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Performance of Receivers and Signal Analyzers Using Broadband Frequency-Sensitive Devices

by

R. C. Cumming and G. A. Myers

March 1967

Technical Report No. 1905-1

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SYSTEMS TECHNIQUES LABORATORY

STANFORD ELECTRONICS LABORATORIES

STANFORD UNIVERSITY · STANFORD, CALIFORNIA

SEL-66-125

PERFORMANCE OF RECEIVERS AND SIGNAL ANALYZERS USING BROADBAND FREQUENCY-SENSITIVE DEVICES

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USING BROADBAND FREQUENCY-SENSITIVE DEVICES

by R. C. Cumming and G. A. Myers

ABSTRACT

This report compares the performance characteristics of a class of receivers intended for spectrum monitoring or signal analysis. A broadband frequency-sensitive element provides the basis of operation of the receivers considered--sometimes called frequency-discriminator receivers. Five different receivers are defined according to the method used to separate amplitude and frequency information. Two of the receivers produce a voltage analog of instantaneous frequency, two produce cathode-ray-tube displays of instantaneous frequency vs instantaneous amplitude, and one produces a binary digital indication of instantaneous frequency. These systems are compared with one another and with an array-of-filters spectrum analyzer. The bases of comparison include dynamic range, power sensitivity, frequency sensitivity, accuracy of measurement, ambiguity, response time, resolution of simultaneous signals, responses to special signals, and complexity.

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I. INTRODUCTION AND SUMMARY

The receivers discussed in this report are intended for the following applications: (1) broadband spectrum monitoring to ascertain which parts of the frequency spectrum contain electromagnetic signals, and (2) signal analysis to learn the properties of particular received signals.

A. DEFINITIONS AND PLAN OF REPORT

The class of receivers studied is based on the response of one, or a small number of, broadband frequency-sensitive devices. Such receivers are sometimes called frequency-discriminator receivers because their operation is similar in some respects to that of certain frequency discriminators or demodulators used in FM communication receivers.

The meaning of the term "broadband frequency-sensitive device" will be clarified by example in the next section. By definition, such a device is a linear circuit or system describable by a transfer function $H(f)$ which relates output amplitude and phase to input amplitude and phase at each frequency f. It is the amplitude characteristic $|H(f)|$ that forms the basis of operation of the systems considered in this report.

The input signal is applied to the frequency-sensitive element, and the amplitude of the output signal is sensed to provide a measure of signal frequency and in some cases of signal amplitude. The various discriminator receiving systems can be distinguished by the devices used to produce the attenuation vs frequency characteristic $|H(f)|$ and by the techniques employed to determine the frequency (and sometimes amplitude) of a signal from the response of these devices. An intriguing analog of the systems studied is the human eye which utilizes three different broadband light sensors to provide color perception over the visible octave.

Frequency-discriminator receivers are distinguished from spectrum-analyzer receivers that use narrowband filters to analyze the Fourier spectrum of a signal. Two types of such spectrum-analyzer receivers can be identified, both of which have received extensive attention in the literature [Refs. 1 and 2]. One type of spectrum analyzer scans the frequency of either an analyzing filter or a heterodyning oscillator. The other type employs an array of contiguous narrowband filters spaced

across the band to be monitored or analyzed. For comparison purposes in this report, we shall sometimes state the performance of the array-of-filters type of spectrum analyzer. We do not consider the scanning type of spectrum analyzer at all.

The purpose of this report is to compare the potential or present advantages of the various frequency-discriminator receiver techniques. Only secondary attention is paid to design procedures or to the present state of the art in components and devices. Special reference is given to the microwave frequency range, but most of the techniques studied are applicable at lower and perhaps higher frequencies as well.

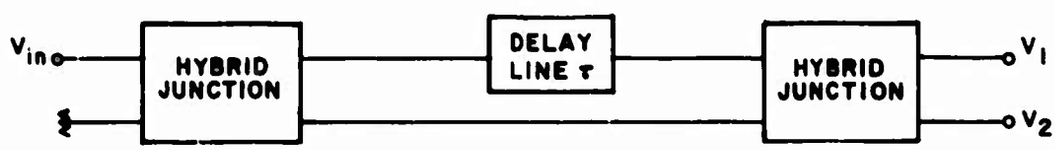
The report is so arranged that Section I can stand by itself as a succinct treatment of the entire subject. The reader's attention is especially directed to Section I.D, which summarizes the performance characteristics of all the receivers considered and which, in effect, constitutes the conclusions to this report. Section I.B treats the concept of a broadband frequency-sensitive device and discusses one particularly useful example in some detail. Section I.C defines the various frequency-discriminator receiver techniques considered, along with block diagrams of each system. The remainder of the report, Sections II through V, provides additional information on each of the systems and supports some of the statements made without proof in Section I.

B. FREQUENCY-SENSITIVE DEVICES

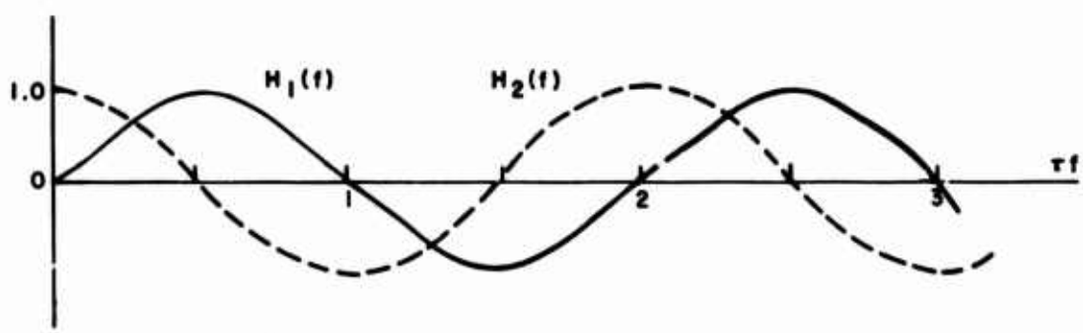
Various devices and circuits have been used to provide the frequency-sensing characteristic $H(f)$. These include transmission lines whose dielectric has a frequency-sensitive dissipation characteristic [Ref. 3]; transmission circuits containing inductors and capacitors [Ref. 4]; transmission circuits containing open or shorted transmission lines [Ref. 5]; traveling-wave tubes operated in their dispersive range of frequencies [Ref. 6]; cathode-ray tubes operating in the frequency region where electron deflection is frequency dependent [Refs. 7-10]; and systems based on wave interference to be described in some detail below.

1. Frequency Sensing by Wave Interference [Refs. 11-13]

The frequency-sensing system shown in Fig. 1, and variations thereof, have proven to be particularly useful in practice. This system consists of two quadrature hybrid junctions and a delay line. The first hybrid junction divides the input signal power equally between two transmission channels, one of which contains the delay line producing a constant time delay τ at all frequencies. The waves in the two channels are recombined in the second hybrid junction, and two output signals are available, as shown in the figure. (The hybrid junctions are four-port devices; one port of the first hybrid junction is terminated by a matched load.)



a. Block diagram



b. Transfer functions

FIG. 1. FREQUENCY-SENSING SYSTEM BASED ON WAVE INTERFERENCE.

Let V_{in} be the complex signal input voltage, let V_1 and V_2 be the two complex output voltages, and define transfer functions $H_1(f)$ and $H_2(f)$ as follows:

$$V_1 = V_{in} H_1(f) \tag{1}$$

$$V_2 = V_{in} H_2(f) \tag{2}$$

It can be shown that the transfer functions (for $f > 0$) are given by

$$H_1(f) = (\sin \pi\tau f) \exp\left(-\frac{j\pi}{2} - j\pi\tau f\right) \tag{3}$$

$$H_2(f) = (\cos \pi\tau f) \exp\left(-\frac{j\pi}{2} - j\pi\tau f\right) \tag{4}$$

The magnitude factors $\sin \pi\tau f$ and $\cos \pi\tau f$ are plotted in Fig. 1. Note that these factors vary sinusoidally with frequency, as would be expected in a wave-interference system. One complete variation (interference "fringe") occurs each time the frequency changes an amount $1/\tau$. Note also that the phase factors $\exp(-j\pi/2 - j\pi\tau f)$ are identical for the two transfer functions; this is important for one of the five systems (the LPD system) to be defined in Section C below.

The practical utility of the foregoing system is due in part to the ease with which the frequency sensitivity can be changed--simply by changing the delay line to change τ.

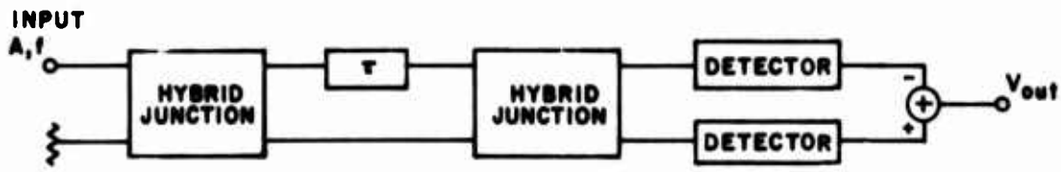
2. A Frequency-Discriminator System

Figure 2a shows a frequency discriminator based on the frequency-sensing system described above. It uses a pair of envelope detectors and a differencing circuit or differential amplifier to produce a dc output voltage V_{out}. If the detectors operate in their square-law regions, the output voltage is given by (constant factor of proportionality omitted for convenience)

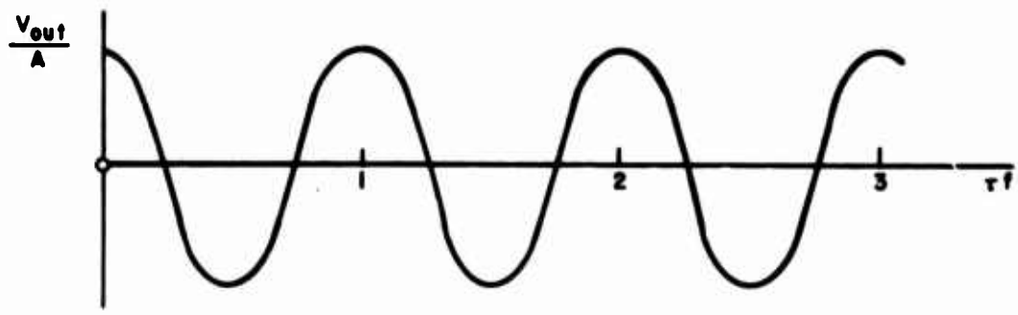
$$V_{out} = A|H_2(f)|^2 - A|H_1(f)|^2 = A \cos 2\pi\tau f \tag{5}$$

where A and f are the amplitude and frequency, respectively, of the input signal. This discrimination characteristic is graphed in Fig. 2b. (If linear rather than square-law detectors were used, the discrimination characteristic would be similar but more linear--more like a triangular function than a cosine function.)

Reference will be made later to this discriminator system. In its present form, it is inadequate for the measurement of frequency because V_{out} depends on signal amplitude A as well as on frequency f.



a. Block diagram



b. Discrimination characteristic

FIG. 2. A DISCRIMINATOR SYSTEM.

C. DEFINITIONS OF FIVE FREQUENCY-DISCRIMINATOR RECEIVERS

1. Amplitude-Sensitivity Problem

In frequency-discriminator receivers, we apply a signal with amplitude A and frequency f to a linear system having transfer function $H(f)$. From the amplitude of the response $A|H(f)|$, we wish to deduce the frequency f of the signal. However, because $A|H(f)|$ is dependent upon A as well as f and since A is unknown, knowledge of $A|H(f)|$ is insufficient to specify f uniquely. The five systems to be defined represent five different solutions to this amplitude-sensitivity problem. Three of them provide a measurement of f only, and the other two measure both A and f.

2. The LIM Frequency-to-Voltage Transducer

An obvious way to cope with the amplitude-sensitivity problem is to place an amplitude limiter ahead of the frequency-sensing system. If the limiter is ideal (constant output amplitude and no AM-to-PM conversion), then the output voltage of the frequency-sensing system provides a unique measure of frequency.† Figure 3 shows a block diagram of such a system (the response of which is graphed in Fig. 2b), in which A is now to be interpreted as the constant limiter output amplitude. For convenience, systems using this amplitude-limiter approach shall be referred to as LIM systems. Section II treats such systems in more detail.

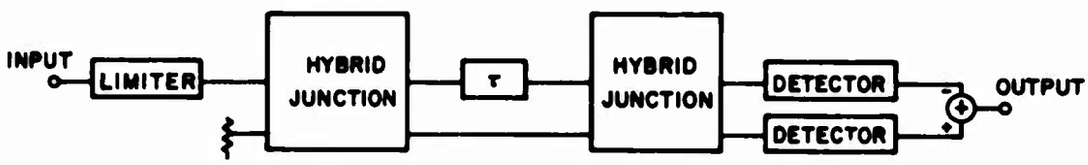


FIG. 3. THE LIM FREQUENCY-TO-VOLTAGE TRANSDUCER.

3. The LOG Frequency-to-Voltage Transducer

Another general approach uses a pair of different transfer functions $H_1(f)$ and $H_2(f)$, detects their outputs, applies the detected output voltages to logarithmic devices, and forms the difference of the resulting output voltages. Thus, the final output voltage is given by

$$V_{out} = \log A|H_1(f)| - \log A|H_2(f)|$$

$$= \log |H_1(f)| - \log |H_2(f)| \qquad (6)$$

---

†This statement assumes also that f is a single-valued function of V_{out}. In systems based on wave interference (Fig. 2), an input filter whose bandwidth is $1/2\tau$ or less should be incorporated in the system; otherwise V_{out} will be an ambiguous measure of frequency. In several instances, it is tacitly assumed that such input filtering is used to prevent ambiguity. Section D6 describes the frequency-ambiguity situation for all of the systems covered by this report.

which is seen to be independent of signal amplitude A and therefore a unique measure of signal frequency f. Systems based on this approach shall be referred to as LOG systems.

Figure 4 shows a block diagram of a particular LOG system. Except for the inclusion of the 6-db attenuator shown in the block diagram, the transfer functions $H_1(f)$ and $H_2(f)$ would be given by Eqs. (3) and (4) which are graphed in Fig. 1b. The 6-db attenuator is included as a practical refinement. It prevents the magnitudes of the transfer functions from becoming zero at any frequency (the logarithm of zero is undefined), and it also leads to a more linear relationship between output voltage and input frequency. The logarithmic amplifiers can be especially designed to match the characteristics of the detectors, in particular, their transition from square-law to linear operation.

LOG systems are discussed further in Section II.

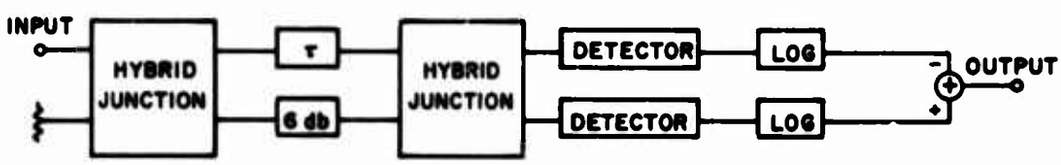


FIG. 4. THE LOG FREQUENCY-TO-VOLTAGE TRANSDUCER.

4. <u>The Linear Polar-Display System (LPD)</u>

In this system, a pair of different transfer functions $H_1(f)$ and $H_2(f)$ is again used, with the additional requirement that their phase functions (phase shift vs f) be identical. The outputs of the transfer systems are applied to the x and y inputs of a cathode-ray tube (CRT). Because the outputs are in-phase, the resulting display will be a straight line. The center of the line will be at the center of the CRT screen, and the length of the line will be directly proportional to A. The angle of the line with respect to horizontal will be independent of A, but will be a unique function of f. This is because (1) the x component of deflection is proportional to $A|H_2(f)|$, while the y component of deflection is proportional to $A|H_1(f)|$, and (2) the angle θ

of the line is an arctangent function of the ratio of y to x deflection amplitudes, giving

$$\theta = \tan^{-1} \frac{A|H_1(f)|}{A|H_2(f)|} = \tan^{-1} \frac{|H_1(f)|}{|H_2(f)|} \tag{7}$$

Shown in Fig. 5 is a particularly attractive system, for which $H_1(f)$ and $H_2(f)$ are given by Eqs. (3) and (4). (Note that the hybrid junctions are of the quadrature type.) Substitution into (7) yields

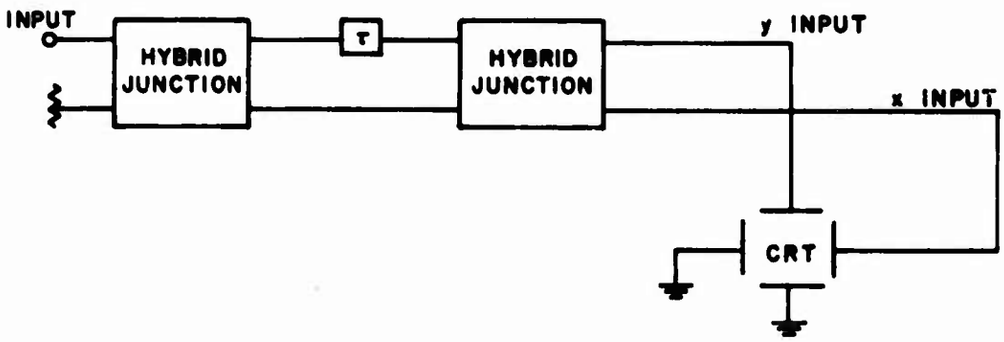
$$\theta = \pi\tau f \tag{8}$$

which states that display angle will be directly proportional to f. Thus, we have a system producing a straight-line display, whose polar coordinates r and θ provide unique and directly proportional indications of A and f, respectively.

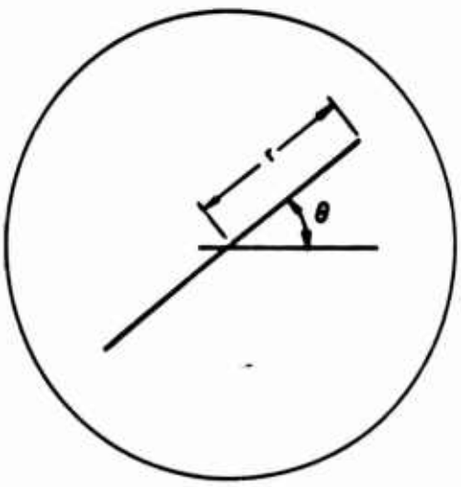
The initials LPD are used to designate the class of systems which produce such polar displays and which are constructed entirely of linear elements. Section III is devoted to further discussion of LPD systems. Next we shall define a class of polar-display systems incorporating envelope detectors which are nonlinear elements.

5. The Postdetection Polar-Display System (DPD)

If the x and y voltages in the foregoing system are envelope detected (and amplified) before applying them to the cathode-ray tube, the resulting polar display of signal amplitude and frequency will be in the form of a point, or spot, rather than a line. Note, however, that the addition of a pair of detectors to the simple system shown in Fig. 5 would yield a display spot confined to one quadrant of the CRT screen, since the detector outputs are unipolar. To provide a four-quadrant display, the system shown in Fig. 6 was devised. This seemingly complicated system can actually be fabricated quite easily and compactly of strip transmission line [Ref. 13].

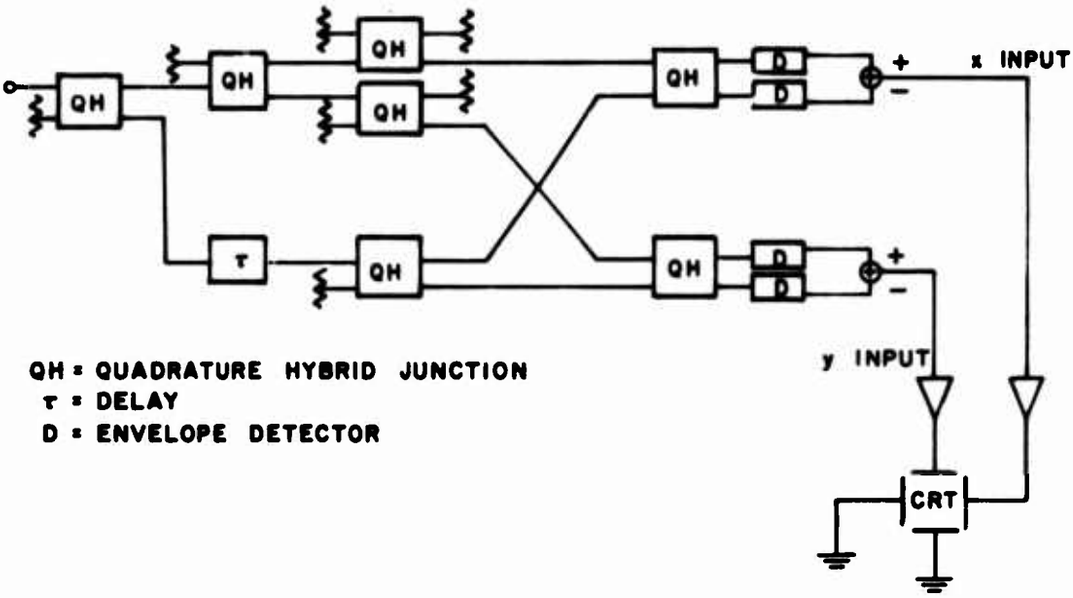


a. Block diagram

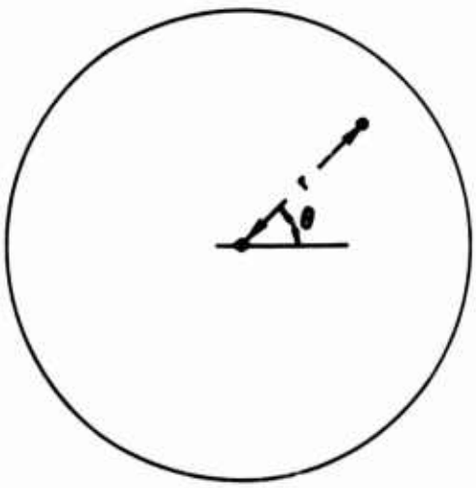


b. Display produced by a sinusoidal signal

FIG. 5. THE LINEAR POLAR-DISPLAY FREQUENCY METER (LPD).



a. Block diagram



b. Display produced by a sinusoidal signal

FIG. 6. THE POSTDETECTION POLAR-DISPLAY FREQUENCY METER (DPD).

We use the initials DPD to designate the class of systems that form a polar display of amplitude vs frequency by using envelope-detected outputs of frequency-sensitive elements. The DPD systems require matched detectors and postdetection amplifiers, but require only an ordinary "video" cathode-ray tube rather than an rf cathode-ray tube as in the LPD systems.

For the system shown in Fig. 6, it is necessary that the four detectors operate in their square-law regions if a linear relationship between display angle θ and signal frequency f is required. If square-law detectors are used, the relationship between θ and f is

$$\theta = 2\pi\tau f \tag{9}$$

and the radial deflection r of the display spot will be proportional to the square of signal amplitude, or proportional to signal power.

Section IV discusses DPD systems in detail.

6. The Polarity-Sensing System (PS)

Another way to solve the amplitude-sensitivity problem is to sense only the output polarity, not the output magnitude, produced by bipolar frequency discriminators such as in Fig. 2. In order to measure frequency accurately with such a system and at the same time have no ambiguity in frequency, it is necessary to use a multiplicity of discriminators, as shown in Fig. 7. The outputs of the n polarity sensors provide a binary digital indication of signal frequency. Systems of this class are designated herein by the initials PS.

In the PS system considered, the delay times τ_1 through τ_n differ from each other by factors of 2. In this case, the frequency accuracy is determined by τ_{max}, and the unambiguous frequency range is determined by τ_{min}.

Section V is devoted to a more complete treatment of PS systems.

D. SUMMARY AND COMPARISON OF THE CHARACTERISTICS OF THE VARIOUS SYSTEMS

The purpose of this section is to define some important operating characteristics of receivers and to compare briefly the characteristics

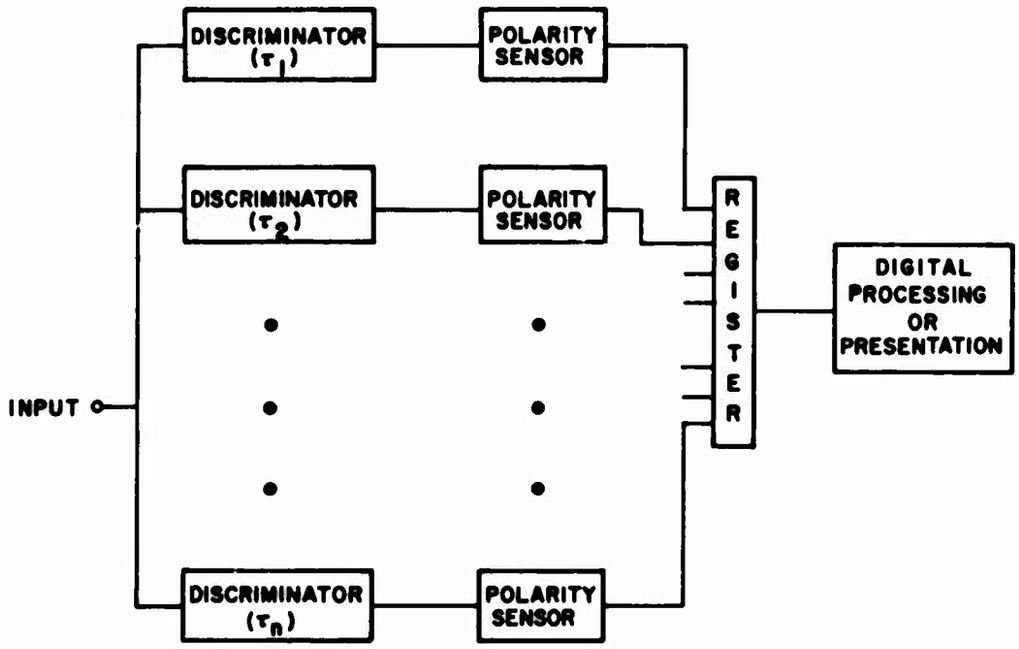


FIG. 7. THE POLARITY-SENSING SYSTEM (PS).

of the various systems treated in this report. The facts presented here are developed more fully in the remainder of the report (Sections II through V).

The array-of-filters spectrum analyzer is considered at various places in this report, including Section V, where its performance is compared with that of the PS system which, like the array-of-filters system, uses a multiplicity of parallel channels to measure frequency. Since it is a well-known method of signal analysis, the array-of-filters performance characteristics are summarized in this section and can be used as a reference in reviewing the performance of the other systems.

In this summary it is assumed that all five systems considered use the interference principle to obtain one or more frequency-sensitive output voltages. In other sections of the report, various alternative frequency-sensing devices or systems are presented briefly.

1. Parameters Measured and Nature of the Output

Here we discuss the signal parameters measured by each system and state the form of the output, which may be either analog or digital and either electrical or visual. Of course, with any of the systems, one can change the form of the output by use of an analog-to-digital converter, a digital-to-analog converter, an electrical-to-visual transducer (cathode-ray tube), or a visual-to-electrical transducer (photoelectric pickups located on the screen of a cathode-ray tube).

The LIM, LOG, and PS systems measure only the instantaneous signal frequency and can present the time variation (waveform) of this quantity. The output is in analog electrical form for LIM and LOG, and in the form of a binary digital code word for PS.

The LPD system measures both instantaneous frequency and instantaneous amplitude but does not indicate their variations with time (their waveforms). However, the brightness distribution of the analog visual display provides some indication of the probability distributions of instantaneous frequency and amplitude. The visual display in polar coordinates (r vs θ) represents a graph of instantaneous amplitude vs instantaneous frequency.

When the signal is a single sinusoid, the LPD indicates signal amplitude and frequency in the form of a straight-line analog visual display in polar coordinates. When the signal is the sum of several sinusoids having incommensurable frequencies, the LPD display indicates the amplitudes and frequencies of all the sinusoids. (In practice, the number of sinusoids must be small or the display becomes complicated and difficult or impossible to use.) The LPD displays produced by modulated sinusoids indicate signal bandwidth in some cases and the distributions of instantaneous amplitude and instantaneous frequency in other cases.

The array-of-filters system produces samples of a smoothed approximation to the signal spectrum, in the form of analog electrical outputs. If the spectrum varies with time, the electrical analogs of the sample values vary accordingly.

2. Dynamic Range

Dynamic range is the range of signal power over which the system is capable of making measurements with acceptable accuracy, without using

either manual or automatic gain control. (Such control--a variable attenuator--could of course be used to reduce the level of a signal to make it lie within a given dynamic range.)

In the LIM system the dynamic range is equal to the dynamic range of the limiter (the signal input range over which the limiter output is reasonably constant). In the LOG system the dynamic range is limited by problems of matching a logarithmic-amplifier nonlinear characteristic to a detector nonlinear characteristic and of matching the characteristics of a pair of detectors and logarithmic amplifiers; experience has shown that 40 db is probably an attainable dynamic range for the LOG system.

The dynamic range of the DPD is limited by the fact that the cathode-ray spot must be deflected at least a few spot diameters, and at most the radius of the screen, in order to measure the polar coordinates of the spot. When square-law detectors are used, as is typically desirable in practice, the measured dynamic range of a representative DPD system is about 10 db. In the LPD system, since the display is a line instead of a spot, there is no requirement for the deflection to remain within the confines of the screen. Further, the system is entirely linear, which improves the dynamic range by a decibel factor of 2 relative to systems employing square-law detectors. A practical limit to dynamic range may be about 40 db, set either by dimness of the display line or by power-handling ability of the cathode-ray deflection structure.

The dynamic range of the PS system and the array-of-filters system is limited by the tangential sensitivity of the detectors for low-level signals and by detector failure for high-level signals. This corresponds to a range of about 60 db for solid-state diodes and about 100 db for vacuum diodes.

3. Power Sensitivity

One kind of sensitivity of an electronic measurement system is its ability to operate satisfactorily with weak signals. To specify this sensitivity we define the threshold signal power as the minimum signal input power that will produce an accurate measurement.

a. Without Preamplification

In the LIM system, the threshold signal power is equal to the threshold input power of the limiter (the power level below which the limiter functions improperly). In the LOG, DPD, and PS systems, the tangential sensitivity of the diode detectors determines the system sensitivity, which is typically about -45 dbm. The LPD system requires a signal power that exceeds the deflection sensitivity of the cathode-ray tube. Typically, this is about 0 dbm for wideband cathode-ray tubes having an impedance of the order of 100 ohms in the deflection circuits.

b. With Preamplification

Preamplification can be used to improve the system sensitivity. The optimum value of gain for the preamplifier makes the output noise power of the preamplifier approximately equal to the threshold input signal power of the measurement system proper. Further increase in gain, assuming a fixed receiving system noise temperature, will not materially improve the sensitivity and will reduce the dynamic range of the system. When the preamplifier gain is equal to or greater than the optimum value, the sensitivity of the system is limited by preamplifier noise, and the threshold signal input power for any of the five discriminator systems becomes approximately

$$S_{min} = 10\ kT_sB \quad \text{watts} \tag{10}$$

where k is Boltzmann's constant, T_s is the prediscriminator noise temperature, and B is the prediscriminator noise bandwidth. In terms of antenna noise temperature T_a and prediscriminator noise figure F,

$$T_s = T_a + 290\ (F - 1) \tag{11}$$

In these equations, the temperatures are in degrees Kelvin, and the noise figure is expressed as a power ratio.

For comparison, the threshold signal input power for an array-of-filters spectrum analyzer is about

$$S_{min} = kT_sB/n \quad \text{watts} \tag{12}$$

where n is the number of equal-bandwidth contiguous filters covering the band B.

4. Frequency Sensitivity

Another kind of sensitivity of a measurement system is the rate of change of the output quantity with respect to the corresponding input quantity. For the polar-display systems, the output quantity corresponding to input signal frequency is the θ polar coordinate of the display line or spot. The variation $d\theta/df$ is an indication of the frequency measurement sensitivity of these systems. The value of this parameter, in radians per hertz, is $\pi\tau$ for the LPD and $2\pi\tau$ for the DPD system, where τ is the differential time delay used in the interference type of frequency-sensitive element (Figs. 5 and 6).

In the LIM and LOG systems, the output quantity corresponding to input signal frequency is voltage. The variation of this voltage with frequency is proportional to the gain of the system as well as to the delay τ. These facts and their effects are apparent in the typical measurement characteristic shown in Fig. 2b.

The PS system and the array-of-filters spectrum analyzer both segment the frequency band into cells. Therefore the concept of measurement sensitivity considered here does not apply to these systems.

5. Accuracy of Measurement

Three limitations to measurement accuracy can be identified. The first limitation relates to the fact that the measurement system represents the quantity to be measured by another quantity, e.g., frequency represented by a voltage level, the θ polar coordinate of a visual display, or a digital code word. Thus, in the LIM and LOG systems, frequency measurement accuracy is limited by the accuracy with which the output voltage can be measured. In the LPD and the DPD systems, the accuracy of determining the orientation or position of a display line or spot limits the accuracy of amplitude and frequency measurement. In the PS system and the array-of-filters system, accuracy is limited by the number of channels or filters used.

Suppose that voltage can be measured to an accuracy of 1 percent of the total voltage range used to indicate frequency. Then the uncertainty in frequency measurement is about $1/(200\tau)$ Hz for LIM or LOG. Similarly, assuming that the θ coordinate of a polar display can be measured to an accuracy of 3.6 deg, it follows that the frequency uncertainty is about $3.6/(180\tau)$ Hz for LPD and about $3.6/(360\tau)$ Hz for DPD. In these expressions, τ is the differential delay in the interference type of frequency-sensing system. The uncertainties in the measurement of frequency will be proportionately less if we can measure voltage or angle more accurately than is assumed above. In the PS system, the frequency uncertainty is about $B/2^n$ Hz, and in the array-of-filters spectrum analyzer, about B/n Hz, where B is the total frequency band monitored, and n is the number of channels or filters used to cover that band.

The second limitation to accuracy comes from the fact that all of the discriminator systems represent different approaches to solving the amplitude-sensitivity problem discussed in Section I.C. In practice, most of the systems do an imperfect job of removing the dependence of frequency indication on signal amplitude. Thus the accuracy of the LIM system is limited by AM-to-PM conversion in practical limiters. In the LOG, DPD, and PS systems, imperfections in the matched detectors and matched amplifiers can lead to frequency inaccuracy caused by amplitude changes. Only the LPD, because it is a completely linear system, is immune to this kind of inaccuracy.

Additive noise is the third limitation to accuracy. In the LIM and LOG systems, the output voltage fluctuates about a mean value representative of the true signal frequency. The standard deviation σ_f of the fluctuation provides a measure of this kind of inaccuracy and can be calculated from the following formula, derived in Section II:

$$\sigma_f = \left(\frac{kT_s b^3}{3S_{in}}\right)^{1/2} \quad \text{hertz} \tag{13}$$

where k is Boltzmann's constant, T_s is the system noise temperature, b is the postdetection bandwidth, and S_{in} is signal input power.

In the PS system, a probability of polarity reversal due to noise will associate an incorrect measurement cell with the input signal and in this manner will reduce the accuracy of the system. The accuracy is a monotonic function of the input signal-to-noise ratio.

In the LPD and DPD systems, the parameter that indicates frequency is angle of the cathode-ray trace. The persistence of the display provides an integrating or memory effect which aids one's ability to estimate the mean angle of a noisy display. The effect is similar to the integration obtained in the postdetection filter of the LIM and LOG systems, represented by the parameter b in Eq. (13). However, reducing b to improve accuracy imposes a proportional penalty in speed of response, while increasing the persistence of the CRT screen does not necessarily reduce the speed of response. Equation (13) can be applied to estimate the frequency accuracy of the LPD and DPD systems if b is interpreted as the reciprocal of the decay time constant of the CRT screen.

6. Unambiguous Bandwidth

Frequency measurement systems based on wave interference provide an output that varies periodically with the signal frequency (Fig. 1). If the input frequency range is sufficiently restricted by a filter, this periodicity will not lead to measurement ambiguity. In terms of the differential delay τ in the interference system, the maximum filter bandwidth that will ensure an unambiguous measurement is $1/2\tau$ for the LIM and LOG systems, $1/\tau$ for the LPD and DPD systems, and $1/\tau_{min}$ for the PS system.

With a given input filter bandwidth, high accuracy without ambiguity can be attained by using a succession of delay times τ. A sufficiently short delay time will give a relatively coarse frequency measurement without ambiguity. Doubling this time delay will improve the accuracy, but will introduce one ambiguity which can be resolved by use of the previous coarser measurement. Successive doubling and use of the previous measurements to resolve ambiguity can give the desired degree of accuracy without ambiguity.

7. Response Time

The response time is the amount of time required for an accurate indication to be produced after some property of the signal (e.g., amplitude or frequency) has been changed abruptly. To be useful with pulsed signals, the response time must be less than the pulse duration.

Systems based on the interference method of frequency sensing (Figs. 2-7) require signals concurrently on two or more channels in order to create a useful interference at the junction of the channels. Since one of the channels contains a delay τ, the systems cannot have a response time less than τ.

For those systems which use detectors (LIM, LOG, DPD, and PS), the bandwidth b of any postdetection amplifier or filter, having response time $1/b$, will determine the system response time if $1/b > \tau$. A further limitation in the case of DPD is set by buildup and decay of illumination on the CRT screen. As discussed in Section IV, the memory or integration effect of the screen may be a necessary factor in achieving a prescribed accuracy of measurement.

The response time of LPD is determined either by τ or by the time necessary for the CRT screen illumination to build up to a useful level.

The PS system output is a binary code word indicative of signal frequency. To respond to a varying frequency, the system must generate a stream of code words. Limitations on the speed of digital generation and presentation systems therefore restrict the speed of response of the PS system.

In the array-of-filters spectrum analyzer, the response time cannot be less than about n/B, where n is the number of contiguous equal-bandwidth filters and B is the total frequency band analyzed. Another limitation equal to $1/b$ is imposed by the postdetection filters, just as in the other systems using detectors, discussed above.

8. Resolution of Simultaneous Signals

The LIM, LOG, DPD, and PS systems all use nonlinear elements (limiter or detectors) that prevent them from determining the frequencies of two or more signals which are received simultaneously. In fact, these

systems will usually give erroneous or misleading indications if the simultaneous-signal amplitudes are comparable to one another. The LPD system and the array-of-filters system both have simultaneous-signal capabilities: they can measure the frequencies and amplitudes of simultaneous signals. In the LPD system, the number of signals that can be resolved is limited by practical rather than theoretical considerations and is usually far less than the number that can be resolved with the array-of-filters system.

A strong pulsed signal received simultaneously with any number of weaker cw or pulsed signals will produce an accurate indication. The combination of the weaker signals acts like background noise; when the strong pulse is present, its instantaneous frequency and amplitude dominate the noise and control the indication of the system.

The case of signals received from several different pulsed systems deserves special comment. Because of the low duty factors and the asynchronous pulse-repetition frequencies of typical systems, most of the pulses will be received separately (i.e., not simultaneously with another pulse) and will produce accurate responses in any of the systems. However, the probability of receiving pulses from two different systems simultaneously is not entirely negligible, and this possibility may have to be taken into account, especially in the design of automatic processing equipment, to eliminate erroneous indications.

Systems that purposely transmit two or more signals simultaneously on different frequencies are another case of interest. When the received signals are of comparable amplitude, only the LPD and the array-of-filters systems are capable of measuring the various signal frequencies and amplitudes. The response of the DPD, LIM, and LOG systems to simultaneous signals is considered in the next section; they may indicate the frequency of the strongest signal or give an entirely erroneous or confusing indication.

9. Recognition and Measurement of Special Signals

Pulse signals having intrapulse FM are readily recognized by the LIM, LOG, LPD, and DPD systems, but not by the PS system. The LIM and LOG systems will give the waveform of the intrapulse FM. The DPD system will display the locus of instantaneous frequency vs instantaneous amplitude as a line trace in polar coordinates--a particularly useful

display. The LPD will indicate the peak-to-peak range of the instantaneous frequency and intrapulse amplitude variations. In principle, the PS system could yield a stream of code words indicating samples of the instantaneous-frequency waveform; however, this would require the generation of a large number of code words during the time of one pulse and would be impractical for typical pulses of interest. The array-of-filters spectrum analyzer can give the spectrum of the pulse but it can neither recognize that it possesses intrapulse FM nor indicate the waveform of the FM.

Systems which transmit simultaneously on two (or more) frequencies are revealed and analyzed exceptionally well by the LPD system which, in response to such signals, produces a characteristic polygonal display whose straight sides indicate the frequencies and the amplitudes of the simultaneously received sinusoids. The DPD will provide this information only in special cases where the received signal amplitudes vary such that one signal is considerably stronger than all the others for a significant fraction of the time. With signals that do not possess such amplitude variation, the DPD will indicate the presence of simultaneous signals by producing a characteristic straight-line or elliptical display if the postdetection bandwidth exceeds the frequency separation of the simultaneous signals. If the postdetection bandwidth is less than this, the DPD will not reveal the presence of simultaneous signals but instead will produce an erroneous display indicative of a single-frequency signal having a frequency somewhere between the two (or more) actually present.

The LIM and LOG systems, if the postdetection bandwidth is as wide as that mentioned above, will respond with a fluctuating output voltage, whose average value will correspond to the stronger of the two sinusoids and whose periodic fluctuation will have a frequency equal to the difference between the two signal frequencies. Hence these systems might be designed to yield useful results with simultaneous signals. A similar statement could be made for the PS system, but again with the qualification regarding high-speed generation of a stream of different code words during the pulse duration. The array-of-filters spectrum analyzer will indicate the two frequencies and amplitudes, but without further processing will not indicate that they occur simultaneously.

10. Frequency Coverage

All of the systems can be made to cover octave or greater bandwidths, and all but LPD can be designed at frequencies up to and including microwaves. With the LPD system, microwave signals must be heterodyned to a frequency range where x-y cathode-ray tubes are available.

11. Complexity or Engineering Challenges

Here we mention the most severe engineering requirements on the various systems with respect to realizability of satisfactory performance or equipment complexity:

1. The LIM system requires a wideband, phase-distortionless limiter.
2. The LOG system requires matched nonlinear elements (diode detectors and logarithmic amplifiers).
3. The DPD system requires two matched pairs of square-law diode detectors and postdetection amplifiers.
4. The PS system requires a matched pair of diode detectors for each of its n channels.
5. The LPD system requires either an rf x-y cathode-ray tube or a frequency down-converter to make the signal frequency compatible with a lower frequency x-y cathode-ray tube.
6. The array-of-filters spectrum analyzer requires one filter and one detector for each of its frequency resolution cells.

II. FREQUENCY-TO-VOLTAGE TRANSDUCERS (LIM AND LOG)

An ideal frequency-to-voltage transducer is defined as a system whose (dc) output voltage is a function only of the instantaneous frequency of the input signal. In particular, the output voltage ideally is independent of signal amplitude. Such systems are familiar in FM demodulators, and it is natural to consider using them for frequency monitoring and signal analysis. The electrical output analog of frequency can be processed automatically, or it can be used to generate various kinds of visual displays.

The purpose of Section II is to discuss, prove, or supplement various assertions made in Section I. Examples of some particular systems will be given, but they are certainly not intended to provide an exhaustive survey of possible approaches or state of the art in all relevant areas. Certain general results applicable to any frequency-to-voltage transducer will be developed, in particular, their performance with additive noise and their response to simultaneous signals.

A. PHYSICAL REALIZATIONS

Two approaches to the realization of a frequency-to-voltage transducer, designated LIM and LOG, were defined in Section I and illustrated in Figs. 3 and 4. Reference 6 describes one example of a LIM system in which the limiter consists of a cascaded pair of traveling-wave tubes operating in their saturation regions. Reference 3 describes an early LOG system in which the dynamic range was relatively poor (about 6 db) and which therefore also used a traveling-wave-tube limiter at the front end; hence this system could be considered to be a combination of the LIM and LOG techniques.

Various frequency-sensing elements or devices other than the one illustrated in Fig. 1 have been used with LIM and LOG systems. Reference 6 treats the use of a traveling-wave tube operating in its dispersive frequency region, where gain is a monotonic function of frequency. Another example [Ref. 3] is the use of a lossy dielectric in a transmission line, which causes the attenuation of the line to increase exponentially with frequency. However, the frequency-sensing system based on wave

interference (Fig. 1) has the important practical merit of adjustable frequency sensitivity, accomplished merely by changing the delay time τ.

Recent developments in detectors and logarithmic amplifiers especially designed to match each other's nonlinear characteristics have greatly extended the dynamic range achievable with the LOG system [Ref. 14]. Figure 8, taken from Ref. 15, illustrates the transducer characteristic and the dynamic range of an experimental LOG system using the circuit shown in Fig. 4 and the logarithmic amplifiers discussed in Ref. 14.

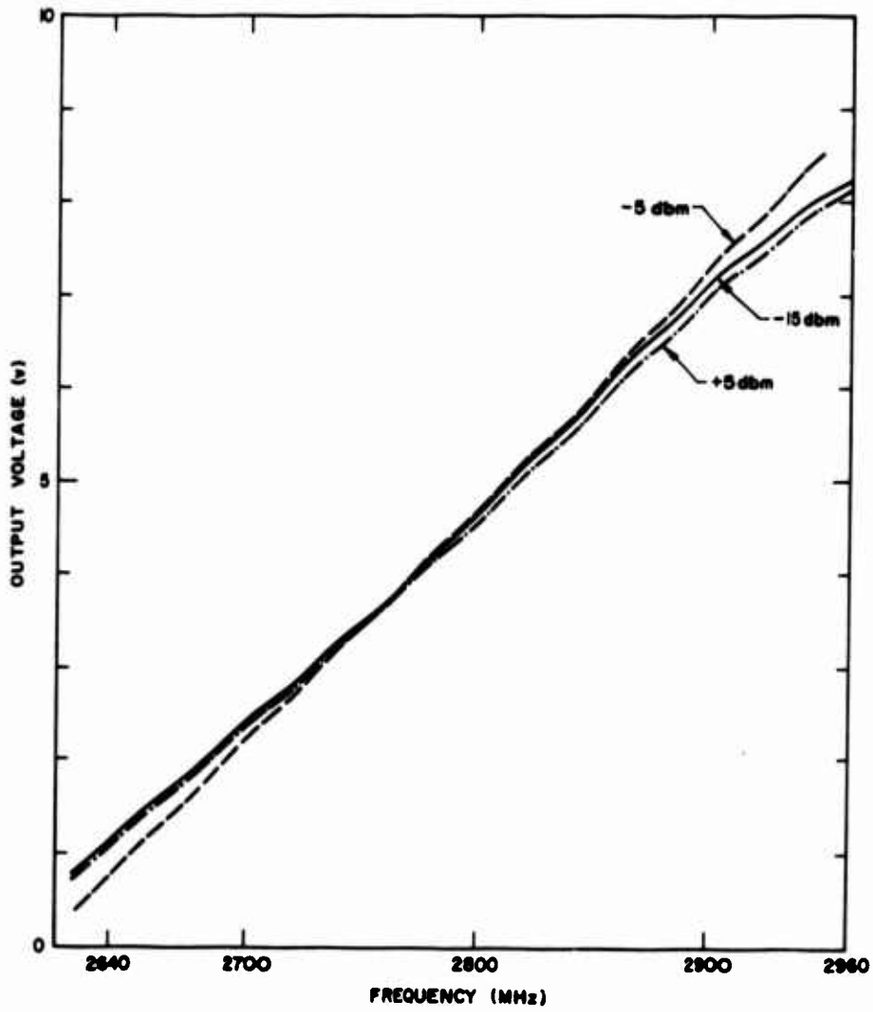


FIG. 8. STATIC CHARACTERISTICS OF AN S-BAND LOG SYSTEM AT THREE VALUES OF SIGNAL INPUT POWER.

With available techniques the dynamic range of the LOG system can probably be extended to about 40 db without the use of front-end limiting.†

B. APPLICATIONS

Many applications for frequency-to-voltage transducers suggest themselves. Some of these involve the generation of visual displays with cathode-ray tubes, while others involve automatic electrical processing, in either case for the purpose of signal monitoring or analysis.

1. Visual Displays

A most useful display is instantaneous frequency vs time, generated by applying the transducer output to cause vertical deflection and using a linear time base for horizontal deflection. The example shown in Fig. 9 utilizes a relatively slow time base and displays the responses produced by several "live" S-band radars with the LOG transducer corresponding to Fig. 8. The various radar frequencies and antenna scan rates are readily obtained from such a display. Displays using a much faster sweep and

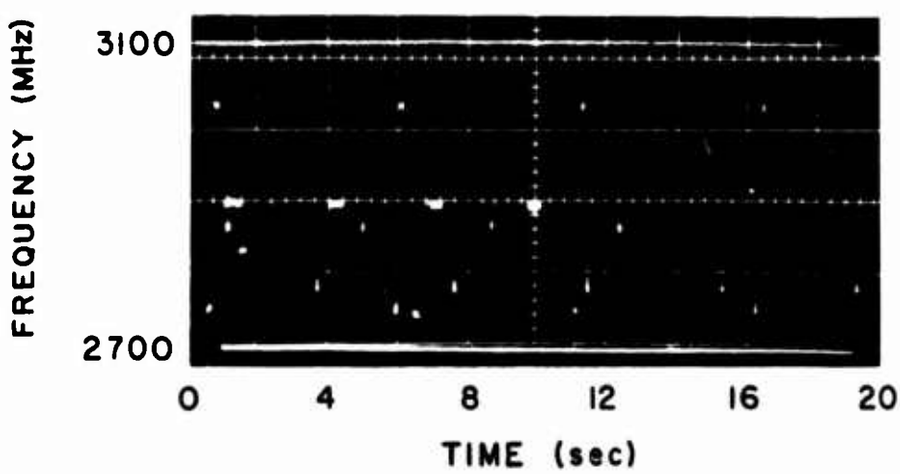


FIG. 9. A DISPLAY OF FREQUENCY VS TIME PRODUCED BY SEVERAL RADAR SIGNALS AND THE LOG SYSTEM CORRESPONDING TO FIG. 8.

†Private communication from J. M. Hunter, Stanford.

synchronized by the radar pulse are discussed and illustrated in Ref. 3. Such displays can be used to analyze intrapulse FM (e.g., Chirp radar or unintentional FM) and to recognize sequential pulses on different frequencies. They also indicate carrier frequency and pulse duration. Single-sweep displays, generated for the purpose of analyzing the instantaneous-frequency waveforms of noiselike signals, are extensively studied in Ref. 16.

Another possible display, illustrated in Ref. 3 and useful for monitoring pulsed signals, is signal amplitude vs signal frequency. In this case, the horizontal deflection is produced by the output of an envelope detector, and the vertical deflection by the output of a frequency-to-voltage transducer.

2. Automatic Electrical Processing

The availability of a voltage analog of frequency facilitates frequency "filtering" or gating on a pulse-by-pulse basis. Double-threshold circuits have been designed which provide an output only when the applied voltage lies in a specified range. These circuits are sensitive to a voltage "window" which is equivalent to a cell or band of frequencies. A signal whose frequency lies within this cell will generate an output from the window circuit. This response can then be used to gate the necessary video circuitry to admit the rf or detected signal for further processing or display. Those signals whose frequencies lie outside this cell are not accepted for processing or display. The input signals are thus sorted on the basis of their frequencies. The output of the window circuit may also be used to unblank a cathode-ray tube and thus provide a display of the activity in any particular frequency band of interest. Another application based on the analog output voltage is the adaptive processing of signals on the basis of frequency [Ref. 17].

Digital processing of the frequency information can be done after analog-to-digital conversion. The transducer output can be quantized into 2^m levels with each level identified by an m-digit binary code word. For an unambiguous monitored bandwidth B, such quantization results in a measurement accuracy of $B/2^m$ Hz.

C. PERFORMANCE CHARACTERISTICS

In this section we treat only some of the performance characteristics; the others are covered in sufficient detail in Section I.D.

1. Noise-Limited Sensitivity and Accuracy

It is assumed here that additive input noise sets the noise performance of the system. This will be the case when the preamplifier gain has at least the amount of gain defined as "optimum" in Section I.D3. Under this assumption, known analytical results for FM demodulators can be applied to the LIM and LOG systems if they act as ideal frequency-to-voltage transducers.

The analysis is based on a study of the instantaneous frequency of the combination of signal and noise at the transducer input. When the signal power is somewhat greater than the noise power, the instantaneous frequency is largely controlled by the signal, while the reverse is true if the noise power exceeds the signal power. This is the angle-modulation capture effect. It imposes the following approximate requirement on minimum useful signal power:

$$S_{min} = 10\ kT_s B \tag{14}$$

where k is Boltzmann's constant, T_s is the pre-transducer system noise temperature, and B is the pre-transducer noise bandwidth. The power S_{min} is measured at the input to the preamplifier.

When the signal power is above the threshold value S_{min}, frequency measurement accuracy can readily be specified in terms of the rms fluctuation, or standard deviation, of the instantaneous frequency about the frequency of the signal. A formula for this standard deviation σ_f of instantaneous frequency is derived in Ref. 18, Eq. (275). In terms of the symbols used in this report, the formula is

$$\sigma_f = \left(\frac{kT_s b^3}{3S_{in}}\right)^{1/2} \quad \text{hertz} \tag{15}$$

where σ_f is the standard deviation of instantaneous frequency, b is the post-transducer bandwidth, and S_{in} is signal power measured at the preamplifier input. For comparison, the symbols of Ref. 18 are defined as follows: $N = 2\pi\sigma_f$, $2\pi W_v = kT_sG/2$, $\omega_a = 2\pi b$, and $W_c = S_{in}C$, where G is the power gain of the preamplifier. The derivation assumes a constant (flat) input noise power spectrum.

Since the monitored band has a width B, the ratio of b to σ_f is approximately the number of distinguishable frequencies, as limited by noise:

$$\frac{B}{\sigma_f} = \frac{B}{b}\left(\frac{3S_{in}}{kT_sb}\right)^{1/2} \tag{16}$$

The word "distinguishable" here refers to frequency accuracy and definitely does not imply a capability to resolve two or more frequencies present simultaneously.

2. Response to Simultaneous Signals

When several signals are present at the input, the response of the LIM and LOG systems is determined by the instantaneous frequency of the combination of signals. In general, this response will give little or no useful information about the frequencies of the individual signals. A simple example is the sum of two unmodulated sinusoids at the input. In practice such a signal might be received from a radar that purposely transmits simultaneously on two different frequencies. Consideration of this case gives some insight into the general simultaneous-signal situation.

It is shown in Ref. 19, pp. 488-492, that the instantaneous frequency f(t) and the instantaneous envelope waveform A(t) of the composite signal are given by

$$f(t) = f_1 + \frac{f_2 - f_1}{2}\left[1 - \frac{1 - a^2}{1 + 2a\cos 2\pi(f_2 - f_1)t + a^2}\right] \tag{17}$$

and

$$A(t) = A_1\left[1 + 2a \cos 2\pi(f_2 - f_1)t + a^2\right]^{1/2} \tag{18}$$

where $a = A_2/A_1$; A_1, A_2 are the amplitudes; and f_1, f_2 are the frequencies of the two signals $(f_1 < f_2)$.

An ideal frequency transducer will respond only to $f(t)$, independent of $A(t)$. Analysis of Eq. (17) shows that the average value of $f(t)$ is exactly equal to the frequency of the stronger input sinusoid. The value of $f(t)$ fluctuates about this average value in a periodic manner with period $(f_2 - f_1)^{-1}$. Hence, provided the post-transducer bandwidth exceeds $f_2 - f_1$, the output of the transducer will fluctuate in a like manner. If the post-transducer bandwidth is less than $f_2 - f_1$, the fluctuations will be filtered out, and the dc output voltage will correspond to the frequency of the stronger input sinusoid.

Further reference to Eqs. (17) and (18) will be made in later sections of the report.

III. LINEAR POLAR-DISPLAY FREQUENCY METERS (LPD)

Section I described how an x-y cathode-ray tube can be used to solve the amplitude-sensitivity problem and at the same time produce a display in polar coordinates of signal frequency and signal amplitude. The rf (or i-f) outputs of two different frequency-sensitive transmission channels--with transfer functions $H_1(f)$ and $H_2(f)$--are applied directly to the cathode-ray deflection plates (Fig. 5). Because $H_1(f)$ and $H_2(f)$ are required to have identical phase characteristics, the resulting display on the CRT screen is a straight-line Lissajous figure when the signal is sinusoidal. The polar angle θ of the line is a function only of signal frequency, and the length $2r$ of the line is directly proportional to signal amplitude only.

Systems of this type are called linear polar-display frequency meters (LPD) because they are composed entirely of linear elements--nonlinear devices such as detectors or postdetection amplifiers are not utilized.

Figure 10 shows a multiexposure calibration photograph of the screen of an LPD system operating in the frequency range 70-130 MHz. Each exposure was made with a sinusoidal input signal, and adjacent exposures correspond to a frequency increment of 10 MHz. This particular system was operated as an i-f indicator in a superheterodyne arrangement to observe signals in the S-band. Photographs of displays produced by live radar signals are shown in Fig. 11. In (a) the system gain was sufficiently high that the two radar signals present saturated the i-f amplifier, giving equal-length, relatively noisefree traces. The gain was reduced in (b) of the figure, permitting observation of the relative strengths of the signals present.

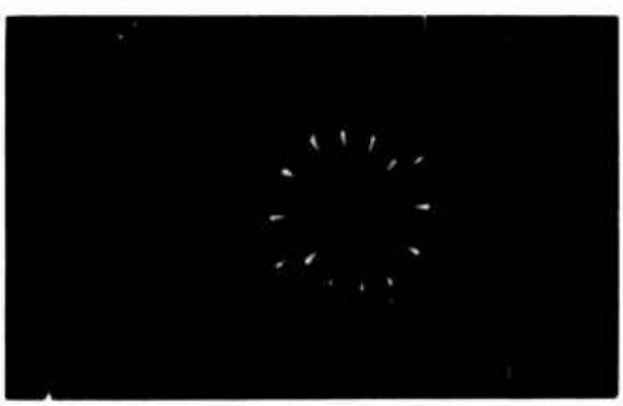


FIG. 10. MULTIEXPOSURE CALIBRATION PHOTOGRAPH FOR AN LPD SYSTEM OPERATING IN THE FREQUENCY RANGE 70-130 MHz.

a. i-f amplifier saturated b. i-f amplifier unsaturated

FIG. 11. DISPLAYS PRODUCED BY S-BAND RADAR SIGNALS.

A. PHYSICAL REALIZATIONS

Various devices have been used to obtain the two transfer functions $H_1(f)$ and $H_2(f)$. Reference 11 describes the use of resistance-capacitance circuits and systems incorporating delay networks and quadrature phase shifters. References 4 and 20 consider inductance-capacitance circuits. Another possibility is to use the frequency-sensitive electron deflection of the cathode-ray deflection plates themselves. When the frequency is sufficiently high, electron transit time in the deflection plates causes the electron deflection to be frequency dependent. This transit-time effect is linear in the sense that it can be described by linear transfer functions $H_1(f)$ and $H_2(f)$. The LPD system based on this transit-time effect in cathode-ray tubes has been called a cathode-ray frequency meter [Refs. 7-10].

Attention in this report is directed to the system shown in Fig. 5, utilizing a pair of quadrature hybrid junctions and a delay device. This system is especially appealing because the frequency sensitivity (rate of change of display angle with frequency) can be changed easily by changing the time delay of the delay device and because the frequency calibration (display angle vs frequency) is linear. These facts are proved in

Section I where the following equation relating display angle θ, delay time τ, and frequency f is derived:

$$\theta = \pi\tau f \tag{19}$$

The system of Fig. 5 can be built at any frequency where broadband quadrature hybrid junctions, delay devices, and x y cathode-ray tubes are available. The hybrid junctions are now available in octave bandwidths at frequencies from the kilohertz region through microwaves. Delay lines in the microwave range and lower can be made from ordinary coaxial cable. When more delay is desired than can be obtained with convenient lengths of cable (the order of 1 μsec), lumped artificial delay lines can be used if the operating frequency is not too high (the order of 30 MHz or lower). At frequencies so high that transit-time effect would be a problem in normal cathode-ray tubes (above about 100 MHz), one should either heterodyne the frequency band of interest down into the range of a normal cathode-ray tube or consider using the cathode-ray frequency meter whose operation makes use of the transit-time effect.

The cathode-ray frequency meter is particularly simple in that the frequency sensitivity is inherent in the deflection process, no frequency-dependent devices external to the cathode-ray tube being required. However, the frequency range is fixed by the tube design (250-500 MHz in present experimental tubes), a significant limitation in some applications. The most flexible LPD system is the one shown in Fig. 5.

B. RESPONSE TO MODULATED SIGNALS AND SIMULTANEOUS SIGNALS

A surprising aspect of the LPD systems is that they give some direct information about the Fourier spectrum of complex waves, in addition to information about the instantaneous frequency and amplitude. The simplest example is a signal consisting of two simultaneous sinusoids. Figure 12 shows the response to such a signal, which is a parallelogram display. The slopes and lengths of the sides of the parallelogram correspond to the frequencies and amplitudes of the two sinusoids in the same way as if the sinusoids were applied one at a time.

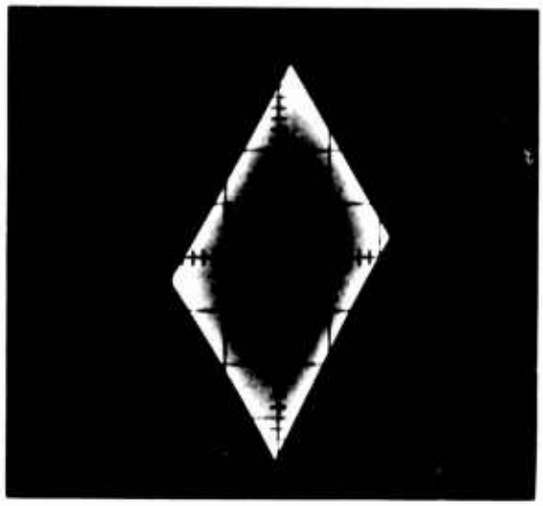


FIG. 12. DISPLAY PRODUCED BY AN LPD SYSTEM IN RESPONSE TO TWO SIMULTANEOUS SINUSOIDAL SIGNALS.

A further example is the display produced by a standard television broadcast signal which consists of modulated sound and picture carriers separated in frequency by 4.5 MHz. Figure 13, taken from Ref. 20, shows eight displays made at slightly different times by a local television signal. The sides of the parallelograms corresponding to the picture carrier are nearly vertical in these photographs. The amplitude modulation of the picture carrier causes the vertical variation in display intensity which varies according to the television picture that was being transmitted when each of the display photographs was made. Radars that transmit simultaneous pulses on different frequencies will produce similar displays, thus providing immediate visual indication of the presence of such a radar along with its two frequencies and relative amplitudes.

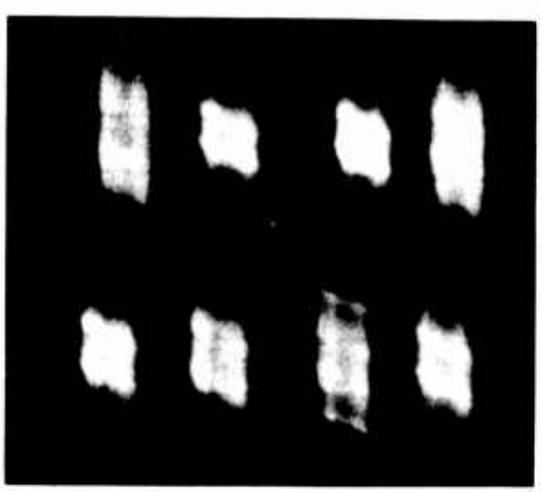


FIG. 13. DISPLAYS PRODUCED BY A STANDARD TELEVISION BROADCAST SIGNAL AT VARIOUS INSTANTS OF TIME SHOWING THE EFFECTS OF DIFFERENT BRIGHTNESS DISTRIBUTIONS IN THE TELEVISION PICTURE. The general parallelogram shape is caused by the picture and sound carriers.

In general, the display produced by M independent simultaneous sinusoids will be a polygon having M pairs of parallel sides indicative of the M frequencies and their corresponding amplitudes. Figure 14 shows an example for three simultaneous sinusoids.

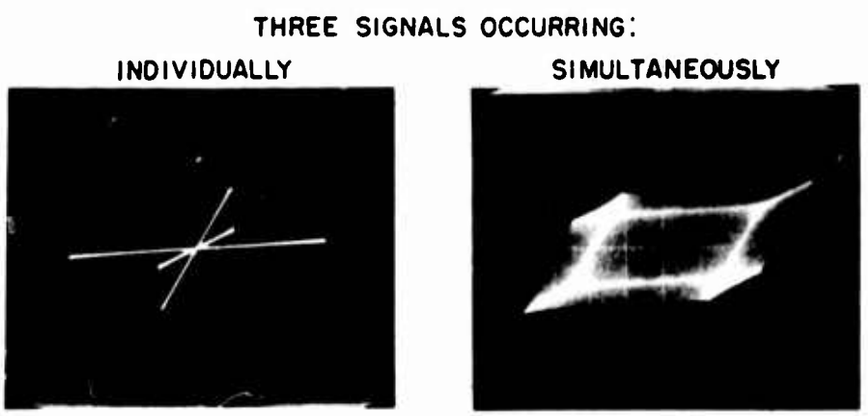


FIG. 14. DISPLAY PRODUCED BY AN LPD SYSTEM IN RESPONSE TO THREE SIMULTANEOUS SINUSOIDAL SIGNALS.

Amplitude-modulated signals and pulses having nonzero rise time yield displays such as that illustrated in Fig. 15. The carrier frequency is indicated by the slope of the major axis of the display, and the bandwidth of the signal is proportional to the included angle of the tip of the display. Unfortunately, however, the included angle of the tip is also influenced by the particular modulating waveform as well as its bandwidth; hence, this angle is not a unique measure of signal bandwidth. In the case of pulse signals having flat tops, the display will be similar to that shown in Fig. 15 except that the major axis will exhibit a bright line corresponding to the top of the pulse which acts like a steady (unmodulated) sinusoid.

Frequency-modulated signals produce displays like that shown in Fig. 16. This form is just what would be expected from the steady-state response of the system. The angles defining the edges of the display correspond to the maximum and minimum values of instantaneous frequency. Hence, the included angle of the central portion of the pattern is a measure of the peak-to-peak frequency deviation of the signal. Chirp radars will produce displays such as this, indicating their presence as well as their parameters.

FIG. 15. LPD DISPLAY PRODUCED BY AN AMPLITUDE-MODULATED SIGNAL.

FIG. 16. LPD DISPLAY PRODUCED BY A FREQUENCY-MODULATED SIGNAL.

C. NOISE-LIMITED SENSITIVITY AND ACCURACY

Since the LPD frequency meters are linear systems, it is relatively easy to describe their performance in the presence of additive noise. When noise alone is present, a more or less round noise spot appears at the center of the screen. As signal is added, the result is as if the noise spot were deflected by the signal, giving a line whose width is proportional to rms noise voltage and whose length is proportional to signal voltage. The minimum detectable signal power corresponds to a signal-to-noise ratio of about unity, which yields a slightly elongated noise spot.

Figure 17 shows the displays produced with various signal-to-noise ratios. The signal was an unmodulated sinusoid (frequency = 123 MHz) and the noise was gaussian. The values of signal power at the input of the preamplifier are indicated in the figure. System noise power (preamplifier unsaturated) referred to the input is about -57 dbm; most of this is excess noise purposely added at the system input to give a sizable noise spot. Saturation (limiting) becomes pronounced in the preamplifier for inputs exceeding about -35 dbm. Thus, from Fig. 17, it can be seen that the input signal is barely discernible when the signal-to-noise ratio is about 57 - 60 = -3 db. When the signal power rises to -45 dbm, the display becomes considerably longer, but its width remains about the same as that of the noise spot. With further increase of the signal power, the preamplifier begins to saturate, giving a limiting action and an angle-modulation capture effect. Consequently, the length of the display line does not continue to increase, but its width decreases because of the noise-suppressing property of the capture effect. To obtain a useful frequency indication, it can be seen that a signal-to-noise ratio of at least 10 db is desirable. In terms of system noise temperature and noise bandwidth, this minimum useful signal power (at the preamplifier input) is given by

$$S_{min} = 10\ kT_s B \tag{20}$$

The accuracy of frequency measurement is determined by the accuracy with which one can determine the angle of the display line. In the presence of noise, the memory or integrating effect of the CRT screen enhances the ability to determine display angle. Because of this integrating effect of the screen (and of the photographic film), the displays shown in Fig. 17 have symmetrical shapes, and therefore the slopes of their major axes can be determined quite accurately. It is to be noted that the displays in Fig. 17 are actually produced by a small spot which varies in position somewhat randomly within the illuminated region of the screen. Hence, the memory effects of the screen, film, or human eye are necessary in producing the symmetrical shapes shown. These memory effects are analogous to the integrating effect of postdetection filtering in

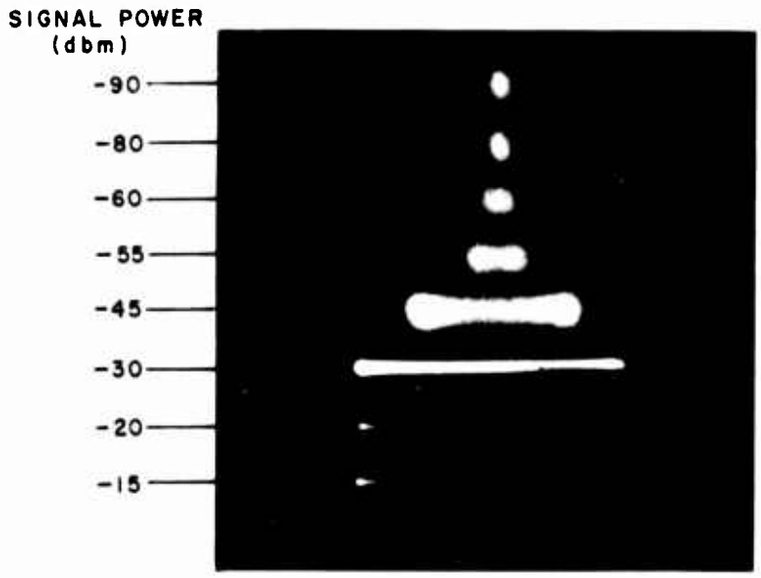


FIG. 17. LPD DISPLAYS PRODUCED BY SIGNAL PLUS NOISE.

systems using detectors. (In fact, one may consider the CRT screen to be a sort of detector in the LPD systems.) It follows that Eq. (15) can be applied to the calculation of frequency accuracy of an LPD system, if the symbol b is interpreted as the reciprocal of the memory time of the CRT screen, the photographic film, or the human eye.

D. DISPLAY BRIGHTNESS

A problem arises when the LPD systems are used to observe pulsed radars because their low duty factor results in a correspondingly dim display. The electron beam is undeflected in the interpulse interval so the average CRT beam current in the display pattern is less by the duty factor than the total beam current. However, live radar signals have been observed with entirely adequate brightness by observing the screen in a darkened room and using blanking circuits to eliminate the undeflected bright spot from the center of the screen. Duty factors as small as 10^{-3} have been observed easily, and 10^{-4} with careful observation.

Display dimness may set the upper limit of the dynamic range of LPD systems. For sufficiently strong signals, the display line will extend

beyond the confines on the CRT screen, and only the relatively dim central portion of the display will remain on the screen.

In the DPD systems, treated in Section IV, a polar display in the form of a dot rather than a line is produced by a sinusoidal signal. Hence, in general, the DPD display will be much brighter because of the higher concentration of electrons on the CRT screen. With respect to dynamic range, however, the dot of the DPD systems obviously must not be deflected off the screen or no indication will be produced; whereas in the LPD system the line display can be deflected beyond the edge of the screen with a sacrifice in brightness, but without loss of the ability to measure frequency.

IV. POSTDETECTION POLAR-DISPLAY FREQUENCY METERS (DPD)

As pofnted out in Section I, another class of polar-display frequency meters detects the outputs of the frequency-sensitive elements and applies the resulting dc voltages to the cathode-ray tube which again serves to separate the amplitude and frequency information. Systems of this class are nonlinear because of the use of detectors (rectifier diodes); the initials DPD are used to distinguish them from the purely linear LPD systems. DPD systems based on wave interference are disclosed in Ref. 11, and the microwave version shown in Fig. 6 is treated in Ref. 13.

The DPD systems have advantages and disadvantages relative to the LPD systems. Advantages include a brighter display on low-duty-factor pulsed signals, a more easily interpretable display of amplitude vs frequency, and the ability to use a video--rather than an rf--cathode-ray tube. Disadvantages include the requirement for matched nonlinear circuits (detectors and video amplifiers) and the lack of a simultaneous-signal capability.

A. DISPLAY CHARACTERISTICS

For a fixed-frequency, constant-amplitude input signal, a dc voltage is produced by the detectors and applied to the cathode-ray-tube deflection plates. As a result, the display is a dot. The angular polar coordinate θ of this dot is a function of signal frequency. In the system of Fig. 18, the video has but one polarity and therefore the dot is confined to one quadrant of the display. This limited use of the display reduces the frequency measurement sensitivity relative to the four-quadrant system to be described next. Further, amplitude variations of the input signal affect the angular coordinate of the display in the one-quadrant system, but not in the four-quadrant system.

For these reasons, the four-quadrant receiver of Fig. 6 has been designed, built, and tested. In this system the dot trace can assume any position on the screen. The radial distance of the dot from the center of the screen is proportional to the square of signal amplitude (proportional to signal power), and the polar angle of the dot is a linear

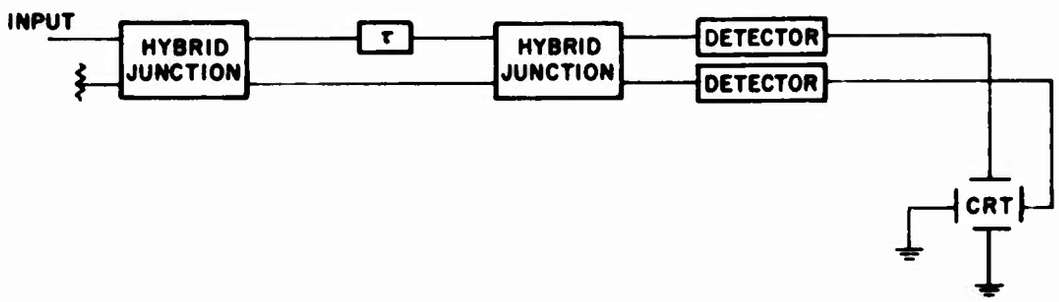


FIG. 18. A ONE-QUADRANT DPD SYSTEM.

function of signal frequency, provided the detectors operate in their square-law regions [Ref. 13].

Figure 19 shows the display produced by an S-band four-quadrant discriminator when the input signal is swept from 1 to 4 GHz. The ripple on the circular trace resulted from small reflections in the rf circuit elements. The linear portion of the receiver (Fig. 6) is a five-port frequency-sensing element (one input, four outputs) which relies on wave interference among the various signal paths. The S-band discriminator used to obtain Fig. 19 was a seven-hybrid stripline device having physical dimensions approximately 8 by 5 by 1/4 in.

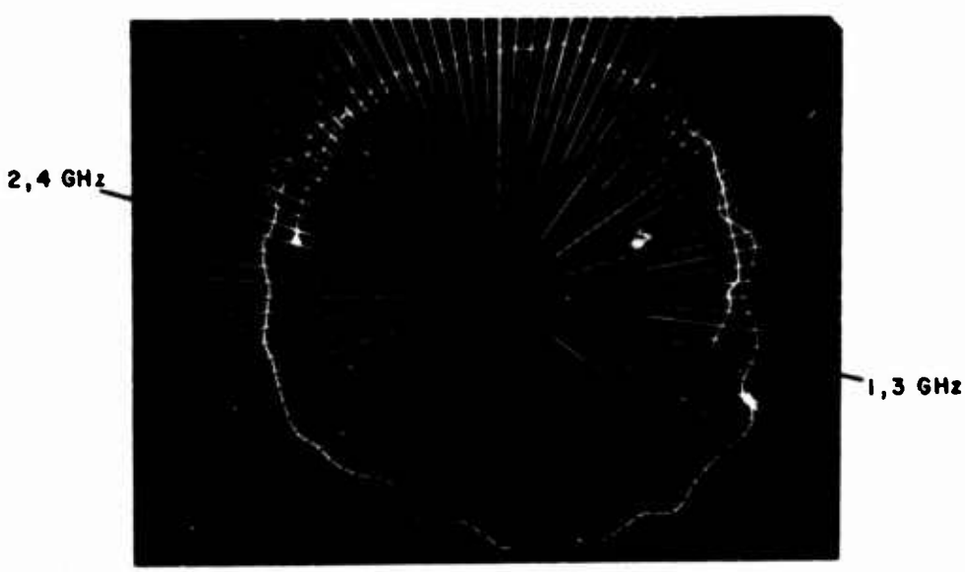


FIG. 19. S-BAND DPD SYSTEM SWEPT FROM 1-4 GHz. Sensitivity is 2 GHz/rev (τ = 0.5 nsec). The radial lines are scribed on the face plate of the scope.

By subtracting the outputs of the detectors and applying these difference voltages to the CRT deflection plates, an indication of frequency independent of signal amplitude is obtained. The radial coordinate of the resulting trace will vary in accordance with the amplitude of the signal, but the angle of the trace is unaffected by amplitude changes. An analysis of the response of this receiver also shows that only pulses of duration exceeding τ sec (Fig. 6) will be displayed. Since τ is usually less than 100 nsec in receivers designed to operate in the microwave range, this does not seriously limit the device as a spectrum monitor.

Frequency-modulated signals appear as an arc on the screen. The peak-to-peak deviation of the carrier frequency is indicated by the excursion of the arc. Chirp radars, for example, provide a characteristic trace as shown in Fig. 20, which also indicates the displays associated with other live and simulated radar signals.

It has been shown analytically and experimentally that DPD systems are not capable of resolving simultaneous signals. The presence of two signals is indicated by the trace, provided their frequency separation is less than the video bandwidth of the receiver. When two such signals are present, the trace is an ellipse* centered in the neighborhood of the vector resultant of the coordinates of the two dots which would be formed if the signals occurred individually. The shape and orientation of the ellipse is a complicated and ambiguous function of the frequency separation of the input signals and their individual amplitudes as well as the circuit delay τ. In general, when several signals are present, neither the number of signals, their individual frequencies, nor their individual amplitudes can be determined from the display.

As indicated in Section I.D8, a number of radar signals can be accurately displayed simultaneously if the pulses received from the various sources are not time coincident. Furthermore, the frequencies of radars that purposely transmit simultaneous pulses on two or more frequencies often can be determined with the DPD system since the various frequencies are typically transmitted on different antenna beams.

The video signal derived to measure frequency can also be used to gate the CRT beam. This results in a more readable display since the screen is

*Theoretically, it should be a straight line, but differences in the phase characteristics of practical detectors and video amplifiers lead to an ellipse.

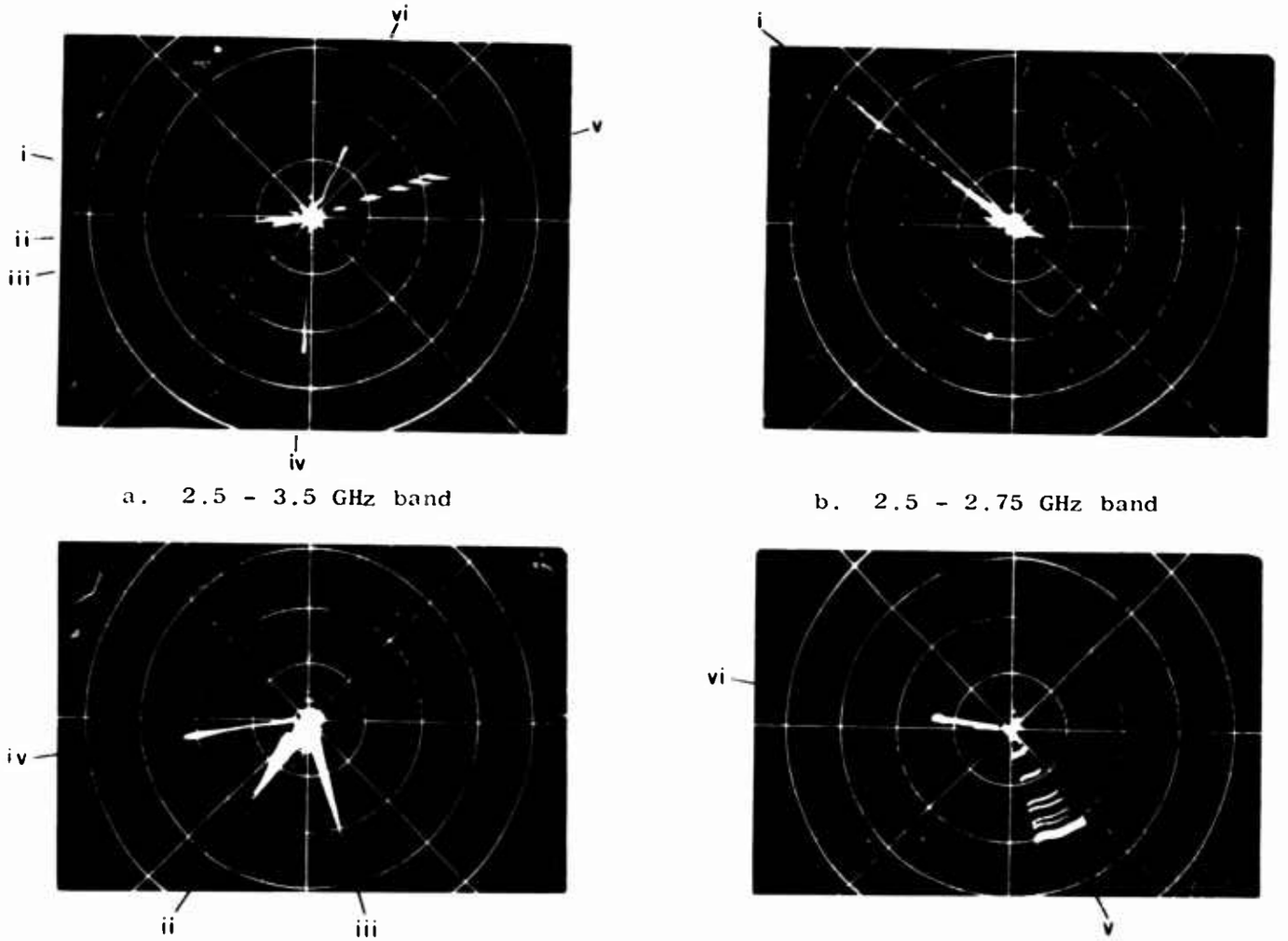


FIG. 20. DISPLAYS OF LIVE AND SIMULATED SIGNALS PRODUCED BY A DPD SYSTEM. The signals are: (i) FAA Air Traffic Control (2.725 GHz); (ii) FAA Air Traffic Control (2.793 GHz); (iii) unidentified (3.100 GHz); (iv) simulated (2.990 GHz); (v) simulated Chirp (center frequency of 3.320 GHz); and (vi) unidentified (3.452 GHz).

not illuminated unless signal energy is present. When the signal is present, the beam current is concentrated at one point on the screen, resulting in a bright, well-defined display even for low-duty-factor radars. This concentration of beam current in the display represents an advantage of the DPD compared with the LPD systems in which the beam current spreads out across the screen giving a line trace. A stripline DPD unit tested in the region 2.5 to 3.5 GHz was capable of measuring pulses as short as 50 nsec (20 MHz video bandwidth). A 1-μsec pulse repeating at a rate of 1 kHz presented a clearly readable trace. Duty factors as low as 10^{-5} or lower provide an adequately bright display.

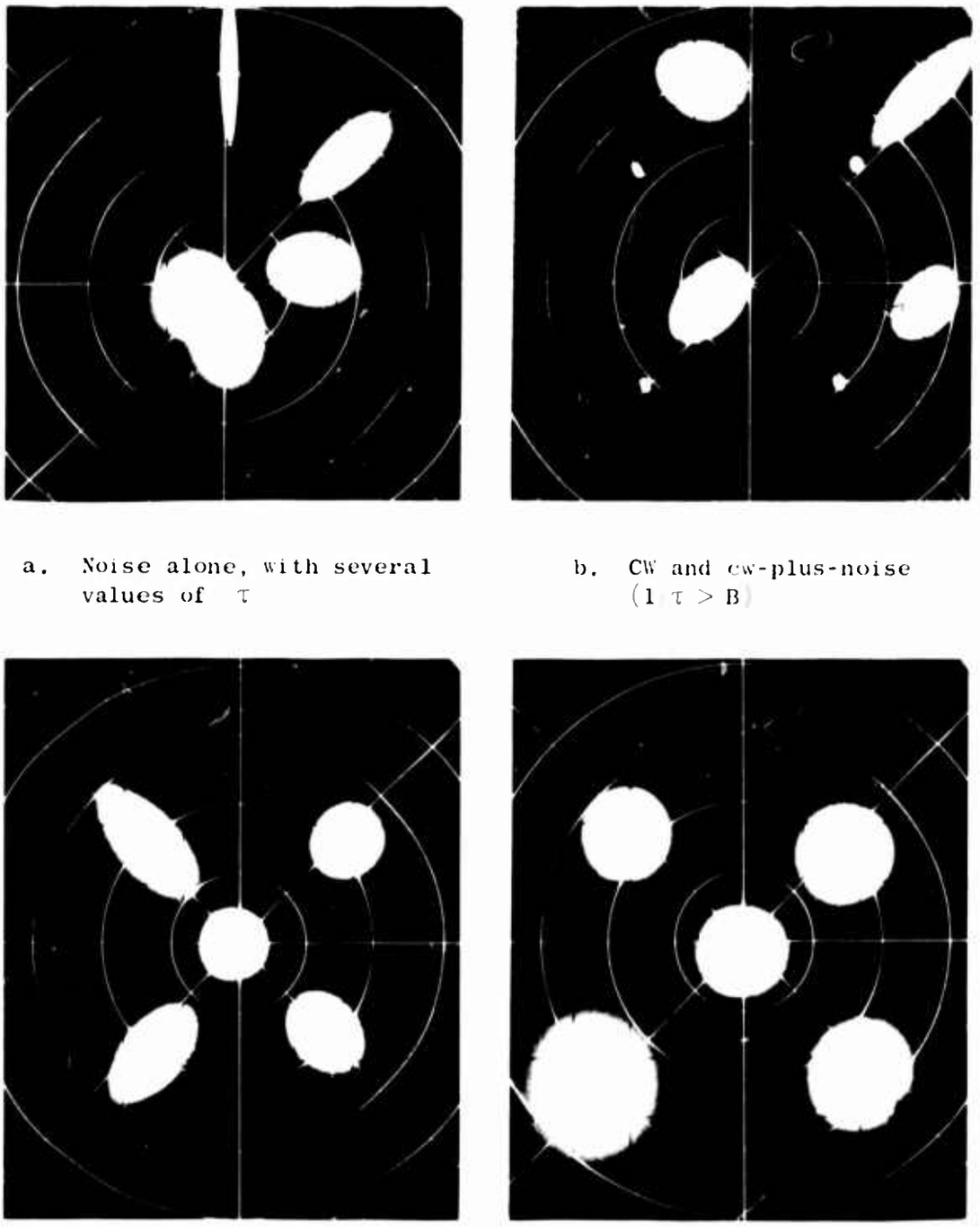
Additive noise at the input transforms the display from a dot to an elliptical area and degrades measurement accuracy. Tests show that a readily interpretable display is achieved when the input signal-to-noise ratio is greater than 10 db. Because of the persistence of the phosphor screen, the polar display presents a time-integrated record of the trace angle which is the measure of frequency. The mean of the record is an estimate of the frequency of the signal component of the input voltage. The accuracy with which the mean can be determined from the record can be calculated from Eq. (15) for the standard deviation σ_f if b is interpreted as the reciprocal of the decay time constant of the screen.

Figure 21 is a series of displays resulting for various values of bandwidth with a signal-to-noise ratio of unity and a sinusoidal input. A statistical analysis of multichannel receivers with applications to broadband DPD frequency discriminators is available in Ref. 21. See Ref. 13 for a more complete discussion of Fig. 21.

B. PERFORMANCE CHARACTERISTICS

The frequency measurement sensitivity of the instrument of Fig. 6 is determined by the delay τ. In general, θ, the angular coordinate of the trace, equals $2\pi f\tau$. The trace, then, makes one revolution on the screen each time f changes by $1/\tau$ hertz, which is therefore the unambiguous frequency range of the system.

As in the LPD system, the frequency range covered in one 360-deg revolution of the trace can be varied from a few megahertz to several gigahertz by simply changing the delay τ.



a. Noise alone, with several values of τ

b. CW and cw-plus-noise ($1/\tau > B$)

c. Noise and cw-plus-noise ($B > 1/\tau > b$)

c. Noise and cw-plus-noise ($B > b > 1/\tau$)

FIG. 21. DPD DISPLAYS PRODUCED BY NOISE AND BY SIGNAL PLUS NOISE. Symbols: B = noise bandwidth of the preamplifier, $1/\tau$ = frequency change that will cause the display spot to rotate 360 deg, b = postdetection bandwidth, and τ = time delay shown in Fig. 6.

Ambiguous readings can be eliminated by restricting the passband of the receiver with an rf filter. Alternatively, two discriminators can be operated in parallel. One discriminator can provide coarse information over a broad frequency band while the second can serve as a vernier to refine the reading in a restricted band. This combination has been used to provide, in the absence of noise, a measure of frequency to an accuracy of 10 MHz over the entire range from 2.5 to 3.5 GHz [Ref. 22]. In this particular unit, the outputs of the two discriminators are switched and presented one at a time on a common display. Using the less sensitive discriminator, the full 1 to 2 GHz band is displayed in the 360-deg sector of the screen. The vernier unit, having four times the measurement sensitivity, resolves the 1 GHz band into four equal subbands. When using the vernier discriminator it is necessary to identify the particular subband being displayed.

One straightforward method of identifying this subband involves filtering the discriminator input signal to pass only the frequencies in the subband of interest. This approach requires rf switching and selective filters to prevent display ambiguities and still provide complete band coverage. Restricting the monitored frequency band in this manner reduces the input noise power and gives the attendant advantages of improved system accuracy and sensitivity to weak signals.

An alternate technique makes use of the polarity-sensing concept presented in Section V. As indicated there, four polarity reversals are required in order to divide the band into four subbands. These are available in the two output voltages of the wideband discriminator shown in Fig. 6. When a signal is received, a code word consisting of two binary digits is then generated by sensing the levels of the outputs of the two differencing circuits. This code word identifies the subband associated with the frequency of the input signal. The code word is available for other logic operations or processing functions, such as selective gating of the video signal or unblanking of the oscilloscope in order to process and display only the signals in a chosen subband.

The remaining performance specifications on the system of Fig. 6 are determined primarily by the diode detectors. To ensure accurate displays and linear variation of the angle of the display trace with frequency,

matching of detector characteristics between diodes and between pairs of diodes, in addition to impedance matching, is necessary.

The dynamic range of the system is also determined by the detectors. The diodes are operated as square-law devices to provide a linear variation of display angle with frequency. Typically, the square-law characteristic of diodes extends over a range of approximately 25 db. With compensation, such as limiting of the rf voltage, a dynamic range in excess of 40 db can be expected. The CRT display further restricts the useful operating range of signal level. With square-law detectors, a change of about 10 db in input signal level will move the radial coordinate of the display over its usable range, which is determined by spot size and screen diameter. Therefore, without manual or automatic gain control, these CRT parameters determine the dynamic range of the system.

The minimum useful signal power of the DPD system without rf preamplification is about equal to the tangential sensitivity of the detectors used. The S-band unit of Ref. 22 used detectors having a tangential sensitivity of -50 dbm when followed by a video filter of 5-MHz bandwidth.

Knowing the tangential sensitivity of the detectors, an optimum value of rf amplifier gain can be determined for a given receiver. The amplifier gain is chosen to provide a noise power at the input to the detectors equal to the diode tangential sensitivity. A larger value of gain would reduce the useful dynamic range of the receiver, since the diodes exhibit a square-law characteristic only over a limited range of input level. A smaller value of gain would result in reduced system sensitivity. The gain is optimum, then, in the sense that noise-limited sensitivity is improved without a reduction in dynamic range.

As an example of the rf gain one would use for a typical receiver, assume a tangential sensitivity of -50 dbm for the detectors, an rf bandwidth B of 1 GHz, a noise figure F of 12 db, and an antenna temperature T_a of 290 °K. From the definition of noise figure, the noise power of the rf preamplifier referred to its input is given by

$$N\ (\text{dbm}) = -114 + F\ (\text{db}) + 10 \log B\ (\text{MHz})$$

which becomes, for the values chosen,

$$N = -114 + 12 + 30 = -72 \text{ dbm}$$

The optimum gain is that value necessary to increase this level at the input to the detectors to their tangential sensitivity level (-50 dbm). The result is

$$\text{Optimum gain} = 72 - 50 = 22 \text{ db}$$

For this example, the noise-limited sensitivity is -72 dbm. If a signal-to-noise ratio of 10 db is required for a good display, then the minimum usable signal power is -72 dbm + 10 db = -62 dbm. In this example, losses in the hybrid junction, the delay line, etc., were not included. To account for these losses in a practical system, a value of rf amplifier gain somewhat greater than that calculated here could be used.

At signal levels that produce other than square-law detection, a departure in the linearity of θ with f is observed. It should be noted, however, that linearity is not necessarily a measure of frequency accuracy, since the CRT screen could be calibrated in angle θ and radius r to give accurate readings. On the other hand, for convenience it is desirable to use a frequency calibration that is a function only of θ and independent of r. When such a calibration is used, an indication of frequency accuracy is the deviation from a radial line experienced by the trace as the signal amplitude varies. To keep this deviation small, the detectors must have a square-law characteristic over the range of amplitudes of their input signals, and all the video circuits must be linear. With the detectors and video amplifiers available, the accuracy can be held to 10 MHz for a sensitivity of 1 GHz per 360-deg revolution of the trace.

Impedance mismatches in the four signal paths of Fig. 6 can also cause θ to be a nonlinear function of f. However, this effect is not amplitude dependent and can be compensated by a screen calibration that is a function only of θ, independent of r.

Stripline DPD systems covering frequency ranges as great as 8.6 to 1 in a single unit have been built. These units have a maximum deviation

of ±3 percent from a linear calibration of θ vs f. Multiple units have been successfully designed to cover 100 to 8000 MHz with typical linearities of ±1 percent over octave bands (20 MHz for S-band when the time delay is set to give a sensitivity of 2 GHz per 360-deg revolution of the trace).

V. POLARITY-SENSING SYSTEMS (PS)

In the frequency discriminator shown in Fig. 2, the level of the output voltage depends on signal amplitude as well as frequency, but the polarity reversals of the output voltage depend only on frequency--occurring at frequencies equal to integral multiples of $1/2\tau$, where τ is the time delay indicated in the figure. Therefore the polarity of the output voltage can be sensed to divide the monitored frequency band B into two unambiguous cells, each of width $1/\tau$. This provides a very coarse measure of frequency. In this section we will show how the number of frequency cells can be increased by providing additional bipolar frequency-dependent voltages.

The technique of using only the polarity of the output signal to measure frequency is distinguished from the methods treated in Section II by the fact that it is not necessary here to remove the dependence of the output voltage on the input signal amplitude. Neither limiting, ratioing, nor gain control are required. For some applications, then, this technique may have definite advantages.

The frequency measurement accuracy is determined by the number of distinct polarity reversals that can be achieved in a bandwidth B. Since any one frequency-sensitive circuit of the type shown in Fig. 2 has but one polarity reversal in its unambiguous bandwidth, additional polarity reversals can be achieved only at the expense of additional circuits. For any frequency, the polarity of the output of any one circuit can be identified by a binary digit. For n circuits, n binary digits are generated and used to form an n-bit word (binary number) which identifies a particular frequency cell. It is possible to obtain such a binary indication of frequency by using n circuits similar to those of Fig. 2.

A. FORM OF THE RECEIVER

The basic receiver configuration is shown in Fig. 7 with the specification that

$$\tau_i = \frac{\tau_{i-1}}{2} \tag{21}$$

where τ_i is the value of the delay in the ith channel.

The variation of voltage with frequency for the n channels assumes the form shown in Fig. 22. The unambiguous bandwidth B is determined by the least sensitive channel (smallest value to τ):

$$B = \frac{1}{\tau_n} \tag{22}$$

The number N of frequency cells is given by

$$N = 2^n \tag{23}$$

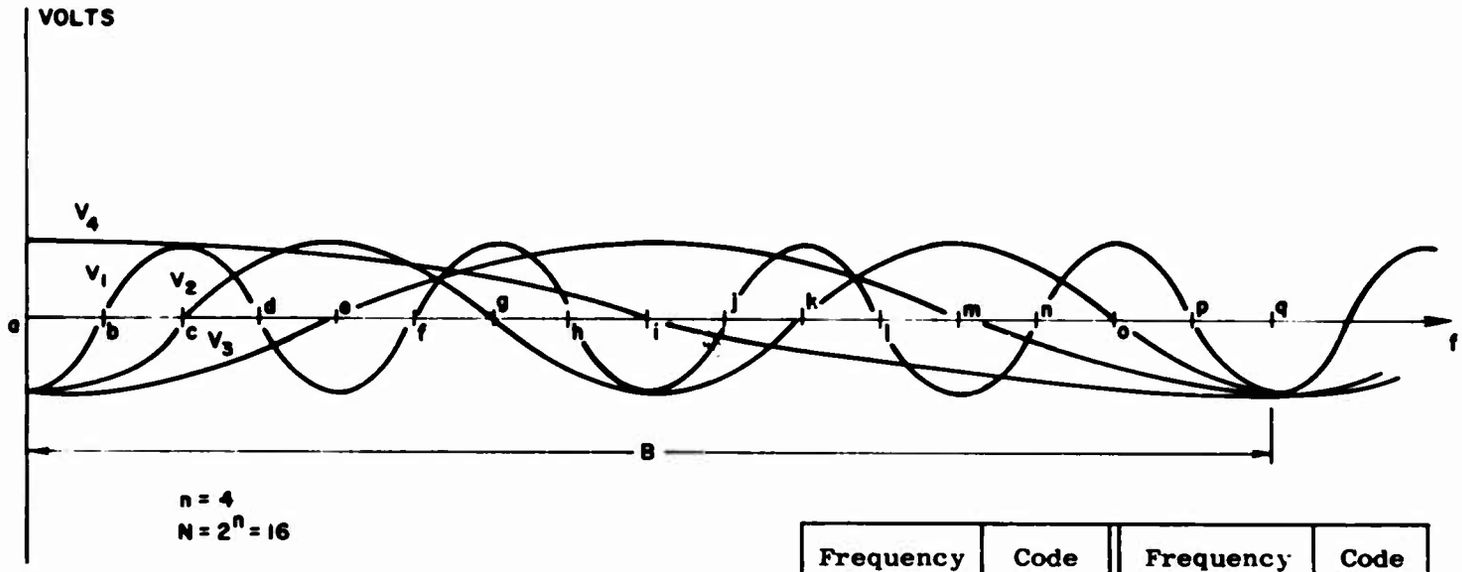
which provides an accuracy of $B/2^n$ Hz. Accuracy is improved exponentially with an increase in the number of channels.

Each cell is identified directly by a binary number (Gray code) as shown in the table in Fig. 22. These code words could be encoded into any other desired form (natural binary, binary-coded decimal, octal, or decimal) which may be convenient for driving indicators, for performing logic operations, or for further automatic digital processing.

The requirement for delay in each channel as expressed by Eq. (21) indicates that for n more than 2 or 3, a considerable difference exists between the least-sensitive channel delay τ_n and the most-sensitive channel delay τ_1. This can be seen by expressing τ_1 in terms of τ_n using Eq. (21):

$$\tau_1 = 2^{(n-1)}\tau_n \tag{24}$$

Typically, the delays necessary for the frequency-sensitive elements considered in this report are realized by using lengths of transmission line such as coaxial cable. Equation (24) indicates that τ_n must be sufficiently small so that a prohibitive length of cable will not be



Frequency Range	Code Word	Frequency Range	Code Word
a-b	0001	i-j	0010
b-c	1001	j-k	1010
c-d	1101	k-l	1110
d-e	0101	l-m	0110
e-f	0111	m-n	0100
f-g	1111	n-o	1100
g-h	1011	o-p	1000
h-i	0011	p-q	0000

FIG. 22. VOLTAGE VARIATION WITH FREQUENCY FOR EACH OF THE (n = 4) CHANNELS OF THE PS SYSTEM.

required to achieve τ_1. Such a small value of τ_n suggests that this polarity-sensing technique is best adapted for use at microwave frequencies.

One realization of a polarity-sensing frequency meter that is described in the literature uses transmission lines terminated in open or short circuits to provide the frequency-sensing devices. This system, named Bess (binary electromagnetic signature sensor), was implemented in two units to cover the frequency ranges 1 to 2 GHz and 55 to 65 MHz [Ref. 23]. The 1 to 2 GHz range was monitored by using the PS system described above.

The design to cover the i-f range of 55 to 65 MHz used a different form of PS system. This lower frequency version uses delays whose values between adjacent channels of Fig. 7 differ but slightly. Typically, the maximum value of delay in the n channels is less than twice the minimum value of delay. The n channels divide the frequency band B into $2n$ unambiguous increments for an accuracy of $B/2n$ which, for the same number of channels, is considerably less than the accuracy of the PS system considered above. For this reason, this latter system is not considered further.

A PS system which has the same value of delay for each of the n channels of Fig. 7 can be defined. The null position in frequency can be controlled by adding a phase-shifting element in series with the delay line or in the line paralleling the delay. In passing through this element, all signals experience the same phase shift ϕ independent of frequency. For equal-width frequency cells, the phase shift ϕ of any channel relative to that of an adjacent channel is π/n radians. Again, the n channels will divide the bandwidth B into $2n$ frequency cells for an accuracy of $B/2n$. Difficulty in realizing wideband phase-shifting elements probably makes this system impractical. For this reason and because of the poorer accuracy of this system compared with the variable-delay system considered initially, this equal-delay system is not considered further.

B. SYSTEM PERFORMANCE AND COMPARISON WITH THE ARRAY-OF-FILTERS SPECTRUM ANALYZER

The n-channel system considered in this section can now be compared with the array-of-filters spectrum analyzer receiver which is another system of n parallel channels as defined in Section I.

1. Accuracy

The array-of-filters receiver increments the band B of frequencies into n cells with n filters each of bandwidth B/n. The PS receiver increments the same band into 2^n cells. Therefore, with the same number of channels, the PS system is capable of greatly increased accuracy.

In the PS system, accuracy is affected by variations in the frequency at which a null occurs. Variation of the response between the two detectors (Fig. 2) required to produce one frequency-sensitive bipolar voltage will cause amplitude changes which produce some variation in the frequency at which polarity reversal occurs. This variation is minimized by choosing diodes with similar characteristics. In the neighborhood of a polarity reversal, each of the diodes experiences the same input signal power; this can be seen by inspecting the diode input voltage variation with frequency in Fig. 1. As a result, the nonlinear response of the diodes is not critical as long as both diodes have the same characteristic.

2. Dynamic Range

In the PS system and the array-of-filters, the dynamic range is determined by the diode detectors. The array-of-filters indicates the distribution of signal energy with frequency on a cell-by-cell basis with the result that the nonlinear response of the diodes is not critical when using this system to measure frequency. As observed in the discussion of accuracy in the previous section, the nonlinear response of the diodes in the PS system is not critical. Therefore, in both systems the diodes can be used in their linear region or as square-law devices. Consequently, there is no fundamental upper limit on the dynamic range of these two systems. In practice, an upper limit is established by diode breakdown.

The lower limit on the dynamic range (minimum usable signal power) is determined by the threshold sensitivity (noise limitation) of

the diodes. Usable dynamic ranges of 60 db (crystal diodes) to 100 db (vacuum diodes) can be expected with these systems.

3. Simultaneous-Signal Response

The array-of-filters spectrum analyzer separates the incoming signals on the basis of frequency and in so doing is capable of indicating the presence and measuring the individual frequencies of several signals occurring simultaneously. (Two or more simultaneous sinusoids whose frequencies all lie in the same measurement cell are interpreted as one signal.) The PS system requires that the input signals be separated in time, to measure their individual frequencies. In the case of two input signals, this requirement of noncoincidence in time can be established by considering the instantaneous frequency $f(t)$ of the composite input signal.

The expression for $f(t)$, Eq. (17), suggests that two or more simultaneous sinusoidal signals applied to the PS system will result in time-varying voltages V_1 through V_n out of the discriminators of Fig. 7. The result is a code word which varies with time. Equation (17) also indicates that the code word defined at any point in time is a function of amplitudes A_1 and A_2 of the individual signals as well as their separation in frequency. Therefore, in the presence of simultaneous signals, this digital output cannot indicate the number of signals present or measure their frequencies and amplitudes.

4. Response Time

These two systems can be compared directly using another parameter, namely, speed of response (minimum duration of an input signal pulse required to produce a usable indication of frequency). The response time τ_f of a filter is often quoted in terms of the bandwidth B_f of the filter. For the array-of-filters, then, the system response time τ_s is determined by the individual filters:

$$\tau_s = \tau_f = \frac{1}{B_f} = \frac{n}{B} \tag{25}$$

For a PS system, the system response time is equal to the value of delay in the most sensitive channel. Using Eq. (21),

$$\tau_s = \tau_1 = 2^{(n-1)}\tau_n \tag{26}$$

Since, from Eq. (22),

$$\tau_n = \frac{1}{B} \tag{27}$$

then

$$\tau_s = \frac{2^{(n-1)}}{B} \tag{28}$$

Compared to the array-of-filters, the PS system provides a considerable increase in accuracy at the expense of a slower response for the same number n of parallel channels. The ratio of the number N of cells to the response time τ_s becomes

$$\frac{N}{\tau_s} = B \qquad \text{for the array-of-filters} \tag{29}$$

and

$$\frac{N}{\tau_s} = 2B \qquad \text{for the PS system} \tag{30}$$

Hence, for the same number of measurement cells, the response times of the two systems are about equal.

5. Form of the Output Voltage

The outputs of the array-of-filters spectrum analyzer and the PS system are markedly different. The former provides continuous (analog) output voltages which are a discrete approximation of the Fourier transform

of the input signal. The output of the polarity-sensing system is a set of two-level voltages (digital code word). Apart from the distinct form of these outputs, their nature suggests that they may be used to advantage in further processing.

For example, in the array-of-filters spectrum analyzer, although the amplitude and phase of an input sinusoidal signal are altered, the output of a filter is still a sinusoidal signal. The filter output signal-to-noise ratio exceeds that of the input by virtue of the filtering action. This filtered analog output is available for further processing if desired.

The PS system with its binary code output may be particularly convenient to use in receiving systems that reduce data by digital means. Such an output can be used directly in conjunction with logic circuits to selectively gate or route signals at various stages in the receiving system.

6. Noise Considerations

Assuming equal rf bandwidths B for the array-of-filters spectrum analyzer and the PS system, the received noise power equals kT_sB. In the array-of-filters the frequency of an input signal in the band B is measured by a filter of bandwidth B/n. Therefore, the received noise power which influences measurement is kT_sB/n.

Each channel of the PS system passes the noise in the entire band B. Therefore the entire input noise power influences the frequency indication. The noise improvement factor of the array-of-filters compared with the PS system is given by n, the number of channels or filters in the system.

7. Complexity

As the name suggests, the array-of-filters spectrum analyzer requires n contiguous bandpass filters to cover the frequency range B. For uniform accuracy over this band B, each filter must have the same bandwidth B/n.

The PS system described in this section requires a pair of matched diode detectors for each of its n channels.

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