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STRIPLINE FERRITE DEVICES

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This report describes the theory and experimental results obtained for an S-Band microstrip three-port ring circulator. Three meander line non-reciprocal phase shifters are tied together in a ring three tee-junctions. The circuit was deposited on a 20 mil thick ferrite disc. A switching wire runs through a hole in the center of the disc and the device is operated in a self-latching mode. The primary reasons for choosing this geometry were its low manufacturing cost and its compatibility with microwave integrated circuits.

The initial circulator that was designed has a 2 db insertion loss and a bandwidth of approximately 2%. Attempts were made to increase the bandwidth by varying various meander line phase shifter parameters. All the devices tested had band widths of approximately 2%

An analysis of the device shows that it has a high Q caused by the large insertion phases (of the phase shifters) required to obtain the required amount of non-reciprocal phase shift. Holding currents or permanent magnets rather than latching operation could increase the bandwidth. It appears doubtful that bandwidths greater than 10% can be obtained from the ring circulator.
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I. INTRODUCTION

This report describes the theory and experimental results obtained for an S-Band microstrip three-port ring circulator. Three meander line non-reciprocal phase shifters are tied together in a ring three tee-junctions. The circuit was deposited on a 20 mil thick ferrite disc. A switching wire runs through a hole in the center of the disc and the device is operated in a self-latching mode. The primary reasons for choosing this geometry were its low manufacturing cost and its compatibility with microwave integrated circuits.

The initial circulator that was designed has a 2 db insertion loss and a bandwidth of approximately 2%. Attempts were made to increase the bandwidth by varying various meander line phase shifter parameters. All the devices tested had bandwidths of approximately 2%. An analysis of the device shows that it has a high Q caused by the large insertion phases (of the phase shifters) required to obtain the required amount of non-reciprocal phase shift. Holding currents or permanent magnets rather than latching operation could increase the bandwidth. It appears doubtful that bandwidths greater than 10% can be obtained from the ring circulator.
2. RING CIRCULATOR ANALYSIS

2.1 Normal Mode Analysis

The scattering parameters of the ring circulator are related to the normal mode reflection coefficients by

\[ S_{11} = \frac{1}{3} \left( s_1 + s_2 + s_3 \right) \]

\[ S_{21} = \frac{1}{3} \left( s_1 + e^{-j\gamma} s_2 + e^{+j\gamma} s_3 \right) \]

\[ S_{31} = \frac{1}{3} \left( s_1 + e^{+j\gamma} s_2 + e^{-j\gamma} s_3 \right) \]

where

- \( S_{11} \) is the circulator reflection coefficient
- \( S_{21}, S_{31} \) are the circulator transmission coefficients
- \( s_1 \) is the zero rotation \((1, 1, 1)\) normal mode reflection coefficient
- \( s_2 \) is the positive rotation \((1, e^{-j\gamma}, e^{+j\gamma})\) normal mode reflection coefficient
- \( s_3 \) is the negative rotation \((1, e^{+j\gamma}, e^{-j\gamma})\) normal mode reflection coefficient

\[ \gamma = \frac{2\pi}{3} \]

The ring circulator performance can be analyzed by finding its normal mode reflection coefficients.

A model for the three-port non-reciprocal ring circuit is shown in Figure 1. The ring circuit segments between the ports are non-reciprocal transmission lines. This is the limiting case of ideal non-reciprocal phase-shifters with perfect match and no loss. The parameters characterizing the circuit are indicated in the figure and are defined below:

- \( Z_0 \): the characteristic impedance of each of the input ports to the ring
- \( Z_+ \), \( Z_- \): the characteristic impedance of the ring in the clockwise and counter-clockwise directions, respectively
Figure 1. Circuit for Determining Reflection Coefficients of the Three-Port Ring Circulator
the insertion phase in the clockwise and counterclockwise directions, respectively, between any two adjacent ports.

Referring again to Figure 1, the ring circuit is assumed excited by its normal modes. For the even zero rotation modes:

\[ a = \beta = 1 \]

For the positive rotation mode:

\[ a = e^{-j \frac{2\pi}{3}} \beta = e^{+j \frac{2\pi}{3}} \]

For the negative rotation mode:

\[ a = e^{+j \frac{2\pi}{3}} \beta = e^{-j \frac{2\pi}{3}} \]

The total voltage \( V \) at port (1) is:

\[ V = 1 + e^{j \phi} = V_+ + V_- \]  \hspace{1cm} (2-1)

where

\[ \phi \] is the angle of the voltage reflection coefficient

\( V_+ \), \( V_- \) are the total travelling waves in the ring circuit in the clockwise and counter-clockwise direction, respectively, immediately to the left of port (1).

The symmetrical component reflection coefficients is of unit magnitude for the lossless circuit. We desire an expression for \( \phi \).

The total current \( I \) flowing into port (1) is:

\[ I = \frac{1 - e^{j \phi}}{Z_o} = \frac{V_+}{Z_o} - \frac{V_-}{Z_o} - \beta \left( \frac{V_+}{Z_+} e^{-j \theta_+} + \frac{V_-}{Z_-} e^{+j \theta_-} \right) \]  \hspace{1cm} (2-2)

The second term on the right side of this equation follows from the relationship between the device symmetry and the normal modes. It represents the current flowing into the junction from the ring segment to the right of the junction.

The voltage equation at port (2) is:

\[ a \left( 1 + e^{j \phi} \right) = V_+ e^{-j \theta_+} + V_- e^{+j \theta_-} \]  \hspace{1cm} (2-3)
Solving equations (I-1) and (I-3) yields:

\[ V_+ = \frac{(1 + e^{j\phi}) (\alpha - e^{+j\theta} -)}{e^{-j\theta} + e^{+j\theta}} \]

\[ V_- = \frac{(1 + e^{j\phi}) (e^{-j\theta} + -\alpha)}{e^{-j\theta} + e^{+j\theta}} \]

(2-4)

It is convenient to express the insertion phase \( \theta_+ \) and \( \theta_- \) in terms of the average insertion phase \( \theta \) and one-half the differential phase-shift \( \epsilon \):

\[ \theta = \frac{\theta_- + \theta_+}{2} \]

\[ \epsilon = \frac{\theta_- - \theta_+}{2} \]

or equivalently

\[ \theta_- = \theta + \epsilon \]

\[ \theta_+ = \theta - \epsilon \]

We also define

\[ Z = 2 \frac{Z_+ Z_-}{Z_+ + Z_-} \]

Using these parameters and substituting equations (I-4) into equation (I-1) gives:

\[ \phi = 2 \tan^{-1}\left\{ \frac{2Z_0}{Z} \left[ \frac{\cos \theta - \frac{1}{2} (\alpha e^{j\epsilon} + \beta e^{j\epsilon})}{\sin \theta} \right] \right\} \]

Inserting the values of \( \alpha \) and \( \beta \) corresponding to the normal modes we finally obtain the expressions for the reflection coefficients. These are:

(1) Even Zero Rotation Mode

\[ \phi_e = 2 \tan^{-1}\left\{ \frac{2 Z_0}{Z} \left[ \frac{\cos \theta - \cos \epsilon}{\sin \theta} \right] \right\} \]
(2) Positive Rotation Mode

\[ \phi_+ = 2 \tan^{-1} \left\{ \frac{2 Z_o}{Z} \left[ \frac{\cos \theta + \sin (30 + \epsilon)}{\sin \theta} \right] \right\} \]

(3) Negative Rotation Mode

\[ \phi_- = 2 \tan^{-1} \left\{ \frac{2 Z_o}{Z} \left[ \frac{\cos \theta + \sin (30 - \epsilon)}{\sin \theta} \right] \right\} \]

The reference planes for these reflection coefficients are at the three junctions of the ring and input ports.

The ring circulates when the three normal mode reflection coefficients are mutually separated by 120°. The three reflection coefficients are plotted in Figure 2 for a ring impedance \( Z_o \) of 50 ohms and a non-reciprocal phase shift \( 2\epsilon \) of 60°. The abscissa is the average insertion phase of the non-reciprocal phase shifter between the tee-junctions. For these values of impedance and phase shift the ring circulates when the average \( \frac{\theta_+ + \theta_-}{2} \) electrical length of the phase shifter is an odd number of quarter wavelengths.

2.2 Bandwidth

A formula for the bandwidth is obtained by taking the variation, about the circulation frequency, of the transmission coefficient to the isolated arm. For circulation in the 1-2-3-1 direction \( s_2 = s_1 \exp ( +j \frac{2\pi}{3} ) \) and \( s_3 = s_1 \exp ( -j \frac{2\pi}{3} ) \). The variation in isolation about the circulation frequency \( \omega_o \) is

\[ \delta S_{31} = \frac{1}{3} \left. \frac{d}{d\omega} \left[ \delta \omega \right] \right|_{\omega = \omega_o} \]

For \( S_1 = e^{j\phi_1} \) \( i = 1, 2, 3 \) (e, +, - modes)

\[ \delta S_{31} = \frac{j}{3} \sum_{i=1}^{3} \left. \frac{d}{d\omega} \left[ \delta \omega \right] \right|_{\omega = \omega_o} \]
Figure 2. Non-Reciprocal Ring Circuit with Circulation at $\theta = 90^\circ$ and $\theta = 270^\circ$
After some manipulation the magnitude is

$$\left| \delta S_{31} \right| = \frac{1}{3} \left[ \sum_{i=1}^{3} \left( \frac{d \phi_i}{d \omega} \right)^2 - \left( \frac{d \phi_1}{d \omega} + \frac{d \phi_2}{d \omega} + \frac{d \phi_3}{d \omega} \right) \right]^{1/2} \left| \delta \omega \right|$$  \hspace{1cm} (2-6)

It is informative to write the derivatives with respect to $\omega$ as

$$\frac{d \phi_i}{d \omega} = \frac{d \phi_i}{d \theta} \frac{d \theta}{d \omega} \hspace{1cm} i = 1, 2, 3$$

$$= K_1 \frac{d \theta}{d \omega}$$

where $K_1$ is just the slope of the reflection coefficient in Figure 2. It may also be determined by taking the derivative of Equation (2-5).

$$K_1 = -\frac{4}{Z} \frac{Z_0}{\frac{Z}{Z_0}} \left\{ 1 + \left[ \frac{\cos \theta - \cos \epsilon}{\sin \theta} \right] \cot \theta \right\}$$

$$K_2 = -\frac{4}{Z} \frac{Z_0}{\frac{Z}{Z_0}} \left\{ 1 + \left[ \frac{\cos \theta + \sin (30 + \epsilon)}{\sin \theta} \right] \cot \theta \right\}$$

$$K_3 = -\frac{4}{Z} \frac{Z_0}{\frac{Z}{Z_0}} \left\{ 1 + \left[ \frac{\cos \theta + \sin (30 - \epsilon)}{\sin \theta} \right] \cot \theta \right\}$$

(2-7)

For the circulation condition with 60° differential phase shift ($\epsilon = 30°$), $Z_0 = 50$ ohms and the phase shifter an odd number of quarter wavelengths long ($\theta = (2n + 1) \pi/2$)

$$K_1 = K_2 = -1 \hspace{1cm} K_3 = -4$$

and Equation (2-6) becomes

$$\left| \delta S_{31} \right| = \left| \frac{d \theta}{d \omega} \right| \left| \delta \omega \right|$$
and the fractional bandwidth $BW$ for a given isolation is

$$BW = 2 \left| \frac{\delta \omega}{\omega_0} \right| = \frac{2 \left| \delta S_{31} \right|}{\omega_0 \left| \frac{d\theta}{d\omega} \right| \omega_0} \quad (2-8)$$

The value of $\theta$, the electrical length of the phase shifter, is determined by the percentage of interaction $P$ in the phase shifter.

$$P = \frac{\text{non-reciprocal phase shift}}{\text{average insertion phase}}$$

Sixty degrees non-reciprocal phase shift is required and the lower the interaction the larger $\theta$ and hence $\left| \frac{d\theta}{d\omega} \right|$. Bandwidth is inversely proportional to this quantity.
3. **Circulator Design and Evaluation**

A microstrip ring circulator was designed from the graph in Figure 2. The circulator is shown in Figure 3. The phase data for the five section meander line is plotted in Figure 4. Non-reciprocal phase (Δθ) greater than the required 60° was obtained. The impedance level of the ring was approximately 50 ohms and it had a 1 db insertion loss. The key to the dimensions on the chart is:

- \( b \) = ground plane spacing
- \( W_e \) = conductor width of uncoupled lines
- \( W_c \) = conductor width of coupled lines
- \( s \) = spacing between coupled lines
- \( l_c \) = length of coupled line sections

There were three frequencies in S-Band at which the device circulated. These were where the phase shifter was an odd number of quarter wavelengths long. A typical circulation characteristic is shown in Figure 5. The 20 db isolation bandwidths is 1-1/2%.

Let us now examine the theoretical bandwidth using Equation (2-8). For a 20 db isolation bandwidth, the formula is

\[
\text{BW} = \frac{0.2}{\omega_o} \left| \frac{d\theta}{d\omega} \right| \bigg|_{\omega = \omega_o}
\]

From Figure 4, \( \frac{d\theta}{dt} \approx 420°/GHz \). The circulation frequency \( f_0 \) is 2.4 GHz. The calculated bandwidth is approximately 1% or 24 MHz which is in good agreement with the experimental value of 36 MHz. The factors causing this narrow bandwidth are (1) the dispersion in the phase shifter and (2) the low interaction (non-reciprocal phase shift/average insertion phase). Both these factors increase \( \frac{d\theta}{d\omega} \) and therefore the bandwidth.

Numerous other meander line configurations were designed and
tested. The coupling, the coupled line lengths, and the number of sections were all varied. No significant improvement in circulation bandwidth was obtained. Because of the large insertion phase required in the actual device, the ring circulator is a high Q narrow bandwidth device.
Figure 3. Microstrip Three-Port Ring Circulator with Meander-Line Phase-Shifters
Figure 4. Phase Characteristics for the 5-Line Meander Line Phase-Shifter

Ferrite: TT1-414

$4\pi M_s = 680$ gauss

$b = 0.20''$

$W_e = W_c = 0.015''$

$s = 0.008''$

$t_c = 0.520''$
Figure 5. Performance of 5-Line Meander Line Ring Circulator