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REPORT NO. RG-TR-69-14

AN ANALYSIS AND SIMULATION OF THE M60A1E2 TANK MAIN GUN'S ELEVATION CONTROL SYSTEM

by

Harold L. Pastrick

August 1969

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DA Project No. MIS98753247381
AMC Management Structure Code No. 4440.15.1111

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Army Inertial Guidance and Control Laboratory and Center
Research and Engineering Directorate (Provisional)
U. S. Army Missile Command
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ABSTRACT

A dynamical analysis of the M60AIE2 Tank main gun's elevation stabilization system is performed. The equations describing the gun, hull, and turret dynamics are derived and computer diagraming is shown for simulation purposes.

CONTENTS

ILLUSTRATIONS

1. Introduction

The M60AIE2 Tank stabilization gun control system was designed to stabilize inertially the turret and cupola in traverse and the main gun and commander's machine gun in elevation. These are depicted in Figure 1. The four stabilization loops including the turret traverse, cupola traverse, main gun elevation, and cupola gun elevation are similar. Each uses electro hydraulic actuators to drive the load based on information derived from the gunner's controls, commander's controls, or rate gyro sensors in the cupola, hull, and turret. Because of a variety of contributing factors, the stabilization system, as originally designed and installed by the contractors, had performance deficiencies.

The task of the Army Inertial Guidance and Control Laboratory and Center was to investigate the system dynamics and to develop fixed gain compensation networks for the stabilization loops. The initial efforts were directed at investigations in the azimuth plane only. In it, the turret and hull assemblies were analyzed, and the model was synthesized on two EAI-22IR analog computers for further study.

At the request of the Program Manager, a scope of work for a follow-on effort was submitted. In essence, the task was to cover the development of improved fixed gain compensation networks for the main gun's elevation loop.

2. Purpose

It is the purpose of this report to show the development of the dynamical equations of motion of the main gun, turret, and hull in the elevation plane. In addition, the synthesis of the mathematical model will be displayed for the analog computer simulation.

3. Main Gun Dynamics in the Elevation Plane

The physical situation of the main gun within the tank turret is shown in Figure 2. The geometry and free body diagram of the forces acting on the gun are shown in Figure 3.

The dynamics are described by summing all the external torques on the main gun and setting them equal to the rate of change of angular momentum of the gun:

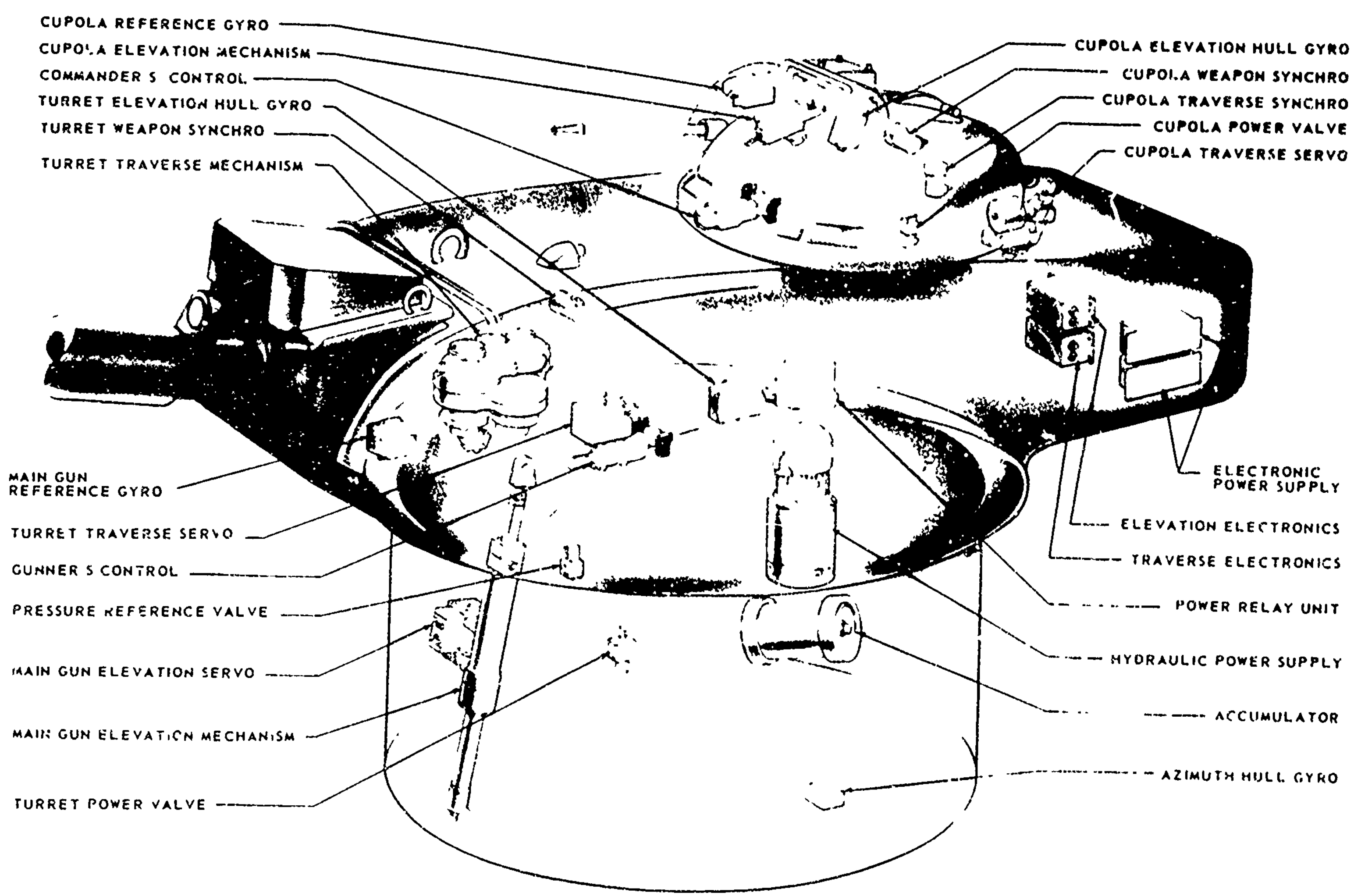

FIGURE 1. THE M60AIE2 TANK STABILIZATION GUN CONTROL SYSTEM

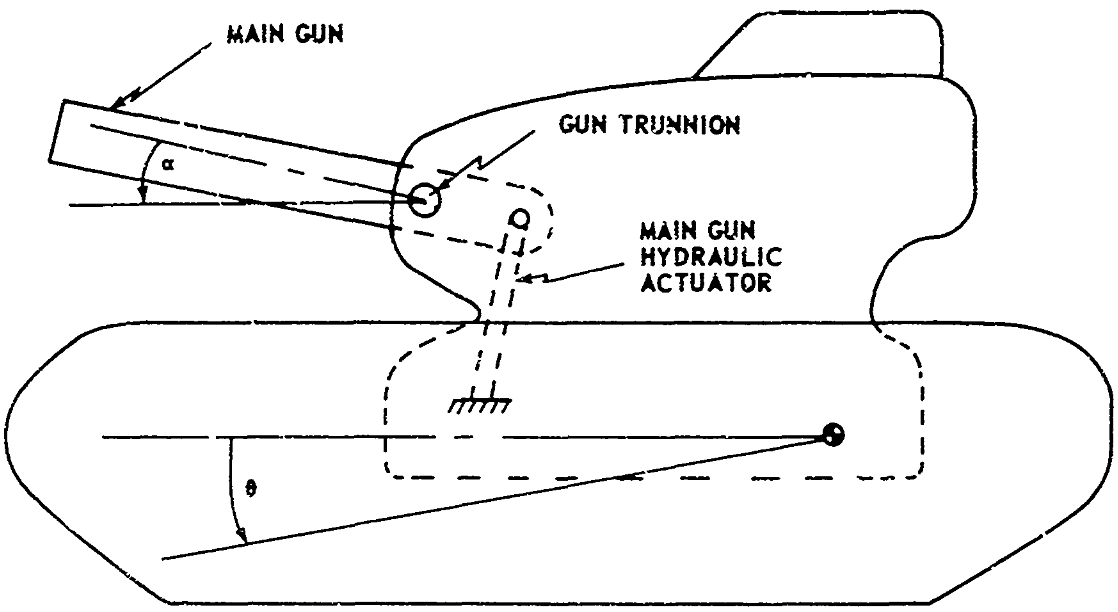

FIGURE 2. SITUATION OF THE MAIN GUN

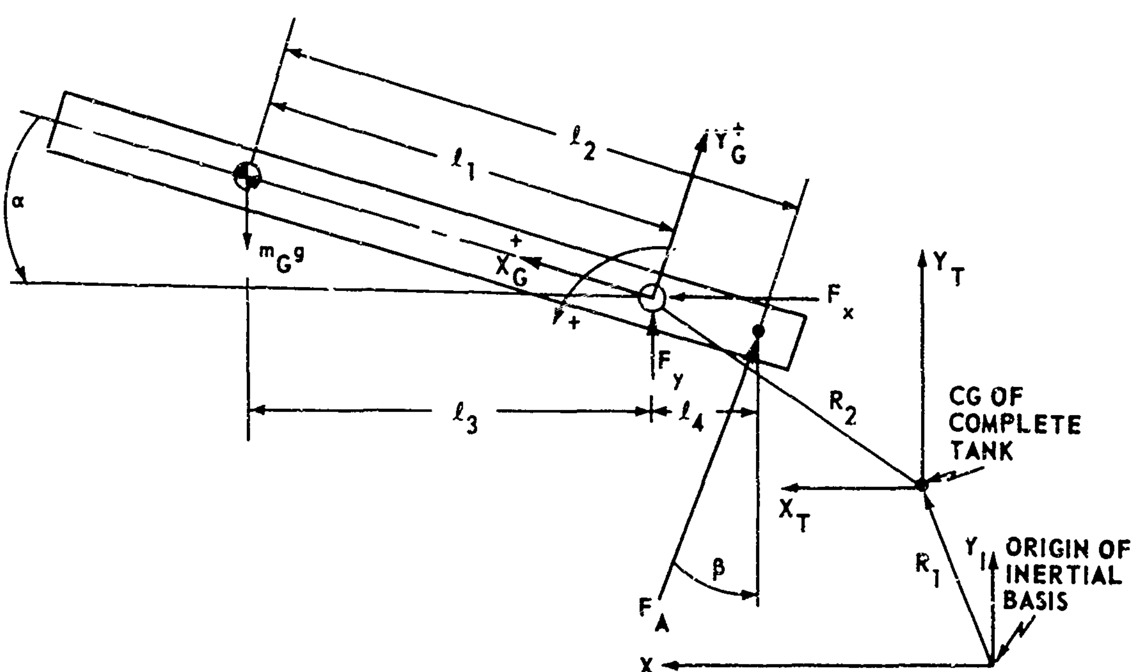

FIGURE 3. FREE BODY FORCE DIAGRAM AND GEOMETRY OF THE MAIN GUN

$$\sum_{i=1}^{n} \vec{M}i_{cm_{gun}} = \overset{I}{\dot{\vec{H}}}_{gun} \quad , \tag{1}$$

where

$\overset{I}{\dot{\vec{H}}} = \left. \frac{d\vec{H}}{dt} \right|_I \triangleq$ First derivative of momentum with respect to the inertial basis $(\vec{X}_I, \vec{Y}_I)$.

$\vec{M}i_{cm_{gun}} \triangleq$ External torques acting on the gun summed about its mass center.

From the right side of equation (1), the momentum is given by:

$$\vec{H}_G = \vec{\vec{I}}_G \cdot \vec{\omega}^{G-I} \tag{2}$$

where

$\vec{\vec{I}}_G \triangleq$ Inertia tensor of the gun

$\vec{\omega}^{G-I} \triangleq$ Angular velocity of the gun with respect to the inertial basis.

Now $\vec{\omega}^{G-I}$ has two components in this analysis; i.e.,

$$\vec{\omega}^{G-I} = \vec{\omega}^{G-H} + \vec{\omega}^{H-I} \quad , \tag{3}$$

where

$\vec{\omega}^{G-H} \triangleq$ Angular velocity of the gun with respect to the hull

$\vec{\omega}^{H-I} \triangleq$ Angular velocity of the hull with respect to the inertial basis.

Since the hull is also described within a rotating reference frame, there is a Coriolis force to be considered.

$$\overset{I}{\vec{H}}_G = \overset{G}{\vec{H}}_G + \vec{\omega}^{G-I} \times \vec{H}_G \,, \tag{4}$$

where

$$\overset{G}{\vec{H}}_G = \left.\frac{d\vec{H}}{dt}\right|_G \triangleq \text{First derivative of momentum with respect to the gun basis } (\vec{X}_G, \vec{Y}_G)$$

Substituting equations (2) and (3) into (4) and performing the indicated differentiation gives the following:

$$\begin{aligned} \overset{I}{\vec{H}}_G &= \left.\frac{d}{dt}\right|_G \left[\overset{\Rightarrow}{I}_G \cdot \vec{\omega}^{G-H} + \vec{\omega}^{H-I} \right] \\ &\quad + \left(\vec{\omega}^{G-H} + \vec{\omega}^{H-I} \right) \times \overset{\Rightarrow}{I}_G \cdot \left(\vec{\omega}^{G-H} + \vec{\omega}^{H-I} \right) \end{aligned} \tag{5}$$

$$\begin{aligned} &= \overset{G}{\overset{\Rightarrow}{I}}_G \cdot \left(\vec{\omega}^{G-H} + \vec{\omega}^{H-I} \right) \\ &\quad + \overset{\Rightarrow}{I}_G \cdot \left(\overset{G}{\vec{\omega}}^{G-H} + \overset{G}{\vec{\omega}}^{H-I} \right) \\ &\quad + \left(\vec{\omega}^{G-H} + \vec{\omega}^{H-I} \right) \times \overset{\Rightarrow}{I}_G \cdot \left(\vec{\omega}^{G-H} + \vec{\omega}^{G-I} \right) . \end{aligned} \tag{6}$$

Thus

$$\overset{I}{\vec{H}}_G = \overset{\Rightarrow}{I}_G \cdot \left(\overset{G}{\vec{\omega}}^{G-H} + \overset{G}{\vec{\omega}}^{H-I} \right) + \left(\vec{\omega}^{G-H} + \vec{\omega}^{H-I} \right) \times \overset{\Rightarrow}{I}_G \cdot \left(\vec{\omega}^{G-H} + \vec{\omega}^{H-I} \right). \tag{7}$$

The first term on the right of the equality sign in equation (6) goes to zero because the term $\overset{G}{\overset{\Rightarrow}{I}}_G$, the derivative of the inertia of the gun, is zero in the gun basis.

Now from equation (1), develop the external force acting on the gun.

$$\sum_{i=1}^{n} Mi_{cm_{gun}} = \sum_{i=1}^{n} Mi_{T_{gun}} + M_G \ell_1^2 (\ddot{\alpha} + \ddot{\theta}) \tag{8}$$

where

$\sum_{i=1}^{n} Mi_{T_{gun}}$ $\triangleq$ Moments of the gun about the trunnion

$M_G \ell_1^2 (\ddot{\alpha} + \ddot{\theta})$ $\triangleq$ The inertia force of the mass center displaced by distance ℓ_1 from the trunnion.

$$\sum_{i=1}^{n} \vec{Mi}_{T_{gun}} = M_G \vec{g} \ell_3 + \vec{F}_A \cos\beta \, \ell_4 \quad . \tag{9}$$

But

$$\ell_3 = \ell_1 \cos\alpha$$

$$\ell_4 = (\ell_2 - \ell_1) \cos\alpha \quad .$$

Thus

$$\sum_{i=1}^{n} \vec{Mi}_{t_{gun}} = M_G \vec{g} \ell_1 \cos\alpha + \vec{F}_A \cos\beta \, (\ell_2 - \ell_1) \cos\alpha$$
$$+ M_G \ell_1^2 (\ddot{\alpha} + \ddot{\theta}) \quad . \tag{10}$$

Combining equations (7) and (10) gives, on the right, the invariant vector form of the gun dynamics.

$$M_G \vec{g} \ell_1 \cos\alpha + \vec{F}_A \cos\beta \, (\ell_2 - \ell_1) \cos\alpha + M_G \ell_1^2 \ddot{\alpha}$$
$$= \vec{\vec{I}}_G \cdot \left(\dot{\vec{\omega}}^{G\text{-}H} + \dot{\vec{\omega}}^{H\text{-}I} \right)$$
$$+ \left(\vec{\omega}^{G\text{-}H} + \vec{\omega}^{H\text{-}I} \right) \times \vec{\vec{I}}_G \cdot \left(\vec{\omega}^{G\text{-}H} + \vec{\omega}^{H\text{-}I} \right) . \tag{11}$$

Equation (11) is still not usable for computer synthesis because the right hand side is still in vector form. It would be desirable to have the vector terms reduced to scalar format in the gun basis. This is done next. The subscript on the inertia term will be dropped to avoid confusion with the notation of a matrix in the gun basis.

$$\omega_G^{G-H} = \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} \end{bmatrix} \qquad (12)$$

$$\omega_H^{H-I} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} . \qquad (13)$$

But to be compatible with the left side of equation (11), the vector-matrix form of equation (13) must be transformed from the H-basis to the G-basis:

$$\omega_G^{H-I} = C_{G/H} \, \omega_H^{H-I} \qquad (14)$$

$$C_{G/H} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} . \qquad (15)$$

Thus,

$$\omega_G^{H-I} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} . \qquad (16)$$

This result is not as striking as it first may appear. It says, in effect, that there is no difference between the H-basis and the G-basis for the angular rate shown. This of course is not true since the two bases are not equivalent. What is actually seen is that the concept of a "pseudo-vector," namely the angular rate, is in a third dimension, but the only concern here is in the X, Y plane for this analysis. Thus, that particular pseudo-vector is not transformed only for this special case.

It remains to reduce the Coriolis term to its scalar equivalent in the G-basis:

$$\left(\vec{\omega}^{G-H} + \vec{\omega}^{H-I}\right) \times \overline{\overline{I}}_G \cdot \left(\vec{\omega}^{G-H} + \vec{\omega}^{H-I}\right) = \left[\underset{G}{\omega}^{G-H} + \underset{G}{\omega}^{H-I}\right]$$

$$\times \left[\underset{G}{I}\right] \cdot \left[\underset{G}{\omega}^{G-H} + \underset{G}{\omega}^{H-I}\right] . \qquad (17)$$

The right side is expanded as follows:

$$\left[\underset{G}{\omega}^{G-H} + \underset{G}{\omega}^{H-I}\right] \times \left[\underset{G}{I}\right] = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & (\dot{\alpha} + \dot{\theta}) \\ I_{xx} & I_{yy} & I_{zz} \end{bmatrix} \qquad (18)$$

$$= -\hat{i}\, I_{yy}(\dot{\alpha} + \dot{\theta}) + \hat{j}\, I_{xx}(\dot{\alpha} + \dot{\theta}) \ , \qquad (19)$$

and the vector inner product is given by

$$\begin{bmatrix} -I_{yy}(\dot{\alpha} + \dot{\theta}) \\ I_{xx}(\dot{\alpha} + \dot{\theta}) \\ 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} + \dot{\theta} \end{bmatrix} = 0 \quad . \qquad (20)$$

Once again the problem of a two-dimensional simulation while working with a three-dimensional real model is evidenced. Equation (20) says that there is no Coriolis term contribution. The purpose here was to verify a relation that is well known. Cannon [1] says,

> For plane (two-dimensional) motion a major simplification occurs in the equations of motion because we are concerned only with axes of rotation and angular-momentum vectors in a single direction, namely, perpendicular to the plane of motion; in this case, of the 36 possible terms in $\overset{I}{\vec{H}}$, only <u>one</u> remains!

To wit:

$$\overset{I}{\dot{\vec{H}}} = \vec{1}_z J_z \dot{\Omega}_z \tag{21}$$

in which $\overset{I}{\dot{\vec{H}}}$ is the rate of change of angular momentum, which is now perpendicular to the plane of motion.

An even more graphical representation of the results of equation (20) is obtained if one uses the notation of Cannon on equation (17),

$$\vec{\omega} \times \vec{\vec{I}} \cdot \vec{\omega} = \omega \hat{1}_z \times J \hat{1}_z \cdot \omega \hat{1}_z = 0 \quad . \tag{22}$$

The strong evidence of the triple scalar product of collinear vectors being zero is proof enough.

Finally, the scalar form of equation (11), in the G-basis, is given by

$$M_G g \ell_1 \cos \alpha + F_A \cos \beta \, (\ell_2 - \ell_1) \cos \alpha + M_G \ell_1^2 (\ddot{\alpha} + \ddot{\theta}) = J_G (\ddot{\alpha} + \ddot{\theta}) \quad . \tag{23}$$

Another dilemma, which is not apparent from equation (23), is the source of the information to obtain $\ddot{\alpha}$ and $\ddot{\theta}$. As previously mentioned, α is the angle between the gun's x reference axis and the hull's x reference axis, and θ is the angle between the hull's x reference axis and the inertial x reference axis. Obtaining $\dot{\theta}$ is straightforward. It is merely picked off of the turret elevation hull gyro. But, $\dot{\alpha}$ is nor measurable directly, since the main gun reference gyro also measures rate with respect to inertial space. However, this can be alleviated with the following consideration:

$$\omega_G^{G-I} - \omega_G^{H-I} = \omega_G^{G-I} + \omega_G^{I-H} = \omega_G^{G-H} = \dot{\alpha} \quad . \tag{24}$$

Thus, the angular rate of the gun with respect to the hull can be instrumented by taking the difference in the outputs of the two reference gyros.

4. Vehicle Dynamics in the Elevation Plane

The turret, cupola, and hull are all considered to be a single rigid body in this analysis. This is not true actually, but the cupola's mass and moment of inertia are so small compared to the overall tank that it can be included without consideration of its change for various hull elevation angles, θ. There is also a negligible effect between the turret and the hull known as "dishpanning." This is of little consequence with respect to the degree of accuracy of this analysis. The geometry of the tank vehicle is shown in Figure 4. The tracks and suspension are considered as a pair of spring and damper combinations. This allows two degrees of freedom in this plane if the forward velocity is not considered. Figure 5 shows the free body diagram of the forces on the tank.

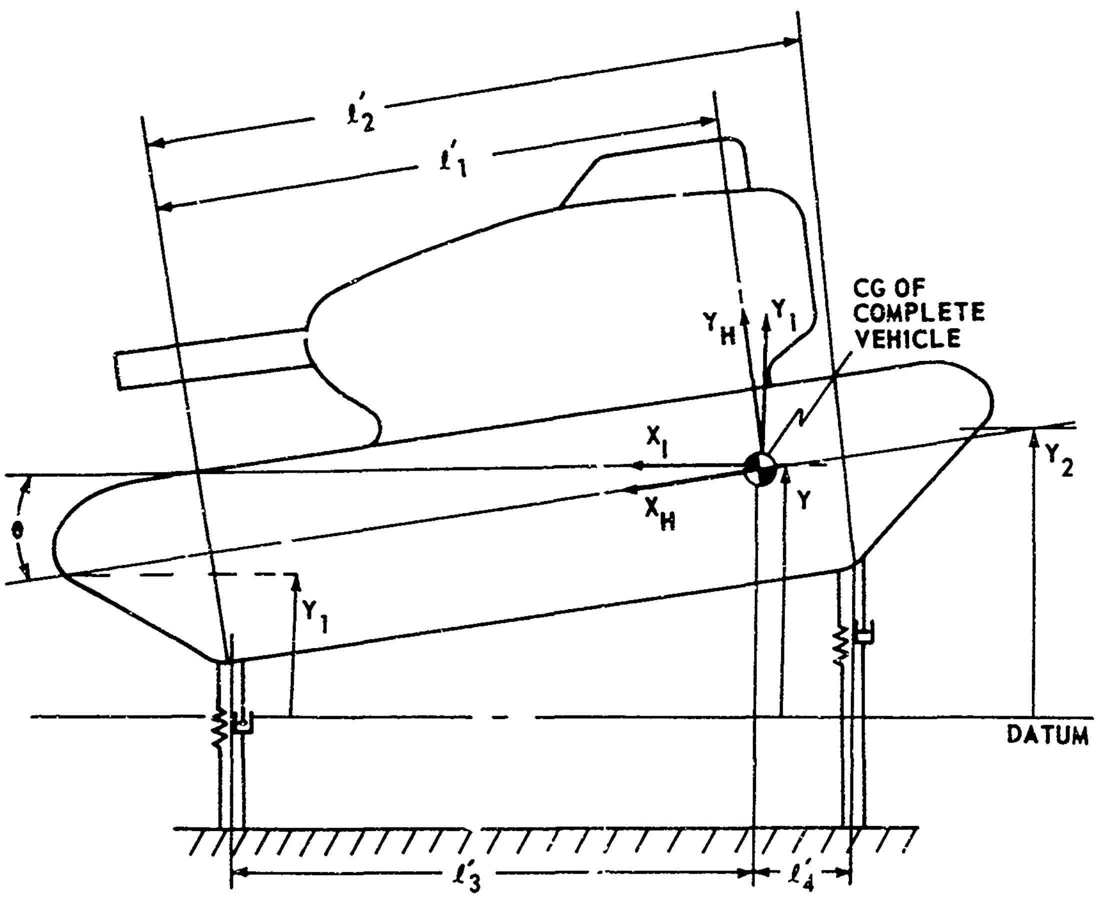

FIGURE 4. GEOMETRY OF THE COMPLETE TANK WITH SUSPENSION

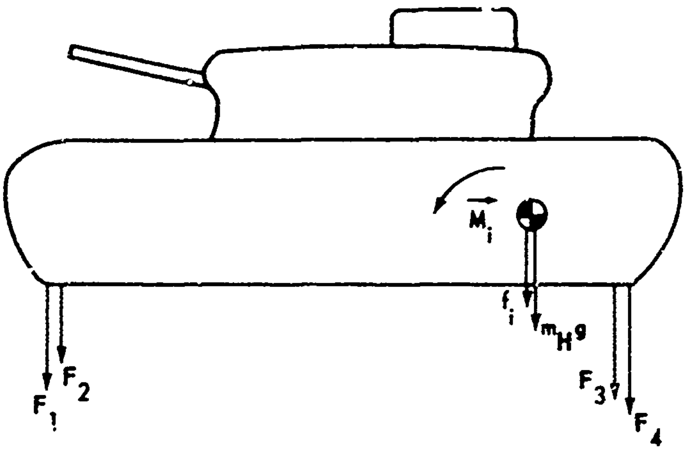

FIGURE 5. FREE BODY FORCE DIAGRAM OF THE TANK AND SUSPENSION

By the principle of D'Alembert, the summation of the forces in the Y direction are given by:

$$\sum_{i=1}^{n} F_{yi} = -F_1 - F_2 - F_3 - F_4 - f_i = 0 \tag{25}$$

$$F_1 = K_1 Y_1 \tag{26}$$

$$F_2 = B_1 \dot{Y}_1 \tag{27}$$

$$F_3 = K_2 Y_2 \tag{28}$$

$$F_4 = B_2 \dot{Y}_2 \tag{29}$$

$$f_i = M_H \ddot{Y} \ . \tag{30}$$

From geometry in Figure 4,

$$Y_1 = Y - \ell_1' \sin\theta \tag{31}$$

$$Y_2 = Y + (\ell_2' - \ell_1') \sin\theta \ . \tag{32}$$

Thus,

$$\sum_{i=1}^{n} F_{yi} = -K_1(Y - \ell_1' \sin\theta) - B_1(\dot{Y} - \ell_1' \cos\theta\dot{\theta}) - M_H\ddot{Y}$$

$$- K_2(Y + \ell_2' \sin\theta - \ell_1 \sin\theta) - B_2(\dot{Y} + \ell_2' \cos\theta\dot{\theta} - \ell_1 \cos\theta\dot{\theta}) \quad , \tag{33}$$

which results in

$$M_H\ddot{Y} + (B_1 + B_2)\dot{Y} + (K_1 + K_2)Y - (K_1\ell_1' + K_2\ell_2' - K_2\ell_1') \sin\theta$$

$$- (B_1\ell_1' + B_2\ell_1' - B_2\ell_2') \cos\dot{\theta} = 0 \quad . \tag{34}$$

The static spring deflection forces have just cancelled the effect of the tank's weight, $M_H g$.

Summing the moments about the center of mass gives the equation for the second degree of freedom for this planar analysis:

$$\sum_{i=1}^{n} \vec{Mi}_{cm_{hull}} = \vec{F}_1\ell_3' + \vec{F}_2\ell_3' - \vec{F}_3\ell_4' - \vec{F}_4\ell_4' + M_{inertial} = 0 \quad . \tag{35}$$

The forces are the same as those described in equations (26) through (29). Also,

$$M_{inertial} = J_H\ddot{\theta} \tag{36}$$

$$\ell_3' = \ell_1' \cos\theta \tag{37}$$

$$\ell_4' = (\ell_2' - \ell_1') \cos\theta \quad . \tag{38}$$

Substituting into equation (35),

$$\sum_{i=1}^{n} Mi_{cm_{hull}} = -K_1(Y - \ell_1' \sin\theta)\ell_1' \cos\theta - B_1(\dot{Y} - \ell_1' \cos\theta\dot{\theta})\ell_1' \cos\theta$$

$$- K_2(Y + \ell_2' \sin\theta - \ell_1' \sin\theta)(\ell_2' - \ell_1') \cos\theta$$

$$- B_2(\dot{Y} + \ell_2' \cos\theta\dot{\theta} - \ell_1' \cos\theta\dot{\theta})(\ell_2' - \ell_1') \cos\theta + J_H\ddot{\theta} = 0 \quad . \tag{39}$$

Finally, simplifying and rearranging yields,

$$J_H \ddot{\theta} + \dot{\theta} \cos^2 \theta (B_1 \ell_1'^2 - B_2 \ell_2'^2 - B_2 \ell_1'^2 + 2B_2 \ell_1' \ell_2')$$
$$+ \cos\theta \sin\theta (K_1 \ell_1'^2 - K_2 \ell_2'^2 - K_2 \ell_1'^2 + 2K_2 \ell_1' \ell_2')$$
$$+ \dot{Y} \cos\theta (B_1 \ell_1' - B_2 \ell_2' + B_2 \ell_1') + Y \cos\theta (K_2 \ell_1' - K_2 \ell_2' - K_1 \ell_1') = 0 \quad . \tag{40}$$

This system is said to be elastically coupled because of the unsymmetric location of the mass center with respect to the suspension system.

It should be noted that equations (34) and (40) are exactly accurate only for the case in which the tank has no roll with respect to inertail basis associated with its motion. This is reasonable only if it is assumed to be traveling on a model of level terrain with only pitch undulations.

5. Actuator and Main Gun Dynamics

For the case of no compliance in the rod or in the fluid, Figure 6 shows that

$$\dot{Y}_1 = \dot{Y}_2 = Y \quad . \tag{41}$$

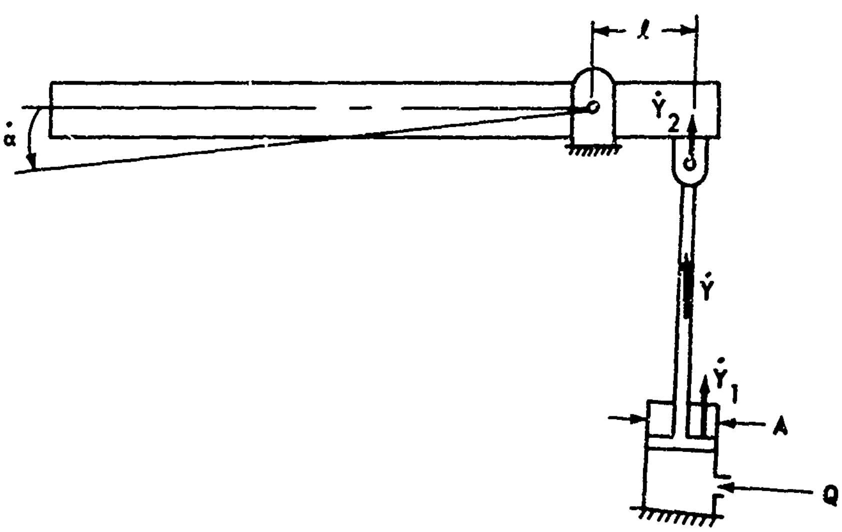

FIGURE 6. MAIN GUN AND HYDRAULIC ACTUATOR FORCES

Thus, fluid flow into the valve gives a force equation

$$Q = \dot{Y}_1 A \quad , \tag{42}$$

where

Q = fluid flow rate

$\dot{Y}_1$ = rate of displacement of the valve

A = area of the valve face.

Similarly, with no compliance, and a small angle approximation,

$$\dot{Y}_2 = \ell \dot{\alpha} \quad . \tag{43}$$

Combining equations (43) and (44) yields

$$Q = \ell \dot{\alpha} A \quad . \tag{44}$$

The general hydraulic equation including losses given by Wroble [2] and Cannon [1] is modified to give

$$Q = KY_0 \sqrt{P_s - P_L - \Delta P} \quad . \tag{45}$$

Thus,

$$\ell \dot{\alpha} A = KY_0 \sqrt{P_s - P_L - \Delta P} \quad . \tag{46}$$

and

$$\dot{\alpha} = K^* Y_0 \sqrt{P_s - P_L - \Delta P} \quad , \tag{47}$$

where

$K^* = \dfrac{K}{\ell A}$

P_s = supply pressure to actuator

P_L = losses in pressure

ΔP = change in pressure measured across the valve .

6. **Model of the Actuator - Equilibrator**

The equilibrator and actuator combination is shown in Figure 7.

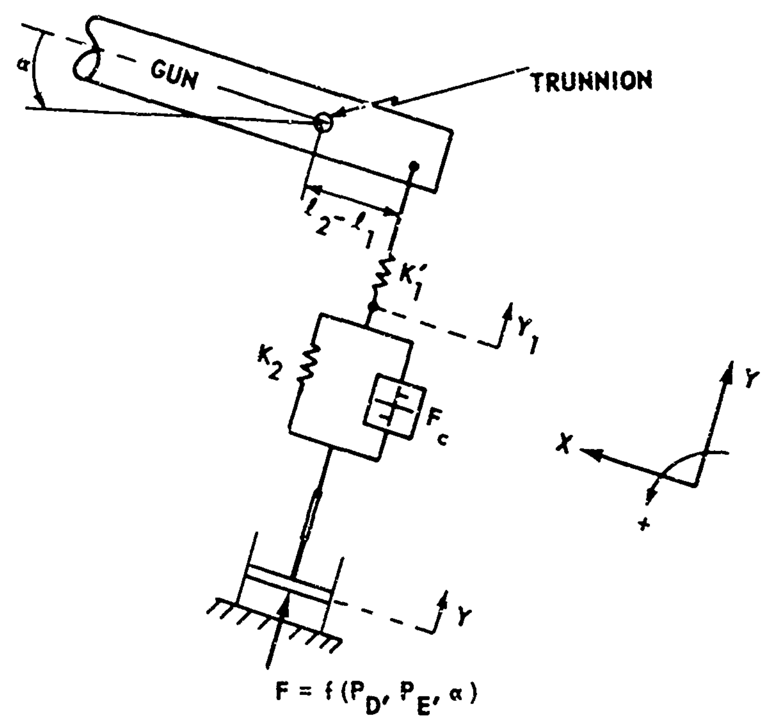

FIGURE 7. EQUILIBRATOR AND ACTUATOR COMBINATION

The free body force diagrams are shown in Figure 8.

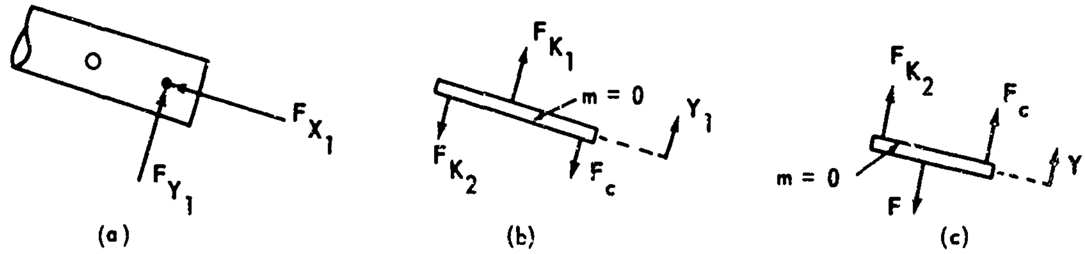

FIGURE 8. FREE BODY FORCE DIAGRAM OF ACTUATOR

The problem is to find the force $F_{Y1} = F_{K1}$ as a function of known quantities including F, Y, and α. As shown in Figure 8, parts (b) and (c), the springs and piston each have no appreciable mass; i.e., $m = 0$.

From the free body diagrams,

$$F_{Y1} = F_{K1} \quad (48)$$

$$F_{K1} = F_{K1} + Fc \quad (49)$$

$$F_{K2} + Fc = F \quad (50)$$

Thus

$$F_{Y1} = F_{K2} + Fc = F \quad . \quad (51)$$

$$F_{K1} = K_1'Y_1 \quad (52)$$

$$F_{K2} = K_2'(Y - Y_1) \quad (53)$$

$$Fc = Fc_{max}\, Sgn(\dot{Y} - \dot{Y}_1) \quad (54)$$

Now, Y is an obtainable quantity, while Y_1 is not. The expression for Y is obtained from solution of equations (34) and (41). Therefore, Y_1 must be replaced by a known value; from equation (53),

$$Y_1 = \frac{F_{K1}}{K_1} \quad . \quad (55)$$

Thus,

$$F_{K2} = \frac{K_2'\left(Y - F_{K1}\right)}{K_1} \quad (56)$$

$$Fc = Fc_{max}\, Sgn\left[\dot{Y} - \frac{d}{dt}\left(\frac{F_{K1}}{K_1'}\right)\right] \quad . \quad (57)$$

Substituting equations (57) and (58) into (51) yields:

$$K_2'\left(Y - \frac{F_{K1}}{K_1}\right) + Fc_{max}\,\text{Sgn}\left(\dot{Y} - \frac{d}{dt}\,\frac{F_{K1}}{K_1'}\right) = F\,. \qquad (58)$$

Solving for F_{K1} :

$$K_2'Y - K_2'\frac{F_{K1}}{K_1'} + Fc_{max}\,\text{Sgn}\left[\dot{Y} - \frac{d}{dt}\left(\frac{F_{K1}}{K_1'}\right)\right] = F \qquad (59)$$

$$F_{K1} = -\frac{K_1'}{K_2'}\left\{F - K_2'Y - Fc_{max}\,\text{Sgn}\left[\dot{Y} - \frac{d}{dt}\left(\frac{F_{K1}}{K_1'}\right)\right]\right\}. \qquad (60)$$

Now the expression, F, is given by ,

$$F = 1.885\left(P_D - P_E\right) - 0.587(1740 + 7.5\alpha) + 20(0.783) \qquad (61)$$

[1]

Thus,

$$F_{K1} = -\frac{K_1'}{K_2'}\left\{1.885\left(P_D - P_E\right) - 0.587(1740 + 7.5\alpha) + 20(0.783)\right.$$

$$\left. - K_2'Y - Fc_{max}\,\text{Sgn}\left[\dot{Y} - \left(\frac{F_{K1}}{K_1'}\right)\right]\right\} \quad . \qquad (63)$$

Differentiating equation (63) with respect to time gives the expression for inclusion in the signum function of the coulomb friction term:

$$\frac{d}{dt}\left(\frac{F_{K1}}{K_1'}\right) = -\frac{K_1'}{K_2'K_1'}\left[-0.587(7.5\dot{\alpha}) - K_2'\dot{Y}\right] ; \qquad (63)$$

so that finally,

$$F_A = F_{K1} = -\frac{K_1'}{K_2'}\left\{1.885\left(P_D - P_E\right) - 0.587(1740 + 7.5\alpha) + 20(0.783)\right.$$

$$\left. - K_2'Y - Fc_{max}\,\text{Sgn}\left[\dot{Y} - \frac{1}{K_2'}\left(0.587(7.5\dot{\alpha}) + K_2'Y\right)\right]\right\} \quad . \qquad (64)$$

[1] Equation (61) was derived by T. G. Wetheral.

Now, equation (65) is exactly the expression needed to complete the moment equation of the gun given by equation (23), where the actuator force F_A in that derivation is the same force as F_{K1} here.

The alternative model of the Actuator-Equilibrator is shown in Figure 9.

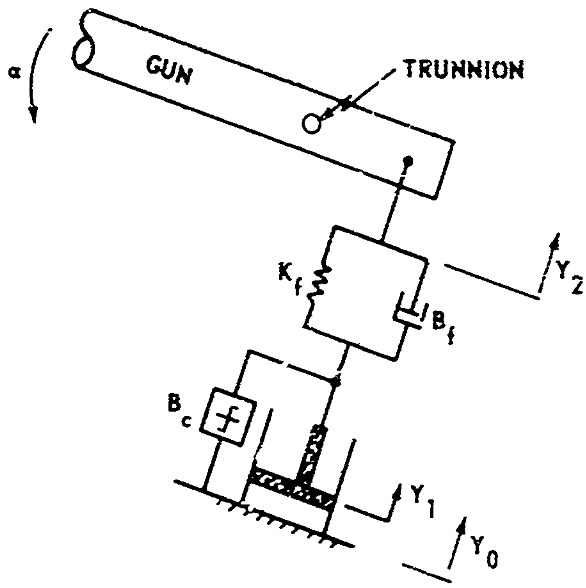

FIGURE 9. ALTERNATIVE ACTUATOR AND EQUILIBRATOR COMBINATION

The free body force diagram is shown in Figure 10.

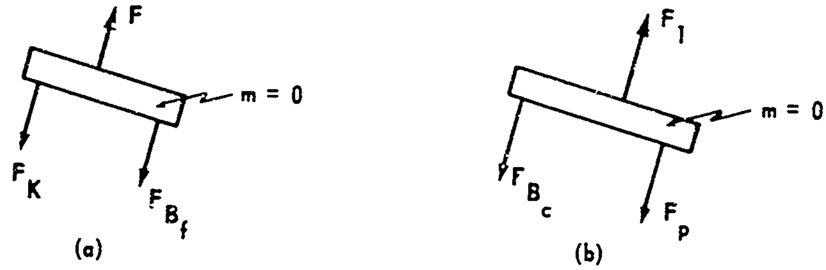

FIGURE 10. FREE BODY FORCE DIAGRAM OF ACTUATOR

From the hydraulic equation this relationship is given

$$\dot{Y}_1 - \dot{Y}_0 = KX\sqrt{F_s - F - B_c \, \mathrm{Sgn}\,(\dot{Y}_1 - \dot{Y}_0)} \qquad (65)$$

and,

$$F = (Y_1 - Y_2)\, K_f + (\dot{Y}_1 - \dot{Y}_2)\, B_f \quad . \qquad (66)$$

This can be visualized in Figure 11.[2]

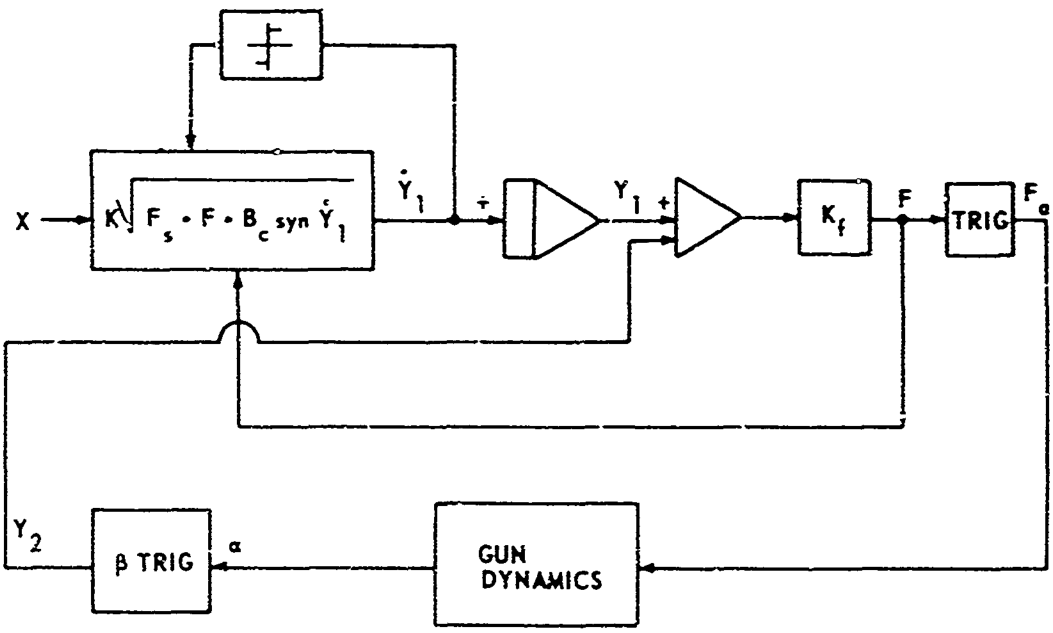

FIGURE 11. ANALOG DIAGRAM OF ALTERNATIVE SCHEME

Now

$$Q = 11 \text{ gal/min} \times \frac{231}{60} \times \frac{\text{in.}^3\text{min}}{\text{gal sec}} = 42.35 \, \frac{\text{in.}^3}{\text{sec}} \qquad (67)$$

at the third stage displacement of the servo spool of $X_0 = 0.125$ inch. Since

$$\dot{Y}_1 = \frac{Q}{A} \, \frac{\text{in.}^3/\text{sec}}{\text{in.}^2} = \frac{42.35}{1.885} \, \frac{\text{in.}}{\text{sec}} = 1.872 \text{ ft/sec} \ . \qquad (68)$$

[2] This derivation was done by W. C. Jordan.

The supply pressure is given,

$$F_s = 2000 \text{ lb/in.}^2 \times 1.885 \text{ in.}^2 = 3770 \text{ lb} \quad ,$$

and

$$F_L = 0 \quad ,$$

since Q is the no load flow. Also, the hydraulic equation is

$$Q = K\bar{\underline{X}}o\sqrt{F_s - F_L} = K\bar{\underline{X}}o\sqrt{F_s} \quad ; \tag{69}$$

thus

$$\dot{Y}_1 = K' \times 0.125 \text{ in.} \times \sqrt{3770 \text{ lb}} \tag{70}$$

or

$$K' = 0.2439 \frac{\text{ft}}{\text{in.-sec}\sqrt{\text{lb}}} \tag{71}$$

7. Analog Computer Synthesis of Main Gun Dynamics

The dynamics of the main gun, given by equation (23), are repeated here as follows:

$$(\ddot{\alpha} + \ddot{\theta}) = \frac{1}{J_G}\left[(\ddot{\alpha} + \ddot{\theta})\, M_G \ell_1^2 + \cos\alpha\left(M_G g \ell_1 + F_A \cos\beta\,(\ell_2 - \ell_1)\right)\right] . \tag{72}$$

This is represented by the circuit in Figure 12.

8. Analog Computer Synthesis of Tank Dynamics in the Pitch Plane

The linear motion of the tank in the pitch plane (with coupling) is given by equation (34), rewritten here

$$-\ddot{Y} = \frac{1}{M_H}\Big[[(B_1 + B_2)\,\dot{Y} + (K_1 + K_2)\,Y - (K_1\ell_1' + K_2\ell_2' - K_2\ell_1')\sin\theta$$

$$- (B_1\ell_1' + B_2\ell_1' - B_2\ell_2')\cos\theta\dot{\theta}]\Big] . \tag{73}$$

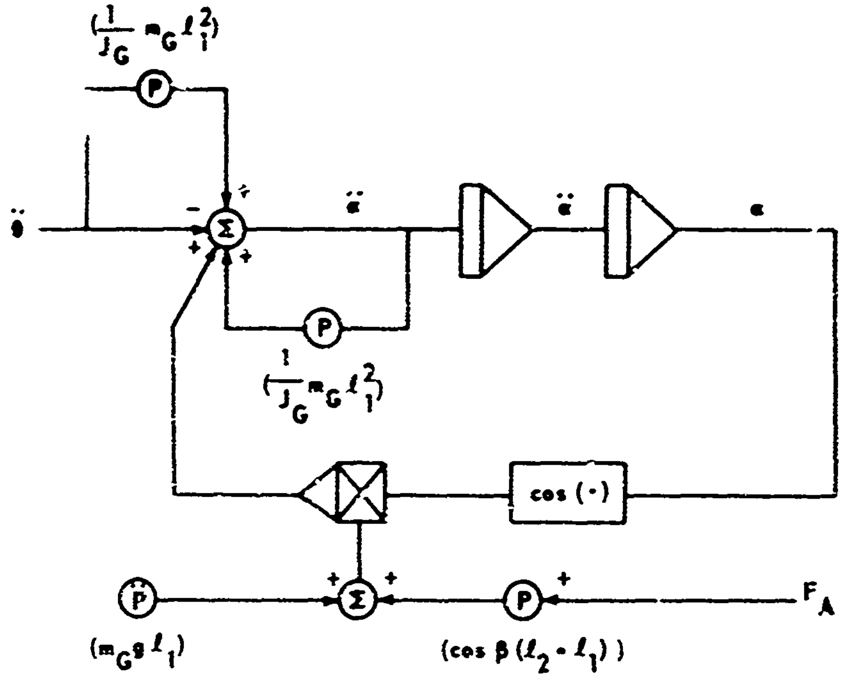

FIGURE 12. ANALOG DIAGRAM FOR MAIN GUN DYNAMICS

This equation is programmed as shown in Figure 13.

The angular motion of the tank is also coupled into the linear motion in the pitch plane. The expression, rewritten from equation (41), is

$$
\begin{aligned}
-\ddot{\theta} = \frac{1}{J_H}\Big[[\dot{\theta} \cos\theta(B_1\ell_1'^2 - B_2\ell_2'^2 - B_2\ell_1'^2 + 2B_2\ell_1'\ell_2') \\
+ \cos\theta \sin\theta(K_1\ell_1'^2 - K_2\ell_2'^2 - K_2\ell_1'^2 + 2K_2\ell_1'\ell_2') \\
+ \dot{Y}\cos\theta(B_1\ell_1' - B_2\ell_2' + B_2\ell_1') \\
+ Y\cos\theta(K_2\ell_1' - K_2\ell_2' - K_1\ell_1')]\Big] = 0 \quad . \qquad (74)
\end{aligned}
$$

This equation is programmed as shown in Figure 14.

These analog diagrams representing the gun and hull dynamics will be incorporated into the system simulation shown in Figure 15.

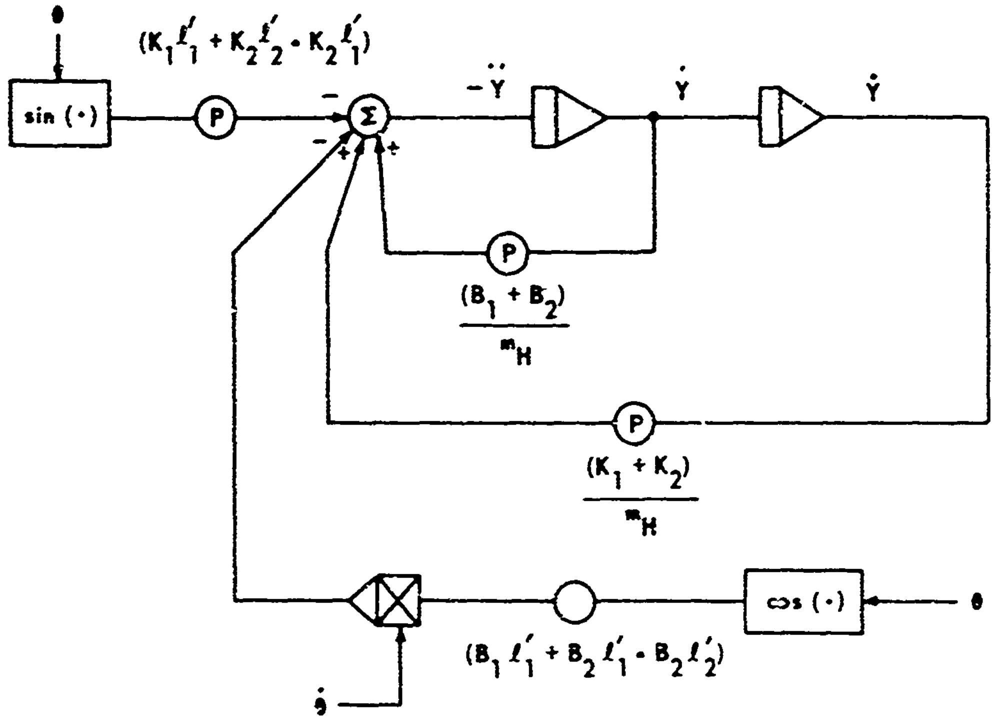

FIGURE 13. ANALOG DIAGRAM OF TANK'S LINEAR MOTION

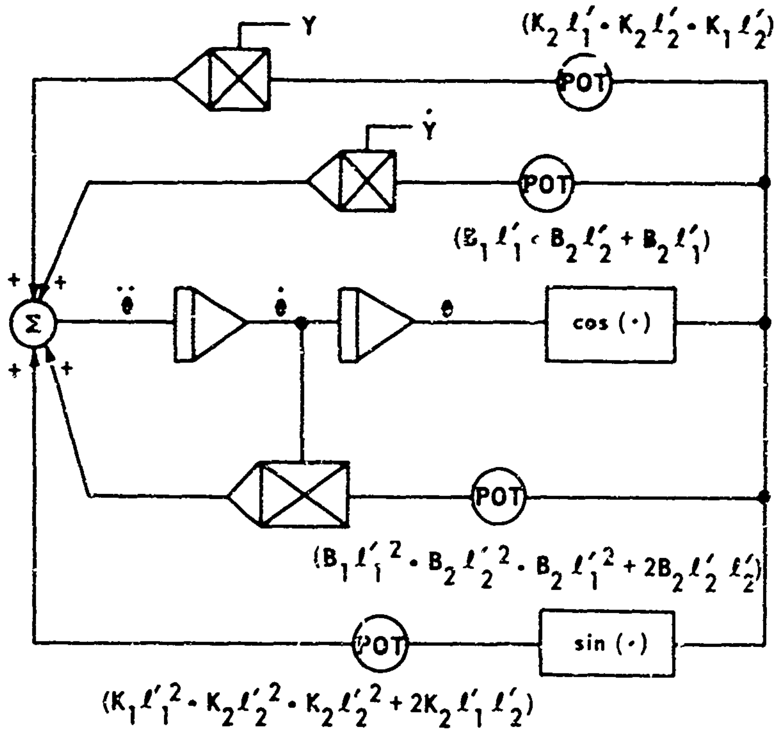

FIGURE 14. ANALOG DIAGRAM OF TANK'S COUPLED ANGULAR MOTION

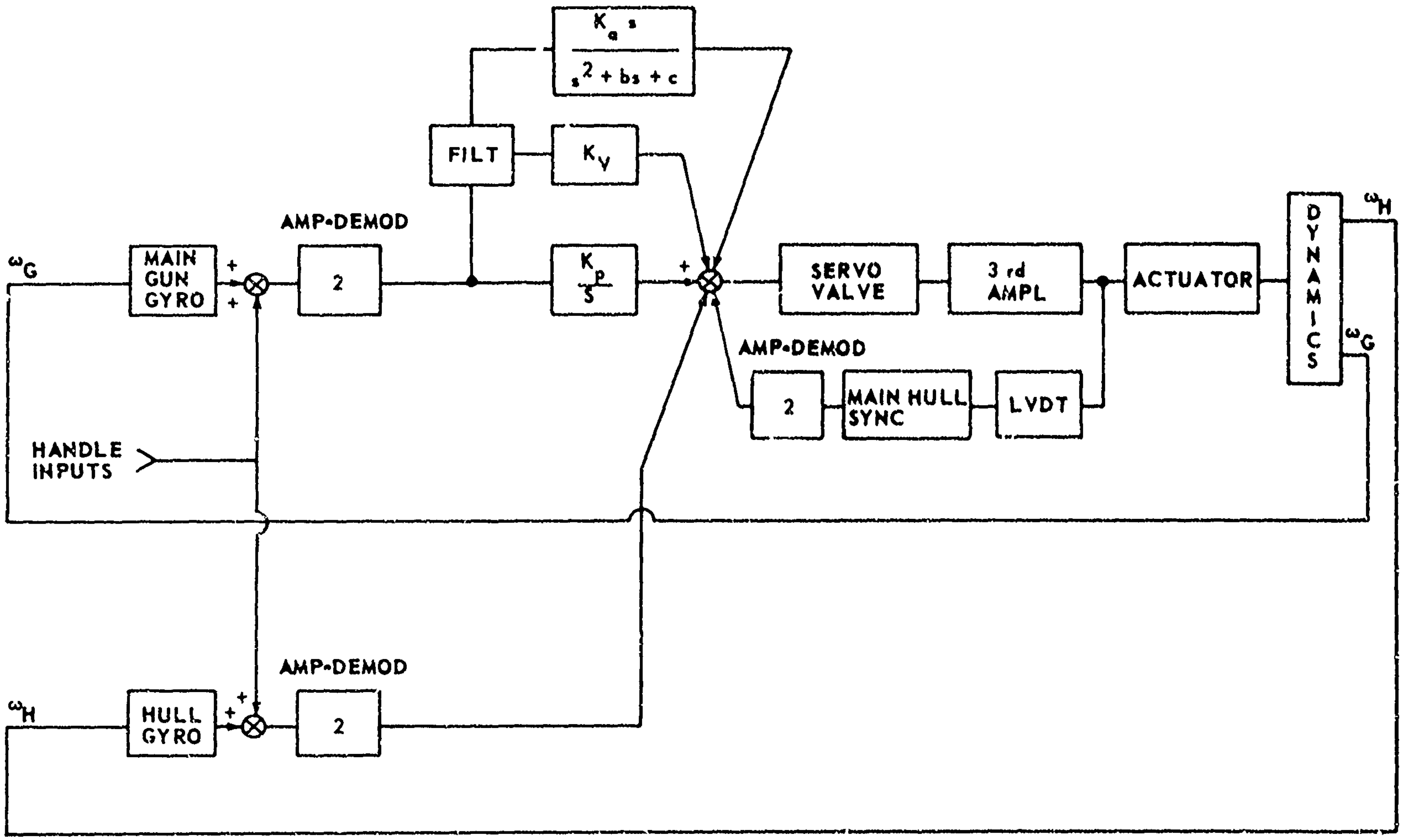

FIGURE 15. ELEVATION SYSTEM BLOCK DIAGRAM

9. Calculation of Angle β

The geometry of angle β is shown in Figure 16.

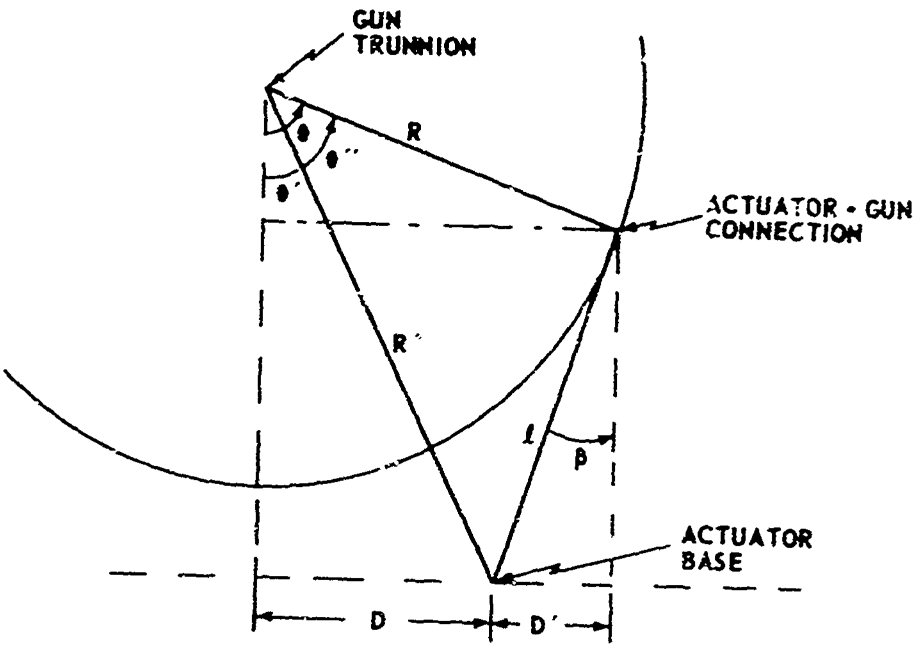

FIGURE 16. GEOMETRY OF ANGLE β

From Figure 16,

$$R \sin\theta - D = D' \tag{75}$$

$$\theta = \theta' + \theta'' \tag{76}$$

$$\beta = \sin^{-1}\frac{\ell'}{\ell} \tag{77}$$

$$\theta' = \sin^{-1}\frac{D}{R} \quad .^{3} \tag{78}$$

Now from trigonometry of oblique triangles,

$$\frac{\theta''}{2} = \tan^{-1}\frac{r}{s - a} \quad , \tag{79}$$

[3]This geometry was largely developed by R. E. Yates.

where

$$r = \sqrt{\frac{(s - a)(s - b)(s - c)}{s}}$$

$a = \ell$

$b = R$

$c = R'$

$S = 1/2\ (a + b + c)$.

Since R and R' are fixed, the angle β is uniquely determined as a function of ℓ .

The list of nominal values used in the simulation is given in Table I.

TABLE I. LIST OF NOMINAL VALUES USED IN THE SIMULATION

J_G	=	585.25 slug-ft^2
M_G	=	189.49 slugs
ℓ_1	=	5.2 in. = 0.4333 ft
$(\ell_2 - \ell_1)$	=	26 in. = 2.1666 ft
F_c	=	230 lb
B	=	10.5° = 0.183 radians, when gun is level
B_1	=	B_2 = 144.68 lb/in./sec = 1736.160 lb/ft/sec
K_1	=	K_2 = 100.000 lb/in. = 1,200,000 lb/ft
M_H	=	3447.20 slugs
ℓ_1'	=	78.55 in. = 6.5458 ft
ℓ_2'	=	166.72 in. = 13,8933 ft
J_H	=	142,609 slug-ft^2
$M_G g$	=	6101.5 lb
K_1'	=	100,000 lb/in. = 1,200,000 lb/ft
K_2'	=	52,000 lb/in. = 624,000 lb/ft

REFERENCES

1. Cannon, R. H., Dynamics of Physical Systems, McGraw-Hill Book Company, Inc., 1967.

2. Wroble, A. J., Analog Computer Simulation Study on the M60AIE2 Turret Stabilization System, Cadillac Gage Company Report No. 68-13, 1968.

BIBLIOGRAPHY

Burington, R. S., Handbook of Mathematical Tables and Formulas, Handbook Publishers, Inc., Third Edition, 1949.

Cadillac Gage Company, Brief Description of the M60AIE2 Tank Stabilized Gun Control System. November 1968.

DelToro, V., and Parker, S. R., Principles of Control System Engineering, McGraw-Hill Book Company, Inc., 1960.

Greenwood, D. T., Principles of Dynamics, Prentice Hall, Inc., 1965.

Lear Siegler, Inc., "Frequency Response of 2157-60 AR Proportional Rate Gyro," Inter Office Correspondence to T. G. Wetheral, March 28, 1969.

Timoshenko, S., and Young, D. H., Vibration Problems in Engineering, D. Van Nostrand, Inc., Third Edition, 1955.

Tse, F. S., Morse, I. E., and Hinkle, R. T., Mechanical Vibrations, Allyn and Bacon Inc., 1963.

Vennard, J. K., Elementary Fluid Mechanics, Wiley, Inc., Fourth Edition, 1965.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Army Inertial Guidance and Control Laboratory and Center Research and Engineering Directorate (Provisional) U. S. Army Missile Command Redstone Arsenal, Alabama 35809	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP N/A

3. REPORT TITLE

AN ANALYSIS AND SIMULATION OF THE M60AIE2 TANK MAIN GUN'S ELEVATION CONTROL SYSTEM

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

Harold L. Pastrick

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
29 August 1969	33	2

8a. CONTRACT OR GRANT NO. b. PROJECT NO. (DA) MIS98753247381 c. AMC Management Structure Code No. d. 4440.15.1111	9a. ORIGINATOR'S REPORT NUMBER(S) RG-TR-69-14 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AD ________

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Same as No. 1

13. ABSTRACT

A dynamical analysis of tne M60AIE2 Tank main gun's elevation stabilization system is performed. The equations describing the gun, hull, and turret dynamics are derived and computer diagraming is shown for simulation purposes.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
M60AIE2 Tank Elevation stabilization system Dynamics equations - gun - hull - turret						