A REVIEW OF SHOCK TUBES AND SHOCK TUNNELS

BY
W. A. MARTIN

CONVAIR-AERONAUTICS
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AND SHOCK TUNNELS

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W. A. MARTIN

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ENGINEERING DEPARTMENT
CONVAIR
A Division of General Dynamics Corporation
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SUMMARY

A review has been made of the basic shock tube along with the various modifications required to produce hypersonic flow of short duration. Modifications to the driver system include multiple diaphragms, area discontinuities at the diaphragm station and combustion drivers. The influence of real gas effects, including shock wave attenuation, has been noted for both the production of strong shocks \((M > 3)\) and the creation of hypersonic flow in an expansion nozzle. Nonreflected, reflected and tailored-interface type shock tunnels are discussed in Section 6 along with their starting problems, real gas effects and Reynolds number and stagnation temperature simulation. Detailed calculations have been omitted for simplicity but many figures have been presented which illustrate the operation of the various shock tube configurations. Further details may be obtained from the references given. The advantages and disadvantages of the various methods of producing hypersonic flow are summarized on the following pages.
## DRIVER SYSTEMS FOR HYPERSONIC SHOCK TUNNELS

<table>
<thead>
<tr>
<th>Type</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td>1. Uniform Tube, Single Diaphragm Station</td>
<td>1. Simplest method</td>
<td>1. Large diaphragm pressure ratio required to produce strong shocks.</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td>1. Overall pressure ratio reduced for a given shock Mach number or a gain in $M_s$ for a given $P_{sh}$</td>
<td>1. Added complexity. Probably not too practical when more than two diaphragms are used.</td>
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<tr>
<td></td>
<td>2. May be combined with buffer gas technique in order to use cold drivers.</td>
<td></td>
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<tr>
<td>2. Multiple Diaphragm Stations</td>
<td>1. Overall pressure ratio less than that of (1) for same conditions.</td>
<td></td>
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<td></td>
<td>2. May be combined with (2) to give an additional gain in performance over (2).</td>
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<td>3. Area Discontinuity at the Diaphragm Station</td>
<td>1. May be used for any of the above methods.</td>
<td>1. Diaphragm pressure ratio required is still relatively high when used with cold drivers.</td>
</tr>
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<td></td>
<td>2. Increases both pressure and internal energy ratio across diaphragm.</td>
<td>2. Driver section must be reinforced due to larger area.</td>
</tr>
<tr>
<td></td>
<td>3. Theoretically, most efficient form of driver</td>
<td></td>
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<tr>
<td></td>
<td>2. May be erratic in operation-detonation is a problem.</td>
<td></td>
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<tr>
<td></td>
<td>3. Difficult to achieve constant volume burning which is most efficient combustion system.</td>
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<td>4. Attenuation greater than cold drivers</td>
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<td>5. Driver section must be reinforced to withstand increased pressure.</td>
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Buffer Gas

1. Enables cold hydrogen to be used as driver without creating combustion at the contact surface.
2. Reduced overall diaphragm pressure ratio for same shock Mach number or increased $M_a$ for same overall $P_{31}$.
3. Combined with (3) to give performance equal to (4).
5. Extends range of tailored interface method for cold hydrogen driver to $M_a = 15$.

1. Added complexity of introducing a third gas.
2. Two diaphragm stations required.
HYPERSOONIC TEST FACILITIES

Type                      Advantages                              Disadvantages

2. Simulate Re and temper- 2. Simulate Re and temperature without any modifications.
   ature without any modifi- 3. Attenuation problems.
   cations.                                                               4. Relatively small models.
2. Divergent Expansion     1. Larger models.                          1. Moderate Re sim-
   Nozzle - Non-reflected    2. Increase in test Mach number for same shock Mach number.
   Method                   3. May be added at exit of type 1 without modifying existing structure.
3. Reflected Method        1. Large models.                          1. Low Re simulation
2. Higher test Mach number for same $M_s$ than types 1 and 2. 2. Tube still relatively long.
3. Attenuation problem only affects production of reservoir conditions so smaller diameter tube may be used.
4. Slightly longer test time than types 1 and 2 for same test Mach number. 3. For same shock Mach number test time reduced due to reflected shock interacting with the contact surface.
Type 3. (contd.)

Advantages

5. Starting pressure ratio less than type 2

Disadvantages

4. Stagnation temperature may be high enough to cause formation of nitric oxide which may be frozen by expansion.

1. Low Re as for type 3.

2. Running times are long enough to cause erosion at nozzle throat.

3. Radiation losses may be important.

4. Driver gas must be heated at higher Mach numbers in order to achieve tailoring (see also Page 14; type 5,)

5. Longer driver section must be longer than for type 3.

6. Driven section must withstand higher pressure than types 2 and 3.

1. Nozzle at diaphragm station must be changed for each test Mach number.

2. Diaphragm pressure ratio is higher than type 4.

3. Other disadvantages as above.

---

Tailored-Interface Method - Unsteady Modification

![Diagram of unsteady modification](unsteady-diagram.png)

- Driver
- Driven
- Nozzle

Channel length may be shorter than for type 2.

Attenuation less due to shorter channel.

---

Tailored-Interface Method - Steady Modification

![Diagram of steady modification](steady-diagram.png)

- Driver
- Driven
- Nozzle

Test times are 25 times that of type 2.

Other advantages as above.

---

1. Large Model.

2. Same test Mach number range as type three.

3. Test times nearly 8 times greater than type 2.

4. Channel length may be reduced.

5. Radiation losses may be important.

6. Driven section must withstand higher pressure than types 2 and 3.
1. INTRODUCTION

During the past five years the shock tube, and its various developments, have achieved prominence as a device for the study of hypersonic flow. This apparatus has an advantage over the wind tunnel in that both Mach number and temperature can be simulated. There is a disadvantage in that total testing times are very short, of the order of one millisecond, so that special instrumentation techniques are required for the measurement of the flow quantities. However, methods have been developed for physical measurements in the short time available and quite acceptable results have been obtained.

It is the purpose of this report to outline some of the more recent developments of the shock tube principle. As these developments are only modifications, a brief resume of the theory and operation of the simple shock tube will also be presented. A more historical introduction is to be found in Refs. 1, and 2 which contain many references to earlier research in this field. A review of more recent advances is given by Hertzberg (Ref. 3) and Glass and Hall (Ref. 4).

2. THE SIMPLE SHOCK TUBE

The simple shock tube consists of a straight tube of uniform cross-section separated by a thin diaphragm which divides the tube into two compartments and enables a pressure ratio to be created between them. One compartment is known as the high pressure chamber, or driver section, the other, the low pressure channel, or driven section. Usually the driver section is pressurized while the driven section may be either
evacuated or at atmospheric pressure. The gases in both sections are normally in thermal equilibrium. Upon rupturing the diaphragm the wave system shown in Fig. 1 is established.

Theoretically, upon removal of the diaphragm, a shock wave is propagated into the low pressure section and the high pressure gas expands into the driven section by means of a rarefaction wave centered at the origin. This is not quite true in practice as the rarefaction wave is not centered at the origin and the shock wave takes a finite time to form from a series of compression waves created when the diaphragm is removed. However, these deviations from the ideal theory are not too important for practical purposes and after the shock has formed it is considered to travel at a constant velocity.

The gas in the driven section is compressed and heated by the shock wave while the driver gas is expanded and cooled by the rarefaction wave. Thus two bodies of gas exist in the shock tube which are brought to the same pressure and have the same particle velocity but due to different thermative processes their temperature, density and entropy are different. These two states are separated by an interface or contact surface which in practice is more of a region rather than a plane surface. This contact surface, or contact front as it is called by some authors, travels at the particle velocity. Therefore, behind the shock there is a region of steady flow at a high pressure and temperature while behind the contact surface the flow is again steady with the same pressure but at a lower temperature. Due to the steady
nature of these states many attempts have been made to utilize them for aerodynamic testing. The particle velocities are identical in each state but due to the lower temperature in (3) the Mach number \(M_3\) is higher than in (2). However, state (2) is more adaptable to hypersonic testing due to temperature requirements and in practice is found to be much more uniform than region (3). \(5\) \(6\) \(7\)

The testing time is determined by the length of the driver and driven sections, the gas combination used and the strength of the initial shock wave. \(2\) If the shock tube is too short the flow will be terminated very quickly by reflected waves from the closed ends. For an infinitely long tube the test time at a station, \(x\) is the interval between the arrival of the initial shock wave and the passing of the contact surface. Therefore, for strong shocks where the contact surface velocity is high, the test section should be as far as possible from the diaphragm station. The total lengths of the driver and driven sections must be chosen so that the reflected shock and rarefaction waves do not return before the arrival of the contact surface. The ideal lengths will be such that the reflected shock and rarefaction meet the contact surface at the test section.

3. **BASIC SHOCK TUBE EQUATIONS**

The states on each side of the rarefaction are related by the isentropic relationships while the normal shock relationships are employed for the transition across the shock front. These two solutions
can be matched at the contact surface and the physical properties of the complete shock tube flow determined. This procedure leads to the following identity which is the basic equation for the shock tube.

\[
P_{i4} = \frac{1}{P_{21}} \left[ 1 - \left( \frac{P_{21}}{P_{i4}} - 1 \right) \frac{\sqrt{\alpha_4 E_{i4}}}{\beta_4} \right]^{\frac{1}{\beta_4}}
\]

Equation (3:1) indicates that the shock pressure ratio \((P_{21})\) is a function of the diaphragm pressure ratio \((P_{i4})\), the internal energy ratio \((E_{i4})\) and the specific heat ratio of the driver and driven gases \(\gamma_4\) and \(\gamma_1\). The requirements for producing strong shock waves, which are necessary in order to achieve temperatures suitable for hypersonic simulation, may be better examined by allowing the diaphragm pressure ratio \((P_{i4})\) to equal zero in the above equation. This gives the following:

\[
(P_{21})_{P_{i4}=0} = 1 + \frac{\alpha_1}{2\beta_4 E_{i4}} + \sqrt{\frac{1}{\beta_4 \beta_1 E_{i4}}} + \left( \frac{\alpha_1}{2\beta_4 E_{i4}} \right)^2
\]

Upon examination of this equation it is found that two methods present themselves for the production of strong shock waves in a simple shock tube. The diaphragm pressure ratio \((P_{i4})\) can be made very large or the energy ratio across the diaphragm \((E_{i4})\) can be made very small, or both. The energy ratio may be reduced by heating the driver and cooling the driven gases, but a more common method is to use a light driver gas and a heavy driven gas. The aerodynamicist usually likes to use
air as a test gas, with possibly nitrogen as a second choice, with helium or hydrogen drivers. Figure 2 gives the diaphragm pressure ratio required for a range of shock pressure ratios for the combination Air/Air, He/Air, and \( \text{H}_2/\text{Air} \) for the simple shock tube case while Fig. 3 presents the variation of shock wave Mach number, \( M_2 \), and particle velocity, \( U_{21} \), with diaphragm pressure ratio, \( P_{41} \) for the three gas combinations.

It can be seen from these two figures that a great advantage is gained by the use of helium or hydrogen as a driver gas. The internal energy ratio can also be reduced by heating the driver gas either by electrical heating or by the ignition of combustible mixtures which give high values of \( T_4 \). The combustible mixture is usually a stoichiometric mixture of hydrogen and oxygen diluted with excess hydrogen or helium.\(^\text{(8)}\) This method has seen widespread use for the production of strong shock waves in shock tubes. Another method used in gun tunnel versions of the shock tube is to compress the driver gas by a piston driven by an explosive process, usually the ignition of a powder charge. This compression raises the pressure and temperature to a predetermined level governed by the strength of the diaphragm which then bursts initiating the flow.

Having established Eq. (3:1) all the other flow quantities can be determined. The most useful equations are listed below in nondimensional notation.\(^\text{(2)}\)
1. Density ratios

\[ \Gamma_{34} = \frac{\rho_3}{\rho_4} = \left[ \frac{P_4}{P_3} \right]^{\gamma_2} \]  \hspace{5cm} (3.3) \]

\[ \Gamma_{21} = \frac{1 + \alpha_1 P_{21}}{\alpha_1 + P_{21}} \]  \hspace{5cm} (3.4) \]

2. Speed of sound and temperature ratio

\[ A_{34} = \frac{T_3}{T_4} = \frac{P_4^{\gamma_2}}{P_3^{\gamma_2}} = \left[ \frac{P_4}{P_3} \right]^{\gamma_2} \]  \hspace{5cm} (3.5) \]

\[ A_{21} = \frac{T_2}{T_1} = \frac{P_1^{\gamma_2}}{P_2^{\gamma_2}} = \left[ \frac{P_1}{P_2} \right]^{\gamma_2} \]  \hspace{5cm} (3.6) \]

3. Velocity of the shock wave

\[ M_s = \left[ \frac{\gamma_1 (1 + \alpha_1 P_{21})}{\gamma_2} \right]^{\frac{1}{2}} \]  \hspace{5cm} (3.7) \]

4. Particle velocity or contact surface velocity

\[ U_{21} = \frac{P_{21} - 1}{\gamma_1 \left[ \frac{\gamma_1 (\alpha_1 P_{21} + 1)}{\gamma_2} \right]^{\frac{1}{2}}} \]  \hspace{5cm} (3.8) \]

\[ U_{34} = \frac{1}{\gamma_4 \gamma_4} \left[ 1 - \left( \frac{P_4}{P_3} \right)^{\gamma_2} \right] = A_{14} U_{21} \]  \hspace{5cm} (3.9) \]

5. Local Mach numbers

\[ M_3 = \frac{U_{34}}{A_{34}} = \frac{1}{\gamma_4 \gamma_4} \left[ \left( \frac{P_4}{P_3} \right)^{\gamma_2} - 1 \right] \]  \hspace{5cm} (3.10) \]

\[ M_2 = \frac{U_{21}}{A_{21}} = \frac{P_{21} - 1}{\gamma_1 \left[ \gamma_1 P_{21} (\alpha_1 + P_{21}) \right]^{\frac{1}{2}}} \]  \hspace{5cm} (3.11) \]
6. Speed of the head and tail of the rarefaction wave

\[ C_{34} = U_{34} - A_{34} \]

\[ C_{44} = -1 \]  

The head of the rarefaction wave travels at the velocity of sound of the driver gas.

It is of interest to examine the limits of these equations for an infinite value of the diaphragm pressure ratio. Table I is a comparison of three different driver gases, air, helium, and hydrogen in combination with air as a driver gas. Table I indicates more than a tenfold increase in shock strength when hydrogen, rather than air, is used as a driver gas. This same increase is obtained in the temperature achieved in the region behind the shock, however, the Mach number in the steady state region (2) is limited by this temperature increase, and even for the ultimate case, where \( P_{14} \) and \( E_{14} \) are both zero, the limiting Mach number \( (M_2) \) is only 1.89. Thus Table I illustrates that although high temperatures may be achieved this Mach number restriction prevents true hypersonic simulation in a simple shock tube. The results shown in Table I are based on perfect gas theory (i.e. \( \gamma = 1.4 \)) which does not hold for the extreme shock pressure ratios calculated.

**4. MODIFICATION OF THE SIMPLE SHOCK TUBE FOR THE PRODUCTION OF STRONG SHOCK WAVES**

4.1 INTRODUCTION

It is possible to improve the performance of the simple shock tube by suitable modifications. These modifications include the installation
or additional diaphragm stations and driver sections whose cross-sectional area is greater than the driven section. The former method is a device which sacrifices pressure ratio in favor of an increased temperature ratio. The latter creates an additional expansion at the diaphragm station which converts thermal energy into kinetic energy. Therefore, in both cases, it is possible to produce the same shock strength as a uniform tube but with a smaller diaphragm pressure ratio, or conversely, to produce a stronger shock for the same diaphragm pressure ratio.

Another method, which increases the temperature ratio across the diaphragm, is to use a combustible mixture as a driver gas. Rather than use combustible mixtures it is sometimes more satisfactory to use either cold helium or hydrogen as a driver, as mentioned in Section (3). A further modification of this technique has been to introduce an inert buffer gas between the hydrogen driver and the air test gas with an area discontinuity at the buffer-air diaphragm station. These methods will now be examined in more detail.

4.2 MULTIPLE DIAPHRAGM SHOCK TUBES

Figure 4 is a sketch of a shock tube of constant area utilizing two diaphragms. \( D_1 \) is the primary diaphragm which is ruptured first producing the shock \( S_6 \). This shock wave is reflected at diaphragm \( D_2 \) bringing the gas in state 5 to rest and raising its temperature and pressure. After a short delay, diaphragm \( D_2 \) is removed and a new wave system is produced resulting in the shock wave \( S_1 \) which is of greater strength than the shock, \( S_6 \). Comparing Fig. 1 and Fig. 4 for the same overall temperature and pressure, and with \( T_1 = T_6 = T_8 \), \( V_1 = \frac{V_6}{V_8} \),
will give an idea of the increase in shock strength for the double diaphragm case. As the gas is brought to rest ahead of \( D_2 \) the pressure and temperature ratios of interest in both cases are \( P_{41} \) and \( T_{41} \). However, \( P_{4d} < P_{4s} \) while \( T_{4d} > T_{4s} \) which gives a shock of greater strength than the system shown in Fig. 1. As an example of the gain to be expected by the addition of diaphragm \( D_2 \) consider the case where the overall pressure ratios, \( P_{41} \) (Fig. 1) and \( P_{61} \) (Fig. 4) are both equal to \( 10^3 \). Then from the Air/Air curve of Fig. 3, \( M_6 = 3.14 \). For the case shown in Fig. 4 let \( P_{61} = 10 \) which means \( P_{66} = 10^2 \). The initial shock wave Mach number, \( M_{66} \) will be found using Fig. 3 and is 2.37. The final overall pressure ratio will be \( P_{61} = P_{46} X P_{61} \). The value of \( P_{46} \) is obtained from interpolation of Table III in Ref. (9) \( (P_{46} = 25.85) \) while \( T_{46} = 3.236 \). Having determined these new initial conditions, the final shock Mach number, \( M_5 \) (=3.77) can be obtained by interpolation from Table I Ref. (9). It is seen that a gain of 20% in shock Mach number is obtained by the addition of the diaphragm, \( D_2 \). The intermediate pressure, \( P_6 \) can be adjusted so as to give a maximum shock Mach number for a given overall pressure ratio, \( P_{61} \).

The above method may be somewhat simplified by making \( D_2 \) weak enough that the initial shock wave, \( M_{36} \) will rupture it on contact. This produces an unsteady expansion from state 6 to state 1 resulting in a less efficient process than the reflected shock type described above, but reducing the complexity of the system caused by the introduction of a mechanical or electrical delay. The wave diagram for this type of shock tube is shown.
in Fig. 5. The analysis for this case is carried out in Ref.

Carrying out the same calculation for the same conditions as used for the example of the reflected shock type of double diaphragm shock tube gives a final Mach number, \( M_{51} \), of 3.73 which is only a decrease of approximately one percent. This variation would be larger at higher Mach numbers but the percentage decrease would probably never be large enough to warrant adopting the delay type installation for a straight shock tube.

Ref. 9 indicates that for the maximum gain in \( M_s \), the pressure must be approximately the geometric mean of the pressures on either side. Then for a multiple diaphragm shock tube the overall pressure ratio is given by \( P_0 = P_1^n \), where \( P_1 \) is the individual pressure ratio across each diaphragm and \( n \) is the number of diaphragms. It can be shown (Ref. 9) that there is a maximum shock Mach number attainable irrespective of the number of diaphragms used and depending only on the overall pressure ratio, across the shock tube. This relationship is \( M_s \text{ Max.} = P_0^{3/14} \) and Table II gives the values of \( M_s \text{ Max.} \) attainable over a range of reasonable overall pressure ratios for the conditions of equal \( \beta \) and \( T \) throughout. It is also shown in Ref. 9 that a multiple diaphragm system, where all the diaphragms except the first have zero pressure difference across them, is another method for creating strong shock waves. If a shock wave of Mach number greater than 2.67 is generated by rupturing the first diaphragm then, theoretically, the maximum obtainable shock speed increases without limit. In this system the shock wave reflecting off each successive diaphragm will increase.
the pressure and temperature ahead of that diaphragm producing a successively stronger shock.

4.3 AREA DISCONTINUITIES

As outlined in Section 2 the basic shock tube comprises a driver and driven section of uniform cross-section separated by a diaphragm. However, many authors have shown that there is a gain in the final shock strength by introducing an area discontinuity at the diaphragm station. This arrangement may also be combined with the double diaphragm method to give a further increase in the final shock strength. This latter method is illustrated in Fig. 6. The area change may also be accomplished by a transition section as shown in Fig. 7.

In Fig. 6 if diaphragm $D_2$ is eliminated a single shock system is set up at $D_1$, with $S_5$ being replaced by a rarefaction wave as in the case of a uniform tube. If the shock $S_4$ is such that $M_3 < 1$ then the wave system will be as shown in Fig. 1, the flow being accelerated subsonically to $M_3$. However, if $M_3 > 1$ the gas in (6) is accelerated to sonic velocity at the diaphragm station and is then further expanded by a rarefaction wave to the supersonic Mach number, $M_3$. The addition of the diaphragm, $D_2$ combines the multiple diaphragm and variable geometry type using a convergent transition section which may or may not contain a convergent-divergent nozzle. However, the only advantage here is a smoother flow from the driver to the driven section as the flow processes are the same whether the area change is gradual or discontinuous.
As in the previous section a comparison will be made between the uniform and the variable geometry shock tubes by determination of the final shock Mach number achieved for identical overall pressure ratios. An analytical expression for such a gain factor is not possible but by use of tables and graphs it is possible to determine these values of \( M_{a1} \). We will only consider the case of \( M_{a1} \) which corresponds to the range of shock strength required for hypersonic testing. Consider first the case where the diaphragm, \( D_2 \) is not present and that \( A_{3b} = A_1 \) is the minimum section such as shown in Fig. 7. Reference (10) develops the following expression for the overall pressure ratio, \( P_{41} \).

\[
P_{41} = \frac{P_{31}}{g} \left[ 1 + \frac{y_4 - 1}{2} M_3 \right] \frac{2y_4}{4} = \frac{P_{31}}{g} \left[ 1 - \frac{\gamma_2}{\gamma_1} \frac{A_2}{A_1} \frac{y_4 - 1}{2} \right] \frac{2y_4}{4} \tag{4:3:1}
\]

where \( g \) is an equivalence factor given by

\[
g = \left( \frac{2 + (y_4 - 1) M_{3b}^2}{2 + (y_4 - 1) M_{3b}^2} \right)^{\frac{1}{2}} \left( \frac{2 + (y_4 - 1) M_{3a}^2}{2 + (y_4 - 1) M_{3a}^2} \right) \frac{2y_4}{4} \tag{4:3:2}
\]

As \( A_{3b} \) is the minimum section, \( M_{3b} \) will be equal to unity but \( M_{3a} \) must be determined in order to calculate \( g \). Knowing the area ratio, \( A_4/A_1 \) then \( M_{3a} \) is determined from the following relationship.

\[
\frac{A_4}{A_1} = \frac{M_{3b}}{M_{3a}} \left[ \frac{2 + (y_4 - 1) M_{3a}^2}{2 + (y_4 - 1) M_{3b}^2} \right]^{\frac{y_4 + 1}{2(y_4 - 1)}} \tag{4:3:3}
\]
It is seen from Eqs. (4:3:2) and (4:3:3) that for a given area ratio $g$ is a constant when $M_3 > 1$. Therefore knowing the area ratio, $g$ can be determined and a plot of $P_{h1}$ vs. $P_{21}$ can be obtained. A comparison of the shock Mach numbers attainable for area ratios of 1, and $\infty$ is presented in Fig. 8. Table 3 is taken from Ref. (10) and gives the maximum shock Mach numbers when $P_{h1} \rightarrow \infty$ for various gas combinations, four area ratios and two temperature ratio. Both Fig. 8 and Table 3 illustrate that the gain in shock strength is not too great even when the area ratio is infinite. Once again, however, the gain achieved by increasing the sound speed in the driver gas is indicated by Table 3.

As an example of the gain to be expected, through the use of an area change at the diaphragm, consider the example used previously. In this instance the area ratio will be 3:1, the diaphragm pressure, $P_{h1} = 10^3$ for an Air/Air combination with a constant initial temperature ratio, $T_{h1}$, which gives a final shock Mach number of 3.35. Combining the multiple diaphragm method (two diaphragms) with the same area discontinuity and total pressure ratio and with $P_{86}$ (Fig. 6) = $10^2$ gives $M_3 = 3.9$. This is an increase of 16 per cent due to the addition of $D_2$ to the system, and an increase of nearly 25 per cent over a simple shock tube with the same $P_{h1}$.

The above example is for an air driver but we have seen from Fig. 3, and Tables 1 and 3, that a light gas such as hydrogen or helium is a more efficient driver. However, when hydrogen is used to produce shocks confliction occurs at the driver-driven interface which sets up severe
flow disturbances. Helium could be substituted as a driver but it is not as efficient. A method to eliminate this combustion problem is the use of a double-diaphragm shock tube with an inert buffer gas between the driving hydrogen and the driven air. There is also an area change incorporated at the diaphragm separating the buffer gas and the air. Figure 9 shows the effect of varying the pressure and molecular weight in various buffer gases for an overall pressure ratio of $10^{14}$. It is seen from the figure that a shock Mach number of approximately 15 can be obtained by the use of any of the inert gases given on the figure, however, the lighter gases such as helium require a large pressure ratio across the downstream diaphragm. Consider as an example a shock tube of this type with argon as the buffer gas. In order to produce a shock Mach number of 13 a simple shock tube using hydrogen as a driver would require a $P_{\text{down}}$ of $7 \times 10^4$. If a large area discontinuity is introduced the diaphragm pressure ratio is reduced to $1.8 \times 10^4$. When an argon buffer is introduced between the driver hydrogen and driven gas an overall pressure ratio of only $3.5 \times 10^3$ is required which is a reduction by a factor of 20. This method seems to offer very good possibilities of producing strong shock waves using cold driver gases.

4.4 CONJUSTION DRIVERS

A common method of producing strong shock waves is by using a combustible mixture which is ignited to give high pressures and temperatures in the driver. A typical composition for a combustible driver is 13.33 per cent hydrogen 6.67 per cent oxygen and 80 per cent helium as a diluent. The ignition of the hydrogen and oxygen raises the
temperature and pressure of the helium which gives a large increase in
the energy and pressure ratios across the diaphragm. Excess hydrogen
has also been used as a diluent. Using this type of driver the diaphragm
is usually burst by the combustion of the mixture. Combustion may be produced
in three ways. A spark plug, or plugs, may be inserted in the driver
section which is fed by a high voltage coil, producing the spark which
causes ignition. A heating wire may be inserted the length of the high
pressure chamber and a large current passed through it which causes a
constant volume burning. The mixture may be allowed to ignite itself
by introducing the combustible gases and then adding the diluent until
the diaphragm ruptures, whereupon the hydrogen-oxygen mixture will ignite.
It has been found that the constant volume process is the most efficient.
There is also a practical limit to which the driver gas can be heated due
to molecular dissociation at the higher temperatures. Figure 10 compares
the theoretical shock Mach numbers obtained over a range of diaphragm
pressure ratios for helium, hydrogen and combustion drivers. Curves are
presented for both the simple shock tube, \( A_1/A_1=1 \) and one where \( A_1/A_1=6.25 \).

5. **DEVIA TIONS FROM IDEAL FLUID THEORY**

5.1 **REAL GAS EFFECTS**

In the previous chapters the discussions and calculations have been
carried out assuming that the specific heat ratio, \( \gamma \) remained constant.
However, for shock Mach numbers greater than three the temperatures pro-
duced are high enough to invalidate this assumption and the variations
from an ideal gas must be considered. \( (14) \) \( (15) \) Above \( M_0 = 3 \) fairly high
temperatures will be achieved in state (2) and very low temperatures in
state (3). For an $M_2$ of 4.0, with an Air/Air combination, the temperature $T_3$ will be 400K which is below the boiling point of nitrogen and oxygen. In state (2) the temperature, $T_2$ will be over 1200K on the basis of the ideal theory. At these temperatures the specific heat ratio, can no longer be taken as 1.4 and above $M_3 = 3$ the gas can no longer be considered an ideal fluid and the real gas effects must be considered.

We are concerned primarily with state (2) behind the shock, as this will be our test gas, so our discussion will be limited to this region. The internal energy content of a diatomic gas is made up as follows:

1) The kinetic energy of translation of a molecule
2) The energy of molecular rotation
3) The energy of molecular vibration
4) The energy of molecular dissociation into atomic groups
5) The energy of electronic excitation
6) The energy of ionization

As the temperature is increased the higher rotational and vibrational levels are excited absorbing heat in the process. This causes an increase in the heat capacity which in turn increases the specific heats, $C_p$ and $C_v$. However, as they both increase about the same amount then the ratio, $\gamma$ will be smaller. Added to this effect the specific heats, and the molecular weight, become pressure dependent. Above 2000 - 3000K dissociation occurs giving nitrogen, oxygen and their atomic states. These temperatures are also high enough to cause
chemical reactions such as the formation of nitric oxides. At still higher temperatures ($\sim 6000^\circ$K) electronic excitation and ionization occur and the air becomes almost completely dissociated giving atomic oxygen and nitrogen as before, small percentages of the various oxides of nitrogen, electrons, negative $O_2$ ions and positive ions of nitric oxide. These conditions would be the same as those produced at the stagnation point of an insulated blunt body traveling at Mach number of 16 at an altitude of 60,000 feet.

The transition through a strong shock wave can be qualitatively described as follows. At the shock front the temperature jumps to a high value as active degrees of freedom, translation and rotation are excited almost immediately in the distance of a few mean free paths. After this initial jump thermal equilibrium will be reached in an exponential manner as energy is transferred to the remaining degrees of freedom during the relaxation period. Therefore, besides the variation in the flow properties, some consideration must be given to the relaxation time to insure that equilibrium conditions exist for the test to be performed.

Figure 11 indicates the variation of the temperature $T_2$ with shock Mach number, $M_2$, for two values of the initial pressure. It is seen that the real gas temperatures are lower at the higher Mach numbers as expected from the preceding discussion. Figure 12 compares the density ratio for a real and an ideal gas over the same range of $M_2$ as used in Fig. 11. Here the real gas density is higher than the
ideal gas case which approaches a limit of 6 while at $M_e = 12$ the real
gas density ratio continues to increase. Due to the increase of density
and decrease of temperature the pressure is found to be very near the
same for the real gas as for the ideal gas. Therefore, Equation 3.1
will give a fairly good approximation for real gas calculations.

The decrease in temperature behind the normal shock in a real gas
is more pronounced than the change in specific heat ratio or the decrease
in molecular weight. Therefore, the speed of sound in a real gas is lower
for the same shock strength than for the ideal case giving a higher
attainable Mach number, $M_p$ behind the shock. This is illustrated in Fig.
13. The maximum value of $M_p$ for the ideal case is 1.89 but Fig. 13 indi-
cates that an $M_p > 3$ can be obtained for a real gas. As the Mach number of the
flow in region (2) is increased while the temperature and speed of sound are
decreased then the particle velocity must be changed very little by the
real gas effects. This is found to be the case with the real values of
$U_2$ being only slightly greater than the ideal value.

The real gas curves shown in Figs. 11, 12 and 13 were computed using
the charts and tables contained in references 18 and 22. The crossover
of the two real gas curves in Fig. 13 is due to the identical behaviour
of the speed of sound ratio with increasing temperature. The ideal gas
curves were computed from Equations 3.4, 3.6 and 3.11. References 18 and 22
are two examples of the many excellent sets of tables and graphs available
for the determination of the flow properties behind strong shock waves. (16-23)
Table III indicated that there was no appreciable gain in performance by introducing an area change at the diaphragm station. However, these calculations were based on the driver gas being ideal. It has been shown that if the driver gas density is increased to a very high value then there is a considerable gain in performance by using a large driver-driven area ratio. This difference is due to the behaviour of the attractive and repulsive intermolecular forces. At normal densities these forces are negligible and the performance of a uniform and non-uniform tube are nearly identical. However, if the driver density is increased to a very high value there is a considerable advantage in increasing the chamber area as the repulsive forces then predominate. It was found in Ref. 24 that an area ratio of five was nearly equivalent to an infinite area ratio, while a chamber twice the area of the channel would give half the performance promised by the infinite value for the case of nitrogen and hydrogen drivers at pressures of 6000 and 2200 atmospheres respectively. It was found that at intermediate densities the performance relative to an ideal gas would be almost the same for the non-uniform tube and considerably reduced for the uniform tube. The test time for identical length chambers is reduced when using high density drivers due to the increase in sound speed, $a_1$, which means that the rarefaction waves reflected from the end at the high pressure section will travel at a higher velocity. Increasing the chamber length would recover this loss in test time.
5.2 SHOCK WAVE ATTENUATION

The ideal shock tube theory predicts that when the diaphragm breaks a flow will be set up as shown in Fig. 1. According to this theory a plane shock wave is produced which travels down the tube at a constant velocity followed by two steady states separated by a contact surface. In practice however, this model is not realized. Instead the shock wave velocity is found to decrease with distance from the diaphragm. In fact the shock does not reach its calculated speed until a short distance from the diaphragm due to the formation process. The shock only attains the ideal value for weak shocks, for as the diaphragm pressure is increased the shock never reaches the theoretically predicted speed, rising to a maximum some distance from the diaphragm and then attenuating as it progresses down the tube. This effect is also more pronounced as the shock tube diameter is decreased.

Due to this shock attenuation, the flow quantities in states (2) and (3) are not steady but vary with time as the shock progresses down the tube thereby making testing in these regions very difficult. The problem of shock attenuation has received a great deal of attention in the last five years. Many of these investigators have focussed their attention on the viscous effects which, of course, are not allowed in the ideal theory. There have been studies made of the boundary layer both theoretical and experimental to determine the causes of the attenuation. A satisfactory explanation has not been arrived at although the theories of Trimi and Cohen and Kirels and Braun give good agreement at lower Mach numbers. There is probably some influence of the diaphragm station especially at the higher shock strengths where
metal diaphragms are used and take a finite time to open. This effect may be instrumental in reducing the velocity \( U_3 \) which must match \( U_2 \) at the contact surface. The complete flow can be considered to be governed by conditions at the contact surface therefore a reduction in the velocity \( U_2 \) will produce a reduction in the shock speed.

The detailed processes of attenuation are not too important in the operation of a shock tunnel but it is desirable to have as little attenuation as possible so that the flow conditions will be fairly steady during the test period. Considerable work has been carried out to determine what initial conditions will give the least attenuation. In addition to the causes given above it has been generally found that a greater attenuation is experienced as the sound speed of the driver gas is increased. Figure 14 shows typical attenuation measurements for several driver gas compositions. The results for helium give the least attenuation but this driver gives the lowest initial shock strength. The combustion driver, which is the most efficient type, produces the highest initial shock strength but at \( L/D \) of 150 the shock has attenuated to a lower speed than the helium driven shock. It is necessary in both straight shock tubes and non-reflected type shock tunnels to have as long a driver section as possible in order to have reasonable test times. The attenuation could be alleviated by increasing the diameter thus reducing the \( L/D \) ratio but there is a limit to this due to structural and handling reasons.

Experimental evidence has shown that the attenuation is inversely proportional to the shock tube Reynolds number \( (\frac{\rho_l D}{\mu_1}) \). For a given shock tube using air as the test gas the Reynolds number can
only be adjusted through the kinematic viscosity or more precisely by adjusting the initial pressure, $P_1$. From the attenuation standpoint, it is desirable to have $P_1$ as large as possible. However, if cold drivers are used, which seems desirable from a study of Figure 14, a low initial pressure will be required to produce the diaphragm pressure ratio to give the same shock strength. These requirements are at cross purposes and, as usual, a suitable compromise must be found in order to reduce the attenuation effects.

6. METHODS OF GENERATING HYPERSONIC FLOW

6.1 EXPANSION NOZZLES

It has been shown in the previous section that, although the proper temperature may be achieved in a straight shock tube, the maximum Mach number is too low for hypersonic simulation. In order to overcome this difficulty the hypersonic shock tunnel was developed. By placing an expansion nozzle at the downstream end of the conventional shock tube the flow behind the shock can be expanded to a higher Mach number. Therefore, by a proper choice of shock strength and expansion ratio it is possible to simulate both temperature and Mach number. Figure 15 presents the shock tube area ratio required over a range of test section Mach numbers.

The simplest type of installation is a single stage expansion nozzle placed at the exit of the driven section of the tube, the final Mach number being determined by the area ratio. This system is known as the nonreflected method. In Ref. (35) a set of mahogany liners were installed in the shock tube with the nozzle located at the
existing test section. Using this simple scheme a study was made of the flow in the nozzle. Figure 16 illustrates the wave system set up in a shock tube of this type. As the initial shock wave, \( S_1 \) enters the nozzle it weakens locally and develops curvature which propagates from the wall to the centerline of the tube as the shock moves into the test section. As the shock weakens the velocity of the particles behind decreases, however, the particles originally set in motion by the passage of the shock down the uniform tube, \( (A_0) \) accelerate as they pass through the expansion waves at the mouth of the nozzle and increase in velocity. Therefore, there is another shock created which separates the two flow regions 2 and 4, (Fig. 16). Due to the entropy being different behind each shock there is also a contact surface formed which separates the regions 2 and 3, (Fig. 16). The wave \( S_2 \) attempts to propagate upstream but as it is moving against a supersonic flow it is swept downstream through the nozzle. As \( S_1 \) moves through the nozzle it weakens which, in turn, causes \( S_2 \) to become stronger. If \( S_2 \) grows sufficiently strong it will not be swept out of the nozzle but will remain standing in the nozzle preventing the tunnel from starting.

Shock tunnels have been built which use double and triple expansion nozzles for the generation of hypersonic flow. This system enables the test section area to be reduced over that required for the system shown in Fig. 16. In order to obtain reasonable testing times at hypersonic Mach numbers it is necessary to have a fairly long shock tube. However, with long tubes the boundary layer built up on the walls may be quite
thick causing severe attenuation problems. A method of circumventing this is to increase the diameter but this in turn means increasing the test section area of the driven section. By introducing a two step nozzle, which gives two stage expansion and also drains off the boundary layer created behind the initial shock wave, the test section area can be decreased. This type of shock tunnel is illustrated in Fig. 17.

6.2 STARTING PROBLEMS

The flow duration is the time between the departure of the starting wave system $S_1$, $C_2$, $S_2$, Fig. 16, from the test section and the arrival of the initial contact surface. In order to make this testing time as long as possible it is necessary to ensure that this wave system is swept through the test section in as short a time as possible. The governing factor is the strength of $S_2$ which for ideal starting must be a sound wave. By placing a diaphragm at the nozzle entrance the test section can be evacuated to a pressure much below the pressure in the uniform tube which will in turn strengthen the initial shock wave when it enters the nozzle. Therefore the velocity difference across $S_2$ will be reduced and in the limit $S_2$ will become a sound wave. If the initial pressure in the nozzle is decreased still further expansion waves will be formed moving towards the nozzle entrance. However since expansion waves move at the local speed of sound relative to the flow there is no gain in reducing the pressure ratio across the nozzle diaphragm below that value required to make $S_2$ a sound wave. Fig. 18 gives the pressure ratio required across the nozzle diaphragm to ensure perfect starting for a range of test section Mach numbers. This figure
indicates that an extremely large pressure ratio is required to obtain a perfect start for Mach numbers greater than 10.

Figure 19 shows the effect of reducing the pressure ratio across the nozzle diaphragm thus giving an imperfect start. In this case the wave $S_2$ (Fig. 16) will stand in the nozzle and suggests the possibility of placing a plenum chamber downstream of the test section. This chamber will create an expansion wave which will then move upstream, weaken the shock $S_2$ and pull it through the test section. A schematic drawing of the arrangement is shown in Fig. 20 while Fig. 21 indicates the pressure ratio required of the plenum chamber for a range of hypersonic Mach numbers in the test section.

A plenum chamber may also be employed when the shock $S_2$ is not standing in the test section. In this case the expansion waves from the plenum will further accelerate the starting process and thus increase the test time. If the pressure in the nozzle and test section were low enough to ensure a perfect start, addition of a plenum chamber would not improve starting, however, the addition of a plenum is advantageous in another way. For a perfect start, $S_1$ (Fig. 16) passes through the test section at a high velocity and a long test section is required to prevent the reflected wave returning and destroying the flow before the arrival of the original contact surface, $C_1$. By adding a plenum chamber at the end of the test section this reflected shock can be greatly weakened. Therefore, by using a plenum of large cross-sectional area and shock length the facility may be made more compact and even reduce the pumping capacity required.
6.3 THE REFLECTED METHOD

If, instead of the straight expansion nozzle, the driven section is terminated by a regular convergent-divergent hypersonic nozzle a reflected type shock tunnel results. At the hypersonic Mach numbers of interest the area ratio is so large and the throat area so small that the nozzle will act as a solid plate and the initial shock wave will be completely reflected. This means that the gas following the shock will be compressed a second time and brought to rest creating a high pressure, high temperature, stagnant reservoir of air to be expanded through the nozzle. The reflected shock governs the duration of steady flow, interacting with the contact surface and returning to the nozzle. A wave diagram and sketch of this method is shown in Fig. 22.

The reflected method has the advantage that hypersonic flow is initiated from a reservoir with almost constant supply conditions. Therefore, the problems of shock attenuation, which would constantly vary the test conditions, and boundary layer build up behind the initial shock wave are not present. For the same temperature requirements the initial shock may be weaker than in the nonreflected case, or conversely, higher stagnation temperatures are obtained for the same shock strength. The testing time for the same Mach number is also slightly increased due to the decrease in required initial shock strength.

The starting problems with this type of tunnel are essentially the same as that of the nonreflected type and a diaphragm at the nozzle is
desirable in order to reduce the starting time. In the nonreflected case the entrance Mach number may be as high as 1.89, based on perfect fluid theory, however, for the reflected type the Mach number will be unity at the throat. Fig. 23 presents the pressure ratio required across the nozzle diaphragm over a range of test section Mach numbers. Comparing this figure with Fig. 19 it is seen that there is a reduction in the pressure ratio required for starting in the reflected case.

6.4 THE TAILORED-INTERFACE MODIFICATION

The test time in the reflected shock type of tunnel is limited to the time interval between the arrival of the initial shock wave at the nozzle and the return of the secondary wave generated by the shock-contact surface interaction. A considerable increase in testing time is possible if the reflected shock can be made to pass through the contact front without producing any additional waves. This type of tunnel derives its name from this impedance matching or "tailoring", and two examples are illustrated in Fig. 24. If a second convergent-divergent nozzle is placed at the initial diaphragm station then the supersonic portion of the rarefaction wave can be eliminated and the reflected shock will traverse the full length of the driven section before returning to the downstream nozzle. In this case the driver section must be long enough to ensure that the reflected rarefaction wave does not reach the downstream nozzle before the reflected shock wave returns. One disadvantage of the method is that the area ratio of the nozzle at the initial diaphragm station must be changed for each Mach number.
If the reflected shock wave passes through the contact front without creating any additional waves the driver and driven gases must be at equal pressure and velocity behind the reflected shock. As the driver and the driven gases already are at equal pressure and velocity ahead of the reflected shock wave, the pressure ratio and velocity difference across the reflected shock are the same for the two gases. This may be stated mathematically with the aid of Fig. 24.

The conditions for nonreflected waves are as follows.\(^{(40)}\)

\[
\frac{P_1}{P_2} = \frac{P_2}{P_3} = \frac{P}{P} \quad (6:4:1)
\]

\[
\frac{u_x - u_d}{a_x} = \frac{u_x - u_d}{a_x} \quad (6:4:2)
\]

In the second of these two equations we have a Mach number and the speed of sound. In this case the strength of the incident and transmitted shocks are equal only if expressed in terms of their pressure ratios. The change in velocity across the shock may be expressed in the following manner.

\[
\Delta u = \left[\frac{2}{y(y-1)}\right]^{\frac{1}{2}} \frac{P - 1}{\left(\frac{y + 1}{y - 1} P\right)} \quad (6:4:3)
\]

Combining \((6:4:1)\) and \((6:4:2)\) gives

\[
\frac{\gamma_x (\gamma_x - 1)}{a_x^3} \left(1 + \frac{\gamma_x + 1}{\gamma_x - 1} P\right) = \frac{\gamma_y (\gamma_y - 1)}{a_y^3} \left(1 + \frac{\gamma_y + 1}{\gamma_y - 1} P\right) \quad (6:4:3)
\]
This equation must be satisfied whenever a shock passes through a contact surface without creating a reflected wave. For equal values of \( \gamma \) the equations then depend solely on the speeds of sound in states (2) and (3). This would be the case for a \( \text{He}_2/\text{Air} \) combination. Therefore knowing the initial pressure \( p_1 \), the shock wave Mach number and the driver gas then the required initial pressure and temperature of the driver gas can be calculated.

Figures 25 and 26 give the initial diaphragm pressure ratio and the initial driver gas temperature required for tailoring over a range of test Mach numbers for helium and hydrogen drivers. It is seen from Fig. 26 that helium at room temperature will tailor at a Mach number of 7.5, hydrogen at 10.6. In order to use the tailored interface technique at higher Mach numbers the driver gas must be heated. Figure 27 gives the variation of flight Mach number with shock Mach number for these conditions. Another disadvantage of the steady configuration, indicated by the figures, is the increased pressure and temperature of the driver gas required for this modification. However the steady tailored interface method produces test times three times greater than the unsteady modification and twenty-five times the nonreflected method. A comparison of testing times is shown in Fig. 28.

6.5 REAL GAS EFFECTS IN A HYPERSONIC SHOCK TUNNEL

In Section 5 the real gas effects in the production of strong shocks were considered. A brief consideration will be given here of the real gas effects on the design of a hypersonic shock tunnel, using an expansion nozzle. Figure 29 indicates the variation of test section
Mach numbers for a range of shock Mach numbers for a real gas, calculated for an equilibrium test section stable temperature of 218°K, corresponding to the isothermal atmosphere. The curve for the ideal gas case is also shown for comparison, and is approximately 6 per cent lower than the real gas results. The density variation is shown in Fig. 30 and the real gas values are decreased compared to the ideal gas. Figure 31 presents the area ratio variation and, as would be expected from the density results, the real gas area ratio is considerably increased. In the reflected type the temperatures ahead of the nozzle may be high enough for dissociation to occur. When the gas is expanded through the nozzle the cooling rate will be very high and some of the constituents such as NO may be frozen.

Another problem encountered using the reflected method is the loss of energy by radiation from the hot gas behind the reflected shock, which becomes more important as testing times are increased. These increased test times are possible using the tailored interface modification and so the radiation losses will have to be considered in a shock tunnel of this type.

6.6 REYNOLDS NUMBER AND STAGNATION TEMPERATURE SIMULATION

In order to do hypersonic testing it is desirable to simulate free flight conditions as closely as possible. This means simulating Mach number, stagnation temperature and Reynolds number. In the straight shock tube it is possible to simulate the temperature and Reynolds number but the Mach number is limited to a value of three as shown in Fig. 13. The use of expansion nozzles at the downstream end of the driven section
enables the flow to expand to a high Mach number but in doing so may limit the other two conditions. The correct Reynolds number and stagnation temperature are controlled by the initial condition before the test gas enters the nozzle and thereby the strength of the initial shock wave. Figures 32 and 33 show the Reynolds number variation with test Mach number for a perfect gas, \( \gamma = 1.4 \), and a test section static temperature of 518.4°R for a range of pressures behind the initial shock wave. Due to the decrease of density caused by the real gas effects (Fig. 30) the actual Reynolds number will be lower than those shown. For a given static pressure in the driven section, higher Reynolds numbers are realized with the nonreflected technique. Even though the Reynolds numbers shown here are lower than those possible in region (2) of a straight shock tube this may not be important as the Reynolds number of a body traveling at a hypersonic Mach number (\( \approx 20 \)), at high altitude (100,000 ft.), is relatively low (2 X 10^6/ft.). Therefore the attainment of a high Reynolds number will not be difficult for this class of problems.

**CONCLUDING REMARKS**

This report has attempted to outline the basic theory of the shock tube and describe most of the modifications to the basic tube that have been devised for hypersonic testing. Detailed calculation and theory have been omitted but these may be found in the many references cited.
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Figure 1 serves to define the flow regions in the basic shock tube. The notation Air/Air, He/Air and \( \text{H}_2/\text{Air} \) denotes a gas combination across the diaphragm, the left-hand symbol representing the gas in the driver section, the right-hand symbol the gas in the driven section.

- **a**: Velocity of sound
- **A**: Area
- **A_{ij}**: Dimensionless speed of sound ratio (i.e. \( A_{ij} = a_i/a_j \))
- **C_{ij}**: Dimensionless rarefaction wave speed
- **D**: Diameter
- **E_{ij}**: Dimensionless energy ratio \( (C_vT)_i/(C_vT)_j \)
- **g**: Equivalence factor (Eq. 4:3:2)
- **L**: Length
- **M_s**: Shock Wave Mach number based on speed of sound of gas into which it travels
- **M**: Mach number
- **p**: Pressure
- **P_0**: Overall pressure ratio (see Section 4.2)
- **P_{ij}**: Dimensionless pressure ratio (i.e. \( P_2/P_1 \))
- **P_{4d}**: Pressure in region \( \text{(4)} \), Fig. 4 for double diaphragm shock tube (Sect. 4.2)
- **P_{hs}**: Pressure in region \( \text{(4)} \) for single diaphragm shock tube (Sect. 4.2)
- **t**: Time
- **T**: Temperature

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\( T_{ij} \) Dimensionless temperature ratio (i.e. \( T_2/T_1 \))

\( T_{h_0} \) Temperature in region \( h \), Fig. 4 for double diaphragm shock tube (Sect. 4.2)

\( T_{h_0} \) Temperature in region \( h \), for single diaphragm shock tube (Sect. 4.2)

\( u \) Particle velocity

\( u_{ij} \) Dimensionless particle velocity (i.e. \( u_2/u_1 \))

\( x \) Position along the shock tube measured from the diaphragm

\( \gamma \) Ratio of specific heats \( (c_p/c_v) \)

\( \alpha \) \( \frac{\gamma+1}{\gamma-1} \)

\( \beta \) \( \frac{\gamma-1}{2\gamma} \)

\( \rho_{ij} \) Dimensionless density ratio (i.e. \( \rho_2/\rho_1 \))

\( \nu \) Kinematic viscosity

\( \rho \) Density

\( \sigma \) Contact surface moving to the right

\( \vec{R} \) Backward facing rarefaction wave (particles enter from left)

\( \vec{S} \) Forward facing shock wave (particles enter from right)

Quantities not included here are defined in the text or on the figures in which they appear.

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FIG. 1 - The Wave System in the Shock Tube. The Bottom Half of the Sketch Indicates the Pressure, Temperature and Mach No. at time, $t = t_1$. 

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Figure 2 - Variation of Shock Pressure Ratio, $P_{21}$ with Diaphragm Pressure Ratio, $P_{41}$, $\gamma = 1.4$. 
Figure 3 - Variation of Shock Mach Number, $M_b$ and Particle Velocity, $U_{21}$, with Diaphragm Pressure Ratio, $P_{41}$ for $\gamma_1 = 1.4$. 
FIG. 4 - The Double Diaphragm Shock Tube - Reflected Shock Type

FIG. 5 - The Double Diaphragm Shock Tube - Unsteady Expansion Type
FIG. 6 - Shock Tube with an Area Discontinuity

FIG. 7 - Shock Tube with a Convergent Transition Section. The dashed lines in the diaphragm section denote a convergent-divergent geometry with minimum area at $3b'$.
Figure 8 - Performance Comparison of a Variable Geometry and Simple Shock Tube, \( \gamma = 1.4 \).
Figure 10 - Performance Comparison of Various Driver Gases for $A_h/A_1 = 1$ and $A_h/A_1 = 6.25$. 
Figure 11 - Variation of Temperature Ratio, $T_{21}$ with Shock Mach Number, $M_s$
Figure 12 - Density Ratio Variation across the Initial Shock Wave.
Figure 13 - Variation of Flow Mach Number with Initial Shock Wave
Figure 14 - Attenuation of strong shock waves in slender shock tubes (Ref. 12)
Figure 15 - Overall area ratio vs. flight Mach number ($\gamma = 1.4$, no reflection) (Ref. 38)
Fig. 17 - The Convair Aeronautical Laboratory 11 - By 15-Inch Supersonic Shock Tunnel
(Ref. 12)
Figure 18: Pressure ratio across diaphragm vs. flight Mach number for perfect starting (no reflection) (Ref. 36)
Figure 19 - Pressure ratio across diaphragm vs. strength of reflected shock ($\gamma = 1.4$, no reflection) (Ref. 39)
Figure 21 - Ratio of pressure in low pressure section to plenum pressure vs. flight Mach number for ideal gas ($\gamma = 1.4$, no reflection) (Ref. 36)
FLOW THROUGH NOZZLE IS ASSUMED TO HAVE A NEGLIGIBLE EFFECT ON THE WAVE DIAGRAM

Figure 22 - Wave diagram for the reflected method. (Ref. 37)
Figure 23 - Pressure ratio across diaphragm vs. strength of reflected shock. (Ref. 38)
Figure 24 - Wave diagrams of shock tube modifications for increased testing time. (Ref. 12)
Figure 25a - Required diaphragm pressure ratio for tailoring at various flight Mach numbers (unsteady configuration). (Ref. 39)
Figure 25b - Required diaphragm pressure ratio for tailoring at various flight Mach numbers (steady configuration). (Ref. 39)
Figure 26a - Required initial driver gas temperature for tailoring at various flight Mach numbers (steady configuration).
(Ref. 39)
Figure 26b - Required initial driver gas temperature for tailoring at various flight Mach numbers (steady configuration). (Ref. 39)
Figure 27 - Variation of flight Mach number with shock Mach numbers (steady configuration). (Ref. 39)
Figure 28a - Testing time for four modifications. (Ref. 37)
Figure 28b - Nominal testing time of hypersonic shock tunnel modifications based on equilibrium real gas calculations. (Ref. 12)
Figure 29 - Variation of Test Section Mach number with driver shock Mach number. (Ref. 14)
Figure 30 - Density in test section of hypersonic shock tunnel vs. test section Mach number. (Ref. 14)
Figure 31 - Expansion of the hypersonic shock tunnel vs. test section Mach number (Ref 31.)
Figure 32 - Performance of the nonreflected method, $\gamma = 1.4$, $T_2 = 518.4^\circ$R,
$T_e = 391.6^\circ$R, $R = 1716$ ft$^2$/SEC$^2$ $^\circ$R, $P_0$ = static pressure
behind the incident shock (lf/in$^2$) (Ref. 37)
Figure 33 - Performance of the reflected shock \( T_1 = 518.4^\circ R \), 
\( T_x = 391.6^\circ R \), \( R = 1716 \text{ FT}^2/\text{SEC}^2 \text{O}^2 \text{R} \), \( P_6 \) = static pressure behind the reflected shock \((\text{LI} / \text{IN}^2)\) (Ref. 37)
\( \text{H}_2/\text{Air, } P_{14} = 0, \ \alpha = 6, \ \beta = 1/7 \)

<table>
<thead>
<tr>
<th>( P_{14} )</th>
<th>( \Gamma_{34} )</th>
<th>( \Lambda_{3} )</th>
<th>( T_{34} )</th>
<th>( \Gamma_{21} )</th>
<th>( \Lambda_{21} )</th>
<th>( T_{21} )</th>
<th>( N_{3} )</th>
<th>( U_{34} )</th>
<th>( U_{21} )</th>
<th>( N_{2} )</th>
<th>( N_{3} )</th>
<th>( C_{34} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \infty )</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>6</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>x</td>
<td>( \infty )</td>
<td>1.69</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>0.1</td>
<td>0.22</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.92</td>
<td>8.45</td>
<td>71.4</td>
<td>19.0</td>
<td>5</td>
<td>15.8</td>
<td>1.87</td>
<td>8</td>
</tr>
<tr>
<td>1.0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.30</td>
<td>2.88</td>
<td>8.30</td>
<td>6.16</td>
<td>5</td>
<td>5.0</td>
<td>1.73</td>
<td>8</td>
</tr>
</tbody>
</table>

\( \text{in determinate} \)

\( \text{He/air, } P_{14} = 0, \ \alpha = 6, \ \beta = 1/7, \ \alpha' = \frac{4}{7}, \ \beta' = 1/5, \ T_{4} = T_{1} \)

\begin{bmatrix}
0.231 \\
132 \\
0 \\
0 \\
0 \\
5.75 \\
4.79 \\
23.0 \\
10.6 \\
3.00 \\
8.83 \\
1.65 \\
\infty \\
3
\end{bmatrix}

\( \text{He}_2/\text{Air, } P_{14} = 0, \ \alpha = 6, \ \beta = 1/7, \ T_{4} = T_{1} \)

\begin{bmatrix}
0.0735 \\
575 \\
0 \\
0 \\
0 \\
5.94 \\
9.82 \\
96.5 \\
22.2 \\
5 \\
18.5 \\
1.88 \\
\infty \\
5
\end{bmatrix}

Table 1 - Flow quantities for very strong shock waves.
<table>
<thead>
<tr>
<th>$P_0$</th>
<th>$10$</th>
<th>$10^2$</th>
<th>$10^3$</th>
<th>$10^4$</th>
<th>$10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_s$ max.</td>
<td>1.64</td>
<td>2.68</td>
<td>4.39</td>
<td>7.20</td>
<td>11.79</td>
</tr>
</tbody>
</table>

**TABLE 2**

**MAXIMUM SHOCK MACH NUMBERS FOR MULTIPLE DIAPHRAGM SHOCK TUBES** (Ref. 9)

<table>
<thead>
<tr>
<th>Driver - Driven Gas Combinations</th>
<th>$A_0/A_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$T_{hl} = 1$</td>
<td>$N_2/N_2$</td>
</tr>
<tr>
<td></td>
<td>$N_2/A$</td>
</tr>
<tr>
<td></td>
<td>$He/N_2$</td>
</tr>
<tr>
<td></td>
<td>$He/A$</td>
</tr>
<tr>
<td></td>
<td>$H_2/N_2$</td>
</tr>
<tr>
<td></td>
<td>$H_2/A$</td>
</tr>
<tr>
<td>$T_{hl} = 2$</td>
<td>$He/N_2$</td>
</tr>
<tr>
<td></td>
<td>$He/A$</td>
</tr>
<tr>
<td></td>
<td>$H_2/N_2$</td>
</tr>
<tr>
<td></td>
<td>$H_2/A$</td>
</tr>
</tbody>
</table>

**TABLE 3**

**MAXIMUM VALUES OF SHOCK MACH NUMBER, $M_s$ FOR $P_{hl} \rightarrow \infty$** (Ref. 10)

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