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A GENERALIZED EQUATION FOR THE ELASTIC MODULI OF COMPOSITE MATERIALS

BY

LAWRENCE E. NIELSEN

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APRIL 1970

MONSANTO/WASHINGTON UNIVERSITY ASSOCIATION
HIGH PERFORMANCE COMPOSITES PROGRAM
SPONSORED BY ONR AND ARPA
CONTRACT NO. N00014-C-0218, ARPA ORDER 876
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MONSANTO RESEARCH CORPORATION
800 NORTH LINDBERGH BOULEVARD
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FOREWORD

The research reported herein was conducted by the staff of Monsanto/Washington University Association under the sponsorship of the Advanced Research Projects Agency, Department of Defense, through a contract with the Office of Naval Research, N00014-67-C-0218 (formerly N00014-66-C-0045), ARPA Order No. 876, ONR contract authority NR 356-484/4-13-66, entitled "Development of High Performance Composites."

The prime contractor is Monsanto Research Corporation. The Program Manager is Dr. Rolf Buchdahl (Phone-314-694-4721).

The contract is funded for $6,000,000 and expires 30 April 1971.

A GENERALIZED EQUATION FOR THE ELASTIC MODULI OF COMPOSITE MATERIALS

Lawrence E. Nielsen

A B S T R A C T

A generalized equation is proposed for the relative elastic moduli of composite materials. By the introduction of a generalized Einstein coefficient and a function which considers the maximum volumetric packing fraction of the filler phase, the moduli of many types of composite systems can be calculated.

(Contribution NPC 70-112 from the Monsanto/Washington University Association sponsored by the Advanced Research Projects Agency, Department of Defense, under ONR Contract N00014-67-C-0218, formerly N00014-66-C-0045).

A GENERALIZED EQUATION FOR THE ELASTIC MODULI OF COMPOSITE MATERIALS

Lawrence E. Nielsen

INTRODUCTION

Halpin and Tsai (1-3) have shown how the Kerner (4) equations and many other equations for the elastic moduli of composite materials can be put into an equation of the general form:

$$\frac{M}{M_1} = \frac{1 + AB\phi_2}{1 - B\phi_2} \tag{1}$$

In this equation M and M_1 are the elastic moduli (shear, Young's, or bulk) of the composite and matrix, respectively, ϕ_2 is the volume fraction of filler phase, while A and B are constants for any given composite. The constant A takes into account such factors as geometry of the filler phase and Poisson's ratio of the matrix. The constant B takes into account the relative moduli of the filler and matrix phases, and it is defined as

$$B = \frac{M_2/M_1 - 1}{M_2/M_1 + A} . \tag{2}$$

The subscripts 1 and 2 refer to the matrix and filler phases, respectively.

A Generalized Equation

Equation 1 omits several important factors, but it can be extended to take them into account. It has long been known in the theory of viscosity of suspensions that the maximum packing fraction of the filler must be considered (5-7). The packing of the filler phase should also be considered in the theory of the moduli of composite systems. In some cases the maximum volumetric packing fraction ϕ_m can be calculated from theory, in other cases it is most easily obtained from sedimentation volumes. Lewis and Nielsen (8, 9) showed how equation 1 can be modified for the shear modulus G to take into account ϕ_m to give

$$\frac{G}{G_1} = \frac{1 + AB\phi_2}{1 - B\psi\phi_2} \tag{3}$$

where ψ is given by such functions as

$$\psi = 1 + \left(\frac{1 - \phi_m}{\phi_m^2}\right)\phi_2 \tag{4}$$

or

$$\psi\phi_2 = 1 - \exp\left(\frac{-\phi_2}{1 - \phi_2/\phi_m}\right). \tag{5}$$

Nielsen also illustrated the applicability of equations 3 to 5 in determining the modulus of systems where phase inversion can occur (9).

The generality of equation 1 can be further enhanced by pointing out the relation between the constant A and the generalized Einstein coefficient k. The generalized Einstein coefficient may be defined as

$$k = \frac{d(M/M_1 - 1)}{d\phi_2} \tag{6}$$

as ϕ_2 approaches zero and M_2/M_1 approaches infinity. The relation is:

$$A = k - 1. \tag{7}$$

Einstein (10) pointed out that k = 2.5 for a suspension of rigid spheres in a matrix with a Poisson's ratio of 0.50. Burgers (11) has tabulated values of k for ellipsoidal or fiber-like particles randomly oriented as a function of the ratio of the length of the particles to their diameter. Lewis and Nielsen (12) theoretically and experimentally determined k as a function of the number of particles in aggregates of spheres immersed in a matrix with a Poisson's ratio of 0.50. The values of k for matrices with Poisson's ratio less than 0.50 has never been determined, however, a good approximation should be the method suggested by Nielsen (13). Thus, the ratio of k at a given value of ν_1 to the value of k at ν_1 = 0.5 is given by Table I for shear moduli.

Thus, an equation which takes into account nearly all the factors which are important in determining the modulus of a composite is:

$$\frac{M}{M_1} = \frac{1 + (k - 1)\, B\phi_2}{1 - B\psi\phi_2} \tag{8}$$

The applicability of equation 8 to predict the modulus of a matrix containing strong rigid aggregates will be illustrated by using the data of Lewis and Nielsen (9) for aggregates of about 20 glass spheres in an epoxy matrix. In a liquid matrix (Poisson's ratio = 0.5), k = 4.4 and ϕ_m = 0.4 (12). In a rigid epoxy matrix where Poisson's ratio is 0.35, the predicted value of k from Table I is k = 3.81. Above the glass transition temperature where the epoxy matrix is a rubber, the modulus ratio G_2/G_1 approaches infinity while below the glass transition temperature this ratio is about 20. Figure 1 compares the experimental values with the predicted values using equations 2, 5, 7, and 8. The experimental values are somewhat low since they have not been corrected for the "skin effect" in which there is an excess of polymer on the surface (8).

A second application of equation 8 can be illustrated by a calculation of the longitudinal shear modulus G_{LT} of fiber-filled composites. Chaffey and Mason (14) and Jeffery (15) give the generalized Einstein coefficient as 2.0 for this case. The complex computer calculations for G_{LT} by Adams and Doner (16) can be approximated by using $\phi_m \triangleq 0.85$ in equations 4 and 8. This value of ϕ_m = 0.85 is about half way between cubic packing (0.785) and hexagonal packing (90.7) for parallel rods — a reasonable value. Figure 2 shows the results using equation 8 along with the equation of Halpin and Tsai for the case where

G_2/G_1 is very large. The calculations of Adams and Doner are also shown. At high fiber concentrations, equation 8 fits the computer calculations much better than the Halpin-Tsai equation. By slightly reducing the value of 2.0 for the Einstein coefficient to correct for Poisson's ratio being less than 0.5 and making minor adjustments in ϕ_m would undoubtedly improve the fit.

Equation 8 has been tested with many other composite systems, and in nearly all cases the agreement with experimental data is good.

References

1. H. W. Teal, U. S. Government Report AD 834851, June 1968.
2. J. E. Ashton, J. C. Halpin, and P. H. Petit, Primer on Composite Analysis, Chap. 5, Technomic Publ. Co., Stamford, 1969.
3. J. C. Halpin, J. Composite Matr., 1, 732 (1969).
4. E. H. Kerner, Proc. Phys. Soc., 69B, 808 (1956).
5. M. Mooney, J. Colloid Sci., 6, 162 (1951).
6. J. V. Robinson, Trans. Soc. Rheol., 1, 15, (1957).
7. R. F. Landel, B. G. Moser, and A. J. Bauman, Proc. 4th Internat. Congr. Rheology, Part 2, p. 661, E. Lee, Ed., Interscience, New York, 1965.
8. T. B. Lewis and L. E. Nielsen, J. Appl. Polymer Sci., to be published.
9. L. E. Nielsen, Appl. Polymer Symposia, No. 12, 249 (1969).
10. A. Einstein, Ann. Physik, 17, 549 (1905), and 19 289 (1906).
11. J. M. Burgers, Second Report on Viscosity and Plasticity, p. 113, Nordemann Publ. Co., New York, 1938.
12. T. B. Lewis and L. E. Nielsen, Trans. Soc. Rheol., 12, 421 (1968).
13. L. E. Nielsen, J. Composite Matr., 2, 120 (1968).
14. C. E. Chaffey and S. G. Mason, J. Colloid Sci., 20, 330 (1965).
15. G. B. Jeffery, Proc. Roy. Soc. (London), A102, 161 (1922).
16. D. F. Adams and D. R. Doner, J. Composite Matr., 1, 4 (1967).

List of Figures

1. Relative shear moduli of aggregates consisting of about 20 glass spheres in a rubbery epoxy matrix and in a rigid epoxy matrix with a Poisson's ratio of 0.35 and $G_2/G_1 = 20$. Crosses represent experimental points. Curves are calculated from equation 8.
2. Relative longitudinal shear modulus of a fiber-filled composite for the case of $G_2/G_1 = \infty$. Curve A — Equation 8. Curve B — the Halpin-Tsai equation. X-Computer calculations of Adams and Doner.

TABLE I

Relative Einstein Coefficient for Different Poisson's Ratios

Poisson's Ratio ν_1	$\frac{k \text{ at } \nu_1}{k \text{ at } \nu_1 = 0.5}$
0.5	1.00
0.4	0.90
0.35	0.867
0.30	0.84
0.2	0.80

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The work described in this paper is part of the research conducted by the Monsanto/Washington University Association sponsored by the Advanced Research Projects Agency, Department of Defense, under Office of Naval Research Contract N00014-67-C-0218, formerly N00014-66-C-0045.

FIG. 1

4/1/70
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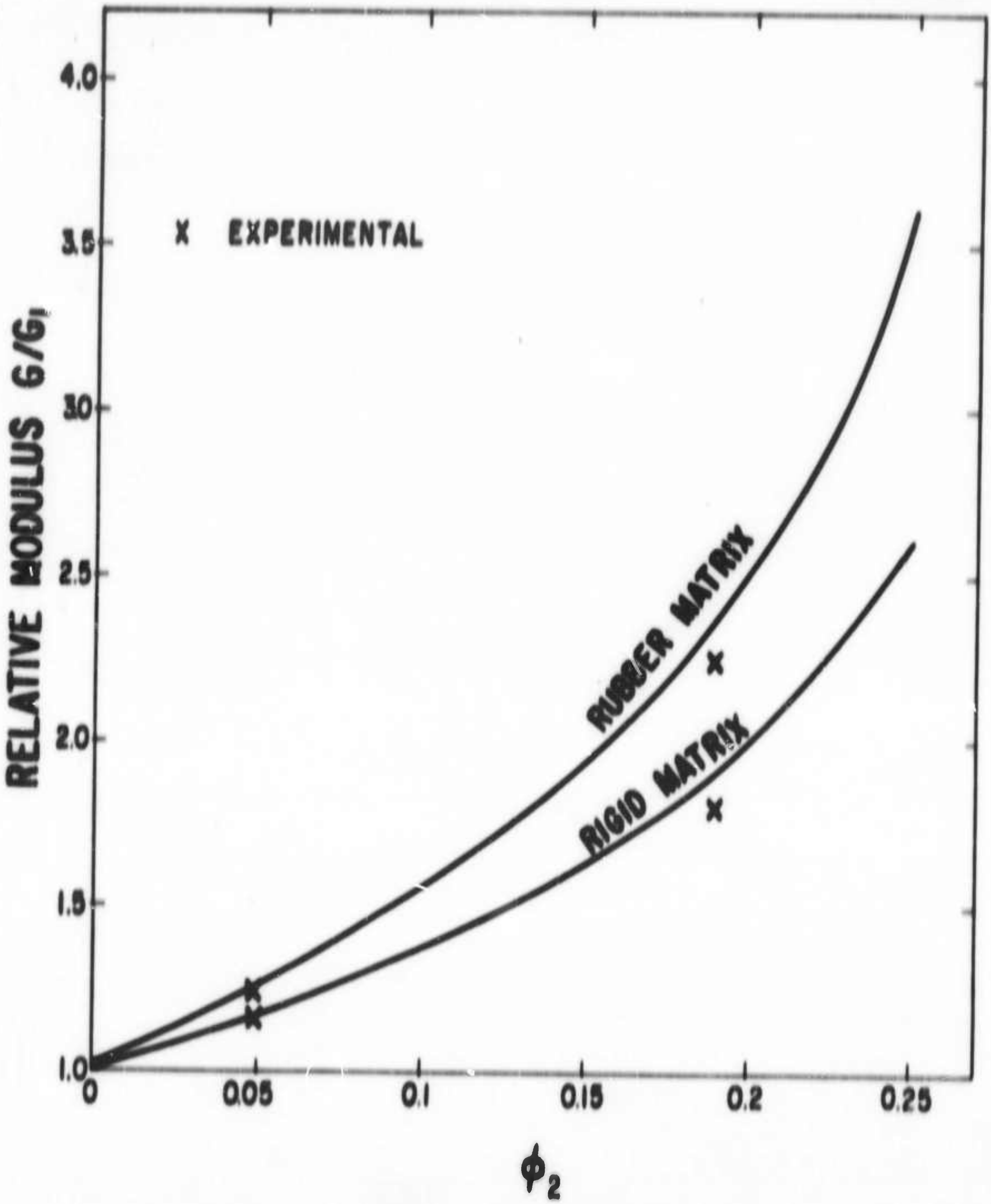
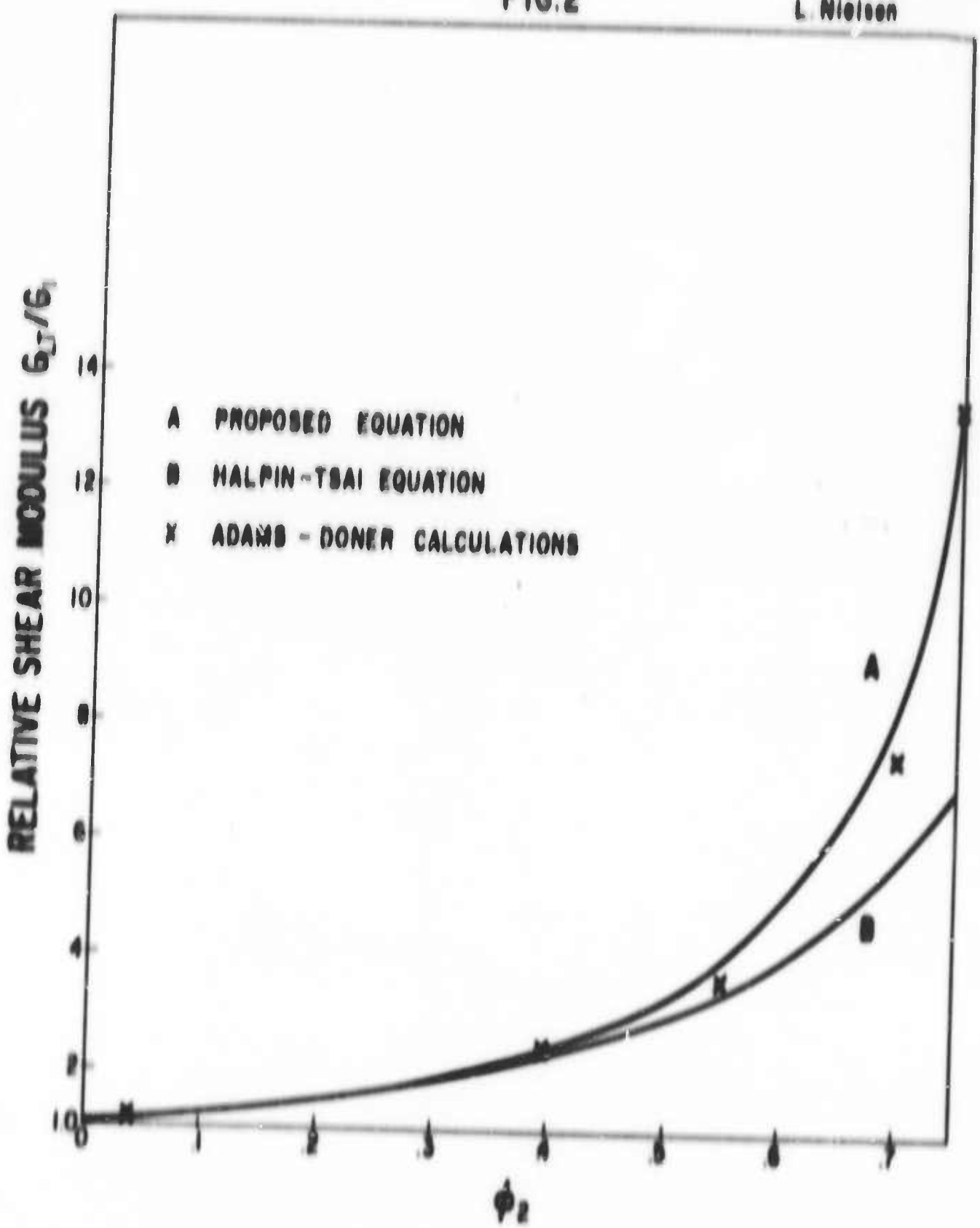


FIG.2

4/1/70
L. Nielsen



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