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BRL D/A ltr, 22 Apr 1981
MEMORANDUM REPORT NO. 1985

DYNAMICS OF LIQUID FILLED SHELL:
LIQUID-CENTRAL BURSTER INTERFERENCE

by

W. P. D'Amico

June 1969

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DYNAMICS OF LIQUID FILLED SHELL:
LIQUID-CENTRAL BURSTER INTERFERENCE

W. P. D'Amico

Exterior Ballistics Laboratory

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RDT&E Project No. 17061102A33D

ABERDEEN PROVING GROUND, MARYLAND
ABSTRACT

Experiments performed in a gyroscope containing liquid-filled, cylindrical cavities were conducted to study the effects of central rod-liquid interference. The results of these tests give evidence that the case of the partially wetted rod can be encompassed by making use of previously reported solutions which consider either an air core or a fully wetted rigid rod.
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I. INTRODUCTION

It is very common for liquid-filled shell to employ central bursters. Military fill ratios are of a magnitude that partial contact between the burster and the liquid is probable. In stability analyses of liquid-filled shell, calculation of the liquid natural frequencies, eigenfrequencies, is essential. At present, however, no method is available for the computation of eigenfrequencies if liquid-central burster interference is assumed. The condition of rod-liquid interference is not hopeless, however. As noted by Frasier and Scott, it might be possible to bound the condition of a partial wetted rod by using the available solutions for an air* and a rigid3 core. Figure 1 represents a cylindrical cavity that is spinning about its longitudinal axis with a constant angular velocity. The cavity has a solid rod mounted concentrically to the spin axis, and it is partially filled with a liquid that is in rigid body rotation. Simple geometric relations can be set-up using Figure 1.

\[
\begin{align*}
\text{Cavity volume} & = V_c = 2c \pi a^2 \\
\text{Air volume} & = V_a = 2c \pi b^2 \\
\text{Rod volume} & = V_r = 2c \pi d^2 \\
\text{Rod fill ratio} & = \frac{V_r}{V_c} = 1-\frac{d^2}{a^2} \\
\text{Liquid fill ratio} & = \frac{V_L}{V_c} = 1-\frac{b^2}{a^2}
\end{align*}
\]

It can be shown that eigenfrequencies, assuming either rigid or air cores, are functions of the slenderness ratio, \(c/a\), and the appropriate fill ratio.1,2 The functional dependence of these non-dimensional frequencies upon the cavity geometry is:

\[
\begin{align*}
\text{Central rod: } \tau_{n1} & = F_s \left[ \frac{d^2}{a^2}, \frac{c/a}{2j+1} \right] \\
\text{Air core: } \tau_{n1} & = F_s \left[ \frac{b^2}{a^2}, \frac{c/a}{2j+1} \right]
\end{align*}
\]

*References are listed on page 19.
where:  
\( n = \text{radial mode number} \)

\( j = \text{longitudinal mode number} \)

From such functional relations, for fixed values of \( n \) and \( \frac{c/a}{2j+1} \), \( b \) and \( d \) can be varied to obtain a plot similar to Figure 2a. Instabilities occur when a shell has a liquid eigenfrequency, \( \tau_n \), hazardously close to its non-dimensional nutational frequency, \( \tau_n \). Figure 2a can be used to select a fill ratio and a burster size such that \( \tau_n \neq \tau_n \). Such a selection results in Figure 2b. The hatched region of Figure 2b, which is marked as the rod volume, has associated with it the particular \( d^2/a^2 \) of the central burster which was chosen on the basis of Figure 2a. The triangular point on the liquid-rod interface is the only possible eigenfrequency for the central rod case. Figure 2b, however, exhibits a discontinuous jump in eigenfrequencies between \( \tau_1 \) and \( \tau_2 \). In this region \( d^2/a^2 \) is approximately equal to the burster volume. This is the region where rod-liquid interference should occur.

Karpov performed experiments where partial liquid contact with a central rod resulted in slower undamping rates than predicted by the air core solution. While looking for viscous corrections to the central rod case, Frasier noted experimental peculiarities that suggested that an unstable condition can be initiated by a partially wetted rod. Mathematically it appears to be an impossible task to handle a rod that has intermittent liquid contact. Can a partially wetted rod condition produce instabilities similar to the air or rigid core cases? Can available theory and design techniques encompass partially wetted rod effects? From an empirical viewpoint it was possible to answer these questions by a series of experiments using a liquid-filled gyroscope. It was expected that for values of \( b^2/a^3 \approx d^2/a^2 \) transition eigenfrequencies will occur between \( \tau_1 \) and \( \tau_2 \), i.e., another Stewartson type instability should occur for fill ratios that induce partial rod-liquid contact.

\(^\text{#Unpublished data.}\)
II. EXPERIMENTAL TECHNIQUES

All data were obtained using cylindrical cavities and a 1 cs oil. A cavity with a central rod was designed such that the gyroscope would be unstable for a range of eigenfrequencies from $\tau_1$ to $\tau_2$, as shown in Figure 2b. First the nutational frequency of the gyroscope, $\tau_n$, was set at $\tau_1$ while the liquid fill ratio was increased. (This is a horizontal movement from left to right on Figure 2b.) The gyroscope became unstable when $\tau_n = \tau_{nj}$, as indicated by Figure 2b. The fill ratio was increased beyond the predicted value of $b^2/a^2$ for the air core instability, and the gyroscope undamped for values of $b^2/a^2 \approx d^2/a^2$. The liquid fill ratio was increased until the available volume was completely filled. $\tau_n$ was then lowered, and the procedure was repeated until $\tau_n < \tau_2$. Results from these experiments are shown in Figure 3.

A second series of experiments was conducted for $\tau_n = 0.055$ while the rod size and liquid fill ratio were varied. Normally central burster diameters are small, but a new stabilization technique uses a cylindrical partition that effectively creates $d^2/a^2$ larger than 0.50. Tests were made to investigate the dependence of partially wetted rod effects upon rod size. Results from these tests are shown in Figures 4a, b, c, d, e, and f.

III. DISCUSSION OF RESULTS

The few series of experiments that were performed have given some insight to the case of rod-liquid interference. Similar to the broken curve in Figure 3, a smooth transition of eigenfrequencies occurs from the air core to the rigid rod case. A discontinuous jump does not occur. Results in Figure 3 were also recorded as a function of the undamping rate. Undamping rates associated with partially wetted rod instabilities are approximately 20% slower than for an air core or central rod instabilities. Since the undamping rates are much slower for the partially wetted rod cases, the associated liquid moments must be significantly reduced.
Consider the case of varying the burster diameter. Results plotted in Figures 4a, b, c, d, e, and f show that the size of a central rod does not produce drastic changes in partially wetted rod effects. For the cavity and $\tau_n$ used in these tests, larger $d^2/a^2$ produced slower undamping rates. Responses similar to Figures 4a, b, and c were obtained with a 1.75 inch diameter rod.

From the experimental results partially wetted rod effects are quite restricted. Why is this so? A quick glance at air core and central rod tables reveals that the longitudinal mode lengths, $c/a_{2j+1}$, are approximately equal for small values of $b^2/a^2$ and $d^2/a^2$. Assume that rod-liquid interference occurs. The orderly boundary conditions needed for either the air core or central rod solutions do not exist. For small rod sizes with rod-liquid interference, the $c/a_{2j+1}$ values for either of the cases are similar, and an eigenfrequency characteristic of the amount of rod-liquid interference and $\tau_n$ occurs. For large rod sizes, the $c/a_{2j+1}$ values diverge, and a disturbance in the boundary conditions will, as shown in Figures 4a, b, c, d, e, and f, produce damping.

IV. CONCLUSION

For military fill ratios partially wetted rod effects can be treated through the use of the air core and central rod cases. Normal fill ratios of 95% of the total available volume would be located at $1-b^2/a^3 = 0.925$ on Figure 3. The original suggestion by Frasier and Scott of bounding the case for rod-liquid interference should be adequate. It was shown that a smooth transition occurs between the air and rodded core solutions and that the region for partially wetted rod effects is quite narrow. Many more experiments could be performed to amass data for empirical relations, but it is felt that this report will give a shell designer insight to the problem such that his designs can avoid instabilities initiated by liquid-central burster interference.
ACKNOWLEDGEMENTS

The author would like to thank Dr. J. T. Frasier who suggested that liquid-rod interference effects needed investigation and Mr. B. McKay who carefully performed the laboratory tests and reduced the raw data.
Figure 1. Partially filled cylindrical cavity with a central burster.
**Figure 2a.** Functional relationships for liquid eigenfrequencies and fill ratios for a cylindrical cavity.

**Figure 2b.** Possible liquid eigenfrequencies after selection of a burster size.
Figure 3. Transition from the air core to the central rod solution.
TEST CONDITIONS:
\[ T_n = 0.055 \]
\[ 2c = 7.63 \text{ in.} \]
\[ 2a = 2.50 \text{ in.} \]
\[ Re = 5.2 \times 10^5 \]

\( \bigcirc \) - AIR CORE (NO ROD)
\( \triangle \) - PARTIALLY WETTED ROD

Figure 4a. Partially wetted rod effects for a 1.11 inch diameter rod.

Figure 4b. Partially wetted rod effects for a 0.99 inch diameter rod.

Figure 4c. Partially wetted rod effects for a 0.78 inch diameter rod.

FILL RATIO: \( 1 - \frac{b^2}{a^2} \) or \( 1 - \frac{d^2}{a^2} \)
TEST CONDITIONS:
\[ T_n = 0.055 \]
\[ 2c = 7.63 \text{ in.} \]
\[ 2a = 2.50 \text{ in.} \]
5000 RPM
1 cs oil

- \( \circ \) AIR CORE (NO ROD)
- \( \triangle \) PARTIALLY WETTED ROD

Rod-liquid interface

Figure 4d. Partially wetted rod effects for a 0.614 inch diameter rod.

Figure 4e. Partially wetted rod effects for a 0.375 inch diameter rod.

Figure 4f. Partially wetted rod effects for a 0.250 inch diameter rod.

FILL RATIO: \( 1 - b^2/a^2 \) or \( 1 - d^2/a^2 \)
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**ABSTRACT**

Experiments performed in a gyroscope containing liquid-filled cylindrical cavities were conducted to study the effects of central rod-liquid interference. The results of these tests give evidence that the case of the partially wetted rod can be encompassed by making use of previously reported solutions which consider either an air core or a fully wetted rigid rod.
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