### UNCLASSIFIED

**AD NUMBER**

| AD840712 |

**LIMITATION CHANGES**

**TO:**

Approved for public release; distribution is unlimited. Document partially illegible.

**FROM:**

Distribution authorized to U.S. Gov't. agencies and their contractors; Critical Technology; JUL 1968. Other requests shall be referred to Air Force Technical Application Center, Washington, DC. Document partially illegible. This document contains export-controlled technical data.

**AUTHORITY**

usaf ltr, 25 jan 1972

**THIS PAGE IS UNCLASSIFIED**
A PRELIMINARY EVALUATION OF THE MATCHED FILTER TECHNIQUE IN THE DETECTION OF LONG-PERIOD BODY WAVE RADIATION

30 JULY 1968

Prepared For
AIR FORCE TECHNICAL APPLICATIONS CENTER
Washington, D.C.

By
P. Glover
TELEDYNE INC.
S. S. Alexander
THE PENNSYLVANIA STATE UNIVERSITY

Under
Project VELA UNI/6XH

Sponsored By
ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Test Detection Office
ARPA Order No. 624
A PRELIMINARY EVALUATION OF THE MATCHED FILTER TECHNIQUE IN THE DETECTION OF LONG-PERIOD BODY WAVE RADIATION

SEISMIC DATA LABORATORY REPORT NO. 222

AFTAC PROJECT NO.: VELA T/6702
Project Title: Seismic Data Laboratory
ARPA Order No.: 624
ARPA Program Code No.: 8F10

Name of Contractor: TELEDYNE INDUSTRIES, INC.

Contract No.: F 33657-68-C-0945
Date of Contract: 2 March 1968
Amount of Contract: $1,251,000
Contract Expiration Date: 1 March 1969
Project Manager: Royal A. Hartenberger
(703) 836-7647

P. O. Box 334, Alexandria, Virginia

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of Chief, AFTAC.
This research was supported by the Advanced Research Projects Agency, Nuclear Test Detection Office, under Project VELA-UNIFORM and accomplished under the technical direction of the Air Force Technical Applications Center under Contract F 33657-68-C-0945.

Neither the Advanced Research Projects Agency nor the Air Force Technical Applications Center will be responsible for information contained herein which may have been supplied by other organizations or contractors, and this document is subject to later revision as may be necessary.
ABSTRACT

Using nine elements of the Montana LASA, long-period body wave radiation was detected at S/N ratios as low as 1 for synthetic test cases. In these instances beamforming the 9 matched filter outputs improved the S/N ratio by a factor of 2 over straightforward phased summation of the raw data. For real data, the results are less satisfactory. However, from a limited sequence of Kurile Island events we establish an approximate threshold of $m_D = 5$ above which we can detect long-period body wave radiation. The results demonstrate the need for a better understanding of long-period body wave excitation as a function of magnitude and focal depth for earthquakes.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>1</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Method of Analysis</td>
<td>3</td>
</tr>
<tr>
<td>Signal Enhancement Criteria</td>
<td>4</td>
</tr>
<tr>
<td>Synthetic Test Cases</td>
<td>4</td>
</tr>
<tr>
<td>a) Method</td>
<td>4</td>
</tr>
<tr>
<td>b) Determination of the &quot;Optimum&quot; Filter</td>
<td>5</td>
</tr>
<tr>
<td>c) Evaluation of the Performance of the Matched Filter Technique</td>
<td>8</td>
</tr>
<tr>
<td>Observed Data</td>
<td>9</td>
</tr>
<tr>
<td>Discussion</td>
<td>14</td>
</tr>
<tr>
<td>Conclusions</td>
<td>17</td>
</tr>
<tr>
<td>Recommendations</td>
<td>18</td>
</tr>
<tr>
<td>References</td>
<td>19</td>
</tr>
<tr>
<td>Table Title</td>
<td>Table No.</td>
</tr>
<tr>
<td>---------------------------------------------------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>Epicenter Data for the Kurile Island Events used in this study</td>
<td>1</td>
</tr>
<tr>
<td>Figure Title</td>
<td>Figure No.</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>Seismograms recorded at WMSO from the nuclear explosion LONG SHOT. LONG SHOT location 5.14°N 179.2°E, depth 0.8km, origin time 21:00.00.1Z Magnitude = 6.1 (USC&amp;GS), 6.0 (NAM)</td>
<td>1a</td>
</tr>
<tr>
<td>Seismograms recorded at WMSO from a nearby earthquake. Earthquake location 50.1°N 178.2°E, Depth = 28km, origin time 04:46:13.1Z Magnitude = 5.5 (USC&amp;GS), 5.2 (NAM)</td>
<td>1b</td>
</tr>
<tr>
<td>Long-period vertical component seismograms of the body waves from the China event Sept. 28, 1966, recorded at the 9 subarrays of LASA used in the synthetic test cases.</td>
<td>2</td>
</tr>
<tr>
<td>Results from a synthetic test case using the signal itself as filter.</td>
<td>3</td>
</tr>
<tr>
<td>Results from a synthetic test case using the first 740 seconds of the buried signal as filter.</td>
<td>4</td>
</tr>
<tr>
<td>Results from a synthetic test case using the first 270 seconds of the buried signal as filter.</td>
<td>5</td>
</tr>
<tr>
<td>Examples of the synthetic seismograms at various S/N ratios from the subarray AO.</td>
<td>6</td>
</tr>
<tr>
<td>Phased sum traces of the matched filter outputs (9 channels) from the synthetic seismograms at various S/N ratios using the 1000 second filter.</td>
<td>7</td>
</tr>
<tr>
<td>Phased sum traces (9 channels) of the synthetic seismograms at various S/N ratios.</td>
<td>8</td>
</tr>
<tr>
<td>Comparison of the matched filter results with those obtained by beamforming.</td>
<td>9</td>
</tr>
<tr>
<td>Long-period vertical-component seismograms from the event April 1, 1967, at 12:23:35.5 which were used as filters for Kurile Island events.</td>
<td>10</td>
</tr>
<tr>
<td>Matched filter results from the double event on April 1, 1967, compared to beamforming the bandpass filtered seismograms.</td>
<td>11</td>
</tr>
<tr>
<td>Figure Title</td>
<td>Figure No.</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>Matched filter results from the double event on April 1, 1967, using the shorter filters.</td>
<td>12</td>
</tr>
<tr>
<td>Matched filter results from the double event on April 1, 1967, using the level and rectified filters.</td>
<td>13</td>
</tr>
<tr>
<td>Matched filter results from the December 22, 1966 event using various filters compared to beamforming the bandpass filtered seismograms.</td>
<td>14</td>
</tr>
<tr>
<td>Matched filter results from the event on June 13, 1967 compared to beamforming the bandpass filtered seismograms.</td>
<td>15</td>
</tr>
<tr>
<td>Matched filter results from the event on March 29, 1967 compared to beamforming the bandpass filtered seismograms.</td>
<td>16</td>
</tr>
<tr>
<td>Matched filter results from the event on April 19, 1967 compared to beamforming the bandpass filtered seismograms.</td>
<td>17</td>
</tr>
<tr>
<td>Graph showing S/N ratios obtained by matched filtering as a function of body wave magnitude ($m_b$).</td>
<td>18</td>
</tr>
</tbody>
</table>
INTRODUCTION

Recently, Alexander and Rabenstine (1967a, 1967b) successfully applied the matched filter technique to the problem of detecting surface wave radiation from small events located at considerable distance from the receiving stations. However, there are no reported applications of this method to the detection of long-period body wave radiation from such events. While body waves are not the most conspicuous phases on long-period seismograms, the presence or absence of long-period body wave radiation may prove to be useful diagnostic in the classification of seismic events. For example, it is well known that negligible long-period body wave energy was radiated by LONG SHOT (long-period seismograms from NMO are shown in Figure 1a) which had a short-period body wave magnitude ($m_b$) of 6.0 for North American stations (NAM) and 6.1 for world-wide station (USC&GS), whereas long-period seismograms recorded at the same station from a nearby earthquake (shown in Figure 1b) exhibit conspicuous body wave arrivals despite the smaller magnitude, 5.2 (NAM) and 5.5 (USC&GS) of the latter event.

Work on the structure of the earth's crust beneath the Montana LASA (Glover and Alexander, 1968) in addition to other investigations has shown that pulse-like body waves incident at the base of the crust are converted into longer trains of arrivals at the surface by the complex structure of the crust. The crustal transfer function for each station is a function of epicentral distance and azimuth. This, in addition to the increasing separation of phase arrival times with increasing distance from the source, suggests that seismograms from a given location may exhibit sufficient character to enable the matched filter technique to be successfully applied. Moreover, Alsop and
Chander (1968) have shown theoretically that coupled PL modes may be generated locally by body waves incident at the base of a plane parallel layered structure such as the earth's crust. Since such waves are themselves dispersive, this provides further motivation for investigating a matched filter approach, because matched filtering takes the energy which is contained in the long wavetrain and compresses it into a short pulse at the initial onset of the event whereas the energy in the noise that is randomly distributed is not compressed. Thus matched filtering effectively increases the S/N ratio of such seismograms.

To determine the effectiveness of this technique, we first set up synthetic test cases by burying a known earthquake signal in long-period noise at various S/N ratios. We then used the known earthquake signal as a matched filter. An optimum filter length was determined from the results of this experiment, along with an estimate of the minimum S/N ratio at which the technique breaks down. The results from the synthetic cases were sufficiently encouraging to warrant proceeding to search for long-period body waves from actual events. For actual events the results are less encouraging. We attribute this in part to the lack of sufficient available data from LASA. Only LASA data was considered because it is already in digital format and is best suited for application of array summing techniques.

The first sections of this report deal with the methods and results for the synthetic test cases, and the latter sections deal with the application of the technique to observed data.
METHOD OF ANALYSIS

The theory of matched filtering is well known and will not be presented in detail here. The technique amounts to filtering a time series \( x(t) = ay(t) + n(t) \) where \( n(t) \) is a random noise process and \( a \) is a constant, with the known signal \( y(t) \). The output of the filter, in the frequency domain, is (Alexander and Rabenstine, 1967b)

\[
C_{xy}(t) = \int_{-\infty}^{\infty} X(w)Y^*(w)e^{i\omega t} \, dw
\]

where \( X(w) = \) frequency spectrum of the test seismogram
\( Y(w) = \) complex conjugate of the spectrum of the reference event.

The cross correlation \( C_{xy}(t) \) is a maximum when the reference signal and the buried signal are aligned exactly.

For the case where the noise \( n(t) \) is stationary but not white, Alexander and Rabenstine (1967a) developed a maximum-likelihood matched filter, which in particular cases is more effective than the usual least-squares filter.

Matched filtering is effective in enhancing signal-to-noise ratios because it compresses a long wavetrain regular in phase into a pulse of short duration, whereas the noise, which is random in phase, is not compressed. Alexander and Rabenstine (1967b) proved that this is indeed the case for surface waves. However body waves per se are not dispersive. Nevertheless the suite of body waves from an event may be considered as a regular wavetrain, analogous to the surface waves. In addition, consideration of the crustal transfer function at LASA (Glover and Alexander, 1968) has shown that each pulse-like arrival at the base of the crust is converted to a longer train of arrivals.
at the surface. Matched filtering should then compress the composite body wave signal into a short pulse at the onset of the initial arrival of this wavetrain as is the result for surface waves.

The filtering programs used in this study are those developed by Alexander and Rabenstine in their work on surface waves.

**Signal Enhancement Criteria**

Throughout this report, we adopted the following definition of signal-to-noise ratio,

\[ S/N = \frac{1}{2} A_{\text{max}} (\text{RMS})^{-1} \]

where \( A_{\text{max}} \) = maximum peak-to-peak amplitude of the signal and \( \text{RMS} \) = root mean square amplitude of the noise. In the case of the results from matched filtering, we applied an additional constraint. The RMS amplitude of the noise on the filtered seismograms was computed over an interval preceding the expected arrival time of the signal, but sufficiently removed from it so as to be unbiased by any ringing of the filter. The signal peak-to-peak amplitude was computed from the maximum amplitude in the filter output in a 100-200 second window around the expected arrival time. Only when the output waveform from the filter (or summed filter outputs) resembled the known autocorrelation function of the reference filter were the computed \( S/N \) ratios considered meaningful.

**Synthetic Test Cases**

a) Method

We set up synthetic test cases by burying a known signal in
LASA noise at selected S/N ratios. The reference signal (see Figure 2) came from a magnitude 5.9 (NAM), 6.2 (USC&GS) event in China.

Date Origin Time Location Depth Location Epicentral
28 Sept 1966 14:00:22.9 27.4°N 100.1°E 33Km 105°

which was well recorded at LASA with S/N ratios of 12, 24, 44 for the phases P, PP and SP respectively. A segment containing the first 1000 seconds of this signal was buried in LASA wintertday noise, at S/N ratios of 0.8, 1, 2, 4, and 8. To do this, we computed the RMS amplitude of 3100 seconds of noise and the signal peak-to-peak amplitude from the maximum in the signal. The signal was then scaled and added point for point over a 1000 point interval of the noise, commencing at point 2001 (the sampling rate of both noise and signal was 1 point/sec). It should be noted that the maximum arrival (the phase SP) which was used for scaling is 2-4 times larger than the other major arrivals included in the filter, so that the indicated signal-to-noise ratios are very conservative.

In all cases the known signal (or parts of it) was used as the filter. Only subarrays in the A, E and F rings were used, partly to save computer time and partly because previous studies (Alexander and Rabenstine, 1967b; Hartenberger, 1967) have shown that beamforming the output from long-period seismometers at LASA which are separated by more than 30 kilometers results in noise reduction of approximately the square of N, where N is the number of seismometers.

b) Determination of the "Optimum" filter

Using the test seismogram with a S/N ratio of 4, we
experimented with filters of various lengths. As predicted by the theory (Turin, 1960), the signal itself proved to be the best filter. Matched filtering the band-passed (15-50 seconds) seismograms increased the mean S/N ratio to 7.1 for each individual trace, whereas beamforming the matched filter output of the nine channels gave a S/N ratio of 21.0, an increase of almost the square root of N over the individual channels matched filter outputs, and an improvement in S/N ratio of 17 over the mean S/N of the individual raw traces. The peak output of the filter, for both the individual channels and the phased sum clearly resembles an autocorrelation function whose location in time corresponds exactly to the known point at which the buried signal commenced (Figure 3).

While the shorter filters did not achieve as large a S/N ratio improvement, they nevertheless detected the buried signal. Figure 4 shows the results of filtering with a 740-point filter and Figure 5 the results from a 270-point filter. In both cases, the matched filter outputs from the individual channels fail to locate unambiguously the onset of the buried signal. However, phased summation of these outputs produces very nearly the square root of N improvement in S/N ratio over the mean of the individual channels, and the onset of the signal is clearly visible. The 270-point filter, which includes only P as a major arrival, correlates not only with itself, but with the later arrivals, PP and SP also (see Figure 5). This suggests that once a signal of interest has been detected using the longer filter, a shorter filter containing the P arrival and the interval prior to the next principle arrival may be used to determine the onset and polarity of the remaining major arrivals in the buried signal.
To investigate the dependence of the matched filter output on the amplitude of the various arrivals which constitute the signal we equalized the amplitude of each peak and trough in the filter series. We did this by dividing each point in the filter by the corresponding point in the envelope of the filter series. The signal from the reference event (i.e., the filter) \( f(t) \) can be represented in terms of its analytic signal (Bracewell, 1965, p. 268)

\[
\hat{f}(t) = f(t) - i F_{Hi}(t)
\]

where \( F_{Hi}(t) \) is the Hilbert transform of \( f(t) \) or simply as

\[
\hat{f}(t) = V(t) e^{i\overline{\omega}t}
\]

where \( \overline{\omega} \) represents the mean frequency of the signal. Then, the envelope function

\[
E(t) = |V(t)| = (f^2(t) + F^2_{Hi}(t))^{1/2}
\]

The equal amplitude or level filter is by definition

\[
Z(t) = \frac{f(t)}{E(t)}
\]

Using the level filter \( Z(t) \) we hoped to adjust for variations in signal amplitude due to differences in relative excitation of the various arrivals between the reference and unknown events which would tend to degrade the performance of the matched filter technique.
When we applied this procedure the S/N ratios of the phased sum outputs from the level filter were lower in all cases than those obtained using the signal itself (see Figure 9), even when the test seismograms were also amplitude equalized. On the other hand, matched filtering using the level filter is superior to simply beamforming the bandpass filtered seismograms, as Figure 9 shows.

Various other filters were tried also; for example, we used the interval between the P and PP arrivals and the interval between PP and SP as filters. These are intervals which should contain PL modes (Alsop and Chander, 1968). Each of these performed less satisfactorily than the filters indicated above.

c) Evaluation of the Performance of the Matched Filter Technique

In an effort to establish some threshold at which the matched filter technique breaks down, we compared the S/N ratios of the phased sum of the matched filter output of nine channels with the phased sum of the individual band-passed (10-50 seconds) traces containing the signal buried at various S/N ratios (8, 4, 2, 1, .75), as shown in Figure 6.

The results from the matched filter program using the 1000 second filter are shown in Figure 7. As can be seen from this figure, the technique breaks down below a S/N ratio of 1. For comparison the results of straightforward beamforming the band-passed traces from the same channels are shown in Figure 8. The shifts applied in the latter case were based on the largest peak in the whole signal as were the peak-to-peak amplitudes.

Figure 9 is a plot of S/N ratio of the matched filter phased sum output versus S/N ratio of the phased sum of the
individual band-passed traces. The figure shows that matched filtering the individual channels prior to beamforming increases the S/N ratio by approximately a factor of 2 over beamforming the raw data. This reflects the factor of 2 gain in S/N obtained by matched filtering a single channel. In addition, the matched filter output serves to identify the initial onset of the body waves, whereas in the case of the phased sum traces (Figure 8) only the largest phase (SP) is clearly visible at the lower S/N ratios.

We also show in Figure 9 the results of matched filtering with the level filter discussed above. On the basis of these results the preferred filter is clearly the signal itself.

OBSERVED DATA

In order to evaluate the performance of the matched filter on real data we looked for a sequence of events meeting the following criteria:

i) A large magnitude event with a high S/N to use as a reference event.

ii) Several other events of lesser magnitudes but with nearly identical epicenters.

From the LASA data available at the SDL, we were only able to select one short sequence of events which met these requirements, and this is only partially complete. These events are from the Kurile Islands region; epicenter data are given in Table 1.

The event on April 1, 1967, at 12:23:35.5 was selected as the filter for the region. The body wave magnitude of this event is given as 5.9 by the USC&GS, and is 6.0 when computed from the North American (NAM) stations. Surprisingly, the body
wave magnitude at LASA, as given in the LASA Bulletin, is only 4.7. This small magnitude is reflected in the long-period seismograms (Figure 10) which show that the amplitude of the initial onset of the P wave is well down in the noise. In spite of the unfavourable S/N ratios we were forced to use the body waves from this event as our filter by the lack of an alternative single event of sufficiently large magnitude.

The events listed in Table 1 included two earthquakes on April 1, 1967, whose origin times were 170 seconds apart. The first of these had a body wave magnitude of 5.7 (USC&GS), 5.3 (NAM), which was reported as 5.3 in the LASA Bulletin; the second 5.5 (USC&GS), 5.2 (NAM) and 5.2 at LASA. The initial onset of the first event is clearly visible (see Figure 11), while the initial onset of the second is not. Even after band pass filtering (10-50 seconds) and beamforming the nine seismograms, the initial onset of the second event is not readily discernible.

Using a 1080-second portion of the reference event as a filter, which interval includes all the body wave arrivals from P through S, matched filtering the individual seismograms from the double event clearly identifies the onset of the first event. On beamforming the nine matched filter outputs, not only is the onset of the first event visible, but a second event is discernible with reversed polarity and lagged by 170 seconds with respect to the first. Although the matched filter detected the onset of the first event, matched filtering and then summing was inferior to plain beamforming in this case, as is shown in Figure 11.

The poor performance of the matched filter suggests that there is considerable mismatch in waveform between the reference
event and the unknown events. In an attempt to test the hypothesis, we shortened the filter, first to 730 seconds and then again in 500 seconds. The 730-point filter, which contains the interval from the initial onset up to and including S, did not improve the signal-to-noise ratio of the phased sum of the matched filter outputs (see Figure 12). However, shortening the filter to 500 seconds, to include the principal P wave arrivals only, improves the signal-to-noise ratio of the phased sum of the nine individual channel matched filter outputs by a factor of 2, though it is still considerably less than the improvement obtained by beamforming the original nine band pass filtered seismograms. Using the shorter filter, it is questionable whether or not we are able to detect the onset of the second event (see Figure 12).

The better performance of the 500-second filter suggests that there are differences between the reference event and the unknown event(s) primarily associated with the later arrivals. Since the later arrivals have larger amplitudes than the initial onsets, we applied the equalized amplitude reference signal as filter. The results are shown in Figure 13. The overall S/N ratio is the same as that for the 1080 second filter. This suggests that there may be significant differences in excitation of the various body wave arrivals between the reference event and the unknown event(s).

In an attempt to minimize the effect of a possible polarity reversal between an arrival in the reference signal and the corresponding arrival in the unknown event we replaced each point in the filter and seismogram traces with its absolute value prior to matched filtering. The original time series \( f(t) \) then becomes \(|f(t)|\).
Hence for cases when an arrival in the unknown event has exactly opposite polarity from the corresponding arrival in the reference event, rectifying both the filter and the raw seismogram trace prior to matched filtering should considerably improve the S/N ratio of the output. Although the rectified filter clearly indentifies to onset of the first event, the S/N ratio of the phases sum filter outputs is much lower than that obtained using the other filters.

The matched filter results for the December 22, 1966, event are shown in Figure 14. The magnitude of this event was variously given as 5.2 (USC&GS), 5.1 (NAM), and 5.4 (LASA). The phased sum of 1080-point filter outputs shows a maximum at the expected arrival time of P. However, there is a certain amount of ringing in of the filter. When the filter is shortened to 730 seconds the output at the expected arrival time of P wave closely resembles the autocorrelation function of the reference signal, although the signal-to-noise ratio of the phased sum matched output is lower. There is also a second autocorrelation-like output some 39 seconds later than the initial onset, with reversed polarity. The sum output from the 500-second filter shows considerable ringing. However, the output peaks at the predicted onset of P. For comparison, we show the results of beamforming the individual band-pass filtered (10-50 second) traces.

Figure 15 shows the matched filter results from the event on June 13, 1967, with body wave magnitude given as 4.6 (USC&GS), 4.5 (NAM), and 4.9 (LASA). All of the filters fail to identify the predicted onset of the event. Included in the figure is the phased sum of the individual band pass filtered traces, which also shows no discernible long-period body waves.
We also tried the various filters on seismograms recorded at LASA from the event on March 29, 1967, which had a body wave magnitude of 4.4 (USC&GS), 4.4 (NAM), and 4.7 (LASA); and that of April 19, 1967, (4.3 (USC&GS), 4.1 (NAM), and 4.0 (LASA)), whose coordinates are listed in Table 1. In all these cases we failed to identify positively long-period body wave radiation coincident with the arrival time for the initial onset, as can be seen from Figures 16 and 17.
DISCUSSION

As can be seen from the preceding sections, the results we obtained in this study are mixed. The synthetic test cases show that when the signal used as the filter is identical with the unknown signal the matched filter technique alone is successful in detecting the unknown signal at a minimum S/N ratio of 4. By beamforming the matched filter outputs from the nine long-period sensors in the A, E, and F rings at LASA, we were able to detect the presence of the unknown signal at S/N ratios as low as 1. The much poorer performance of the matched filter on observed data may be attributed to differences within the wavetrain of the reference event with respect to the unknown event, and/or the relative absence of long-period body wave radiation from the smaller events. In this section we examine these alternatives.

The matched filter technique worked best on the seismograms from the Kurile Island double event. Using the whole suite of arrivals P through S, we were able to detect both events, the second with an apparent polarity reversal. However, the S/N ratio of the matched filter sum trace for the first event was considerably less than that obtained by straightforward beamforming the band passed seismograms. This suggests that there is considerable mismatch in waveform between the reference signal and the unknown signals, even though the events occurred close together within the same source region. Also, the marked improvement in S/N ratio using the shorter 500-second filter suggests that the mismatch becomes more important when the later phases, particularly S arrivals, are included in the filter. The second event is not easily detected from the individual traces, but after beamforming the matched filter outputs it is.

The three events on April 1, 1967, have epicenters that are within 55 km of one another. Furthermore, the epicenters reported by the USCGS are all at the same depth, the P wave delays across LASA being almost identical. Therefore the observed signal mismatch is probably due to differences in relative excitation of the various
arrivals between the events. Theoretically, we would expect this difference to be greatest for the S wave arrivals. However, we were not able to eliminate the effect of the differences using the level or rectified filters, which suggest that the differences may be more complex than simple variations in amplitude or polarity reversals of the arrivals.

The matched filter results from the December 22, 1966, event are interesting. The focal depth for this event is 77 km. It is located some 350 km from the reference event; although the P wave delays across LASA for the two events are equal to within 0.5 seconds. Using the 730-second filter, which includes the P through SKS arrivals, we were able to detect long-period body wave radiation at the appropriate times for both P and PcP. This is probably due to the fact that the P-S interval is of very nearly the same duration as the PcP-SKS interval at this particular distance and focal depth. The S/N ratio of the matched filter outputs is low, and without prior knowledge of the expected arrival time it is doubtful whether the onset would have been recognized, particularly using the 500-second filter.

In the case of the three events with magnitudes less than 5, we were unable to detect any long-period body wave radiation, either by matched filtering or beamforming. This in part may be due to differences in epicenter location, particularly as they are at different depths from the reference events. However, the partial success of the matched filter on the 77 km deep event suggests that more probably these events radiated only small amounts of energy in the pass band of 10-50 seconds.

The failure of the matched filter below magnitude 5 leads to the question of threshold. Obviously the main problems involved are radiation pattern differences and differences in polarity from one event to another. Relative time shifts of arrivals between the reference event and the unknown event will also tend to degrade the matched filter performance, since the wavetrains no longer match exactly. Differences in radiation pattern result in not only mismatch of the
wavetrains, but also in differences in reported body wave magnitudes. Throughout this report we have given magnitudes based on the USC&GS Bulletin, the North American stations reported in this Bulletin, and the magnitudes reported from LASA. The variation indicates the difficulty in assigning a threshold. Our results (see Figure 18) indicate that the threshold for detection is somewhere around $m_b = 5$ for the Kurile Islands in terms of USC&GS magnitudes. It is impossible to express this result in terms of LASA magnitudes since the LASA magnitude ($m_b$) of the reference event as reported is less than 5.

Finally we used the portions of the filter between the principle arrivals as our filter in several instances. These intervals are the ones in which PL modes are most likely to occur. We found no evidence that these sections alone improved our ability to detect long-period energy. Nor did simply shortening this interval to bring the principle phases in the filter into alignment with the predicted arrival times of the unknown event produce any significant improvement in S/N ratio for the smaller ($m_b < 5$) events.

The long-period body wave matched filter technique, although it is not as effective as the surface wave technique in terms of detecting whether or not an event occurred in a particular region, may constitute an important tool for the classification of events. If any long-period body waves are found, their presence would indicate that the event is an earthquake, since long-period teleseismic P waves have not been observed for explosions. Moreover, the method is cheap in terms of computer time and could be incorporated in an on-line detection system. The ultimate goal should be a system in which, once an event has been detected and located by other means, a long-period body wave matched filter could be quickly and inexpensively applied to determine the presence or absence of long-period body wave radiation. Then if any long-period body wave radiation was found the event could immediately be classified as "probable earthquake".
CONCLUSIONS

From the results of this study we conclude:

1. For cases where the unknown signal and the reference signal are very similar in waveform the matched filter works well.

2. For input S/N ratios less than 4, array summing of the individual match filter outputs is required to observe long-period body wave radiation.

3. Ideally, beamforming the matched filter outputs improves the S/N ratio by a factor of 2 (6db) over beamforming the bandpass filtered seismograms.

4. Using observed data, the threshold for detection of long-period body waves from events in the Kurile Islands at depths less than 80 km is probably around magnitude 5 (mL), using 9 elements of LASA. The best results are likely to be obtained when the depth of the reference event and the unknown event are the same.

5. The matched filter technique works best when only the P wave arrivals are used.

6. It would not appear legitimate to include portions of the seismogram after the S arrival time, and attribute the output to the presence of long-period body waves, since we know that higher mode surface waves have S wave velocities as an upper bound and may be present in the record anywhere after the S arrival.

7. Despite several attempts to use only "interphase" (PL) portions as a reference filter we found no evidence that using these PL portions of the record alone improved our ability to discern the presence of long-period energy.

8. Amplitude equalization is not an effective means of improving the performance of the matched filter, probably because weaker portions of the reference signal contain noise which we build up by this procedure.

9. Many more events must be analyzed before a reliable threshold for detection of long-period body waves at LASA can be established.
RECOMMENDATIONS

In view of the partial success of the matched filter technique reported here we recommend:

i) Events from other seismic areas should be studied. For example, we believe that suitable sequences of events could probably be found from Mongolia, the Rat Islands, and possibly Hokkaido.

ii) Other arrays, such as TFO, and the LRSM stations should be used and incorporated into a continental-size array.

iii) A study should be made to determine the relative excitation of long-period body waves as a function of depth and magnitude for particular regions of interest.
REFERENCES


Table 1
Epicenter Data for the Kurile Island Events used in this study

<table>
<thead>
<tr>
<th>Date</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Origin Time</th>
<th>Depth (km)</th>
<th>Magnitude (USC&amp;GS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Apr 1967</td>
<td>45.7°N</td>
<td>151.8°E</td>
<td>12:23:35.5</td>
<td>40</td>
<td>5.9</td>
</tr>
<tr>
<td>1 Apr 1967</td>
<td>45.8°N</td>
<td>151.8°E</td>
<td>05:54:19.1</td>
<td>40</td>
<td>5.7</td>
</tr>
<tr>
<td>1 Apr 1967</td>
<td>46.3°N</td>
<td>152.0°E</td>
<td>05:57:09.0</td>
<td>40</td>
<td>5.5</td>
</tr>
<tr>
<td>22 Dec 1967</td>
<td>48.6°N</td>
<td>154.3°E</td>
<td>19:24:06.5</td>
<td>77</td>
<td>5.2</td>
</tr>
<tr>
<td>13 Jun 1967</td>
<td>47.6°N</td>
<td>154.3°E</td>
<td>02:42:45.1</td>
<td>32</td>
<td>4.6</td>
</tr>
<tr>
<td>29 Mar 1967</td>
<td>44.4°N</td>
<td>148.4°E</td>
<td>10:01:10.2</td>
<td>26</td>
<td>4.4</td>
</tr>
<tr>
<td>19 Apr 1967</td>
<td>45.6°N</td>
<td>150.8°E</td>
<td>10:46:49.0</td>
<td>33</td>
<td>4.3</td>
</tr>
</tbody>
</table>
Figure 1b. Seismograms recorded at WMSO from a nearby earthquake. Earthquake location 50.10°N, 178.2°E. Depth = 28 km. Origin time 04:46:13.1.2. Magnitude = 5.5 (USGS).
Signal Buried in Noise
\[ x(t) = a y(t) + n(t) \text{ S/N} = 4.0 \]

Matched Filter Output
\[ C_{xy}(t) \text{ S/N} = 7.1 \]

Sum Of 9 Matched Filter Outputs \( S/N = 21.0 \)

\[ \text{Onset of Signal} \]

Figure 3. Results from a synthetic test case using the signal itself as filter.
Figure 4. Results from a synthetic test case using the first 740 seconds of the buried signal as filter.
Figure 5. Results from a synthetic test case using the first 270 seconds of the buried signal as filter.
Figure 6. Examples of the synthetic seismograms at various S/N ratios from the subarray AO.
Input S/N of 0.75

Input S/N of 1.0

Input S/N of 2.0

Input S/N of 4.0

Input S/N of 8.0

Figure 7. Phased sum traces of the matched filter outputs (9 channels) from the synthetic seismograms at various S/N ratios using the 1000 second filter.
Figure 8. Phased sum traces (9 channels) of the synthetic seismograms at various S/N ratios.
Figure 9. Comparison of the matched filter results with those obtained by beamforming.
Figure 10. Long-period vertical-component seismograms from the event April 1, 1967, at 12:23:35.5 which were used as filters for Kurile Island events.
Figure 11. Matched filter results from the double event on April 1, 1967, compared to beamforming the bandpass filtered seismograms.
Figure 12. Matched filter results from the double event on April 1, 1967, using the shorter filters.
Figure 13. Matched filter results from the double event on April 1, 1967, using the level and rectified filters.
Figure 14. Matched filter results from the December 22, 1966 event using various filters compared to beamforming the bandpass filtered seismograms.
Sum Of 9, 1080 Point Filter Outputs S/N=2.7

Sum Of 9, 730 Point Filter Outputs S/N=1.9

Sum Of 9, 500 Point Filter Outputs S/N=2.7

Sum Of 9, Level Filter Outputs S/N=3.2

Phased Sum Of 9 Band Pass Filtered S/N=2.5

Figure 15. Matched filter results from the event on June 13, 1967 compared to beamforming the bandpass filtered seismograms
Figure 16. Matched filter results from the event on March 29, 1967 compared to beamforming the bandpass filtered seismograms.
Figure 17. Matched filter results from the event on April 19, 1967 compared to beamforming the bandpass filtered seismograms.
Figure 18. Graph showing S/N ratios obtained by matched filtering as a function of body wave magnitude ($m_h$).
A PRELIMINARY EVALUATION OF THE MATCHED FILTER TECHNIQUE IN THE DETECTION OF LONG-PERIOD BODY WAVE RADIATION

Using nine elements of the Montana LASA, long-period body wave radiation was detected at S/N ratios as low as 1 for synthetic test cases. In these instances beamforming the 9 matched filter outputs improved the S/N ratio by a factor of 2 over straightforward phased summation of the raw data. For real data, the results are less satisfactory. However, from a limited sequence of Kurile Island events we establish an approximate threshold of $m_b = 5$ above which we can detect long-period body wave radiation. The results demonstrate the need for a better understanding of long-period body wave excitation as a function of magnitude and focal depth for earthquakes.