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EXTERNAL TECHNICAL MEMORANDUM NO. 74

RAY TRACING IN THE IONOSPHERE

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EXTERNAL TECHNICAL MEMORANDUM NO. 74

RAY TRACING IN THE IONOSPHERE

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ABSTRACT

The problems of tracing rays in the ionosphere for radio propagation, primarily for high frequencies (3 to 30 Mc/s) and above, are discussed. The limitations on "classical" geometrical optics are described, and their applications to ionospheric problems are indicated.

Some "simplified" processes of ray tracing, which apply well at the earth's surface, but not in the ionosphere itself, are first presented. Next, ray-tracing methods which apply in an isotropic ionosphere, when the effects of the earth's magnetic field are negligible, are discussed. The results are then extended to more general cases of an isotropic medium, and then to an anisotropic medium (such as the ionosphere in the presence of the earth's magnetic field).

Certain special problems which must be considered in ray-tracing studies are described next and finally, a few of the important areas for the application of ray tracing are noted.

This report has been accepted for publication in the journal RADIO SCIENCE.

1. INTRODUCTION

All students of elementary physics are introduced to the concept of representing the direction of energy flow of light in terms of rays, and to the use of Snell's law to compute changes in the ray direction as the light crosses the boundary surface between two media of differing refractive index. The application of such techniques to a medium in which the refractive index changes continuously is, perhaps, slightly less well known, but not because the idea is new; for several hundred years (at least back to the time of Newton), astronomers have corrected their observations for the effects of atmospheric refraction using ray-tracing methods. The objective of the present paper is to review the use of similar procedures, and their extensions, in studying radio transmission in the ionosphere.

The use of ray tracing is generally confined to conditions contained within the limitations imposed on geometrical optics; however, there are at least two special conditions in which ray-tracing procedures can be shown to give acceptable results, even when geometrical optics is not properly applicable. Both the limitations on geometrical optics, and the special cases when ray tracing can be performed outside these limitations, are discussed in a later section. For ionospheric problems, ray tracing can be used to determine such basic parameters as the direction of energy flow; the phase path, group path, and actual path lengths; and the integrated absorption. From these basic data, many other derived quantities can then be determined, as will be discussed later.

Certain forms of ray tracing have been used for ionospheric problems for more than forty years, since the first years of ionospheric research. However, within the past decade or so, the increasing availability of high-speed digital computers has greatly stimulated the development and application of ray-tracing processes by permitting extensive use of voluminous and complex calculations which earlier could be done only in rare cases and at the expense of much time and labor. This report attempts to review the fundamental background of ray-tracing methods; to describe the major classes of available methods - from elementary to complex types; to take note of important problem areas in applying ray tracing; and to note briefly some of the applications for which ray tracing is used.

2. GEOMETRICAL OPTICS AND RAY TRACING

2.1. GENERAL CONDITIONS ON VALIDITY OF GEOMETRICAL OPTICS

If one examines the basis of radio wave propagation in the ionosphere from a position of considerable generality, the foundation for ray tracing seems to rest upon an almost unending sequence of approximations. Fortunately, however, it can be shown that for frequencies in the HF spectrum, (and, of course, at higher frequencies), these approximations are generally well founded. The applications to lower frequencies (less than 3 Mc/s) must be examined with care. For examples of applications at very low frequencies, see Yabroff (1961) and Kimura (1966).

Försterling (1942) derived equations for vertical propagation of a radio wave in an inhomogeneous, anisotropic, and dispersive ionosphere. These relations were expressed as a pair of coupled, second-order differential equations (or, equivalently, as single fourth-order, or four first-order, differential equations). The extension to oblique incidence was given by Heading and Whipple (1954); an excellent discussion of these two cases is provided by Budden (1961). According to these coupled differential equations, the classical ordinary and extraordinary modes do not propagate independently, but instead, they are coupled; that is, energy from one mode is fed into the other in a very complicated manner.* Fortunately, in the HF range, and at higher frequencies, this effect is generally negligible, and the two modes can be generally treated as if they satisfy two independent differential equations of the form:

$$\nabla^2 E + k_o^2 \eta^2 E = 0, \quad \eta = \eta\,(x, y, z) \tag{2.1}$$

*This is not the most general possible form for expressing ionospheric propagation since it does not include the even more general combination of electromagnetic theory and hydrodynamics that is included in hydromagnetics (magnetohydrodynamics); however, it is more than adequate for the conditions of interest here.

where E is the electric field, $k_o = \omega/c = 2\pi/\lambda_o$, $\omega = 2\pi f$, f = frequency, c = velocity of light in a vacuum, λ_o = vacuum wavelength, and η is the complex refractive index for the mode in question. The validity of the Differential Eq. (2.1) depends on negligibility of a "coupling parameter," ψ; the necessary conditions are discussed in detail by Budden, but for frequencies above 2 Mc/s, the conditions reduce basically to a slow variation of electron density with distance in the direction of propagation.

As a wave equation, (2.1) can be solved analytically only in very special cases, and numerically in greater generality, but at the expense of great effort. It is necessary, therefore, to seek still more tractable relations involving further approximations.

The following discussion of the limitations of geometrical optics is intended only to acquaint the reader with some of the principles involved. In particular, our attention is confined to "classical" geometrical optics as expressed in the equivalent forms of the eiconal equation, Fermat's principle, Snell's law, or Hamiltonian optics, and is restricted to the case of an isotropic medium. An excellent, and much more extensive, discussion is given by Bremmer (1958), who also considers higher order approximations, especially as applied to the calculation of field intensities. The reader will also find it informative to consult Born and Wolf (1965). An even broader view would be obtained by consideration of the generalized geometrical optics begun in the 1940's by Luneburg (see Luneburg, 1966), and extended in the very informative book by Klein and Kay (1965). However, an attempt here to provide even a sketchy summary of these more general treatments would carry the discussion well beyond what is needed for the purposes of the ray-tracing processes of interest in this report.

Therefore, we confine our attention to "classical geometrical optics," and illustrate the basic limitations by consideration of the eiconal equation for an isotropic medium.

We start by noting that if the complex refractive index η were constant, the solution of Eq. (2.1) could be written as a plane wave, in the form

$$E = A \exp \left[-j\, k_o \eta \; (\vec{r} \cdot \vec{n}) \right] \tag{2.2}$$

where $j = \sqrt{-1}$, A = constant, $\vec{r}$ is a vector defining the point where the field is to be evaluated, and $\vec{n}$ is a unit vector perpendicular to the wavefront in the direction of motion of the planes of constant phase.

When η varies slowly with position, the constants A and $\eta\, (\vec{r} \cdot \vec{n})$ of Eq. (2.2) are replaced by functions of position in the form

$$E = u(x, y, z) \exp \left[-j\, k_o \; S(x, y, z) \right] \tag{2.3}$$

The phase function S is termed the "eiconal function;" the wavefronts are given by the surfaces on which S = constant and the rays are the orthogonal trajectories to these surfaces,* i.e., the rays are in the directional of S. The substitution of E from Eq. (2.3) into Eq. (2.1) leads to

$$u \left[\eta^2 - |\vec{\nabla} S|^2 \right] + (1/k_o{}^2) \nabla^2 u - (j/k_o) \left[u \nabla^2 S + 2(\vec{\nabla} u) \cdot (\vec{\nabla} S) \right] = 0 \tag{2.4}$$

If we now apply the conditions

$$\nabla^2 u \ll k_o{}^2$$

$$u \nabla^2 S + 2(\vec{\nabla} u) \cdot (\vec{\nabla} S) \ll k_o \tag{2.5}$$

and note that the amplitude factor u cannot be everywhere zero, we obtain

$$|\vec{\nabla} S|^2 = \eta^2 \tag{2.6}$$

which is the "equation of the eiconal." This equation is one of several equivalent formulations of the basic expressions for geometrical optics; other formulations are those of Snell's law, Fermat's principle, and

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*Generally true only for an isotropic medium.

Hamilton's equations of characteristics [Budden (1961), for detailed discussions of the relation among these formulations]. As noted, the applicability of Eq. (2.6) depends upon the two conditions of Eq. (2.5).

The first of these two conditions is violated in any region containing point sources, or even strong distributed sources (see the well known Poisson's equation). At a shadow boundary, across a caustic surface, and at a focal point, the amplitude changes rapidly, i.e., $\vec{\nabla} u$ is large, so that the second condition is violated. It can also be shown [Stratton (1941)] that the validity of Eq. (2.6) also requires that the fractional change of refractive index in the direction of a ray must be small in a distance of $\lambda_o/2\pi$.

All of the above conditions must be satisfied if the normal methods of ray tracing are to be applicable. Fortunately, this is usually true for most conditions of interest in the HF and higher frequency spectral ranges; however, several exceptions are discussed below. The extension of ray-tracing methods to cases of lower frequencies in ionospheric propagation is sometimes valid, but must always be approached with care.

The wave amplitude may be studied by following the procedures used by Bremmer (1958). First, the result in Eq. (2.6) may be considered as the definition of the eiconal S, so that Eq. (2.6) is exact rather than approximate. Then, since the first term of Eq. (2.4) is zero, we are left with

$$u \nabla^2 S + 2(\vec{\nabla} u) \cdot (\vec{\nabla} S) + (j/k_o) \nabla^2 u = 0 \qquad (2.7)$$

As the frequency is increased, the factor $(1/k_o)$ becomes small. In the limit, the third term of Eq. (2.7) vanishes. According to the equation of the eiconal, the quantity $(\vec{\nabla} S)/\eta$ is a unit vector. Therefore the scalar product $(\vec{\nabla} u) \cdot (\vec{\nabla} S)$ may be replaced by $\eta\, du/ds$, where du/ds is a directional derivative along the direction of $\vec{\nabla} S$, i.e., the ray direction. Then Eq. (2.7) can be written

$$\frac{d(\log u)}{ds} = \frac{\nabla^2 S}{2n} \tag{2.8}$$

This differential equation, together with the equation of the eiconal make it possible to compute the variations of the amplitude u that result from geometrical spreading. This procedure was applied in a ray-tracing study by Grossi and Longworthy (1966).

2.2. HF PROBLEMS FOR WHICH RAY TRACING IS INVALID

There are several important conditions in HF propagation in which geometrical optics is no longer valid. One of these arises in the neighborhood of the shadow region (caustic surface) formed by the lower boundary of the ray family whenever a skip distance exists. The appearance of such a shadow region is clearly illustrated in the sample ray tracing shown in Figure 1. (The rays are traced for a frequency of 12 Mc/s in an ionosphere that is almost parabolic in profile; the regularly spaced "tick" marks correspond to intervals of group delay time of 0.25 millisecond.) Another caustic surface of interest is formed by the envelope of ray apogees.

As noted above, the ray amplitude changes discontinuously across a caustic surface; hence, the conditions of Eq.(2.5) do not hold. Although it is not unreasonable to attempt to represent the rays geometrically along such a surface, it is not possible to compute the amplitude along the surface by means of simple geometrical optics.

A still higher order violation of the conditions of Eq. (2.5) is made at focal points, which can be considered as cusps of caustic surfaces.

Bremmer (1949, 1958) has computed the amplitudes in the vicinity of two such focal points, using higher-order saddle-point approximations. The first case is in the vicinity of the skip distance, where the high and low rays produce a focal point. The second case

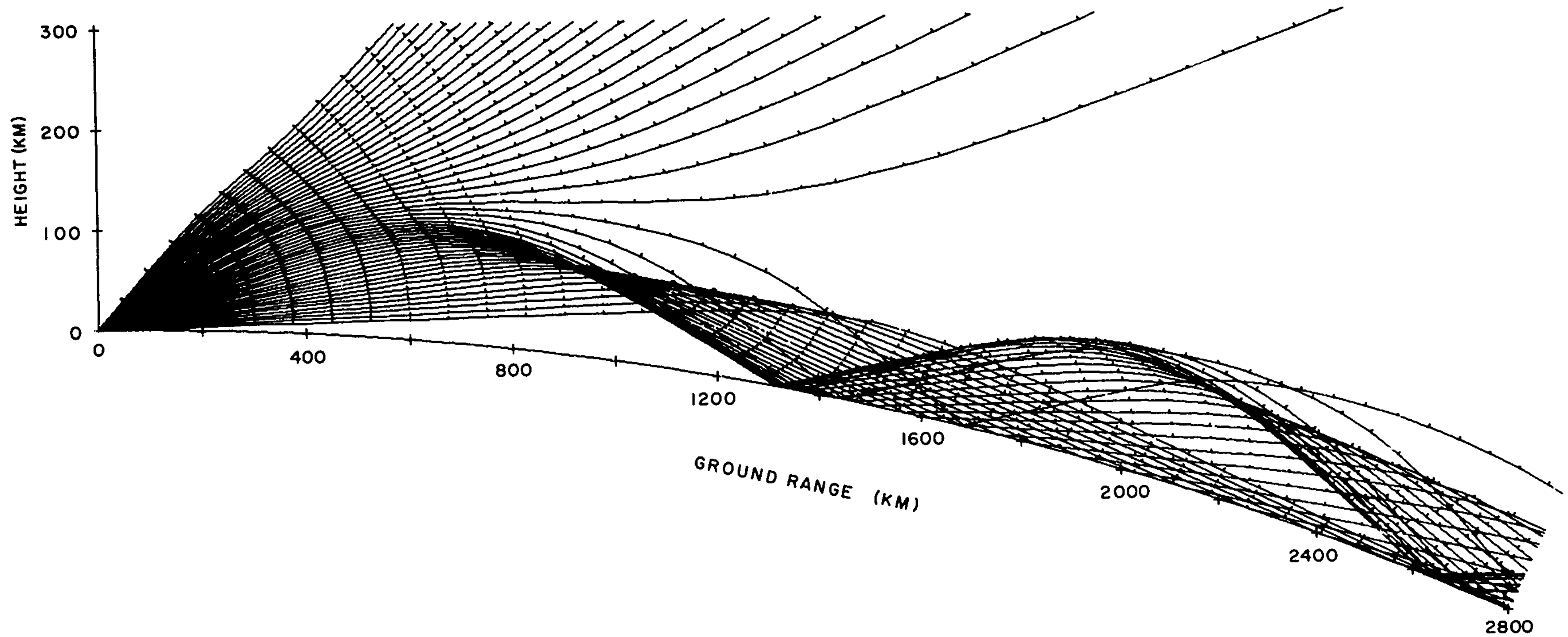

Figure 1. Family of rays in a quasi-parabolic ionosphere computed for a frequency of 12 Mc/s. Group delay increments of 0.25 ms are marked on each ray.

is for antipodal focusing. Budden (1961) also considers the field intensity in the neighborhood of a caustic, and, in particular, discusses the caustic formed by the envelope of ray apogees. Brekhovskikh (1960) discusses the amplitude variations of fields near caustics in a waveguide. As shown by these treatments, the amplitude undergoes rapid oscillations with position across the caustic surface. The regions where horizon focusing is predicted might call for similar treatment, but the problem here is complicated because true horizon rays can be produced in the HF spectrum only by elevated antennas, which, therefore, introduce further problems of the diffraction at the region of tangency to the horizon.

One further problem area exists at or near the region of vertical incidence reflection. Since (in simple approximations) the condition for reflection is the vanishing of the refractive index, it is clear that the fractional change of refractive index in a distance of $\lambda_0/2\pi$ becomes infinite. Fortunately, more exact treatments show that elementary methods can still give acceptable accuracy, despite this violation of geometrical optics; see, for example, Hartree (1931) and Rydbeck (1942).

The presence of absorption (i. e., a complex value of refractive index) means that the methods above are not fully applicable. However, Epstein (1930) has shown that in the presence of modest absorption, the ray directions can be computed adequately from calculations neglecting the magnetic field.

Finally, Titheridge (1967) has shown that the virtual heights in an absorbing ionospheric layer can, under some circumstances, be calculated more accurately by a simple theory ignoring the absorption than by an apparently more accurate technique that includes the absorption.

3. "SIMPLIFIED" METHODS FOR RAY TRACING

This section notes briefly rapid methods for obtaining some of the information desired in ray tracing. They are denoted as "simplified" because they do not describe correctly the ray structure in the ionosphere, although they are valid for the regions below the ionosphere, and properly represent a number of propagation parameters, such as ground distance, radiation angle, and group path.

The most obvious of these methods is the treatment of the ionosphere as a discrete, horizontal, mirror reflector, usually at a fixed height. A single mirror height can give results valid over a modest range of vertical radiation angles. The use of such methods can be aided by simple nomographic techniques, such as those provided by Laitenen and Haydon (1962). It is, of course, not necessary that the mirror be horizontal. Croft and Fenwick (1963) considered the effect of tilting such a mirror reflector, both about an axis along the direction of propagation and about an axis normal to the plane of propagation, and provided a number of charts from which values can be read. Since the apparent height of reflection varies somewhat with radiation angle (very rapid variations arise in some cases), it would be desirable to allow the height of the mirror (still assumed horizontal) to vary with radiation angle (i. e., with distance from the point of transmission).

The use of more complete ray-tracing methods allows one to compute the locus of such mirror positions, called the "reflectrix" by Lejay and Lepechnisky (1950). Croft (1967a) has provided a recent demonstration of the use of the reflectrix.

The utility of such methods for finding the ray geometry is obvious, but their value is much enhanced by the simple relation to group propagation time (or the group path length) as specified by the theorem of Breit and Tuve (1926). In a horizontally stratified ionosphere, this theorem states that the group path length along the triangular path to the apparent reflection point is equal to that along the true ray path. In a curved ionosphere the results must be corrected by a small factor evaluated approximately by Smith (1938, 1939).

When the calculations are to be based on ionogram data, the use of standard transmission curves, which can be placed as overlays on the ionograms, provides a very convenient method for obtaining the relationships among frequency, vertical radiation angle, propagation distance, and group path length. Such transmission curves, originally developed by Smith, are correct in principle only for a specific ionospheric layer, but the present standard curves have been adjusted empirically to yield useful results over a wide range of conditions.

Another simple procedure for using ionogram data is through application of the well known theorem of Martyn (1935), in which oblique-incidence delay time and absorption are obtained from vertical-incidence values at a transformed frequency.

4. GENERAL RAY-TRACING METHODS

This section provides a brief description of a number of the basic ray-tracing methods available. Since many of these methods are discussed in general references on ionospheric propagation such as Budden (1961) and Kelso (1964), and since some are also treated in considerable detail in other documentation, the descriptions will be kept relatively brief, but are assembled here to provide the reader with a unified summary of a number of techniques. Methods applicable when the earth's magnetic field can be neglected (isotropic media) are presented first, and then those extensions that permit the inclusion of the magnetic field are discussed.

4.1. METHODS WITHOUT MAGNETIC FIELD

In the absence of a magnetic field, the ray and the wave normal are parallel; in the presence of a magnetic field this parallelism is no longer generally true. Basically then, the principal restriction on the present discussion is that the methods described here do not make provisions for non-coincidence of the ray and the wave normal directions. Another difference that usually applies is the simpler form for the refractive index in the absence of the magnetic field; however, there are occasions where it is useful to calculate propagation parameters using a refractive index which includes the magnetic field effects, but with the ray direction established by a method appropriate to an isotropic medium. An example of such a case arises in some calculations of Faraday rotation, mentioned in a later section.

A further subdivision of the methods is based upon whether or not the method is restricted to spherically symmetric ionospheres. The simpler, spherically symmetric cases will be considered first.

4.1.1. Use of the Radius of Curvature

We discuss first a technique that does not actually provide ray tracings, but which yields a very convenient qualitative description of the overall ray characteristics. Following the general lines of a derivation given by Born and Wolf (1965), it can be shown that the radius of curvature ρ of a ray in a general (within the limits of ray theory) isotropic ionosphere can be written as

$$\frac{1}{\rho} = \frac{1}{\mu}\left|\vec{\nabla}\mu - \vec{t}\,\frac{d\mu}{ds}\right| \tag{4.1}$$

where μ is the refractive index, $\vec{t}$ is the unit tangent vector to the ray at the point of interest, and s is the arc length of the ray. At reflection, the ray must be tangent to a surface μ = constant; thus $d\mu/ds = 0$. Then

$$\frac{1}{\rho} = \frac{1}{\mu}\left|\vec{\nabla}\mu\right| \quad \text{(at reflection)} \tag{4.2}$$

For the case of reflection in a spherically symmetric medium where μ is a function of r (the radial distance) only, Eq. (4.2) becomes still simpler, resulting in an expression given by Bremmer (1949). First, we note that $\left|\vec{\nabla}\mu\right|$ becomes $d\mu/dr$ in a spherically stratified medium. Applying the well known Bouguer's rule, which is the expression of Snell's law in a spherically stratified medium, i.e.,

$$\mu r \sin\Theta = \text{constant} \tag{4.3}$$

where Θ is the angle between the ray direction and the local vertical, the condition at "reflection" is that $\sin\Theta = 1$, and we have μr = constant at reflection. Then at a position where the ray is reflected either upward, or downward, we have μr = constant, and Eq. (4.2) can be written

$$\frac{1}{\rho} = \frac{1}{\mu r}\left[\mu - \frac{d(\mu r)}{dr}\right] \tag{4.4}$$

Thus, at the point of reflection we have the conditions listed in Table 4-1.

Table 4-1. Reflection conditions.

Condition	Implication
$\frac{d}{dr}(\mu r) < 0$	Downward reflection ($\rho < r$)
$\frac{d}{dr}(\mu r) = 0$	Concentric circle ($\rho = r$)
$0 < \frac{d}{dr}(\mu r) < \mu$	Upward reflection (ray concave downward) ($\rho > r$)
$\frac{d}{dr}(\mu r) = \mu$	Straight ray (or inflection point) ($\rho = \infty$)
$\mu < \frac{d}{dr}(\mu r)$	Upward reflection (Ray concave upward) ($0 \leq \rho \leq \infty$)

At first glance, it might seem that the condition for reflection downward is simply that the rays must have a downward curvature. However, further consideration will indicate that a downward curvature with a radius of curvature greater than the radial distance r corresponds to a ray whose height above the earth increases as the ground position moves away from the point vertically below reflection; thus, this case actually corresponds to an upward reflection.

If the quantity μr is plotted as a function of r as shown in Figure 2, and the vertical line corresponding to $\mu r \sin \theta_0 =$ constant is superimposed, it is clear from Table 4-1 that a ray reaching the points G' and E' would be reflected upward, and a ray reaching F' would be reflected downward. Thus, the figure shows that a ray launched horizontally at E' curves upward until it is reflected at a height F', whereupon it descends, to be reflected upward again at E'. Thus, if a mechanism for launching this ray exists, the ray would be trapped, and continue to propagate until horizontal variations of the ionosphere or absorption mechanisms removed it.

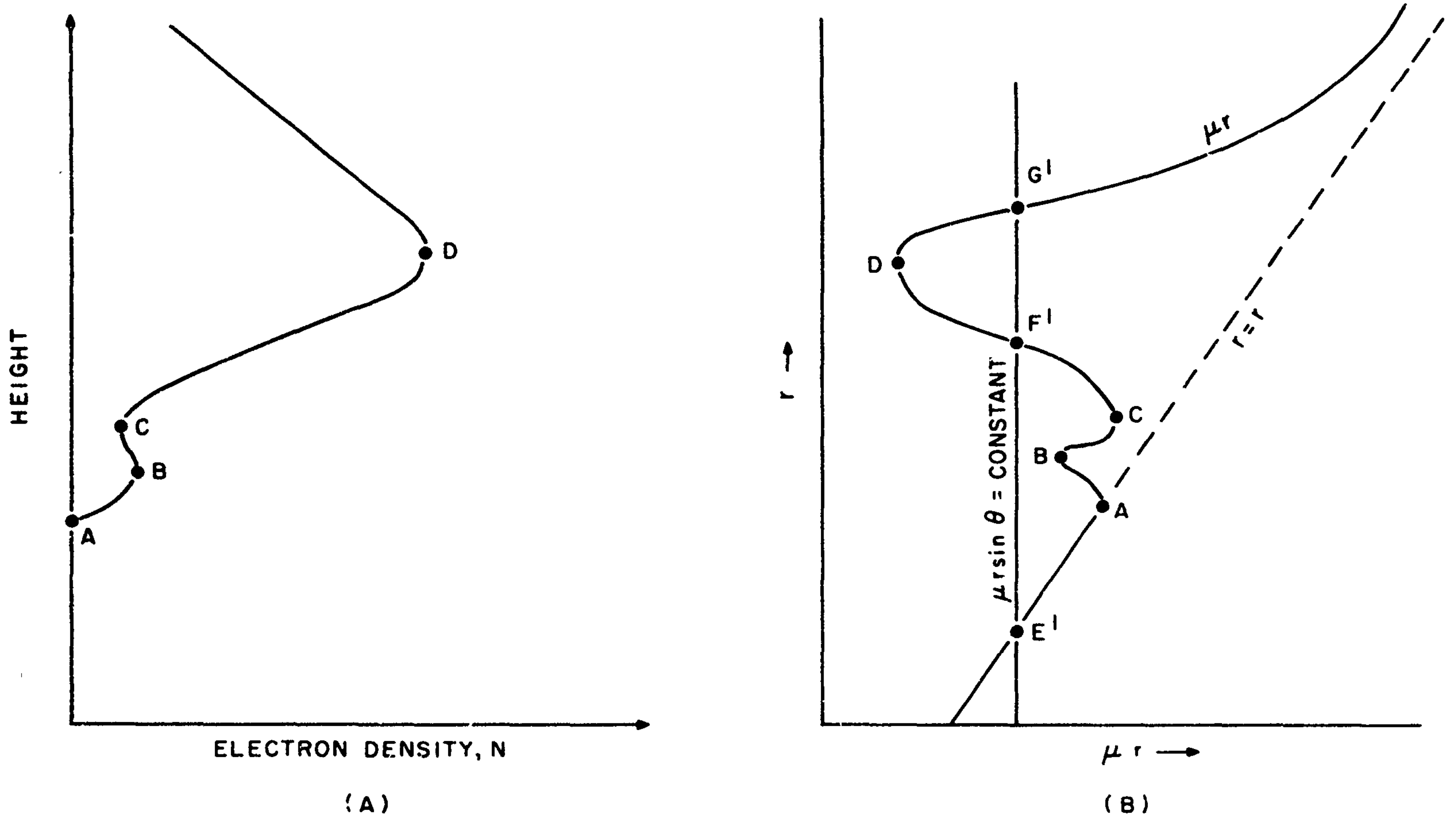

Figure 2. (A) Electron density as a function of height for a two-layer ionosphere; (B) corresponding plot of μr as a function of height.

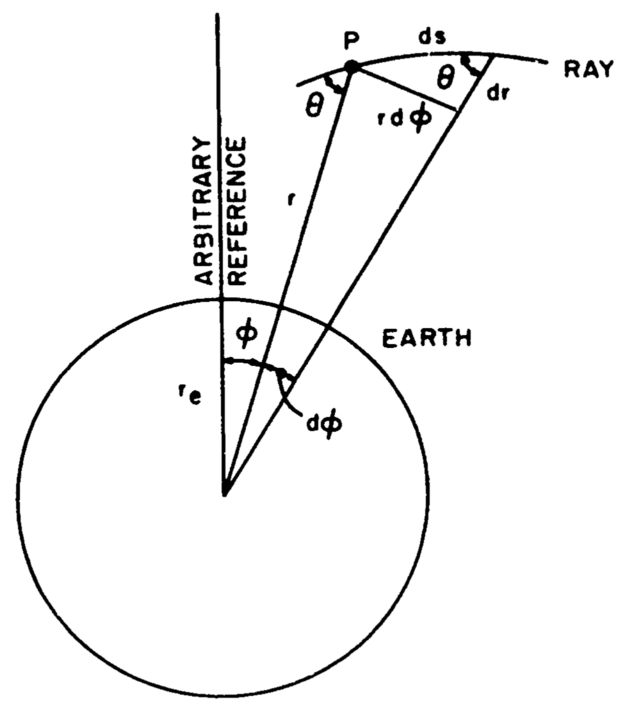

Figure 3. Sketch defining coordinates for ray calculations in a spherically stratified ionosphere.

If the vertical line $\mu r \sin \Theta$ = constant is moved left toward the vertical line $\mu r = 0$, it is seen that the point E' descends in height until E' drops below the surface of the earth. This would correspond to a mode which is transmitted from the earth, propagates upward to some height h, is reflected downward, and then strikes the earth. If the earth's surface did not reflect the ray, it would continue until it was tangent to a sphere which passed through the point E'.

By pursuing such considerations, one can obtain general estimates of the type of propagation modes which can exist in a spherically stratified ionosphere having a specified variation of μ with radial distance r. Such considerations have been pursued by Woyk (1959).

Results equivalent to those in Table 4-1 have been presented in an interesting graphical form by Croft (1967a).

4.1.2. Analytic Solutions (Exact and Approximate)

The conditions for a ray obeying Snell's law (or the equivalent relation of Bouguer's rule) in a spherically stratified ionosphere are readily obtained from the geometry shown in Figure 3. Here, at the point P, the ray makes an angle Θ from the vertical. In polar coordinates (since the ray is confined to a plane, it is possible to replace spherical coordinates by a polar system), it is clear from the figure that

$$\tan \Theta = \frac{r d\Phi}{dr}$$

Then, solving this differential equation for Φ, and introducing Bouguer's rule $\mu r \sin \Theta = \mu_0 r_0 \sin \Theta_0$, where the subscript "o" denotes an initial condition, we may write for the angular displacement Φ of the ray from its initial position

$$\phi = r_o \sin \theta_o \int_{r_o}^{r} \frac{dr}{\sqrt{\mu^2 r^4 - r^2 r_o^2 \sin^2 \theta_o}} \tag{4.5}$$

Corresponding equations can readily be written for the phase path, $P = \int \mu ds$, and the group path $= P' = \int (1/\mu) ds$ (for the special case of zero magnetic field), as indicated by Budden (1961) and Kelso (1964).

An analytic solution for ray tracing in a spherically symmetric ionosphere is obtained whenever the radial variation of refractive index $\mu(r)$ permits the integration of Eq. (4.5).

For vertical-incidence propagation, analytical results can be obtained for the phase path and group path using a number of simple forms of ionospheric height variation, including a linear layer, parabolic layer (concave upward or downward), cosine layer, and other simple forms. Corresponding oblique-incidence results can be obtained directly from the theorem of Breit and Tuve.

Similarly, the ray can be traced through these simple layers when the layer is horizontally (plane) stratified. Since there is little present need for the restriction to horizontal stratification, specific reference lists are omitted, but the interested reader can refer to Budden or Kelso. The remaining discussions of this section will be based on a curved earth and ionosphere.

An approximate solution of Eq. (4.5) was provided by Forsterling and Lassen (1933) for a parabolic variation of electron density with height; the approximation was obtained by noting that the quantity r/r_e, where r = radius of the earth, differs only slightly from unity. The electron density is given by

$$N = N_m \left[1 - \frac{(r-r_m)^2}{y_m^2} \right] \qquad (4.6)$$

where N is the electron density, N_m is the maximum value of N, r_m is the height at which $N = N_m$, and y_m is the semithickness of the layer.

More recently, Muldrew (1959) has defined a layer, which has very nearly a linear variation of electron density with height, in a form permitting an exact evaluation of Eq. (4.5). Similarly, de Voogt (1953) defined a layer having a very nearly parabolic variation with height that is similarly subject to exact evaluation. This layer defined by de Voogt is the subject of a separate document by Croft and Hoogasian (1968).

The parabolic layer and the very similar layer defined by de Voogt are of special importance because they clearly provide at least a zero-order fit to the overall ionospheric structure, having the particularly important property of exhibiting a maximum of electron density. In particular, the simple Chapman layer [Chapman (1931)] can be very well approximated by a parabolic layer in the region of the maximum electron density. For many years, the parabolic layer was the most convenient analytical representation of the ionosphere. Many basic properties of ionospheric propagation were investigated through calculations based on the parabolic layer.

Using a high-speed digital computer, it is about as easy to use the exact expressions corresponding to the quasi-linear layer of Muldrew or the quasi-parabolic layer of de Voogt as it is the approximate expressions for the parabolic layer. Thus, fairly general ray-tracing methods could be based on such layers. In the main, however, the most important present use of layers leading to analytical solutions is probably as a test of approximate, but more flexible, methods of ray tracing. Such use is made by Inston and Curtis (1967). A second important use for exact solutions occurs when it is desired to study the effect of very small changes in the parameters, for example, in computing the doppler shifts of a moving target by the difference in phase path at successive vehicle positions. The outputs of some approximate methods have a basic fine-structure which would not permit a more-or-less "microscopic" inspection of variations. By this, it is meant that some processes may show small discontinuities in output parameters (such as ground range) when the input quantities (frequency, vertical radiation angle) are varied slightly; in other cases, the effects may be continuous, but not monotonic. This makes it difficult to study effects involving differences (such as phase differences to obtain doppler shifts) or differentiation (say, the derivative of range as a function of vertical radiation angle, for use in computing focusing). In such problems, analytic solutions can be very valuable.

4.1.3. Numerical Quadrature

Another approach to the ray tracing is to use numerical quadrature techniques to evaluate the integral in Eq. (4.1). An important application of this technique was made by Jaeger (1948), who provided numerical results appropriate to a spherically stratified Chapman layer.

In general, the direct evaluation of Eq. (4.1), and the corresponding integrals for phase path and group path, would be an obvious step, except for the difficulty that the integrand of Eq. (4.1) becomes infinite at the point of reflection, when the denominator vanishes. As far as the present author is aware, no general application of such numerical integration has yet been made for ionospheric propagation, but Blake (1968) describes such a process for application to radar propagation in the troposphere. However, for the tropospheric case considered by Blake, ray reflection is not involved and the integrand becomes singular only in one special case.

In the ionospheric case, the singularity in the integrand can be removed by the following pair of transformations

$$q = \frac{r_o}{r} \sin\Theta_o$$
$$u = \sin^{-1}\left[\frac{KN(q)}{f^2} + q^2\right]^{1/2} \tag{4.6}$$

$N = N(q)$ is the electron density, f is the frequency, and K is a constant such that the refractive index is given by $\mu = \sqrt{1-(KN/f^2)}$. Then Eq. (4.4) becomes

$$= -2 \int_{\Theta_o}^{u} \frac{\sin u \, du}{\left[\frac{K}{f^2} \frac{dN}{dq} + 2q\right]} \tag{4.7}$$

Some very preliminary tests involving Eq. (4.7) have been made for a quasi-parabolic ionosphere of the type described earlier. The ground ranges computed in this way agree with the exact analytical results to within a small fraction of a kilometer. It appears, therefore, that numerical integration of Eq. (4.7) can form the basis for a convenient ray-tracing method applicable to spherically stratified ionospheres of rather arbitrary vertical structure.

4.1.4. Ray Tracing Through Homogeneous Shells

An obvious technique for computing the propagation of a ray in the ionosphere is to trace the ray through a succession of homogenous shells in each of which the ray travels along a straight line. It is common to set up the relations for ray propagation on the basis of finite strata before passing to the limit of vanishing thickness [for example, see Smart (1931)]. It is clear that at least a rough approximation to the ray tracing can be obtained by retaining the shells, and not passing to the continuous limit. However, formerly, when numerical results could be obtained only at the expense of tedious arithmetic on a desk calculator, such procedures could be used only in relatively rare cases where the importance of obtaining the results outweighed the difficulty of the computations.

The availability of high-speed digital computers has changed the situation dramatically, since such computers are well suited to making repetitive calculations through a large number of such homogenous strata. Such a method has been applied for some time to tropospheric problems [see Millman (1959)].

Probably the most extensive use of these calculation procedures for ionospheric problems has been made by Croft, who developed several basic methods for computing ray propagation in homogeneous shells. The first, and most obvious method, uses Snell's law (or the equivalent Bouguer's rule) to compute the passage of a ray through a succession of a large number (from dozens to hundreds) or prespecified, homogeneous shells. Using this method, it is relatively easy to calculate the ray path, the phase path, the group path, and the absorption in an ionosphere that varies with height in a rather arbitrary manner. Croft has not yet published these procedures in open literature, but the earlier stages of the work were described in a report, Croft and Gregory (1963).

One difficulty with this process is that the pre-specifications of the shell thicknesses means that there is no convenient way to differentiate between the rather coarse steps actually needed when the ray climbs steeply, and the very thin steps required in the region of ray apogee (reflection) where the ray makes a long horizontal excursion. There are at least two basically different methods for making better allowances for these two ray conditions. One, also due to Croft (private communication), defines the overall vertical variations of the layer in some general form such as interpolation in a table of stored values. In the tracing operation, the thickness of an individual homogeneous shell is defined by projecting the ray forward by some pre-specified distance in the direction specified by Snell's law. At the end of this distance, a new shell boundary is defined, and Snell's law is re-applied.

An alternative procedure currently under investigation by Kolesar (private communication) achieves a similar objective by establishing the thickness of the next shell as a function of the ray angle from the vertical. The thicknesses of the shells are reduced as the ray approaches a horizontal orientation. In both Croft's and Kolesar's methods, the ray is carried through the lower ionosphere, where the refraction effects are slight, in large vertical steps, but reduce the vertical steps in the reflection region where the rays approach a horizontal position.

The condition for reflection requires special treatment. The simplest approach would be to follow the passage of the ray through successive shells, until the ray reaches an interface where Snell's law cannot offer a real angle of refraction. Then, a mirror reflection at the interface could be used. However, the height of reflection would remain constant for small ranges of frequency or radiation angle, and then change discontinuously. Similar discontinuities would appear in other ray-tracing parameters, such as ground range and group path. Therefore, unless the shells are very thin, this simple procedure will not give satisfactory results.

Another possibility is to replace the last shell which the ray can enter by a continuously varying layer, such as a linear variation.

A third, and perhaps the most satisfactory, solution is to use a semi-empirical relation that assigns a new upper boundary for the last shell at some height depending on the amount by which the ray angle at incidence on the interface just passed would have to be changed, first to permit its entry into the next higher shell, and second to prevent its entry into the shell it presently occupies. The new shell boundary can then be set at a height proportional to the position of the ray angle between the two extreme angles just noted, to permit mirror reflection at the new boundary.

Either of these last two procedures, or a number of variants of them, will yield smooth variations of the ray-tracing parameters with angle-of-incidence or frequency.

The shell procedure provides a method for introducing the effects of horizontal variations. A particular shell must be treated as homogeneous (to permit a straight ray segment) only in the region containing the ray segment. In another region of the same shell, a different value of refractive index can be assigned. This procedure does not, of itself, properly represent the effects of horizontal variations, since one can see that Bouger's rule applies continuously within a narrow region enclosing a particular ray. Then, at the earth's surface, where $\mu = 1$ and $r = r_e$ (earth radius), the vertical angle for the downcoming ray must be exactly equal to that for the upgoing ray.

Kolesar (private communication) is currently investigating a more general variation in which Snell's law is applied across a boundary tilted along the surfaces of constant electron density. In its present stage of development, the method provides useful results, but it is not yet clear how its accuracy compares with other methods using equivalent amounts of computation time.

4.1.5. Analog Computation

The relative ease with which ray propagation in an isotropic medium can be expressed as a set of differential equations makes the use of analog computers attractive for certain purposes not requiring precise results. An extensive investigation of such possibilities for tropospheric propagation was carried out by Wong [for example, see Wong (1958, 1966)]. Also, the paper by Inston and Curtis (1968) discusses computation on an analog computer as a means for obtaining very rapidly an approximate representation of the ray structure which aids in the application of more accurate, but slower, digital computations.

Generally speaking, however, the greater flexibility and accuracy obtainable by digital methods would appear to limit the use of analog methods.

4.1.6. Numerical Integration of Differential Equations

There are a number of ways in which the mathematics of ray tracing can be put into the form of differential equations, which can be solved numerically. The paper by Inston and Curtis (1968) describes one such procedure. Another method may be derived from the very general equations of Haselgrove (1955), but the generality of those relations places it more properly in the discussion of methods which include the earth's magnetic field.

Typically, methods based on numerical integration of differential equations make it relatively easy, in principle at least, to permit rather arbitrary variations of the medium properties in two or even three dimensions. Two problem areas, that of supplying the input data and that of finding an efficient manner of displaying the outputs are discussed in a later section. Additional problems arise when the electron density contains discontinuous derivatives, as discussed in a later section.

4.2. METHODS WITH MAGNETIC FIELD

4.2.1. Various Early Developments

As noted earlier, the inclusion of the earth's magnetic field means that the medium is anisotropic, and implies that the ray direction does not generally coincide with the wave-normal direction. Since the refractive index depends upon the direction of the wave normal, which in turn is not fully known without knowledge of the previous history of the ray, it is obvious that the presence of the magnetic field poses serious problems. Budden and Daniell (1965) presented an interesting procedure for finding the refractive index, given the ray direction, but the method is difficult to use.

Early methods for including the effects of the magnetic field were developed by Booker (1936, 1938, 1949) and Millington (1951, 1954) but since these methods could be applied, at the time they were first presented, only to special cases and with rather exhausting effort, they are not treated further here. However, this disregard of them should not be taken as a casual dismissal, since they were heroic efforts for their time.

The next method to be noted is that of Poeverlein (1948, 1949, and 1950), which again is not recommended for current use, but which supplied some interesting knowledge of special effects. In a plane-stratified ionosphere, Poeverlein made use of the relation that $\mu \sin \theta = \text{constant}$ (Snell's law in such a medium) to provide a graphical technique for following the course of a ray. Using this method, he displayed the rather interesting horizontal deviations of the ordinary and extraordinary rays in vertical propagation. He also showed that the ordinary ray in oblique propagation could exhibit a surprising cusp-like effect in the reflection region.

We next wish to pass to some methods more suited for present day use with available digital computation facilities.

4.2.2. Spherical Strata

Lawrence and Posakony (1961) prepared a ray-tracing program (including the magnetic field) for use in propagation measurements involving earth satellites. This method, a spherical shell procedure, used an iterative technique to eliminate the difficulty caused by the dependence of the refractive index on the previous ray history. The method, as described in the cited reference, does not make provisions for a reflected ray, but it is described so completely that it could be readily implemented by those interested in the higher-frequency regime where reflections do not occur.

4.2.3. Integration of Differential Equations

The most general method for ray tracing in the ionosphere is that presented by Haselgrove (1955, 1957), and Haselgrove and Haselgrove (1960). This method, based on Hamiltonian optics, uses a set of 6 first-order differential equations, and permits the inclusion of arbitrary spatial variations (to within the limits of geometrical optics) of the refractive electron density and geomagnetic field. Two additional differential equations permit the calculation of group path and absorption. Obviously, a variety of simplifications would permit more rapid computations of special cases, such as propagation in an isotropic medium or a spherically stratified medium.

An extensive discussion of the Haselgrove method is given by Brandstatter (1963), and Jones (1968) presents the results of a particular implementation of this technique in a recent issue of Radio Science Journal.

5. SPECIAL PROBLEMS OF RAY TRACING

There are a number of problems connected with ray tracing, not all of which are immediately obvious. These are the concern of this section.

The first such problem concerns the ionospheric data available for introduction into ray-tracing calculations. This problem is not mentioned as one which could be missed by any reasonably intelligent observer; however, the question of what to do about it does indeed merit some discussion.

First, it is clear that our knowledge of the instantaneous state of the ionosphere in any region can never be complete. Does this mean that it is never useful to work toward ray-tracing methods which are more accurate than our knowledge of the ionosphere? This author must take the position that such is not the case. There are many instances for which useful results can be obtained from this fine-structure of the ray-tracing results, while the overall characteristics are no better than the ionospheric inputs. Examples of this were given earlier in this report.

A further point might be made that the ionospheric uncertainties are large enough so that it would be a pity if the mathematical errors in ray tracing added to the limits imposed by our ignorance of the physical situation.

It must be recognized, however, that the availability of ray-tracing techniques which make provisions for two-, or, even three-, dimensional ionospheric variations does indeed put a premium on the current capability to define a reasonable ionospheric input to the ray-tracing process.

One difficulty arises in some of the frequently used methods for defining the vertical variations of ionospheric electron density using "true height analysis" techniques. Some such processes define individually the electron densities at a series of heights. Without some further provision for smoothing, it is possible that the use of the results from such true-height analyses could lead to the input of an ionosphere whose vertical gradient of electron density changes discontinuously. That such changes in gradient produce anomalous variations of virtual reflection height is rather well known (see Kelso (1954) for an illustration of the effect of a simple inflection of the profile). The theorems of Breit and Tuve (1926) and Martyn (1935) also show that such effects in the virtual heights of reflection can readily produce irregularities in oblique ray tracing.

One method for deriving the vertical electron-density profile from ionosonde records, developed by Titheridge (1961), takes special pains to produce a continuous representation of the vertical profile of electron density with height, and, in fact, manages to preserve the continuities of the vertical derivative of the electron density. For the purposes of accurate ray tracing, the ionospheric inputs should be derived from a method which preserves such important properties.

In fact, the continuity of the vertical and horizontal derivatives of electron density are of especial importance in methods based on numerical integration of differential equations, since those methods usually require the introduction of horizontal and vertical derivatives of refractive index. If such derivatives are not continuous, the entire process of ray tracing is subject to abrupt changes as the integration passes a region where the derivative changes suddenly.

The problems of disposing of the output are nearly as vexing. A digital computer can generate a large quantity of data in the course of tracing a single ray. What are the best forms in which to store and present the data? An obvious step is to plot the ray trajectories, perhaps with additional marks indicating the way in which some parameter (e.g., the group delay) varies along the ray; this approach is illustrated in Figure 1. The use of the reflectrix can store a great quantity of the information, as was shown by Croft (1967a). Since the purposes of various ray-tracing tasks are different, it is clear that several convenient ways of presentation and storage are needed. It is to be hoped that they will be forthcoming before the world is buried under computer printouts.

6. APPLICATIONS OF RAY TRACING

For those who have already expended substantial efforts on the use of ray tracing, the value of such efforts must surely be clear. However, those who have not had occasion to apply such techniques might reasonably wonder what they can gain by doing so. A number of examples are included in this report, and a few additional comments may be in order.

Among the problems particularly amenable to solution by ray-tracing techniques are those involving transmissions between the ground and an earth satellite. This particular utility is grounded in two basic facts; first, the frequencies involved in such transmissions are usually sufficiently high so that there is no doubt as to the validity of ray-tracing methods; second, the accuracy necessary to utilize the resulting measurements is often sufficient to require a detailed ray-tracing treatment. Among such measurements are those related to the determination of ionospheric properties, based on the effects of Faraday rotation or doppler shift. The literature concerning such observations is very large, but the interested reader may gain access to it by consulting Mass (1963).

The processes of ray tracing are of great utility in a variety of other problems of radio propagation. The computation of the spatial distribution of field strength from a given transmitter and ionospheric conditions is an obvious candidate for such calculations. Several techniques for using such a method have been introduced, but a truly useful one will require a better understanding of the processes for including realistic ionosphere data. However, in a number of special cases, the introduction of ray-tracing methods can be of considerable assistance in signal intensity calculations. An effect directly amenable to solutions based on ray tracing is the change in signal intensity caused by focusing or de-focusing. These effects are readily determined by computing the area over which the energy in a transmitted solid angle is spread upon reception at a given position. Calculations of this type for a simple ionosphere were carried out analytically more than thirty years ago by Forsterling and Lassen (1933); a similar approach was later pursued extensively by Rawer (1948, 1952). A recent paper, Croft (1967a), applies computer ray tracing to demonstrate the focusing effects produced by the ionization in the region between ionospheric layers.

The interpretation of oblique, swept-frequency ionograms may be aided greatly by using ray-tracing methods to transform vertical ionograms into the corresponding oblique form. Such a process is not new, of course; Appleton and Beynon (1940) discussed a variety of methods for computing the oblique ionogram, including the use of Martyn's theorem or Smith's transmission curves (both discussed earlier in this report) and analytic calculations (ray tracing) based on a parabolic layer whose parameters were determined from the vertical ionogram. The use of computer ray tracing permits the rapid processing of large quantities of such information, and can provide greater accuracy, at least in the sense of establishing the differences between modes, if not in the absolute values assigned to either. Kopka and Moller (1968) show the extension of such considerations to the case of ionospheres with horizontal variations.

The problem of computing the amplitude of ground backscatter resulting from transmissions from a pulse transmitter operating in the HF spectrum are considered in the paper by Surtees (1968); such processes have been treated previously by Shearman (1956), Phillips (1960), Croft (1967b), and others.

7. ACKNOWLEDGEMENTS

The author is much indebted to many members of the ITT Electro-Physics Laboratories, Inc. for their contributions to the information presented in this report. In particular he must acknowledge the contributions made by J. D. Kolesar, who has been responsible for a number of ray-tracing developments under the author's guidance, and who was of considerable assistance in the preparation of this report, including the tests of the numerical-integration ray-tracing procedure described in the text.

The preparation of this report was supported by the Office of Naval Research under Contract Nonr-4204(00).

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UNCLASSIFIED

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author) ITT Electro-Physics Laboratories, Inc. 3355 52nd Avenue Hyattsville, Maryland 20781	2a REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b GROUP 3

3 REPORT TITLE

RAY TRACING IN THE IONOSPHERE

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

External Technical Memorandum No. 74

5 AUTHOR(S) (Last name, first name, initial)

Kelso, John M.

6. REPORT DATE	7a TOTAL NO. OF PAGES	7b NO OF REFS
August 1967	46	63

8a CONTRACT OR GRANT NO. 4204(00) b. PROJECT NO NR 088-024 c d	9a ORIGINATOR'S REPORT NUMBER(S) ETM No. 74 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) None

10 AVAILABILITY/LIMITATION NOTICES

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13 ABSTRACT

The problems of tracing rays in the ionosphere for radio propagation, primarily for high frequencies (3 to 30 Mc/s) and above, are discussed. The limitations on "classical" geometrical optics are described, and their applications to ionospheric problems are indicated.

Some "simplified" processes of ray tracing, which apply well at the earth's surface, but not in the ionosphere itself, are first presented. Next, ray-tracing methods which apply in an isotropic ionosphere, when the effects of the earth's magnetic field are negligible, are discussed. The results are then extended to more general cases of an isotropic medium, and then to an anisotropic medium (such as the ionosphere in the presence of the earth's magnetic field).

Certain special problems which must be considered in ray-tracing studies are described next and finally, a few of the important areas for the application of ray tracing are noted.

This report has been accepted for publication in the journal RADIO SCIENCE.

DD FORM 1 JAN 64 1473

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14 KEY WORDS	LINK A		LINK B		LINK C	
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There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

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