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**MONSANTO/WASHINGTON UNIVERSITY
ONR/ARPA ASSOCIATION**

CRACK GROWTH IN UNIDIRECTIONAL COMPOSITES
UNDER REPEATED LOADINGS

Edward M. Wu

September 1968

**PROGRAM MANAGER
ROLF BUCHDAHL**

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MONSANTO/WASHINGTON UNIVERSITY ASSOCIATION

Sponsored by ONR and ARPA

Development of High Performance Composites

CRACK GROWTH IN UNIDIRECTIONAL COMPOSITES
UNDER REPEATED LOADINGS

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September 1968

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FOREWORD

The research reported herein was conducted by the staff of the Monsanto/Washington University Association under the sponsorship of the Advanced Research Projects Agency, Department of Defense, through a contract with the Office of Naval Research, N00014-67-C-0218 (formerly N00014-66-C-0045), ARPA Order No. 873, ONR contract authority NR 356-484/4-13-66, entitled "Development of High Performance Composites."

The prime contractor is Monsanto Research Corporation. The Program Manager is Dr. Rolf Buchdahl (phone 314-694-4721).

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CRACK GROWTH IN UNIDIRECTIONAL COMPOSITES
UNDER REPEATED LOADINGS

Edward M. Wu*

A B S T R A C T

A phenomenological description of the slow growth of longitudinally oriented cracks in an orthotropic plate is explored. Detail microscopic mode of crack extension is dominated by crack skipping around the fibers. This crack skipping contributes to the increase of resistance to subsequent crack growth. Mathematical models are used to relate experimental results of crack growth under repeated loadings.

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Introduction

The rupture process of structural material due to repeated loadings (fatigue) is generally classified into three stages. These stages are: nucleation of flaws, slow growth of flaws to cracks of Griffith dimension, and finally, rapid propagation of cracks to catastrophic failure. It is the general consensus that a majority of the life time of a structure is consumed in the second stage of quasi-equilibrium slow growth of cracks [1]. Hence, an understanding and characterization of the slow growth of cracks under repeated loading will contribute to a more reliable prediction of the life of a structure. Previous investigators have attempted to characterize this crack growth stage by employing concepts equivalent to limiting stress (Frost & Dugdale [2], Liu [3]), limiting strain (Head [4]), or energy stability (Paris [5], Liu [6], Cotterell [7]). These investigations are limited to statistically homogeneous isotropic materials. In view of the recent increase of composite materials in structural application, it is desirable to extend these approaches to anisotropic materials. As an initial attempt, the slow growth of cracks in orthotropic plates is explored. It is hoped that this work will provide a foundation toward an understanding of general classes of anisotropic and laminated bodies.

Among the aforementioned approaches in characterizing slow crack growth in isotropic materials, the energy stability or crack driving force concept appeared to provide the most satisfactory description over a broad spectrum of crack growth

velocities. Furthermore, this approach also provides a link between crack growth by repeated loadings and the Griffith theory of fracture. This general approach is adopted for our analysis of crack growth in orthotropic plates.

Phenomenological Description of Crack Growth

In this investigation, a longitudinally oriented crack in a unidirectional fiber-reinforced composite is examined. An attempt is made to relate the slow growth in static fracture to that under repeated loadings. The Griffith-type condition, generally employed in linear fracture mechanics, essentially assumes that the onset of catastrophic crack propagation is a function of critical energy (or driving forces as interpreted by Irwin [8]). Time effect is ignored and crack geometry is assumed to be independent of the magnitude and history of loading. In the fracture of composites, these assumptions must be examined critically and modified whenever necessary.

The propagation pattern of a longitudinally oriented crack in a unidirectional fiber-reinforced composite differs significantly from that in a homogeneous isotropic medium [9]. The gross propagation trajectory in a unidirectional composite is collinear with the original crack both under symmetric and skew-symmetric loadings. However, the latter has not been observed in isotropic materials. In fact, these patterns uniquely coincide with the opening and forward shear modes envisioned by Irwin. Treating a material as statistically homogeneous and orthotropic, its strain energy release rates or the equivalent parameters—stress intensity factors—can be easily evaluated. The critical stress intensity factors can be determined experimentally and they

proved to form a very satisfactory criteria [9]. To extend this treatment to crack growth under repeated loadings, a closer examination of crack growth resistance is required.

Fracture experiments in composites [10] indicated that a significant amount of stable crack growth took place prior to fast propagation. Inasmuch as the slow stable growth and fatigue are time-dependent processes, the associated anisotropic viscoelastic rupture process must be characterized and the experimentally determined parameters clearly identified.

The effect of inhomogeneity due to the fibers must also be better understood. Although the gross propagation trajectory of the crack is straight, microscope examination revealed that the crack propagation pattern is dominated by crack skipping across the fibers [11]. A consequence of this crack skipping phenomenon is that the amount of absolute crack extension increases the resistance of subsequent crack growth and thus introduces a strong geometric effect on the crack growth history.

In repeated loadings, another important consideration is the local modification of material in the neighborhood of the crack tip. This material modification may take the form of strain hardening in the case of crystalline materials and of cold drawing in the case of amorphous polymers. Thus, the three primary effects to be considered in the description of crack propagation under repeated loading in composites are: first, the viscoelastic rupture process; second, the crack growth history; and third, the local modification of the material near the crack tip. The viscoelastic rupture of isotropic homogeneous material was studied by Halpin [12]. The geometric effect

of crack growth history is treated in detail here, while the remaining two effects for composites are left for future investigation.

Analysis of the Geometric Effect of Crack Growth

In this section, the geometric effect of crack skipping cumulated in the course of slow growth is first examined. Its role in relation to the rate of crack growth under repeated loading is then explored.

The crack skipping phenomena is attributable to the strong orthotropy in strength and stiffness of the composite [10]. In isotropic materials, subcritical flaws can be envisioned to distribute randomly over the body. Under the intensified stress field in the neighborhood of the main crack tip, the flaw which is normal to the maximum tensile stress ($\sigma_{g \max}$ Fig. 1a) will extend. The main crack then propagates by joining with flaws in this direction. Under many loading conditions, e.g., skew-symmetric loads, this direction of propagation is not co-linear with the crack. In unidirectionally reinforced composites, however, the subcritical flaws are randomly distributed but oriented along the direction of the reinforcing fibers. In this case, the flaws which are oriented normal to the maximum transverse stress ($\sigma_{y \max}$ Fig. 1b) are most likely to extend. Due to elastic orthotropy, the direction maximum transverse stress is in general not co-linear with the crack. Ergo, when these flaws are joined with the main crack, it then gives the appearance of the crack skipping, while leaving strings of fiber in the path of propagation. On the other hand, because the strength of the fiber is usually much higher than the matrix, the crack is channeled in the general direction of the matrix. According to the elastic stress analysis, this

crack skipping should be random when subjected to pure tension (symmetric load) and it should be oriented in the direction of shear when subjected to pure shear (skew-symmetric load) [10].

Figs. 2 are closed-up photographs of the crack skipping modes. Fig. 2a shows a crack which skipped randomly under tension. Fig. 2b shows a crack which skipped in the direction of the shear force and Fig. 2c shows a crack which reversed its direction of skipping when the shear force was reversed. Figs. 2d and 2e are further evidences of crack skipping. Fig. 2d shows a specimen broken in tension with randomly oriented strings of fiber still attached to both sides. Fig. 2e shows a specimen broken under a combined tension and shear now with strings of fibers oriented in the direction of shear force.

To assess the effect of these stringers formed by unbroken fibers, one can conceptually remove these stringers and replace them by singular forces [10]. In the course of summing up the effect of these forces, several difficulties arise. First of all, the number of stringers formed cannot be conveniently evaluated experimentally. Secondly, the total effect of the stringers cannot be obtained by simply using the singular force solution as Green's function integrating over the length of crack extension because a singular force exerted at the crack tip is also involved. Both of these problems can be alleviated by assuming that the effect of the stringers is equivalent to normal and shear forces distributed over the length of crack growth. These distributed forces are functions of the stiffness of the reinforcing fibers as well as the relative displacement between the top and bottom of the crack surfaces. These forces can be

evaluated if the precise geometry of crack skipping is known. However, since these can, at best, be predicted in a statistical manner, it would be far more practical to assume that they are uniform as illustrated schematically in Fig. 3. Their effective magnitude p and t shall be determined experimentally. The effect of these stringer forces are analyzed first and their combined effect with externally applied forces shall be arrived at by superposition. The linear elastic boundary value problem, shown in Fig. 4, can be solved by the method described in Ref. [13]. Consider the shear force alone first in the mapped plane (Fig. 4b), the boundary conditions are:*

$$\begin{aligned}
 f_1 &= 0 & \sigma_1 \bar{\sigma}_1 \\
 f_2 &= -\frac{t_1 a}{2} [(\bar{\sigma}_1 + \sigma_1) - (\sigma_1 + \bar{\sigma}_1)] & \sigma_1 \bar{\sigma}_2 \\
 f_3 &= 0 & \sigma_2 \bar{\sigma}_3 \\
 f_4 &= -\frac{t_2 a}{2} [(\bar{\sigma}_2 + \sigma_2) - (\sigma_2 + \bar{\sigma}_3)] & \sigma_3 \bar{\sigma}_4 \\
 f_5 &= 0 & \sigma_4 \bar{\sigma}_5
 \end{aligned} \tag{1}$$

where σ is a point on the unit circle and σ_1 is arbitrarily chosen as the starting point.

Substituting Eq. (1) into Eq. (30) in Ref. [13] and upon evaluating the integrals, the following result can be arrived at:

$$\begin{aligned}
 \Phi_1(\zeta) &= \pm \frac{it_1 a}{4\pi(\zeta_1 - \zeta_2)} \left\{ \left(1 - \frac{1}{\zeta^2}\right) \left[\log \left(\frac{\bar{\sigma}_1^2 - \zeta^2}{\zeta^2 - \bar{\sigma}_1^2} \right) + \frac{2}{\zeta^2} \log \frac{\bar{\sigma}_1}{\zeta} \right] - 2(\zeta_1^2 + 1) \left[\frac{\bar{\sigma}_1^2}{(1 - \zeta_1^2 \zeta^2)} - \frac{1}{(\zeta_1^2 - \zeta^2)} \right] \right. \\
 &\quad \left. + 2(\zeta_2^2 + 1) \left[\frac{1}{(1 - \sigma_1^2 \zeta^2)} - \frac{1}{(\zeta_1^2 - \zeta^2)} \right] \right\} \\
 &\quad \begin{array}{l} \rightarrow f_1 = \zeta^{-2} \\ - f_2 = \zeta^{-2} \end{array}
 \end{aligned} \tag{2}$$

*The same notations as in Ref. [13] are used here.

Proceeding in a similar manner, the results, for uniformly distributed normal forces (Fig. 3b), are:

$$\Phi_j(\zeta) = \mp \frac{\sigma_0 \sqrt{a}}{4\pi(\zeta_1 - \zeta_2)} \left\{ \left(1 - \frac{1}{\zeta^2}\right) \left[\operatorname{Re} \left(\frac{\zeta_1^2 - \zeta^2}{(\zeta_1^2 - \zeta^2)^2} \right) + \frac{2}{\zeta^2} \operatorname{Im} \left(\frac{\zeta_1^2}{\zeta_1^2} \right) - 2(\zeta_1^2 - 1) \left[\frac{\zeta_1^2}{(\zeta_1^2 - \zeta^2)} - \frac{1}{(\zeta_1 - \zeta)} \right] \right. \right. \\ \left. \left. + 2(\zeta_1^2 - 1) \left[\frac{1}{(\zeta_1^2 - \zeta^2)} - \frac{1}{(\zeta_1 - \zeta)} \right] \right\} \quad \begin{array}{l} - \frac{1}{\zeta^2} \\ - \frac{1}{\zeta^2} + 2 \end{array} \quad (3)$$

Using these complex potentials $\Phi_j(\zeta)$, the stress distribution can be determined for standard relations given by Lekhnitskii [14].

Since both flaw nucleation and crack growth initiate from the vicinity of the crack tip, the solution can be specialized for this region by taking the limit of $\Phi_j(\zeta)$ when ζ approaches unity ($\zeta = 1$ being the crack tip). Performing the limiting process on Eq. (2) and (3), and noting that

$$\operatorname{Re} \frac{\zeta_1^2}{\zeta_1^2} = \cos^{-1} \left(\frac{a - \Delta a}{a} \right)$$

the following results can be obtained, for a distributed shear, t , over Δa :

$$\lim_{\zeta \rightarrow 1} \Phi_j(\zeta) = \pm \frac{t a}{\pi(\zeta_1 - \zeta_2)} \cos^{-1} \left(\frac{a - \Delta a}{a} \right) \quad (4)$$

and for a distributed tension, p , over Δa :

$$\lim_{\zeta \rightarrow 1} \Phi_j(\zeta) = \mp \frac{p a}{\pi(\zeta_1 - \zeta_2)} \cos^{-1} \left(\frac{a - \Delta a}{a} \right) \quad (5)$$

From these results, together with the relationship given by Eq. (3) in Ref. [8], the stress intensity factors can be obtained directly. Using the subscript s to denote the

stress intensity factors associated with the stringers, for the distributed tension and shear, respectively, they are:

$$\begin{aligned} k_{1s} &= -\frac{2\tau}{\pi} a^{\frac{1}{2}} \cos^{-1}\left(\frac{a-\Delta a}{a}\right) \\ k_{2s} &= -\frac{2\tau}{\pi} a^{\frac{1}{2}} \cos^{-1}\left(\frac{a-\Delta a}{a}\right) \end{aligned} \quad (6)$$

As stated earlier, the macroscopic crack propagation modes are precisely mode I and mode II. Consequently, the strain energy release rates can be directly related to the stress intensity factors by (Cf. Ref. [15]):

$$\begin{aligned} G_1 &= k_1^2 \left[\pi \left(\frac{C_{11} C_{22}}{2} \right) \left(\frac{C_{22}}{C_{11}} + \frac{2C_{12} + C_{66}}{2C_{11}} \right) \right]^{\frac{1}{2}} \\ G_2 &= k_2^2 \left[\frac{C_{11}}{2} \right] \left[\pi \left(\frac{C_{22}}{C_{11}} + \frac{2C_{12} + C_{66}}{2C_{11}} \right) \right]^{\frac{1}{2}} \end{aligned} \quad (7)$$

Thus, for the cases under consideration, the strain energy release G_1 and k_1^2 are equivalent parameters. For the sake of simplicity in computation, stress intensity factors are used in this study.

If k'_i are used to denote stress intensity factors when the effect of stringers is ignored, then the actual stress intensity factors at the crack tip can be obtained by superposition in the form:

$$\begin{aligned} k_1 &= k'_1 + k_{1s} \\ k_2 &= k'_2 + k_{2s} \end{aligned} \quad (8)$$

Since both k_{1s} and k_{2s} are negative (Eq. 6), the stringers formed by crack skipping tend to retard the crack growth, or tantamountly, they increase the material resistance to crack growth. From Eq. (6) and (8) it can be seen that the longer the crack growth (Δa), the greater the resistance to crack growth. Insofar as crack propagation by repeated loading is a quasi-equilibrium crack growth process, the growth resistance in such composite should increase with the loading cycles. This phenomena can be characterized by modifying the Cotterell-Krafft's [6, 16] model for fatigue crack growth in isotropic materials.

In Krafft's model [16], the instability of a crack is defined when the slope of the crack resistance denoted as R is tangent to the driving force denoted as ζ_j .

Since ζ_j and $(k)^2$ differ by a constant which is a function of the material (Eq. 7), the same relationship exists as indicated schematically in Fig. 5. In the first cycle of loading, the critical state is k_{c1}^2 . In the second cycle, due to crack growth Δa_1 the growth resistance increases as indicated as R_2 which raises the critical point to k_{c2}^2 . In a similar manner, the cumulated crack growth Δa_2 raises the critical state to a still higher point k_{c3}^2 . This phenomena differs from those observed in the isotropic case in that the experimentally observed crack resistance curve in isotropic materials and the critical stress intensity factor, for all practical purposes, are independent of crack length and growth history. Although in metals the crack resistance curve changes somewhat in the initial cycling drive to strain hardening, it stabilizes in subsequent cycles.

In order to modify Cotterell's [6] model for fatigue crack propagation for this class of composites, it is necessary to hypothesize that the shape of the crack resistance curve R is unique and that it is independent of crack length but is a function of the mode of the externally applied load $P(\theta)^*$, history of crack growth Δa , time t , and temperature T , i.e.,

$$R = f(R(\theta), \Delta a, t, T) \quad (9)$$

Projecting the shape of the crack resistance curve R schematically in Fig. 6 for one mode of loading ($\theta = \text{constant}$) a crack propagation model for repeated load can be obtained. For a crack of initial length a_0 under a repeated load of constant amplitude, so the crack growth would follow the appropriate crack resistance curve to length a_1 until the load is released. In the second cycle, the crack would follow a higher resistance curve (higher because of the growth ($a_1 - a_0$)) and grows to a_2 . Similarly, in the third cycle the crack would follow a still higher resistance curve and grows to a_3 . This process is continued until in the last cycle is tangent to the load curve at which time rapid fracture occurs. Accordingly, if the function suggested by Eq. (9) is known, the number of cycles to fracture can be estimated.

Since the time-temperature effect is kept constant in this study, only the first two parameters in Eq. (9) are considered. Since crack growth resistance is a material property, the hypothesis that the crack resistance curve is unique has to be tested experimentally. If the resistance curve is indeed unique, then the growth history

* θ is the angle between the load vector and the crack; i.e., $\theta = \frac{\pi}{2}$ is tension
 $\theta = 0$ shear.

effect can be determined using the parameters k_{is} given in Eq. (6).

Experimental Observations

The goals in the experimental observation are: first, to investigate the increase in the critical point of instability both under tension and under shear ($\theta = \frac{\pi}{2}$ and $\theta = 0$, respectively, under the notation used in Eq. 9); second, to explore whether the crack resistance curve is unique when the mode of load is the same ($\theta = \text{constant}$); third, to check whether the mathematical models are adequate in characterizing the crack growth history; and finally, to estimate the effective tension and shear forces (p and t in Eq. 6) induced by the stringers.

The materials used in the experiments are 3M Scotch-ply Type 1002 unidirectional fiberglass reinforced plastic plates 0.05 inch thick. In each specimen, one straight crack is cut parallel to the reinforcing fibers. Rectangular specimens 6 inches wide by 10 inches long were used for symmetric loading tests in which tension is applied perpendicular to crack and fiber. Cantilever beam specimens 4 inches high by 10 inches long were used for skew-symmetric loading tests. It was established in Ref. [14] that the cantilever beam was equivalent to a pure shear fracture test when a crack was located along the neutral axis. The experimental set-up and procedure are similar to those described in Ref. [9]. Briefly, the procedure consists of using a motor-driven camera to record the crack length in the process of loading. An event marker in the load recording chart is synchronized with the load to the crack length recorder on the film. From these records, point wise crack growth resistance can be computed from which a continuous curve can be drawn. The specimens were subjected

to approximately four cycles as shown schematically in Fig. 5. In the first cycle, load was increased while the crack growth was closely monitored. When the combination of load and crack propagation appeared to have reached or slightly passed the instability point (the point of tangency of the driving force and the resistance curve), the specimen was rapidly unloaded. This was repeated for the second and third cycles and in the fourth cycle the specimen was loaded all the way to fracture. In the symmetric load tests, the loading program was zero to maximum tension (up to instability) for four cycles. In the skew-symmetric load tests the loading program was zero to maximum shear in alternate directions for four cycles. The typical crack resistance curves thus obtained for tension and shear respectively are shown in Figs. 7 and 8. The stress intensity factors k_1 are computed where the stringer forces are not taken into consideration. They are computed by the relationships [8, 16]

$$k_1 = \sqrt{\sigma a} \quad \text{and} \quad k_2 = \frac{3Q}{2A} \sqrt{a}$$

where σ is the tension, a is the half crack length and Q the shear force A the transverse cross section area.

The critical values plus other pertinent information is tabulated in tables I and II for tension and shear, respectively. From Figs. 7 and 8 it can be noted that, with experience, the specimens were loaded very close to or sometimes past the critical point in the repeated loading process. This experience, however, was not gained without cost; the missing specimen numbers in tables I and II were broken in this learning process.

Both of the figures and the tabulated values show clearly that the critical point rose in repeated loadings.

To explore whether the crack resistance curve is unique, the crack resistance curves are normalized and compared. Fig. 9 represents the normalized data of the crack resistance data from four specimens, subjected to tension. Fig. 10 represents the data from four specimens subject to shear. Within the realm of our experimental scatter, the crack growth resistance curves appear to be unique. However, significant differences exist between the resistance curve for tension and shear which are apparent when they are compared in Fig. 11. It can be concluded then that the form of the crack growth resistance curve is independent of crack length but dependent on the mode of loading.

To examine the validity of the mathematical model for characterizing crack skipping, Eq. (8) can be rearranged; together with Eq. (6) we have,

$$\begin{aligned} k_1' &= k_1 + \frac{2\sigma}{\pi} a^{\frac{1}{2}} \cos^{-1}\left(\frac{a-\Delta a}{a}\right) \\ k_2' &= k_2 + \frac{2\tau}{\pi} a^{\frac{1}{2}} \cos^{-1}\left(\frac{a-\Delta a}{a}\right) \end{aligned} \quad (10)$$

If the mathematical models are adequate, the stress intensity factors k_1 and k_2 should be constants and Eq. (10) in a graph of k_1 versus $a^{\frac{1}{2}} \cos^{-1}\left(\frac{a-\Delta a}{a}\right)$ should be straight lines. Data from tables I and II presented in this form are shown in Figs. 12 and 13. Here too, within our experimental scatter, the mathematical models provide very close characterization indeed. The intercepts in the ordinates should be the critical k_{1c} and k_{2c} for static fracture tests. This compared very favorably with previously obtained results [8]. Finally, the effective tension and shear, p and t , can be evaluated easily from Figs. 12 and 13. The slopes of the

straight lines are $\frac{2p}{\pi} = 570$ and $\frac{2t}{\pi} = 1560$ respectively, which give the values $p = 900$ psi and $t = 2500$ psi. It is worth noting that p is limited by the "peel strength" of the fiber from the matrix. The value evaluated above is certainly not far from the "peel strength" reported in other investigations. Furthermore, from the same model, the maximum value of t is limited by the tensile strength of the fiber. In this case, t is substantially less than the strength of glass fiber which does not contradict the observation that the glass fiber stringers remained intact after fracture (Fig. 1 and Fig. 2).

Conclusions

This investigation is an initial step in the attempt to better characterize the fatigue of fibrous composite materials. One of the primary effects, the geometric history effect of crack growth, was explored.

It was observed that the crack resistance curve appeared to be independent of crack length and of crack growth history for a given mode of loading. However, the crack resistance curves for tension and for shear are distinctly different indicating that they are functions of the mode of loading.

It was also observed that under small number of repeated loadings deliberately designed to provide large amounts of crack growth, the crack growth resistance increased with the number of cyclic loads. This increase in crack growth resistance was successfully characterized by a mathematical model in which the growth history can be related to static fracture strength. The magnitude of the distributed stringer forces

estimated by means of this model also agreed well with other findings. With this model, the geometrical effect of crack growth can be characterized and Cotterell's mechanics of isotropic fatigue crack growth can be modified for unidirectionally reinforced composites.

Acknowledgment

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TABLE I
REPEATED TENSION

Spec. No.	a_0	a_{c1}	k_1	a_{c1}	a_{c2}	k_1	a_{c2}	a_{c3}	k_1	a_{c3}	a_{c4}	k_1	a_{c4}
T7	.375	.415	920	.040	.435	940	.060	.462	980	.087	.553	1080	.178
T8	.367	.390	1020	.023	.410	1110	.043	.450	1150	.083	.602	1300	.235
T9	.301	.362	1030	.061	.390	1070	.089	.412	1090	.111	.467	1120	.166
T10	.316	.370	1020	.054	.430	1050	.114	.463	1070	.147	.525	1100	.209
T11	.328	.365	1040	.037	.393	1100	.065	.420	1170	.092	.467	1210	.139
T12	.312	.325	820	.013	.346	890	.034	.384	930	.072	.460	1010	.148
T13	.246	.289	1010	.043	.327	1060	.081	.360	1070	.114	.394	1090	.148

TABLE II

REPEATED SHEAR

Spec. No.	a_0	a_{c1}	k_{2c}	a_{c1}	a_{c2}	k_{2c}	a_{c2}	a_{c3}	k_{2c}	a_{c3}	a_{c4}	k_{2c}	a_{c4}
1	.387	.540	3410	.153	.600	3420	.213	.878	4110	.491			
2	.442	.580	3490	.158	.710	3640	.288	.865	3940	.443			
3	.499	.665	3770	.166	.765	4000	.266						
4	.254	.340	3300	.086	.480	3940	.226						
5	.265	.430	3740	.165	.630	4150	.365						
7	.226	.305	3290	.079	.420	3590	.194	.520	4000	.294	.780	4290	.554
8	.254	.370	3690	.116	.580	4050	.326	.670	3710	.416	.920	4630	.666
9	.328	.495	3990	.167	.580	3970	.252						
10	.263	.465	3810	.202									
11	.231	.310	2980	.079	.380	3390	.149	.560	3870	.329	.840	4440	.609
12	.246	.290	2920	.044	.430	3620	.184	.575	3700	.329	.700	3920	.454
13	.220	.285	2900	.065	.345	3160	.125	.460	3440	.240	.750	4540	.530
14	.250	.290	3160	.040	.385	3420	.135	.530	3690	.280	.690	4220	.440
15	.287	.382	3450	.095	.470	3860	.183	.590	4180	.303			
16	.232	.273	3070	.041	.365	3700	.133	.460	4000	.228	.615	4360	.383
17	.238	.285	3130	.047	.485	4000	.247	.650	3940	.412	1.015	4360	.777
18	.245	.330	3370	.085									
19	.257	.340	3150	.083	.537	3690	.280	.935	4190	.678			
20	.247	.340	3230	.093	.455	3580	.208	.592	3870	.345	.710	4470	.463
21	.247	.350	3160	.103	.495	3870	.248	.585	4070	.338			

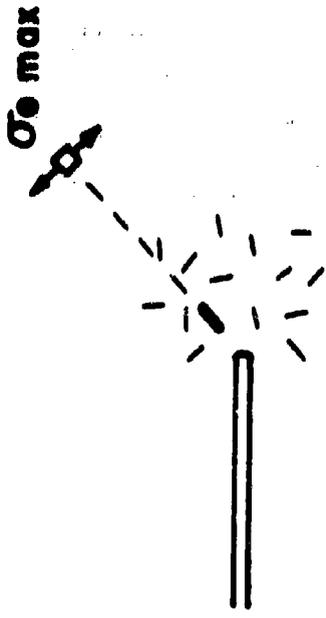
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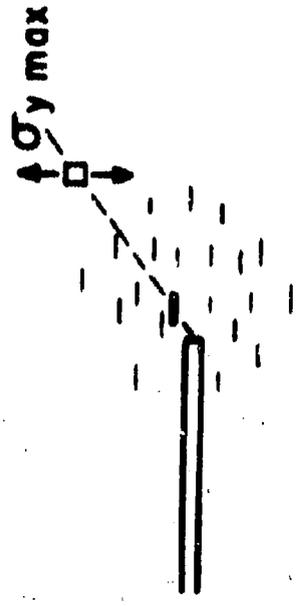


NUCLEATION



PROPAGATION

ORTHOTROPIC



NUCLEATION



PROPAGATION

Fig. 1



d



e



Fig. 2

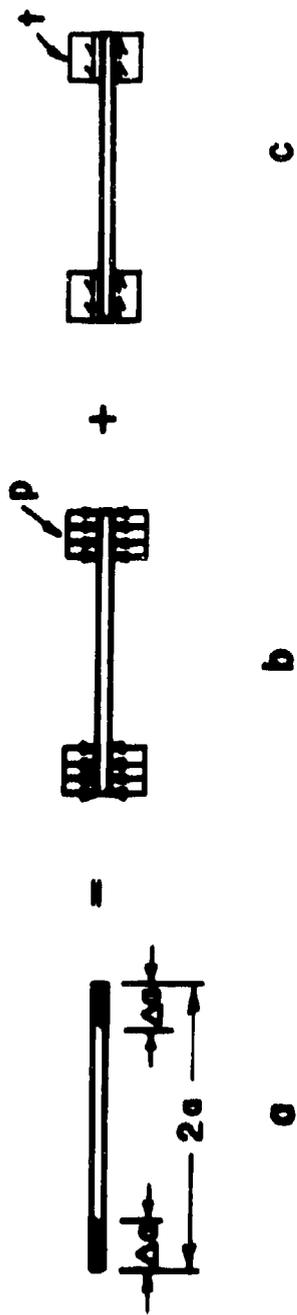
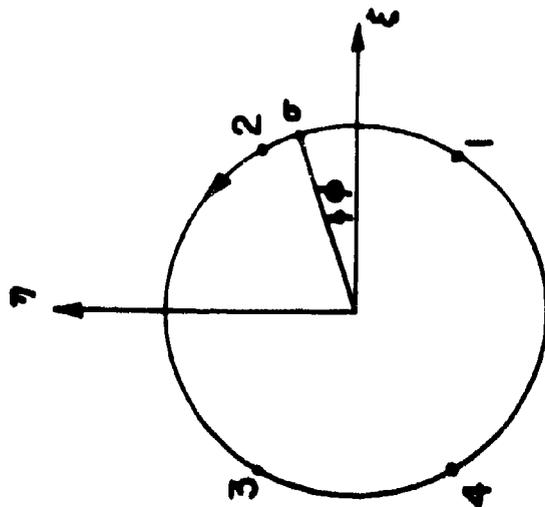
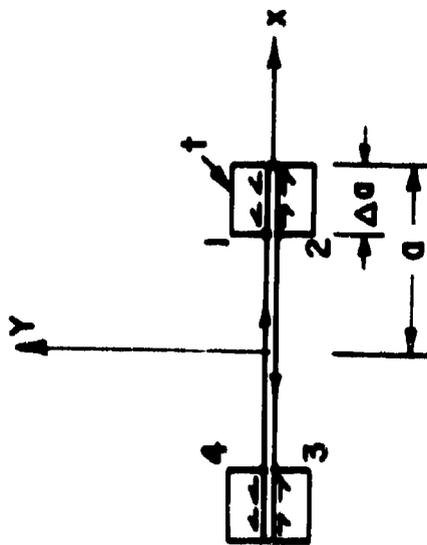


Fig. 3



(b)



(a)

Fig. 4

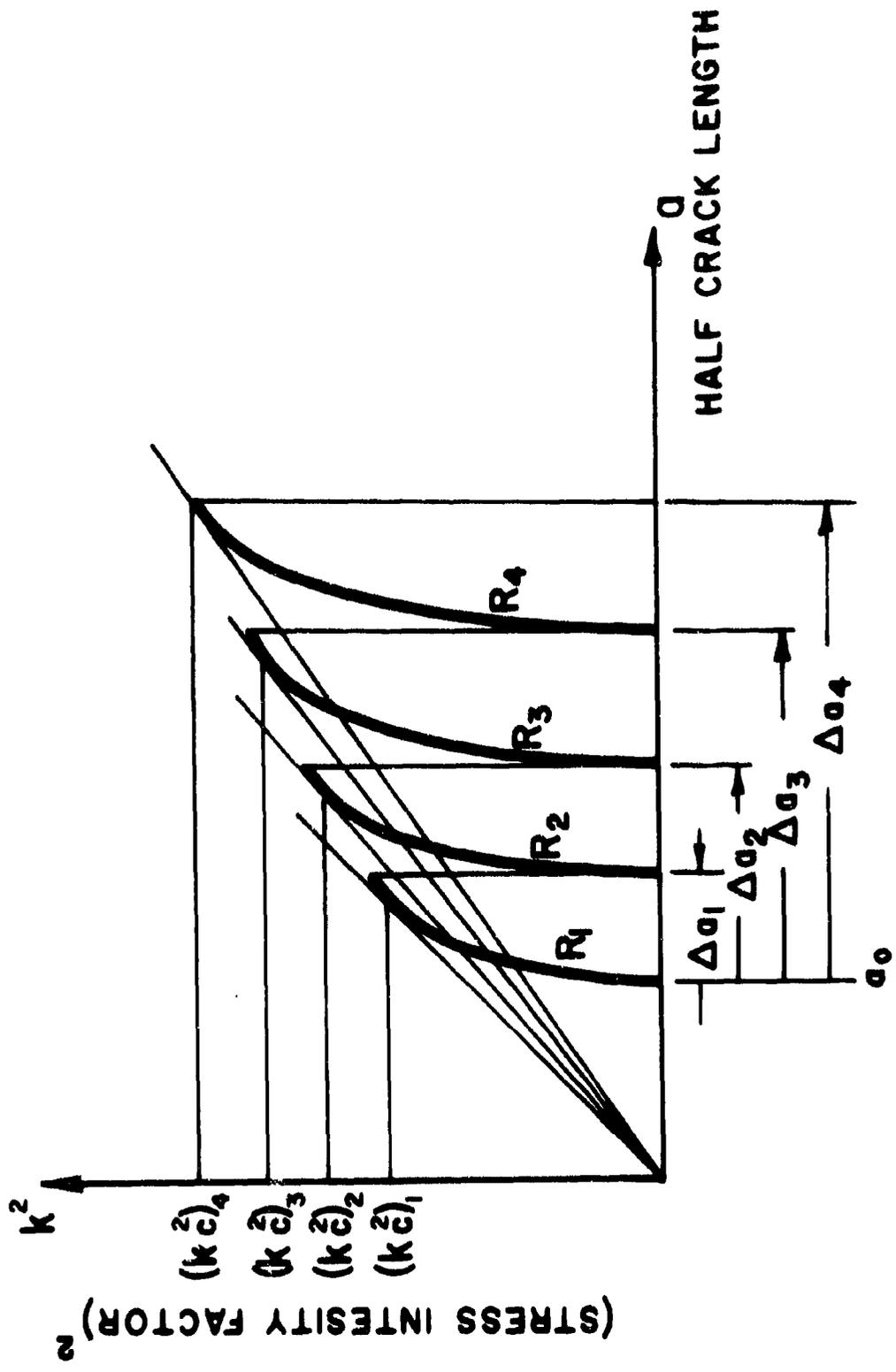


Fig. 5

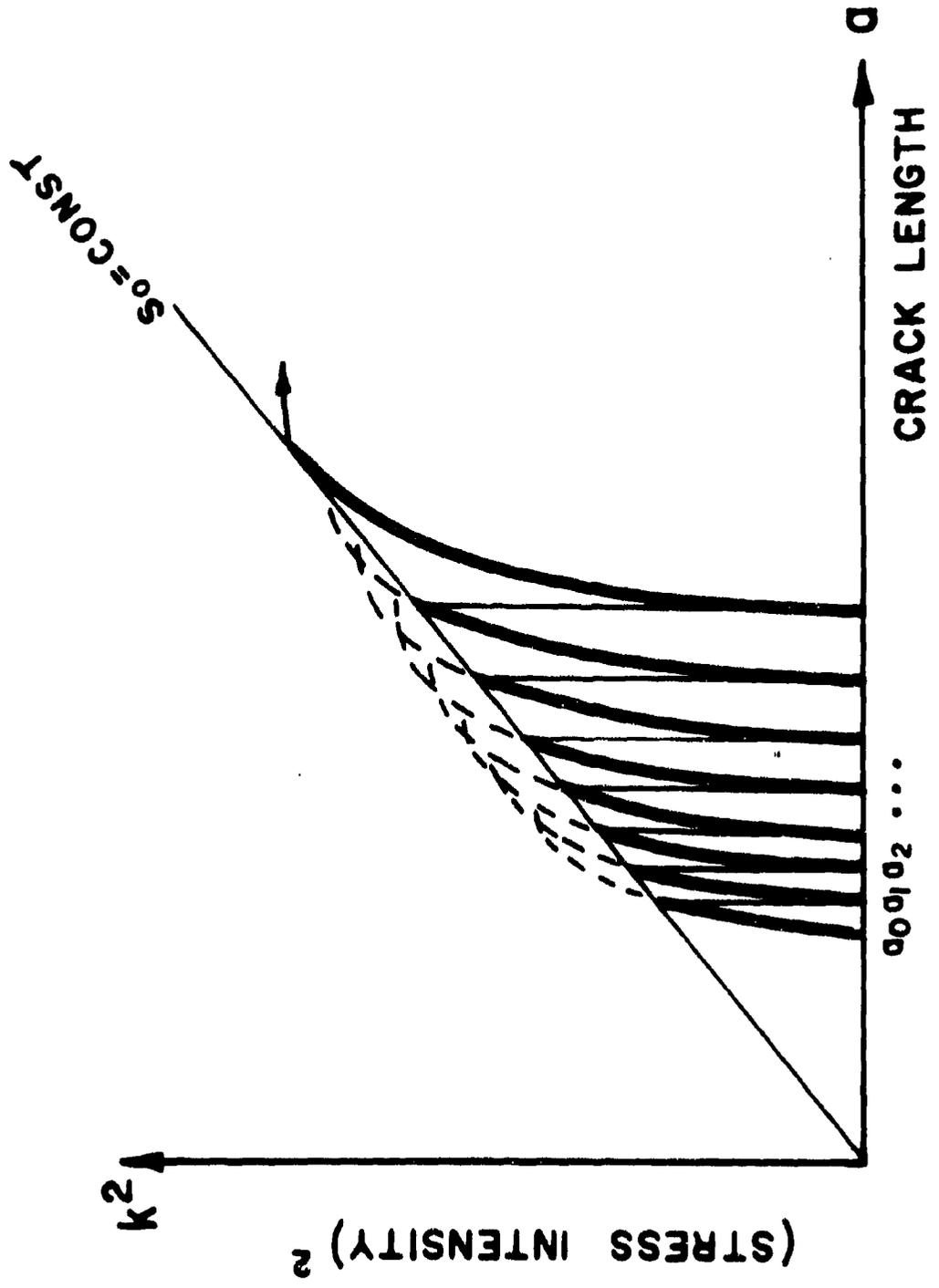


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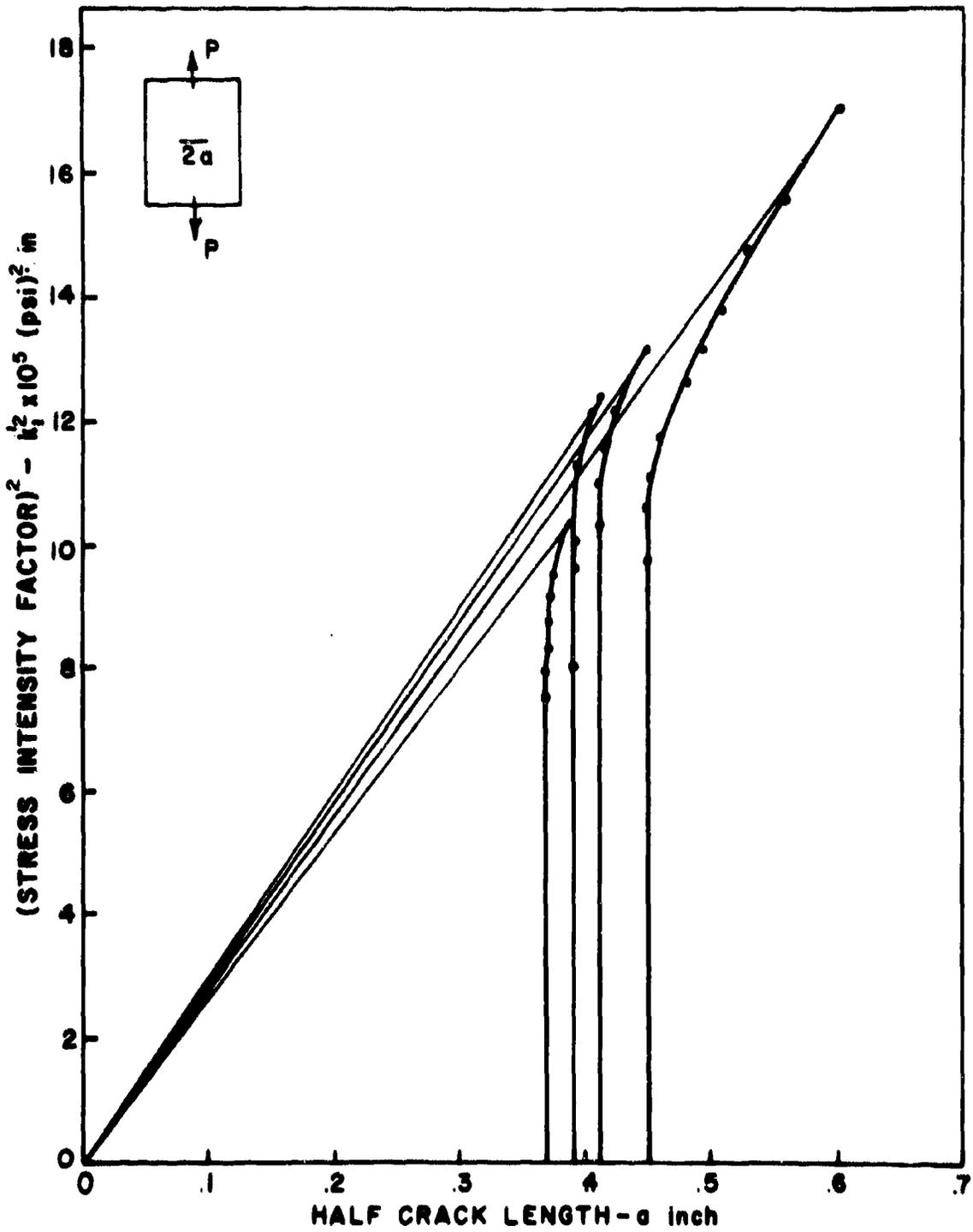


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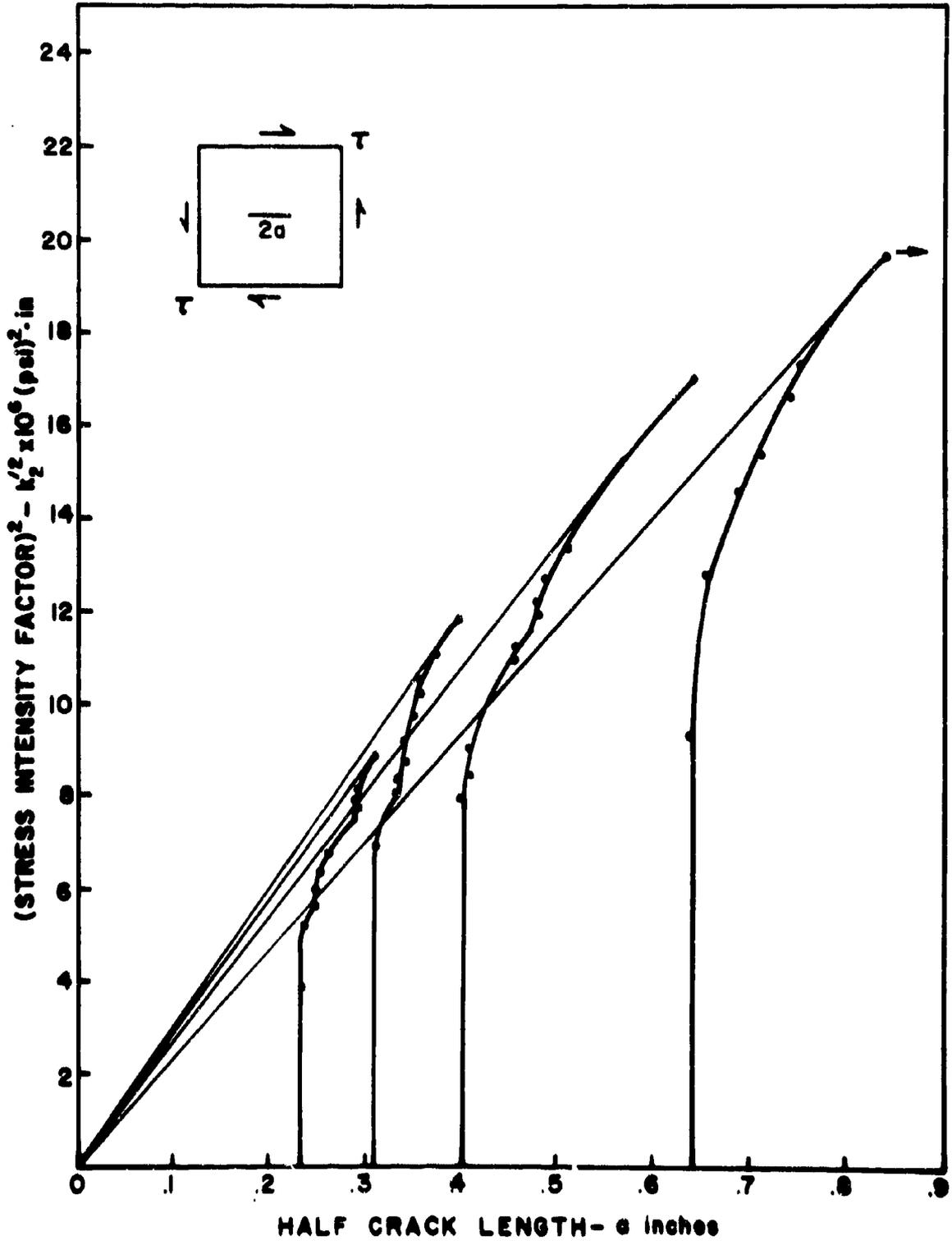


Fig. 8

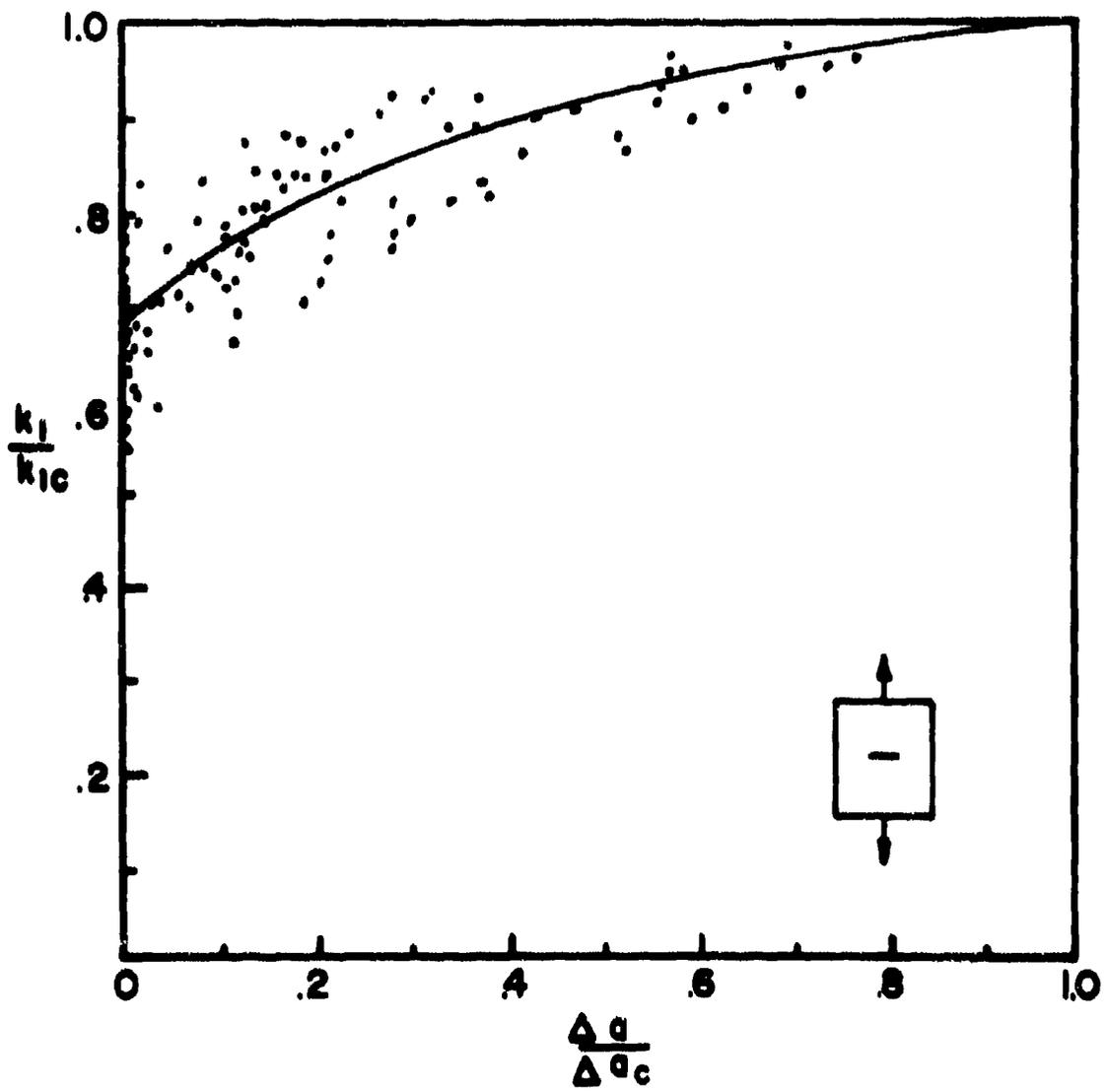


Fig. 9

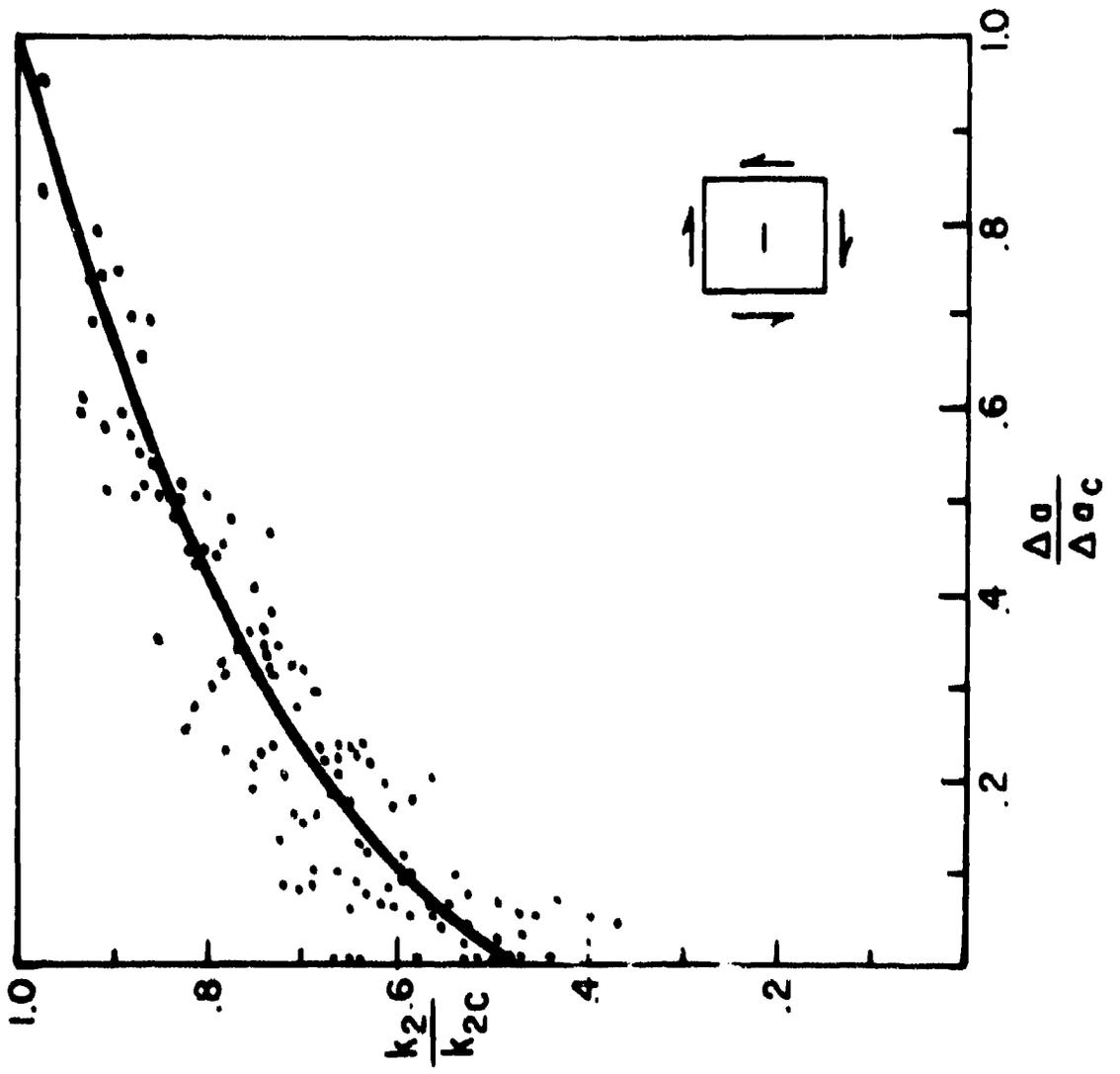
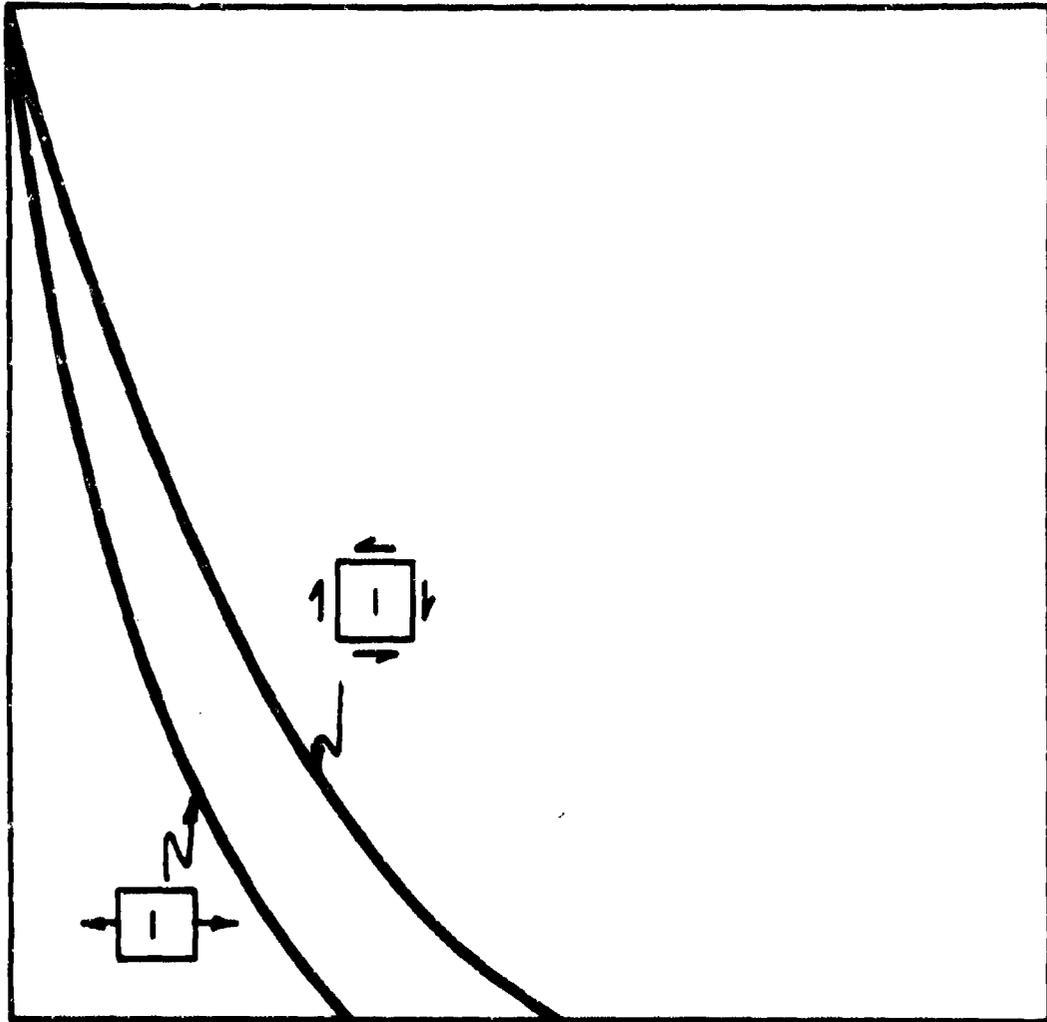


Fig. 10



$$\frac{\Delta \sigma}{\Delta \sigma_c}$$

Fig. 11

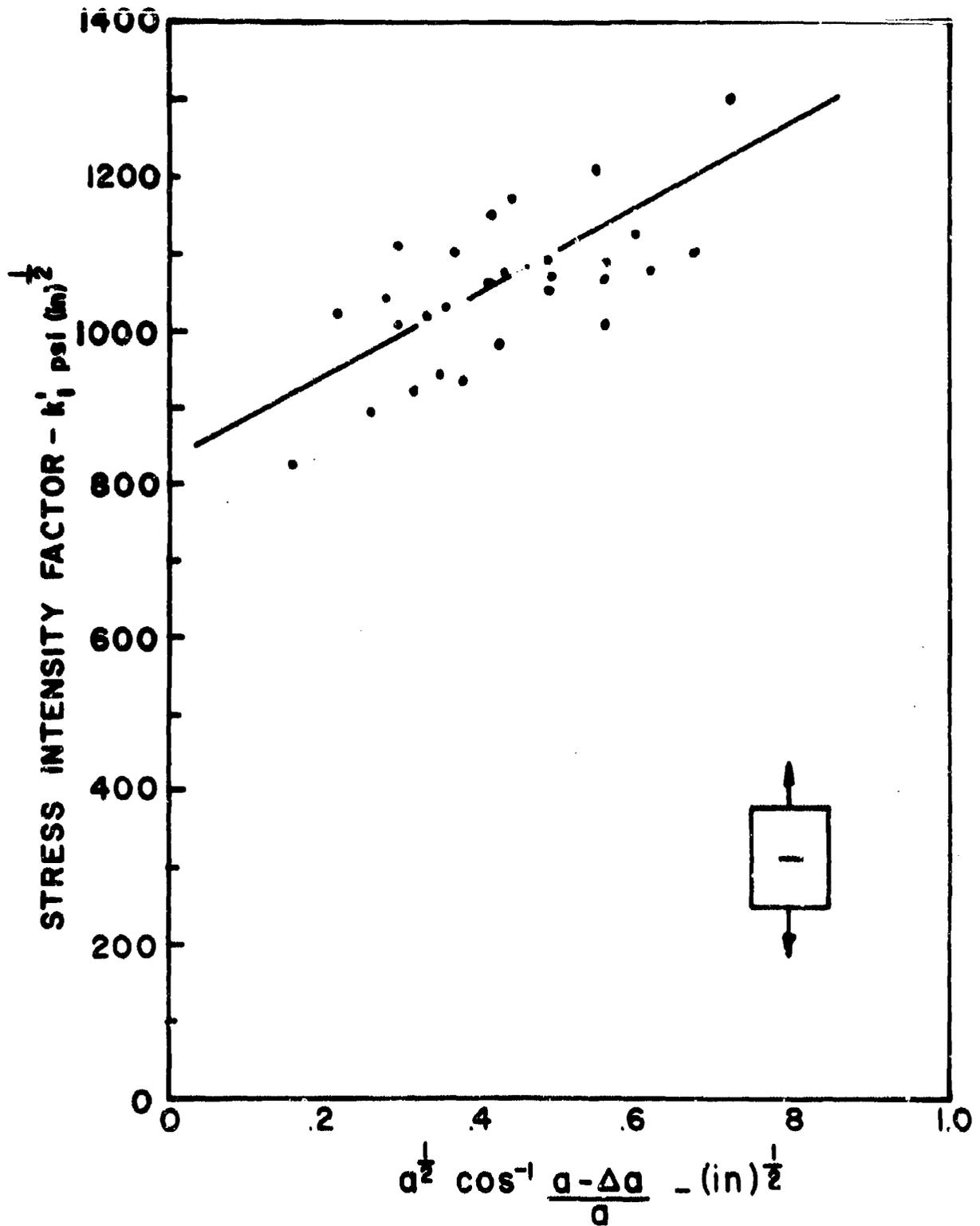


Fig. 12

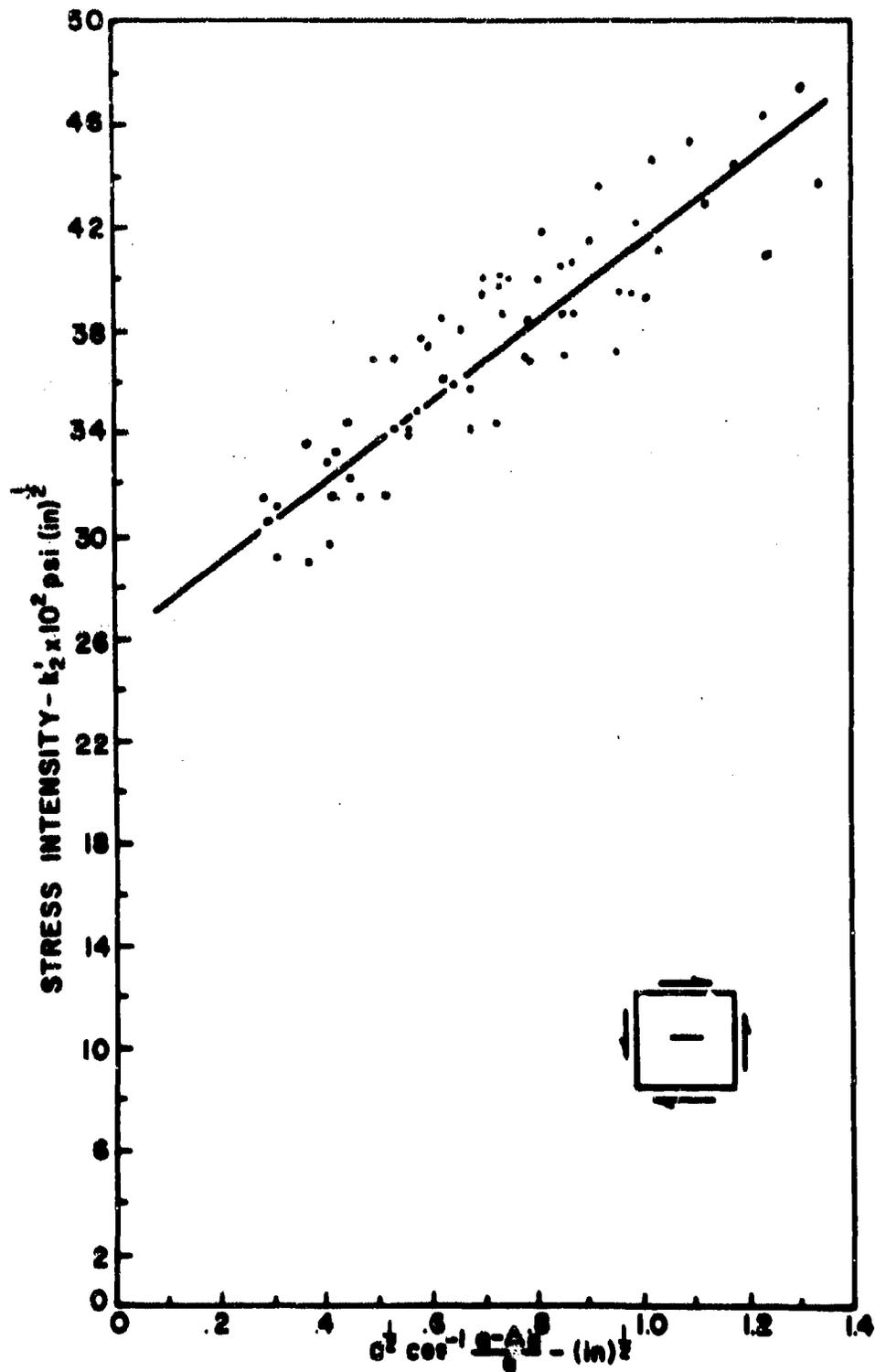


Fig. 13

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13. ABSTRACT A phenomenological description of the slow growth of longitudinally oriented cracks in an orthotropic plate is explored. Detail microscopic mode of crack extension is dominated by crack skipping around the fibers. This crack skipping contributes to the increase of resistance to subsequent crack growth. Mathematical models are used to relate experimental results of crack growth under repeated loadings.			

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
fiberglass-reinforced plastic crack propagation slow crack growth repeated loadings stress intensity factors fiber stringers						