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SOUND VELOCITY, ELASTICITY, AND RELATED PROPERTIES OF MARINE SEDIMENTS, NORTH PACIFIC

II. Elasticity and Elastic Constants

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October 1969

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THE PROBLEM

Determine and study the acoustic properties of sea-floor sediments and acoustic models of the sea floor; specifically, use measured values of compressional-wave (sound) velocity and density, with computed values of sediment bulk modulus to compute other elastic constants, and determine appropriate elastic models for the sea floor.

RESULTS

1. Marine sediments can be considered as elastic media and the equations of Hookean elasticity can be used to compute unmeasured elastic constants. When sound attenuation must be considered, a viscoelastic model is favored in which the rigidity modulus, μ, and Lamé's constant, λ, in the equations of elasticity are replaced by complex Lamé constants ($\mu + i\mu'$ and $\lambda + i\lambda'$) which are independent of frequency. In this model, μ and λ represent elastic response, and $i\mu'$ and $i\lambda'$ represent damping of wave energy. This model implies that wave velocities and the dissipation function, $1/Q$, are independent of frequency, and attenuation in dB per unit length varies linearly with frequency in the range from a few Hz to the megahertz range.

2. Density and compressional-wave velocity were measured in the present study. A computed value for the system bulk modulus (following Gassmann, 1951) was used as the third constant required to compute the other elastic constants with the equations of elasticity. These computations were thus based on theory without empirical factors or constants.

The components of the computed system bulk modulus are porosity, the bulk modulus of pore water, an aggregate bulk modulus of mineral grains, and a bulk modulus of the structure, or frame, formed by the mineral grains. Good values for the bulk moduli of distilled and seawater, and most of the common minerals of sediments, have been established in recent years; this leaves only a value for the frame-bulk modulus to compute a bulk modulus for the water-mineral system of the sediment.

Curves and regression equations relating porosity and dynamic frame bulk modulus were derived and used in computations of a system bulk modulus, which, with measured density and compressional-wave velocity, was used to compute other elastic constants.

3. The computations of elastic constants as discussed in this report, compared with other laboratory and *in situ* measurements (such as shear-wave velocity) indicate that the equations of elasticity can be used to derive reasonable values for those elastic constants not measured. If so, the listed values in the tables, and the method, can be used to predict these constants for the major sediment types.

RECOMMENDATIONS

1. Continue to measure and study the least-known acoustic properties of marine sediments—the attenuation of compressional (sound) waves, the velocities of shear waves, and elastic and viscoelastic constants. These measurements and studies should include variations of properties with depth in the sea floor.

2. Direct these studies toward improvement of acoustic models of the sea floor to support under-water sound propagation and geophysical studies.

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PREFACE

This report is Part II in a series of Technical Publications on sound velocity, elasticity, and related properties of marine sediments from three major environments of the North Pacific: the continental terrace (shelf and slope), abyssal plain (turbidite), and abyssal hill (pelagic).

Part I (TP 143) detailed the measurement and computation of mass physical properties of the sediments, including density, velocity, grain size, impedance, reflection coefficients, and bottom loss.

The present report (TP 144) presents discussions of elastic and viscoelastic models for water-saturated porous media, and measurements and computations of elastic constants.

Part III (TP 145) will be concerned with predictions and computations of *in situ* physical properties.

Throughout each report, references are made to the other two studies, as appropriate.

INTRODUCTION

The characteristics of the sea floor as they affect the propagation of acoustic energy in the sea are the subject of a major continuing study. The phase of this study with which the present series of reports is concerned is an investigation of sound velocity, elasticity, and related properties of marine sediments from three major elevation elements of the North Pacific—the continental terrace (shelf and slope), abyssal plain (turbidite), and abyssal hill (pelagic). Each of the major sediment types except calcareous ooze is represented.

NUC Technical Publication 143 (Part I of the series) detailed the properties and interrelationships of mass physical properties of the sediments—mean and median grain diameters, percentages of sand, silt, and clay, saturated bulk densities, bulk densities of mineral grains, porosities, sound velocities, impedances, density ratio (sound velocity in the ratio of sound velocity in sediment to that in seawater), and Rayleigh reflection coefficients and bottom losses at normal incidence. Part III will be concerned with prediction of *in situ* sediment properties.

The present report is concerned with the elastic properties of the sediments, including elastic models. The following sections present measurements and computations of elastic constants and their interrelationships with other physical properties. The discussions cover compressibility, bulk modulus, rigidity (shear) modulus, Lamé's constant, Poisson's ratio, density, and shear and compressional wave velocities.

ROCKS AND SEDIMENTS AS ELASTIC MEDIA

Introduction

Many papers in the literature of geology, geophysics, physics, and soil mechanics are directed to the question of whether water-saturated rocks, sands, and clays are elastic bodies (in a classical sense) or viscoelastic, or other, media, and whether, if they are viscoelastic, these media are closer to Hookean, Kelvin-Voigt, Maxwell, or other models. During the past [illegible] years a great deal of theoretical and experimental work (mostly on rocks) has dealt with this or associated questions. In this period, especially since about 1960, careful experimental work with modern electronic measuring equipment, together with theoretical studies, has established that rocks (including water-saturated rocks) are, for all practical purposes of geophysics and underwater sound, elastic bodies which can be studied with the Hookean equations of classical elasticity, insofar as the velocity of compressional and shear waves and associated elastic constants are concerned.

The author is of the opinion that the body of theoretical and experimental evidence concerning water-saturated sediments shows that these media are, like rocks, elastic bodies which can be studied, and their properties computed, using the equations of Hookean elasticity.

Many of the basic theories of soil mechanics are based on the premise that a water-saturated sediment is, for small strains, a linearly elastic, porous medium which is macroscopically isotropic. Terzaghi (1925) adopted this point of view in his development of the theory of consolidation. In 1941, Biot wrote a classic paper on three-dimensional consolidation in which he adopted the same viewpoint. This paper has not

received the attention it deserves. Biot and Willis (1957)* showed how the elastic constants, derived in 1941, could be experimentally determined by static tests on jacketed and unjacketed samples of porous media. In 1956, Biot extended his analyses to compressional and shear waves by adding inertial terms to his theory of three-dimensional consolidation; he later extended his concepts to viscoelastic and anisotropic bodies. In a recent paper (Biot, 1962) he summarized and extended much of his work of the previous 20 years.

In his various papers, Biot discussed the full range of systems in which water within pore spaces does or does not move with the solids upon imposition of a small stress such as that of a sound wave. In some acoustic models and theoretical studies, this movement or flow of water through the sediment mineral structure has been considered to be of the Poiseuille type. In the last several decades it has been determined that the simple flow equations of the Poiseuille type (derived from flow of water through tubes) do not hold for real, *in situ* sediments. These equations have to be considerably altered, even for clean sands, and are not applicable to relatively impermeable clays (Yong and Warkentin, 1966). The question of relative water movement is a critical key to whether or not the equations of elasticity (*i.e.*, Hookean responses) can be used in studies of rocks and sediments. If the pore water does not move significantly with respect to the solids, then the effective density of the medium is the sum of the mass of the water and solids in a unit volume, water viscosity need not be considered, and the equations of ideal elasticity can be used. This is the "closed system" of Gassmann (1951).

To avoid an extensive review of the elastic relationships in porous media (sediments and rocks), the following concepts (believed by the author to be true) are stated, with references which support or have an important (affirmative) bearing on the concept.

1. If a saturated porous medium is a "closed system" it responds to small stresses (such as that of a sound wave) as a Hookean body (strain is recoverable upon removal of stress) in a linearly elastic system; any deviations from Hookean response are so small that they can be neglected for the purposes of underwater acoustics and geophysics, and the system can be studied with the equations of classical elasticity (Biot, 1941, 1956, 1962; Biot and Willis, 1957; Gassmann, 1951; Morse, 1952; White and Sengbush, 1953; Zwikker and Kosten, 1949; Brandt, 1955; Hamilton *et al.*, 1956; Paterson, 1956; Wyllie, Gregory, and Gardner, 1956; Laughton, 1957; Jones, 1958; Knopoff and MacDonald, 1958; McDonal *et al.*, 1958; Birch, 1960, 1961, 1966; Barkan, 1962; Geertsma and Smit, 1961; Nafe and Drake, 1963; Brutsaert, 1964; Simmons and Brace, 1965; White, 1965).

2. Most rocks, composed of randomly-oriented mineral crystals, are macroscopically isotropic to compressional and shear waves in a frequency range from a few Hz to the megahertz range if the grain sizes are much smaller than wavelengths; the elastic constants of such systems can be computed with the Voigt-Reuss-Hill averaging method (Hill, 1963), discussed below. A recent excellent summary of the evidence for the Voigt-Reuss-Hill averaging method has been published by Anderson and Liebermann (1968). In addition to references cited in the previous paragraph, attention is called to reports by Brace (1965), Peselnick (1962), and Christensen (1965, 1966).

3. Static measurements of the elastic constants of a saturated, porous rock or sediment are essentially isothermal; dynamic constants are adiabatic. In rocks and minerals, isothermal and adiabatic measurements can be experimentally and theoretically related in a linearly elastic system (Biot and Willis, 1957; Biot, 1962; Anderson *et al.*, 1968; Brutsaert, 1964; Trietel, 1959; Anderson and Schreiber, 1965; and others). In connection

*See list of references at end of report.

with this concept and for discussions of the fact that elastic constants measured by static methods are smaller than those measured by dynamic methods, attention is directed to reports by Simmons and Brace (1965) and Walsh (1965).

4. Liquid saturation of porous rocks and sediments increases the velocity of compressional waves, and decreases the velocity of shear waves (Gassmann, 1951; White and Sengbush, 1953; Biot, 1956; King, 1966). Figures 12 and 13 illustrate these phenomena (Shell Development Co., private communication, 1965).

5. It is not necessary to postulate movement of viscous pore-water relative to mineral grains to explain attenuation in saturated porous media. Other sound-attenuation factors not involving sediment-structural permeability, water viscosity and time are apparently present — for example, thermal losses and losses due to internal friction (see Bradley and Fort, 1966, for a recent resumé).

6. The velocities of compressional and shear waves are independent of frequency (*i.e.*, no dispersion or change of velocity with frequency) from a few hertz into the megahertz range; or dispersion is negligibly small. Experimental evidence on this subject is discussed in a later section.

Elastic and Viscoelastic Models for Rocks and Sediments

In the above discussion of theoretical development, it was stated or assumed that the numbered statements were true only for small stresses. In the field of soil mechanics, especially, large static or dynamic stresses have to be considered, and over the full range of stresses, sediments are both elastic and viscoelastic. Yong and Warkentin (1966, p. 80-94) have a good discussion of the various models and elements within the models which describe this behavior. These writers favored the Burger model for mechanical simulation of actual soil behavior.

The Burger model is formed by coupling the Kelvin-Voigt model in series with the Maxwell model (Yong and Warkentin, 1966, p. 90-93, figs. 4, 12-15). In the Burger model very small stresses are apt to be recoverable (*i.e.*, Hookean), but if the magnitude of a stress is sufficient, the medium behaves as a viscoelastic body. In the fields of soil mechanics and foundation engineering, the Hookean model and equations are commonly used for derivations of dynamic elastic constants and studies of vibrating loads (*e.g.*, Barkan, 1962; Heukelom, 1961; Jones, 1958; Evison, 1956; Hardin and Richart, 1963; Hall and Richart, 1963; Richart and Whitman, 1967).

In the fields of physics and geophysics, studies of the elasticity of minerals and rocks have demonstrated that the elastic equations of the Hookean system adequately define the velocities of compressional and shear waves; these equations are conveniently interrelated in a table by Birch (1961, p. 2206). This field has been summarized by Birch (1966) and by Anderson and Liebermann (1968); papers of special interest are by Christensen (1966a, b), Brace (1965), and Simmons and Brace (1965).

Although the elastic equations of the Hookean model adequately account for wave velocities in earth materials, they do not provide for wave-energy losses (attenuation) in these media. To account for attenuation, various viscoelastic or "near-elastic" models and equations have been proposed. One of these, which has been studied in connection with rocks and sediments, is the Kelvin-Voigt model, in which, as originally defined, compressional-wave velocity varies with frequency; and attenuation, at frequencies of interest in underwater acoustics and geophysics, increases with the square of frequency. White (1965, p. 110-112) has a thorough discussion of theory and experimental evidence on this subject, and concludes (p. 112) that neither velocity nor attenuation shows this

frequency dependence, and the Voigt solid cannot be considered an adequate model of earth materials. The experimental evidence of this report (and of reports in preparation) is in accord with this conclusion.

A growing body of experimental evidence in the field and laboratory has tentatively placed several probable restrictions on nonelastic models and equations. For example, velocities and energy damping are independent of frequency, and spatial attenuation increases approximately linearly with frequency in the range of frequencies of most interest in engineering, marine geophysics, and underwater acoustics (a few hertz to the megahertz range).

A viscoelastic or "near-elastic" model within the limits of the above restrictions, without specification of the mechanics of wave attenuation, has been discussed by White (1965, p. 79-134), Bucker (1964), Hamilton *et al.* (1969), and others. In this model the Lamé constants, μ and λ, in the equations of elasticity are replaced by complex Lamé constants $(\mu + i\mu')$ and $(\lambda + i\lambda')$ in which μ and λ represent elastic response, and μ' and λ' represent damping of wave energy. This approach is based on the assumption that all elements of the complex Lamé constants are independent of frequency in the range of frequencies of interest in most problems of engineering, geophysics, and underwater acoustics. The basic equations of this and associated models are presented in papers by Ewing *et al.* (1957), Bucker (1964), White (1965), and Hamilton *et al.* (1969).

The assumption that the complex Lamé constants are independent of frequency from a few hertz to the megahertz range implies that their elastic, or real, parts, μ and λ, are independent of frequency. Because these constants, and density, govern wave velocities it is further implied that both compressional- and shear-wave velocities are independent of frequency (in literature: "no dispersion of velocity with frequency"). The subject of possible dispersion will be discussed in a later section.

If the complex Lamé constants are independent of frequency, their imaginary parts, which govern energy losses of wave propagating through a medium, must be independent of frequency. Biot (1962, p. 1498) noted that the imaginary part of complex rigidity (and other operators involved in energy losses) represent a damping which varies very little within a relatively large range of frequency. The independence of the dissipative parts of the complex Lamé constants from frequency is implied in the studies of energy losses in earth materials, where the specific dissipation function, $1/Q$, and the logarithmic decrement, Δ, have been shown to be independent of frequency over a range of at least 10^8 Hz (Knopoff and Macdonald, 1958; White, 1965; Bradley and Fort, 1966; Attwell and Ramana, 1966).

Important evidence that the complex Lamé constants are independent of frequency in high-porosity sediments was recently furnished by Cohen (1968). He measured both μ and $i\mu'$ in complex rigidity $(\mu + i\mu')$ in artificial, laboratory sediments composed of kaolinite and bentonite in distilled water, with and without a deflocculating agent. Cohen demonstrated that both μ and $i\mu'$ were independent of frequency in the range 8.6 to 43.2 kHz in flocculated clay, but when a deflocculating agent was added, the flocculated structure of the clay sediment dispersed, the material lost all rigidity, μ, and behaved as a Newtonian viscous fluid in which $i\mu'$ was "viscosity," and was linearly dependent on frequency. The addition of 35.5 ppt of NaCl caused reflocculation, and complex rigidity was the same as before. Because $i\mu'$ governs energy dissipation, this demonstration that $i\mu'$ was independent of frequency extends the range of frequency-independence of the dissipation constant, $1/Q$, from rock to flocculated clay structures. Other aspects of this study will be discussed in later sections.

Another implication of the assumption that the complex Lamé constants are independent of frequency is that attenuation in dB/unit length, α, increases linearly with frequency, f (White, 1965, p. 98). Recent summaries of work in this field indicate that, for most earth materials, there is a small variation around linearity in the range of

frequencies of interest in underwater acoustics and geophysics; that is, in the relationship, $\alpha = \kappa f^n$ the exponent "n" is about one. In clean sands, however, there is some evidence that the exponent may be nearer 0.5 (Nolle, *et al.*, 1963; Hampton, 1967), but this matter requires further study in natural marine sands. A linear relation of attenuation with frequency has been successfully used by Cole (1965) in extrapolating resonant-chamber attenuations in marine sediments at 20 to 40 kHz down to 100 Hz; and Wood and Weston (1964), in an experiment in a tidal mud flat, found an approximate linear dependence in the frequency range from 4 to 50 kHz.

The elements of the complex Lamé constants (μ, μ', λ, and λ') can be computed if the velocity and attenuation of the compressional wave, velocity of the shear wave, and density are known, and bulk viscosity is assumed to be zero. "Bulk viscosity" is a phenomenon related to the time lag which may occur between pressure and volume change (Jaeger, 1962, p. 71; Biot, 1962, p. 1492); it is analogous to the elastic bulk modulus, $\kappa = \lambda + 2\mu/3$. Setting bulk viscosity equal to zero ($\lambda' = -2\mu'/3$) is a simplification which has been used by Jaeger (1962, p. 71), Ewing, *et al.* (1957, p. 273), Biot (1962), and others.

Recently, *in situ* measurements were made (from a research submersible) in sands and silt-clays of the velocities and attenuations of compressional waves, and velocities of Stoneley waves (from which shear waves were computed). These measurements allowed tentative evaluations of the above viscoelastic model, and computations of both elastic (Hookean) constants and the complex Lamé constants of the viscoelastic model (Hamilton *et al.*, 1969). These computations affirmed that either Hookean-elastic or viscoelastic equations can be used to derive the same compressional- and shear-wave velocities and associated elastic constants.

The author believes that the evidence of the present report, against the theoretical and experimental background outlined above, strongly supports the use of the Hookean ("closed system") elastic model and its equations in computations and predictions of the velocities of compressional and shear waves, and other elastic constants, in natural marine sediments.

Pertinent Equations of Elasticity

If the Hookean model, or the Hookean component in more complex models, is approximately correct for studying the velocities of compressional and shear waves in saturated sediments, and classical equations of elasticity can be used to compute those constants not measured, density, and any two of the other elastic constants are required. In the case of water-saturated sediments, the density and compressional-wave velocity can be easily measured or can be closely predicted (as discussed in Parts I and III). The best third constant for computations of the elastic constants of marine sediments is the velocity of shear waves, at least for purposes of underwater sound and geophysics.

Information on the velocity of shear waves in marine sediments is rare, in fact, it is so rare that one of the principal contributions of this report could be in the computations of shear-wave velocities (which, in effect, constitute predictions of the values expectable in natural marine sediments). Comparison of experimental with computed values of shear-wave velocities will be made in a later section.

Lacking sufficient information on shear-wave velocities, the third constant used here for computations of elastic constants is the bulk modulus, or incompressibility (the reciprocal of compressibility). This constant was selected because it is possible to compute, in a logical manner, the bulk modulus of the sediment (water-mineral) system from

its components without estimations; this subject will be discussed in detail in the next section. Prior to these discussions, the pertinent equations of elasticity should be reviewed. The equations favored are those involving the two measured constants (density and compressional-wave velocity) and the bulk modulus.

The basic equation for the velocity of a compressional wave, V_p, is

$$V_p = \left(\frac{\kappa + 4/3\,\mu}{\rho}\right)^{1/2} \tag{1}$$

where

κ = incompressibility or bulk modulus = $1/\beta$

μ = shear (rigidity) modulus

ρ = density

β = compressibility.

When a medium lacks rigidity, μ, equation 1 becomes

$$V_p = \left(\frac{\kappa}{\rho}\right)^{1/2}, \text{ or } \kappa = \rho V_p^{\,2} \tag{2a}$$

$$V_p = \left(\frac{1}{\beta\rho}\right)^{1/2} \tag{2b}$$

Equation 2 applies to any liquid, emulsion, or suspension which lacks rigidity.

For a unit volume of a suspension or porous material, lacking rigidity and composed of water and mineral grains, compressibility, β, and density, ρ, in equation 2b have been expanded into

$$\beta_{sw} = n\beta_w + (1-n)\beta_s \tag{3a}$$

or, expressed as κ_{sw}

$$\kappa_{sw} = \frac{\kappa_w \kappa_s}{n\,(\kappa_s - \kappa_w) + \kappa_w} \tag{3b}$$

$$\rho = n\rho_w + (1-n)\rho_s \tag{4}$$

where

n = volume of pore space occupied by water (fractional porosity), subscripts "s" and "w" indicate mineral solids and water, β_w is computed with equation 2a, and β_s with the Voigt-Reuss-Hill averaging method (discussed below); and "sw" is used to indicate system moduli computed with these two components alone, and to differentiate such moduli from the system bulk moduli, κ (as in equations 7-10).

The result, when expanded β and ρ (eqs. 3 and 4) are used in equation 2,

$$V_p = \left(\frac{1}{[n\,\beta_w + (1-n)\,\beta_s][n\,\rho_w + (1-n)\,\rho_s]}\right)^{1/2} \tag{5}$$

is known as the Wood equation (Wood, 1941); its experimental and theoretical justification was affirmed by Urick (1947) and Chambré (1955).

The basic equation for the velocity of a shear wave, V_s, is

$$V_s = (\mu/\rho)^{1/2} \tag{6}$$

Substituting $\mu = \rho V_s^2$ into equation 1 yields

$$\kappa = \rho\left(V_p^2 - 4/3\, V_s^2\right) \tag{7}$$

which is the best equation for the determination of the bulk modulus, κ, when V_p, V_s, and ρ are known.

In the present study, V_p and ρ were measured, and κ computed (as discussed in the next section). These three constants were then favored in computations of the other constants (when V_s was not known). The equations used were

$$\mu = \left(\rho V_p^2 - \kappa\right) 3/4 \tag{8}$$

$$V_s = \left[\frac{\left(\rho V_p^2 - \kappa\right) 3/4}{\rho}\right]^{1/2} \tag{9}$$

$$\text{Poisson's ratio, } \sigma = \frac{3\kappa - \rho V_p^2}{3\kappa + \rho V_p^2} = \frac{(V_p/V_s)^2 - 2}{2\left[(V_p/V_s)^2 - 1\right]} \tag{10}$$

$$\text{Lamé's constant, } \lambda = \kappa - 2/3\,\mu \tag{11}$$

RESULTS AND CONCLUSIONS

The Bulk Modulus of Saturated Sediment

INTRODUCTION

The best values for dynamic bulk moduli of water-saturated sediments are obtained when densities and compressional- and shear-wave velocities are known (eq. 7), $\kappa = (V_p^2 - 4V_s^2/3)$. Lacking shear-wave velocities, the problem is to compute in a logical manner, without empirical estimations, or constants, values for bulk moduli which can then be used with measured densities and compressional-wave velocities to compute the other elastic constants.

As noted in a previous section, it has been demonstrated that compressibilities and their reciprocals, incompressibilities or bulk moduli, can be computed for a unit volume of a rock or mineral aggregate from the volumetric contributions of its components. The question is: can an aggregate theory be applied to relatively highly porous saturated sediments? As a first approach to this question, experimental work done by Urick (1947) proved that an aggregate theory in the form of equation 3a could be applied to suspensions of kaolinite in distilled water. Urick used a deflocculent in these experiments, which prevented flocculation of the kaolinite (to form clay-mineral structures); thus, the material was a true suspension without rigidity and the Wood equation (5) applied.

Hamilton *et al.* (1956) computed elastic constants, using equations 1-11, for natural marine sediments using measured densities and compressional-wave velocities, and values for aggregate compressibilities, β_{sw}, computed with equation 3a, except that mineral compressibilities were computed with the Reuss method. Such computations imply (see eq. 1) that if $\rho V_p^2 > \kappa$, all the excess is due to dynamic rigidity, μ, which is theoretically present (as noted by Nafe, and Drake, 1963, p. 808) but apt to be very small. Lacking shear-wave velocities (or other independently measured constants), this method could not, at that time, be evaluated.

There are now sufficient data from laboratory and field experiments where shear-wave velocities and other constants were measured to allow evaluation of aggregate theories, as applied to saturated sediments, and to improve the methods for computing bulk moduli. In the cases where compressional- and shear-wave velocity and density have been measured in a saturated, porous sediment or rock, a true value of the system bulk modulus, κ (eq. 7), is always greater than an aggregate bulk modulus, κ_{sw}, determined from equations 3a, b. The difference lies in a necessary additive component of the bulk modulus of the system: the "skeletal, structural, or frame bulk modulus." With this increment, the aggregate theory appears to be valid for computations of bulk moduli and compressibilities of saturated sediments.

In figure 1, ρV_p^2 is plotted against an aggregate bulk modulus (κ_{sw}, eq. 3b) which does not include the frame-bulk modulus. If the Wood equation (5) applied for these deep-sea sediments, they would be suspensions, lacking rigidity and frame-bulk moduli; consequently, the line, $\rho V_p^2 = \kappa$, shown on figure 1 would adequately define these sediments. The fact that they all fall above the line indicates that κ is not adequately defined or that rigidity is present, or both.

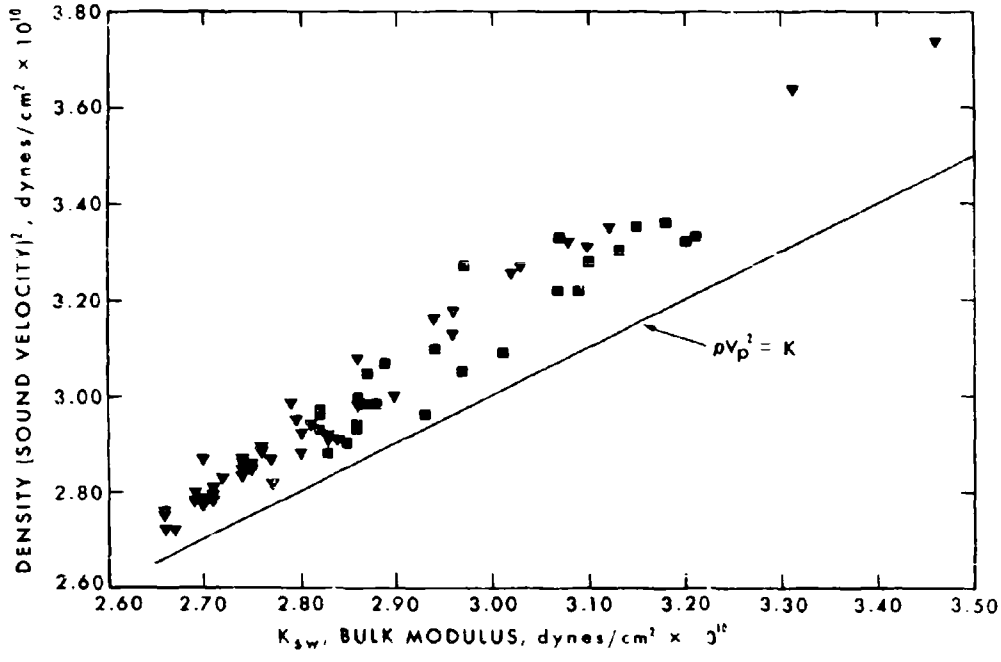

Figure 1. Bulk modulus, κ_{sw} (without a frame bulk modulus, κ_f) vs. ρV_p^2, abyssal hill (squares) and abyssal plain (triangles) environments.

It has become apparent (resumé below) that almost all marine sediments have measurable rigidities and transmit shear waves. This fact, plus the need for a frame-bulk modulus as an increment to aggregate bulk moduli (discussed above), indicates that deviations from the Wood equation (as in fig. 1) are due to the presence of both rigidity and a frame bulk modulus. For computations of better values of both rigidity and shear-wave velocities (and other constants), the frame-bulk modulus has to be considered in computations of the bulk modulus of the system.

COMPONENTS OF THE BULK MODULUS

In his study of the elasticity of porous media (rocks and sediments), Gassmann (1951) considered the "closed system" in which the water within pore spaces does not "circulate" under the small stress of an elastic wave, and the elastic moduli are calculated with the equations of elasticity. Although Biot (1941) must be credited with the early discussion of the "closed system" (from static considerations), Gassmann (1951) has a particularly clear discussion of the factors involved in computations of dynamic elastic constants in water-saturated, porous media.

Gassmann (1951) demonstrated the possibility and utility, of deriving a bulk modulus for saturated rocks from measurements of compressional- and shear-wave velocities and density on dry material. His equations (discussed below) are used in a different way, in this report, to derive an essential component of the system bulk modulus (a "frame bulk modulus"), and then to compute bulk moduli for the sediments in this report.

In deriving his equation, Gassmann used the principle of effective pressure from the field of soil mechanics; for details see any text book on soil mechanics (Skempton, 1961, has a particularly good discussion):

$$P = P_h + P_e \tag{12}$$

where

P = total pressure

P_h = hydrostatic pressure

P_e = effective, or intergranular, pressure

Upon application of a total pressure, P, upon all sides of a unit volume of a porous, "closed-system" medium, there are three pressure effects: (1) the effect of hydrostatic pressure, P_h, on the pore water, (2) the effect of hydrostatic pressure, P_h, on the mineral grains which form the frame, structure, or skeleton, of the sediment, and (3) the effective, or intergranular, pressure, P_e, on the frame. These pressures, when related to the compressibilities of the mineral and water components of the unit volume, result in three component compressibilities (or their reciprocals, bulk moduli). At any given pressure, these bulk moduli are dependent on porosity, and are related to the system bulk modulus, κ, by Gassmann's basic equations (Gassmann, 1951, p. 15):

$$\kappa = \kappa_s \frac{\kappa_f + Q}{\kappa_s + Q}, \qquad Q = \frac{\kappa_w (\kappa_s - \kappa_f)}{n (\kappa_s - \kappa_w)} \tag{13a}$$

where

κ_s = aggregate bulk modulus of mineral solids

κ_w = bulk modulus of pore water

κ_f = frame-bulk modulus ("skeletal" bulk modulus of Gassmann)

n = fractional porosity.

Gassmann's paper (1951) is in German and he did not show the complete derivation of equation 13a; therefore, it is fully developed in Appendix A.

When solved for κ_f, equation 13a becomes

$$\kappa_f = \frac{\kappa[n(\kappa_s - \kappa_w) + \kappa_w] - \kappa_s \kappa_w}{n(\kappa_s - \kappa_w) + \kappa_w(\kappa/\kappa_s - 1)} \tag{13b}$$

When the bulk modulus of the skeleton or frame, κ_f, is zero, equation 13a reduces to the Wood equation (5) for a suspension of mineral grains in water. When porosity is zero (*i.e.*, a solid cube of minerals), equations 13a and b reduce to

$$\kappa = \kappa_f = \kappa_s$$

White and Sengbush (1953) and Wyllie, Gregory, and Gardner (1956), with proper reference to Gassmann (1951), came to the same conclusions regarding the relationships of the bulk moduli as shown in equation 13a. Laughton's (1957) "structural bulk modulus" is, of course, fundamentally the same as Gassmann's skeletal bulk modulus.

The only difference between Gassmann's skeletal bulk modulus, κ_{skel}, and Laughton's structural bulk modulus, κ_c, is that in the fundamental equations relating pressures and compressibilities, Gassmann included effective pressure on mineral compressibilities (*i.e.*, $P_e B_s$, or P_e/κ_s), and Laughton omitted this effect. As a result, at high porosities, κ_{skel} and κ_c are close together, but diverge at lower porosities because, as porosity approaches zero, hydrostatic pressure approaches zero, and total pressure approaches effective pressure (eq. 12). Laughton's equation ($\kappa = \kappa_m + \kappa_c$, wherein κ_m is κ_{sw} in equation 3b) results in the unacceptable solution, at zero porosity, that $\kappa_c = \kappa - \kappa_s$. Thus, in a rigorous solution, Gassmann's prior equations should be used, especially with the lower porosities of sands and rocks.

Using Gassmann's equation (13a) to derive the system bulk modulus for saturated sediment (without a gas phase), one needs to know four components: porosity, n; the bulk modulus of the pore water, κ_w; the aggregate bulk modulus of the mineral solids, κ_s; and the frame bulk modulus, κ_f.

The basic assumption underlying computations of the bulk modulus of pore water in sediments (in this report) is that pore water and bottom water have the same salinity. Siever *et al.* (1965) and Friedman *et al.* (1968) have shown that this assumption is valid within small limits. Any changes of sound velocity and density caused by these salinity differences can be disregarded. Thus, the values of the bulk modulus of distilled and seawater can be found in tables or computed with equation 2a, b and used as the bulk modulus of sediment pore water, κ_w. Some typical values for seawater are listed in Appendix B.

In recent years, the elastic constants of the more common minerals have either been determined or revised by a number of specialists. Attention is especially directed to two recent compilations which also include excellent discussions and numerous references (Birch, 1966, Anderson and Liebermann, 1968). Because of their common occurrence in marine sediments, attention is also directed to reports on calcite (Peselnick, 1962), on quartz (McSkimin *et al.*, 1965; Soga, 1968), on micas, hornblende, and feldspars (Christensen, 1965, 1966), on obsidian (Manghnani *et al.*, 1968), and the bulk moduli of oxides (Anderson and Nafe, 1965).

The availability of these recent, accurate measurements of the elastic constants of minerals allows use of the Voigt-Reuss-Hill averaging method (hereafter, the "VRH method"), to compute the bulk modulus of a randomly-oriented mineral aggregate with some confidence (references under paragraph numbered 2, above). As Hill (1952, 1963) pointed out, the Reuss average assumes uniform stress within the aggregate, and a lower limit to incompressibility, κ_R, is computed with

$$1/\kappa_R = V_1/\kappa_1 + V_2/\kappa_2 + \ldots$$

The Voigt averaging method assumes uniform strain with the aggregate, and an upper limit to incompressibility, κ_V, is computed with

$$\kappa_V = V_1\kappa_1 + V_2\kappa_2 + \ldots$$

where

V_1 and V_2 = fractional concentration of phases (mineral species) in a unit volume

κ_1 and κ_2 = incompressibilities (bulk moduli) of the phases

Hill showed that the correct average lies halfway between the Reuss and Voigt averages; thus, the computed bulk modulus, κ, of the aggregate is

$$\kappa = (\kappa_R + \kappa_V)/2$$

Hill (1963, p. 361) noted that both the Reuss and Voigt averages were "rather poor when the phase moduli differ by more than a factor of 2 or so." This is shown to be true, for example, if the VRH method is tried for a two-phase suspension composed of water and mineral particles.

The aggregate bulk modulus of mineral grains, κ_s, was computed in this study by using the VRH averaging method.

The only remaining constant needed to compute the bulk modulus of a saturated sediment (using eq. 13a) is the frame bulk modulus, κ_f. The recommended way to derive this value (for use in eq. 13a) is to enter a curve relating this constant to porosity or void ratio (volume of voids/volume of solids). These curves have been derived in two ways: by Laughton (1957) using drained, static compression tests, and in this report by solving equation (13b) with carefully selected data which include values for density, and compressional- and shear-wave velocities, as described below.

Laughton (1957) derived two curves relating static frame-bulk moduli to void ratios through a series of drained compression tests on two deep-sea sediment types a calcareous ooze and a "terrigenous mud." As Laughton noted, a static test, because of various laboratory testing difficulties, has wide margins of error (he estimated plus or minus 2.5×10^{10} dynes/cm^2 for his tests). Although theoretically usable for dynamic computations, a static compression test for the frame bulk modulus is less apt

to be as correct as a dynamically determined modulus, especially for the purposes of computing other dynamic elastic constants which are usable in underwater sound or geophysical studies.

DYNAMIC FRAME BULK MODULUS

A "dynamic frame bulk modulus," κ_f, as used in this report, is one determined with equation 13b, using dynamic values for the various bulk moduli required to solve the equation.

The accumulation of measurements of the elastic constants of minerals, and compressional- and shear-wave velocities and densities in rocks and sediments during the past 15 years, now permits, in the author's opinion, computations of system-bulk moduli (κ in eqs. 7 and 13a), and values for the bulk moduli of pore water and minerals (κ_w and κ_s) for enough rocks and sediments to define, approximately, relationships between dynamic frame-bulk moduli and porosity (κ_f and n in eq. 13a, b).

Data which can be used in computations of a dynamic frame-bulk modulus, using equation 13b, must meet the following requirements.

1. Measurements must include both compressional- and shear-wave velocities in water-saturated sediments or rocks.

2. Density and porosity are known, or can be computed from known data.

3. The mineralogy of the medium is known (including percentage of volume of species), or can be reasonable estimated.

4. The nature of the pore fluid is known, or can be reasonably inferred.

5. The temperature within the medium at the time of the velocity measurements is known, or can be reasonably inferred.

6. The pressure on the sample, or medium, is known or can be computed (because any bulk modulus varies with pressure).

A selection of data (from the literature and from field work by the author and Bucker) which met the above requirements was used to compute dynamic frame-bulk moduli for a variety of saturated rocks and sediments. These moduli were then plotted against porosity (fig. 2). All data in figure 2 were referred to an approximate, common pressure which would be expectable with 1 meter or less of overburden pressure, and a common temperature of 23°C; temperature is particulary important in computing compressibilities of pore fluids.

In a previous section it was noted that in Gassmann's equation (13b), when porosity is zero, the bulk modulus of the frame equals the bulk modulus of the solid mineral (or the aggregate of minerals); this allows a valid point to be plotted at zero porosity through which any regression line for media of the same mineralogy should pass. Four such points are noted at zero porosity in figure 2: quartz (Soga, 1968), calcite (Peselnick, 1962), and two aggregate values for sediments off San Diego (Appendix B).

A single regression line is shown in figure 2 for all data. However, this line should be considered as approximate because of the sparse data and because there should be individual lines for each sediment-rock mineral type. Examples of these lines should be (1) relatively pure calcitic limestones, sands, and calcareous oozes, (2) relatively pure

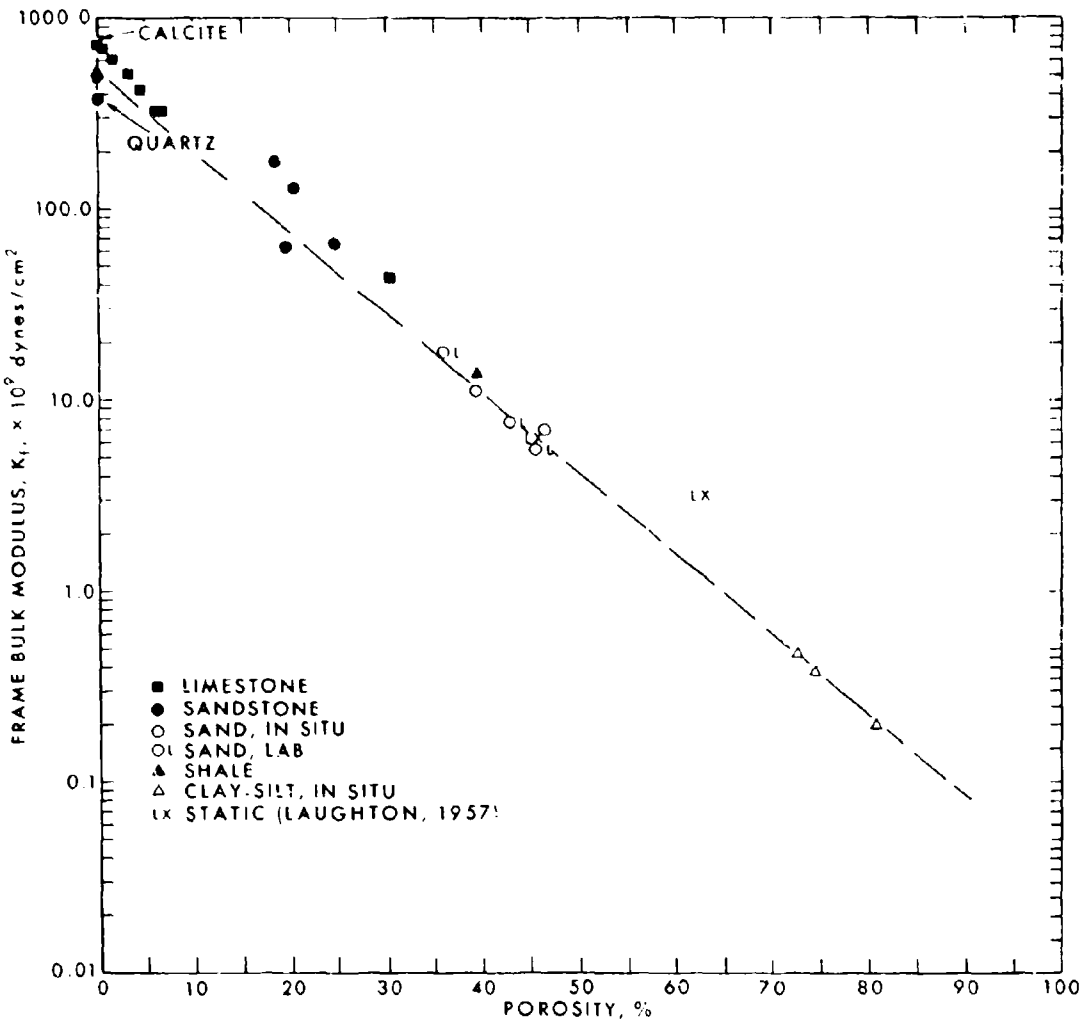

Figure 2. Porosity vs. dynamic frame bulk modulus, κ_f. See text for discussion, see Appendix B for regression equations for sands and silt-clays.

quartz sandstones and sands, and (3) natural, multimineralic sedimentary rocks, sands, and finer-grained terrigenous sediments. However, it is surprising how well the generalized regression line defines the available data, for example, the trend of the data from zero porosity to about 45 percent porosity would have predicted reasonable values of κ_f for the high porosities above 70 percent. Appendix C provides regression equations for sands and silt-clays.

Because the frame-bulk modulus varies with porosity and effective pressure (Appendix A), a line relating log κ_f and porosity for pure calcitic rocks and sediments should lie above those for pure quartz and normal, multimineral earth materials, as porosity increases from zero. This is indicated in figure 2 by the values for calcite at zero porosity, limestone (Peselnick, 1962), and Austin Chalk (White and Sengbush, 1953), compared with the values for quartz (Soga, 1968) and quartz sands (Shell Development Co., private comm., 1965). In Laughton's static tests, the line relating κ_f with void ratio for calcareous ooze was above and roughly parallel to that for

terrigenous mud (1957, fig. 14, p. 258). The lowest pressures at which Laughton obtained static measurements of frame-bulk moduli (his κ_f) calcareous ooze (an estimate at P = 8 kg/cm^2) and terrigenous mud (P = 16 kg/cm^2) are included in figure 2 for comparison with the dynamic moduli. Other sources of data for points in figure 2 are: sandstone (King, 1966), Eagle Ford shale (White and Sengbush, 1953), and sands and silty clays off San Diego (Hamilton *et al.*, 1969).

Computations and Discussions of Elastic Constants

INTRODUCTION

The elastic constants for various sediment types within the three environments were computed with equations 7 to 11. The input data were (1) measured values of saturated bulk densities and compressional-wave velocities (Part I, tables 1 and 2; reproduced in this report as tables D-1 and D-2, Appendix D) and (2): computed values of bulk moduli, κ, using equation 13a with: (a) measured values of porosities, n (Part I, tables 1 and 2); (b) computed values for the bulk moduli of the pore water, κ_w, using equation 2a,b; (c) computed values for the aggregate bulk moduli of the mineral grains, κ_s, using literature values for the bulk moduli of individual minerals and summing their individual volumetric contributions to the aggregate bulk moduli using the VRH method; and (d) values computed for the frame-bulk moduli, κ_f, by entering the regression equations for sands and clay-silt (fig. 2, Appendix C).

The computed elastic constants are listed in tables 1 and 2. The relationships of several constants with other physical properties are shown in the figures.

All of the input data noted above have margins of error, some known and some unknown. All of the computed constants should be considered approximations. Consequently, no attempt was made to statistically estimate variances or errors in the final computations. The numbers of decimal places shown in the tables are for purposes of comparison between the various computations and should not be taken as the author's estimate of accuracy.

Computations of elastic constants are strongly dependent on accurately measured values of density, porosity, and compressional velocity. An examination of equations 1 and 13a indicate why this is true. If ρV_p^2 is not greater than the computed bulk modulus, κ (which is strongly dependent on porosity), there is no rigidity or shear wave, and Poisson's ratio is 0.50. Because the rigidity modulus, μ, is so small in most marine sediments, inaccurate values of ρ, V_p, and n, especially, may result in computations indicating no, or too great, rigidity. For these reasons, all attempts should be made in the laboratory to improve measurements of these properties. For example, "saturated densities" should be truly, fully saturated (not always easy in sands), and properties should be "salt-free" (*i.e.*, no dried salt weighed with the dried minerals; see Part I). Accurate measurement of bulk grain density, without salt, aids in cross-checking density and porosity values (Part I). In the computed elastic constants listed in tables 1 and 2, $\kappa > \rho V_p^2$ in some principal sediment types, as follows: 3 out of of 40 cases in continental-terrace clayey silts, 1 out of 52 cases in abyssal-plain silt-clays, none out of 41 in abyssal-hill silt-clays, and 2 out of 9 in fine sand; these cases were not included in the averages of μ, V_s, or σ. For this reason, but mostly because of rounding off at a lower number of decimal places than used in the computations, the average values in the tables cannot usually be exactly interrelated with the elastic equations.

It should be emphasized that all values for the elastic constants are for a temperature of 23°C and 1 atmosphere of pressure. Methods for relating these constants to *in situ* values will be discussed in Part III (Prediction).

TABLE 1: CONTINENTAL TERRACE (SHELF AND SLOPE) ENVIRONMENT: COMPUTED ELASTIC CONSTANTS IN SEDIMENTS

Sediment Type	κ		μ		λ		σ		V_s	
	Avg.	SE	Avg.	SE	Avg.	SE	Avg.	SE	Avg.	SE
Sand										
Coarse	6.6859	...	0.1289	...	6.6000	...	0.491	...	250	...
Fine	5.6877	0.195	0.3212	0.064	5.3044	0.247	0.469	0.007	382	45
Very Fine	5.1182	...	0.5035	...	4.7825	...	0.453	...	503	...
Silty sand	4.6812	0.145	0.3926	0.051	4.3330	0.152	0.457	0.006	457	31
Sandy silt	3.4152	...	0.2809	...	3.2279	...	0.461	...	379	...
Sand-silt-clay	3.5781	0.095	0.2731	0.023	3.3995	0.090	0.463	0.003	409	18
Clayey silt	3.1720	0.040	0.1427	0.011	3.0735	0.040	0.478	0.002	364	57
Silty clay	3.1476	0.037	0.1323	0.029	3.0592	0.027	0.480	0.004	287	22

Notes: Laboratory values: 23°C, 1 atmosphere pressure

κ = bulk modulus, dynes/cm^2 $\times$ 10^{10}

μ = rigidity (shear) modulus, dynes/cm^2 $\times$ 10^{10}

λ = Lamé's constant, dynes/cm^2 $\times$ 10^{10}

σ = Poisson's ratio

V_s = velocity of shear wave, m/sec

SE = Standard error of the mean

TABLE 2: ABYSSAL PLAIN (TURBIDITE) AND ABYSSAL HILL (PELAGIC) ENVIRONMENTS: COMPUTED ELASTIC CONSTANTS IN SEDIMENTS

Environment Sediment Type	κ		μ		λ		σ		V_s	
	Avg.	SE	Avg.	SE	Avg.	SE	Avg.	SE	Avg.	SE
Abyssal Plain (Turbidite)										
Sandy silt	4.2572	- - -	0.0668	- - -	4.2127	- - -	0.492	- - -	201	- - -
Silt	3.9516	- - -	0.2362	- - -	3.7941	- - -	0.471	- - -	384	- - -
Clayey silt	3.0561	0.060	0.1435	0.015	2.9604	0.056	0.477	0.002	312	18
Silty clay	2.7772	0.019	0.0773	0.007	2.7245	0.020	0.486	0.001	240	11
Clay	2.7805	- - -	0.0483	- - -	2.7483	- - -	0.491	- - -	196	- - -
Abyssal Hill (Pelagic)										
Clayey silt	3.1213	- - -	0.1408	- - -	3.0274	- - -	0.478	- - -	312	- - -
Silty clay	3.0316	0.030	0.0795	0.007	2.9786	0.030	0.487	0.001	232	11
Clay	3.0781	0.055	0.0544	0.006	3.0419	0.058	0.491	0.001	195	12

Notes: Laboratory values: 23°C, 1 atmosphere pressure

κ = bulk modulus, dynes/cm$^2 \times 10^{10}$

μ = rigidity (shear) modulus, dynes/cm$^2 \times 10^{10}$

λ = Lamé's constant, dynes/cm$^2 \times 10^{10}$

σ = Poisson's ratio

V_s = velocity of shear wave, m/sec

SE = Standard error of the mean

EFFECTS OF USING A FRAME-BULK MODULUS

A value for a "system bulk modulus," κ, can be computed from its components (and porosity) in two ways. (1) using the moduli for pore water, κ_w, and minerals, κ_s, in equation 3b, and assuming that κ_{sw} is κ, or (2) using moduli for pore water, minerals, and a frame-bulk modulus, κ_f, in equation 13a. The effect of using a frame-bulk modulus is to increase the value of the system-bulk modulus by a small, but significant amount. When measured values of density and compressional-wave velocity are used with computed values for κ to compute rigidity, μ (eq. 8), an increase in κ reduces the value of rigidity. Lower rigidities cause lower shear-wave velocities (eq. 6). Numerical examples of the differences in κ, μ, and V_s when κ is computed with and without κ_f are given below.

System Bulk Modulus Used* (dynes/cm^2 × 10^{10})		μ (dynes/cm^2 × 10^{10})**	V_s (m/sec)***
1. Fine sand (n=45%)			
κ_{sw} (without κ_f):	5.0333	0.6997	594
κ (with κ_f):	5.5418	0.3183	401
2. Abyssal-hill silty clay (n=84.3%)			
κ_{sw} (without κ_f):	2.8165	0.1151	296
κ (with κ_f):	2.8292	0.1056	284

*κ_{sw}: eq. 3b; κ, eq. 13a

**$\mu = (\rho V_p^2 - \kappa)3/4$

***$V_s = (\mu/\rho)^{1/2}$

Differences in rigidities and shear-wave velocities are greater in the lower-porosity sands (above) because κ_f increases with decreasing porosity (fig. 2), causing a larger difference between the two bulk moduli (κ_{sw} and κ).

COMPRESSIBILITY AND INCOMPRESSIBILITY (BULK MODULUS)

The theoretical and experimental evidence concerning compressibility and the bulk modulus have been discussed in previous sections; therefore, this section will be confined to empirical relationships.

Because the bulk modulus is favored in the computations, values of compressibility are not listed in tables 1 and 2. However, if desired, they can be easily computed ($\beta = 1/\kappa$).

At any given temperature and pressure, values of compressibility and the bulk modulus vary within a small range in both pore water and mineral solids (Appendix B). The volumetric contributions of both of these moduli, and the frame-bulk modulus, are strongly dependent on porosity, n, space occupied by water, or $(1 - n)$, the space occupied by mineral solids in a unit volume. Therefore, it is no surprise to find that the plots of computed values of β or κ vs. n, or density, ρ, show a good correlation (figs. 3-10).

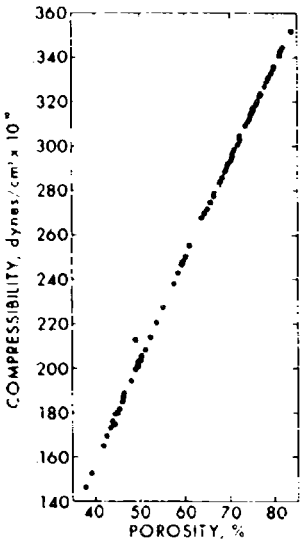

Figure 3. Porosity vs. compressibility, continental terrace.

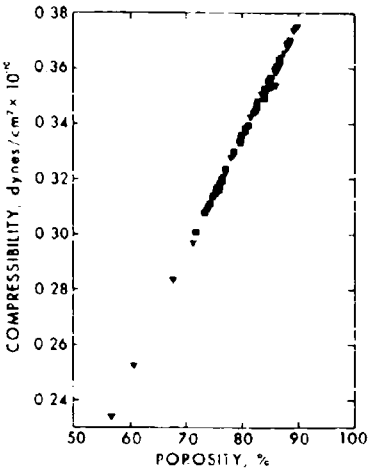

Figure 4. Porosity vs. compressibility, abyssal hill (squares) and abyssal plain (triangles) environments.

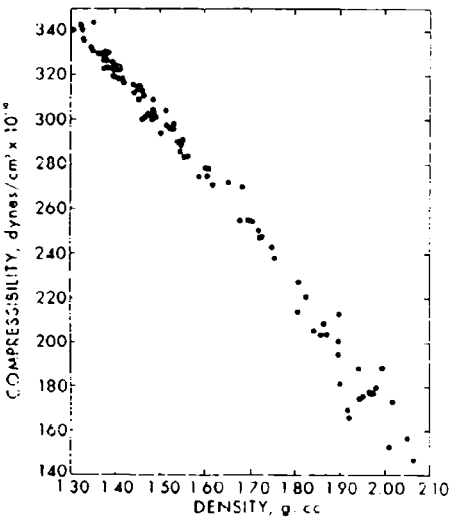

Figure 5. Density vs. compressibility, continental terrace.

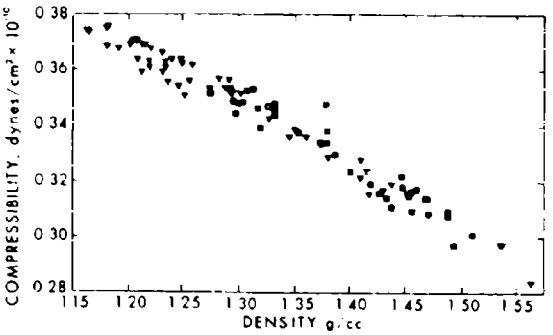

Figure 6. Density vs. compressibility, abyssal hill (squares) and abyssal plain (triangles) environments.

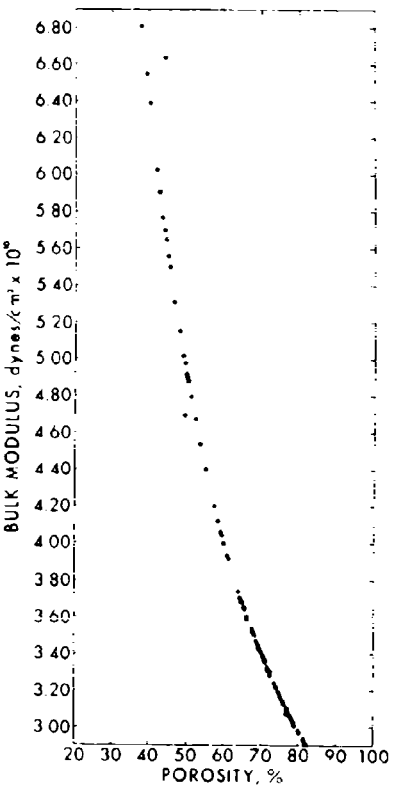

Figure 7. Porosity vs. bulk modulus, continental terrace.

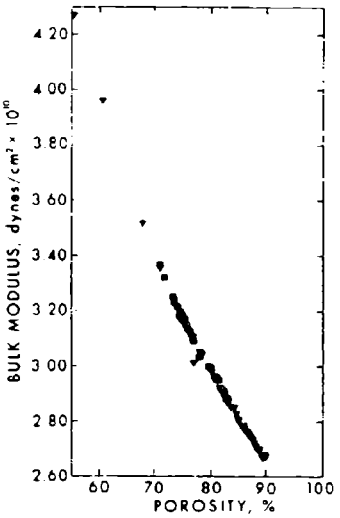

Figure 8. Porosity vs. bulk modulus, abyssal hill (squares) and abyssal plain (triangles) environments.

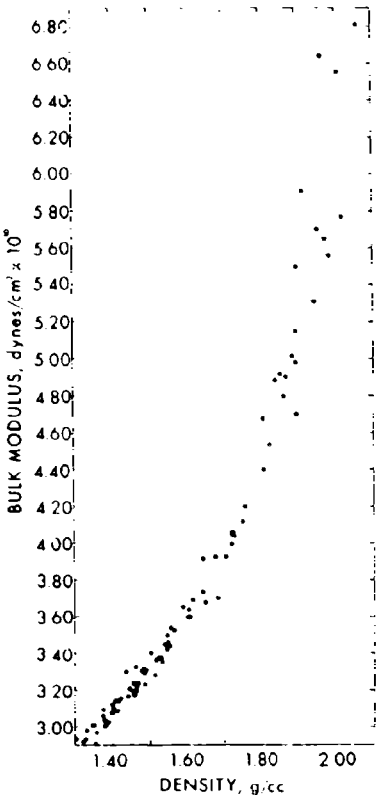

Figure 9. Density vs. bulk modulus continental terrace.

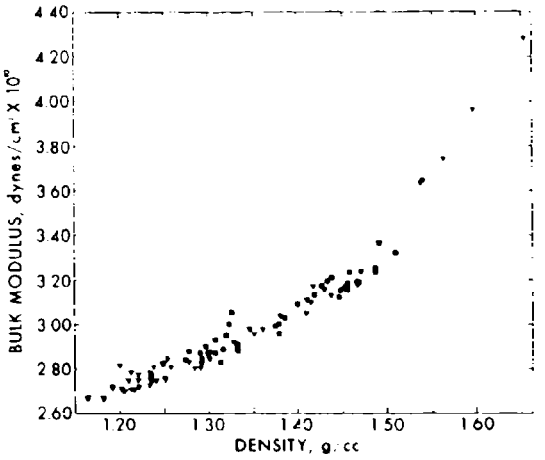

Figure 10. Density vs. bulk modulus, abyssal hill (squares) and abyssal plain (triangles) environments.

Values for β or κ at 23°C and 1 atmosphere can be derived by entering figures 3 to 10 with density or porosity, or by use of the regression equations for the bulk modulus shown in the figures (see Appendix C).

The bulk moduli of the deep-water sediments are plotted against ρV_p^2 in figure 11, together with a line, $\rho V_p^2 = \kappa$. In this figure the values of κ contain the frame-bulk modulus, and are assumed to represent "true" values of κ (as compared with fig. 1, in which κ_{sw} lacked frame-bulk moduli). If these are true values of κ, the divergence of the data points from the line indicate the presence (and values) of rigidity ($\rho V_p^2 = \kappa + 4/3\mu$).

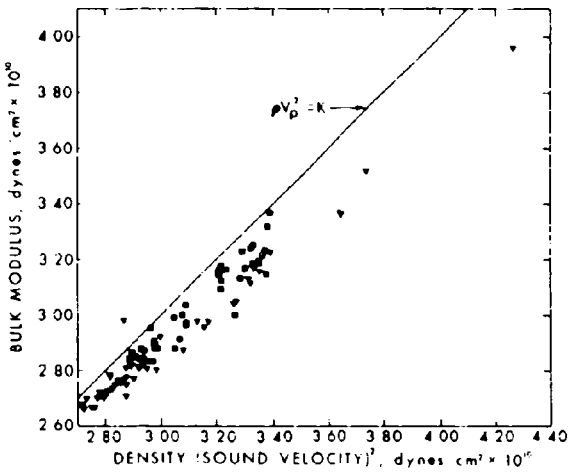

Figure 11. ρV_p^2 vs. bulk modulus, abyssal hill (squares) and abyssal plain (triangles) environments. $\rho V_p^2 = \kappa$ indicates relationship without rigidity, μ (see eq. 1).

RIGIDITY (SHEAR) MODULUS AND SHEAR-WAVE VELOCITY

INTRODUCTION

There has long been a question in marine geophysics and underwater acoustics: do surficial, natural, marine sediments have enough rigidity to allow transmission of shear waves; and, if so, what are reasonable values of these moduli? The answer appears to be that (1) in some bays and estuaries, near river deltas, and in a few other localities, sediments are deposited at a fast rate, lack appreciable structural strength, and for all practical purposes are little more than suspensions; however, (2) almost all of the remainder of continental-terrace and deep-sea sediments should possess enough rigidity to allow transmission of shear waves.

The evidence for the above statements can be divided into experimental evidence in the laboratory and *in situ* on land and on the sea floor, and theoretical evidence. Because the presence of a shear wave is dependent on rigidity (eq. 6), and most measurements have been of shear waves, the following discussion will be largely confined to shear waves. Values of shear-wave velocities from the discussions below are assembled in table 3.

LABORATORY MEASUREMENTS OF SHEAR-WAVE VELOCITIES

In the laboratory, shear waves can be transmitted through dry and saturated sands at relatively low normal pressures. Experimental difficulities have, so far, prevented low-pressure, laboratory measurements of shear-wave velocities in high-porosity, natural marine sediments. For example, Laughton (1957) was unable to measure shear waves in a sample of calcareous ooze until the material was compacted to a porosity of about 33 percent under a pressure of 512 kg/cm^2.

Much of the laboratory work on shear waves in sands has come from soil-mechanics and oil-industry research. Recent papers which summarize (and add to) much of this work are by Barkan (1962), Hardin and Richart (1963, including discussions), and Shell Development Company (private communication, 1965; figs. 12 and 13, this

TABLE 3. SUMMARY OF SHEAR-WAVE VELOCITIES IN FULLY-SATURATED SEDIMENTS.

Sediment	V_s (m/sec)	Ref.*	Remarks
Laboratory			
Coarse sand	95	1	$P = 0.07$ kg/cm^2; resonant col.; Ottawa sand
Coarse sand	133	1	$P = 0.12$ kg/cm^2; resonant col.; crushed sand
Coarse sand	285	2	$P = 1.4$ kg/cm^2; pulse tech.; St. Peters sand
Fine sand	260	2	$P = 1.4$ kg/cm^2; pulse tech.; St. Peters sand
In Situ (Land)			
Sand (dense)	250	3	Russian soil-mechanics lit.
Medium sand	160	3	Russian soil-mechanics lit.
Fine sand	110	3	Russian soil-mechanics lit.
Clay	150	3	Russian soil-mechanics lit.
Sand	130	3	Russian soil-mechanics lit.; vibration: 10-35 Hz.
Sand	534	4	50-ft depth; explosive
Clay-silt	244	4	10-ft depth; explosive
Clay-silt	131	5	Sediment sfc; vibration: 35-400 Hz
Fine sand	190	6	Sediment sfc; vibration: 8-60 Hz
Sand	255	6	Sediment sfc; vibration: 8-60 Hz
Sand	230	6	Sediment sfc; vibration: 8-60 Hz
Sand	227	6	Sediment sfc; vibration: 8-60 Hz
Clay-silt	150	6	Sediment sfc; vibration: 8-60 Hz
Clay-silt	180	6	Sediment sfc; vibration: 8-60 Hz
Clay-silt	200	6	Sediment sfc; vibration: 8-60 Hz
Clay	230	7	Bore hole; 3-m depth; explosive (detonator)
Clay	120	8	0 to 20 m depth (seismic logging); from V_p/V_s
Clay	170	8	0 to 10 m depth (seismic logging); from V_p/V_s

TABLE 3 (continued)

Sediment	V_s (m/sec)	Ref.*	Remarks
In Situ (Sea Floor); computed from Stoneley waves			
Silt-clay	50-190	9	Deep Indian Ocean; explosive
Silt-clay	101	10	San Diego Trough; explosive
Silt-clay	107	10	
Silt-clay	87	10	
Medium sand	197	11	Cont. shelf off San Diego; explosive
Fine sand	101	11	
Theoretical			
Pacific Basin	250	12	Rayleigh wave model (*in situ* values)
Pacific Basin	250	13	
Atlantic Basin	150	14	
Atlantic Basin	30-120	15	
Argentine Basin	200-400	16	
Computed average values, this report (23°C; 1 atmosphere)			
Fine sand	382		Continental shelf
Silty clay	287		Continental shelf
Silty clay	232		Abyssal hills, Pacific Basin
Clay	195		Abyssal hills, Pacific Basin
Silty clay	240		Deep abyssal plains, adjacent to Pacific Basin

*References

1. Hardin and Richart (1963; and discussions)
2. Shell Development Co., personal communication, 1965)
3. Barkan (1962)
4. White and Sengbush (1953)
5. Jones (1958)
6. Heukelom (1961)
7. Molotova (1963)
8. Zhadin, V. V. (*in* Molotova and Vassil'ev, 1960)
9. Davies (1965)
10. Hamilton *et al.* (1969)
11. Bucker *et al.* (1964)
12. Kovach and Press (1961)
13. Oliver and Dorman (1961)
14. Latham and Sutton (1966)
15. Anderson and Latham (1969)
16. Sykes and Oliver (1964)

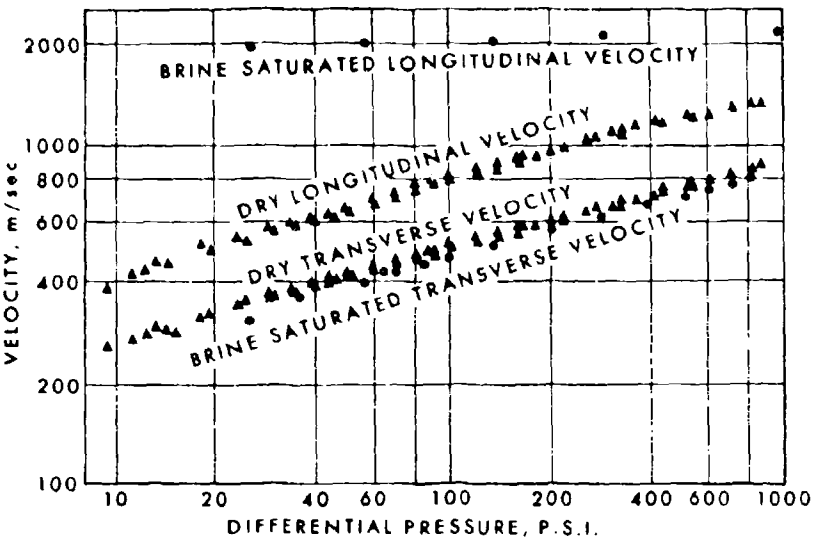

Figure 12. Velocity of compressional and shear waves vs. pressure in dry and brine-saturated (25 g/liter NaCl) St. Peters coarse sand (20-30 mesh, 0.59-0.84 mm). By permission of the Shell Development Co., Houston, Texas (private communication, 1965).

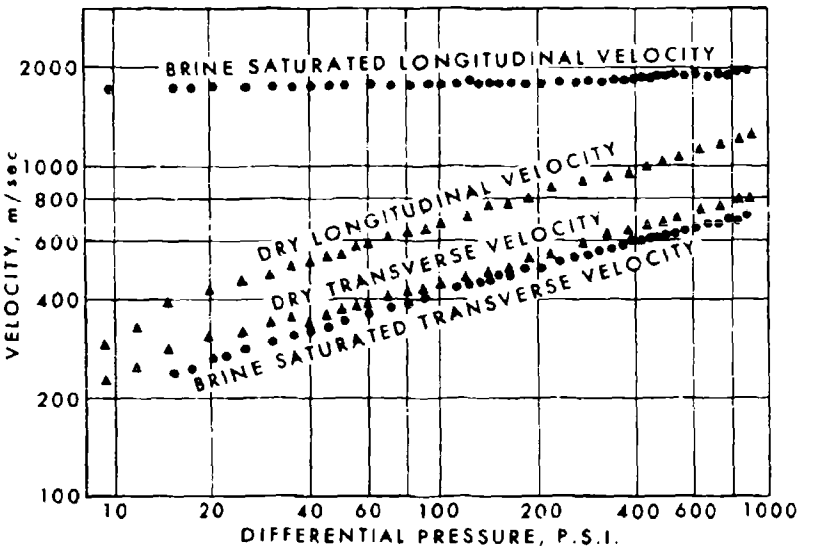

Figure 13. Velocity of compressional and shear waves vs. pressure in dry and brine saturated (25 g/liter NaCl) St. Peters fine sand (100-120 mesh, 0.125-0.149 mm). By permission of the Shell Development Co., Houston, Texas (private communication, 1965).

report). Many of the studies referenced in these reports also include values for the velocity of compressional waves in artificial sands at "full saturation" (usually with distilled water). Most of these values must be disregarded. The soils engineer must know the physical properties of soils from the dry to fully-saturated state, and the author does not question those values at less than full saturation. However, unless extreme care is taken to evacuate all air from the "saturated" samples, the values for compressional-wave velocities are far too low. Any values for compressional-wave velocities at 100 percent saturation should be well above velocities in water. Schon (1963) illustrated the marked increase in compressional-wave velocity as saturation nears 100 percent. Examples of reasonable values for compressional velocities in artificial sands at full saturation are in reports by Brandt (1960), Schon (1963), Nolle *et al.* (1963), and Shell Development Co. (see figs. 12 and 13). These values for artificial sands are consistent with those measured in natural sands in the laboratory, and *in situ* by Hamilton *et al.* (1956), Shumway (1960), and with the measurements of this report. The velocity of shear waves in fully-saturated sands is slightly lower than in dry or partially saturated sands (Hardin and Richart, 1963; Shell Development Co., figs. 12 and 13).

Wilson and his students at the U. S. Naval Postgraduate School have a continuing program to measure dynamic rigidity in artificially sedimented clays and natural marine sediments, using a torsional oscillator resonance technique (Hutchins and Wilson, 1968; Hutchins, 1967; Cohen, 1968). Typical values of rigidity have been: 2.16 dynes/cm^2 $\times$ 10^5 for a 32-percent concentration (by weight) of kaolinite in distilled water, and 4.6 dynes/cm^2 $\times$ 10^5 for a 19-percent concentration of bentonite in distilled water (Cohen, 1968). Using equation 6 and values for density and rigidity from Cohen's report, the computed values of shear-wave velocities in four selected samples were 5 to 7 m/sec. Some additional aspects of this work will be discussed below.

IN SITU MEASUREMENTS OF SHEAR-WAVE VELOCITIES (LAND)

In situ experimental work on shear-wave velocities on land has largely been by oil-industry and soil-mechanics laboratories. White and Sengbush (1953) measured shear and compressional velocities in saturated sands and other materials using explosive charges (blasting caps) and dropped weights in a borehole. Heukelom (1961) used dynamic deflection techniques to measure rigidity and shear-wave velocities in a variety of natural sediments in the Netherlands. Russian work in soil mechanics in this field was assembled by Barkan (1962). Jones (1958) has described measurements of dynamic properties of soils at the Road Research Laboratory, Harmondsworth, England, in which an electromagnetic vibrator was used to generate elastic waves. These measurements are of particular interest because the technique and theory were similar to those of Davies (1965) and of Bucker *et al.* (1964) for *in situ* measurements on the sea floor.

IN SITU MEASUREMENTS OF SHEAR-WAVE VELOCITIES (SEA FLOOR)

Davies (1965), Bucker *et al.* (1964), and Bucker and his colleagues (in Hamilton *et al.*, 1969) have measured values of Stoneley-wave velocities from which shear wave velocities can be computed. Davies' measurements were in the deep Indian Ocean, and Bucker's were in sands and clay-silts of the continental shelf. The conversion of compressional waves to shear waves at refraction boundaries within deep-sea sediments was reported by Nafe and Drake (1957).

THEORETICAL VALUES OF SHEAR-WAVE VELOCITIES

The theoretical values of shear-wave velocities noted in this section come from the reconciliation of experimental data with theory in connection with the propagation of Love and Rayleigh waves across ocean basins, and from the computations of this report.

To get quantitative agreement between theory and observations of the dispersion of Love and Rayleigh waves across ocean basins, Kovach and Press (1961), and Oliver and Dorman (1961), used shear-wave velocities of 250 m/sec in their models. Latham and Sutton (1966), for an area near Bermuda, used a value of 150 m/sec, and Sykes and Oliver (1964) used 200 to 400 m/sec for the Argentine Basin.

Anderson and Latham (1969) studied the dispersion of Rayleigh waves caused by sediment layers in the sea floor between the Mid-Atlantic Ridge and Bermuda. They derived values of shear-wave velocities for various cases. Because sediment properties in the area are reasonably well known, a value for shear-wave velocity in the upper 1 meter of sediments was computed using the method described in this report. All values were corrected to *in situ* values using the methods of Part III. The necessary data for the sediments were taken from Horn *et al.* (1968c): average, laboratory values of density, porosity, and velocity for the upper 1 meter of five cores taken on the Mid-Atlantic Ridge and Sohm Abyssal Plain (Stations AS7-1, 2, 3, 4, and 5) were averaged and corrected to *in situ* values. The velocity of bottom water at 5000 m (1545 m/sec) was taken from NAVOCEANO TR-171 (1965); bottom-water density was assumed to be 1.05 g/cc. The computed *in situ* value for the average shear-wave velocity (148 m/sec) lies between Anderson and Latham Cases C and D (160 and 70 m/sec). The authors prefer Case D, but cannot eliminate Case F which corresponds most nearly to seismic profiler results and calls for shear-wave velocities of 30 m/sec at the top and 120 m/sec at the bottom of a layer 150 m thick. Similar computations for a generalized red clay in the Pacific Basin at a water depth of 5000 m indicate a sediment-surface shear-wave velocity of about 220 m/sec, which can be compared with the value of 250 m/sec used by Kovach and Press (1961) and Oliver and Dorman (1961) to reconcile experiment with theory in their studies of Rayleigh-wave dispersion.

Computed values of shear-wave velocities for each sediment type within each environment in this report are listed in tables 1 and 2. For comparison with the values from the studies noted above, average laboratory values for several major sediment types are also shown in table 3. The correction of laboratory values of sediment mass physical properties to *in situ* values is a subject in Part III (TP 145). In the case of shear-wave velocity in surficial sediments, such computations indicate a progressive decrease in velocity with water depth. This velocity decrease from laboratory to 5000 m seawater depth, for a high-porosity silty clay, is of the order of 10 to 15 percent.

An examination of table 3 indicates that values computed in this report are within the range of (and consistent with) a wide range of laboratory and *in situ* measurements of shear waves in saturated sediments.

CAUSES OF RIGIDITY

In saturated sediments, rigidity is related to sediment structure and the complex factors restricting relative interparticle movements under shear stresses. In Part I, the common sediment structures (Part I, fig. 1) were reviewed; in this section, those aspects of sediment structural strength relating to rigidity will be discussed.

Shear strength is one of the critical engineering properties of sediments or soils. As a result, there is a voluminous literature on this subject in the fields of soil mechanics and foundation engineering. Fully referenced discussions appear in recent textbooks

(*e.g.*, Jumikis, 1964; Yong and Warkentin, 1966) and in recent papers of particular interest to the present discussion (Rosenqvist, 1960; Lambe, 1960; Schmertmann and Osterberg, 1960; Hvorslev, 1960; Seed and Chan, 1961; Mitchell, 1964; Grim, 1962). The following outline is derived from these sources and those referenced below.

The shear strength of a sediment at failure, τ_f, is represented by

$$\tau_f = c + (\sigma - \mu) \tan \phi \tag{14}$$

where

c = cohesion

σ = normal stress on the plane of failure

μ = excess pressure in pore water

ϕ = angle of internal friction

$(\sigma - \mu)$ = effective stress

Equation 14 has two components [cohesion, c, and friction, $(\sigma - \mu) \tan \phi$]; it can be applied to all sediments. Shear strength in sands without significant amounts of fine silt and clay is defined by the friction component (*i.e.*, these are "cohesionless" sediments). Most silt-clay sediments have both cohesion and friction (under normal stress). A few clays may have no angle of internal friction, in which case the shear strength is defined by cohesion alone.

Equation 14 is partly empirical in that the exact mechanics of failure are not completely understood, and it is not always possible to separate the contributions of the two components to ultimate shear strength. However, Schmertmann and Osterberg (1960), and others, have shown by careful testing that true cohesion and friction are mechanically independent, and that equation 14 is valid over the entire range of strain, as well as at failure.

In sands and most clays, shear strength and dynamic rigidity will increase with effective pressure, as indicated in equation 14. The effects of pressure on rigidity, however, will not be considered below, although very light effective pressures exist even in a small laboratory sample. The propagation of elastic waves in sands under pressure is well understood (Brandt, 1960; Hardin and Richart, 1963; Shell Development Co., figs. 12 and 13; review by White, 1965). Because this report deals with surface sediments (0 to 30 cm; see Part I), the discussion here will be concerned with the effects of cohesion as the source of rigidity in high-porosity silt-clays.

Cohesion is the resistance to shear stresses which can be mobilized between adjacent, fine particles which stick, or cohere, to each other. Cohesion is considered to be an inherent property of fine-grained, clayey sediments which is independent of stress; it is caused by physio-chemical forces of an interparticle, intermolecular, and intergranular nature. Some important components and aspects of cohesion which affect rigidity are as follows.

1. Clay particles are surrounded by layers of adsorbed water through which they interact with other particles. The amount of pore water, the distance between particles, and the number of interparticle contacts are important influences on cohesion.

2. Interparticle forces of the London-van der Waal and Coulombic type, the positive and negative charges on the faces and edges of clay particles, and the type of

ions adsorbed on the clay surfaces and in the diffuse ion layer in the adjacent adsorbed water are important contributors to cohesion.

3. At points of near-contact between clay particles there is often bonding of the nature of cementation, especially in the presence of iron oxides, calcium, silica, and other minerals in solution in interstitial waters. Where sediments have been exposed to overburden pressures there is apt to be pressure-point solution and redeposition.

4. The structure of the mass of clay particles is important; for example, it has been demonstrated that the flocculated, or "cardhouse," structure (Part I, fig. 1d) is the strongest; these structures are largely determined by interparticle forces and the number of interparticle contacts.

5. Differing clay minerals affect cohesion because of particle size and differing interparticle forces; for example, Na-montmorillonite has stronger cohesive bonds than kaolinite.

6. Shear stress in clayey sediments occurs between particles and not through them; near-contact points will deform elastically, or plastically (depending on stress), by an amount sufficient to sustain the effective stress.

7. The rate of deposition, or age, of a clayey deposit is an important factor in cohesive strength. It has been shown that when the interparticle bonds of a slowly deposited sediment are broken, only a part of the original strength is regained (thixotropic regain), especially when the original strength is considered to be of the nature of interparticle cementation.

8. Cohesion is mobilized at very small strains relative to the frictional component of ultimate shear strength; friction may be negligible at the strain of maximum cohesion.

9. Cohesive strength decreases with increasing temperature. Lambe (1960) has diagrammatically explained the components of shear strength (fig. 14).

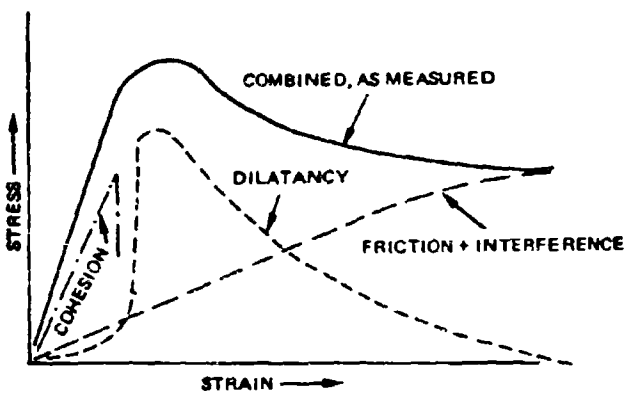

Figure 14. Components of shear strength of sediments (from Lambe, 1960). See text for discussion.

The value reported as the shear strength of a tested sediment is the highest point on the upper stress-strain curve ("combined, as measured" in fig. 14). This value is the additive result of the various components of shear strength. Cohesion is mobilized at very small strains; after it is destroyed it ceases to contribute to overall shear strength. In this connection, some investigators (*e.g.*, Schmertmann and Osterberg, 1960; Seed *et al.*, 1960) indicate a more gradual decline in the curve for cohesion, which is logical because increasing pressures force particles closer together, thus increasing some interparticle forces. Dilatancy, which results from particle interference, causes a

tendency to volume increase and more shear force to overcome. After additional strain, there is no further tendency toward volume increase; thus, the dilatancy component is overcome, and the important component is friction. When the overall stress-strain curve becomes horizontal, the only component of shear strength is friction, including particle interference, which lessens as the clay platelets tend to align themselves with their long axis parallel to the direction of shear.

To destroy the sediment structure, shear stress must first break down the complex of interparticle forces and cementing bonds outlined above. When a clay sediment structure and cohesion are destroyed by stress, porosity is reduced under additional pressure, so that further tests of sound velocity are not realistic and results should not be applied to real sediments. This is why velocity measurements of silt-clay sediment samples under laboratory pressures (as in the consolidation test) cannot be directly related to the same pressure levels in a natural sediment. Conversely, any artificial clay-silt sediment composed in the laboratory will not have the interparticle bonds (cohesion) of a natural sediment, especially those sediments of the deep sea which have accumulated slowly over geologic time.

A recent study with artificially sedimented clays dramatically demonstrated some of the effects of sediment structure, interparticle bonding, thixotropic regain of strength, and mineralogy on dynamic rigidity (Cohen, 1968). Some other important aspects of this study were discussed in the section "Rocks and Sediments as Elastic Media." Cohen used a torsional oscillator resonance technique to measure complex dynamic rigidities in mixtures of kaolinite and bentonite in distilled water at various frequencies. His results included the following:

1. Kaolinite

   a. Kaolinite concentrations of about 32 percent (by weight) in distilled water formed flocculated structures with densities and porosities comparable to high-porosity sediments. The elastic portion of complex rigidity, μ, in one experiment increased from 0.8 dynes/cm^2 $\times$ 10^5 after one day, to 2.16 dynes/cm^2 $\times$ 10^5 after 5 days.

   b. When Calgon, a deflocculating agent, was added to the above mixture, the structure became dispersed, no elastic rigidity was measured, and the mixture behaved as a Newtonian fluid.

   c. When 35.5 ppt of NaCl was added to the Calgon mixture, the clay flocculated and rigidities were about the same as before.

2. Bentonite

   a. Bentonite concentrations of about 19 percent in distilled water formed flocculated structures in which densities were lower and porosities higher than in natural sediments. Elastic rigidities increased in a typical sample from 1.33 dynes/cm^2 $\times$ 10^5 after one day, to 4.87 dynes/cm^2 $\times$ 10^5 after 3 days (with little or no increase in the next few days).

Cohen's results demonstrated:

1. the quantitative rigidities mobilized in flocculated clays, after a few days, as the result of interparticle bonding and thixotropic regain of strength (after deflocculation), and the increase of rigidity with time,

2. the destruction of interparticle bonding by addition of a deflocculating agent, and resumption of bonding in the presence of an electrolyte; and that flocculated

structures of this type have measurable dynamic rigidities, and dispersed structures behave as fluids or suspensions,

3. that bentonite (dominantly montmorillonite) forms stronger structures than does kaolinite, a well-known fact in soil-mechanics research (*e.g.*, Warkentin and Yong, 1962; review by Meade, 1964).

Slow rates of deposition and great age of deposits cause increases in sediment structural strength due to increased intergrain bonding having the nature of cementation (Leonards and Ramiah, 1959; Bjerrum and Wu, 1960; Bjerrum and Lo, 1963; Meade, 1963; Leonards and Altschaeffl, 1964). Such increase in strength is reflected in the lack of appreciable reduction in porosity with overburden pressure in deep-water silt-clays (reviewed in Part I), and in the relatively high shear strengths in these sediments (Moore, 1961, 1964; Hamilton, 1964; Richards and Hamilton, 1967). The maximum rigidities measured by Cohen (1968) were about 2 dynes/cm^2 $\times$ 10^6. In the San Diego Trough, *in situ* determinations of shear-wave velocities and densities resulted in computed least values of about 1 dyne/cm^2 $\times$ 10^8 (Hamilton *et al.*, 1969). In deep-sea clays, computed values of rigidity were about 5 dynes/cm^2 $\times$ 10^8. The increased values in the natural sediments probably include additional rigidity, resulting from the effects of age and bonding, which has the nature of cementation. Schreiber (1968) has demonstrated the effects of cementation in volcanic glass-ash layers in a natural marine sediment from the deep Caribbean Sea. These layers had unusually high compressional velocities and sediment strength (cohesion). The addition of HCl, or loss of water by drying, resulted in marked loss of strength.

Hardin and Richart (1963) have demonstrated that in sands, shear-wave velocities are independent of grain size and size distribution, except as they affect porosity; and that dynamic rigidity decreases with increasing porosity. However, sands composed of angular grains have higher rigidities (at the same porosities) than do those composed of round grains, apparently because of increased resistance of the angular grains to intergrain movements.

Shumway (1960) related the amount of sand (fraction of a unit volume) in his samples to rigidity under the assumption that these grains were in contact and that the greater their volume, the greater the elastic rigidity of the sediment structure. This is an invalid assumption; in the higher porosity sediments (even if present in significant amounts) such larger grains are not in contact (Part I, fig. 1f) as pointed out (in the context of elasticity) by Kozlov (1962). Shumway also assumed that rigidity was zero at 80 percent porosity, which is incorrect in the laboratory (Cohen, 1968) or *in situ* Hamilton *et al.*, 1969; discussions above).

QUANTITATIVE EFFECTS OF RIGIDITY ON COMPRESSIONAL-WAVE VELOCITY

The effects of rigidity on values of compressional-wave velocity in natural marine sediments are small, but significant. Computations were made which involved equation 1, measured values of compressional-wave velocities and densities, and either

values of μ computed from equation 6, when shear-wave velocities were known, or those computed after an initial computation of the bulk modulus. The results were:

Sediment Type	V_p (m/sec)		Remarks
	with μ	without μ	
Fine sand (laboratory)	1740*	1714	Shell Development Co. data
Medium sand (*in situ*)	1798*	1784	Hamilton *et al.* (1969)
Fine sand (cont. terrace)	1742**	1696	
Silty clay (abyssal hills)	1507**	1486	

*V_p, V_s, ρ measured; μ computed

**V_p, ρ measured; κ, and then μ computed

In general, the computations indicate that the presence of dynamic rigidity in sands is apt to raise compressional-wave velocities on the order of 1 to 3 percent; in deep-sea clays the average increases should be about 1 to 2 percent.

CONCLUSIONS REGARDING RIGIDITY

The data presented in previous sections indicate that almost all marine sediments which have mineral-to-mineral contacts (as in sands), or flocculated clay structures, have small but significant rigidities which allow transmission of shear waves. This is true even in artificial clays in distilled water (Cohen, 1968). Thus any equation for elastic-wave propagation in natural marine sediments which does not provide for rigidity (as the Wood equation, 5) should be abandoned.

It is apparent from the discussion of cohesion that dynamic rigidity cannot be computed for a system such as deep-sea clay from its physical components (*i.e.*, given porosity, mineralogy, and pore-fluid composition). One might be able to compute some of the interparticle forces using clay-mineral technology, but the bonding resulting from age of the deposit and "cementing" effects could not be computed. However, enough information is at hand to reasonably predict values of dynamic rigidity, given the sediment type and environment, plus physical properties such as density, porosity, and mineralogy.

Although there are "usable" empirical relationships between rigidity and other physical properties (figs. 15, 16), a better procedure for deriving a value for rigidity (lacking shear-wave velocity) is to use equation 8 and values for density, compressional velocity, and a computed value for the bulk modulus – especially for the continental-terrace environment where scatter is great between rigidity and any common physical property. Lacking values for any physical properties, the average values of rigidity for the estimated sediment type in the particular environment should be used. If grain-size data, density, or porosity are known, missing values of density, porosity, and velocity can be determined for 23°C and 1 atmosphere by using procedures discussed in Part I; *in situ* values should be computed according to the procedures of Part III.

Empirically, the best index to rigidity is ρV_p^2 in both the continental-terrace and abyssal-plain environments (fig. 16). In the abyssal-hill environment, the best index is percent clay size (or sand plus silt; see fig. 15). Regression equations are included in Appendix C for these three relationships.

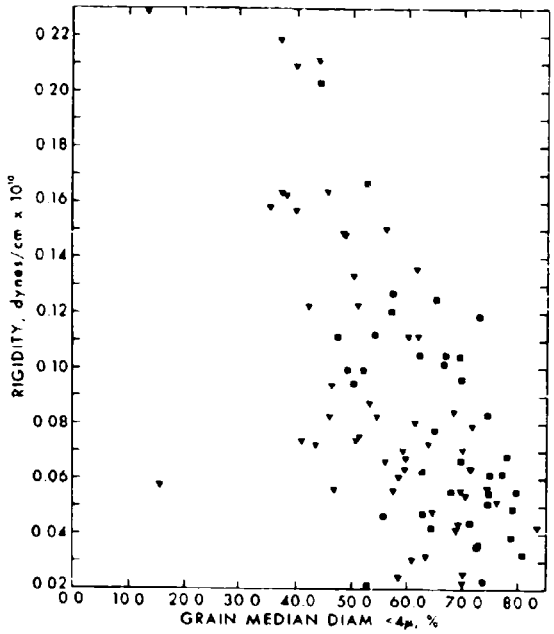

Figure 15. Percent clay size (< 0.004 mm) vs. rigidity modulus, μ; abyssal hill (squares) and abyssal plain (triangles) environments.

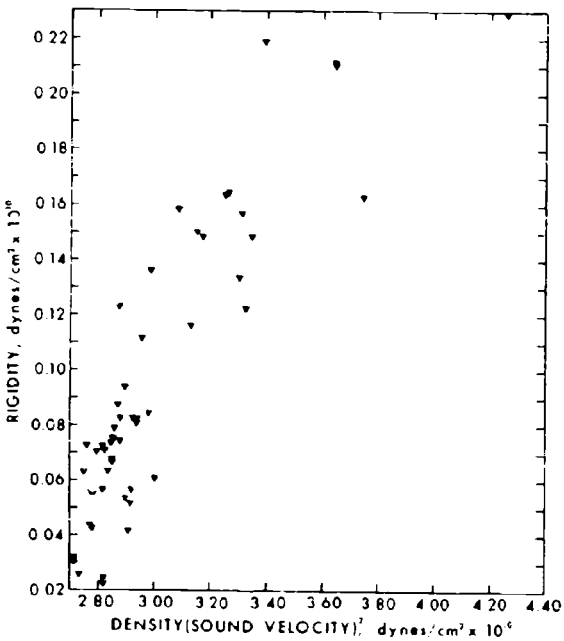

Figure 16. ρV_p^2 vs. rigidity modulus, μ; abyssal plain environment.

POISSON'S RATIO

When rigidity is zero, no shear wave can be transmitted, and Poisson's ratio is that of a fluid or suspension, 0.50 (eq. 10). As discussed in the preceding section, most natural marine sediments possess rigidity and transmit shear waves; therefore, most sediments have values of Poisson's ratio less than 0.50.

As noted previously, many literature values of compressional-wave velocities in "saturated" sediments (especially sands) must be disregarded because the measurements were, apparently, made in sediments at less than full saturation, where shear-wave velocities are slightly higher, but compressional velocities are far too low. As a result, many published values for Poisson's ratio, especially in soil mechanics literature, are too low because the ratio, V_p/V_s, determines Poisson's ratio (eq. 10).

In the references to table 3 there are 11 cases where compressional- and shear-wave velocities in the laboratory or *in situ* allow computations of Poisson's ratio; these are within the range of this ratio as computed in the present study (tables 1 and 2).

Average values and standard errors of Poisson's ratio are listed in tables 1 and 2. Maximum and minimum values (not listed) indicate the following ranges and averages in some principal sediment types and environments.

Environment Sediment	Poisson's Ratio			No. of Samples
	Max.	Min.	Avg.	
Continental Terrace:				
Sands (all grades)	0.496	0.416	0.470	13
Clayey silt	0.499	0.447	0.478	40
Abyssal Plain (Turbidite):				
Silt-clays	0.496	0.466	0.484	52
Abyssal Hill (Pelagic):				
Silt-clays	0.499	0.467	0.487	41

The data in the table above are for laboratory conditions. Anderson and Schreiber (1965), Anderson and Liebermann (1968), and Nafe and Drake (1967) have noted that Poisson's ratio does not change substantially with pressure and temperature in rocks. Using the methods discussed in Part III, computations comparing laboratory and *in situ* values of Poisson's ratio for a given high-porosity deep-sea silty clay indicate this is also true of marine sediments. With temperature and pressure the only variables, Poisson's ratio in the laboratory at 23°C and 1 atmosphere is 0.486; at a water depth of 6000 m (pressure: 626 kg/cm^2; temperature: 1.5°C), Poisson's ratio is 0.490.

Velocity-Frequency Relationship

The subject of velocity dispersion (dependence of velocity on frequency) is important in geophysics and underwater acoustics. Questions concerning basic theories and models for propagation of elastic waves in various media cannot be resolved until various questions are answered: does velocity dispersion exist; and, if so, to what extent, in which media, and over what frequency range? The subject of velocity dispersion in saturated sediments was recently raised, again, by Hampton (1967) who reported dispersion in laboratory measurements in artificial sediments.

Because well-known and valid theoretical models exist on both sides of the question of dispersion in earth materials, the answer must lie in experimental work which will set parameters for these models. It is the opinion of the author that the experimental evidence strongly indicates that in the frequency range from a few hertz to the megahertz range, there is no measurable dependence of velocity on frequency in earth materials, including almost all natural sediments and rocks.

In any discussion of the evidence concerning velocity dispersion, a clear distinction should be made between various media. The broad categories are:

1. rocks with little or no porosity,

2. porous, saturated, and unsaturated rocks,

3. fully-saturated sands with mineral-to-mineral grain contacts,

4. higher-porosity, fully saturated sediments with a structure possessing cohesion,

5. partially saturated sediments (natural or laboratory) containing gas or air in pore spaces, and

6. clay-silt "sediments" which are suspensions, especially those formed artificially in the laboratory with a deflocculating agent.

The experimental evidence on velocity dispersion falls into the categories of work in the laboratory on rocks and on natural and artificial sediments, and *in situ* on rocks and sediments. Measurements of both velocity and sound absorption are involved. The references cited below are recent examples of this experimental work; no comprehensive review is intended.

A number of investigators have measured compressional- and shear-wave velocity and absorption in rocks (laboratory and *in situ*). Their common conclusion is that there is no (or negligible) measurable velocity dispersion in the range from seismic frequencies into the megahertz range, and that the specific dissipation constant, $1/Q$, is independent of frequency. Examples include work by Wyllie *et al.* (1956), Birch (1961), Peselnick and Outerbridge (1961), McDonal *et al.* (1958), White (1965), Press (1966). Bradley and Fort (1966) have good resumes of much of the evidence.

No velocity dispersion was measured in artificial sands in the laboratory by Hardin and Richart (1963), Nolle *et al.* (1963), and Schon (1963); these measurements included a frequency range from 200 Hz to 1 MHz.

In soil-mechanics investigations *in situ* in sands, low-frequency vibrations were used in studies by Barkan (1962) and Jones (1958) to measure shear-wave velocities (Jones' measurements also included clay-silt). No velocity dispersion was measured in the frequency range from 10 to 400 Hz.

Compressional-velocity studies in high-porosity, deep-sea clay in the North Pacific by the writer at 200 kHz (laboratory), by Schreiber (1968b) and Horn *et al.* (1968b) at 400 kHz (laboratory), and by Fry and Raitt (1961; seismic measurements at sea) are all in reasonable accord (less than 1 percent differences). The average of compressional velocity for abyssal-hill, silty clay (Part I, table 2) is 1507 m/sec. An average velocity of the top of 10 cores off Hawaii (Schreiber, 1968a), in the same material, is 1504 m/sec. The average ratio, velocity in sediment/velocity in water, at 200 kHz (Part I, table 2) for abyssal-hill silty clay is 0.985 (max 1.006, min 0.973). The ratio computed by Fry and Raitt (1961, table 1) for this sediment type was 0.974 (see discussion in Part I).

In a continuing program to measure compressional velocity and attenuation *in situ*, the writer measured no velocity dispersion in shallow-water sands (three stations), or silt-clay (one station) at 14, 7, and 3.5 kHz.

In a laboratory study of complex rigidity ($\mu + i\mu'$) in artificially sedimented clays, Cohen (1968) demonstrated that both components of complex rigidity, μ and $i\mu'$,

were independent of frequency in the range 8.6 to 43.2 kHz. When a deflocculating agent was added, the flocculated structure was dispersed, the material lost all rigidity, μ, and the mixture behaved as a Newtonian viscous fluid in which $i\mu'$ was linearly dependent on frequency. The addition of 35.5 ppt of NaCl caused reflocculation and both elements of complex rigidity were the same as before.

There are several important conclusions to be drawn from Cohen's study (in addition to those discussed in previous sections):

1. Suspensions (without structure) do not behave as flocculated clay structures, and almost all natural, high-porosity silt-clays have this type of structure (see section on "Causes of Rigidity").

2. If the elastic portion of complex rigidity, μ, is independent of frequency, there is no shear velocity dispersion in the flocculated clay.

Hampton (1967), in a laboratory study of artificial sediments, reported a 4 to 6 percent increase in compressional velocity from 3 to 200 kHz (this amounts to 60 to 90 m/sec in silt-clays); his ratios, velocity in sediment/velocity in water (1967, fig. 11), indicated sediment velocities as low as 7 percent below that in water at lower frequencies. Hampton referred to Ament (1953) for theoretical support of his conclusions.

Ament's (1953) theoretical approach to sound propagation in gross mixtures involved a true, viscous suspension in which particles were not in contact and in which permeability, viscosity, and scattering of sound by the particles were involved. He compared his approach with experiments by Urick, who measured compressional velocities through suspensions of mercury, bromoform, and kaolinite (with a deflocculating agent) in water. Sutton *et al.* (1957) and Laughton (1957) discussed Ament's equations and concluded that they could not be applied to natural marine sediments; the author agrees with this conclusion. As discussed above, subsequent measurements of elastic-wave velocities in the laboratory and field do not support any theory involving significant velocity dispersion from low (seismic) frequencies to the megahertz range.

Hampton's values of the ratio, velocity in sediment to velocity in water (1967, fig. 11) not only varied with frequency but were anonymously low (at lower frequencies). His values for artificial kaolinite "sediments," for example, in the frequency range 8 to 40 kHz (0.93 to 0.97), can be compared (at similar concentrations) to those of Cohen (1968) in kaolinite suspensions and flocculated clay in the same frequency range (0.97 to 0.99), and those of Urick (1947) in a kaolinite suspension at 1 MHz (0.97 to 0.99). Shumway (1958, 1960) measured a ratio of 0.97 in a red-clay slurry. An average ratio for six of Shumway's (1960) high-porosity sediments in the San Diego Trough (20 to 40 kHz) was 0.98, his equation 7 (1960, fig. 1, p. 161) predicts an average least value of 0.97 for his sediments. The author (1956) measured, *in situ*, in the San Diego Trough, average values of 0.98 at 100 kHz. Fry and Raitt (1967) computed an average value of 0.974 for deep Pacific surface sediments at seismic frequencies. In the Atlantic, Houtz and Ewing (1964) used the same measuring techniques that were used by Fry and Raitt to obtain sediment surface velocities. At three stations (Stas. 4, 5, and 8) favored in later discussions, Houtz and Ewing measured values of 1517, 1532, and 1517 m/sec, which when divided by appropriate values for bottom-water velocity yield an average ratio of 0.98. Table 2 in Part I lists average ratio values in deep-water silt-clays (at 200 kHz) in the range 0.98 to 0.99. In summary, the evidence from laboratory and field, over a wide range of frequencies, does not support Hampton's low ratio values.

When air or gas bubbles (from decaying organic material) are trapped in pore spaces within any sediment they have a marked effect on both compressional velocity and attenuation, depending on the concentration and size of bubbles, and the frequency (*e.g.*, Meyer, 1957): velocity varies with frequency (velocity is usually too low) and attenuation is apt to be high. Although Hampton recognized these facts, and attempted

to remove gas and air bubbles from his artificial sediments, his data indicate the probability of air and/or gas bubbles within his materials.

Summary of Factors Affecting Compressional-Wave Velocity

To summarize the effects of many complex, variable factors on compressional velocity, it is instructive to separate equation 1 into two components and to expand density, ρ, as per equation 4, and κ as per equation 13a.

$$V_p^2 = \frac{\kappa}{\rho} + \frac{4/3\,\mu}{\rho} \tag{1}$$

where

$$\kappa = \kappa_s \frac{\kappa_f + Q}{\kappa_s + Q} \qquad Q = \frac{\kappa_w (\kappa_s - \kappa_f)}{n(\kappa_s - \kappa_w)} \qquad \rho = n\rho_w + (1-n)\rho_s \tag{13a}$$

The variable factors contributing to compressional-wave velocity are thus:

Bulk moduli of pore water (κ_w), minerals (κ_s), and frame (κ_f).

Porosity (n); in a unit volume, porosity equals the volume of pore water and ($1-n$) equals the volume, or concentration, of mineral grains.

Density of pore water (ρ_w), and minerals (ρ_s)

Rigidity modulus (μ)

Given laboratory conditions of constant temperature and pressure, and assuming that pore water is the same (true within narrow limits; see Appendix B), the effects of changes in porosity and mineralogy are thus (summarizing previous discussions):

1. Changes in mineralogy *only* cause changes in
   a. the bulk modulus, κ, through changes in
      (1) the bulk modulus of minerals, κ_s
      (2) the frame-bulk modulus, κ_f
   b. bulk density, ρ, through: $(1-n)\rho_s$
   c. the rigidity modulus, μ; in sediments have cohesion, some minerals have stronger interparticle bonds (see "Causes of Rigidity")
2. Changes in porosity *only* cause changes in
   a. the bulk modulus, κ, through changes in
      (1) the frame-bulk modulus, κ_f (fig. 2)
      (2) the denominator of Q (eq. 13a): $n(\kappa_s - \kappa_w)$
   b. bulk density, ρ, through the products (eq. 4): $n\rho_w$ and $(1-n)\rho_s$
   c. the rigidity modulus, μ, because higher porosities lead to fewer interparticle contacts in sands and in silt-clays (having cohesion; see "Causes of Rigidity")

The causes of variations in rigidity were discussed in a previous section, but the results of these variations can be readily seen when equation 1 is separated into its two components, as above. When rigidity, μ, is zero or negligibly small, the component

$$\frac{4/3\,\mu}{\rho}$$

drops out and the remainder of equation 1 is the equation for compressional velocity in a liquid, or any medium without rigidity. It is the same as the Wood equation (5) if the medium is a true suspension with no sediment structure, in which case κ_{sw} (eq. 3b) can be used for the bulk modulus, κ. If there is a sediment structure, κ should include a frame-bulk modulus, κ_f. Because almost all marine sediments have a definite structure and possess rigidity, the Wood equation, which served to approximately define compressional velocity in much early work, should be abandoned in favor of equation 1, the classic equation for compressional velocity.

The interaction of all of the above factors results in almost all higher-porosity sediments having compressional velocities less than in water (pore water or bottom water above the sea floor) unless rigidity is unusually high. This low-velocity phenomenon is mostly due to the low rigidities in high-porosity sediments, and the dominant effects of high water compressibilities (or low incompressibilities) relative to mineral moduli (see Appendix B for typical values).

Summary Statement Regarding Elasticity of Marine Sediments

To prove that a medium responds as an elastic body, it is necessary to use the theory of elasticity and its equations (eqs. 1, 2, 5-11) to compute unmeasured elastic constants (without empirical constants or assumptions) and verify these by measurements. Further, values for elastic constants should be computable from the basic components of the medium, and static and dynamic values should be theoretically related. All of these requirements have been satisfied in the case of rocks (*e.g.*, Birch, 1961; Simmons and Brace, 1965; Anderson and Liebermann, 1968). It is a conclusion of this report that these requirements can also be satisfied in the case of saturated, porous sediments.

Density and compressional-wave velocity were measured in the present study. A computed value for the bulk modulus was used as the third constant required to compute the other elastic constants with the equations of elasticity. This bulk modulus was computed from the porosity and the bulk moduli of the components of the medium (pore fluid, minerals, and a frame-bulk modulus) following Gassmann's equations (13a; Appendix A). This method is thus based on theory without empirical factors or constants.

The computations of elastic constants as discussed in this report, compared with other laboratory and *in situ* measurements (such as shear velocity, table 3), indicate that the equations of elasticity can be used to derive reasonable values for those elastic constants not measured. If so, the computed values in tables 1 and 2 predict these constants for the major marine-sediment types.

APPENDIX A: DERIVATIONS OF GASSMANN'S EQUATIONS

The derivation of Gassmann's equation (1951, p. 15, paragraph No. 59)

$$\kappa = \kappa_s \frac{\kappa_f + Q}{\kappa_s + Q} \qquad Q = \frac{\kappa_w (\kappa_s - \kappa_f)}{n (\kappa_s - \kappa_w)}$$

is indicated, but not completely shown in his original (German) publication. The following derivation (by the author) is completed because of its importance to the computations of this report, and to make it more readily available to interested readers.

Gassmann's equation, above, is not empirical; it follows from the basic assumptions that a unit volume of a saturated porous medium responds elastically under the light pressures of a sound wave. In this "closed system" no pore water leaves or is added to the unit volume during passage of the sound wave.

The notations of Gassmann are changed as follows:

Item	Gassmann	Changed To
Total pressure	P	No change
Hydrostatic pressure	$\tilde{P}$	P_h
Effective pressure, or pressure on frame	$\bar{P}$	P_e
Bulk modulus of system	κ	No change
Bulk modulus of pore water	$\tilde{\kappa}$	κ_w
Bulk modulus of mineral solids	$\hat{\kappa}$	κ_s
Bulk modulus of mineral frame	$\bar{\kappa}$	κ_f
Total volume	V	No change
Volume of pore water	$\tilde{V}$	V_w
Volume of mineral solids	$\hat{V}$	V_s

1. With these changes in notation, pertinent equations and "running" translations from Gassmann (1951, p. 15) are:

> "(58) In order to determine κ, the unit volume, V, will be exposed to an additional total (hydrostatic) pressure, ΔP, which is effective on all surfaces . . . ΔP is resolved into two components. ΔP_h is that part of ΔP which is effective on all surfaces of the volume V, and determines the additional hydrostatic pressure in the pores. ΔP_e is the remainder of ΔP which is effective on those portions of the surfaces occupied by the mineral solids of the frame.

a. $$\Delta P = \Delta P_h + \Delta P_e \qquad \text{(A-1a)}$$

The desired bulk modulus, κ, of the closed system is defined by the equation.

b. $$\frac{\Delta V}{V} = -\frac{\Delta P}{\kappa} \qquad \text{(A-1b)}$$

c. $$\frac{\Delta V_w}{V_w} = -\frac{\Delta P_h}{\kappa_w} \qquad \text{(A-1c)}$$

describes the compression of the pore water (or, from f):

$$\frac{\Delta V_w}{V} = -\Delta P_h \, n/\kappa_w$$

d. $$\frac{\Delta V}{V} = -\frac{\Delta P_h}{\kappa_s} - \frac{\Delta P_e}{\kappa_f} \qquad \text{(A-1d)}$$

describes the compression of the frame, proceeding from the two components ΔP in (a).

e. $$\frac{\Delta V_s}{V} = -\frac{(1-n)\,\Delta P_h}{\kappa_s} - \frac{\Delta P_e}{\kappa_s} \qquad \text{(A-1e)}$$

describes the compression of the mineral solids . . . with the application of $V_s = (1-n)V$.

f. $$V_w = nV \qquad \text{(A-1f)}$$

defines porosity.

g. $$\Delta V = \Delta V_s + \Delta V_w \qquad \text{(A-1g)}$$

which indicates that the system is closed, therefore, no pore water can be added nor can any escape."

Using the above basic relationships furnished by Gassmann, the final equation for computing the system bulk modulus, κ, can be derived as follows:

2. Equating volume changes

$$\frac{\Delta V}{V} = \frac{\Delta V_s}{V} + \frac{\Delta V_w}{V}$$

from b, c, and e, above, and multiplying by (-1)

$$\frac{\Delta P}{\kappa} = (1-n)\frac{\Delta P_h}{\kappa_s} + \frac{\Delta P_e}{\kappa_s} + \frac{\Delta P_h n}{\kappa_w}$$

from a $\Delta P_e = \Delta P - \Delta P_h$

$$\frac{\Delta P}{\kappa} = \frac{(1-n)\,\Delta P_h}{\kappa_s} + \frac{\Delta P_h n}{\kappa_w} + \frac{\Delta P - \Delta P_h}{\kappa_s}$$

reduces to

$$\frac{\Delta P}{\kappa} = \frac{\Delta P_h\,[n(\kappa_s - \kappa_w)]}{\kappa_s \kappa_w} + \frac{\Delta P}{\kappa_s}$$

Divide by ΔP

$$\frac{1}{\kappa} = \frac{\Delta P_h}{\Delta P}\,\frac{[n(\kappa_s - \kappa_w)]}{\kappa_s \kappa_w} + \frac{1}{\kappa_s}$$

$$\frac{1}{\kappa} - \frac{1}{\kappa_s} = \frac{\Delta P_h}{\Delta P} \frac{[n(\kappa_s - \kappa_w)]}{\kappa_s \kappa_w}$$

$$\frac{\kappa_s - \kappa}{\kappa \kappa_s} = \frac{\Delta P_h}{\Delta P} \frac{[n(\kappa_s - \kappa_w)]}{\kappa_s \kappa_w}$$

reduces to

$$\frac{\Delta P}{\Delta P_h} = \frac{\kappa [n(\kappa_s - \kappa_w)]}{\kappa_w (\kappa_s - \kappa)} \qquad \text{(A-2)}$$

3. Equating pressure-volume-moduli relationships.

a. Compression of frame (decreases n) . . . . . Eq. A-1(d)

b. Compression of solids (increases n) . . . . . Eq. A-1(e)

c. Compression of water . . . . . Eq. A-1(c)

In the closed system no pore water is added or escapes, therefore

$$3a - 3b = 3c, \text{ or}$$

$$\frac{\Delta P_h}{\kappa_s} + \frac{\Delta P_e}{\kappa_f} - \left[\frac{(1-n)\Delta P_h}{\kappa_s} + \frac{\Delta P_e}{\kappa_s}\right] = \frac{\Delta P_h n}{\kappa_w}$$

$$\frac{\Delta P_h}{\kappa_s} + \frac{\Delta P_e}{\kappa_f} - \frac{\Delta P_h}{\kappa_s} + \frac{\Delta P_h n}{\kappa_s} - \frac{\Delta P_e}{\kappa_s} = \frac{\Delta P_h n}{\kappa_w}$$

$$\frac{\Delta P_e}{\kappa_f} - \frac{\Delta P_e}{\kappa_s} = \frac{\Delta P_h n}{\kappa_w} - \frac{\Delta P_h n}{\kappa_s}$$

$$\frac{\Delta P_e (\kappa_s - \kappa_f)}{\kappa_s \kappa_f} = \frac{\Delta P_h n (\kappa_s - \kappa_w)}{\kappa_s \kappa_w}$$

$$\Delta P_e = \frac{\Delta P_h n (\kappa_s - \kappa_w) \kappa_f}{\kappa_w (\kappa_s - \kappa_f)}$$

from eq. 1a, $\Delta P_e = \Delta P - \Delta P_h$

$$\Delta P - \Delta P_h = \frac{\Delta P_h n (\kappa_s - \kappa_w) \kappa_f}{\kappa_w (\kappa_s - \kappa_f)}$$

$$\Delta P = \frac{\Delta P_h n (\kappa_s - \kappa_w) \kappa_f + \Delta P_h [\kappa_w (\kappa_s - \kappa_f)]}{\kappa_w (\kappa_s - \kappa_f)}$$

Divide by ΔP_h

$$\frac{\Delta P}{\Delta P_h} = \frac{n (\kappa_s - \kappa_w) \kappa_f + \kappa_w (\kappa_s - \kappa_f)}{\kappa_w (\kappa_s - \kappa_f)} \qquad \text{(A-3)}$$

4. Equating equations A-2 and A-3

$$\frac{\kappa\,[n(\kappa_s-\kappa_w)]}{\kappa_w(\kappa_s-\kappa)} = \frac{n\,(\kappa_s-\kappa_w)\,\kappa_f + \kappa_w\,(\kappa_s-\kappa_f)}{\kappa_w(\kappa_s-\kappa_f)}$$

let

$$\kappa_w(\kappa_s-\kappa_f) = A$$

$$n\,(\kappa_s-\kappa_w) = B$$

$$Q = \frac{A}{B}$$

then

$$\frac{\kappa B}{\kappa_w\,(\kappa_s-\kappa)} = \frac{B\kappa_f + A}{A}$$

which, after dividing by B, reduces to

$$\kappa\left(A + \kappa_f\kappa_w + \kappa_w\frac{A}{B}\right) = \kappa_f\kappa_s\kappa_w + \kappa_s\kappa_w\frac{A}{B}$$

Substitute $Q = \frac{A}{B}$ and solve for κ

$$\kappa = \frac{\kappa_w\kappa_s\,(\kappa_f + Q)}{A + \kappa_f\kappa_w + \kappa_w Q} \qquad A = \kappa_w(\kappa_s-\kappa_f)$$

$$= \frac{\kappa_w\kappa_s\,(\kappa_f+Q)}{\kappa_s\kappa_w - \kappa_w\kappa_f + \kappa_w\kappa_f + \kappa_w Q}$$

which reduces to

$$\kappa = \frac{\kappa_s\,(\kappa_f + Q)}{\kappa_s + Q}$$

which is Gassmann's equation (1951, p. 15, para. 59).

APPENDIX B:
BULK MODULI OF WATER AND MINERALS

Some values of bulk moduli (incompressibilities) of seawater, κ_w, and minerals, κ_s, used in the computations in the main text, are listed below. In addition, values for the aggregate bulk moduli for some of the more important types of sediment mineral aggregates are listed. These were computed by the Voigt-Reuss-Hill method (see discussion, main text).

A. Seawater*

Salinity (ppt)	Density (g/cc)	V_p (m/sec)	Bulk Modulus, κ_w (dynes/cm^2 × 10^{10})
33.50	1.0228	1528.3	2.388955
34.00	1.0232	1528.9	2.391766
34.50	1.0236	1529.4	2.394266
35.00	1.024[illegible]	1530.0	2.397082

*Density and velocity from NAVOCEANO SP-68 (1966); κ_w computed from: $\rho V_p^2 = \kappa$.

B. Mineral Species**

Mineral	Bulk Modulus, κ_s (dynes/cm^2 × 10^{10})	Reference
Calcite	72.940	Peselnick (1962)
Microcline	51.813	Anderson and Nafe (1965)
Orthoclase	47.393	Anderson and Nafe (1965)
Albite	52.910	Brace (1965); Anderson and Nafe (1965)
Labradorite	66.667	Birch (1966)
Quartz	37.726	Soga (1968); McSkimin *et al.* (1965)
Obsidian	37.800	Manghnani *et al.* (1968)
Hornblende	84.175	Brace (1965)
Biotite	41.152	Brace (1965)
Apatite	91.743	Birch (1966)
Magnetite	181.818	Birch (1966)
Olivine	126.582	Birch (1966)
Enstatite	99.010	Birch (1966)
Hypersthene	101.010	[illegible] (1966)
Augite	98.039	Birch (1966)

**Computed from $\kappa = 1/\beta$ where β given, references are to recent, readily available sources which frequently cite other sources; for recent, general compilations see Birch (1966) [illegible], Anderson and Liebermann (1968).

C. Computed Aggregate Bulk Moduli, κ_s*

Sediment	Bulk Modulus, κ_s (dynes/cm^2 × 10^{10})	Reference
Medium sand	51.206	San Diego (nearshore)
Fine gray sand	52.326	San Diego (nearshore)
Clayey silt	54.425	San Diego Trough
"Clay"	50.- - -	Skempton (1961); used for deep-sea silt-clays.

*Some typical values.

APPENDIX C: EQUATIONS FOR REGRESSION LINES AND CURVES (ILLUSTRATED DATA)

Regression lines and curves were computed for those illustrated sets of (x,y) data which constitute the best indices (x) to obtain desired properties (y). Separate equations are listed, where appropriate, for each of the three general environments, as follows: continental terrace (shelf and slope), (T); abyssal hill (pelagic), (H); abyssal plain (turbidite), (P). The equations are keyed by figure numbers to the related scatter diagrams in the main text. The Standard Errors of Estimate, σ, opposite each equation, are applicable only near the mean of the (x,y) values, and accuracy of the (y) values, given (x), falls off away from this region (Griffiths, 1967, p. 448).

It is important that the regression equations be used only between the limiting values of the index property (x values), as noted below. These equations are strictly empirical and apply only to the (x,y) data points involved.

The limiting values of (x) in the equations below are:

1. Porosity, n, percent

   (T), 35 to 82 percent

   (H) and (P), 70 to 90 percent

2. Density, ρ, g/cc

   (T), 1.25 to 2.10 g/cc

   (H), 1.25 to 1.50 g/cc

   (P), 1.15 to 1.45 g/cc

3. Density × (Velocity)^2, ρV_p^2, dynes/cm^2 × 10^{10}

   (H), 2.9 to 3.4 dynes/cm^2 × 10^{10}

   (P), 2.7 to 3.4 dynes/cm^2 × 10^{10}

4. Clay size grains, C, percent

   (H), 45 to 80 percent

<u>Frame Bulk Modulus, κ_f (dynes/cm^2) vs. Porosity, n (fraction) (Figure 2)</u>

(T) For sands: κ_f, dynes/cm^2 × 10^9

$$\log \kappa_f = 2.71405 - 4.12135\,(n)$$

(T), (H), (P) For silt-clays: κ_f, dynes/cm^2 × 10^8

$$\log \kappa_f = 3.73807 - 4.25571\,(n)$$

<u>Bulk Modulus, κ (dynes/cm^2 × 10^{10}) vs. Porosity, n (percent) (Figures 7, 8)</u>

(T) $\kappa = 22.73698 - 0.679982\,(n) + 0.008265\,(n)^2 - 0.00003566\,(n)^3$

$$\sigma = 0.1379$$

(H) (P) $\kappa = 9.05546 - 0.114167\,(n) + 0.000478\,(n)^2$

$$\sigma = 0.0141$$

<u>Bulk Modulus, κ (dynes/cm^2 × 10^{10}) vs. Density, ρ (g/cc) (Figures 9, 10)</u>

(T) $\kappa = -22.16073 + 49.936116\,(\rho) - 34.144205\,(\rho)^2 + 8.119349\,(\rho)^3$

$$\sigma = 0.1812$$

(H) $\kappa = 6.69991 - 7.428005\,(\rho) + 3.445399\,(\rho)^2$

$\sigma = 0.0362$

(P) $\kappa = 3.42494 - 2.486986\,(\rho) + 1.596811\,(\rho)^2$

$\sigma = 0.1525$

Bulk Modulus, κ (dynes/cm^2 $\times$ 10^{10}) vs. Density $\times$ (Velocity)^2, $\rho V_p^{\,2}$ (dynes/cm^2 $\times$ 10^{10}) (Figure 11)

(H) $\kappa = 0.24037 + 0.891027\left(\rho V_p^{\,2}\right)$

$\sigma = 0.0528$

(P) $\kappa = 0.66633 + 0.742309\left(\rho V_p^{\,2}\right)$

$\sigma = 0.1446$

Rigidity Modulus, μ (dynes/cm^2 $\times$ 10^{10}) vs. Clay Size, C (percent) (Figure 15)

(H) $\mu = 2.38655 - 0.105307\,(C) + 0.001611\,(C)^2 - 0.00000828\,(C)^3$

$\sigma = 0.0334$

Rigidity Modulus, μ (dynes/cm^2 $\times$ 10^{10}) vs. Density $\times$ (Velocity)^2, $\rho V_p^{\,2}$ (dynes/cm^2 $\times$ 10^{10}) (Figure 16)

(P) $\mu = 4.78616 - 5.154326\left(\rho V_p^{\,2}\right) + 1.797785\left(\rho V_p^{\,2}\right)^2$

$- 0.200143\left(\rho V_p^{\,2}\right)^3$

$\sigma = 0.0229$

APPENDIX D: ADDITIONAL SEDIMENT PROPERTIES

Tables 1 and 2 from Part I (TP 143) are included in this appendix as a convenience to the reader. These tables furnish additional information on properties of the sediments discussed in this report (TP 144). Tables D-1 and D-2 can be correlated with tables 1 and 2 in the main text.

TABLE D-1: SEDIMENT PROPERTIES, CONTINENTAL TERRACE (SHELF AND SLOPE) ENVIRONMENT

Sediment Type	No. Samples	Grain Diameter		Sand (%)	Silt (%)	Clay (%)	Bulk Grain Density (g/cc)	Density (g/cc)		Porosity (%)		Velocity (m/sec)		Ratio	
		Mean (mm)	Median (mm)					Avg.	SE	Avg.	SE	Avg.	SE	Avg.	SE
Sand															
Coarse	2	0.530	0.520	100.0	- - -	- - -	2.71	2.03	- - -	38.6	- - -	1836	- - -	1.201	- - -
Medium	12	0.376	0.356	99.8	0.2	- - -	2.70	2.01	0.009	39.7	0.46	1749	6	1.144	0.004
Fine	9	0.153	0.171	88.1	6.3	7.1	2.70	1.98	0.024	43.9	1.29	1742	10	1.139	0.006
Very fine	3	0.090	0.094	83.9	13.0	2.9	2.74	1.91	- - -	47.4	- - -	1711	- - -	1.121	- - -
Silty sand	11	0.073	0.126	65.0	21.6	13.4	2.71	1.83	0.025	52.8	1.55	1677	9	1.096	0.006
Sandy silt	6	0.036	0.051	34.5	51.2	14.3	2.75	1.56	- - -	68.3	- - -	1552	- - -	1.015	- - -
Sand-silt-clay	17	0.018	0.041	32.6	41.2	26.1	2.71	1.58	0.030	67.5	1.66	1578	9	1.032	0.006
Clayey silt	40	0.006	0.011	6.1	59.2	34.8	2.71*	1.43	0.016	75.0	0.87	1535	3	1.004	0.002
Silty clay	17	0.003	0.004	5.3	41.5	53.6	2.69	1.42	0.013	76.0	0.74	1519	3	0.994	0.002

*5 samples

Notes

Laboratory values: 23°C, 1 atmosphere

Density: saturated, bulk density; porosity: salt free; ratio: velocity in sediment/velocity in seawater at 23°C, 1 atmosphere, and salinity of sediment pore-water.

SE: standard error of the mean.

TABLE D-2 SEDIMENT PROPERTIES, ABYSSAL PLAIN (TURBIDITE) AND ABYSSAL HILL (PELAGIC) ENVIRONMENTS

Environment Sediment Type	No. Samples	Grain Diameter		Sand (%)	Silt (%)	Clay (%)	Bulk Grain Density (g/cc)	Density (g/cc)		Porosity (%)		Velocity (m/sec)		Ratio Avg.	SE
		Mean (mm)	Median (mm)					Avg.	SE	Avg.	SE	Avg.	SE		
Abyssal Plain (Turbidite)															
Sandy silt	1	0.017	0.017	19.4	65.0	15.6	2.46	1.65	---	56.6	---	1622	---	1.061	---
Silt	1	0.016	0.018	7.2	79.5	13.3	2.47	1.60	---	60.6	---	1634	---	1.069	---
Clayey silt	15	0.005	0.006	7.6	50.3	42.1	2.61	1.38	0.029	78.6	1.53	1535	2	1.003	0.001
Silty clay	35	0.002	0.003	2.9	36.1	61.3	2.55	1.24	0.010	85.8	0.49	1521	2	0.994	0.001
Clay	2	0.001	0.001	0.1	20.3	79.6	2.67	1.26	---	85.8	---	1505	---	0.985	---
Abyssal Hill (Pelagic)															
Clayey silt	3	0.0035	0.0053	3.3	50.0	46.7	2.58	1.41	---	76.4	---	1531	---	1.000	---
Silty clay	32	0.0026	0.0023	2.6	32.9	65.2	2.71	1.37	0.014	79.4	0.77	1507	2	0.985	0.001
Clay	6	0.0015	0.0013	0.6	20.7	78.9	2.76	1.42	0.023	77.5	1.35	1491	1.4	0.975	0.001

Notes

Laboratory values: 23°C, 1 atmosphere.

Density: saturated, bulk density; porosity: salt-free; ratio: velocity in sediment/velocity in seawater at 23°C, 1 atmosphere, and salinity of sediment pore water

SE: standard error of the mean

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11. SUPPLEMENTARY NOTES

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13. ABSTRACT

A study of the elastic properties of marine sediments from three major environments of the North Pacific: the continental terrace (shelf and slope), abyssal plain (turbidite), and abyssal hill (pelagic). Elastic constants are measured and computed. Discussions cover elastic and viscoelastic models, compressibility, bulk modulus, rigidity (shear) modulus, Lamé's constant, Poisson's ratio, density, shear- and compressional-wave velocities, and interrelationships between these and other physical properties.

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