ESD-TR-69-373 MTR-971

SIMPLIFIED ERROR PROBABILITY CURVES FOR "NON-FADING" AND RAYLEIGH-FADING DIGITAL LINKS, AND THEIR APPLICATION TO DESIGN MARGINS

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DECEMBER 1969

Prepared for

DIRECTORATE OF PLANNING AND TECHNOLOGY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 600D

Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts

Contract AF19(628)-68-C-0365

FOREWORD

This report was prepared by MITRE Corporation, Bedford, Mass., as part of MITRE Project 600D support to Headquarters Electronic Systems Division, L. G. Hanscom Field, Bedford, Massachusetts on the TACSATCOM System 626A under Air Force contract F19 (628)-68-C-0365.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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ABSTRACT

Straight line relationships (plus added constants for various pulse code modulations) for error probability vs. carrier to noise ratio are shown to be possible for both "non-fading" and Rayleigh-fading radio links. These are used to show the effects of fades, relative to the nominal "non-fading" design, independently of the specific pulse code modulation technique. Common causes and levels of such fades are listed; particularly those experienced on spacecraft/aircraft to ground/sea links. The relative performance of "non-fading" and Rayleigh-fading links (with any number of diversities) is discussed. Some general conclusions and guidance on "fade margins" are given.

TABLE OF CONTENTS

LIST OF FIGURES

SECTION I

INTRODUCTION AND SUMMARY

When data is transferred over a noisy communication link by pulse (bit) sequences, the link performance is usually quoted as an Error Rate (or, preferably, an Error Probability), which is expressed as a negative exponent of 10 (e.g. 10^{-6}). A 10^{-6} (bit) error probability (rate) means a probability of 1 error in 10^{6} successive bits, on the average; that is, over a time interval long enough to enable measurement of the average rate with sufficient accuracy (say, over 10 times the period during which 1 bit error occurs on the average).

Bit error probability, P_e, versus some ratio of the signal and noise, is usually shown as a pronounced curve for each and every modulation/demodulation (mod/demod) method. A family of such curves is presented as the only format available to the communication link/system analyst/designer.

For radio links/systems, there are several families of such curves; one family for the "non-fading" link,[1.] and the others for specific types of fading both without and with "diversity"[2.]

1. In this paper "non-fading" will be kept in quotes, because there is no such thing as a radio link that is not subject to fading at some time. "Non-fading" links are typically (or nominally) "line-of-sight." See Appendix II for some causes and values of fading on such links.
2. Diversity is discussed in Section 3.2.

In attempting to use these data for basic design and optimization tradeoffs, the system analyst/designer is faced with multiple mod/demod choices for each link, and the estimation of P_e values and changes, for various signal/noise values and changes, from curves of varying shapes and slopes. Such optimal choices are particularly difficult to make for the so-called "line-of-sight" radio link,[1.] because such links are in the broad class known as "non-fading" and this class has the most curved characteristics of all classes.

Furthermore, the careful system analyst/designer knows only too well that "non-fading" links are subject to fading (see Section 5 and Appendix II), and that the first requirement of good design is to ensure an adequate "fade-margin" <u>per link</u>. However, what is adequate on one link may be too much or too little on another link, and if the system has tandem radio links then the optimal design does not necessarily use the same fade margins even on similar (tandem) links, nor, for that matter, does the optimal design operate the links at the same P_e, or the same signal/noise ratio, or use the same mod/demod method. These choices are all dependent on the individual and total environments and uses of the links and system. Hence the effects of various choices ought to be readily obtainable if any significant and timely progress toward system optimization is to be made. These effects are <u>not</u> readily obtainable or ascertainable from the usual P_e vs. signal/noise families of curves.

1. This applies especially to links with small elevation angles; e.g., aircraft/spacecraft to ground/sea links.

This paper does present the P_e versus signal/noise data in a form simple enough to enable the above link and systems analysis/design and optimization to be achieved easily and quickly. In summary, it is shown that two "universal" straight-line curves plus respective lists of added constants (Figures 1 and 2) can replace all of the conventional families of curves for the various mod/demod methods, both baseband and carrier, without and with fading, and with multiple diversity in the latter case. These two straight lines cope with all P_e values down to 10^{-8}, can be further extrapolated, and in particular Figure 1 enables the simple derivation of a new group of curves (Figure 3) for P_e vs. fading levels that apply to all "non-fading" mod/demod methods. (A similar group could be derived from Figure 2, for fading links, but is not shown because their form is obvious, by inspection). Some general comparisons are then possible as between fading and "non-fading" conditions, and are made in Section 4. Finally, and because of this simplification, it is possible to draw some general and simple conclusions and guidance on the choice of "fade-margin" for all the varieties of mod/demod methods and for both "non-fading" and fading links carrying pulse-code data. (See Section 5).

SECTION II

ERROR PROBABILITIES FOR "NON-FADING" PULSE-CODE CHANNELS (Figure 1)

A start is made with what may be termed the basic case[1.] of (Carrier) Coherent Binary Phase Shift Keying (PSK, Binary Coherent), since this technique gives the smallest error probability for a given value of the ratio function of signal and noise. (The word "coherent" means that the start and stop phases of the carrier phase shift are fixed and that the demodulator carrier is in phase with the received (unmodulated) carrier).

The ratio function of signal and noise chosen for this paper is the ratio of carrier signal power, C, to thermal noise power, N, at a given point in the link (i.e., in the given bandwidth). This ratio is much more widely used and understood than other ratios.[2.]

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1. Baseband Pulse-Code Modulation is the non-carrier (DC) analog, and has the same error probability versus ratio function of signal and noise.
2. Othe ratios use the (thermal) spectral noise density, N_o, as the divisor (which "normalizes" the channel bandwidth at 1 Hz) and either the energy per bit, E (i.e., signal power x time, per bit), or P_r (the received signal power), as the numerator. Use of any of these ratios assumes that each of these quantities exist simultaneously at the instant of signal (symbol) detection. However C&N are more easily measured than E or N_o; (P_r = C, and is redundant). Particular C/N values are also readily available (e.g., receiver threshold C/N values). Use of the ratio E/N_o leads to an assumption about the bandwidth-time product, when conversion to C/N is required (i.e., E/N_o = C/N x bandwidth-time product).

It is now common practice to plot the "non-fading" probability of error, P_e, and the C/N both on logarithmic orthogonal scales. This results in a number of curves whose slope changes from almost zero at low C/N values to very high values around the smaller P_e values of most interest. Such a plot does not demonstrate the "natural" law for these smaller P_e values. This present practice therefore makes a comparison of such curves for different mod/demod coding techniques very difficult, especially in overall systems work where trends and tradeoffs have to be manipulated.

2.1 The (Carrier) Coherent Binary[1] PSK Error Probability, $P_e(2)$

This is given by[2]

$$P_e(2) = \tfrac{1}{2} \operatorname{erfc} (C/N)^{\frac{1}{2}} \qquad (1)$$

and erf(x) and erfc(x) are defined in Appendix I.

Using the final approximation given in Appendix I (equation A.I.4) in equation 1, gives,

$$P_e(2) \approx \frac{\sqrt{\pi}}{8} \; \frac{e^{-C/N}}{(C/N)^{\frac{1}{2}}} \text{, to} \leqslant 5\% \text{ for } C/N \geqslant 3 \qquad (2)$$

---

1. Pulse-Code Modulation (PCM) can be effected by binary, ternary, quaternary, etc. signal "levels;" these will be called 2-ary, 3-ary, 4-ary, and in general, M-ary methods, in this paper.
2. See, for example, "Communications Systems and Techniques," Schwartz et al, McGraw-Hill, Page 303. The baseband binary PCM case is given on page 16 (where $A/2 \equiv$ carrier amplitude).

Now take 10 $\log_{10}$ of each side of equation 2, giving

$$10 \log_{10} P_e(2) \approx - [C/N \; 10 \log_{10} e + 5 \log_{10} (C/N) + 10 \log_{10} (8/\sqrt{\pi})]$$
$$\approx - [4.34 (C/N) + 5 \log_{10} (C/N) + 6.55] \quad (3)$$

By inspection, it is clear that 4.34 (C/N) increases much faster than 5 $\log_{10}$ (C/N), so that a plot of P_e in dB versus $(C/N)_{ratio}$ ought to rapidly join an asymptote straight line of slope -4.34.

Furthermore, if the slope value and the constant term are increased slightly, it ought to be possible to eliminate the 5 $\log_{10}$ (C/N) term.

Figure 1 shows a plot of the true values for 10 $\log_{10} P_e$ (2) using equation 1 (crosses). A straight line of the following type,

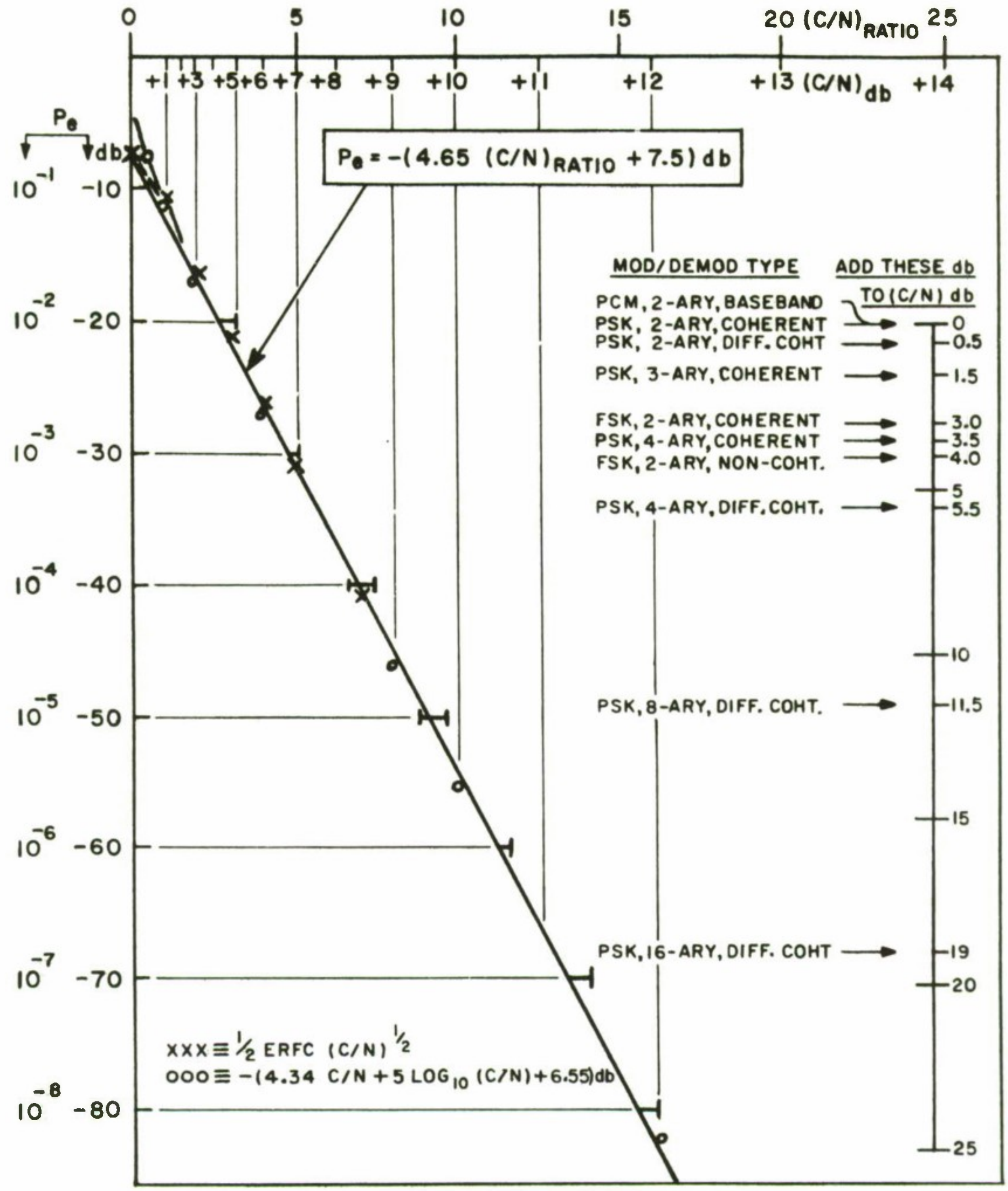
$$(P_e(2))_{dB} = - (4.65 \; C/N + 7.5) \quad (4)$$

is also shown, giving a maximum error of about 2 dB <u>in the value of</u> P_e, at a C/N value of about +1 dB;[1.] that is, at a P_e value of about 10^{-1}. This error is tolerable, in this region, because it will be the objective of most system designs to achieve P_e values less than 10^{-4}. The consequences of <u>not</u> achieving P_e less than 10^{-4} are discussed in Section 5.1.

2.2 Use of The PSK, 2-ary, Coherent Straight Line As A Universal Non-Fading Characteristic

Eight other mod/demod pulse-code methods are listed on Figure 1, all of them requiring greater C/N values for given P_e values. By use

1. The error is indicated by the short and long dash lines over the C/N range 0 to 2. As can be seen, equation 4 is much closer to equation 1 than is predicted in Appendix 1, because of the change in the constant term.



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1. "COMMUNICATIONS SYSTEMS AND TECHNIQES", SCHWARTZ ET AL, McGRAW-HILL
2. "HIGH SPEED DATA TRANSMISSION", J.W. CONARD, INSTRUMENTS & CONTROL SYSTEMS, APRIL 1967
3. "PSK ERROR PERFORMANCE WITH GAUSSIAN NOISE & INTERFERENCE", A.S. ROSENBAUM, B.S.T.J. FEB. 1969

IB-28,368

Figure 1 - UNIVERSAL ERROR PROBABILITY (P_e) FOR "NON-FADING" PCM CHANNELS

of the vertical listing of required additional $(C/N)_{dB}$ for these other methods, it has been found that all lie within 2 dB of the Figure 1 straight line approximation for the basic PSK, 2-ary, coherent case.[1] (The lateral spreads indicated by the short horizontal lines at 10 dB steps in P_e (Figure 1) give the maximum errors found as compared to published (conventional) plots of P_e versus (C/N)[2].

Hence the first advantage of this form of presentation is that the penalties in C/N value are not primarily dependent on the required P_e value, but are given by the list of added C/N values shown in Figure 1. The high C/N penalties for increasing the number of PSK Differentially Coherent "levels" from 4 through 8 to 16 should be noted.

Another advantage of the Figure 1 presentation is the simpler demonstration of the superiority in using pulse-coded data as compared to analog data; that is, a small (constant) increase in $(C/N)_{ratio}$ gives a constant ratio decrease in P_e. This means, of course, that

1. The basic reason why the same straight line can be used merely by applying the specified constants, is due to the fact that the FSK Non-Coht. & PSK Diff. Coht. expression are inverse exponentials, while the coht. forms are erfc functions, and the latter translate into inverse exponentials for large C/N (See Appendix 1 and Schwartz et al, ibid. p. 299).
2. The spread of these errors appears to be about 0.7 in terms of C/N_{ratio}. Hence, applied as P_e errors, these would be about 3 dB, independent of P_e value, and this is tolerable for P_e values less than 10^{-4}. Applied in terms C/N they are again tolerable for $(C/N)_{ratio}$ values equal or greater than 3.

a given decrease in $(P_e)_{dB}$ needs smaller increase in $(C/N)_{dB}$, as the given value of P_e decreases.[1] (This tendency is clear in conventional plots, but owing to their high slope at low P_e values it is difficult to estimate the required $(C/N)_{dB}$ change, and it is especially difficult to compare these high slopes as between different mod/demod methods). All that has to be remembered (from Figure 1) for any "non-fading" link and or mod/demod method, is that the slope of $(P_e)_{dB}$ versus $(C/N)_{ratio}$ is -4.65.

In general, Figure 1 states that if the C/N can be kept above, say 10 (or +10 dB), then the P_e can be almost ignored, because an average error rate less than 10^{-6} is not only good, but it is also easy to handle by simple error-correction techniques.

Furthermore, it is now obvious that there is good reason to choose between the various pulse-code techniques, for a "non-fading" channel. For example, the additional 3 dB needed by Coherent FSK (binary) over Coherent PSK (binary) means sacrificing (approximately) a potential 10^{-10} error probability (at (10 + 3) dB) for a 10^{-5} error probability (at + 10 dB). This is not so for fading channels (see Section 3).

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1. The inverse argument is also too true; see Section 5.1.

SECTION III

ERROR PROBABILITIES FOR RAYLEIGH-FADING BINARY PULSE-CODE CHANNELS (Fig. 2)

3.1 <u>Without Diversity</u>

The basic equations, for 2-ary coding, taken from the Schwartz et al reference (see Figure 2), are,

For large C/N

$$P_e(2) = \tfrac{1}{2}\left[1 - (1 + 1/(C/N))^{-\frac{1}{2}}\right. \longrightarrow \frac{1}{4\ C/N}, \quad \text{for PSK-Coht.} \tag{5a}$$

$$P_e(2) = \tfrac{1}{2}(1 + C/N)^{-1} \longrightarrow \frac{1}{2\ C/N}, \quad \text{for PSK-Diff. Coht.} \tag{5b}$$

$$P_e(2) = \tfrac{1}{2}\left[1 - (1 + 2/(C/N)^{-\frac{1}{2}}\right] \longrightarrow \frac{1}{2\ C/N}, \quad \text{for FSK-Coht.} \tag{5c}$$

$$P_e(2) = (2 + C/N)^{-1} \longrightarrow \frac{1}{C/N}, \quad \text{for FSK-Non-Coht.} \tag{5d}$$

More compactly, and for <u>large</u> C/N,

$$(P_e(2))_{dB} = -\ (C/N + 6,\ 3,\ 3,\ 0)\ \text{dB} \tag{6}$$

for PSK Coherent, PSK-Differentially Coherent, FSK-Coherent, and FSK-Non-Coherent, respectively.

Equation 6 is plotted in Figure 2, using the best mod/demod. type (PSK-Coherent), with a mean curve (dotted) between 0 and +10 dB.[1]

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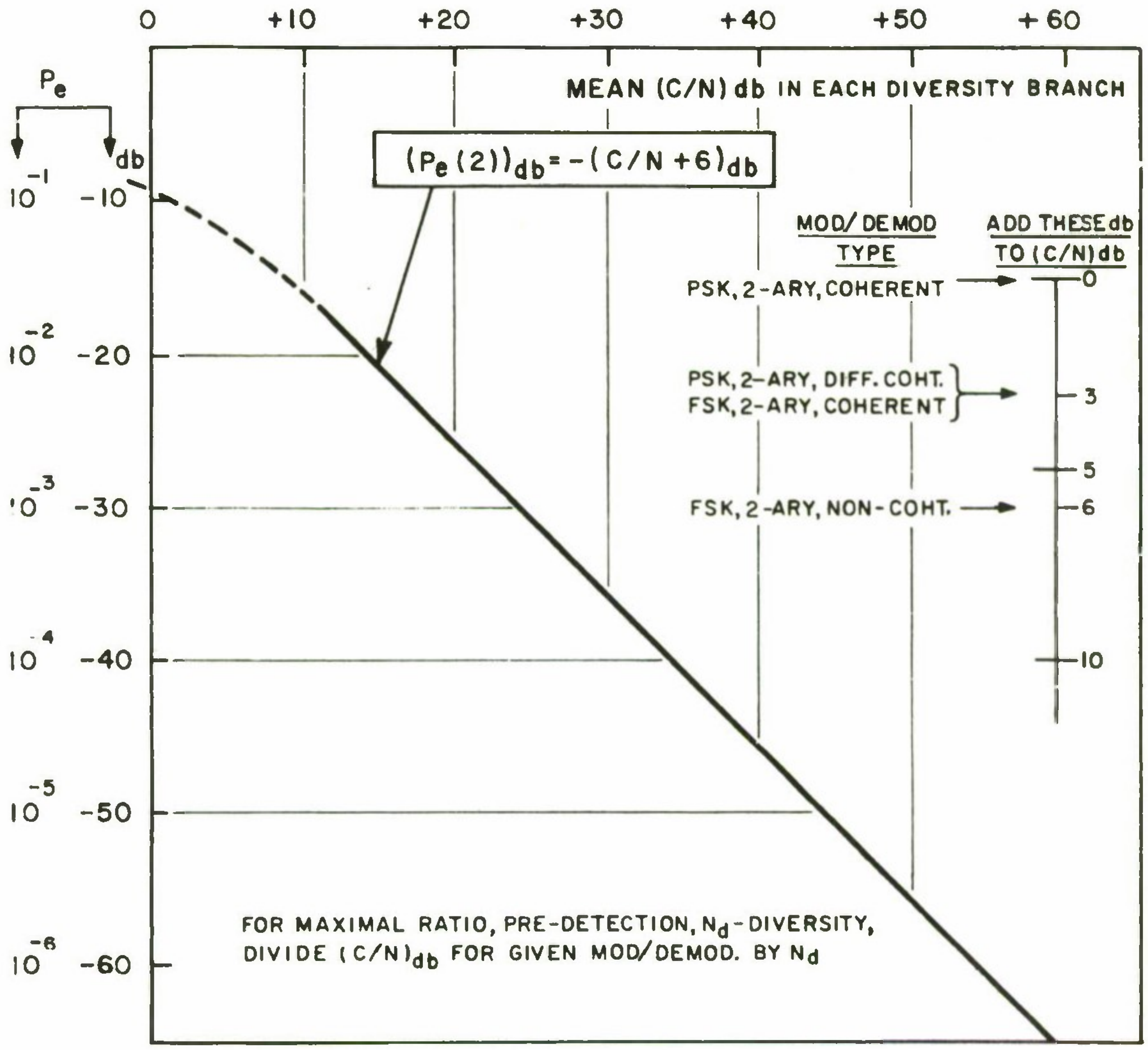
1. A few calculations show that large C/N means C/N $\geq$ + 10 dB. For C/N values +10 to 0 dB, the dotted line in Figure 2 is a mean curve for all four coding methods, with an error $\not> \pm$ 2 dB.



REFERENCE:
"COMMUNICATIONS SYSTEMS AND TECHNIQUES," SCHWARTZ ET AL, McGRAW-HILL (P.407 NO DIVERSITY ($N_d = 1$); P.461 & P.464 FOR N_d-DIVERSITY)

IB-23,369

Figure 2 - UNIVERSAL ERROR PROBABILITY (P_e) FOR RAYLEIGH FADING BINARY P.C.M. CHANNELS

At the right side of Figure 2 is another descending order list of the added dB necessary for the four mod/demod types.[1.] In this Figure, both scales are in dB, since the (large) C/N functions are all hyperbolic.

It should be noticed that the C/N values are defined as the "mean C/N in each diversity branch," using the words "mean C/N" in the Schwartz et al (ibid) sense of the "mean C/N (over fading) at the sampling instants at the filter output" (page 399), and adding the words "in each diversity branch" so that the same figure can be used for the diversity cases to be discussed in the next sub-section.

The marked difference in the P_e theoretical law for this fading case, as compared to the non-fading case, is due to the analytical process involved. In all of the non-fading cases, the P_e functions are substantially inverse exponentials of the C/N.[2.] The fading cases are calculated by integrating the product of these inverse exponential P_e functions with the particular form of fading function (in this case the Rayeligh probability distribution function), and

1. In both Figures 1 and 2, the sharp-eyed reader will note the distinction made between PSK differentially coherent and FSK non-coherent. In FSK non-coherent operation, there is literally no provision made for using the start and stop phases of the frequency shift. In PSK differentially coherent, the phase shift is the signal and it is either detected by reference to the known (a priori) start and stop phases (coherent), or is detected with reference to the (a posteriori) phases of the preceding received bit.
2. They are either explicit inverse exponential functions, or they are error function complements which translate into exponential functions for "large" C/N (see also Appendix 1).

taking the integration over limits of 0 and infinity for C/N. As soon as the latter operation is performed, all the exponential terms disappear, to be replaced by constants, and (for Rayleigh fading) inverse linear functions of C/N (e.g., see equations 5a through 5d). (The same effect is likely to occur using other fading distribution functions; hence the Rayleigh model is a useful guideline for most fading-prone channels).[1.]

This simple fact is the cause of the virtually catastrophic increase in bit errors per unit time; as experienced, for example, by troposcatter channels at C/N levels of say, + 10 dB. This C/N value would produce tolerable bit error rates less than 10^{-5} in a "non-fading" environment. This fact is also a major part of the reason for the large antennas, high powers and relatively small bandwidth (bit rate) of troposcatter systems.[2.]

However, some of this deficiency can be made up by use of diversity transmission and reception.[3.]

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1. The Schwartz et al book was published in 1965. Other papers since then have also assumed other forms of fading; for example, M. Nesenbergs, "Error Probability for Multipath Fading-The Slow and Flat Idealization," IEEE. Trans. Comm. Techno., Dec., 1967. See also Section 5.2 footnote.
2. Another major part of the reason is, of course, the low value of the mean C/N due to the non-line-of-sight propagation path.
3. Diversity means the use of more than one transmission and/or reception mode, where each mode is designed to produce received signals whose fading is not time-correlated with the other modes.

3.2 With Diversity

In general, the diversity case can be boiled down to P_e being proportional to $1/(C/N)^{N_d}$, where N_d is the number of diversity branches available at the receiver for maximal ratio[1] pre-detection combining.

In terms of Figure 2, this can be interpreted as shown in the instruction. This is not mathematically accurate, but it works to a maximum error in P_e of about $\pm$ 2 dB over the range of P_e values (10^0 to 10^{-4}), N_d values (1 to 10), and mod/demod types shown in the reference's Figure 10-8-1 (page 461).[2] This order of accuracy in P_e is adequate for most basic design or analysis purposes.

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1. Maximal-ratio predetection combining infers that the "the instantaneous C/N is the sum of all the instantaneous C/N's in the individual branches" (see Schwartz et al, page 442), and application of this results in the $1/(C/N)^{N_d}$ component of P_e (ibid. page 460). As is pointed out by the reference (pages 452, 453), relatively complex circuitry is needed for maximal-ratio combining, but in practice the relatively simple "equal-gain" (phase-locked) pre-detection summing method gives about equal performance.
2. In using this referenced figure, it should be noted that the Γ scale values (i.e., C/N values) should be reduced by 3 dB for PSK-coherent as regards the solid line curves, and also by 3 dB for PSK-differential coherent as regards the dotted line curves. That is, the sequence for the added "constant" is still 0, 3, 3, 6 dB for PSK-coherent, PSK-differentially coherent, FSK-coherent, FSK-non-coherent, as it is in the "non-fading" case of Figure 1.

SECTION IV

COMPARISON OF "NON-FADING" AND RAYLEIGH-FADING P_e VERSUS C/N

The great difference in P_e values at a given C/N for these two cases is due to the "non-fading" case being an inverse exponential function of C/N (for C/N greater than 3), and the Rayleigh-fading case being an inverse linear function of C/N (for C/N greater than 10). This basic difference is the reason for plotting the C/N values linearly in Figure 1, and logarithmically in Figure 2.

It is interesting to note, by comparing Figures 1 and 2, that the P_e values are nearly the same for $(C/N)_{dB}$ values around 0 dB. However for almost any positive $(C/N)_{dB}$ value, the "non-fading" P_e reduction rate rapidly outstrips that for the Rayleigh-fading case.

Adding diversity to the Rayleigh-fading case helps, of course, although the resulting basic "law" still has a linear relationship for $(P_e)_{dB}$ versus $(C/N)_{dB}$. For example and for the PSK-coherent case, suppose the "non-fading" P_e of 10^{-5} at + 9.5 dB C/N is to be obtained for the Rayleigh-fading case by using diversity. It would take five diversity branches to do this.[1] Hence diversity can help, provided P_e values more than, say 10^{-5}, are satisfactory, and if the cost and complexity of multiple-diversity installations is acceptable.

1. From Figure 2, $P_e = 10^{-5}$ is obtained with a no-diversity C/N of +44 dB, which, divided by 9.5 gives a 4 + diversity requirement.

No known installation uses more than 4 diversities.

The above Rayleigh-fading P_e versus C/N handicap is typical of all channels required to function in a fading environment, whether the fading be characteristically continuous (as in troposcatter links) or whether it be due to non-continuous environmental phenomena.

It is therefore of prime importance that any radio link be analyzed for fading conditions, and that a adequate system C/N margin be allowed on each such link, if literally continuous availability is desired. Some typical causes of fading are discussed in Appendix II.

SECTION V

EFFECTS OF NON-RANDOM, NON-CONTINUOUS FADING ON DESIGN C/N MARGINS

5.1 Nominally "Non-Fading" Channels

Figure 3 shows the same vertical P_e scale as in Figure 1, but with a horizontal scale ranging from 3 dB boost (the opposite of a fade), to 12 dB fade, plotted on a linear dB scale because the discussion will be expressed in dB changes of C/N.

Each curve is obtained by assuming a given P_e value at "0 dB", $(P_e)_{0dB}$, where "0 dB" is the normal "non-fading" C/N value that gives this particular P_e. For each such $(P_e)_{0dB}$ the corresponding $(C/N)_{dB}$ value is found on Figure 1, and from this value is subtracted 1, 2, 3, etc. dB, in turn. For each of these reduced $(C/N)_{dB}$ values the P_e is read off Figure 1 and plotted in Figure 3 as points on a curve for the particular $(P_e)_{0dB}$. Each curve is extended into the boost region 1 dB, so that the 0 dB slope of the curves may be better estimated (as shown on each curve in the 0 dB region).

In addition, Figure 3 also has shaded regions for certain P_e bounds, with a notation characterizing P_e quality in these regions.[1]

1. By way of explanation, a 10^{-1} bit error rate means a 1 in 2 to 1 in 1 word error rate, for words of 5 bits or more. This is virtually an "impossible" situation, since most or all of the data samples (words) are then wrong. A 10^{-1} to 10^{-2} bit error rate is "bad," because of the possibility of error grouping wiping out a significant sequence of data samples; and so on.

Figure 3 therefore shows pictorially the change in P_e for any (slow) change in $(C/N)_{dB}$, whether such change be due to change in C or in N, or in both.[1]

Figure 3 can be used regardless of the mod/demod type, because Figure 1 demonstrates that mod/demod types can be coped with by added $(C/N)_{dB}$ constants, and the process for obtaining Figure 3 is differential in nature. For example, a $(P_e)_{0dB}$ of 10^{-8} is obtained at a C/N of +12 dB for PSK coherent, and at +15 dB for FSK coherent. A reduction of 1 dB in C/N will produce a P_e of 3×10^{-7} for both mod/demod types.

This feature, which is the essence of this paper, permits the following generalized statements to be made, using Figure 3.

1. The lower the channel design value for P_e, the more vulnerable the channel is to small changes in $(C/N)_{dB}$. For example, in a precision data link designed for $(P_e)_{0dB} = 10^{-8}$, a $\pm$ 1 dB change in C/N will give about $\pm$ 2 decades of change in P_e; that is from about 10^{-10} to 10^{-6}. Since $\pm$ 1 dB changes of C/N are common and would hardly be called fading (or boosting), a careful designer should therefore base a precision data system design on a likely minimum P_e value of 10^{-6}, or even 10^{-5}, for each link.

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1. The most common cause of reduced C/N is a reduction in C. Automatic Gain Control can partially restore the output C level (which is why a slow change is stipulated) but it cannot alter the lower C/N value.

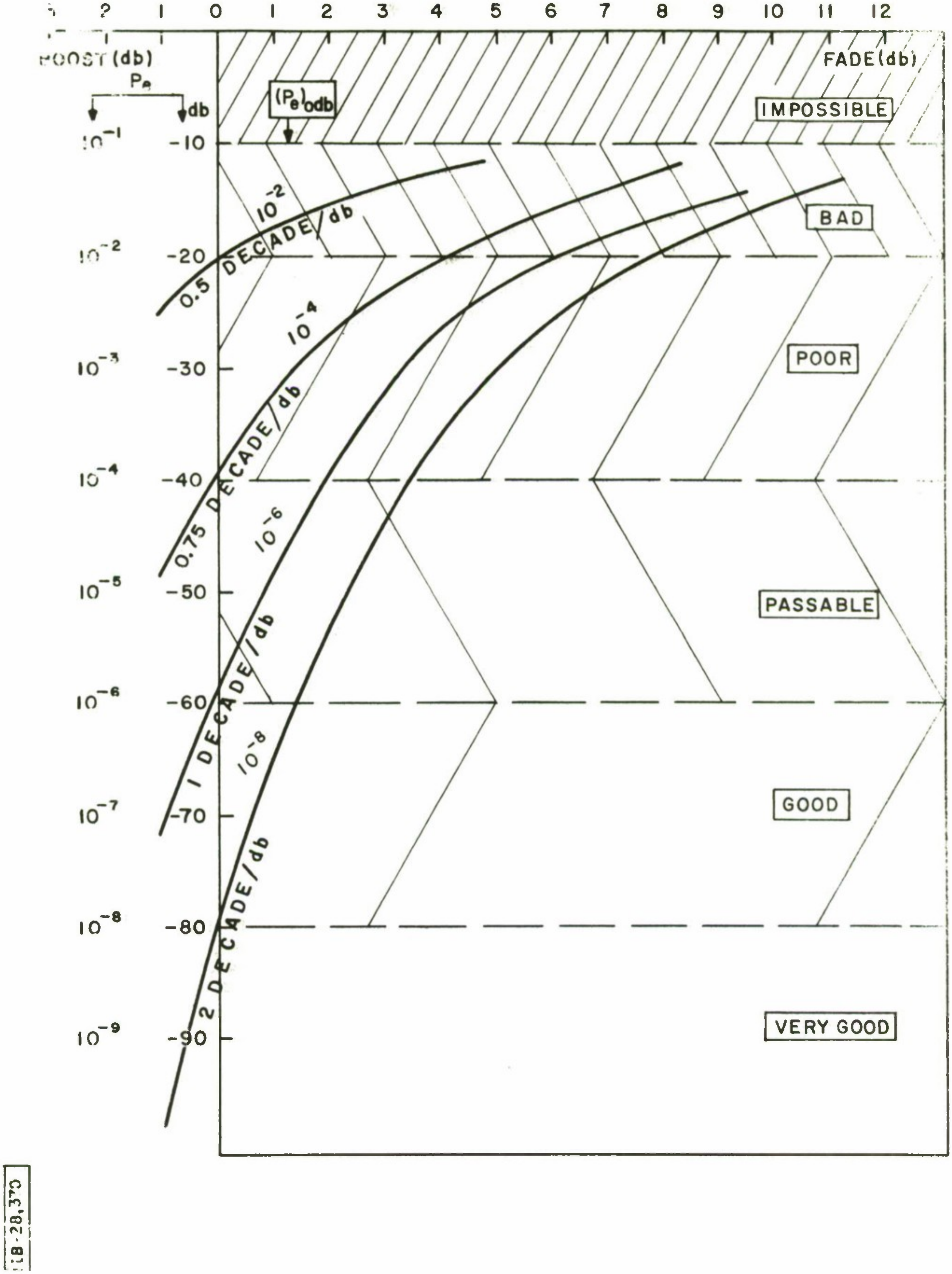

Figure 3 - UNIVERSAL (NON-FADING) ERROR PROBABILITY (P_e) vs SLOW FADE/BOOST LEVELS

In other words, the transition from a "very good" to "good" P_e, to a "passable" P_e, takes place with about 2 dB reduction in C/N, for precision data channels designed on a "non-fading" basis. This is why any channel design for carrying vital (non-redundant) data should include an adequate C/N or P_e "fade-margin." Examples of such data are telemetry data, vocoded-voice (because of removal of normal redundancy), and numerical data (especially that carrying addresses).

2. Even for a "moderate" P_e "non-fading" design, say $P_e = 10^{-4}$, a $\pm$ 1 dB change in C/N gives $\pm$ 1 decade change in P_e. A -2 dB change takes the channel from the "passable" region to the "poor" region, and any further small reduction in $(C/N)_{dB}$ takes it to the "bad" region.

   In other words, such a "moderate" design must include error detection, and should include error correction. Furthermore it is now necessary to find out the characteristic sequence of bit errors; e.g., are they still to be considered as occurring randomly with time, or should they be classed as "burst" error sequences? Both the amount of data degradation and the appropriate error correction means depend entirely on the type of error sequence.

3. About the only "good" feature displayed by Figure 3, is that the <u>rate of increase in P_e</u> reduces quickly once a $(C/N)_{dB}$ reduction of about 4 dB (from the nominal "non-fading" value) is exceeded. For example, for a $(P_e)_{0dB}$ of 10^{-8}, the P_e remains in the "poor" region (10^{-4} to 10^{-2}) from -3.5 dB to -8 dB change in C/N.

   In other words, once the channel P_e is more than 10^{-4}, whether by design or by natural causes, it takes about 4 dB more reduction in C/N to get into the "bad" region.

For a summary of some common <u>causes</u> of fades on a nominally "non-fading" (line-of-sight) link, see Appendix II. Such causes regularly give fades of $\pm$ 2 dB in C or N, while some can give +3 to -10 dB (or more) $(C/N)_{dB}$ changes. As shown above, such fades are very detrimental to the error rate of any "non-fading" (e.g., line-of-sight) data link. It is suggested that unless error detection and correction methods are used, the "fade margin" should be not less than +10 dB.[1] Such fade margins, and more, are provided on ground microwave (radio relay) links. Appendix II suggests that space/air to ground links need similar fade margins, and Figures 1 and 3 show these required margins are independent of mod/demod type.

1. That is, the link power/noise budget should show a surplus of 10 dB in C/N after all other factors have been specified or calculated.

5.2 Fading Channels

The chief example of this type of channel is troposcatter propagation, although HF radio is almost in the same category.[1.] The former is quite accuractly modeled by the assumption of a Rayleigh amplitude distribution (with time) for C, and its P_e is given by Figure 2.

As mentioned before, the C/N value used here is the mean value in one or more branches of the usual diversity-type terminals. Consequently, there is no allowance made for "slow" fading in the sense used in Section 5.1; in fact, the mean C/N value of troposcatter is fairly stable and seems to be affected only on a seasonal basis.

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1. HF radio C/N fading, although persistent to the point of being systematic, has not been properly modeled as yet, probably because the noise component is mainly due to "sferics" (or impulses) from thunderstorms occurring in the equatorial belt that easily propagate globally. This would tend to give errors in bursts, and considerable work has been done on this aspect, in terms of error detection/correction; e.g., K. Brayer, O. Cardinale, "HF Channel Data Error Statistics Description," MTR-171-4/5/66; K. Brayer, O. Cardinale, "The Application of HF Channel Data To The Development of Block Error Correctors," MTP-32, 6/13/66; K. Brayer, W. F. Longchamp, "Error Control For Digital Communications In The Tactical Environment" MTP-87, Aug. 1968; all from The MITRE Corp.

   However, the C component fading could probably be modeled as a Rayleigh distribution (e.g., "Communications Systems Handbook," D. H. Hamsher, McGraw-Hill, page 15-23, which quotes from "ionospheric Radio Propagation," of the National Bureau of Standards Circular 462, 1948); also Schwartz et al, ibid. p. 381.

Even so, it is clear from Figure 2 that the slope of $(P_e)_{dB}$ vs. mean $(C/N)_{dB}$ is unity, regardless of mod/demod type, or of C/N value.

Hence, the <u>change</u> in $(P_e)_{dB}$ for slow changes in mean C/N is independent of the nominal C/N (above +10 dB), and is much less than for the "non-fading" channel.

It is quite hard to design a troposcatter link with a mean $(C/N)_{dB}$ of +35 dB (or more) that is necessary to give a "moderate" P_e value of 10^{-4}, unless up to 4 diversity branches are used (when the economics of the proposition begin to be painful).

However it is interesting to note that, to achieve the same "nominal 0 dB" sensitivities displayed in Figure 3, the following would have to be done in the case of the Rayleigh fading channel,

$(P_e)_{0dB}$	10^{-2}	10^{-4}	10^{-6}
Fading Sensitivity from Figure 3	1 decade/2dB	1 decade/1.3 dB	1 decade/1 dB
Equivalent Number Diversities needed on a Rayleigh fading link	5	7.5	10

$(P_e)_{0dB}$ Sensitivity = rate of change of $(C/N)_{dB}$ vs. $(P_e)_{0dB}$

= 1 decade per 10 dB for no diversity Rayleigh-fading, or 1 decade per 5 dB for 2-diversity Rayleigh-fading, and so on.

Even if such equivalent sensitivity were to be attained (by diversity methods), the Rayleigh-fading channel would still be inferior, because it <u>would more rapidly degenerate</u> to the "poor," "bad," and "impossible", P_e regions (because its sensitivity is constant, see above).

Fortunately this type of problem (a non-line-of-sight radio link) and its design/cost problems is becoming academic with the increasing availability of true long-distance line-of-sight links via satellites. However these still need a fade margin (see Section 5.1).

APPENDIX I

SIMPLIFIED EXPRESSIONS FOR ERROR FUNCTION AND ITS COMPLEMENT

1. GENERAL

The standard form for the error function of x, erf (x), is given by

$$\text{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}\, dy \qquad \text{(A.I.1)}$$

and its complement, erfc (x), by

$$\text{erfc}\,(x) = 1 - \text{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-y^2}\, dy \qquad \text{(A.I.2)}$$

The integration in equation A.I.1 is not straight-forward; therefore the form of erf (x) and its complement is not obvious and neither can it be manipulated easily.

However, according to the latest (fifth) edition of Reference Data For Radio Engineers (page 44-37, under the subtitle "Error Function"), some industrious mathematician has come up with the following inequality,

$$\frac{\sqrt{\pi}/2 . e^{-x^2}}{x + \sqrt{x^2+2}} \leqslant \text{erfc}\,(x) \leqslant \frac{\sqrt{\pi}/2 . e^{-x^2}}{x + \sqrt{x^2 + 4/\pi}}, \quad x \geqslant 0. \qquad \text{(A.I.3)}$$

and clearly for x large

$$\text{erfc}\,(x) \rightarrow \frac{\sqrt{\pi}\, e^{-x^2}}{4x} \qquad \text{(A.I.4)}$$

2. TWO CLOSELY APPROXIMATE EXPRESSIONS

2.1 Use of One Bound

In inequality A.I.3, the numerators are equal and the inequality is due to the denominators. If the ratio of the denominators is taken, and evaluated for various values of x, the following results,

x	0.5	1.0	2.0	3.0
Ratio of Denoms.	1.15	1.09	1.03	1.02

Therefore either denominator can be taken, to any required accuracy. For example,

$$\mathrm{erfc}\,(x) = \frac{\sqrt{\pi}/2.e^{-x^2}}{x + \sqrt{x^2 + 2}} \qquad \text{(A.I.5)}$$

to less than 15%, 9%, 3%, 2%, for x values $\geqslant$ 0.5, 1, 2, 3, respectively.

2.2 Use of Equation A.I.4

Taking ratios of the two denominator bounds to their value (2x) when x is large, the following results

x	0.5	1.0	2.0	3.0
L. H. Denom/$_{(2x)}$	2	1.37	1.11	1.05
R. H. Denom/$_{(2x)}$	1.74	1.26	1.08	1.04

Therefore the equation,

$$\mathrm{erfc}\ (x) = \frac{\sqrt{\pi}\cdot e^{-x^2}}{4x} \tag{A.I.4}$$

can be used to -11%, +8% for $x \geq 2$; to -5%, +4% for $x \geq 3$, etc.

3. THE ERROR FUNCTION EXPRESSION

From equation A.I.2,

$$\mathrm{erf}\ (x) = 1-\mathrm{erfc}\ (x) \tag{A.I.2}$$

By inspection, any one of the approximate erfc (x) expressions is less than 0.164 for $x \geq 1$. Hence using any one of them in equation A.I.2 will give results of even greater accuracy than those for erfc (x), provided $x \geq 1$.

APPENDIX II

SOME CAUSES OF SLOW C/N FADES UP TO 10 dB

The following illustrations refer generally to the use of radio carrier frequencies above 100 MHz, because of the increasing importance of this region for communications and/or data transfer. (Causes of C/N fading for radio carrier frequencies below 100 MHz are better known, and usually there is more provision made for their mitigation; or perhaps there is more tolerance of C/N fading because of the redundant nature of the information, e.g., commercial broad-case radio, "commercial-quality" radio-telephony and telegraphy).

a. Aircraft Transmitter Antenna Patterns.[1] At carrier frequencies up to 500 MHz, such antennas are usually non-directional in principle. In practice the design pattern is considerably distorted, in azimuth and elevation, by multiple nulls averaging about -10 dB in depth. As the aircraft banks, several nulls will be traversed.

b. Ground Transmitter Antenna Patterns. Objects in the vicinity of the ground antenna can produce similar

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1. For examples of patterns, see "Antenna Engineering Handbook," H. Jasik, McGraw-Hill, Pages 27-31 through 27-46; "Radio Antennas for Aircraft and Aerospace Vehicles," AGARD Conf. Proc. #15, Circa Publications Inc., New York.

nulls (usually in azimuth),[1.] which are, of course, effective if transmissions over 360° azimuth is required (either beamed or non-beamed), for example to satellites and aircraft.

c. Multipath Nulls/"Blackout" At Relatively Low Ground/ Sea Antennas

The words "relatively low ground/sea antennas" mean relative to the other antenna in a radio link; such as, when the other antenna is on an aircraft, or a satellite, or is a 1,000 foot broadcast/TV station antenna as compared to the average home antenna height of 30 feet or a car antenna height of 6 feet.

The propagation versus distance performance for these very common antenna conditions warrants a separate paper; in fact, many papers have been written dealing with specific link tests[2.] (because of the generally much poorer performance than is

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1. For example, see P. Night, "Radiation From Masts and Similar Obstacles At Radio Frequencies," Proc. IEE, (London), Jan. 1967; D. E. Troy, K. W. Kirk, "Radiation Pattern Effects of Antenna Site Obstructions for An Air-to-Ground UHF Propagation Path" IEEE Intl. Conf. Comm. Paper 19CP67-1110, 1967, which shows multiple 15 to 30 dB nulls due to an "obstacle" 140 feet away from the ground antenna.
2. For example, A. N. Ince, H. P. Williams, "Design Studies for Reliable Long-Range Ground To Air Communications," IEEE Trans. Comm. Techn. October 1967; B. J. Starkey, L. G. Rollinson, G. A. Fatum, "Cold Lake Radio Propagation and Meteorological Experiments, etc." MTR-118, The MITRE Corp. June 1967.

even indicated by theory), but there appears to be no general write-up available.

In summary, the signal strength versus increasing distance (from the lower antenna) shows a number of nulls, of decreasing "frequency" and increasing depth (from about 3 dB to about 20 dB), up to the last null occurring at the "radio horizon." For the lower UHF band (around 400 MHz) the peak preceding the radio horizon null extends (roughly, dependent on the actual antenna heights) over the last half to one-third of the radio horizon distance, and it is in this region that the classical 12 dB/octave of distance, multipath/free-space propagation is effective;[1] the loss per distance ratio is usually higher in practice. For the upper UHF band (above 1600 MHz), the number of nulls is proportionately more in the same radio horizon distance, so that many nulls of 10 to 20 dB are now encountered over the last half to one-third of the radio horizon distance, and there is practically no classical 12 dB/octave of distance region before the radio

1. This is the reason why aircraft UHF works at all, since the preceding nulls (closer to the ground station) are shallow (about 6 dB) and are offset by the normal increase in "free space" signal level.

horizon[1]. All of these statements apply over the radio horizon distance, and this distance can be shortened drastically by very small positive foreground grades; e.g., the radio horizon is reduced to two-thirds by a $+1^{o}$ foreground slope, to one-half by a $+2^{o}$ slope, etc. In this context, the foreground ranges from 0.1 to 1 mile radially from the ground station.

d. Mid-Path Obstacles and Reflectors Within The Receiving Antenna Beamwidth.

These effects are similar to those discussed in c, above, but now apply to both ends of the link, and are, of course, most noticeable for ground radio-relay links or for air or space to ground/sea links at low elevation angles. A common illustration of the effect is the picture-wobble on TV sets when an airplane flies over the mid-path.

e. Man-Made Noise

This subject would also justify a separate paper. The latest, readily available, information is summarized in "Reference Data For Radio Engineers," 5th Edition, October 1968, Howard W. Sams Co. (I.T.T.) and comes from

a. Composite curves from measurements by Work

1. This is one reason why aircraft radio link operation becomes much less reliable over the same distance range, when using S-band (about 1.6 to 4.5 gHz).

Group 3, F.C.C. Advisory Committee, Land Mobile Radio Service; the Institute for Telecommunications Sciences and Aeronomy, E. S. S. A.; New York City Noise Levels Reports in P.S.T.J., Nov. 1952, and

b. The CCIR Report 322, 10th Plenary Assembly, Geneva 1963.

In line with modern practice, all the noise levels have been expressed in terms of dB above thermal receiver noise level, kT_oB, (where k is Boltzman's constant, T_o is the "standard" temperature of 290° Kelvin, and B is the receiver bandwidth at the point of measurement).[1.]

The antenna used for these noise level measurements is either the isotropic, with an area $\lambda^2/4\pi$ where λ is the wavelength, or a (grounded) half-wave dipole (i.e., a monopole) with an area of 1.64 $\lambda^2/4\pi$.

Many of these curves therefore show, and in any event include, the $1/f^2$ function of the antenna area. Hence any dB gain (above isotropic) of the actual receiving antenna <u>must be added on to these levels</u>.

1. The product KT_o is 4×10^{-21} watts/herz.

It is certain that these noise levels have not decreased in the intervening six years. Roughly, for man-made noise the urban and suburban[1.] noise levels vary from +40/+25, to +18/0 dB above KT_oB respectively, for radio frequencies from 100 to 1,000 MHz. Above 1,000 MHz there are no external noise levels from any source (as received by an isotropic antenna) that are significant compared to KT_oB; but, of course, KT_oB is now multiplied by the significant noise factor of the receiver front end itself. C/N values calculated on KT_oB will, of course, be reduced proportionately by man-made noise levels.

The above partial listing of non-random, non-continuous fading causes should be sufficient to justify the use of quotes about the word "non-fading," and to warrant skepticism on all conventional C/N calculations using line-of-sight attenuation for C and front-end thermal resistor noise for N, for all earth-bound or low elevation-angle links to and from the earth.

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1. "Suburban" areas stretch for hundreds of miles; e.g., the whole U.S. Atlantic and Pacific coastlines to an interior depth of at least 200 miles is "surburban." Most of non-asiatic, non-African Europe can also be classed as suburban.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* The MITRE Corporation Bedford, Massachusetts	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

SIMPLIFIED ERROR PROBABILITY CURVES FOR "NON-FADING" AND RAYLEIGH-FADING DIGITAL LINKS, AND THEIR APPLICATION TO DESIGN MARGINS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

N/A

5. AUTHOR(S) *(First name, middle initial, last name)*

Bertram M. Hadfield

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1969	38	0

8a. CONTRACT OR GRANT NO. AF 19(628)-68-C-0365	9a. ORIGINATOR'S REPORT NUMBER(S) ESD-TR-69-373
b. PROJECT NO. 600D	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* MTR-971
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

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ystems

11. SUPPLEMENTARY NOTES N/A	12. SPONSORING MILITARY ACTIVITY Directorate of Planning & Technology, Electronic Systems Division, Air Force Systems Command, L. G. Hanscom Field, Bedford, Massachusetts

13. ABSTRACT

Straight line relationships (plus added constants for various pulse code modulations) for error probability vs. carrier to noise ratio are shown to be possible for both "non-fading" and Rayleigh-fading radio links. These are used to show the effects of fades, relative to the nominal "non-fading" design, independently of the specific pulse code modulation technique. Common causes and levels of such fades are listed; particularly those experienced on spacecraft/aircraft to ground/sea links. The relative performance of "non-fading" and Rayleigh-fading links (with any number of diversities) is discussed. Some general conclusions and guidance on "fade margins" are given.

DD FORM 1 NOV 65 1473

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Simplified Error Probability Non-Fading Digital Links Rayleigh-Fading Digital Links Digital Link Power Design Margin						

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