingress and egress points by fording, even though it has inherent floating capability. In this case, performance depends upon the bottom soils, the depth of water, and the stream velocity. Where the stream velocity is appreciable, the stream

bottom will be hard. Sinkage will be small but traction may be limited by water lubrication of, and reduced loading on, the contact surfaces--the latter as a result of partial buoyancy. Partial buoyancy can be particularly troublesome for vehicles with inherent floating capability when stream depth approaches vehicle draft and stream velocity is high. In this particular circumstance, the path across a stream may be totally out of effective driver control. Nonfloating vehicles often have an advantage in these circumstances.

219. When water depth exceeds the fording depth of the vehicle (with or without special snorkeling arrangements), the vehicle must either have floating capability, inherent or added, or the situation obviously becomes no go for it. Once free of the bottom, a floating vehicle is simply a boat, usually a very bad one. Modeling the primary performance of a floater while fully afloat is accordingly a relatively straightforward problem in hydrodynamics, complicated when in a swift current by the vehicle's relatively slow still-water speed. Note that, because of the bank exit window situation, the ability to land the vehicle at a precise point along the far bank is of great importance to its effective egress capability.

220. Existing water-crossing models predict only still-water speed, although this can readily be converted to speed-made-good across a stream on a tacking course, of course. Modeling water-crossing performance, at the moment, ignores serious potential control and stability problems which beset these poor boats in a current. A reasonable estimate can be made of the still-water speed of floaters and amphibians of the current general style and performance, using the relation given in table 16.

Ingress model

221. In most instances, vehicle entrance into a stream is not considered critical, the presumption being that any stream that a vehicle can get out of, it can more easily get into. By and large, this is a practical simplification for most purposes, but cases where it would be of interest to know that a vehicle might have difficulty entering a stream come readily to mind. As one example, it might be desirable to enter a stream with the purpose of using it as a trail for some distance, exiting miles away in a totally different terrain situation.

Table 16

Still-Water Speed of Floaters and Amphibians

$$V = K \sqrt[3]{L \cdot (HP/W)}, \text{ mph}$$

where

K = value tabulated below

L = vehicle waterline length, ft

HP = gross installed power available for propulsion, hp

W = gross vehicle weight, short tons

Propulsion	Hull	K
Propeller	Rough*	0.95
Propeller	Clean*	1.10
Special tracks**	Clean	0.95
Normal tracks**	Clean	0.60
Wheels	Clean	0.45

* "Rough" = hulls extensively cut up for wheels, axles, drive lines, etc., as on the DUKW and its immediate successors.

"Clean" = similar to the 4x4 LARC's with fewer cutouts, fewer wheels, close-fitting wheel wells made possible by elimination of the suspension, etc.

** "Special tracks" = tracks designed specifically for effective water propulsion, as on the LVT's.

"Normal tracks" = land tracks as on the M113, M116, etc.

222. The principal factor that must be added to a downhill obstacle model (incorporating vehicle stability model) to convert it to an ingress model is the consideration of possible swamping. This will often be a dynamic splash situation, however, in which simple static waterline examination will prove conservative.

Egress models

223. There are three egress models in current use. All three predict either go or no go only.

224. WES egress model. The WES egress model simply applies the WES soil-slope model, by using values for the soil cone index appropriate to the bank soil conditions and the effective vehicle obstacle angle as determined by using the WES obstacle-geometry model.

225. LLD/Stevens Institute of Technology (SIT) model. The LLD/SIT model is essentially a regression equation relating observed and estimated bank exiting performance of the M113 to bank slope and height. It is simple, but its rationale does not tempt one to use it for other vehicles.

226. WNRE egress model. The WNRE egress model accepts bank geometry in terms of the two-dimensional MERS linearizations and subsequently treats the problem as an obstacle negotiation prediction, testing both for geometric interference and adequate traction. Static buoyancy at various points in the exit regimen is considered in its effects upon nominal unit ground pressure of the vehicle and hence its net traction. This model is totally unsubstantiated by any tests.

227. Other egress model. A fourth model, still in an early state of development, includes both some simple rigid-body dynamics and static buoyancy in treating a two-dimensional vehicle exiting up a sloped bank. Development of this model, by Davidson Laboratory, ceased during 1968 at an early stage and results from it are trivial at the moment. Accordingly, it is not a candidate although its potential is considerable.

PART III: CROSS-COUNTRY PERFORMANCE MODELS

228. In this part of the study, comprehensive cross-country performance models that are currently operational are identified and analyzed. Following the analyses, a single comprehensive model is outlined. This model, termed the NOW model, utilizes the best of the currently available single feature models in an optimal, fully computerized configuration. The NOW model represents the current state-of-the-art of calculating off-road vehicle performance, and accordingly shares the limitations of the state-of-the-art. The NOW model is thus incomplete in some important respects, lacks realism in others, and has not been fully validated in some parts or as a whole. Despite this, the NOW model is considered to be useful and usable in current and near-future simulations, provided only that its basic limitations are not completely overlooked in the interpretation of final results.

229. By definition, the NOW model can be assembled without further terrain-vehicle research, using only straightforward computer programming. On the basis of the flow charts, details, and references given herein, the critical portion of the NOW model that computes performance of a given vehicle in a fully specified unit terrain can be programmed, in a user-oriented language such as FORTRAN, and fully debugged well within a six man-month effort.

230. A model for simulating cross-country operation is a series of mathematical expressions and analytical processes such as the single feature models briefed in Part II which relate measures of vehicle performance to quantitatively expressed vehicle, operational, and terrain factors. A comprehensive cross-country performance model should be able to satisfy the needs both of vehicle designers and of vehicle evaluators. To this end, it must be able to predict reliably the performance of any vehicle operating on any path in any properly described environment, and it should be suitable for developing the characteristics of vehicles capable of achieving a stated level of mobility performance within any specified environment.

231. Despite the use of ever larger computers, all models must for practical purposes involve extensive simplifications of the vehicle and

terrain and of their interrelations. An optimal model for a given purpose will be one that is no more detailed than necessary for that purpose. In the present context, however, a good overall or general purpose model is sought. By definition, such a model cannot in its completely general form, be thus finely tuned to its usage. Accordingly, a good general purpose model might be so structured that in its complete form it will yield the highest available precisions, while by an orderly series of theoretical simplifications and commensurate reductions in computational complexity it may be made less precise to suit cases in which maximum precision is not required, which may be often. As an alternative to a single "detunable" model, it might prove easier to structure a series of alternate models, each of which meets the special requirements of a more limited set of uses. In this case, the overall model becomes essentially a chassis into which detailed modules may be plugged according to the needs of the moment.

232. There are two essentially independent types of precision. In design use, which implies inputting the vehicle entirely in parametric form, it is desirable that the model produce reliable relative performance in various types of terrain with good sensitivity to design changes. Current models reflecting the state-of-the-art in the terrain-vehicle research are almost totally insensitive to a variety of relatively small design details that are known from field experience to be of critical importance in specific, actual terrain situations. For evaluation purposes, it is more important that the model produce results whose absolute accuracy correlates well on a statistical basis with real world experience. The ideal model, of course, would have both kinds of accuracy. In structuring the NOW model, primary emphasis is placed on achieving maximum reliability in evaluating vehicles in real world situations.

Elements of a Cross-Country Performance Model

233. A complete cross-country performance model comprises the following basic elements:

a. Means to specify the vehicle in meaningful parametric form.

b. Means to depict the terrain as an areal phenomenon.

c. Criteria and procedures for generating and/or selecting paths through the terrain (including, as an almost trivial end point, fully specifying a path).

d. Means to quantify terrain factors for all segments of any selected path.

e. Means to predict desired vehicle performance measures along each segment of any selected path.

f. Rules for joining performances along individual segments into a continuous traverse.

g. Means to aggregate predictions for individual route segments into totals, statistical measures, maps, etc., according to the needs of the model user or of the larger simulation of which the model is a part.

234. The heart of such a model is the calculation of vehicle performance on a homogeneous path segment once all terrain factors pertinent to that segment are specified. The resolution of terrain data available and pragmatic computing considerations, plus terrain complexity, will dictate the number of distinguishable homogeneous unit terrains by which a given area will be represented in practice. Elements a through d above, which precede prediction of vehicle performance in a given unit terrain, are for present purposes considered as preprocessing of the data; steps f and g which follow, as output processing. The logic and functioning of both will be determined largely by the intended use of the complete model. In the remainder of Part III, attention will be directed solely to the structure of the central performance calculation: its completeness, the adequacy of the single factor models it uses, and the rationale and logic of couplings it uses to deal with multifactored terrain situations.

Model Uses

235. A total terrain-vehicle model can be used:

a. To analyze terrain in context of vehicle operations.

b. To optimize new vehicle configurations for stated missions and terrains.

c. To assess the operational capability of new vehicle designs or alterations to existing designs.

d. As part of a more comprehensive study, to make comparative evaluations among a number of operational or mechanical concepts that need not be closely related.

e. To generate "going" maps for use either at the strategic or at the tactical level.

236. When a terrain-vehicle model is used to develop, from samples of a given terrain, testable performance specifications for vehicles adapted in various degrees to that terrain, it becomes in effect a terrain analyzer, in which vehicles function as "filters" or realistic terrain factor weighting devices. In this use the output is a specifications table rather than the performance of a particular vehicle, and high precision in predicting actual vehicle performance is not critical.

237. For use as a design tool, the model must be responsive to "paper" changes in the vehicle. That is, it must provide means to calculate realistically the results of changing vehicle dimensions, loads, pressures, speeds, etc., entirely from the vehicle mechanical and dimensional parameters. It should be sensitive to the order of design changes being investigated (assuming these are themselves potentially significant) and its indications of relative merit must be valid at least as to simple ranking of competitive designs and preferably on a scalar ranking.

238. For vehicle and terrain evaluation and strategic planning, the simulation should be sufficiently precise to discriminate among the performances of different vehicle and operational concepts in a wide variety of terrains. Point-by-point absolute accuracy of prediction in the terrain is not essential provided that, as in the design case, the vehicles are properly ranked in each situation on a reasonably linear scale of merit. In this application, the model may, if it improves overall precision, employ performance data measured during tests on existing vehicles. Such data must still be generalized on a sound basis (as through the use of validated nondimensional coefficients, and they do not eliminate the need for rational coupling of simpler terrain-vehicle interactions) in order to make realistic predictions for complex terrain situations. The more usual

use for good performance data, which for practical reasons will always be limited, will be to improve the overall realism and accuracy of the simulation by fine-tuning single feature models and verifying particular combining procedures.

239. Generation of tactical "going" maps for a given group of vehicles requires the greatest precision and realism. Ideally, all parts of the performance model should be so precise that the quality of the output is essentially dependent only on the resolution and reliability of the terrain data used. The potential accuracy of computer-generated tactical maps based upon proper terrain-vehicle modeling, which conceivably might be prepared on a daily or even hourly basis as terrain data are updated, is far beyond that of the "going" maps with which the Army is presently living.

240. In each use, the most general form of the output will be in terms of maps of individual vehicle performance measures, such as speed, fuel consumption, go or no go, or no go diagnostics. Such maps may be expressed graphically or stored in a matrix that represents the maps as a digitized network, and may ultimately be compressed into statistics of varying degrees of sophistication. Alternatively, the output might be limited to the tabulation of performance measures, segment by segment, along paths selected by various criteria, and the accumulation of related statistics.

241. A route might be selected, for example, to minimize total travel time between two points, to minimize fuel consumption, or to minimize exposure to enemy hazards, all within further constraints such as a specified maximum deviation from a straight-line course, or boundaries defined by potential enemy reaction. Any route selection that does not reflect the extensive influence of moment-by-moment driver decision will be unrealistic to some degree, however, because such a route will be selected from a relatively omniscient viewpoint. Such omniscience may only be approached in practice, and then only where good terrain intelligence permits close route selection prior to the operation and good navigation equipment and good navigators are able to hew to the chosen course.

242. Route preselection in a tactical situation is not to be confused with path specification when the problem is to evaluate one or more vehicles. In the latter case, route specification may be tantamount to

preselection of the most suitable vehicle, because an arbitrarily specified route, in effect, provides an arbitrary, fixed weighting of the relative importance and frequency of various terrain factors. To treat real terrain so simply is to risk nullifying the usefulness of the entire exercise. Vehicles with different performance profiles, each optimally used, will each see the same terrain in terms of quite different weightings of individual and combined factors.

Operational Comprehensive Models

243. Published literature, including many U. S. Army and NASA reports, was reviewed and various DOD agencies known to have used comprehensive cross-country performance models in the past were surveyed to identify significant, extant cross-country performance models.

244. In light of the objectives of ANAMOB, attention was concentrated upon those models meeting the following criteria:

a. The model should be fully quantitative; i.e. able to predict quantitative performance in specific, quantified terrain situations.

b. It should be reasonably comprehensive; i.e. be structured to predict vehicle performance in a variety of multi-factored unit terrains.

c. It should be operational, i.e. developed to the point where it is more-or-less immediately usable.*

Ten models were considered to meet these requirements. They are listed in table 17.

245. Two other cross-country vehicle evaluation procedures in current use which are not comprehensive performance models may be mentioned. The first of these is a computation used by USATECOM at APG to calculate a single index of cross-country performance based upon a subjective weighting of vehicle characteristics and measured performance. The second, REVAL WHEELS, places vehicles in various mobility categories on the basis of

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* See second footnote, p 11.

measured and/or computed go-no go, speed, and range limits in a small number of relatively simple situations involving soils, obstacle negotiation, and obstacle avoidance. Terrain-vehicle relations used in the computations are similar to those used in the other, truly comprehensive models. The several incommensurate factors considered are combined by a standardized but subjective scoring procedure, as in the APG calculations.

246. The design procedures proposed by Bekker, most recently in his book Introduction to Terrain-Vehicle Systems, are treated in further discussions as a comprehensive performance model even though they are not organized into a functioning computer model for predictions in full multi-featured terrain units as defined herein. Most of the single factor models required are available in his system, as is some of the coupling rationale needed.

Strategies available

247. All of the models listed in table 17, except the WNRE model (which treats only go or no go), predict vehicle performance in a unit terrain primarily in terms of speed. Most predict

a. Maximum average speed due to available propelling force and offsetting terrain impedances, V_{prop}.
b. Maximum instantaneous speeds during longitudinal acceleration and deceleration resulting from excess or deficit in available propelling forces, or in presumed driver election, $V_{acc-dec}$.
c. Maximum speed as limited by ride dynamics, V_{ride} and/or V_{fire}.
d. Maximum average speed, V_{limav}.

In addition, some models predict: maximum speed as limited by need for obstacle avoidance, V_{avoid}; as limited by driver vision, V_{vis} ($V_{degraded}$ or V_{night}); and/or while crossing streams, V_{st}. When the unit terrain is such that more than one concurrent speed limit may be calculated (i.e. both V_{prop} and V_{ride}, for example), the assigned vehicle speed is taken to be the least of these. If, however, two or more calculated speed maxima are mutually exclusive (speeds when overriding or avoiding the same array of obstacles, for example), the optimum speed is sometimes selected.

Table 17

Quantitative Comprehensive Operational Cross-Country Performance Models

Agency	Model Designation	Principal Reference
1. U.S. Army Engineer Waterways Experiment Station	WES	"An Analytical Model for Predicting Cross-Country Vehicle Performance," USAEWES Report No. 3-783 (8 vols.), 1969-1970.
2. U.S. Army Combat Developments Command	TATAWS	"Tank, Antitank and Assault Weapons Requirements Study (U)," Dec 1968 (Final Draft SECRET Report).
3. U.S. Army Tank-Automotive Command	OMEGA	"USATACOM Using Mathematical Models for Mobility Evaluation," Army Research and Development Newsmagazine, Jan 1969, pp 20-21.
		"Status Report: Systems Analysis Models at U.S. Army Tank-Automotive Command," Technical Report No. 10308, Oct 1968, Warren, Mich.
4. Cornell Aeronautical Laboratory, Inc.	CAL	"Analysis and Simulation of Dynamical Vehicle-Terrain Interaction," Technical Memorandum V-J-2330-G-56, May 1969, Buffalo, N. Y.
5. Booz-Allen Applied Research, Inc.	Booz-Allen	"Armored Reconnaissance Scout Vehicle (XM800) Parametric Design/Cost Effectiveness Study (U)."
6. GM Defense Research Laboratories	McKenzie	"Evaluation of Counterinsurgency Mobility in Relation to Environment," Final Report, Jun 1966, Santa Barbara, California.
7. Ohio State University	Ohio State	Systems Research Group, "The Tank Weapon System," Annual Progress Report, Report No. RF-573 AR 65-1 (U), Jun 1965, Columbus, Ohio.
8. U.S. Army Tank-Automotive Command	Bekker	"Engineering Optimization of Terrain-Vehicle Systems, A Case Study of a Concept Analysis of a New Amphibian," Technical Report No. 10421, Feb 1969, Warren, Michigan.
		Introduction to Terrain-Vehicle Systems, The University of Michigan Press, Ann Arbor, Mich. 1969.
9. Chrysler Corporation	Chrysler	Test notes inclosed with letter dated 19 Sept 69 to Mr. A. A. Rula from Mr. B. D. Van Deusen, Manager, Mobility Research, Chrysler Corporation.
10. WNRE, Inc.	WNRE	"A First-Cut at Developing Testable Off-Road Performance Specifications for Vehicles Adapted to Specific Terrain," Report No. 169-1, July 1968, WNRE, Inc., Chestertown, Maryland.

Common elements and differences

248. All ten of the full cross-country models analyzed employ a number of single feature models that in general may be applied independently or coupled together. Tables 18 and 19 identify the specific single-feature models incorporated in each, and the flow charts given in figs. 57-66 show the basic logic and primary couplings involved. Note that several detailed types of dynamic models are listed that occur only in the WES comprehensive model (No. 1). Most of the other models shown to have a ride dynamics single-feature model can handle such discrete obstacles as well, so that lack of a citation means only that the published model does not include specific inputs or speed limiting criteria for using the dynamic model in these situations.

249. Input data requirements for each model listed in tables 18 and 19 are fully determined by the data needs of its several single-feature models. All models studied require terrain inputs in deterministic form, except for those used in "ride" computations where either deterministic or probabilistic data usually may be used.

250. Driver modeling is in all cases limited essentially to the driver using his discomfort as the primary determinant for maximum speed as limited by continuous terrain roughness, or to estimate single obstacle negotiation speed. Driver reaction time is also included, explicitly or implicitly, in some of the acceleration-deceleration models. None of the models yet incorporate the effects of driver competence and judgment or the influence upon mean performance of his detailed path selection.

251. The several comprehensive models differ among themselves primarily in one or more of the following:

a. Number of single-feature models included.
b. Specific single-feature model utilized.
c. Coupling involved.
d. Input required.
e. Output yielded.

Brief description of each model

252. Following is a brief description of each extant comprehensive model based on information in the reports cited in table 17.

Table 18

Key to Single-Feature Models Used in Comprehensive Models Examined

No.	Identification	References: Main Report Par. No.	References: Main Report Fig. No.	References: Appendix No.	References: Appendix Par.
1.0	Soil Models				
1.1	WES VCI	75-77			
1.2	WES mobility numeric	78-79			
1.3	Bekker/LLD	80-83			
1.4	Perloff				
2.0	Vehicle Models				
2.1	Automotive engineering	84-87			
3.0	Soil-Vehicle Models				
3.1	WES VCI	89-90		A	7-16
3.2	WES mobility numeric	91-93		A	22-25
3.3	Bekker/LLD	94-97		A	30-33
3.4	WES-WNRE	98			
3.5	WNRE-CAL	99			
3.6	Perloff	100		A	40-43
4.0	Obstacle-Vehicle Models				
4.1	Ride dynamic models				
4.1.1	TACOM		18		
4.1.2	Chrysler		19		
4.1.3	GM (McKenzie)		20		
4.1.4	Ohio State		21		
4.1.5	WES-FMC		22		
4.1.6	CAL		25		
4.2	Slope models	181-184			
4.3	Obstacle-vehicle geometry, interference models				
4.3.1	WES, GM, Chrysler, Others	188			
4.3.2	LLD	189			

(Continued)

Table 18 (Concluded)

No.	Identification	References: Main Report Par. No.	Main Report Fig. No.	Appendix No.	Appendix Par.
4.3.3	WNRE	190			
4.4	Obstacle-traction models				
4.4.1	WES	192			
4.4.2	WNRE	193			
4.5	Obstacle override speed models				
4.5.1	WES	194			
4.6	Maneuver models				
4.6.1	WES	195-199			
4.7	Vegetation models				
4.7.1	Impact models				
4.7.1.1	WES	202-206			
4.7.2	Override speed models				
4.7.2.1	WES	207			
4.7.3	Traction model				
4.7.3.1	WES	208			
4.7.4	Visibility models				
4.7.4.1	WES	209			
5.0	Acceleration-Deceleration Models	211-213			
6.0	Stream Crossing Models				
6.1	Ingress models	221-222			
6.2	Water crossing models	218-220			
6.3	Egress models	223			
6.3.1	WES	224		B	26-33
6.3.2	LLD/Stevens Institute of Technology	225		B	13-25
6.3.3	WNRE	226		B	34-43
6.3.4	Chrysler				
7.0	Driver Decision (Except in Ride Dynamics)				

Table 19

Comprehensive Model Components

Key No.	Single-Feature Model	Comprehensive Model* 1	2	3	4	5	6	7	8	9	10
1.0	Soil	1.1,1.2	1.3	1.3	P	1.3	1.3	1.4	1.3	1.3	1.1,1.3
2.0	Vehicle	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1
3.0	Soil-Vehicle	3.1,3.2	3.3	3.3	P	3.3	3.3	3.6	3.3	3.3	3.4,3.5
4.1	Ride dynamics	4.1.5	4.1.1	4.1.1	4.1.6	4.1.1	4.1.3	4.1.4	4.1.1	4.1.2	X
4.2	Slope	4.2	4.2	4.2	4.2	4.2	4.2	1.4,4.2	4.2	4.2	4.2
4.3	Obstacle-vehicle geometry, interference	4.3.1	X	4.3.2	X	4.3.2	X	X	4.3.2	4.3.2	4.3.3
4.4	Obstacle traction	4.4.1	X	X	X	X	X	X	X	X	4.4.2
4.5	Obstacle override speed	4.5.1	**	X	X	X	X	X	X	X	X
4.6	Maneuver	4.6.1	**	X	X	X	4.6.1	X	X	X	4.6.1
4.7.1	Vegetation impact	4.7.1.1	X	X	X	X	X	X	X	X	X
4.7.2	Vegetation override speed	4.7.2.1	X	X	X	X	X	X	X	X	X
4.7.3	Vegetation traction	4.7.3.1	X	X	X	X	X	X	X	X	4.7.3.1
4.7.4	Visibility	4.7.4.1	**	X	X	X	X	X	X	X	X
5.0	Acceleration-Deceleration	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	X
6.1	Stream ingress	6.1	**	6.1	X	X	X	X	X	X	X
6.2	Water crossing	6.2	**	6.2	X	X	X	X	X	X	6.2
6.3	Stream egress	6.3.1	**	6.3.2	X	X	X	X	X	6.3.2	6.3.3
7.0	Driver Decision (except in ride dynamics)	X	X	X	P	X	P	X	X	X	X

Note: X, not considered; P, proposed but not verified.

* 1 designates WES; 2, TATAWS; 3, OMEGA; 4, CAL; 5, Booz-Allen; 6, GM; 7, Ohio State; 8, Bekker, 9, Chrysler; 10, WNRE.

** Effect on vehicle performance accounted for by assigning a constant speed or a fixed time penalty.

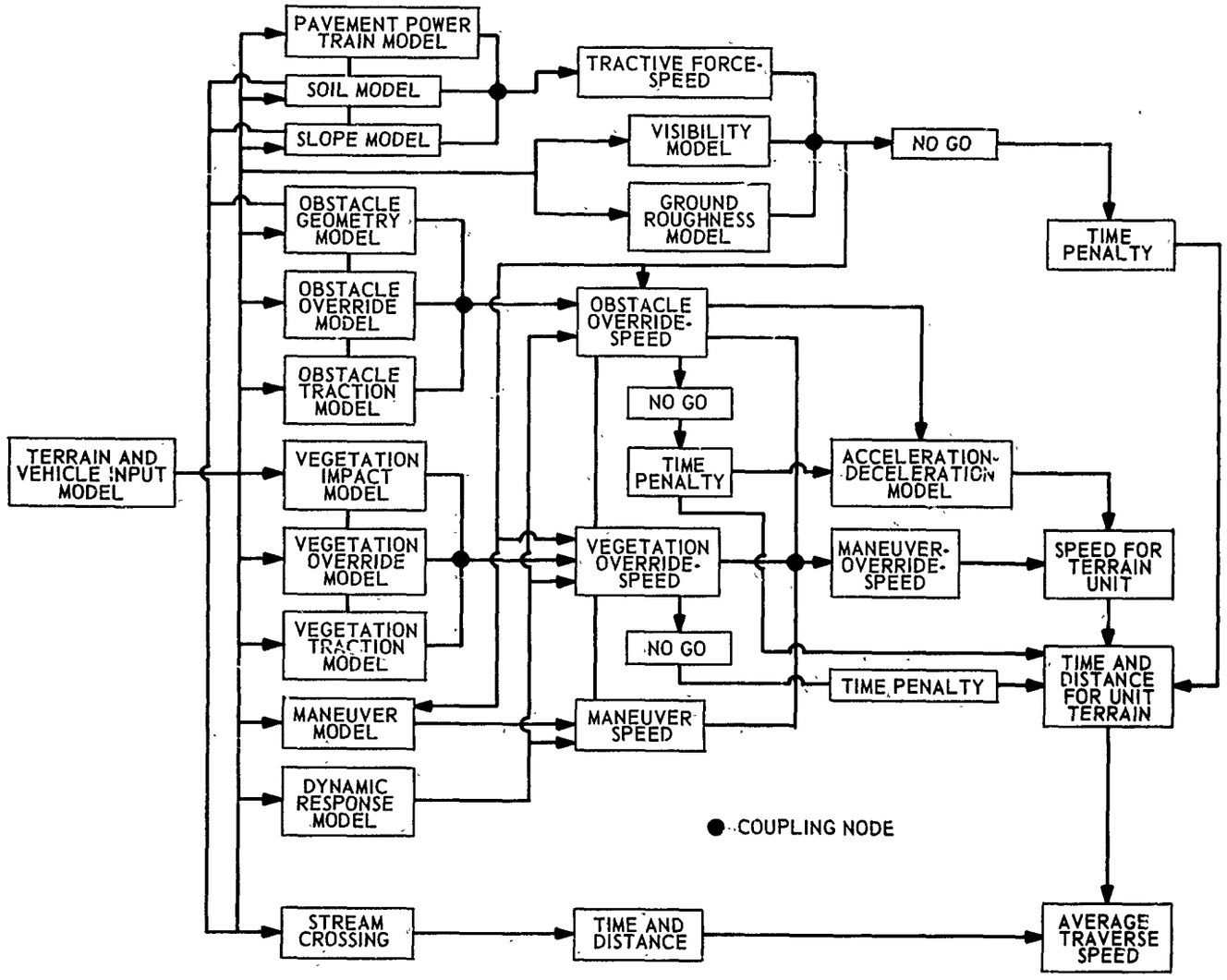


Fig. 57. Flow diagram showing coupling of single-feature models used in WES comprehensive model

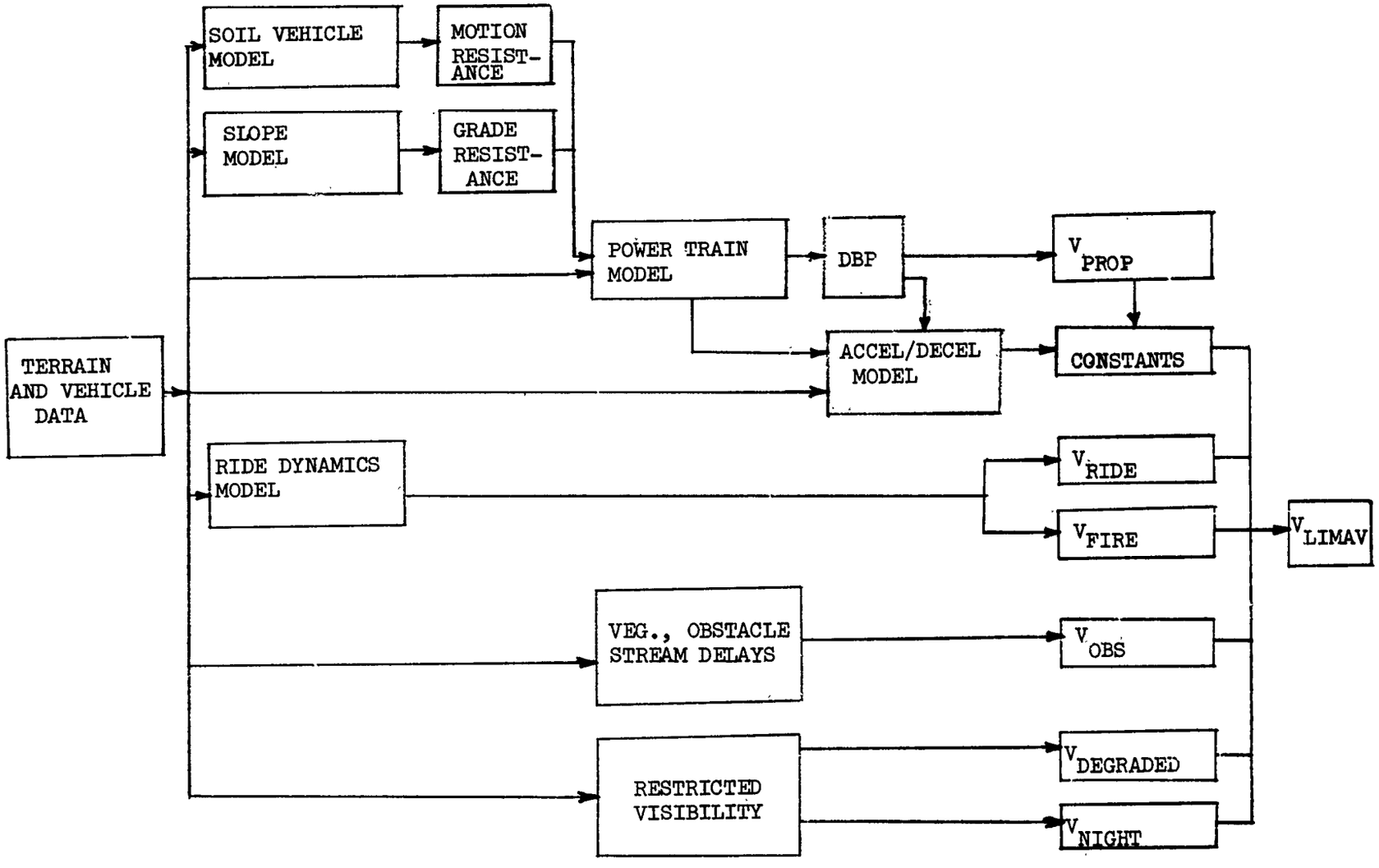


Fig. 58. Primary coupling of single-feature models and arbitrary speed restrictions and time assessments in the TATAWS comprehensive model

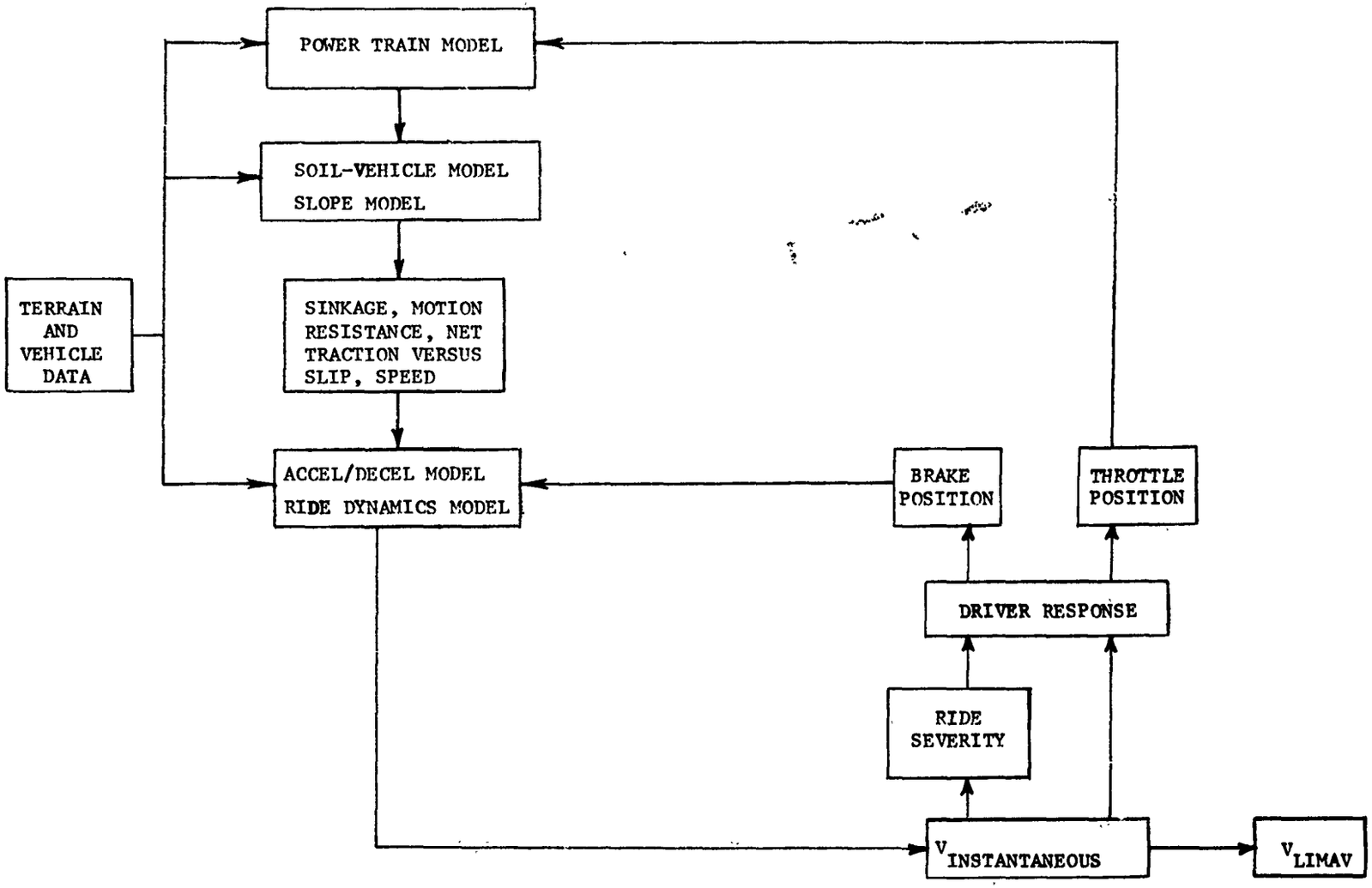


Fig. 59. Primary coupling of single-feature models, and feedback loops in OMEGA comprehensive model

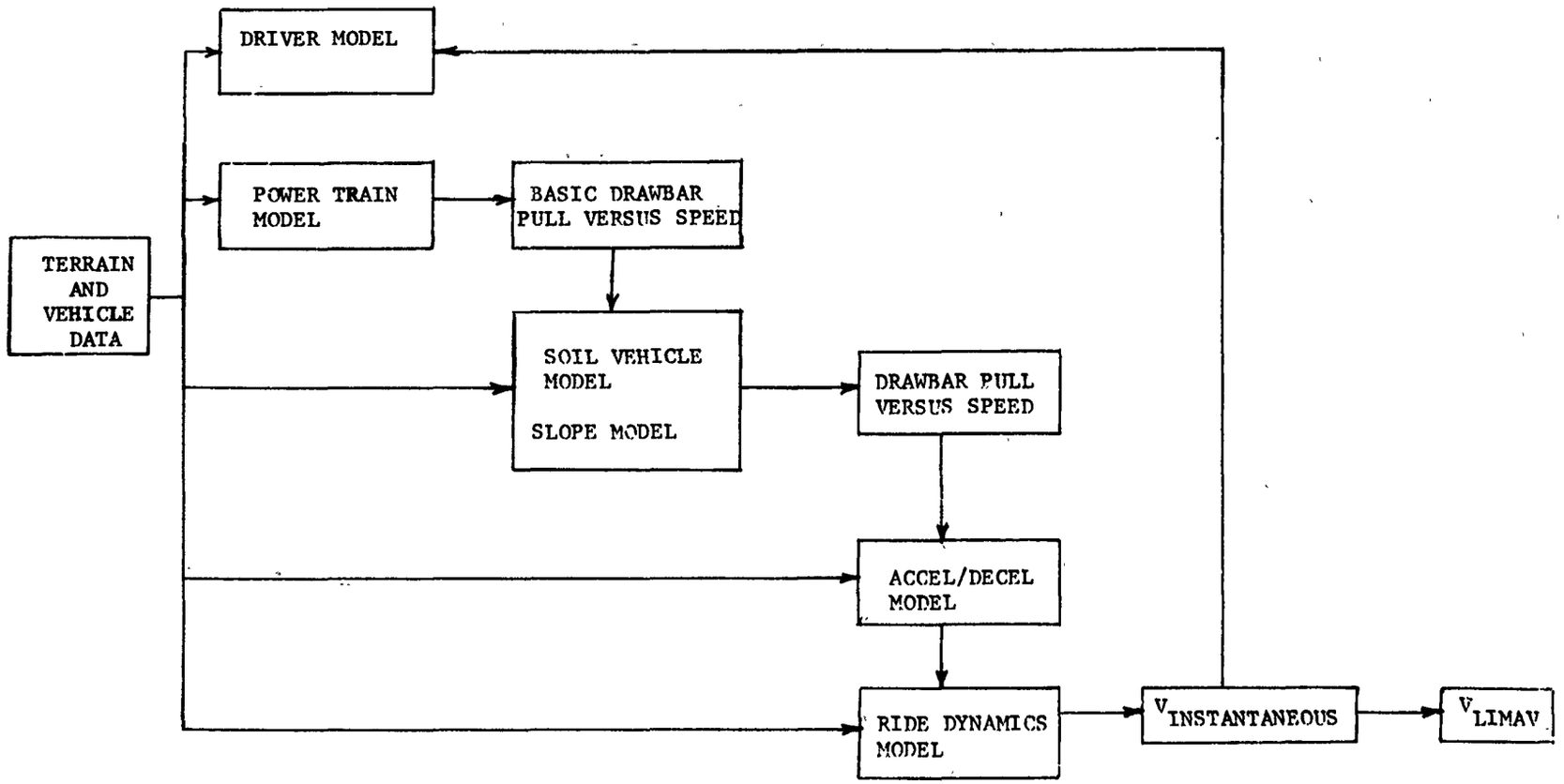


Fig. 60. Primary coupling of single-feature models, and feedback loops in CAL comprehensive model

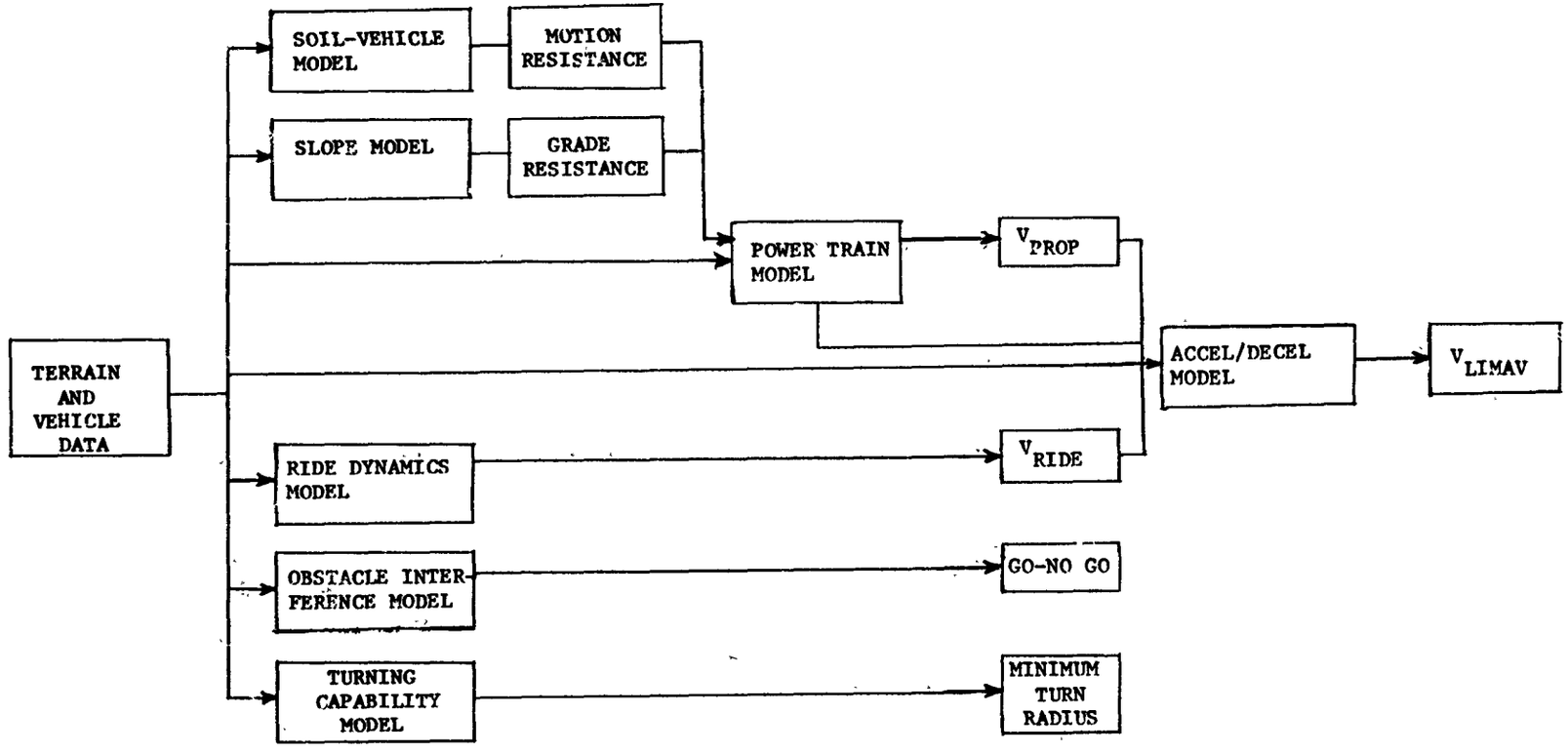


Fig. 61. Primary coupling of single-feature models used in Booz-Allen comprehensive model

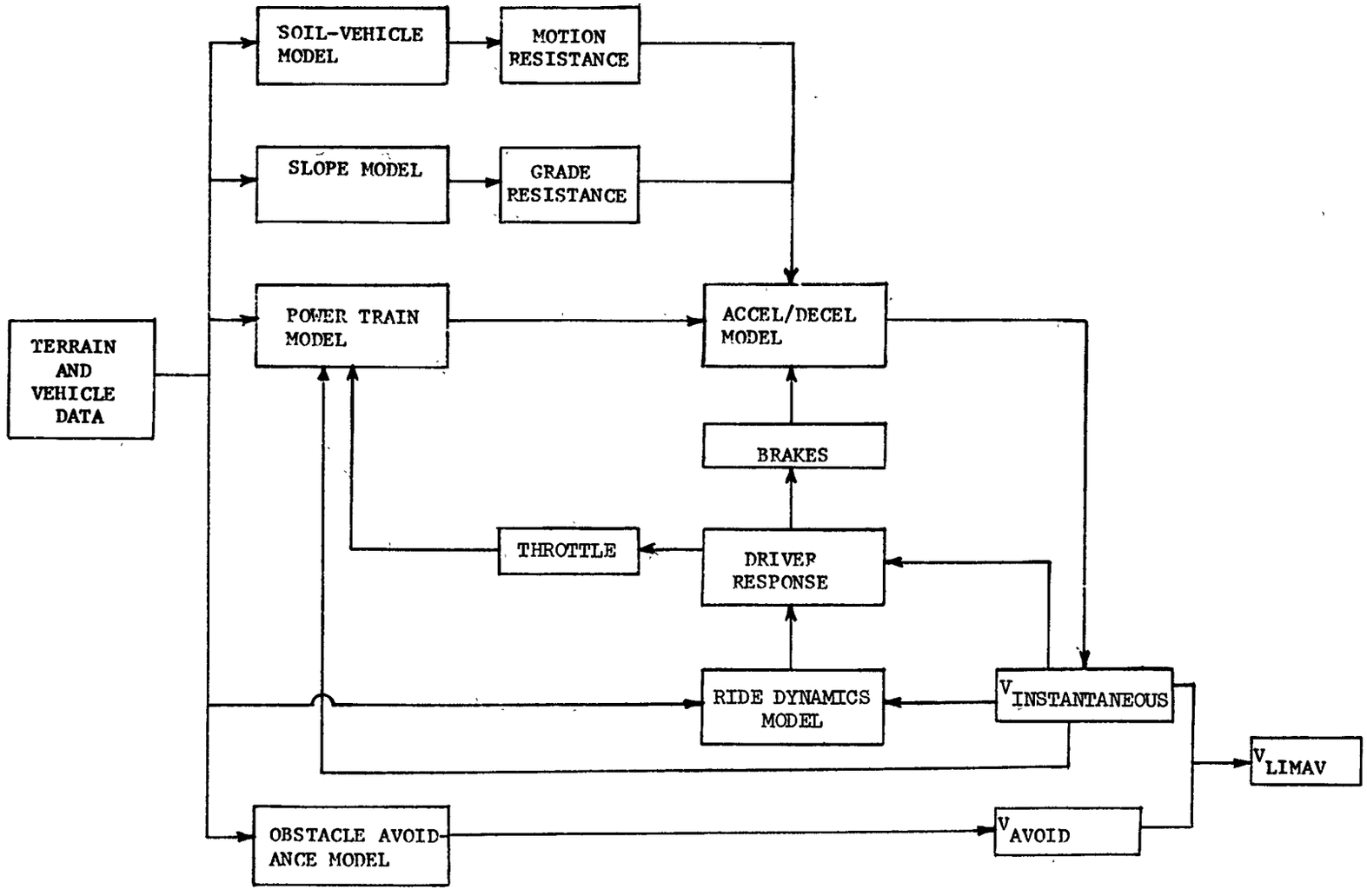


Fig. 62. Primary coupling of single-feature models, and feedback loops used in McKenzie comprehensive model

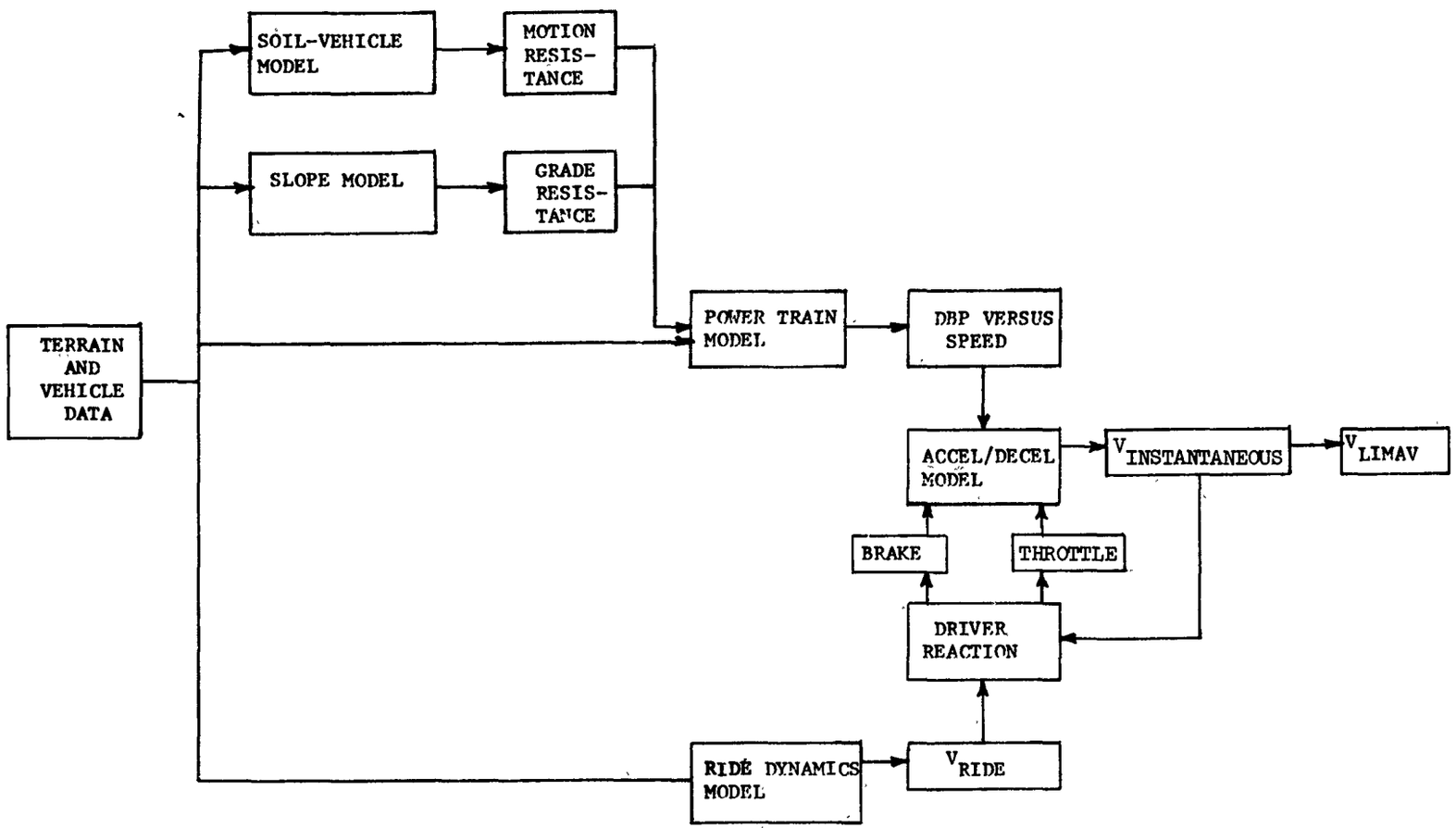


Fig. 63. Primary coupling of single-feature models in Ohio State University comprehensive model

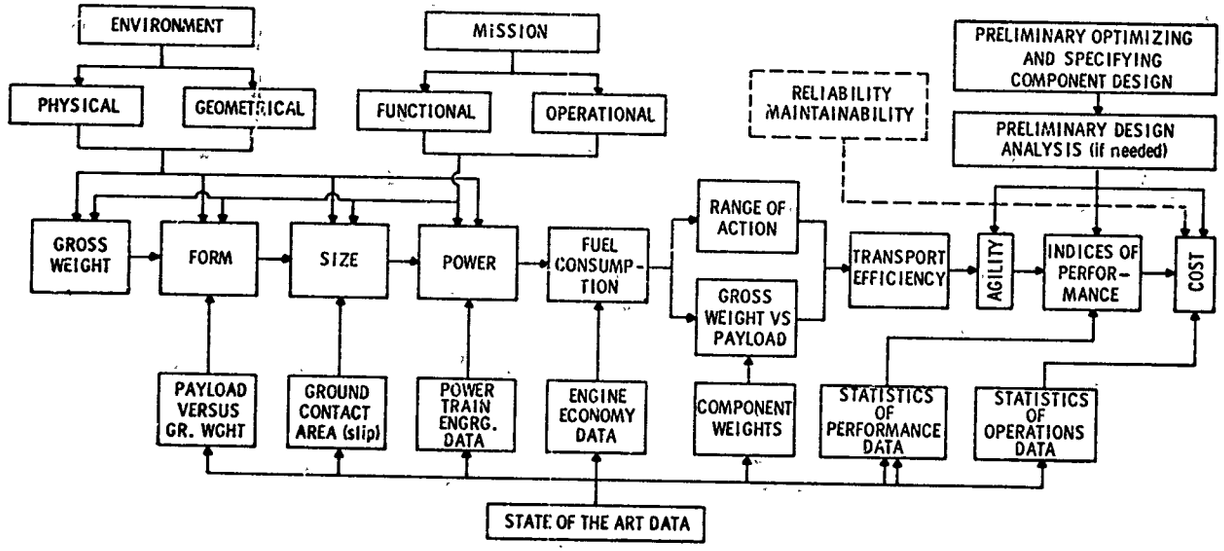


Fig. 64. Design optimization procedure outlined by Bekker incorporates many single-feature terrain-vehicle relations that could be readily used in a comprehensive cross-country vehicle performance model

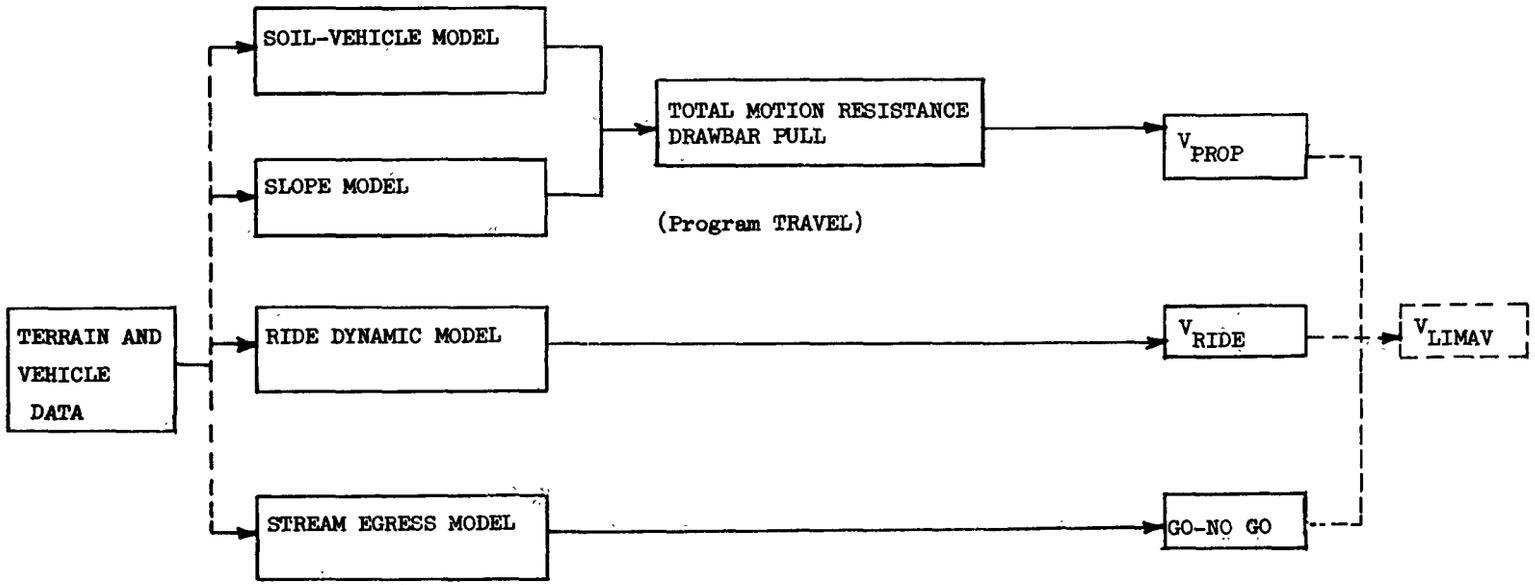


Fig. 65. Single-feature models and coupling in Chrysler comprehensive model. Dashed lines signify that model is not fully coupled, but could readily be coupled

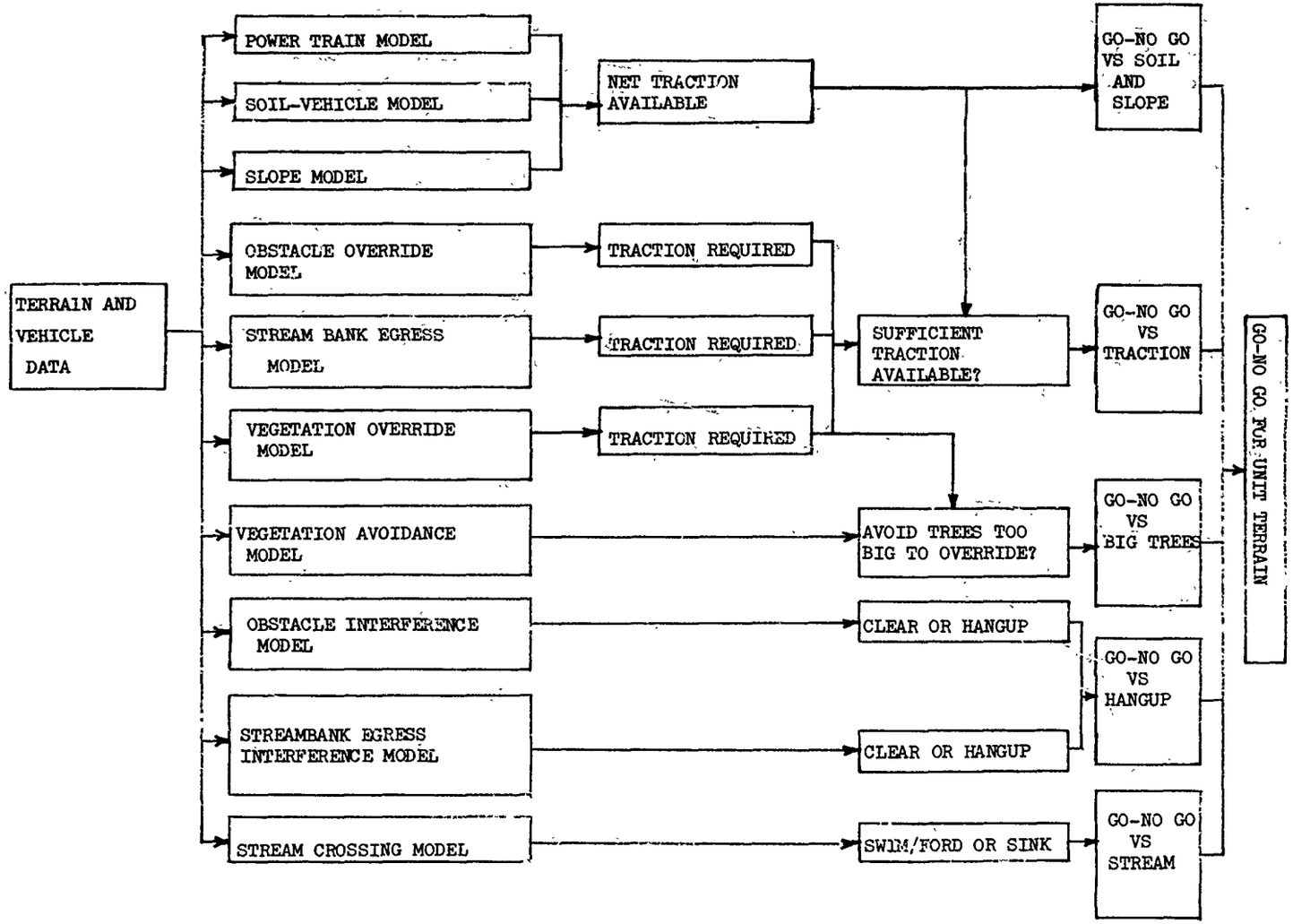


Fig. 66. Primary coupling and single-feature models used in WNRE go-no go comprehensive terrain-vehicle performance model

253. WES comprehensive model. The WES comprehensive cross-country performance model (fig. 57) was designed specifically as a flexible procedure to predict the performance of ground vehicles in any terrain configuration. Its primary prediction is average vehicle speed in a discrete terrain unit and in a traverse through a series of discrete units. Other related outputs are fuel consumption rate and cargo delivery rate.

254. The WES model was formed by coupling together single- and multiple-feature models schematically as shown in fig. 57. For an areal terrain unit, the model computes an average speed. When crossing a linear terrain feature, such as a stream or a dry wash, time and distance are computed. Whenever the computations indicate an immobilization (no go), an arbitrary time penalty is assessed. Aggregating the unit terrain computations to predict an average speed along selected (sometimes several alternative) traverses through a variegated terrain is made manually by accumulating transit times plus immobilization time penalties for consecutive path segments across mapped unit terrains or linear features.

255. The WES model will presently handle wheeled or tracked vehicles in all terrain except muskeg (organic soils). At present, terrain data suitable for direct use in the WES model are available in maps or on punched cards for significant areas in Thailand, Canada, Germany, and the United States.

256. At the time of this writing (1970) the WES model is approximately 80 percent programmed for a digital computer, but it is expected that this figure will be raised to about 95 percent within the next year. A more important limitation, however, is that it utilizes in several places measured vehicle performance data, generalized on a simple regression basis only. As a result, the WES model is not suitable to investigate the value of new designs that involve unusual mobility characteristics. By the same token, actual performance data on a particular vehicle in-being may readily be incorporated, so that the model is particularly suited to evaluating new machines in a broad range of terrain and mission situations, once a reasonable amount of relevant, reliable performance data become available.

257. The most fundamental deficiency of the WES model, which it shares with all others presently in operation, is its lack of anything but

the most crude modeling of driver influences. Because this leads to omniscient path selection, point-to-point average speeds predicted by it must be expected to be high.

258. Limited validation studies in which predicted travel times were subsequently compared with actual times of real vehicles in the same (but real) terrain have indicated that the overall accuracy of the model is reasonably good. Far more validation with a wider range of vehicle types in a broader array of terrain types is evidently required before the true reliability of the model can be assessed.

259. The WES comprehensive performance model has been used in several studies to evaluate the cross-country capabilities of a variety of vehicles. These studies are listed in table 20.

260. TATAWS (III) model. The TATAWS comprehensive mobility model (fig. 58) was designed as part of an ad hoc tank combat simulation to predict the speed of tanks in selected terrains. It was formed by coupling several of the single-feature models as shown in fig. 58. Specific single-feature models included are identified in tables 18 and 19.

261. The TATAWS model determines limiting vehicle speed in a discrete unit terrain as the minimum of the following:

a. Speed as limited by propulsion, V_{prop}.

b. Speed as limited by weapon firing, V_{fire}.

c. Speed as limited by ride, V_{ride}.

d. Speed as limited by generally degraded visibility, $V_{degraded}$.

e. Speed as limited by night visibility problems, V_{night}.

V_{ride} and V_{fire} are determined separately by the TACOM ride dynamic single-feature model and form part of the input data for the main program. $V_{degraded}$, V_{night}, and obstacle crossing time delays are arbitrary restrictions imposed on vehicle performance to make some allowance for missing single-feature models. Average speed for a traverse or a segment of a traverse is determined using the limiting speed for individual traverse segments adjusted for acceleration and deceleration, plus time delays caused by obstacles or mission requirements.

262. The computed average speeds for each traverse segment and for

Table 20

Comprehensive Models Utilized for Design or Evaluation of Military Vehicles Through 1970

Comprehensive Model	References
WES	1. Grabau, W. E., Stoll, J. K. and Stinson, B. G., "A Plan for Quantitative Evaluation of the Cross-Country Performance of Prototype Vehicles," Miscellaneous Paper M-70-7, Sept 1970, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Mississippi.
	2. Stoll, J. K., Randolph, D. D., and Rula, A. A., "Relative Off-Road Mobility Performance of Six Wheeled and Four Tracked Vehicles in Selected Terrain," Technical Report No. M-70-4, Mar 1970, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Mississippi.
	3. Randolph, D. D., "Evaluation of the Relative Off-Road Performance of 15 Vehicles in Synthalogous Theaters of Operation (STOP) Terrain Factor Complexes," Technical Report No. M-70-7, May 1970, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Mississippi.
	4. Hutto, T. D., Decell, J. L., and Rula, A. A., "Limited Study of Effects of Vegetation Characteristics on Performance of Selected Self-Propelled Vehicles," Miscellaneous Paper M-69-5, Report 2, Oct 1969, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Mississippi.
	5. Stinson, B. G., "Evaluation of WES Analytical Model in Selected Terrains (XM559E1 GOER Tests at Camp Gagetown, New Brunswick, Canada)," Technical Report No. M-70-3, Mar 1970, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Mississippi.
TATAWS	U. S. Army Combat Developments Command, "Tank, Antitank and Assault Weapons Requirements Study (U)," Dec 1968 (Final Draft SECRET Report).
Booz-Allen	Booz-Allen Applied Research Inc., "Armored Reconnaissance Scout Vehicle (XM800) Parametric Design/Cost Effectiveness Study (U),"
Bekker	Bekker, M. G., "Engineering Optimization of Terrain-Vehicle Systems, A Case Study of a Concept Analysis of a New Amphibian," Technical Report No. 10421, Feb 1969, U. S. Army Tank-Automotive Command, Warren, Michigan.

completed traverses are optimum speeds. The model would be improved if a driver model could be added to simulate the ability of the driver to interpret the terrain and adjust vehicle speed and course accordingly.

263. There are no restrictions upon terrain types in which the TATAWS model can be used, but its employment of arbitrary vehicle performance restrictions in place of some single-feature models is clearly undesirable. The model is of moderate complexity and requires the use of both analog and digital computers. Storage required is dependent upon the number of discrete terrain segments for which predictions are to be made.

264. No part of the TATAWS comprehensive model has been validated and its use has been limited to studies of one class of tanks plus related combat support vehicles (also on tracks). The present terrain data bank for the TATAWS model includes selected terrain from three areas in Germany.

265. OMEGA model. The OMEGA comprehensive model is designed to predict the performance of any given wheeled or tracked vehicle configuration in a variety of simplified environmental conditions. It consists of several submodels, each of which can be operated individually. Fig. 59 outlines the overall model.

266. Limiting controllable speed, V_{ride}, is determined by the vehicle ride and severity submodels and maximum limiting propulsion speed, V_{prop}, by engine and transmission characteristics (power train single-feature model), soil-vehicle interactions (soil and slope single-feature models), and the vehicle acceleration and deceleration single-feature model. Driver response (not to be confused with proper driver modeling) establishes a desired vehicle velocity and compares this with V_{ride} and V_{prop} and sets throttle or brake positions according to the sensed error.

267. Terrain data input for the OMEGA model includes ground profile, slope, and soils data for successive segments along a selected route. The primary vehicle performance output is instantaneous speed, which may be accumulated to derive V_{limav}

268. The OMEGA comprehensive model is limited in that, due to omission of appropriate single-feature models, it cannot predict performance in terrains containing significant vegetation, streams, degraded visibility, or large vertical obstacles. The model has not been formally assembled as

a single computer program at the present time. However, TACOM has operational a vehicle dynamic model, an automotive model, and a soft-soil model, which together contain most of the submodels required.

269. Coupling of the single-feature models in the OMEGA model follows the pattern utilized by most models which output average vehicle speed and is considered to be realistic. Accuracy of the model's performance predictions cannot be commented on since the model has had no validation.

270. CAL model. The CAL comprehensive model is outlined in fig. 60. Its primary output is wheeled or tracked vehicle speed along a specific traverse, but the model was planned in such fashion that vehicle behavior could be modeled and studied almost to the component level. This level of detail was not achieved at the time work on this model was halted in 1969. The model has not been used in any design evaluation studies.

271. This CAL model was designed to combine the advantages of rapid analog computations of vehicle dynamics with versatile digital representation of terrain. A digital model using the same equations as the analog model was programmed for backup but requires much more running time.

272. This model has many missing single-feature models and therefore cannot be used to predict performance in terrain containing significant vegetation, degraded visibility, large obstacles, or streams.

273. Booz-Allen model. The Booz-Allen comprehensive model predicts vehicle average speed over highways or soft soil. The basic speed model is coupled from a group of single factor models and features a single-feature soils model that differentiates between single-unit and articulated vehicles. Turning capability and obstacle negotiation capability are determined from single-feature models applied independently. A simplified flow diagram showing the coupling of the single-feature models in the speed mode is shown in fig. 61. The Booz-Allen model was used in the comparative evaluation of a number of designs for scout vehicles (see table 20). Its principal limitations are (a) that it has no single-feature model to determine the effects of vegetation, visibility, or shock due to crossing discrete vertical obstacles at some speed and (b) that its output is not a single performance value. The model also has a limited real terrain data bank.

274. McKenzie model (General Motors). This comprehensive model

considers the combined effects on vehicle performance of terrain roughness, soil traction-slip characteristics, vegetation override and avoidance, and vehicle power. It includes a simple driver response time element similar to that in the OMEGA model. The primary output of the McKenzie model is vehicle speed. Single-feature models incorporated and their coupling are shown in fig. 62. The obstacle avoidance single-feature model determines the minimum nearest neighbor spacing among large trees for a go situation.

275. This model does not include obstacle geometry, obstacle traction, stream entrance, stream crossing, stream egress, or visibility single-feature models. In addition, the vegetation avoidance and override models it uses are not adequate for determining the optimum speed due to override and maneuver. Accordingly, terrain types containing vegetation, streams, degraded visibility conditions, and/or large obstacles cannot be adequately handled.

276. The McKenzie model has not been validated or used in any previous studies. The data bank for the model presently consists of three sets of terrain and soil conditions chosen to approximate the gross spectrum of environmental features found in Southeast Asia.

277. Ohio State University. This comprehensive model was developed to predict the average speed of tracked vehicles in discrete unit terrains and for traverses made up of segments of rough terrain, soft soils, secondary roads, and/or highways. A basic flow diagram identifying the single-feature models used and their coupling is shown in fig. 63. Note that the simple driver response time element feeds back both throttle and brake signals directly to the acceleration-deceleration model. The Ohio State University model has not been used in any actual vehicle evaluation studies and has not been validated. The model can handle tracked vehicles only, and cannot be used for terrain types containing significant vegetation, streams, large vertical obstacles, or conditions of degraded visibility due to omission of appropriate single-feature models.

278. Bekker design optimization model. Bekker has outlined a formal design optimization procedure using state-of-the-art data to approximate performance and cost of feasible wheeled or tracked vehicle concepts that will lead to cost-effective mission accomplishment. Vehicle concepts are

defined basically in terms of form-size-weight-power relations. Fig. 64 diagrams the main coupling in the Bekker procedure. Bekker's "state-of-the-art data" include soils, obstacle geometry, power train, and ride dynamic single-feature models along with other available important terrain-vehicle data. The effects of vegetation or stream crossing are not considered.

279. Chrysler model. The Chrysler comprehensive model for predicting vehicle performance uses single-feature models both coupled and applied independently. The program with the most coupling, designated "TRAVEL," computes soil-vehicle interaction primarily according to the Bekker/LLD soil-vehicle system. Some relations for predicting wheel sinkage proposed by Schuring (CAL) are also utilized. Input to the Chrysler model consists of engine torque-speed characteristics, vehicle geometry, center of gravity position, and an optional towing resistance curve and a mission profile that is characterized by a distribution of slopes and another of the Bekker/LLD soil parameters.

280. Within the model, curves of available torque versus slip and of required torque versus slip are generated as a function of the given soil parameters. Superimposed, these generate slip curves as a function of speed and drawbar pull. Resistances computed include bulldozing and compaction resistances of the soil, air resistance, track or wheel losses as towing resistances, and transmission and drive-train efficiencies.

281. The output of the Chrysler model is vehicle sinkage, ground pressure, maximum slope capability as a function of speed, acceleration time on a level surface, maximum-drawbar-pull-to-weight ratio as a function of speed, and total time and average speed to complete a mission involving the given distribution of slopes and soils.

282. TRAVEL is a sophisticated model for determining the effects of soil and slope on vehicle performance, but missing single-feature models for predicting effects of vegetation, vertical obstacles, visibility, and streams prevent it from being applicable to many terrains. Inclusion of Chrysler's "stream egress" and "ride dynamic" single-feature models would increase TRAVEL'S applicability. No validation or evaluation studies have been conducted with this model. Fig. 65 diagrams the single-feature

models and coupling in the Chrysler comprehensive model.

283. WNRE model. The WNRE comprehensive model predicts go-no go vehicle performance only and contains no dynamic simulation. The model uses soil, slope, power train, vegetation override and avoidance, vertical obstacle override and interference, stream crossing, and streambank egress and interference single-feature models. These single-feature models are identified and their couplings shown in fig. 66.

284. Since this model predicts only go-no go, it has a relatively simple structure. Lack of all dynamics limits the usefulness of this model to the study of large areas for such purposes as vehicle oriented terrain analysis. Figs. 67 and 68, which illustrate the range of terrain-vehicle configurations examined in this simple static model only begin to suggest the complexity of a realist model for general purpose uses.

The NOW Model

285. The diagrams of the several comprehensive models presented in figs. 57-66 reveal the considerable similarity in structure among all of the models (except that of Bekker's whose objective is somewhat different). This not unexpected broad consensus is useful at the present juncture.

286. The primary variation in structure is in the completeness with which terrain is represented, reflected in the number of single-feature models employed and in the selection of possible couplings among them. This diversity is for the most part deliberate. In the several ad hoc models this arises either from the limits of the problem for which the model was constructed and/or lack of available single-feature models at the time the model was constructed. Clearly, the most complete comprehensive model is that put together by WES, which has had the benefit of almost immediate results from on-going research.

287. All of the models (except the WNRE go-no go model) consider ride dynamics as an element of the problem, but two distinct approaches are taken to develop a ride-limited speed. In one, vehicle responses for an entire homogeneous terrain segment are accumulated into one or another statistic describing the mean vibration environment before applying limiting

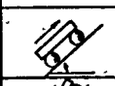
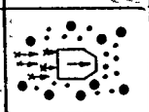
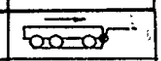
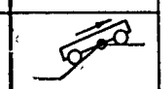
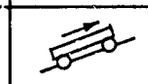
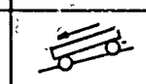
TEST (AND FAILURE) CODE	TEST DIRECTION	DIAGNOSTIC COMPARISON	BASIC TERRAIN FACTORS EXAMINED				MEANING WHEN FAILURE (CODE) IS INDICATED	SCHEMATIC REPRESENTATION
			Basic Slope	Soil Strength	Vege-tation	Obstacle Geom.		
A	Uphill	Basic slope vs. torque-limited gradeability.	X	-	-	-	Vehicle low-gear torque limited gradeability is insufficient to surmount basic grade.	
	Downhill*	Basic slope vs. downhill control stability.	X	-	-	-	Slope is greater than 100%.	
B	Uphill	Basic soil strength vs. VCI.	-	X	-	-	Basic soil strength is inadequate for operation (including maneuver) in smooth level terrain.	
	Downhill*	No test.*						
C	Uphill	Spacing of large trees vs. vehicle maneuver requirement.	-	-	X	-	Single trees too large to push over, based on vehicle low-gear torque, are too close together.	
	Downhill	No test.						
D	Uphill	Spacing of small trees vs. vehicle torque-limited thrust.	-	-	X	-	Trees small enough to be overridden singly, based on low-gear torque, are so closely spaced that vehicle thrust is nonetheless inadequate.	
	Downhill	No test.						
E	Uphill	Vertical obstacle height vs. basic vehicle capability.	-	-	-	X	Vertical obstacle height is greater than basic vehicle limit. (Vertical obstacles = α >75°)	
	Downhill	No test.						
F	Uphill	Vehicle angle of approach vs. obstacle angle.	-	-	-	X	Vehicle nose interferes with obstacle before running gear can make contact.	
	Downhill	No test.						
G	Uphill	Male obstacle configuration vs. belly clearance.	-	-	-	X	Vehicle bellies on male obstacle (side elevation only -- no test for normal tracked vehicles).	
	Downhill	No test.						
H	Uphill	Vehicle belly clearance vs. ditch edge.	-	-	-	X	Vehicle bellies on ditch edge (side elevation only -- no test for normal tracked vehicles).	
	Downhill	No test.						
I	UphilT	Dummy test.	-	-	-	X	Intended to check for obstacle spacing and dimensions which might trap a vehicle, but no terrain dimensions of this order were present in the study region so test was not detailed.	
	Downhill	No test.						
J	Uphill	Soil-limited traction** vs. grade.	X	X	-	-	Insufficient net traction developed at soil-vehicle interface** to negotiate basic grade.	
	Downhill	Soil-limited traction** vs. grade.	X	X	-	-	Traction plus grade assist insufficient for forward progress down the slope.	
K	Uphill	Spacing of large trees vs. vehicle maneuver requirement.	X	X	X	-	Single trees too large to push over, based on soil-limited vehicle traction less grade load, are too close together.	As test "C," but operating upgrade, under soil strength constraints.

Fig. 67. Structure of WNRE go-no go model, illustrating types of terrain-vehicle configurations tested and iterative test approach used to develop diagnostic keys for immobilizations (sheet 1 of 2)

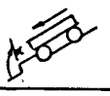
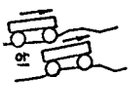
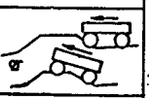
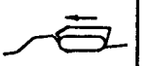
TEST (AND FAILURE) CODE	TEST DIRECTION	DIAGNOSTIC COMPARISON	BASIC TERRAIN FACTORS EXAMINED				MEANING WHEN FAILURE (CODE) IS INDICATED	SCHEMATIC REPRESENTATION
			Basic Slope	Soil Strength	Vegetation	Obstacle Geom.		
	Downhill	Spacing of large trees vs. vehicle maneuver requirement.	X	X	X	-	Single trees too large to push over, based on soil-limited vehicle traction aided by the grade, are too close together.	As test "C," but operating downgrade, under soil strength constraints.
L	Uphill	Spacing of small trees vs. vehicle soil and grade limited thrust.	X	X	X	-	Trees small enough to be overridden singly, based on soil-limited vehicle traction less grade load, are so closely spaced that vehicle thrust is nonetheless inadequate.	As test "D," but operating upgrade, under soil strength constraints.
	Downhill	Spacing of small trees vs. vehicle soil and grade limited thrust.	X	X	X	-	Trees small enough to be overridden singly, based on soil-limited vehicle traction aided by the grade, are so closely spaced that vehicle thrust is nonetheless inadequate.	As test "D," but operating downgrade, under soil strength constraints.
TESTS M_V, N_V, AND O_V FOR WHEELED VEHICLES ONLY								
M_V	Uphill	Vertical obstacle height vs. vehicle capability revised to reflect soil strength and grade effects on traction.	X	X	-	X	Vertical obstacle height is greater than vehicle limit as determined by soil-limited vehicle traction less grade load (Vertical obstacles = α >75°)	
	Downhill	Vertical obstacle height vs. vehicle capability revised to reflect soil strength and grade effects on traction.	X	X	-	X	Vertical obstacle height is greater than vehicle limit as determined by soil-limited vehicle traction aided by grade. (α >75°)	
N_V	Uphill	Non-vertical obstacle traction requirement vs. soil and grade limited vehicle traction.	X	X	-	X	Available soil-limited traction, reduced by the grade load, is insufficient for obstacle negotiation.	
	Downhill	Non-vertical obstacle traction requirement vs. soil and grade limited vehicle traction.	X	X	-	X	Available soil-limited traction, aided by the grade, is insufficient for obstacle negotiation.	
O_V	Uphill	Combined grade and obstacle face slope vs. soil-limited vehicle traction.	X	X	-	X	The obstacle face is long enough to accommodate the vehicle, and the combined slope of the obstacle face and the base grade requires more traction than available.	
	Downhill	Combined grade and obstacle face slope vs. soil-limited vehicle traction.	X	X	-	X	The obstacle face is long enough to accommodate the vehicle, and the combined slope of the obstacle face and the base grade requires more traction than available.	
TESTS M_T AND N_T FOR TRACKED VEHICLES ONLY								
M_T	Uphill	Initial obstacle traction requirement vs. soil and grade limited vehicle traction.	X	X	-	X	Available soil-limited traction, reduced by the grade load, is insufficient to begin obstacle negotiation.	
	Downhill	Initial obstacle traction requirement vs. soil and grade limited vehicle traction.	X	X	-	X	Available soil-limited traction, aided by the grade, is insufficient to begin obstacle negotiation.	
N_T	Uphill	Final obstacle traction requirement vs. soil and grade limited vehicle traction.	X	X	-	X	Available soil-limited traction, reduced by the grade load, is insufficient to complete obstacle negotiation.	
	Downhill	Final obstacle traction requirement vs. soil and grade limited vehicle traction.	X	X	-	X	Available soil-limited traction, aided by the grade, is insufficient to complete obstacle negotiation.	

Fig. 67 (sheet 2 of 2)

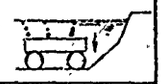
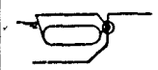
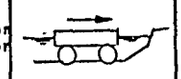
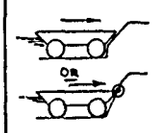
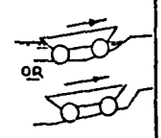
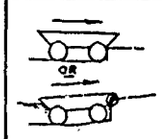
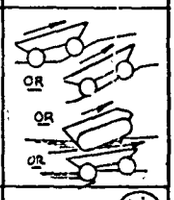
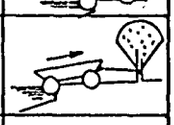
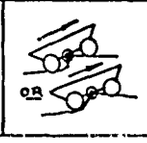
TEST (AND FAILURE) CODE	DIAGNOSTIC COMPARISON	BASIC TERRAIN FACTORS EXAMINED				MEANING WHEN FAILURE (CODE) IS INDICATED	SCHEMATIC REPRESENTATION
		Water Depth	Soil Strength	Lower Bank Geom.	Upper Bank Geom.		
R	Water depth vs. vehicle fording ability (non-swimming vehicles only).	X	-	-	-	Water depth exceeds vehicle fording capability.	
S	Upper bank step vs. vehicle floating attitude (swimming vehicles only).	X	-	-	X	Vehicle running gear cannot make useful contact with the bank.	
T	Basic soil strength vs. VCI.	X	X	-	X	Basic soil strength is inadequate for simple operation on smooth level ground (with or without buoyancy effects, as appropriate).	
U	Lower bank geometry vs. vehicle configuration	-	-	X	X	Lower bank presents either a vertical obstacle beyond basic vehicle capability, or there is angle of approach interference.	
V	Lower bank configuration vs. net vehicle traction.	X	X	X	-	Traction required to surmount lower bank exceeds soil-limited traction available (including buoyancy effects where present).	
W	Upper bank geometry vs. vehicle configuration.	-	-	X	X	Upper bank presents either a vertical obstacle beyond basic vehicle capability, or there is angle of approach interference.	
X	Upper bank configuration vs. net vehicle traction.	X	X	X	X	Traction required to surmount upper bank exceeds soil-limited traction available (including buoyancy effects where present).	
Y	Upper bank configuration vs. net traction plus 50% GVW outside thrust assist.	X	X	X	X	Deficit in required traction is more than 50% of gross vehicle weight.	
Z	Upper bank configuration vs. vehicle belly clearance.	-	-	X	X	Vehicle bellies on ditch edge (side elevation only -- no test for normal tracked vehicles).	

Fig. 68. Structure of WNRE go-no go model for linear terrain feature (streams and dry washes)

criteria based upon driver, cargo, and/or weapon serving tolerances. In the other, vibration based upon instantaneous speeds is continually monitored by a tolerance criterion, and driver corrections are continuously fed back in a loop to alter vehicle speed in response, usually through the power train and/or acceleration/deceleration model. Driver reaction time is usually incorporated in this feedback loop. This is probably an unwarranted refinement at this time but it does serve to remind all that more complete driver modeling is required.

288. At this stage, it is likely that most available, realistic terrain roughness data will be in terms of estimates of power spectral densities. Accordingly, a statistical input/output treatment appears most suitable even when the transfer operation involves the intermediate generation of discrete input and output rather than a direct statistic-to-statistic transfer function. In the present absence of such a transfer function, this obviously affects the manner in which the comprehensive models interpret the ride dynamic data generated, rather than the ride dynamic model itself.

289. Associated with this general difference in philosophy on ride is a somewhat similar difference in the handling of soil-vehicle and power train model interrelations. In some models, the relations are converted to a pull-slip-speed relation for the particular soil and slope in a unit terrain before proceeding to other parts of the program. In others, the program continuously generates the information on a moment-by-moment basis. The latter approach is appropriate where instantaneous speed values are needed. In cases where there are a number of individual speed or go-no go tests run before selecting a single controlling speed, as in the WES comprehensive model, the drawbar-slip-speed amalgamation appears more suitable.

290. While the WES model is the only one that specifically includes a speed limit while crossing a discrete obstacle, such as a rice paddy dike, any of the other ride dynamic models could generate similar speed versus acceleration information upon which selection of a possible maximum crossing speed could be based.

291. At present, obstacle avoidance speeds, where used at all, are based essentially upon a static viewpoint incorporated in regression

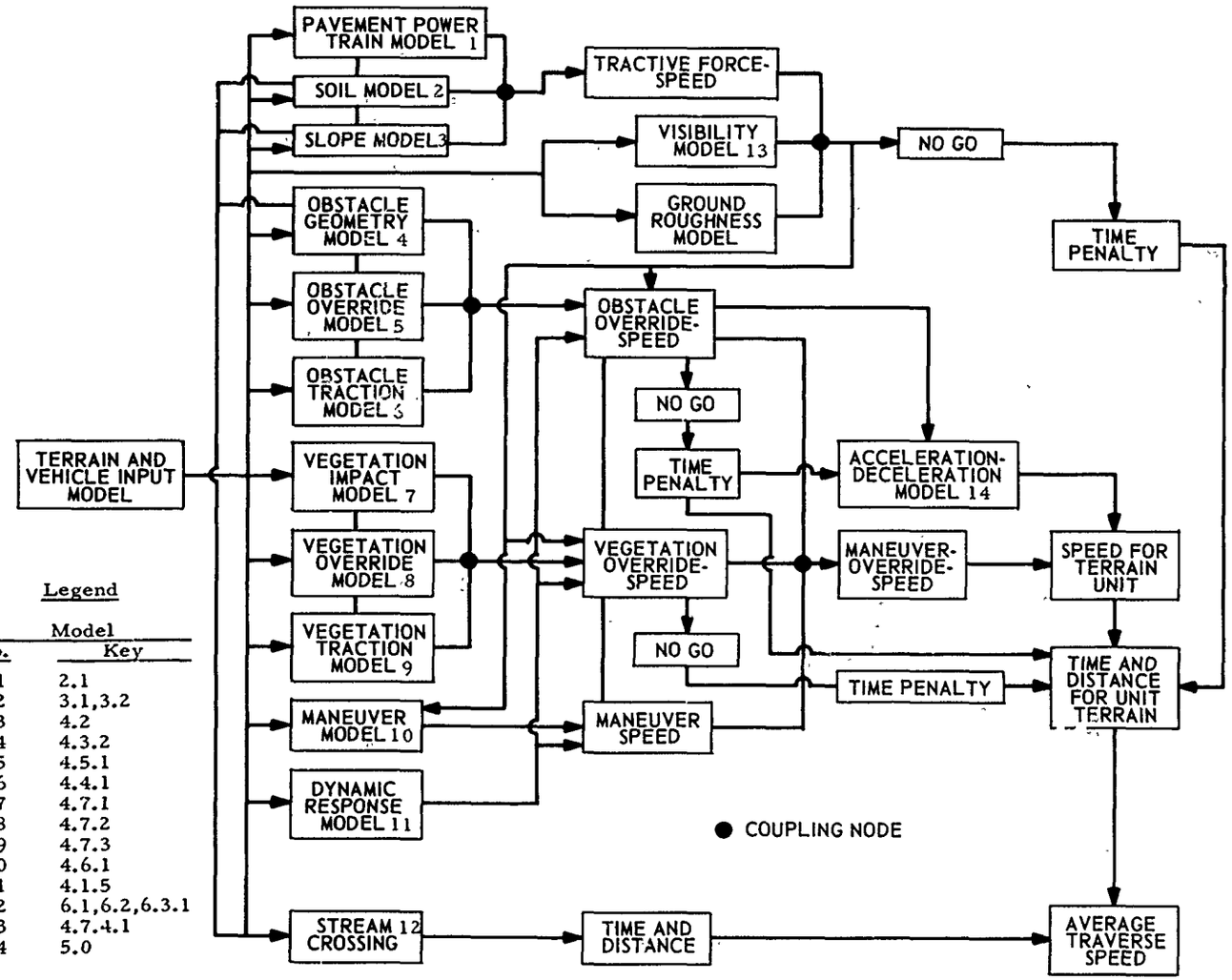
equations developed from limited test results. The present problem is one of closely spaced insurmountable obstacles, such as large trees, and there is no specific modeling of driver's route selection in real terrain. None of the models attempt to account for driver route selection on any larger scale.

292. In most areas of the world, stream crossing is a major impediment to off-road vehicle movement. Despite this, only four of the ten comprehensive models looked at included any modeling of this critical operation. Even these use modeling which is suspect, unrealistic, and/or totally unvalidated. The consensus is that the bank egress problem is the most critical for actual vehicles and is the most difficult to model. The WES approach treats the streambank as a simple soil-slope problem. The WNRE static model considers the bank as a complex, two-dimensional obstacle upon which a vehicle may fail through geometric interference or lack of sufficient traction at any one of several critical encounter points. A first-order attempt to approximate transient contact forces during the time when the vehicle is partially buoyant is included. In the Davidson Laboratory model, which is currently being elaborated by the University of Detroit under a TACOM/LLD contract, vehicle momentum and buoyancy effects upon ground contact pressure (and hence ground reaction) are being incorporated, but hydrodynamic wave forces will still be missing. However, this appears to be an area where available or near available modeling from several sources could be immediately combined to provide a substantial improvement.

Selection of NOW model

293. The proposed NOW model is essentially the WES comprehensive model as shown in fig. 69, with some suggested additions and changes in the specific submodels that should be used. The numbers indicated in the blocks in fig. 69 are the submodels suggested according to the listing given in table 21.

294. The WES comprehensive model is proposed primarily because it already makes provisions for the maximum array of single-feature and multiple-feature terrain situations, which most of the comprehensive models examined cannot handle at the present time without the expenditure of



Legend

Model No.	Key
1	2.1
2	3.1,3.2
3	4.2
4	4.3.2
5	4.5.1
6	4.4.1
7	4.7.1
8	4.7.2
9	4.7.3
10	4.6.1
11	4.1.5
12	6.1,6.2,6.3.1
13	4.7.4.1
14	5.0

Fig. 69. Suggested primary couplings of single-feature models for the NOW comprehensive model

Table 21

Single Feature Models for NOW Comprehensive Model

Model	Model Identification (See Table 19)
1. Soil-vehicle - for evaluation of vehicle where adequate performance data are available for all	3.1,3.2
2. - for new design studies	3.3
3. Slope	4.2
4. Obstacle interference	4.3.2,4.3.3*
5. Obstacle override-avoidance	4.4.1,4.4.2 4.5.1 4.6.1
6. Vegetation override-avoidance	4.7.1.1 4.7.2.1 4.7.3.1 4.6.1
7. Visibility	4.7.4
8. Ride dynamics - continuous roughness	4.1**
9. - discrete obstacles	4.1**
10. Power train	2.1
11. Acceleration/deceleration	5.0
12. Stream - entrance	6.1
13. - crossing	6.2
14. - egress	6.3.1,6.3.2,6.3.3*

* Merge.

** As suitable for form of input data and vehicle type.

considerable effort. Furthermore, WES terrain-vehicle submodels accounting for the effects of soil, vegetation, and discrete obstacles were developed from field test data, which makes them more realistic than if they were derived from a purely theoretical standpoint. Preliminary pilot field tests also indicate that predictions made with the WES comprehensive model are reasonably accurate. Another factor that weighed heavily in selecting the framework of the WES comprehensive model for the NOW model is the fact that WES has developed a method for describing terrain for ground mobility purposes that is geared to satisfy the terrain input requirement for the WES cross-country model.

Suggested additions to NOW model

295. In addition to considering the dynamics effects of discrete and continuous rough terrain on the driver, shock and vibration limits for cargo and firing should also be included. For the moment, an arbitrary limit should be assigned for cargo tolerance, and the shock and vibration limits established by TACOM and used in the OMEGA comprehensive model should be used for ride tolerance while firing. The basic dynamic model can be used to obtain the speed over specified terrain at which those limits will be reached. The ride dynamic model used may be selected from many others available according to what is most suitable for the form of input data, the type of vehicle involved, etc.; however, once selected for a given study, the same dynamic model should be used for making close internal comparison as in a design study. This study did not deal critically with the criteria upon which ride data developed by the dynamic models are reduced to a speed level. Part of this problem is associated with the fact that there is little evidence in the form of test data to arrive at an objective decision. The WES ride performance criteria are based upon vertical and horizontal acceleration limits, and root mean square acceleration at the driver's seat. The absorbed power criterion developed by TACOM for driver and firing tolerances appears reasonable, and since an output of absorbed power can be obtained by modifying the WES dynamic model, it should be included as a ride performance parameter.

296. For the NOW model, the WES manual operation for determining obstacle-vehicle geometry interference should be replaced by the

computerized method proposed by TACOM and/or those incorporated in the WNRE cross-country model, possibly including extension of these latter methods contained in a more recent go-no go study in lunar terrain.

297. Particular attention should be given to the stream egress model used in the NOW model. It is suggested that this be a reasonable merger of the best features of the several available models, since no egress model by itself represents the best that can be done at the present time.

PART IV: GUIDELINES FOR FUTURE DEVELOPMENT OF GROUND MOBILITY MODELS

298. As has been shown in the previous sections of this report, ground mobility models have been recently devised and most of them have been developed for a particular application, used perhaps once, and have remained dormant ever since. The comprehensive models reviewed usually differed in objective, scope, application, submodels incorporated, and level of sophistication. It is also obvious that the information available is limited insofar as the user is concerned in assisting him in evaluating, selecting, or constructing a model that meets his requirements.

299. This study has not established any direct relations between the comprehensive cross-country models examined and the real world because model verification tests have not been run. On the other hand, a few relations have been established between several submodels that predict a particular kind and degree of terrain, vehicle, driver interaction. Furthermore, it has been recognized during this study that without an analysis of existing models and related submodels and preparing a model for current use and future development, development of comprehensive models would continue along parallel or slowly converging paths. Moreover, without standardization and continued use of an appropriate ground mobility model, it is not likely that correlation between an acceptable comprehensive model and the real world would be available any time soon. A unified approach is a necessity if stagnation in ground mobility modeling is to be prevented and a means is to be provided for the user and the research community to communicate in the solution of common problems.

300. Although the study has shown that considerable effort has been spent in mobility research and that it is apparent that considerable effort must be expended in the future before a comprehensive ground mobility model will be available that will meet acceptable standards of accuracy, flexibility, and realism for all potential users, past research is only beginning to pay off. The reason for the slow rate in payoff of ground mobility knowledge can probably be attributed to the fact that what is known concerning off-road ground mobility has not been organized in a form that

lends itself to straightforward application to common user problems. A common approach in defining requirements and missions is needed before available knowledge can be arranged in a form that will produce timely direct answers to questions frequently asked. Ongoing research must also be geared to problem areas for which suitable answers are not available.

Past Accomplishments

301. Some of the knowledge available has been used in the design and evaluation of design and prototype combat vehicles that have performed reasonably well in most environments if the vehicle design and environment were adequately described within the boundaries of available knowledge. Extrapolation to remote area environments or quantifying the effects of vehicle characteristics on performance, such as wagon steer employed in GOER vehicles or limited slip differentials, has been less successful. Other recent achievements are: the Army's adoption of standard tools and techniques for the measurement of soil strength for mobility purposes; QMR's for new vehicles are now beginning to include mobility specifications in terms of an ability to operate in soil of a specific strength (rather than in "deep soft mud"); vehicles have been designed and built to attain a specified degree of mobility using mobility research findings and tests have conclusively demonstrated the validity of these findings; AMC cost effectiveness personnel have been assisted in evaluation of the relative performance of existing and proposed vehicles; a DOD-level ad hoc group, the Intratheater Transportation Requirements Study Group, was recently rendered assistance in its study of transportation requirements of the future; the Army Materiel Systems Analysis Agency was significantly aided in its comparative evaluation of the MBT70 and M60 tanks; imaginative new vehicle concepts, e.g. walking machines, hubless wheels, etc., have been proposed for the consideration of Army planners; mobility research findings are currently being used in DASA studies of the effectiveness of craters as barriers to ground vehicles; and mobility research engineers are serving as consultants to NASA lunar trafficability panels and are engaged in test programs to assist in the selection of the proper vehicles for lunar travel

(for NASA). The examples cited illustrate the useful achievements stemming primarily from vehicle mobility research and the wide spectrum of military purposes to which the results of this research can, and undoubtedly will, be put in the future.

302. A rapid catalysis of these products occurred mainly because of a special project sponsored and funded by ARPA. In 1963, ARPA, under its Project AGILE, initiated a research program to determine the effects of the various features of the physical environment on the performance of cross-country ground vehicles and to provide data that could be used to improve both the design and employment of such vehicles. A condition of the project was that the data be interpretable in terms of vehicle requirements for Southeast Asia. This program, called Mobility Environmental Research Study (MERS), was begun in July 1963 following a preliminary study made in Thailand. This additional funding made it possible for the various mobility research laboratories to pull together, synthesize, and partially validate all relevant data and capabilities, and to conduct testing on obstacle effects that might otherwise have not been conducted for several years. Although circumstances forced ARPA to withdraw its support before the project had been completed, the progress made before this occurred constituted a quantum step in the scope of vehicle mobility research.

Approach to Future Plans

303. A more recent catalyzer of ground-mobility research has been the demand by operations and cost effectiveness analysts for mobility evaluation performed by the application of a realistic comprehensive ground-mobility model for predicting vehicle performance.

304. The general course for future mobility research is now quite clear, namely, efforts should be directed toward the development of a comprehensive analytical model of vehicle performance. Such a model is now generally envisaged as a coupling of appropriate terrain-vehicle-driver interactions expressed in mathematical form to predict the performance of any vehicle (in terms of go-no go, speed, fuel consumption, and cargo delivery capability) in any terrain. The component equations of the model

may vary from purely theoretical to purely empirical. Fortunately, the existence of objective statements of the relations existing between terrain and vehicles makes it possible to test and validate those statements by appropriately designed field experiments. Such experiments can obviously progress from the simple to the complex, and this attribute makes it practical to identify a number of well-defined tasks. These tasks can then be scheduled into a manageable sequence of activities and programs which in sum constitute a comprehensive research program.

305. Development of a comprehensive model of vehicle performance must be accompanied by a concurrent effort toward improving the current ability to measure, describe, store, and retrieve terrain data. It is axiomatic that these functions must be compatible with the vehicle performance model. While considerable progress has been made in this subject (again, fruition was hastened by special funding), much remains to be done.

306. To ensure continued usage of a model or submodels as a tool, the compilation of a complete set of instructions for using them is a necessity. The ideal model would be a complete simulation model defining the interactions between terrain, vehicle, and driver. Complete validity can be assured only when the effect of every significant component of the vehicle, as well as of the terrain and driver, can be isolated and independently examined. This means that vehicle inputs could be "modified," inserted in the general model, and the modified vehicle "tested" with the model in such a way that the effect of the modification on performance could be evaluated. It is apparent that an extension of this procedure could be used to evaluate the almost limitless numbers of possible engineering trade-offs that might be made in any given conceptual vehicle. Thus, the comprehensive mathematical simulation model becomes a basic and extraordinarily powerful and flexible design and evaluation tool. It is clearly the ultimate responsibility of the research community to provide the design and engineering community with appropriate instructions for using it to maximum advantage. The consummation of this task must, however, await the results of research to develop a comprehensive model of vehicle performance and improved terrain description methods. Once these two tasks have been accomplished, or perhaps even after incomplete but substantial

accomplishment, it should be a relatively simple matter to prepare a useful instruction handbook.

307. The goals outlined are unquestionably worthy and every indication exists that they can be accomplished. However, that accomplishment can only result from systematic, continuous application of experienced personnel in a properly funded, carefully prepared, and executed research program.

308. These goals for future ground mobility research have been recently considered by the Research. Development and Engineering Directorate, AMC, and mobility and terrain research programs conducted by various AMC laboratories have been reprogrammed to ensure that a comprehensive ground-mobility model will be compiled in the near future and that future research has been geared to the improvement of the first-generation model. The guidelines used in developing and implementing the AMC ground-mobility research program are sound, and the plan satisfies the known requirements for ground-mobility modeling. The plans presented herein are thus an abbreviated form of the AMC program plan.

AMC Ground-Mobility Research General Plan

309. The AMC mobility research program has been designed with several conditions in mind. One of the conditions is that most of the research will be accomplished and directed primarily by Army laboratories at TACOM and WES, with assistance from the Natick and Cold Regions Research and Engineering Laboratories as required. Another condition is the fact that personnel of TACOM and WES have either made or contributed significantly to most of the major advances in the state-of-the-art of mobility research for the past several years. Since they are at the forefront of the discipline, it seems most unlikely that any significant benefits could accrue from further surveys of the state-of-the-art. Such studies would inevitably consist primarily only of bringing the surveyors up to the levels of sophistication already reached by the scientific and engineering staffs of TACOM and WES. The general plan presumes that the state-of-the-art is well known. The proposed research is designed only to extend it;

no resources are devoted in the plan to surveys or similar excursions into futility. The last condition is that scientific knowledge consists of approximations; absolute truth or total reliability is never completely reached. This is as true for knowledge about mobility as for any other discipline. Thus, there will never be a "final" absolutely complete and totally reliable mathematical mobility simulation model. The general plan is not premised on any such unattainable goal; however, it is based on the firm conviction that a widely useful and acceptably reliable model can be achieved within about five years. It will not be perfect, but it will nevertheless be a powerful tool.

310. The AMC mobility research program consists of two phases (Phase I: Vehicle Performance and Phase II: Terrain Description) that will be prosecuted concurrently. If at the end of the first cycle, the research program has not resulted in satisfactory solutions, the program will be reevaluated and a new plan developed.

Phase I: Vehicle Performance

311. Phase I consists of five tasks, conducted largely but not completely in sequence (see fig. 70). Task I consists of a consideration of all the mathematical models of vehicle performance that are currently available and the compilation from the best of the submodels a general mobility model. It is expected that the results of the present ANAMOB study will in large measure fulfill the purposes of Task I. Upon completion of a first-generation model, a set of experiments will be designed and conducted (Task II) with the objective of revealing all deficiencies in the first-generation model. Upon completion of the tests and analysis of the results, Task III can be initiated; it consists of an identification of all of the research tasks required to deal with deficiencies revealed by the field tests and the design of appropriate research programs for each. Task IV consists of the conduct of those research tasks, and Task V completes the cycle; it consists of the compilation of the second-generation simulation model. The subtasks associated with each task are briefly described in table 22.

Phase II: Terrain Description

312. Phase II consists of six tasks, one each to establish the

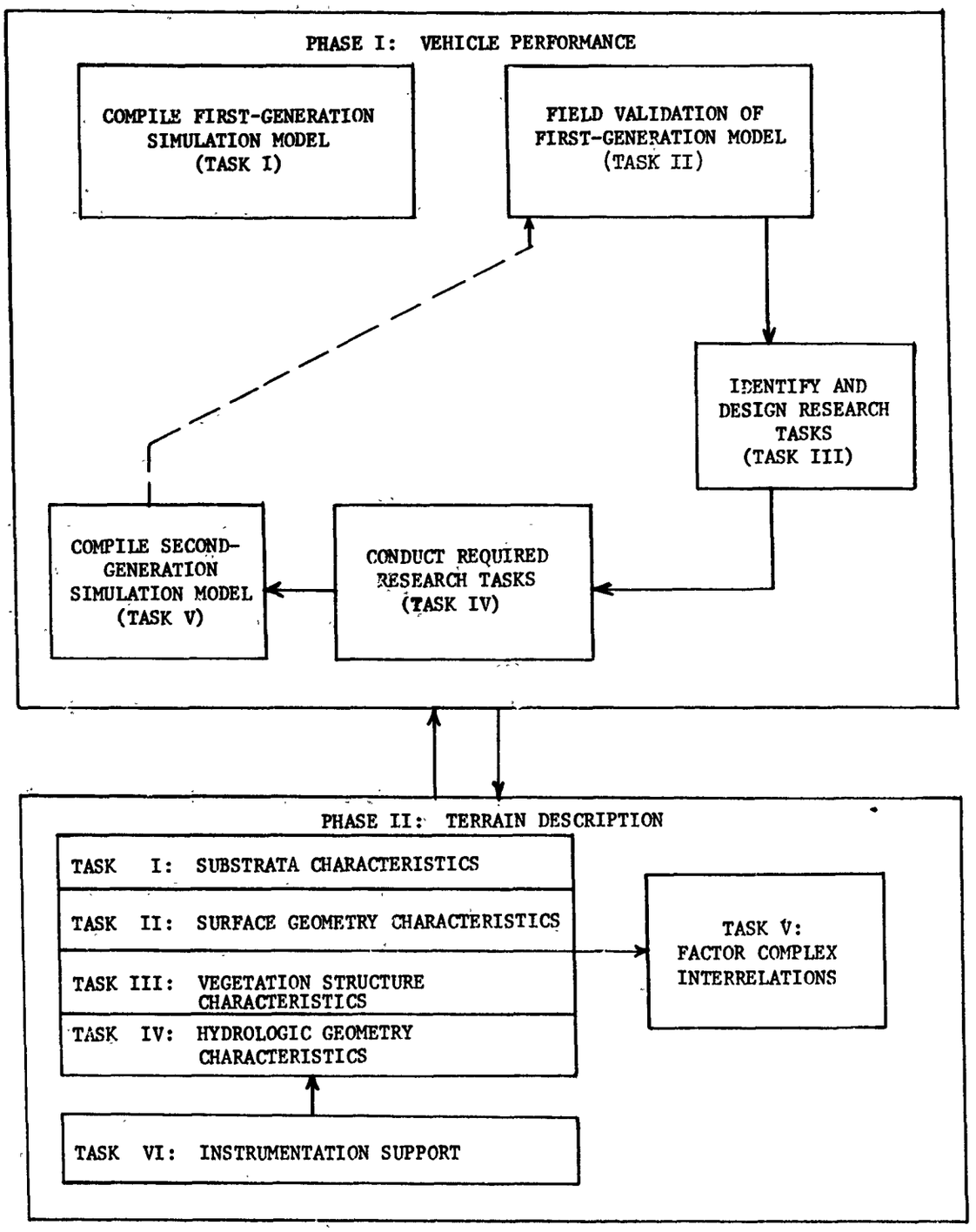


Fig. 70. General research plan, phase and task relations

Table 22

Outline of Plans for Phase I: Vehicle Performance

Task I: Completion of First-Generation Simulation Model

Purpose: Compile the most comprehensive and reliable mathematical simulation model of terrain-vehicle-driver interactions possible from existing sources.

Subtask No.	Title	Brief Description of Work
A	Examine Existing Models	
	1. Literature search	Obtain and conduct literature search to fill knowledge gaps in existing U.S. and foreign ground mobility models. Select comprehensive models worthy of further study. Group models into pertinent categories.
	2. Critical analysis	Identify submodels or single-feature terrain vehicle relations that constitute the building blocks of comprehensive models selected for further study. Determine commonality in single-feature submodels. Identify assumptions, rationale, theory, or basis upon which submodel was founded. List input parameters, equations, and output performance parameters. Evaluate existing test data and compare measured and predicted performance results. Determine submodel applicability and status of verification. Program submodels for analysis purposes if required. Identify relationships between submodels. Establish common terminology. Prepare in handbook form how each submodel is used, citing examples of computations, etc. (The handbook form will be used to prepare test plans for validation testing and instructions for computer programmers.)
B	Compile General Model	
	1. Develop flow chart and logic	Prepare flow chart of first-order analytical model and identify the submodels for which the critical analysis revealed potentially usable submodels.

(Continued)

(1 of 5 sheets)

Table 22 (Continued)

Subtask No.	Title	Brief Description of Work
	2. Write quantitative model	Develop procedures for compiling or integrating submodels into necessary computational processes required to predict vehicle speed performance for off-road operation. Identify the submodels grouped together in each computational process. Establish common mathematical notations.
	3. Program for computer solution	Select computer which will be used to obtain computer solutions. Program for computer solutions the submodels accepted as elements of the first-order analytical model and test for acceptable solutions. Debug model. Prepare report on computer program.
C	Restructure Existing In-House Program	
	1. Identify programs to be continued	Evaluate TACOM/WES mobility research programs for relevance and application to first-generation analytical model. Identify those studies that are needed and can be incorporated into the second-generation model.
	2. Identify new problem areas	List new problems identified in preparing the first-generation model that require research and can be completed in several years.
	3. Prepare research programs	Prepare research plans for each of the problem areas identified in previous item.

Task II: Field Validation of First-Generation Model

Purpose: Validate both the general first-generation performance prediction model and its component submodels or modules.

A	Design of Experiments	
	1. Identify testable model components	Upon completion of critical analysis (Task I, subtask A2) of a submodel or a group of related submodels, examine input requirements and determine the measurement method and technique that will satisfy the input requirements. If measurement of

(Continued)

(2 of 5 sheets)

Table 22 (Continued)

Subtask No.	Title	Brief Description of Work
		input data is beyond current field testing capabilities, identify for further study. Identify which submodel outputs are used as input to other submodels and which submodels must be tested independently and which related submodels can be grouped and tested in a common test program.
	2. Identify terrain, vehicle, and driver parameters	From critical analysis, identify range of variation over which input parameters may occur in nature, and identify the range of each input parameter for which predictions are desired. This latter information is a prerequisite for validation testing. For example, soil strength of fine-grained soil has little or no effect on vehicle traction performance when it exceeds the vehicle cone index (VCI) about 60 points; therefore, most of the field testing should be conducted in the VCI_1 + 60 range. Prepare test specifications for each submodel.
	3. Select test conditions	Identify model to be tested singly and prepare testing procedures to be followed in testing them. If testing is to be initiated in laboratory with a vehicle component under controlled conditions, indicate the manner in which model verification will be achieved for prototype vehicles in natural terrain conditions. Prepare testing procedures for the submodels that will be grouped for testing in concert. Select test conditions necessary to meet above requirements.
	4. Select and prepare test instrumentation	Examine each submodel or group of submodels to determine instrumentation requirements for verification testing. Determine for which submodels suitable instrumentation is available. Prepare sketch plans identifying the instrumentation to be used for testing each submodel or group of related submodels. Prepare designs for instrumentation not available to perform desired test procedures. Acquire component parts, assemble instrumentation system, and check out system.

(Continued) (3 of 5 sheets)

Table 22 (Continued)

Subtask No.	Title	Brief Description of Work
	5. Write test plans	Prepare test plans for validating each submodel after relevant terrain, vehicle, and driver parameters have been specified and instrumentation requirements are known. Select suitable test areas and prepare test schedule. Include in test plans test procedures and data analysis to be performed.
B	Conduct Experiments	
	1. Mobilize for tests	Mobilize for test and train teams in testing procedure, utilization of instrumentation and other measuring equipment, and reduction and preliminary analysis of data.
	2. Execute tests	Conduct sufficient tests to validate individual submodels or combinations of submodels incorporated in the general model. Identify gaps in knowledge for use as basis for conducting research programs.
C	Analysis for Deficiencies	
	1. Analysis	Compare predicted and measured submodel results for level of accuracy. If comparative results are poor, state why and whether modification would improve prediction accuracy. Evaluate for consideration as a research task.
	2. Report preparation	Prepare interim report on each test program.

Task III: Identify and Define Research Programs

Purpose: Identify and define research programs as required to improve to an acceptable level of reliability submodels and submodel combinations which are revealed as deficient by the test and validation program (Task II).

A	Identify Deficiencies on Model	Identify, define, and prepare research programs as required to improve reliability of submodels and coupling processes revealed deficient by tests.

(Continued)

(4 of 5 sheets)

Table 22 (Concluded)

Subtask No.	Title	Brief Description of Work
B	Write State-of-the-Art Models of Each Problem Area	Prepare state-of-the-art of models and submodels to elucidate the relations existing among some combinations of terrain, vehicle, and driver factors and to insure that the submodels are realistic and the interaction can be described by mathematical processes.
	Task IV: Conduct Required Research Tasks	
Purpose:	Develop a basic understanding of terrain-vehicle-driver interactions in sufficient detail to make it possible to compile a highly reliable second-generation mathematical performance prediction model.	
A	Design Research Programs	Develop research plans for each task as soon as research requirements are identified and defined. Establish research task priorities.
B	Conduct Research Programs	Conduct research on updated programs which were being pursued prior to the initiation of the AMC ground mobility program and new research programs.
	Task V: Compile Second-Generation General Model	
Purpose:	Develop a reliable second-generation mathematical model for predicting the performance of military ground-contact vehicles on a real-world combination of terrain conditions.	
A	Develop Flow Chart and Logic	Compile a complete and detailed flow chart of the general comprehensive model logic.
B	Write Quantitative Model	Prepare quantitative mathematical expressions (or simulation routines) as required by the model logic.
C	Program for Computer Solution	Program comprehensive model for computer solution and document for use.

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characteristics of substrates, surface geometry, vegetation structure, and hydrologic geometry, a fifth to consolidate these into a total environment, and a sixth on instrumentation support.

313. The tasks propose research effort concentrated on providing the detailed terrain descriptions required by vehicle designers, vehicle test agencies, evaluators, and planners who are preparing specifications for new vehicle families. The proposed tasks do not incorporate any research effort devoted expressly to those activities more closely related to military geographic intelligence, such as mapping significantly large areas in terms of terrain descriptors for the purpose of constructing cross-country locomotion maps, operations planning maps, and the like. Such activities are primarily the responsibility of agencies other than AMC.

314. The plan is also based on the assumption that all six tasks will be conducted simultaneously, so that information and experience gained in work related to one can be immediately transferred to the others.

315. It is further assumed that some effort in Phase I (Vehicle Performance) will be devoted to terrain description in direct support of those activities. For example, the test courses for the validation of performance prediction models developed as a product of Phase I research will have to be carefully described. These descriptions will be made using the most sophisticated methods and techniques developed by Phase II research up to that time, but such "production descriptions" will be obtained by Phase I researchers. The subtasks to be pursued under each task are briefly described in table 23.

Table 23

Outline of Plans for Phase II: Terrain Description

Task I: Substrata Characteristics

Purpose: Develop rapid, efficient, and reliable methods for acquiring data on substrata (soil, snow, muskeg, etc.) characteristics in a form usable by the mathematical performance prediction submodels compiled in Phase I, and to establish the significant ranges of variations of the relevant properties of such material on a worldwide basis.

Subtask No.	Title	Brief Description of Work
A	Identify Significant Descriptors	Examine submodels to be used and identify terrain factors as input.
B	Establish Sampling Techniques	Develop and refine sampling theory; develop methods of measuring the parameters or factors required in above subtask.
C	Establish Descriptor Variations	Establish the ranges of variations on a world scale by terrain descriptions required, both independently and in combination.

Task II: Surface Geometry Characteristics

Purpose: Develop rapid, efficient, and reliable methods for acquiring data on surface geometry characteristics in a form usable by the mathematical performance prediction submodels compiled as a product of Phase I research, and to establish the significant ranges of variations of surface geometry configurations on a worldwide basis.

A	Identify Significant Descriptors	Examine submodels to be used to ensure that all relevant properties of surface geometry required are appropriately considered and described.
B	Establish Sampling Techniques	Develop and refine sampling theory. Improve data acquisition methods.
C	Establish Descriptor Techniques	Establish ranges of variation exhibited on a world scale of surface geometry features that affect vehicle locomotion.

(Continued)

(1 of 3 sheets)

Table 23 (Continued)

Task III: Vegetation Structure Characteristics

Purpose: Develop rapid, efficient, and reliable methods for acquiring data on vegetation structures in a form usable by the mathematical performance prediction models compiled in Phase I, and to establish the significant ranges of variations of the relevant vegetation factors on a worldwide basis.

Subtask No.	Title	Brief Description of Work
A	Identify Significant Descriptors	Examine submodels to determine the vegetation structure factors required to satisfy input requirements.
B	Establish Sampling Techniques	Develop adequate descriptors and field procedures for making measurements relevant to vegetation factors. Develop methods for rapidly acquiring vegetation data.
C	Establish Descriptor Variations	Establish ranges of variations exhibited on a world scale by vegetation structure factors and descriptions, both independently and in combination.

Task IV: Hydrologic Geometry Characteristics

Purpose: Develop rapid, efficient, and reliable methods for acquiring data on the characteristics of hydrologic features in a form usable by the mathematical performance prediction models compiled in Phase I, and to establish the significant ranges of variations of the relevant hydrologic geometry factors on a worldwide basis.

A	Identify Significant Descriptors	Examine submodels and assemble the descriptions relevant to the problem.
B	Establish Sampling Techniques	Develop methods of measuring the descriptors or factors relevant to entry, fording, and exiting processes. Improve methods for rapid measurement. Develop and refine theory of sampling.
C	Establish Descriptor Variations	Establish ranges of variations on a world scale by the factors and descriptors used in submodels.

(Continued)

(2 of 3 sheets)

Table 23 (Concluded)

Task V: Factor Complex Interrelations

Purpose: Determine those combinations of factors and factor values that significantly affect vehicle performance, define their general geographic distribution on a worldwide scale, and develop methods of describing such factor interrelations so that they may be readily used as input to the mathematical performance prediction models developed as a product of Phase I research.

Subtask No.	Title	Brief Description of Work
A	Data Compilation	Assemble data collected in all program tasks into a single data store in a form that can be examined for a variety of purposes.
B	Identification of Significant Factor Value Combinations	Evaluate the effect produced by specific combinations on terrain factors to provide insight as to which terrain descriptors should be presented for utilization in other military activities.

(3 of 3 sheets)

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APPENDIX A: SOIL-VEHICLE MODELS

Introduction

1. The relations in this appendix, whether equations or graphs, are shown in their current form without reference to prior evolutionary steps. Assumptions, inputs, outputs, etc., involved in each model are presented along with the explicit relations. Symbols common to all models are listed immediately below. Definitions and symbols peculiar to a particular model are given with the listings for that model.

W = load on traction element, lb

b = width of traction element contact; for tires, undeflected section width, in.

ℓ = length of traction element contact, in. (detailed definition varies in the several models, especially for wheels and tires)

d = unloaded tire diameter, in.

h = unloaded tire section height, in.

δ = tire deflection, in.

s = slip, ratio $\left(s = 1 - \frac{V_v}{V_t}\right)$

V_v = velocity of the vehicle, mph

V_t = velocity of traction element, ft per sec

p = pressure, psi

z = sinkage or depth of penetration of traction elements, in.

A = area, sq in.

c = cohesion, psi (as defined by the instrument with which the measurement is obtained)

ϕ = angle of internal friction, deg (as defined by the instrument with which the measurement is obtained)

γ = soil density, lb/in.^{3}

τ = shear stress, psi

θ = terrain-slope angle, deg

α = trim angle of running gear of vehicles, deg

N = number of major traction element assemblies supporting the vehicle

P = net traction (drawbar pull), lb

WES VCI Model

2. The WES VCI model for predicting vehicle performance for fine- and coarse-grained inorganic soils includes determination of minimum soil strength requirements in terms of vehicle cone index (VCI), maximum towing force, and towed motion resistance while a vehicle is traveling in a straight line in unaccelerated motion on unobstructed level and sloping soil surfaces. All of the performance parameters are related to rating cone index (RCI) for fine-grained soils and cone index (CI) for coarse-grained soils. The pertinent soil-vehicle performance relations were empirically derived from field test data that included a range in vehicle characteristics and soil strengths. Initially performance predictions in fine-grained soils were for 50-pass traffic; however, subsequently a one-pass performance prediction method was required. The VCI one-pass performance prediction scheme was developed by adapting techniques used in the 50-pass prediction. For coarse-grained soils, one-pass performance only is determined because tests have shown that the minimum soil strength for go on the first pass is adequate for go on all subsequent passes as well.

3. Definitions peculiar to this model are listed below.

Critical layer. The layer of soil that is most pertinent to establishing relations between soil strength and vehicle performance. For 50-pass performance in fine-grained soils and sands with fines, poorly drained, it is usually the 6- to 12-in. layer; however, it varies with weight and type of vehicle and with soil strength profile. For one-pass performance, it is usually closer to the surface.

Fine-grained soil. A soil of which more than 50 percent of the grains, by weight, will pass a No. 200 sieve (smaller than 0.74 mm in diameter).

Coarse-grained soil. A soil of which more than 50 percent of the grains, by weight, will be retained on a No. 200 sieve.

Sand. A coarse-grained soil with the greater percentage of the coarse portion (larger than 0.74 mm) passing the No. 4 sieve (4.76 mm).

Sand with fines, poorly drained. A sand that contains some fines and

layer" (between depths z_1 and z_2), selected according to vehicle weight and type, soil type, and cone index profile character, adequately represents the full CI profile.

(2) There is no large discontinuity between mass soil strength and surface soil strength.

(3) Nonwoody surface vegetation has no effect on performance.

(4) Soil stickiness has no effect on performance.

b. Vehicle.

(1) The vehicle moves only in straight, unaccelerated motion.

(2) Sufficient torque is available for self-propulsion in all soil conditions.

(3) Individual tractive elements (tires or tracks) share the gross load equally.

(4) All wheels in contact with the ground are powered.

(5) Tire deflection on self-propelled, wheeled vehicles is assumed constant at the level determined by the inflation pressure recommended for the vehicle in cross-country operation.

5. Soil parameters used in this model are:

a. CI or RCI.

b. Gross slope.

c. Classification of soil (fine-grained, coarse-grained, or organic).

d. Critical layer.

(1) It may vary with weight and type of vehicle and soil strength profile.

(2) For freely draining or clean sands, it is usually the 0- to 6-in. layer.

(3) For fine-grained soils and sands with fines, poorly drained, it is usually the 0- to 6-in. layer for one-pass performance and the 6- to 12-in. layer for 50-pass performance.

6. Vehicle parameters pertinent to this model are:

a. Tracked vehicles.

(1) Gross weight.

(2) Track width.

(3) Track ground contact area.

(4) Track pitch.

(5) Grouser height.

(6) Total number of bogies or road wheels for all tracks.

(7) Ground clearance to lowest point on hull.

(8) Engine horsepower.

(9) Transmission type (manual or automatic).

b. Wheeled vehicles.

(1) Gross weight.

(2) Number of tires.

(3) Tire nominal width.

(4) Undeflected tire outside diameter.

(5) Tire inflation pressure.

(6) Tire ply rating.

(7) Number of axles.

(8) Ground clearance to lowest point on chassis.

(9) Engine horsepower.

(10) Transmission type (manual or automatic).

Fine-grained soil and
sands with fines, poorly drained

7. VCI, towed motion resistance, and drawbar pull performance curves were derived from actual measurements in over 1600 tests run in prepared and natural soil materials with self-propelled and towed wheeled and tracked vehicles. The complete test data cover a wide range of vehicle characteristics and types of fine-grained soils. Test vehicle weights ranged from about 4000 lb to more than 100,000 lb; tire diameters ranged from about 30 to 60 in.; and soil conditions included all fine-grained soil types, each in strengths ranging from unquestionable go to unquestionable no go for each vehicle tested in it.

8. In fine-grained soils

$$VCI = RCI = \frac{1}{n+1} \sum_{j=n}^{j=0} CI_j \times RI_j \qquad (A1)$$

where

CI_j = before-traffic CI for soil at a depth z ;

$$z = z_1 + j \frac{(z_2 - z_1)}{n}$$

z_1 and z_2 = depth boundaries of the critical layer

RI_j = RI at depth z

n = number of equal divisions in critical layer used for measurement purposes.

9. The fundamental relations and empirically derived equations and graphs that are component parts of the WES VCI model are used for predicting vehicle performance in fine-grained soils and are outlined below.

a. MI is determined from one of two equations, according to whether the vehicle is tracked or wheeled. The equations (A2 and A3) for self-propelled vehicles are given for the appropriate vehicle type in tables A1 and A2. Equations for computing MI for towed wheeled and tracked trailers are available, but since they are not used in any analytical ground-mobility model, they are not included herein.

b. VCI is a function of vehicle type, MI, and the number of passes to be completed. It is obtained from the curves in fig. A1.

c. The net maximum drawbar pull coefficient on level ground and the corresponding maximum slope negotiable are determined as functions of the excess RCI over VCI (i.e., RCI - VCI), classification of vehicle type, and number of passes to be completed. The relations used for 50- and one-pass traffic are given in figs. A2 and A4,* respectively.

d. Towed motion resistance coefficient on level ground may be

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* Curves in fig. A4 are presently being tested and have not yet been validated. They should be considered as preliminary and subject to some changes as more data become available.

Table A1

Mobility Index Equation for Self-Propelled Tracked Vehicles (Equation A2)

$$\text{Mobility index}^{*} = \left(\frac{\text{contact pressure factor} \times \text{weight factor}}{\text{track factor} \times \text{grouser factor}} + \text{bogie factor} - \text{clearance factor} \right) \times \text{engine factor} \times \text{transmission factor}$$

where

Contact pressure factor = $\dfrac{\text{gross weight, lb}}{\text{area of tracks in contact with ground, sq in.}}$

Weight factor:
- Less than 50,000 lb = 1.0
- 50,000 to 69,999 lb = 1.2
- 70,000 to 99,999 lb = 1.4
- 100,000 lb or greater = 1.8

Track factor = $\dfrac{\text{track width, in.}}{100}$

Grouser factor:
- Grousers less than 1.5 in. high = 1.0
- Grousers more than 1.5 in. high = 1.1

Bogie factor = $\dfrac{\text{gross weight, lb, divided by 10}}{\text{(total number of bogies on tracks in contact with ground)} \times \text{(area, sq in., of 1 track shoe)}}$

Clearance factor = $\dfrac{\text{clearance, in.}}{10}$

Engine factor:
- ≥10 hp/ton of vehicle wt = 1.00
- <10 hp/ton of vehicle wt = 1.05

Transmission factor: Automatic = 1.0; manual = 1.05

* The MI index obtained is converted to VCI from the curves shown in fig. A1.

Table A2

Mobility Index Equation for Self-Propelled Wheeled (All-Wheel Drive) Vehicles (Equation A3)

$$\text{Mobility index}^{*} = \left(\frac{\text{contact pressure factor} \times \text{weight factor}}{\text{tire factor} \times \text{grouser factor}} + \text{wheel load factor} - \text{clearance factor} \right) \times \text{engine factor} \times \text{transmission factor}$$

where

$$\text{Contact pressure factor} = \frac{\text{gross weight, lb}}{\text{nom tire width, in.} \times \frac{\text{outside diam of tire, in.}}{2} \times \text{No. of tires}}$$

Weight factor:

Weight Range, lb**	Weight Factor Equations†
<2000	Y = 0.553X
2000 to 13,500	Y = 0.033X + 1.050
13,501 to 20,000	Y = 0.142X - 0.420
>20,000	Y = 0.278X - 3.115

Tire factor: $\frac{10 + \text{tire width, in.}}{100}$

Grouser factor: With chains = 1.05
Without chains = 1.00

Wheel load factor: $\frac{\text{gross weight, kips}}{\text{No. of axles}/2}$

Clearance factor: $\frac{\text{clearance, in.}}{10}$

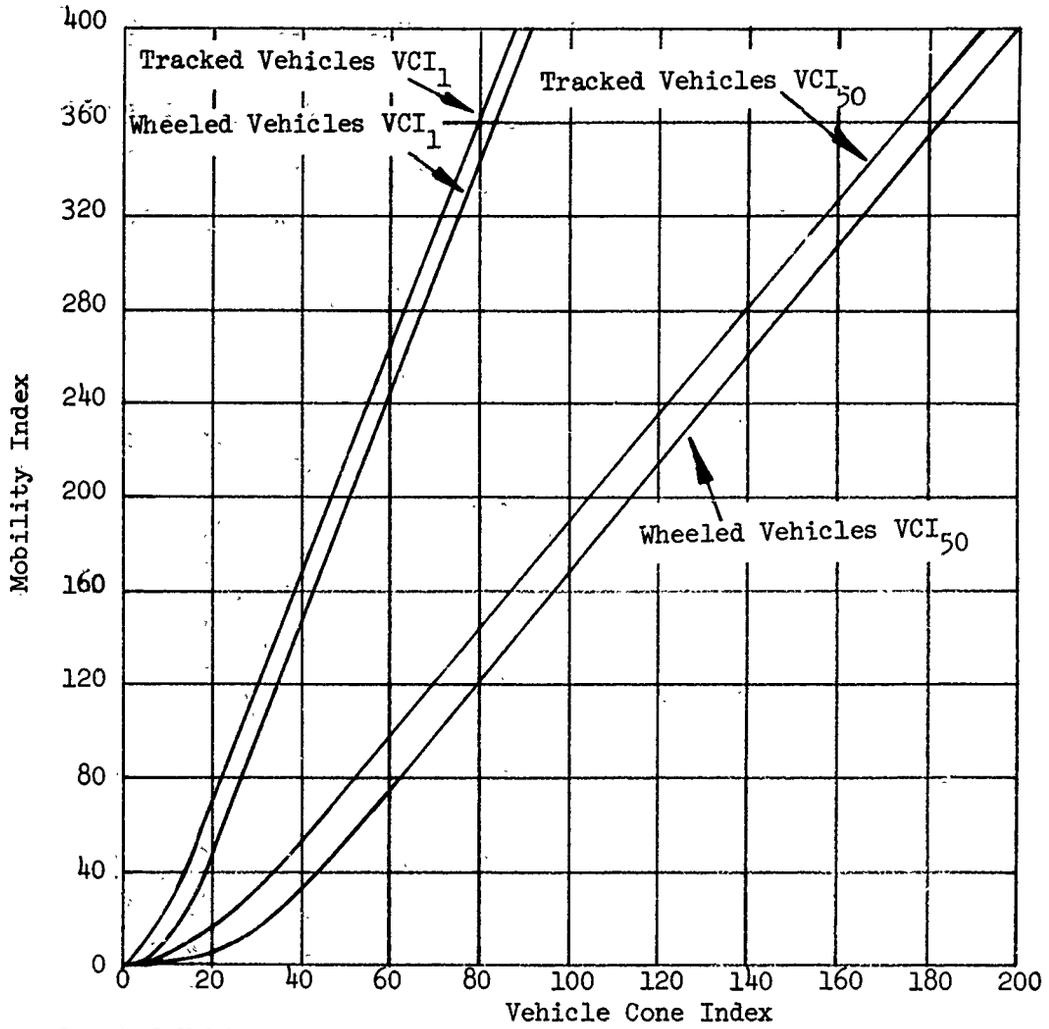
Engine factor: ≥10 hp/ton = 1.00
<10 hp/ton = 1.05

Transmission factor: Automatic = 1.00; manual = 1.05

* The MI obtained is converted to VCI from the curves shown in fig. A1.

** $\frac{\text{Gross weight, lb}}{\text{No. of axles}}$.

† Y = weight factor, $X = \frac{\text{gross weight, kips}}{\text{No. of axles}}$.



Tracked Vehicles

$$VCI_1 = 7.0 + 0.2\ MI - \left(\frac{39.2}{MI + 5.6}\right)$$

$$VCI_{50} = 19.27 + 0.43\ MI - \left(\frac{125.79}{MI + 7.08}\right)$$

Wheeled Vehicles

$$VCI_1 = 11.48 + 0.2\ MI - \left(\frac{39.2}{MI + 3.74}\right)$$

$$VCI_{50} = 28.23 + 0.43\ MI - \left(\frac{92.67}{MI + 3.67}\right)$$

Fig. A1. Relation of MI to VCI for self-propelled vehicles

$$Y = 0.0146X + 0.419 - \sqrt{(0.0146X + 0.419)^2 - 0.021X} \qquad (1)$$

$$Y = 0.0146X + 0.425 - \sqrt{(0.0146X + 0.425)^2 - 0.0198X} \qquad (2)$$

$$Y = 0.0109X + 0.366 - \sqrt{(0.0109X + 0.366)^2 - 0.013X} \qquad (3)$$

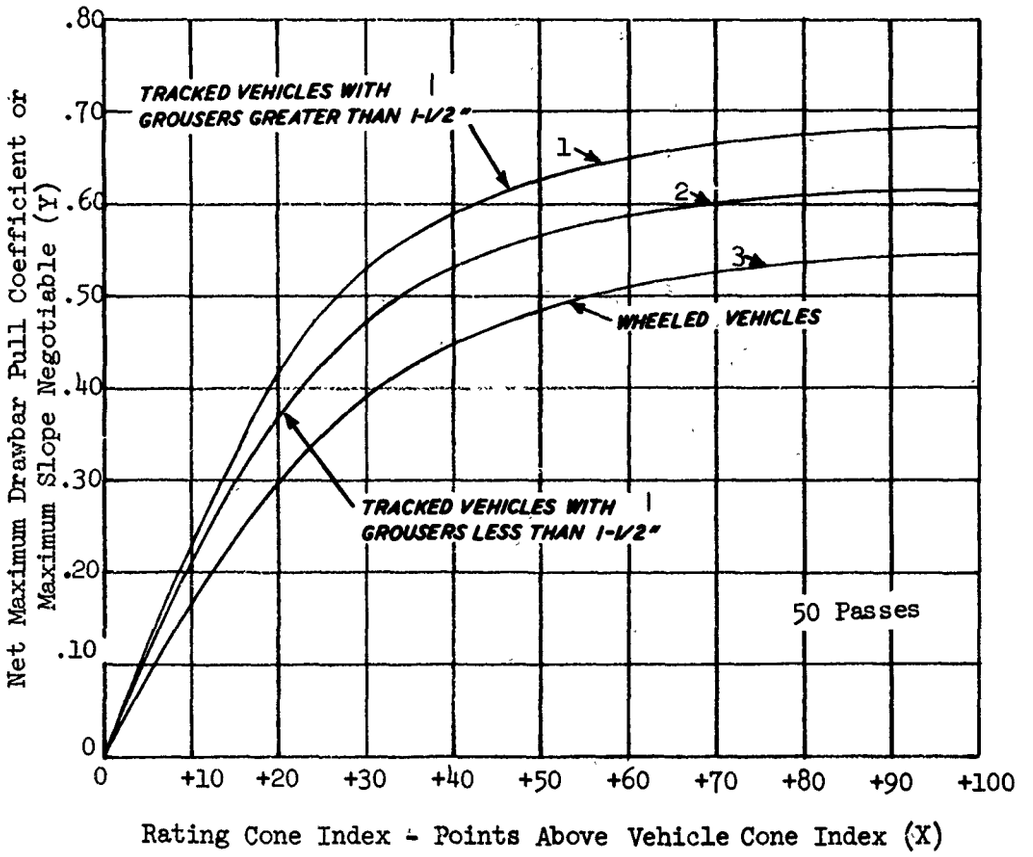


Fig. A2. Relation of net maximum drawbar pull on level ground and maximum slope negotiable to RCI in excess of VCI

estimated from its relation to RCI or excess RCI, the classification of the vehicle by load and type, and the number of passes to be completed. The relations used for 25 passes and one pass are shown in figs. A3 and A5,* respectively.

$$Y = 0.1858 - 0.01453X + \sqrt{(0.1858 - 0.01453)^2 - 0.00224X + 0.8623} \quad (1)$$

$$Y = 0.1609 - 0.00595X + \sqrt{(0.1609 - 0.00595X)^2 - 0.001X + 0.2838} \quad (2)$$

$$Y = 0.6 - 0.00885X + \sqrt{(0.6 - 0.00885X)^2 + 0.001X + 0.027} \quad (3)$$

$$Y = 0.4167 - 0.01052X + \sqrt{(0.4167 - 0.01052X)^2 + 0.1886} \quad (4)$$

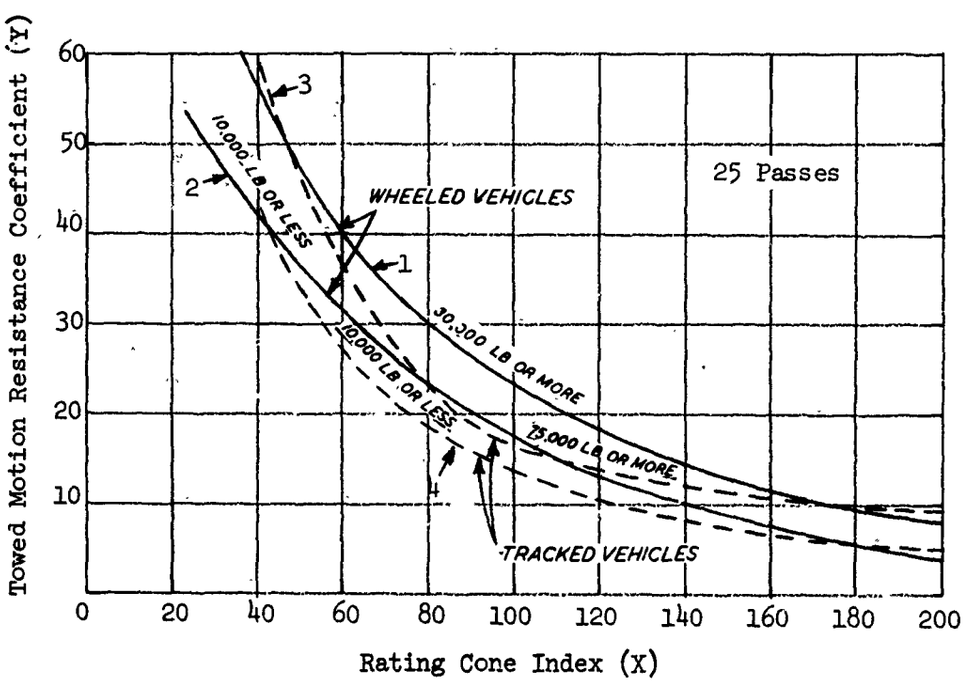


Fig. A3. Relation of towed motion resistance on level ground to RCI

* Curves in fig. A5 are presently being tested and have not yet been validated. They should be considered as preliminary and subject to some changes as more data become available.

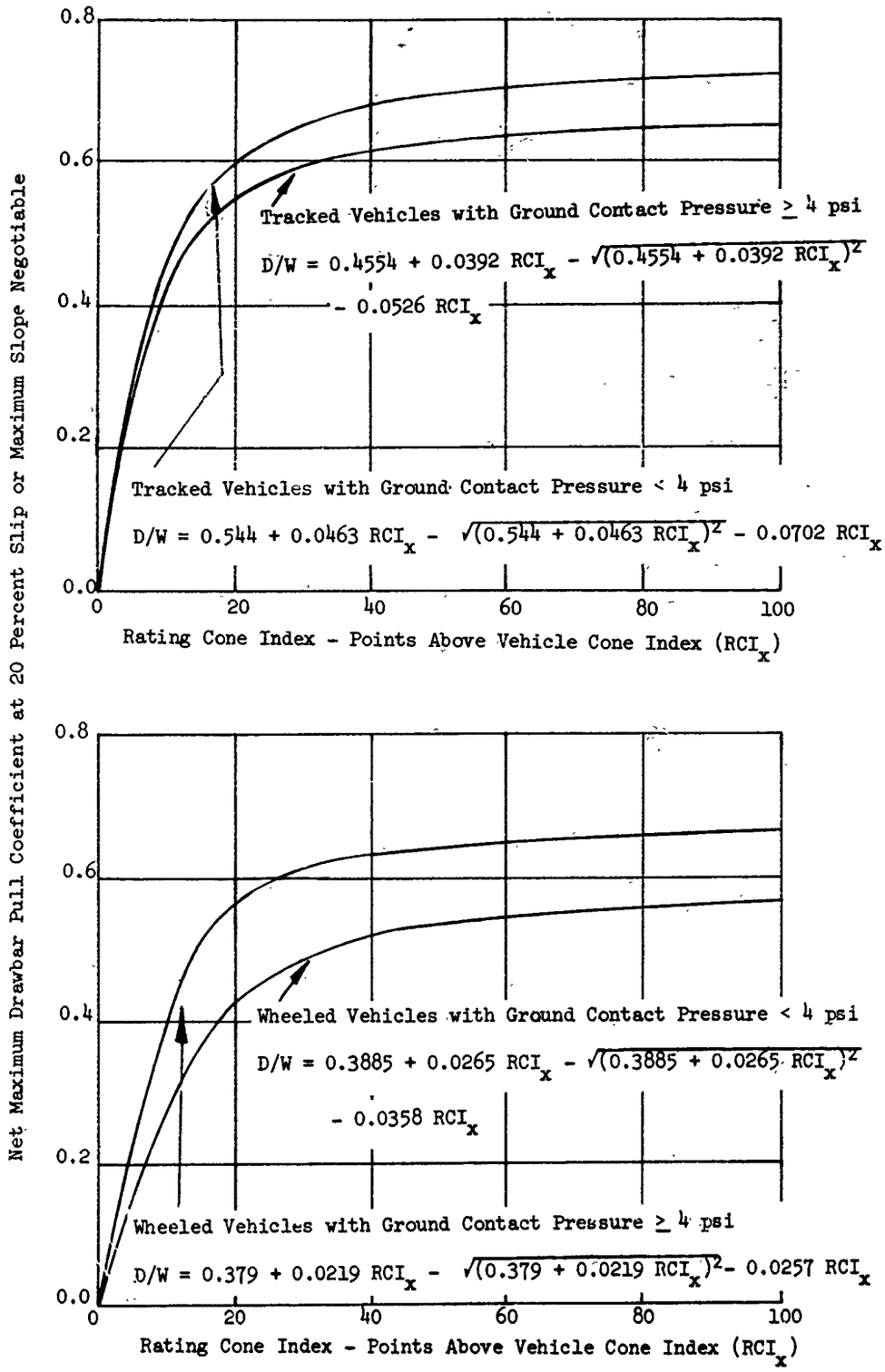


Fig. A4. Net maximum drawbar pull, maximum slope negotiable-soil strength relations for tracked and wheeled vehicles operating on fine-grained soils

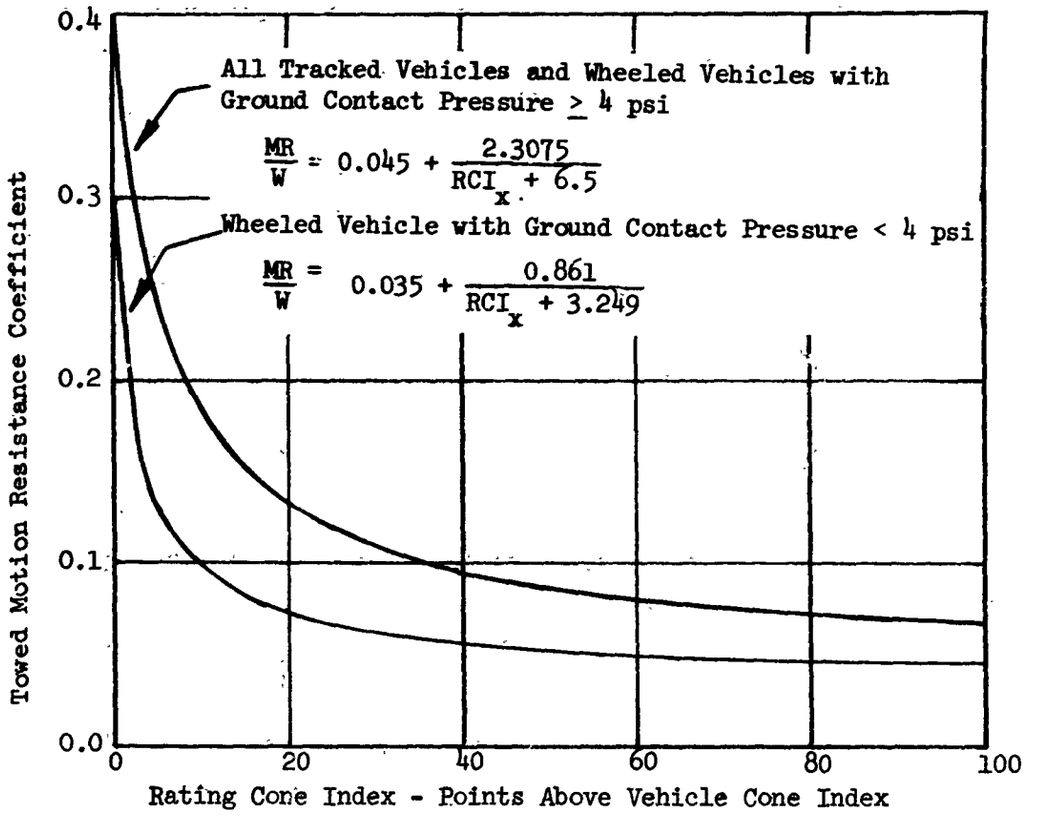


Fig. A5. First-pass motion resistance-soil strength relations for tracked and wheeled vehicles operating on fine-grained soils

Coarse-grained soils

10. The same basic soil and vehicle characteristics are used to predict vehicle performance in coarse-grained soils in terms of VCI, towed motion resistance, drawbar pull, and maximum slope negotiable for tracked and wheeled vehicles. The relevant data were derived from some 1400 tests of self-propelled wheeled vehicles (all-wheel drive) and about 200 tests of self-propelled tracked vehicles run mostly in natural sand conditions. Test site data are characteristic of dry and moist sands commonly found in inland deserts of the United States and of continental and river beaches of the United States and foreign countries. Origins of the sands were quartz, coral, and volcanic ash. The available test data represent a reasonable range in vehicle and soil characteristics. The soil types tested include sand and gravel, and sand and gravel mixtures. Wheeled

vehicle test weights ranged from 2600 to 33,000 lb, tire diameters ranged from about 30 to 65 in., and tire pressure ranged from 10 to 60 psi. Tracked vehicles tested included girderized and flexible tracks, and weights ranged from 4500 to about 36,000 lb.

11. In clean sands

$$VCI_1 = \overline{CI}_{z_1,z_2} \tag{A4}$$

where

$\overline{CI}$ = average before-traffic cone index

z_1,z_2 = depth boundaries of critical layer, usually 0- to 6-in. depth

12. Wheeled vehicles. From the individual tests, 352 separate empirical soil-vehicle performance relations (see fig. A6a for examples) were established for specific vehicles, and data from these relations were input to a statistical analysis to develop regression equations relating the performance parameters to soil and vehicle factors. Multiple curvilinear regression techniques were used to establish equations for predicting VCI, maximum drawbar pull, and maximum slope negotiable; and linear regression techniques were used to develop an equation for towed motion resistance. An example of the soil, vehicle-performance parameter relations used in the analysis is shown in fig. A6. The vehicle and soil characteristics used, together with forms for computing performance, are given in tables A3 and A4.

13. The equations for vehicle performance in coarse-grained soil resulting from this statistical analysis are as follows:

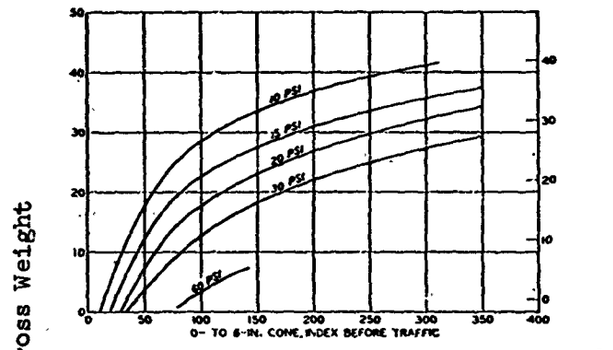
a. VCI_1 = antilogarithm* $(-0.350X_1 + 0.0526X_2 + 0.0211X_3 + 1.5870)$. (A5)

b. Maximum net drawbar pull, percent of gross weight = $(28.87X_1 + 10.10X_2 - 1.52X_3 - 0.61X_4 - 43.82) + 100$. (A6)

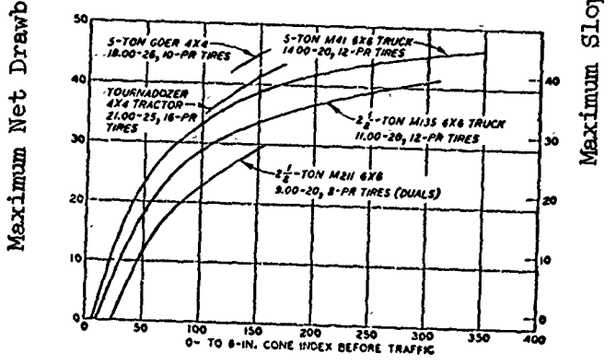
c. Maximum slope negotiable, percent = $(28.87X_1 + 10.10X_2 - 1.52X_3 - 0.61X_4 - 45.82) + 100$. (A7)

d. Towed motion resistance = $\left\{(22.20 + 0.92 \times X_4) + \left[(-88 - 0.37 \times X_4)(\log X_5)\right]\right\} + 100$. (A8)

* Logarithm to the base 10.



a. Effect of tire pressure on vehicle performance on dry-to-moist sand. M34 and M135, 2-1/2-ton, 6x6 trucks; 11.00-20, 12-PR, NDCC tires



b. Effect of tire size on vehicle performance on dry-to-moist sand. Vehicles operated at 10-psi tire pressure

Fig. A6. Soil, vehicle-performance relations used in coarse-grained soil prediction analysis

Table A3

Data Form for Computing Vehicle Cone Index (VCI_1) for Wheeled Vehicles in Sands (All-Wheel Drive)(Equation A5)

Vehicle _____

Equation: Vehicle cone index (VCI_1) = antilogarithm* (-0.350 (contact area factor, X_1) + 0.0526 (number of tires powered, X_2) + 0.0211 (tire pressure, X_3) + 1.5870)

Vehicle and Soil Characteristics

(1) Gross vehicle wt, lb = _____

(2) Nominal tire width, in. = _____

(3) Rim diameter, in. = _____

(4) Number of tires powered = X_2 = _____

(5) Tire ply rating = _____

(6) Tire pressure, psi = X_3 = _____

Factors

(7) $\frac{\text{Nominal tire width, in.}}{\text{Rim diameter, in.}}$ = _____; if ≥ 2.4, factor (7) = 2.0
if < 2.4, factor (7) = 5.0

(8) Wheel diameter factor = (7)** × (2) + (3) = _____
= (7) × _____ + _____ = _____

(9) Contact pressure factor $= 0.607 \times (6) + 1.35 \left(\frac{117.0 \times (5)}{(8)}\right) - 4.93 =$ _____

$= 0.607 \times$ _____ $+ 1.35 \left(\frac{117.0 \times ___}{___}\right) - 4.93 =$ _____

(10) Contact area factor = $X_1 = \log \frac{(1)}{(9)} = \log$ (_____) = _____

(11) Strength factor = -0.350 × (10) + 0.0526 × (4) + 0.0211 × (6)
+ 1.5870 = _____ = -0.350 × _____ + 0.0526 × ___
+ 0.0211 × _____ + 1.5870 = _____

Vehicle cone index (VCI_1) = antilogarithm (11) = antilogarithm (_____)
= _____

* Logarithm to the base 10.
** Number in parentheses indicates value assigned to that factor number.

Table A4

Data Form for Computing Maximum Net Drawbar Pull (DBP_{max}) and Maximum Slope Negotiable (S_{max}) for Wheeled Vehicles in Sands

Vehicle _____

Basic Equations

DBP_{max}(% of vehicle wt) = $28.87X_1 + 10.10X_2 - 1.52X_3 - 0.61X_4 - X_5$ = _____

S_{max}(%) = $28.87X_1 + 10.10X_2 - 1.52X_3 - 0.61X_4 - X_6$ = _____

Vehicle Characteristics and Cone Index

(1) Gross wt, lb _____ (2) Nominal tire width, in. _____

(3) Rim diameter, in. _____ (4) No. of powered tires _____

(5) Tire ply rating _____ (6) Tire inflation pressure, psi ____

(7) Cone index of 0- to 6-in. layer _____

X Factors

X_1 = strength factor = log (7)* = _____

X_2 = contact area factor = $\log \frac{(1)}{(X_a)}$ = _____

X_a = contact pressure factor = $0.607 \times (6) + 1.35 \left(\frac{117.0 \times (5)}{(X_b)}\right) - 4.93$ = ___

X_b = wheel diameter factor = $X_7 \times (2) + (3)$ = _____

X_3 = same as (4)

X_4 = same as (6)

X_5 = 43.82 for maximum net drawbar pull computations

X_6 = 45.82 for maximum slope negotiable

$X_7 = \frac{\text{Nominal tire width, in.}}{\text{Rim diameter, in.}}$ = _____ if ≥ 2.4 factor (7) = 2.0
if < 2.4 factor (7) = 5.0

DBP_{max} = 28.87(____) + 10.10(____) - 1.52(____) - 0.61(____) - 43.82

= _____

S_{max} = 28.87(____) + 10.10(____) - 1.52(____) - 0.61(____) - 45.82 = __

* Number in parentheses indicates the vehicle characteristic, cone index, or X factor to use.

where for equations A5, A6, A7, and A8

X_1 = slope, percent

X_2 = log of estimated total contact area in square inches (see tables A3 and A4)

X_3 = number of tires powered

X_4 = tire pressure, psi

X_5 = log of cone index

45.82 and 43.82 = equation constants

14. The equations above were derived from available data, and after-the-fact field tests have not been conducted to determine their prediction accuracy. However, on the basis of the go and no go performance data from which the relations were derived, the equations for maximum slope negotiable (equation A7) and VCI_1 (equation A5) predicted vehicle performance with an accuracy of 78 percent. The quality of the relations established, in terms of absolute deviation,* was the same (0.02) for both maximum drawbar pull (equation A6) and towed motion resistance (equation A8).

15. Tracked vehicles. Data available on tracked vehicle performance in coarse-grained soils indicate that tracked vehicles usually experience little or no difficulty traversing level, clean sands. The effect of soil strength and ground contact pressure on the performance (drawbar pull and slope climbing) of a given tracked vehicle is small. Track type appears to be the principal factor influencing performance, as shown in fig. A7. On the basis of these data, average drawbar pull values of 50 and 56 percent of gross vehicle weight are used for prediction purposes for flexible and girderized tracks, respectively. Motion resistance has not been well defined. For the time being, however, the following equations are used.

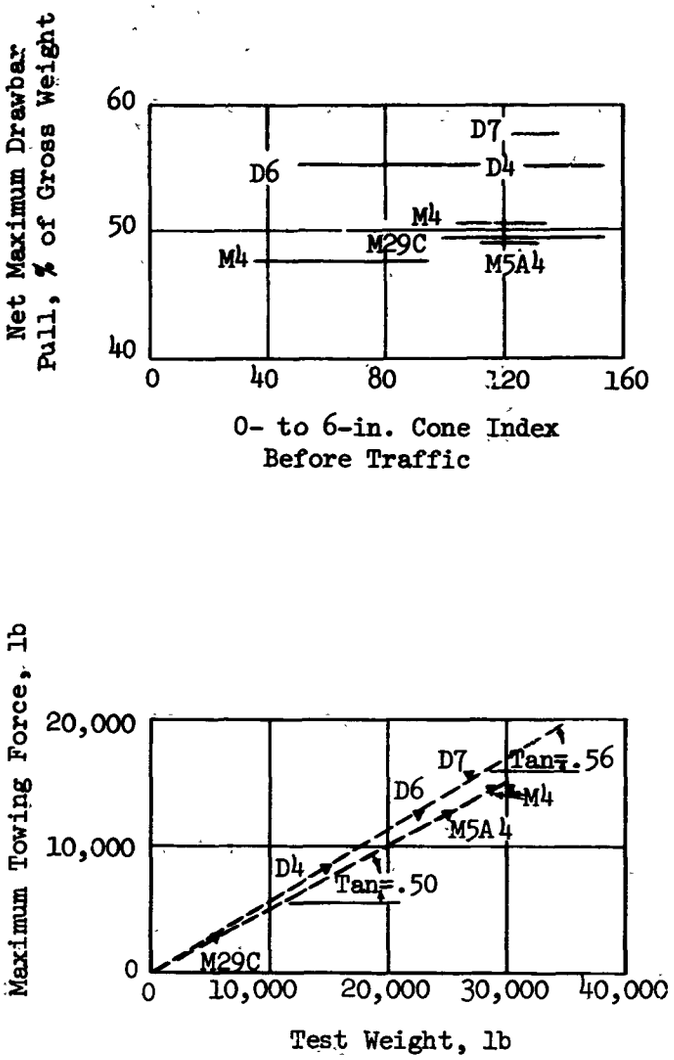
Flexible tracks

$$MR_s = 0.10W \tag{A9}$$

Girderized tracks

$$MR_s = 0.074W \tag{A10}$$

* Without regard to algebraic signs.



Legend

D4,D6,&D7 Engineer Tractors (Girderized Tracks)
M4 & M5A4 Hi-Speed Tractors } (Flexible Tracks)
M29C Weasel

Fig. A7. Performance of tracked vehicles in dry-to-moist sand

where

MR_s = towed motion resistance (soil), lb

W = gross vehicle test weight, lb

16. Fundamental relations. The fundamental relations employed in the WES VCI model for predicting performance of vehicles operating in coarse-grained soils are summarized on the following page.

a. VCI for wheeled vehicles is determined using a regression equation (equation A5) involving vehicle type, soil slope, and vehicle characteristics. This equation is for dry-to-moist sands only; investigations of wet and inundated sands, which may present some problems for heavy vehicles because of load liquefaction, have not been completed.

b. Means for computing a VCI for tracked vehicles have not as yet been developed.

c. The net maximum drawbar pull coefficient and the corresponding maximum slope negotiable are determined from empirical relations of CI and vehicle type classifications (equations A6 and A7).

d. Motion resistance coefficient or percent of gross vehicle weight on level ground may be estimated for a wheeled vehicle by its relation to tire pressure and CI (equation A8) and to track type for tracked vehicles (equations A9 and A10).

WES Mobility Numeric Model (Wheeled Vehicles Only)

17. The WES mobility numeric model consists of one set of empirical relations for predicting the performance of a single tire moving at 5 to 6 fps in a straight line in a fully remolded clay, and a second set for performance in a dry sand. These relations were developed by the consolidation of systematic laboratory test data by means of dimensional analysis, extended by regression techniques. After the basic clay numeric was developed, analysis of data from field tests with prototype wheeled vehicles indicated that the use of RCI rather than CI would increase prediction accuracy. Selected performance parameters (minimum soil strength required to complete the first pass--VCI_1, drawbar pull, input torque at 20 percent slip, and towed motion resistance) are presented in proper dimensionless form as unique functions of a "mobility numeric," which is a dimensionless load coefficient expressing the ratio of soil strength (in RCI terms) to a nominal unit load exerted by the deflected tire upon the soil.

18. The basic mobility numeric relations were developed from laboratory results of more than 184 single-wheel tests in a heavy clay soil and over 122 tests in sand. Each test involved soil strength and tire behavior measurements during three to five passes of a wheel in the same path and under a constant load. During each pass, the slip was slowly increased linearly from -20 to 100 percent. In the remolded clay soil the drawbar pull and torque of a wheel as related to slip were essentially the same from pass to pass, but differences were found in sand because its strength may increase or decrease under the action of traffic, depending essentially on its before-traffic density.

19. The soil was so prepared in the laboratory that reasonably uniform soil conditions were achieved. The mass soil strength for the clay soils in which tires were tested ranged from 7 to 45 CI (0- to 6-in. depth), while sand strengths were from 8 to 77 CI (0- to 6-in. depth) or, in terms of the strength gradient (G) over a depth equal to the tire width, were from 2.6 to 25.7 psi/in.

20. Tires used in the development of the basic mobility numeric data were treadless. The largest was 41.3 in. in diameter and the smallest was 8 in. Ratios of width to diameter ranged from 0.15 to 0.92.

21. Assumptions currently involved in this model are listed below.

a. General.

(1) The basic dimensional analysis of the tire-soil relation using a single cone index (C), rating cone index (RCI), or cone index gradient (G) as the soil strength parameter is valid and adequate.

(2) Contributions of soil dynamics and shear-rate-dependent effects to net tire performance are negligible in the soil-tire systems under study.

(3) Empirically determined relations accounting for changes in tire proportions (b/d, δ/h, h/d), which enter the final form of the mobility numeric, are generally applicable to similar soil-tire systems.

(4) The vehicle moves in equilibrium in a straight line on

level soil at no more than normal vehicle speeds off-road.

(5) The performance of a (multiwheeled) vehicle is the sum of the performance (+) of its individual tires.

(6) It is adequate for design and simulation to represent a slip-dependent relation in terms of performance at 20 percent slip, which is near optimum in terms of drawbar efficiency.

b. Soil.

(1) The soil body is essentially uniform and is either purely cohesive or entirely frictional (c/ϕ soils cannot yet be handled).

(2) The mean CI of a uniform clay soil body in the laboratory, measured with standard cone penetrometer, adequately describes those properties of the soil which relate to vehicle performance.

(3) For field performance predictions in clay soils, mean RCI within the critical layer, as defined by the WES VCI method, is used in place of the laboratory mean CI to calculate the controlling mobility numeric.

(4) There is no significant discontinuity between surface and mass soil strengths.

(5) The gradient (G) of CI with penetration distance adequately describes the properties of both natural and laboratory beds of purely frictional soils as these relate to slow-speed-vehicle performance.

c. Vehicle.

(1) Tire geometry is suitably described by measurements of undeflected tire diameter, section width and section height, plus, as an index of tire flexibility in the soil, static deflection under the test load, when on an unyielding surface.

(2) The primary effects of tire pressure on performance are fully accounted for by the deflection measurement.

(3) Tread pattern has only a negligible influence on tire performance in uniform soils.

d. Soil parameters.

(1) For natural clay, RCI is the only soil parameter.

(2) For natural sand, CI gradient is the only soil parameter.

e. Vehicle parameters. Tire parameters are wheel load (W), undeflected section width (b), unloaded diameter (d), unloaded section height (h), and hard surface deflection (δ).

Mobility numerics

22. Predictions are made from empirical relations of dimensionless performance numbers (listed below) and dimensionless mobility numerics.

P_{20}/W* = drawbar pull coefficient

P_T/W = towed motion resistance coefficient

Z_{20}/d = sinkage coefficient

Two of the performance parameters measured during tests that relate to the performance numbers listed above are pull (P_{20}) and pull required for a free-rolling wheel (P_T). When it is necessary to estimate motion resistance at 20 percent slip, the towed force is used as a reasonable approximation.

23. The WES clay mobility numeric for natural soil is

$$\frac{RCI\ bd}{W}\left(\frac{\delta}{h}\right)^{1/2}\left(\frac{1}{1+\frac{b}{2d}}\right) \qquad \text{(A11)}$$

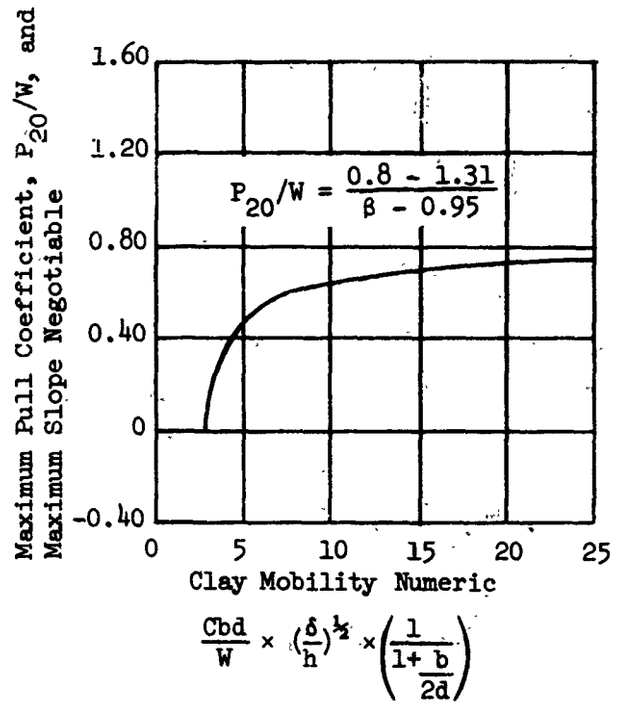
where

RCI = rating cone index for the 0- to 6-in. depth

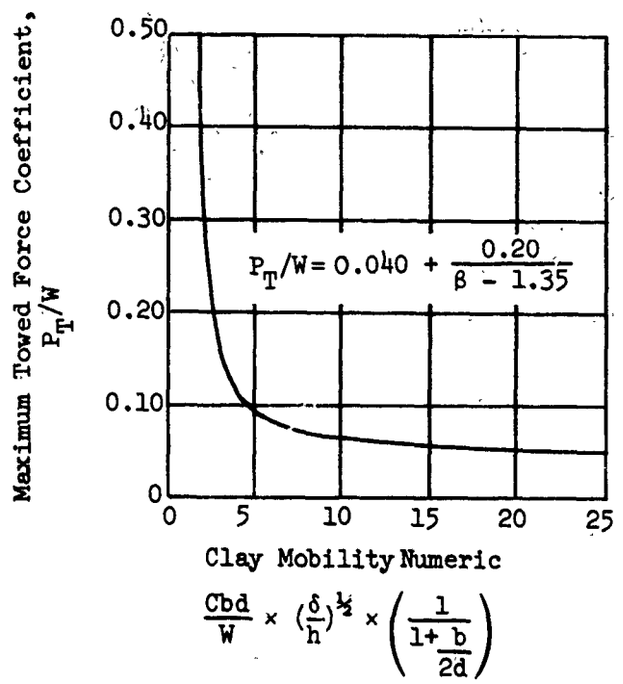
(See paragraph 1 for identification of other symbols.)

Relations of performance coefficients for the first pass of a single wheel and revised clay mobility numeric are given in fig. A8. Relations of pull

* Subscript, 20, denotes slip, in percent.



a. Pull coefficient vs clay mobility numeric from field data



Note:

$$\beta = \frac{(RCI)bd}{W}\left(\frac{\delta}{h}\right)^{\frac{1}{2}}\left(\frac{1}{1+\frac{b}{2d}}\right)$$

b. Towed force coefficient vs clay mobility numeric from field data

Fig. A8. Relations of performance coefficients for first pass of single wheel to clay mobility numeric

and towed force coefficients and the original WES numeric for clay are given in fig. A9.

24. The WES sand mobility numeric is

$$\frac{G(bd)^{3/2}}{W} \times \frac{\delta}{h} \tag{A12}$$

where

G = the slope of the curve of penetration resistance versus depth, averaged over a 6-in. depth. (See paragraph 1 for identification of other symbols.)

Relations of performance coefficients for the first pass of a single wheel, slope negotiable, and towed force and sand mobility numeric are given in fig. A10.

Fundamental relations

25. The fundamental relations pertinent to this model are as follows:

a. The clay mobility numeric is determined from the soil and vehicle parameters and subsequently used in the relations (fig. A9) to obtain performance coefficients for a single wheel or wheeled vehicle. Actual wheel performance values are calculated by multiplying the coefficients by W or Wd for the wheel, as appropriate. The performance of a complete vehicle is obtained by summing the performance of each of its wheels (including a net motion resistance or negative performance when a tire is not powered or has insufficient net traction to propel itself). Performance measured in the laboratory during the second through fifth passes varied but little from that during the first pass. Thus, although the graphs were developed by the consolidation of first-pass test results, they can be used to calculate pull, torque, and towed force for every individual wheel on a vehicle, and even several passes of the vehicle.

b. The sand mobility numeric, calculated from the soil and vehicle parameters, is used to determine corresponding performance coefficients from the curves shown in fig. A10.

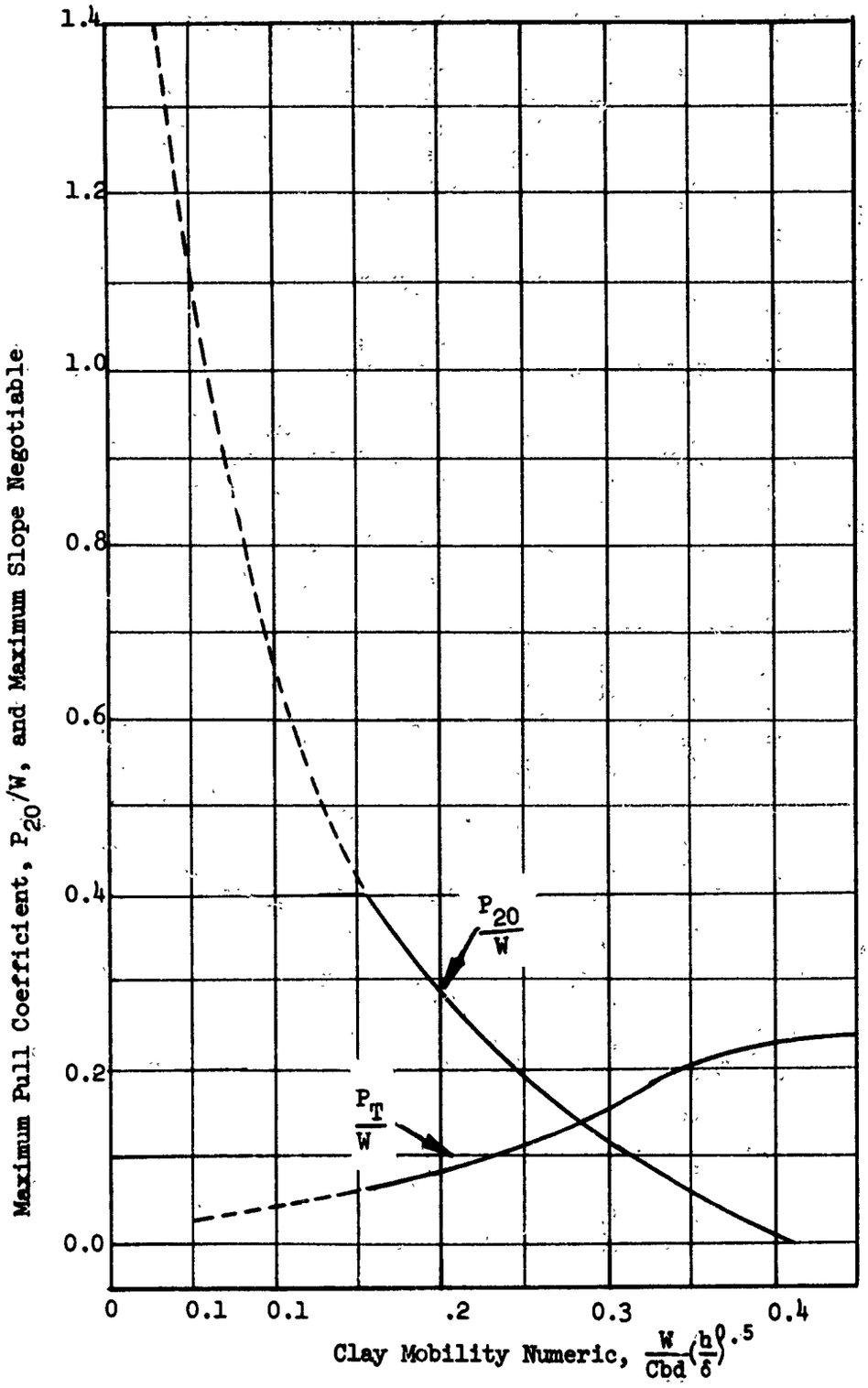
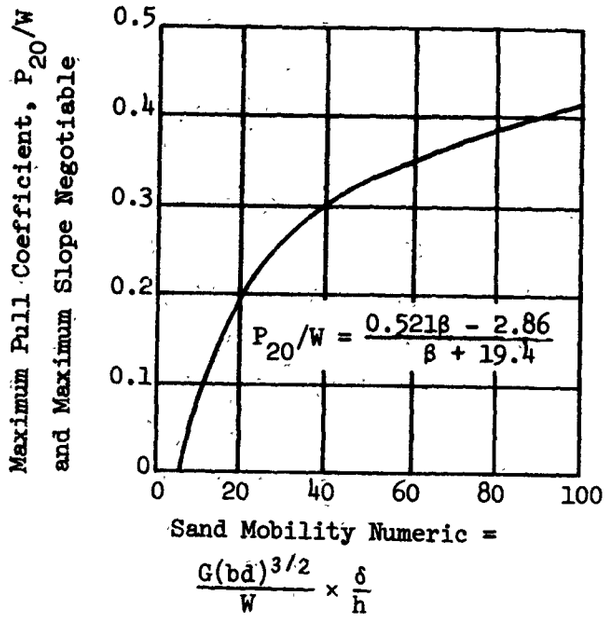
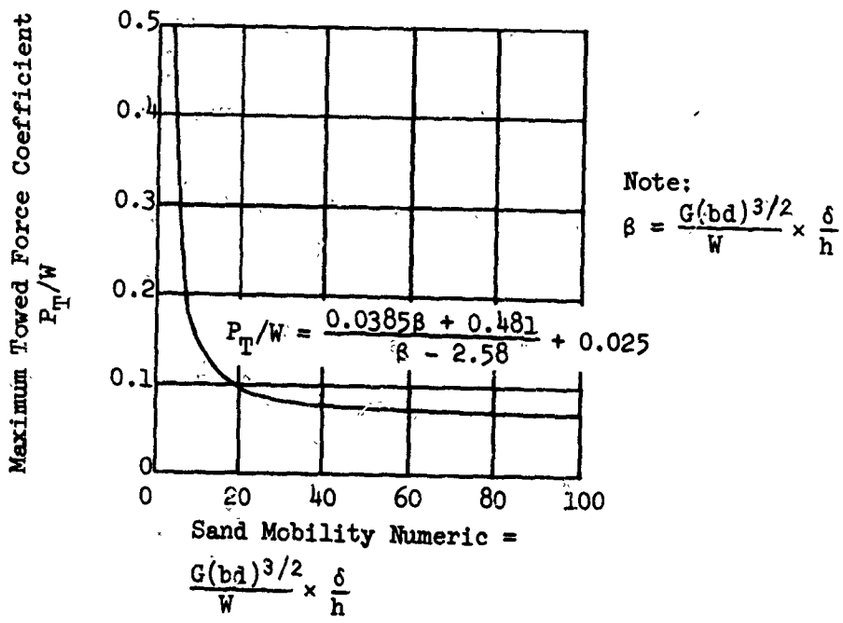


Fig. A9. Relation of pull, slope, and towed force coefficients to WES numeric for clay



a. Pull coefficient vs sand mobility numeric



b. Towed force coefficient vs sand mobility numeric

Fig. A10. Relation of performance coefficients for first pass of single wheel to sand mobility numeric

Bekker/LLD Soil-Vehicle Model

26. The Bekker/LLD soil-vehicle model was first assembled by M. G. Bekker and has subsequently been modified by U. S. Army Tank Automotive Command (USATACOM) Land Locomotion Division (LLD) in various minor ways on the basis of experimental data. Still other refinements have been proposed by other researchers (Reece, Firth, etc.). The model has both theoretical and empirical aspects. It includes methods to predict vehicle sinkage, motion resistance, and drawbar pull versus slip in soft soils. In firm soils, where motion resistance is small, it is usually neglected. Equations are available for predicting the performance of tracked and wheeled vehicles, often in several forms. The forms differ according to the detailed assumptions made, the level of mathematical treatment, and the overall framework within which the results are to be used.

27. Six soil values measured by means of a bevameter are employed in the various equations. Values for soil cohesion (c), friction angle (ϕ), and tangent modulus (K), used in the equation for gross tractive effort (H), are determined from tests upon a small grousered annular ring loaded to several normal loads. The ring is rotated under each load until the soil beneath is sheared, and a record of torque versus angular deformation obtained. c, ϕ, and K are computed from the records for several normal loads. Three soil constants are used in the equations for track or wheel sinkage: soil moduli of cohesion (k_c) and friction (k_ϕ), and n, an exponent of sinkage. These are obtained by analysis of curves plotted for results of pressure-sinkage tests with two or more small-diameter flat plates forced vertically into the ground. Three further soil values are used in a now little used equation for "bulldozing" resistance: two coefficients of bulldozing (K_ϕ and K_b) and soil bulk density (γ). Other symbols used in various current equations are as follows:

s = slip ratio (dimensionless)

D = undeflected tire diameter, in.

z_m = maximum sinkage, in.

τ = shear stress, psi

28. The Bekker/LLD model was created by combining and amplifying simple models whose beginnings may be traced to Bernstein (1913), Nichols (1931), Goriatchkin (1936), and Mickelthwaite (1944). Bekker consolidated the earlier approximations into the first coherent engineering system for calculating vehicle-soil interactions (1956). Since their first publication, Bekker's equations have been more-or-less continuously modified in various details, largely by LLD, on the basis of data obtained from several hundred scale-model vehicle tests conducted in the laboratory in several soil types (clay, sand, and mixtures thereof) prepared to various consistencies. Several hundred field tests in a number of different representative soil conditions were also conducted with full-scale vehicles to verify the predictions. Details of laboratory experiments and full-scale vehicle tests are not well documented in the literature, however.

29. Assumptions pertinent to this model as currently used are listed below.

a. General.

(1) Soil parameters derived from analyses of soil response data obtained with the bevameter are soil invariants, not significantly affected by instrument configuration.

(2) Values for the soil parameters at points disturbed by the passage of a vehicle are unaffected.

(3) Pressure distributions under moving tires and tracks are two-dimensional, are effectively analogous to the axisymmetric pressure distributions under small circular plates, and are adequately represented at all points by the power expression $p = kz^n$.

(4) Response of the soil body to vertical and horizontal forces imposed on it by the vehicle may be calculated independently and subsequently combined by simple superposition; i.e., vehicle sinkage is independent of track or wheel traction and slip.

(5) Gross vehicle traction is developed by shearing soil along a surface defined by the instantaneous position of wheel or tire grouser tips.

(6) Shear stress (τ) developed at any point in the shear failure surface can be expressed as a function of cumulative shear deformation (j) along that surface by the expression

$$\tau = (c + p \tan \phi)(1 - e^{-s\ell/K})$$

(7) A vehicle track is treated as a rigid, flat plate, whose sinkage is bounded between that calculated considering sinkage to be uniform along the nominal contact length, and sinkage calculated by considering that it be zero at the front of the contact area, increasing linearly to a maximum at the rear.

(8) Pneumatic tire performance is bounded between that of a rigid, cylindrical wheel having the same overall diameter and width as the undeflected tire, and that of a short track whose length equals the contact length of the deflected tire on a hard surface, and whose width equals the undeflected tire section width.

(9) The ground contact surface of a vehicle tire or track is defined by the assumed running gear geometry connecting the point where this intersects the undisturbed soil level with the point of its greatest penetration into the soil, and by overall width of the tire or track.

(10) The vehicle moves in static equilibrium in a straight line at a slow uniform speed on terrain with a plane surface.

(11) Traction and support developed by track or tire contact with rut sidewalls are negligible.

(12) Vehicle sinkage on sloped soil surfaces is identical with that on level surfaces.

(13) The net traction of a (multiwheeled or multitracked) vehicle is the algebraic sum of the net traction of

all tires or tracks, each calculated as though operating in level soil unrutted by the prior passage of any running gear ahead.

b. Soil. The soil mass is homogeneous and semi-infinite with no hardpan at any depth from the surface less than twice the maximum sinkage.

c. Vehicle.

(1) Slip of all wheels or tracks is the same.

(2) The weight distribution on the ground contact elements does not change under applied external loads.

(3) Power train torque available at the running gear is sufficient to ensure that soft-soil performance is limited by soil-vehicle interactions rather than available torque.

Soil parameters

30. Soil parameters obtained from results of tests with the bevameter are k_c , k_ϕ , n , c , ϕ , and k , where

$$k = \frac{k_c}{b} = k_\phi$$

b = smallest dimension (inches) of penetrating element (for a circular plate penetration test, b is the radius).

Vehicle parameters

31. Vehicle parameters are as follows:

a. Wheeled vehicle parameters are W , D , b , s , and δ .

b. Tracked vehicle parameters are W , ℓ , s , and b .

Fundamental relations

32. The fundamental relations pertinent to this model are as follows:

a. Resistance for all vehicle types.

(1) Compaction resistance.

$$R_c = \left(\frac{bk}{n + 1}\right) z_m^{n+1} \qquad \text{(A13)}$$

(This equation is often written with one of the sinkage expressions substituted for z, which changes its appearance grossly without in any way altering its actual meaning.)

(2) Gravity resistance.

$$R_\theta = W \sin \theta \tag{A14}$$

(3) Trim angle resistance.

$$R_\alpha = W\left[\sin (\theta + \alpha) - \sin \alpha\right] \tag{A15}$$

(4) Mechanical rolling resistance.

$$R_r \tag{A16}$$

(From manufacturer's data engineering tests or generalized handbook information.)

(5) Total motion resistance.

$$R = R_c + R_\theta + R_\alpha + R_r \tag{A17}$$

(6) Motion resistance coefficient.

$$\frac{R}{W} = \frac{z}{\ell(n + 1)} + \sin \theta + \sin (\theta + \alpha) - \sin \alpha + \frac{R_r}{W} \tag{A18}$$

b. Net traction for all vehicle types.

(1) Drawbar pull or net traction.

$$DBP = H - R \tag{A19}$$

where

H = gross traction

(2) Traction coefficient.

$$\frac{DBP}{W} = \frac{H}{W} - \frac{R}{W} \tag{A20}$$

c. Rigid wheels.

(1) Sinkage.

$$z = \left[\frac{3W}{bK\sqrt{D(3 - n)}}\right]^{\frac{2}{2n+1}} \tag{A21}$$

(2) Gross traction for high slip only.

$$H = Ac + W \tan \phi \tag{A22}$$

where

$A = b\ell$

$\ell = \sqrt{z(D - z)}$

(3) Gross traction for any slip value (cycloid method).

$$H = H_1 - H_2 \tag{A23}$$

where

$\theta_v = 2\pi - \cos^{-1}(1 - 2z/D)$

$\theta_o = 2\pi - \cos^{-1}(1 - s)$

θ = angle that varies between θ_v and 2π

$\theta_o \leq \theta_v$ Case 1

$\theta_o > \theta_v$ Case 2

$$p = k\left[\frac{D}{2}(\cos\theta - \cos\theta_v)\right]^n$$

Case 1: $H_1 = \frac{bD}{2}\int_{\theta_v}^{2\pi} (c + p \tan \phi)(1 - e^{-j_1/k}) \cos\theta \, d\theta$

$H_2 = 0$

$$j_1 = \frac{D}{2}\left[(1 - s)(\theta_v - \theta) + \sin\theta - \sin\theta_v\right]$$

$$j_2 = 0$$

Case 2: $$H_1 = \frac{bD}{2}\int_{\theta_v}^{2\pi} (c + p\tan\phi)\left(1 - e^{-j_1/K}\right)\cos\theta\, d\theta$$

$$H_2 = \frac{bD}{2}\int_{\theta_v}^{\theta_o} (c + p\tan\phi)\left(1 - e^{j_2/K}\right)\cos\theta\, d\theta$$

$$j_1 = \frac{D}{2}\left[(1 - s)(\theta_o - \theta) + \sin\theta - \sin\theta_o\right]$$

$$j_2 = \frac{D}{2}\left[(1 - s)(\theta_v - \theta) + \sin\theta - \sin\theta_v\right]$$

d. Deflected tire and track that sinks uniformly.

(1) Sinkage.

$$z = \left(\frac{W}{Ak}\right)^{1/n} \tag{A24}$$

(2) Gross traction.

$$H = (Ac + W\tan\phi)\left[1 - \frac{K}{s\ell}\left(1 - e^{-s\ell/K}\right)\right] \tag{A25}$$

where

for wheel, $\ell = 2\sqrt{\delta(D - \delta)}$

for track, ℓ = length of contact area of track

$A = b\ell$

(3) Gross traction with grouser effects.

$$H = \left\{Ac\left(1 + \frac{2h}{b}\right) + W\tan\phi\left[1 + 0.64\left(\frac{h}{b}\cot^{-1}\frac{h}{b}\right)\right]\right\}\left[1 - \frac{K}{s\ell}\left(1 - e^{-s\ell/K}\right)\right] \tag{A26}$$

where

h = grouser height

e. Track that sinks nonuniformly.

(1) Sinkage at rear of track.

$$z_m = \left[\frac{(n + 1)W}{Ak}\right]^{1/n} \tag{A27}$$

(2) Gross traction.

$$H = A\left\{c\left[1 - \frac{K}{s\ell}\left(1 - e^{-s\ell/K}\right)\right] + kz_m^n \tan\phi\left(\frac{1}{n + 1} - \frac{1}{\ell^{n+1}}\int_0^{\ell} x^n e^{-sx/K}\,dx\right)\right\} \tag{A28}$$

f. Power required (HP) to overcome compaction resistance.

$$N_e = \frac{v}{550(1 - s)\,\eta\,(n + 1)(bk)^{1/n}}\left(\frac{W}{\ell}\right)^{\frac{n+1}{n}} \tag{A29}$$

where

v = velocity, fps

η = overall efficiency of power transfer between running gear and engine

g. Other. Relations that have been replaced or are seldom used are given below. These equations are included because they were used in at least one of the comprehensive ground-mobility models considered in the ANAMOB study.

(1) Bulldozing resistance R_b.

$$R_b = b\left(2K_b zc + K_\theta \gamma z^2\right) \tag{A30}$$

where

$$K_b = (N_c - \tan\phi)\cos^2\phi\,(1 + \cot\alpha\tan\phi)$$

$$K_\theta = \left(\frac{2N_\gamma}{\tan\phi} + 1\right)\cos^2\phi\,(1 + \cot\alpha\tan\phi)$$

N_c and N_γ = Terzaghi's bearing capacity factors

For wheels $\alpha = \cos^{-1}\left(1 - \frac{2z}{D}\right)$

(2) Bulldozing resistance B_x.

$$P_x = \frac{1}{2}\gamma z^2 \left[\frac{(\cot x_c - \tan\beta)}{\cot(x_c + \phi) - \tan\beta}\right] + cz\left[\frac{\tan(x_c + \phi) + \cot x_c}{1 - \tan\beta\tan(x_c + \phi)}\right] \quad \text{(A31)}$$

where

β = angle of approach

x_c = angle of rupture plane

If

$\beta = 0$

then

$x_c = 45^\circ - \frac{\phi}{2}$

and

$$B_x = \frac{1}{2}\gamma z^2 \tan^2\left(45^\circ + \frac{\phi}{2}\right) + 2cz\tan\left(45^\circ + \frac{\phi}{2}\right)$$

(3) Drawbar pull.

$$DBP = H - (R_c + R_b + R_\theta) \quad \text{(A32)}$$

where

$R_\theta = W \sin\theta$

(4) Bulldozing of front wheels.

$$p = k(z_1 + z_2)^n \quad \text{(A33)}$$

where

z_1 = sinkage of front wheels

z_2 = sinkage of second wheels relative to z_1

(5) Motion resistance of rigid wheel.

$$R_c = \frac{1}{(3 - n)^{\frac{2n+2}{2n+1}}(n + 1)(bk)^{\frac{1}{2n+1}}}\left(\frac{3W}{\sqrt{D}}\right)^{\frac{2n+2}{2n+1}} \quad \text{(A34)}$$

Predictions

33. Predictions are straightforward after the running gear of the vehicles has been defined and the soil and vehicle parameters have been determined. One of the problem areas in this system is to differentiate between rigid wheels and wheels mounted with pneumatic tires. A rigid pneumatic tire is defined by any inflation pressure above the critical inflation pressure and the critical inflation pressure is determined from the following equation.

$$p_{ic} = \frac{(n + 1)W}{b\sqrt{z(D - z)}} \tag{A35}$$

where

$$z = \left[\frac{3W}{(3 - n)bk\sqrt{D}}\right]^{\frac{2}{2n+1}}$$

In the equation above, however, the determination of z requires the assumption that the wheel is rigid.

Reece-modified Bekker/LLD equations

34. Pressure-sinkage relation. Reece's modification of the pressure-sinkage relation, which he considers as tentative, is as follows:

$$p = \left(ck'_c + \gamma \frac{b}{2} k'_\phi\right)\left(\frac{z}{b}\right)^n \tag{A36}$$

where

k'_c , k'_ϕ , and n = dimensionless soil constants

35. Gross traction of grousered track. The equation for gross traction H proposed by Reece is:

$$H = (H_{base} + H_{side})\left(1 + \frac{K}{s\ell} e^{-s\ell/K} - \frac{K}{s\ell}\right) \tag{A37}$$

where

$$H_{base} = b^2cN_c + b^2c_aN_a + b^2p_oN_q$$

$$H_{side} = 4ch_g\ell \sin^2\left(45 + \frac{\phi}{2}\right) + 2\gamma h_g\ell z_m \tan\left(45 + \frac{\phi}{2}\right) \cos(90 - \phi)$$

where

$N_c = (1 - m)\left(\frac{\ell}{b}\right)$

c_a = adhesion

$N_a = m\left(\frac{\ell}{b}\right)$

$p_o = \frac{W}{(b\ell)}$

$N_q = \frac{\ell}{b} \tan \phi$

h_g = height of track grouser

m = ratio of the area of the grouser tips to the total contact area

Reece hopes to replace K, a purely empirical constant, in the near future. He also is working on a modification of the slip-sinkage relation.

Perloff Model

36. The Perloff model is essentially an unvalidated procedure for predicting the effect of soil strength on the performance of tracked vehicles only. It is complex and any precision gained by the computation involved has not been demonstrated. The only application of the model has been in a digital computer simulation of a tank engagement to predict tank performance in soft soil from design characteristics. The output of the complete simulation was intended to show the effect of individual design features, including mobility elements, on the overall effectiveness of tank-weapon systems. Soil parameters used in the Perloff model are the Mohr-Coulomb failure parameters c and ϕ, and shear stress/deformation data measured in a standard direct shear test, and a pressure-sinkage relation. Specific equipment and procedures that should be used to obtain the pressure-sinkage relation to be used are not identified.

37. The model first finds, by a computer iterated process, vehicle sinkage and trim, which satisfy the assumed pressure-sinkage assumptions, plus static equilibrium equations which embrace terrain slope. It then computes tractive force and track slip on the basis of some broad assumptions relating soil displacement under a track to the unit shearing strain experienced by the soil. By applying the shear stress/deformation data, a distribution of shear stress under the track is developed as a function of

the track slip and integrated to obtain corresponding total and net tractive efforts.

38. Then calculating track sinkage and attitude, the model assumes the track to be a flat, rigid plate, plus a front ramp treated as a passive earth pressure problem, and deals with it as a two-dimensional problem only. In common with similar but less complex models, the unspecified pressure-sinkage relation is taken to be independent of the size and shape of the penetrating surface, and of the path of any part of that surface in reaching a final sinkage. Again, as is commonly done, the effects of normal and tangential loadings are computed separately and considered to be totally independent. External motion resistance due to soil failure is considered to be reflected completely in the final vehicle trim angle.

39. Assumptions and specific steps involved in calculating traction (Perloff, 1966) are:

> In order to relate the soil displacement at the surface to the unit strain experienced by the soil, the total shear distortion is assumed linear with depth. It is further assumed that above some critical depth the soil is affected by the shear stresses, and below that depth the soil is unstressed. An estimate of this depth has been obtained by analysis of an analogous elastic problem. The result given by Scott (1963, p. 500) for the shear stress distribution due to a point shear force on the surface of an elastic half space has been integrated twice to yield the stress distribution due to an applied strip shear loading. The results of the analysis are shown as a plot of shear stress along the center line of the track, τ_{xz} , as a function of the depth [Fig. A11]. Note that at a depth of twice the track width, the shear stress diminishes to 15 percent of the applied stress. Examination of many stress-strain curves for soils indicates that the strains corresponding to a stress level of 15 percent of the maximum shear stress are negligible. Therefore, a depth of twice the track width was chosen as the depth above which the soil is affected by the shear stress applied by the track, and below which the soil remains undisturbed. Knowing this depth, the unit shear strain is obtained by dividing the soil displacement at the track surface by the depth of soil affected.
>
> It is then necessary to know the relationship between shear stress and shear strain for each given soil. The direct shear test provides the closest analogy to the shearing effect produced by a tank track of all the

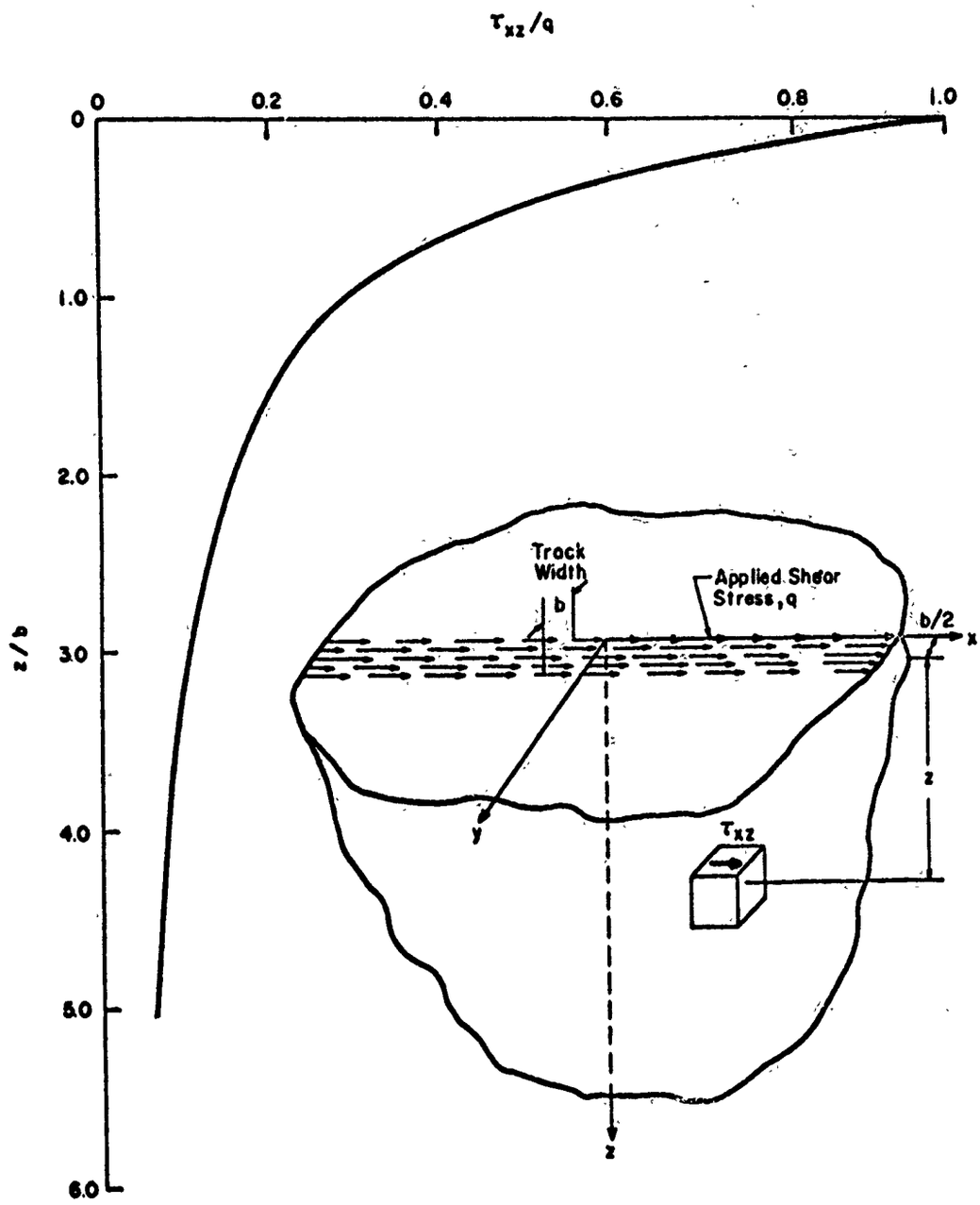


Fig. A11. Shear stress (τ_{xy}) under center line of track as a function of depth for an applied strip shear loading

standard engineering tests on soils. Results presented by Roscoe (1953) indicate that the shear strains in a direct shear test are distributed over approximately the middle one-quarter of the specimen. Assuming the strains are uniformly distributed over this height, it is possible to determine the unit shear strain, and to relate the direct shear test results to the conditions under a tank track.

For each value of slip, the displacement at various points under the track, d , is computed and converted to the displacement, d_{ds} , in the direct shear test which corresponds to the same unit strain, i.e.,

$$d_{ds} = d \frac{TS/4}{TT} \qquad [A38]$$

where TS is the thickness of the direct shear test specimen, and TT is the depth of soil affected by the shear stresses imposed by the track (TT = 2b).

Computation of tractive effort is complicated by the fact that, in general, the shear stress in the soil depends not only upon the induced strain but also upon the normal pressure. The results of direct shear tests on a variety of soils suggest that the Coulomb (1776) equation can reasonably be approximated by:

$$\tau(\epsilon) = A(\epsilon) + B(\epsilon)\sigma_n \qquad [A39]$$

where (τ) is the shear stress at any strain, ϵ . $A(\epsilon)$ and $B(\epsilon)$ are empirical parameters determined from the results of two or more direct shear tests at different normal stresses, and σ_n is the normal pressure. When the soil strength is fully utilized, equation [A39] reduces to the Coulomb equation for which $A(\epsilon) = c$ and $B(\epsilon) = \tan \phi$.

Direct shear test results for the soils being considered are read into the computer in terms of $A(\epsilon)$ and $B(\epsilon)$, from equation [A39], for various values of shear displacement.

The values of $A(\epsilon)$ and $B(\epsilon)$ which correspond to the various d_{ds} at different points under the track are determined by Lagrange interpolation (Milne, 1949) between the input data points.

Using the normal pressure distribution under the track determined previously, and substituting into equation [A39], the shear stress distribution under the track is computed.

The total tractive force per unit track width is

determined by numerically integrating the shear stress distribution under the track by Simpson's rule (Milne, 1949).

Input data

40. Vehicle and terrain data inputs required for this model are listed below.

a. Tank. (A schematic representation of a tank in the soil is given in fig. A12.)

(1) Weight, per unit of total track width, W_1 .

(2) Location of center of gravity.

$\overline{X}$ = distance to front of track base

$\overline{Y}$ = distance above track base

(3) Length of track base in contact with ground, ℓ .

(4) Angle (Δ) between the track base plane and the forward inclined portion of tank track, ℓ_2 .

(5) Undercarriage clearance.

(6) Values of maximum power/torque limited velocity, V'_{max} , on a rigid surface for various tank inclinations, α .

(7) Track width, b .

b. Terrain.

(1) Ground surface inclination, θ .

(2) Cohesive component of soil strength, c .

(3) Angle of shearing resistance, ϕ .

(4) Unit weight of the soil, γ .

(5) Data points from the assumed pressure-sinkage relation for the soil.*

(6) Data points from the expression for shear stress-displacement relation for the soil from direct shear test results.

(7) Thickness of the direct shear test specimen.

---

* No generally accepted method for extrapolation of pressure-sinkage data from small-scale models to prototype vehicles is currently available; therefore, the general shape of curves obtained experimentally from model studies have been assumed.

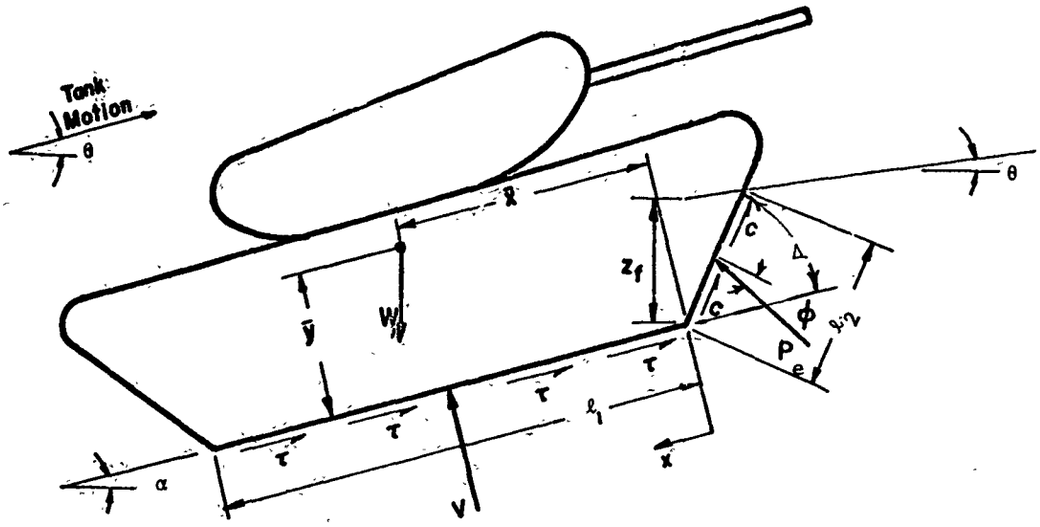


Fig. A12. Schematic representation of a tank in the soil

Fundamental relations

41. The fundamental equations and general procedures used in this model are as follows:

a. Sinkage (a rigid track is assumed).

(1) At any point, x , along the track, the sinkage, z , is expressed by

$$z = z_f + \frac{x}{\cos\theta} \sin(\alpha - \theta) \tag{A40}$$

where

z_f = vertical sinkage at front of track base

α = total trim angle of track base length relative to the horizon

(2) Trial values of z_f and α are selected.

(3) Normal pressures along the track contact length are determined to suit the resulting $z = x$ line according to the assumed input pressure-sinkage relation.

(4) Total normal force (V) per unit of track width is obtained by integrating

$$V = \int_0^{\ell_1} p_v(x)d_x \tag{A41}$$

where

$p_v(x)$ = normal pressure at point x

(5) Trial values of z_f and α are tested by examining whether or not the resulting force system is in static equilibrium, i.e. whether or not

$$\int_0^{\ell_1} p_v(x)dx = W \cos \alpha - P_e \cos (\Delta - \phi) - c\ell_2 \sin \Delta \tag{A42}$$

$$\int_0^{\ell_1} \frac{x}{\overline{X}} p_v(x)dx = W_1\left(\cos \alpha + \frac{\overline{Y}}{\overline{X}} \sin \alpha\right) + \frac{0.4\ell_2}{\overline{X}} P_e \cos \phi \tag{A43}$$

where

W_1 = weight per unit track width, lb

P_e = passive earth pressure, psi

The passive pressure force per unit width is determined by iteration for each value of z_f. If equilibrium conditions are not satisfied, inclination, α, is incremented and the process repeated. Values of α are increased until the rear end clearance of the track is reached before z_f is incremented to begin a new cycle. Iteration is continued until suitable values of z and α are obtained or until the tank bellies out.

b. Slip.

$$S = 1 - \frac{V_v}{V_T} \tag{A44}$$

where

V_v = actual forward speed of tank relative to the ground

V_T = velocity of track relative to tank hull

c. Slip velocity between track and soil.

$$V_s = V_T - V_v \tag{A45}$$

d. Soil displacement.

$$d = V_s t \tag{A46}$$

e. Time.

$$t = \frac{x}{V_T} \tag{A47}$$

f. Actual soil displacement at track rear.

$$d_m = sx \tag{A48}$$

g. Maximum soil displacement at track rear.

$$d'_m = s\ell_1 \tag{A49}$$

d'_m may be greater than d_m

h. Shear stress.

$$\tau_x = f(d) \tag{A50}$$

i. Thrust force per unit width of track.

$$T = \int_0^{\ell_1} f(d)dx \tag{A51}$$

j. Tractive force required to just initiate or maintain motion.

$$T_{req} = W_1 \sin \alpha + P_e \sin (\Delta - \phi) - cz'_f \frac{\cos \Delta}{\sin (\Delta + \alpha)} \tag{A52}$$

where

$$z'_f = z_f \left[1 + \frac{\tan (\Delta + \alpha) \sin (\Delta + \alpha) \sin \theta}{\sin (\Delta + \alpha - \theta)}\right]$$

k. Maximum available thrust.

$$T_{max} = T \text{ at some optimum slip value}$$

l. Mobility factor.

$$k'' = \frac{T_{req}}{T_{max}} \tag{A53}$$

m. Drawbar pull.

$$T_{max} - T_{req} = T_{max}(1 - k'') \tag{A54}$$

n. Maximum velocity.

$$V_{max} = V'_{max}(1 - s_{min}) \tag{A55}$$

where

s_{min} = minimum slip at which actual thrust is equal to required thrust

V'_{max} = maximum velocity on hard surface for the slope angle, α , as determined by power and power train characteristics

Output values

42. Output values obtained from this model are as follows:

a. Maximum track speed, V'_{max} , as limited by the soil at the point which is being studied.

b. Actual track speed.

c. The tractive force (thrust), T , per unit of track width exerted between the soil and the tank. When taken with track speed and track width, corresponding horsepower output and fuel consumption rate may be computed.

Predictions

43. To predict vehicle speed with the Perloff model involves a large number of computations, each requiring many iterations. In a

discussion of the application of the model in a computer simulation, Perloff (1966) stated that

> At this time it is difficult to assess the degree of validity of the results of this analysis. As with the development of any model, many simplifying assumptions were required. Although it is believed that the analysis leads to results which are reasonable, and which will serve to distinguish the mobility characteristics of tank design candidates, there is not yet any experimental verification of its predictive capability.

A part of the problem is to specify appropriate instruments and procedures for obtaining the required pressure-sinkage relations. There is still no generally accepted method for extrapolating to prototype vehicles pressure-sinkage data from field tests of small plates or other track analogs.

British Model

44. British investigations of the general relation between off-road performance of vehicles and the physical environment have been the responsibility of the Fighting Vehicles Research and Development Establishment (FVRDE). On several occasions research sponsored by the Ministry of Defense has been conducted by British universities. Generally, the work in soil-vehicle research has been to improve a unified theory based on relations developed using methods of soil mechanics and making appropriate allowances or adjustments to account for soil-vehicle interactions in a manner that would yield the best agreement with experimental test results in the laboratory with running gear elements or in the field with prototype vehicles. The basic types of vehicle running gear investigated are the wheel and track, with special emphasis also given to wheel locomotion with traction devices.

45. In the review of British literature, considerable attention was given to a paper prepared as a basis for discussion at the Fourth Quadripartite Meeting of the Working Group on Ground Mobility held in Britian in 1963. The paper presents a summary of the past and recent British thinking on the soil-vehicle relations required to predict the performance of vehicle running gear operating in a variety of soil conditions. The

information presented is a digest of British experiences gained through comparing the application of theory to measured results for separate relations required to account for the total soil-vehicle interactions.* The report also presents a method for predicting the effect of soil on the running gear of a vehicle by coupling the separate effects of sinkage, motion resistance, slope resistance, and maximum net traction (drawbar pull). Evidence of the quality of some of the single relations is available, but no results are available in the literature by which the prediction accuracy for net traction, which is the resultant prediction of the separate relations coupling process, can be determined.

46. In the referenced paper, it is apparent that different soil descriptors are used as input to the various relations used to predict an element of vehicle performance. For example, in the relation used to predict motion resistance of wheeled and tracked vehicles operating on sand, the Bekker/LLD soil parameters are used to obtain a value for one of the unknowns, whereas the conventional soil mechanics parameters, c and ϕ, are used to predict sinkage of tracked vehicles.

47. The assumptions, soil and vehicle parameters, and fundamental relations common to the British soil-vehicle model are listed below.

a. Assumptions.

(1) Plastic soil deformation without recovery.

(2) For small sinkages, the uniform pressure, q, is identified with the strip load bearing capacity of the soil.

(3) Soil is generally described in terms of conventional soil mechanic parameters.

b. Fundamental relations.

(1) Sinkage in cohesive soils.

(a) Wheel (rigid).

$$\frac{z}{d} = \left(\frac{W \cos \theta}{CN_c bd}\right)^2 \qquad \text{(A56)}$$

* The referenced paper does not include soil-vehicle research contributions made by Dr. A. R. Reece and Dr. B. M. D. Wills at the University of Newcastle-upon-Tyne sponsored by the Ministry of Defense. The contributions made by Reece are discussed under the Bekker/LLD soil-vehicle model.

where

C = mean value of shear strength measured over the depth of disturbance

N_c = bearing capacity factor

(b) Track.

$$z = -\frac{b}{5}\log_e\left(\frac{q \sec\theta - \frac{W}{b\ell}}{q_1 - \frac{W}{b\ell}}\right) \tag{A57}$$

where

q = bearing capacity of soil = 8C at deep sinkages

$q_1 = \frac{W}{n_1 b d_1}$ or $\frac{1}{2}\left(\frac{WE}{N_1 br}\right)^{1/2}$, whichever is smaller

n_1 = number of bogie stations per track

d_1 = track pitch

E = elastic modulus for soil, 1000 psi

(2) Motion resistance.

(a) Cohesive soils. Wheel (rigid) and track.

$$R_c = qbz \tag{A58}$$

(Subscript c for cohesive soil.)

(b) Loose, dry sand.

1. Wheel (rigid).

$$R_s = \frac{0.529}{\left(bd^2\right)^{1/3}}(W\cos\theta)^{4/3} \tag{A59}$$

2. Track.

$$R_s = \frac{0.0582}{b\ell^2}(W\cos\theta)^2 \tag{A60}$$

(Subscript s for sand.)

(3) Drawbar pull.

(a) Cohesive soils.

1. Wheel (rigid).

$$P = \frac{Cbd}{4N_c} \quad \text{when} \quad \frac{W}{bd} = C \quad \text{and} \quad \frac{z}{d} = 0.0077 \tag{A61}$$

2. Track.

$$P = Ac - R \tag{A62}$$

(b) Loose, dry sand.

1. Wheel (rigid).

$$P = W \tan \phi - R_s \tag{A63}$$

2. Track.

$$P = W \tan \phi - \frac{0.0582W^2}{b\ell^2} \tag{A64}$$

(c) Firm, frictional slopes. Wheel (rigid) and track.

$$P = W \tan \phi \tag{A65}$$

(4) Slope-climbing ability.

(a) Cohesive soils. Wheel (rigid) and track.

$$\sin \theta = \frac{Ac - R}{W} \tag{A66}$$

(b) Loose, dry sand.

1. Wheel (rigid).

$$\tan \theta = \left[\tan \theta - 0.529 \left(\frac{W \cos \theta}{bd^2}\right)^{1/3}\right] \tag{A67}$$

2. Track.

$$\tan \theta = \left[\tan \phi - 0.0582 \left(\frac{W \cos \theta}{b\ell^2}\right)\right] \qquad (A68)$$

(c) Firm, frictional slopes. Wheel (rigid) and tracks.

$$\theta = \phi \qquad (A69)$$

APPENDIX B: STREAM-CROSSING MODELS

Introduction

Background

1. Of all terrain features, probably the least effort has been devoted toward modeling vehicle performance in stream crossing. The reasons for this are allied with the development of amphibious vehicles, which first made their appearance, in a military sense, during World War II. The amphibious vehicles were originally intended to transport cargo and personnel from ship to shore, and early research efforts were devoted to increasing water speed and cargo capacity. In the years following World War II, a new line of vehicles was developed, the DRAKE, LARC, and BARC. This same period witnessed a quantum advance in off-road mobility, except for the problem of stream crossings. For, while the performance of the exotic amphibians in making the ship-to-shore transition was nothing short of spectacular, their stream-crossing capabilities left much to be desired. The LLD began work on the stream-crossing problem in 1960 with the construction of a river simulator tank. The early work with this facility led to the conclusion that the stream-crossing problem had not been adequately defined. A program was initiated to evaluate the basic reasons for an amphibious vehicle's difficulty in negotiating streams, and field surveys were made to develop basic river environment data. These studies indicated that the primary cause of difficulty was the steep riverbank slopes and they led to the development of two models for predicting performance in stream exiting in go-no go terms.

2. Concurrently, through the MERS and MEGA programs WES was establishing a bank of basic stream data and conducting experiments with full-size vehicles, which came to fruition with the development of empirical relations between maximum bank slope negotiable and soil strength.

Strategies available

3. The strategies available for predicting stream crossing all hinge on the capability of the vehicle to negotiate the exit bank slope. The approaches have been either theoretical or empirical. The systems differ

somewhat in the terrain and vehicle parameters used and in degree of sophistication. Fig. B1 graphically illustrates the main elements of the overall procedures. The terrain parameters used in each model and the method of measurement are given in table B1; the vehicle parameters and the vehicle types for which each model is applicable are also given in table B1. The vehicle performance output for each of the current models is given in table B2, and the utilization of the current models and appropriate references are also given.

The problem

4. The fundamental terrain conditions affecting the performance of a vehicle attempting to cross a stream may be divided into three general categories--configuration, soil conditions, and water conditions--as shown in fig. B1. Any one of the three may prevent passage of the vehicle, or,

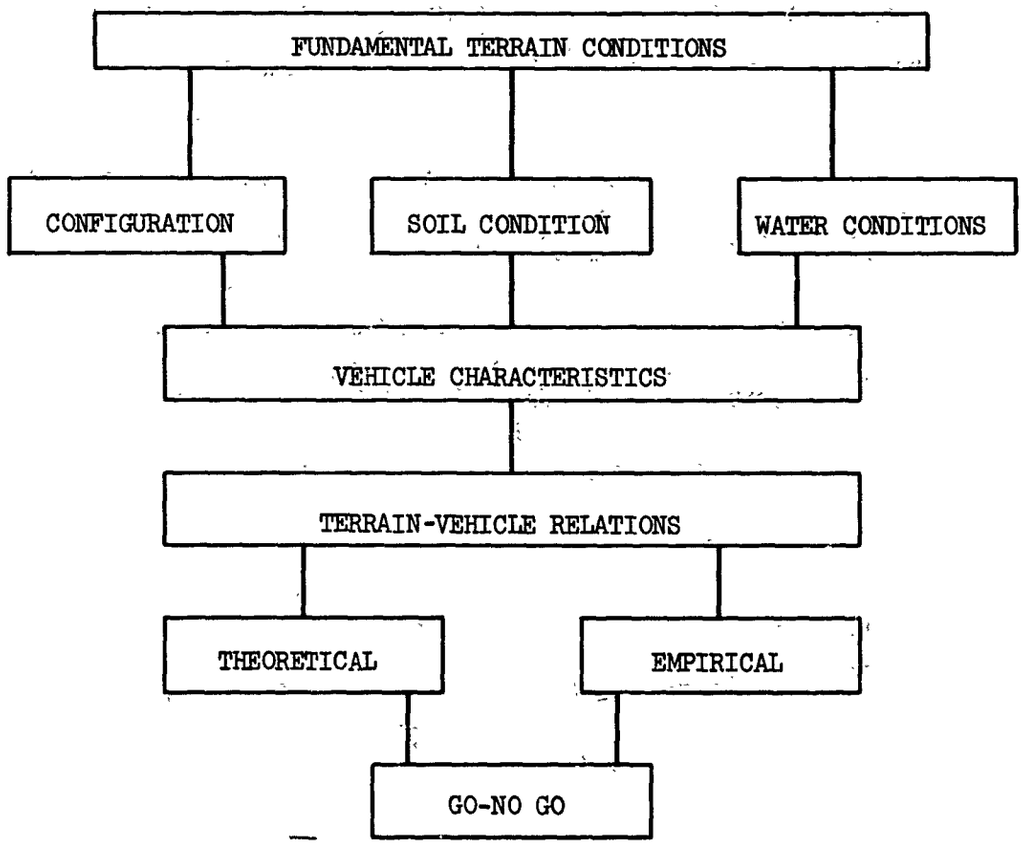


Fig. B1. Flow chart illustrating possible strategies for coupling stream-crossing models

Table B1

Terrain and Vehicle Parameters Used in Current Stream-Crossing Models

Model Identification	Terrain: Parameter	Terrain: Surrogate for Property	Terrain: How Measured	Vehicle: Type	Vehicle: Parameters
Dugoff	Bank angle	Configuration	Specific angle assumed	4x4 wheeled	Length Freeboard Vehicle center of gravity
	Tire-soil friction coefficient	--	Assumed value		Suspension spring, damping constants Bank contact velocity
Sloss	Bank height Bank angle	Configuration	Inclinometer	All	Maximum vertical wall vehicle can climb
Baker	Bank angle Tire-track friction	Configuration --	Inclinometer Assumed value	All	Complete vehicle scale model
LLD	Bank angle ϕ , c	Configuration	Inclinometer	All	Two-dimensional configuration
WES	Bank profile Cone index	-- --	Survey technique Cone penetrometer	All	One-pass vehicle cone index
WES hydrostatic	Bank angle	Configuration	Inclinometer	Tracked	Vehicle center of gravity Center of buoyancy Plane of transient waterline Plane of loaded waterline Hull draft Freeboard Vehicle weight Bank contact velocity
WNRE	Bank angle Step heights Rating cone index	Configuration	Inclinometer	4x4 wheeled tracked	Two-dimensional configuration Center of gravity Vehicle cone index

Table B2

Vehicle Performance Output and Utilization of Current Stream-Crossing Models

Model Identification	Vehicle Performance Output	Utilization	
		By	Reason for Use
Dugoff	A. Full vehicle trajectory as function of time from bank contact B. Parametric examination of vehicle input lists effects, presented in terms of distance vehicle travels upslope before coming to rest at traction failure	Not used	
Sloss	Go, marginal, no go	LLD	To estimate performance of M113 (a) at 82 sites on Black and Huron Rivers, and (b) at 115 sites on 43°N and 36°N latitude traverses, and (c) in 12 tests in Panama
Baker	Normal and tangential bank contact forces as function of bank angle and friction constant, and distance vehicle travels upslope in dimensionless form	Not used	
LLD	Go, no go	Not used	
WES	Go, no go, maximum bank slope negotiable	WES	To predict performance of M290, M113, and XM561 in 40 water-to-land transitions
WES hydrostatic	A. Relation of pitch angle and effective vehicle weight B. Kinetic energy available at bank contact C. Maximum bank angle vehicle will conform to as a result of kinetic energy	Not used	
WNRE	Go, no go, reason for no go	WNRE CAL	To estimate performance of M113, M290, and M41 in 18- by 27-km section of Thailand

acting in concert, any combination may prevent passage. Moreover, when acting individually, each category requires consideration of exclusive vehicle parameters; hence it is apparent that a complete stream model must be reasonably complex even to yield a simple go-no go answer. For example, if the configuration of the streambank is such that the leading edge of the vehicle strikes the face of the bank, the vehicle obviously cannot pass; if the soil permits excessive sinkage, the vehicle is immobilized, and if the water depth exceeds fording depth (for nonamphibious vehicles) or if the stream velocity exceeds the vehicle's water speed (for amphibians), the stream is impassable. While the configuration of the stream may be expected to remain constant for reasonable periods of time, the water depth and velocity may be subject to rapid change in short periods of time as may the soil conditions in certain soil types.

5. There are some redeeming factors, however. Streams that have appreciable gradients lack soft soil banks and bottoms. And in areas where the stream meanders, frequent "exit windows" may be found, which suggest that reconnaissance might change many a no go condition into a go condition. In addition, some of the more recently developed vehicles have positive articulation, "inching capabilities," self-carried bridges, underwater fording capabilities, and improved winching abilities for self-recovery.

Description and Analysis of Stream-Crossing Models

Description

6. A complete model for predicting the stream-crossing capabilities of ground vehicles is not available; however, various researchers have investigated various parts of the overall problem. The particular studies reviewed are discussed in the following paragraphs and are identified according to the researcher or agency. All the models discussed are egress models.

7. Dugoff. This model determines the trajectory of an amphibious vehicle making the transition from water to land. Equations are used to account for the weight, buoyancy, auxiliary propulsion, wheel torque, and

body-suspension interaction components. Two sets of equations represent the tire-soil reaction at the bank. One set is based on the Bekker system employing five soil parameters, and the other set is based on the WES system employing two soil parameters (neither set of equations has been programmed for use in the model at this time). Provision is made in the model for the effects of auxiliary egress assistance (e.g. winches, rockets, propellers, hydrojets, etc.).

8. At the present time, only a simplified version of the model has been programmed for computer solution. The simplifications consist of a restriction to nonyielding, purely frictional soil banks, and the assumption that the vehicle body is a sealed rectangular parallelepiped. With these simplifications, an egress trajectory of 10-sec duration required about 4 hours computer time for the IBM 1130. Work is currently in progress to develop a more reasonable computer-to-real-time ratio.

9. The initial application of the model consisted of a study of a 4-wheeled, box-shaped vehicle in the approximate proportions of a 1/4-scale model of a 1/4-ton truck (jeep) egressing a uniform, rigid bank. The following vehicle parameters were varied one value more and one value less than the basic vehicle.

a. Vehicle length.
b. Freeboard.
c. Vertical center of gravity (CG).
d. Horizontal CG.
e. Suspension spring constants.
f. Suspension damping constants.
g. Initial velocity.

10. The bank-slope angle and the tire-bank coefficient of friction were also varied within ranges so that:

$$\text{Coefficient friction} < \tan \theta$$

Under this condition no vehicle, regardless of configuration or initial velocity, could proceed indefinitely up the slope. Hence, for any set of vehicle and bank parameters considered, the total distance traveled up

the bank before stopping became an index of vehicle performance.

11. Using this performance index, the results of the computer runs indicated the following minor changes in performance: (a) performance increased with an increase in vehicle length, (b) performance decreased with an increase in freeboard, and (c) performance increased with higher center of gravity. Major effects on performance were as follows: (a) increase in initial velocity increased performance, (b) increase in coefficient of friction of bank increased performance, and (c) increase in bank slope angle decreased performance.

12. At the present time there is no verification for this model, and no serious attempt has been made to develop any empirical relations on the basis of trajectory data. It is possible that this model may be of some value to the vehicle designer, but it appears to require an exorbitant amount of computer time in its early format, which admittedly is grossly simplified. Nevertheless, it does represent a first small step toward the ambitious goal of developing a general mathematical model describing the egress maneuver, and the mere exercise of fabricating after-the-fact explanations of the computer results has helped to define and establish limits to the stream-crossing problem. If the results indicating minor importance of some factors and major importance of other factors can be confirmed with real vehicles in real stream egress tests, the way might be shown toward a practical, realistic model based on only those factors of significant value.

13. Sloss. This model estimates vehicle performance in exiting streams by using a slope severity factor determined by reducing the bank height by a trigonometric function of the bank slope angle. From examination of a number of cross-section profiles of riverbanks in the United States, Sloss concluded that the bank configuration could be represented by a series of slope angles and bank step heights, and that vehicle performance could be qualitatively estimated by reducing the measured bank height to account for the slope and comparing this computed value, termed slope severity factor, to the height of vertical wall the vehicle can climb. The functional static relation used to compute the slope severity factor is

$$S = h \sin^2 \alpha$$

where

S = slope severity factor

h = height of bank

α = simple bank slope

Slope severity limits for the M113 were qualitatively established as follows:

Severity Factor	Performance
0-2	Go, will negotiate all banks
2-4	Marginal, will negotiate some banks
> 4	No go, will not negotiate any banks

14. In this model, the terrain and the vehicle inputs are grossly oversimplified. The most important omission from terrain input is the character of the bank. The prediction accuracy is as yet undetermined but is probably low. The model is applicable only to tracked vehicles. The go, marginal, and no go output has very limited uses.

15. Baker. Using a simple bank angle, α, and a tire/track friction constant, f, Baker determines the normal and tangential bank contact forces as a function of bank angle and friction constant, and the distance traveled up the bank. The functional relations used are Archimedes' Law, the general laws of static mechanics, and simple hydrostatic scaling.

16. The terrain inputs reflect the simplest possible geometry and traction. The constant, f, used as a surrogate for true traction is not really existent except on paved banks. The vehicle input is complete to the extent that the scale model is complete. A scale model of any vehicle is possible, and if the scale model is complete, there can be neither omissions nor assumptions of vehicle data.

17. The functional relations used are theoretical. Within the limits tested, validation is 100 percent. At this time no further validation is needed as this is a simple matter of hydrostatics and static moment; by this same token, however, the static case may prove to be of little value once the full dynamics are understood.

18. The principal limitations are the grossly oversimplified terrain

input and the assumption that the vehicle always approaches the bank at 90 deg. The accuracy is limited to experimental accuracy only. The procedure is applicable to basically all vehicle types.

19. This procedure makes no specific vehicle performance predictions, and is useful only as a part of a more complex model and only when scale model results are available.

20. LLD. LLD's stream-exiting model grew out of the scale model studies of amphibious vehicles by W. J. Baker, D. A. Sloss, and C. F. Miranda. The fundamental philosophy was that any system of external forces could be reduced to one vertical force, one horizontal force, and a moment acting at the center of gravity. The vertical downward force is gradually transferred to the ground as the vehicle moves out of the water, and the change in vertical ground reaction should then be equal to the change in buoyancy between any two points. The horizontal force will directly affect the motion of the vehicle and will act for or against the motion depending on the direction of the net force and will occur only in the equation of motion. The moment acting at the center of gravity of the vehicle will affect the trim angle directly.

21. During egress the motion of the vehicle is assumed to be slow (creeping) and all of the dynamic effects are ignored. Then the vertical ground reaction, as a function of the distance traveled, can be related to the vehicle parameters, the net vertical force, and the moment acting at the center of gravity by the equations of statics. An equation of compatibility can be defined to relate the geometry of the ground to the number of wheels in contact to the trim angle. The equations are transcendental in nature and are solved by trial and error. With the use of a digital computer, values of the trim angle, β , and values of the total vertical reaction and reactions of each of the wheels are determined for each incremental distance traveled. By iteration the resistance is compared with the propelling forces to determine go or no go.

22. The vehicle input parameters assume an idealized box-shaped vehicle.* The terrain input parameters reflect a simple bank angle and

* Wheels, axles, and other appurtenances are considered as additional boxes.

an assumed angle of internal friction and cohesion for the soil.

23. The model yields a go-no go output, and in addition, the amount of additional effort required in event a no go condition is indicated. Nevertheless, there are some rather severe limitations inherent, especially in any attempt to predict cross-country performance. Obtaining reliable cohesion and angle of internal friction values for soil is difficult, per se. Next, no provision is made for change in tire-soil friction under traffic, and slip is excluded.

24. Nevertheless, this model has considerable potential for development. LLD plans to program the four LLD soil values, bank angle, and bank length, and for any area use a statistical analysis based on random sampling to generate a probability distribution curve. The evaluation of vehicle performance would require a combination of the environmental characteristics that give rise to a no go condition. This would be accomplished by repeated analysis for increasing bank angle until the no go condition is reached. The combination of environmental factors giving rise to a no go condition would then be compared with the probability distribution curve to indicate the probability of a particular vehicle negotiating the environmental conditions in the area.

25. Thus, it seems that the model shows promise for comparison of vehicles, but only in a probablistic sense.

26. WES. This model relates bank slope negotiable to soil strength in terms of the average cone index (CI) of the submerged portion of the water-land interface. It was developed by empirical analysis of 40 water-to-land transitions (or attempted transitions) of three vehicles in the United States and in Thailand. These tests were conducted as a part of the WES MERS program. The terrain inputs are the simple bank angle, α, and the average 0- to 6-in. CI for submerged banks. Vehicle input is VCI_1, the minimum soil strength required for a vehicle to complete one pass on a level surface. The functional relations consist of curves relating maximum slope negotiable to the average soil strength of the 0- to 6-in. soil layer of that portion of the streambank that is submerged. If the bank angle is greater than the maximum bank angle negotiable as shown by the curve, a no go condition is indicated; if the bank angle is equal to or less than

the maximum slope negotiable shown by the curve, a go condition is indicated.

27. The use of the submerged soil strength implies the assumption that the dry, i.e. above-waterline, portion of the bank is stronger than the submerged portion--an assumption that can hardly be faulted. The empirical nature of the relation adequately accounts for all pertinent vehicle parameters.

28. This model is considered to be adequately validated for three vehicles: the M29C, the M113, and the XM561. Field tests are required to establish the soil-slope or soil-drawbar pull curve for each vehicle of interest, but there are indications that vehicles may be grouped into classes, thus permitting a single curve for each class of vehicle.

29. The procedures in using the model are very simple and it is well adapted to field use. The prediction accuracy for the tests conducted to date appears to be well within the limits of experimental error. The growth potential for this model appears excellent. It may well turn out that results from field tests with a few representative vehicles can be extrapolated to represent all vehicles.

30. Another WES model uses equations for a simplified floating vehicle to determine the maximum bank slope to which the vehicle will conform as a result of vehicle momentum at the point of contact with the bank. From the background of the WES water-land interface studies, Stinson concluded that quite frequently the ability of an amphibious vehicle to negotiate a streambank was dependent upon the assistance received from momentum of the vehicle as it contacted the bank.

31. The terrain input is the simple bank angle, α. This implies the assumptions of a hard bank, water depth such that the vehicle will float until the bank slope is contacted by the vehicle, and no current. The vehicle is assumed to have a rectangular receptacle shape that may be described in the following terms:

Center of gravity
Center of buoyancy
Plane of transient waterline
Plane of loaded waterline
Hull draft

Freeboard
Vehicle weight
Bank contact velocity
Location of center of front bogie wheel

The functional relations used are simplified dynamics allowing two degrees of freedom. The graphical output presents the relation of pitch angle and effective vehicle weight, the kinetic energy available at time of contact with the bank, and the maximum bank angle the vehicle will conform to as a result of kinetic energy.

32. The terrain input is grossly oversimplified; the vehicle shape is admittedly simplified, but not to an unreasonable extent. However, the assistance that the vehicle might receive from its water-propulsion elements is ignored. The functional relations are theoretically sound and dimensionally adequate within the limits considered, although there has been no validation. Indeed, the model needs expanding before any validation is attempted. The introduction of wave effects and a yielding soil might change the general results.

33. The procedures involved in using this model are quite complex and it is not, at this time, suitable for field use. The principal limitations are the terrain oversimplification, the two-dimensional behavior, and the omission of traction and water-propulsion assistance forces. The prediction accuracy, as yet undetermined, is probably low. The development of this model is static at this time.

34. WNRE. The WNRE model treats bank egress as an obstacle problem with soil strength constraints and possible buoyancy effects on vehicle attitude and net ground loading. Lower and upper slope angles with respective step heights define the bank profile. Soil strength is considered in terms of rating cone index (RCI).

35. Essentially, the go-no go criteria presuppose only two basic independent reasons for immobilizations: geometric interference or lack of adequate traction, usually due to soil shear limitation.

36. The potential for geometric interference is examined by geometrically matching the parametrically described obstacle and vehicle (in side elevation only). Both the vehicle and the obstacle are assumed to be

rigid in this encounter, and the bank is assumed to be defined by straight lines with sharp, angular corners.

37. Calculation of bank-climbing ability is based upon a static frictional model of a grossly simplified rigid vehicle on a rigid obstacle. The vehicle is considered to approach this type of obstacle at right angles only.

38. For most purposes, all vehicles are treated as 4x4's by making appropriate adjustments to the center of gravity location and rear axle treatment. For example, to determine whether a tracked vehicle could make the initial start up an obstacle face, it is considered to be a 4x4 having a like weight distribution relative to the ground contact length. However, in testing its final negotiation of the obstacle, the 4x4 rear axle climbing criterion is replaced by a test that treats vehicle attitude as the moment the vertical projection of its center of gravity crosses the upper edge of the bank.

39. The equations used to estimate net traction are given below:

$$D/W = COH/NUGP + TNP - RES$$

where

D = net available traction, lb

W = gross vehicle weight, lb

COH = apparent soil cohesion, psi

NUGP = nominal unit ground pressure, psi

For wheeled vehicles:

$$NUGP = \frac{W}{nb\,\frac{d}{2}}$$

where n = No. of tires
b = tire section width, in.
d = tire outside diameter, in.

For tracked vehicles:

$$NUGP = \frac{W}{(b\ell)}$$

where b = track width, in.
ℓ = total length of track on the ground (all tracks, both sides), in.

TNP = tangent of soil apparent angle of internal friction

$RES = 0.3\ (VCI/RCI)^3$

For wheeled vehicles:

$$VCI = 3 + 3 \times NUGP$$

For tracked vehicles:

$$VCI = 6 + 3 \times NUGP$$

40. This model has been programmed for computer solution. An added feature of the computer program is a printout of the reason for failure if a no go prediction is made. An outline of the test sequence of the model is given in fig. B2.

41. Because the bank egress problem is intrinsically so severe, one intermediate level of failure is calculated (test Y) that is intended conceptually to indicate a conditional go when a reasonable degree (arbitrarily set at 50 percent of gross vehicle weight) of winch or towing assistance is utilized. Test Z was made the final test with the same assistance potential implicit in case of failure.

42. The model has been used to predict performance of three military vehicles across the streams occurring in a small (18 x 27 km) area of Thailand that had been previously mapped for mobility purposes as a part of the MERS program. No validation has been attempted.

43. The growth potential of this model appears limited. The procedures are too complex for field use, and the oversimplification of terrain and vehicle input are not consistent with computer use. The procedures used for estimating net traction are not well validated. The recent development by WES of a VCI for one pass has superseded the former VCI as used in this model.

Analysis of egress models

44. From tables B1 and B2 it can be seen that the current

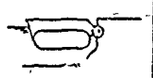
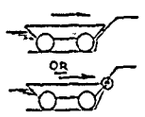
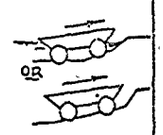
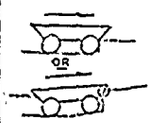
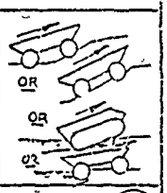
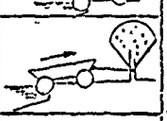
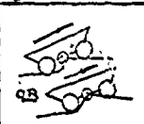
TEST (AND FAILURE) CODE	DIAGNOSTIC COMPARISON	BASIC TERRAIN FACTORS EXAMINED				MEANING WHEN FAILURE (CODE) IS INDICATED	SCHEMATIC REPRESENTATION
		Water Depth	Soil Strength	Lower Bank Geom.	Upper Bank Geom.		
R	Water depth vs. vehicle fording ability (non-swimming vehicles only).	X	-	-	-	Water depth exceeds vehicle fording capability.	
S	Upper bank step vs. vehicle floating attitude (swimming vehicles only).	X	-	-	X	Vehicle running gear cannot make useful contact with the bank.	
T	Basic soil strength vs. VCI.	X	X	-	X	Basic soil strength is inadequate for simple operation on smooth level ground (with or without buoyancy effects, as appropriate).	
U	Lower bank geometry vs. vehicle configuration	-	-	X	X	Lower bank presents either a vertical obstacle beyond basic vehicle capability, or there is angle of approach interference.	
V	Lower bank configuration vs. net vehicle traction.	X	X	X	-	Traction required to surmount lower bank exceeds soil-limited traction available (including buoyancy effects where present).	
W	Upper bank geometry vs. vehicle configuration.	-	-	X	X	Upper bank presents either a vertical obstacle beyond basic vehicle capability, or there is angle of approach interference.	
X	Upper bank configuration vs. net vehicle traction.	X	X	X	X	Traction required to surmount upper bank exceeds soil-limited traction available (including buoyancy effects where present).	
Y	Upper bank configuration vs. net traction plus 50% GVW outside thrust assist.	X	X	X	X	Deficit in required traction is more than 50% of gross vehicle weight.	
Z	Upper bank configuration vs. vehicle belly clearance.	-	-	X	X	Vehicle bellies on ditch edge (side elevation only -- no test for normal tracked vehicles).	

Fig. B2. Go-no go test sequence for stream crossing

stream-crossing models share some similarity, yet have some significant differences. Both are discussed in the following paragraphs.

45. Terrain input. The terrain parameters used in the models, as shown in table B1, are quite similar, ranging from a simple bank angle to a complete profile with soil strength measurements. Simplest is the WES's hydrostatic model which uses only the bank angle and includes no soil consideration. The dynamic trajectory model of Dugoff and the scale model studies of Baker also use the bank angle as a surrogate for the configuration of the streambank and assume a tire/soil and tire/track friction coefficient, respectively. The modified Baker model developed by LLD inputs the bank angle of friction and soil cohesion. Sloss's slope severity factor ignores the problem of yielding soil but considers the bank height in addition to the bank angle. The WES model considers the cross-section profile of the stream and uses the 0- to 6-in. average CI to account for both traction and yielding soil. The WNRE model considers a break in the bank slope to yield upper and lower approach angles and uses the RCI with some semiempirical equations to account for traction. For all models the bank angle, bank height, or profile (as the case may be) is easily measured without elaborate instrumentation. As regards the soil parameters, all except the CI are somewhat difficult to secure.

46. Vehicle parameters. The vehicle parameters used in the current models (as shown in table B2) range from WES's simple two-dimensional scale model for obstacle interference to complete scale models (Baker) and even complete vehicles (WES).

47. It may be noted that the Dugoff and WES hydrostatic models use somewhat similar vehicle parameters, considering that Dugoff's dynamic trajectory is limited to 4x4 wheeled vehicles and the WES's hydrostatic model applies only to tracked vehicles. Baker's scale-model programs are of course applicable to any vehicle for which a complete scale model is constructed; LLD's modification is applicable only to wheeled vehicles at this time. The WNRE two-dimensional configuration is applicable to 4x4 wheeled vehicles and single-unit tracked vehicles. The empirical models of Sloss and WES apply to all vehicles.

48. Vehicle performance output. The vehicle performance outputs of

the current stream-crossing models are listed in table B2. The outputs of the Dugoff, WES hydrostatic, and Baker and LLD models are useful only as a part of a more complex model and are not of themselves a meaningful prediction of vehicle performance in stream egress. The output of Sloss's slope-severity factor model in terms of go, no go, and marginal is indeed a prediction of stream egress, but marginal is hardly acceptable. The WES model and the LLD modification of Baker yield a straightforward go-no go output that is a meaningful prediction and poses no interface problems. The added reason for no go as output from the WNRE go is a valuable aid in cross-country predictions.

49. Utilization. From table B2 it can be seen that only three of the stream-egress models have been used. Sloss's model was used to estimate M113 performance at a total of 197 sites in the United States; however, no actual test results were available to evaluate the estimates. This model was also used to estimate performance of three vehicles on four bank slopes in Panama for which test results were available, and some inconsistencies appeared in the evaluation. The WNRE model was used to estimate performance in one small sector in Thailand; however, no actual test results were available for comparison with the predictions. The WES model was developed from 40 tests with real full-size vehicles.

Summary

50. Each of the stream-egress models has some disadvantages. The Dugoff and WES hydrostatic models do not consider yielding soil, are restricted to certain types of vehicles, do not of themselves yield a meaningful prediction, and have no validation; thus these models are considered as currently unacceptable. The Bekker and LLD scale-model programs can be applied to any vehicle for which a complete scale model is available. Unfortunately, the results of scale-model tests pose difficult interface problems, especially when soil is a factor; and frequently a large number of tests with full-size vehicles are required before the scale-model test results can be effectively used. LLD modification of the Baker equations is interesting, but is lacking in validation. Sloss's slope severity

factor has no provision for soil characteristics and at best yields an incomplete result. The WNRE model lacks validation, and relies upon equations using a VCI for 50 passes developed by WES that has been superseded by a recently developed VCI for one pass. The WES model considers all pertinent vehicle factors, and the CI has been repeatedly shown to serve as an adequate surrogate for soil characteristics important to mobility. Admittedly, the empirical approach does require extensive testing with full-size vehicles, but the answers are real answers in a real-world situation. It is judged that the WES model in its present form has perhaps the greatest ability for field application.

SUPPLEMENTARY

INFORMATION

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WATERWAYS EXPERIMENT STATION, CORPS OF ENGINEERS
P. O. BOX 631
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IN REPLY REFER TO: WESAR

27 July 1972

AD 886513L

Errata Sheet

AN ANALYSIS OF GROUND MOBILITY MODELS (ANAMOB)

Technical Report M-71-4

July 1971

1. Please replace pages A8, A12, A24, and A26 with the revised pages A8, A12, A24, and A26 attached hereto.

Table A2

Mobility Index Equation for Self-Propelled Wheeled (All-Wheel Drive) Vehicles (Equation A3)

$$\text{Mobility index}^{*} = \left(\frac{\text{contact pressure factor} \times \text{weight factor}}{\text{tire factor} \times \text{grouser factor}} + \text{wheel load factor} - \text{clearance factor} \right) \times \text{engine factor} \times \text{transmission factor}$$

where

$$\text{Contact pressure factor} = \frac{\text{gross weight, lb}}{\text{nom tire width, in.} \times \frac{\text{outside diam of tire, in.}}{2} \times \text{No. of tires}}$$

	Weight Range, lb**	Weight Factor Equations†
Weight factor:	<2000	Y = 0.553X
	2000 to 13,500	Y = 0.033X + 1.050
	13,501 to 20,000	Y = 0.142X - 0.420
	>20,000	Y = 0.278X - 3.115

Tire factor: $\frac{10 + \text{tire width, in.}}{100}$

Grouser factor: With chains = 1.05
Without chains = 1.00

Wheel load factor: $\frac{\text{gross weight, kips}}{\text{No. of axles x 2}}$

Clearance factor: $\frac{\text{clearance, in.}}{10}$

Engine factor: ≥10 hp/ton = 1.00
<10 hp/ton = 1.05

Transmission factor: Automatic = 1.00; manual = 1.05

* The MI obtained is converted to VCI from the curves shown in fig. A1.

** $\frac{\text{Gross weight, lb}}{\text{No. of axles}}$.

† Y = weight factor , $X = \frac{\text{gross weight, kips}}{\text{No. of axles}}$.

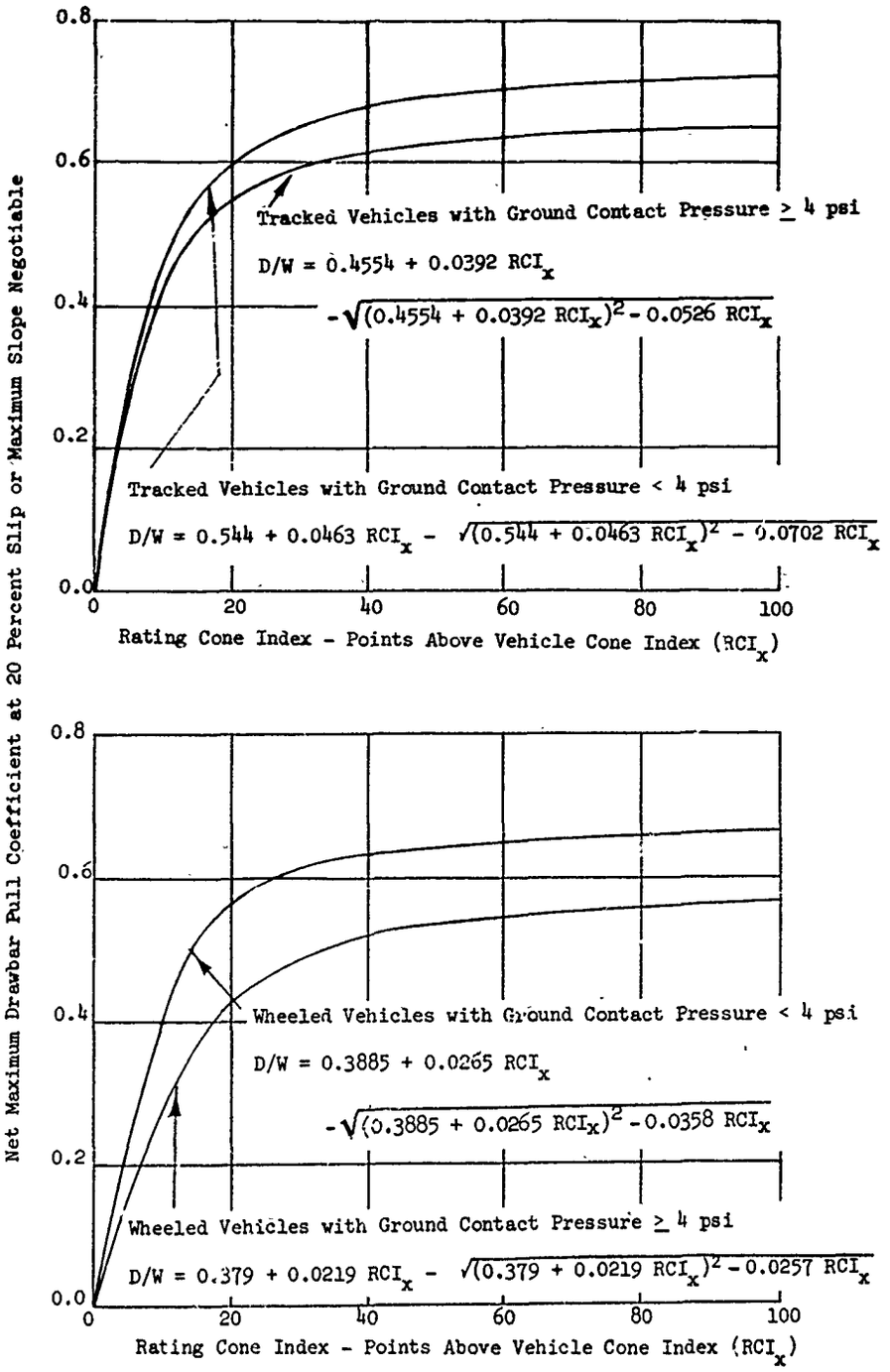
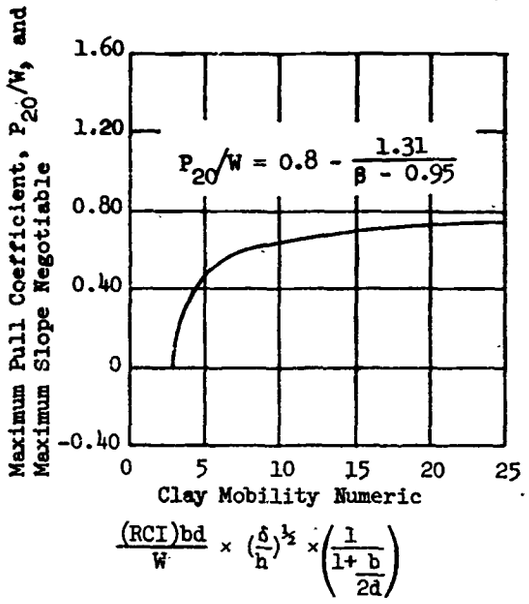
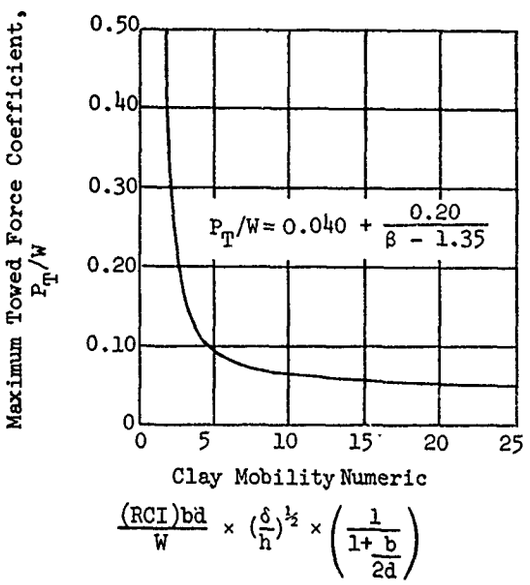


Fig. A4. First-pass net maximum drawbar pull, maximum slope negotiable-soil strength relations for tracked and wheeled vehicles operating on fine-grained soils



a. Pull ccefficient vs clay mobility numeric from field data



Note:

$$\beta = \frac{(RCI)bd}{W}\left(\frac{\delta}{h}\right)^{\frac{1}{2}}\left(\frac{1}{1+\frac{b}{2d}}\right)$$

b. Towed force coefficient vs clay mobility numeric from field data

Fig. A8. Relations of performance coefficients for first pass of single wheel to clay mobility numeric

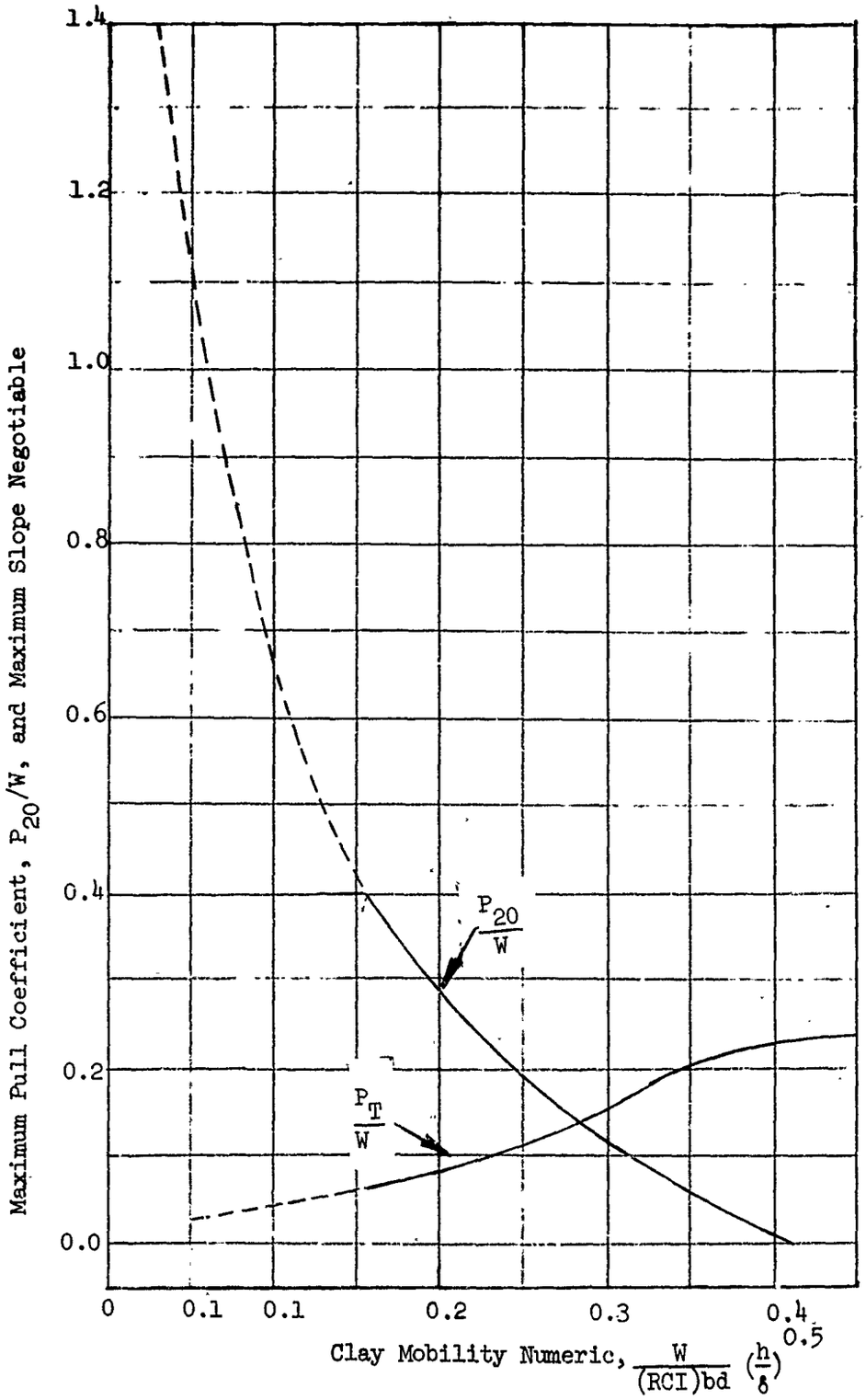


Fig. A9. Relation of pull, slope, and towed force coefficients to WES numeric for clay