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AFAPL-TR-65-45
Part VII

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY

Part VII: The Three Lobe Bearing and Floating Ring Bearing

J. Lund

Mechanical Technology Incorporated

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TECHNICAL REPORT AFAPL-TR-65-45, PART VII

February 1968

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Part VII

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Air Force Aero Propulsion Laboratory
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio

FOREWORD

This report was prepared by Mechanical Technology Incorporated, 968 Albany-Shaker Road, Latham, New York 12110 under USAF Contract No. AF33(615)-3238. The contract was initiated under Project No. 3048, Task No. 304806. The work was administered under the direction of the Air Force Aero Propulsion Laboratory, with Mr. Micheal R. Chasman (APFL) acting as project engineer.

This report covers work conducted from 1 August 1966 to 1 August 1967.

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This technical report has been reviewed and is approved.

Arthur V. Churchill

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ABSTRACT

This volume treats three special bearing types selected for study because of their favorable stability characteristics and, hence, their potential for use in high speed rotating machinery applications. The three bearing types are:

a. The Three Lobe Journal Bearing
b. The Floating Sleeve Bearing with an Incompressible Lubricant
c. The Floating Sleeve Bearing with a Compressible Lubricant.

In the floating sleeve bearings, the ring is prevented from rotating but is otherwise free to move. The ring is floated by pressurizing the outer film of the bearing. In the case of a compressible lubricant, the inner film is pressurized as well.

The volume gives extensive design data in form of charts and tables from which the bearing dimensions can be obtained for a given application. Data are given for bearing flow, friction power loss and the speed at which hydrodynamic instability sets in. In addition, two computer programs accompany the volume, and instructions and listings of the programs are included. The programs may be used to obtain data for cases not covered by the presented design data.

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TABLE OF CONTENTS

		Page
INTRODUCTION		1
THE THREE LOBE JOURNAL BEARING		5
THE HYDRODYNAMIC-HYDROSTATIC RING BEARING		15
THE HYBRID-HYDROSTATIC RING BEARING		25
APPENDIX I:	The Static and Dynamic Performance of a Partial Arc Bearing with Turbulent Film	95
APPENDIX II:	The Static and Dynamic Performance of the Three Lobe Bearing with Turbulent Film	119
APPENDIX III:	The Stiffness and the Damping of a Hydrostatic Bearing with an Incompressible Lubricant	127
APPENDIX IV:	The Stability of the Hydrodynamic - Hydrostatic Ring Bearing with an Incompressible Lubricant	147
APPENDIX V:	The Stiffness and the Damping of a Hybrid Journal Bearing with a Compressible Lubricant	155
APPENDIX VI:	The Stability of the Hybrid-Hydrostatic Ring Bearing with a Compressible Lubricant	177
APPENDIX VII:	Computer Program - The Static and Dynamic Performance of a Lobed Bearing with Turbulent Film	179
APPENDIX VIII:	Computer Program - The Performance and Stability of a Hybrid Journal Bearing with Flexible, Damped Support	231
REFERENCES		267

ILLUSTRATIONS

Figure		Page
1	The 3 Lobe Bearing, Schematic	42
2	The Hydrodynamic-Hydrostatic Ring Bearing, Schematic	43
3	The Hybrid-Hydrostatic Ring Bearing, Schematic	44
4	3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = \frac{1}{2}$, Stability Map	45
5	3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = 1$, Stability Map	46
6	3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = \frac{1}{2}$, Friction	47
7	3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = 1$, Friction	48
8	3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Stability Map	49
9	3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Stability Map	50
10	3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Friction	51
11	3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Friction	52
12	3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Minimum Film Thickness	53
13	3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Minimum Film Thickness	54
14	3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Flow	55
15	3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Flow	56

Illustrations (Continued)

Figure		Page
16	Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 0$, $\frac{CK_o}{W} = 0.7$, Stability Map	57
17	Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 0$, $\frac{CK_o}{W} = 0.4$, Stability Map	58
18	Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 0$, $\frac{CK_o}{W} = 0.1$, Stability Map	59
19	Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 30{,}000$, $\frac{CK_o}{W} = 0.7$, Stability Map	60
20	Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 30{,}000$, $\frac{CK_o}{W} = 0.4$, Stability Map	61
21	Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 30{,}000$, $\frac{CK_o}{W} = 0.1$, Stability Map	62
22	Hydrostatic Bearing, Incompressible, Single Plane Admission, Stiffness	63
23	Hydrostatic Bearing, Incompressible, Double Plane Admission, Stiffness	64
24	Hydrostatic Bearing, Incompressible, Single Plane Admission, Damping	65
25	Hydrostatic Bearing, Incompressible, Double Plane Admission, Damping	66

Illustrations (Continued)

Figure		Page
26	Hydrostatic Bearing, Incompressible, Flow	67
27	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Stability Map	68
28	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Instability Frequency	69
29	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Stability Map	70
30	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Instability Frequency	71
31	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Stability Map	72
32	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Instability Frequency	73
33	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Stability Map	74
34	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Instability Frequency	75
35	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Stability Map	76

Illustrations (Continued)

Figure		Page
36	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Instability Frequency	77
37	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Stability Map	78
38	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Instability Frequency	79
39	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Stability Map	80
40	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Instability Frequency	81
41	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Stability Map	82
42	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Instability Frequency	83
43	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Stability Map	84
44	Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Instability Frequency	85
45	Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 1.25$, Stiffness	86

Illustrations (Continued)

Figure		Page
46	Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 2$, Stiffness	87
47	Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 5$, Stiffness	88
48	Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 10$, Stiffness	89
49	Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 1.25$, Damping	90
50	Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 2.$ Damping	91
51	Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 5$, Damping	92
52	Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 10$, Damping	93
53	Hydrostatic Bearing, Compressible, Flow	94
54	Geometry of Partial Arc Bearing	97
55	Finite Difference Grid	103
56	Finite Difference Grid Lines	104
57	Three Lobe Bearing	119
58	Geometry of Single Lobe	119
59	Axial Strip	131
60	Axial Strip	134
61	Hydrodynamic-Hydrostatic Ring Bearing	147

Illustrations (Continued)

Figure		Page
62	Axial Strip	164
63	Feeding Hole	205

LIST OF TABLES

		Page
Table 1:	3 Lobe Bearing, Vertical Rotor	37
Table 2:	3 Lobe Bearing, Preload: δ =0.5	39
Table 3:	Plain Cylindrical Bearing	41

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SYMBOLS

a	Orifice radius (or radius of laminar restrictor tube), inch
B	Damping coefficient, lbs.sec/inch
B_o	Damping coefficient for outer film, lbs.sec/inch
$B_{xx}, B_{xy}, B_{yx}, B_{xx}$	Damping coefficients for bearing, lbs.sec/inch
$B_{rr}, B_{rt}, B_{tr}, B_{tt}$	Damping coefficients for bearing, lbs.sec/inch
$\bar{B}_B$	Dimensionless effective bearing film damping
C	Radial clearance, inch
C_o	Radial clearance for outer film, inch
C_D	Discharge coefficient or vena contracta coefficient
C_f	Coefficient of friction
D	Journal diameter, inches
D_o	Diameter of outer bearing, inch
d	Feeder hole diameter, inch
e	Journal center eccentricity, inch
e_B	Eccentricity of journal center with respect to bearing center, inch
e_0	Journal center eccentricity, steady-state, inch
F_f	Friction force, lbs.
F_r, F_t	Radial and tangential components of bearing force, lbs.
F_x, F_y	Vertical and horizontal components of bearing force, lbs.
F_{x0}, F_{y0}	Static components of F_x and F_y, lbs.
f_r	$= F_r/SW$ (Appendix I) or $= F_r/P_aLD$ (Appendix V)
f_t	$= F_t/SW$ (Appendix I) or $= F_t/P_aLD$ (Appendix V)
G_x, G_z	Turbulent flow coefficients, see Ref. 1

G	$= G_z/G_x$
G_0, G_1, G_2	Defined by Eq.(A-19), Appendix I
G_0, G_1, G_2	Defined through Eqs.(E-17) to (E-19), Appendix V
H	$= h^{3/2}\ G_x^{\ 1/2}$
H_0, H_1, H_2	Defined by Eq. (A-18), Appendix I
H_1, H_2	Defined by Eq. (C-56), Appendix III
$\bar{h}$	Film thickness, inch
h	$= \bar{h}/C$, dimensionless film thickness
i	$= \sqrt{-1}$
i, j	Finite difference indices, Appendix I
j	$= \sqrt{-1}$, Appendix V
K	Spring coefficient, lbs/inch
K_o	Spring coefficient for outer film, lbs/inch
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	Spring coefficients for bearing, lbs/inch
$K_{rr}, K_{rt}, K_{tr}, K_{tt}$	Spring coefficients for bearing, lbs/inch
$\bar{K}_B$	Dimensionless effective bearing film stiffness
K_R	Stiffness of shaft, lbs/inch
K_{eff}	Effective bearing film stiffness, lbs/inch
L	Bearing length, inch
L_o	Length of outer bearing, inch
L_1	Distance between admission planes, inch
L_2	$= L - L_1$, inch
ℓ	Length of feeder hole, inch
M	Journal mass (half rotor mass for rigid rotor), lbs.sec^2/inch
M_B	Mass flow into bearing from one feeder hole, lbs.sec/inch
M_c	Mass flow through feeder hole orifice, lbs.sec/inch

$\bar{M}$	Total bearing mass flow, lbs.sec/inch
M_R	Mass of rotor disc, lbs.sec^2/inch
m	Dimensionless mass flow (Eq. (C-13), App. III or Eq. (E-31), App. V)
m	Number of finite difference increments
N	Rotor speed, rps
$\bar{N}$	Rotor speed, rpm
n	Number of feeder holes
n	Number of finite difference increments on half bearing length
$\bar{P}$	Film pressure, psi
P	Dimensionless film pressure
P_s	Supply pressure, psi
P_a	Ambient pressure, psi
$\bar{P}'_c$	Film pressure at rim of feeder hole, psi
Q	Bearing flow, inch^3/sec
Q_x	Flow in circumferential direction, inch^3/sec.
Q_z	Side flow, inch^3/sec.
q	Dimensionless flow (Eq. (C-47), App. III or Eq. (E-40), App. V)
R	Journal radius, inch
R_o	Radius of outer bearing, inch
R_e	$= \rho R\omega C/\mu$, Reynolds number for bearing
R_h	$= \rho R\omega\bar{h}/\mu = hR_e$, local Reynolds number for film
$\mathcal{R}$	Gas constant, inch^2/sec^2 °R
r	Distance of lobe centers from bearing center, 3 lobe bearing, inch

r_p	Distance of lobe center from bearing center, inch
S	$= \mu NDL(\frac{R}{C})^2/W$, Sommerfeld number
T	Total temperature of film, °F.
t	Time, seconds
W	Static load on bearing, lbs
V	$= P_s/P_a$, supply pressure ratio
V_c	Feeder hole volume
x,y	Journal center amplitudes, inch
z	Axial coordinate, inch
γ	$= \frac{\nu}{\omega}$, frequency ratio
δ	$= a^2/dC$, inherent compensation factor
δ	$= r/C$, preload
δ_p	$= r_p/C$, preload for lobe p
ϵ	$= e/C$, eccentricity ratio
ϵ_B	$= e_B/C$, eccentricity ratio for bearing
ϵ_p	Eccentricity ratio for lobe p
ϵ_0	Eccentricity ratio, steady-state
ζ	$= z/R$
θ	Angular coordinate, radians
θ_1,θ_2	Angular coordinates for leading and trailing edges of lobe, radians
θ_{in}	Angle from load vector to leading edge of lobe, radians
θ_{out}	Angle from load vector to trailing edge of lobe, radians
Λ	$= 6\mu\omega(\frac{R}{C})^2/P_a$, compressibility number

Λ_s	Restrictor coefficient for incompressible flow, defined by Eq. (C-12), App.III For compressible flow, defined by Eq. (E-33), App.V
λ	Spacing factor (pressure correction factor), Eqs. (E-49)and (E-50), App.V
μ	Lubricant viscosity, lbs.sec/inch^2
ν	Frequency, radians/sec
ξ_1	$= L_1/D$
ξ_2	$= L_2/D$
ρ	Mass density of lubricant, lbs.sec^2/inch^4
σ	Squeeze number (Eq. (C-8), App.III or Eq. (E-10), App.V)
τ	Dimensionless time
φ	Attitude angle, radians
φ_B	Attitude angle for bearing, radians
φ_0	Attitude angle, steady-state, radians
ψ_p	Angle between load vector and preload direction for lobe p, radians
ψ	= PH, Appendix I
ψ	Defined by Eq. (C-39), Appendix III
$\psi_1, \psi_2, \psi_3, \psi_4$	Perturbations of ψ = PH, Appendix I
ψ_0', ψ_1'	Defined by Eqs. (E-41) and (E-42), Appendix V
ψ_0, ψ_1	Defined by Eqs. (E-60) and (E-61), Appendix V
ω	Angular speed of rotor, radians/sec.

Subscripts

0	Steady-state or static condition
o	Outer film or outer bearing
p	Lobe number
c	Conditions in bearing film at rim of feeder hole
x	In x-direction
y	In y-direction
i,j	Finite difference coordinates

INTRODUCTION

The current trend towards high-speed rotating machinery in many applications has focused attention on the bearings supporting the rotor. Most conventional bearings are limited to relatively low speeds for a variety of reasons such as operating life, high friction power loss and stability, and at the present time there is no universal bearing which will eliminate all the problem areas.

The present volume analyzes three special journal bearing types which are capable of insuring stable rotor operation to rather high speeds. As conceived, the bearings assume low viscosity lubricants such as a gas or a low viscosity fluid (first. a liquid metal) in order to hold the friction power loss to an acceptable level. The three bearings are intended to provide the designer of high speed machinery with a greater variety of bearing types to choose from, a choice which at present is mostly limited to the tilting pad journal bearing.

The volume gives extensive design data in form of charts and tables from which the dimensions and performance data of the bearings can be obtained. Several numerical examples are included to illustrate the actual use of the data in making a design. Furthermore, two computer programs accompany the volume, and instructions for using the programs and listings of the programs are given.

The bearing considered first is the three lobe bearing which is shown schematically in fig. 1. This bearing has been used for many years because of its known ability to suppress instability, but up to this time, no information has been available on its actual stability limit. Also, data has been lacking on the friction power loss and flow of the bearing. The influence on the bearing performance of turbulence in the bearing film is included in the investigation to cover lubricants such as liquid metals which are frequently used in space power plants or in machinery for nuclear reactors.

The major advantages of the three lobe bearing are that it is relatively simple in construction, it has no moving parts like a tilting pad bearing or a floating

sleeve bearing, it can be designed to operate stable at rather high speeds and it is also able to run with a vertical rotor or a rotor in a zero-g field which is not possible with conventional cylindrical bearings. The limitations of the bearing are its inability to accommodate any appreciable shaft misalignment unless specially mounted, and its relatively high friction power loss.

The second bearing type is of the floating sleeve variety and is shown in Figure 2. It is given the name of the hydrodynamic-hydrostatic ring bearing. The ring inserted between the journal and the outer bearing is prevented from rotating but is otherwise unrestrained. The ring is floated by pressurizing the outer film through feeder holes in the outer wall. By proper selection of bearing dimensions and operating parameters, this bearing can be designed to have a very high stability limit (theoretically, the bearing can be inherently stable if the mass of the sleeve is ignored). Furthermore, because of its construction the bearing can accommodate more shaft misalignment than the three lobe bearing. It is found, on the other hand, that to achieve the improved stability it is necessary to have a relatively big clearance in the outer film which means that the flow requirements of the bearing are quite high compared to non-pressurized bearings. Also, the friction power loss of the bearing becomes prohibitive at speeds considerably lower than the speed at which the bearing would otherwise become unstable such that the bearings high speed potential cannot be utilized fully. Even so, the bearing is a more truly high-speed bearing than the three lobe bearing.

The third bearing type is also a floating sleeve bearing and quite similar to the one considered above except that both the inner film and the outer film are pressurized and the lubricant is a gas instead of a liquid. The bearing is called the hybrid-hydrostatic ring bearing and is shown in fig. 3. It has the same desirable stability characteristics as in the previous case and because of the gaseous lubricant, the friction power loss is not nearly as serious. Hence, the hybrid-hydrostatic bearing can operate at very high speeds, say 100,000 rpm or more, depending on the application. The bearing, however, requires somewhat tighter clearances than the liquid lubricated ring bearing which means that its ability to accommodate misalignment is more restricted.

In the following, each of the three bearing types are treated in separate sections. The sections are devoted to a detailed discussion of the design procedure for the bearings and explain the use of the design charts contained in figs. 4 to 53. In addition, complete numerical examples have been worked out for each bearing type. The theoretical analyses from which the data have been obtained, are given for reference in 6 appendices in back of the volume. Two computer programs have been written, one for the three lobe bearing which also applies to the inner film of the hydrodynamic-hydrostatic ring bearing, and one for the hybrid-hydrostatic ring bearing. Manuals for the programs are given in Appendices VII and VIII.

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THE THREE LOBE JOURNAL BEARING

The geometry of the 3 lobe bearing is shown schematically in figure 1. The bearing is composed of 3 circular arcs whose centers of curvature are removed from the center of the bearing by the distance r. Thus, even when the journal is centered in the bearing, the pads are loaded. In this way, the stability threshold of the bearing is raised and even a vertical rotor, or a rotor operating in a "zero-g field," can run stably which is not possible with a conventional full circular bearing. However, the improved stability threshold is paid for by an increase in friction power loss and a smaller operating minimum film thickness which makes the bearing more sensitive to impurities in the lubricant.

The journal diameter is D, its radius is R and the bearing length is L. The lobes are separated by axial grooves with an arc width of 20 degrees. The machined radial clearance of the lobes, common to all three lobes, is C such that the radius of curvature of the lobes is R + C. The preload is defined as:

$$\text{Preload:} \quad \delta = \frac{r}{C} \tag{1}$$

Thus, when $\delta = 0$, the bearing becomes cylindrical and when $\delta = 1$, the journal touches all three lobes.

The preload is an important design parameter for the 3 lobe bearing. It strongly influences the stability threshold and the friction power loss of the bearing.

The other design parameters are the length-to-diameter ratio $\frac{L}{D}$, the Sommerfeld number S and the Reynolds number R_e:

$$\text{Sommerfeld Number:} \quad S = \frac{\mu NDL}{W}\left(\frac{R}{C}\right)^2 \tag{2}$$

$$\text{Reynolds Number:} \quad R_e = \frac{2\pi\rho RNC}{\mu} \tag{3}$$

where:

W = bearing load, lbs

N = rotor speed, rps

μ = lubricant viscosity, lbs.sec/inch^2

ρ = lubricant mass density, lbs.sec^2/inch^4

For a given bearing geometry (i.e. for known values of $\frac{L}{D}$ and δ) and for a given operating condition (i.e. for known values of S and R_e), the bearing performance can be calculated as shown in the analyses in Appendices I and II. The actual calculations are carried out on a computer by means of the program described in Appendix VII where a listing and the instructions for using the program are given.

For known values of the four parameters, the bearing performance is defined by a set of dimensionless quantities:

the bearing eccentricity ratio: $\epsilon_B = \frac{e_B}{C}$ (4)

the bearing attitude angle: φ_B (5)

flow parameter: $\frac{Q_z}{NDLC}$ (6)

the friction factor: $\frac{R}{C} \frac{F_f}{W}$ (7)

the stability mass parameter: $\frac{CM\omega^2}{W}$ (8)

the instability frequency ratio: $\gamma = \frac{\nu}{\omega}$ (9)

the dimensionless spring coefficients: $\frac{CK_{xx}}{W}, \frac{CK_{xy}}{W}, \frac{CK_{yx}}{W}, \frac{CK_{yy}}{W}$ (10)

the dimensionless damping coefficients: $\frac{C\omega B_{xx}}{W}, \frac{C\omega B_{xy}}{W}, \frac{C\omega B_{yx}}{W}, \frac{C\omega B_{yy}}{W}$ (11)

where:

e_B = the distance between the bearing center and the journal center, inch

Q_z = the total hydrodynamic leakage flow, inch^3/sec

F_f = the total friction force, lbs

M = the mass of the journal, lbs.sec^2/inch

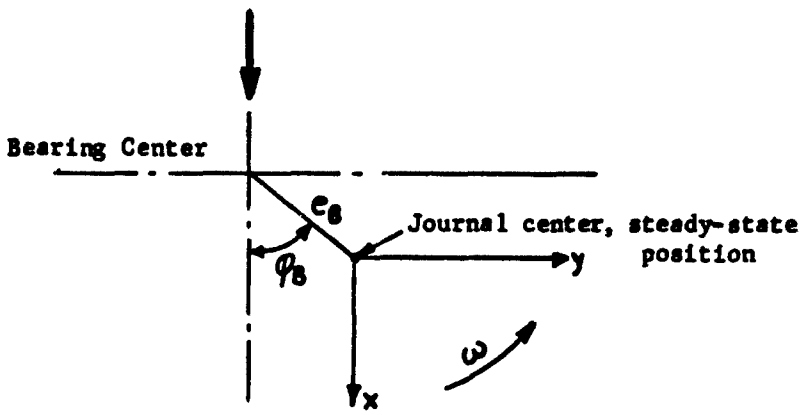
$\omega = 2\pi N$, the angular speed of rotation, radians/sec

ν = the whirl frequency at onset of instability, radians/sec

K_{xx}, K_{xy}, K_{yx}, K_{yy} = spring coefficients, lbs/inch

B_{xx}, B_{xy}, B_{yx}, B_{yy} = damping coefficients, lbs.sec/inch

The x-y-coordinate system has its origin in the steady-state position of the journal center with the x-axis in the direction of the applied static load:



Under dynamic load, the journal center motion is described by the amplitudes x and y. Then the dynamic forces acting on the journal are:

$$F_x = -K_{xx}x - B_{xx}\frac{dx}{dt} - K_{xy}\,y - B_{xy}\frac{dy}{dt}$$
$$F_y = -K_{yx}x - B_{yx}\frac{dx}{dt} - K_{yy}\,y - B_{yy}\frac{dy}{dt} \qquad (12)$$

The 8 dynamic coefficients are used to represent the bearing in a rotor response calculation as described in Volume 5 (see also reference 3).

The hydrodynamic leakage flow, Q_g, is the sum of the end leakage flows from the lobes and also includes any net excess flow into the grooves (see Appendix VII.) To get the flow in gallons per minute (gpm), multiply Q_g by 60 and divide by 231.

Usually, the lubricant is supplied to the bearing under some pressure. The flow induced in this way (the so called "zero speed flow") must be added to the hydrodynamic leakage flow to obtain the total bearing flow.

The friction power loss can be calculated from the friction force, F_f as:

$$\text{Friction power loss} = \frac{\omega F_f}{6600} \quad \text{HP} \tag{13}$$

If it is assumed that all of the heat generated by the friction power loss goes into the lubricant film, the corresponding temperature rise of the lubricant can be calculated based on the obtained flow. With this temperature the operating viscosity of the lubricant can be determined.

The eccentricity ratio ϵ_B and the attitude angle φ_B defines the steady-state position of the journal center relative to the bearing center and the static load line (the x-axis). From this the minimum film thickness in the bearing can be determined (see Appendix II). Under almost all conditions, the minimum film thickness occurs at the bottom lobe (see fig. 1) in which case:

$$\text{Minimum Film Thickness, inch} = \text{min. of} \begin{cases} C\left[1-\sqrt{\varepsilon_B^2+\delta^2+2\delta\varepsilon_B\cos\varphi_B}\,\right] \\ C\left[1+\cos\left(230^\circ-\tan^{-1}\left(\frac{\varepsilon_B\sin\varphi_B}{\delta+\varepsilon_B\cos\varphi_B}\right)\right)\sqrt{\varepsilon_B^2+\delta^2+2\varepsilon_B\cos\varphi_B}\,\right] \end{cases}$$

The minimum film thickness gives a relative measure of how heavily the bearing is loaded. The acceptable lower value for the minimum film thickness depends on the condition of the lubricant.

The threshold of instability is defined through the stability mass parameter in eq. (8). It applies to a rigid rotor with total mass 2M supported in two similar bearings, and the parameter defines that mass the rotor must have in order for the rotor-bearing system to be on the threshold of instability for the specified operating condition. If the actual rotor mass is bigger, the bearing is unstable. At the threshold of instability the steady-state equilibrium position of the journal is neutrally stable and the journal center whirls in an infinitessimal small closed orbit with a frequency ν, given through the whirl frequency ratio = $\gamma = \frac{\nu}{\omega}$.

The stability mass parameter can also be used to determine the threshold of instability for a flexible rotor. Let the rotor consist of a shaft with stiffness

K_R on which is mounted a central disc with mass $2M_R$. K_R and M_R are defined such that the natural frequency of the motor simply supported at the bearing centerlines is equal to $\sqrt{K_R/M_R}$. Then the threshold of stability is defined by the parameter:

Stability Mass Parameter for Flexible Rotor: $$\frac{CM_R\omega^2}{W} = \frac{\frac{CK_R}{W}}{\frac{CK_R}{W} + \gamma^2\left[\frac{CM\omega^2}{W}\right]}\left[\frac{CM\omega^2}{W}\right]$$

where $\frac{CM\omega^2}{W}$ and γ have the values obtained for the rigid motor. It is seen that the flexibility of the rotor lowers the threshold of instability.

If the rotor is not symmetric, the computer program described in Volume 5 can be used to calculate the speed at onset of instability. In this calculation, the bearing is represented by the 8 dynamic coefficients as obtained above.

The performance of the 3 lobe bearing with a preload of: $\delta = 0.5$, and for two values of the length-to-diameter ratio: $L/D = 0.5$ and 1, is given in Table The columns of the table give the values of the parameters defined above. For those conditions where no value is given for the instability frequency ratio or the stability mass parameter, the bearing is stable.

The table contains two additional quantities, namely $Q_x/NDLC$ which is the dimensionless circumferential flow (i.e. the sum of the flows entering the three lobes from the grooves), and $CMW/\mu DL\left(\frac{R}{C}\right)^2$ which is another dimensionless form of the stability mass parameter (it is equal to $\frac{CM\omega^2}{W} / 4\pi^2 S$).

The most significant results are plotted in figs. 8 to 15. Figs. 8 and 9 give the stability mass parameter. Figs. 10 and 11 show the friction factor, figs. 12 and 13 give the minimum film thickness normalized with respect to C, and figs 14 and 15 give the flow parameter. The abcissa is in all cases the Sommerfeld number S, and each graph contains four curves corresponding to Reynolds numbers of 0, 2000, 10000 and 30,000. The curves for $R_e=0$ apply to laminar flow (i.e. when $R_e < 2{,}000$).

The use of the graphs is best illustrated by an example. Let a rigid, symmetric rotor of 200 lbs. be supported in two similar 3 lobe bearings with a length-to-diameter ratio of 1/2. The lubricant is liquid potassium at 500°F. The data are:

bearing diameter: $D = 3$ inch

bearing length: $L = 1.5$ inch

radial clearance: $C = 0.002$ inch

preload: $b = 0.5$

bearing load: $W = 100$ lbs.

journal mass: $M = 100/386 = 0.259$ lbs.sec^2/inch

lubricant viscosity: $\mu = 3.85 \cdot 10^{-8}$ lbs.sec/inch^2

lubricant mass density: $\rho = 7.3 \cdot 10^{-5}$ lbs.sec^2/inch^4

Denote the rotor speed in rpm as $\bar{N}$. Then:

$$R_e = \frac{2\pi \cdot 7.3 \cdot 10^{-5} \cdot 1.5 \cdot 0.002}{3.85 \cdot 10^{-8}} \cdot \frac{\bar{N}}{60} = 0.596 \cdot \bar{N}$$

$$S = \frac{3.85 \cdot 10^{-8} \cdot 3 \cdot 1.5}{100} \left(\frac{1.5}{0.002}\right)^2 \cdot \frac{\bar{N}}{60} = 1.625 \cdot 10^{-5} \cdot \bar{N}$$

$$\frac{CM\omega^2}{W} = \frac{0.002 \cdot 0.259}{100} \left(\frac{2\pi \bar{N}}{60}\right)^2 = 5.68 \cdot 10^{-8}\, \bar{N}^2$$

$$Q_x = 3.896 \cdot 10^{-5} \left(\frac{Q_z}{NDLC}\right) \bar{N} \quad \text{gpm}$$

$$\text{friction power loss} = 3.17 \cdot 10^{-6} \left(\frac{RF_f}{CW}\right) \bar{N} \quad \text{HP}$$

With these relationships the bearing performance can be obtained from figs. 10, 12 and 14:

Rotor speed $\bar{N}$, rpm	R_e	S	From fig. 10 $\frac{R}{C}\frac{F_f}{W}$	From fig.12 $\frac{h_{min}}{C}$	From fig.14 $\frac{Q_z}{NDLC}$	Power loss HP	Flow gpm	Min. Film Thickness, inch
5,000	2,980	0.0812	5.8	0.2	1.66	0.092	0.32	0.0004
10,000	5,960	0.1625	14	0.335	0.84	0.44	0.33	0.00067
15,000	8,940	0.2438	31	0.4	0.59	1.48	0.34	0.0008
18,000	10,710	0.2925	47	0.428	0.51	2.68	0.36	0.00086

To determine the threshold of instability, plot the stability limit for the rotor in fig. 8:

R_e	$\bar{N}$, rpm	S	$\frac{CM\omega^2}{W}$
2,000	3,360	0.0546	(stable)
10,000	16,800	0.273	17.9
30,000	50,400	0.819	81.5

The operating line of the rotor is given by: $\frac{CM\omega^2}{W} = 215 \cdot S^2$ which is a straight line with a slope of 2 in fig. 8 because of the logarithmic scales. The operating line intersects the stability limit curve at $\frac{CM\omega^2}{W} = 19$ from which the speed at onset of instability is found to be 18,300 rpm. For comparison, the instability speed of a plain cylindrical bearing can be determined from Table 3 as:

<u>Plain Cylindrical Bearing</u> 10,700 rpm for C = 0.002
14,260 rpm for C = 0.001

This gives an indication of the stability improvement obtainable with a three lobe bearing.

If the rotor instead of being rigid is flexible and consists of a shaft on which is mounted a central disc, the threshold speed will be lowered. Let the disc weigh 200 lbs and let the shaft have a stiffness of 10^6 lbs/inch. The natural frequency of the rotor simply supported is:

$$\text{Natural frequency} = \sqrt{\frac{1,000,000}{200/386}} = \sqrt{\frac{500,000}{100/386}} = 1,390 \ \frac{\text{radians}}{\text{sec}} = 13,300 \text{ rpm}$$

The shaft stiffness to be used in eq. (15) is: $K_R = 500,000$ lbs/inch and the half disc mass is: $M_R = 100/386 = 0.259$ lbs.sec^2/inch. Hence:

$$\frac{CK_R}{W} = \frac{0.002 \cdot 500,000}{100} = 10$$

$$\frac{CM_R\omega^2}{W} = 215.S^2$$

The flexible rotor has the same operating line as the rigid rotor since the mass is unchanged. The stability limit, however, is lowered. From eq. (15)

$$\frac{CM_R\omega^2}{W} = \frac{10 \cdot \frac{CM\omega^2}{W}}{10 + (0.5)^2 \frac{CM\omega^2}{W}}$$

where the instability frequency ratio has been set equal to 0.5 (for a more accurate value, use Table 2). Hence, by using fig. 8:

R_e	$\bar{N}$, rpm	S	$\frac{CM\omega^2}{W}$ (from fig. 8)	$\frac{CM_R\omega^2}{W}$
2,000	3,360	0.0546	(stable)	(stable)
10,000	16,800	0.273	17.9	12.35
30,000	50,400	0.819	81.5	26.35

The operating line intersects the stability curve at $\frac{CM_R\omega^2}{W} = 12$ to which corresponds a threshold speed of 14,500 rpm.

When the rotor is vertical or operates in a "zero g field", the bearing is unloaded, i.e. W = 0. Hence, the eccentricity ratio ϵ_B is zero, and the Sommerfeld number becomes infinite and can no longer be employed as a parameter. In this case the performance parameters of eqs. (7), (8), (10) and (11) are redefined.

<u>Vertical Rotor</u>

friction factor: $$\frac{R}{C}\ \frac{F_f}{\mu NDL(\frac{R}{C})^2} = \frac{1}{S}\ \frac{R}{C}\ \frac{F_f}{W} \tag{16}$$

stability mass parameter: $$\frac{CMN}{\mu DL(\frac{R}{C})^2} = \frac{1}{4\pi^2 S}\ \frac{CM\omega^2}{W} \tag{17}$$

dimensionless spring coefficients: $$\frac{CK_{xx}}{\mu NDL(\frac{R}{C})^2} = \frac{1}{S}\ \frac{CK_{xx}}{W} \tag{18}$$

dimensionless damping coefficients: $$\frac{C\omega B_{xx}}{\mu NDL(\frac{R}{C})^2} = \frac{1}{S}\ \frac{C\omega B_{xx}}{W} \tag{19}$$

and similarly for K_{xy}, K_{yx}, K_{yy}, B_{xy}, B_{yx} and B_{yy}. It should be noted that for a vertical rotor:

$$K_{yy} = K_{xx}$$

$$K_{yx} = -K_{xy}$$

$$B_{yy} = B_{xx}$$

$$B_{yx} = -B_{xy}$$

Calculated performance data are given in Table 1 for a 3 lobe bearing with $\frac{L}{D} = \frac{1}{2}$ and 1, and for 4 values of the Reynolds number. 6 preloads are considered. The stability mass parameter and the friction factor are plotted in figs. 4 to 7 as a function of the preload. Hence, for a given application the preload required to ensure stable operation is readily determined. To illustrate, consider the same rigid rotor as in the previous example. From the given rotor and bearing data:

$$R_e = 0.596 \cdot \bar{N}$$

$$\frac{CMN}{\mu DL\left(\frac{R}{C}\right)^2} = \frac{MN}{2\mu L\left(\frac{R}{C}\right)^3} = \frac{0.259}{2 \cdot 3.85 \cdot 10^{-8} \cdot 1.5 \cdot \left(\frac{1.5}{0.002}\right)^3} \frac{\bar{N}}{60} = 0.886 \cdot 10^{-4}\,\bar{N}$$

$$\text{friction power loss} = 5.15 \cdot 10^{-11}\left(\frac{R}{C}\,\frac{F_f}{\mu NDL\left(\frac{R}{C}\right)^2}\right) \cdot \bar{N}^2 \quad HP$$

Thus, by using figs. 4 and 6:

R_e	$\bar{N}$, rpm	$\frac{CMN}{\mu DL\left(\frac{R}{C}\right)^2}$	From Fig.4 Preload: δ	From Fig. 6 $\frac{R}{C}$	From Fig. 6 $\frac{F_f}{\mu NDL\left(\frac{R}{C}\right)^2}$	Power Loss, HP
2,000	3,360	0.297	0.398	54		0.03
10,000	16,800	1.49	0.52	156		2.27
30,000	50,400	4.46	0.575	360		47.1

Hence, the rotor can operate stably at 50,000 rpm if the preload is approximately 0.6 but the friction power loss is then close to 50 HP.

It is seen that the threshold speed for the vertical rotor with a bearing preload of 0.5 is approximately 16,000 rpm which is a slight reduction of the speed determined for a horizontal rotor where the bearing is loaded.

THE HYDRODYNAMIC-HYDROSTATIC RING BEARING

The bearing is shown schematically in figure 2. It actually consists of two bearings (two lubrication films) separated by a floating ring. The ring is restrained from rotating by a pin but is otherwise free to move. In the inner film, pressures are developed by the hydrodynamic action from the rotating journal. The outer film, which is a hydrostatic bearing, is supplied with pressurized lubricant through restricted feeder holes. In this way the bearing can be designed such that the damping from the outer film stabilizes the inner bearing thereby raising the stability limit of the bearing significantly. Furthermore, by this construction the bearings ability to accommodate misalignment is appreciably improved. On the other hand, the flow requirements of the bearing are large as discussed in the following.

The detailed analysis of the bearing is given in Appendices III and IV. The computer program described in Appendix VII is used to calculate the performance characteristics of the inner film and some typical data are given in Table 3.

The stability characteristics of the bearing are given by figures 16 to 21. The length-to-diameter ratio is equal to 1, based on the journal diameter.

The charts are based on a given outer film stiffness, K_o, and the curves give the stability mass parameter $CM\omega^2/W$ as a function of the damping, B_o, of the outer film. The curves are plotted for several values of the Sommerfeld number S of the inner film, and two Reynolds numbers are considered: 0 and 30,000.

The outer film stiffness and damping are used in the dimensionless forms:

for outer film stiffness: $\frac{CK_o}{W}$ (20)

for outer film damping: $\frac{C\omega B_o}{W}$ (21)

where:

C = radial clearance of inner film, inch

W = bearing load, lbs

ω = angular speed of rotation, radians/sec

K_o = outer film spring coefficient, lbs/inch

B_o = outer film damping coefficient, lbs.sec/inch

The stability mass parameter has been defined and discussed in the previous section on the three lobe bearing (see eq. (8)). It applies to a rigid, symmetric rotor with mass 2M.

The charts, figs. 16 to 21, show that for a certain range of the outer film damping, the bearing is inherently stable. This "corridor" of inherent stability is seen to have a slope of 2 in the charts. Since the charts employ logarithmic scales, this means that for the bearing to operate in the corridor the ratio $\left(\frac{C\omega B_o}{W}\right)^2 / \frac{CM\omega^2}{W}$ is constant. This ratio is independent of speed.

In designing the bearing, the objective is to select the outer film stiffness and damping such that the bearing operates in the stable corridor. The outer film coefficients are given by the charts in figs. 22 to 25. In these charts the coefficients have the dimensionless form:

Dimensionless stiffness: $\frac{1+\delta^2}{1+\frac{2}{3}\delta^2} \quad \frac{C_o K_o}{P_s L_o D_o} \quad \xi_2$

Dimensionless damping: $\frac{CoBo}{\mu L_o D_o \left(\frac{Ro}{C_o}\right)^2}$

(note: subscript "o" is left out on the charts)

where:

D_o = bearing diameter, inch

L_o = bearing length, inch

R_o = bearing radius, inch

C_o = radial clearance of outer film, inch

K_o = spring coefficient, lbs/inch

B_o = damping coefficient, lbs.sec/inch

P_s = lubricant supply pressure, psig

μ = lubricant viscosity, lbs.sec/inch^2

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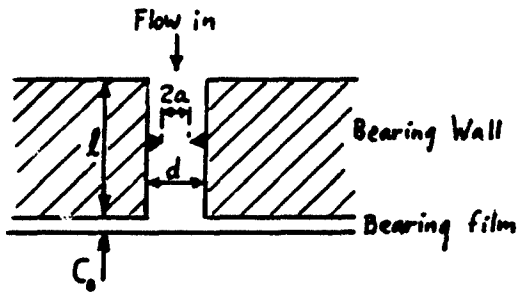
The feeder holes of the bearing may either be in the centerplane of the bearing, called "single plane admission", or arranged in two planes symmetric with respect to the centerplane, called "double plane admission." If the distance between the admission planes is L_1, define:

$$L_2 = L_o - L_1$$
$$\xi_1 = \frac{L_1}{D_o} \qquad (22)$$
$$\xi_2 = \frac{L_2}{D_o}$$

Hence, for single plane admission, $\xi_1=0$ and $\xi_2=L_o/D_o$, whereas for double plane admission the above definitions apply.

The flow restriction may be provided by orifices or by the feeder holes themselves



If there is no orifice and the feeder hole acts as a narrow tube, the bearing is said to be laminar restricted. When the predominant flow restriction is in the orifice, the bearing is orifice restricted. Finally, the flow may be restricted in the "curtain" area formed between the rim of the feeder hole and the surface of the ring. This is called "inherent compensation." The inherent compensation factor:

$$\delta = \frac{a^2}{dC_o} \qquad (24)$$

gives the ratio between the orifice area and the curtain area. When $\delta=0$, the feeder hole is purely orifice restricted, and when $\delta=0$, the flow restriction takes place in the curtain area only. For intermediate values of δ, both restrictors are acting. For the laminar restrictor, set $\delta=0$.

The spacing factor λ accounts for the effect of the spacing of the feeder holes. Let there be n feeder holes in total (i.e. for double plane admission there are $\frac{n}{2}$ holes per plane). Then λ is given by:

$$\lambda = 1 + \frac{2}{n\xi_2} \log_e \left(\frac{D}{n_p d}\right) \tag{25}$$

where n_p is the number of holes per admission plane ($n_p = n$ or $n_p = \frac{1}{2} n$.) For more accurate expressions, see eqs. (C-28), (C-29), (C-33) and (C-34), Appendix III. A typical value is: $\lambda = 1.5$.

In the charts, figs. 16 to 21, the abcissa is the restrictor coefficient Λ_s which gives the ratio between the flow resistance of the bearing film and the feeder holes. It is defined as:

$$\text{Restrictor Coefficient: } \Lambda_s = \begin{cases} \dfrac{3\mu C_D n a^2}{C_o^3 \sqrt{\rho P_s} \sqrt{1+\delta^2}} \lambda\xi_2 & \text{(Orifice restriction + inherent compensation)} \\[2ex] \dfrac{3}{8} \dfrac{n a^4}{C_o^3 \ell} \lambda\xi_2 & \text{(Laminar restriction)} \end{cases}$$

where:

C_D = discharge coefficient ($C_D \cong 0.6$)

a = radius of orifice or feeder hole, inch

ρ = mass density of lubricant, $\text{lbs}\cdot\text{sec}^2/\text{inch}^4$

ℓ = length of feeder hole, inch

and the other symbols have been defined above. It is seen that the bearing is optimized with respect to stiffness when $\Lambda_s = 1$.

The charts assume that the bearing is operating with no eccentricity. This assumption is valid in most cases. Furthermore, the load displacement characteristic is close to being linear such that:

$$W \cong C_o \epsilon_o K_o \tag{27}$$

where:

W = bearing load, lbs.

ϵ_o = bearing eccentricity ratio

This relationship is valid for eccentricities up to: $\epsilon_o \cong 0.4$ to 0.5.

The bearing flow can be determined from fig. 26 where the ordinate is the dimensionless flow:

$$\frac{3\mu Q}{P_s C_o^3} \cdot \lambda \, \xi_2$$

Here:

Q = flow, inch^3/sec.

The chart is valid for both single plane admission and double plane admission, and for any length-to-diameter ratio.

The use of the design charts is best illustrated by an example. First, it is seen from eq. (27) that:

$$\frac{C_o K_o}{W} = \frac{1}{\epsilon_o}$$

or:

$$\frac{C}{C_o} = \epsilon_o \frac{CK_o}{W} \tag{28}$$

ϵ_o should not exceed 0.4 in order for the bearing to have some load margin. Hence, the dimensionless stiffness $\frac{CK_o}{W}$ should be chosen as large as possible in order for C_o not to become too large which would lead to unreasonable flow requirements. On the other hand, the stability corridor narrows down with increasing outer film stiffness and eventually disappears. Thus, a compromise is necessary. In the present case, choose the largest dimensionless stiffness value given in the charts, namely $CK_o/W = 0.7$, which means that figs. 16 and 19 apply. To operate in the stability corridor, set $C\omega B_o/W = 0.6$ for $CM\omega^2/W = 1$ whereby the rotor will operate along a straight line with a slope of 2 in the chart. Hence:

$$B_o \sqrt{\frac{C}{MW}} = \frac{C\omega B_o/W}{(CM\omega^2/W)^{1/2}} = \frac{0.6}{\sqrt{1}} = 0.6 \tag{29}$$

Let the rotor be rigid and symmetric with a weight of 200 lbs. The data for the rotor and its two bearings are:

bearing diameter: $D = 3$ inch

bearing length: $L = 3$ inch

outer diameter of ring: $D_o = 3.5$ inch

lubricant viscosity: $\mu = 3.85 \cdot 10^{-8}$ lbs·sec/inch^2

lubricant mass density: $\rho = 7.3 \cdot 10^{-5}$ lbs·sec^2/inch^4

bearing load: $W = 100$ lbs.

journal mass: $M = 100/386 = 0.259$ lbs·sec^2/inch

The lubricant is liquid potassium at 500°F.

The outer bearing is chosen as a hydrostatic bearing with single plane admission and laminar restricted feeder holes. Hence, fig. 24 applies where the dimensionless damping is:

$$\frac{C_o B_o}{\mu L_o D_o \left(\frac{R_o}{C_o}\right)^2} = \frac{B_o}{2\mu L_o \left(\frac{R_o}{C_o}\right)^3} = \frac{1}{2\mu L_o \left(\frac{R_o}{C_o}\right)^3} \sqrt{\frac{MW}{C}} \cdot \frac{0.6}{\sqrt{1}} \tag{30}$$

or:

$$\left(\frac{R_o}{C_o}\right)^3 = \frac{1}{2\mu L_o} \cdot \sqrt{\frac{MW}{C}} \cdot \frac{0.6}{\sqrt{1}} \Bigg/ \frac{C_o B_o}{\mu L_o D_o \left(\frac{R_o}{C_o}\right)^2} \tag{31}$$

Now, from eq.(28) it is seen that $C/C_o < 0.4 \cdot 0.7 = 0.28$, i.e. even if it is desired to have a small value of C_o to keep the flow down, C_o cannot be too small if C shall have a practical value. Set $C = 0.0007$ inch and try with a dimensionless damping value of 4 (corresponds to $\Lambda_s = 0.25$). Hence, from eq. (31):

$$\left(\frac{R_o}{C_o}\right)^3 = \frac{0.6}{2 \cdot 3.85 \cdot 10^{-8} \cdot 3} \cdot \sqrt{\frac{0.259 \cdot 100}{0.0007}} \Big/ 4.0 = 1.25 \cdot 10^8$$

or:

$$\frac{R_o}{C_o} = 5 \cdot 10^2$$

i.e.

$$C_o = \frac{1.75}{500} = 0.0035 \text{ inch}$$

Since $C/C_o = 0.0007/0.0035 = 0.2 < 0.28$, this value of C_o can be accepted. The chosen dimensionless damping corresponds to a restrictor coefficient of: $\Lambda_s = 0.25$. Thus, from fig. 22:

$$\frac{C_o K_o}{P_s L_o D_o} \lambda \xi_2 = 0.28$$

Now:

$$\frac{C_o K_o}{P_s L_o D_o} \lambda \xi_2 = \frac{C_o}{C} \frac{W}{P_s L_o D_o} \lambda \xi_2 \frac{CK_o}{W} = 0.28$$

or:

$$P_s = \frac{C_o}{C} \frac{W}{L_o D_o} \lambda \xi_2 \frac{CK_o}{W} \frac{1}{0.28} \tag{32}$$

Here: $\xi_2 = L_o/D_o$ and λ shall be set equal to 1.5. Thus:

$$P_s = \frac{0.0035}{0.0007} \frac{100}{3 \cdot 3.5} \cdot 1.5 \cdot \frac{3}{3.5} \cdot 0.7 \frac{1}{0.28} = 150 \text{ psig.}$$

which establishes the required supply pressure.

From the definition of the restrictor coefficient:

$$\Lambda_s = 0.25 = \frac{3}{8} \frac{na^4}{C_o^3 \ell} \lambda \xi_2$$

or:

$$\frac{na^4}{\ell} = \frac{8}{3} \frac{(0.0035)^3}{1.5 \cdot 3/3.5} = 8.89 \cdot 10^{-8}$$

Let the feeder hole have a diameter: $d = 2a = 0.020$ inch and let the number of holes be: $n = 10$ whereby:

$$\ell = 1.125 \text{ inch}$$

Check the spacing factor:

$$\lambda = 1 + \frac{2}{n\xi_2} \log_e \left(\frac{D_o}{nd}\right) = 1 + \frac{2}{10 \cdot 3/3.5} \cdot \log_e \left(\frac{3.5}{10 \cdot 0.02}\right) = 1.67$$

which is close enough to the estimated value of 1.5. It should be noted that if the obtained feeder hole dimensions are not acceptable for various reasons, the bearing can be made orifice restricted instead or a different Λ_s-value can be chosen which would have little influence on the selection of C and C_o. Going to a value of $\Lambda_s = 0.5$ would reduce the required feeder hole length to $\ell = 0.56$ inch, the supply pressure would become $P_s = 105$ psig whereas the outer film clearance only would change from 0.0035 inch to 0.0034 inch. In other words, it is very important to select the proper value of the outer film clearance but the bearing is not too sensitive to even quite larger changes in the other parameters. This is quite readily deduced from eq. (31) where, for a given rotor weight, lubricant viscosity and overall bearing dimensions, the clearance C_o is pretty well defined once it is required that the bearing must operate in the stable corridor. Thus, eq. (31) can be considered to be the governing design equation with eq. (28) as a necessary condition.

From fig. 26 the dimensionless flow is found to be 0.2. Hence, the flow becomes:

$$Q = \frac{\pi \cdot P_s C^3}{3\mu\lambda\xi_2} \cdot 0.2 = \frac{\pi \cdot 150 \cdot (0.0035)^3}{3 \cdot 3.85 \cdot 10^{-8} \cdot 1.5 \cdot 3/3.5} \cdot 0.2 = 27.2 \ \frac{\text{inch}^3}{\text{sec}} = 7.1 \text{ gpm}$$

This is seen to be 20 times the flow required for the three lobe bearing for the same application. However, whereas the three lobe bearing becomes unstable at 18,000 rpm, the hydrodynamic-hydrostatic bearing is stable to much higher speeds.

The actual stiffness of the hydrostatic bearing is calculated to be:

$$K_o = \frac{150 \cdot 3 \cdot 3.5}{0.0035} \cdot 0.28 = 126{,}000 \text{ lbs/inch}$$

The damping coefficient becomes:

$$B_o = \frac{3.85 \cdot 10^{-8} \cdot 3 \cdot 3.5}{0.0035} \cdot \left(\frac{1.75}{0.0035}\right)^2 \cdot 4.0 = 116 \ \frac{\text{lbs.sec}}{\text{inch}}$$

Assuming a linear load-displacement relationship, the eccentricity ratio becomes:

$$\epsilon_o = \frac{W}{C_o K_o} = \frac{100}{0.0035 \cdot 126000} = 0.227$$

which is considerably less than 0.4. Hence, the bearing should be able to withstand dynamic loads of at least the same magnitude as the static load.

From Table 3, the stability mass parameter of the inner film is found to be approximately equal to 6 with a whirl frequency ratio of 0.5. Now, the onset of instability can be likened to a resonance where:

$$\nu^2 = \frac{K'_{eff}}{M}$$

where K_{eff} represents the effective stiffness of the inner film. Hence:

$$K_{eff} = \nu^2 M = \left(\frac{\nu}{\omega}\right)^2 \frac{CM\omega^2}{W} \cdot \frac{W}{C} = (0.5)^2 \cdot 6 \cdot \frac{100}{0.0007} = 214{,}000 \text{ lbs/inch}$$

The ring has an inner diameter of 3 inches and an outer diameter of 3.5 inches. Hence, its mass is:

$$\frac{\pi}{4}(3.5^2 - 3^2) \cdot 3 \cdot 0.283 = 2.17 \text{ lbs} = 0.0056 \text{ lbs}\cdot\text{sec}^2/\text{inch}$$

The natural frequency of the ring is:

$$\sqrt{\frac{214{,}000 + 126{,}000}{0.0056}} = 7{,}800 \ \frac{\text{radians}}{\text{sec}} = 75{,}000 \text{ rpm}$$

With an estimated instability frequency ratio of 0.5, the ring may become unstable at 150,000 rpm. This, then should be considered the top speed of the rotor to be on the safe side.

Making use of Table 3, the friction power loss of the bearing can be computed:

Rotor Speed rpm	R_e	S	Power Loss HP
20,000	4,200	5.35	9.7
40,000	8,300	10.7	64.4
60,000	12,500	16.1	195
80,000	16,700	21.4	432

Thus, even if the bearing is stable up to at least 150,000 rpm, the power loss becomes prohibitive at half that speed because of turbulence in the film.

In summary, the calculated bearing dimensions and the bearing performance data are:

journal diameter:	$D = 3$ inch
outer diameter of ring:	$D_o = 3.5$ inch
bearing length:	$L = 3$ inch
radial clearance of inner film:	$C = 0.0007$ inch
radial clearance of outer film:	$C_o = 0.0035$ inch
number of feeder holes for hydrostatic bearing:	$n = 10$
feeder hole diameter:	$d = 0.020$ inch
feeder hole length:	$\ell = 1.25$ inch
supply pressure for hydrostatic bearing:	$P_s = 150$ psig
bearing flow:	$Q = 7.1$ gpm
maximum stable speed (conservative estimate):	150,000 rpm
friction power loss: at 70,000 rpm	9.7 HP
at 40,000 rpm	64.4 HP
at 60,000 rpm	195 HP
at 80,000 rpm	432 HP

THE HYBRID-HYDROSTATIC RING BEARING

The bearing is shown schematically in figure 3. It differs from the previously considered hydrodynamic-hydrostatic ring bearing by employing a gas as a lubricant instead of a liquid. Furthermore, both the inner and the outer film are pressurized thereby enhancing the load carrying capacity which would otherwise be rather limited with gas as a lubricant. The ring separating the inner and the outer film, is restrained from rotating but is otherwise free to follow the motions of the journal. As shown in fig. 3, the inner film is supplied with pressurized gas through feeder holes in the centerplane of the ring. The load carrying capacity of the inner film is then produced by both hydrostatic and hydrodynamic action which is known as a hybrid bearing. The outer film is purely hydrostatic and, as shown in fig. 3, consists of two bearings supplied with pressurized gas through feeder holes in a central plane. The ends of the bearings are vented to atmosphere. Figure 3 is only intended to show one possible arrangement and other designs would be possible.

The major objective of this type of bearing design is to improve the stability limit of the bearing. Furthermore, the bearing offers the advantage of being able to accommodate a larger amount of misalignment than more conventional bearing types.

The analysis of the bearing is given in Appendices V and VI. Furthermore, a computer program for calculating the bearing has been written, and the instructions for using the program and a listing of the program are given in Appendix VIII.

The stability characteristics of the bearing are given by the charts in figs. 27 to 44. Three values of the supply pressure for the inner film have been considered:

$$\frac{P_s}{P_a} = 2,\ 5 \text{ and } 10$$

where:

P_s = supply pressure for inner film, psia

P_a = ambient pressure, psia

Furthermore, three values of the stiffness of the outer film are considered. In dimensionless form:

$$\frac{CK_o}{(P_s-P_a)LD} = 0.24,\ 0.18 \text{ and } 0.09 \tag{33}$$

where:

D = journal diameter, inch

L = bearing length, inch

C = radial clearance of inner film, inch

K_o = stiffness of outer film, lbs/inch

Curves are given for 7 values of the compressibility number Λ:

$$\Lambda = \frac{6\mu\omega}{P_a}\left(\frac{R}{C}\right)^2 = 0.3,\ 1,\ 2,\ 5,\ 10,\ 30 \text{ and } 100 \tag{34}$$

where:

μ = gas viscosity, lbs·sec/inch^2

ω = angular speed of journal, radians/sec

The charts, figs. 27, 29, 31 --- 43, give the value of the stability mass parameter:

$$\text{Stability Mass Parameter: } \frac{CM\omega^2}{(P_s-P_a)LD} \tag{35}$$

as a function of the dimensionless damping of the outer film:

$$\frac{C\omega B_o}{P_a LD} \tag{36}$$

where:

M = journal mass (half the rotor mass), lbs·sec^2/inch

B_o = damping coefficient of outer film, lbs·sec/inch

As in the case of the hydrodynamic-hydrostatic ring bearing, it is seen that for a certain range of the outer film damping the bearing is inherently stable. The

objective when designing the bearing is to select the bearing dimensions such that the bearing operates in the "stability corridor" established by the charts.

The charts, figs. 28, 30, 37,--44, give the whirl frequency ratio:

Whirl frequency ratio: $\frac{\nu}{\omega}$ (37)

as a function of the stability mass parameter. Here:

ν = whirl frequency at onset of instability, radians/sec

These charts are used in calculating the stiffness and damping of the outer film as discussed later.

All the charts, figs. 27 to 44, are based on the following data:

length-to-diameter ratio: $\frac{L}{D} = 1$

restrictor coefficient: $\Lambda_s = 0.7$

Inherent compensation factor: $\delta = 1000$

spacing factor: $\lambda = 1.5$

eccentricity ratio: $\epsilon = 0.02$

For $\Lambda_s = 0.7$, the bearing is optimized with respect to the hydrostatic stiffness. The bearing is inherently compensated (i.e. there are no orifices) to eliminate the possibility of pneumatic hammer instability. The journal is assumed to operate essentially in its concentric position but since the load is reasonably linear with displacement, the charts should be valid up to an eccentricity ratio of approximately 0.4 which covers the acceptable design range.

Figures 45 to 52 give the dimensionless stiffness and dimensionless damping of the hydrostatic outer film in the form:

Dimensionless Stiffness: $\frac{C_c K_o}{(P_s - P_a)_o L_o D_o}$ (38)

Dimensionless Damping: $\frac{B_o}{\mu L_o \left(\frac{R_o}{C_o}\right)^3}$ (39)

(Note: in the charts, subscript "o" has been left out).

The abcissa in the charts is the Squeeze number σ:

$$\text{Squeeze Number:} \quad \sigma = \frac{12\mu\nu}{P_{so}} \left(\frac{R_o}{C_o}\right)^2 \tag{40}$$

The symbols are:

D_o = outer diameter of ring, inch

R_o = outer radius of ring, inch

L_o = length of outer bearing, inch

C_o = radial clearance of outer film, inch

$(P_s)_o$ = supply pressure for outer film, psia

$(P_a)_o$ = ambient pressure for outer film, psia

μ = gas viscosity, lbs·sec/inch^2

ν = vibratory frequency, radians/sec

K_o = spring coefficient of outer film, lbs/inch

B_o = damping coefficient of outer film, lbs·sec/inch

The charts are valid for a single admission plane of feeder holes in the center-plane of the bearing. The feeder holes are inherently compensated (δ=1000) to avoid pneumatic hammer and the spacing factor is λ=1.5. The length-to-diameter ratio is 0.4 and four supply pressure ratios have been considered: P_s/P_a=1.25, 2, 5 and 10. A wide range of the restrictor coefficient is covered: $\Lambda_s \xi_2$=0.01, 0.02, 0.05, 0.1, 0.2, 0.5 and 1. The restrictor coefficient is defined as:

$$\Lambda_s = \frac{6\mu n d \sqrt{\mathcal{R} T}}{P_{so} C_o^2} \qquad \text{(inherent compensation)} \tag{41}$$

where:

n = number of feeder holes

d = feeder hole diameter, inch

$\mathcal{R}$ = gas constant, inch^2/sec^2 ^oR

T = total temperature, ^oR

The gas flow of the outer film is given in dimensionless form in the chart, fig. 53:

$$\frac{6\mu \mathcal{R} T \bar{M}}{\pi P_{so}^2 C_o^3} \frac{L_o}{D_o} \tag{42}$$

where:

$\bar{M}$ = mass flow, lbs·sec/inch

The chart is actually valid for all length-to-diameter ratios, for both single and double plane admission, and for both orifice restriction and inherent compensation. Hence, $\frac{L_o}{D_o}$ is replaced by $\frac{L_2}{D} = \xi_2$ and the restrictor coefficient is given in its general form (see Appendix V).

The use of the charts is best illustrated by an example. The or is assumed to be rigid and symmetric and it weighs 90 lbs. The given data re:

bearing load: $W = 45$ lbs.

journal mass: $M = 45/386 = 0.117$ lbs·sec^2/inch

journal diameter: $D = 3$ inch

bearing length: $L = 3$ inch

radial clearance, inner film: $C = 0.0015$ inch

outer diameter of ring: $D_o = 3.75$ inch

length of one outer bearing: $L_o = 1.25$ inch

ambient pressure: $P_a = 14.7$ psia

gas viscosity (air at 120°F): $\mu = 2.8 \cdot 10^{-9}$ lbs·sec/inch^2

gas constant (air): $\mathcal{R} = 2.472 \cdot 10^5$ inch^2/sec^2 °R

total temperature: $120 + 460 = 580$°R

Hence:

$$\mathcal{R}\,T = 1.434 \cdot 10^8 \text{ inch}^2/\text{sec}^2$$

or:

$$\sqrt{\mathcal{R}T} = 1.198 \cdot 10^4 \text{ inch/sec}$$

The load on the bearing is 45 lbs and the bearing area is: $L \cdot D = 3 \cdot 3 = 9$ inch^2. The load per square inch is then 5 psi which requires that the available pressure across the bearing be at least 20 psi and preferably more, i.e.:

$$P_s - P_a > 20$$

$$\text{or } \frac{P_s}{P_a} > 2.4$$

The stability charts have the dimensionless support stiffness in the form: $CK_o/(P_s-P_a)LD$. The eccentricity ratio of the outer film is determined from the relationship:

$$C_o \epsilon_o K_o = W \tag{43}$$

from which:

$$\epsilon_o = \frac{W}{C_o K_o} = \frac{W}{(P_s-P_a)LD} \quad \frac{C}{C_o} \quad \frac{(P_s-P_a)LD}{CK_o} \tag{44}$$

ϵ_o should be small and should not exceed 0.4. Thus, in order to keep the clearance ratio around 1, the dimensionless outer film stiffness should be chosen as large as possible and the supply pressure should also be kept high. Choose the largest value of the dimensionless support stiffness in the charts, namely:

$$\frac{CK_o}{(P_s-P_a)LD} = 0.24$$

Estimate that $C/C_o \cong 1$ and set $\epsilon_o = 0.2$ to get:

$$(P_s-P_a) = \frac{1}{\epsilon_o} \quad \frac{C}{C_o} \quad \frac{W}{LD} \quad \frac{1}{0.24} = 100 \text{ psi} \tag{45}$$

or:

$$P_s = 120 \text{ psia}$$

which means:

$$\frac{P_s}{P_a} = \frac{120}{14.7} \cong 8$$

The stability charts to use for this case are then fig. 33 and fig. 39. To operate in the stability corridor it is found from the charts that the dimensionless outer film damping should be:

$$\frac{C\omega B_o}{P_a LD} = 2.5 \quad \text{at} \quad \frac{CM\omega^2}{(P_s-P_a)LD} = 1$$

For constant support damping, the rotor will operate along a straight line with a slope of 2 in the charts as the speed increases. The corresponding damping can be found from the relationship:

$$\frac{C\omega B_o/P_a LD}{\sqrt{C\omega^2/(P_s-P_a)LD}} = \frac{1}{P_a}\sqrt{\frac{C(P_s-P_a)}{MLD}} \cdot B_o = \frac{2}{\sqrt{1}} \tag{46}$$

From figs. 50 to 53 it is seen that the dimensionless outer film damping will be around 1. Set:

$$\frac{B_o}{\mu L_o\left(\frac{R_o}{C_o}\right)^3} = 1$$

Since there are two outer film bearings, the damping per bearing is half the required damping given in eq. (46). The damping per bearing can be expressed by means of eq. (46) as:

$$\frac{B_o}{\mu L_o\left(\frac{R_o}{C_o}\right)^3} = 1 = \frac{1}{2}\,\frac{P_a}{\mu L_o\left(\frac{R_o}{C_o}\right)^3} \cdot \sqrt{\frac{MLD}{C(P_s-P_a)}} \cdot \frac{2}{\sqrt{1}}$$

or:

$$\left(\frac{R_o}{C_o}\right)^3 = \frac{1}{2}\,\frac{P_a}{\mu L_o} \cdot \sqrt{\frac{MLD}{C(P_s-P_a)}} \cdot \frac{2}{\sqrt{1}}\,\frac{1}{1} = \frac{1}{2}\,\frac{14.7}{2.8\cdot 10^{-9}\cdot 1.25} \cdot \sqrt{\frac{0.117\cdot 3\cdot 3}{C\cdot(120-14.7)}} \cdot \frac{2}{\sqrt{1}} \cdot \frac{1}{1} \tag{47}$$

Set C = 0.0015 inch whereby:

$$\left(\frac{R_o}{C_o}\right)^3 = 10.84\cdot 10^9$$

or:

$$\frac{R_o}{C_o} = 2.21\cdot 10^3$$

i.e.

$$C_o = \frac{1.875}{2.21} \cdot 10^{-3} = 0.00085 \text{ inch}$$

Eq. (47) (together with eqs. (44 or (45)) must be considered the key design equation. For a given rotor-bearing system, the clearance of the outer film can only vary within a very limited range if the rotor is to operate within the stability corridor. Hence, C_o is the critical design dimension whereas the other design parameters can vary appreciably without having serious effects.

To check the estimated outer film damping, the following relationship is readily deduced:

$$\frac{CM\omega^2}{(P_s-P_a)LD} = \frac{\Lambda^2}{72} \quad \frac{MP_a^2}{\mu^2 L(P_s-P_a)\left(\frac{R}{C}\right)^5} \tag{48}$$

Here:

$$\frac{MP_a^2}{\mu^2 L(P_s-P_a)\left(\frac{R}{C}\right)^5} = \frac{0.117\cdot(14.7)^2}{(2.8\cdot 10^{-9})^2 .3\cdot(120-14.7)\left(\frac{1.5}{0.0015}\right)^5} = 10.21$$

whereby:

$$\frac{CM\omega^2}{(P_s-P_a)LD} = 0.142\cdot\Lambda^2$$

From the definition of Λ and σ (eqs. (34) and (40)) it is seen that:

$$\sigma = 2\cdot\frac{\nu}{\omega}\cdot\Lambda \tag{49}$$

Making use of these relationships, figs. 34 and 40 can be employed to set up the following table:

		$B_o \to 0$		$B_o \to \infty$	
Λ	$\frac{CM\omega^2}{(P_s-P_a)LD}$	$\frac{\nu}{\omega}$	σ	$\frac{\nu}{\omega}$	σ
2	0.567	0.5	2.0	0.5	2.0
5	3.54	0.202	2.02	0.31	3.1
10	14.2	0.105	2.10	0.185	3.7
30	128	0.036	2.16	0.075	4.5
100	1420	0.0112	2.24	0.0245	4.9

In this table, $\frac{\nu}{\omega}$ is the instability frequency ratio obtained by interpolation between figs. 34 and 40. As seen from figs. 33 and 39, the stability corridor passes between two regions of instability, and the instability frequency is different for the two regions as shown by figs. 34 and 40. The left hand region is identified in figs. 34 and 40 as the region where $B_o \to 0$, and the right hand region is identified as the region where $B_o \to \infty$.

At the onset of instability, the journal whirls in closed orbit with frequency ν. Hence, ν becomes the vibratory frequency as seen by the outer film and the

stiffness and damping of the outer film must, therefore, be evaluated at the corresponding values of the Squeeze number as given in the table. Since the outer film decreases with increasing squeeze number it is only necessary to consider the Squeeze number corresponding to the left hand instability region ($B_o \to 0$).

The charts, figs. 33 and 39, are based on a dimensionless support stiffness of 0.24. Hence:

$$K_o = \frac{(120-14.7)\cdot 3\cdot 3}{0.0015} = 151{,}600 \text{ lbs/inch}$$

Since there are two bearings, the required stiffness per bearing is:

$$(K_o)_{\text{per bearing}} = 75{,}800 \text{ lbs./inch}$$

Assuming a restrictor coefficient value such that:

$$\Lambda_s \xi_2 = 0.05$$

the dimensionless stiffness per bearing is found by interpolation between figs. 47 and 48 as ($\sigma \cong 2.2$):

$$\frac{C_o K_o}{(P_s - P_a)_o L_o D_o} = 0.15$$

or:

$$(P_s - P_a)_o = \frac{0.00085\cdot 75{,}800}{1.25\cdot 3.75\cdot 0.15} = 91.6$$

Therefore:

$$P_{so} = 106.3 \text{ psia} \cong 120 \text{ psia}$$

whereby:

$$\left(\frac{P_s}{P_a}\right)_o = 8$$

Thus, both films have the same supply pressure.

At this calculated pressure ratio, with $\Lambda_s \xi_2 = 0.05$ and $\sigma = 2.2$ the dimensionless damping per bearing is found from figs. 51 and 52 to be equal to the earlier assumed value of 1. Hence, the assumed value of $\Lambda_s \xi_2 = 0.05$ is correct. Now:

$$\xi_2 = \frac{L_o}{D_o} = \frac{1.25}{3.75} = \frac{1}{3}$$

(Note: even if the charts, figs. 45 to 52, are based on ξ_2=0.4, the dimensionless stiffness and damping are not too sensitive to variations in ξ_2 and the charts may, therefore, also be used for $\xi_2 = \frac{1}{3}$). Hence:

$$\Lambda_s = 0.05 \cdot 3 = 0.15$$

From the definition of Λ_s in eq. (41):

$$n_o d_o = \frac{P_{so} C_o^2 \Lambda_s}{6\mu \sqrt{\mathcal{R} T}} = \frac{120 \cdot (0.00085)^2 \cdot 0.15}{6 \cdot 2.8 \cdot 10^{-9} \cdot 1.198 \cdot 10^4} = 0.0646 \text{ inch}$$

set:

$$n_o = 10$$

whereby:

$$d_o = 0.0065 \text{ inch}$$

The spacingfactor becomes:

$$\lambda = 1 + \frac{2}{n_o \xi_2} \log_e \left(\frac{D_o}{n_o d_o} \right) = 3.4$$

which is higher than the value of 1.5 for which the charts are valid. This, however, has only very minor influence on the damping but the stiffness is somewhat reduced. On the other hand, the supply pressure has been set 14 psi larger than actually required which should offset the reduction due to the spacing factor.

Even so, the calculated feeder hole diameter is small and should preferably be larger (0.01 to 0.02 inches) although the obtained value can be used as is.

The mass of the ring is:

$$\frac{\pi}{4} (3.75^2 - 3.0^2) \cdot 3 \cdot 0.283 = 3.38 \text{ lbs} = 0.008755 \text{ lbs} \cdot \text{sec}^2/\text{inch}$$

If the ring was rigidly supported, figs. 33 and 39 give:

$$\text{at} \quad B_o = \infty : \quad \frac{CM\omega^2}{(P_s - P_a) LD} = 1.4$$

The corresponding frequency ratio is: $\frac{\nu}{\omega} = 0.5$. Define an effective stiffness of the inner film by:

$$\nu^2 = \frac{K_{eff}}{M}$$

or:

$$\frac{CK_{eff}}{(P_s - P_a) LD} = \left(\frac{\nu}{\omega}\right)^2 \frac{CM\omega^2}{(P_s - P_a) LD} = 0.25 \cdot 1.4 = 0.35 \qquad (50)$$

whereby:

$$K_{eff} = \frac{(120 - 14.7)\cdot 3\cdot 3}{0.0015} \cdot 0.35 = 221{,}000 \text{ lbs/inch}$$

With a total outer film stiffness of: K_o = 151,600 lbs/inch, the natural frequency of the ring is:

$$\text{Natural frequency of ring} = \sqrt{\frac{221{,}000 + 151{,}600}{0.00875}} = 6{,}530 \text{ radians/sec} = 62{,}400 \text{ rpm}$$

Assuming an instability frequency ratio of 0.5, the ring may become unstable by itself at 125,000 rpm. This is a conservative estimate but to be safe, this speed should be considered the maximum speed for the bearing.

With $\Lambda_s \xi_2 = 0.05$, the dimensionless flow for the outer film is found from fig. 53 to be 0.034. Hence, the flow per bearing becomes:

$$\bar{M} = \frac{\pi\cdot(120)^2\cdot(0.00085)^3}{6\cdot 2.8\cdot 10^{-9}\cdot 1.434\cdot 10^8\cdot 0.33} \cdot 0.034 = 1.172\cdot 10^{-6}\ \frac{\text{lbs}\cdot\text{sec}}{\text{inch}} = 4.52\cdot 10^{-4}\ \frac{\text{lbs}}{\text{sec}}$$

$$= 0.33 \text{ scfm}$$

For the two outer bearings, the flow is 0.0009 lbs/sec = 0.66 scfm.

For the inner film, $\Lambda_s \xi_2 = 0.7$. Since $\xi_2 = 1$, $\Lambda_s = 0.7$ or:

$$nd = \frac{120\cdot(0.0015)^2}{6\cdot 2.8\cdot 10^{-9}\cdot 1.198\cdot 10^4} \cdot 0.7 = 0.938$$

Set n = 12 whereby:

d = 0.076 inch

The corresponding spacing factor is: $\lambda = 1.22$ which is close to the value of $\lambda = 1.5$ on which the charts are based.

The dimensionless flow for the inner film is determined from fig. 53 to be 0.39. Hence, the flow becomes:

$$\bar{M} = \frac{\pi\cdot(120)^2\cdot(0.0015)^3}{6\cdot 2.8\cdot 10^{-9}\cdot 1.434\cdot 10^8} \cdot 0.39 = 2.475\cdot 10^{-5}\ \frac{\text{lbs}\cdot\text{sec}}{\text{inch}} = 9.55\cdot 10^{-3}\ \frac{\text{lbs}}{\text{sec}} = 7.1 \text{ scfm}$$

The calculated bearing dimensions and performance data are summarized below:

total rotor weight:	2W = 90 lbs.
journal diameter:	D = 3 inch

outer diameter of ring:	D_o = 3.75 inch
length of ring:	L = 3 inch
length of one outer bearing:	L_o = 1.25 inch
radial clearance of inner film:	C = 0.0015 inch
radial clearance of outer film:	C_o = 0.00085 inch
number of feeder holes for inner film:	n = 12
diameter of feeder holes for inner film:	d = 0.076 inch
number of feeder holes for one outer bearing:	n_o = 10
diameter of feeder holes for outer bearing:	d_o = 0.0065 inch
supply pressure for both films:	P_s = 120 psia
flow for inner film:	7.1 scfm
flow for two outer bearings:	0.66 scfm
maximum stable speed (conservative estimate):	125,000 rpm

TABLE

3 LOBE BEARING, VERTICAL ROTOR

$\frac{L}{D} = \frac{1}{2}$

Preload δ	R_e	$\frac{R}{C}\frac{F_f}{SW}$	$\frac{Q_z}{NDLC}$	$\frac{Q_x}{NDLC}$	$\frac{\nu}{\omega}$	$\frac{CMN}{\mu DL(\frac{R}{C})^2}$	$\frac{CK_{xx}}{SW}$ $\frac{CK_{yy}}{SW}$	$\frac{CK_{xy}}{SW}$ $-\frac{CK_{yx}}{SW}$	$\frac{C\omega B_{xx}}{SW}$ $\frac{C\omega B_{yy}}{SW}$	$\frac{C\omega B_{xy}}{SW}$ $-\frac{C\omega B_{yx}}{SW}$
	0	18.04	.0812	4.345	.4978	.01503	.1499	1.585	3.184	-.00577
	2000	44.55	.0892	4.351	.4983	.02122	.2150	2.536	5.090	-.0139
0.1	10000	133.8	.0946	4.355	.4986	.04833	.4893	6.179	12.39	-.0300
	30000	306.3	.0964	4.357	.4987	.1011	1.027	13.24	26.56	-.0678
	0	21.11	.1994	3.787	.4935	.07009	.6783	2.542	5.152	-.00912
	2000	47.48	.2185	3.803	.4952	.09369	.9174	3.815	7.703	-.0207
0.25	10000	139.6	.2331	3.814	.4962	.2039	2.004	8.983	18.10	-.0432
	30000	317.7	.2381	3.818	.4965	.4206	4.143	18.96	38.18	-.0992
	0	24.66	.2933	3.313	.4886	.1935	1.832	3.986	8.159	-.0171
	2000	50.77	.3207	3.337	.4918	.2464	2.367	5.732	11.66	-.0307
0.375	10000	145.8	.3445	3.357	.4938	.5111	4.952	17.98	26.28	-.0635
	30000	329.7	.3528	3.363	.4944	1.038	10.08	27.11	54.84	-.1413
	0	29.74	.3810	2.828	.4817	.5339	4.905	6.912	14.35	-.0301
	2000	55.38	.4154	2.860	.4869	.6456	6.067	9.391	19.29	-.0497
0.5	10000	154.0	.4502	2.890	.4909	1.258	12.02	20.22	41.19	-.1015
	30000	345.2	.4628	2.900	.4921	2.501	24.01	41.66	84.65	-.2183
	0	41.48	.4817	2.156	.4673	2.521	21.78	17.61	37.69	-.1025
	2000	66.00	.5223	2.198	.4762	2.836	25.44	22.29	46.80	-.1032
0.6667	10000	170.72	.5769	2.243	.4861	4.876	45.61	44.57	91.68	-.2331
	30000	376.2	.5984	2.261	.4891	9.264	87.73	88.79	181.5	-.4849
	0	52.27	.5198	1.807	.4560	6.646	54.63	32.89	72.13	-.1817
	2000	75.92	.5617	1.849	.4667	7.198	62.01	40.07	85.86	-.2182
0.75	10000	189.6	.6301	1.904	.4832	11.27	104.1	76.51	158.3	-.4248
	30000	401.0	.6585	1.926	.4883	20.58	194.2	150.1	307.5	-.8336

TABLE 1 (Continued)

3 LOBE BEARING, VERTICAL ROTOR

$$\frac{L}{D} = 1$$

Preload δ	R_e	$\frac{R}{C}\frac{F_f}{SW}$	$\frac{Q_z}{NDLC}$	$\frac{Q_x}{NDLC}$	$\frac{\nu}{\omega}$	$\frac{CMN}{\mu DL(\frac{R^2}{C})}$	$\frac{CK_{xx}}{SW}$ $\frac{CK_{yy}}{SW}$	$\frac{CK_{xy}}{SW}$ $-\frac{CK_{yx}}{SW}$	$\frac{C\omega B_{xx}}{SW}$ $\frac{C\omega B_{yy}}{SW}$	$\frac{C\omega B_{xy}}{SW}$ $-\frac{C\omega B_{yx}}{SW}$
	0	18.04	.0434	4.318	.4968	.02665	.2584	3.343	6.729	.00256
	2000	44.55	.0500	4.323	.4976	.03991	.3912	5.788	11.63	-.00230
0.1	10000	133.8	.0549	4.326	.4979	.09622	.9486	14.89	29.90	-.0141
	30000	306.3	.0567	4.328	.4980	.2059	2.035	32.47	65.21	-.0389
	0	21.12	.1054	3.718	.4908	.1230	1.171	5.194	10.58	-.00222
0.25	2000	47.50	.1209	3.730	.4930	.1732	1.667	8.496	17.23	-.00881
	10000	139.7	.1338	3.740	.4942	.4003	3.875	21.20	42.90	-.03069
	30000	317.8	.1386	3.744	.4945	.8447	8.186	45.79	92.59	-.0619
	0	24.67	.1533	3.207	.4840	.3358	3.110	7.985	16.50	-.0115
	2000	50.84	.1747	3.226	.4882	.4469	4.214	12.35	25.29	-.0200
0.375	10000	146.0	.1953	3.242	.4906	.9847	9.381	29.92	60.99	-.0477
	30000	330.1	.2031	3.248	.4913	2.054	19.63	04.01	130.3	-.1217
	0	29.85	.1960	2.682	.4747	.9105	8.112	13.22	27.85	-.02706
0.5	2000	55.57	.2219	2.708	.4816	1.140	10.45	19.37	40.23	-.0374
	10000	154.4	.2512	2.731	.4866	2.362	22.13	45.23	92.96	-.0968
	30000	346.2	.2628	2.740	.4880	4.833	45.54	95.89	196.5	-.1957
	0	41.89	.2405	1.954	.4560	4.141	34.01	31.05	68.09	-.0670
	2000	66.61	.2691	1.985	.4676	4.741	40.96	41.92	89.65	-.0839
0.6667	10000	172.2	.3122	2.017	.4807	8.586	78.42	92.94	193.3	-.1821
	30000	379.5	.3308	2.031	.4846	16.82	156.1	193.7	399.9	-.4056
	0	53.07	.2545	1.576	.4414	10.58	81.38	54.72	124.0	-.03635
0.75	2000	77.07	.2824	1.605	.4554	11.57	94.80	71.23	156.4	-.1660
	10000	187.4	.3336	1.638	.4776	18.75	169.0	152.7	319.8	-.3023
	30000	407.2	.3573	1.653	.4848	35.18	326.7	315.0	649.7	-.6616

TABLE 2

3 LOBE BEARING, PRELOAD: $\delta = 0.5$

R_s	$\epsilon_s \cos\phi_s$	ϵ_s	ϕ_s	Min. Film Thickn.	S	$\frac{R}{C}\frac{F_r}{W}$	$\frac{Q_s}{WDLC}$	$\frac{Q_x}{WDLC}$	$\frac{V}{\omega}$	$\frac{C\omega_n^2}{V}$	$\frac{CW}{\mu DL\left(\frac{R}{C}\right)^2}$	$\frac{CK_{XX}}{V}$	$\frac{CK_{XY}}{V}$	$\frac{CK_{YX}}{V}$	$\frac{CK_{YY}}{V}$	$\frac{C\omega_{XX}}{V}$	$\frac{C\omega_{XY}}{V}$	$\frac{C\omega_{YX}}{V}$	$\frac{C\omega_{YY}}{V}$
$\frac{L}{D} = \frac{1}{2}$	0.02	.0363	56.57	.4791	3.127	93.20	.3833	2.827	.4810	68.00	.5509	17.49	21.28	-22.43	14.23	47.61	-1.429	-1.313	43.13
	0.1	.1910	58.43	.3783	.4952	15.73	.6014	2.809	.4559	19.43	.9941	6.261	4.761	- 4.275	2.754	12.07	.0415	.0360	6.975
0	0.15	.2806	57.68	.3081	.2638	9.092	.8061	2.787	.4022	20.77	1.994	6.301	4.023	- 2.548	2.172	9.796	.6652	.6812	4.311
	0.2	.3482	54.94	.2442	.1557	5.901	.5883	2.764	.2511	46.38	7.545	7.125	4.035	-1.485	1.885	9.158	1.106	1.107	2.941
	0.25	.4023	51.58	.1865	.09285	3.933	1.154	2.741	-	-	-	8.639	4.390	- .6330	1.658	9.279	1.425	1.428	2.037
	0.3	.4373	46.68	.1391	.0561	2.668	1.343	2.773	-	-	-	10.94	4.788	.1774	1.358	9.756	1.438	1.461	1.021
	0.02	.0384	58.60	.4790	2.231	123.7	.4179	2.860	.4866	58.78	.6673	15.44	20.71	-21.72	12.60	45.57	-1.216	-1.113	41.53
	0.1	.2009	60.14	.3752	.3585	20.59	.6253	2.843	.4683	16.93	1.196	5.648	4.701	- 4.149	2.563	11.47	.0986	.1015	6.867
2000	0.15	.2919	59.08	.3034	.1951	11.72	.8317	2.824	.4246	17.60	2.285	5.766	3.999	- 2.490	2.087	9.301	.6741	.6870	4.309
	0.2	.3589	56.14	.2392	.1184	7.518	1.015	2.804	.3094	29.55	6.323	6.600	4.066	- 1.478	1.874	8.798	1.133	1.138	3.010
	0.25	.4129	52.74	.1812	.07210	4.907	1.186	2.782	-	-	-	8.158	4.400	- .6389	1.665	8.834	1.411	1.406	2.145
	0.3	.4469	47.83	.1341	.04462	3.277	1.364	2.802	-	-	-	10.53	4.851	.2204	1.381	9.392	1.451	1.474	1.084
	0.02	.0386	58.78	.4790	1.058	163.0	.4527	2.889	.4909	54.11	1.296	14.29	21.25	-22.03	11.92	45.86	- .8953	-.8568	42.25
	0.1	.2099	61.55	.3723	.1649	25.90	.6527	2.874	.4813	14.49	2.226	4.964	4.628	-4.049	2.369	10.92	.1398	.1472	6.782
10000	0.15	.3041	60.44	.2983	.09153	14.74	.8653	2.858	.4554	13.97	3.866	5.019	3.946	-2.478	1.972	8.807	.6760	.6883	4.331
	0.2	.3708	57.36	.2335	.05731	9.501	1.053	2.840	.3858	17.66	7.804	5.696	4.000	-1.532	1.805	8.238	1.091	1.081	3.078
	0.25	.4774	54.20	.1738	.03552	6.115	1.235	2.821	.0820	343.6	245.0	6.990	4.386	- .7362	1.642	8.258	1.376	1.384	2.254
	0.3	.4677	49.58	.1259	.02234	4.009	1.415	2.832	--	--	--	9.032	4.890	.1469	1.359	8.673	1.390	1.399	1.163
	0.02	.0382	58.47	.4790	.5285	182.5	.4653	2.900	.4920	53.56	2.567	14.17	21.82	-22.54	11.88	46.76	-.8353	-.7939	43.34
	0.1	.2130	61.99	.3712	.08014	28.12	.6627	2.886	.4852	13.75	4.347	4.729	4.592	-4.021	2.310	10.73	.1499	.1579	6.761
30000	0.15	.3083	60.88	.2964	.04487	16.07	.8772	2.870	.4650	12.96	7.316	4.750	3.902	-2.483	1.935	8.619	.6639	.6779	4.346
	0.2	.3758	57.85	.2311	.02836	10.40	1.068	2.854	.4102	15.17	13.55	5.343	3.945	-1.560	1.784	8.020	1.066	1.059	3.112
	0.25	.4331	54.74	.1708	.01777	6.718	1.253	2.836	.2219	45.98	55.53	6.484	4.323	-.8115	1.629	7.995	1.340	1.348	2.328
	0.3	.4695	50.28	.1223	.01123	4.390	1.436	2.844	--	--	--	8.247	4.849	.0666	1.342	8.345	1.344	1.350	1.201
$\frac{L}{D} = 1$	0.02	.0406	60.47	.4788	1.571	47.02	.1977	2.680	.4740	57.97	.9350	14.72	21.02	-20.87	11.61	47.46	-.9866	-.8697	40.93
	0.1	.2038	60.62	.3743	.2444	7.901	.5409	2.633	.4545	16.06	1.665	5.477	4.986	-3.545	2.335	12.51	.4165	.4397	6.030
0	0.15	.2843	58.16	.3066	.1392	4.887	.7480	2.591	.4171	15.64	2.847	5.544	4.234	-2.095	1.917	10.40	.9740	1.080	3.741
	0.2	.3513	55.30	.2428	.08550	3.325	.9381	2.545	.3261	21.93	6.497	6.252	4.086	-1.213	1.693	9.642	1.344	1.367	2.582
	0.25	.4029	31.65	.1862	.05422	2.366	1.094	2.503	--	--	--	7.543	4.255	- .7520	1.511	9.588	1.570	1.590	2.290
	0.3	.4355	46.46	.1400	.03521	1.727	1.318	2.562	--	--	--	9.557	4.469	.1307	1.247	9.913	1.454	1.457	.8990

TABLE 2 (continued)

R_e	$\epsilon_B \cos\phi_B$	ϵ_B	ϕ_B	Min. Film Thickn.	S	$\frac{R}{C}\frac{F_f}{W}$	$\frac{Q_z}{NDLC}$	$\frac{Q_x}{NDLC}$	$\frac{V}{\omega}$	$\frac{CM\omega^2}{W}$	$\frac{CMW}{\mu DL\left(\frac{R}{C}\right)^2}$	$\frac{CK_{xx}}{W}$	$\frac{CK_{xy}}{W}$	$\frac{CK_{yx}}{W}$	$\frac{CK_{yy}}{W}$	$\frac{C\omega B_{xx}}{W}$	$\frac{C\omega B_{xy}}{W}$	$\frac{C\omega B_{yx}}{W}$	$\frac{C\omega B_{yy}}{W}$
	0.02	.0445	63.32	.4785	1.003	55.84	.2242	2.705	.4813	46.40	1.172	12.26	19.84	-19.34	9.503	[illegible] 90	-.8903	-.7660	37.57
	0.1	.2166	62.50	.3700	.1611	9.383	.5806	2.658	.4720	12.99	2.043	4.747	4.988	-3.288	2.095	1[illegible].85	.5000	.5393	5.675
2000	0.15	.2987	59.86	.3006	.09359	5.730	.7988	2.614	.4450	12.26	3.317	4.869	4.265	-1.949	1.783	9.822	1.003	1.016	3.568
	0.2	.3646	56.73	.2365	.05935	3.870	.9752	2.567	.3759	15.10	6.446	5.564	4.128	-1.151	1.624	9.107	1.333	1.346	2.528
	0.25	.4158	53.04	.1797	.03882	2.729	1.140	2.523	.1237	138.3	90.22	6.833	4.290	-.7608	1.486	9.041	1.553	1.567	2.333
	0.3	.4468	47.82	.1341	.02618	1.991	1.347	2.562	--	--	--	8.817	4.528	.1506	1.239	9.442	1.451	1.465	.9452
	0.02	.0476	65.16	.4782	.4083	63.12	.2543	2.729	.4868	39.07	2.424	10.60	19.05	-18.28	8.166	41.44	-.7[illegible]90	-.63[illegible]	35.32
	0.1	.2275	63.92	.3662	.06666	10.60	.6207	2.681	.4885	10.52	3.996	4.062	4.921	-3.087	1.863	11.16	.5233	.5515	5.355
10000	0.15	.3123	61.30	.2946	.03911	6.410	.8456	2.636	.4786	9.077	5.878	4.075	4.274	-1.848	1.601	9.217	.9606	.9691	3.[illegible]
	0.2	.3792	58.17	.2294	.02504	4.264	1.033	2.586	.4574	8.574	8.672	4.472	4.205	-1.141	1.495	8.529	1.275	1.291	2.4[illegible]
	0.25	.4306	54.51	.1721	.01661	2.961	1.217	2.535	.3951	10.17	15.51	5.261	4.414	-.6449	1.408	8.398	1.473	1.480	1.929
	0.3	.4628	49.59	.1259	.01135	2.124	1.426	2.[illegible]48	.2938	11.89	26.54	6.631	4.669	.0264	1.145	8.556	1.314	1.337	.9387
	0.02	.0485	65.66	.4781	.1900	65.84	.2661	2.738	.4884	37.17	4.956	10.13	18.82	-17.99	7.834	40.76	-.6924	-.5630	34.69
	0.1	.2310	64.34	.3649	.03123	11.08	.6330	2.691	.4929	9.904	8.033	3.866	4.885	-3.051	1.797	10.93	.5127	.5403	5.301
30,000	0.15	.3179	61.84	.2922	.01829	6.666	.8659	2.645	.4890	8.232	11.40	3.824	4.240	-1.798	1.547	8.970	.9394	.9466	3.289
	0.2	.3844	58.65	.2268	.01178	4.436	1.054	2.596	.4313	7.285	15.66	4.098	4.208	-1.152	1.454	8.315	1.243	1.260	2.412
	0.25	.4358	55.00	.1694	.00783	3.064	1.245	2.543	.4556	7.015	22.70	4.625	4.445	-.7021	1.367	8.165	1.435	1.437	1.927
	0.3	.4683	50.16	.1229	.00533	2.176	1.460	2.547	.4764	3.946	18.76	5.473	4.750	-.0926	1.104	8.294	1.274	1.294	.9394

TABLE 3

PLAIN CYLINDRICAL BEARING

	$\frac{L}{D} = \frac{1}{2}$				$\frac{L}{D} = 1$			
R_e	ε	S	$\frac{R}{C}\frac{F_f}{W}$	$\frac{CM\omega^2}{W}$	ε	S	$\frac{R}{C}\frac{F_f}{W}$	$\frac{CM\omega^2}{W}$
	0.0907	4.778	94.74	7.202	0.0964	1.353	26.88	6.418
	0.1578	2.654	53.14	7.039	0.1672	0.7594	15.29	6.381
	0.2902	1.278	26.50	6.719	0.3068	0.3757	7.937	6.300
0	0.4149	0.7377	16.19	6.483	0.4366	0.2267	5.168	6.368
	0.5138	0.4791	11.23	6.701	0.5379	0.1549	3.850	6.767
	0.5995	0.3198	8.111	7.592	0.6246	0.1095	3.009	7.698
	0.6777	0.2107	5.884	10.75	0.7017	0.07730	2.388	10.41
	0.8182	0.07786	2.884	- -	0.8365	0.03433	1.466	- -
	0.0969	2.878	148.7	6.318	0.1019	0.7948	41.13	5.716
	0.1680	1.609	83.47	6.190	0.1766	0.4458	23.21	5.639
	0.3075	0.7835	41.24	5.968	0.3230	0.2202	11.72	5.439
	0.4366	0.4610	24.84	5.846	0.4578	0.1327	7.331	5.305
2,000	0.5365	0.3061	16.97	6.058	0.5617	0.09076	5.238	5.473
	0.6213	0.2091	12.00	7.222	0.6495	0.06441	3.921	6.158
	0.6977	0.1415	8.493	9.920	0.7266	0.04583	2.982	7.780
	0.8314	0.05618	3.899	- -	0.8562	0.02139	1.699	- -
	0.1013	1.149	180.3	5.785	0.1058	0.3133	49.20	5.288
	0.1756	0.6440	101.3	5.667	0.1833	0.1759	27.75	5.197
	0.3209	0.3165	50.26	5.469	0.3346	0.08720	13.98	4.952
	0.4541	0.1891	30.48	5.381	0.4727	0.05279	8.684	4.714
10,000	0.5560	0.1277	20.95	5.549	0.5780	0.03624	6.146	4.532
	0.6414	0.08900	14.91	6.464	0.6662	0.02579	4.535	4.676
	0.7170	0.06171	10.61	8.053	0.7430	0.01836	3.377	5.031
	0.8470	0.02595	4.830	- -	0.8714	0.00851	1.791	- -
	0.1028	0.5313	191.5	5.664	0.1072	0.1442	51.98	5.144
	0.1783	0.2979	107.6	5.493	0.1857	0.08102	29.33	5.056
	0.3257	0.1469	53.46	5.302	0.3388	0.04021	14.77	4.799
	0.4604	0.08818	32.52	5.213	0.4781	0.02441	9.174	4.543
30,000	0.5632	0.05992	22.43	5.357	0.5840	0.01679	6.486	4.369
	0.6492	0.04204	16.02	6.183	0.6723	0.01197	4.777	4.314
	0.7243	0.02941	11.45	7.458	0.7488	0.008535	3.544	4.271
	0.8532	0.01269	5.270	- -	0.8764	0.003933	1.839	4.648

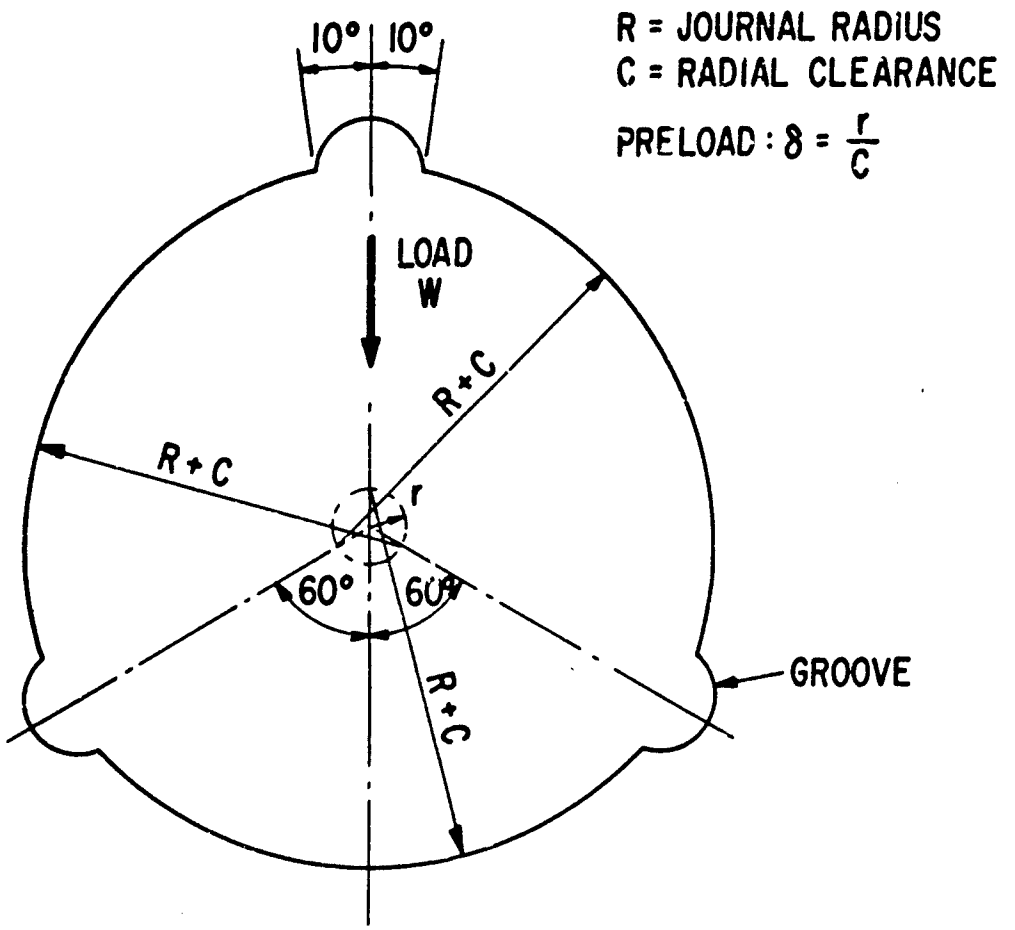


Figure 1. The 3 Lobe Bearing, Schematic

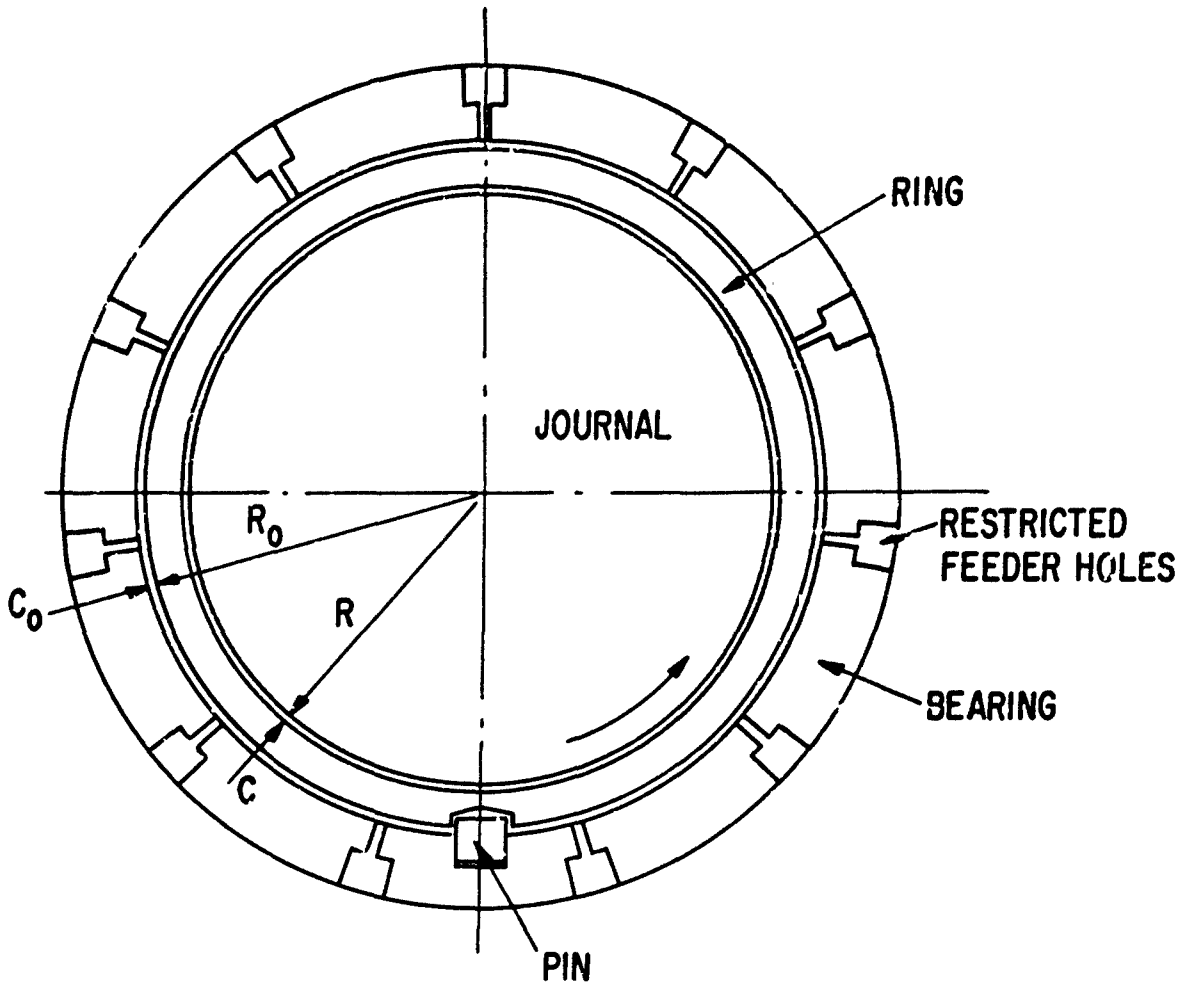


Figure 2. The Hydrodynamic-Hydrostatic Ring Bearing, Schematic

MTI-5033

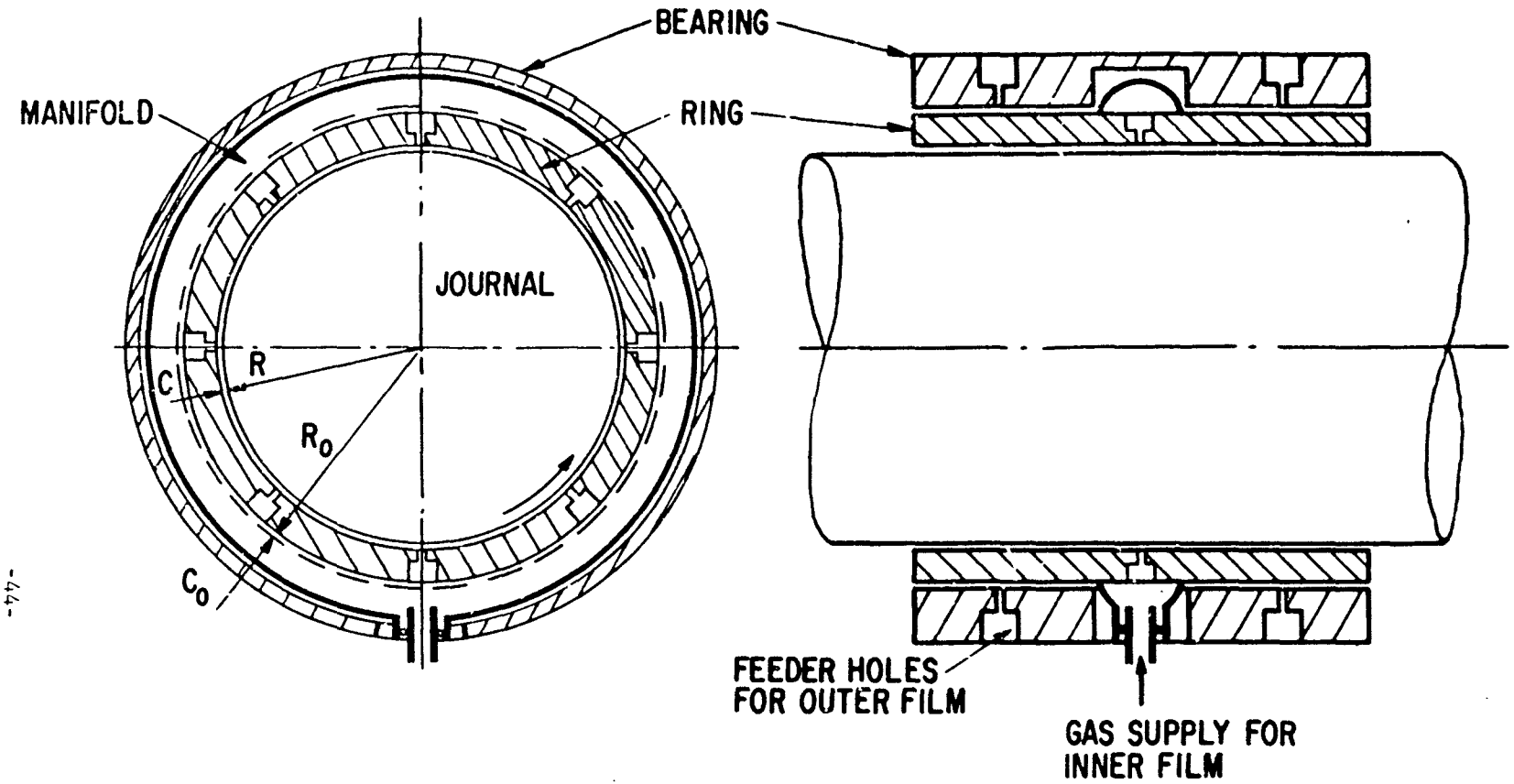


Figure 3. The Hybrid-Hydrostatic Ring Bearing, Schematic

MTI-5034

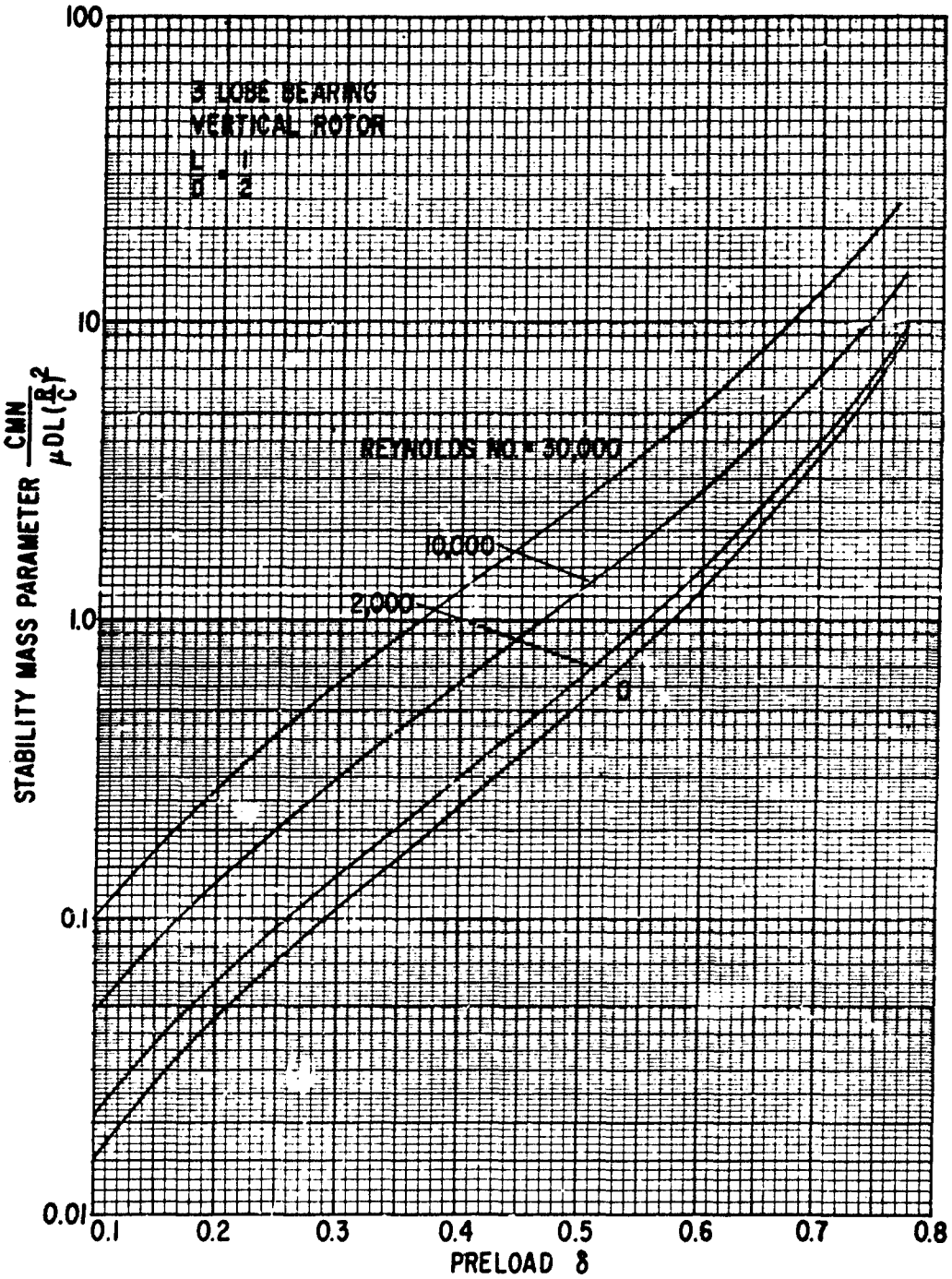


Figure 4. 3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = \frac{1}{2}$, Stability Map

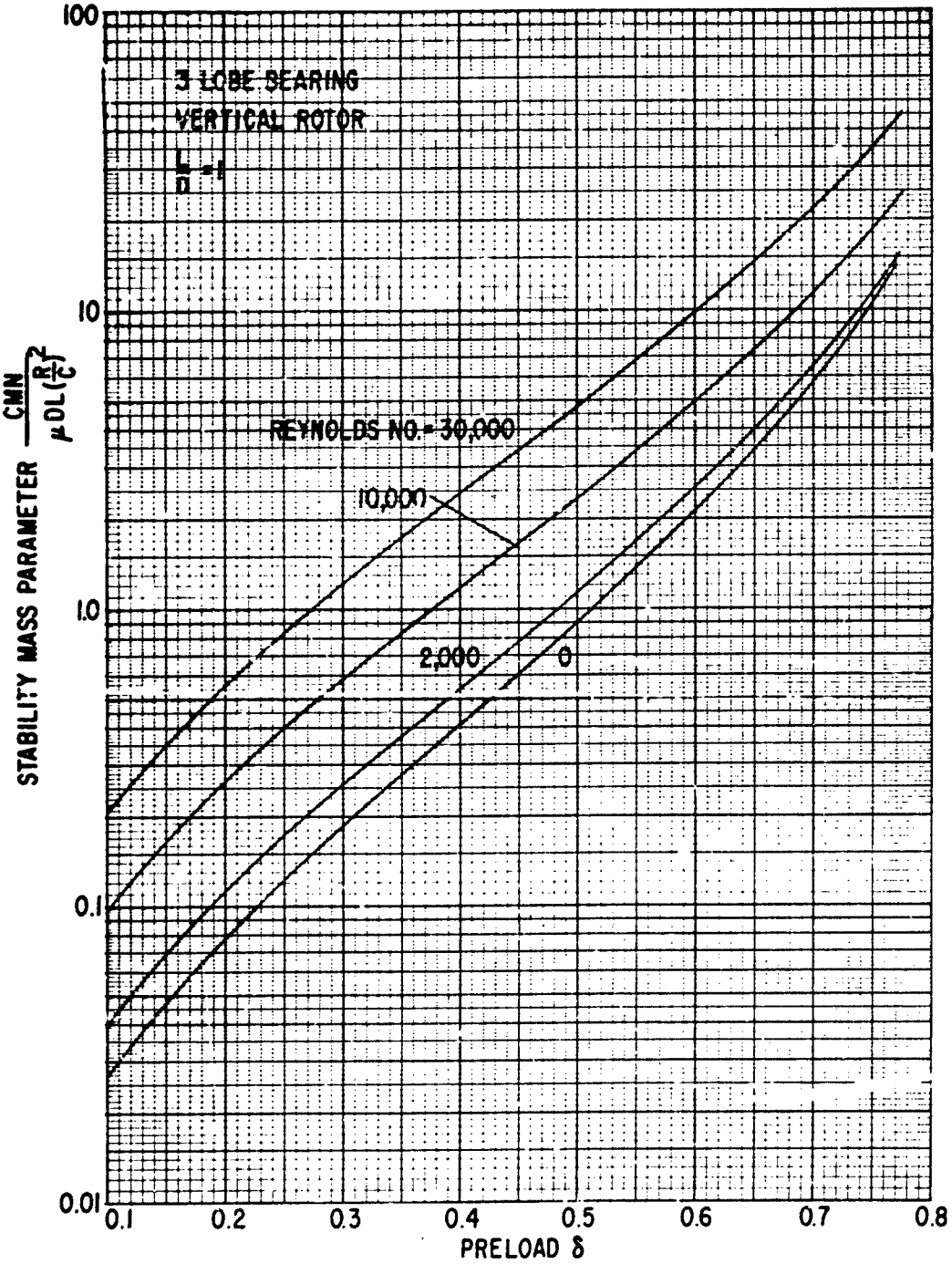


Figure 5. 3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = 1$, Stability Map

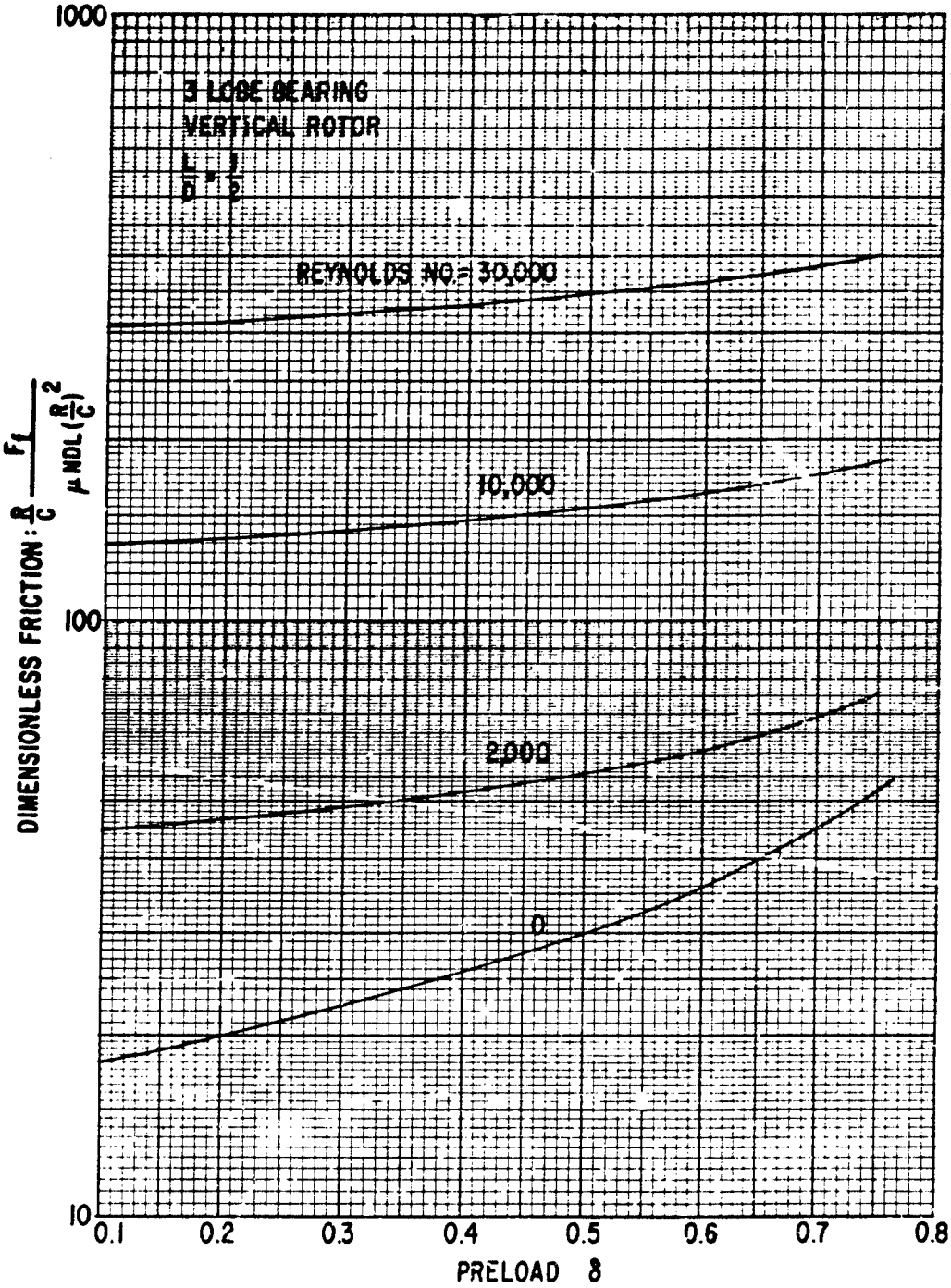


Figure 6. 3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = \frac{1}{2}$, Friction

MTI-5116

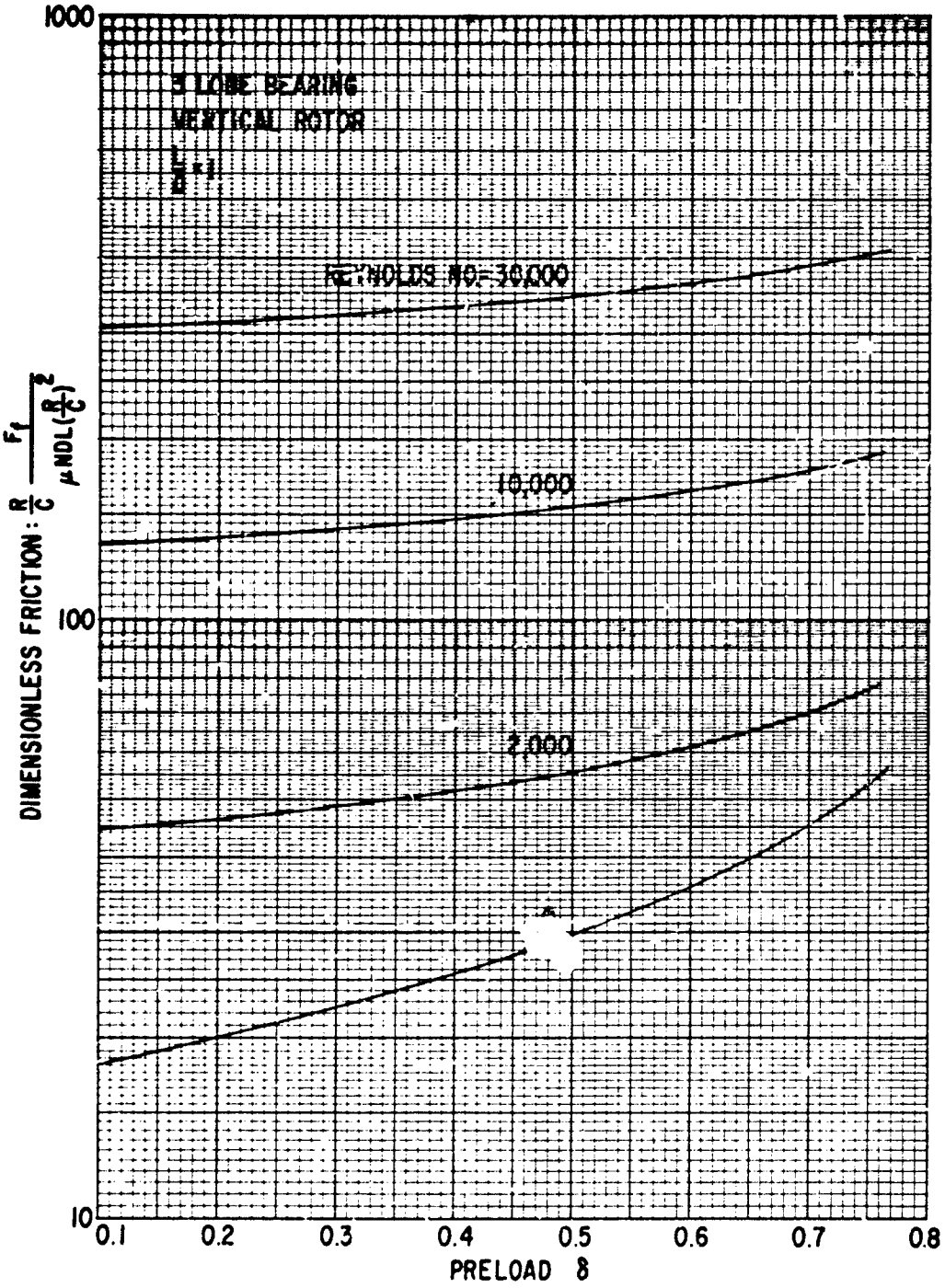


Figure 7. 3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = 1$, Friction

MTI-5119

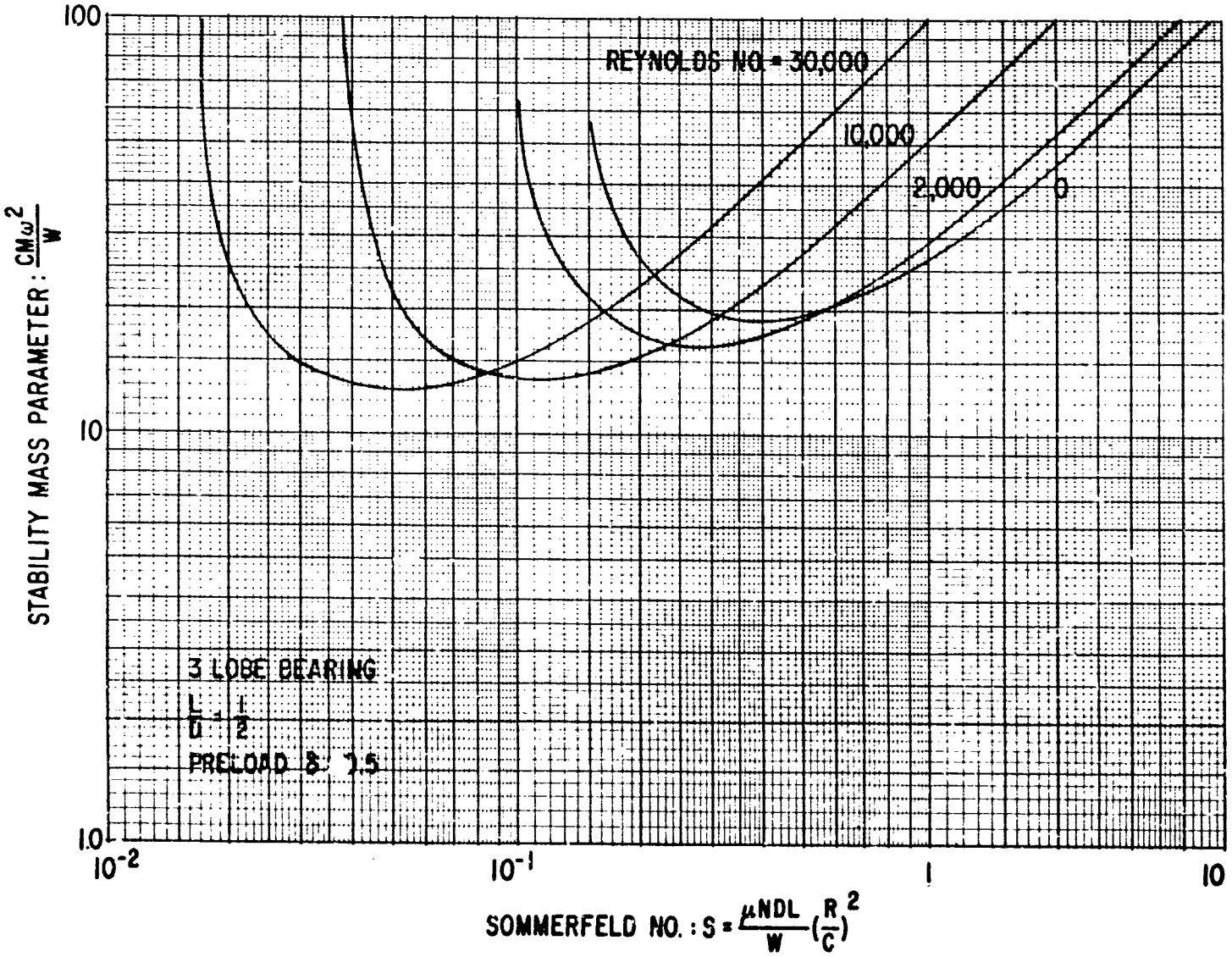


Figure 8. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Stability Map

MTI-5035

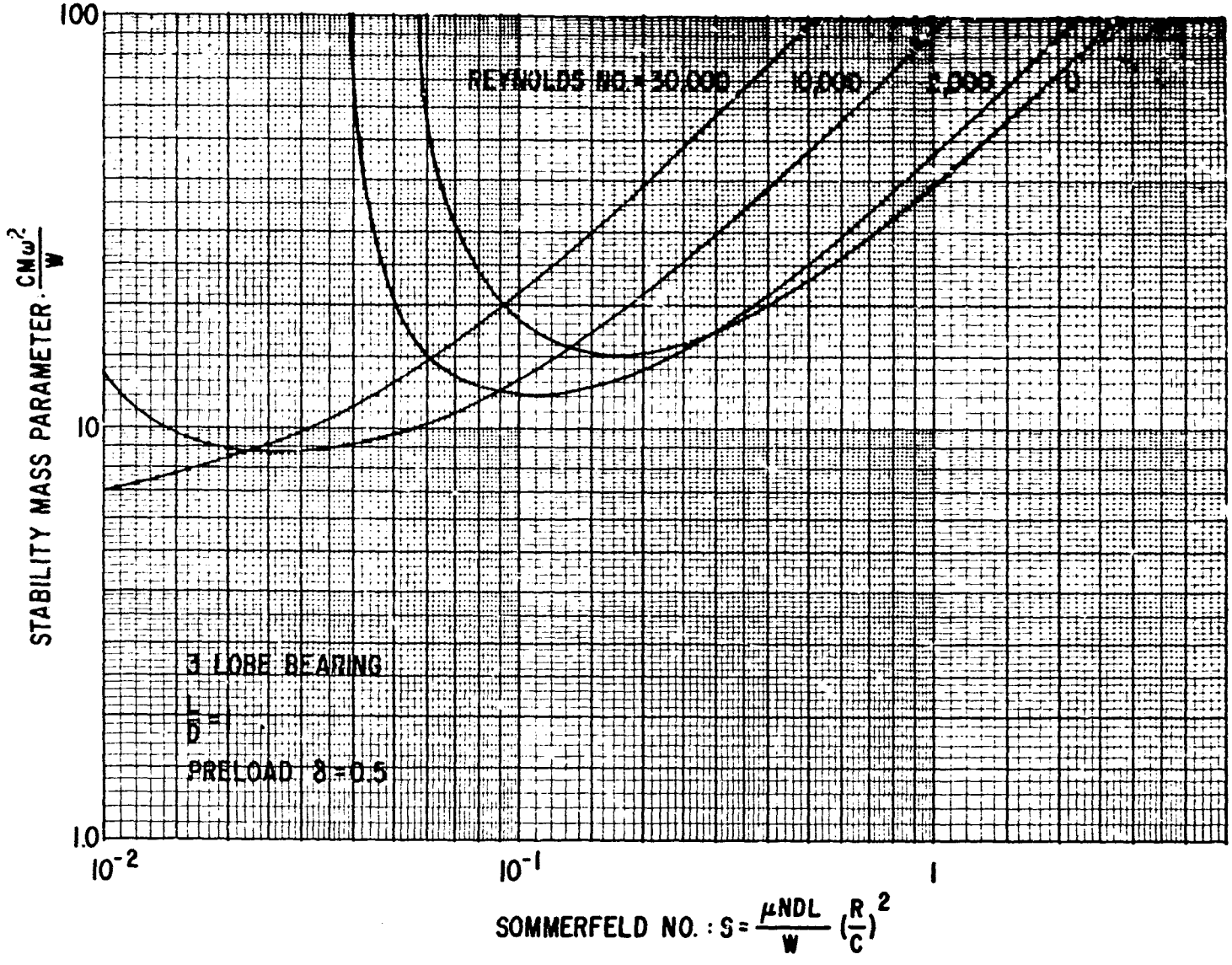


Figure 9. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D}$ = 1, Stability Map

MTI-5036

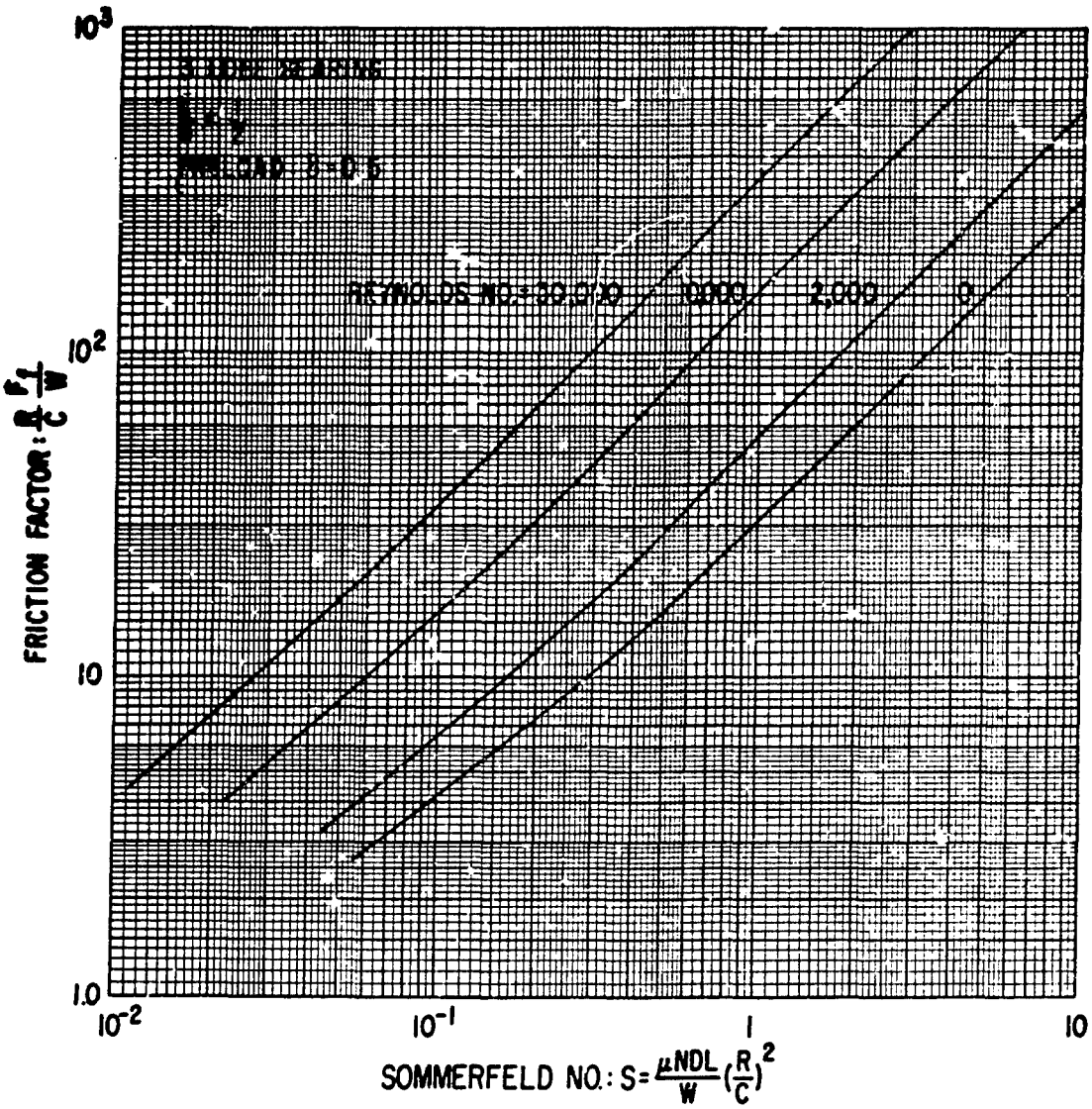


Figure 10. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Friction

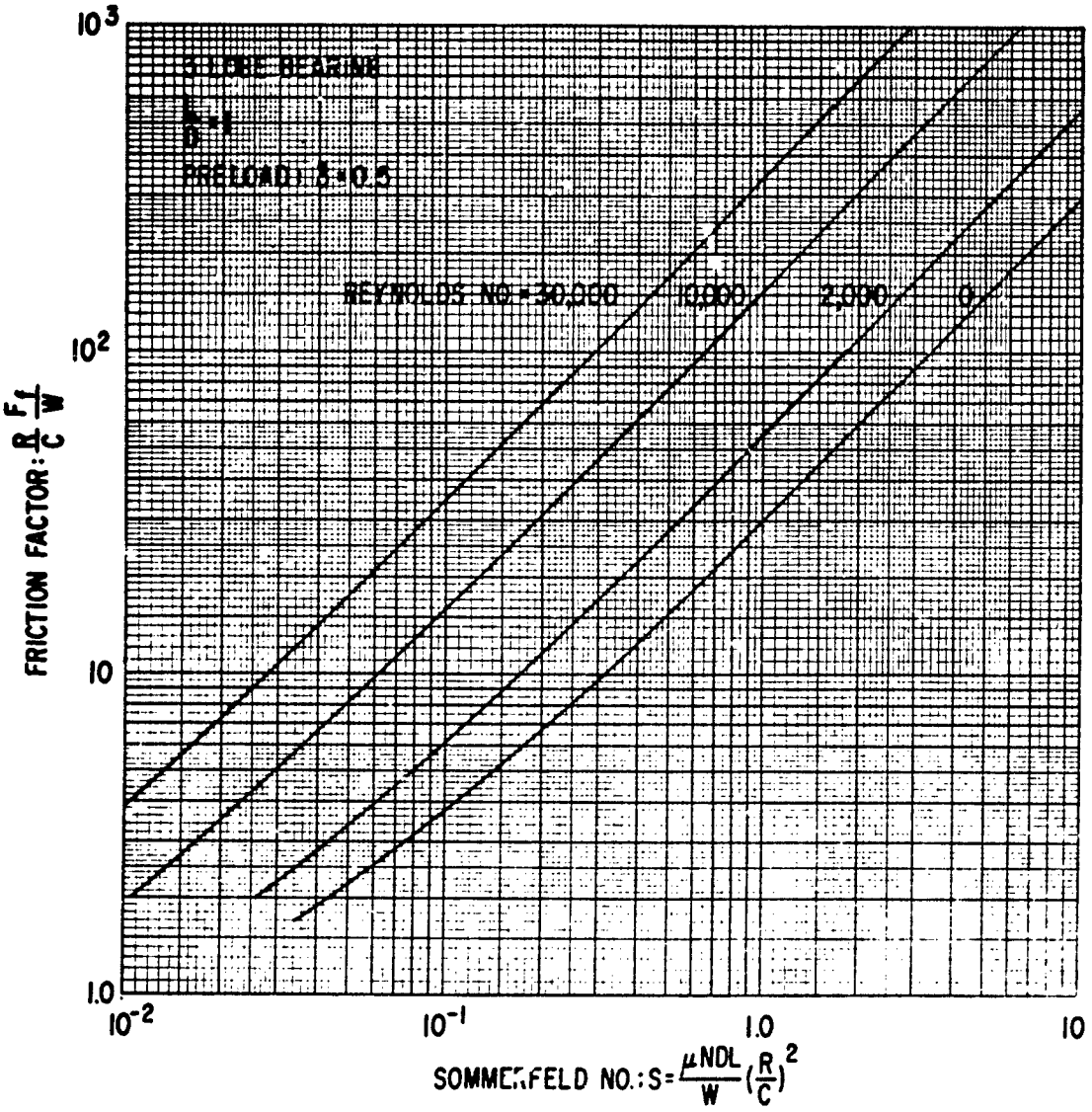


Figure 11. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D}$ = 1, Friction

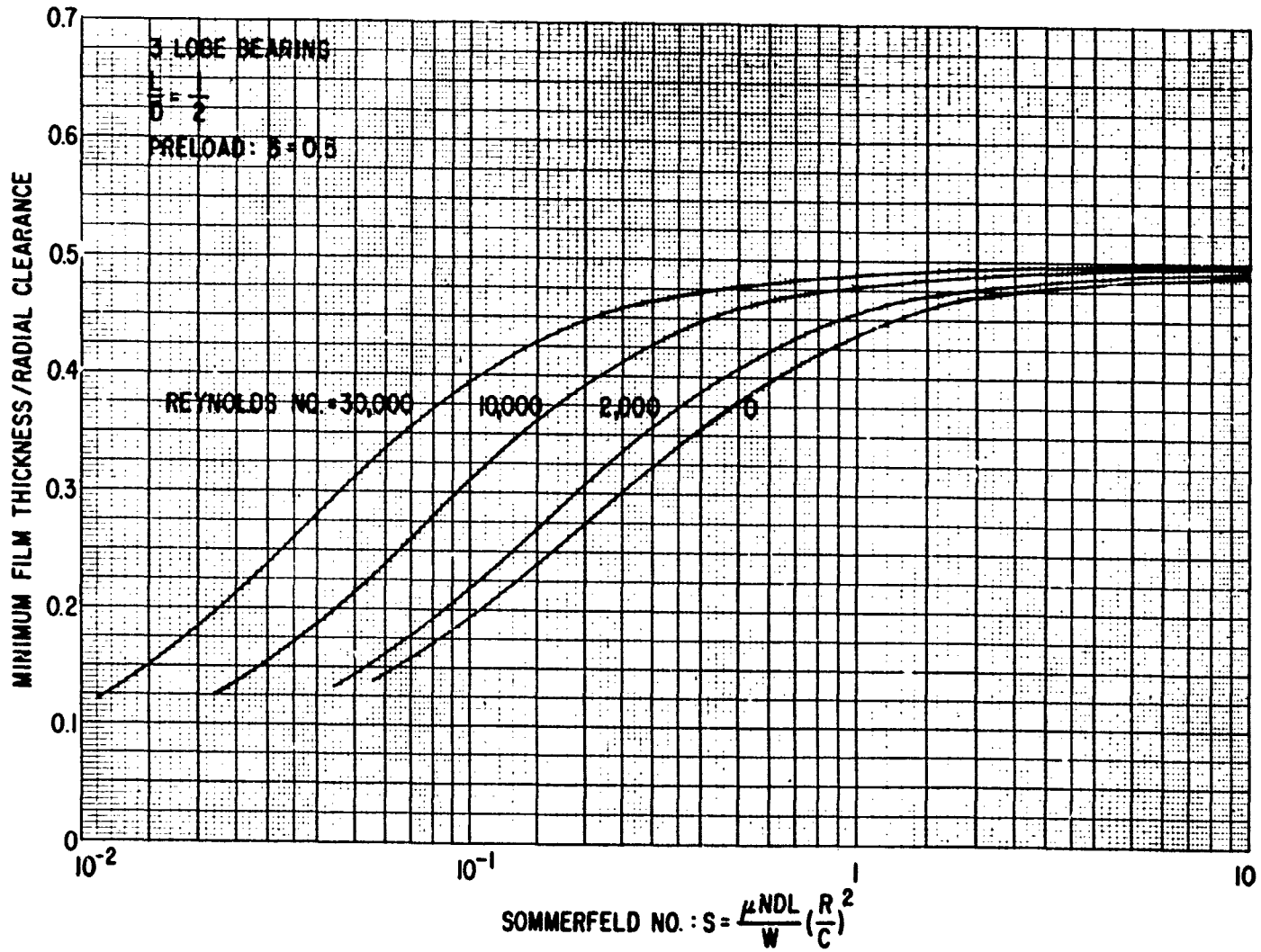


Figure 12. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Minimum Film Thickness

MTI-3121

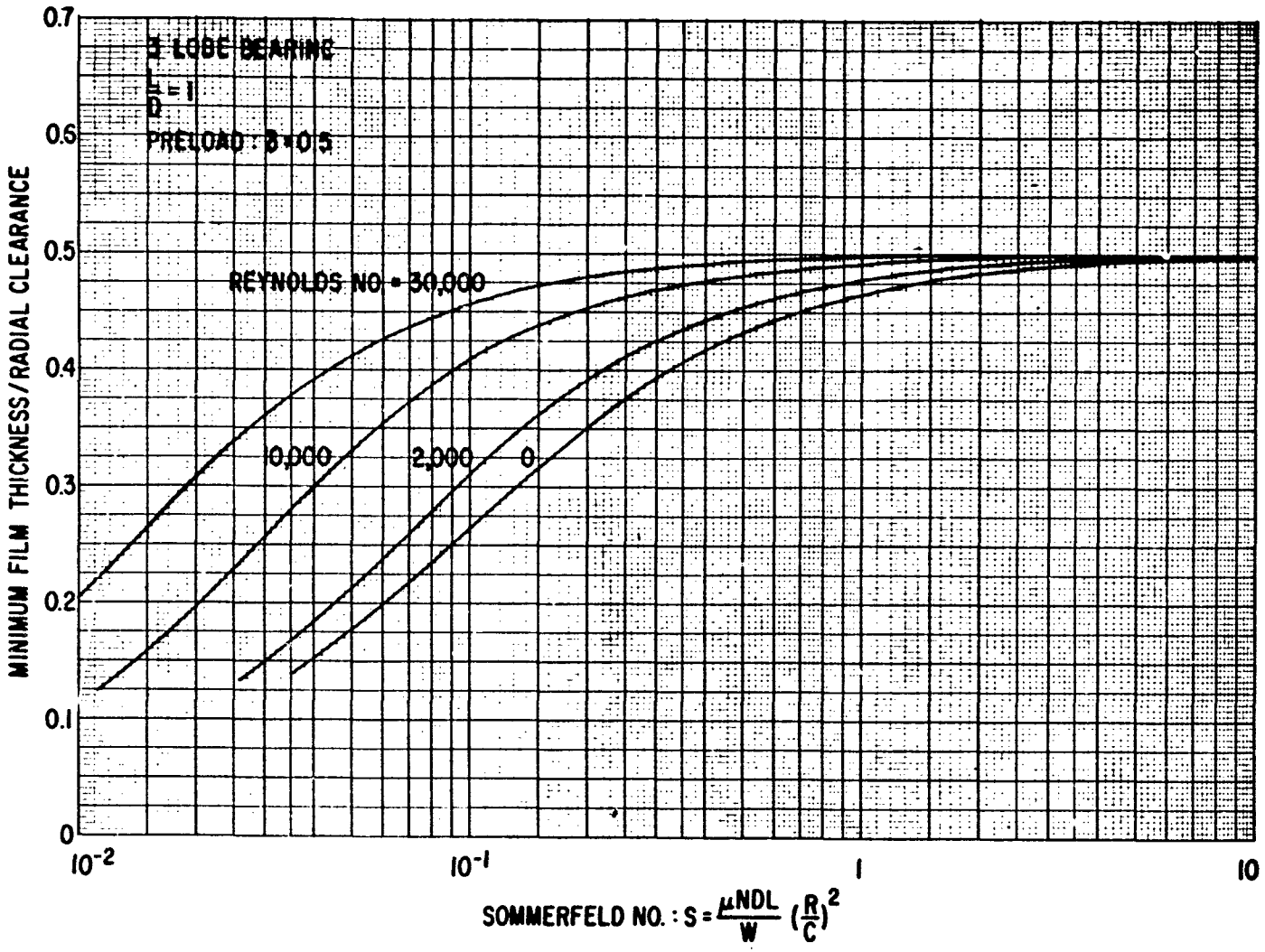


Figure 13. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Minimum Film Thickness

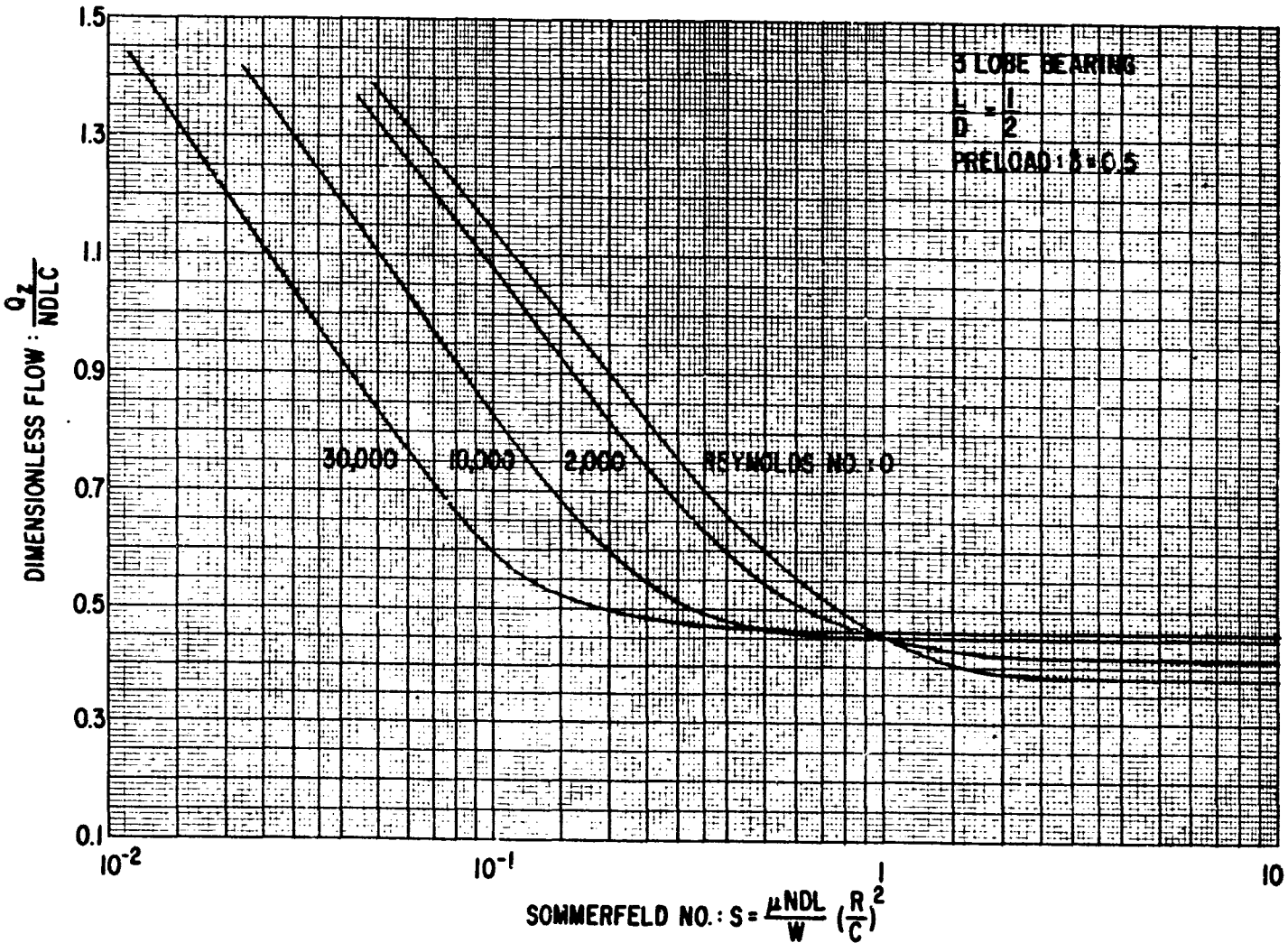


Figure 14. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Flow

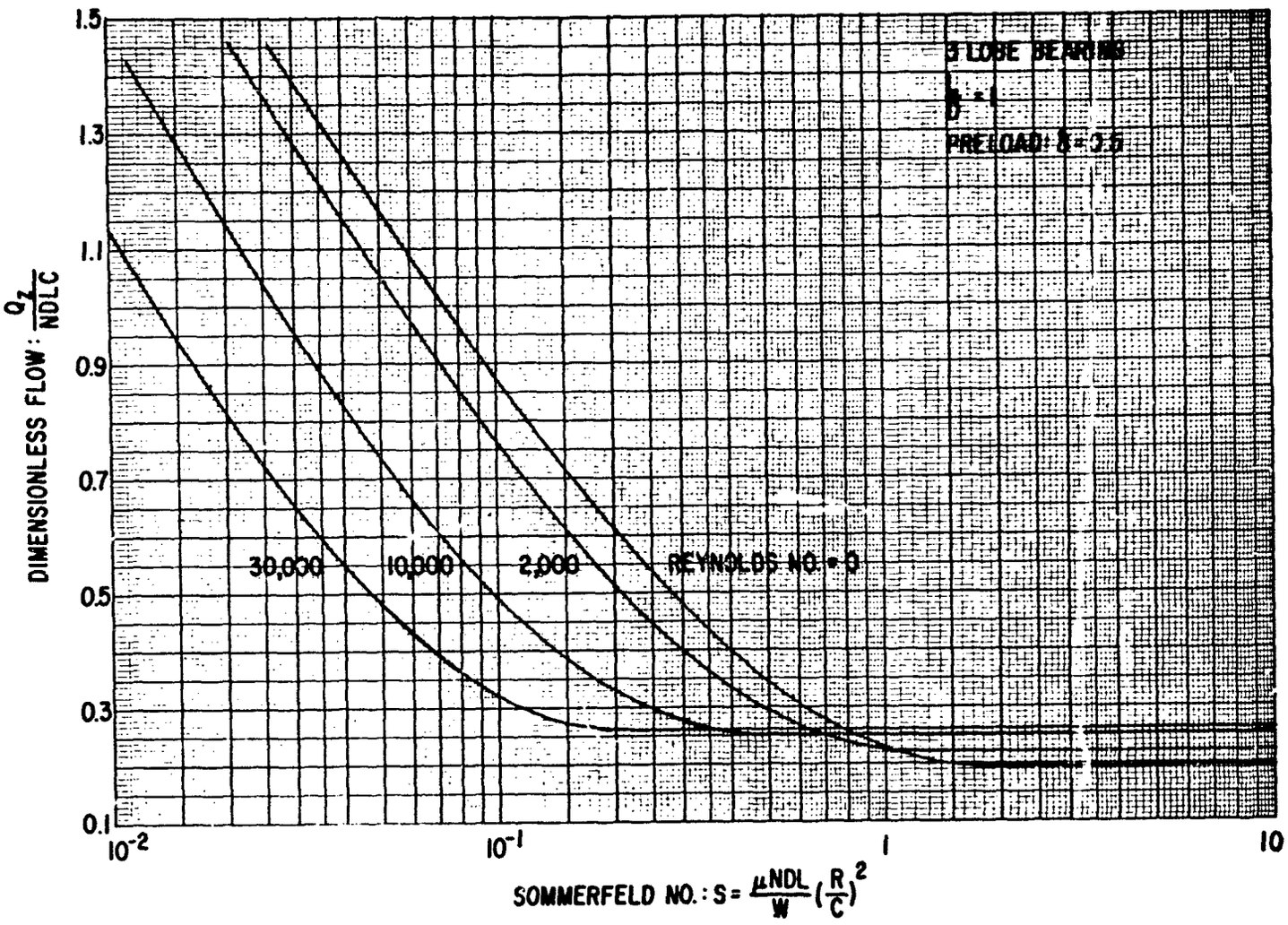


Figure 15. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D}$ = 1, Flow

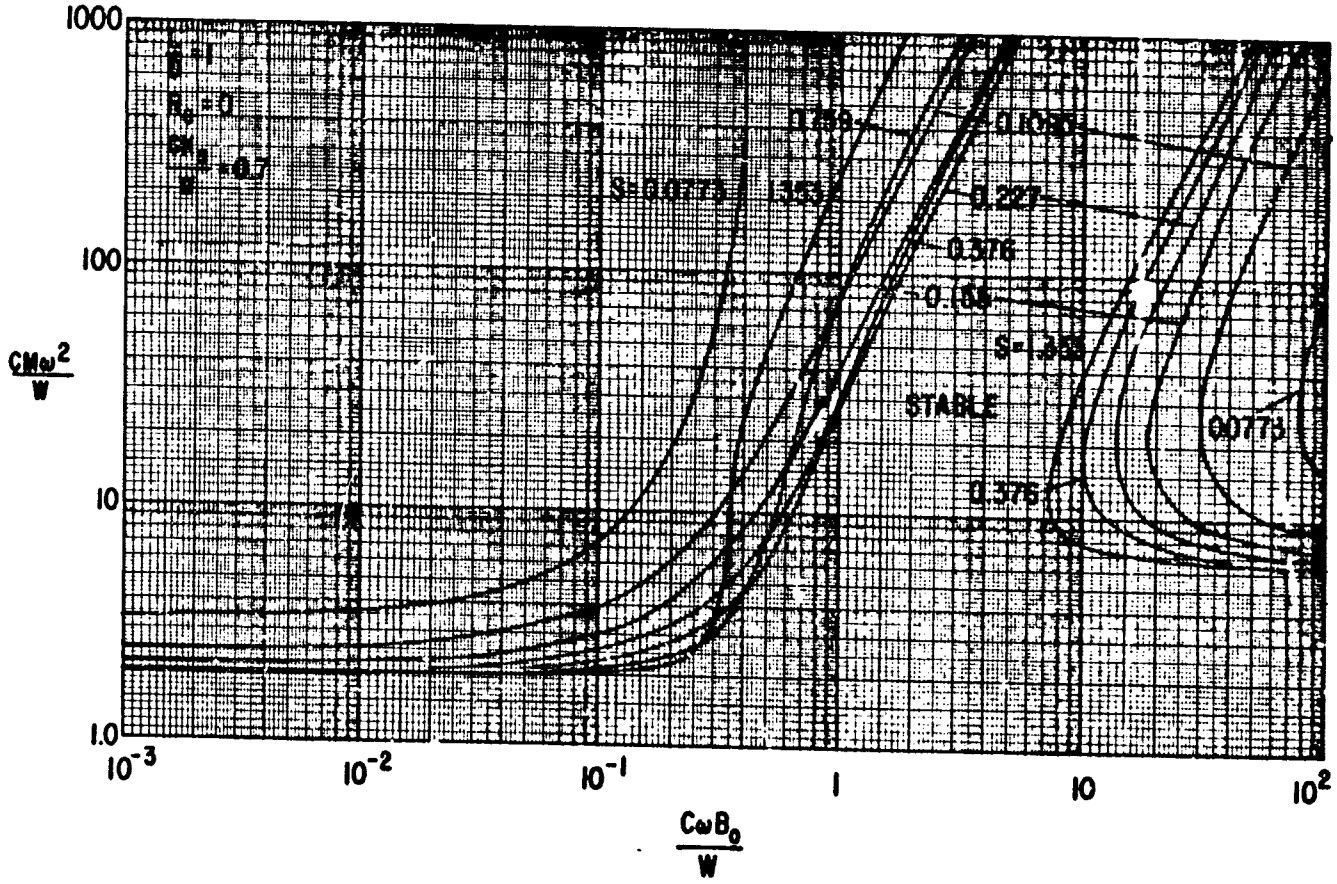


Figure 16. Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 0$, $\frac{CK_o}{W} = 0.7$, Stability Map

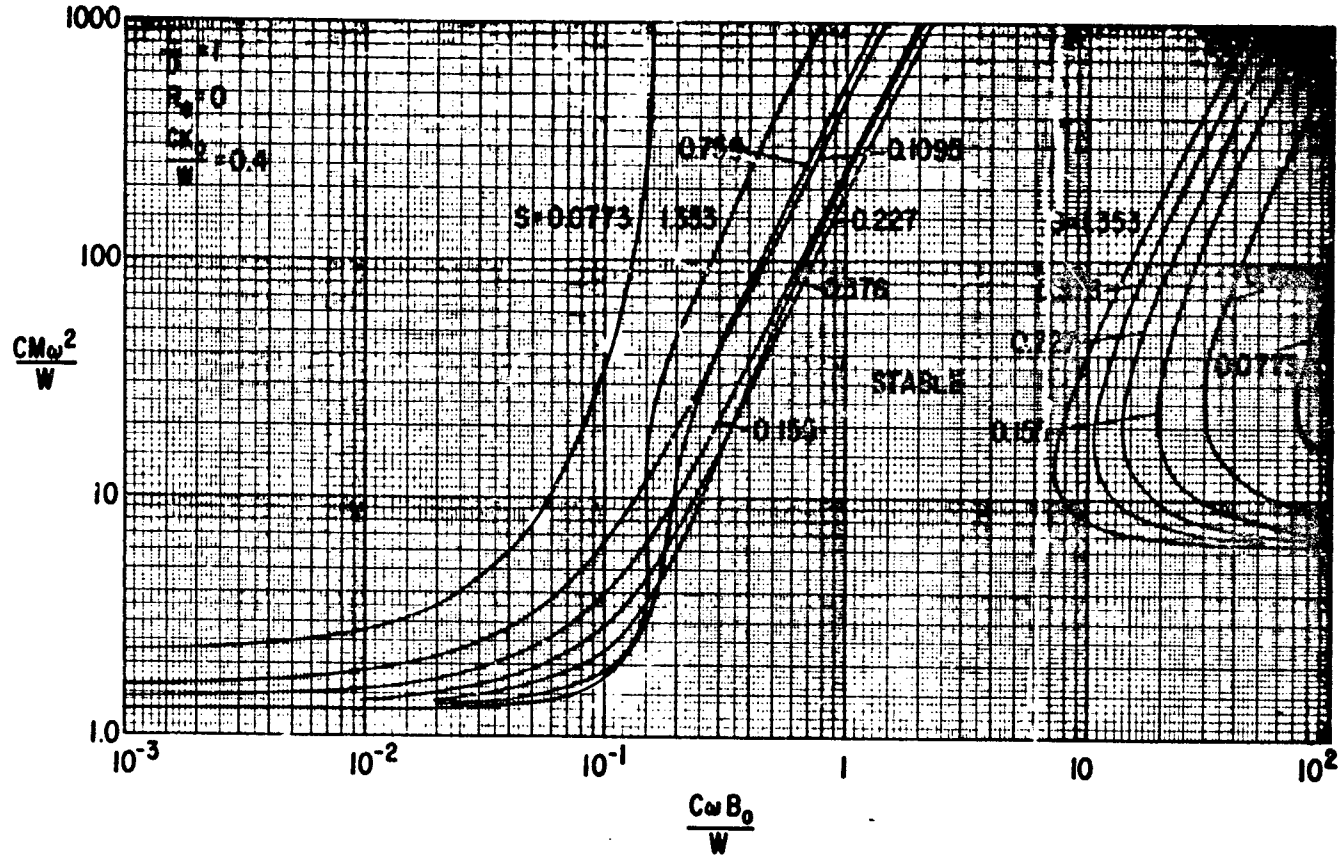


Figure 17. Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 0$, $\frac{CK_o}{W} = 0.4$, Stability Map

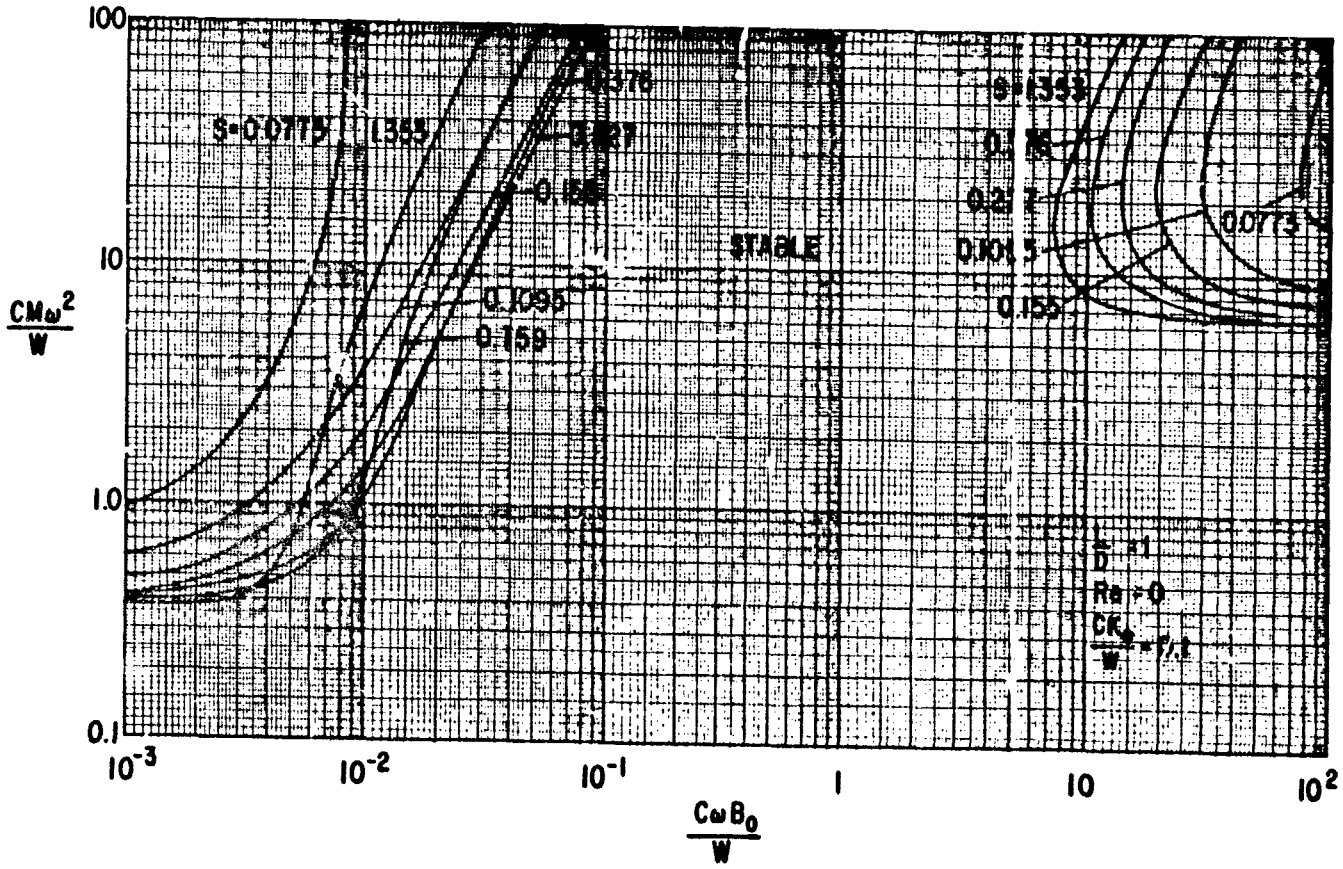


Figure 18. Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 0$, $\frac{CK_o}{W} = 0.1$, Stability Map

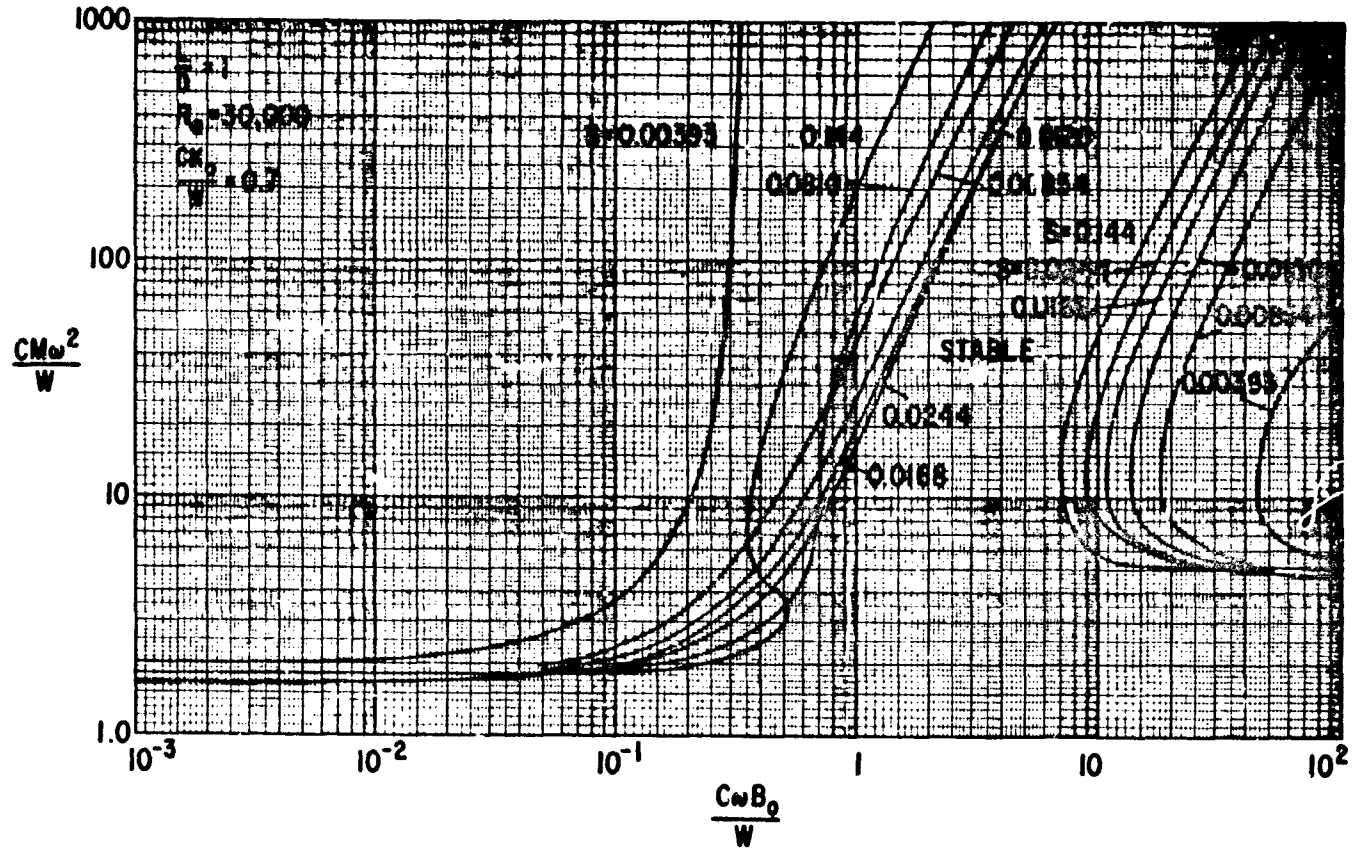


Figure 19. Hydrodynamic-Hydrostatic Ring Bearing, R_e = 30,000, $\frac{CK_o}{W}$ = 0.7, Stability Map

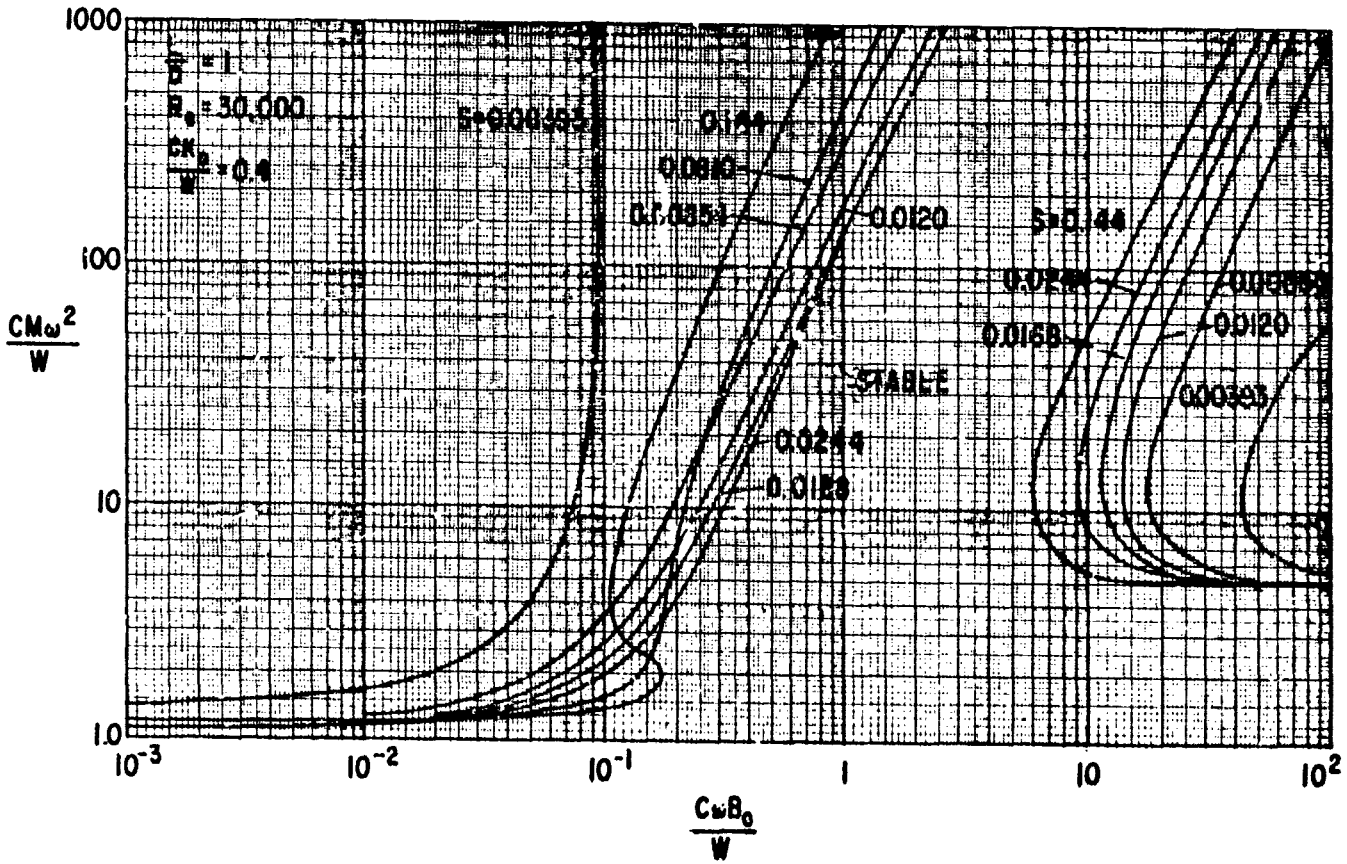


Figure 20. Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 30,000$, $\frac{CK_0}{W} = 0.4$, Stability Map

MTI-5127

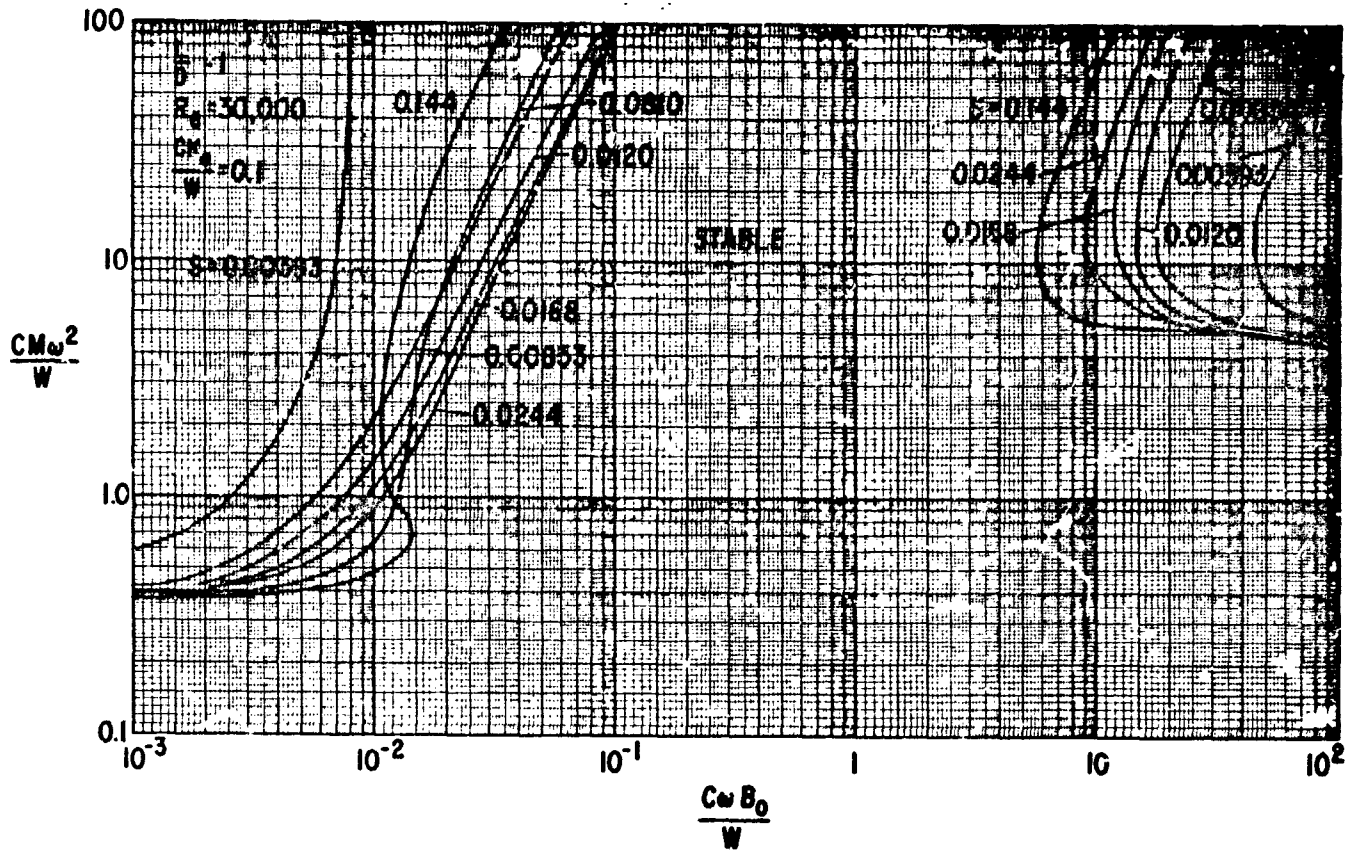


Figure 21. Hydrodynamic-Hydrostatic Ring Bearing, R_e = 30,000, $\frac{CK_o}{W}$ = 0.1, Stability Map

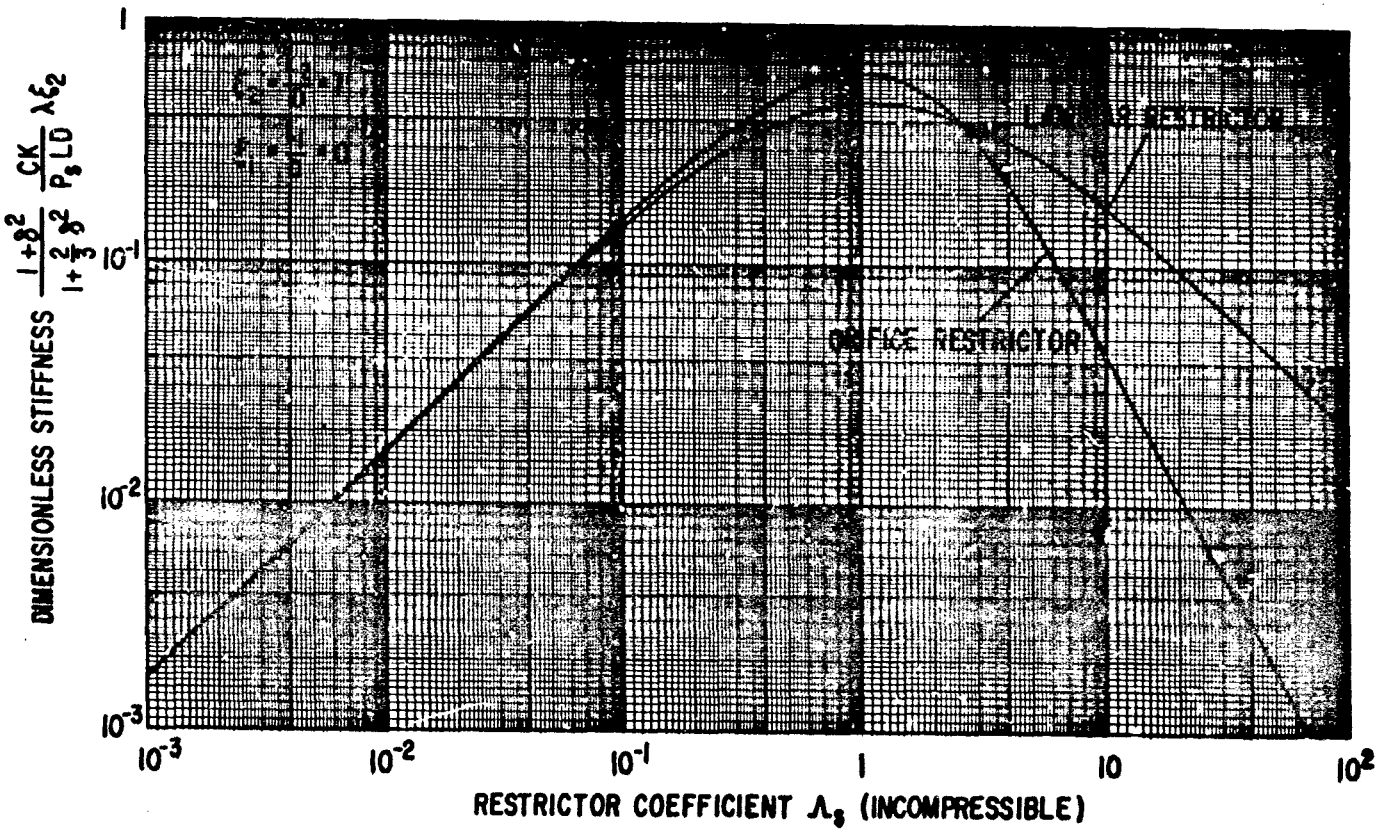


Figure 22. Hydrostatic Bearing, Incompressible, Single Plane Admission, Stiffness

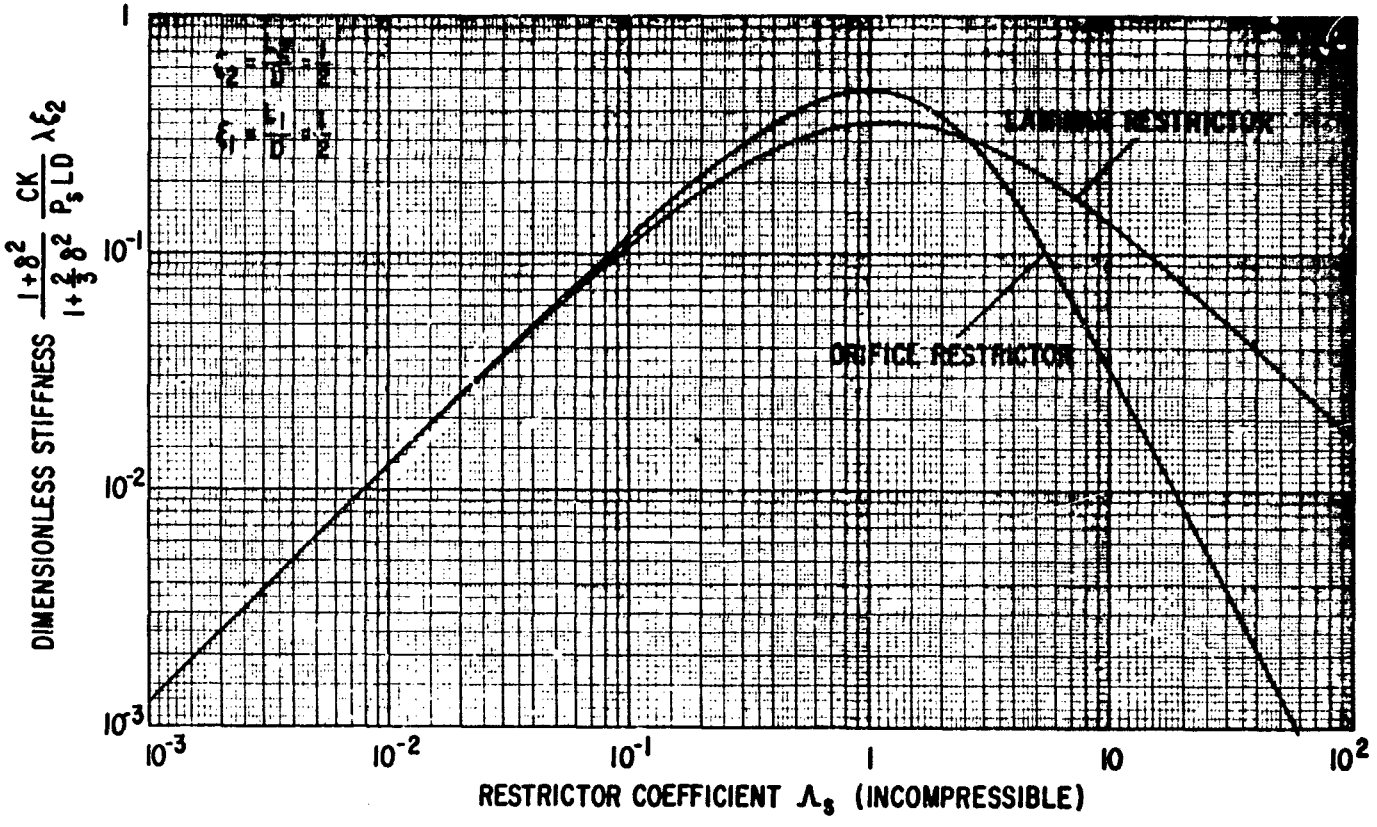


Figure 23. Hydrostatic Bearing, Incompressible, Double Plane Admission, Stiffness

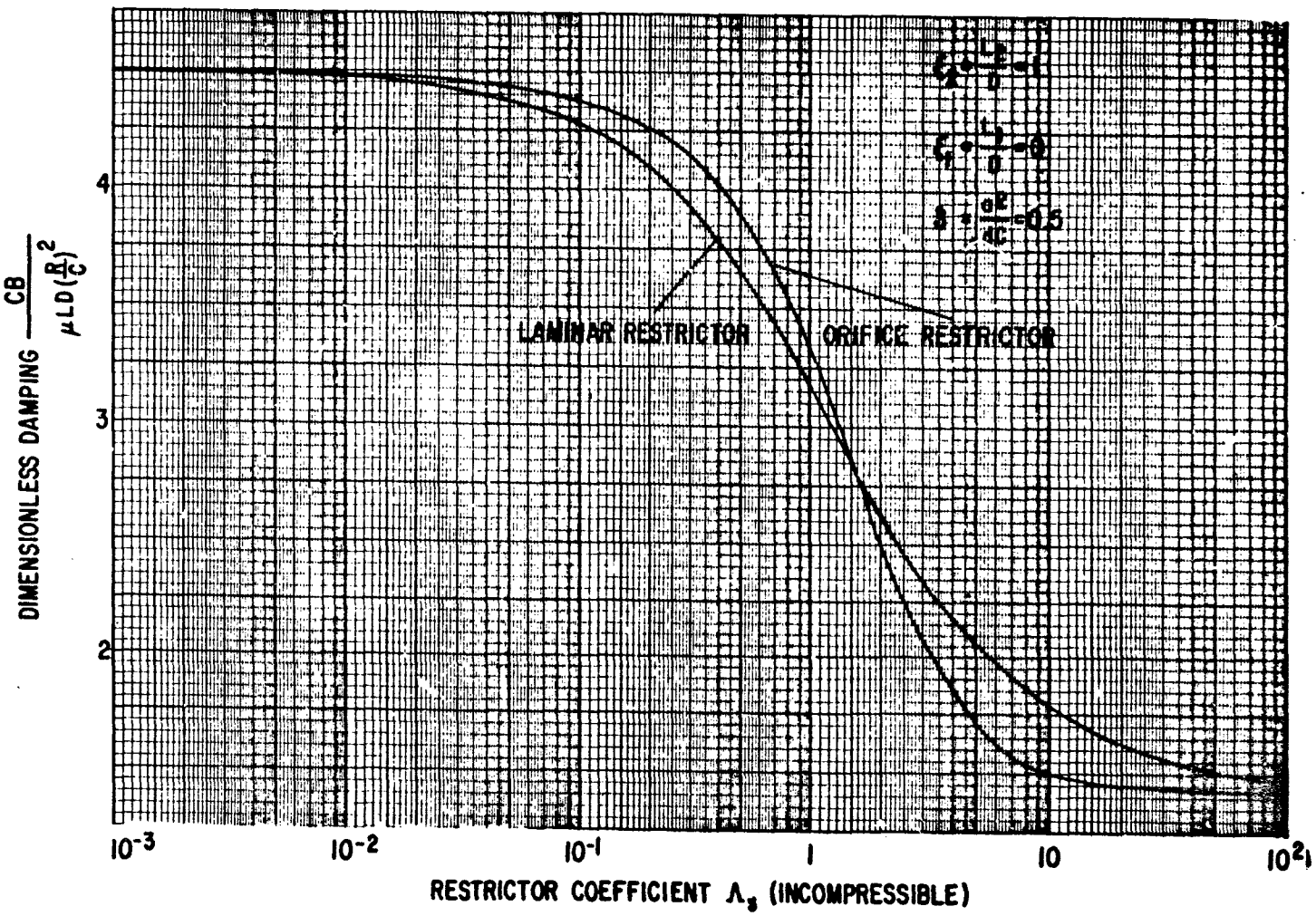


Figure 24. Hydrostatic Bearing, Incompressible, Single Plane Admission, Damping

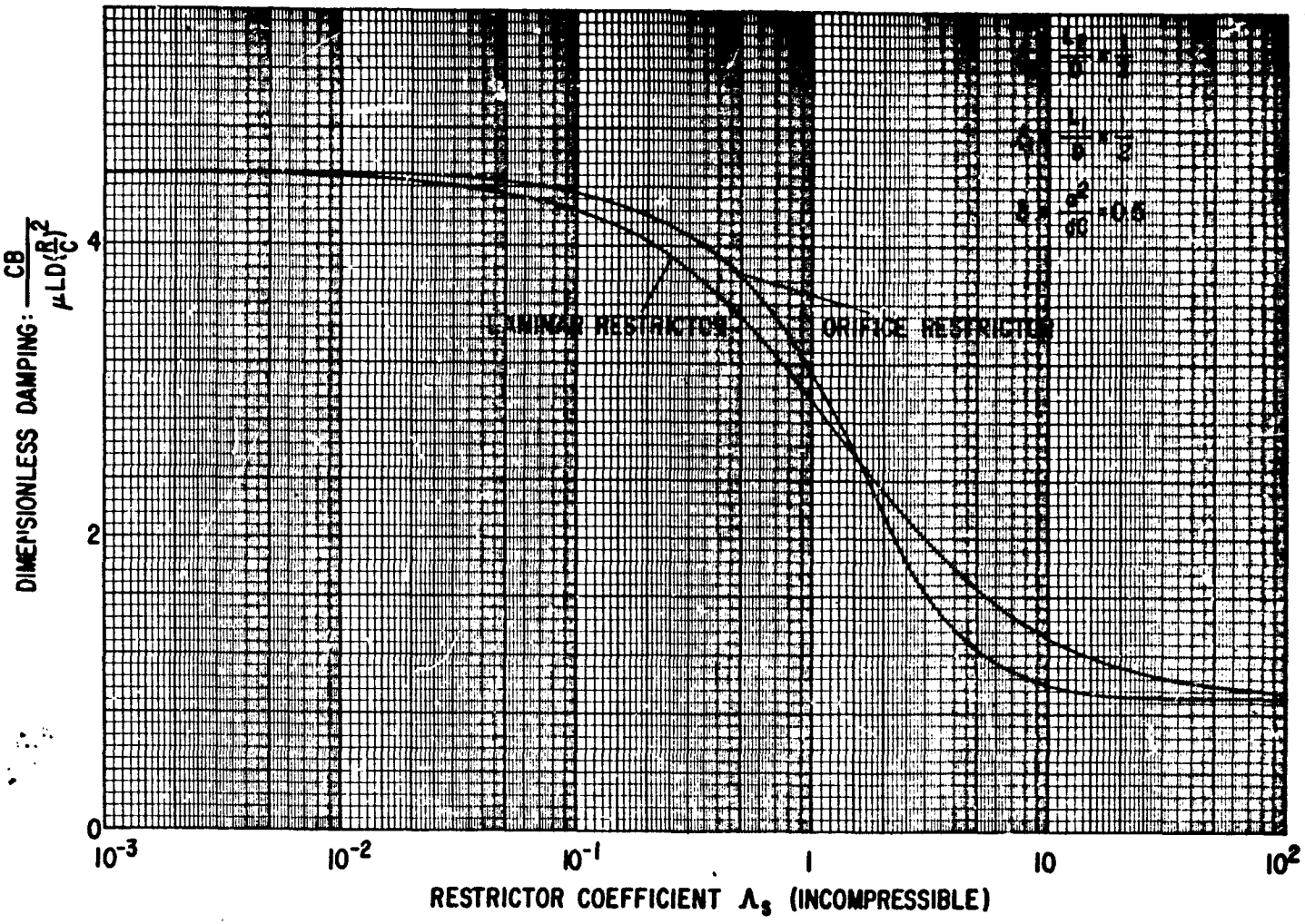


Figure 25. Hydrostatic Bearing, Incompressible, Double Plane Admission, Damping

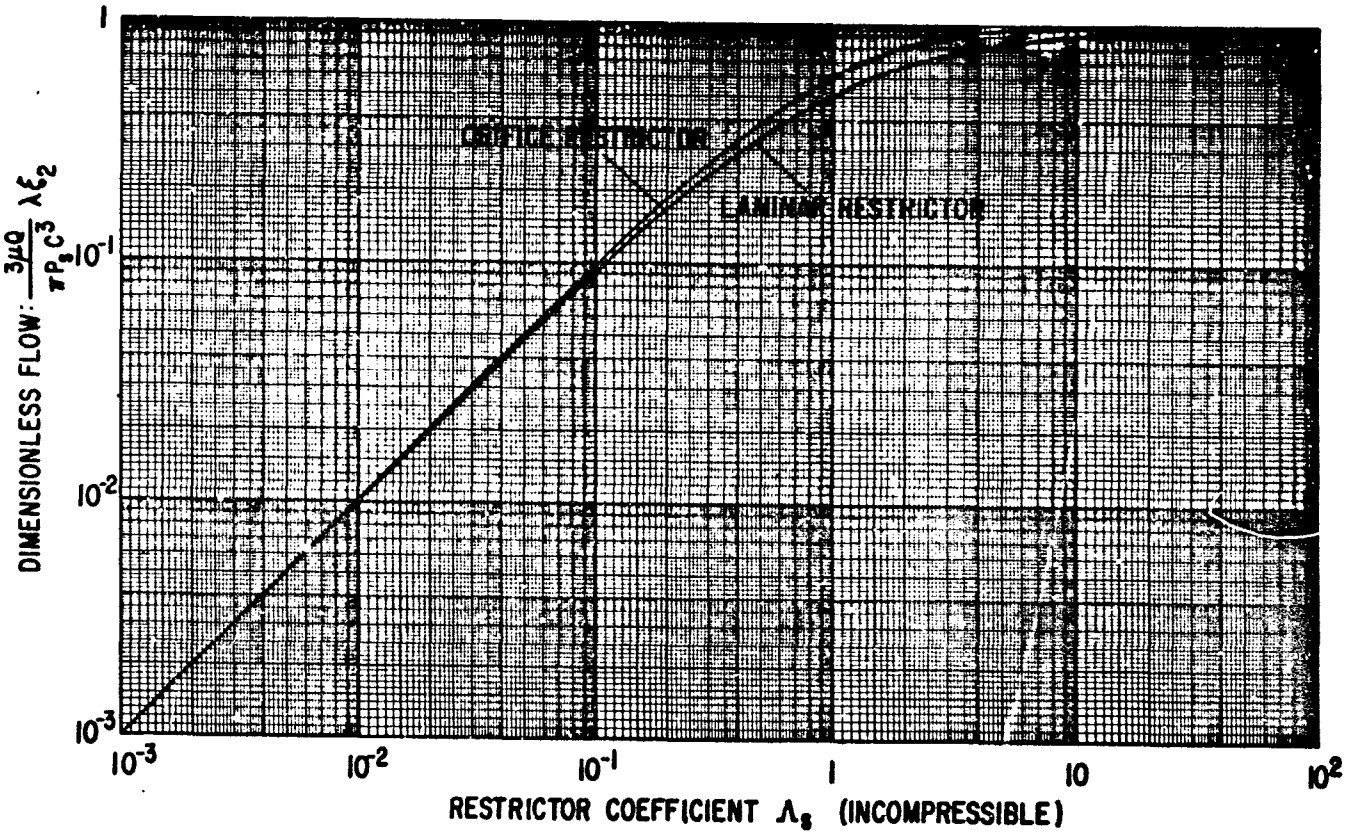


Figure 26. Hydrostatic Bearing, Incompressible, Flow

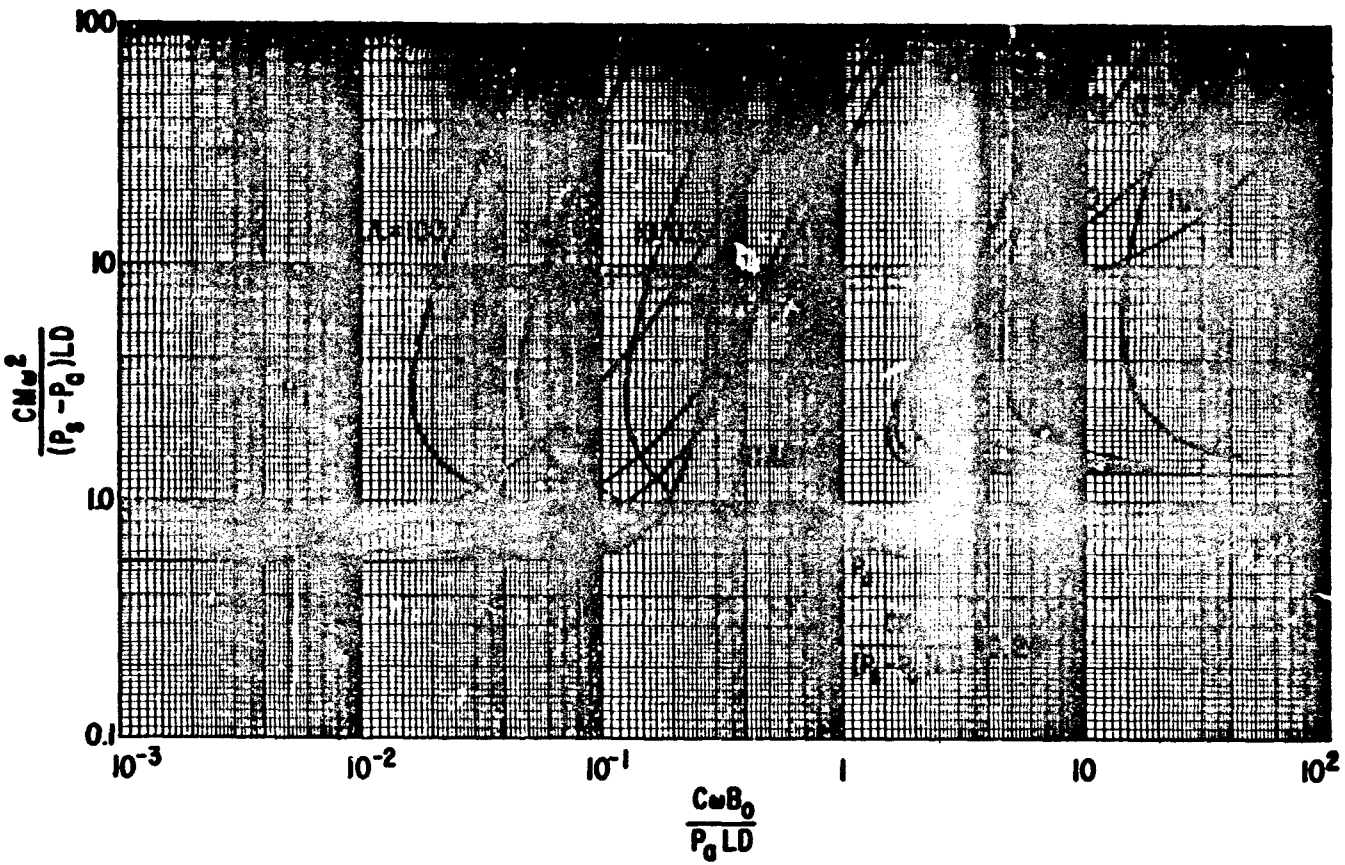


Figure 27. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Stability Map

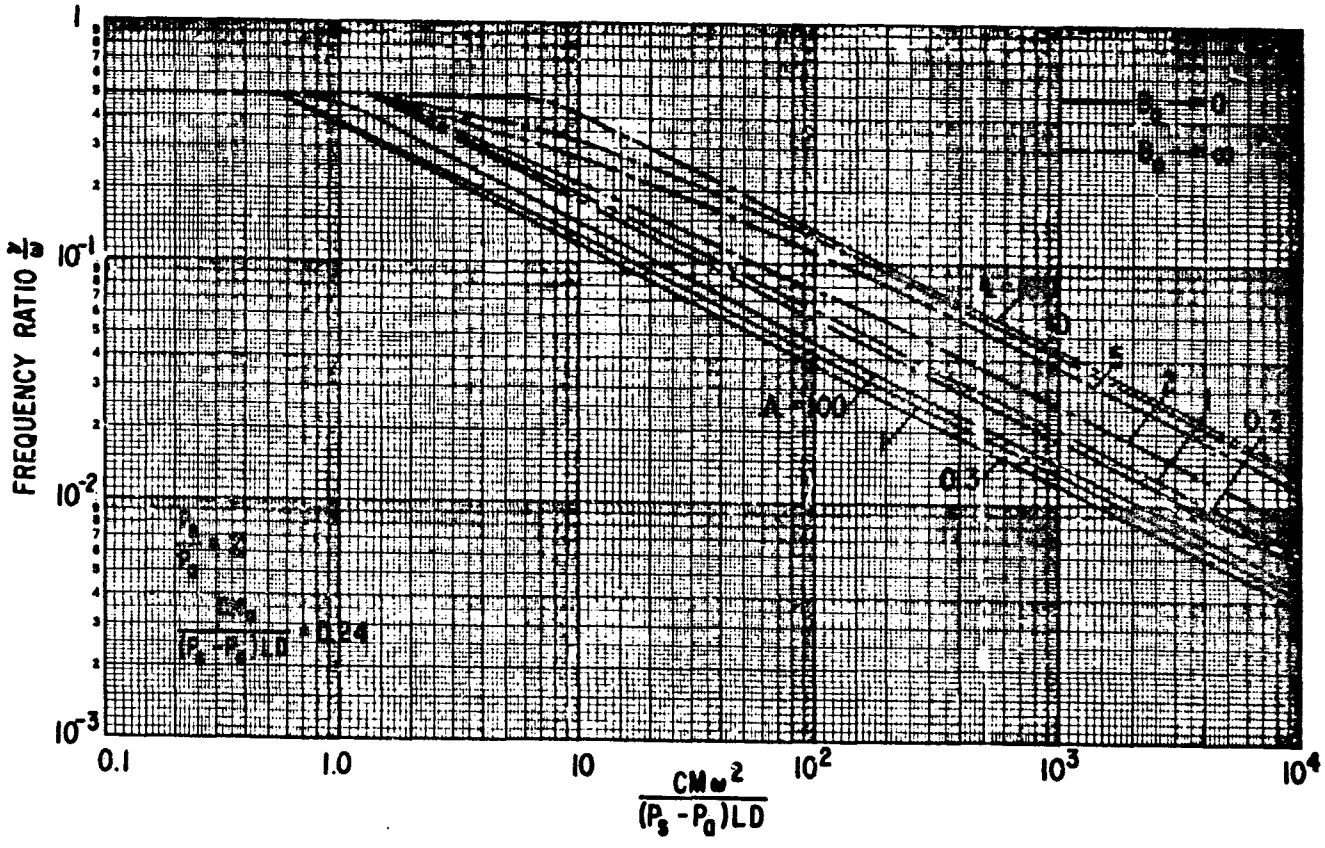


Figure 28. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Instability Frequency

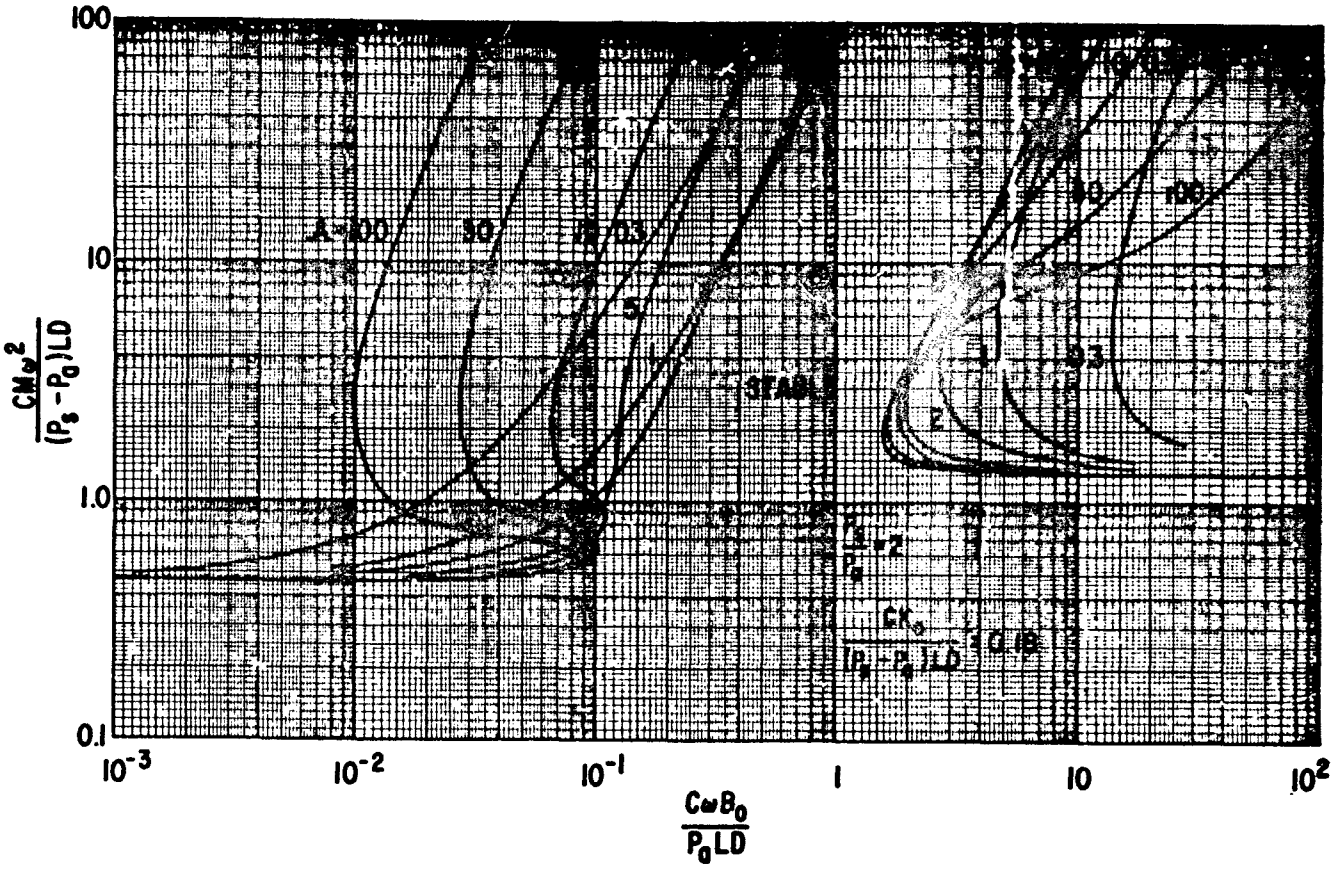


Figure 29. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s-P_a)LD} = 0.18$, Stability Map

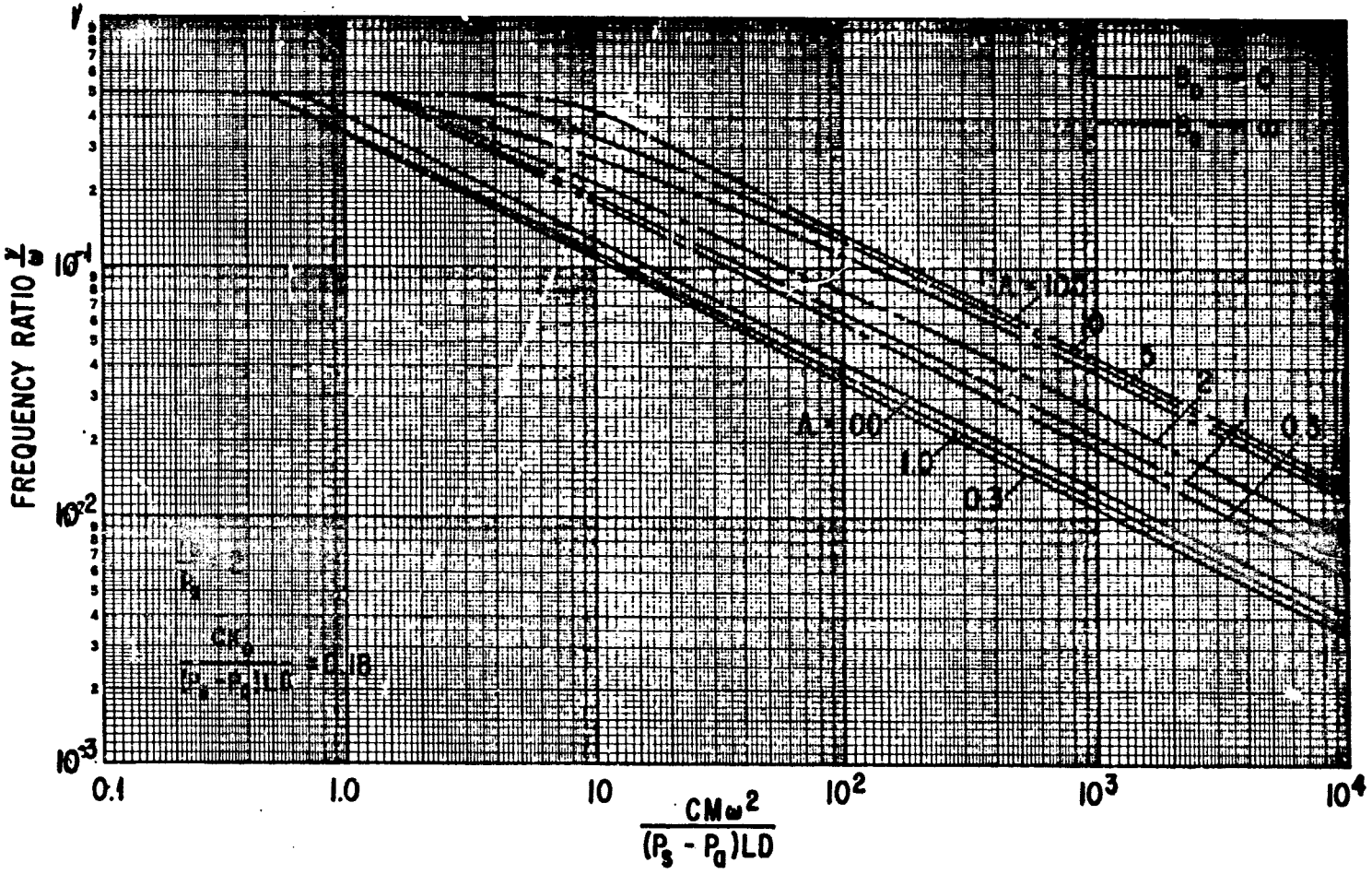


Figure 30. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Instability Frequency

MTI-5037

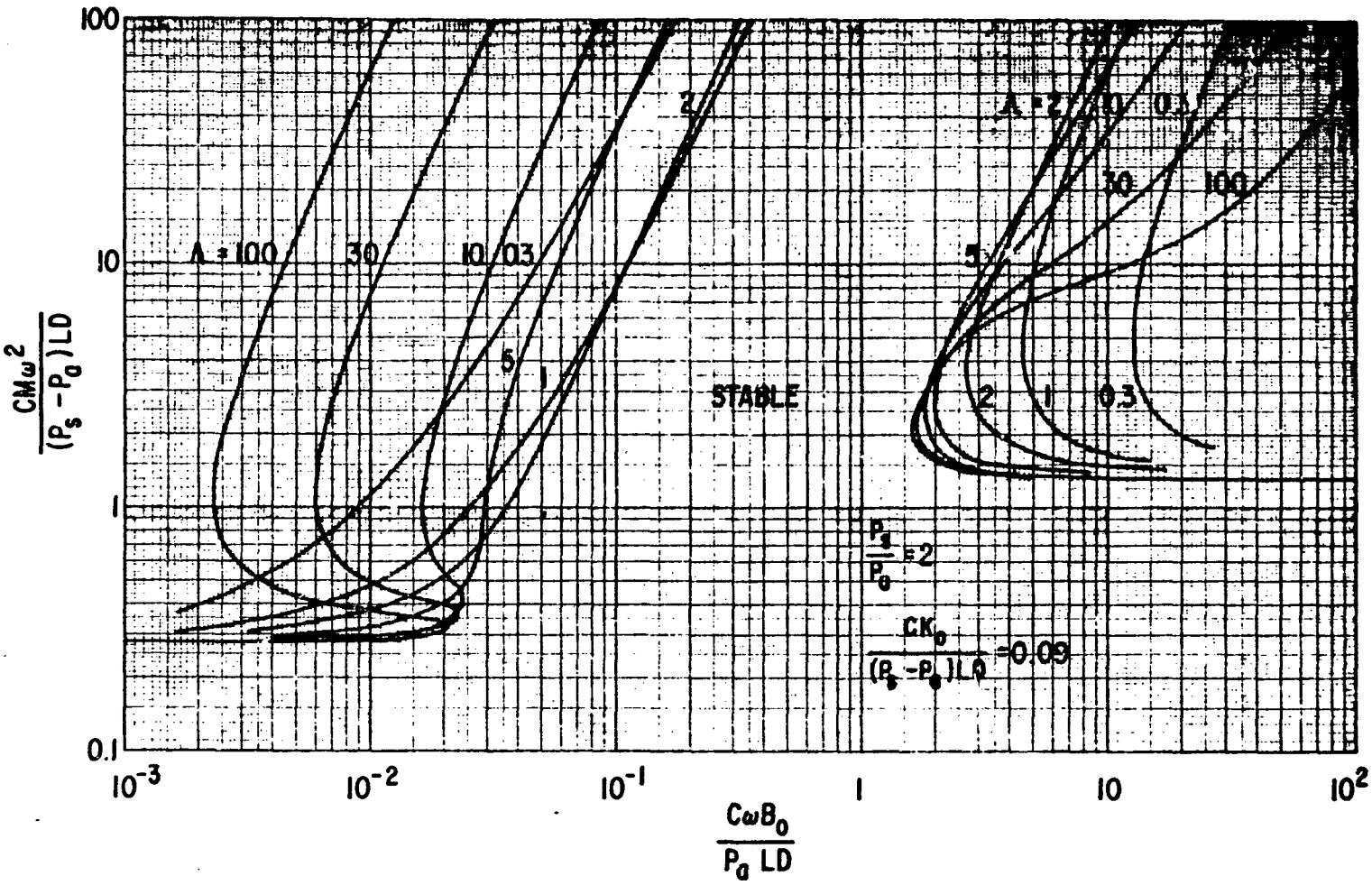


Figure 31. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Stability Map

NTI-5638

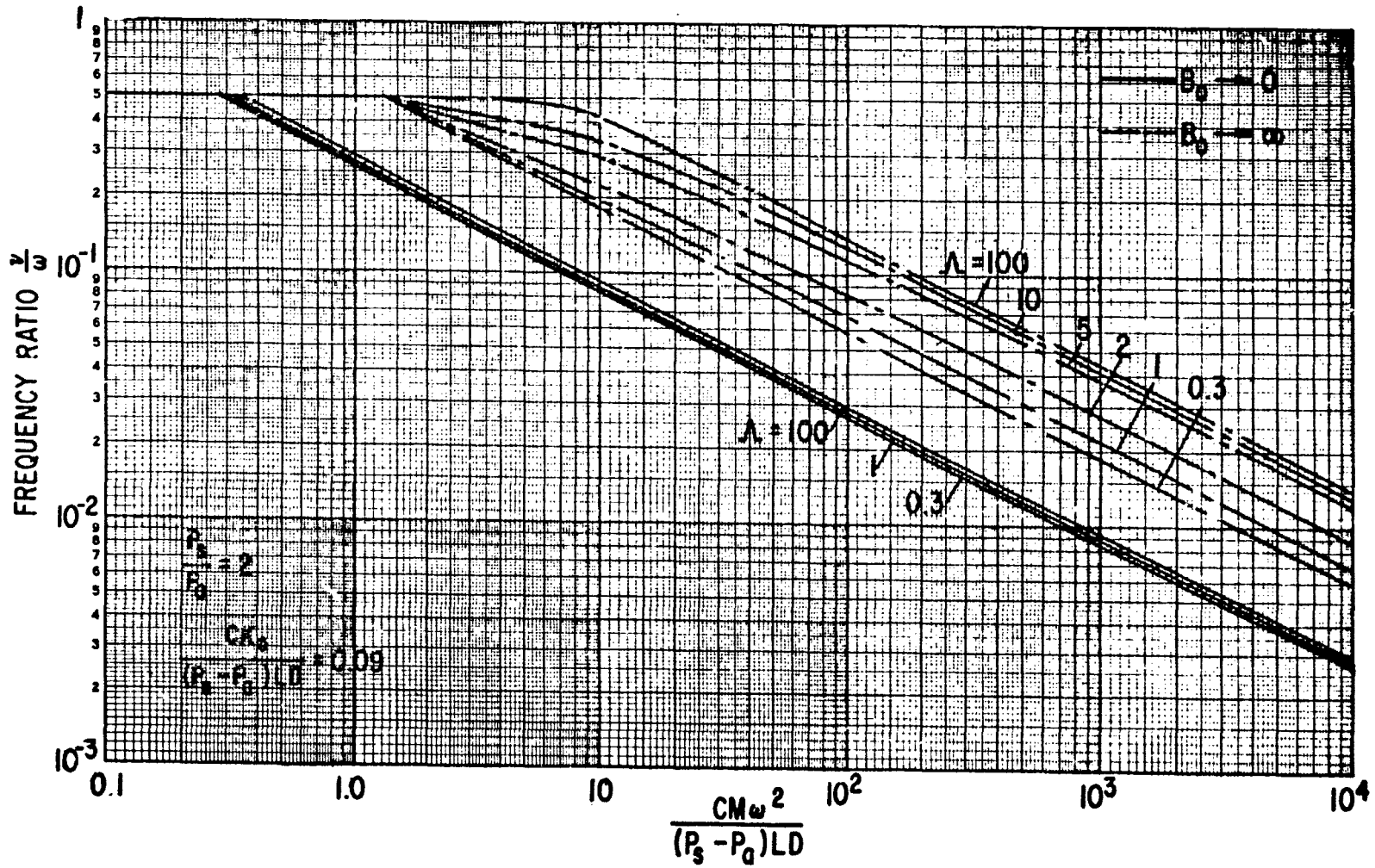


Figure 32. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Instability Frequency

MTI-5029

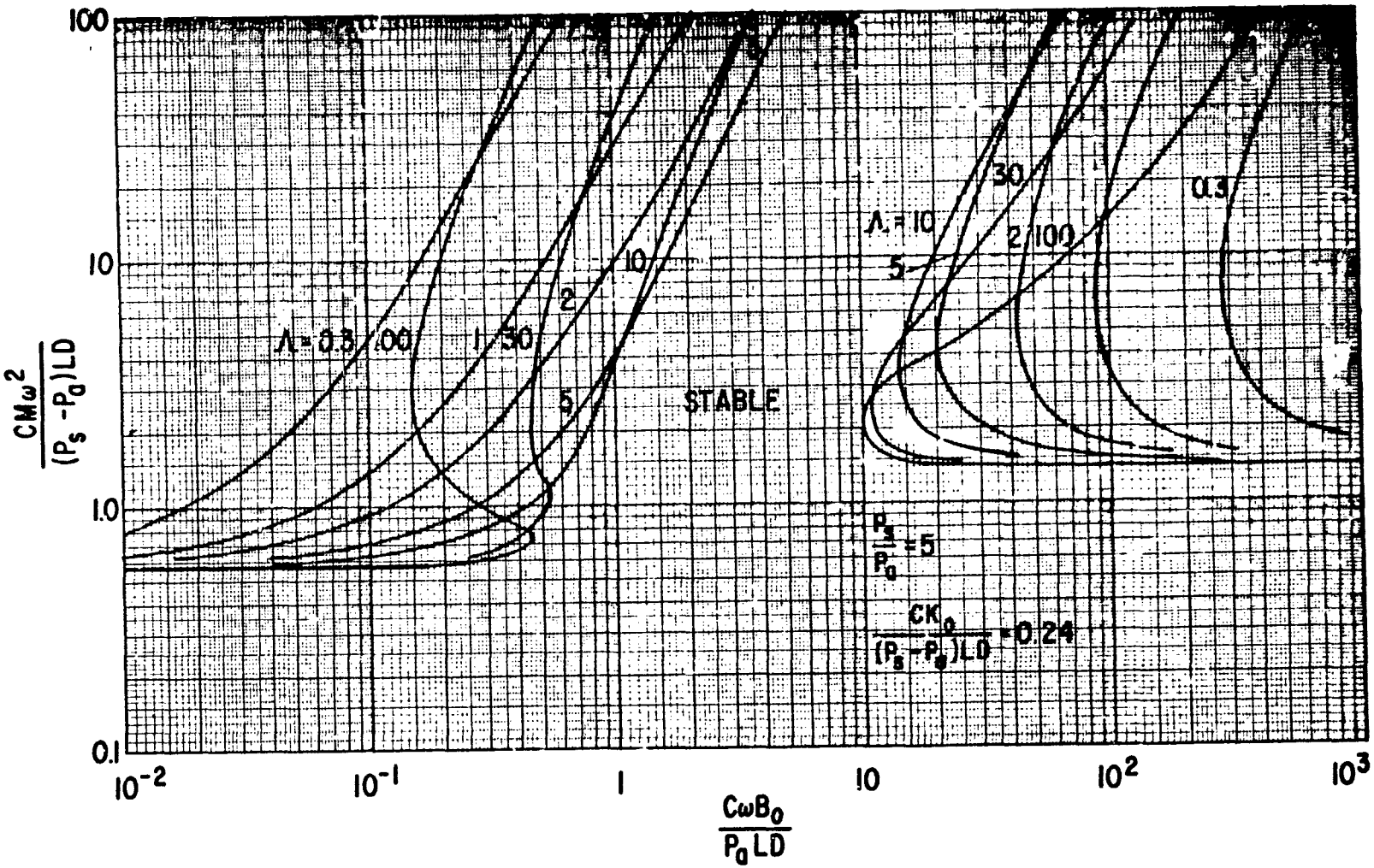


Figure 33. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.24$, Stability Map

MTI-5040

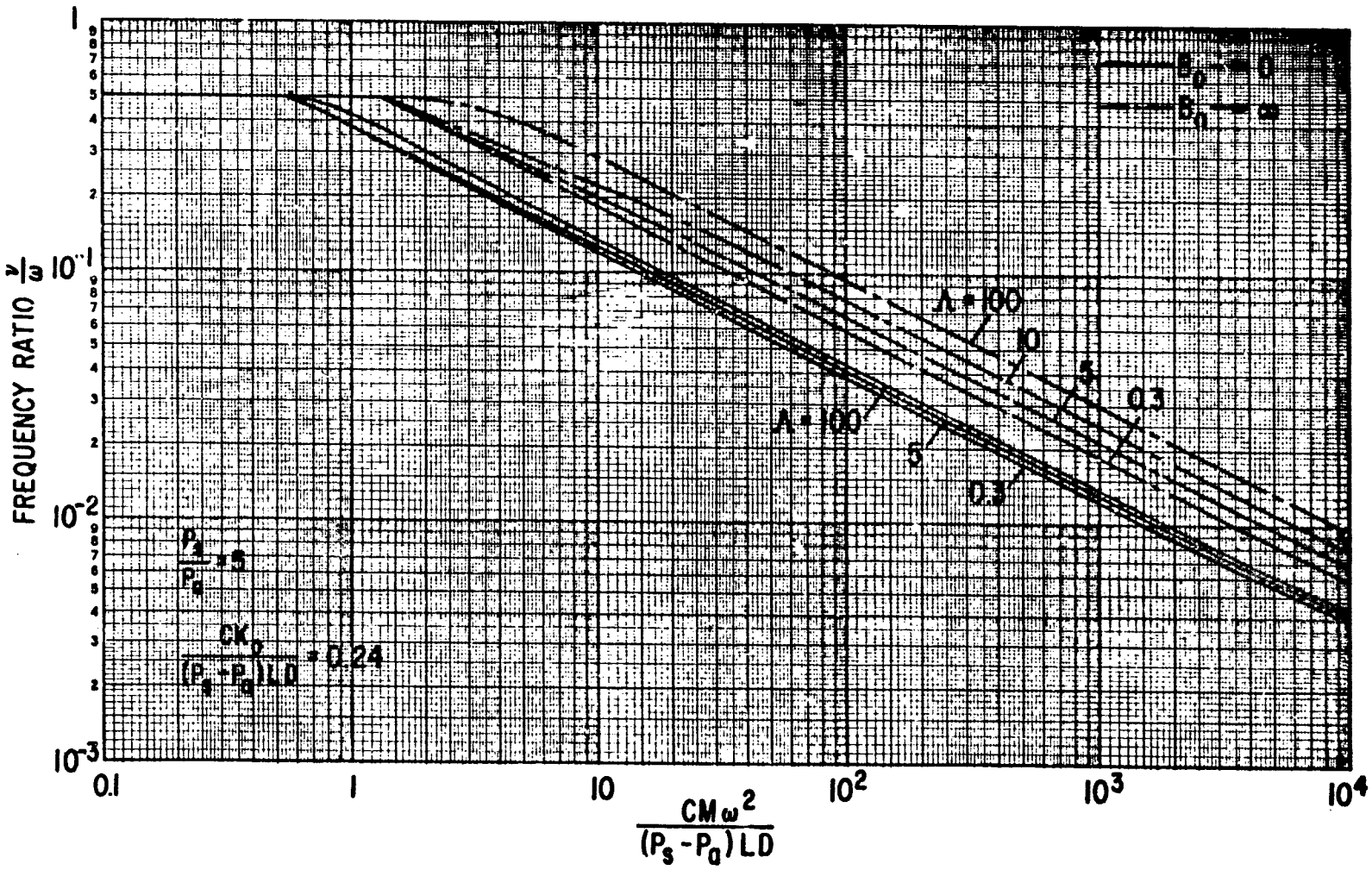


Figure 34. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s-P_a)LD} = 0.24$, Instability Frequency

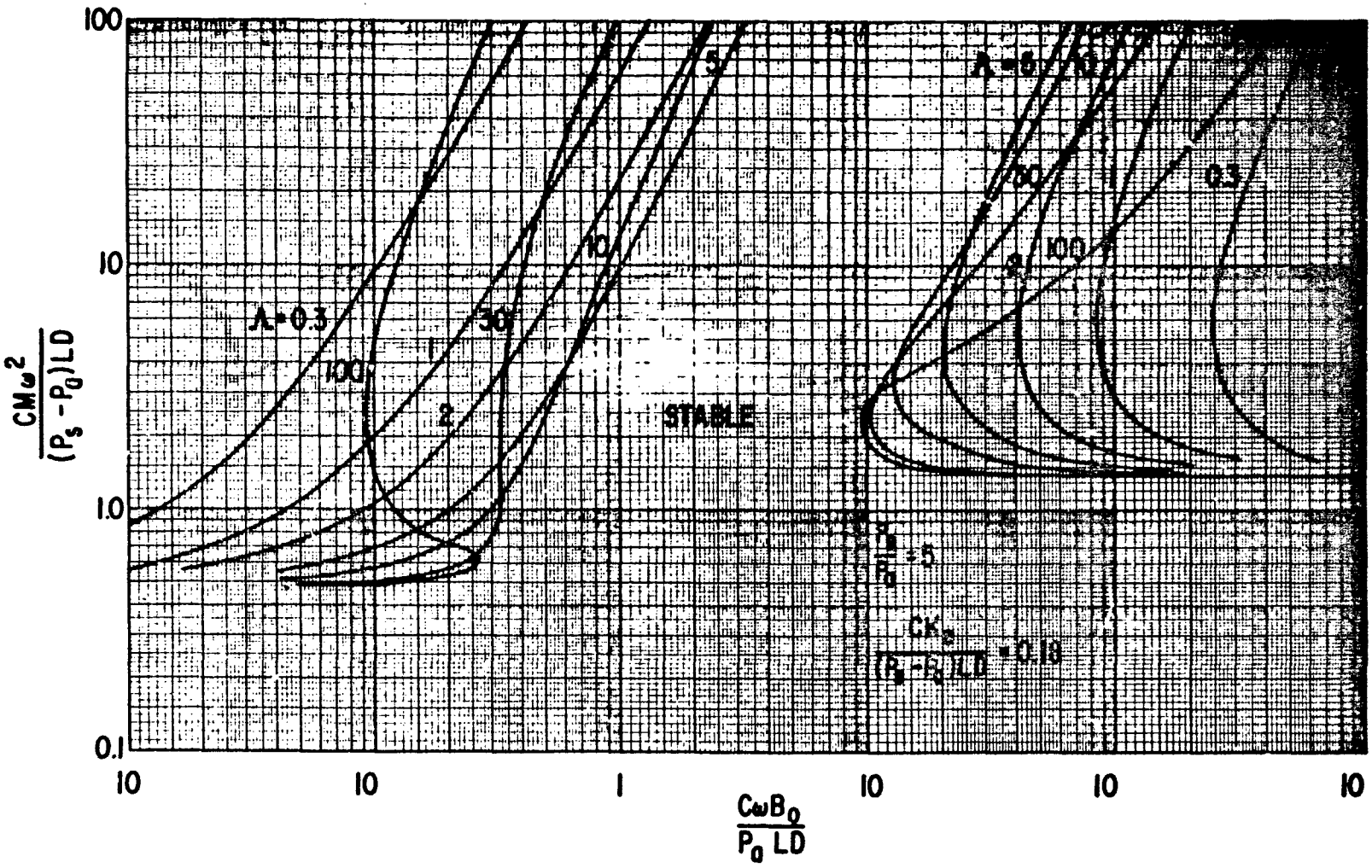


Figure 35. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Stability Map

MT1-5042

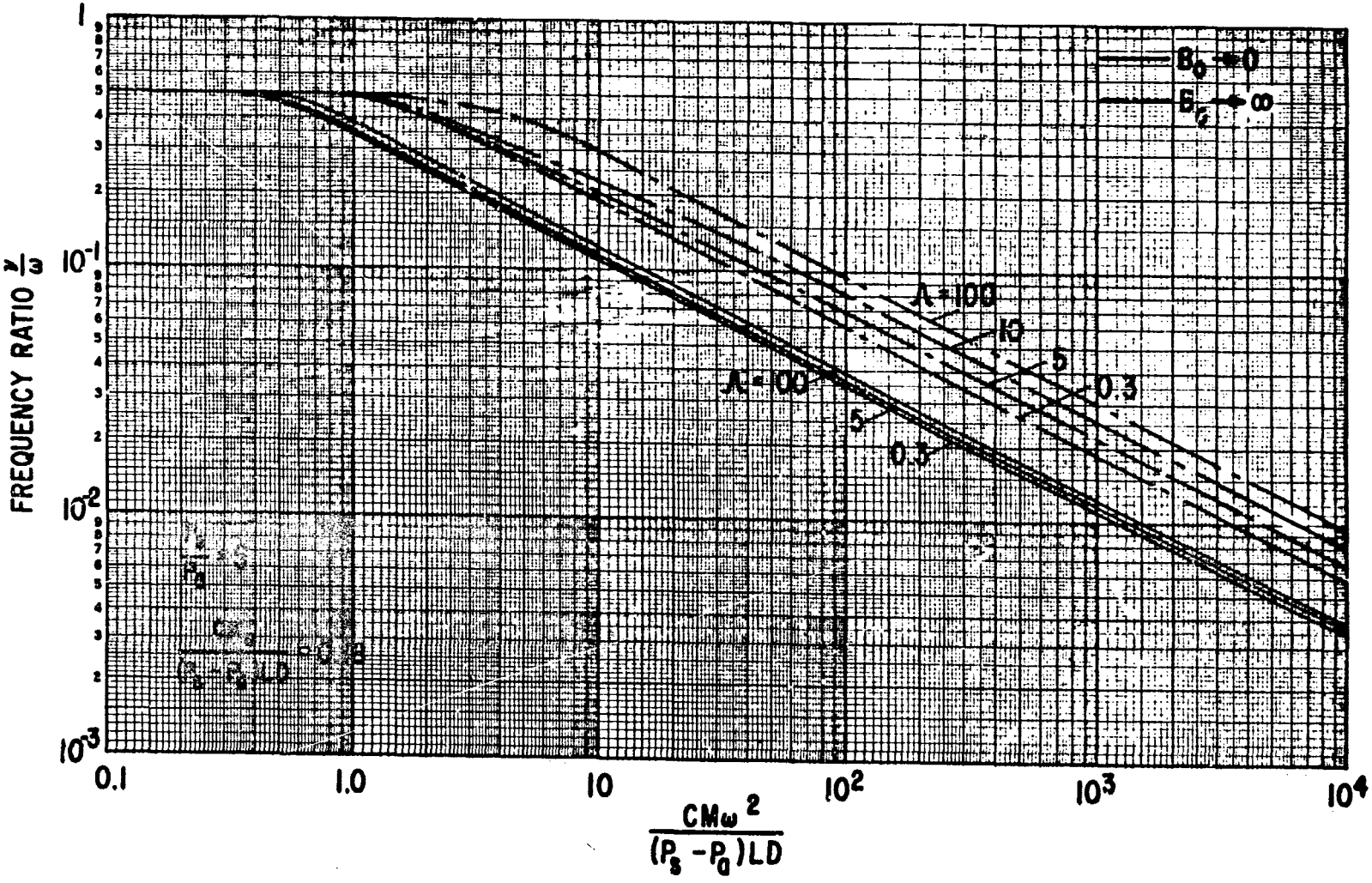


Figure 36. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Instability Frequency

MT1-3643

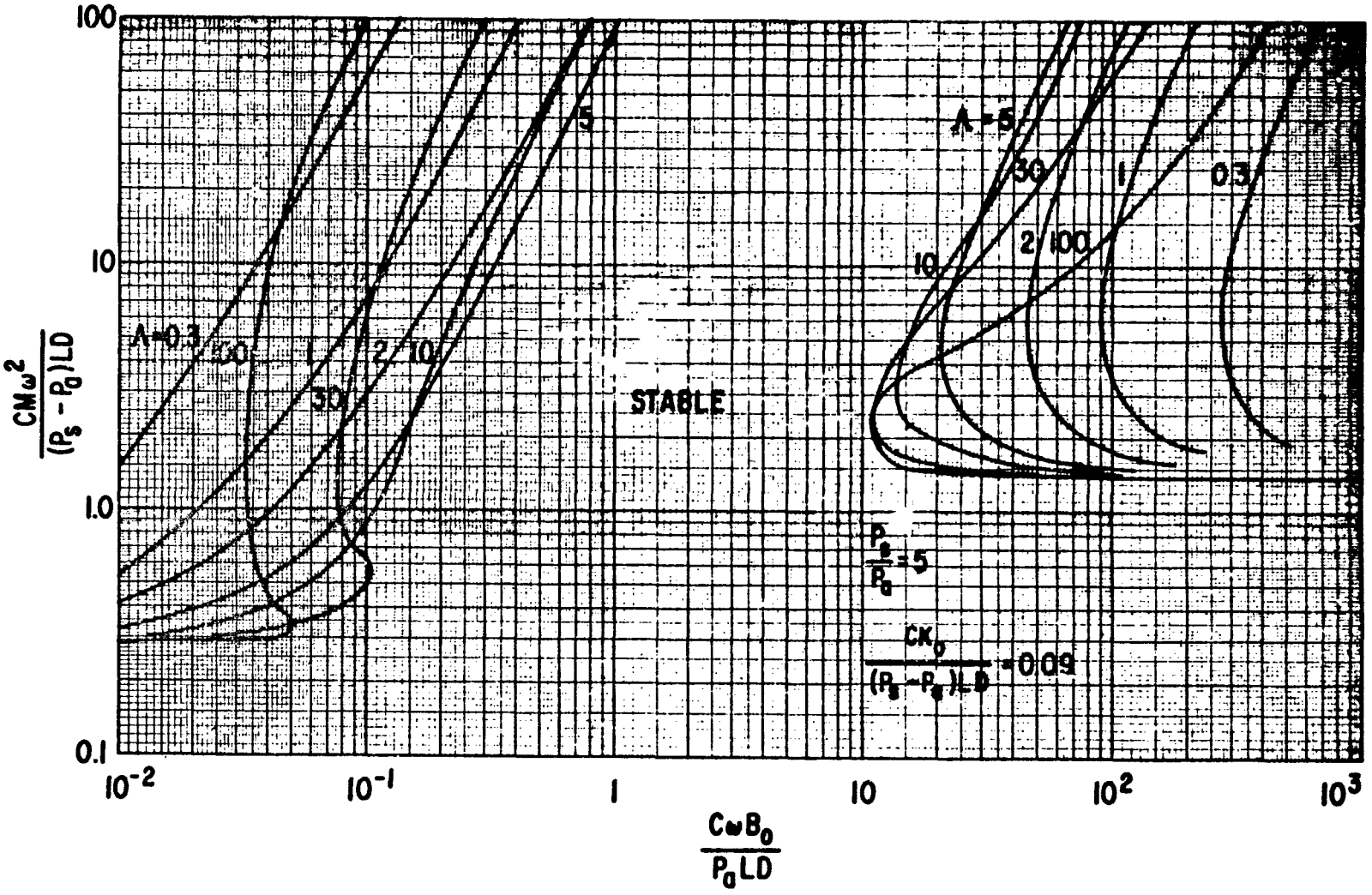


Figure 37. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Stability Map

MTI-3044

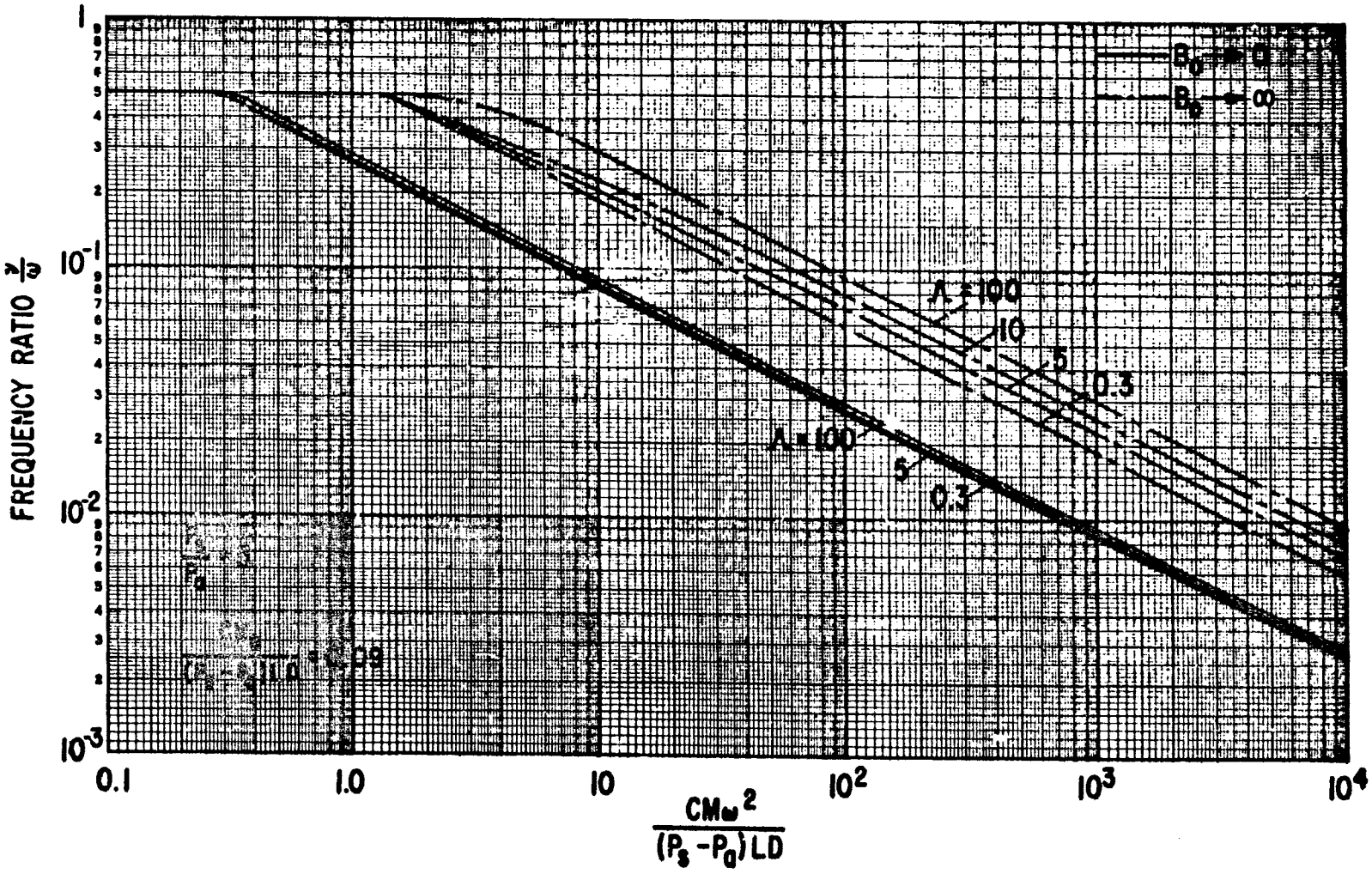


Figure 38. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 5$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Instability Frequency

MTI-3043

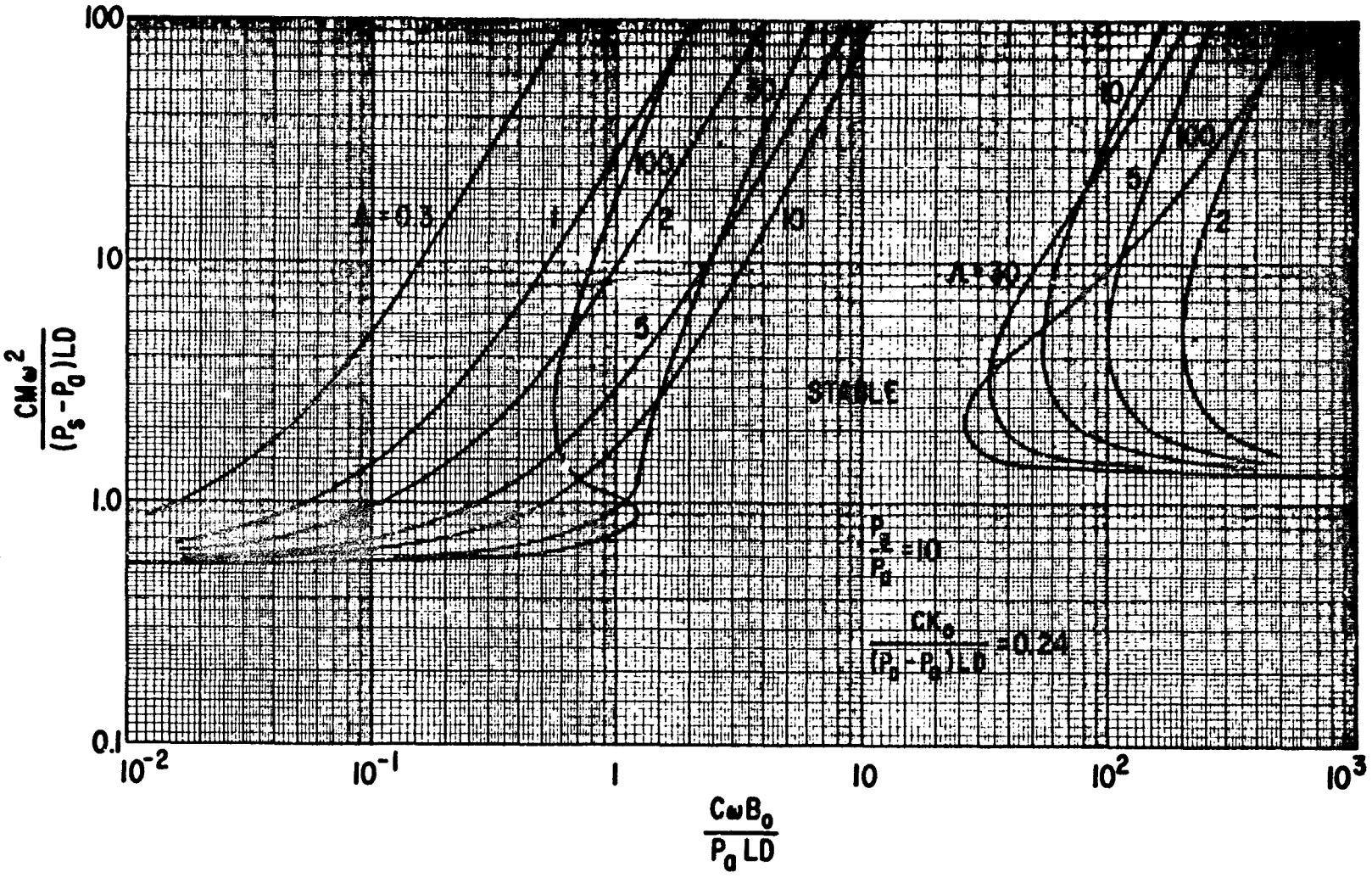


Figure 39. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s-P_a)LD} = 0.24$, Stability Map

MTI-5046

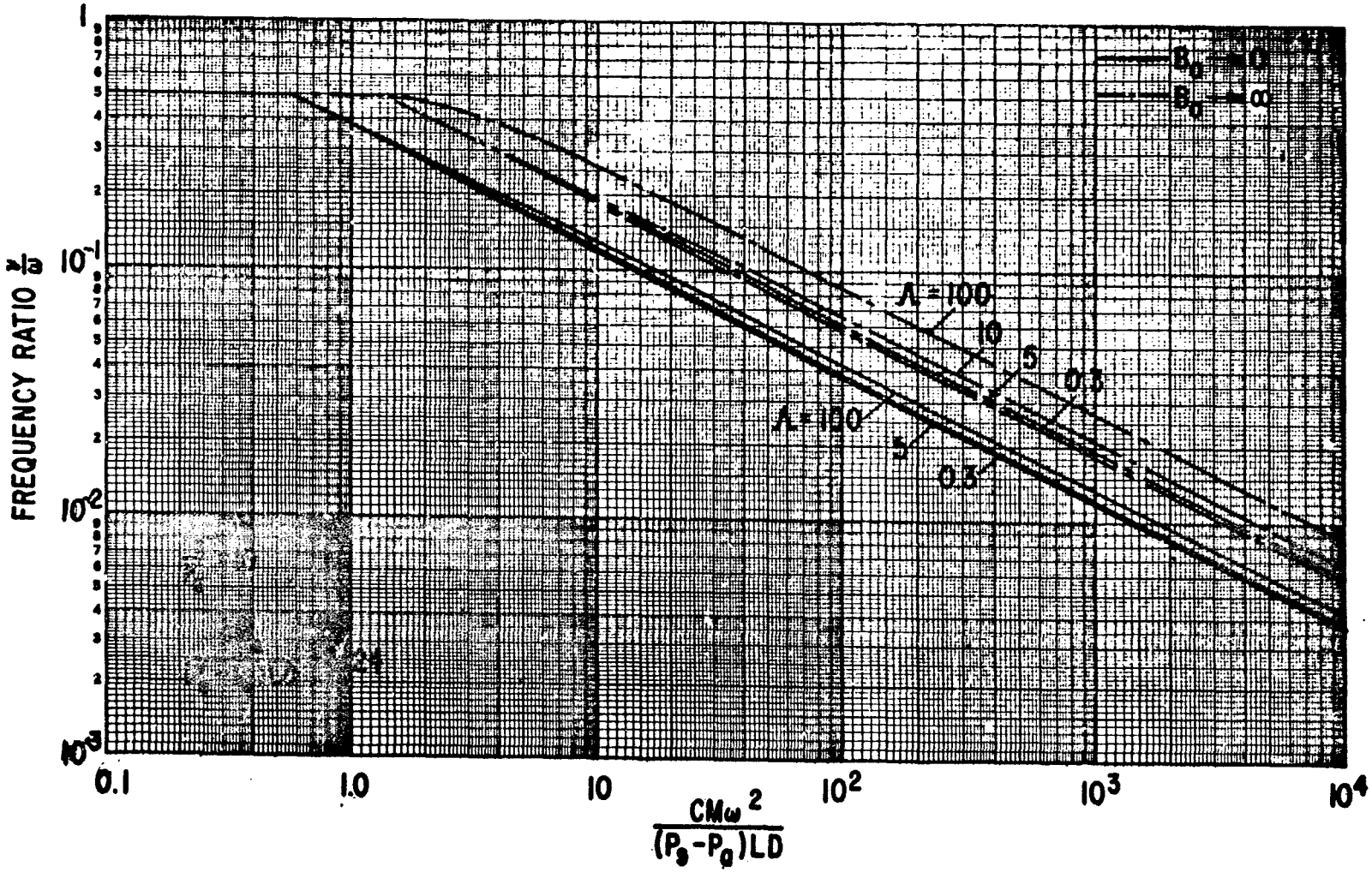


Figure 40. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s-P_a)LD} = 0.24$, Instability Frequency

MTI-5047

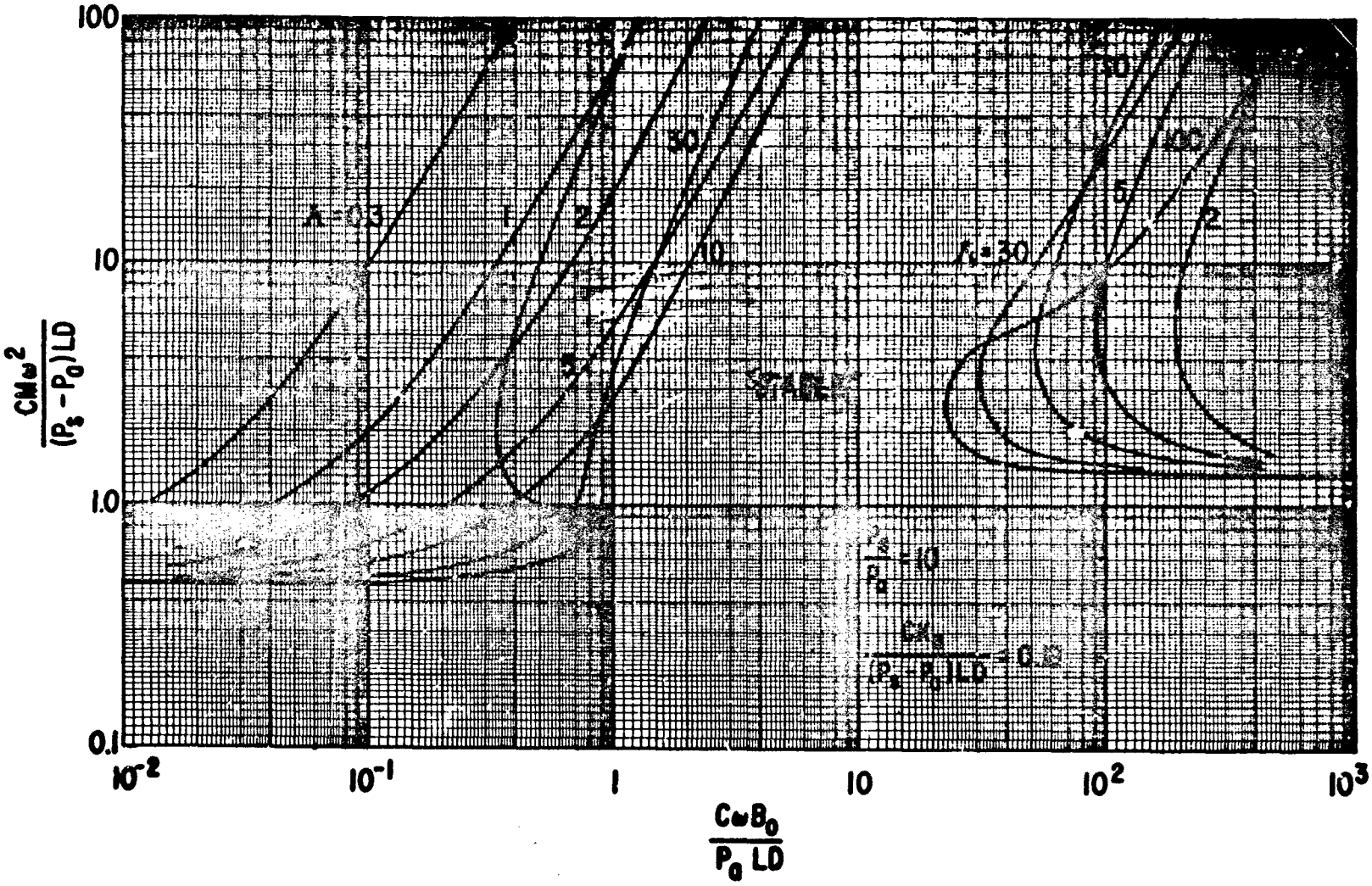


Figure 41. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Stability Map

MTI-5048

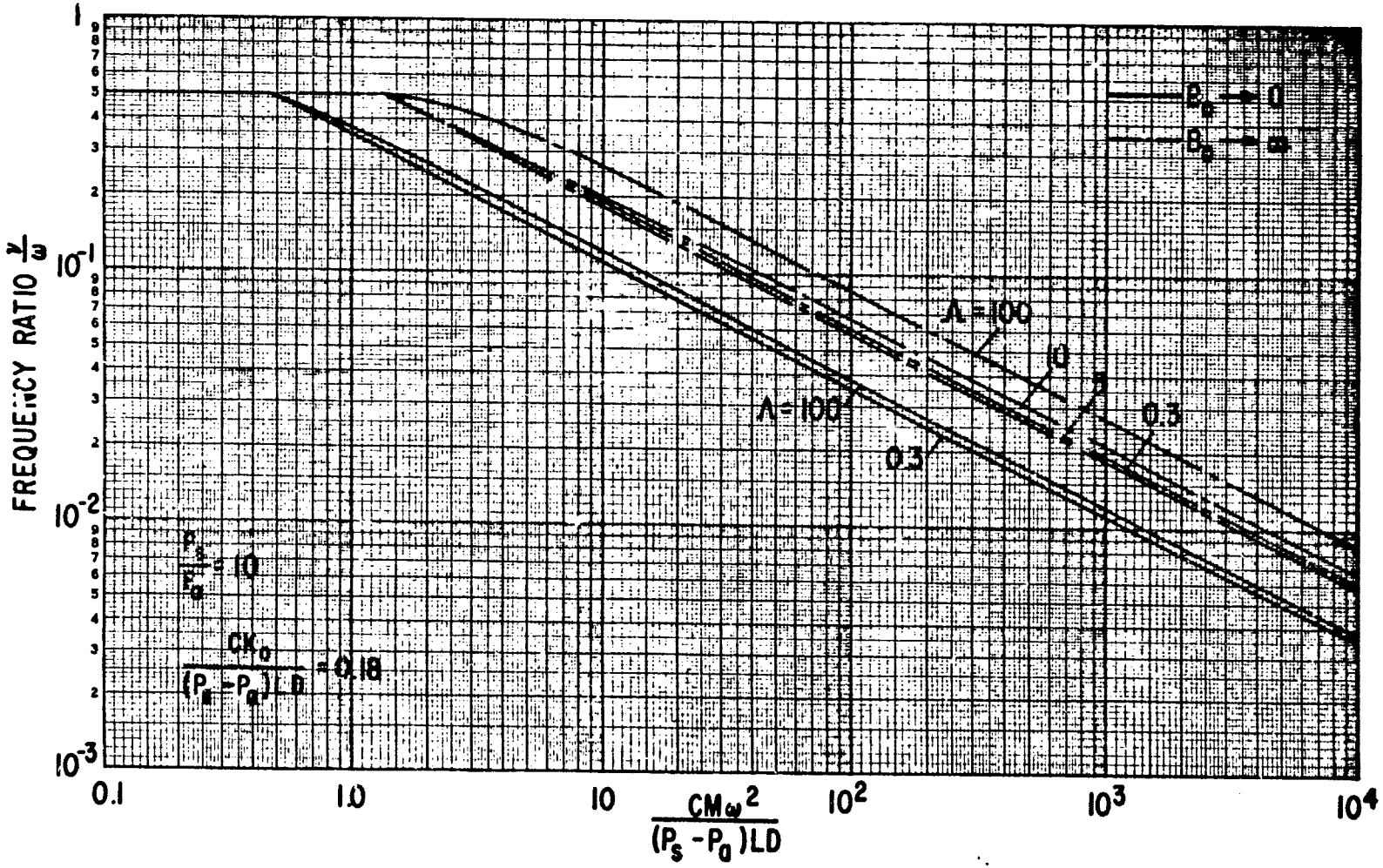


Figure 42. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.18$, Instability Frequency

RTI-5049

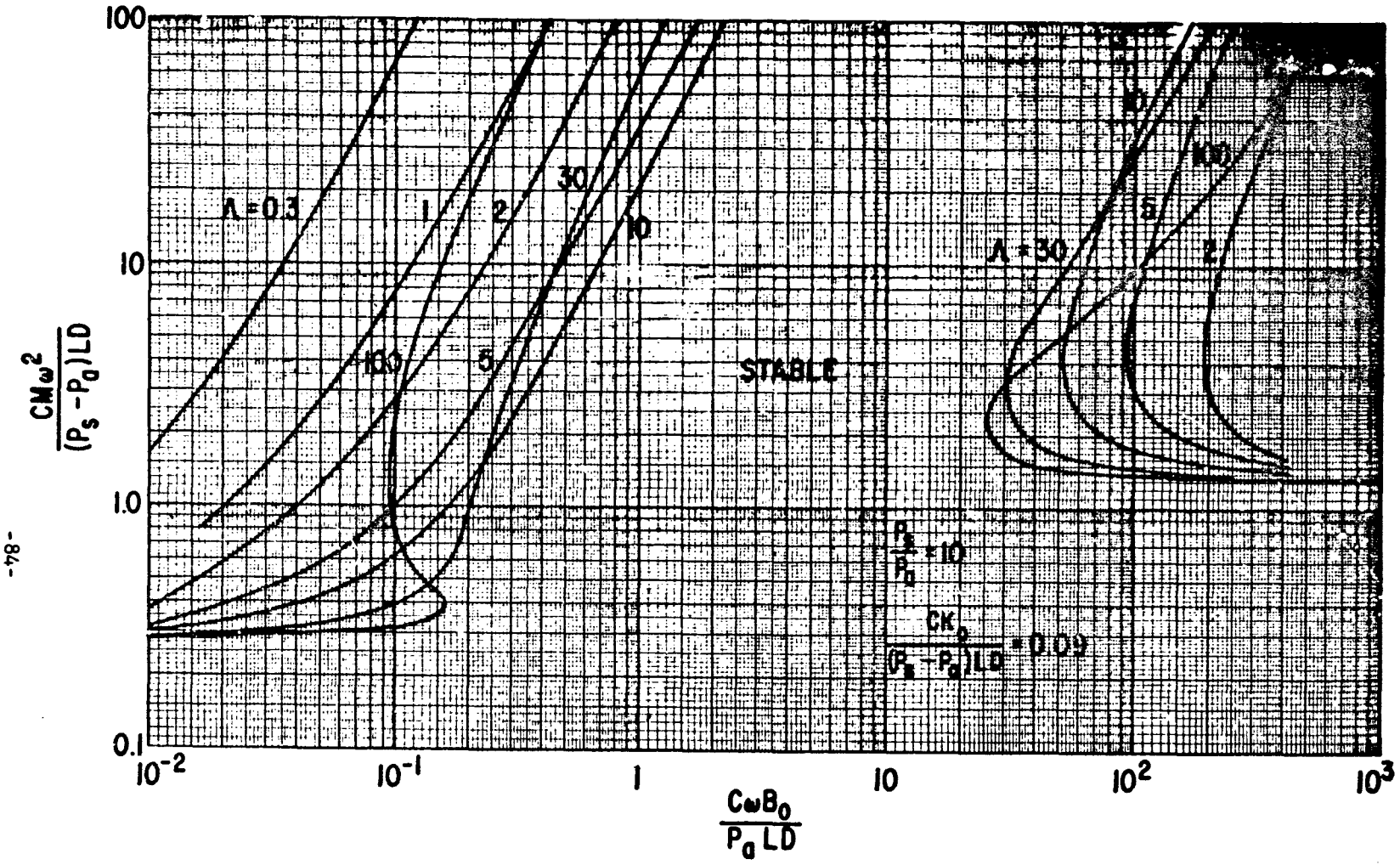


Figure 43. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Stability Map

MTI-5050

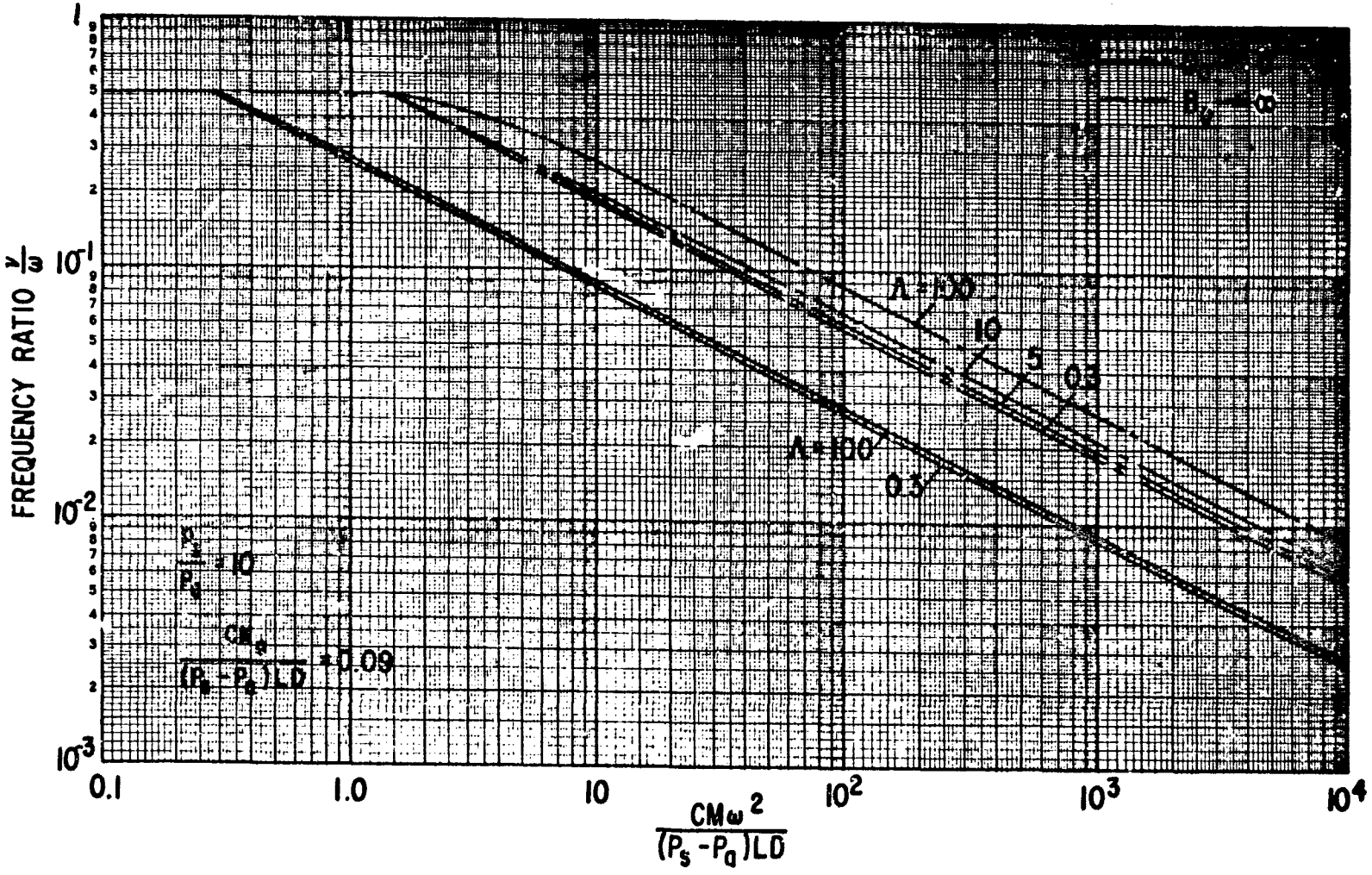


Figure 44. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Instability Frequency

MTI-5051

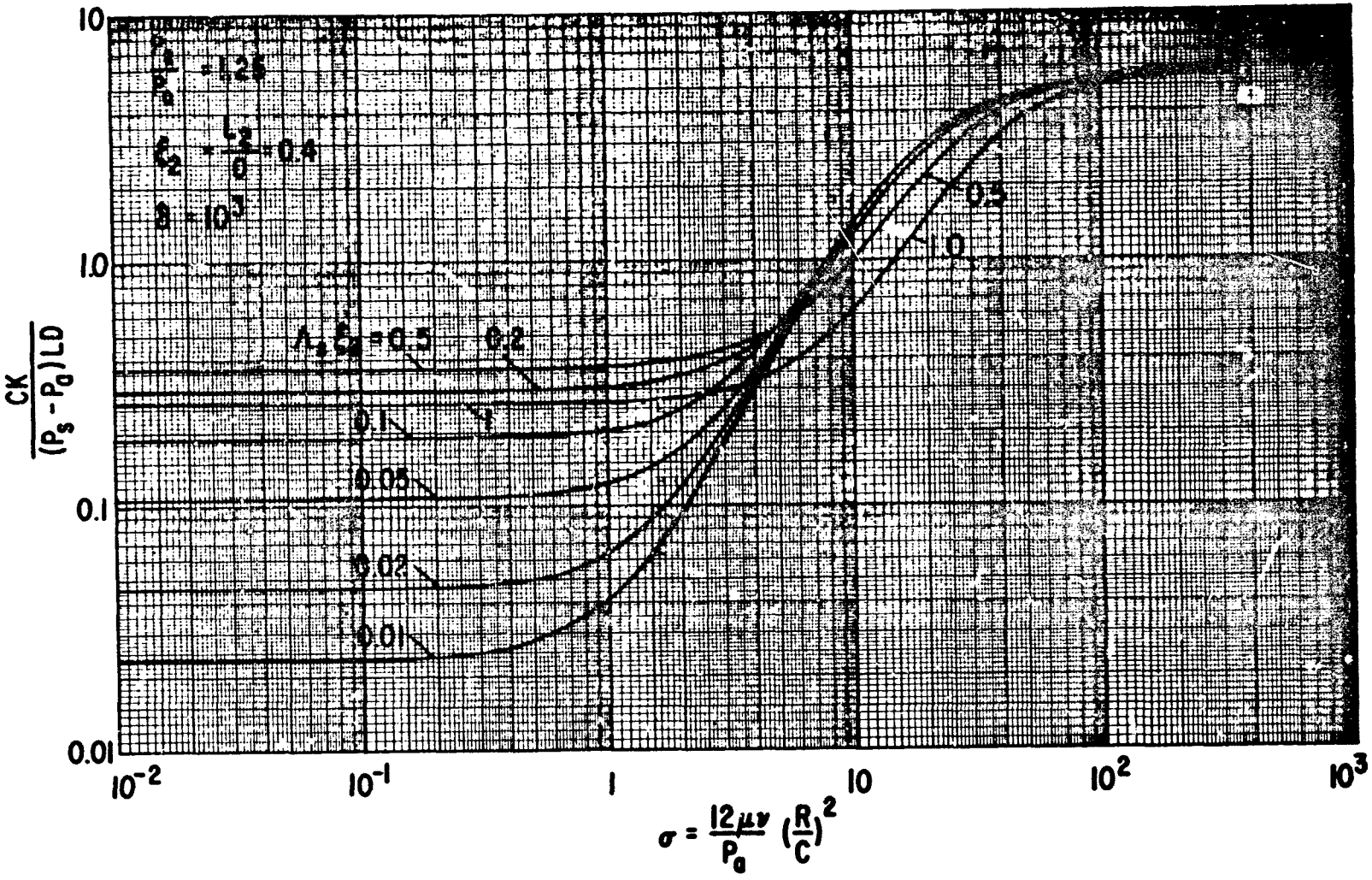


Figure 45. Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 1.25$, Stiffness

MTI-5052

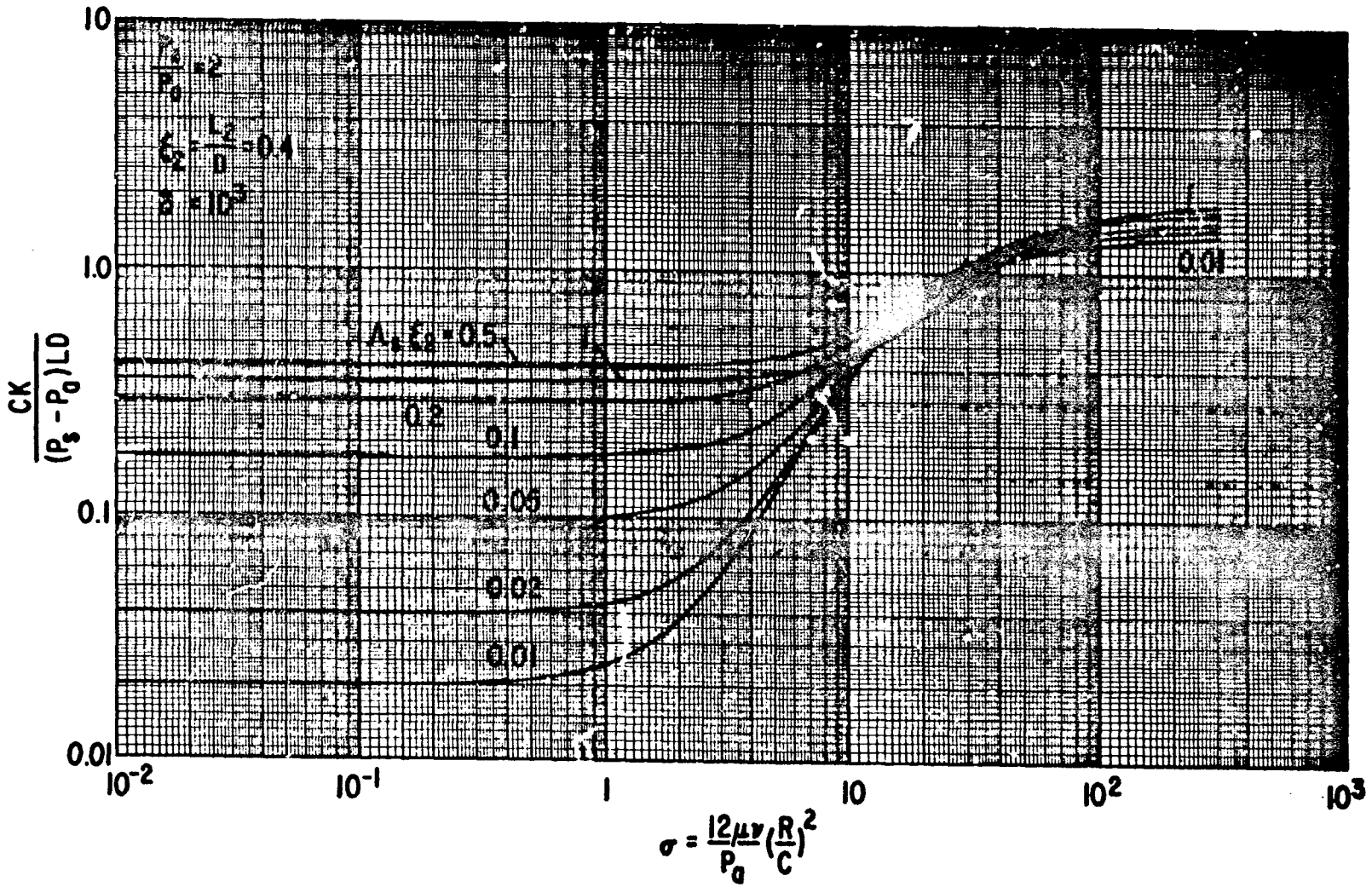


Figure 46. Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 2$, Stiffness

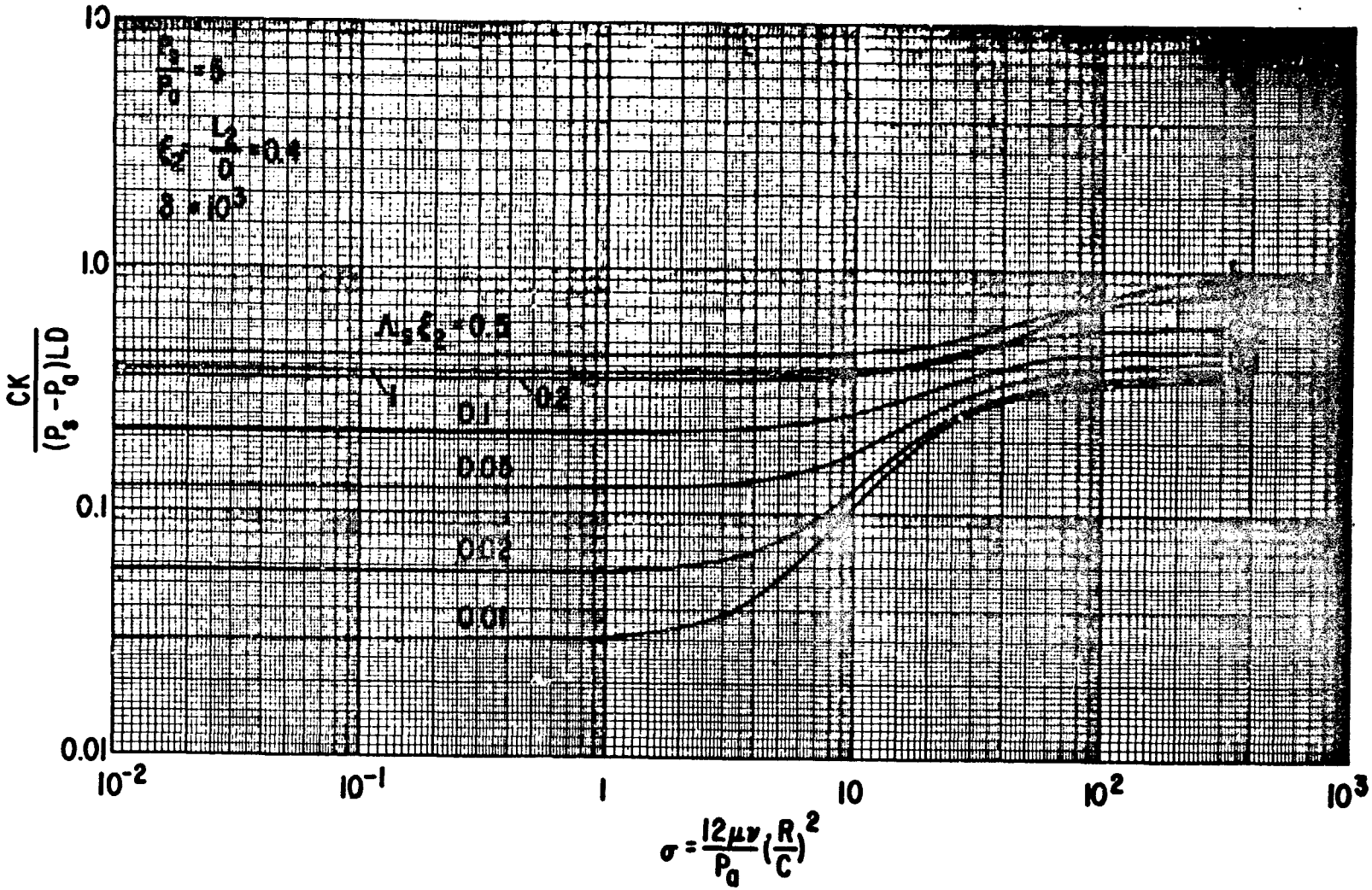


Figure 47. Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 5$, Stiffness

MTI-5054

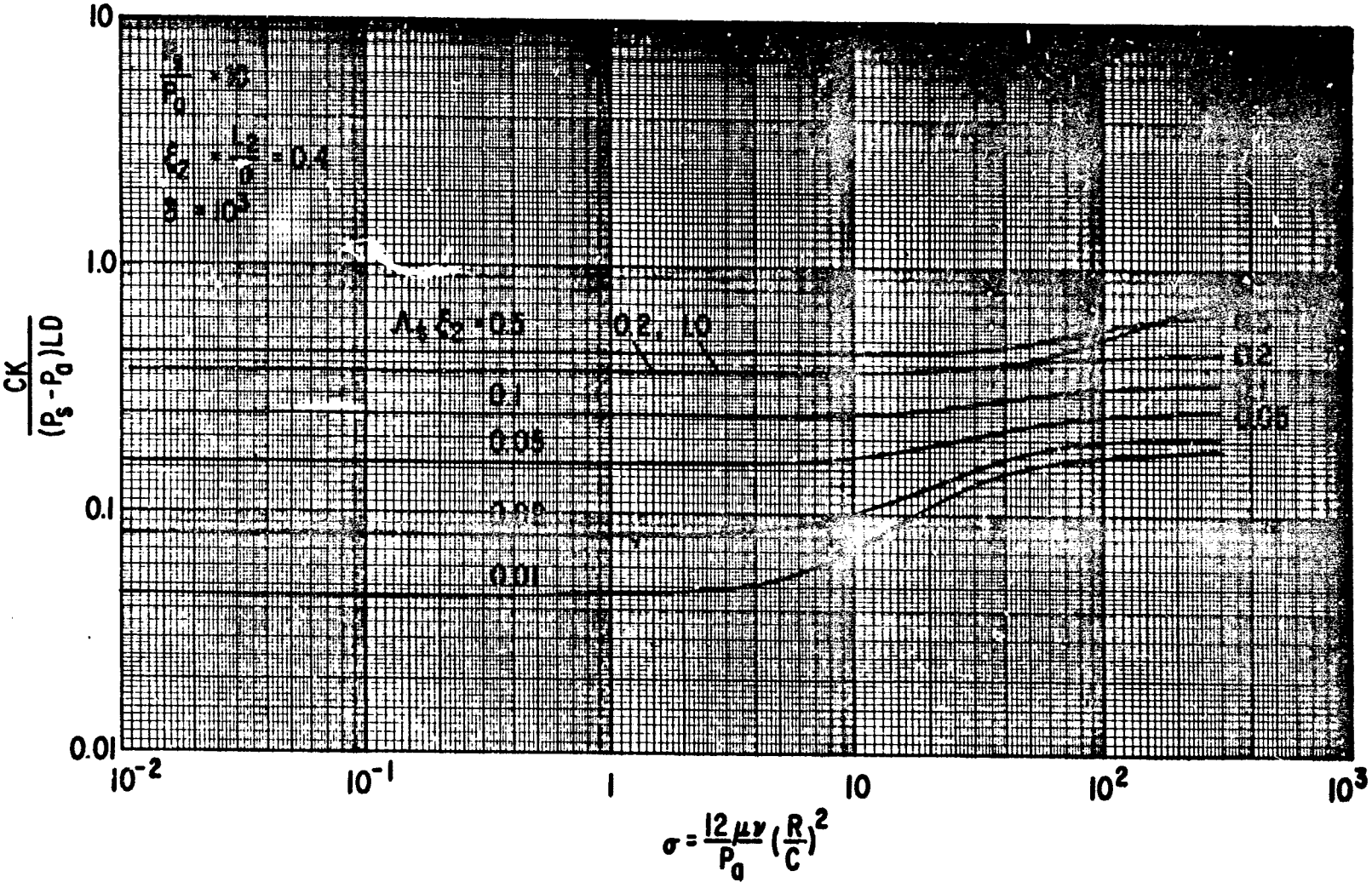


Figure 48. Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 10$, Stiffness

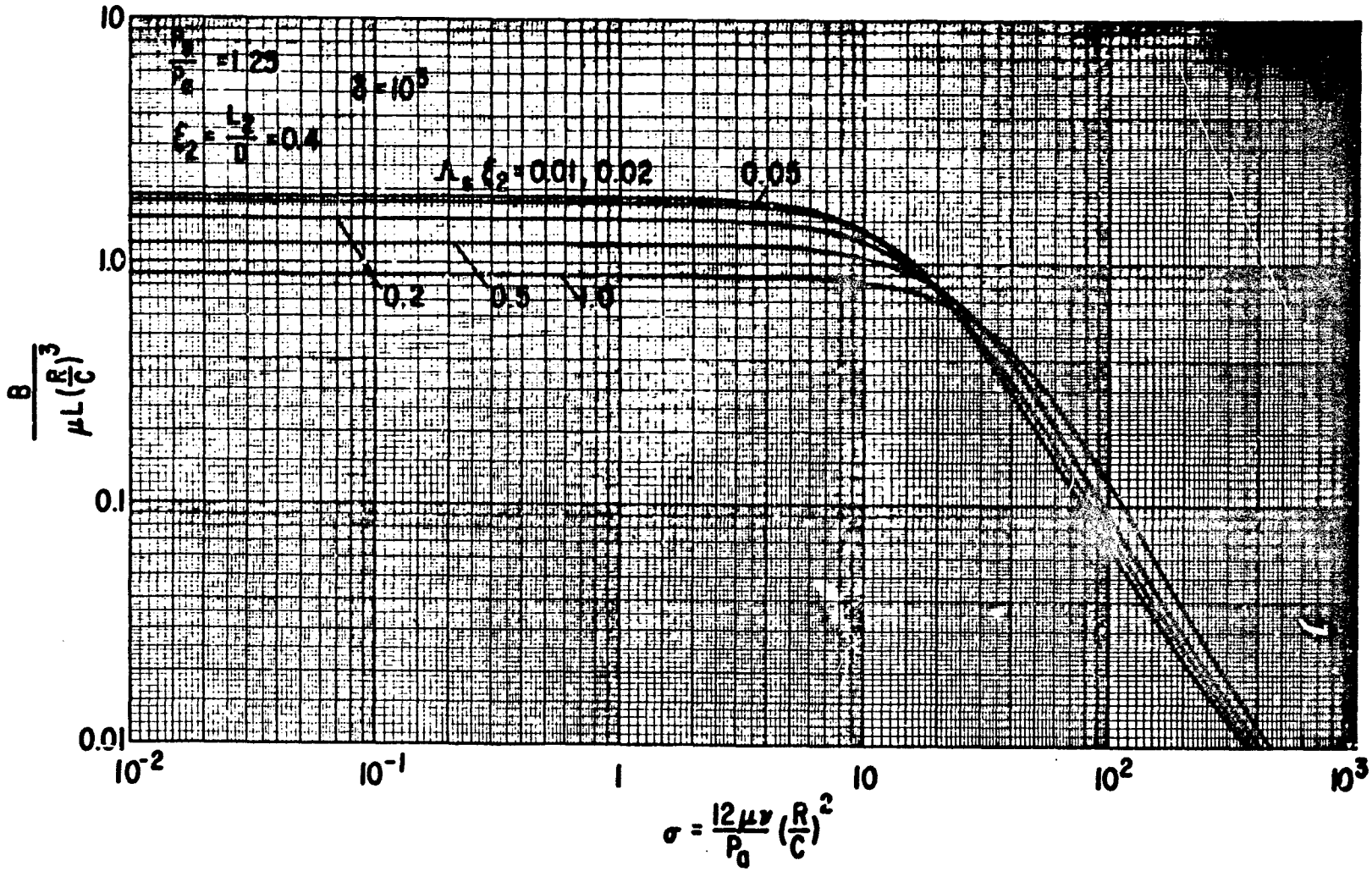


Figure 49. Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 1.25$, Damping

MTI-5056

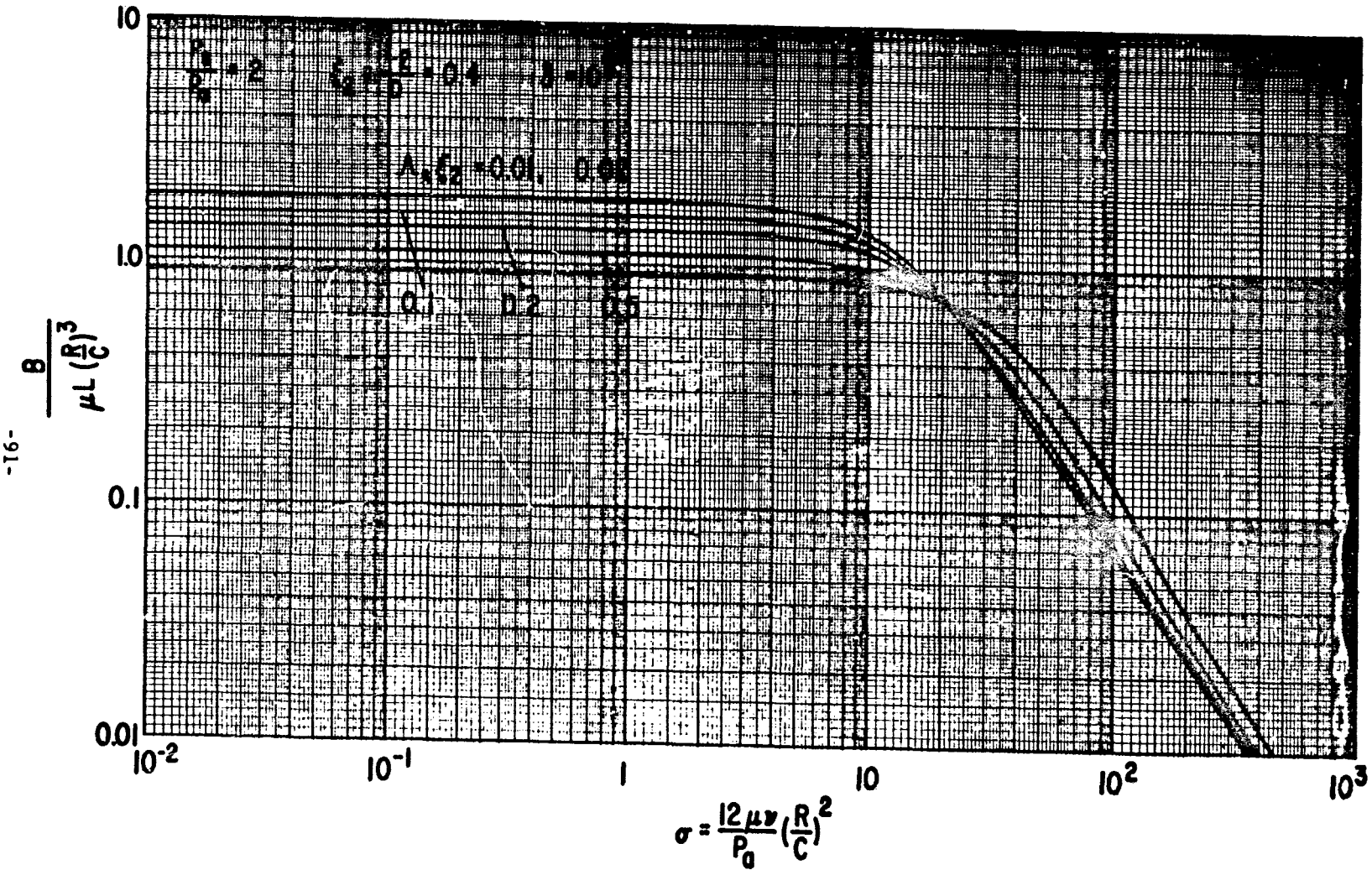


Figure 50. Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 2$, Damping

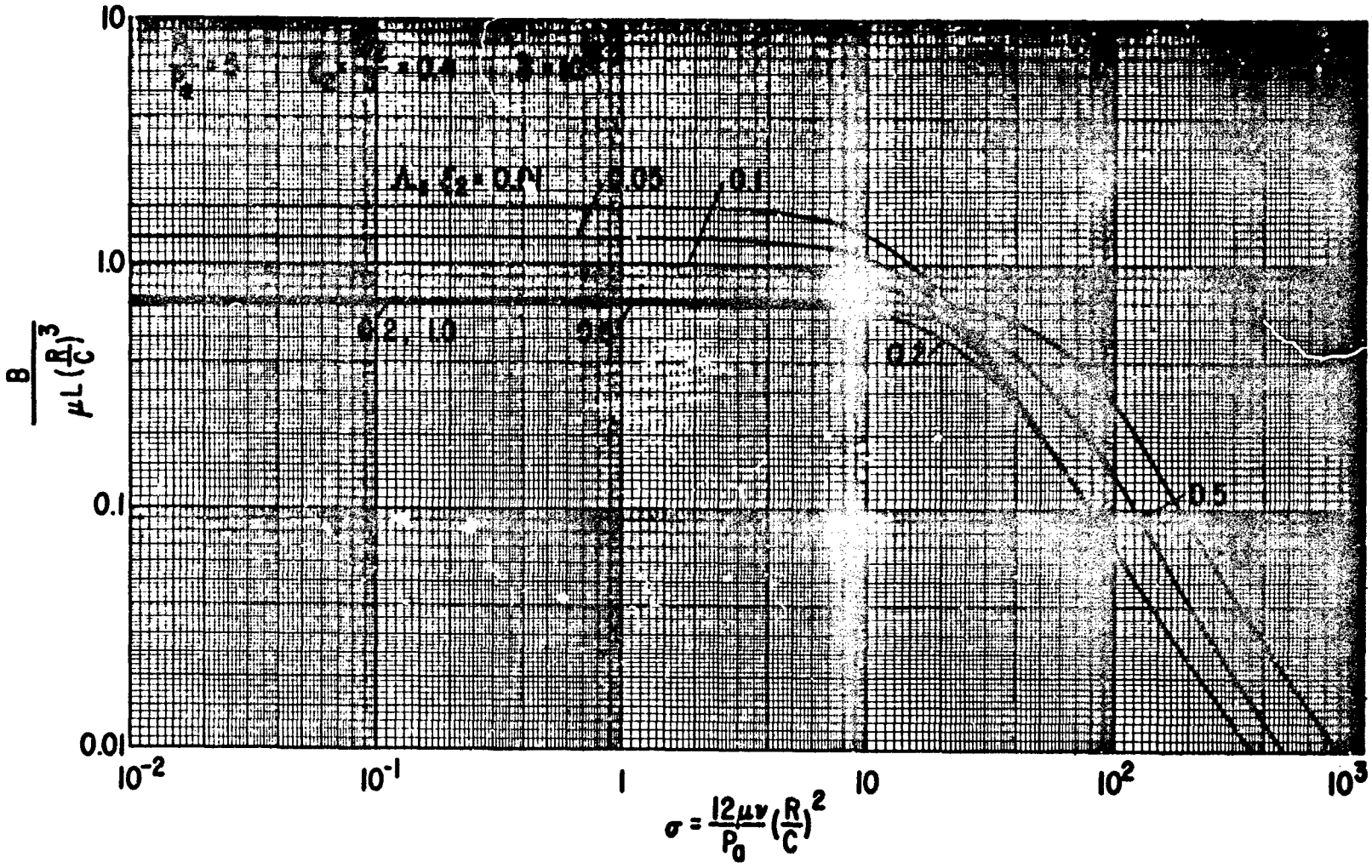


Figure 51. Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 5$, Damping

MTI-5058

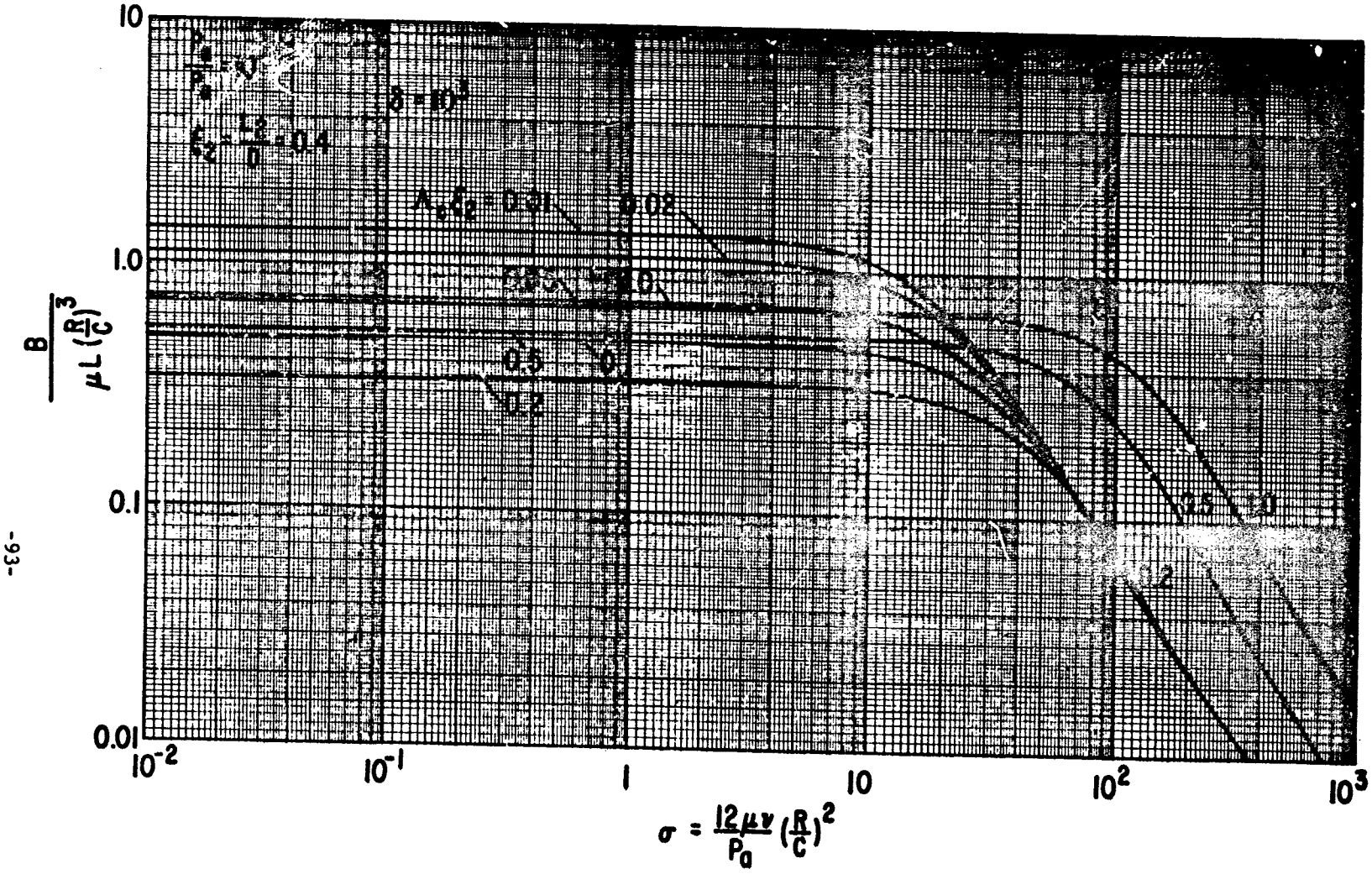


Figure 52. Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 10$, Damping

MTI-5059

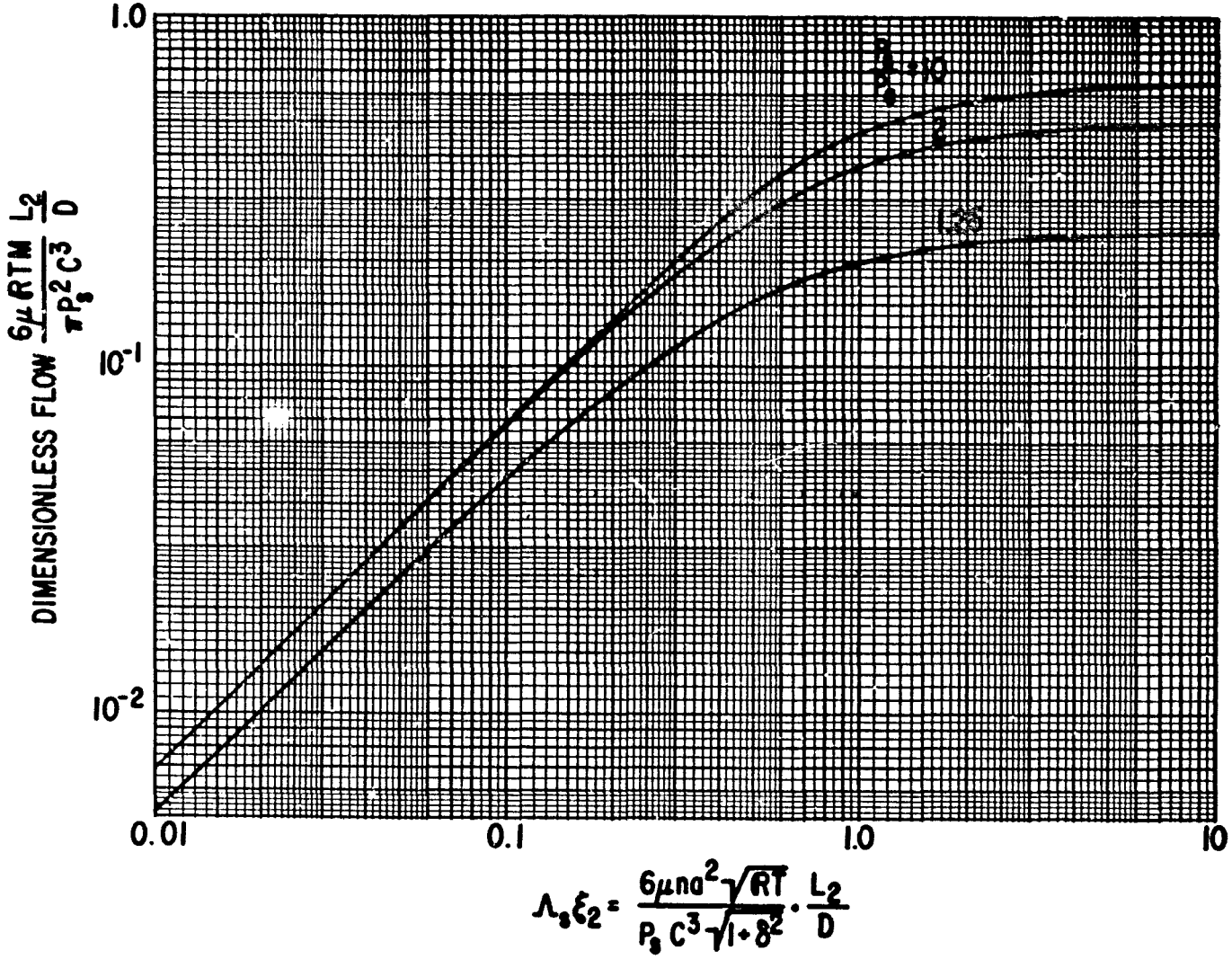


Figure 53. Hydrostatic Bearing, Compressible, Flow

MTI-5060

APPENDIX I: The Static and Dynamic Performance of a Partial Arc Bearing with Turbulent Film

To calculate the load carrying capacity, the friction, the flow and the dynamic coefficients of a turbulent lubricant film it is necessary to solve Reynolds equation. This is done numerically as described in this appendix.

In a turbulent lubricant film Reynolds equation is given by (see reference 1):

$$\frac{\partial}{R\partial\theta}\left[G_x \frac{\bar{h}^3}{12\mu}\frac{\partial\bar{P}}{R\partial\theta}\right] + \frac{\partial}{\partial z}\left[G_z \frac{\bar{h}^3}{12\mu}\frac{\partial\bar{P}}{\partial z}\right] = \frac{1}{2}R\omega\frac{\partial\bar{h}}{R\partial\theta} + \frac{\partial\bar{h}}{\partial t} \qquad \text{(A-1)}$$

where $\bar{P}$ is the film pressure, μ is the lubricant viscosity, R is the journal radius, ω is the angular speed of the journal, θ is the angular coordinate, (see fig. 54), z is the axial coordinate, t is time, G_x and G_z are turbulent flow coefficients and $\bar{h}$ is the local film thickness:

$$\bar{h} = C + e\cdot\cos\theta \qquad \text{(A-2)}$$

Here, C is the radial clearance (the difference between the radius of curvature of the bearing surface, and the radius of the journal, see fig. 54) and e is the eccentricity (the distance between the journal center and the center of curvature of the bearing).

Reynolds equation, eq. (A-1), is made dimensionless by setting:

$$\zeta = \frac{z}{R} \qquad \text{(A-3)}$$

$$\tau = \omega t \qquad \text{(A-4)}$$

$$h = \frac{\bar{h}}{C} = 1 + \varepsilon\cos\theta \qquad \text{(A-5)}$$

$$\varepsilon = \frac{e}{C} \qquad \text{(A-6)}$$

$$P = \frac{\bar{P}}{\mu\omega\left(\frac{R}{C}\right)^2} = \frac{1}{2\pi S}\frac{\bar{P}}{W/LD} \qquad \text{(A-7)}$$

where W is the load carried by the bearing, L is the bearing length, D is the journal diameter and:

$$\text{Sommerfeld Number:} \quad S = \frac{\mu N D L}{W}\left(\frac{R}{C}\right)^2 \tag{A-8}$$

(N is the journal speed in rps, i.e. $\omega = 2\pi N$)

Substituting eqs. (A-3) to (A-7) into eq. (A-1) yields Reynolds equation in dimensionless form:

$$\frac{\partial}{\partial\theta}\left[G_x h^3 \frac{\partial P}{\partial\theta}\right] + \frac{\partial}{\partial\zeta}\left[G_z h^3 \frac{\partial P}{\partial\zeta}\right] = 6\frac{\partial h}{\partial\theta} + 12\frac{\partial h}{\partial\tau} \tag{A-9}$$

The dimensionless turbulent flow coefficients G_x and G_z are functions of the local Reynolds number R_h:

$$R_h = h R_e \tag{A-10}$$

where:

$$\text{Reynolds Number:} \quad R_e = \frac{\rho R \omega C}{\mu} \tag{A-11}$$

ρ is the mass density of the lubricant.

The actual functional relationships between G_x, G_z and R_h are given in reference 1. When comparing with ref. 1, however, it should be noted that G_x and G_z as used in the present analysis are larger than the values given in ref. 1 by a factor of 12 such that for laminar flow ($R_h = R_e = 0$), G_x and G_z are equal to unity rather than 1/12 as in ref. 1.

From eq. (A-10) it is seen, that for a given Reynolds number R_e, G_x and G_z are functions of θ only (since they are functions of h) and do not depend on the axial coordinate ζ.

Before solving eq. (A-9) it is convenient to give it another form. Introduce the new variables:

$$G = \frac{G_z}{G_x} \tag{A-12}$$

$$H = h^{3/2} G_x^{1/2} \tag{A-13}$$

$$\psi = PH = Ph^{3/2} G_x^{1/2} \tag{A-14}$$

whereby eq. (A-9) can be written:

$$\frac{\partial^2 \psi}{\partial \theta^2} + G \frac{\partial^2 \psi}{\partial \zeta^2} - \frac{1}{H} \frac{\partial^2 H}{\partial \theta^2} \psi = \frac{1}{H}\left[6 \frac{\partial h}{\partial \theta} + 12 \frac{\partial h}{\partial \tau}\right] \tag{A-15}$$

Assume that the journal operates under static conditions with an eccentricity ratio ε_o and an attitude angle φ_o where, in the present analysis, the attitude angle φ is the angle from a fixed x-axis to the line connecting the center of the bearing with the center of the journal:

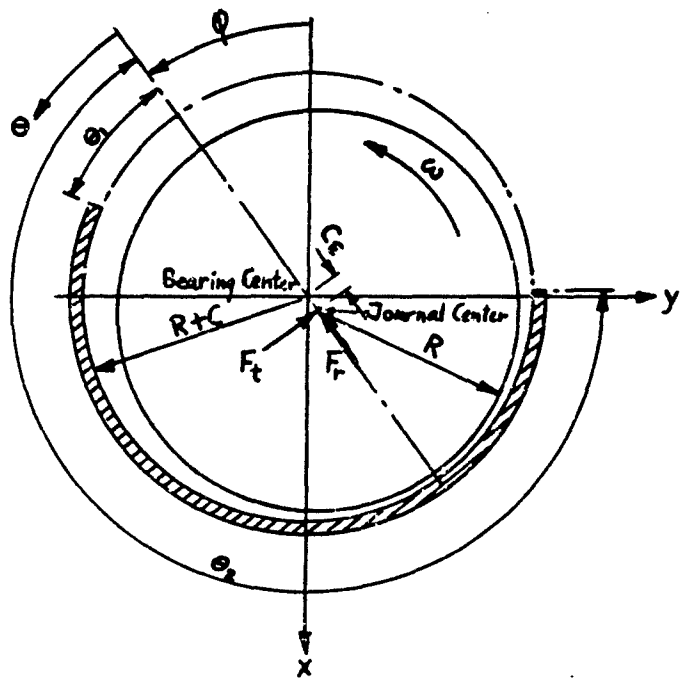


Figure 54: Geometry of Partial Arc Bearing

The static equilibrium position (ε_o, φ_o) is perturbed by giving the journal center a small amplitude motion with amplitudes (ε_1, φ_1) and corresponding velocities ($\dot{\varepsilon}_1, \dot{\varphi}_1$) where "dot" refers to $d/d\tau$. Hence, the instantaneous position of the journal center is defined by:

$$\varepsilon = \varepsilon_o + \varepsilon_1$$

$$\varphi = \varphi_o + \varphi_1 \tag{A-16}$$

where $\varepsilon_1 \ll \varepsilon_o$ and $\varphi_1 \ll \varphi_o$. This motion causes similar perturbations in the film pressure and the film thickness such that:

$$\psi = \psi_o + \varepsilon_1 \psi_1 + \varepsilon_o \varphi_1 \psi_2 + \dot{\varepsilon}_1 \psi_3 + \varepsilon_o \dot{\varphi}_1 \psi_4 \tag{A-17}$$

$$H = H_o + \varepsilon_1 H_1 + \varepsilon_o \varphi_1 H_2 \tag{A-18}$$

$$G = G_o + \varepsilon_1 G_1 + \varepsilon_o \varphi_1 G_2 \tag{A-19}$$

$$h = 1 + \varepsilon_o \cos\theta + \varepsilon_1 \cos\theta + \varepsilon_o \varphi_1 \sin\theta = h_o + \varepsilon_1 \cos\theta + \varepsilon_o \varphi_1 \sin\theta \tag{A-20}$$

Substitution of eqs. (A-17) to (A-20) into eq. (A-15) yields 5 equations to determine the static pressure and its four perturbation components:

$$\frac{\partial^2 \psi_o}{\partial \theta^2} + G_o \frac{\partial^2 \psi_o}{\partial \zeta^2} - \frac{1}{H_o} \frac{\partial^2 H_o}{\partial \theta^2} \psi_o = \frac{6}{H_o} \frac{\partial h_o}{\partial \theta} = -\frac{6 \sin\theta}{H_o} \varepsilon_o \tag{A-21}$$

$$\frac{\partial^2 \psi_1}{\partial \theta^2} + G_o \frac{\partial^2 \psi_1}{\partial \zeta^2} - \frac{1}{H_o} \frac{\partial^2 H_o}{\partial \theta^2} \psi_1 = \frac{1}{H_o}\left(\frac{\partial^2 H_1}{\partial \theta^2} - \frac{H_1}{H_o} \frac{\partial^2 H_o}{\partial \theta^2}\right)\psi_o - G_1 \frac{\partial^2 \psi_o}{\partial \zeta^2} - \frac{6}{H_o}\left(\sin\theta + \frac{H_1}{H_o} \frac{\partial h_o}{\partial \theta}\right) \tag{A-22}$$

$$\frac{\partial^2 \psi_2}{\partial \theta^2} + G_o \frac{\partial^2 \psi_2}{\partial \zeta^2} - \frac{1}{H_o} \frac{\partial^2 H_o}{\partial \theta^2} \psi_2 = \frac{1}{H_o}\left(\frac{\partial^2 H_2}{\partial \theta^2} - \frac{H_2}{H_o} \frac{\partial^2 H_o}{\partial \theta^2}\right)\psi_o - G_2 \frac{\partial^2 \psi_o}{\partial \zeta^2} - \frac{6}{H_o}\left(-\cos\theta + \frac{H_2}{H_o} \frac{\partial h_o}{\partial \theta}\right) \tag{A-23}$$

$$\frac{\partial^2 \psi_3}{\partial \theta^2} + G_o \frac{\partial^2 \psi_3}{\partial \zeta^2} - \frac{1}{H_o} \frac{\partial^2 H_o}{\partial \theta^2} \psi_3 = \frac{12 \cos\theta}{H_o} \tag{A-24}$$

$$\frac{\partial^2 \psi_4}{\partial \theta^2} + G_0 \frac{\partial^2 \psi_4}{\partial \zeta^2} - \frac{1}{H_0} \frac{\partial^2 H_0}{\partial \theta^2} \psi_4 = \frac{12 \sin\theta}{H_0} \tag{A-25}$$

Comparing eqs. (A-21) and (A-25), it is seen that:

$$\psi_0 = -\frac{1}{2} \varepsilon_0 \psi_4 \tag{A-26}$$

The boundary conditions are that the pressure is ambient (i.e. zero) along the peripheri of the film. Hence, at the sides of the bearing, $\overline{P} = 0$ or:

$$\underline{\zeta = \pm \frac{L}{D} \quad (z = \pm \tfrac{L}{2}):} \qquad \psi = 0$$

Since there is symmetry with respect to $\zeta = 0$, this condition can also be written:

$$\underline{\zeta = \frac{L}{D}:} \qquad \psi = 0 \qquad \therefore \quad \psi_0 = \psi_1 = \psi_2 = \psi_3 = \psi_4 = 0 \tag{A-27}$$

$$\underline{\zeta = 0:} \qquad \frac{\partial \psi}{\partial \zeta} = 0 \qquad \therefore \quad \frac{\partial \psi_0}{\partial \zeta} = \frac{\partial \psi_1}{\partial \zeta} = \frac{\partial \psi_2}{\partial \zeta} = \frac{\partial \psi_3}{\partial \zeta} = \frac{\partial \psi_4}{\partial \zeta} = 0 \tag{A-28}$$

If the film is complete over the entire bearing surface, the two remaining boundary conditions are that the pressure is zero at the leading edge and at the trailing edge of the bearing arc:

$$\left.\begin{array}{l} \underline{\theta = \theta_1} \\ \\ \underline{\theta = \theta_2} \end{array}\right\}: \qquad \psi = 0 \qquad \therefore \quad \psi_0 = \psi_1 = \psi_2 = \psi_3 = \psi_4 = 0 \tag{A-29}$$

<u>No film rupture</u>

On the other hand, the film is frequently not complete. If any portion of the bearing arc is in the diverging part of the film (i.e. $180 < \theta < 360$, see fig. A-1), Reynolds equation predicts subambient film pressures in this region. These subambient pressures cannot, in general, exist unless the bearing is submerged completely in its own lubricant. Instead, the film contracts such that it does not take up the full length of the bearing, and the pressure in the contracted film is zero. This is called film rupture. On the boundary between the complete film and the contracted film, the pressure is zero (i.e. eq. (A-29) is valid when θ_1 and θ_2 are adjusted to give the angular location of the boundary). In addition, a second conditon is:

$$\frac{\partial \bar{P}}{\partial \theta} = \frac{\partial \bar{P}}{\partial \zeta} = 0$$

or since $\psi = 0$ on the boundary:

$$\frac{\partial \psi}{\partial \theta} = \frac{\partial \psi}{\partial \zeta} = 0$$

Let a point on the boundary have the coordinates (θ, ζ), i.e.:

on free boundary: $\psi(\theta,\zeta) = 0 \qquad \left.\frac{\partial \psi}{\partial \theta}\right]_{\theta,\zeta} = \left.\frac{\partial \psi}{\partial \zeta}\right]_{\theta,\zeta}$ (A-30)

Under static conditions the boundary has the coordinates (θ_0, ζ_0). For small scale motion of the journal the boundary is perturbed such that:

$$\theta = \theta_0 + \delta\theta$$
$$\zeta = \zeta_0 + \delta\zeta \tag{A-31}$$

where $\delta\theta$ and $\delta\zeta$ are small quantities. This results in a similar perturbation of the pressure variable ψ :

$$\psi = \psi_0 + \delta\psi \tag{A-32}$$

Expand ψ in a Taylor series from a point on the boundary:

$$\psi(\theta,\zeta) = \psi(\theta_0,\zeta_0) + \left.\frac{\partial\psi}{\partial\theta}\right]_0 \delta\theta + \left.\frac{\partial\psi}{\partial\zeta}\right]_0 \delta\zeta = 0$$

where terms of order δ^2 have been ignored. Introduce eq. (A-32) to get:

$$\psi(\theta,\zeta) = \psi_0(\theta_0,\zeta_0) + \delta\psi(\theta_0,\zeta_0) + \left.\frac{\partial\psi_0}{\partial\theta}\right]_0 \delta\theta + \left.\frac{\partial\psi_0}{\partial\zeta}\right]_0 \delta\zeta = 0 \qquad \text{(A-33)}$$

With $\psi_0(\theta_0,\zeta_0) = \left.\frac{\partial\psi_0}{\partial\theta}\right]_0 = \left.\frac{\partial\psi_0}{\partial\zeta}\right]_0 = 0$ as the given boundary condition, this equation results in:

$$\delta\psi(\theta_0,\zeta_0) = 0 \qquad \text{(A-34)}$$

Comparing eq. (A-32) with eq. (A-17) it is seen that:

$$\delta\psi(\theta_0,\zeta_0) = \left[\varepsilon_1\psi_1 + \varepsilon_0\varphi_1\psi_2 + \dot{\varepsilon}_1\psi_3 + \varepsilon_0\dot{\varphi}_1\psi_4\right]_{\theta_0,\zeta_0}$$

Since ε_1, φ_1, $\dot{\varepsilon}_1$ and $\dot{\varphi}_1$ are independent variables, each of the four pressure perturbations must vanish independently on the same free boundary as obtained under static conditions. Hence, the complete boundary conditions at the free boundary becomes:

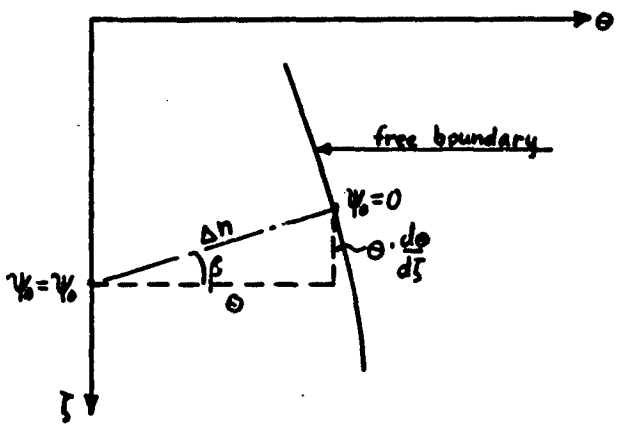
$$\left.\begin{array}{l}\underline{\text{on "zero order"}} \\ \\ \underline{\text{free boundary}}\end{array}\right\} \quad \begin{array}{l}\psi_0 = \frac{\partial\psi_0}{\partial\theta} = \frac{\partial\psi_0}{\partial\zeta} = 0 \qquad \text{(A-35)} \\ \\ \psi_1 = \psi_2 = \psi_3 = \psi_4 = 0 \qquad \text{(A-36)}\end{array}$$

Hence, the location of the free boundary is determined from the solution of ψ_0 (i.e. the steady-state solution). Once the boundary has been established, the perturbations ψ_1, ψ_2, ψ_3 and ψ_4 are to be solved over the same domain such that they vanish along the peripheri of the domain.

To determine the coordinates of the free boundary, substitution of eq. (A-35) into eq. (A-21) yields:

on free boundary: $$\frac{\partial^2 \psi_0}{\partial \theta^2} + G_0 \frac{\partial^2 \psi_0}{\partial \zeta^2} = \frac{6}{H_0} \frac{\partial h_0}{\partial \theta} \tag{A-37}$$

Let the free boundary have the coordinates (θ, ζ) where θ is considered a function of ζ :



The inward directed normal to the boundary is n . Since all derivatives along the boundary are zero, ψ_0 can be expanded in a Taylor series from a point on the boundary as:

$$\psi_0 \cong \frac{1}{2} \frac{\partial^2 \psi_0}{\partial n^2} (\Delta n)^2 \tag{A-38}$$

From the above figure:

$$(\Delta n)^2 = \theta^2 \left[1 + \left(\frac{d\theta}{d\zeta}\right)^2 \right] = \theta^2 (1 + \tan^2 \beta) \tag{A-39}$$

By a coordinate transformation, $\frac{\partial^2 \psi_0}{\partial n^2}$ can be found from eq. (A-37) as:

$$\frac{\partial^2 \psi_0}{\partial n^2} \left[\cos^2 \beta + G_0 \sin^2 \beta \right] = \frac{\partial^2 \psi_0^2}{\partial \theta^2} + G_0 \frac{\partial^2 \psi_0^2}{\partial \zeta^2} = \frac{6}{H_0} \frac{\partial h_0}{\partial \theta} \tag{A-40}$$

Combining eqs. (A-38) to (A-40) yields:

$$\frac{3}{H_0} \frac{\partial h_0}{\partial \theta} \frac{1 + \tan^2 \beta}{\cos^2 \beta + G_0 \sin^2 \beta} \theta^2 - \psi_0 = 0 \tag{A-41}$$

or since $\frac{1}{\cos^2\beta} = 1+\tan^2\beta = 1+\left(\frac{d\varphi}{d\zeta}\right)^2$:

$$\frac{3}{H_0}\frac{\partial h_0}{\partial \theta}\frac{\left[1+\left(\frac{d\varphi}{d\zeta}\right)^2\right]^2}{1+G_0\left(\frac{d\varphi}{d\zeta}\right)^2}\theta^2 - \psi_0 = 0 \qquad \text{(A-42)}$$

This equation must be solved together with eq. (A-21) as described later.

Equations (A-21) to (A-25) are solved by finite difference methods. A finite difference grid is introduced with i-axis along the ζ-direction and j-axis along the θ-direction:

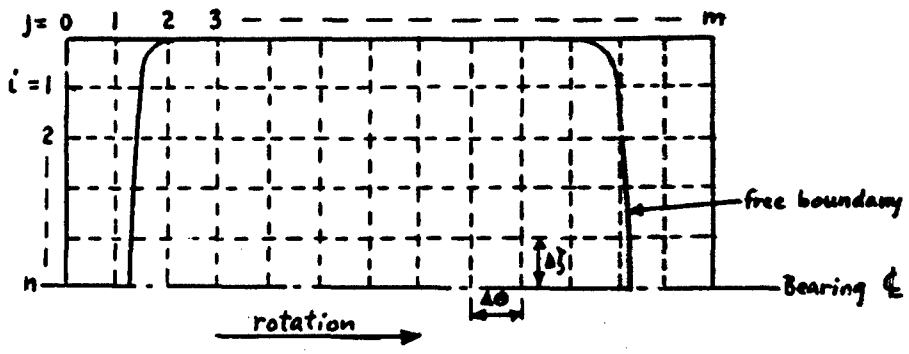


Figure 55: Finite Difference Grid

There are n increments in the ζ-direction such that the length of an increment becomes:

$$\Delta\zeta = \frac{1}{n}\frac{L}{D} \qquad \text{(A-43)}$$

In the θ-direction, the arc length of the bearing pad is subdivided into equal increments of length $\Delta\theta$.

In order to write eqs. (A-21) to (A-25) as finite difference equations, consider the grid point (i,j):

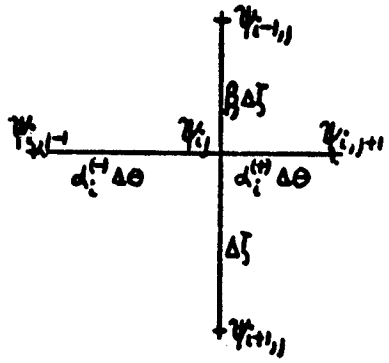


Figure 56: Finite Difference Grid Lines

Here, $\alpha_i^{(+)}$, $\alpha_i^{(-)}$ and β_j are introduced to take into account the location of the free boundary. Hence, for most points $\alpha_i^{(+)} = \alpha_i^{(-)} = \beta_j = 1$, and only for the j - grid lines closest to the boundaries do $\alpha_i^{(+)}, \alpha_i^{(-)}$ or β_j differ from 1.

Eqs. (A-21) to (A-25) can be written in the general form:

$$\frac{\partial^2 \psi}{\partial \theta^2} + G_0 \frac{\partial^2 \psi}{\partial \zeta^2} + f_1 \psi = f_2 + f_3 \psi_0 + f_4 \frac{\partial^2 \psi_0}{\partial \zeta^2} \tag{A-44}$$

where:

$$f_1 = -\frac{1}{H_0} \frac{\partial^2 H_0}{\partial \theta^2} \tag{A-45}$$

and where f_2, f_3 and f_4 are only functions of θ which can be determined by comparing eq. (A-44) with any one of eqs. (A-21) to (A-25). Thus, $f_3 = f_4 = 0$ for $\psi = \psi_0, \psi_3$ and ψ_4.

Referring to fig. 56. eq. (A-44) can be written in finite difference form as:

$$\frac{2G_{0j}}{\beta_j(1+\beta_j)\Delta\zeta^2}\psi_{i-1,j} + \left[f_{1j} - \frac{2}{\alpha_i^{(+)}\alpha_i^{(-)}\Delta\theta^2} - \frac{2G_{0j}}{\beta_j\Delta\zeta^2}\right]\psi_{ij} + \frac{2G_{0j}}{(1+\beta_j)\Delta\zeta^2}\psi_{i+1,j}$$

$$+ \frac{2}{\alpha_i^{(-)}(\alpha_i^{(+)}+\alpha_i^{(-)})\Delta\theta^2}\psi_{i,j-1} + \frac{2}{\alpha_i^{(+)}(\alpha_i^{(+)}+\alpha_i^{(-)})\Delta\theta^2}\psi_{i,j+1} = \tag{A-46}$$

$$f_{2j} + \left[f_{3j} - \frac{2}{\beta_j\Delta\zeta^2} f_{4j}\right](\psi_0)_{ij} + f_{4j}\frac{2}{(1+\beta_j)\Delta\zeta^2}\left[\frac{1}{\beta_j}(\psi_0)_{i-1,j} + (\psi_0)_{i+1,j}\right]$$

Introduce the n-dimensional vector ϕ_j:

$$\phi_j = \begin{Bmatrix} \psi_{1j} \\ \psi_{2j} \\ \vdots \\ \psi_{nj} \end{Bmatrix} \tag{A-47}$$

whereby eq. (A-46) can be written as:

$$A_j \phi_j + B_j \phi_{j-1} + C_j \phi_{j+1} = F_j \tag{A-48}$$

Here, F_j is an n-dimensional vector whose n elements are given by the right hand side of eq. (A-46), A_j is an nxn - matrix, and B_j and C_j are nxn - diagonal matrices. Noting that $\psi_{0j} = 0$ and $\psi_{n+1,j} = \psi_{n-1,j}$ from the given boundary conditions, the elements of the matrix A_j become (first index gives row number, second index gives column number):

$$\begin{aligned}
(A_j)_{ii} &= f_{1j} - \frac{2}{\alpha_i^{(+)}\alpha_i^{(-)}\Delta\theta^2} - \frac{2G_{0j}}{\beta_j \Delta\zeta^2} && \underline{1 \le i \le n} \\
(A_j)_{i,i-1} &= \frac{2G_{0j}}{\beta_j(1+\beta_j)\Delta\zeta^2} && \underline{2 \le i \le n} \\
(A_j)_{n,n-1} &= \frac{2G_{0j}}{\beta_j \Delta\zeta^2} && \underline{i = n} \\
(A_j)_{i,i+1} &= \frac{2G_{0j}}{(1+\beta_j)\Delta\zeta^2} && \underline{1 \le i \le n-1}
\end{aligned} \tag{A-49}$$

All other elements are zero such that A_j is a tri-diagonal matrix. The elements of the B_j -matrix and C_j -matrix are:

$$\begin{aligned}
(B_j)_{ii} &= \frac{2}{\alpha_i^{(-)}(\alpha_i^{(+)}+\alpha_i^{(-)})\Delta\theta^2} && \underline{1 \le i \le n} \\
(C_j)_{ii} &= \frac{2}{\alpha_i^{(+)}(\alpha_i^{(+)}+\alpha_i^{(-)})\Delta\theta^2} && \underline{1 \le i \le n}
\end{aligned} \tag{A-50}$$

All other elements are zero.

To solve eq. (A-48), define an nxn-matrix, D_j , and an n-dimensional vector E_j

such that:

$$\phi_{j-1} = D_{j-1}\,\phi_j + E_{j-1} \tag{A-51}$$

Substitution of this expression into eq. (A-48), yields the following recurrence relationships:

$$D_j = -(A_j + B_j D_{j-1})^{-1} C_j = -G_j C_j \tag{A-52}$$

$$E_j = (A_j + B_j D_j)^{-1}(F_j - B_j E_{j-1}) = G_j (F_j - B_j E_{j-1}) \tag{A-53}$$

In figure 55, the points on the free boundary should be at j=0 and j=m. Since $\psi = 0$ on the boundary, the boundary condition becomes:

$$\phi_0 = \phi_m = 0 \tag{A-54}$$

Hence, from eq. (A-51):

$$D_0 = E_0 = 0$$

after which eqs. (A-52) and (A-53) can be used to calculate D_j and E_j step by step, starting with j = 1, until j = m-1. Then eq. (A-51) is employed, letting j go from j = m , where $\phi_m = 0$, to j = 2.

In this way the solution for ψ has been obtained. It should be noted, that since A_j, B_j and C_j are the same for all the ψ -equations (i.e. eq. (A-21) to (A-25)), then the D_j-matrice and the G_j-matrices are also the same. Thus, D_j and G_j can be stored in the first ψ-calculation and used unchanged in all the remaining ψ -calculations.

In order to avoid a singularity when $\varepsilon_0 = 0$, it is most convenient to solve for ψ_4 first from eq. (A-25). With $\psi_0 = -\frac{1}{2}\varepsilon_0 \psi_4$ from eq. (A-26), the equation to determine the free boundary, eq. (A-42), becomes:

$$\left(\frac{6\sin\theta}{H_0}\right)_j \frac{\left[1+\left(\frac{d\theta}{d\zeta}\right)^2\right]^2}{1+G_0\left(\frac{d\theta}{d\zeta}\right)^2}\Theta^2 - (\psi_4)_{ij} = 0 \qquad \text{(A-55)}$$

To solve this equation it is necessary to have a first estimate of the ψ_4 distribution available. For this purposes eqs. (A-52) and (A-53) are used directly such that j=0 corresponds to the leading edge of the bearing arc and j = m corresponds to the trailing edge of the arc. In the "back-sweep", employing eq. (A-51), any negative value in a ϕ_j -vector is set equal to zero before computing the next ϕ_j -vector. Based on the ψ_4 -distribution determined in this way, eq. (A-55) is solved step by step, letting i go from i=n to i=1. At i=n, $\frac{d\theta}{d\zeta} = 0$ such that eq. (A-55) is readily solved for the corresponding Θ -value. At subsequent i-values, $\frac{d\theta}{d\zeta}$ can be expressed in terms of Θ and the previous Θ -values, allowing the complete boundary to be established.

Once the free boundary is known, $\alpha_i^{(+)}, \alpha_i^{(-)}$ and β_j can be found for use in the finite difference equations (see fig. 55), a new ψ_4 -distribution can be computed and the corresponding new boundary can be found. This process is repeated until the relative difference between two subsequent boundaries is smaller than some preassigned error limit. When the final boundary has been established, the solutions for ψ_1, ψ_2 and ψ_3 are obtained simply by matrix multiplications.

The film forces have two components acting in the journal center: a radial component F_r directed towards the bearing center and a tangential component positive in the direction of rotation, (see fig. 54). These forces are obtained by integration of the film pressures $\bar{P}$:

$$\left.\begin{matrix} F_r \\ \\ F_t \end{matrix}\right\} = 2\int_0^{\frac{L}{2}}\int_{\theta_1}^{\theta_2} \bar{P} \left\{\begin{matrix} -\cos\theta \\ \\ \sin\theta \end{matrix}\right\} R\,d\theta\,dz \qquad \text{(A-56)}$$

or by substitution fr. m eqs. (A-3), (A-7) and (A-8):

$$\left.\begin{aligned} f_r &= \frac{F_r}{SW} = \frac{F_r}{\mu NDL\left(\frac{R}{C}\right)^2} \\ f_t &= \frac{F_t}{SW} = \frac{F_t}{\mu NDL\left(\frac{R}{C}\right)^2} \end{aligned}\right\} = \frac{\pi}{\frac{L}{D}} \int_0^{\frac{L}{D}} \int_{\theta_1}^{\theta_2} P \begin{Bmatrix} -\cos\theta \\ \sin\theta \end{Bmatrix} d\theta\, d\zeta \tag{A-57}$$

Combining eqs. (A-14), (A-17) and (A-18):

$$P = \frac{\Psi}{H} = \frac{\Psi_0}{H_0} + \left[\frac{\Psi_1}{H_0} - \frac{H_1}{H_0}\frac{\Psi_0}{H_0}\right]\varepsilon_1 + \left[\frac{\Psi_2}{H_0} - \frac{H_2}{H_0}\frac{\Psi_0}{H_0}\right]\varepsilon_0\varphi_1 + \frac{\Psi_3}{H_0}\dot{\varepsilon}_1 + \frac{\Psi_3}{H_0}\varepsilon_0\dot{\varphi}_1 \tag{A-58}$$

At the same time, f_r and f_t can be expanded as:

$$f_r = f_{r0} + \frac{\partial f_r}{\partial \varepsilon}\varepsilon_1 + \frac{\partial f_r}{\varepsilon_0 \partial \varphi}\varepsilon_0\varphi_1 + \frac{\partial f_r}{\partial \dot{\varepsilon}}\dot{\varepsilon}_1 + \frac{\partial f_r}{\varepsilon_0 \partial \dot{\varphi}}\varepsilon_0\dot{\varphi}_1 \tag{A-59}$$

and similarly for f_t . Thus, substituting eq. (A-58) into eq. (A-57) and comparing with eq. (A-59) results in:

$$\left.\begin{aligned} \frac{\partial f_r}{\partial \varepsilon} \\ \frac{\partial f_t}{\partial \varepsilon} \end{aligned}\right\} = \frac{\pi}{\frac{L}{D}} \int_0^{\frac{L}{D}} \int_{\theta_1}^{\theta_2} \left[\frac{\Psi_1}{H_0} - \frac{H_1}{H_0}\frac{\Psi_0}{H_0}\right] \begin{Bmatrix} -\cos\theta \\ \sin\theta \end{Bmatrix} d\theta\, d\zeta \tag{A-60}$$

$$\left.\begin{aligned} \frac{\partial f_r}{\varepsilon_0 \partial \varphi} \\ \frac{\partial f_t}{\varepsilon_0 \partial \varphi} \end{aligned}\right\} = \frac{\pi}{\frac{L}{D}} \int_0^{\frac{L}{D}} \int_{\theta_1}^{\theta_2} \left[\frac{\Psi_2}{H_0} - \frac{H_2}{H_0}\frac{\Psi_0}{H_0}\right] \begin{Bmatrix} -\cos\theta \\ \sin\theta \end{Bmatrix} d\theta\, d\zeta \tag{A-61}$$

$$\left.\begin{matrix}\dfrac{\partial f_r}{\partial \dot{\varepsilon}} \\ \\ \dfrac{\partial f_t}{\partial \dot{\varepsilon}}\end{matrix}\right\} = \frac{\pi}{\frac{L}{D}} \int_0^{\frac{L}{D}} \int_{\theta_1}^{\theta_2} \frac{\psi_3}{H_0} \begin{Bmatrix} -\cos\theta \\ \sin\theta \end{Bmatrix} d\theta \, d\zeta \tag{A-62}$$

$$\left.\begin{matrix}\dfrac{\partial f_r}{\varepsilon_0 \partial \dot{\varphi}} \\ \\ \dfrac{\partial f_t}{\varepsilon_0 \partial \dot{\varphi}}\end{matrix}\right\} = \frac{\pi}{\frac{L}{D}} \int_0^{\frac{L}{D}} \int_{\theta_1}^{\theta_2} \frac{\psi_4}{H_0} \begin{Bmatrix} -\cos\theta \\ \sin\theta \end{Bmatrix} d\theta \, d\zeta \tag{A-63}$$

$$\left.\begin{matrix} f_{ro} \\ \\ f_{to}\end{matrix}\right\} = -\frac{1}{2}\varepsilon_0 \begin{Bmatrix} \dfrac{\partial f_r}{\varepsilon_0 \partial \dot{\varphi}} \\ \\ \dfrac{\partial f_t}{\varepsilon_0 \partial \dot{\varphi}} \end{Bmatrix} \tag{A-64}$$

For convenience in combining several bearing lobes and also to eliminate the changing polar coordinate system (i.e. φ_0 changes with applied static load), a fixed cartesian x-y-coordinate system is introduced as shown in fig. 54. The following coordinate transformations are readily deduced:

$$-F_x = F_r \cos\varphi_0 + F_t \sin\varphi_0 \tag{A-65}$$

$$-F_y = F_r \sin\varphi_0 - F_t \cos\varphi_0 \tag{A-66}$$

$$\frac{x}{C} = \varepsilon \cos\varphi \tag{A-67}$$

$$\frac{y}{C} = \varepsilon \sin\varphi \tag{A-68}$$

from which:

$$\frac{\partial\varepsilon}{\partial x} = \frac{1}{C}\cos\varphi_0 \qquad \frac{\partial\varphi}{\partial x} = -\frac{1}{C\varepsilon_0}\sin\varphi_0$$

$$\frac{\partial\varepsilon}{\partial y} = \frac{1}{C}\sin\varphi_0 \qquad \frac{\partial\varphi}{\partial y} = \frac{1}{C\varepsilon_0}\cos\varphi_0 \tag{A-69}$$

Thus, from eqs. (A-65) and (A-66):

$$\frac{CK_{xx}}{SW} = -\frac{C\partial}{\partial x}\left(\frac{F_x}{SW}\right) = \left(\frac{\partial f_r}{\partial\varepsilon}\cos\varphi_0 + \frac{\partial f_t}{\partial\varepsilon}\sin\varphi_0\right)\cos\varphi_0 - \left(\frac{\partial f_r}{\varepsilon_0\partial\varphi}\cos\varphi_0 + \frac{\partial f_t}{\varepsilon_0\partial\varphi}\sin\varphi_0\right)\sin\varphi_0 \tag{A-70}$$

$$\frac{CK_{xy}}{SW} = -\frac{C\partial}{\partial y}\left(\frac{F_x}{SW}\right) = \left(\frac{\partial f_r}{\partial\varepsilon}\cos\varphi_0 + \frac{\partial f_t}{\partial\varepsilon}\sin\varphi_0\right)\sin\varphi_0 + \left(\frac{\partial f_r}{\varepsilon_0\partial\varphi}\cos\varphi_0 + \frac{\partial f_t}{\varepsilon_0\partial\varphi}\sin\varphi_0\right)\cos\varphi_0 \tag{A-71}$$

$$\frac{CK_{yx}}{SW} = -\frac{C\partial}{\partial x}\left(\frac{F_y}{SW}\right) = \left(\frac{\partial f_r}{\partial\varepsilon}\sin\varphi_0 - \frac{\partial f_t}{\partial\varepsilon}\cos\varphi_0\right)\cos\varphi_0 - \left(\frac{\partial f_r}{\varepsilon_0\partial\varphi}\sin\varphi_0 - \frac{\partial f_t}{\varepsilon_0\partial\varphi}\cos\varphi_0\right)\sin\varphi_0 \tag{A-72}$$

$$\frac{CK_{yy}}{SW} = -\frac{C\partial}{\partial y}\left(\frac{F_y}{SW}\right) = \left(\frac{\partial f_r}{\partial\varepsilon}\sin\varphi_0 - \frac{\partial f_t}{\partial\varepsilon}\cos\varphi_0\right)\sin\varphi_0 + \left(\frac{\partial f_r}{\varepsilon_0\partial\varphi}\sin\varphi_0 - \frac{\partial f_t}{\varepsilon_0\partial\varphi}\cos\varphi_0\right)\cos\varphi_0 \tag{A-73}$$

$$\frac{C\omega B_{xx}}{SW} = -\frac{C\,\partial}{\frac{1}{\omega}\partial\left(\frac{dx}{dt}\right)}\left(\frac{F_x}{SW}\right) = \left(\frac{\partial f_r}{\partial\dot{\varepsilon}}\cos\varphi_0 + \frac{\partial f_t}{\partial\dot{\varepsilon}}\sin\varphi_0\right)\cos\varphi_0 - \left(\frac{\partial f_r}{\varepsilon_0\partial\dot{\varphi}}\cos\varphi_0 + \frac{\partial f_t}{\varepsilon_0\partial\dot{\varphi}}\sin\varphi_0\right)\sin\varphi_0 \tag{A-74}$$

$$\frac{C\omega B_{xy}}{SW} = -\frac{C\,\partial}{\frac{1}{\omega}\partial\left(\frac{dy}{dt}\right)}\left(\frac{F_x}{SW}\right) = \left(\frac{\partial f_r}{\partial\dot{\varepsilon}}\cos\varphi_0 + \frac{\partial f_t}{\partial\dot{\varepsilon}}\sin\varphi_0\right)\sin\varphi_0 + \left(\frac{\partial f_r}{\varepsilon_0\partial\dot{\varphi}}\cos\varphi_0 + \frac{\partial f_t}{\varepsilon_0\partial\dot{\varphi}}\sin\varphi_0\right)\cos\varphi_0 \tag{A-75}$$

$$\frac{C\omega B_{yx}}{SW} = -\frac{C\partial}{\frac{1}{\omega}\partial\left(\frac{dx}{dt}\right)}\left(\frac{F_y}{SW}\right) = \left(\frac{\partial f_r}{\partial \dot{\varepsilon}}\sin\varphi_0 - \frac{\partial f_t}{\partial \dot{\varepsilon}}\cos\varphi_0\right)\cos\varphi_0 - \left(\frac{\partial f_r}{\varepsilon_0 \partial \dot{\varphi}}\sin\varphi_0 - \frac{\partial f_t}{\varepsilon_0 \partial \dot{\varphi}}\cos\varphi_0\right)\sin\varphi_0 \qquad \text{(A-76)}$$

$$\frac{C\omega B_{yy}}{SW} = -\frac{C\partial}{\frac{1}{\omega}\partial\left(\frac{dy}{dt}\right)}\left(\frac{F_y}{SW}\right) = \left(\frac{\partial f_r}{\partial \dot{\varepsilon}}\sin\varphi_0 - \frac{\partial f_t}{\partial \dot{\varepsilon}}\cos\varphi_0\right)\sin\varphi_0 + \left(\frac{\partial f_r}{\varepsilon_0 \partial \dot{\varphi}}\sin\varphi_0 - \frac{\partial f_t}{\varepsilon_0 \partial \dot{\varphi}}\cos\varphi_0\right)\cos\varphi_0 \qquad \text{(A-77)}$$

$$-\frac{F_{x0}}{SW} = f_{r0}\cos\varphi_0 + f_{t0}\sin\varphi_0 \qquad \text{(A-78)}$$

$$-\frac{F_{y0}}{SW} = f_{r0}\sin\varphi_0 - f_{t0}\cos\varphi_0 \qquad \text{(A-79)}$$

The forces acting on the journal can, therefore be written:

$$F_x = F_{x0} - K_{xx}x - B_{xx}\frac{dx}{dt} - K_{xy}y - B_{xy}\frac{dy}{dt} \qquad \text{(A-80)}$$

$$F_y = F_{y0} - K_{yx}x - B_{yx}\frac{dx}{dt} - K_{yy}y - B_{yy}\frac{dy}{dt} \qquad \text{(A-81)}$$

The static load on the bearing pad is W where:

$$W = \sqrt{F_{r0}^2 + F_{t0}^2} \qquad \text{(A-82)}$$

or, with the definition of f_r and f_t from eq. (A-57):

$$S = \frac{1}{\sqrt{f_{ro}^2 + f_{t\theta}^2}} \tag{A-83}$$

The friction force F_f acting on the journal is determined by integrating the shear stress over the wetted area of the journal. The shear stress is:

$$\bar{\tau} = \frac{1}{8} R_h C_f \frac{\mu R \omega}{\bar{h}} + \frac{1}{2} \bar{h} \frac{\partial \bar{P}}{R \partial \theta} \tag{A-84}$$

where C_f is the turbulent friction coefficient. In laminar flow with $R_e = 0$, $\frac{1}{8} R_h C_f = 1$. For details, see reference 1.

The total friction force then becomes:

$$F_f = 2 \int_0^{\frac{L}{2}} \int_{\theta_1}^{\theta_2} \bar{\tau} \, R d\theta \, dz$$

Introducing $\bar{\tau}$ from eq. (A-84) and making the equation dimensionless by means of eqs. (A-3), (A-5), (A-7) and (A-8) results in:

$$\frac{R}{C} \frac{F_f}{SW} = \pi \int_{\theta_1}^{\theta_2} \frac{1}{8} R_h C_f \frac{1}{\bar{h}} \, d\theta + \frac{1}{2} \frac{\pi}{\frac{L}{D}} \int_0^{\frac{L}{D}} \int_{\theta_1}^{\theta_2} h \frac{\partial P}{\partial \theta} \, d\theta \, d\zeta \tag{A-85}$$

where $R \cdot F_f$ is the total friction torque. The last integral in this equation can be reduced as follows:

$$\int_{\theta_1}^{\theta_2} h \frac{\partial P}{\partial \theta} d\theta = \int_{\theta_1}^{\theta_2} h \, dP = hP \Big]_{\theta_1}^{\theta_2} - \int_{\theta_1}^{\theta_2} P \frac{\partial h}{\partial \theta} d\theta = \varepsilon_0 \int_{\theta_1}^{\theta_2} P \sin\theta \, d\theta$$

whereby eq. (A-85) becomes:

$$\frac{R}{C}\,\frac{F_f}{SW} = \pi\int_{\theta_1}^{\theta_2}\frac{\frac{1}{8}R_h C_f}{h}\,d\theta + \frac{\varepsilon_p}{2}f_{to} \tag{A-86}$$

Turning next to the flow, the volume flow per inch in the circumferential direction is:

$$q_x = \frac{1}{2}R\omega\bar{h} - G_x\frac{\bar{h}^3}{12\mu}\frac{\partial\bar{P}}{R\partial\theta}$$

The volume flow per inch in the axial direction is:

$$q_z = -G_z\frac{\bar{h}^3}{12\mu}\frac{\partial\bar{P}}{\partial z}$$

The total flow is found by integration which, in dimensionless form becomes:

$$\left[\frac{Q_x}{NDLC}\right]_{lead.edge} = \left[\frac{\pi}{2}h - \frac{\pi}{12}G_x h^3\frac{1}{\frac{L}{D}}\int_0^{\frac{L}{D}}\frac{\partial P}{\partial\theta}\,d\zeta\right]_{\theta=\theta_1} \tag{A-87}$$

$$\left[\frac{Q_x}{NDLC}\right]_{trail.edge} = \left[\frac{\pi}{2}h - \frac{\pi}{12}G_x h^3\frac{1}{\frac{L}{D}}\int_0^{\frac{L}{D}}\frac{\partial P}{\partial\theta}\,d\zeta\right]_{\theta=\theta_2} \tag{A-88}$$

$$\left[\frac{Q_z}{NDLC}\right]_{sides} = -\frac{\pi}{12}\frac{1}{\frac{L}{D}}\int_{\theta_1}^{\theta_2}G_z h^3\left(\frac{\partial P}{\partial\zeta}\right)_{\zeta=\frac{L}{D}}d\theta \tag{A-89}$$

where Q_z is the total side flow (i.e. for both sides of the bearing). For flow continuity:

$$(Q_x)_{lead.edge} = (Q_x)_{trail.edge} + (Q_z)_{sides} \tag{A-90}$$

If there is film rupture at either the leading edge, at the trailing edge or at both edges, the integrals in eqs. (A-87) and/or (A-88) are zero, and the remaining term, $\frac{\pi}{2} h$ must be integrated along the boundary.

Having described the calculation of the dynamic coefficients, the load carrying capacity, the friction force and the flow, it remains to determine the various functions in the basic eqs. (A-21) to (A-25). With:

$$h_0 = 1 + \varepsilon_0 \cos\theta \tag{A-91}$$

the derivatives of H_0 become:

$$\frac{1}{H_0}\frac{\partial H_0}{\partial \theta} = \frac{1}{H_0}\frac{dH_0}{dh_0}\frac{dh_0}{d\theta} \tag{A-92}$$

$$\frac{1}{H_0}\frac{\partial^2 H_0}{\partial \theta^2} = \frac{1}{H_0}\frac{dH_0}{dh_0}\frac{d^2h_0}{d\theta^2} + \frac{1}{H_0}\frac{d^2H_0}{dh_0^2}\left(\frac{dh_0}{d\theta}\right)^2 \tag{A-93}$$

where (see eq. (A-13)):

$$H_0 = h_0^{3/2} G_x^{1/2} \tag{A-94}$$

Expansion of H yields:

$$H = H_0 + \frac{dH_0}{dh_0}(h - h_0) = H_0 + \varepsilon_1 \frac{dH_0}{dh_0}\cos\theta + \varepsilon_0 \varphi_1 \frac{dH_0}{dh_0}\sin\theta \tag{A-95}$$

or, by comparison with eq. (A-18):

$$\frac{1}{H_0} H_1 = \frac{1}{H_0}\frac{dH_0}{dh_0}\cos\theta \tag{A-96}$$

$$\frac{1}{H_0} H_2 = \frac{1}{H_0}\frac{dH_0}{dh_0}\sin\theta \tag{A-97}$$

from which:

$$\frac{1}{H_o}\frac{\partial^2 H_1}{\partial\theta^2} = -\cos\theta\,\frac{1}{H_o}\frac{dH_o}{dh_o} + \left[\frac{d^2h_o}{d\theta^2}\cos\theta - 2\frac{dh_o}{d\theta}\sin\theta\right]\frac{1}{H_o}\frac{d^2H_o}{dh_o^2} + \left(\frac{dh_o}{d\theta}\right)^2\cos\theta\,\frac{1}{H_o}\frac{d^3H_o}{dh_o^3} \tag{A-98}$$

$$\frac{1}{H_o}\frac{\partial^2 H_2}{\partial\theta^2} = -\sin\theta\,\frac{1}{H_o}\frac{dH_o}{dh_o} + \left[\frac{d^2h_o}{d\theta^2}\sin\theta + 2\frac{dh_o}{d\theta}\cos\theta\right]\frac{1}{H_o}\frac{d^2H_o}{dh_o^2} + \left(\frac{dh_o}{d\theta}\right)^2\sin\theta\,\frac{1}{H_o}\frac{d^3H_o}{dh_o^3} \tag{A-99}$$

These equations contain the first three derivatives of H_o with respect to h_o. The derivatives are found from Eq. (A-94):

$$\frac{1}{H_o}\frac{dH_o}{dh_o} = \frac{3}{2}\frac{1}{h_o} + \frac{1}{2}\frac{1}{G_x}\frac{dG_x}{dh_o} \tag{A-100}$$

$$\frac{1}{H_o}\frac{d^2H_o}{dh_o^2} = \frac{3}{4}\frac{1}{h_o^2} + \frac{3}{2}\frac{1}{h_o}\frac{1}{G_x}\frac{dG_x}{dh_o} - \frac{1}{4}\left(\frac{1}{G_x}\frac{dG_x}{dh_o}\right)^2 + \frac{1}{2}\frac{1}{G_x}\frac{d^2G_x}{dh_o^2} \tag{A-101}$$

$$\begin{aligned}\frac{1}{H_o}\frac{d^3H_o}{dh_o^3} = &-\frac{3}{8}\frac{1}{h_o^3} + \frac{9}{8}\frac{1}{h_o^2}\frac{1}{G_x}\frac{dG_x}{dh_o} - \frac{9}{8}\frac{1}{h_o}\left(\frac{1}{G_x}\frac{dG_x}{dh_o}\right)^2 + \frac{3}{8}\left(\frac{1}{G_x}\frac{dG_x}{dh_o}\right)^3 \\ &-\frac{3}{4}\frac{1}{G_x}\frac{dG_x}{dh_o}\frac{1}{G_x}\frac{d^2G_x}{dh_o^2} + \frac{9}{4}\frac{1}{h_o}\frac{1}{G_x}\frac{d^2G_x}{dh_o^2} + \frac{1}{2}\frac{1}{G_x}\frac{d^3G_x}{dh_o^3}\end{aligned} \tag{A-102}$$

where

$$\frac{1}{G_x}\frac{dG_x}{dh_o} = R_e\left(\frac{1}{G_x}\frac{dG_x}{dR_h}\right)$$

$$\frac{1}{G_x}\frac{d^2G_x}{dh_o^2} = R_e^2\left(\frac{1}{G_x}\frac{d^2G_x}{dR_h^2}\right) \qquad \text{(A-103)}$$

$$\frac{1}{G_x}\frac{d^3G_x}{dh_o^3} = R_e^3\left(\frac{1}{G_x}\frac{d^3G_x}{dR_h^3}\right)$$

Since G_x is a given function of the local Reynolds number R_h, the three derivatives of G_x are known. For laminar flow, these derivatives are zero.

Similarly, eq. (A-12) can be expanded:

$$G = G_o + \frac{dG_o}{dh_o}(h-h_o) = G_o + \varepsilon_1\frac{dG_o}{dh_o}\cos\theta + \varepsilon_o\varphi_1\frac{dG_o}{dh_o}\sin\theta \qquad \text{(A-104)}$$

where:

$$G_o = G_x \qquad \text{(A-105)}$$

Comparing eqs. (A-104) and (A-1[illegible]):

$$G_1 = \frac{dG_o}{dh_o}\cos\theta = R_e\frac{d}{dR_h}\left(\frac{G_z}{G_x}\right)\cos\theta$$

$$\text{(A-106)}$$

$$G_2 = \frac{dG_o}{dh_o}\sin\theta = R_e\frac{d}{dR_h}\left(\frac{G_z}{G_x}\right)\sin\theta$$

In this way, all the coefficients in eqs. (A-24) to (A-25) can be evaluated and the equations can be solved as previously discussed.

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APPENDIX II: <u>The Static and Dynamic Performance of the Three Lobe Bearing with Turbulent Film</u>

The three lobe bearing is made up of three arcs:

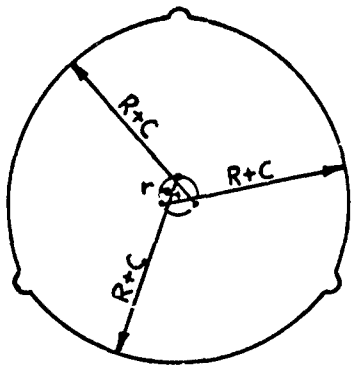

Figure 57: Three Lobe Bearing

The centers of curvature of the arcs do not coincide with the center of the bearing. Instead, the centers lie on a small circle with radius r. In this way the lobes are pre-loaded.

For the purpose of generality, assume that each lobe has its own pre-load radius $r=r_p$. Introduce a cartesian x- y - coordinate system with origin in the bearing center and the x-axial in the direction of the applied static load:

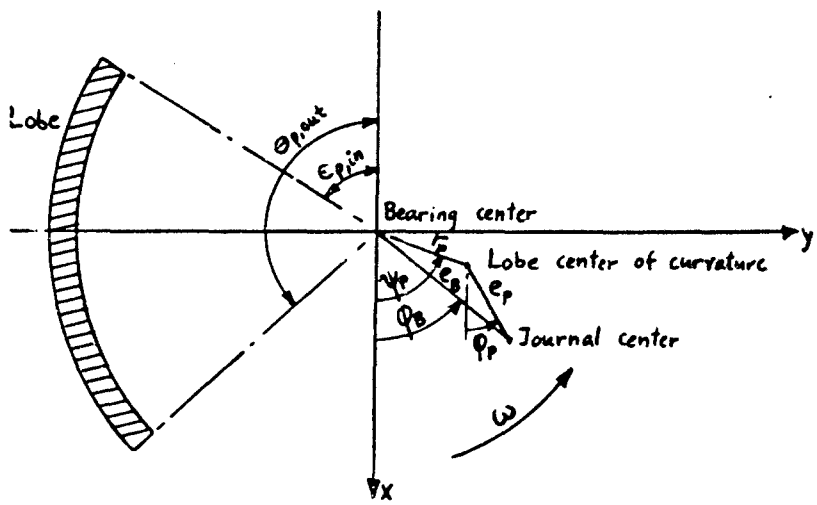

Figure 58: Geometry of Single Lobe

The journal radius is R and the lobes have the common radius of curvature: R+C, where C is the radial clearance. The center of the lobe is removed from the bearing center by a distance r_p, and the angle between the x-axis and the line connecting the two centers is ψ_p . The distance between the journal center and the lobe center is the lobe eccentricity e_p , and φ_p is the angle between the x-axis and the line connecting the two centers. With respect to the bearing center, the journal center has the eccentricity e_B and the attitude angle φ_B . From fig. 58.

$$e_B \cos\varphi_B = e_p \cos\varphi_p + r_p \cos\psi_p$$

$$e_B \sin\varphi_B = e_p \sin\varphi_p + r_p \sin\psi_p \tag{B-1}$$

Introduce the eccentricity ratios:

$$\varepsilon_B = \frac{e_B}{C}$$

$$\varepsilon_p = \frac{e_p}{C} \tag{B-2}$$

and the preload:

$$\delta_p = \frac{r_p}{C} \tag{B-3}$$

whereby eq. (B-1) becomes:

$$\varepsilon_p \cos\varphi_p = \varepsilon_B \cos\varphi_B - \delta_p \cos\psi_p$$

$$\varepsilon_p \sin\varphi_p = \varepsilon_B \sin\varphi_B - \delta_p \sin\psi_p \tag{B-4}$$

Hence:

$$\varepsilon_p = \sqrt{(\varepsilon_B \cos\varphi_B - \delta_p \cos\psi_p)^2 + (\varepsilon_B \sin\varphi_B - \delta_p \sin\psi_p)^2} \tag{B-5}$$

$$\varphi_p = \tan^{-1}\left[\frac{\varepsilon_B \sin\varphi_B - \delta_p \sin\psi_p}{\varepsilon_B \cos\varphi_B - \delta_p \cos\psi_p}\right] \tag{B-6}$$

Thus, for given coordinates of the journal center with respect to the bearing center, the corresponding lobe eccentricity ratio and attitude angle can be determined. The angles from the line of centers to the beginning and the end of the lobe become:

$$\theta_1 = \theta_{p,in} - \varphi_p$$

$$\theta_2 = \theta_{p,out} - \varphi_p \tag{B-7}$$

Thereby all the data required for calculating the lobe are known. The calculation is described in Appendix I. Performing calculations for each of the n lobes making up the bearing, the properties of the composite bearing are obtained by a simple summation over the lobes:

$$-\frac{F_{xo}}{\mu NDL\left(\frac{R}{C}\right)^2} = \sum_1^n \left(-\frac{F_{xo}}{SW}\right)_p \tag{B-8}$$

$$\frac{F_{yo}}{\mu NDL\left(\frac{R}{C}\right)^2} = \sum_1^n \left(\frac{F_{yo}}{SW}\right)_p \tag{B-9}$$

$$\frac{R}{C}\,\frac{F_f}{\mu NDL\left(\frac{R}{C}\right)^2} = \sum_1^n \left(\frac{R}{C}\,\frac{F_f}{SW}\right)_p \tag{B-10}$$

$$\frac{Q_z}{NDLC} = \sum_1^n \left(\frac{Q_z}{NDLC}\right)_p \tag{B-11}$$

$$\frac{CK_{xx}}{\mu NDL\left(\frac{R}{C}\right)^2} = \sum_1^n \left(\frac{CK_{xx}}{SW}\right)_p \tag{B-12}$$

$$\frac{C\omega B_{xx}}{\mu NDL\left(\frac{R}{C}\right)^2} = \sum_1^n \left(\frac{C\omega B_{xx}}{SW}\right)_p \tag{B-13}$$

and similarly for the 6 remaining dynamic coefficients.

The total hydrodynamic bearing flow is the side flow Q_z. To this should be added, however, any "surplus" flow from the grooves between the lobes. To illustrate, the circumferential flow from lobe p into the groove separating lobe p and lobe (p+1), is $(Q_x)_{p,\text{trail. edge}}$ (see Appendix I, eq. (A-88)). Similarly, the circumferential flow out of the groove is $(Q_x)_{p+1,\text{ lead. edge}}$. If $(Q_x)_{p,\text{trail. edge}}$ is greater than $(Q_x)_{p+1,\text{ lead. edge}}$, the difference should be added to Q_z. Otherwise, there is no surplus flow.

If in eqs. (B-8) to (B-13), the x-direction is considered to be the direction of the applied load W, then for static equilibrium:

$$F_{y_0} = 0 \tag{B-14}$$

$$-F_{x_0} = W \tag{B-15}$$

This condition is satisfied by performing the calculations with a fixed value of $\varepsilon_B \cos\varphi_B$ and vary $\varepsilon_B \sin\varphi_B$ until eq. (B-14) is fulfilled. Then, combining eqs. (B-15) and (B-8):

$$S = \frac{\mu NDL}{W}\left(\frac{R}{C}\right)^2 = \left(-\frac{F_{x_0}}{\mu NDL\left(\frac{R}{C}\right)^2}\right)^{-1} \tag{B-16}$$

where S is the Sommerfeld number for the bearing, Thereafter the bearing friction and the dynamic coefficients can be given in a different dimensionless form than in eqs. (B-10), (B-12) and (B-13):

$$\frac{R}{C}\,\frac{F_f}{W} = S\left(\frac{R}{C}\,\frac{F_f}{\mu N D L \left(\frac{R}{C}\right)^2}\right) \tag{B-17}$$

$$\frac{C K_{xx}}{W} = S\left(\frac{C K_{xx}}{\mu N D L \left(\frac{R}{C}\right)^2}\right) \tag{B-18}$$

$$\frac{C \omega B_{xx}}{W} = S\left(\frac{C \omega B_{xx}}{\mu N D L \left(\frac{R}{C}\right)^2}\right) \tag{B-19}$$

and similarly for the 6 remaining coefficients. This latter form is to be preferred except when the bearing is unloaded in which case the original expressions are employed.

In order to express the stability properties of the bearing, a symmetrical, rigid rotor with a mass of 2M is considered. The rotor is supported in two identical bearings. Assuming the translatory critical speed to be the lowest (corresponds approximately to requiring that the transverse radius of gyration of the rotor is less than half the bearing span which is normally the case), the motions of the two journals in their bearings will be in phase and be the same. Hence, the mass of the rotor can be divided in two equal parts, each of mass M, and lumped at the journals. Hence, the equations of motion for a journal become:

$$\begin{aligned} M\frac{d^2x}{dt^2} &= -K_{xx}\,x - B_{xx}\frac{dx}{dt} - K_{xy}\,y - B_{xy}\frac{dy}{dt} \\ M\frac{d^2y}{dt^2} &= -K_{yx}\,x - B_{yx}\frac{dx}{dt} - K_{yy}\,y - B_{yy}\frac{dy}{dt} \end{aligned} \tag{B-20}$$

At the threshold of instability, x and y are pure harmonic motions with frequency ν, i.e.:

$$x = x_c \cos(\nu t) - x_s \sin(\nu t) = (x_c + i x_s) e^{i\nu t} \tag{B-21}$$

and similarly for y where:

$$i = \sqrt{-1} \tag{B-22}$$

Hence, eq. (B-20) can be written:

$$\begin{Bmatrix} (K_{xx} - M\nu^2 + i\nu B_{xx}) & (K_{xy} + i\nu B_{xy}) \\ (K_{yx} + i\nu B_{yx}) & (K_{yy} - M\nu^2 + i\nu B_{yy}) \end{Bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = 0 \tag{B-23}$$

These two equations only have a non-trivial solution when the determinant is zero. Equating the real part and the imaginary part of the determinant to zero results in:

$$M\nu^2 = \frac{K_{xx} B_{yy} + K_{yy} B_{xx} - K_{xy} B_{yx} - K_{yx} B_{xy}}{B_{xx} + B_{yy}} \tag{B-24}$$

$$\nu^2 = \frac{(K_{xx} - M\nu^2)(K_{yy} - M\nu^2) - K_{xy} K_{yx}}{B_{xx} B_{yy} - B_{xy} B_{yx}} \tag{B-25}$$

Computing eq. (B-24) first, $M\nu^2$ can be substituted into eq. (B-25) whereby the instability frequency ν is determined. Thereafter the corresponding journal mass M is readily obtained.

The results are most conveniently represented in dimensionless form. For this

purpose, eqs. (B-24) and (B-25) are written:

$$4\pi^2 \frac{CMN}{\mu DL\left(\frac{B}{C}\right)^2}\left(\frac{\nu}{\omega}\right)^2 = \frac{\frac{CK_{xx}}{SW}\cdot\frac{C\omega B_{yy}}{SW} + \frac{CK_{yy}}{SW}\frac{C\omega B_{xx}}{SW} - \frac{CK_{xy}}{SW}\cdot\frac{C\omega B_{yx}}{SW} - \frac{CK_{yx}}{SW}\cdot\frac{C\omega B_{xy}}{SW}}{\frac{C\omega B_{xx}}{SW} + \frac{C\omega B_{yy}}{SW}} \tag{B-26}$$

$$\left(\frac{\nu}{\omega}\right)^2 = \frac{\left[\frac{CK_{xx}}{SW} - 4\pi^2\frac{CMN}{\mu DL\left(\frac{B}{C}\right)^2}\left(\frac{\nu}{\omega}\right)^2\right]\left[\frac{CK_{yy}}{SW} - 4\pi^2\frac{CMN}{\mu DL\left(\frac{B}{C}\right)^2}\left(\frac{\nu}{\omega}\right)^2\right] - \frac{CK_{xy}}{SW}\frac{CK_{yx}}{SW}}{\frac{C\omega B_{xx}}{SW}\frac{C\omega B_{yy}}{SW} - \frac{C\omega B_{xy}}{SW}\frac{C\omega B_{yx}}{SW}} \tag{B-27}$$

where the 8 bearing coefficients are in dimensionless form as defined by equations (B-12) and (B-13).

The ratio $\frac{\nu}{\omega}$ between the instability frequency and the rotational speed is known as the frequency ratio. Under most conditions, $\frac{\nu}{\omega} \cong \frac{1}{2}$. The instability mass can either be expressed in the form given by eq. (B-26) or in the form:

$$\frac{CM\omega^2}{W} = 4\pi^2 S \frac{CMN}{\mu DL\left(\frac{B}{C}\right)^2} \tag{B-28}$$

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APPENDIX III: The Stiffness and the Damping of a Hydrost tic Bearing with an Incomprussible Lubricant

In the hydrodynamic-hydrostatic ring bearing where the lubricant is a liquid (incompressible), the outer bearing is a purely hydrostatic bearing. This appendix describes the analysis for calculating the flow, the stiffness and the damping of a hydrostatic bearing.

The basic equation governing the flow through the bearing film is Reynolds equation in which there is no contribution from journal rotation:

$$\frac{\partial}{R\partial\theta}\left[\frac{\bar{h}^3}{12\mu}\frac{\partial\bar{P}}{R\partial\theta}\right]+\frac{\partial}{\partial z}\left[\frac{\bar{h}^3}{12\mu}\frac{\partial\bar{P}}{\partial z}\right]=\frac{\partial\bar{h}}{\partial t} \qquad \text{(C-1)}$$

Here, $\bar{P}$ is the film pressure, μ is the lubricant viscosity, t is time, R is journal radius (outer radius of ring), θ is the angular coordinate, z is the axial coordinate and $\bar{h}$ is the film thickness:

$$\bar{h} = C + e\cos\theta \qquad \text{(C-2)}$$

where C is the radial clearance and e is the eccentricity between the bearing center and the journal center.

To make Reynolds equation dimensionless, set:

$$P = \frac{\bar{P}}{P_s} \qquad \text{(C-3)}$$

$$h = \frac{\bar{h}}{C} \qquad \text{(C-4)}$$

$$\zeta = \frac{z}{R} \qquad \text{(C-5)}$$

$$\tau = \nu t \tag{C-6}$$

Thereby eq. (C-i) becomes:

$$\frac{\partial}{\partial \theta}\left(h^3 \frac{\partial P}{\partial \theta}\right) + \frac{\partial}{\partial \xi}\left(h^3 \frac{\partial P}{\partial \xi}\right) = \sigma \frac{\partial h}{\partial \tau} \tag{C-7}$$

where:

$$\sigma = \frac{12 \mu \nu}{P_s}\left(\frac{R}{C}\right)^2 \tag{C-8}$$

P_s is the supply pressure of the lubricant to the bearing and ν is the frequency of the motion of the journal.

The flow is supplied to the bearing through n restricted feeder holes. The flow restriction may either be accomplished by an orifice or by a thin tube, the latter method denoted as the laminar restrictor. In the case of an orifice the mass flow through the feeder hole is given by:

<u>Orifice Restrictor</u>

$$M_B = C_D \pi a^2 \frac{\sqrt{\rho (P_s - \bar{P}_c')}}{\sqrt{1 + \left(\frac{\delta}{h}\right)^2}} \tag{C-9}$$

where M_B is the mass flow in lbs.sec/inch, C_D is a discharge coefficient, a is the radius of the orifice, ρ is the mass density of the lubricant, $\bar{P}_c'$ is the pressure downstream of the feeder hole at the inlet to the bearing film, h is the dimensionless film thickness at the feeder hole and δ is the inherent compensation factor:

$$\delta = \frac{a^2}{dC}$$

d is the diameter of the feeding hole. Thus eq. (C-9) assumes a pressure drop both through the orifice and through the "curtain" area between the rim of the feeder hole and the journal surface. When $\delta \cong 0$, the flow restriction takes place only in the orifice whereas $\delta \to \infty$ means that the curtain area dominates as the restricting mechanism.

It is also possible to restrict the flow by simply relying on the feeder hole itself to provide the restriction. The viscous drag in the hole causes a drop in pressure as the flow passes through. The relationship is given by:

<u>Laminar Restrictor</u>

$$M_B = \frac{\pi \rho a^4}{8 \mu \ell} \left(P_s - \bar{P}_c' \right) \tag{C-10}$$

where ℓ is the length of the feeder hole, a is the radius of the feeder hole and the other symbols are defined above.

The two restrictor equations can be written in dimensionless form as:

$$\frac{3 \mu n M_B}{\pi \rho C^3 P_s} \lambda \xi_2 = \Lambda_s m \tag{C-11}$$

where:

$$\Lambda_s = \begin{cases} \dfrac{3 \mu C_D n a^2}{C^3 \sqrt{\rho P_s} \sqrt{1+\delta^2}} \lambda \xi_2 & \text{Orifice Restrictor} \\[2ex] \dfrac{3}{8} \dfrac{n a^4}{C^3 \ell} \lambda \xi_2 & \text{Laminar Restrictor} \end{cases} \tag{C-12}$$

$$m = \begin{cases} \dfrac{\sqrt{1-P_c'}}{\sqrt{\left(1+\left(\frac{\delta}{h}\right)^2\right)/(1+\delta^2)}} & \underline{\text{Orifice Restrictor}} \\ \\ (1-P_c') & \underline{\text{Laminar Restrictor}} \end{cases} \qquad \text{(C-13)}$$

Here, $P_c' = \bar{P}_c'/P_s$ which defines the pressure ratio across the feeder hole. The symbols λ and ξ_2 will be defined later.

Λ_s is known as the restrictor coefficient and it is the governing parameter for the performance of the hydrostatic bearing. It defines the ratio between the flow resistance of the bearing film and the flow resistance of the feeder holes.

To calculate the bearing performance, Reynolds equation, eq. (C-7), must be solved together with the feeder hole flow equation, eq. (C-11). The present solution will be based on the fact that the journal in a hydrostatic bearing normally operates close to its concentric position. This means that the journal center eccentricity e is small compared to the radial clearance C, or in terms of the eccentricity ratio ε :

$$\varepsilon = \frac{e}{C} \qquad \text{(C-14)}$$

ε is much smaller than 1 (in practice the solution is a valid approximation for ε- values as large as 0.4 to 0.5). Thus, the dimensionless pressure can be written as:

$$P = P_0 + \varepsilon P_1 + \dot{\varepsilon} P_2 \qquad \text{(C-15)}$$

where:

$$\dot{\varepsilon} = \frac{d\varepsilon}{d\tau} = \frac{1}{C\nu}\frac{de}{dt} \qquad \text{(C-16)}$$

The dimensionless film thickness is found from eqs. (C-2) and (C-4) as:

$$h = 1 + \varepsilon \cos\theta \tag{C-17}$$

Substitute eqs. (C-15) and (C-17) into eq. (C-7) and collect terms according to the perturbation variables:

$$\frac{\partial^2 P_0}{\partial \theta^2} + \frac{\partial^2 P_0}{\partial \zeta^2} = 0 \tag{C-18}$$

$$\frac{\partial^2 P_1}{\partial \theta^2} + \frac{\partial^2 P_1}{\partial \zeta^2} = 3 \sin\theta \frac{\partial P_0}{\partial \theta} \tag{C-19}$$

$$\frac{\partial^2 P_2}{\partial \theta^2} + \frac{\partial^2 P_2}{\partial \zeta^2} = \sigma \cos\theta \tag{C-20}$$

Consider first eq. (C-18) which gives the solution for the pressure in the film when the journal is concentric in the bearing. Under that condition each feeder hole has the same flow and in the analysis it is only necessary to consider an axial strip belonging to one feeder hole:

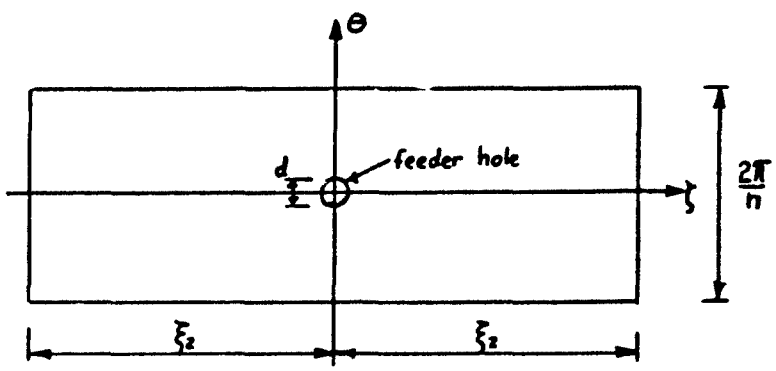

Figure 59: Axial Strip

The strip extends over the length L_z of the bearing and its width is $2\pi R/n$. Hence, the ranges of the coordinates are:

$$-\frac{L_z}{2} \leq z \leq \frac{L_z}{2}$$

$$-\frac{\pi R}{n} \leq R\theta \leq \frac{\pi R}{n}$$

or dimensionless:

$$-\xi_z \leq \zeta \leq \xi_z$$

$$-\frac{\pi}{n} \leq \theta \leq \frac{\pi}{n} \tag{C-21}$$

where:

$$\xi_z = \frac{L_z}{D} \tag{C-22}$$

and D = 2R is the diameter of the bearing.

The boundary conditions are that the pressure is ambient, (i.e. it is zero) at the ends of the bearing and there is no flow across the sides of the strip:

$$\text{for} \quad \zeta = \pm \xi_z : \qquad P_o = 0$$

$$\text{for} \quad \theta = \pm \frac{\pi}{n} : \qquad \frac{\partial P_o}{\partial \theta} = 0 \tag{C-23}$$

The solution of eq. (C-18) is then the solution of the potential equation for a rectangle with a source in the center. If the source strength in $\bar{C}$, the solution becomes:

$$P_o = \frac{1}{2} \bar{C} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[\frac{\cosh(n\xi_z(2k+1)) - 1}{\cosh((\zeta + 2k\xi_z)n) - \cos(n\theta)} \right] \tag{C-24}$$

(see reference 2). To determine the source strength, the flow out of the strip which, of course, is the same as the flow M_{BO} through the feeder hole, can be computed:

$$M_{BO} = 4\int_0^{\frac{\pi}{n}} \rho \frac{C^3}{12\mu}\left(-\frac{\partial \bar{P}_o}{\partial z}\right)_{z=\frac{1}{2}} R\, d\theta$$

or:

$$\frac{3\mu M_{BO}}{\rho C^3 P_s} = -\int_0^{\frac{\pi}{n}} \left(\frac{\partial P_o}{\partial \zeta}\right)_{\zeta=\xi_z} d\theta = \frac{1}{2}\bar{C}\sum_{k=-\infty}^{\infty}(-1)^k \int_0^{\pi} \frac{\sinh((\zeta+2k\xi_z)n)\, d(n\theta)}{\cosh((\zeta+2k\xi_z)n)-\cos n\theta} = \frac{\pi}{2}\bar{C} \quad \text{(C-25)}$$

Hence:

$$\frac{1}{2}\bar{C} = \frac{3\mu M_{BO}}{\pi\rho C^3 P_s} \quad \text{(C-26)}$$

The pressure P'_{oc} at which the flow enters the film, is taken at a point on the rim of the feeder hole with the coordinates $\zeta = 0,\ \theta = d/D$ where d is the feeder hole diameter. Thus, from eq. (C-24):

$$P'_{oc} = \frac{1}{2}\bar{C}\, n\lambda\xi_z = \frac{3\mu n M_{BO}}{\pi\rho C^3 P_s}\lambda\xi_z \quad \text{(C-27)}$$

where:

$$\lambda = \frac{1}{n\xi_z}\sum_{k=-\infty}^{\infty}(-1)^k \log\left[\frac{\cosh(n\xi_z(2k+1))-1}{\cosh(2kn\xi_z)-\cos(n\frac{d}{D})}\right] \quad \text{(C-28)}$$

<u>Single Plane Admission</u>

Since $\frac{d}{D} << 1$, eq. (C-28) can be written with good approximation as:

$$\lambda \cong \frac{1}{n\xi_2} \log\left[\frac{\cosh(n\xi_2)-1}{1-\cos(n\frac{d}{D})}\right] \cong 1+\frac{2}{n\xi_2}\log\left(\frac{D}{nd}\right) \tag{C-29}$$

Single Plane Admission

If there are two admission planes, there are two feeder holes per axial strip and fig. 59 is modified to:

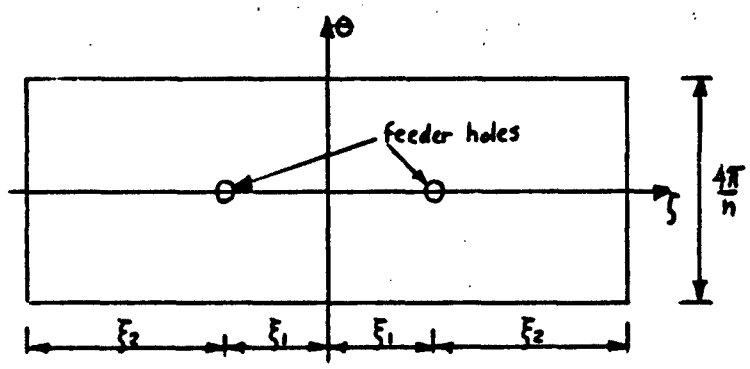

Figure 60: Axial Strip

With a total of n feeder holes, there are 1/2n holes per admission plane. The length between admission planes is L_1, and the total bearing length is: $L = L_1 + L_2$ such that L_2 becomes the combined length outside the admission planes. The ranges of the dimensionless coordinates for the strip are:

$$-(\xi_1+\xi_2) \leq \zeta \leq (\xi_1+\xi_2)$$

$$-\frac{2\pi}{n} \leq \theta \leq \frac{2\pi}{n} \tag{C-30}$$

where:

$$\xi_1 = \frac{L_1}{D} \tag{C-31}$$

The solution for the pressure distribution is found from the method of sources and sinks to be:

$$P_o = \bar{C} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[\frac{\left[\cosh\left(\frac{\eta}{2}(\xi_2+2k(\xi_1+\xi_2))\right)-1\right]\left[\cosh\left(\frac{\eta}{2}(\xi_2+2\xi_1+2k(\xi_1+\xi_2))\right)-1\right]}{\left[\cosh\left(\frac{\eta}{2}(\zeta-\xi_1+2k(\xi_1+\xi_2))\right)-\cos\left(\frac{\eta}{2}\theta\right)\right]\left[\cosh\left(\frac{\eta}{2}(\zeta+\xi_1+2k(\xi_1+\xi_2))\right)-\cos\left(\frac{\eta}{2}\theta\right)\right]} \right] \tag{C-32}$$

Defining the pressure, P'_{oc} downstream of the feeder hole to be the pressure at: $\left(\zeta=\xi_1, \theta=\frac{d}{D}\right)$, eq. (C-27) can be employed with:

$$\lambda = \frac{1}{n\xi_2} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[\frac{\left[\cosh\left(\frac{\eta}{2}(\xi_2+2k(\xi_1+\xi_2))\right)-1\right]\left[\cosh\left(\frac{\eta}{2}(\xi_2+2\xi_1+2k(\xi_1+\xi_2))\right)-1\right]}{\left[\cosh\left(nk(\xi_1+\xi_2)\right)-\cos\left(\frac{nd}{2D}\right)\right]\left[\cosh\left(n(\xi_1+k(\xi_1+\xi_2))\right)-\cos\left(\frac{nd}{2D}\right)\right]} \right] \tag{C-33}$$

<u>Double Plane Admission</u>

In practice, $\frac{nd}{2D} \ll 1$ whereby an approximate expression for λ becomes:

$$\lambda \cong \frac{1}{n\xi_2} \log \left[\frac{\left[\cosh\left(\frac{\eta}{2}\xi_2\right)-1\right]\left[\cos\left(\frac{\eta}{2}(\xi_2+2\xi_1)\right)-1\right]}{\left[1-\cos\left(\frac{nd}{2D}\right)\right]\left[\cosh(n\xi_1)-\cos\left(\frac{nd}{2D}\right)\right]} \right] \cong 1+\frac{2}{n\xi_2}\log\left(\frac{2D}{nd}\right) \tag{C-34}$$

The perturbation of the film pressure defined by eq. (C-15) causes a similar perturbation in the flow. From eq. (C-13):

$$m = m_o + \varepsilon\left[\left(\frac{\partial m}{\partial P'_c}\right)_o P'_{kc} + \left(\frac{\partial m}{\partial h}\right)_o \cos\theta\right] + \dot{\varepsilon}\left(\frac{\partial m}{\partial P'_c}\right) P'_{\dot{c}} \tag{C-35}$$

$$m_o = \begin{cases} \sqrt{1-P'_{oc}} & \text{Orifice Restrictor} \\ 1-P'_{oc} & \text{Laminar Restrictor} \end{cases} \tag{C-36}$$

$$\left(\frac{\partial m}{\partial P_c'}\right)_0 = \begin{cases} -\dfrac{1}{2m_0} & \underline{\text{Orifice Restrictor}} \\ -1 & \underline{\text{Laminar Restrictor}} \end{cases} \tag{C-37}$$

$$\left(\frac{\partial m}{\partial h}\right)_0 = \begin{cases} \dfrac{\delta^2}{1+\delta^2}\, m_0 & \underline{\text{Orifice Restrictor}} \\ 0 & \underline{\text{Laminar Restrictor}} \end{cases} \tag{C-38}$$

Define:

$$\psi = -\frac{\Lambda_s}{\xi_z}\left(\frac{\partial m}{\partial P_c'}\right)_0 \tag{C-39}$$

whereby eq. (C-35) can be written:

$$m = m_0 + \varepsilon\left[\left(\frac{\partial m}{\partial h}\right)_0 \cos\theta - \frac{\xi_z}{\Lambda_s}\psi P_{1c}'\right] - \dot{\varepsilon}\,\frac{\xi_z}{\Lambda_s}\psi P_{2c}' \tag{C-40}$$

In solving eqs. (C-18) to (C-20), only for the first equation is an exact solution readily obtained as shown by eqs. (C-24) and (C-32). To solve the two remaining equations it is necessary to perturb the source strength. In the present analysis this will be done by an approximate method.

Assume that there are infinitely many feeder holes. Thereby, the feeder holes form a continuous line feed from which the flow is purely axial. The mass flow per inch of circumference becomes:

$$\frac{\rho \bar{h}^3}{12\mu}\left[\frac{\partial \bar{P}}{\partial z} - \frac{\partial \bar{P}}{\partial z_2}\right]_{z=\frac{1}{2}L_1} = \frac{\frac{1}{2} n M_B}{2\pi R} \tag{C-41}$$

where $z_2 = z - 1/2\ L_1$. In dimensionless form:

$$\left[\frac{\partial P}{\partial \zeta} - \frac{\partial P}{\partial \zeta_2}\right]_{\zeta=\xi_1} = \frac{3\mu n M_B}{\pi \rho P_s C^3}\frac{1}{h^3} = \frac{\Lambda_s}{\lambda \xi_2}\frac{m}{h^3} \tag{C-42}$$

where:

$$\zeta_2 = \frac{z_2}{R} = \zeta - \xi_1 \tag{C-43}$$

Substitute from eqs. (C-15), (C-17) and (C-40) into eq. (C-42) to get:

$$\left[\frac{\partial P_0}{\partial \zeta} - \frac{\partial P_0}{\partial \zeta_2}\right]_{\zeta=\xi_1} = q \tag{C-44}$$

$$\left[\frac{\partial P_1}{\partial \zeta} - \frac{\partial P_1}{\partial \zeta_2}\right]_{\zeta=\xi_1} = -3q\,\frac{1+\frac{3}{2}\delta^2}{1+\delta^2}\cos\theta - \psi\frac{1}{\lambda}P_{1c}' \tag{C-45}$$

$$\left[\frac{\partial P_2}{\partial \zeta} - \frac{\partial P_2}{\partial \zeta_2}\right]_{\zeta=\xi_1} = -\psi \frac{1}{\lambda} P_{2c}' \tag{C-46}$$

where:

$$q = \frac{\Lambda_s m_o}{\lambda \xi_2} = \frac{3\mu n M_{Bo}}{\pi \rho C^3 P_s} \tag{C-47}$$

and δ is defined to be zero for the laminar restrictor. Making use of eqs. (C-47) and (C-37), eq. (C-39) becomes:

$$\psi = \begin{cases} \dfrac{\Lambda_s^2}{2q\lambda\xi_2} & \underline{\text{Orifice Restrictor}} \\ \dfrac{\Lambda_s}{\xi_2} & \underline{\text{Laminar Restrictor}} \end{cases} \tag{C-48}$$

With the assumption of a line feed instead of discrete feeder holes, there is no variation circumferentially in the pressure when the journal is concentric in the bearing. Therefore:

$$\frac{\partial P_o}{\partial \theta} = 0 \tag{C-49}$$

Hence, the solution for P_o can be found directly from eqs. (C-18) and (C-44) as:

$$P_o = \begin{cases} q(\xi_2 - \zeta_2) & \underline{0 \le \zeta_2 \le \xi_2 \quad (\text{or}: \xi_1 \le \zeta \le (\xi_1 + \xi_2))} \\ q\xi_2 & \underline{0 \le \zeta \le \xi_1} \end{cases} \tag{C-50}$$

The pressure at the line feed is:

$$P_{oc} = q\xi_2 = \frac{\Lambda_s m_o}{\lambda} \tag{C-51}$$

Comparing eqs. (C-51) and (C-27):

$$P'_{oc} = \Lambda_s m_o = \lambda P_{oc} \tag{C-52}$$

which gives the actual pressure downstream of the feeder holes in terms of the approximate line feed pressure. Inserting eq.(C-36) into eq. (C-52), the feeder hole downstream pressure is found to be:

$$P'_{oc} = \begin{cases} \frac{1}{2}\Lambda_s\left[-\Lambda_s + \sqrt{\Lambda_s^2 + 4}\right] & \text{Orifice Restrictor} \\ \\ \dfrac{\Lambda_s}{1+\Lambda_s} & \text{Laminar Restrictor} \end{cases} \tag{C-53}$$

Then:

$$q = \frac{1}{\lambda\xi_2} P'_{oc} \tag{C-54}$$

Thus, the exact solution for P_o, eq. (C-24) or eq. (C-32) can be replaced by the approximate solution of eq. (C-50). The total flow is the same in the two cases which means that only in the immediate neighborhood of the feeder holes is there any significant difference between the two solutions. This localized effect can be ignored in computing the load carrying capacity of the bearing.

Turning to the solution of the perturbed pressures, it is again assumed that the feeder holes can be represented by a line feed but with a correction introduced for the pressure downstream of the feeder holes to insure a correct flow. Actually, this correction factor must be determined from a perturbation of the source strength in eqs. (C-24) and (C-32). However, in the present analysis it shall be assumed that the same correction factor as derived for P_o, namely λ, also applies to the perturbed pressure. In other words, it is assumed that:

$$P'_{1c} = \lambda P_{1c}$$
$$P'_{2c} = \lambda P_{2c} \tag{C-55}$$

The solutions are taken in the form:

$$P_1 = H_1 \cos\theta$$
$$P_2 = \delta H_2 \cos\theta \tag{C-56}$$

where H_1 and H_2 are functions of ζ only. Thereby eqs. (C-19), (C-20), (C-45) and (C-46) become:

$$\frac{d^2 H_1}{d\zeta^2} - H_1 = 0 \tag{C-57}$$

$$\frac{d^2 H_2}{d\zeta^2} - H_2 = 1 \tag{C-58}$$

$$\left[\frac{dH_1}{d\zeta} - \frac{dH_1}{d\zeta_2}\right]_{\zeta=\zeta_1} = -3q\,\frac{1+\frac{2}{3}\delta^2}{1+\delta^2} - \psi H_{1c} \tag{C-59}$$

$$\left[\frac{dH_1}{d\zeta} - \frac{dH_2}{d\zeta_2} \right]_{\zeta=\xi_1} = -\psi H_{2c} \tag{C-60}$$

where eqs. (C-59) and (C-60) serve as boundary conditions to eqs. (C-57) and (C-58). The other boundary conditions are:

$$\underline{\text{at } \zeta=0:} \qquad \frac{dH_1}{d\zeta} = \frac{dH_2}{d\zeta} = 0 \tag{C-61}$$

$$\underline{\text{at } \zeta_2=\xi_2 \ (\zeta=\xi_1+\xi_2):} \qquad H_1 = H_2 = 0 \tag{C-62}$$

The solutions are obtained directly as:

$$H_1 = \begin{cases} H_{1c} \dfrac{\cosh\zeta}{\cosh\xi_1} & \underline{0 \le \zeta \le \xi_1} \\ \\ H_{1c} \dfrac{\sinh(\xi_2-\zeta_2)}{\sinh\xi_2} & \underline{0 \le \zeta_2 \le \xi_2} \end{cases} \tag{C-63}$$

$$H_2 = \begin{cases} (H_{2c}+1) \dfrac{\cosh\zeta}{\cosh\xi_1} - 1 & \underline{0 \le \zeta \le \xi_1} \\ \\ (H_{2c}+1) \dfrac{\sinh(\xi_2-\zeta_2)}{\sinh\xi_2} + \dfrac{\sinh\zeta_2}{\sinh\xi_2} - 1 & \underline{0 \le \zeta_2 \le \xi_2} \end{cases} \tag{C-64}$$

where:

$$H_{1c} = \frac{-3q\,\frac{1+\xi\delta^2}{1+\delta^2}\sinh\xi_2}{\cosh\xi_2 + [\psi + \tanh\xi_1]\sinh\xi_2} \tag{C-65}$$

$$H_{2c} + 1 = \frac{\psi\sinh\xi_2 + 1}{\cosh\xi_2 + [\psi + \tanh\xi_1]\sinh\xi_2} \tag{C-66}$$

The force F exerted on the journal by the film is found by integrating the pressure $\bar{P}$ in the film:

$$F = -\int_{-\frac{L}{2}}^{\frac{L}{2}} \int_0^{2\pi} \bar{P}\cos\theta\, R\, d\theta\, dz \tag{C-67}$$

Making use of eqs. (C-3), (C-5), (C-22), (C-31) and (C-15), eq. (C-67) becomes:

$$F = -2R^2 P_s \int_0^{(\xi_1+\xi_2)} \int_0^{2\pi} (P_0 + \varepsilon P_1 + \dot{\varepsilon} P_2)\cos\theta\, d\theta\, d\zeta \tag{C-68}$$

With P_o given by eq. (C-50), it is seen that P_o, of course, gives no contribution to the force, i.e. $F_o = 0$. Substituting for P_1 and P_2 from eq. (C-56) yields:

$$F = -2\pi R^2 P_s \int_0^{(\xi_1+\xi_2)} (\varepsilon H_1 + \dot{\varepsilon} H_2)\, d\zeta \tag{C-69}$$

Since the solution has been linearized, the force can be expressed in terms of a spring coefficient K and a damping coefficient B:

$$F = Ke + B\frac{de}{dt} = CK\varepsilon + C\nu B\dot{\varepsilon} \tag{C-70}$$

where e is the displacement (the eccentricity) of the journal center, see eqs. (C-14) and (C-16). Comparing eqs. (C-69) and (C-70):

$$CK = -2\pi R^2 P_s \int_0^{\xi_1+\xi_2} H_1 \, d\zeta \tag{C-71}$$

$$C\nu B = -2\pi R^2 P_s \sigma \int_0^{\xi_1+\xi_2} H_2 \, d\zeta \tag{C-72}$$

In dimensionless form:

$$\frac{CK}{(L_1+L_2)DP_s} = -\frac{\pi/2}{\xi_1+\xi_2}\int_0^{\xi_1+\xi_2} H_1 \, d\zeta \tag{C-73}$$

$$\frac{CB}{\mu(L_1+L_2)D\left(\frac{R}{C}\right)^2} = -\frac{6\pi}{\xi_1+\xi_2}\int_0^{\xi_1+\xi_2} H_2 \, d\zeta \tag{C-74}$$

With substitution from eqs. (C-63) and (C-65), eq. (C-73) yields:

$$\frac{1+\delta^2}{1+\frac{2}{3}\delta^2}\frac{CK}{(L_1+L_2)DP_s}\lambda = \frac{3\pi}{2(\xi_1+\xi_2)}\frac{q\lambda}{\cosh\xi_2+[\psi+\tanh\xi_1]\sinh\xi_2}\left[\cosh\xi_2-1+\sinh\xi_2\tanh\xi_1\right] \quad \text{(C-75)}$$

In this form, the dimensionless stiffness is a function of three parameters only: Λ_s, ξ_1 and ξ_2. The factor: $q\lambda\xi_2$ depends on Λ_s only (see eqs. (C-53) and (C-54) and the parameter $\psi\xi_2$, therefore, is also a function of Λ_s only (see eq. (C-48)).

With substitution from eqs. (C-64) and (C-66), eq. (C-74) yields:

$$\frac{CB}{\mu(L_1+L_2)D\left(\frac{R}{C}\right)^2} = 6\pi\left[1-\frac{1}{\xi_1+\xi_2}\frac{\sinh\xi_2+\cosh\xi_2\tanh\xi_1+\psi(2\cosh\xi_2-2+\sinh\xi_2\tanh\xi_1)}{\cosh\xi_2+(\psi+\tanh\xi_1)\sinh\xi_2}\right] \quad \text{(C-76)}$$

The dimensionless damping is a function of three parameters only: Λ_s, ξ_1 and ξ_2.

The total volume flow to the bearing, Q inch^3/sec, can be defined in dimensionless form from eq. (C-47) as:

$$\frac{3\mu Q}{\pi P_s C^3}\lambda\xi_2 = q\lambda\xi_2 = P'_{oc} \quad \text{(C-77)}$$

which depends only on Λ_s and is independent of ξ_1 and ξ_2.

Eqs. (C-75) to (C-77) are used to calculate the design charts for the stiffness, the damping and the flow for a hydrostatic journal bearing with an incompressible lubricant.

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APPENDIX IV: The Stability of the Hydrodynamic - Hydrostatic Ring Bearing with an Incompressible Lubricant

In the hydrodynamic-hydrostatic ring bearing there is a ring between the journal and the bearing. The ring is prevented from rotating but is otherwise free to move. The inner diameter of the ring is D and the outer diameter is D_o. The radial clearance of the inner film is C and that of the outer film is C_o. The bearing length is L.

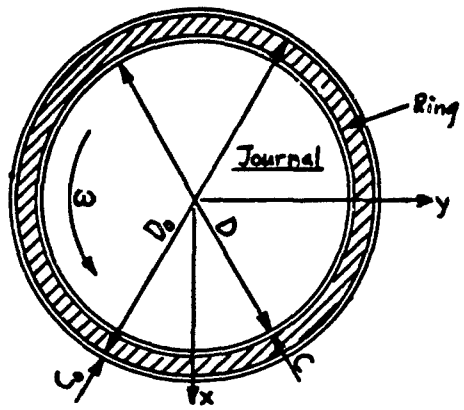

Figure 61: Hydrodynamic-Hydrostatic Ring Bearing

As shown in Appendix I, the dynamic forces of the inner film can be expressed in terms of 8 coefficients: K_{xx}, K_{xy}, K_{yx}, K_{yy}, B_{xx}, B_{xy}, B_{yx} and B_{yy}. The outer film, on the other hand, requires only two coefficients, a spring coefficient K_o and a damping coefficient B_o, as given in Appendix III.

In calculating the stability properties of this bearing, the rotor is assumed to be rigid and symmetrical with a mass of 2M. As discussed in connection with eq. (B-20) in Appendix II, the rotor mass can be lumped at the journals whereby each journal is assigned a mass of M.

Then the equations of motion become:

$$M\frac{d^2x}{dt^2} = -K_{xx}(x-x_o) - B_{xx}\frac{d(x-x_o)}{dt} - K_{xy}(y-y_o) - B_{xy}\frac{d(y-y_o)}{dt} = -K_o x_o - B_o\frac{dx_o}{dt}$$

$$M\frac{d^2y}{dt^2} = -K_{yx}(x-x_o) - B_{yx}\frac{d(x-x_o)}{dt} - K_{yy}(y-y_o) - B_{yy}\frac{d(y-y_o)}{dt} = -K_o y_o - B_o\frac{dy_o}{dt} \quad \text{(D-1)}$$

where x and y are the amplitudes of the journal center, and x_o and y_o are the amplitudes of the center of the ring. In these equations, the mass of the ring itself has been ignored because it is small compared to the journal mass. Eq. (D-1) is made dimensionless by setting:

$$\bar{x} = \frac{x}{C} \qquad \bar{y} = \frac{y}{C} \tag{D-2}$$

$$\bar{x}_o = \frac{x_o}{C} \qquad \bar{y}_o = \frac{y_o}{C} \tag{D-3}$$

$$\tau = \omega t \tag{D-4}$$

$$\bar{K}_o = \frac{CK_o}{\mu NDL\left(\frac{R}{C}\right)^2} = \left(\frac{C}{C_o}\right)\frac{P_s L D_o}{SW}\,\frac{C_o K_o}{P_s L D_o} \tag{D-5}$$

$$\bar{B}_o = \frac{C\omega B_o}{\mu NDL\left(\frac{R}{C}\right)^2} = 2\pi\left(\frac{C}{C_o}\right)^3\left(\frac{D_o}{D}\right)^3\frac{C_o B_o}{\mu L D_o\left(\frac{R_o}{C_o}\right)^2} \tag{D-6}$$

$$\bar{M} = \frac{C\omega^2 M}{SW} = 4\pi^2\,\frac{CNM}{\mu DL\left(\frac{R}{C}\right)^2} \tag{D-7}$$

$$\bar{K}_{xx} = \frac{CK_{xx}}{SW} = \frac{CK_{xx}}{\mu DL\left(\frac{R}{C}\right)^2} \tag{D-8}$$

$$\bar{B}_{xx} = \frac{C\omega B_{xx}}{SW} = \frac{C\omega B_{xx}}{\mu NDL\left(\frac{D}{2}\right)^2} \tag{D-9}$$

and similarly for $\bar{K}_{xy}$, $\bar{K}_{yx}$, $\bar{K}_{yy}$, $\bar{B}_{xy}$, $\bar{B}_{yx}$ and $\bar{B}_{yy}$. Here, ω is the angular speed of the journal and S is the Sommerfeld number. Thereby eq. (D-1) becomes:

$$\bar{M}\ddot{\bar{x}} = -\bar{K}_{xx}(\bar{x}-\bar{x}_o) - \bar{B}_{xx}(\dot{\bar{x}}-\dot{\bar{x}}_o) - \bar{K}_{xy}(\bar{y}-\bar{y}_o) - \bar{B}_{xy}(\dot{\bar{y}}-\dot{\bar{y}}_o) = -\bar{K}_o\bar{x}_o - \bar{B}_o\dot{\bar{x}}_o$$

$$\bar{M}\ddot{\bar{y}} = -\bar{K}_{yx}(\bar{x}-\bar{x}_o) - \bar{B}_{yx}(\dot{\bar{x}}-\dot{\bar{x}}_o) - \bar{K}_{yy}(\bar{y}-\bar{y}_o) - \bar{B}_{yy}(\dot{\bar{y}}-\dot{\bar{y}}_o) = -\bar{K}_o\bar{y}_o - \bar{B}_o\dot{\bar{y}}_o \tag{D-10}$$

where "dot" refers to $\frac{d}{d\tau}$.

At the threshold of instability, the motion is purely harmonic with frequency ν such that:

$$\bar{x} = \bar{x}_c \cos(\nu t) - \bar{x}_s \sin(\nu t) = \mathcal{R}e\left\{(\bar{x}_c + i\bar{x}_s)e^{i\nu\tau}\right\} \tag{D-11}$$

and similarly for $\bar{y}$, $\bar{x}_o$ and $\bar{y}_o$, where:

$$\gamma = \frac{\nu}{\omega} \tag{D-12}$$

Then eq. (D-10) can be written as:

$$\bar{M}\gamma^2\bar{x} = (\bar{K}_{xx} + i\gamma\bar{B}_{xx})(\bar{x}-\bar{x}_o) + (\bar{K}_{xy} + i\gamma\bar{B}_{xy})(\bar{y}-\bar{y}_o) = (\bar{K}_o + i\gamma\bar{B}_o)\bar{x}_o$$

$$\bar{M}\gamma^2\bar{y} = (\bar{K}_{yx} + i\gamma\bar{B}_{yx})(\bar{x}-\bar{x}_o) + (\bar{K}_{yy} + i\gamma\bar{B}_{yy})(\bar{y}-\bar{y}_o) = (\bar{K}_o + i\gamma\bar{B}_o)\bar{y}_o \tag{D-13}$$

Solve for $\bar{x}_0$ and $\bar{y}_0$:

$$\bar{x}_0 = \frac{\bar{M}\gamma^2}{\bar{K}_0 + i\gamma\bar{B}_0}\,\bar{x}$$

$$\bar{y}_0 = \frac{\bar{M}\gamma^2}{\bar{K}_0 + i\gamma\bar{B}_0}\,\bar{y} \qquad \text{(D-14)}$$

and substitute into eq. (D-13) to get:

$$\begin{Bmatrix} \left(\bar{K}_{xx} - \frac{(\bar{K}_0 + i\gamma\bar{B}_0)\bar{M}\gamma^2}{\bar{K}_0 - \bar{M}\gamma^2 + i\gamma\bar{B}_0} + i\gamma\bar{B}_{xx}\right) & \left(\bar{K}_{xy} + i\gamma\bar{B}_{xy}\right) \\ \left(\bar{K}_{yx} + i\gamma\bar{B}_{yx}\right) & \left(\bar{K}_{yy} - \frac{(\bar{K}_0 + i\gamma\bar{B}_0)\bar{M}\gamma^2}{\bar{K}_0 - \bar{M}\gamma^2 + i\gamma\bar{B}_0} + i\gamma\bar{B}_{yy}\right) \end{Bmatrix} \begin{Bmatrix} \bar{x} \\ \bar{y} \end{Bmatrix} = 0 \qquad \text{(D-15)}$$

At the threshold of instability, the determinant of the coefficient matrix vanishes. For convenience, introduce the abbreviations:

$$E = \frac{(\bar{K}_0 + i\gamma\bar{B}_0)\bar{M}\gamma^2}{\bar{K}_0 - \bar{M}\gamma^2 + i\gamma\bar{B}_0} \qquad \text{(D-16)}$$

$$Z_{xx} = \bar{K}_{xx} + i\gamma\bar{B}_{xx} \qquad (\text{similarly for } Z_{xy}, Z_{yx}, Z_{yy}) \qquad \text{(D-17)}$$

With this notation, the determinant of eq. (D-15) can be equated to zero to give:

$$E^2 - (Z_{xx} + Z_{yy})E + (Z_{xx}Z_{yy} - Z_{xy}Z_{yx}) = 0 \qquad \text{(D-18)}$$

with the solution:

$$E = \tfrac{1}{2}(Z_{xx}+Z_{yy}) \pm \sqrt{\tfrac{1}{4}(Z_{xx}-Z_{yy})^2 + Z_{xy}Z_{yx}} \tag{D-19}$$

where only the root with the minus sign in front of the square root is of interest when studying stability. Set:

$$G = \mathcal{R}e\left\{\tfrac{1}{4}(Z_{xx}-Z_{yy})^2 + Z_{xy}Z_{yx}\right\} = \tfrac{1}{4}(\bar{K}_{xx}-\bar{K}_{yy})^2 + \bar{K}_{xy}\bar{K}_{yx} - \tfrac{1}{4}\gamma^2(\bar{B}_{xx}-\bar{B}_{yy}) - \gamma^2\bar{B}_{xy}\bar{B}_{yx} \tag{D-20}$$

$$H = \tfrac{1}{2}\mathcal{I}m\left\{\tfrac{1}{4}(Z_{xx}-Z_{yy})^2 + Z_{xy}Z_{yx}\right\} = \gamma\left[\tfrac{1}{4}(\bar{K}_{xx}-\bar{K}_{yy})(\bar{B}_{xx}-\bar{B}_{yy}) + \tfrac{1}{2}(\bar{K}_{xy}\bar{B}_{yx}+\bar{K}_{yx}\bar{B}_{xy})\right] \tag{D-21}$$

whereby eq. (D-19) becomes:

$$E = \tfrac{1}{2}(Z_{xx}+Z_{yy}) - \left\{\sqrt{\tfrac{1}{2}(G+\sqrt{G^2+4H^2})} + i\sqrt{\tfrac{1}{2}(-G+\sqrt{G^2+4H^2})}\right\} \tag{D-22}$$

Set:

$$E = \bar{K}_B + i\gamma\bar{B}_B \tag{D-23}$$

where:

$$\bar{K}_B = \tfrac{1}{2}(\bar{K}_{xx}+\bar{K}_{yy}) - \sqrt{\tfrac{1}{2}(G+\sqrt{G^2+4H^2})} \tag{D-24}$$

$$\gamma\bar{B}_B = \tfrac{1}{2}\gamma(\bar{B}_{xx}+\bar{B}_{yy}) + \sqrt{\tfrac{1}{2}(-G+\sqrt{G^2+4H^2})} \tag{D-25}$$

Combining eqs. (D-16) and (D-23) yields:

$$\bar{K}_B = \bar{M}\gamma^2 \left[1 + \bar{M}\gamma^2 \frac{\bar{K}_0 - \bar{M}\gamma^2}{(\bar{K}_0 - \bar{M}\gamma^2)^2 + (\gamma \bar{B}_0)^2} \right] \tag{D-26}$$

$$\gamma \bar{B}_B = -(\bar{M}\gamma^2)^2 \frac{\gamma \bar{B}_0}{(\bar{K}_0 - \bar{M}\gamma^2)^2 + (\gamma \bar{B}_0)^2} \tag{D-27}$$

Eliminate $\bar{M}\gamma^2 / [(\bar{K}_0 - \bar{M}\gamma^2)^2 + (\gamma \bar{B}_0)^2]$ between the two equations to get:

$$\bar{M}\gamma^2 = \frac{\bar{K}_B \gamma \bar{B}_0 + \bar{K}_0 \gamma \bar{B}_B}{\gamma \bar{B}_0 + \gamma \bar{B}_B} \tag{D-28}$$

Substitute for $M\gamma^2$ into eq. (D-27) to obtain an equation which contains γ as the only unknown:

$$\gamma \bar{B}_0 \left[\bar{K}_B^2 + (\gamma \bar{B}_B)^2 \right] + \gamma \bar{B}_B \left[\bar{K}_0^2 + (\gamma \bar{B}_0^2) \right] = 0 \tag{D-29}$$

Since $\bar{K}_B$ and $\gamma \bar{B}_B$, as seen from eqs. (D-24) and (D-25), are rather complicated functions of γ, eq. (D-29) is a higher order polynominal in γ which cannot be solved in closed form. Instead, the solution is found numerically. Once the roots for γ have been obtained, the corresponding dimensionless journal mass $\bar{M}$ is found from eq. (D-28).

If $\bar{M}$ is plotted as a function of $\bar{B}_0$ for a fixed value of $\bar{K}_0$, it is found that under certain circumstances the rotor is inherently stable, i.e. $\bar{M} \to \infty$. When $\bar{M} \to \infty, \gamma \to 0$ such that eqs. (D-24) and (D-25) yield:

$$\begin{aligned} \bar{K}_B &\to \tfrac{1}{2}(\bar{K}_{xx} + \bar{K}_{yy}) \\ \gamma \bar{B}_B &\to -\sqrt{-\left[\tfrac{1}{4}(\bar{K}_{xx} - \bar{K}_{yy})^2 + \bar{K}_{xy}\bar{K}_{yx}\right]} \end{aligned} \tag{D-30}$$

where $\left(\frac{1}{4}(\bar{K}_{xx}-\bar{K}_{yy})^2+\bar{K}_{xy}\bar{K}_{yx}\right)<0$ which holds true except at large eccentricity ratios. Solve eq. (D-29) for $\gamma\bar{B}_o/\gamma\bar{B}_B$:

$$\frac{\gamma\bar{B}_o}{\gamma\bar{B}_B} = -\frac{1}{2}\,\frac{\bar{K}_B^2+\gamma\bar{B}_B^2}{\gamma\bar{B}_B^2} \pm \sqrt{\frac{1}{4}\left(\frac{\bar{K}_B^2+\gamma\bar{B}_B^2}{\gamma\bar{B}_B^2}\right)^2 - \frac{\bar{K}_o^2}{(\gamma B_B)^2}} \qquad \text{(D-31)}$$

For a given value of $\bar{K}_o$ and with $\bar{K}_B$ and $\gamma\bar{B}_B$ defined by eq. (D-30), this equation allows calculation $\gamma\bar{B}_o/\gamma\bar{B}_B$ for $\gamma=0$.
Noting that:

$$\gamma^2 = \left(\frac{\gamma\bar{B}_o}{\gamma B_B}\right)^2 \frac{(\gamma B_B)^2}{\bar{B}_o^2}$$

eq. (D-28) yields:

$$\bar{M} = \frac{1}{\gamma^2}\,\frac{\bar{K}_B\left(\frac{\gamma B_o}{\gamma B_B}\right)+\bar{K}_o}{1+\left(\frac{\gamma\bar{B}_o}{\gamma\bar{B}_B}\right)} = \frac{\bar{K}_o+\frac{\gamma\bar{B}_o}{\gamma\bar{B}_B}\bar{K}_B}{(\gamma\bar{B}_B)^2\left(\frac{\gamma\bar{B}_o}{\gamma\bar{B}_B}\right)^2\left[1+\frac{\gamma\bar{B}_o}{\gamma\bar{B}_B}\right]}\,\bar{B}_o^2 \qquad \text{(D-32)}$$

Corresponding to the two roots for $\gamma\bar{B}_o/\gamma\bar{B}_B$ from eq. (D-31), eq. (D-32) yields two possible values for $\bar{M}$. The rotor is stable between these two values.

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APPENDIX V: THE STIFFNESS AND THE DAMPING OF A HYBRID JOURNAL BEARING WITH A COMPRESSIBLE LUBRICANT

In the hybrid-hydrostatic ring bearing which is gas lubricated, the outer bearing is purely hydrostatic whereas the inner bearing, supplied with pressurized gas from the outer bearing, is a hybrid bearing. This appendix describes the analysis for calculating the load carrying capacity, the flow, the stiffness and the damping of the hybrid journal bearing but the analysis applies equally well to the hydrostatic bearing.

For a compressible lubricant under isothermal conditions, Reynolds equation becomes:

$$\frac{\partial}{R\partial\theta}\left[\frac{\bar{h}^3}{12\mu}\bar{P}\frac{\partial\bar{P}}{R\partial\theta}\right]+\frac{\partial}{\partial z}\left[\frac{\bar{h}^3}{12\mu}\bar{P}\frac{\partial\bar{P}}{\partial z}\right]=\frac{1}{2}R\omega\frac{\partial(\bar{P}\bar{h})}{\partial\theta}+\frac{\partial(\bar{P}\bar{h})}{dt} \qquad \text{(E-1)}$$

where the symbols are defined in connection with eq. (A-1), Appendix I. The equation is made dimensionless by setting:

$$\zeta=\frac{z}{R} \qquad \text{(E-2)}$$

$$\tau=\nu t \qquad \text{(E-3)}$$

$$h=\frac{\bar{h}}{C}=1+\varepsilon\cos\theta \qquad \text{(E-4)}$$

$$\varepsilon=\frac{e}{C} \qquad \text{(E-5)}$$

$$P=\frac{\bar{P}}{P_a} \qquad \text{(E-6)}$$

whereby eq. (E-1) can be written in dimensionless form as:

$$\frac{\partial}{\partial\theta}\left[h\frac{\partial(Ph)^2}{\partial\theta}-2\frac{\partial h}{\partial\theta}(Ph)^2\right]+\frac{\partial}{\partial\zeta}\left[h\frac{\partial(Ph)^2}{\partial\zeta}\right]=2\Lambda\frac{\partial(Ph)}{\partial\theta}+4\gamma\Lambda\frac{\partial(Ph)}{\partial\tau} \quad \text{(E-7)}$$

where:

Compressibility Number: $$\Lambda=\frac{6\mu\omega}{P_a}\left(\frac{R}{C}\right)^2 \quad \text{(E-8)}$$

Frequency Ratio: $$\gamma=\frac{\nu}{\omega} \quad \text{(E-9)}$$

In the purely hydrostatic bearing, $\omega = 0$ which means that $\Lambda = 0$. In that case:

Squeeze Number: $$\sigma=2\gamma\Lambda=\frac{12\mu\nu}{P_a}\left(\frac{R}{C}\right)^2 \quad \text{(E-10)}$$

Eq. (E-7) is non-linear. It shall be solved under the assumption that the eccentricity ratio ε is small whereby the pressure variable can be expanded in a perturbation:

$$Ph=P_0+\varepsilon P_1 \quad \text{(E-11)}$$

In making use of eq. (E-11) it shall be assumed that the pressure, P_o, which is the film pressure when the journal is concentric in the bearing, does not depend

on θ $(i.e.\ \partial P_0/\partial\theta = 0)$. This assumption implies that the flow to the bearing is considered to be supplied by a line feed rather than discrete feeding holes. A correction factor will be added later to adjust the flow.

With this assumption, eqs. (E-11) and (E-4) can be substituted into eq. (E-7) whereby two equations are obtained. The first equation is simply:

$$\frac{\partial^2 P_0^2}{\partial \zeta^2} = 0 \tag{E-12}$$

In deriving the second equation it should be noted that eq. (E-4) is based on θ being measured from the line connecting the bearing center with the journal center. The angle between this line and the static load line is the attitude angle φ . Therefore:

$$\frac{\partial(Ph)}{\partial\tau} = -\frac{\partial(Ph)}{\partial\theta}\frac{d\varphi}{d\tau} + \left.\frac{\partial(Ph)}{\partial\tau}\right]_{\theta\ \text{constant}} \tag{E-13}$$

Thereby the second perturbation equation becomes:

$$\frac{\partial^2(\varepsilon P_0 P_1)}{\partial\theta^2} + \frac{\partial^2(\varepsilon P_0 P_1)}{\partial\zeta^2} - \frac{\Lambda}{P_0}(1-2\gamma\dot{\varphi})\frac{\partial(\varepsilon P_0 P_1)}{\partial\theta} - 2\gamma\frac{\Lambda}{P_0}\frac{\partial(\varepsilon P_0 P_1)}{\partial\tau} = -\varepsilon\cos\theta\, P_0^2 \tag{E-14}$$

Under static conditions, the journal center operates at an eccentricity ratio ε_0 and an attitude angle φ_0 . Letting the journal center perform a harmonic small amplitude motion ($\varepsilon_1 e^{i\nu t}$, $\varphi_1 e^{i\nu t}$) with frequency ν around this equilibrium position, the journal center coordinates become:

$$\varepsilon = \varepsilon_0 + \varepsilon_1 e^{i\tau}$$

$$\varphi = \varphi_0 + \varphi_1 e^{i\tau} \tag{E-15}$$

The corresponding changes in the pressure variable can be expressed by:

$$\varepsilon P_0 P_1 = \varepsilon_0 q_0 + \varepsilon_1 e^{i\tau} q_1 + \varepsilon_0 \varphi_1 e^{i\tau} q_2 \tag{E-16}$$

Set:

$$q_0 = \mathcal{R}e\left\{ G_0(\zeta)\, e^{j\theta} \right\} \tag{E-17}$$

$$q_1 = \mathcal{R}e\left\{ e^{i\tau}\, \mathcal{R}e\left\{ G_1(\zeta) e^{j\theta} \right\} \right\} \tag{E-18}$$

$$q_2 = \mathcal{R}e\left\{ e^{i\tau}\, \mathcal{R}e\left\{ G_2(\zeta) e^{j\theta} \right\} \right\} \tag{E-19}$$

where: $i = \sqrt{-1}$ and $j = \sqrt{-1}$ With these definitions, eq. (E-14) gives rise to 3 equations:

$$\frac{d^2 G_0}{d\zeta^2} - \left(1 + j\frac{\Lambda}{P_0}\right) G_0 = -P_0^2 \tag{E-20}$$

$$\frac{d^2 G_1}{d\zeta^2} - \left(1 + j\frac{\Lambda}{P_0} + i2\gamma\frac{\Lambda}{P_0}\right) G_1 = -P_0^2 \tag{E-21}$$

$$\frac{d^2 G_2}{d\zeta^2} - \left(1 + j\frac{\Lambda}{P_0} + i2\gamma\frac{\Lambda}{P_0}\right) G_2 = -ij2\gamma\frac{\Lambda}{P_0} G_0 \tag{E-22}$$

The length of the bearing is L. There are two admission planes separated by the distance L_1, and the combined length outside the admission planes is $L_2 = L - L_1$. Since there is symmetry with respect to the centerplane of the bearing, the dimensionless axial coordinate ranges from $\zeta = 0$ to $\zeta = (\xi_1 + \xi_2)$

where:

$$\xi_1 = \frac{L_1}{D}$$

$$\xi_2 = \frac{L_2}{D} \tag{E-23}$$

It is convenient to introduce an additional axial coordinate:

$$\zeta_2 = \zeta - \xi_1$$

so that $0 \leq \zeta_2 \leq \xi_2$.

At the end of the bearing the pressure is ambient and equal to P_a, i.e. $P = \overline{P}/P_a = 1$ at $\zeta_2 = \xi_2$ which means:

$$\underline{\zeta_2 = \xi_2:} \quad \begin{cases} P_0 = G_0 = G_1 = 1 \\ G_2 = 0 \end{cases} \tag{E-24}$$

At the centerplane of thebearing, $\partial P / \partial \zeta = 0$ whereby:

$$\underline{\zeta = 0:} \quad \frac{\partial P_0}{\partial \zeta} = \frac{dG_0}{d\zeta} = \frac{dG_1}{d\zeta} = \frac{dG_2}{d\zeta} = 0 \tag{E-25}$$

There are $\frac{1}{2} n$ feeder holes per admission plane. The mass flow through a hole is M_B. Thus, when the admission plane is represented by a line feed, the flow per inch is $n M_B / 4\pi R$. Hence, the boundary condition at the admission plane becomes:

$$\underline{\zeta = \xi_1, \zeta_2 = 0:} \quad \frac{6\mu R T}{\pi P_a^2 C^3} n M_B = h \left[\left(\frac{\partial (Ph)^2}{\partial \zeta} \right)_{\zeta = \xi_1} - \left(\frac{\partial (Ph)^2}{\partial \zeta_2} \right)_{\zeta_2 = 0} \right] \tag{E-26}$$

where $\mathcal{R}$ is the gas constant and T is the total temperature. Expand the right hand side by means of eqs. (E-11), (E-16) and (E-4):

$$\frac{6\mu\mathcal{R}T}{\pi P_A^2 C^3} n M_B = (1+\varepsilon\cos\theta)\left(\frac{\partial}{\partial\zeta} - \frac{\partial}{\partial\zeta_2}\right)_c \left[P_0^2 + 2\varepsilon P_0 P_1\right] = \left(\frac{\partial}{\partial\zeta} - \frac{\partial}{\partial\zeta_2}\right)_c \left[P_0^2 + 2\left(\varepsilon P_0 P_1 + \tfrac{1}{2}\varepsilon P_0^2\cos\theta\right)\right]$$

$$= \left(\frac{\partial}{\partial\zeta} - \frac{\partial}{\partial\zeta_2}\right)\left[P_0^2 + 2\varepsilon_0\left(q_0 + \tfrac{1}{2}P_0^2\cos\theta\right) + 2\varepsilon_1 e^{i\tau}\left(q_1 + \tfrac{1}{2}P_0^2\cos\theta\right) + 2\varepsilon_0\varphi_1 e^{i\tau} q_2\right] \quad \text{(E-27)}$$

where subscript c refers to the condition at the admission plane.

Next, the left hand side of eq. (E-27) must be expanded. Let the volume of a feeder hole be represented by V_c and let the flow into the feeder hole be M_c. The pressure in the feeder hole is $\bar{P}_c'$. Then a flow balance for the feeder hole volume can be set up:

$$M_B = M_c - \frac{\partial(V_c \bar{\rho}_c')}{\partial t} \quad \text{(E-28)}$$

where $\bar{\rho}_c' = \bar{P}_c' / \mathcal{R}T$ is the mass density of the gas in the feeder hole. To make eq. (E-28) dimensionless, set:

$$\frac{M_c\sqrt{\mathcal{R}T}}{\pi a^2 P_s} = \frac{m}{\sqrt{1+\left(\frac{\delta}{h}\right)^2}} \quad \text{(E-29)}$$

where δ is the inherent compensation factor:

$$\delta = \frac{a^2}{dC} \quad \text{(E-30)}$$

Here, P_s is the supply pressure, a is the radius of the orifice and d is the diameter of the feeding hole. The dimensionless orifice flow m is given by the standard equation:

$$m = \begin{cases} C_D \sqrt{\frac{2k}{k+1}} \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} & \frac{\bar{P}_c'}{P_s} \le \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \\ C_D \sqrt{\frac{2k}{k-1}} \left(\frac{\bar{P}_c'}{P_s}\right)^{1/k} \sqrt{1-\left(\frac{\bar{P}_c'}{P_s}\right)^{\frac{k-1}{k}}} & \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \le \frac{\bar{P}_c'}{P_s} \le 1 \end{cases} \tag{E-31}$$

where C_D is the vena contracta coefficient. Actually, C_D is a function of the pressure ratio $\bar{P}_c'/P_s$.

The dimensionless form of eq. (E-28) becomes:

$$\frac{6\mu RT}{\pi P_a^2 C^3} n M_B = \Lambda_s V^2 \frac{\sqrt{1+\delta^2}}{\sqrt{1+\left(\frac{d}{h}\right)^2}} m - 4\gamma \Lambda \frac{n V_c}{\pi D L C} (\xi_1 + \xi_2) \frac{\partial P_c'}{\partial \tau} \tag{E-32}$$

where:

Restrictor Coefficient: $$\Lambda_s = \frac{6\mu n a^2 \sqrt{RT}}{P_s C^3 \sqrt{1+\delta^2}} \tag{E-33}$$

Pressure Ratio: $$V = \frac{P_s}{P_a} \tag{E-34}$$

$$P_c' = \frac{\bar{P}_c'}{P_a} \tag{E-35}$$

The restrictor coefficient, Λ_s, is, together with Λ, the governing performance parameter. It gives the ratio between the flow resistance of the bearing film and the flow resistance of the feeder hole restrictor.

From eqs. (E-11) and (E-16):

$$P_c = \frac{(Ph)_c}{h} = (1-\varepsilon\cos\theta)(P_{0c}+\varepsilon P_{1c}) = P_{0c} + \frac{1}{P_{0c}}\left[(\varepsilon P_0 P_1)_c - \varepsilon P_{0c}^2\cos\theta\right]$$

$$= P_{0c} + \frac{1}{P_{0c}}\left[\varepsilon_0(q_{0c} - P_{0c}^2\cos\theta) + \varepsilon_1 e^{i\tau}(q_{1c} - P_{0c}^2\cos\theta) + \varepsilon_0\varphi_1 e^{i\tau} q_{2c}\right] \qquad \text{(E-36)}$$

On this basis and making use of eq. (E-13), it is found that:

$$\frac{\partial P_c'}{\partial\tau} = \frac{i}{P_{0c}}\left[\varepsilon_1 e^{i\tau}(q_{1c}' - P_{0c}'^2\cos\theta) + \varepsilon_0\varphi_1 e^{i\tau}\left(q_{2c}' - \left(\frac{\partial q_0}{\partial\theta}\right)_c' - P_{0c}'^2\sin\theta\right)\right] \qquad \text{(E-37)}$$

Furthermore, m must also be expanded:

$$m = m_0 + \left(\frac{\partial m}{\partial\left(\frac{P_c'}{P_s}\right)}\right)_0 \frac{1}{V}(P_c' - P_{0c}') = m_0 + \left(\frac{\partial m}{\partial\left(\frac{P_c'}{P_s}\right)}\right)_0 \frac{1}{VP_{0c}'}\left[\varepsilon_0(q_{0c}' - P_{0c}'^2\cos\theta) + \varepsilon_1 e^{i\tau}(q_{1c}' - P_{0c}'^2\cos\theta) + \varepsilon_0\varphi_1 e^{i\tau} q_{2c}'\right] \qquad \text{(E-38)}$$

where:

$$m_0 = m\left(\frac{P_{0c}'}{V}\right) \qquad \text{(E-39)}$$

Introduce the abbreviations:

$$q = \Lambda_s V^2 m_0 \qquad \text{(E-40)}$$

$$\psi_0' = -\frac{\Lambda_s V}{2P_{oc}'}\left(\frac{\partial m}{\partial\left(\frac{P_f'}{P_s}\right)}\right)_0 \tag{E-41}$$

$$\psi_1' = \frac{nV_c}{\pi DLC}\,\frac{\xi_1+\xi_2}{P_{oc}'} \tag{E-42}$$

Eqs. (E-37) to (E-42) are substituted into eq. (E-32) which is then compared to eq. (E-27). Collecting terms according to ε_0, ε_1 and φ_1, four equations are obtained

$$\left(\frac{\partial P_0^2}{\partial \zeta_2}\right)_c = -q + \left(\frac{\partial P_0^2}{\partial \zeta}\right)_c \tag{E-43}$$

$$\left(\frac{\partial q_0}{\partial \zeta_2}\right)_c = \frac{q}{2(1+\delta^2)}\cos\theta + \psi_0'(q_{oc}' - P_{oc}'^2\cos\theta) + \left(\frac{\partial q_0}{\partial \zeta}\right)_c \tag{E-44}$$

$$\left(\frac{\partial q_1}{\partial \zeta_2}\right)_c = \frac{q}{2(1+\delta^2)}\cos\theta + (\psi_0' + i2\gamma\Lambda\psi_1')(q_{1c}' - P_{oc}'^2\cos\theta) + \left(\frac{\partial q_1}{\partial \zeta}\right)_c \tag{E-45}$$

$$\left(\frac{\partial q_2}{\partial \zeta_2}\right)_c = (\psi_0' + i2\gamma\Lambda\psi_1')q_{2c}' - i2\gamma\Lambda\psi_1'\left(\left(\frac{\partial q_0'}{\partial \theta}\right)_c + P_{oc}'^2\sin\theta\right) + \left(\frac{\partial q_2}{\partial \zeta}\right)_c \tag{E-46}$$

In these equations, the primed quantities refer to the actual conditions at the feeder holes whereas the unprimed quantities derive from the solution based on the line feed assumption. Thus, it is necessary to establish a relationship between the two conditions in order to apply the equations. This is done by the same method as employed in Appendix III. In other words, consider the case where the journal is concentric in the bearing ($\varepsilon = 0$). Because of symmetry, it is then only necessary to consider an axial strip:

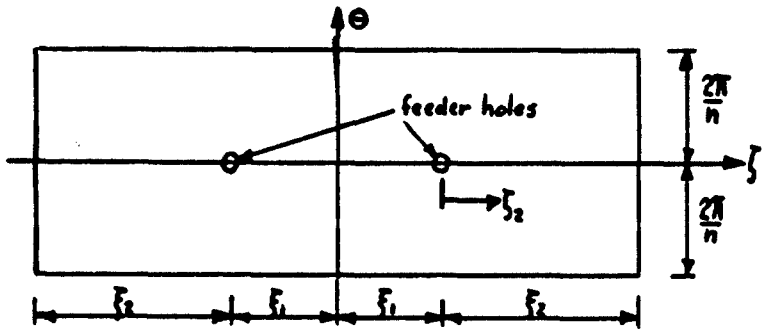


Figure 62: Axial Strip

The solution for the gas film pressure, P_o', can be obtained from the method of sources and sinks as:

$$P_o'^2 = 1 + \frac{6\mu RT}{\pi P_a^2 C^3} M_{Bo} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[\frac{\left[\cosh\left(\frac{n}{2}(\xi_2 + 2k(\xi_1+\xi_2))\right) - 1\right]\left[\cosh\left(\frac{n}{2}(\xi_2+2\xi_1+2k(\xi_1+\xi_2))\right) - 1\right]}{\left[\cosh\left(\frac{n}{2}(\xi_2+2k(\xi_1+\xi_2))\right) - \cos\left(\frac{n}{2}\theta\right)\right]\left[\cosh\left(\frac{n}{2}(\xi_2+\xi_1+2k(\xi_1+\xi_2))\right) - \cos\left(\frac{n}{2}\theta\right)\right]} \right] \quad \text{(E-47)}$$

If there is only one admission plane ($\xi_1 = 0$) with n feeder holes, the solution becomes (see reference 2):

$$P_o'^2 = 1 + \frac{6\mu RT}{\pi P_a^2 C^3} M_{Bo} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[\frac{\cosh(n\xi_2(2k+1)) - 1}{\cosh(n(\xi_2+2k\xi_2)) - \cos(n\theta)} \right] \quad \text{(E-48)}$$

In these two equations, M_B is the mass flow from one feeder hole (concentric journal). The pressure, P_{oc}', downstream of the feeder hole is defined as the pressure on the rim of the feeder hole where $\xi_2 = 0$ (i.e. $\zeta = \xi_1$) and $\theta = d/D$ (d = feeder hole diameter, D= journal diameter). Since d << D the dominant term in the series in eqs. (E-47) and (E-48) is the one where k = 0, which is the only term that needs to be considered. Define:

Double Plane Admission

$$\lambda = \frac{1}{n\xi_2}\sum_{k=-\infty}^{\infty}(-1)^k \log\left[\frac{\left[\cosh\left(\frac{\pi}{2}(\xi_2+2k(\xi_1+\xi_2))\right)-1\right]\left[\cosh\left(\frac{\pi}{2}(\xi_2+2\xi_1+2k(\xi_1+\xi_2))\right)-1\right]}{\left[\cosh(nk(\xi_1+\xi_2))-\cos\left(\frac{\pi}{2}\frac{d}{D}\right)\right]\left[\cosh(n(\xi_1+k(\xi_1+\xi_2))-\cos\left(\frac{\pi}{2}\frac{d}{D}\right)\right]}\right] \tag{E-49}$$

$$\cong \frac{1}{n\xi_2}\log\left[\frac{\left[\cosh\left(\frac{\pi}{2}\xi_2\right)-1\right]\left[\cosh\left(\frac{\pi}{2}(\xi_2+2\xi_1)\right)-1\right]}{\left[1-\cos\left(\frac{\pi}{2}\frac{d}{D}\right)\right]\left[\cosh(n\xi_1)-\cos\left(\frac{\pi}{2}\frac{d}{D}\right)\right]}\right] \cong 1+\frac{2}{n\xi_2}\log\left(\frac{2D}{nd}\right)$$

Single Plane Admission

$$\lambda = \frac{1}{n\xi_2}\sum_{k=-\infty}^{\infty}(-1)^k \log\left[\frac{\cosh(n\xi_2(2k+1))-1}{\cosh(2nk\xi_2)-\cos\left(n\frac{d}{D}\right)}\right] \cong \frac{1}{n\xi_2}\log\left[\frac{\cosh(n\xi_2)-1}{1-\cos\left(n\frac{d}{D}\right)}\right] \cong 1+\frac{2}{n\xi_2}\log\left(\frac{D}{nd}\right) \tag{E-50}$$

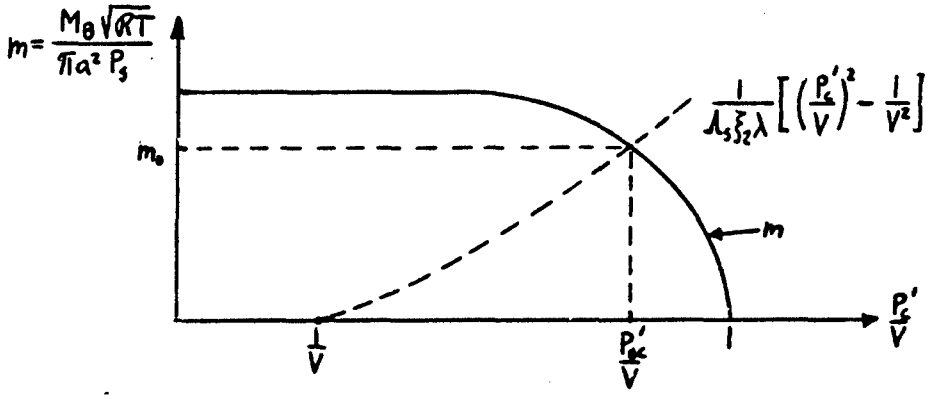
Then the downstream feeder hole pressure becomes:

$$P_{oc}'^2 = 1+\frac{6\mu RTnM_{Bo}}{\pi P_a^2 C^3}\lambda\xi_2 = 1+\Lambda_s V^2 m_o \xi_2\lambda = 1+q\xi_2\lambda \tag{E-51}$$

Now, m_o is a function of $\frac{P'_{oc}}{V}$ (see Eqs. (E-31) and(E-39)). From Eq. (E-51):

$$\frac{1}{\Lambda_s\xi_2\lambda}\left[\left(\frac{P'_{oc}}{V}\right)^2-\frac{1}{V^2}\right] = m_o = m\left(\frac{P'_{oc}}{V}\right) \tag{E-52}$$

This equation can be solved graphically:



Knowing $\frac{P'_{oc}}{V}$, q can be calculated directly from Eq. (E-51). In addition, the slope $\left(\frac{\partial m}{\partial(\frac{P'_{oc}}{V})}\right)_0$ of m at $\frac{P'_{oc}}{V}$ can be computed whereby ψ'_0 is given through Eq. (E-41).

The corresponding line feed solution can be determined from solving Eq. (E-12) with boundary conditions specified by Eqs. (E-24), (E-25) and (E-43):

$$\underline{0 \leq \zeta \leq \xi_1} \qquad P_0^2 = 1 + q\xi_2$$

$$\underline{0 \leq \zeta_2 \leq \xi_2} \qquad P_0^2 = 1 + q(\xi_2 - \zeta_2) \tag{E-53}$$

from which

$$P_{oc}^2 = 1 + q\xi_2 \tag{E-54}$$

A comparison between Eqs. (E-54) and (E-51) yields:

$$\frac{P_{oc}'^2 - 1}{P_{oc}^2 - 1} = \lambda \tag{E-55}$$

This establishes the relationship between the actual feeder hole downstream pressure and the one obtained from the line feed solution, based on a concentric journal. When the journal is eccentric, the flow from the feeder holes is no longer confined to the axial strip shown in Fig. 62 and, furthermore, the source strength changes. However, to get an approximate solution, consider a rectangular strip as in Fig. 62 with a uniform film thickness. In that case, the ratio:

$$\lambda = \frac{P_c'^2 - 1}{P_c^2 - 1} \tag{E-56}$$

stays constant, independent of the film thickness. Assuming that this relationship is valid even under dynamic conditions, Eq. (E-36) can be used to expand Eq. (56):

$$\lambda = \frac{P_{oc}'^2 - 1 + 2\varepsilon\left((P_0P_1)'_c - P_{oc}'^2\cos\theta\right)}{P_{oc}^2 - 1 + 2\varepsilon\left((P_0P_1)_c - P_{oc}^2\cos\theta\right)} = \frac{1}{P_{oc}^2 - 1}\left[P_{oc}'^2 - 1 + 2\varepsilon\left((P_0P_1)'_c - P_{oc}'^2\cos\theta\right)\right]\left[1 - \frac{2\varepsilon}{P_{oc}^2 - 1}\left((P_0P_1)_c - P_{oc}^2\cos\theta\right)\right]$$

from which

$$\frac{P_{oc}'^2 - 1}{P_{oc}^2 - 1} = \lambda$$

$$(\varepsilon P_0 P_1)_c' - \varepsilon P_{oc}'^2 \cos\theta = \lambda\left[(\varepsilon P_0 P_1)_c - \varepsilon P_{oc}^2 \cos\theta\right]$$

The first equation checks with Eq. (E-55). The second equation can be expanded further by means of Eqs. (E-15) and (E-17) to yield:

$$q_{0c}' - P_{oc}'^2 \cos\theta = \lambda\left[q_{0c} - P_{oc}^2 \cos\theta\right] \tag{E-57}$$

$$q_{1c}' - P_{oc}'^2 \cos\theta = \lambda\left[q_{1c} - P_{oc}^2 \cos\theta\right] \tag{E-58}$$

$$q_{2c}' = \lambda q_{2c} \tag{E-59}$$

Define the parameters:

$$\psi_0 = \lambda \psi_0' = -\frac{\lambda \Lambda_s V}{2P_{oc}'}\left(\frac{\partial m}{\partial\left(\frac{P_c'}{P_s}\right)}\right) \tag{E-60}$$

$$\psi_1 = \lambda \psi_1' = \frac{nV_c}{\pi DLC}\,\frac{\lambda(\xi_1+\xi_2)}{P_{oc}'} \tag{E-61}$$

Thus, Eqs. (E-57) to (E-61) can be substituted into Eqs. (E-44) to (E-46). Noting that:

$$P_{oc}^2 \cos\theta = Re\{P_{oc}^2 e^{j\theta}\}$$

$$P_{oc}^2 \sin\theta = Re\{-jP_{oc}^2 e^{j\theta}\}$$

and making use of Eqs. (E-17) to (E-19), the resulting equations become:

$$\left(\frac{dG_0}{d\zeta_2}\right)_c = \frac{q}{2(1+\delta^2)} + \psi_0\left(G_{oc} - P_{oc}^2\right) + \left(\frac{dG_0}{d\zeta}\right)_c \tag{E-62}$$

$$\left(\frac{dG_1}{d\zeta_2}\right)_c = \frac{q}{2(1+\delta^2)} + \left(\psi_0 + i2\zeta\Lambda\psi_1\right)\left(G_{1c} - P_{oc}^2\right) + \left(\frac{dG_1}{d\zeta}\right)_c \tag{E-63}$$

$$\left(\frac{dG_2}{d\zeta_2}\right)_c = (\psi_0 + i2\gamma\Lambda\psi_1)G_{2c} - ij2\gamma\Lambda\psi_1(G_{0c} - P_{0c}^2) + \left(\frac{dG_1}{d\zeta}\right)_c \tag{E-64}$$

These equations together with Eqs. (E-24) and (E-25) are the boundary conditions for Eqs. (E-20) to (E-22).

It is seen directly that:

$$G_2 = j(G_0 - G_1) \tag{E-65}$$

It remains to determine the solutions for G_0 and G_1.

P_0 is constant between the feeding planes, i.e. in the range $0 \leq \zeta \leq \xi_1$ as given by the first of Eqs. (E-53). Under these circumstances, G_0 and G_1 have closed form solutions:

$$G_0 = \frac{P_{0c}^2}{1 + j\frac{\Lambda}{P_{0c}}} + \left[G_{0c} - \frac{P_{0c}^2}{1 + j\frac{\Lambda}{P_{0c}}}\right]\frac{\cosh(\alpha_0 + j\beta_0)\zeta}{\cosh(\alpha_0 + j\beta_0)\xi_1} \tag{E-66}$$

$$G_1 = \frac{P_{0c}^2}{1 + j\frac{\Lambda}{P_{0c}} + i2\gamma\frac{\Lambda}{P_{0c}}} + \left[G_{1c} - \frac{P_{0c}^2}{1 + j\frac{\Lambda}{P_{0c}} + i2\gamma\frac{\Lambda}{P_{0c}}}\right]\frac{\cosh(\alpha + j\beta + i(\alpha_1 + j\beta_1))\zeta}{\cosh(\alpha + j\beta + i(\alpha_1 + j\beta_1))\xi_1} \tag{E-67}$$

where:

$$\alpha_0 + j\beta_0 = \sqrt{1 + j\frac{\Lambda}{P_{0c}}} \tag{E-68}$$

$$\alpha + j\beta + i(\alpha_1 + j\beta_1) = \sqrt{1 + j\frac{\Lambda}{P_{0c}} + i2\gamma\frac{\Lambda}{P_{0c}}} \tag{E-69}$$

$$\alpha_0 = \left[\frac{1}{2}\left(1 + \sqrt{1 + \frac{\Lambda^2}{P_{0c}^2}}\right)\right]^{1/2} \tag{E-70}$$

$$\beta_0 = \left[\frac{1}{2}\left(-1 + \sqrt{1 + \frac{\Lambda^2}{P_{0c}^2}}\right)\right]^{1/2} \tag{E-71}$$

$$\alpha = \left\{\frac{1}{4}\left[\chi_1 + 1 + \sqrt{(\chi_1 + 1)^2 + \left(\chi_2 + \frac{\Lambda}{P_{0c}}\right)^2}\right]\right\}^{1/2} \tag{E-72}$$

$$\beta = \left\{ \frac{1}{4} \left[-\varkappa_1 - 1 + \sqrt{(\varkappa_1+1)^2 + (\varkappa_2 + \frac{\Delta}{P_{oc}})^2} \right] \right\}^{1/2} \tag{E-73}$$

$$\alpha_1 = \left\{ \frac{1}{4} \left[\varkappa_1 - 1 + \sqrt{(\varkappa_1-1)^2 + (\varkappa_2 - \frac{\Delta}{P_{oc}})^2} \right] \right\}^{1/2} \tag{E-74}$$

$$\beta_1 = \left\{ \frac{1}{4} \left[-\varkappa_1 + 1 + \sqrt{(\varkappa_1-1)^2 + (\varkappa_2 - \frac{\Delta}{P_{oc}})^2} \right] \right\}^{1/2} \tag{E-75}$$

$$\varkappa_1 = \left\{ \frac{1}{2} \left[1 - \left(\frac{\Delta}{P_{oc}}\right)^2 + \left(2\gamma \frac{\Delta}{P_{oc}}\right)^2 + \sqrt{\left(1 - \left(\frac{\Delta}{P_{oc}}\right)^2 + \left(2\gamma \frac{\Delta}{P_{oc}}\right)^2\right)^2 + 4\left(\frac{\Delta}{P_{oc}}\right)^2} \right] \right\} \tag{E-76}$$

$$\varkappa_2 = \left\{ \frac{1}{2} \left[-\left(1 - \left(\frac{\Delta}{P_{oc}}\right)^2 + \left(2\gamma \frac{\Delta}{P_{oc}}\right)^2\right) + \sqrt{\left(1 - \left(\frac{\Delta}{P_{oc}}\right)^2 + \left(2\gamma \frac{\Delta}{P_{oc}}\right)^2\right)^2 + 4\left(\frac{\Delta}{P_{oc}}\right)^2} \right] \right\}^{1/2} \tag{E-77}$$

Hence:

$$\left(\frac{dG_o}{d\xi}\right)_c = \left[G_{oc} - \frac{P_{oc}^2}{1 + j\frac{\Delta}{P_{oc}}} \right] (\alpha_o + j\beta_o) \tanh(\alpha_o + j\beta_o)\xi_1 \tag{E-78}$$

$$\left(\frac{dG_1}{d\xi}\right)_c = \left[G_{1c} - \frac{P_{oc}^2}{1 + j\frac{\Delta}{P_{oc}} + i2\gamma\frac{\Delta}{P_{oc}}} \right] (\alpha + j\beta + i(\alpha_1 + j\beta_1)) \tanh(\alpha + j\beta + i(\alpha_1 + j\beta_1))\xi_1 \tag{E-79}$$

These expressions can be substituted into Eqs (E-62) and (E-63) to eliminate the two derivatives.

Eqs. (E-20) and (E-21) are solved by numerical integration. They have the general form:

$$\frac{d^2G}{d\xi_2^2} - EG = F \tag{E-80}$$

where:

$$E = \begin{cases} 1 + j\frac{\Delta}{P_o} & \text{for } G = G_o \\ 1 + j\frac{\Delta}{P_o} + i2\gamma\frac{\Delta}{P_o} & \text{for } G = G_1 \end{cases} \tag{E-81}$$

$$F = -P_o^2 = -(1 + q(\xi_2 - \zeta_2)) \tag{E-82}$$

The boundary conditions are:

$$\underline{\zeta_2 = \xi_2} \qquad G = 1 \tag{E-83}$$

$$\underline{\zeta_2 = 0} \qquad \frac{dG}{d\zeta_2} = a + b\,G_c \tag{E-84}$$

where:

$$a = \begin{cases} \dfrac{q}{2(1+\delta^2)} - \left[\psi_0 + \dfrac{(\alpha_0 + j\beta_0)\tanh(\alpha_0 + j\beta_0)\xi_1}{1 + j\frac{\Lambda}{P_{ac}}}\right](1 + q\xi_2) & \underline{G = G_0} \\ \dfrac{q}{2(1+\delta^2)} - \left[\psi_0 + i2_j\Lambda\psi_1 + \dfrac{(\alpha + j\beta + i(\alpha_1 + j\beta_1))\tanh(\alpha + j\beta + i(\alpha_1 + j\beta_1))\xi_1}{1 + j\frac{\Lambda}{P_{ac}} + i2_\gamma\frac{\Lambda}{P_{ac}}}\right](1 + q\xi_2) & \underline{G = G_1} \end{cases} \tag{E-85}$$

$$b = \begin{cases} \psi_0 + (\alpha_0 + j\beta_0)\tanh(\alpha_0 + j\beta_0)\xi_1 & G = G_0 \\ \psi_0 + i2_j\Lambda\psi_1 + (\alpha + j\beta + i(\alpha_1 + j\beta_1))\tanh(\alpha + j\beta + i(\alpha_1 + j\beta_1))\xi_1 & G = G_1 \end{cases} \tag{E-86}$$

Integrate Eq. (E-80) twice to get:

$$G = \int_0^{\zeta_2} (\zeta_2 - \zeta_2')(EG + F)\,d\zeta_2' + a\zeta_2 + (b\zeta_2 + 1)G_c \tag{E-87}$$

Subdivide the length of integration into m increments of length $\Delta\zeta = \xi_2/m$ and write Eq. (E-87) in finite difference form:

$$G_{n+1} = G_n + (\Delta\zeta)^2\left[\tfrac{1}{2}H_0 + H_1 + \cdots + H_n\right] + \Delta\zeta(a + bG_c) \qquad \underline{0 \le n \le m-1} \tag{E-88}$$

where $H = EG + F$. Setting:

$$G = t_c + jt_s + i(u_c + ju_s) + \left[v_c + jv_s + i(w_c + jw_c)\right]G_c \tag{E-89}$$

eq. (E-88) can be used to calculate G_n step by step, keeping G_c as an unknown constant ($G_0 = G_c$). At $\zeta_2 = \xi_2$, $G_m = 1$ from which G_c can be computed. Thereafter, G can be determined at each point by back substitution.

The force acting on the journal is found by integration of the film pressure. The force has a radial component F_r and a tangential component F_t :

$$\left.\begin{matrix} F_r \\ F_t \end{matrix}\right\} = 2\int_0^{\frac{L}{2}}\int_0^{2\pi} \bar{P}\left\{\begin{matrix} -\cos\theta \\ \sin\theta \end{matrix}\right\} R\,d\theta\,dz = 2R^2 P_a \int_0^{(\bar{\xi}_1+\bar{\xi}_2)}\int_0^{2\pi} P\left\{\begin{matrix} -\cos\theta \\ \sin\theta \end{matrix}\right\} d\theta\,d\zeta \tag{E-90}$$

or, in dimensionless form, making use of Eqs. (E-16), (E-4) and (E-15):

$$\left.\begin{matrix} f_r = \dfrac{F_r}{P_a L D} \\ f_t = \dfrac{F_t}{P_a L D} \end{matrix}\right\} = \frac{1}{2(\bar{\xi}_1+\bar{\xi}_2)}\int_0^{(\bar{\xi}_1+\bar{\xi}_2)}\int_0^{2\pi} \frac{P_0+\varepsilon_0\frac{q_0}{P_0}+\varepsilon_1 e^{i\tau}\left[\frac{q_1}{P_0}-\frac{\cos\theta}{1+\varepsilon_0\cos\theta}\left(P_0+\varepsilon_0\frac{q_0}{P_0}\right)\right]+\varepsilon_0\varphi_1 e^{i\tau}\frac{q_2}{P_0}}{1+\varepsilon_0\cos\theta}\left\{\begin{matrix} -\cos\theta \\ \sin\theta \end{matrix}\right\} d\theta\,d\zeta \tag{E-91}$$

The following integrals can be derived:

$$\int_0^{2\pi}\frac{d\theta}{1+\varepsilon_0\cos\theta} = \frac{2\pi}{\eta} \tag{E-92}$$

$$\int_0^{2\pi}\frac{\cos\theta\,d\theta}{1+\varepsilon_0\cos\theta} = -\frac{2\pi\varepsilon_0}{\eta(1+\eta)} \tag{E-93}$$

$$\int_0^{2\pi}\frac{\sin\theta\,d\theta}{1+\varepsilon_0\cos\theta} = 0 \tag{E-94}$$

$$\int_0^{2\pi}\frac{\cos^2\theta\,d\theta}{1+\varepsilon_0\cos\theta} = \frac{2\pi}{\eta(1+\eta)} \tag{E-95}$$

$$\int_0^{2\pi}\frac{\sin^2\theta\,d\theta}{1+\varepsilon_0\cos\theta} = \frac{2\pi}{(1+\eta)} \tag{E-96}$$

$$\int_0^{2\pi}\frac{\cos\theta\cdot\sin\theta\,d\theta}{1+\varepsilon_0\cos\theta} = 0 \tag{E-97}$$

$$\int_0^{2\pi}\frac{d\theta}{(1+\varepsilon_0\cos\theta)^2} = \frac{2\pi}{\eta^3} \tag{E-98}$$

$$\int_0^{2\pi}\frac{\cos\theta\,d\theta}{(1+\varepsilon_0\cos\theta)^2} = -\frac{2\pi\varepsilon_0}{\eta^3} \tag{E-99}$$

$$\int_0^{2\pi} \frac{\sin\theta \, d\theta}{(1+\varepsilon_0\cos\theta)^2} = 0 \tag{E-100}$$

$$\int_0^{2\pi} \frac{\cos^2\theta \, d\theta}{(1+\varepsilon_0\cos\theta)^2} = \frac{2\pi}{\eta^3(1+\eta)}\left(1+\eta-\eta^2\right) \tag{E-101}$$

$$\int_0^{2\pi} \frac{\sin^2\theta \, d\theta}{(1+\varepsilon_0\cos\theta)^2} = \frac{2\pi}{\eta(1+\eta)} \tag{E-102}$$

$$\int_0^{2\pi} \frac{\cos\theta\sin\theta \, d\theta}{(1+\varepsilon_0\cos\theta)^2} = 0 \tag{E-103}$$

$$\int_0^{2\pi} \frac{\cos^3\theta \, d\theta}{(1+\varepsilon_0\cos\theta)^2} = -\frac{2\pi}{\varepsilon_0\eta^3(1+\eta)}(1-\eta)(1+2\eta) \tag{E-104}$$

$$\int_0^{2\pi} \frac{\cos^2\theta\sin\theta \, d\theta}{(1+\varepsilon_0\cos\theta)^2} = 0 \tag{E-105}$$

$$\int_0^{2\pi} \frac{\cos\theta\sin^2\theta \, d\theta}{(1+\varepsilon_0\cos\theta)^2} = -\frac{2\pi(1-\eta)}{\varepsilon_0\eta(1+\eta)} \tag{E-106}$$

where:

$$\eta = \sqrt{1-\varepsilon_0^2} \tag{E-107}$$

From Eqs. (E-17) and (E-18):

$$\begin{aligned} g_0 &= Re_j\{G_0\}\cos\theta - Im_j\{G_0\}\sin\theta \\ g_1 &= Re_i\left\{\left[Re_j\{G_1\}\cos\theta - Im\{G_1\}\sin\theta\right]e^{i\tau}\right\} \end{aligned} \tag{E-108}$$

Thus, the integrals in Eq. (E-91) can be computed as:

$$f_r = \frac{2\varepsilon_0}{\eta(1+\eta)}H_{0r} + \varepsilon_1 e^{i\tau}\left[\frac{2(1-\eta)(2\eta+1)}{\eta^3(1+\eta)}H_{0r} + \frac{2}{\eta(1+\eta)}\left(H_{1r}+iJ_{1r}\right)\right] + \varepsilon_0\varphi_1 e^{i\tau}\frac{2}{\eta(1+\eta)}\left[-H_{0j}+H_{1j}+iJ_{1j}\right] \tag{E-109}$$

$$f_t = \frac{2\varepsilon_0}{1+\eta} H_{0j} + \varepsilon_1 e^{i\tau}\left[\frac{2(1-\eta)}{\eta(1+\eta)} H_{0j} + \frac{2}{1+\eta}\left(H_{1j} + iJ_{1j}\right)\right] - \varepsilon_0 \rho_1 e^{i\tau} \frac{2}{1+\eta}\left[-H_{0r} + H_{1r} + iJ_{1r}\right] \tag{E-110}$$

where:

$$H_{0r} = \frac{\pi}{2(\xi_1+\xi_2)} \int_0^{\xi_1+\xi_2} \left(P_0 - \frac{Re\{G_0\}}{P_0}\right) d\zeta \tag{E-111}$$

$$H_{0j} = -\frac{\pi}{2(\xi_1+\xi_2)} \int_0^{\xi_1+\xi_2} \frac{Im\{G_0\}}{P_0}\, d\zeta \tag{E-112}$$

$$H_{1r} + jH_{1j} + i\left(J_{1r} + jJ_{1j}\right) = \frac{\pi}{2(\xi_1+\xi_2)} \int_0^{\xi_1+\xi_2} \left(P_0 - \frac{G_1}{P_0}\right) d\zeta \tag{E-113}$$

With substitution from Eqs. (E-53), (E-66) and (E-67) these integrals become:

$$H_{0r} + jH_{0j} = \frac{\pi}{2(\xi_1+\xi_2)} \left\{ \xi_1\sqrt{1+q\xi_2} + \frac{2}{3q}\left[(1+q\xi_2)^{3/2} - 1\right] - \xi_1 \frac{\sqrt{1+q\xi_2}}{1+j\frac{\Delta}{P_{oc}}} - \left[\frac{G_{0c}}{P_{oc}} - \frac{\sqrt{1+q\xi_2}}{1+j\frac{\Delta}{P_{oc}}}\right] \frac{\tanh(\alpha_0+j\beta_0)\xi_1}{\alpha_0+j\beta_0} - \int_0^{\xi_2} \frac{G_0}{\sqrt{1+q(\xi_2-\zeta_2)}}\, d\zeta_2 \right\} \tag{E-114}$$

$$H_{1r} + jH_{1j} + i\left(J_{1r} + jJ_{1j}\right) = \frac{\pi}{2(\xi_1+\xi_2)} \left\{ \xi_1\sqrt{1+q\xi_2} + \frac{2}{3q}\left[(1+q\xi_2)^{3/2} - 1\right] - \xi_1 \frac{\sqrt{1+q\xi_2}}{1+j\frac{\Delta}{P_{oc}} + i2\eta\frac{\Delta}{P_{oc}}} - \left[\frac{G_{1c}}{P_{oc}} - \frac{\sqrt{1+q\xi_2}}{1+j\frac{\Delta}{P_{oc}} + i2\eta\frac{\Delta}{P_{oc}}}\right] \frac{\tanh(\alpha+j\beta+i(\alpha_1+j\beta_1))\xi_1}{\alpha+j\beta+i(\alpha_1+j\beta_1)} - \int_0^{\xi_2} \frac{G_1}{\sqrt{1+q(\xi_2-\zeta_2)}}\, d\zeta_2 \right\} \tag{E-115}$$

where P_{oc} is given by:

$$P_{oc} = \sqrt{1+q\xi_2} \tag{E-116}$$

The two integrals on the right hand side are evaluated by numerical integration from the previously determined values of G_0 and G_1 .

When the static load on the bearing is W, the total force components acting on the journal in the radial and tangential directions are:

Radial direction (opposite ε_0): $F_r^* = F_r - W\cos\varphi$

Tangential direction (opposite to $\varepsilon_0\varphi_1$): $F_t^* = -F_t + W\sin\varphi$ (E-117)

With φ given by Eq. (E-15), these components can be written in dimensionless form as:

$$f_r^* = f_r - \frac{W}{P_a LD}\cos\varphi = f_r - \frac{W}{P_a LD}\left(\cos\varphi_0 - \varphi_1 e^{i\tau}\sin\varphi_0\right)$$

$$f_t^* = -f_t + \frac{W}{P_a LD}\sin\varphi = -f_t + \frac{W}{P_a LD}\left(\sin\varphi_0 + \varphi_1 e^{i\tau}\cos\varphi_0\right) \quad \text{(E-118)}$$

Set:

$$f_{r0} = \frac{2\varepsilon_0}{\eta(1+\eta)} H_{or} \quad \text{(E-119)}$$

$$f_{t0} = \frac{2\varepsilon_0}{1+\eta} H_{oj} \quad \text{(E-120)}$$

$$f_r^* = f_r - \frac{W}{P_a LD}\cos\varphi = (\bar{K}_{rr} + i\gamma\bar{B}_{rr})\varepsilon_1 e^{i\tau} + (\bar{K}_{rt} + i\gamma\bar{B}_{rt})\varepsilon_0\varphi_1 e^{i\tau} \quad \text{(E-121)}$$

$$f_t^* = -f_t + \frac{W}{P_a LD}\sin\varphi = (\bar{K}_{tr} + i\gamma\bar{B}_{tr})\varepsilon_1 e^{i\tau} + (\bar{K}_{tt} + i\gamma\bar{B}_{tt})\varepsilon_0\varphi_1 e^{i\tau} \quad \text{(E-122)}$$

Then:

$$\frac{W}{P_a LD}\cos\varphi_0 = f_{r0} \quad \text{(E-123)}$$

$$\frac{W}{P_a LD}\sin\varphi_0 = f_{t0} \quad \text{(E-124)}$$

$$\frac{W}{P_a LD} = \sqrt{f_{r0}^2 + f_{t0}^2} \quad \text{(E-125)}$$

$$\varphi_0 = \tan^{-1}\left(\frac{f_{t0}}{f_{r0}}\right) \quad \text{(E-126)}$$

$$Z_{rr} = \bar{K}_{rr} + i\gamma\bar{B}_{rr} = \frac{2(1-\eta)(2\eta+1)}{\eta^3(1+\eta)} H_{or} + \frac{2}{\eta(1+\eta)}\left(H_{ir} + iJ_{ir}\right) \quad \text{(E-127)}$$

$$Z_{rt} = \bar{K}_{rt} + i\gamma \bar{B}_{rt} = -\frac{2(1-\eta)}{\eta(1+\eta)} H_{0j} + \frac{2}{\eta(1+\eta)} (H_{1j} + i J_{1j}) \tag{E-128}$$

$$Z_{tr} = \bar{K}_{tr} + i\gamma \bar{B}_{tr} = -\frac{2(1-\eta)}{\eta(1+\eta)} H_{0j} - \frac{2}{1+\eta} (H_{1j} + i J_{1j}) \tag{E-129}$$

$$Z_{tt} = \bar{K}_{tt} + i\gamma \bar{B}_{tt} = \frac{2(1-\eta)}{\eta(1+\eta)} H_{0r} + \frac{2}{1+\eta} (H_{1r} + i J_{1r}) \tag{E-130}$$

Here, $\bar{K}_{rr}, \bar{B}_{rr}, \bar{K}_{rt}, \bar{B}_{rt}$ etc. are the dimensionless spring and damping coefficients:

$$\begin{aligned} \bar{K}_{rr} &= \frac{\partial f_r^*}{\partial \varepsilon} = \frac{CK_{rr}}{P_a L D} \\ \bar{B}_{rr} &= \frac{\partial f_r^*}{\gamma \partial \dot{\varepsilon}} = \frac{C\omega B_{rr}}{P_a L D} \\ \bar{K}_{rt} &= \frac{\partial f_r^*}{\varepsilon_0 \partial \varphi} = \frac{CK_{rt}}{P_a L D} \\ \bar{B}_{rt} &= \frac{\partial f_r^*}{\gamma \varepsilon_0 \partial \dot{\varphi}} = \frac{C\omega B_{rt}}{P_a L D} \end{aligned} \tag{E-131}$$

and analogously for the four remaining coefficients. If it is desired, the coefficients can readily be expressed in an x-y-coordinate system with the x-axis in the direction of the static load W as in the preceeding appendices. This is done by making use of Eqs. (A-70) to (A-77), Appendix I where SW is replaced by $P_a LD$, f_r by f_r^* and $(-f_t)$ by f_t^*.

For the purely hydrostatic bearing, $\Lambda = 0$ but such that $2\gamma\Lambda = \sigma$ where σ is the squeeze number (see Eq. (E-10)). In that case $H_{0j} = H_{1j} = J_{1j} = 0$ and $\beta_0 = \beta = \beta_1 = 0$ which considerably simplifies the calculations. Furthermore, for the hydrostatic bearing, the static load increases almost proportional with displacement and, therefore, η should be set equal to 1 in the above equations. In total, then, for a hydrostatic bearing:

<u>Purely Hydrostatic Bearing</u>

$$\begin{aligned} \frac{W}{P_a L D} &= f_{r0} = \varepsilon_0 H_{0r} \\ f_{t0} &= 0 \qquad \varphi_0 = 0 \end{aligned} \tag{E-132}$$

$$\bar{K}_{rr} + i\gamma \bar{B}_{rr} = \bar{K}_{tt} + i\gamma \bar{B}_{tt} = H_{tr} + i J_{tr}$$

$$\bar{K}_{rt} + i\gamma \bar{B}_{rt} = \bar{K}_{tr} + i\gamma \bar{B}_{tr} = 0 \tag{E-133}$$

Thus, for a purely hydrostatic bearing there is only one spring coefficient and one damping coefficient which shall be denoted as K_0 and B_0. It is convenient to express these coefficients and the load carrying capacity in the dimensionless form:

$$\frac{W}{(P_s - P_a)LD} = \frac{1}{V-1}\,\frac{W}{P_a LD} \tag{E-134}$$

$$\frac{CK_0}{(P_s - P_a)LD} = \frac{1}{V-1}\,\bar{K}_{rr} \tag{E-135}$$

$$\frac{B_0}{\mu L \left(\frac{B}{C}\right)^3} = \frac{24}{\sigma}\,\gamma\,\bar{B}_{rr} \tag{E-136}$$

APPENDIX VI: THE STABILITY OF THE HYBRID-HYDROSTATIC RING BEARING WITH A COMPRESSIBLE LUBRICANT

The analysis of the stability of the hybrid-hydrostatic ring bearing is almost identical to the analysis of the hydrodynamic-hydrostatic ring bearing given in Appendix IV. Therefore, the present appendix will only describe the differences between the two analyses and otherwise refer to Appendix IV.

The equations of motion are given by Eqs. (D-1) where K_{xx}, B_{xx}, K_{xy} and so on are the dynamic coefficients of the inner film, and K_o and B_o are the coefficients of the outer film. They are determined from the analysis in Appendix V (note: instead of K_{xx}, B_{xx}, K_{xy}, etc can be used K_{rr}, B_{rr}, K_{rt}, etc. or the latter set of coefficients can be transformed to the first set of coefficients as discussed in Appendix V). Eq. (D-1) is made dimensionless by setting:

$$\bar{K}_o = \frac{CK_o}{P_a L D} = \left(\frac{C}{C_o}\right)\left(\frac{D_o}{D}\right)\left(\frac{P_s}{P_a}-1\right)\frac{C_o K_o}{(P_s-P_a)LD_o} \tag{F-1}$$

$$\gamma\bar{B}_o = \frac{C\nu B_o}{P_a L D} = \left(\frac{C}{C_o}\right)\left(\frac{D_o}{D}\right)\frac{6_o}{24}\frac{B_o}{\mu L\left(\frac{R_o}{C_o}\right)^3} \tag{F-2}$$

$$\bar{M} = \frac{C\omega^2 M}{P_a L D} \tag{F-3}$$

$$\bar{K}_{xx} = \frac{CK_{xx}}{P_a L D} \tag{F-4}$$

$$\bar{B}_{xx} = \frac{C\omega B_{xx}}{P_a L D} \tag{F-5}$$

and similarly for $\bar{K}_{xy}$, $\bar{K}_{yx}$, $\bar{K}_{yy}$, $\bar{B}_{xy}$, $\bar{B}_{yx}$ and $\bar{B}_{yy}$. The symbols are defined in Appendix IV and Appendix E.

The stability analysis is performed for fixed values of the compressibility number Λ and the static eccentricity ratio ε_o (i.e. for a constant static load W and a given speed ω). Hence, the dynamic coefficients are only functions of the frequency ratio $\gamma = \frac{\nu}{\omega}$. Then the analysis given in

Appendix N can be used directly as long as it is remembered, that whereas the dynamic coefficients in Appendix D are independent of γ because the lubricant is incompressible, they now depend implicitly on γ. The equations, however, are the same in the two cases.

APPENDIX VII: Computer Program - The Static and Dynamic Performance of a Lobed Bearing with Turbulent Film

This appendix describes the computer program PN0375: "The Static and Dynamic Performance of a Lobed Bearing with Turbulent Film" and gives the detailed instructions for using the program. The program is based on the analyses contained in Appendices I and II. It calculates the Sommerfeld number, the flow, the coefficient of friction, the 8 dynamic bearing coefficients and the critical journal mass, for a journal bearing with up to 12 lobes whose lubricant film may be turbulent. Film rupture is included.

COMPUTER INPUT

An input data form is given in back of this appendix for quick reference when preparing the computer input. In the following, the more detailed instructions are given.

Card 1 (I5)

This card contains one value:

NRET which gives the number of film Reynolds numbers in the table of turbulent flow coefficients ($1 \leq NRET \leq 250$)

Table of Turbulent Flow Coefficients (4E15.7)

In a bearing film with fully developed turbulence where the Couette flow is the dominating flow component, the effect of turbulence on the lubrication action can be accounted for by means of two coefficients, G_x and G_z. These coefficients modify the Poiseuille flow (the pressure induced flow) and they are functions of the film Reynolds number R_h:

$$R_h = \frac{\rho R \omega \bar{h}}{\mu} = h R_e$$

where h is the local film thickness, normalized with respect to the radial clearance C, and R_e is the Reynolds number for the bearing. G_x and G_z are given in reference 1 and can also be found in the table in the input data for the sample calculation later in this appendix. It should be noted, that the values used for G_x and G_z in the present program are twelve times the values found in reference 1. By doing this, G_x and G_z are equal to 1 when $R_h=0$ (ie. for laminar flow).

For use in the calculation of the dynamic performance of the bearing it is necessary to specify the first three derivatives of G_x with respect to R_n. Also, the first derivative of G_z/G_x is required. Finally, to determine the friction loss of the bearing, the coefficient of friction, C_f must be given. In the table, C_f is specified by giving the product $\frac{1}{8}R_h C_f$ which is equal to 1 for $R_h=0$. In total, then, there are 7 quantities for each film Reynolds number. They are given on two cards:

<u>First Card (4E15.7)</u>

1. $R_h = \frac{\rho\omega R\bar{h}}{\mu} = hR_e$, the films Reynolds number. The first value in the table should preferably be 0. In any case, the table must span over the range: $(1-\varepsilon_p)R_e \le R_h \le (1+\varepsilon_p)R_e$ where R_e is the input value of the bearing Reynolds number (see later input list) and ε_p is the maximum eccentricity ratio for any lobe in the particular calculation. Since the largest possible value of ε_p is 1, it is safest to let the range of R_h be: $0 \le R_h \le 2R_e$.

To use the table, the program employs linear interpretation. The local dimensionless film thickness h is computed from which the film Reynolds number is determined as $R_h = hR_e$ where R_e is given in the input. Then the table is scanned, starting with the first table value of R_h, until $(R_h)_{Table} \ge (R_h)_{Calculated}$. Assume this to be at the k'th table value (ie. $(R_h)_{Table} = (R_h)_k$). The proper value of G_x is then computed as:

$$G_x = \frac{(R_h)_k - (R_h)_{calc.}}{(R_h)_k - (R_h)_{k-1}}(G_x)_{k-1} + \frac{(R_h)_{calc.} - (R_h)_{k-1}}{(R_h)_k - (R_h)_{k-1}}(G_x)_k$$

and similarly for the remaining 6 quantities. If $(R_h)_{calcul.}$ is greater than the last value in the table, the program sets $(R_h)_{calcul.} = (R_h)_{last\ value}$. If $(R_h)_{calcul.}$ is smaller than the first value in the table, the program will be in error.

2. $\frac{1}{8} R_h C_f$, the dimensionless friction factor. For R_h=0 (laminar flow), $\frac{1}{8} R_h C_f = 1$.

3. G_x, the dimensionless Poiseuille flow coefficient for the circumferential direction. For R_h=0 (laminar flow), G_x=1.

4. $\frac{1}{G_x} \frac{dG_x}{dR_h}$, dimensionless. This is the first derivative of G_x divided by G_x. For R_h=0 (laminar flow), $\frac{1}{G_x} \frac{dG_x}{dR_h} = 0$.

Second Card(4E15.7)

1. $\frac{1}{G_x} \frac{d^2G_x}{dR_h^2}$, dimensionless. This is the second derivative of G_x divided by G_x. For R_h=0 (laminar flow), $\frac{1}{G_x} \frac{d^2G_x}{dR_h^2} = 0$.

2. $\frac{1}{G_x} \frac{d^3G_x}{dR_h^3}$, dimensionless. This is the third derivative of G_x divided by G_x. For R_h=0 (laminar flow), $\frac{1}{G_x} \frac{d^3G_x}{dR_h^3} = 0$.

3. $\frac{G_z}{G_x}$, dimensionless. This is the ratio between the Poiseuille flow coefficient, G_z, for flow in the axial direction, and G_x. For R_h=0 (laminar flow), $\frac{G_z}{G_x} = 1$.

4. $\frac{d}{dR_h}\left(\frac{G_z}{G_x}\right)$, dimensionless. This is the first derivative of $\frac{G_z}{G_x}$. For R_h=0 (laminar flow), $\frac{d}{dR_h}\left(\frac{G_z}{G_x}\right) = 0$.

In total the table contains NRET entry values of R_h arranged in sequence, starting with the smallest value. Thus, the table consists of 2 NRET cards. There can be a maximum of 250 entry values.

Card (72H)

This card is used as a title card, identifying the calculation. Any descriptive text can be given in columns 2 to 72. If a 1 is punched in Column 1, the output listing will start on a new page for each case.

Control Card (12I5)

This card controls the input and the output. It has 12 values:

1. M. This is the number of finite difference increments in the circumferential direction per lobe, see Fig. 55, Appendix I. When the lobe arc is $(\theta_{out} - \theta_{in})$, degrees, the circumferential increment is:

$$\Delta\theta = \frac{(\theta_{out} - \theta_{in})}{M} \quad degrees$$

It is suggested to let $\Delta\theta$ be 10 to 15 degrees, depending on the eccentricity ratio (the larger the eccentricity ratio, the smaller should $\Delta\theta$ be). The largest allowable value of M is 36. Also, M must be larger than or equal to 3.

2. N. This is the number of finite difference increments in the axial direction on half of the bearing length (see Fig. 55, Appendix I). If the increment is ΔZ and the bearing length is L, then:

$$\Delta Z = \frac{L/2}{N}$$

In dimensionless form as used by the program:

$$\Delta\zeta = \frac{1}{R}\Delta Z = \frac{L/D}{N}$$

N should be chosen such that $\Delta\zeta$ is approximately the same as $\Delta\theta$ when $\Delta\theta$ is measured in radians. However, since there are pressure gradients in the circumferential direction which are considerably larger than the axial pressure gradients, $\Delta\zeta$ can be made somewhat larger than $\Delta\theta$. Furthermore, it is of great importance to keep N as small as possible since the calculation time is roughly proportional to N^2 or N^3.

Usual values for N are 5 to 7, depending, of course, on the L/D- ratio. The allowable range of N is: $3 \leq N \leq 13$.

3. NLD Specifies the number of L/D-values in the designated input list (L/D is the length-to-diameter ratio of the bearing). The allowable range is: $1 \leq NLD \leq 20$.

4. NRE Specifies the number of bearing Reynolds numbers R_e in the designated input list. The allowable range is: $1 \leq NRE \leq 30$.

5. NPAD Specifies the number of bearing lobes. The allowable range is: $1 \leq NPAD \leq 12$.

6. NECA Specifies the number of $\varepsilon_B \cos\varphi_B$ values in the designated input list. (ε_B is the bearing eccentricity ratio, φ_B is the bearing attitude angle) The allowable range is: $1 \leq NECA \leq 20$.

7. NAB Specifies the maximum allowable number of calculations to be performed in determining that bearing attitude angle for which the horizontal force F_y is zero. A detailed discussion is given later in connection with the input list for $\varepsilon_B \cos\varphi_B$ and $\varepsilon_B \sin\varphi_B$. NAB should be greater than NAT (the number of $\varepsilon_B \sin\varphi_B$-values in designated input list).

8. NRP The program provides for the case of film rupture where no sub-ambient film pressures are permitted. In the regions where the pressure otherwise would have been less than ambient, the film contracts such that the pressure in the contracted film becomes ambient. The boundary separating the full film and the contracted portion is determined by iterations as discussed later in connection with the convergence limit for the rupture calculation. NRP specifies the maximum allowable number of such iterations to be performed. A usual value for NRP is 5 to 7. If it is not desired to include the effect of rupture, set NRP=-1.

9. NPW In most cases, only the final results of the calculations for the composite bearing are of interest. Then NPW is set equal to zero. On occasions, however, it may be desired to get the results for each bearing lobe before they are combined into the composite bearing. In that case, set NPW=1 or NPW = -1. If NPW = -1, the results will include the pressure distribution. If NPW = 1, the pressure distribution will be omitted from the output.

10. NRW Under certain conditions the calculations of the boundary between the complete film and the contracted film does not readily converge. In that case it may be desired to explore the matter in more detail. If NRW = 1, the output lists the "error" (the relative difference between two successive calculations) for each rupture iteration of each lobe. If NRW = -1, the output gives, in addition to the "error", the coordinates of the rupture boundary for each interation. If NRW = -2, the output gives, in addition to the output for NRW = -1, the finite difference coefficients $\alpha_i^{(+)}$, $\alpha_i^{(-)}$ and β_j (see Fig. 56, Appendix I) for each iteration..

When the calculations experience difficulties, the "error" does not converge smoothly. It will normally be found that the reason is that the rupture boundary intersects a j-gridline (see Fig. 55, Appendix I). Thus, the trouble can frequently be eliminated by changing the gridline spacing (ie. change M, item 1, control card).

Only under unusual circumstances is it of interest to explore the rupture boundary calculations. By setting NRW = 0, the output will not include any results of these calculations. Hence, the normal value of NRW is zero.

11. NPUN As discussed in connection with Item 7, the program may perform several calculations in order to determine the attitude angle for which the horizontal force is zero. If it is desired to get the results from each calculation, set NPUN = -1 or NPUN = -2. In the latter case, the

results for the composite bearing are punched on cards after each attitude angle calculation but only if there is no more than 1 lobe. No punched card output is given if NPUN = -1. If it is desired to get the final results only after the correct attitude angle has been determined, set NPUN =0 or NPUN =1. In the latter case, the output is also punched on cards for bearings with 1 lobe only. This is omitted when NPUN = 0.

Thus, the usual value of NPUN is zero (or 1, if punched card output is also desired).

12. INP When INP = 0, a new set of input data follows the present set of data. The new set of data starts with the text card (Card (72H)). The table of turbulent flow coefficients should not be repeated and can only be given with the first set of input. If INP = 1, the present set of data is the last or only set.

Card (1P5E14.6)

This card contains two values:

1. CVLA This is the convergence limit for the bearing attitude angle φ_B. After each complete bearing calculation, the total horizontal force F_{y0} and the total vertical force F_{x0} are computed by a summation over all bearing lobes, see eqs. (B-8) and (B-9), Appendix II. The correct attitude angle is the one for which $F_{y0}=0$. This condition is accepted to be satisfied by the program when:

$$\left|\frac{F_{y0}}{F_{x0}}\right| \leq CVLA$$

or when the total number of attitude angle calculations exceeds NAB (Item 7, control card).

The attitude angle is φ_B. If the deviation between the attitude angle for which $F_{y0}=0$, and the calculated attitude angle is called $\Delta\varphi_B$, then:

$$\Delta\varphi_B = \tan^{-1}\left(\frac{F_{y0}}{F_{x0}}\right)$$

When F_{y0} is small this is equivalent to:

$$\Delta\varphi_B \cong \frac{F_{y0}}{F_{x0}}$$

Hence, the given convergence criteria corresponds to requiring:

$$|\Delta\varphi_B|, radians < CVLA$$

or

$$|\Delta\varphi_B|, degrees < 57.3 \cdot CVLA$$

If it is desired to obtain the attitude angle within 0.01 degrees, then set $CVLA \cong 1.7 \cdot 10^{-4}$. A typical value for CVLA is 10^{-4}.

2. CVLR This is the convergence limit for the calculation of the rupture boundary for each bearing lobe. The program calculates the angular coordinates of the boundary for each i gridline (see fig. 55, Appendix I) let these coordinates be θ_i, $i=1,2,---n$. After the k'th rupture iteration, the program tests the convergence of the calculations by:

$$\frac{\sum|\theta_i^{(k)} - \theta_i^{(k-1)}|}{\sum|\theta_i^{(k-1)}|} \leq CVLR$$

When this condition is satisfied or when the total number of iterations is equal to NRP (Item 8, control card), i.e. when $k=NRP$, the program assumes that the calculations have converged. A typical value for CVLR is 10^{-5}.

When NRP = -1, set CVLR = 100.

List of Bearing Length-to-Diameter Ratios (1P5E14.6)

This list gives the input values of the length-to-diameter ratio, L/D, where L is the bearing length and D is the journal diameter. In total there are NLD values (Item 3, control card), maximum 20.

List of Bearing Reynolds Numbers (1P5E14.6)

This list gives the input values for the bearing Reynolds number:

$$R_e = \frac{\rho R \omega C}{\mu}$$

where ρ = mass density of lubricant, $lbs \cdot sec^2/inch^4$, μ = viscosity of lubricant, $lbs \cdot sec/inch^2$, ω = angular speed of journal, radians/sec, R = journal radiaus, inch, and C = radial clearance of lobes, inch. When $R_e = 0$, the film is laminar. In total, there are NRE values of R_e (item 4, control card), maximum 30.

Data for Journal Center Position

Referring to fig. 58, Appendix II, the coordinates of the journal center with respect to the bearing center can be expressed by means of the eccentricity ratio ε_B and the attitude angle φ_B. ε_B is the ratio between the journal center eccentricity, e_B, and the machined radial clearance, C, of the lobes:

$$\varepsilon_B = \frac{e_B}{C}$$

The attitude angle, φ_B, is the angle between the static load line (the x-axis) and the line connecting the bearing center and the journal center, measured in the direction of journal rotation. The corresponding coordinates in the x-y-coordinate system are: $\varepsilon_B \cos\varphi_B$ and $\varepsilon_B \sin\varphi_B$.

A given bearing calculation is performed for a fixed value of $\varepsilon_B \cos\varphi_B$. With this value fixed, $\varepsilon_B \sin\varphi_B$ is varied over a range in specified increments to determine when the horizontal force F_{y0} becomes zero. For each value of $\varepsilon_B \sin\varphi_B$, the program calculates the static horizontal force F_{y0}. When F_{y0} changes sign, F_{y0} is calculated at the midpoint of the last $\varepsilon_B \sin\varphi_B$-interval and quadratic interpolation is employed to obtain the first "guess" of the value of $\varepsilon_B \sin\varphi_B$ for which F_{y0} should be zero. If the corresponding value of F_{y0} is not sufficiently small, as tested by means of the convergence criteria explained above, the latest obtained value of F_{y0} is used together with the two closest previously obtained values to calculate a new value of $\varepsilon_B \sin\varphi_B$ by quadratic interpolation. This procedure is repeated until either the convergence criteria is satisfied or the total number of calculations exceeds the

allowable limit NAB (Item 7, control card).

In total, there are NECA values of $\varepsilon_B \cos\varphi_B$ (Item 6, control card), maximum 20. The card specifying $\varepsilon_B \cos\varphi_B$, contains two items:

Card (1PE14.6, I5)

1. $\varepsilon_B \cos\varphi_B$. It can have any value as long as the corresponding eccentricity ratios, ε_p, for any of the bearing lobes do not equal or exceed 1, see eq. (B-5), Appendix II.

2. NAT If NAT $\geq$ 1, the program determines the attitude angle for which the horizontal force is zero. Then NAT gives the number of values of $\varepsilon_B \sin\varphi_B$ in the following input list. Item 7, control card: NAB should be greater than NAT to allow for several interpolations in determining φ_B (suggested value: NAB = NAT + 3).

   If NAT $\leq$ -1, no interpolation is performed to determine when the horizontal force is zero. The program calculates the bearing for each specified value of $\varepsilon_B \sin\varphi_B$ and gives the results of each calculation (set NPUN = -1 or -2, item 11, control card). The absolute value of NAT, |NAT|, specifies the number of values of $\varepsilon_B \sin\varphi_B$ in the following input list. Item 7, control card: NAB should be set equal to |NAT|.

   Note: NAT cannot be zero. The maximum value of NAT (or |NAT|) is 25.

   This card is followed by:

List of $\varepsilon_B \sin\varphi_B$ Values (1P5E14.6)

This list gives the input values of $\varepsilon_B \sin\varphi_B$ for the above specified $\varepsilon_B \cos\varphi_B$ value. In total there are NAT values, maximum 25. $\varepsilon_B \sin\varphi_B$ may have any value as long as the eccentricity ratio, ε_p, of any bearing lobe does not equal or exceed 1, see eq. (B-5), Appendix II.

If it is desired to calculate the bearing with a concentric journal, set $\varepsilon_B \cos\varphi_B = 0$ and give only one value of $\varepsilon_B \sin\varphi_B$, namely $\varepsilon_B \sin\varphi_B = 0$. Furthermore, set NAT = 1 and NAB = 0.

In the case where $\varepsilon_B \cos\varphi_B = 0$ and the journal is not concentric in the bearing, no value of $\varepsilon_B \sin\varphi_B$ should be zero.

If it is desired to calculate the attitude angle for which the horizontal force is zero (i.e. NAT is positive), the $\varepsilon_B \sin\varphi_B$ values should be listed in sequence, starting with the lowest value (it may be zero when $\varepsilon_B \cos\varphi_B \neq 0$). The range of $\varepsilon_B \sin\varphi_B$ should be large enough that it includes the point where $F_{y_0} = 0$. Otherwise, the program cannot obtain a solution.

The preceeding input, namely the card with $\varepsilon_B \cos\varphi_B$ and the list of $\varepsilon_B \sin\varphi_B$ values, must be repeated together NECA times (Item 6, control card).

<u>Bearing Geometry Data (1P5E14.6)</u>

The bearing is made up of NPAD lobes (Item 5, control card). The geometry of the lobe is specified by giving the angles from the static load line (the x-axis) to the leading edge and the trailing edge of the lobe, and by giving the coordinates of the center of curvature of the lobe relative to the bearing center and the static load line, see figure 56, Appendix II. The input consists of NPAD cards, maximum 12, and on each card are four values:

1. $\Theta_{p,in}$, degrees. This is the angle from the load line (the negative x-axis) to the leading edge of the lobe, measured in the direction of journal rotation (see fig. 58, Appendix II). The value for $\Theta_{p,in}$ should be such that: $0 \leq \Theta_{p,in} \leq 360$. $\Theta_{p,in}$ must be smaller than $\Theta_{p,out}$.

2. $\Theta_{p,out}$, degrees. This is the angle from the load line (the negative x-axis) to the trailing edge of the lobe, measured in the direction of journal rotation (see Fig. 58, Appendix II). The value for $\Theta_{p,out}$ should lie in

range: $\theta_{p,in} < \theta_{p,out} \leq 360 + \theta_{p,in}$ $(\theta_{p,out} > 0)$.

3. δ_p . This is the preload of the lobe:

$$\delta_p = \frac{r_p}{C}$$

where r_p is the distance between the bearing center and the center of curvature of the lobe. C is the machined radial clearance of the lobe (i.e. the differenc between the lobe radius of curvature and the journal radius. C is the same for all lobes). For a cylindrical bearing, $\delta_p = 0$. For the elliptical bearing or a three-lobe bearing, δ_p is the same for all lobes and equal to the preload of the bearing in which case $\delta_p < 1$.

4. ψ_p degrees. This is the angle from the static load line (the positive x-axis) to the line connecting the bearing center and the center of curvature of the lobe, measured in the direction of journal rotation (see Fig. 58 , Appendix II). For a cylindrical bearing, set $\psi_p = 0$. In the elliptical bearing or the three lobe bearing, set $\psi_p = \frac{1}{2}(\theta_{p,out} + \theta_{p,in})$.

COMPUTER OUTPUT

The output first repeats the input data for checking purposes (the table of turbulent flow coefficients is not included). Then follow the results of the calculations. Because of the several output format options, a complete sequential description of the output will not be given. Instead, the output for the normal type of calculation will be described where only the final results for the composite bearing are given. The final results are in five lines where the items are identified by the titles:

L/D is the specified length - to-diameter ratio of the bearing.

REYN.NO. is the specified bearing Reynolds number R_e.

ECC*COS(ATT) is the specified fixed value of $\varepsilon_B \cos\varphi_B$.

ECC*SIN(ATT) is the final value of $\varepsilon_B \sin\varphi_B$.

ECC.RATIO is the bearing eccentricity ratio $\varepsilon_B = \sqrt{(\varepsilon_B \cos\varphi_B)^2 + (\varepsilon_B \sin\varphi_B)^2}$

ATT. ANGLE is the bearing attitude angle $\varphi_B = \tan^{-1}(\varepsilon_B \sin\varphi_B / \varepsilon_B \cos\varphi_B)$, degrees.

1/S is the inverse Sommerfeld number for the bearing : $\frac{1}{S} = \frac{W}{\mu N D L (\frac{D}{C})^2}$

F-Y/S*W is the residual dimensionless horizontal force: $F_{yo}/\mu NDL(\frac{R}{C})^2$, eq. (B-9)

1/S*(R/C*F/W) is the friction factor: $\frac{R}{C}\frac{F_f}{SW} = \frac{R}{C}\frac{F_f}{\mu NDL(\frac{R}{C})^2}$, eq. (B-10)

(Q-Z/NDLC)SIDE is the dimensionless flow out of both bearing sides: $Q_z/NDLC$, see eq.

(Q/NDLC)TOTAL is the total dimensionless flow consumption of the bearing: $Q/NDLC$

For a lobed bearing with NPAD lobes, the total flow is:

$$Q = \sum_{p=1}^{NPAD}\left[(Q_z)_p + \left\{\begin{array}{l}\left((Q_x)_{p,trail.edge} - (Q_x)_{p,lead.edge}\right)_{when\ positive} \\ 0 \quad (when\ above\ quantity\ is\ negative\end{array}\right\}\right]$$

(see remarks following eq. (B-13), Appendix II)

When NPAD = 1: $Q = Q_z + (Q_x)_{trail.edge}$

(Q-X/NDLC)CIRC is the dimensionless total flow into the bearing lobes from the grooves:

$$\sum_{p=1}^{NPAD}\left(\frac{Q_x}{NDLC}\right)_{p,lead.edge}$$

F-X/S*W is the dimensionless vertical bearing reaction: $-F_{xo}/\mu NDL(\frac{R}{C})^2$, eq. (B-8). When $F_{yo}=0$, this is equal to $1/S$, see eq. (B-16)

CMN/MUDL(R/C)2 is the dimensionless critical journal mass at the threshold of instability: $\frac{CMN}{\mu DL(\frac{R}{C})^2}$, calculated from eqs. (B-26) and (B-27).

FREQ/W is the frequency ratio: $\gamma = \frac{\nu}{\omega}$, at the threshold of instability.

(FREQ/W)**2 is the square of the instability frequency ration: $\gamma^2 = (\frac{\nu}{\omega})^2$, calculated from eqs. (B-27). When γ^2 is negative, there is no instability threshold.

CKXX/SW is the dimensionless bearing spring coefficient for the x-direction: $\frac{CK_{xx}}{SW} = \frac{CK_{xx}}{\mu NDL(\frac{R}{C})^2}$, see eq (B-12)

CKXY/SW is the dimensionless bearing cross-coupling spring coefficient for the x-direction: $\frac{CK_{xy}}{SW} = \frac{CK_{xy}}{\mu NDL(\frac{R}{C})^2}$

CKYX/SW is the dimensionless bearing cross-coupling spring coefficient for the y-direction: $\frac{CK_{yx}}{SW} = \frac{CK_{yx}}{\mu NDL(\frac{R}{C})^2}$

CKYY/SW is the dimensionless bearing spring coefficient for the y-direction: $\frac{CK_{yy}}{SW} = \frac{CK_{yy}}{\mu NDL(\frac{R}{C})^2}$

CWBXX/SW is the dimensionless bearing damping for the x-direction: $\frac{C\omega B_{xx}}{SW} = \frac{C\omega B_{xx}}{\mu NDL(\frac{R}{C})^2}$, see eq. (B-13).

CWBXY/SW is the dimensionless bearing cross-coupling damping for the x-direction: $\frac{C\omega B_{xy}}{SW} = \frac{C\omega B_{xy}}{\mu NDL\left(\frac{R}{C}\right)^2}$

CWBXY/SW is the dimensionless bearing cross-coupling damping for the y-direction: $\frac{C\omega B_{yx}}{SW} = \frac{C\omega B_{yx}}{\mu NDL\left(\frac{R}{C}\right)^2}$

CWBYY/SW is the dimensionless bearing damping for the y-direction: $\frac{C\omega B_{yy}}{SW} = \frac{C\omega B_{yy}}{\mu NDL\left(\frac{R}{C}\right)^2}$

SOMMERFELD NO is the Sommerfeld number: $S = \frac{\mu NDL}{W}\left(\frac{R}{C}\right)^2$

F-X/W is a second form of the dimensionless vertical bearing reaction: $-F_{xo}/W$ $(= 1 \text{ when } F_{yo} = 0)$

F-Y/W is a second form of the dimensionless residual horizontal force: F_{yo}/W

(R/C)*(F/W) is the most usual form of the friction factor: $\frac{R}{C}\frac{F_f}{W}$, see eq. (B-17)

CMW2/W** is a second form of the dimensionless critical journal mass: $\frac{CM\omega^2}{W}$ see eq. (B-28)

CKXX/W is a second form of the dimensionless bearing spring coefficient in the x-direction: $\frac{CK_{xx}}{W}$, see eq. (B-18)

CKXY/W $= \frac{CK_{xy}}{W}$

CKYX/W $= \frac{CK_{yx}}{W}$

CKYY/W $= \frac{CK_{yy}}{W}$

CWBXX/W is a second form of the dimensionless bearing damping in the x-direction: $\frac{C\omega B_{xx}}{W}$, see eq. (B-19)

CWBXY/W $= \frac{C\omega B_{xy}}{W}$

CWBYX/W $= \frac{C\omega B_{yx}}{W}$

CWBYY/W $= \frac{C\omega B_{yy}}{W}$

The nomenclature is:

B_{xx}, B_{xy}, B_{yx}, B_{yy}	Damping coefficients for the bearing, lbs sec/inch
C	The machined radial clearance of the lobes, inch
D	The journal diameter, inch
e_B	The eccentricity between the journal center and the bearing center, inch
F_{xo}	The static bearing reaction in the x-direction, lbs.
F_{yo}	The static bearing reaction in the y-direction, lbs.
F_f	The friction force, lbs (the friction torque = RF_f)

K_{xx}, K_{xy}, K_{yx}, K_{yy}	Spring coefficients for the bearing, lbs/inch
L	The bearing length, inch
M	The critical mass of the journal at the threshold of instability,
N	The rotor speed, rps
Q	The total hydrodynamic bearing flow, cub.inch/sec
Q_x	The hydrodynamic flow in the circumferential direction, cub.inch/sec
Q_z	The hydrodynamic flow out of both bearing sides, cub.inch/sec
R	The journal radius, inch
R_e	The Reynolds number for the bearing: $R_e = \frac{\rho R \omega C}{\mu}$ dimensionless
S	The Sommerfeld number for the bearing: $S = \frac{\mu N D L}{W} \left(\frac{R}{C}\right)^2$ dimensionless
W	The static bearing load, lbs
γ	The instability frequency ratio: $\gamma = \frac{\nu}{\omega}$,dimensionless
ε_B	The eccentricity ratio for the bearing, $\varepsilon_B = e_B/C$ dimensionless
μ	The lubricant viscosity, lbs.sec/inch^2
ν	The frequency at the threshold of instability, radians/sec
ρ	The mass density of the lubricant, lbs.sec^2/inch^4
ϕ_B	The attitude angle for the bearing. It is the angle from the X-axis to the line connecting the bearing center and the journal center, measured in the direction of journal rotation, degrees.
ω	The angular speed of the journal, radians/sec.

When it is desired to obtain the data for the individual lobes in addition to the composite bearing results, $NPW \neq 0$ in the input (Item 9, control card). Then there will be output for each lobe for each attitude angle iteration. Each set of output is identified by the title: BEARING PAD NO., followed by the number of the particular lobe (the lobes are numbered consecutively in the

sequence as given in the input). The results are identified by titles:

SOMMERFELD NO is the Sommerfeld number for the lobe, $S_p = \frac{\mu N D L}{W_p}\left(\frac{R}{C}\right)^2$

ECC. RATIO is the lobe eccentricity ratio ε_p, see eq. (B-5)

ATT. ANGLE is the lobe attitude angle φ_p, see eq. (B-6), degrees.

CALC. ATT. ANG is the angle: $\tan^{-1}(f_{to}/f_{ro})$, see Appendix I, eqs (A-64), (A-78) and (A-79)

F-R/S*W is the dimensionless radial static force component for the lobe: $f_{ro} = \frac{F_{ro}}{S_p W_p}$

F-T/S*W is the dimensionless tangential static force component for the lobe: $f_{to} = \frac{F_{to}}{S_p W_p}$

1/S*(R/C*F/W) is the dimensionless friction factor for the lobe: $\frac{R}{C}\frac{F_{f,p}}{S_p W_p} = \frac{R}{C}\frac{F_{f,p}}{\mu N D L\left(\frac{R}{C}\right)^2}$

(Q-X/NDLC)IN is the dimensionless flow into the lobe across the leading edge: $\left(\frac{Q_x}{NDLC}\right)_{lead}$

(Q-X/NDLC)OUT is the dimensionless flow out of the lobe across the trailing edge: $\left(\frac{Q_x}{NDLC}\right)_{tra}$

(Q-Z/NDLC)SIDE is the dimensionless flow out of the lobe across both sides of the lobe: $\frac{Q_z}{NDLC}$

F-X/S*W is the dimensionless x-component of the static force on the lobe: $-\frac{F_{xp}}{S_p W_p}$, see eq (A-78)

F-Y/S*W is the dimensionless y-component of the static force on the lobe: $-\frac{F_{yp}}{S_p W_p}$, see eq. (A-79)

DFR/SDE is the dimensionless radial stiffness: $\frac{\partial f_r}{\partial \varepsilon}$, see eqs. (A-59) and (A-60)

DFT/SDE is the dimensionless tangential cross-coupling stiffness: $\frac{\partial f_t}{\partial \varepsilon}$, see eq. (A-60)

DFR/SDA is the dimensionless radial cross-coupling stiffness: $\frac{\partial f_r}{\varepsilon_0 \partial \varphi}$, see eqs. (A-59) and A-61)

DFT/SDA is the dimensionless tangential stiffness: $\frac{\partial f_t}{\varepsilon_0 \partial \varphi}$, see eq. (A-61)

DFR/SDEDT is the dimensionless radial damping: $\frac{\partial f_r}{\partial \dot{\varepsilon}}$, see eqs. (A-59) and (A-62)

DFT/SDEDT is the dimensionless radial damping: $\frac{\partial f_t}{\partial \dot{\varepsilon}}$, see eqs. (A-62)

DFR/SDADT is the dimensionless radial cross-coupling damping: $\frac{\partial f_r}{\varepsilon_0 \partial \dot{\varphi}}$, see eqs. (A-59 and (A-63)

DFT/SDADT is the dimensionless tangential damping: $\frac{\partial \delta_\phi}{\varepsilon_0 \partial \phi}$, see eq. (A-63)

CKXX/SW is the dimensionless stiffness, x-direction: $\frac{CK_{xx}}{S_p W_p}$, see eq. (A-70)

CKXY/SW = $\frac{CK_{xy}}{S_p W_p}$, eq. (A-71)

CKYX/SW = $\frac{CK_{yx}}{S_p W_p}$, eq. (A-72)

CKYY/SW = $\frac{CK_{yy}}{S_p W_p}$, eq. (A-73)

CWBXX/SW = $\frac{C\omega B_{xx}}{S_p W_p}$, eq. (A-74)

CWBXY/SW = $\frac{C\omega B_{xy}}{S_p W_p}$, eq. (A-75)

CWBYX/SW = $\frac{C\omega B_{yx}}{S_p W_p}$, eq. (A-76)

CWBYY/SW = $\frac{C\omega B_{yy}}{S_p W_p}$, eq. (A-77)

CMN/MUDL(R/C)2 is the dimensionless critical journal mass for the lobe: $\frac{CM_p N}{\mu DL (\frac{R}{C})^2}$

CMW**2/W is another form of the dimensionless critical journal mass: $CM_p \omega^2 / W_p$

FREQ/W is the instability frequency ratio for the lobe: $\gamma_p = \nu_p / \omega$

(FREQ/W)**2 is the square of the instability frequency ratio for the lobe: γ_p^2.

If γ_p^2 is negative, there is no instability threshold for the lobe.

In addition to these data, the output for a lobe also includes the results from the last rupture boundary calculation. First is a line giving the number of iterations and the final error (the left hand side of the equation given in the discussion of the input data for CVLR). This is followed by a 5 column list, giving the coordinates of the rupture boundary. The five columns are identified by titles:

I is the i-index for the gridline (see fig. 55, Appendix I).

BEGIN, INDEX gives the j-coordinate (see fig. 55) of the rupture boundary at the leading edge. If j=1, leading edge rupture does not occur.

END INDEX gives the j-coordinate (see fig. 55) of the rupture boundary at the trailing edge. If j = M +1 (item 1, control card in input), trailing edge rupture does not occur.

BEGIN, ANGLE gives the angular coordinate in degrees of the rupture boundary at the leading edge. The angle is measured from the line connecting the bearing center and the journal center.

END ANGLE gives the angular coordinate in degrees of the rupture boundary at the trailing edge. The angle is measured in the same way as for the leading edge angle. When NRW $\leq$ -1 (item 10, control card in input), the above data are repeated for each rupture calculation.

When NPW = -1, the output includes the pressure distribution for each lobe. It is identified in the output by the title: PRESSURE DISTRIBUTION. The dimensionless pressure $P = \bar{P}/\mu\omega\left(\frac{R}{C}\right)^2 = \frac{1}{2\pi S_p}\frac{\bar{P}}{W/LD}$ (see eq. (A-7)), where $\bar{P}$ is the actual pressure in psi, is listed starting at the leading edge of the lobe (i.e. at j = 0 in figure 55.). The first value in each line is at i = 1 and the last value is at the centerline (i = N), see fig. 55.

When NPUN = 1 or -2 (item 11, control card in input), punched card output will be given. The first card is given at the start of a calculation for a new value of the L/D-ratio or new value of the Reynolds number (it is only given if NPAD = 1). The card contains four values:

Card (4E13.5)

1. L/D
2. The lobe arc, degrees ($\theta_{p,out} - \theta_{p,in}$)
3. The relative angular location of the center of pressure ($(180^\circ - \theta_{p,in})/(\theta_{p,out} - \theta_{p,in})$)
4. The Reynolds number R_e

This card is followed by the results of the composite bearing for each value of $\varepsilon_B \cos\varphi_B$ (and if NPUN = -2, also for each value of $\varepsilon_B \sin\varphi_B$), there will be two cards:

Card (5E13.5)

1. $\varepsilon_B \cos\varphi_B$
2. $1/S = W/\mu NDL\left(\frac{R}{C}\right)^2$, the inverse Sommerfeld number
3. The friction factor: $\frac{R}{C}\frac{F_f}{SW} = \frac{R}{C}\frac{F_f}{\mu NDL(R/C)^2}$
4. The dimensionless flow across the leading edge: $\frac{Q_x}{NDLC}$
5. The bearing eccentricity ratio ε_B.

Card (5E13.5)

1. The dimensionless radial stiffness for an inertialess tilting pad:

$$\frac{C}{SW}\left[K_{xx} - \mathcal{R}e\left\{\frac{(K_{xy}+i\omega B_{xy})(K_{yx}+i\omega B_{yx})}{K_{yy}+i\omega B_{yy}}\right\}\right]$$

2. The dimensionless radial damping for an inertialess tilting pad:

$$\frac{C}{SW}\left[\omega B_{xx} - \mathcal{I}m\left\{\frac{(K_{xy}+i\omega B_{xy})(K_{yx}+i\omega B_{yx})}{K_{yy}+i\omega B_{yy}}\right\}\right]$$

3. The dimensionless stiffness for the y-direction: $\frac{CK_{yy}}{SW}$
4. The dimensionless damping for the y-direction: $\frac{C\omega B_{yy}}{SW}$
5. The attitude angle φ_B, degrees

INPUT FORM FOR COMPUTER PROGRAM
PN0375: THE STATIC AND DYNAMIC PERFORMANCE OF A LOBED BEARING WITH TURBULENT FILM

Card 1 (I5)

NRET = Number of film Reynolds numbers in table for turbulent flow coefficients

Table of Turbulent Flow Coefficients (4E15.7)

For each Reynolds number, give two cards with four values per card.

First Card:

1. $R_h = \rho R \omega \bar{h} / \mu$, the film Reynolds number (dimensionless)
2. $\frac{1}{8} C_f R_h$, the friction factor (dimensionless). For $R_h = 0$, $\frac{1}{8} C_f R_h = 1$
3. G_x, the turbulent flow coefficient for the circumferential direction (dimensionless). For $R_h = 0$, $G_x = 1$
4. $\frac{1}{G_x} \frac{dG_x}{dR_h}$, the first derivative of G_x divided by G_x (dimensionless) For $R_h = 0$, $\frac{dG_x}{dR_h} = 0$

Second Card:

1. $\frac{1}{G_x} \frac{d^2G_x}{dR_h^2}$, the second derivative of G_x divided by G_x (dimensionless). For $R_h = 0$, $\frac{d^2G_x}{dR_h^2} = 0$
2. $\frac{1}{G_x} \frac{d^3G_x}{dR_h^3}$, the third derivative of G_x divided by G_x (dimensionless). For $R_h = 0$, $\frac{d^3G_x}{dR_h^3} = 0$
3. $\frac{G_z}{G_x}$, where G_z is the turbulent flow coefficient for the axial direction (dimensionless). For $R_h = 0$, $\frac{G_z}{G_x} = 1$
4. $\frac{d}{dR_h}\left(\frac{G_z}{G_x}\right)$, the first derivative of $\frac{G_z}{G_x}$ (dimensionless). It is zero for $R_h = 0$.

Card (72)) Text

Control Card (12I5)

1. M = Number of finite difference increments in the circumferential direction per lobe ($3 \leq M \leq 36$)
2. N = Number of finite difference increments in the axial direction on half the bearing length ($3 \leq N \leq 13$)
3. NLD = Number of L/D-ratios in input list ($1 \leq NLD \leq 20$)
4. NRE = Number of bearing Reynolds numbers, R_e, in input list ($1 \leq NRE \leq 30$)
5. NPAD = Number of lobes ($1 \leq NPAD \leq 12$)
6. NECA = Number of $\varepsilon_B \cos\varphi_B$ - values in input list ($1 \leq NECA \leq 20$)
7. NAB = Maximum allowable number of attitude angle iterations.
8. NRP; If NRP $\geq$ 1: Maximum allowable number of iterations to determine rupture boundry.

   If NRP = 0: Film rupture tables place, but there are no iterations.

   If NRP = -1: No film rupture, and no iterations.
9. NPW; If NPW = 0: Only the composite bearing results are given, not the results for the individual lobes.

   If NPW = 1: The results for the individual pads and the composite bearing are given, but not the pressure distribution.

   If NPW = -1: Same as for NPW =1, but with pressure distribution.
10. NRW; If NRW = 0: No results are given from the rupture boundary iteration.

    If NRW = 1: The convergence error for each rupture boundary iteration is given.

    If NRW = -1: Same as for NRW =1, but output also includes the coordinates of the rupture boundary for each iteration.

    If NRW = -2: Diagnostic for rupture boundary calculation.
11. NPUN; If NPUN = 0: The composite bearing results are only given after the final attitude angle calculation, no cards are punched.

    If NPUN = 1: Same as NPUN = 0, but the results are also punched on cards.

    If NPUN = -1: The composite bearing results are given after each attitude angle calculation, no cards are punched.

If NPUN = -2: Same as NPUN = -1, but the results are also punched on cards.

12. INP; If INP = 0: More input data follows the present set of data, starting with the text card.

If INP = 1: This is the last set of input data.

Card (1P5E14.6)

1. CVLA = Convergence limit for attitude angle calculation.
2. CVLR = Convergence limit for rupture boundary iteration.

List of L/D-Ratios (1P5E14.6)

Give NLD-values of the length-to-diameter ratio, L/D, 5 values per card.

List of Bearing Reynolds Numbers (1P5E14.6)

Give NRE-values of the bearing Reynolds number, R_e, 5 values per card.

Data for Journal Center Position

The following two input items, namely the card with $\varepsilon_B \cos\varphi_B$ and the list of $\varepsilon_B \sin\varphi_B$, must be repeated together NECA times (Item 6, control card).

Card (1PE14.6, I5)

1. $\varepsilon_B \cos\varphi_B$
2. NAT; If NAT $\geq$ 1: Number of $\varepsilon_B \sin\varphi_B$-values in input list which follows. The program iterates on the attitude angle to make horizontal force zero ($1 \leq$ NAT ≤ 25)

If NAT ≤ -1: The absolute value of NAT gives the number of $\varepsilon_B \sin\varphi_B$ values in input list which follows ($1 \leq |\text{NAT}| \leq 25$). No attitude angle iteration takes place. Set NAB = |NAT| , Item 7, control card.

List of $\varepsilon_B \sin\varphi_B$ Values (1P5E14.6)

Give |NAT| -values of $\varepsilon_B \sin\varphi_B$, 5 values per card.

Bearing Geometry Data (1P5E14.6)

Give NPAD cards (Item 5, control card). Each card contains 4 values:

1. Θ_{in} , degrees. The angle from the load line to the leading edge of the lobe

2. Θ_{out} , degrees. The angle from the load line to the trailing edge of the lobe

3. $\delta_p = \frac{r_p}{C}$. The lobe preload ($0 \leq \delta_p < 1$), dimensionless.
4. ψ_p . Preload angle, degrees.

```
$SETUP A(6)     DISK,PUNCH
$IBJOB          MAP,ALTIO ,NOGO
$IBFTC MAIPRO   DECK
C               MECHANICAL TECHNOLOGY INC., J. LUND, 5-25-67
C               PN0375 - PARTIAL ARC OR LOBED BRG, TURBULENT
      DIMENSION D(37,13,13),G(37,13,13),B(37,13),E(37,13),F(37,13),
     1P(37,13),ALFAP(37,13),ALFAM(37,13),H32(37),THA(37),AG1(37),
     2AG4(37),CS(37),SS(37),BT(37),KI(37),JSX(13),JEX(13),SX(13),EX(13),
     3THS(13),THE(13)
      COMMON D,G,B,E,F,P,ALFAP,ALFAM,H32,THA,AG1,AG4,CS,SS,BT,KI,JSX,
     1JEX,SX,EX,THS,THE,DX,DZ,CSII,ALPH
      DIMENSION RH(250),CFR(250),GXC(250),GX1(250),GX2(250),GX3(250),
     1GX4(250),GX5(250),ALDS(20),REYN(30),ECAST(20),NATS(20),THN(12),
     2THT(12),PRLD(12),PRAN(12),ESAS(25),AG2(37) ,AG3(37),AF1(37),
     3AF2(37),AF3(37),AF4(37),AF5(37),AF6(37),AF7(37),AF8(37),AF9(37),
     4CFP(12),       C(13),FXP(12),FYP(12),QZT(12),QXL(12),QXT(12),
     5SXXP(12),SXYP(12),SYXP(12),SYYP(12),BXXP(12),BXYP(12),BYXP(12),
     6BYYP(12),X(3),Y(3),HMN(12),IHM(12)
      DIMENSION ESAST(20,25),A(13,13),AK(13,13),GK(13,1)
      DO 10 I=1,12
   10 IHM(I)=I
      READ(5,98) NRET
      DO 11 I=1,NRET
      READ(5,99) RH(I),CFR(I),GXC(I),GX1(I)
   11 READ(5,99) GX2(I),GX3(I),GX4(I),GX5(I)
      NPUN1=1
   12 READ(5,100)
      READ(5,101) M,N,NLD,NRE,NPAD,NECA,NAB,NRP,NPW,NRW,NPUN,INP
      READ(5,102) CVLA,CVLR
      WRITE(6,100)
      WRITE(6,104)
      WRITE(6,105)M,N,NLD,NRE,NPAD,NECA,NAB,NRP,NPW,NRW,NPUN,INP
      WRITE(6,106)
      WRITE(6,102)CVLA,CVLR
      WRITE(6,107)
      READ(5,102) (ALDS(I),I=1,NLD)
      WRITE(6,102)(ALDS(I),I=1,NLD)
      READ(5,102) (REYN(I),I=1,NRE)
      WRITE(6,108)
      WRITE(6,102)(REYN(I),I=1,NRE)
      WRITE(6,109)
      DO 13 I=1,NECA
      READ(5,103) ECAST(I),NATS(I)
      J1=IABS(NATS(I))
      READ(5,102) (ESAST(I,J),J=1,J1)
      WRITE(6,110)ECAST(I),NATS(I)
      WRITE(6,111)
   13 WRITE(6,102)(ESAST(I,J),J=1,J1)
      WRITE(6,112)
      WRITE(6,113)
      DO 14 I=1,NPAD
      READ(5,102) THN(I),THT(I),PRLD(I),PRAN(I)
   14 WRITE(6,102)THN(I),THT(I),PRLD(I),PRAN(I)
      M3=M+1
      M1=M-1
      M2=M-2
      NC=N+1
      HM=M
      NLD1=1
   20 ALD=ALDS(NLD1)
      C2=N
      DZ=ALD/C2
```

```
      DELX3=1.0/DZ/DZ
      DZ2=DELX3+DELX3
      SPD2=1.0471976/C2
      SPD3=0.021816616/C2
      SPD4=0.78539816/C2
      SPD5=0.043633231/ALD*C2/ALD
      SPD6=0.17453293/C2
      NRE1=1
   21 RE=REYN(NRE1)
      NEC1=1
   22 ECA=ECAST(NEC1)
      NAT=NATS(NEC1)
      IF(NAT) 23,24,24
   23 NAB1=-NAT
      J1=NAB1
      GO TO 25
   24 NAB1=NAB
      J1=NAT
   25 DO 26 J=1,J1
   26 ESAS(J)=ESAST(NEC1,J)
      ESA=ESAS(1)
      NA=1
      NA1=0
      NA2=1
   32 IPD=1
   34 DX=(THT(IPD)-THN(IPD))/HM*0.017453293
      DELY3=1.0/DX/DX
      DX2=DELY3+DELY3
      SPD=SPD6*DX
      IF(NA-1) 30,27,30
   27 IF(NPUN) 28,30,29
   28 IF(NPUN+1) 29,30,30
   29 C1=THT(IPD)-THN(IPD)
      C2=(180.0-THN(IPD))/C1
      WRITE(10,114)ALD,C1,C2,RE,NPUN1
      NPUN1=NPUN1+1
   30 C1=0.017453293*PRAN(IPD)
      C2=SIN(C1)
      C1=COS(C1)
      C1=ECA-C1*PRLD(IPD)
      C2=ESA-C2*PRLD(IPD)
      ECC=SQRT(C1*C1+C2*C2)
      IF(1.0-ECC) 15,15,18
   15 WRITE(6,140)IPD
      IF(NA-NAB1) 16,17,17
   16 NA=NA+1
      GO TO 443
   17 WRITE(6,134)
      GO TO 485
   18 ATP=TANV(C1,C2)
      C1=ATP+180.0
      KRB=1
      KRE=1
      KBE=1
      C2=THN(IPD)
      C3=C2+360.0
      C4=THT(IPD)
      C5=C4-360.0
   35 IF(C1-360.0) 37,36,36
   36 C1=C1-360.0
      GO TO 35
   37 IF(ATP-C4) 39,38,38
```

```
 38 IF(C1-C4) 39,43,43
 39 IF(ATP-C2) 40,40,44
 40 IF(ATP-C5) 44,41,41
 41 IF(C2-C1) 45,42,42
 42 IF(C1-C5) 45,43,43
 43 KBE=-1
    GO TO 49
 44 KRB=0
    KBE=0
 45 IF(C1-C4) 46,49,49
 46 IF(C1-C2) 47,47,48
 47 IF(C1-C5) 48,49,49
 48 KRE=0
    KBE=0
 49 D13=0.0
    CSII=C2-ATP
    ALPH=C4-C2
    C3=0.017453293*CSII
    DO 60 J=1,MB
    C1=COS(C3)
    C2=SIN(C3)
    CS(J)=C1
    SS(J)=C2
    THA(J)=C3
    C4=ECC*C1
    C5=ECC*C2
    C6=1.0+C4
    C7=C5*C5
    C8=SQRT(C6)
    C9=C6*C8
    IF(RE) 50,50,51
 50 D1=1.0
    D2=1.0
    D3=0.0
    D4=0.0
    D5=0.0
    D6=0.0
    D7=1.0
    D12=1.0
    GO TO 56
 51 D8=RE*C6
    DO 55 K=1,NRET
    IF(D8-RH(K)) 52,52,53
 52 D9=RH(K)-RH(K-1)
    D6=(RH(K)-D8)/D9
    D7=(D8-RH(K-1))/D9
    D1=D6*GXC(K-1)+D7*GXC(K)
    D12=D6*CFR(K-1)+D7*CFR(K)
    D14=D6*GX4(K-1)+D7*GX4(K)
    C2=SQRT(D1)
    D3=RE*(D6*GX1(K-1)+D7*GX1(K))
    D4=RE*(D6*GX2(K-1)+D7*GX2(K))
    D5=RE*(D6*GX3(K-1)+D7*GX3(K))
    D6=RE*(D6*GX5(K-1)+D7*GX5(K))
    D7=D14
    GO TO 56
 53 IF(NRET-K) 54,54,55
 54 D8=RH(K)
    GO TO 52
 55 CONTINUE
 56 D11=C9*D2
    H32(J)=D11
```

```
      D10=1.0/(C6*C6)
      D8=1.5/C6+0.5*D3
      D9=0.75*D10+(1.5/C6-0.25*D3)*D3+0.5*D4
      D10=-0.375/C6*D10+0.375*D3*(3.0*D10-D4-D4+D3*(D3-3.0/C6))
     1+2.25/C6*D4+0.5*D5
      AG1(J)=D7
      AG2(J)=C1*D6*D11
      AG3(J)=C2*D6*D11
      D4=C1*D8
      D5=C2*D8
      D6=C7*D9-C4*D8
      AG4(J)=D1
      AF1(J)=(-C1*D8+(2.0*C5*C2-C4*C1)*D9+C7*C1*D10-D4*D6)*D11
      AF2(J)=(D4*C5-C2)/D11*6.0
      AF3(J)=12.0/D11*C1
      AF4(J)=-D4
      AF5(J)=-D6
      AF6(J)=-6.0/D11*C2
      AF7(J)=(-C2*D8-(2.0*C5*C1+C4*C2)*D9+C7*C2*D10-D5*D6)*D11
      AF8(J)=(D5*C5+C1)/D11*6.0
      AF9(J)=-D5
      D14=D12/C6*DX
      IF(J-1) 67,67,57
   67 HLE=C6
      GO TO 58
   57 IF(MB-J) 68,68,59
   68 HTE=C6
   58 D14=0.5*D14
   59 D13=D13+D14
   60 C3=C3+DX
      IF(KRE) 70,69,70
   69 HMN(IPD)=1.0-ECC
      GO TO 73
   70 IF(HLE-HTE) 72,72,71
   71 HMN(IPD)=HTE
      GO TO 73
   72 HMN(IPD)=HLE
   73 C5=MB
      DO 63 J=1,MB
      BT(J)=1.0
      NI(J)=1
      DO 62 I=1,N
      JSX(I)=2
      JEX(I)=M
      SX(I)=1.0
      EX(I)=C5
      THS(I)=CSII
      THE(I)=CSII+ALPH
      ALFAM(J,I)=1.0
   62 ALFAP(J,I)=1.0
   63 CONTINUE
      JS=1
      JF=MB
      JS1=2
      JF1=MB-1
      NIT=0
      ERR=1.0
      IF(NPW) 65,64,65
   64 IF(NRW) 65,66,65
   65 WRITE(6,116)IPD
   66 IF(KBE) 423,301,301
  301 DO 303 I=1,N
```

```
      DO 302 K=1,N
      D(JS,I,K)=0.0
  302 G(JS,I,K)=0.0
      G(JS,I,I)=1.0
      GK(I,1)=0.0
      DO 303 J=1,MB
      P(J,I)=0.0
      E(J,I)=0.0
  303 F(J,I)=0.0
      DO 326 J=JS1,JF1
      C1=AF5(J)
      C2=AF6(J)
      C3=AG1(J)
      C4=C3*DELX3
      DO 305 I=1,N
      P(J,I)=0.0
      DO 304 K=1,N
  304 A(I,K)=0.0
      B(J,I)=0.0
      C(I)=0.0
  305 F(J,I)=0.0
      KJ=KI(J)
      KJ1=KJ-1
      BT1=1.0
      IF(KJ1) 306,327,311
  306 DO 310 I=1,N
      A(I,I)=C1-DELY3-DELY3-C4-C4
      B(J,I)=DELY3
      C(I)=DELY3
      F(J,I)=C2
      IF(N-I) 307,307,308
  307 A(I,I-1)=C4+C4
      GO TO 310
  308 A(I,I+1)=C4
      IF(I-1) 310,310,309
  309 A(I,I-1)=C4
  310 CONTINUE
      GO TO 320
  311 BT1=BT(J)
      DO 312 I=1,KJ1
  312 A(I,I)=1.0
  327 DO 319 I=KJ,N
      C5=ALFAP(J,I)
      C9=ALFAM(J,I)
      D1=C5+C9
      D1=DX2/D1
      D2=D1/C5
      D1=D1/C9
      C9=DX2/C5/C9
      F(J,I)=C2
      IF(I-KJ1) 313,313,314
  313 C6=C1-C9-DZ2/BT1*C3
      C8=DZ2/(1.0+BT1)*C3
      C7=C8/BT1
      GO TO 315
  314 C6=C1-C9-DZ2*C3
      C8=DELX3*C3
      C7=C8
  315 A(I,I)=C6
      IF(N-I) 317,317,316
  316 A(I,I-1)=C7
      A(I,I+1)=C8
```

```
      GO TO 318
  317 A(I,I-1)=C7+C8
  318 B(J,I)=D1
      C(I)=U2
  319 CONTINUE
  320 DO 322 I=1,N
      DO 321 K=1,N
  321 AK(I,K)=A(I,K)+B(J,I)*D(J-1,I,K)
  322 CONTINUE
      CALL MATINV(AK,N,GK,0,DET,ID)
      GO TO (324,323),ID
  323 WRITE(6,139)IPD
      GO TO  425
  324 DO 325 I=1,N
      DO 325 K=1,N
      C5=AK(I,K)
      G(J,I,K)=C5
  325 D(J,I,K)=-C5*C(K)
  326 CONTINUE
      CALL PRESS(JS1,JF1,N,NRP)
      IF(NRW) 341,344,344
  341 WRITE(6,117)NIT
      WRITE(6,118)
      DO 342 I=1,N
  342 WRITE(6,119)I,SX(I),EX(I),THS(I),THE(I)
      IF(NRW+1) 343,344,344
  343 WRITE(6,120)
  344 DO 347 J=JS,JF
      C3=H32(J)
      DO 345 I=1,N
      C4=E(J,I)/C3
      E(J,I)=C4
  345 P(J,I)=C4*ECC
      IF(NRW+1) 346,347,347
  346 WRITE(6,121)(P(J,I),I=1,N)
  347 CONTINUE
      CALL SIMP(N,FR,FT)
      FR=SPD*FR
      FT=SPD*FT
      BRDA=-FR-FR
      BTDA=-FT-FT
      FR=ECC*FR
      FT=ECC*FT
      SMFV=SQRT(FR*FR+FT*FT)
      IF(SMFV) 423,423,349
  349 SMF=1.0/SMFV
      ATT=TANV(FR,FT)
      CFP(IPD)=3.1415927*D13+0.5*ECC*FT
      C1=0.017453293*ATP
      CATP=COS(C1)
      SATP=SIN(C1)
      FXP(IPD)=FR*CATP+FT*SATP
      FYP(IPD)=FT*CATP-FR*SATP
      IF(KRB) 351,362,351
  351 IF((N/2)*2-N) 352,353,353
  352 QXS=16.0*P(2,N)-4.0*P(3,N)
      J1=N-2
      GO TO 354
  353 QXS=2.0*P(3,N)-8.0*P(2,N)
      J1=N-1
  354 DO 355 I=1,J1,2
  355 QXS=QXS+32.0*P(2,I)-8.0*P(3,I)+16.0*P(2,I+1)-4.0*P(3,I+1)
```

```
      QXS=1.5707963*(1.0+ECC*CS(1))-SPD3*QXS/DX*(H32(1)**2)
  356 IF(KRE) 357,367,357
  357 IF((N/2)*2-N) 358,359,359
  358 QXE=16.0*P(M,N)-4.0*P(M1,N)
      J1=N-2
      GO TO 360
  359 QXE=2.0*P(M1,N)-8.0*P(M,N)
      J1=N-1
  360 DO 361 I=1,J1,2
  361 QXE=QXE+32.0*P(M,I)-8.0*P(M1,I)+16.0*P(M,I)-4.0*P(M1,I)
      QXE=1.5707963*(1.0+ECC*CS(MB))+SPD3*QXE/DX*(H32(MB)**2)
      GO TO 372
  362 QXS=0.0
      DO 366 I=1,N
      C1=THS(I)/57.29578
      C1=1.0+ECC*COS(C1)
      IF(I-2) 363,366,364
  363 C1=4.0*C1
      GO TO 366
  364 IF(N-I) 366,366,365
  365 C1=C1+C1
  366 QXS=QXS+C1
      QXS=SPD4*QXS
      GO TO 356
  367 QXE=0.0
      DO 371 I=1,N
      C1=1.0+ECC*COS(THE(I)/57.29578)
      IF(I-2) 368,371,369
  368 C1=4.0*C1
      GO TO 371
  369 IF(N-I) 371,371,370
  370 C1=C1+C1
  371 QXE=QXE+C1
      QXE=SPD4*QXE
  372 QZ2=AG1(M)*(H32(M)**2)*(16.0*P(M,1)-4.0*P(M,2))
      J1=2
      IF((M/2)*2-M) 373,374,374
  373 QZ2=QZ2+3.0*AG1(2)*(H32(2)**2)*(4.0*P(2,1)-P(2,2))
     1-0.25*AG1(3)*(H32(3)**2)*(4.0*P(3,1)-P(3,2))
      J1=3
  374 DO 375 J=J1,M2,2
  375 QZ2=QZ2+4.0*AG1(J)*(H32(J)**2)*(4.0*P(J,1)-P(J,2))
     1+2.0*AG1(J+1)*(H32(J+1)**2)*(4.0*P(J+1,1)-P(J+1,2))
      QZT(IPD)=SPD5*DX*QZ2
      QXL(IPD)=QXS
      QXT(IPD)=QXE
      ERR=0.0
      IF(NRP) 387,387,380
  380 IF(KBE) 387,381,387
  381 ERR=ECC
      CALL RUPT(M,N,KRB,KRE,NRW,JS,JF,JS1,JF1,ERR)
      NIT=NIT+1
      IF(NRW) 382,383,382
  382 WRITE(6,122)NIT,ERR
      J1=1
  383 IF(NRW) 384,385,385
  384 WRITE(6,124)
      WRITE(6,123)SMF,ECC,ATP,ATT,FR,FT
      WRITE(6,125)
      WRITE(6,123)CFP(IPD),QXS,QXE,QZT(IPD),FXP(IPD),FYP(IPD)
      IF(J1) 388,388,385
  385 IF(CVLR-ERR) 386,387,387
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```
  386 IF(NRP-NIT) 387,301,301
  387 J1=0
      IF(NPW) 384,392,384
  388 WRITE(6,122)NIT,ERR
      WRITE(6,118)
      DO 389 I=1,N
  389 WRITE(6,119)I,SX(I),EX(I),THS(I),THE(I)
      IF(NPW) 390,392,392
  390 WRITE(6,120)
      DO 391 J=1,MB
  391 WRITE(6,121)(P(J,I),I=1,N)
  392 DO 393 J=1,MB
      DO 393 I=1,N
      E(J,I)=0.0
  393 F(J,I)=0.0
      DO 402 J=JS1,JF1
      C1=AF1(J)
      C2=AF2(J)
      C3=-AG2(J)
      KJ=KI(J)
      IF(KJ) 394,394,395
  394 KJ=1
  395 DO 401 I=KJ,N
      C4=P(J,I)
      IF(I-KJ) 396,396,397
  396 C5=0.0
      GO TO 398
  397 C5=P(J,I-1)
  398 IF(N-I) 399,399,400
  399 C6=C5
      GO TO 401
  400 C6=P(J,I+1)
  401 F(J,I)=C1*C4+C2+C3*DELX3*(C5+C6-C4-C4)
  402 CONTINUE
      CALL PRESS(JS1,JF1,N,-1)
      DO 404 J=JS,JF
      C1=H32(J)
      C2=AF4(J)
      DO 403 I=1,N
  403 E(J,I)=E(J,I)/C1+C2*P(J,I)
  404 CONTINUE
      CALL SIMP(N,C1,C2)
      DRDE=SPD*C1
      DTDE=SPD*C2
      DO 413 J=JS1,JF1
      C1=AF7(J)
      C2=AF8(J)
      C3=-AG3(J)
      KJ=KI(J)
      IF(KJ) 405,405,406
  405 KJ=1
  406 DO 412 I=KJ,N
      C4=P(J,I)
      IF(I-KJ) 407,407,408
  407 C5=0.0
      GO TO 409
  408 C5=P(J,I-1)
  409 IF(N-I) 410,410,411
  410 C6=C5
      GO TO 412
  411 C6=P(J,I+1)
  412 F(J,I)=C1*C4+C2+C3*DELX3*(C5+C6-C4-C4)
```

```
  413 CONTINUE
      CALL PRESS(JS1,JF1,N,-1)
      DO 415 J=JS,JF
      C1=H32(J)
      C2=AF9(J)
      DO 414 I=1,N
  414 E(J,I)=E(J,I)/C1+C2*P(J,I)
  415 CONTINUE
      CALL SIMP(N,C1,C2)
      DRDA=SPD*C1
      DTDA=SPD*C2
      DO 419 J=JS1,JF1
      C1=AF3(J)
      KJ=KI(J)
      IF(KJ) 416,416,417
  416 KJ=1
  417 DO 418 I=KJ,N
  418 E(J,I)=C1
  419 CONTINUE
      CALL PRESS(JS1,JF1,N,-1)
      DO 421 J=JS,JF
      C1=H32(J)
      DO 420 I=1,N
  420 E(J,I)=E(J,I)/C1
  421 CONTINUE
      CALL SIMP(N,C1,C2)
      BRDE=SPD*C1
      BTDE=SPD*C2
      C1=DRDE*CATP+DTDE*SATP
      C2=DRDA*CATP+DTDA*SATP
      SXXP(IPD)=C1*CATP-C2*SATP
      SXYP(IPD)=C1*SATP+C2*CATP
      C1=DRDE*SATP-DTDE*CATP
      C2=DRDA*SATP-DTDA*CATP
      SYXP(IPD)=C1*CATP-C2*SATP
      SYYP(IPD)=C1*SATP+C2*CATP
      C1=BRDE*CATP+BTDE*SATP
      C2=BRDA*CATP+BTDA*SATP
      BXXP(IPD)=C1*CATP-C2*SATP
      BXYP(IPD)=C1*SATP+C2*CATP
      C1=BRDE*SATP-BTDE*CATP
      C2=BRDA*SATP-BTDA*CATP
      BYXP(IPD)=C1*CATP-C2*SATP
      BYYP(IPD)=C1*SATP+C2*CATP
      C1=(SXXP(IPD)*BYYP(IPD)+SYYP(IPD)*BXXP(IPD)-SXYP(IPD)*BYXP(IPD)
     1-SYXP(IPD)*BXYP(IPD))/(BXXP(IPD)+BYYP(IPD))
      C2=((SXXP(IPD)-C1)*(SYYP(IPD)-C1)-SXYP(IPD)*SYXP(IPD))/
     1(BXXP(IPD)*BYYP(IPD)-BXYP(IPD)*BYXP(IPD))
      C1=C2/C1
      C3=SQRT(ABS(C2))
      C4=SMF*C1
      C1=C1/39.478418
      IF(NPW) 422,425,422
  422 WRITE(6,126)
      WRITE(6,123)DRDE,DTDE,DRDA,DTDA,BRDE,BTDE,BRDA,BTDA
      WRITE(6,127)
      WRITE(6,123)SXXP(IPD),SXYP(IPD),SYXP(IPD),SYYP(IPD),BXXP(IPD),
     1BXYP(IPD),BYXP(IPD),BYYP(IPD)
      WRITE(6,128)
      WRITE(6,123)C1,C4,C3,C2
      GO TO 425
  423 CFP(IPD)=3.1415927*D13
```

```
      QXL(IPD)=1.5707963*(1.0+ECC*CS(1))
      QXT(IPD)=QXL(IPD)
      QZT(IPD)=0.0
      FXP(IPD)=0.0
      FYP(IPD)=0.0
      SXXP(IPD)=0.0
      SXYP(IPD)=0.0
      SYXP(IPD)=0.0
      SYYP(IPD)=0.0
      BXXP(IPD)=0.0
      BXYP(IPD)=0.0
      BYXP(IPD)=0.0
      BYYP(IPD)=0.0
      IF(NPW) 424,425,424
  424 WRITE(6,129)
      WRITE(6,130)
      WRITE(6,123)CFP(IPD),QXL(IPD)
  425 IPD=IPD+1
      IF(NPAD-IPD) 426,34,34
  426 FXB=0.0
      FYB=0.0
      SXXB=0.0
      SXYB=0.0
      SYXB=0.0
      SYYB=0.0
      BXXB=0.0
      BXYB=0.0
      BYXB=0.0
      BYYB=0.0
      RCFW=0.0
      QXLB=0.0
      QZTB=0.0
      QBT=0.0
      DO 432 I=1,NPAC
      IF(NPAD-I) 427,427,428
  427 C1=QXT(I)-QXL(1)
      GO TO 429
  428 C1=QXT(I)-QXL(I+1)
  429 IF(C1) 430,430,431
  430 C1=0.0
  431 QBT=QBT+C1
      QXLB=QXLB+QXL(I)
      QZTB=QZTB+QZT(I)
      RCFW=RCFW+CFP(I)
      FXB=FXB+FXP(I)
      FYB=FYB+FYP(I)
      SXXB=SXXB+SXXP(I)
      SXYB=SXYB+SXYP(I)
      SYXB=SYXB+SYXP(I)
      SYYB=SYYB+SYYP(I)
      BXXB=BXXB+BXXP(I)
      BXYB=BXYB+BXYP(I)
      BYXB=BYXB+BYXP(I)
  432 BYYB=BYYB+BYYP(I)
      QBT=QBT+QZTB
      IF(NPAD-1) 436,436,437
  436 QBT=QZTB+QXT(1)
  437 SMFV=SQRT(FXB*FXB+FYB*FYB)
      IF(NPW) 435,433,435
  433 IF(NAT) 435,435,434
  434 IF(NPUN) 435,440,440
  435 WRITE(6,132)NA
```

```
      WRITE(6,131)ESA,FYB
      WRITE(6,133)
      WRITE(6,123)FXB,RCFW,QZTB,QBT,QXLB,SMFV
      WRITE(6,127)
      WRITE(6,123)SXXB,SXYB,SYXB,SYYB,BXXB,BXYB,BYXB,BYYB
  440 IF(NA-NAB1) 442,441,441
  441 WRITE(6,134)
      GO TO 475
  442 NA=NA+1
      IF(NAT) 443,441,444
  443 ESA=ESAS(NA)
      GO TO 32
  444 IF(FXB) 446,445,447
  445 FXB=1.0
  446 IF(NA1) 447,452,447
  447 IF(CVLA-ABS(FYB/FXB)) 448,475,475
  448 IF(NA1) 455,449,454
  449 IF(NA2) 451,451,450
  450 NA2=0
      GO TO 452
  451 IF(FYB*X(1)) 453,475,452
  452 X(1)=FYB
      Y(1)=ESA
      GO TO 443
  453 NA1=1
      X(3)=FYB
      Y(3)=ESA
      ESA=(Y(1)+ESA)/2.0
      GO TO 32
  454 NA1=-1
      X(2)=FYB
      Y(2)=ESA
      GO TO 465
  455 IF(ESA-Y(2)) 461,456,456
  456 IF(X(2)*X(3)) 457,457,458
  457 X(1)=X(2)
      X(2)=FYB
      Y(1)=Y(2)
      Y(2)=ESA
      GO TO 465
  458 IF(FYB*X(1)) 459,459,460
  459 X(3)=FYB
      Y(3)=ESA
      GO TO 465
  460 X(2)=FYB
      Y(2)=ESA
      GO TO 465
  461 IF(X(1)*X(2)) 462,462,463
  462 X(3)=X(2)
      X(2)=FYB
      Y(3)=Y(2)
      Y(2)=ESA
      GO TO 465
  463 IF(FYB*X(3)) 464,464,460
  464 X(1)=FYB
      Y(1)=ESA
  465 C1=Y(2)-Y(1)
      C2=Y(3)-Y(2)
      C3=Y(3)-Y(1)
      C4=(X(3)-X(2))/(C2*C3)
      C5=(X(1)-X(2))/(C1*C3)
      C3=C4+C5
```

```
      C4=C1*C4-C2*C5
      C5=X(2)
      IF(C3) 467,466,467
  466 C6=-C5/C4
      GO TO 470
  467 C6=0.5*C4/C3
      C7=SQRT(ABS(C6*C6-C5/C3))
      IF(C6) 468,469,469
  468 C7=-C7
  469 C6=C7-C6
  470 ESA=Y(2)+C6
      GO TO 32
  475 ECC=SQRT(ECA*ECA+ESA*ESA)
      ATT=TANV(ECA,ESA)
      IF(NPAD-1) 476,476,477
  476 QBT=QZT(1)+QXT(1)
  477 C1=(SXXB*BYYB+SYYB*BXXB-SXYB*BYXB-SYXB*BXYB)/(BXXB+BYYB)
      C2=((SXXB-C1)*(SYYB-C1)-SXYB*SYXB)/(BXXB*BYYB-BXYB*BYXB)
      C1=C1/C2
      C3=SQRT(ABS(C2))
      C5=C1/39.478418
      WRITE(6,135)
      WRITE(6,123)ALD,RE,ECA,ESA,ECC,ATT,SMFV,FYB
      WRITE(6,136)
      WRITE(6,123)RCFW,QZTB,QBT,QXLB,FXB,C5,C3,C2
      WRITE(6,127)
      WRITE(6,123)SXXB,SXYB,SYXB,SYYB,BXXB,BXYB,BYXB,BYYB
      IF(SMFV) 479,479,478
  478 SMF=1.0/SMFV
      RCFWS=SMF*RCFW
      FXBS=SMF*FXB
      FYBS=SMF*FYB
      C1=SMF*C1
      SXXS=SMF*SXXB
      SXYS=SMF*SXYB
      SYXS=SMF*SYXB
      SYYS=SMF*SYYB
      BXXS=SMF*BXXB
      BXYS=SMF*BXYB
      BYXS=SMF*BYXB
      BYYS=SMF*BYYB
      WRITE(6,137)
      WRITE(6,123)SMF,FXBS,FYBS,RCFWS,C1
      WRITE(6,138)
      WRITE(6,123)SXXS,SXYS,SYXS,SYYS,BXXS,BXYS,BYXS,BYYS
  479 WRITE(6,141)
      WRITE(6,143)(IHM(I),I=1,NPAD)
      WRITE(6,142)(HMN(I),I=1,NPAD)
      IF(NPUN) 480,485,481
  480 IF(NPUN+1) 481,485,485
  481 C6=SXYB*SYXB-BXYB*BYXB
      C7=SXYB*BYXB+SYXB*BXYB
      C8=SYYB*SYYB+BYYB*BYYB
      C9=BXXB-(SYYB*C7-BYYB*C6)/C8
      C8=SXXB-(SYYB*C6+BYYB*C7)/C8
      WRITE(10,115)ECA,SMFV,RCFW,QXLB,ECC,NPUN1
      NPUN1=NPUN1+1
      WRITE(10,115)C8,C9,SYYB,BYYB,ATT,NPUN1
      NPUN1=NPUN1+1
      IF(NPUN-1) 485,485,482
  482 WRITE(10,114) SXXB,SXYB,SYXB,SYYB
      NPUN1=NPUN1+1
```

```
      WRITE(10,114) BXXB,BXYB,BYXB,BYYB
      NPUN1=NPUN1+1
  485 NEC1=NEC1+1
      IF(NECA-NEC1) 486,22,22
  486 NRE1=NRE1+1
      IF(NRE-NRE1) 487,21,21
  487 NLD1=NLD1+1
      IF(NLC-NLD1) 488,20,20
  488 IF(INP) 489,12,489
  489 STOP
   98 FORMAT(I5)
   99 FORMAT(4E15.7)
  100 FORMAT(72H
     1                    )
  101 FORMAT(12I5)
  102 FORMAT(1P5E14.6)
  103 FORMAT(1PE14.6,I5)
  104 FORMAT(1H0,117HCIRC.DIV    AX.DIV     NO.L/D     NO.REYN    NO.PAD   N
     1O.E*COSA  NO.A-IT  NO.RUP.IT PAD.DATA  RUPT.DATA ATT.A.DATA  INPUT
     1)
  105 FORMAT(I6,11I10)
  106 FORMAT(1H0,27HCONV.LIM.ATT.A CONV.LIM.RUP)
  107 FORMAT(1H0,10HL/D RATIOS)
  108 FORMAT(1H0,16HREYNOLDS NUMBERS)
  109 FORMAT(1H0,17HECC*COS(ATT) DATA)
  110 FORMAT(1H0,13HECC*COS(ATT)=1PE13.6,14H  NO.E*SIN(A)=I3)
  111 FORMAT(21H LIST OF ECC*SIN(ATT))
  112 FORMAT(1H0,21HBEARING GEOMETRY DATA)
  113 FORMAT(55H  INLET ANGLE   OUTLET ANGLE    PRELOAD    ANGLE PRLD-X)
  114 FORMAT(4E13.5,I28)
  115 FORMAT(5E13.5,I15)
  116 FORMAT(1H1,15HBEARING PAD NO.,I2)
  117 FORMAT(///1H0,21HRUPTURE ITERATION NO.,I2)
  118 FORMAT(1H0,60H I    BEGIN.INDEX     END INDEX     BEGIN ANGLE     E
     1ND ANGLE)
  119 FORMAT(I3,1P4E15.7)
  120 FORMAT(1H0,21HPRESSURE DISTRIBUTION)
  121 FORMAT(1PE12.4,1P9E11.4)
  122 FORMAT(22H RUPTURE ITERATION NO.I2,9H   ERROR=1PE13.6)
  123 FORMAT(1P8E15.7)
  124 FORMAT(1H0,86HSOMMERFELD NO    ECC.RATIO      ATT.ANGLE    CALC.AT
     1T.ANG.       F-R/S*W        F-T/S*W)
  125 FORMAT(1H0,86H 1/S*(R/C*F/W)  (Q-X/NDLC)IN   (Q-X/NDLC)OUT (Q-Z/ND
     1LC)SIDE     F-X/S*W        F-Y/S*W)
  126 FORMAT(1H0,12H     DFR/SDE,8X,7HDFT/SDE,8X,7HDFR/SDA,8X7HDFT/SDA,7
     1X,9HDFR/SDEDT,6X,9HDFT/SDEDT,6X,9HDFR/SDADT,6X,9HDFT/SDADT)
  127 FORMAT(1H0,4X,7HCKXX/SW,8X,7HCKXY/SW,8X,7HCKYX/SW,8X,7HCKYY/SW,7X,
     18HCHBXX/SW,7X,8HCWBXY/SW,7X,8HCWBYX/SW,7X,8HCWBYY/SW)
  128 FORMAT(1H0,58HCMN/MUDL(R/C)2     CMW**2/W        FREQ/W       (FREQ
     1/W)**2)
  129 FORMAT(1H0,15HPAD IS UNLOADED)
  130 FORMAT(1H0,28H 1/S*(R/C*F/W)  (Q-X/NDLC)IN)
  131 FORMAT(1H0,13HECC*SIN(ATT)=1PE13.6,17H  HORIZ.FORCE/SW=1PE13.6)
  132 FORMAT(///1H0,33HBEARING RESULTS,ATT.ANGLE IT.NO.,I2)
  133 FORMAT(1H0,4X,7HF-X/S*W,5X,12H1/S(R/C*F/W),2X,14H(Q-Z/NDLC)SIDE,2X
     1,13H(Q/NDLC)TOTAL,1X,14H(Q-X/NDLC)CIRC ,7X,3H1/S)
  134 FORMAT(1H0,40HNUMBER OF ATT.ANGLE ITERATIONS TOO LARGE)
  135 FORMAT(///1H0,6X,3HL/D,9X,9HREYN.NO.     ECC*COS(ATT)   ECC*SIN(A
     1TT)      ECC.RATIO      ATT.ANGLE          1/S            F-Y/S*W)
  136 FORMAT(1H0,116H 1/S*(R/C*F/W) (Q-Z/NDLC)SIDE  (Q/NDLC)TOTAL Q-X/ND
     1LC)CIRC     F-X/S*W    CMN/MUDL(R/C)2      FREQ/W       (FREQ/W**2)
  137 FORMAT(1H0,72H SOMMERFELD NO        F-X/W          F-Y/W        (R/C
```

```
      1)*(F/W)      CMW**2/W)
  138 FORMAT(1H0,5X,6HCKXX/W,9X,6HCKXY/W,9X,6HCKYX/W,9X,6HCKYY/W,8X,7HCW
      1BXX/W,8X,7HCWBXY/W,8X,7HCWBYX/W,8X,7HCWBYY/W)
  139 FORMAT(1H0,26HMATRIX IS SINGULAR,PAD NO.,I2)
  140 FORMAT(1H0,41HECCENTRICITY RATIO GREATER THAN 1,PAD NO.,I2)
  141 FORMAT(1H0,34HMINIMUM FILM THICKNESS FOR PAD NO.)
  142 FORMAT(1P10E12.4)                                                  211
  143 FORMAT(I7,9I12)                                                    212
      END                                                                213
$IBFTC SURPT    DECK
      SUBROUTINE RUPT(M,N,KRB,KRE,MP,JS,JF,JS1,JF1,ERR)                   2
      DIMENSION D(37,13,13),G(37,13,13),B(37,13),E(37,13),F(37,13),       3
     1P(37,13),ALFAP(37,13),ALFAM(37,13),H32(37),THA(37),AG1(37),         4
     2AG4(37),CS(37),SS(37),BT(37),KI(37),JSX(13),JEX(13),SX(13),EX(13),  5
     3THS(13),THE(13)                                                     6
      COMMON D,G,B,E,F,P,ALFAP,ALFAM,H32,THA,AG1,AG4,CS,SS,BT,KI,JSX,     7
     1JEX,SX,EX,THS,THE,DX,DZ,CSII,ALPH                                   8
      M=M                                                                 9
      N=N                                                                10
      KRB=KRB                                                            11
      KRE=KRE                                                            12
      MP=MP                                                              13
      JS=JS                                                              14
      JF=JF                                                              15
      JF1=JF1                                                            16
      JS1=JS1                                                            17
      ECC=ERR                                                            18
      N1=N-1                                                             19
      MB=M+1                                                             20
      NC=N+1                                                             21
      N2=NC+2                                                            22
      SER1=1.0                                                           23
      SER2=0.0                                                           24
      SER3=1.0                                                           25
      SER4=0.0                                                           26
      DR=0.1*DX                                                          27
      JS=1                                                               28
      JS1=2                                                              29
      JF=MB                                                              30
      JF1=M                                                              31
      DO 502 J=1,MB                                                      32
      BT(J)=1.0                                                          34
      KI(J)=0                                                            33
      DO 501 I=1,N                                                       35
      ALFAM(J,I)=1.0                                                     36
  501 ALFAP(J,I)=1.0                                                     37
  502 CONTINUE                                                           38
      C1=MB                                                              39
      DO 503 I=1,N                                                       40
      THS(I)=CSII                                                        41
      THE(I)=CSII+ALPH                                                   42
      JSX(I)=2                                                           43
  503 JEX(I)=M                                                           44
      DO 592 I=1,N                                                       45
      J1=NC-I                                                            46
      K7=I-1                                                             47
      C1=K7                                                              48
      Y4=C1*DZ                                                           49
      K7=0                                                               50
      IF(J1-N1) 509,508,507                                              51
  507 K5=1                                                               52
      GO TO 510                                                          53
```

```
  508 K5=-1
      GO TO 510
  509 K5=0
  510 DO 591 J=2,M
      PR= P(J,J1)
      IF(PR) 511,511,515
  511 IF(K7) 591,591,512
  512 K7=0
      IF(KRE) 591,513,591
  513 X4=1.0
      THC=THA(J-1)
      P2= P(J-1,J1)/3.0
      GX1=AG4(J-1)
      GX2=AG4(J)
      GZ1=AG1(J-1)
      GZ2=AG1(J)
      X2=X2E
      X3=X3E
      Z2=Z2E
      Z3=Z3E
      Y2=Y2E
      K2=K2E
      K3=K3E
      KX=J-1
      KJ=KX
      JP=KX
      KX1=1
      KX2=0
      Y1=MB-KX
      GO TO 525
  515 IF(K7) 591,516,591
  516 K7=1
      IF(KRB) 591,517,591
  517 IF(J-2) 591,518,520
  518 KRB=1
      DO 519 K=1,N
      ALFAM(J,K)=1.0
      SX(K)=1.0
      THS(K)=C9II
  519 JSX(K)=2
      SER2=0.0
      SER1=1.0
      BT(J)=1.0
      GO TO 591
  520 X4=-1.0
      THC=THA(J)
      GX1=AG4(J)
      GX2=AG4(J-1)
      GZ1=AG1(J)
      GZ2=AG1(J-1)
      P2=PR/3.0
      X2=X2B
      X3=X3B
      Z2=Z2B
      Z3=Z3B
      Y2=Y2B
      K2=K2B
      K3=K3B
      KX=MB-J
      KJ=J
      JP=KJ
      KX1=-1
```

```
      KX2=0
      Y1=J-1
  525 KI(KJ)=1
      GX2=(GX2-GX1)/DX
      GZ2=(GZ2-GZ1)/DX
      IF(K5) 527,528,526
  526 K2=KX
      Y1=DX*Y1
      GO TO 534
  527 K3=K2
      GO TO 530
  528 KQ=K2-KX
      IF(KQ) 530,530,529
  529 C2=KQ
      C2=C2*DX
      Z3=Z3+C2
      X3=X3+C2
  530 Y1=X2
      IF(Z2-X2) 531,532,532
  531 Y1=Z2
  532 KQ=K2-KX
      IF(KQ) 534,534,533
  533 C2=KQ
      C2=C2*DX
      Z2=Z2+C2
      X2=X2+C2
      Y1=Y1+C2
  534 C6=0.0
      C9=(Y2+DZ-Y4)/DR
      C3=-P2
      K4=0
  535 C6=C6+DR
  536 C7=C6*C9+Y4
      IF(K5) 538,539,537
  537 C7=0.0
      C9=0.0
      GO TO 545
  538 C9=(X2-C6)/C7*2.0
      GO TO 545
  539 D2=X3-X2
      D1=X3-C6
      IF(D1) 540,540,541
  540 C9=0.0
      GO TO 545
  541 IF(D2) 542,542,543
  542 C9=D1/C7*2.0
      GO TO 545
  543 C9=D1/D2
      IF(C9-1.0) 540,540,544
  544  C9= ALOG(ABS(C9))/ALOG(ABS(C7/Y2))*D1/C7
  545 C8=THC+X4*C6
      D3=COS(C8)
      D4=-ECC*SIN(C8)
      D5=1.0+ECC*D3
      D1=GZ1+C6*GZ2
      D2=GX1+C6*GX2
      D6=D5*D5*D5
      C5=(1.0+C9*C9)*C6
      C5=(C5/D2)*(C5/D6)*(D4/(1.0+D1*C9*C9))-P2
      C7=C6*C9+Y4
      IF(K4) 556,546,550
  546 C1=C3
```

```
      C3=C5
      IF(C1*C5) 549,556,547
  547 IF(C6-Y1) 535,548,548
  548 C6=Y1
      C7=Y4
      C8=THC+X4*C6
      KX2=1
      GO TO 557
  549 K4=1
      C9=C6-0.5*DR
      GO TO 536
  550 K4=-1
      D3=(C1+C3-C5-C5)/DR*2.0/DR
      D4=(C3-C1)/DR
      IF(D3) 552,551,552
  551 D5=-C5/D4
      GO TO 555
  552 D6=D4/D3*0.5
      D7=SQRT(ABS(D6**2-C5/D3))
      IF(D6) 553,554,554
  553 D5=-D6-D7
      GO TO 555
  554 D5=-D6+D7
  555 C6=C6+D5
      GO TO 536
  556 IF(C6-Y1) 557,558,548
  557 K4=0
      Y1=C7
      THC=C8
      IF(K5) 559,560,558
  558 Z1=C6
      Y1=0.0
      GO TO 561
  559 D3=Y1*Y1
      D3=(C6-Z2)/D3
      D4=0.0
      Z1=D3*DZ*DZ+Z2
      GO TO 561
  560 D5=Y1-Y4+DZ
      D6=Y4-DZ
      D7=(Z3-Z2)/D6
      D8=(C6-Z2)/D5
      D4=D5+D6
      D3=(D7+D8)/D4
      D4=(D8*D6-D7*D5)/D4
      Z1=DZ*(D3*DZ+D4)+Z2
  561 IF(Z1-DX) 565,565,562
  562 KI(KJ)=0
      JP=KJ
  563 KX=KX+1
      JP=JP+KX1
      Z1=Z1-DX
      Z2=Z2-DX
      C6=C6-DX
      IF(Z1) 564,564,563
  564 Z1=Z1+DX
      Z2=Z2+DX
      C6=C6+DX
      KX=KX-1
      JP=JP-KX1
  565 IF(K5) 566,566,580
  566 IF(Z1-Z2) 568,567,567
```

```
  567 Z1=Z2
      GO TO 580
  568 IF(Z2-DX) 580,569,570
  569 KQ=JP+KX1
      KI(KQ)=J1+1
      BT(KQ)=1.0
      GO TO 580
  570 KQ=KX+1
      JQ=JP
      DO 579 L=KC,K2
      JQ=JQ+KX1
      IF(KX1) 571,572,572
  571 IF(JQ-1) 579,579,573
  572 IF(JQ-MM) 573,579,579
  573 KI(JQ)=J1+1
      D5=L-KX
      C5=Z2-D5*DX
      IF(D3) 575,574,575
  574 D5=-D5/D4
      GO TO 576
  575 C6=D4/D3*0.5
      C5=-D6+SQRT(ABS(D6**2-D5/D3))
      D5=-D6+SQRT(ABS(D6**2-D5/D3))
  576 D5=D5/DZ
      IF(D5-1.0) 578,578,577
  577 D5=1.0
  578 BT(JQ)=D5
  579 CONTINUE
  580 D5=Z1/DX
      IF(K7) 586,581,586
  581 IF(K5) 585,585,582
  582 IF(KX2) 583,584,583
  583 KRE=1
      JF=MM
      JF1=M
      GO TO 591
  584 JF1=JP
      JF=JP+1
      K3E=KX
      X3E=Z1
      Z3E=Z1
  585 Y2E=Y1
      X2E=C6
      Z2E=Z1
      K2E=KX
      THE(J1)=57.29578*THC
      ALFAP(JP,J1)=D5
      D6=JP
      D6=D6+D5
      SER3=SER3+ABS(EX(J1))
      SER4=SER4+ABS(EX(J1)-D6)
      EX(J1)=D6
      JEX(J1)=JP
      GO TO 591
  586 IF(K5) 590,590,587
  587 IF(KX2) 588,589,588
  588 KRB=1
      JS=1
      JS1=2
      GO TO 591
  589 JS1=JP
      JS=JP-1
```

```
      K3B=KX
      X3B=Z1
      Z3B=Z1
  590 Y2B=Y1
      X2B=C6
      Z2B=Z1
      K2B=KX
      THS(J1)=57.29578*THC
      ALFAM(JP,J1)=D5
      D6=JP
      D6=D6-D5
      SER1=SER1+ABS(SX(J1))
      SER2=SER2+ABS(SX(J1)-D6)
      SX(J1)=D6
      JSX(J1)=JP
  591 CONTINUE
  592 CONTINUE
      ERR=SER2/SER1+SER4/SER3
      IF(MP+1) 593,596,596
  593 WRITE(6,599)
      WRITE(6,597)((ALFAP(J,I),I=1,N),J=1,MB)
      WRITE(6,597)((ALFAM(J,I),I=1,N),J=1,MB)
      WRITE(6,597)(BT(J),J=1,MB)
      WRITE(6,598)(KI(J),J=1,MB)
  596 RETURN
  597 FORMAT(1PE12.4,1P9E11.4)
  598 FORMAT(20I5)
  599 FORMAT(1H0,32HRUPTURE DIAGN.=ALFAP,ALFAM,BT,KI)
      END
$IBFTC SUPRES  DECK
      SUBROUTINE   PRESS(J1T,J2T,NCT,NZT)
      DIMENSION D(37,13,13),G(37,13,13),B(37,13),E(37,13),F(37,13),
     1P(37,13),ALFAP(37,13),ALFAM(37,13),H32(37),THA(37),AG1(37),
     2AG4(37),CS(37),SS(37),BT(37),KI(37),JSX(13),JEX(13),SX(13),EX(13),
     3THS(13),THE(13)
      COMMON D,G,B,E,F,P,ALFAP,ALFAM,H32,THA,AG1,AG4,CS,SS,BT,KI,JSX,
     1JEX,SX,EX,THS,THE,DX,DZ,CSII,ALPH
      J1=J1T
      J2=J2T
      NC=NCT
      NZ=NZT
      DO 503 J=J1,J2
      DO 502 I=1,NC
      C3=0.0
      DO 501 K=1,NC
  501 C3=C3+(F(J,K)-B(J,K)*E(J-1,K))*G(J,I,K)
  502 E(J,I)=C3
  503 CONTINUE
      DO 508 J=J1,J2
      J3=J1+J2-J
      DO 507 I=1,NC
      C3=E(J3,I)
      DO 504 K=1,NC
  504 C3=C3+D(J3,I,K)*E(J3+1,K)
      IF(NZ) 507,505,505
  505 IF(C3) 506,506,507
  506 C3=0.0
  507 E(J3,I)=C3
  508 CONTINUE
      RETURN
      END
$IBFTC SUBSIM  DECK
```

```
      SUBROUTINE SIMP(NCT,FRT,FTT)
      DIMENSION D(37,13,13),G(37,13,13),B(37,13),E(37,13),F(37,13),
     1P(37,13),ALFAP(37,13),ALFAM(37,13),H32(37),THA(37),AG1(37),
     2AG4(37),CS(37),SS(37),BT(37),KI(37),JSX(13),JEX(13),SX(13),EX(13),
     3THS(13),THE(13)
      COMMON D,G,B,E,F,P,ALFAP,ALFAM,H32,THA,AG1,AG4,CS,SS,BT,KI,JSX,
     1JEX,SX,EX,THS,THE,DX,DZ,CSII,ALPH
      DIMENSION SR(13),ST(13)
      NC=NCT
      DO 710 I=1,NC
      JS=JSX(I)
      JE=JEX(I)
      AS=ALFAM(JS,I)
      AE=ALFAP(JS,I)
      KX=JE-JS
      J2=JE-1
      C1=E(JS,I)
      C2=(1.0-AE)*E(JE,I)
      C3=C2*CS(JE)
      C4=-C2*SS(JE)
      IF((KX/2)*2-KX)   704,703,703
  703 C1=(1.0+AS)*C1
      C3=C3-C1*CS(JS)
      C4=C4+C1*SS(JS)
      J1=JS+1
      GO TO 708
  704 C5=(1.0+AS)*(3.0-AS)/4.0
      IF(AS-0.01) 705,705,706
  705 C6=0.0
      C5=C5/3.0*4.0
      GO TO 707
  706 C6=1.0+AS
      C6=C6/AS*C6/2.0
      C6=C6*C6
  707 C5=(1.0+C5)*E(JS+1,I)
      C6=C6*C1
      C3=C3-C6*CS(JS)-C5*CS(JS+1)
      C4=C4+C6*SS(JS)+C5*SS(JS+1)
      J1=JS+2
  708 DO 709 J=J1,J2,2
      C3=C3-4.0*E(J,I)*CS(J)-2.0*E(J+1,I)*CS(J+1)
  709 C4=C4+4.0*E(J,I)*SS(J)+2.0*E(J+1,I)*SS(J+1)
      SR(I)=C3
  710 ST(I)=C4
      C1=SR(NC)
      C2=ST(NC)
      IF((NC/2)*2-NC) 711,712,712
  711 J1=NC-2
      FR=4.0*C1
      FT=4.0*C2
      GO TO 713
  712 J1=NC-1
      FR=-2.0*C1
      FT=-2.0*C2
  713 DO 714 I=1,J1,2
      FR=FR+8.0*SR(I)+4.0*SR(I+1)
  714 FT=FT+8.0*ST(I)+4.0*ST(I+1)
      FRT=FR
      FTT=FT
      RETURN
      END
$IBFTC MAINVR  DECK
```

```
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
C     NOVEMBER 1692   S GOOD  DAVID TAYLOR MODEL BASIN   AM MAT1
C
      SUBROUTINE MATINV(A,N1,B,M1,DETERM,ID)
C
C     GENERAL FORM OF DIMENSION STATEMENT
C
      DIMENSION A(13,13),B(13,1)
      DIMENSION INDEX(13,3)
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)
C
C     INITIALIZATION
C
      M=M1
      N=N1
      DO 8 I=1,N
      K1=1
      K2=1
      DO 6 J=1,N
      IF(A(I,J)) 3,4,3
    3 K1=0
    4 IF(A(J,I)) 5,6,5
    5 K2=0
    6 CONTINUE
      IF(K1+K2) 8,8,7
    7 ID=2
      DETERM=0.0
      GO TO 740
    8 CONTINUE
   10 DETERM=1.0
   15 DO 20 J=1,N
   20 INDEX(J,3) = 0
   30 DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
      IF(INDEX(J,3)-1) 60, 105, 60
   60 DO 100 K=1,N
      IF(INDEX(K,3)-1) 80, 100, 715
   80 IF(AMAX-ABS(A(J,K)))  85,100,100
   85 IROW=J
   90 ICOLUM=K
      AMAX=ABS(A(J,K))
  100 CONTINUE
  105 CONTINUE
      INDEX(ICOLUM,3) = INDEX(ICOLUM,3) +1
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLUM) 140, 310, 140
  140 DETERM=-DETERM
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      IF(M) 310, 310, 210
  210 DO 250 L=1, M
  220 SWAP=B(IROW,L)
```

```
  230 B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
 310  PIVOT   =A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
  355 IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+I-I
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
  640 JCOLUM=INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLUM)
  700 A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
      DO 730 K = 1,N
      IF(INDEX(K,3) -1) 715,720,715
  715 ID =2
      GO TO 740
  720 CONTINUE
  730 CONTINUE
      ID=1
  740 RETURN
C     LAST CARD OF PROGRAM
      END
$IBFTC ARCROU  DECK
C     ARCTAN ROUTINE, TANV GREATER OR EQUAL ZERO,LESS THAN 360.
      FUNCTION TANV(CSS,SNN)
      CS=CSS
      SN=SNN
      IF(CS) 57,50,54
   50 IF(SN) 53,51,52
   51 TANV=0.0
      GO TO 61
   52 TANV=90.0
      GO TO 61
   53 TANV=270.0
```

```
      GO TO 61
   54 IF(SN) 56,55,55
   55 A=0.0
      B=57.29578
      GO TO 60
   56 A=360.0
      B=-57.29578
      SN=-SN
      GO TO 60
   57 CS=-CS
      A=180.0
      IF(SN) 59,58,58
   58 B=-57.29578
      GO TO 60
   59 B=57.29578
      SN=-SN
   60 TANV=A+B*ATAN(SN/CS)
   61 RETURN
      END
$IBMAP UNIT-P  8,DECK
       ENTRY   .UN10.
.UN10. PZE     UNIT10
UNIT10 FILE    ,A(6),READY,LIST,INOUT,BCD,BLK=14
       END
```

THREE LOBE BEARING, M=0.5

```
 15    6    2    4    3    1   10    8    0    0    0    0
0.0001         0.0001
0.5            1.0
0.0            2000.0        10000.0        30000.0
0.02            10
0.0            0.05          0.1            0.15           0.2
0.25           0.3           0.35           0.4            0.45
10.0           110.0         0.5            60.0
130.0          230.0         0.5            180.0
250.0          350.0         0.5            300.0
```

THREE LOBE BEARING, M=0.5

CIRC.DIV	AX.DIV	NO.L/D	NO.REYN	NO.PAD	NO.E*COSA	NO.A-IT	NO.RUP.IT	PAD.DATA	RUPT.DATA	ATT.A.DATA	INPUT
15	6	2	4	3	1	10	8	0	0	0	0

CONV.LIM.ATT.A CONV.LIM.RUP
1.000000E-04 1.000000E-04

L/D RATIOS
5.000000E-01 1.000000E 00

REYNOLDS NUMBERS
0. 2.000000E 03 1.000000E 04 3.000000E 04

ECC*COS(ATT) DATA

ECC*COS(ATT)= 2.000000E-02 NO.E*SIN(A)= 10
LIST OF ECC*SIN(ATT)
0. 5.000000E-02 1.000000E-01 1.500000E-01 2.000000E-01
2.500000E-01 3.000000E-01 3.500000E-01 4.000000E-01 4.500000E-01

BEARING GEOMETRY DATA

INLET ANGLE	OUTLET ANGLE	PRELOAD	ANGLE PRLD-X
1.000000E 01	1.100000E 02	5.000000E-01	6.000000E 01
1.300000E 02	2.300000E 02	5.000000E-01	1.800000E 02
2.500000E 02	3.500000E 02	5.000000E-01	3.000000E 02

L/D	REYN.NO.	ECC*COS(ATT)	ECC*SIN(ATT)	ECC.RATIO	ATT.ANGLE	1/S	F-Y/S*W
5.0000000E-01	0.	2.0000000E-02	3.0301647E-02	3.6306883E-02	5.6573987E 01	3.1983138E-01	-2.0861625E-06

1/S*(R/C*F/W)	(Q-Z/NDLC)SIDE	(Q/NDLC)TOTAL	(Q-X/NDLC)CIRC	F-X/S*W	CHN/NUDL(R/C)2	FREQ/W	(FREQ/W)**2
2.9309359E 01	3.8327824E-01	3.8327824E-01	2.8274795E 00	3.1983139E-01	5.5090043E-01	4.8104757E-01	2.3140676E-01

CKXX/SW	CKXY/SW	CKYX/SW	CKYY/SW	CWBXX/SW	CWBXY/SW	CWBYX/SW	CWBYY/SW
5.5947423E 00	6.8068700E 00	-7.1739856E 00	4.5513619E 00	1.5227733E 01	-4.5704956E-01	-4.1995123E-01	1.3793705E 01

SOMMERFELD NO	F-X/W	F-Y/W	(R/C)*(F/W)	CHW**2/W
3.1266475E 00	1.0000000E 00	-6.5226949E-06	9.3203356E 01	6.8000447E 01

CKXX/W	CKXY/W	CKYX/W	CKYY/W	CWBXX/W	CWBXY/W	CWBYX/W	CWBYY/W
1.7492787E 01	2.1282683E 01	-2.2430524E 01	1.4230504E 01	4.7611753E 01	-1.4290329E 00	-1.3130395E 00	4.3128054E 01

MINIMUM FILM THICKNESS FOR PAD NO.

1	2	3
5.3624E-01	4.7912E-01	4.8274E-01

L/D	REYN.NO.	ECC*COS(ATT)	ECC*SIN(ATT)	ECC.RATIO	ATT.ANGLE	1/S	F-Y/S*W
5.0000000E-01	2.0000000E 03	2.0000000E-02	3.2770680E-02	3.8391632E-02	5.8604215E 01	4.4820513E-01	-1.1928929E-07

1/S*(R/C*F/W)	(Q-Z/NDLC)SIDE	(Q/NDLC)TOTAL	(Q-X/NDLC)CIRC	F-X/S*W	CHN/NUDL(R/C)2	FREQ/W	(FREQ/W)**2

SOMMERFELD NO	F-X/W	F-Y/W	(R/C)*(F/W)	CMW**2/W
2.9314212E-08	9.9999999E-01	-2.6597837E-07	1.3374363E 02	3.8773488E 01

CKXX/W	CKXY/W	CKYX/W	CKYY/W	CWBXX/W	CWBXY/W	CWBYX/W	CWBYY/W
1.5439686E 01	2.0712738E 01	-2.1715473E 01	1.2600928E 01	4.5572996E 01	-1.2157892E 00	-1.1130386E 00	4.1530459E 01

MINIMUM FILM THICKNESS FOR PAD NO.

1	2	3
5.3838E-01	4.7897E-01	4.8053E-01

L/D	MEYN.NO.	ECC*COS(ATT)	ECC*SIN(ATT)	ECC.RATIO	ATT.ANGLE	1/S	F-Y/S*W
5.0000000E-01	1.0000000E 04	2.0000000E-02	3.2997570E-02	3.8585484E-02	5.8779727E 01	9.4516373E-01	1.6435981E-05

1/S*(R/C*F/W)	(Q-Z/NDLC)SIDE	(Q/NDLC)TOTAL	(Q-X/NDLC)CIRC	F-X/S*W	CMN/MUDL(R/C)2	FREQ/W	(FREQ/W)**2
1.5486673E 02	4.5269322E-01	4.5269322E-01	2.8891387E 00	9.4516374E-01	1.2055189E 00	4.9889548E-01	2.4897837E-01

CKXX/SW	CKXY/SW	CKYX/SW	CKYY/SW	CWBXX/SW	CWBXY/SW	CWBYX/SW	CWBYY/SW
1.3509874E 01	2.0086497E 01	-2.0820897E 01	1.1764417E 01	4.3349007E 01	-8.4515945E-01	-8.0984800E-01	3.9931128E 01

SOMMERFELD NO	F-X/W	F-Y/W	(R/C)*(F/W)	CMW**2/W
1.0580177E 00	9.9999999E-01	1.7389559E-05	1.6380533E 02	5.4112357E 01

CKXX/W	CKXY/W	CKYX/W	CKYY/W	CWBXX/W	CWBXY/W	CWBYX/W	CWBYY/W
1.4293686E 01	2.1251870E 01	-2.2028879E 01	1.1917953E 01	4.5864017E 01	-8.9525171E-01	-8.5683355E-01	4.2247833E 01

MINIMUM FILM THICKNESS FOR PAD NO.

1	2	3
5.3858E-01	4.7895E-01	4.8032E-01

L/D	MEYN.NO.	ECC*COS(ATT)	ECC*SIN(ATT)	ECC.RATIO	ATT.ANGLE	1/S	F-Y/S*W
5.0000000E-01	3.0000000E 04	2.0000000E-02	3.2600393E-02	3.8246380E-02	5.3471320E 01	1.8922900E 00	-4.9769878E-06

1/S*(R/C*F/W)	(Q-Z/NDLC)SIDE	(Q/NDLC)TOTAL	(Q-X/NDLC)CIRC	F-X/S*W	CMN/MUDL(R/C)2	FREQ/W	(FREQ/W)**2
3.4536464E 02	4.6528737E-01	4.6528737E-01	2.8997768E 00	1.8922900E 00	2.5673522E 00	4.9199498E-01	2.4205987E-01

CKXX/SW	CKXY/SW	CKYX/SW	CKYY/SW	CWBXX/SW	CWBXY/SW	CWBYX/SW	CWBYY/SW
2.6866761E 01	4.1285703E 01	-4.2651574E 01	2.2488966E 01	8.8487424E 01	-1.5825709E 00	-1.5823808E 00	8.2021237E 01

SOMMERFELD NO	F-X/W	F-Y/W	(R/C)*(F/W)	CMW**2/W
5.9846023E-01	9.9999999E-01	-2.6301402E-06	1.8251148E 02	5.3562088E 01

CKXX/W	CKXY/W	CKYX/W	CKYY/W	CWBXX/W	CWBXY/W	CWBYX/W	CWBYY/W
1.4166307E 01	2.1817852E 01	-2.2539661E 01	1.1884524E 01	4.6762084E 01	-8.3632579E-01	-7.9394853E-01	4.3344962E 01

MINIMUM FILM THICKNESS FOR PAD NO.

1	2	3
5.3823E-01	4.7896E-01	4.8068E-01

L/D	MEYN.NO.	ECC*COS(ATT)	ECC*SIN(ATT)	ECC.RATIO	ATT.ANGLE	1/S	F-Y/S*W
1.0000000E 00	0.	2.0000000E-02	3.5310706E-02	4.0581349E-02	6.0472764E 01	6.3667449E-01	-1.2665987E-07

1/S*(R/C*F/W)	(Q-Z/NDLC)SIDE	(Q/NDLC)TOTAL	(Q-X/NDLC)CIRC	F-X/S*W	CMN/MUDL(R/C)2	FREQ/W	(FREQ/W)**2
2.9937620E 01	1.9769897E-01	1.9769897E-01	2.6799314E 00	6.3667450E-01	9.3495516E-01	4.7401897E-01	2.2468641E-01

CKXX/SW	CKXY/SW	CKYX/SW	CKYY/SW	CWBXX/SW	CWBXY/SW	CWBYX/SW	CWBYY/SW
9.3730096E 00	1.3382045E 01	-1.3286647E 01	7.3947384E 00	3.0214236E 01	-6.2815594E-01	-5.4990440E-01	2.6059344E 01

SOMMERFELD NO	F-X/W	F-Y/W	(R/C)*(F/W)	CMW**2/W
1.5706613E 00	1.0000000E 00	-1.9893976E-07	4.7021861E 01	5.7973973E 01

CKXX/W	CKXY/W	CKYX/W	CKYY/W	CWBXX/W	CWBXY/W	CWBYX/W	CWBYY/W
1.4721823E 01	2.1018660E 01	-2.0868823E 01	1.1614630E 01	4.7456331E 01	-9.8662023E-01	-8.6371357E-01	4.0938435E 01

MINIMUM FILM THICKNESS FOR PAD NO.

1	2	3
5.4058E-01	4.7880E-01	4.7825E-01

L/D	MEYN.NO.	ECC*COS(ATT)	ECC*SIN(ATT)	ECC.RATIO	ATT.ANGLE	1/S	F-Y/S*W
1.0000000E 00	2.0000000E 03	2.0000000E-02	3.9799411E-02	4.4542037E-02	6.3319557E 01	9.9694223E-01	-3.7242903E-07

1/S*(R/C*F/W)	(Q-Z/NDLC)SIDE	(Q/NDLC)TOTAL	(Q-X/NDLC)CIRC	F-X/S*W	CMN/MUDL(R/C)2	FREQ/W	(FREQ/W)**2
5.5668815E 01	2.2422257E-01	2.2422257E-01	2.7053781E 00	9.9694224E-01	1.1718127E 00	4.8130690E-01	2.3165633E-01

CKXX/SW	CKXY/SW	CKYX/SW	CKYY/SW	CWBXX/SW	CWBXY/SW	CWBYX/SW	CWBYY/SW
1.2222727E 01	1.9782748E 01	-1.9279074E 01	9.4737666E 00	4.3763401E 01	-8.8758796E-01	-7.6365053E-01	3.7450740E 01

SOMMERFELD NO	F-X/W	F-Y/W	(R/C)*(F/W)	CMW**2/W
1.0030671E 00	9.9999999E-01	-3.7367163E-07	5.5839558E 01	4.6403200E 01

CKXX/W	CKXY/W	CKYX/W	CKYY/W	CWBXX/W	CWBXY/W	CWBYX/W	CWBYY/W
1.2260216E 01	1.9843424E 01	-1.9338206E 01	9.5628238E 00	4.3897629E 01	-8.9031032E-01	-7.6599275E-01	3.7565606E 01

MINIMUM FILM THICKNESS FOR PAD NO.

1	2	3
5.4446E-01	4.7848E-01	4.7421E-01

L/D	MEYN.NO.	ECC*COS(ATT)	ECC*SIN(ATT)	ECC.RATIO	ATT.ANGLE	1/S	F-Y/S*W
1.0000000E 00	1.0000000E 04	2.0000000E-02	4.3197331E-02	4.7602619E-02	6.5156261E 01	2.4491436E 00	6.3545109E-07

1/S*(R/C*F/W)	(Q-Z/NDLC)SIDE	(Q/NDLC)TOTAL	(Q-X/NDLC)CIRC	F-X/S*W	CMN/MUDL(R/C)2	FREQ/W	(FREQ/W)**2
1.5460118E 02	2.5431949E-01	2.5431949E-01	2.7290307E 00	2.4491436E 00	2.4240457E 00	4.8680555E-01	2.3697964E-01

CKXX/SW	CKXY/SW	CKYX/SW	CKYY/SW	CWBXX/SW	CWBXY/SW	CWBYX/SW	CWBYY/SW
2.5950587E 01	4.0647465E 01	-4.4780384E 01	1.9999987E 01	1.0149782E 02	-1.8589430E 00	-1.5438983E 00	8.6509560E 01

SOMMERFELD NO	F-X/W	F-Y/W	(R/C)*(F/W)	CMW**2/W
4.0830599E-01	1.0000000E 00	2.6770627E-07	6.3124586E 01	3.9073905E 01

CKXX/W	CKXY/W	CKYX/W	CKYY/W	CWBXX/W	CWBXY/W	CWBYX/W	CWBYY/W
1.0595780E 01	1.9046439E 01	-1.8284099E 01	8.1161143E 00	4.1441168E 01	-7.5901755E-01	-5.3038293E-01	3.5322371E 01

MINIMUM FILM THICKNESS FOR PAD NO.

1	2	3
5.4739E-01	4.7821E-01	4.7116E-01

NUMBER OF ATT.ANGLE ITERATIONS TOO LARGE

L/D	REYN.NO.	ECC*COS(ATT)	ECC*SIN(ATT)	ECC.RATIO	ATT.ANGLE	1/S	F-Y/S*W
1.0000000E 00	3.0000000E 04	2.0000000E-02	4.4219224E-02	4.8531822E-02	6.5663142E 01	5.2634884E 00	1.4278293E-03

1/S*(R/C*F/W)	(Q-Z/NDLC)SIDE	(Q/NDLC)TOTAL	(Q-X/NDLC)CIRC	F-X/S*W	CMN/MUDL(R/C)2	FREQ/W	(FREQ/W)**2
3.4654565E 02	2.6612027E-01	2.6612027E-01	2.7380186E 00	5.2634882E 00	4.9555764E 00	4.8837462E-01	2.3850977E-01

CKXX/SW	CKXY/SW	CKYX/SW	CKYY/SW	CWBXX/SW	CWBXY/SW	CWBYX/SW	CWBYY/SW
5.3318968E 01	9.9060295E 01	-9.4669126E 01	4.1235590E 01	2.1455264E 02	-3.6445462E 00	-2.9632985E 00	1.8260455E 02

SOMMERFELD NO	F-X/W	F-Y/W	(R/C)*(F/W)	CMN**2/W
1.8998807E-01	9.9999996E-01	2.7127052E-04	6.5859539E 01	3.7168945E 01

CKXX/W	CKXY/W	CKYX/W	CKYY/W	CWBXX/W	CWBXY/W	CWBYX/W	CWBYY/W
1.0129968E 01	1.8820274E 01	-1.7986004E 01	7.8342699E 00	4.0762441E 01	-6.9242030E-01	-5.6299137E-01	3.4697685E 01

MINIMUM FILM THICKNESS FOR PAD NO.

1	2	3
5.4827E-01	4.7812E-01	4.7024E-01

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APPENDIX VIII: Computer Program - The Performance and Stability of a Hybrid Journal Bearing with Flexible, Damped Support

This appendix describes the computer program PN0144: "The Performance and Stability of a Hybrid Journal Bearing with Flexible, Damped Support." It gives the detailed instructions for using the program, for preparing the input data and a listing of the Fortran instructions is also given. The analysis for the program is contained in Appendices V and VI. The program calculates the load carrying capacity, the flow, the 8 dynamic coefficients and the critical journal mass at the onset of instability. The program applies to a purely hydrostatic, a purely hydrodynamic or a hybrid bearing with a compressible lubricant. The bearing is cylindrical with a single row of feeder holes in the center plane of the bearing, and the support for the bearing has flexibility, damping and mass.

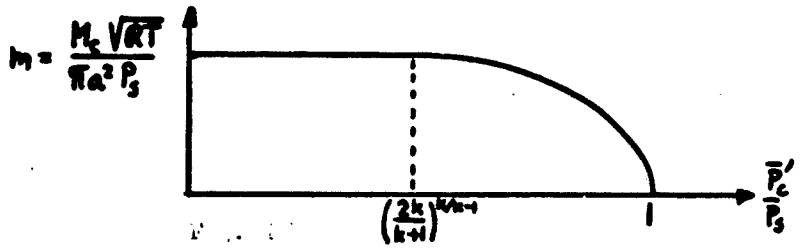
The analysis assumes that the bearing operates with a small eccentricity ratio

COMPUTER INPUT

An input data form is given in the back of this appendix for quick reference when preparing the computer input. In the following, the more detailed instructions are given.

Table of Vena Contracta Coefficients (5E14.6)

This table consists of 5 cards with a total of 25 values of the vena contracta coefficient C_D for the feeder hole restrictor. C_D is defined through eq. (E-31), Appendix V. The program assumes $C_D = 1$ when the feeder hole is choked, and for unchoked conditions, C_D is specified by the present input list. The dimensionless flow through the restrictor is a function of the pressure ratio across the restrictor:



where M_c is the mass flow, lbs.sec/inch, $\mathcal{R}T$ is the product of the gas constant and the total temperature, inch^2/sec^2 , a is the orifice radius, inch, Ps is the supply pressure, psia, $\bar{P}_c'$ is the pressure downstream of the orifice,psia, and k is the ratio of specific heats ($k = 1.4$ for air).

The vena contracta coefficient C_D is the ratio between the actual flow (made dimensionless) and the ideal isentropic flow such that $C_D \leq 1$ (in general, $0.6 \leq C_D \leq 1$). The 25 input values of C_D are taken at those pressure ratios which result from subdividing the range $\left(\frac{2k}{k+1}\right)^{k/k-1} \leq \frac{\bar{P}_c'}{\bar{P}_s} \leq 1$ into 25 equal parts. The first value of C_D applies to the first pressure ratio after $\frac{\bar{P}_c'}{\bar{P}_s} = \left(\frac{2k}{k+1}\right)^{k/k-1}$, and the last value of C_D is taken at $\frac{\bar{P}_c'}{\bar{P}_s} = 1$.

Card (5E14.6)

This card contains one item, the ratio of specific heats, k. For air, k = 1.4.

Card (72H)

Any descriptive text may be given to identify the calculation.

Card (8I5)

This card is the "control" card which describes the subsequent input data. The card has 8 values:

1. NVL gives the number of pressure ratios, P_s/P_a , in the designated list. The maximum value of NVL is 25. For a purely hydrodynamic bearing, set NVL = 0 (then NLB $\neq$ 0).

2. NLSL gives the number of restrictor coefficients, Λ_s , in the designated list. The maximum value of NLSL is 25. For a purely hydrodynamic bearing, set NLSL=0 (when NLSL = 0, NVL must be zero).

3. NLB gives the number of compressibility numbers, Λ , in the designated list. The maximum value is 25. For a purely hydrostatic bearing, set NLB = 0 (then NLV $\neq$ 0).

4. NEP gives the number of eccentricity ratios, ε , in the designated list. The maximum value is 25 (NEP $\neq$ 0).

5. MNS gives the number of values in the list of frequency ratios (when NLB=0, the list contains values of the squeeze number instead of the frequency ratio). The maximum value of MNS is 95 (MNS $\neq$ 0).

6. MT. As described in Appendix E, the program calculates the pressure in the gas film by numerical integration. To this end, the length between the admission plane and the end of the bearing is subdivided into increments. MT specifies the number of subdivisions. MT should be large for high values of Λ and/or high values of the squeeze number σ . A typical value for MT is 10 to 15 for moderate Λ or σ values (say up to $\Lambda \cong 5$ and $\sigma \cong 5$). The maximum value for MT is 30.

7. INT. If INT = 0, the bearing is rigidly supported and no input data can be given for the three support parameters. If INT = 1, the bearing is flexibly supported.

8. INP. If INP = 0, the program returns to read in more input after completion of the calculations for the present set of input data. A new set of input data starts with the identification card (i.e. with the card (72H)). The table of vena contracta coefficients and the card with the ratio of specific heats are not to be repeated.

If IMP = 1, the present set of input data is the last set.

Card (5E14.6)

This card contains 4 values:

1. L/D, the ratio between the total bearing length and the journal diameter.

2. λ , the source correction factor. λ is given by eq. (E-50), Appendix V, or in approximate form:

$$\lambda \cong 1 + \frac{2}{n\delta} \cdot \log_e \left(\frac{D}{nd}\right)$$

where n is the number of feeder holes, d is the feeder hole diameter (not the orifice diameter), D is the journal diameter and L is the total bearing length. Typically, λ = 1.3 to 1.5.

3. δ , the inherent compensation factor. Even when the feeder holes are provided with orifices, an additional flow restriction normally occurs where the gas leaves the feeder hole and enters the bearing film:

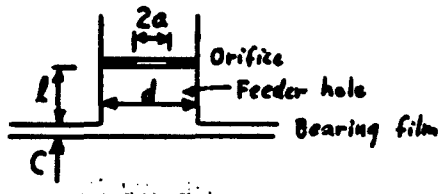


Figure 63: Feeding Hole

The orifice area is πa^2 where a is the orifice radius. The "curtain" area at the rim of the feeder hole has the area $\pi d C$ where d is the feeder hole diameter and C is the radial bearing clearance. The ratio between the two areas is called the inherent compensation factor:

$$\delta = \frac{a^2}{dC}$$

When there is no orifice, $\delta = \infty$ and the bearing is called inherently compensated (for the inherently compensated bearing, set δ = 1000 or some other large value in the input). If the bearing is purely orifice restricted, set δ = 0.

4. $nV_c/\pi DLC$, the ratio between the combined volume of all feeder holes and the volume of the gas film. Here, D is the journal diameter, L is the total bearing length, C is the radial bearing clearance, n is the number of feeder holes and V_c is the "capacitative" volume of one feeder hole. The effect of a feeder hole volume is to introduce a time lag between a change in the downstream feeder hole pressure and the corresponding change in the flow through the hole, see eq. (E-28), Appendix V. This time lag tends to reduce the damping capacity of the bearing and, in this way, adversely affects the stability margin. At present, there is no method to determine what this "capacitative" volume is, but it is suggested to set V_c equal to the volume of the feeder hole below the orifice, i.e. set $V_c = \frac{\pi}{4} d^2 \ell$ in fig. 63 where ℓ is the length of the feeder hole below the orifice. For an inherently compensated bearing in which there are no orifices, V_c may be set equal to zero. However, it must be remembered that V_c, properly interpreted, gives a measure of the time lag between pressure change and flow change. If there are other factors than the feeder hole volume which contributes to such a time lag, their effect must be reflected in the value for V_c according to eq. (E-28).

List of Supply Pressure Ratios (5E14.6)

This list consists of up to five cards with values of the ratio between the supply pressure P_s, psia, and the ambient pressure P_a, psia. In total, there are NVL values (item 1, control card), maximum 25 values. In the analysis in Appendix V, the pressure ratio is denoted by the symbol V. The ratio must be greater than 1. If NVL = 0, the list is omitted in the input and the bearing is purely hydrodynamic.

List of Restrictor Coefficients (5E14.6)

This list consists of up to five cards, giving the values of the restrictor coefficient Λ_s:

$$\Lambda_s = \frac{6\mu n a^2 \sqrt{RT}}{P_s C^3 \sqrt{1+\delta^2}}$$

where μ is the viscosity of the gas, lbs.sec/inch, RT is the product of the gas constant and the total temperature, $inch^2/sec^2$, P_s is the supply

pressure, psia, C is the radial clearance, inch, n is the number of feeder holes, a is the orifice radius, inch, and δ is the inherent compensation factor. In the inherently compensated bearing where there is no orifice, the restrictor coefficient becomes:

<u>Inherently Compensated Bearing</u>: $\Lambda_s = \frac{6\mu n d\sqrt{RT}}{P_s C^2}$

where d is the feeder hole diameter, inch.

In the purely hydrodynamic bearing, where NVL = 0 and, therefore, NLSL = 0, this list is omitted. For NLSL $\neq$ 0, give a total of NLSL values of Λ_s, maximum 25.

<u>List of Compressibility Numbers (5E14.6)</u>

This list consists of up to five cards, giving a total of NLB values of the compressibility number Λ (see item 3, control card). Λ is defined as:

$$\Lambda = \frac{6\mu\omega}{P_a}\left(\frac{R}{C}\right)^2$$

where μ is the viscosity of the gas, lbs.sec/inch^2, P_a is the ambient pressure, psia, ω is the angular speed of the journal, radians/sec, R is the journal radius, inch and C is the radial clearance, inch. A maximum of 25 values can be given.

For the purely hydrostatic bearing, set NLB = 0 and omit this list.

<u>List of Eccentricity Ratios (5E14.6)</u>

This list consists of up to five cards, giving a total of NEP values of the eccentricity ratio ε (see item 4, control card). ε is defined as:

$$\varepsilon = e/C$$

where e is the distance between the journal center position and the bearing center, inch, and C is the radial clearance, inch. ε cannot be zero and the input list cannot be omitted. A maximum of 25 values can be given.

<u>List of Frequency Ratios or Squeeze Numbers (5E14.6)</u>

This list consists of up to 19 cards, giving a total of MNS values (item 5,

control card). A maximum of 95 values can be given.

In the hybrid bearing or the purely hydrodynamic bearing (NLB $\neq$ 0, item 3, control card), the list gives values of the frequency ratio γ :

$$\gamma = \frac{\nu}{\omega}$$

where ν is the frequency for the journal center motion, radians/sec, and ω is the angular speed of the roter, radians/sec. γ cannot be zero. The input values of γ are used by the program in searching for the threshold of instability. For each value of γ , the program calculates the left hand side of eq. (D-29), Appendix IV (as modified per Appendix VI), which represents the overall effective damping of the gas film and the bearing support. The threshold of instability occurs at those values of γ for which the effective damping is zero. The program uses the values of γ in the same sequence as given in the input. Thus, the input list should start with a small value of γ (say, γ = 0.0001) and thereafter, give successively larger values of γ with the last value of $\gamma \cong$ 0.51. Whenever the effective damping changes sign as the program goes through the γ list, the program interpolates to determine accurately the γ values at which the damping is zero.

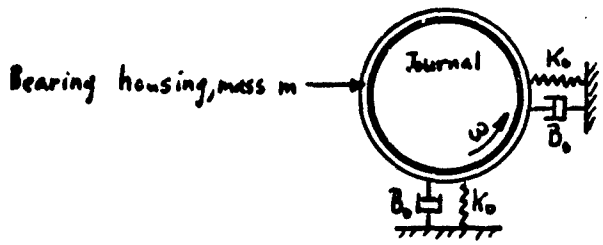
The very first value of γ is not used in the stability calculation and, therefore, can be any value. It is used as that value for which the program output gives the 8 dynamic coefficients. Thus, it is suggested to set the first value of γ equal to 1.

For the purely hydrostatic bearing (NLB = 0), this input list gives values of the squeeze number σ :

$$\sigma = \frac{12\mu\nu}{P_a}\left(\frac{R}{C}\right)^2$$

where the symbols have been defined previously. The input is prepared in the same way as described above except that, whereas the range of interest for γ is known to be from 0 to 0.5, the range for σ is not known in advance. It is suggested to let σ go from a small value (say 0.001) to approximately 100. It should be noted, however, that the program is basically a hybrid bearing program and it is not very efficient in calculating a purely hydrostatic bearing although the answers are correct.

If INT = 0 (item 7, control card), the bearing is rigidly supported and the last input data is the preceeding list of frequency ratios (or squeeze numbers). If INT = 1, the bearing is flexibly mounted and it is necessary to specify the support parameters. Schematically, the bearing and its support can be shown as:



Card (8I5)

This card is the "control" card for the support parameters. It contains 3 items:

1. NK gives the number of values of the dimensionless support stiffness in the designated list ($1 \leq NK \leq 25$).

2. NM gives the number of values of the dimensionless bearing boundary mass in the designated list ($1 \leq NM \leq 25$).

3. ND gives the number of values of the dimensionless support damping in the designated list ($ND \leq 25$). If ND = 0, the support is undamped and no input can be given for the support damping.

List of Support Stiffness Values (5E14.6)

This list consists of up to five cards, giving a total of NK values of the dimensionless support stiffness:

$$\bar{K}_o = \frac{C K_o}{P_a L D}$$

where K_o is the support spring coefficient, lbs/inch, C is the radial bearing clearance, inch, L is the bearing length, inch, D is the journal diameter, inch, and P_a is the ambient pressure, psia. There can be a maximum of 25 values.

List of Support Mass Values (5E14.6)

This list consists of up to five cards, giving a total of NM values of the dimensionless bearing housing mass:

$$\bar{m} = \frac{C m \omega^2}{P_a L D}$$

where m is the bearing housing mass, lbs.sec^2/inch, ω is the angular speed of the journal, radians/sec, and the other symbols are defined above. There can be a maximum of 25 values. The mass may be zero if desired.

List of Support Damping Values (5E14.6)

If ND = 0, this list is omitted and the support has no damping. Otherwise, the list consists of up to five cards, giving a total of ND values of the dimensionless support damping:

$$\bar{B}_0 = \frac{B_0}{\mu L \left(\frac{R}{C}\right)^3}$$

where B_0 is the support damping coefficient, lbs.sec/inch, and the other symbols are defined above.

The support damping has a strong influence on the stability of a flexible mounted bearing. Properly chosen, the damping may vastly improve the stability limit of the bearing or even remove the tendency for instability all together. To determine which damping to provide, a whole range of damping values should be tried (say, from 10^{-3} to 10). At the same time, the support stiffness should be considerably smaller than the bearing stiffness (of the order of one half or less of the bearing stiffness).

COMPUTER OUTPUT

The first page of output is a repetition of the input for checking purposes. Therefore, follow the output for each case. The output values are identified by text as follows:

L/D: the L/D-ratio

PR.RATIO: the supply pressure ratio $V = P_s/P_a$

LAMBDA-S: the restrictor coefficient Λ_s

LAMBDA: the compressibility number Λ

ECCENTR: the eccentricity ratio ε

LAMBDA-T: $\Lambda_s V$

Q: the dimensionless flow $q = \frac{6\mu R T \bar{M}}{\pi P_a^2 C^3}$ where $\bar{M}$ is the total mass flow to the bearing, lbs.sec/inch, and the other symbols have been defined previously. Thus, knowing q, the bearings gas consumption can be determined.

M-O: the dimensionless orifice flow $m_o = q/\Lambda_s V^2$ which is given by the right hand side of eq.(E-31), Appendix V.

PSI: ψ_o, the dimensionless rate of change of flow with changing downstream pressure (defined by eq. (E-60), Appendix V)).

ORIF. PR. RATIO: the pressure ratio $\bar{P}_c'/P_s$ across the feeder hole ($\bar{P}_c'$ is the pressure downstream of the feeder hole).

W/PALD: W/PaLD, the dimensionless bearing load where W is the bearing load, lbs.

W/DPLD: $W/(P_s - P_a)LD$

ATT. ANG: the attitude angle φ, degrees.

FR/PALD: $f_{r_o} = F_r/P_a LD$, the dimensionless radial component of the bearing reaction, eq. (E-119), Appendix V.

FT/PALD: $f_{t\theta} = F_{t\theta}/P_a LD$, the dimensionless tangential component of the bearing reaction, eq. (E-120), Appendix V.

CKXX/PALD: $\bar{K}_{rr} = CK_{rr}/P_a LD$, the dimensionless radial spring coefficient, eq. (E-131), Appendix V.

CWCXX/PALD: $\bar{B}_{rr} = C\omega B_{rr}/P_a LD$, the dimensionless radial damping, eq. (E-131), Appendix V.

CKXY/PALD: $\bar{K}_{rt} = CK_{rt}/P_a LD$, the dimensionless radial cross-coupling spring coefficient, eq. (E-131), Appendix V.

CWCXY/PALD: $\bar{B}_{rt} = C\omega B_{rt}/P_a LD$, the dimensionless radial cross-coupling damping, eq. (E-131) Appendix V.

CKYX/PALD: $\bar{K}_{tr} = CK_{tr}/P_a LD$, the dimensionless tangential cross-coupling spring coefficient.

CWCYX/PALD: $\bar{B}_{tr} = C\omega B_{tr}/P_a LD$, the dimensionless tangential spring coefficient.

CWCYY/PALD: $\bar{B}_{tt} = C\omega B_{tt}/P_a LD$, the dimensionless tangential damping.

EFF. STIFF-1: the major dimensionless, effective stiffness $\bar{K}_\theta$ (eq. (D-24), Appendix IV, with plus in front of the square root instead of minus.

EFF. STIFF-2: the minor dimensionless, effective stiffness $\bar{K}_\theta$, eq. (D-24), Appendix IV,

EFF. DAMP-1: the major dimensionless, effective damping $\gamma \bar{B}_\theta$ (eq. (D-25), Appendix IV, with plus in front of the square root instead of minus.

EFF. DAMP-2: the minor dimensionless, effective damping $\gamma \bar{B}_\theta$, eq. (D-25) Appendix IV.

FREQ/W: $\gamma = \frac{\omega}{\Omega}$, the frequency ratio.

(B*GAM)2: $M\gamma^2 = (\bar{K}_0 \gamma \bar{B}_0 + \bar{K}_0 \gamma \bar{B}_B)/(\gamma \bar{B}_0 + (1+\bar{m}) \gamma \bar{B}_B)$, see eq. (D-28), Appendix IV.

CMW2/PALD: $\bar{M} = CM\omega^2/P_a LD$, the dimensionless journal mass.

CMW2/DPLD: $CM\omega^2/(P_s - P_a) LD$, dimensionless expression for the journal mass.

MPA/MU2L(R/C)5: $MP_a/\mu^2 L \left(\frac{R}{C}\right)^5$, dimensionless expression for the journal mass.

ERROR: the left hand side of eq. (D-29), Appendix IV.

PEDEST. STIFFN.: $\bar{K}_0 = CK_0/P_a LD$, the dimensionless support stiffness.

PEDEST. DAMPING: $\bar{B}_0 = B_0/\mu L \left(\frac{R}{C}\right)^3$, the dimensionless support damping.

EFF. STIFFNESS: the minor dimensionless, effective stiffness $\bar{K}_B$, eq. (D-24), Appendix IV.

EFF. DAMPING: the negative of the minor dimensionless, effective damping $\gamma \bar{B}_B$ eq. (D-25), Appendix IV.

In each combination of $\frac{L}{D}$, V , $\Lambda_s, \Lambda, \varepsilon$, there is a separate output which begins by specifying the appropriate values of $\frac{L}{D}, V, \Lambda_s, \Lambda$ and ε , and then gives the corresponding bearing flow, the load carrying capacity, the attitude angle, the 8 dynamic coefficients and the four effective coefficients as identified by the labels explained above. The dynamic and the effective coefficients are calculated for the first frequency ratio given in the input data. Thereafter, follow the result of the stability calculation for a rigid support where eqs. (D-28) and (D-29) ((B*GAM)2 and ERROR) are listed for the specified values of the frequency ratio. The threshold of instability occurs when the "error" is zero and the program determines this by interpolation. For the threshold value of the frequency

ratio, the dimensionless journal mass is given in three different forms, identified by the labels explained above.

When the bearing is flexibly supported, the preceeding results are followed by the calculations of the stability of the flexibly supported bearing. First, the dimensionless support stiffness is given together with the corresponding threshold value of the dimensionless journal mass for an undamped support. Thereafter, for each specified value of the dimensionless support damping is a 6 column list with the results of the calculations. The first column gives the frequency ratio γ, the second column gives $\bar{M}\gamma^2$, the third column gives the dimensionless minor effective bearing stiffness, the fourth column gives the dimensionless minor effective bearing damping and the sixth column gives the "ERROR". When an instability threshold is found, the corresponding dimensionless journal mass is given in three different forms in columns 3, 4, and 5.

For any given support damping, there may be up to three instability thresholds. At a threshold, the "error" must be very small (in theory it should be zero). This condition should be used to "weed out" a false root which frequently occurs. At such a false root, $\bar{M}\gamma^2$ goes to plus-minus infinity on either side of the frequency ratio value and the corresponding dimensionless journal mass will tend to be very large (in theory, it is infinite). The solution should be ignored. It should be noted, however, that there are proper solutions where the dimensionless journal mass also is very large which must not be ignored.

SAMPLE CALCULATION

In back of this Appendix is shown the input for calculating the stability of a flexible supported bearing ($L/D = 1$, $\epsilon = 0.02$, $\Lambda_s = 0.7$, $\lambda = 1.5$, $\delta = 1000$, 4 values of P_s/P_a, 7 values of Λ, 3 values of support stiffness and 15 values of support damping). Also shown are the first couple of pages of output. The results are most conveniently plotted on logarithmic paper with the support damping as abcissa and the dimensionless journal mass as ordinate for fixed values of the supply pressure ratio and the support stiffness. The

compressibility number is used as a parameter. A typical plot is shown in figure 27. From such a plot it can be determined what support damping to provide to improve the stability limit of the bearing. A more detailed discussion is given in the body of the report.

INPUT FORM FOR COMPUTER PROGRAM

PN0144: THE PERFORMANCE AND STABILITY OF A HYBRID JOURNAL BEARING WITH FLEXIBLE, DAMPED SUPPORT

Table of Vena Contracta Coefficients (5E14.6)

Give 25 values of the orifice vena contracta coefficient C_D

Card (5E14.6)

k, the ratio of specific heats

Card (72H)

Text

Card (8I5)

1. NVL — Number of P_s/P_a -values in input (NVL $\leq$ 25). If NVL = 0, the bearing is purely hydrodynamic (NLB $\neq$ 0).

2. NLSL — Number of Λ_s - values in input (NLSL $\leq$ 25). When NLSL = 0, NVL must be zero.

3. NLB — Number of Λ-values in input (NLB $\leq$ 25). If NLB = 0, the bearing is purely hydrostatic (NVL $\neq$ 0).

4. NEP — Number of ε -values in input (NEP 25).

5. MNS — If NLB $\neq$ 0, MNS gives number of χ-values in input. If NLB = 0, MNS gives number of σ -values in input (MNS $\leq$ 95).

6. MT — Number of finite difference increments (MT $\leq$ 30).

7. INT 0: Bearing support is rigid. 1: Bearing support is flexible.

8. INP 0: More input follows. 1: Last set of input.

Card (5E14.6)

1. L/D, the length-to-diameter ratio.

2. λ , the source correction factor (see eq. (E-50), Appendix V).

3. $\delta = a^2/dC$, the inherent compensation factor.

4. $nV_c/\pi DLC$, the feeder hold volume ratio.

List of Supply Pressure Ratios (5E14.6)

Give NVL values of $V = P_s/P_a$. Omit the list when NVL = 0.

List of Restrictor Coefficients (5E14.6)

Give NLSL values of Λ_s . Omit the list when NVL = 0 (and, hence, NLSL = 0).

List of Compressibility Numbers (5E14.6)

Give NLB values of Λ . Omit the list when NLB = 0.

List of Eccentricity Ratios (5E14.6)

Give NEP values of ε $(\varepsilon \neq 0)$

List of Frequency Ratios or Squeeze Numbers

If NLB $\neq 0$ (hybrid or hydrodynamic bearing), give MNS values of the frequency ratio γ.

If NLB = 0 (hydrostatic bearing), give MNS values of the squeeze number σ .

Note: If INT = 0, omit the following input data.

Card (8I5)

1. NK — Number of support stiffness values in input ($1 \leq NK \leq 25$)

2. NM — Number of support mass values in input ($1 \leq NM \leq 25$)

3. ND — Number of support damping values in input ($ND \leq 25$)

List of Support Stiffness Values (5E14.6)

Give NK values of the dimensionless support stiffness.

List of Support Mass Values (5E14.6)

Give NM values of the dimensionless support mass (may be zero).

List of Support Damping Values (5E14.6)

Give ND values of the dimensionless support damping. Omit the list when ND = 0.

```
$JOB            0,20,70000      66-217   CHASMAN  APFL
$IBJOB MTI
$IBFTC MTI
*O ())))))))))))          ())))))))))))      (ZZZZZZZZZZZZZ(
C     MECHANICAL TECHNOLOGY INC.   JORGEN W. LUND 6-22-1966
C     PN0144 HYBRID JOURNAL BEARING STABILITY WITH FLEX-DAMPED SUPPORT
      DIMENSION TFR(31),TFE(31),VFR(31),VFE(31),EPSL(25),FRR(31),FJR(31)
      DIMENSION  FJE(31),VCF(26),BSIG(95)
      DIMENSION FF(95),EF(95),PDKL(25),PDL(25),PDML(25)
      DIMENSION VLST(99),FLSL(99),FLBL(99),                  GST(99),PZ2(
     199),PZN(99)
      Y1=0.0
      READ(5,301)(VCF(I),I=2,26)
      READ(5,301)ADC
      VCF(1)=1.0
      WRITE(6,310)
      WRITE(6,810)(VCF(I),I=2,26)
      WRITE(6,315)
   13 READ(5,300)
      READ(5,302)NVL,NLSL,NLB,NEP,MNS,MT,INT,INP
      READ(5,301)FLD,PRC,CMP,FDVL
      WRITE(6,300)
      WRITE(6,303)
      WRITE(6,304)NVL,NLSL,NLB,NEP,MNS,MT,INP
      WRITE(6,309)
      WRITE(6,810)     PRC,CMP,FDVL,ADC
      IF(NVL) 15,14,15
   14 VLST(1)=2.0
      FLSL(1)=0.0
      GO TO 20
   15 WRITE(6,305)
      READ(5,301)(VLST(I),I=1,NVL)
      WRITE(6,810)(VLST(I),I=1,NVL)
      WRITE(6,306)
      READ(5,301)(FLSL(I),I=1,NLSL)
      WRITE(6,810)(FLSL(I),I=1,NLSL)
      IF (NLB) 20,19,20
   19 FLBL(1)=1.0
      GO TO 22
   20 WRITE(6,307)
      READ(5,301)(FLBL(I),I=1,NLB)
      WRITE(6,810)(FLBL(I),I=1,NLB)
   22 WRITE(6,316)
      READ(5,301)(EPSL(I),I=1,NEP)
      WRITE(6,810)(EPSL(I),I=1,NEP)
      WRITE(6,308)
      READ(5,301)(BSIG(I),I=1,MNS)
      WRITE(6,810)(BSIG(I),I=1,MNS)
      IF(INT) 24,29,24
   24 READ(5,302)NK,NM,ND
      READ(5,301)(PDKL(I),I=1,NK)
      READ(5,301)(PDML(I),I=1,NM)
      IF(ND) 29,29,27
   27 READ(5,301)(PDL(I),I=1,ND)
   29 EX6=2.0/(ADC+1.0)
      EX1=1.0/(ADC-1.0)
      EX7=(EX6**EX1)*SQRT (EX6*ADC)
      EX4=2.0*EX1
      EX1=ADC*EX1
      CRP=EX6**EX1
      C1=SQRT (2.0*EX1)
```

```
      EX6=1.0/EX6
      EX7=1.0/ADC
      DIT=(1.0-CRP)/50.0
      CX=CRP
      DO 51 I=1,50
      CX=CX+DIT
      C5=CX**EX1
   51 QST(I)=C1*C5*SQRT (1.0-CX/C5)
      C1=1.0+CMP*CMP
      CMP=0.5/C1
      C1=MT
      DLT=FLD/C1
      DL2=DLT*DLT
      DL2H=0.5*DL2
      SFACT=1.5707963/C1
      MT1=MT+1
      DO 285 NV=1,NVL
      V=VLST(NV)
      VMW=V-1.0
      V2=V*V
      DO 284 NLS=1,NLSL
      FLAS=FLSL(NLS)
      FLAT=FLAS*V
      C1=FLAT*V*FLD*PRC
      CX=SQRT (1.0+C1*EX7)/V
      IF (CRP-CX) 152,151,151
  151 Q=FLAT*V*EX7
      SLP=0.0
      PZ=V*CX
      VC=1.0
      DVC=0.0
      GO TO 200
  152 C2=V*CRP
      C2=C2*C2-1.0-C1*EX7
      C4=DIT+DIT
      CX=CRP+C4
      KS=0
      L=2
      M=2
  154 VC=VCF(M)
  155 C5=CX*CX*V2-1.0-C1*VC*QST(L)
      IF (KS) 161,158,161
  158 C6=C5*C2
      IF (C6) 160,160,159
  159 L=L+2
      M=M+1
      C2=C5
      CX=CX+C4
      IF (L-50) 154,154,284
  160 KS=1
      C7=C5
      L=L-1
      C5=VCF(M-1)
      DVC=(VC-C5)/C4
      VC=(VC+C5)/2.0
      CX=CX-DIT
      GO TO 155
  161 C1=C4*C4
      C1=2.0*(C2+C7-2.0*C5)/C1
      C3=(C7-C2)/C4
      IF (C1) 163,162,163
```

```
162 C6=-C5/C3
    GO TO 166
163 C4=0.5*C3/C1
    C6=SQRT (C4*C4-C5/C1)
    IF (C4) 164,165,165
164 C6=-C6
165 C6=C6-C4
166 CX=CX+C6
167 PZ=V*CX
    Q=(PZ*PZ-1.0)/FLD/PRC
    VC=VC+C6*DVC
    C4=VC*FLAT*V
    C4=Q/C4
    C5=-DVC/VC
    C6=CX**EX1
    SL1=VC/2.0*FLAT/PZ*(C5*C4+EX4*C6/C4*(EX6-C6/CX))
200 PZSQ=1.0+Q*FLD
    PSC=PZ
    PZ=SQRT(PZSQ)
    C4=SL1*FLD*PRC
    C4=(PRC+C4)/(1.0+C4)
    FDV1=C4*FDVL/PSC*FLD*DLT
    SLP=C4*SL1
    BCZ=(CMP*Q-SLP*PZSQ)*DLT
    BCZ1=DLT*SLP
    IF(NVL) 192,191,192
191 FLWZ=0.0
    SPZ=1.5707963
    GO TO 193
192 FLWZ=Q/(FLAT*V)
    SPZ=(PZSQ*PZ-1.0)/FLD*1.0471976/Q
193 PZ2(1)=PZSQ
    C1=1.0+Q*FLD
    C2=Q*DLT
    PZN(1)=PZ
    DO 201 I=2,MT1
    C1=C1-C2
    PZ2(I)=C1
201 PZN(I)=SQRT (C1)
    DO 283 NL=1,NLB
    FLA=FLBL(NL)
    IF (NLB) 198,197,198
197 FLA=0.0
    FLX=1.0
    GO TO 199
198 FLX=2.0*FLA
    FLA2=FLA*FLA
199 FLFZ=FLA/PZ
    PR=-DL2H*PZSQ
    PE=0.0
    RR=DL2H
    RE=DL2H*FLFZ
    TFR(1)=0.0
    TFE(1)=0.0
    VFR(1)=1.0
    VFE(1)=0.0
    C1=0.0
    C2=0.0
    C3=0.5/PZ
    C4=0.0
    DO 205 I=2,MT1
```

```
      TFR(I)=TFR(I-1)+PR+BCZ
      TFE(I)=TFE(I-1)+PE
      VFR(I)=VFR(I-1)+RR+BCZ1
      VFE(I)=VFE(I-1)+RE
      C5=PZN(I)
      IF (I-MT1) 203,202,202
  202 C5=2.0*C5
      C6=1.0-TFR(I)
      C7=TFE(I)
      C8=VFR(I)
      C9=VFE(I)
      GO TO 204
  203 C6=FLA/C5
      PR=PR+DL2*(TFR(I)-C6*TFE(I)-C5*C5)
      PE=PE+DL2*(TFE(I)+C6*TFR(I))
      RR=RR+DL2*(VFR(I)-C6*VFE(I))
      RE=RE+DL2*(VFE(I)+C6*VFR(I))
  204 C1=C1+TFR(I)/C5
      C2=C2+TFE(I)/C5
      C3=C3+VFR(I)/C5
  205 C4=C4+VFE(I)/C5
      C5=C8*C8+C9*C9
      HZR=(C6*C8-C7*C9)/C5
      HZE=(-C6*C9-C7*C8)/C5
      SFZR=SFACT*(C1+C3*HZR-C4*HZE)
      SFZE=SFACT*(C2+C3*HZE+C4*HZR)
      DO 206 I=1,MT1
      TFR(I)=(TFR(I)+HZR*VFR(I)-HZE*VFE(I))/PZN(I)
  206 TFE(I)=(TFE(I)+HZE*VFR(I)+HZR*VFE(I))/PZN(I)
      FRSW=SPZ-SFZR
      FTSW=-SFZE
      DO 282 NE=1,NEP
      EPS=EPSL(NE)
      MC=MNS
      GAM=0.0
      EP2=EPS*EPS
      E7=1.0-EP2
      C1=SQRT (E7)
      C2=1.0+C1
      E1=2.0/C2
      E2=E1*C2
      E3=E1/C1
      E4=E3*EPS
      E5=E4*EPS/C2
      E6=2.0/(C1*E7)-E3
      E7=E4*EPS/E7+E5/C1
      FRS=E4*FRSW
      FTS=E2*FTSW
      WPLD=SQRT (FRS*FRS+FTS*FTS)
      WDPLD=WPLD/VMW
      IF (FRS) 230,226,230
  226 IF (FTS) 227,228,229
  227 ANGZ=-90.0
      GO TO 232
  228 ANGZ=0.0
      GO TO 232
  229 ANGZ=90.0
      GO TO 232
  230 ANGZ=FTS/FRS
      ANGZ=57.295780*ATAN (ANGZ)
      IF (FRS) 231,232,232
```

```
231 ANGZ=ANGZ+180.0
232 WRITE(6,311)
    WRITE(6,810)FLD,V,FLAS,FLA,EPS
    WRITE(6,312)
    WRITE(6,810)FLAT,Q,FLWZ,SLP,CX
    WRITE(6,313)
    WRITE(6,810)WPLD,WDPLD,ANGZ,FRS,FTS
    FRS=FRS/EPS
    FTS=FTS/EPS
    MS=2
    KC=0
    LC=0
    KK=0
    DO 281 NS=1,MNS
    SIG=BSIG(NS)
    SIG2=SIG*SIG
    IF(NS-2) 250,550,250
550 WRITE(6,318)
250 KS=0
    SIG2=SIG*SIG
    AS=BCZ
    FDV2=FLX*SIG*FDV1
    AS1=FDV2*PZSQ
    BS=1.0
    DO 251 I=1,MT1
    FRR(I)=-PZ2(I)
    FJR(I)=0.0
251 FJE(I)=0.0
252 PR=DL2H*FRR(1)
    PE=0.0
    QR=DL2H*FJR(1)
    QE=DL2H*FJE(1)
    RR=DL2H
    RE=DL2H*FLFZ
    SR=DL2H*FLX/PZ*SIG
    SE=0.0
    TR=0.0
    TE=0.0
    UR=0.0
    UE=0.0
    VE=0.0
    WR=0.0
    WE=0.0
    VR=1.0
    C1=0.0
    C2=0.0
    C3=0.0
    C4=0.0
    C6=0.0
    C7=0.0
    C8=0.0
    C5=0.5/PZ
    DO 256 I=2,MT1
    TR=TR+PR+AS
    TE=TE+PE
    UR=UR+QR-AS1
    UE=UE+QE
    VR=VR+RR+BCZ1
    VE=VE+RE
    WR=WR+SR+FDV2
    WE=WE+SE
```

```
      A1=PZN(I)
      IF (I-MT1) 254,253,253
  253 A1=2.0
      GO TO 255
  254 A2=FLA/A1
      A3=FLX/A1*SIG
      PR=PR+DL2*(TR-A2*TE-A3*UR+FRR(I))
      PE=PE+DL2*(TE+A2*TR-A3*UE)
      QR=QR+DL2*(UR-A2*UE+A3*TR+FJR(I))
      QE=QE+DL2*(UE+A2*UR+A3*TE+FJE(I))
      RR=RR+DL2*(VR-A2*VE-A3*WR)
      RE=RE+DL2*(VE+A2*VR-A3*WE)
      SR=SR+DL2*(WR-A2*WE+A3*VR)
      SE=SE+DL2*(WE+A2*WR+A3*VE)
  255 C1=C1+TR/A1
      C2=C2+TE/A1
      C3=C3+UR/A1
      C4=C4+UE/A1
      C5=C5+VR/A1
      C6=C6+VE/A1
      C7=C7+WR/A1
  256 C8=C8+WE/A1
      TR=BS-TR
      A1=TR*VR+TE*VE-UR*WR+UE*WE
      A2=TR*VE-TE*VR-UR*WE-UE*WR
      A3=-TR*WR-TE*WE-UR*VR+UE*VE
      A4=-TR*WE+TE*WR-UR*VE-UE*VR
      A5=VR*VR-VE*VE+WR*WR-WE*WE
      A6=2.0*(VR*VE+WR*WE)
      A7=A5*A5+A6*A6
      HZR=(A1*A5+A2*A6)/A7
      HZE=(A2*A5-A1*A6)/A7
      HZJR=(A3*A5+A4*A6)/A7
      HZJE=(A4*A5-A3*A6)/A7
      IF(INP) 401,402,402
  401 WRITE(6,810)A1,A2,A3,A4,A5
      WRITE(6,810)A6,A7,HZR,HZE,HZJR
  402 G2RR=SFACT*(C1+HZR*C5-HZE*C6-HZJR*C7+HZJE*C8)
      G2RE=SFACT*(C2+HZR*C6+HZE*C5-HZJR*C8-HZJE*C7)
      G2JR=SFACT*(C3+HZR*C7-HZE*C8+HZJR*C5-HZJE*C6)
      G2JE=SFACT*(C4+HZR*C8+HZE*C7+HZJR*C6+HZJE*C5)
      IF (KS) 259,257,259
  257 G1RR=G2RR
      G1RE=G2RE
      G1JR=G2JR
      G1JE=G2JE
      KS=1
      AS=0.0
      AS1=0.0
      BS=0.0
      A1=SIG*FLX
      DO 258 I=1,MT1
      FRR(I)=0.0
      FJR(I)=A1*TFE(I)
  258 FJE(I)=-A1*TFR(I)
      GO TO 252
  259 CRR=E6*SPZ-E7*SFZR-E3*G1RR
      CRJ=-E3*G1JR
      DRR=-E3*G2RR+FTS
      DRJ=-E3*G2JR
      CTR=-E5*SFZE-E1*G1RE
```

```
      CTJ=-E1*G1JE
      DTR=-E1*G2RE-FRS
      DTJ=-E1*G2JE
      A1=(CRR+DTR)/2.0
      A2=CRJ-DTJ
      A3=0.5*A2
      A4=DRR*CTJ+CTR*DRJ
      A5=DRJ*CTJ-CRJ*DTJ-DRR*CTR
      C3=(A1*A1-A3*A3+A5)/2.0
      C4=(A1*(CRJ+DTJ)-A4)/2.0
      C4=C3*C3+C4*C4
      C4=SQRT (C4)
      FFC=-C3+C4
      FFC=SQRT (FFC)-A3
      C3=CRJ+FFC
      C4=FFC-DTJ
      C5=C3+C4
      EFC=(CRR*C4-DTR*C3+A4)/C5
      IF(LC) 719,188,719
  188 IF(KC) 190,189,190
  189 FF(NS)=FFC
      EF(NS)=EFC
  190 WRT=(A4-CRR*DTJ-DTR*CRJ)/A2
      C4=WRT
      FEJL=(CRR-WRT)*(DTR+WRT)+A5
      IF(NS-1) 510,510,511
  510 CTR=-CTR
      CTJ=-CTJ
      DTR=-DTR
      DTJ=-DTJ
      WRITE(6,321)
      WRITE(6,810)CRR,CRJ,DRR,DRJ
      WRITE(6,322)
      WRITE(6,810)CTR,CTJ,DTR,DTJ
      C3=A3*A3-FEJL
      IF(C3) 512,513,513
  512 C1=-1.0
      C2=-1.0
      C3=-1.0
      C4=-1.0
      GO TO 514
  513 C3=SQRT(C3)
      C4=A3+C3
      C5=A3-C3
      C3=2.0*C3
      C3=(A1*(CRJ-DTJ)-A4)/C3
      C2=(CRR+DTR)/2.0
      C1=C2+C3
      C2=C2-C3
  514 WRITE(6,323)
      WRITE(6,810)C1,C2,C4,C5
      GO TO 281
  511 IF (NLB) 195,194,195
  194 WRT1=WRT
      WRT2=WRT/VMW
      WRT3=WRT/SIG2*288.0
      GO TO 196
  195 WRT1=WRT
      WRT=WRT1/SIG2
      WRT2=WRT/VMW
      WRT3=WRT/FLA2*72.0
```

```
196 IF(KK) 520,521,520
521 WRITE(6,804)SIG,WRT1,FEJL
    GO TO 522
520 WRITE(6,803)SIG,WRT1,WRT,WRT2,WRT3,FEJL
    KK=0
522 IF (KC) 273,260,254
260 IF(NS-2) 262,262,261
261 C1=Y1*FEJL
    IF (C1) 263,273,262
262 Y1=FEJL
    X1=SIG
    W1=WRT
    MS=MS+1
    IF(MNS-MS) 571,281,281
571 IF(INT) 699,282,699
263 Y3=FEJL
    W3=WRT
    X3=SIG
    KC=1
    DSIG=X3-X1
    SIG=SIG-0.5*DSIG
    GO TO 250
264 Y2=FEJL
    W2=WRT
    X2=SIG
    C3=X3-X2
    C4=X2-X1
    C5=X3-X1
    C6=(Y3-Y2)/C5
    C6=C6/C3
    C7=(Y1-Y2)/C5
    C7=C7/C4
    C5=C6+C7
    C4=C4*C6-C3*C7
    D6=(W3-W2)/C5
    D6=D6/C3
    D7=(W1-W2)/C5
    D7=D7/C4
    A1=D6+D7
    A2=C4*D6-C3*D7
    A3=W2
    IF(C5) 266,265,266
265 C3=-Y2/C4
    GO TO 271
266 C4=C4/C5*0.5
    C5=C4*C4-Y2/C5
    C5=SQRT(C5)
    IF(C4) 269,270,270
269 C5=-C5
270 C3=-C4+C5
271 SIG =X2+C3
    WRT=(A1*C3+A2)*C3+A3
    KC=-1
    WRITE(6,810)SIG,WRT
    KK=1
    GO TO 250
273 KC=0
    FFC1=FFC
    EFC1=EFC
    FKAP=C4
    GAM=SIG
```

```
      GAM2=SIG2
      MC=NS
      IF(INT) 290,282,699
  290 Y1=Y3
      W1=W3
      X1=X3
      GO TO 281
  699 IK=1
  700 PDK=PDKL(IK)
      PDKQ=PDK*PDK
      IF(GAM) 703,704,703
  703 C1=FKAP+PDK
      C1=FKAP/C1*PDK/GAM2
      WRITE(6,811)PDK,C1
  704 IM=1
  701 PDM=PDML(IM)
      PDM1=1.0+PDM
      PDM2=PDM1*PDM1
      IF(PDM) 705,706,705
  705 IF(GAM) 698,706,698
  698 C1=(PDK+FKAP*PDM1)/PDM*0.5
      C2=PDK/PDM*FKAP
      C3=C1*C1-C2
      C3=SQRT (C3)
      C2=( C1-C3)/GAM2
      C1=( C1+C3)/GAM2
      WRITE(6,814)
      WRITE(6,802)PDM,PDK,C1,C2
  706 IF(ND) 741,741,707
  707 ID=1
  702 PDP1=PDL(ID)
      PDP=PDP1/12.0*FLA
      PDPQ=PDP*PDP
      WRITE(6,812)PDP1
      WRITE(6,314)
      NC=2
      JC=0
      LC=1
  718 FFC=FF(NC)
      EFC=EF(NC)
      X=BSIG(NC)
  719 C2=X*X
      C1=C2*PDPQ
      C4=PDP*X
      S3=C4-PDM1*FFC
      S3=(C4*EFC-PDK*FFC)/S3
      C3=PDK-PDM1*S3
      C3=C3*C3+C1
      ERR3=FFC-C4*S3/C3*S3
      IF(NLB) 501,502,501
  502 RT1=S3
      RT2=S3/VMW
      RT3=S3/C2  *288.0
      GO TO 503
  501 RT1=S3/C2
      RT2=RT1/VMW
      RT3=RT1/FLA2*72.0
  503 IF(KK) 523,524,523
  524 WRITE(6,804)X,S3,ERR3
      WRITE(6,805)EFC,FFC
      GO TO 525
```

```
523 WRITE(6,803)X,S3,RT1,RT2,RT3,ERR3
    WRITE(6,805)EFC,FFC
    KK=0
525 IF(JC) 760,728,720
720 JC=0
    GO TO 733
728 IF(NC-2) 729,729,730
729 X3=X
    Y3=ERR3
    NC=NC+1
    GO TO 718
730 X1=X3
    X3=X
    Y1=Y3
    Y3=ERR3
    C3=Y1*Y3
    IF(C3) 731,720,733
731 JC=-1
    X=(X1+X3)/2.0
    SIG=X
    GO TO 250
733 NC=NC+1
    IF(MC-NC) 740,734,718
734 IF(GAM) 735,718,735
735 X=GAM
    FFC=FFC1
    EFC=EFC1
    GO TO 719
760 X2=X
    Y2=ERR3
    C3=X3-X2
    C4=X2-X1
    C5=X3-X1
    C6=(Y3-Y2)/C5
    C6=C6/C3
    C7=(Y1-Y2)/C5
    C7=C7/C4
    C5=C6+C7
    C4=C4*C6-C3*C7
    IF(C5) 762,761,762
761 C3=-Y2/C4
    GO TO 765
762 C4=C4/C5*0.5
    C5=C4*C4-Y2/C5
    C5=SQRT (C5)
    IF(C4) 763,764,764
763 C5=-C5
764 C3=-C4+C5
765 X=X2+C3
    SIG=X
    JC=1
    KK=1
    GO TO 250
740 ID=ID+1
    IF(ND-ID) 741,702,702
741 IM=IM+1
    IF(NM-IM) 742,701,701
742 IK=IK+1
    IF(NK-IK) 281,700,700
281 CONTINUE
282 CONTINUE
```

```
283 CONTINUE
284 CONTINUE
285 CONTINUE
    IF(INP) 508,18,508
 18 WRITE(6,315)
    GO TO 13
508 STOP
300 FORMAT(72H1
   1                    )
301 FORMAT(5E14.6)
302 FORMAT(8I5)
303 FORMAT(6H0 NO.V7X5HN.L-S6X5HN.LAM6X4HN.EP7X5HN.SIG6X4HDIVS8X3HINP)
304 FORMAT(1XI4,7XI4,7XI4,7XI4,7XI4,7XI4,7XI4)
305 FORMAT(16H0PRESSURE RATIOS)
306 FORMAT(14H0LAMBDA-S LIST)
307 FORMAT(12H0LAMBDA LIST)
308 FORMAT(26H0LIST  OF FREQUENCY RATIOS)
309 FORMAT(13H0   OR.RA.FCT5X8HINH.COMP6X8HFEED.VOL4X11HSP.HE.RATIO)
310 FORMAT(28H0VENA CONTRACTA COEFFICIENTS)
311 FORMAT(1H15X3HL/D9X8HPR.RATIO6X8HLAMBDA-S7X6HLAMBDA7X8HECCENTR.)
312 FORMAT(12H0   LAMBDA-T9X1HQ12X3HM-011X3HPSI6X13HORIF.PR.RATIO)
313 FORMAT(1H04X6HW/PALD8X6HW/DPLD7X7HATT.ANG7X7HFR/PALD7X7HFT/PALD)
314 FORMAT(11H0     FREQ/W7X8H(S*GAM)25X9HCMW2/PALD5X9HCMW2/DPLD3X14HMP
   1A/MU2L(R/C)54X5HERROR/29X13HEFF.STIFFNESS2X11HEFF.DAMPING)
315 FORMAT(1H1)
316 FORMAT(20H0ECCENTRICITY RATIOS)
318 FORMAT(11H0     FREQ/W7X8H(S*GAM)25X9HCMW2/PALD5X9HCMW2/DPLD3X14HMP
   1A/MU2L(R/C)54X5HERROR)
321 FORMAT(55H0   CKXX/PALD    CWCXX/PALD     CKXY/PALD    CWCXY/PALD)
322 FORMAT(55H0   CKYX/PALD    CWCYX/PALD     CKYY/PALD    CWCYY/PALD)
323 FORMAT(55H0 EFF.STIFF-1   EFF.STIFF-2    EFF.DAMP-1    EFF.DAMP-2)
802 FORMAT(4(1PE15.7))
803 FORMAT(6(1PE14.6))
804 FORMAT(1PE14.6,1PE14.6,42X1PE14.6)
805 FORMAT(28X1PE14.6,1PE14.6)
810 FORMAT(5(1PE14.6))
811 FORMAT(/16H0PEDEST.STIFFN.=,1PE13.6,14H  MCW**2/PALD=,1PE13.6)
812 FORMAT(/20H0  PEDESTAL DAMPING=,1PE13.6)
814 FORMAT(/14H0  PEDEST.MASS5X9HPED.STIFF6X9HMASS RT.16X9HMASS RT.2)
    END
```

```
0.9985          0.9955          0.9915          0.9871          0.9824
0.9776          0.9726          0.9675          0.9623          0.957
0.9516          0.946           0.9402          0.9341          0.9276
0.9206          0.913           0.9046          0.895           0.8836
0.869           0.8477          0.8105          0.736           0.6
1.4
HYBRID-HYDROSTATIC RING BEARING , PS/PA=1.25 , 7-7-1967
  1    1    7    1   33   15    1    0
1.0             1.5             1000.0          0.0
1.25
0.7
0.3             1.0             2.0             5.0             10.0
30.0            100.0
0.02
1.0             0.00001         0.00005         0.0001          0.0005
0.001           0.003           0.006           0.01            0.03
0.06            0.1             0.15            0.2             0.25
0.3             0.35            0.4             0.42            0.44
0.46            0.48            0.485           0.49            0.492
0.494           0.496           0.497           0.498           0.499
0.5             0.501           0.51
  3    1   10
0.05            0.038           0.019
0.0
0.2             0.5             2.0             5.0             20.0
50.0            100.            200.            500.            1000.
```

```
HYBRID-HYDROSTATIC RING BEARING , PS/PA=1.25 , 7-7-1967

 NO.V        N.L-S       N.LAM       N.EP        N.SIG       DIVS        INP
   1           1           7           1          33          15          0

   OR.RA.FCT      INH.COMP       FEED.VOL     SP.HE.RATIO
 1.500000E 00  1.000000E 03  0.             1.400000E 00

PRESSURE RATIOS
 1.250000E 00

LAMBDA-S LIST
 7.000000E-01

LAMBDA LIST
 3.000000E-01  1.000000E 00  2.000000E 00  5.000000E 00  1.000000E 01
 3.000000E 01  1.000000E 02

ECCENTRICITY RATIOS
 2.000000E-02

LIST  OF FREQUENCY RATIOS
 1.000000E 00  1.000000E-05  5.000000E-05 10.000000E-05  5.000000E-04
10.000000E-04  3.000000E-03  6.000000E-03 10.000000E-03  3.000000E-02
 6.000000E-02  1.000000E-01  1.500000E-01  2.000000E-01  2.500000E-01
 3.000000E-01  3.500000E-01  4.000000E-01  4.200000E-01  4.400000E-01
 4.600000E-01  4.800000E-01  4.850000E-01  4.900000E-01  4.920000E-01
 4.940000E-01  4.960000E-01  4.970000E-01  4.980000E-01  4.990000E-01
 5.000000E-01  5.010000E-01  5.100000E-01
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IBJOB VERSICN 5 HAS CONTROL.
$IBJOB MTI
$IBLDR MTI                                                    11/21/66    MTI 0000

VENA CCNTRACTA COEFFICIENTS
 9.985000E-01  9.955000E-01  9.915000E-01  9.871000E-01  9.824000E-01
 9.776000E-01  9.726000E-01  9.675000E-01  9.623000E-01  9.570000E-01
 9.516000E-01  9.460000E-01  9.402000E-01  9.341000E-01  9.276000E-01
 9.206000E-01  9.130000E-01  9.046000E-01  8.950000E-01  8.836000E-01
 8.690000E-01  8.477000E-01  8.105000E-01  7.360000E-01  6.000000E-01
```

L/D	PR.RATIO	LAMBDA-S	LAMBDA	ECCENTR.
1.000000E 00	1.250000E 00	7.000000E-01	3.000000E-01	2.000000E-02

LAMBDA-T	Q	M-O	PSI	ORIF.PR.RATIO
8.750000E-01	2.762114E-01	2.525362E-01	1.332548E 00	9.514005E-01

W/PALD	W/DPLD	ATT.ANG	FR/PALD	FT/PALD
2.022866E-03	8.091465E-03	4.370880E 01	1.462251E-03	1.397788E-03

CKXX/PALD	CWCXX/PALD	CKXY/PALD	CWCXY/PALD
8.666243E-02	1.377865E-01	6.786915E-02	-1.350596E-02

CKYX/PALD	CWCYX/PALD	CKYY/PALD	CWCYY/PALD
-6.788353E-02	1.350326E-02	8.661582E-02	1.377589E-01

EFF.STIFF-1	EFF.STIFF-2	EFF.DAMP-1	EFF.DAMP-2
1.003508E-01	7.292743E-02	2.046240E-01	7.092146E-02

FREQ/W	(S*GAM)2	CMW2/PALD	CMW2/DPLD	MPA/MU2L(R/C)5	ERROR
1.000000E-05	6.621407E-02				-4.933395E-03
5.000000E-05	6.621407E-02				-4.933395E-03
10.000000E-05	6.621407E-02				-4.933394E-03
5.000000E-04	6.621407E-02				-4.933390E-03
10.000000E-04	6.621408E-02				-4.933375E-03
3.000000E-03	6.621416E-02				-4.933216E-03
6.000000E-03	6.621457E-02				-4.932681E-03
10.000000E-03	6.621547E-02				-4.931412E-03
3.000000E-02	6.622655E-02				-4.915548E-03
6.000000E-02	6.626411E-02				-4.862009E-03
1.000000E-01	6.635310E-02				-4.735124E-03
1.500000E-01	6.652693E-02				-4.487395E-03
2.000000E-01	6.677020E-02				-4.140781E-03
2.500000E-01	6.708286E-02				-3.695483E-03
3.000000E-01	6.746478E-02				-3.151759E-03
3.500000E-01	6.791589E-02				-2.509956E-03
4.000000E-01	6.843601E-02				-1.770424E-03
4.200000E-01	6.866333E-02				-1.447346E-03
4.400000E-01	6.890169E-02				-1.108731E-03
4.600000E-01	6.915176E-02				-7.546133E-04
4.800000E-01	6.941139E-02				-3.850241E-04
4.850000E-01	6.947822E-02				-2.902138E-04
4.900000E-01	6.954571E-02				-1.944393E-04
4.920000E-01	6.957288E-02				-1.558591E-04
4.940000E-01	6.960014E-02				-1.171258E-04
4.960000E-01	6.962758E-02				-7.823831E-05
4.970000E-01	6.964126E-02				-5.873647E-05
4.980000E-01	6.965506E-02				-3.919589E-05
4.990000E-01	6.966892E-02				-1.961708E-05
5.000000E-01	6.968270E-02				3.224159E-10
4.995000E-01	6.967579E-02				-9.813994E-06
5.000000E-01	2.787861E-01				
5.000000E-01	6.968270E-02	2.787308E-01	1.114923E 00	2.229847E 02	-1.368790E-10

PEDEST.STIFFN.= 5.000000E-02 MCW**2/PALD= 1.164457E-01

PEDESTAL DAMPING= 2.000000E-01

FREQ/W	(S*GAM)2	CMW2/PALD EFF.STIFFNESS	CMW2/DPLD EFF.DAMPING	MPA/MU2L(R/C)5	ERROR
1.000000E-05	4.999998E-02				-4.494735E 04

		7.313440E-02	6.989497E-02		
5.000000E-05	4.999992E-02				-9.009510E 03
		7.313385E-02	6.988941E-02		
10.000000E-05	4.999983E-02				-4.502197E 03
		7.313316E-02	6.988245E-02		
5.000000E-04	4.999917E-02				-9.010618E 02
		7.312765E-02	6.982681E-02		
10.000000E-04	4.999834E-02				-4.503558E 02
		7.312078E-02	6.975726E-02		
3.000000E-03	4.999501E-02				-1.499771E 02
		7.309331E-02	6.947903E-02		
6.000000E-03	4.998998E-02				-7.487353E 01
		7.305239E-02	6.906166E-02		
10.000000E-03	4.998320E-02				-4.483033E 01
		7.299818E-02	6.850510E-02		
3.000000E-02	4.994799E-02				-1.478160E 01
		7.273359E-02	6.572138E-02		
6.000000E-02	4.989048E-02				-7.259640E 00
		7.235728E-02	6.154291E-02		
1.000000E-01	4.980264E-02				-4.235936E 00
		7.189382E-02	5.596658E-02		
1.500000E-01	4.966765E-02				-2.700306E 00
		7.137624E-02	4.898883E-02		
2.000000E-01	4.948962E-02				-1.901080E 00
		7.092734E-02	4.200385E-02		
2.500000E-01	4.923927E-02				-1.380375E 00
		7.054729E-02	3.501268E-02		
3.000000E-01	4.885526E-02				-9.775532E-01
		7.023618E-02	2.801635E-02		
3.500000E-01	4.818385E-02				-6.177224E-01
		6.999411E-02	2.101588E-02		
4.000000E-01	4.669986E-02				-2.789018E-01
		6.982114E-02	1.401231E-02		
4.200000E-01	4.544253E-02				-1.610079E-01
		6.977129E-02	1.121025E-02		
4.400000E-01	4.300706E-02				-6.730940E-02
		6.973254E-02	8.407920E-03		
4.600000E-01	3.628871E-02				-1.036437E-02
		6.970487E-02	5.605391E-03		
4.800000E-01	-6.733018E-02				2.012720E-03
		6.968825E-02	2.802725E-03		
4.700000E-01	2.503681E-02				1.860958E-03
		6.969517E-02	4.204076E-03		
4.675185E-01	2.920510E-02	1.336168E-01	5.344673E-01	1.068935E 02	-1.417531E-06
		6.969735E-02	4.551807E-03		
4.850000E-01	1.978179E-01				-2.239775E-03
		6.968585E-02	2.102047E-03		
4.825000E-01	-1.140688E 00				2.382574E-04
		6.968693E-02	2.452388E-03		
4.827751E-01	1.270730E 03	5.452098E 03	2.180839E 04	4.361678E 06	-2.273591E-07
		6.968678E-02	2.413838E-03		
4.900000E-01	9.598949E-02				-9.241067E-03
		6.968413E-02	1.401367E-03		
4.920000E-01	8.616499E-02				-1.277892E-02
		6.968361E-02	1.121090E-03		
4.940000E-01	7.904167E-02				-1.672000E-02
		6.968319E-02	8.408194E-04		
4.960000E-01	7.543107E-02				-2.105232E-02
		6.968294E-02	5.605491E-04		
4.970000E-01	7.369077E-02				-2.336124E-02
		6.968278E-02	4.204111E-04		

4.980000E-01	7.217921E-02				-2.576278E-02
		6.968274E-02	2.802722E-04		
4.990000E-01	7.085406E-02				-2.825516E-02
		6.968276E-02	1.401352E-04		
5.000000E-01	6.968270E-02				-3.083686E-02
		6.968270E-02	0.		

PEDESTAL DAMPING= 5.000000E-01

FREQ/W	(S*GAM)2	CMW2/PALD EFF.STIFFNESS	CMW2/DPLD EFF.DAMPING	MPA/MU2L(R/C)5	ERROR
1.000000E-05	4.999996E-02				-1.801912E 04
		7.313440E-02	6.989497E-02		
5.000000E-05	4.999979E-02				-3.605347E 03
		7.313385E-02	6.988941E-02		
10.000000E-05	4.999959E-02				-1.802223E 03
		7.313316E-02	6.988245E-02		
5.000000E-04	4.999793E-02				-3.603328E 02
		7.312765E-02	6.982681E-02		
10.000000E-04	4.999586E-02				-1.800926E 02
		7.312078E-02	6.975726E-02		
3.000000E-03	4.998753E-02				-5.992807E 01
		7.309331E-02	6.947903E-02		
6.000000E-03	4.997494E-02				-2.988602E 01
		7.305239E-02	6.906166E-02		
10.000000E-03	4.995796E-02				-1.786884E 01
		7.299818E-02	6.850510E-02		
3.000000E-02	4.966954E-02				-5.850193E 00
		7.273359E-02	6.572138E-02		
6.000000E-02	4.972418E-02				-2.842366E 00
		7.235728E-02	6.154291E-02		
1.000000E-01	4.949983E-02				-1.633696E 00
		7.189382E-02	5.596658E-02		
1.500000E-01	4.914928E-02				-1.019420E 00
		7.137624E-02	4.898883E-02		
2.000000E-01	4.867561E-02				-6.980372E-01
		7.092734E-02	4.200385E-02		
2.500000E-01	4.798636E-02				-4.856614E-01
		7.054729E-02	3.501268E-02		
3.000000E-01	4.687280E-02				-3.175511E-01
		7.023618E-02	2.801635E-02		
3.500000E-01	4.474341E-02				-1.662452E-01
		6.999411E-02	2.101588E-02		
4.000000E-01	3.900330E-02				-3.811148E-02
		6.982114E-02	1.401231E-02		
4.200000E-01	3.258475E-02				-5.637898E-03
		6.977129E-02	1.121025E-02		
4.400000E-01	1.267814E-02				7.786740E-03
		6.973254E-02	8.407920E-03		
4.300000E-01	2.605859E-02				3.746990E-03
		6.975052E-02	9.809118E-03		
4.252980E-01	2.960905E-02	1.636959E-01	6.547835E-01	1.309567E 02	-2.791046E-05
		6.975993E-02	1.046793E-02		
4.600000E-01	8.335150E-01				-9.015465E-04
		6.970487E-02	5.605391E-03		
4.500000E-01	-3.027189E-02				6.210613E-03
		6.971728E-02	7.006675E-03		
4.590629E-01	7.155573E 01	3.395471E 02	1.358188E 03	2.716377E 05	-9.609095E-06
		6.970591E-02	5.736705E-03		
4.800000E-01	8.694693E-02				-2.957141E-02

		6.968825E-02	2.802725E-03		
4.850000E-01	8.013430E-02				-3.910153E-02
		6.968585E-02	2.102047E-03		
4.900000E-01	7.552384E-02				-4.930543E-02
		6.968413E-02	1.401367E-03		
4.920000E-01	7.407165E-02				-5.354350E-02
		6.968361E-02	1.121090E-03		
4.940000E-01	7.278582E-02				-5.785708E-02
		6.968319E-02	8.408194E-04		
4.960000E-01	7.163938E-02				-6.223691E-02
		6.968294E-02	5.605491E-04		
4.970000E-01	7.111142E-02				-6.444930E-02
		6.968278E-02	4.204111E-04		
4.980000E-01	7.061071E-02				-6.667459E-02
		6.968274E-02	2.802722E-04		
4.990000E-01	7.013513E-02				-6.891187E-02
		6.968276E-02	1.401352E-04		
5.000000E-01	6.968270E-02				-7.116072E-02
		6.968270E-02	0.		

PEDESTAL DAMPING= 2.000000E 00

FREQ/W	(S*GAM)2	CMW2/PALD EFF.STIFFNESS	CMW2/DPLD EFF.DAMPING	MPA/MU2L(R/C)5	ERROR
1.000000E-05	4.999983E-02				-4.504705E 03
		7.313440E-02	6.989497E-02		
5.000000E-05	4.999917E-02				-9.010617E 02
		7.313385E-02	6.988941E-02		
10.000000E-05	4.999834E-02				-4.504810E 02
		7.313316E-02	6.988245E-02		
5.000000E-04	4.999172E-02				-9.000745E 01
		7.312765E-02	6.982681E-02		
10.000000E-04	4.998342E-02				-4.494466E 01
		7.312078E-02	6.975726E-02		
3.000000E-03	4.995003E-02				-1.490265E 01
		7.309331E-02	6.947903E-02		
6.000000E-03	4.989942E-02				-7.392185E 00
		7.305239E-02	6.906166E-02		
10.000000E-03	4.983091E-02				-4.388043E 00
		7.299818E-02	6.850510E-02		
3.000000E-02	4.946902E-02				-1.384065E 00
		7.273359E-02	6.572138E-02		
6.000000E-02	4.885431E-02				-6.327760E-01
		7.235728E-02	6.154291E-02		
1.000000E-01	4.785214E-02				-3.306548E-01
		7.189382E-02	5.596658E-02		
1.500000E-01	4.613578E-02				-1.752784E-01
		7.137624E-02	4.898883E-02		
2.000000E-01	4.346099E-02				-9.030746E-02
		7.092734E-02	4.200395E-02		
2.500000E-01	3.859127E-02				-2.998548E-02
		7.054729E-02	3.501268E-02		
3.000000E-01	2.667990E-02				1.412863E-02
		7.023618E-02	2.801635E-02		
2.750000E-01	3.422371E-02				-5.257838E-03
		7.038309E-02	3.151510E-02		
2.811421E-01	3.276569E-02	4.145918E-01	1.658367E 00	3.316734E 02	1.284359E-04
		7.034542E-02	3.065562E-02		
3.500000E-01	-4.951887E-02				1.681304E-02
		6.999411E-02	2.101588E-02		

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13. ABSTRACT

~~This~~ volume treats three special bearing types selected for study because of their favorable stability characteristics and, hence, their potential for use in high speed rotating machinery applications. The three bearing types are:

a. The Three Lobe Journal Bearing
b. The Floating Sleeve Bearing with an Incompressible Lubricant
c. The Floating Sleeve Bearing with a Compressible Lubricant.

In the floating sleeve bearings, the ring is prevented from rotating but is otherwise free to move. The ring is floated by pressurizing the outer film of the bearing In the case of a compressible lubricant, the inner film is pressurized as well.

The volume gives extensive design data in form of charts and tables from which the bearing dimensions can be obtained for a given application. Data are given for bearing flow, friction power loss and the speed at which hydrodynamic instability sets in. In addition, two computer programs accompany the volume, and instructions and listings of the programs are included. The programs may be used to obtain data for cases not covered by the presented design data.

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