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OPTICAL PROCESSOR FOR SYNTHETIC SPECTRUM DATA

FINAL REPORT
CONTRACT AF30(602)4287
ARPA ORDER 837

T. R. Hughes
C. A. McGrew
B. S. Smith
et al

May, 1967

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Air Force Systems Command
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OPTICAL PROCESSOR

FOR

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May, 1967

MDE 2142

FOREWORD

This is the Final Report on contract AF 30(602) 4287, ARPA Order 837, entitled, "Optical Processor for Synthetic Spectrum Data". The sponsoring agency for this work was the Air Force Systems Command, Research and Technology Division, Rome Air Development Center, Griffiss Air Force Base, New York 13440, and Project Defender, Advanced Research Projects Agency (ARPA), Department of Defense, Washington, D.C. 20301. Mr. Robert MacMillan (EMASS) was the RADC Project Engineer and Mr. Morris Witow was the ARPA monitor for this work. Work was performed on this contract between May, 1966, and May, 1967, at the Westinghouse Defense and Space Center, Surface Division, P. O. Box 1897, Baltimore, Maryland 21203.

Personnel who contributed to the analysis, design and documentation performed on this contract were Messrs. B. L. Drummond, H. S. Fitzhugh II, T. C. Griskey, T. R. Hughes, G. S. Ley, C. A. McGrew, L. A. Nix, M. W. Partin, B. S. Smith, J. Szostak, and J. E. Thompson. In addition, the processing and analysis of recorded data was performed by Dr. R. B. Crane and Mr. R. Fedorowicz of the Institute of Science and Technology, University of Michigan, Ann Arbor, Michigan.

ABSTRACT

A film recorder for high resolution radar echoes has been designed and fabricated. It employs a modulated cathode ray tube trace which is imaged onto a moving film. Individual pulse echoes are written across one dimension of the film, corresponding to the direction of the CRT sweep. Successive echoes are accumulated side-by-side in the orthogonal direction. The data is recorded in a format suitable for subsequent processing in a coherent optical system. The purpose is to obtain doppler resolution by integration over a sequence of pulses. This is to be carried out simultaneously for each range resolution element, giving an output with two dimensions of resolution.

The specifications, design, fabrication, and check-out of the film recording equipment are described in this report. Simulated data has been employed to verify the overall performance of the device. A review is given of the data frames recorded and the test results obtained from them. The system can record data in two different formats, to allow some flexibility in implementing the coherent imaging processor.

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SECTION I

REPORT SUMMARY

1. INTRODUCTION

This is the final report on contract AF 30 (602) 4287, ARPA Order 837. The purpose of this contract was to determine the feasibility of optically processing synthetic spectrum radar data to yield a high resolution range/doppler map. On this contract, computer software was written to utilize the Univac 1108 and the Scientific Data Systems, Inc, SDS-910 digital computers to format the radar data for subsequent film recording. Digital to analog conversion equipment and a timing oscillator for system synchronization were bought from SDS for use with the SDS-910 computer. A film recorder was designed and built by Westinghouse which takes video information and synchronization signals from the SDS equipment and displays these on the face of a high resolution cathode ray tube. A special camera was fabricated to Westinghouse specifications by Photogrammetry, Inc. Rockville, Maryland. This camera contains a precision film drive motor and associated control electronic purchased from Sequential Electronics Systems, Inc., New York.

2. EQUIPMENT DESCRIPTION

A photograph of the film recorder is shown in Figure 1, in which the various sub-systems are identified. The digital-to-analog conversion equipment and the timing oscillators for system synchronization are located in the SDS-910 computer chassis and are not shown in Figure 1.

The equipment utilized in performing optical processing of synthetic spectrum data is shown in Figure 2. A brief description of the functions performed in each sub-system of the equipment will be given here. Detailed descriptions of the functions will be found in the body of this report.

The UNIVAC 1108 digital computer is used to perform several operations on the radar data and to assemble it into a format compatible for the SDS-910 computer. The radar data is calibrated, the responses are aligned, pre-compensation for film recorder non-linearities can be applied and, finally, the proper format is selected. Additional operations on the data required for processing in data format II (described in section I-3) are also performed in the Univac 1108 computer.

The SDS-910 digital computer is used to read, store and output the data in synchronism with the sweep of the cathode ray tube (CRT) beam. The DIGITAL-TO-ANALOG (D/A) CONVERTER, housed in the SDS-910 computer, accepts the 9-bit digital output and converts it to an analog voltage which is the video signal to the CRT. The TIMING UNIT, also housed in the SDS-910 Computer, generates the trigger signals and control frequencies used to synchronize system operation. A pulse of data is read into the SDS-910 core, stored there for a short time, and then read out and written onto film, prior to the next pulse being used.

The FILM RECORDER consists of four subsystems, the CRT, the CRT CONTROL ELECTRONICS, the CAMERA and the FILM DRIVE UNIT with its control electronics. Timing and control signals and video informations signals are supplied to the FILM RECORDER. The film is moved continuously at a precisely controlled rate and the CRT beam is swept repetitively across the same trace on the CRT face. Once the information is recorded on film, the film (after development) is ready for processing in an optical correlator. On this contract, the film was processed by the Institute of Science and Technology (IST) of the University of Michigan.

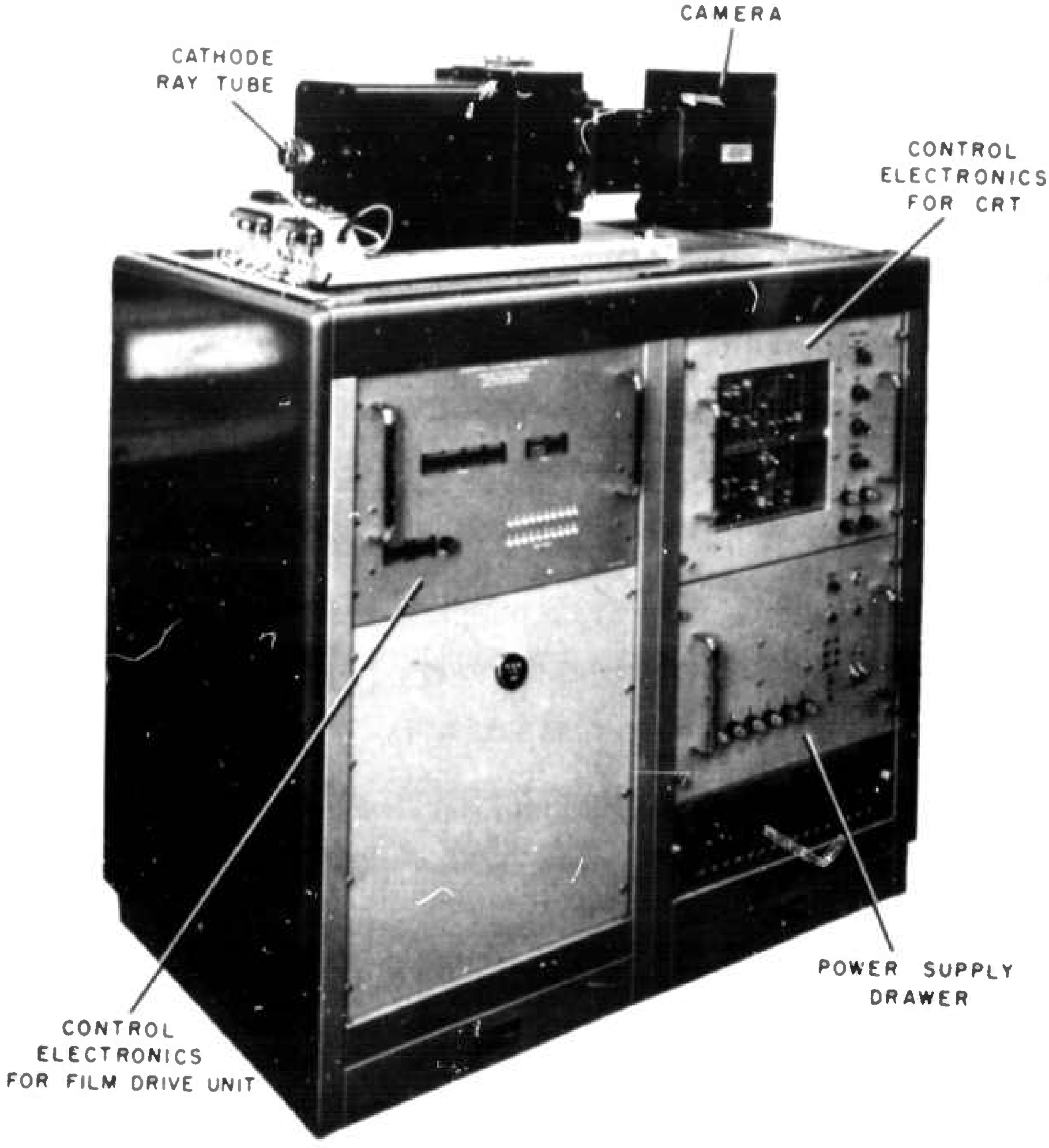


Figure 1. Film Recorder for Synthetic Spectrum Data

3. BRIEF DESCRIPTION OF FORMATS

The data film transparency employed with an optical processor (using coherent light) must form a diffraction pattern. Particular fringes of this pattern are isolated by spatial filtering and further transformed by lens elements, to obtain a processed output. The recording format is determined largely by the requirement of forming an interference fringe which is modulated by the radar echo data.

To give the required fringes, a "spatial carrier" is introduced into the recorded data. This appears as an underlying sinusoidal oscillation of the film transmissivity. It acts as a diffraction grating when the film in placed in a coherent optical correlator. The presence of the spatial carrier also allows a radar signal having both amplitude and phase dependence to be recorded. The amplitude is used to control the magnitude of the spatial carrier oscillations (e.g. the envelope of the waveform). The phase is used to shift the position of the carrier oscillations by a fraction of a cycle (with a full cycle representing 360° of phase). This is the natural meaning of a phase difference, namely the shift of an oscillatory waveform with respect to a standard oscillation of the same frequency. In the case of the radar data film, the standard (phase reference) signal would be a uniform sinusoidal diffraction grating with a spacing constant equal to one cycle of the carrier on the film.

In addition to the carrier, a bias light level is used when exposing the film in the recorder. This bias is set to an operating point on the linear region of the film response characteristic. It allows the carrier oscillations to appear as fluctuations in transmissivity about the bias level. There is no need for negative values of transmissivity, which are not, in fact, achievable in practice.

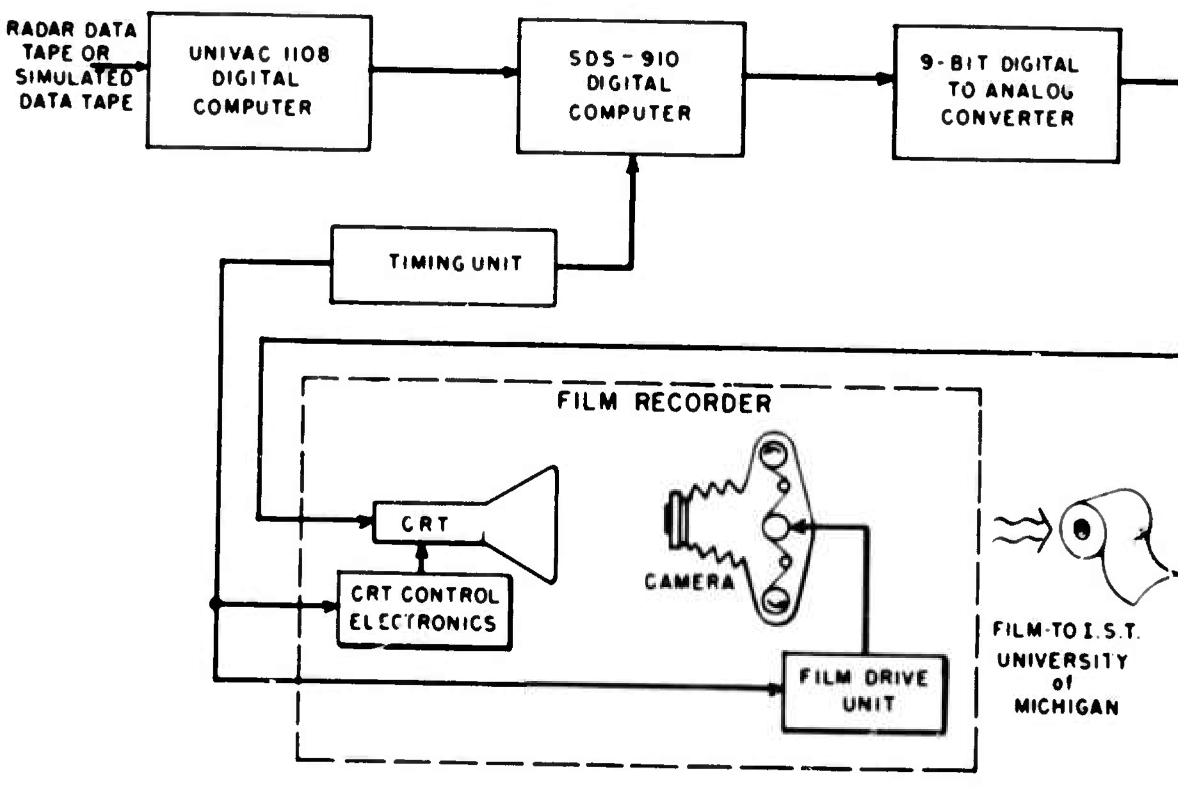


Figure 2. Optical Processor for Synthetic Spectrum Data

The recorder can produce data in either of two formats. In both, each line on the film (corresponding to a CRT sweep) represents the echo from one radar pulse. The difference lies in the nature of the data written along each sweep. In Format I, the outputs of range channels in the radar directly form the modulation on the sweep. That is, distance along the sweep is proportional to range across the tracking window of the radar. In Format II, the fourier Format I data) is recorded instead. Here, distance along the sweep is proportional a frequency location across the radar signal band.

A typical film recording and processed output for two scatterers is shown in Figure 3. A sample of the recorded film (Format II) in normal size is shown in 3(a). In 3(b) a 3ox enlargement of this recorded film is shown in which the recording process is more apparent. The diagonal bands are due to the changes in spectrum due to rotation of the target, which was a dumbell configuration. In 3(c) the processed output of the optical correlator is shown in which both the zeroth order fringe (on the optical axis of the correlator) and the first order fringe are shown. The zeroth order fringe is normally removed by a spatial filter and is included only for illustration. The offset of the target image from the zero order is caused by the presence of the spatial carrier. The light from the background bias level on the film is concentrated in the bright zero order.
Additional processed results are given in Section V.

4. SUMMARY OF CAPABILITIES

The equipment has been used to record simulated data in both Formats I and II. The tests which have been made on the equipment and the film which has been processed, have determined the recorder capabilities which are listed in Table A. In two instances, the values of the parameters are significantly different from the design goals and improved data recording is possible. First, the film drive unit has operated with a control frequency of less than

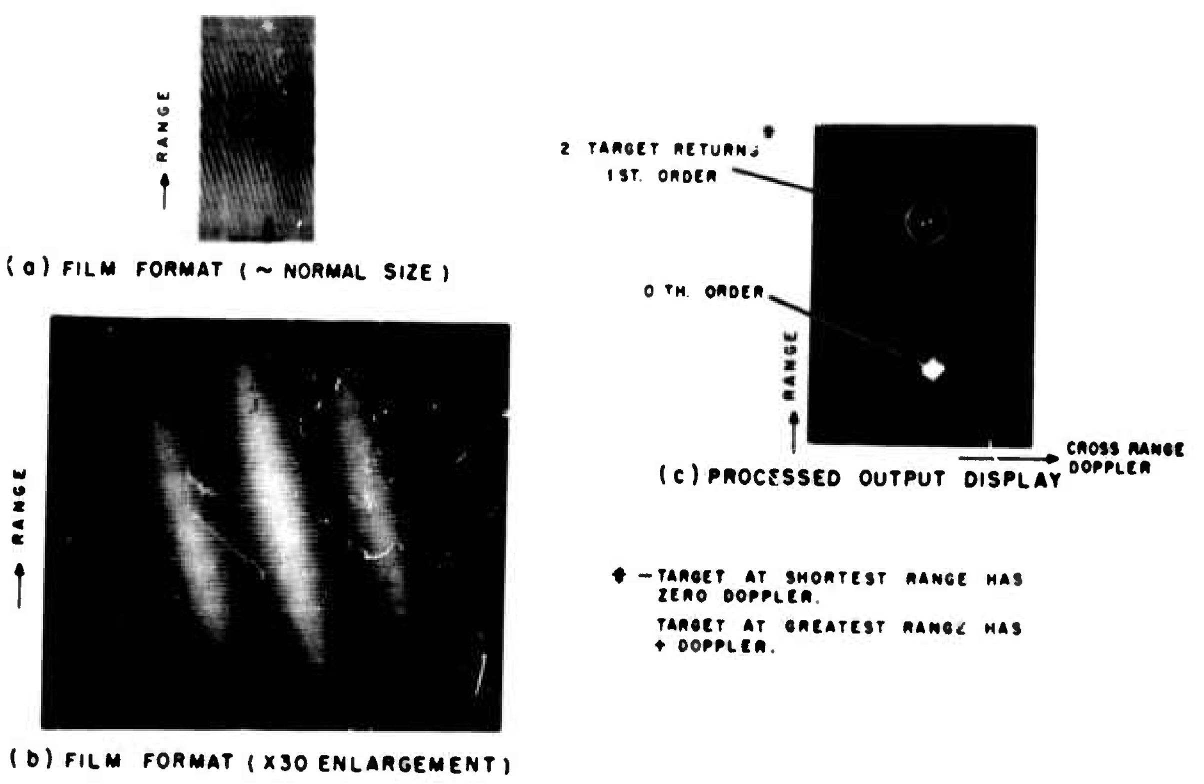


Figure 3. Typical Film Recording and Processed Output-Format II

TABLE I. SUMMARY OF EQUIPMENT CAPABILITIES

CRT spot size	0.0015 inches
Number of "spots" per line of recorded data	1024-2048 (selectable)
Number of lines (pulses) per 50 mm frame	500-1250
CRT line sweep time	27-81 msec
Spatial carrier	10-20 lpr/mm
Maximum number of pulses which can be recorded continuously (magnetic tape limited)	3550
Input control frequency for film drive	600-60,000

600 Hz where 714 Hz was the design value. By virtue of this, the maximum density of lines along the film was approximately 25 lines per millimeter rather than the design goal of 20 lines per millimeter (which permits 1250 lines per 50 mm. frame of recorded data.) To achieve this required an external signal source be used instead of the signal from the TIMING UNIT.

In all categories, except one the equipment has performed at or beyond design specifications. The one exception is in the behavior of the spatial carrier in Format II recording. In this format, excessive range sidelobes in the processed data are produced due to a variation in the recorded spatial carrier with distance along the trace.

Although the recording in Format II has corrections to be made to it in order to give best processed output, the recordings which have been made have demonstrated conclusively that optical processing of synthetic spectrum data is feasible. The results given in Section V and the conclusions drawn in Section VI elaborate on this.

5. APPLICATIONS TO SPACE OBJECT IDENTIFICATION (SOI)

To apply this device to the SOI problem, the equipment corrections to give recording in Format II should be made. Additional processing of simulated data using more complicated grometrical configurations typical of targets of interest should be performed. This, and the processing of radar data, should be done both to have data on more complex targets and to have more data from which to make observations on the quality of processed outputs (especially in having a time record available on which to base observations). These recommendations are discussed in Section VI.

SECTION II

THEORY OF OPERATION

A target, when illuminated by a microwave radar, modifies the incident signal in the backscattering process. The radar receiver may be designed to sense certain of these changes or modulations in order to give information about the target. The most commonly used parameters are echo time delay and doppler shift; these are linearly related to target range and range rate, respectively. The received signal power, proportional to target radar cross-section, is often processed as well. In observing space objects, one would like to extract much more detailed information about the target. Increasing the resolution capability of the radar can provide more target-induced data, especially when the resolution cell becomes smaller than the target extent. When the inherent resolution cell is smaller than the body, the possibility of forming a "radar image" of the configuration arises. This section discusses radar range/doppler imaging and the applicability of optical techniques in the signal processor. Suitable film recording formats and a summary of critical error types are also given.

1. CHARACTERISTICS OF HIGH RESOLUTION ECHOES

At short radar wavelengths, rigid bodies (such as satellites) exhibit so-called "scattering centers". That is, the radar echo can be associated with one or more localized regions of the target. When illuminated with a very short pulse, these scattering centers produce individual echoes with different time delays (due to their differences in range). When illuminated with a single frequency, the individual echoes superimpose vectorially since they are not resolved in time. If the target orientation changes, the relative phases of the echoes are altered and the vector sum exhibits a beat pattern. The rapidly lobing cross-section of a rocket body as it tumbles is a common example of this.

A range/doppler imaging radar makes use of both the range differences between scatterers and the relative phase changes occurring as the target changes orientation by a small amount. A wide-band transmission is employed to obtain a range resolution cell small compared to total target extent. The phase information can be gathered with a single radar site by observing over an interval of time, during which the target changes aspect. The phase information could also be gotten with a multisite system, where the separate receivers each view the object at a different angle, at the same time. The first (monostatic) type of data has been considered on this contract.

While range resolution is inherent in the single pulse transmissions, an equivalent doppler resolution is obtained only after processing a sequence of echo pulses. The spectrum (Fourier transform) of the amplitude and phase behavior of a scatterer gives its doppler resolution pattern. When a small fraction of a total target revolution period is processed, the doppler offset of a scatterer is proportional to its location along an axis perpendicular to the line-of-sight. This property aids in utilizing the radar image as a spatial map or projection of the echo centers. An analytical model for this high resolution data will now be given.

a. Form of Echo Pulses

A pulse stretch/collapse system would undoubtedly be used in a high resolution radar designed for observing space vehicles. The pulse modulation will be denoted by p (t) and the r-f center frequency by f_c. The transmitted signal will be represented by the phasor

$$s(t) = p(t) \exp (2 \pi i f_c t) \tag{1}$$

For example, if the modulation were a linear f-m ramp, the function p (t) would be

$$p(t) = \text{rect}\ \frac{t}{T}\ \exp(\pi i k t^2)\ ;\ \text{rect}\ (X) = \begin{cases} 1, & |X| \le 1/2 \\ 0, & |X| > 1/2 \end{cases}$$

The instantaneous frequency of s(t) is defined, as usual, by

$$f_i = \frac{1}{2\pi} \frac{d\left[\text{Arg } s(t)\right]}{dt} = \frac{1}{2\pi} \frac{d}{dt} (2\pi f_c t + \pi k t^2) = f_c + kt \qquad (2)$$

Although such an f-m ramp is a common modulation, we will not restrict the signal to be of this form.

A single point scatterer would produce an echo of the form (neglecting the range factor R^{-2}):

$$a e^{i\phi} s(t-\tau)$$

where
- a is the amplitude of the scatterer
- ϕ is the phase shift in scattering
- τ is the delay of the scatterer (2R/C).

When the scattering center has frequency dependence, the shape of the echo pulse is not exactly that transmitted, since the scatterer introduces some weighting of the spectrum. However, this distortion is slight for a radar pulse of less than 20% bandwidth, and will not be considered here.

An extended solid body usually exhibits several scattering centers, whose characteristics (amplitude, phase, polarization and frequency dependence) depend on the direction of the line-of-sight. The total echo, e(t), for a given orientation is a summation of single scatterer terms:

$$e(t) = \sum_{n=1}^{N} a_n e^{i\phi_n} s(t-\tau_n)$$

$$= e^{2\pi i f_c t} \sum_{n=1}^{N} a_n e^{i(\phi_n - 2\pi f_c \tau_n)} p(t-\tau_n) \qquad (3)$$

where
- a_n is the amplitude of the n^{th} scattering center
- ϕ_n is the phase shift in scattering for the n^{th} scatterer
- τ_n is the delay of the n^{th} scatterer
- N is the total number of scattering centers
- S is the transmitted signal, $S(t) = p(t)e^{2\pi i f_c t}$
- f_c is the center frequency of the pulse
- p is the (baseband) modulation of the radar pulse.

Each term in Equation (3) can usually be associated with a localized discontinuity in the target surface curvature or a section of surface normal to the line-of sight.

In the radar receiver, this echo is processed to achieve pulse collapse. For a "matched" receiver, the echo is correlated with the conjugate of the transmitted signal. Mathematically, the radar response $r(\tau)$ is given by

$$r(\tau) = \int e(t)s^*(t-\tau)\,dt \qquad (4)$$

This reduces to

$$r(\tau) = \sum_{n=1}^{N} a_n e^{i\phi_n} e^{2\pi i f_c(\tau - \tau_n)} \int p(t-\tau_n)\,p^*(t-\tau)\,dt$$

The integral in the last equation is the matched output response, $r_o(\tau - \tau_n)$, of the radar. There are N such terms, one for each scattering center. Therefore, the radar response of the extended target is

$$r(\tau) = \sum_{n=1}^{N} a_n e^{i\phi_n} e^{2\pi i f_c(\tau - \tau_n)} r_o(\tau - \tau_n) \qquad (5)$$

where r_o is the matched output response, defined by

$$r_o(\chi) \equiv \int p(t)\,p^*(t-\chi)\,dt$$

f_c is the radar center frequency

a_n, ϕ_n, τ_n, N are as defined above (Equation (3))

τ is the running delay variable. The individual response peaks occur when $\tau = \tau_n$; n=1, . . . N.

The Synthetic Spectrum radar, which is the data source for the optical recording and processing systems, measures the target response in a bank of range channels. These are spaced at equal intervals in delay (or range). A sample of the output response signal is obtained in each of these channels. This sample is a complex number in polar form; that is, the amplitude and phase are measured and recorded.

$$\text{Output: } r(\tau_m) = |r_m| e^{i \text{Arg} r_m}; \quad \tau_m = \tau_o + m\,\tau_s, \quad m = -43, \ldots, 42$$

τ_o = gross target delay established by the real time tracker

τ_s = sampling increment

The single pulse information (84 amplitudes, 84 phases, and much auxiliary data like range, range rate, azimuth, elevation, etc.) is multiplexed out of boxcar stores onto magnetic tape during the interpulse period. This data is in digital, not analogue, form. Amplitudes are recorded on a logarithmic scale with about 80 db range using nine bits. Phases are recorded in the interval (0, 2π) using eight bits.

In the past, data processing operations have been carried out on a large digital computer (IBM 7094 or UNIVAC 1108). The amplitude and phase information is first corrected using calibration data (to compensate for wide-band channel differences and log-amp and phase detector non-linearities). The relative phases between radar echoes are then modified to remove as much of the phase modulation due to target translation as possible. Three techniques for doing this are discussed in References 1 and 2.* After a good "pulse alignment" by one of these methods, the pulse-to-pulse modulation of the high resolution echoes appears to arise from target rotation about a fixed point. This modulation is discussed in the next section.

b. Pulse-To-Pulse Phase Modulation

Consider one of our target scattering centers moving with respect to the fixed body phase reference. Its delay, τ_n, changes between pulses. Over a small interval of target rotation, say 0.1 radian (5.7°), the scatterer delay may be approximated by an average value and a term linear in time.

$$\tau_n \simeq T_n + \upsilon_n t \qquad \text{(time origin at middle of small rotation interval)} \qquad (6)$$

Our moving scatterer is represented by one term in Equation (5), namely

$$a e^{i\emptyset} e^{2\pi i f_c(\tau - T)} r_o(\tau - T - \upsilon t) e^{-2\pi i f_c \upsilon t} \qquad (7)$$

where the subscript n labeling the scatterer has been dropped. Suppose the scatterer moves by one tenth resolution cell, i. e. by a delay $\upsilon t = (10B)^{-1}$ where B is the signal bandwidth. Letting the center frequency $f_c = \frac{B}{\Delta}$ the phase change in the measured echo (contained in the last factor of expression (7) is

$$-2\pi f_c \upsilon t = -\frac{2\pi B}{10\Delta B} = -2\pi \frac{1}{\Delta 10} \;;\; \Delta = \%\ \text{bandwidth}.$$

For example, if Δ is 10% the phase change is a full cycle. Associated with this remarkable phase change is a small shift in the "short pulse envelope" $r_o(\tau - T - \upsilon t)$ which amounts to about one tenth its 3 db width.

The other factors, $a e^{i(\emptyset + 2\pi f_c(\tau - T))}$, are sensibly constant over the small rotation interval. Therefore, the dominant effect of the relative rotational motion of the individual echo centers is a rapid r-f phase change. This progressive phase shift is equivalent to a doppler frequency. A second dimension of resolution can be obtained by frequency analysis of pulse-to-pulse echo data, as described in the next section.

c. Processing for Range/Doppler Resolution

The echo data is inherently two-dimensional. In Equation (7) the two variables are:

τ : representing single pulse delay across the 84 channel "range window"

t : contained in each pulse-to-pulse scatterer delay term, $\tau_n = T_n + \upsilon_n t$, measuring time relative to the pulse in the middle of a data block, (t=0 for this pulse).

There are also two "variables" or parameters associated with each echo center:

T_n : the delay of the scatterer relative to the fixed phase reference point, at t = 0.

υ_n : the rate of change of delay of the scatterer, at t = 0.

We would like to obtain a display with coordinates delay and delay rate in which the energy from the n^{th} scatterer is concentrated near the point (T_n, υ_n). The coordinate τ (relative

* References are given on page 26.

delay) already has this property in that the "short pulse envelope", $r_o(\tau - T_n)$ has a peak when its argument is zero, i. e. when $\tau = T_n$. The Fourier transform of the data in the t-direction (a one-dimensional transform) concentrates the echo energy in the new variable f. This is easily seen by considering the expression:

$$u(\tau, f) = \sum_{l=-L}^{L} \left[a e^{i\phi} e^{2\pi i f_c(\tau - T)} r_o(\tau - T - \upsilon t_\ell) \right] e^{2\pi i (f - f_c \upsilon) t_\ell} \tag{8}$$

with $t_l = \ell(\Delta t)$, Δt is the pulse repetition interval.

A sum is used rather than an integral because the radar transmits discrete pulses. The slight amplitude modulation of the term in brackets will be neglected (this modulation is caused by the shift of the envelope function r_o, as υt_l changes values). The result is

$$u(\tau, f) \simeq a e^{i\phi} e^{2\pi i f_c(\tau - T)} r_o(\tau - T) \frac{\sin\left[(2L+1)\pi(f - f_c \upsilon)\Delta t\right]}{\sin\left[\pi(f - f_c \upsilon)\Delta t\right]} \tag{9}$$

Suppose that the radar has a rectangular power spectrum. An explicit form then results for the range resolution factor $r_o(\tau - T)$.

$$r_o(t) = \int_{-B/2}^{B/2} e^{2\pi i f t} \, df = \frac{\sin \pi B t}{\pi t} \tag{10}$$

Thus, the product of two "resolution patterns" occurs for the typical echo center:

$$\underbrace{u(\tau, f) \simeq a e^{i\phi} e^{2\pi i f_c(\tau - T)}}_{\text{amplitude \& phase}} \cdot \underbrace{\frac{\sin \pi B(\tau - T)}{\pi(\tau - T)}}_{\text{range resolution}} \cdot \underbrace{\frac{\sin\left[(2L+1)\pi(f - f_c \upsilon)\Delta t\right]}{\sin \pi(f - f_c \upsilon)\Delta t}}_{\text{doppler resolution}}$$

The peak occurs at $\tau = T$ and $f = f_c \upsilon$. The first zero in the range pattern is at $(\tau - T) = \frac{1}{B}$. The first zero in the doppler pattern is at $(f - f_c \upsilon) = \frac{1}{(2L+1)\Delta t} = \frac{1}{\mathcal{T}}$. Here $\mathcal{T}$ is the total processing interval since it is the number of pulses times their spacing. The result above is perfectly obvious, namely that the resolution in delay time is roughly the reciprocal of the signal bandwidth and that the resolution in doppler frequency is roughly the reciprocal of the integration time.

The purpose of the optical processor is to perform the doppler frequency analysis (one-dimensional Fourier transform) simultaneously for all range channels. It is easily recognized that, if the spectra of the echo pulses are used in lieu of the pulses themselves, a processor employing a two-dimensional transform gives simultaneous range and doppler resolution. Both these configurations are well-known in optical signal processing and are briefly described in the next section. The data film for the processor requires a special format, since the radar data initially has both amplitude and phase modulation. To record this information on film use is made of a "spatial carrier", similar to the holographic recording technique developed at University of Michigan. The format and functional form of the writing beam intensity are discussed in a following section. Finally, the effects of errors in recording are summarized.

2. BASIC OPTICAL PROCESSOR

a. Introduction

The optical processor for S0I radar data should transform echo information, recorded on a film transparency to a range/doppler image. The radar data derives from the echo pattern for each radar pulse. The transparency is made by a CRT film recording system. The information from a single pulse is modulated as brightness variations along a sweep of the writing beam. Successive pulses are stacked side by side along the film. The developed transparency is processed in a coherent optical system. In order to carry out the operations outlined in the last section, the Fourier transforming properties of lenses are used. The configuration chosen for the processor has a bearing on the film format and is discussed here for this reason.

b. Processor Configuration

The processor consists of a source and collimating assembly, a input film plane, a two-dimensional transforming lens, a spatial filter, a one-dimensional transforming lens, a relay lens and the output data plane. The one-dimensional transform can be removed by pre-transforming the radar data before the input transparency is written. These systems are shown schematically in Figure 4 .

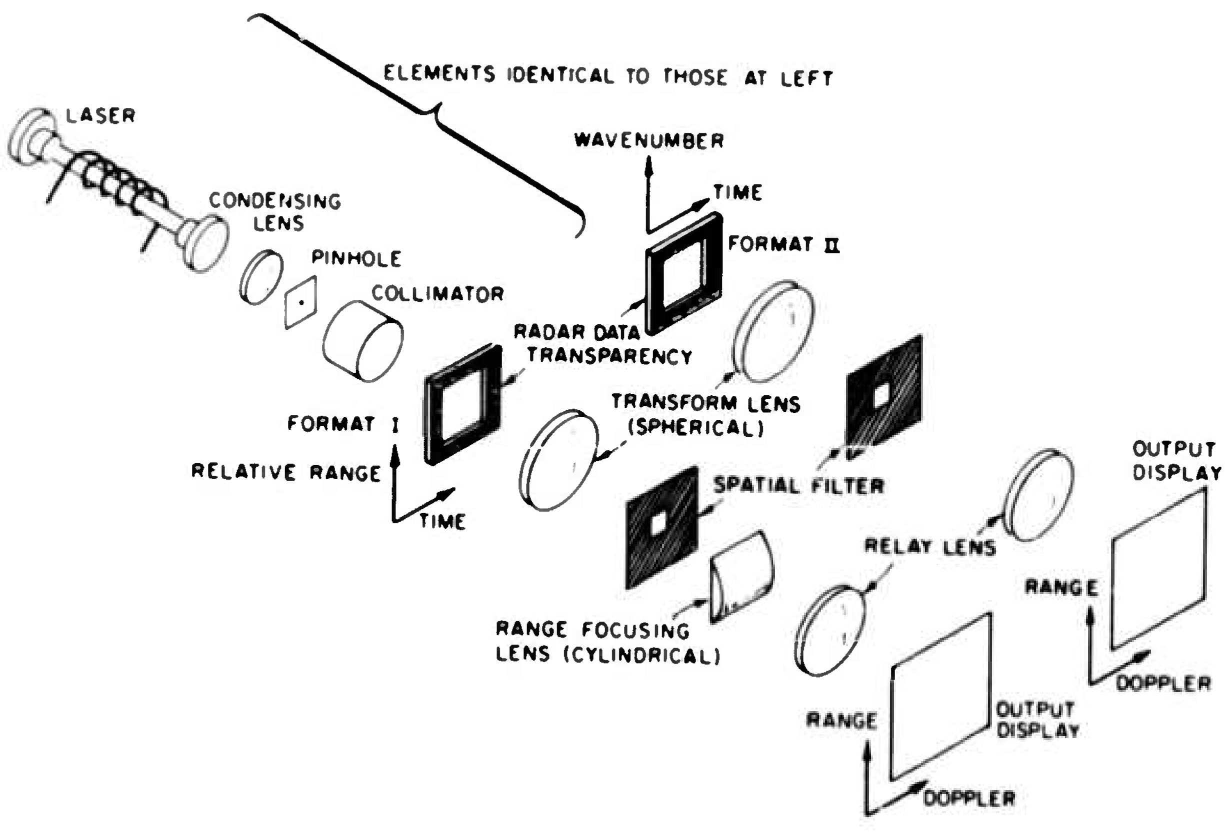


Figure 4. Basic Optical Processor Schematic

The source is conveniently a laser, which provides suitable coherence with reasonable intensity. The collimating assembly includes a condensing lens followed by a pinhole and a collimator. The pinhole is used to remove "noise" in the intensity distribution of the laser output caused by reflections from the mirrors and walls of the discharge tube. The collimator produces a plane wave, covering the input film aperture, from the light leaving the pinhole.

The input film plane contains the radar dara transparency immersed in an index-matching liquid cell or "liquid gate". This is done to compensate for optical path differences caused by film thickness variations. Successive radar pulses are recorded on separate traces, side-by-side across the film. They accumulate in the direction marked "time" in Figure 4 . The coordinate in the other direction can be relative range (as in an A-scope display) or the Fourier transform variable, wave number. These two possibilities are Formats I and II, respectively.

The spherical lens following the input data plane produces the Fourier transform of the information on the transparency. This two-dimensional transform appears in the back focal plane of the lens. The data must be recorded in a way which permits one to select the desired information from the background light by means of a spatial filter. To accomplish this a spatial carrier is introduced into the brightness variations along the data sweep. The spherical lens then produces fringes corresponding to this sinusoidal diffraction grating.

The spatial filter in the back focal plane of the transforming lens blocks the zero order light which represents the background transmitted by the transparency. An opening in this "filter" allows a region about the first order fringe of the spatial carrier to pass through. The Fourier transform of the pulse-to-pulse phase modulation into its doppler components is achieved in this plane. If the film is in Format I, an additional one-dimensional transform to reconstitute the range resolution is done by the cylindrical lens. Format II does not require this lens. In Format II, the spherical lens forms the range pattern as the Fourier transform of the wavenumber information on the input film.

The relay lens may be included to enlarge the output image and focus it on the final observation plane. This might be a ground glass screen or a film in a camera, if a permanent record is desired.

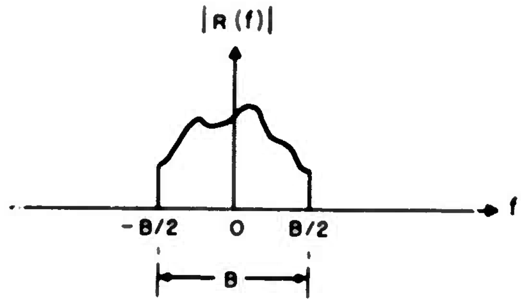
3. DATA FILM FORMAT REQUIREMENTS

a. General Considerations

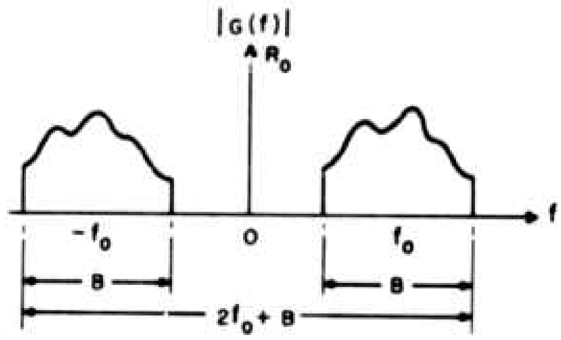
The transmissivity of the film is controlled by the CRT trace exposure. This allows only intensity variations of the light through the transparency. The phase variations due to the film itself cannot be modulated and are, in fact, smoothed out by the liquid gate. The radar data contains both amplitude and phase modulation and both are necessary in processing for a range/doppler image. Thus, the first consideration is to convert the combined amplitude/phase modulation of the data to amplitude or intensity fluctuations only.

It is also necessary to record using a section of the film transfer curve that does not severely saturate. Saturation, being a non linearity, will produce intermodulation between the individual echo centers and give rise to ghost scatterers

Finally, an underlying grating structure (i. e. a periodicity) should be introduced. This allows the background light to be spatially filtered leaving a diffraction fringe which carries the radar data. All three of these considerations can be achieved by recording the data as modulated on a spatial carrier with a bias level added to prevent negative amplitudes. A combination of setting the bias and sealing the maximum amplitude is used to obtain a desirable operating range on the film transfer curve.



(a)
ECHO PULSE MODULATION SPECTRUM



(b)
SPECTRUM WITH SPATIAL CARRIER AND BIAS

Figure 5. Frequency Specta For Data Traces

b. Effect of Spatial Carrier and Bias Level

As pointed out above, the film can reproduce only an amplitude modulated non-negative signal. Simple modulation theory allows one to analyze a method for converting the radar signal (having both A-M and P-M) to a real unipolar waveform. The radar echo to be recorded will be denoted by $r(\tau)$. This has a spectrum centered on zero frequency of bandwidth B, as represented in Figure 5. The echo amplitude $|r(\tau)|$ and phase Arg $r(\tau)$ are used to modulate a carrier. The carrier, denoted by f_0, is chosen greater than B/2, giving rise to a waveform:

$$g(\tau) = |r(\tau)| \left[\cos \ 2\pi f_0 \tau + \text{Arg}\, r(\tau)\right] \tag{11}$$

This real function has positive and negative frequency bands of width B centered on $\pm f_0$ as in Figure 5. It can change sign because of the cosine function. To make a positive signal, one may add a d. c. bias R_0 greater than the maximum value of the signal envelope.

$$f(\tau) = R_0 + |r(\tau)| \cos\left[2\pi f_0 \tau + \text{Arg}\, r\, \tau\right]; \ R_0 > \max|r(\tau)| \tag{12}$$

In the frequency domain, R_0 appears as a line at zero frequency.

Suppose a film is written with a trace whose amplitude transmission function varies as Equation 12. Let it be illuminated at normal incidence with coherent light. According to simple physical optics, the Fraunhoffer pattern of the film is formed by the emerging light. This pattern has three parts which correspond to the three sections of the spectrum of f (t). They emerge at different angles as in Figure 6 and can be separated by a spatial filter in the focal plane of an objective lens behind the film. The first order spectrum represents the function

$$\frac{1}{2} \ |R(y)| \exp \left[2\pi i f_0 y + i \text{ Arg } R(y)\right] = \frac{1}{2} \ R(y) \ e^{\ 2\pi i f_0 y}$$

$$y = c_2 \tau \qquad\qquad R(y) = r\left(\frac{y}{C_2}\right)$$

where the delay variable τ has been replaced by a proportional distance variable y. This function has the desired factor, r (y), which carries the amplitude and phase of the radar data in the correct form. This part of the focal plane distribution enters the remaining section of the optical processor.

The film is, of course, two-dimensional and the carrier can be introduced in either the time (x-dimension) or the single pulse variable (y-dimension) or even in a skewed direction. The single pulse radar echoes are defined by 34 or less samples (complex numbers) depending on the number of radar range channels used. However, more samples are needed in writing the film to represent the spatial carrier. The desirable number of pulses to be used in one integration frame is from about a hundred up to one thousand. A 50x50 mm square on 70 mm film contains one of these frames of radar data. A spatial carrier frequency of 10 line-pair per mm (cycles/mm) or greater is desirable to get the data-carrying fringe out of the "processor noise" region around the direct beam.

The coherent light used in processing the radar data must interact in a linear (amplitude) manner rather than as addition of intensities (power). For proper operation, the amplitude transmission coefficient, T_a, of the film (after developing) should be proportional to the function in equation (12).

$$T_a = \left(\frac{I'}{I_0}\right)^{1/2} = R_0 + |R(X, y)| \cos\left[2\pi f_0 y + \text{Arg } R(x, y)\right]$$

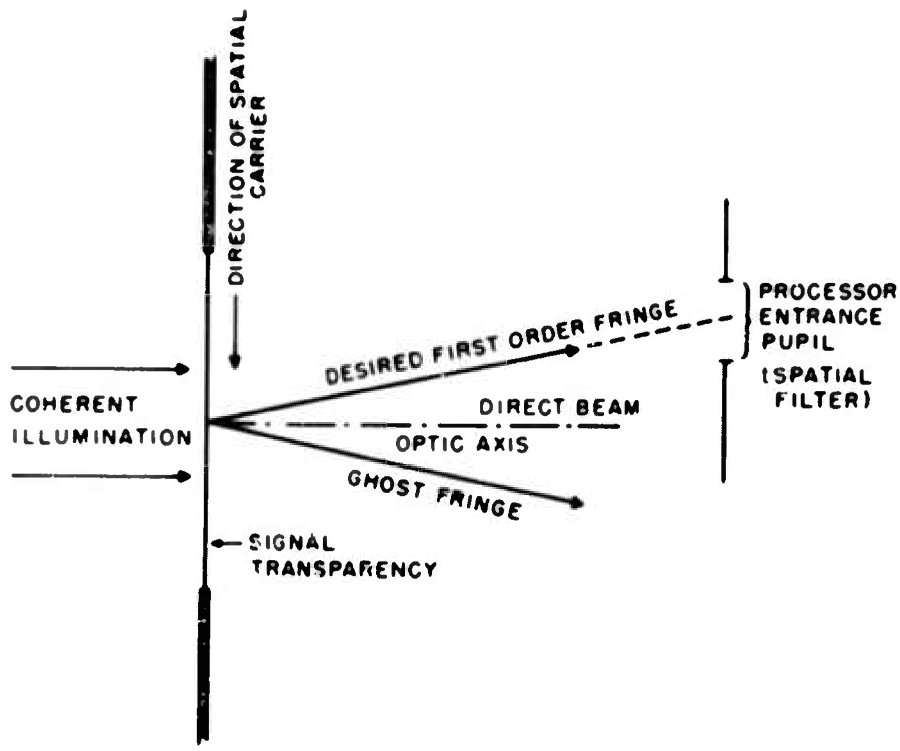


Figure 6. Diffraction Pattern Introduced by the Spatial Carrier

where I' : light intensity transmitted by the recording

I_0' : intensity of the coherent read-out beam in the optical processor

R_0 : bias level

$|R(x, y)|$: envelope of the radar data translated to film coordinates

$\text{Arg } R(x, y)$: phase of the radar data translated to film coordinates

f_0 : spatial carrier

y : distance coordinate on the film proportional to fine range (or delay) on a single radar echo

X : distance coordinate on the film, orthogonal to y, which takes on sucessive values for successive radar pulses (i. e. proportional to time).

If the spectra of the radar pulses are recorded, the variable y is proportional to frequency measured across the radar signal band, rather then to delay with in the echo response.

The transfer characteristic between the grid drive of the CRT and the film transmission coefficient determines the operating points. This is discussed, with reference to experimental data, in Section IV.

c. Sampling and Spot Size Effects

The grid drive of the CRT is modulated by digital information which has been D/A converted from a magnetic tape. The individual data samples are nine-bit quantities. The actual echo data shows symmetric deviations about a central bias level. Thus, the maximum amplitude is half the quantizing range, and equal to 255 times the quantizing step. This "contrast" is equivalent to 48db. A signal 30db below the maximum would be 18 db above the quantizing unit. Its oscillation would be represented by the first 8 quanitzing levels. The rather fine subdivision afforded by nine bits gives some freedom in operating with a markedly non-linear transfer characteristics, when pre-compensation of the tape data is used.

A given CRT-trace (e g. radar pulse) contains many of the individual nine-bit samples the sample density must be greater than that of the original radar data because the information is now in sidebands of the spatial carrier. The "sampling theorem" gives a maximum spacing of $\Delta y = \frac{1}{2W}$ where W is the highest spatial frequency in the signal. However, this rate is not suitable when a finite section of a signal is to be represented. A higher density is needed to reduce the "truncation errors" as developed in Reference 3.

A representation of this effect in the frequency domain is shown in Figure 7. The spectrum of the band-limited waveform to be recorded is represented in Figure 7. When this waveform is sampled with a spacing Δy, the associated spectrum becomes an infinite set of replicas separated by $\frac{1}{\Delta y}$, as in Figure 7. The space between successive replicas is referred to as the guard band. Because the waveform is band-limited, its time representation cannot be identically zero outside some finite interval. Recording only a segment

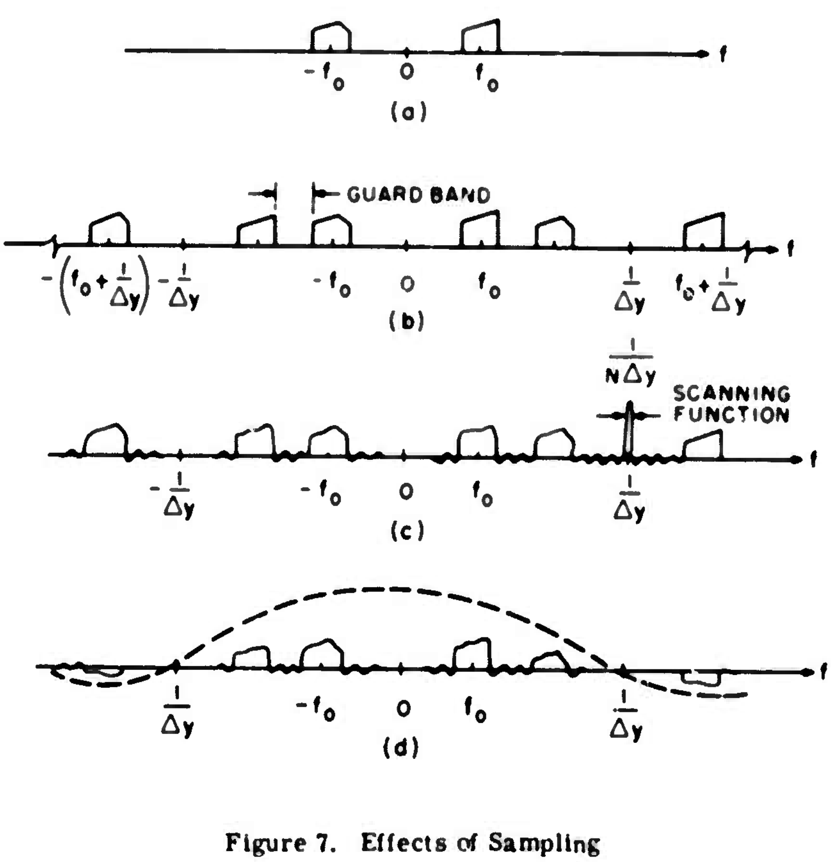


Figure 7. Effects of Sampling

of this signal thus changes the spectrum again. If N samples are recorded, the signal is effectively gated by a rectangular wave $N\Delta y$ in width. The frequency domain effect is that the set of replicas (in M3b) is convolved with the transform of the gate. The latter "scanning" function is $\frac{\sin \pi f N\Delta y}{\pi f N\Delta y}$ which has first zero crossings at $f = \frac{1}{N\Delta y}$. It is important that the guard band be several times wider than the main lobe on this $\frac{\sin X}{X}$ function. For example, the third sidelobe of the scanning function is some 20 db down while the fifth is suppressed about 25 db. Sampling at the rate $3f_o$ (f_o being the spatial carrier) gives a wide guard band. The convolution effect is shown in Figure 7.

Actually, point samples are not produced by the D/A converter which drives the CRT intensity control. Instead, the voltage adjusts to the digital input rapidly (5 μsec or less) and remains at the steady level until the next sample. The effect is of a "box car" where each sample is convolved with a rectangle of width Δy, the space between samples. This means that the spectrum is multiplied by an envelope $\frac{\sin \pi f\Delta y}{\pi f\Delta y}$ as in Figure 7. The spot size of the CRT, the camera lens, and the film itself introduce further frequency transfer functions. These should be maintained as broad as possible (i. e. resolution not degraded) to minimize spectral distortion. The spot size appearing on the film and the associated spatial frequency transfer function are reviewed in Section IV.

To recapitulate, the spatial carrier f_o must be chosen larger than the inherent data bandwidth so that the sidebands (fringes) will be separated. The digital sampling rate must be significantly greater than $2f_o$ is more than adequate.

The maximum inherent bandwidth of the single pulse data (when on film) is easily found. The actual radar response has a bandwidth not greater than the reciprocal of the resolution cell length. When this cell length is expressed in mm on the film, its reciprocal is the spatial bandwidth of the envelope and phase modulation of the data trace. There will be, at most, 84 cells in 50 mm. Thus the data band width is

$$\frac{84}{50 \text{ mm}} = 1.68 \text{ cycles/mm (line pair/mm)}$$

Because an actual target will shift about somewhat in the real time range gate, some channels on the ends will not contain useful information. Thus, 84 range cells is an upper limit, with 60-75 cells being more likely. The data bandwidth reduces to only 1.2 cycles/mm with 60 range cells. If the spectrum form of the data is used, one would have the same inherent bandwidth. In this case, variations having n-42, n-41, . . . , n+40, n+41 cycles in 50 mm (n being the number of spatial carrier cycles) are required to represent channels -42, -41, . . . , 40, 41. Again the maximum difference frequency is 84 cycles in 50 mm or 1.68 cycles/mm. This inherent bandwidth is for the single pulse (y) dimension or direction or direction on the film.

In the x-direction, the bandwidth of the doppler variations of the radar signals appears as spatial frequencies. This is the direction of film motion in the camera. The data bandwidth in this dimension depends on the target as well as on the radar waveform and prf. It is possible (indeed, a common occurance) that the radar prf is too low for unambigous data samples. Thus, the doppler (x-direction) bandwidth will be taken as the maximum allowed by the prf. A phase change lying between -180° and +180 is allowable from one pulse to the next. The relative difference is 180-(-180) or one cycle. Thus, if 1000 pulse traces are written in the 50 mm aperture, one could have a spatial doppler frequencies spanning 20 cycles/mm. Lower frequency excursions on the film can be obtained by increasing the distance between traces or by repeating the same pulse data in a sequence of redundant lines.

The spatial carrier has been introduced in the single pulse trace since the data bandwidth there is rather low. In synthetic aperture radar processors the carrier usually appears in the time or doppler dimension. These systems have many more range cells and hence more cross-film bandwidth than in the present case. In the present case, the spatial carrier, if inserted in the x-direction, would be unrealistically high, considering the resulting sample density and the tube spot size. We thus restrict further format consideration to a carrier in the y-direction with either the time domain or frequency domain representation of the echoes.

4. SUMMARY OF ERROR SENSITIVITIES

a. Introduction

Positional errors in the training beam, film drive deviations from constant speed, phosphor and film noise, and the transfer characteristics of the CRT, lens, and film will all cause errors in the transparency. When the film is used in the coherent optical processor, it acts as an extended, two-dimensional source. That is, the plane wave of coherent light impinging on it is modulated by the local transmissivity of each region of the film. The emerging light superimposes coherently (at a large distance from the film) to form an interference pattern. This pattern is the two-dimensional Fourier transform of the amplitude transmissivity of the film.

The presence of the spatial carrier allows one to separate a fringe, carrying the radar data modulation, from background light in the direct beam. However, the spatial carrier has the effect of converting small errors in the location of the writing beam to relatively large phase fluctuations in the echo data. For example, if the carrier is 10 line pair per mm, one cycle (360°) occupies $\frac{1}{10}$ mm, and a 3.60 phase error corresponds to only $\frac{1}{1000}$ mm (one micron). The phase error due to a positional error is proportional to the spatial carrier. This carrier frequency cannot be arbitrarily reduced, however, becasue the data-bearing fringe must be deviated away from the direct beam and the processor noise region near zero spatial frequency.

The film acts somewhat like a two-dimensional antenna composed of individual elements. Both have radiation patterns given by the Fourier transform of their aperture distribution. Analysis of error effects can be based on this analogy. The film is regarded as consisting of cells (elements) each with amplitude, phase, and positional errors. There are two ways of picking these elementary cells. The first is to regard each sample point (9-bit number) transfered from tape to film as a separate element. There are between 1000 and 2000 of these points in a trace and between 100 and 1000 traces or lines in a data frame. In Appendix II, an error analysis is carried out from this standpoint.

The second viewpoint is to think of each trace as composed of radar resolution cells or independent spectral components, depending on the format. There are, at most 84 such cells in a trace, with 60 being a more likely number. Again, there are 100 to 1000 traces. In this second approach, the true fine structure within the cell is by-passed and a resultant amplitude and phase error applied to the cell as a whole. If the spatial carrier is 10 line per/mm, and 60 cells are recorded in a trace there will be

$$\frac{50\text{ mm}}{60\text{ cells}} \times \frac{10\text{ cycles}}{\text{mm}} = 8.3\text{ cycles/cell}$$

There is little difference in the two view points for Format I data, since there is no integration across sub-elements of one range cell. The transformation is in the orthogonal direction. The difference arises in Format II where both direction are transformed. In this format, one should use the number of independent samples per range cell in defining the resultant amplitude and phase error for the cell. By independent samples, we mean those chosen far enough apart so that the random sweep errors are not correlated. There is correlation between successive positions of the actual 9-bit data words so that using these data points as independent (in the sense of positional errors) gives an optimistic estimate of the performance in Format II. Using the range cells as the basic independent elements is more pessimistic, since there are fewer samples superimposing in the transform pattern.

b. Format I Errors

Random errors will be considered first, then systematic errors which are of lesser importance. In this format, a range cell swath along the film (x-direction) is being transformed. It acts as a linear array of elements, one for each radar pulse trace. Amplitude errors arise from phosphor and film irregularities and sweep errors which stretch out or compress the 9-bit data samples. Phase errors (which are more critical) arise from sweep positional errors and from film drive jitter. The amplitude errors and the first type of phase errors are similar to "element current errors" in an antenna array while the second type of phase error in similar to "element placement error" in the array. The former affect all beams (or positions along the output doppler axis) equally. The latter have no effect for the broadside beam (zero doppler) and maximum effect on the most deviated beam (maximum doppler). The worst case will be considered. Because maximum (positive) doppler corresponds to data having a phase shift of π radians between traces, a fractional error in the trace spacing $\frac{\Delta x}{x_o}$ = (positional error)/(trace spacing) results in a worst phase error of $\frac{\Delta x}{x_o}\pi$. Actual errors for a beam corresponding to a smaller doppler are reduced by the ratio of this doppler to the maximum doppler. Jitter in the trace sweep produces positional errors in the y-direction. If this error is Δy, the associated phase error is $2\pi f_o \Delta y$ where f_o is the spatial carrier.

The one-dimensional transform used to process Format I data has the form:

$$\mu(\tau_m, f) = \sum_{\ell=-L}^{L} T(m,\ell)\, e^{-2\pi i f x_\ell} \quad m \text{ fixed} \tag{13}$$

where
- $(2L+1)$: number of radar pulses integrated
- μ : output light distribution (first fringe)
- f : transform variable in output plane, proportional to doppler frequency
- τ_m : delay coordinate of the m^{th} range swath, measured across the film.
- X_ℓ : positional coordinate of the ℓ^{th} sample point in the range swath, measured along the length of the film.
- $T(m,\ell)$: complex transmission coefficient for the film, corresponding to the upper sideband (first fringe) produced by the transmissivity expressed in Equation 12.

For a given echo center, the τ_m- coordinate (delay) is nearly constant. The integration or summation is done for a fixed value of delay so the dependence on this coordinate will not be carried explicity. The notation can be simplified to:

$$w(\eta) = c \sum_{\ell=-L}^{L} a_\ell\, e^{i\theta_\ell}\, e^{i\ell\eta} \tag{14}$$

where
- $\eta = -2\pi f x_o$: the cross-range variable
- x_o: the nominal spacing of the traces (pulses) on the film
- c: a complex constant, $\exp(2\pi i f_o m \tau_o)$ representing the nominal phase factor for the m^{th} range cell
- τ_o : spacing of the range cells on the film.

a_ℓ : amplitude, a random quantity with mean equal to the nominal (correct) amplitude and standard deviation ($\sigma_a \, a_\ell$). Thus, the standard deviation of the fractional amplitude error is σ_a.

θ_ℓ : phase, a random quantity with mean zero and standard deviation σ_θ. It is composed of the sweep and trace spacing error terms discussed above.

Error analysis which apply to this problem have been done many times - J. Ruse "Physical Limitations on Antennas" TR248 Res. Lab. of Elect MIT Oct. 30, 1952 contains an early example. We outline the error analysis but do not carry out all the steps. One defines the real and imaginary parts of W (τ) :

$$X = \sum_{\ell=-L}^{L} a_\ell \cos(\ell\tau + \theta_\ell) \qquad (15a)$$

$$Y = \sum_{\ell=-L}^{L} a_\ell \sin(\ell\tau + \theta_\ell) \qquad (15b)$$

the envelope $|W|$ and phase Φ:

$$|W| = (X^2 + Y^2)^{1/2} \quad ; \quad \Phi = \tan^{-1} (Y/X) \qquad (15c)$$

For large N (say greater than 10) and mild assumptions on the distribution of a_l and θ_l, one can invoke the central limit theorem and treat X and Y as Gaussian variables. The probability distributions of X and Y are

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-(x-\bar{x})^2 / 2\sigma_x^2} \quad ; \quad P(Y) = \frac{1}{\sqrt{2\pi}\sigma_y} e^{-(Y-\bar{Y})^2/2\sigma_y^2}$$

When the amplitudes have an even distribution ($\bar{a}_l = \bar{a}_{-l}$) and when the phase errors are symmetrical about zero so that $\overline{\sin \theta_l} = 0$, one has the mean of the imaginary component equal to zero.

$$\overline{Y} = 0$$

Under these conditions, one can also show that the correlation coefficient of X and Y vanishes. Since they are also nearly Gaussian, they are almost independent, and their joint probability p (X, Y) will be taken as the product of p(X) and p(Y). In order to find the statistics of the envelope $|W|$ and phase Φ one converts to polar coordinates.

The distribution for the envelope becomes the modified Rayleigh distribution

$$p(|W|) = \frac{|W|}{\sigma^2} e^{-\frac{1}{2\sigma^2}(|W|^2 + \overline{X}^2)} I_0\left(\frac{\overline{X}\,|W|}{\sigma^2}\right) \qquad (16)$$

The approximation $\sigma_x^2 = \sigma_y^2 = \sigma_z^2$ has been used. It turns out that this is valid in the sidelobe region of the integrated pattern W (τ) . When $\sigma \gg \overline{X}$ (noise power much greater than sidelobe power), the distribution is nearly Rayleigh:

$$p(|W|) = \frac{|W|}{\sigma^2} e^{-|W|^2/2\sigma^2} \quad ; \sigma \gg \overline{X} \qquad (17)$$

When $\sigma \ll \overline{X}$ (noise power much less than nominal sidelobe power), the distribution has a Gaussian shape about the mean sidelobe pattern.

$$p(|W|) = \sqrt{\frac{|W|}{\overline{X}}} \; \frac{1}{\sqrt{2\pi}\sigma} \; e^{-\frac{1}{2\sigma^2}(|W|/X^2)} \qquad ;\sigma \ll \overline{X} \tag{18}$$

The limiting form for noise larger than nominal sidelobes will be used for estimating allowable tolerances. In this case, the mean is approximately $\sqrt{\frac{\pi}{2}}\,\sigma$ and the root mean square $\sqrt{2}\,\sigma$. There is a 99% probability of all values being less than the 3σ value.

The parameters of the envelope distribution, $\overline{X}$ and σ, can be simply expressed in terms of the individual phasor errors. For simplicity, the phase errors are considered to be normally distributed with mean zero and common variance σ_θ^2. The percent or fractional amplitude errors are taken to be normally distributed with zero mean and common variance σ_a^2.

We have

$$\overline{X} = \sum_{\ell=-L}^{L} \overline{a}_\ell \,(1-\tfrac{1}{2}\sigma_\theta^2)\cos \ell\tau$$

$$\overline{Y} = 0$$

The variances of $\overline{X}$ and $\overline{Y}$ may be calculated using the expressions

$$\overline{a_\ell^2} = \overline{a}_\ell^{\,2}(1+\sigma_a^2); \quad \overline{\cos\theta_\ell}^{\,2} \simeq 1-\sigma_\theta^2 \;; \quad \overline{\cos 2\theta_\ell} \simeq 1-2\sigma_\theta^2$$

The average of σ_X^2 and σ_Y^2 is independent of τ.

$$\sigma^2 = \frac{1}{2}(\sigma_X^2 + \sigma_Y^2) = \frac{1}{2}(\sigma_a^2 + \sigma_\theta^2) \sum_{\ell=-L}^{L} \overline{a}_\ell^{\,2} \tag{19}$$

The mean square sidelobe power level (due to errors) is $2\sigma^2$. The ratio of this mean square power level to the expected main lobe peak power is:

$$\frac{2\sigma^2}{\overline{X}^2(o)} = \frac{(\sigma_a^2 + \sigma_\theta^2)}{1-\sigma_\theta^2} \; \frac{\sum_{\ell=-L}^{L} \overline{a}_\ell^{\,2}}{\left(\sum_{\ell=-L}^{L} \overline{a}_\ell\right)^2} = R \cdot \frac{(\sigma_a^2 + \sigma_\theta^2)}{1-\sigma_\theta^2} \tag{20}$$

The $(1-\sigma_\theta^2)$ term in the denominator arises from the factor $\overline{\cos\theta_\ell}^{\,2}$ in the expected peak. The summation in the denominator is the nominal peak of the pattern. The factor R, the ratio of the two summations in the last equation, is equal to $\frac{1}{2L+1}$ for a flat amplitude distribution of (2L+1) components. When taper is employed this factor is somewhat greater than $(2L+1)^{-1}$; for example, it is 0.025 for 51 lines with 33 db sidelobe suppression. A curve of R is given in Appendix II.

The side power has 37% probability of exceeding the mean square level. However, there is only 1% probability that a sidelobe exceeds the mean square level by more than 6.5 db. If we set the mean square level at 30 db, the 99% confidence level is 23.5 db and the parameter $2\sigma^2 / \bar{x}^2(0)$ is 0.001. The number of pulses integrated is not entirely our choice since it depends on the rotation rate of the object. We will take the conservative estimate of 100 and make R = 0.01. Taking the major error source to be in the phase (trace and carrier positioning), one has

$$\frac{\sigma^2}{1-\sigma_\theta^2} = \frac{0.001}{0.01} \quad \text{or} \quad \sigma_\theta \simeq 0.3 \text{ radian} = \frac{1}{20} \text{ cycle.}$$

When the phase error is dominated by the sweep velocity jitter, the tolerance of 1/20 cycle results in the following positioning tolerances.

Spatial Carrier	Film Position Error	Fractional Error (50 mm trace)
15 line pr/mm	(1/300) mm = 3.3 micron	6.7×10^{-5}
10 line pr/mm	(1/200)mm = 5 micron	10^{-4}

(100 pulses integrated)

If the phase error is attributed solely to film drive non-linearities, the 1/20 cycle tolerance results in a fractional trace position error of 10%. The film drive is much better than this, so this error source may be neglected in practice. To be comparable to the phase error of 1/20 cycle, the amplitude error would be about 30%. Thus, the positional jitter due to sweep velocity will dominate the error sources.

In Figure 8, the expected sidelobe level is plotted versus the design level (which is achieved by a certain amplitude taper across the aperture) for various values of the parameter $\sigma / \bar{X}(0)$. These curves show a saturation level when the error power dominates the design sidelobes.

In the region beyond the knee in the curves (where the error power dominates the design level), the rms sidelobe is 1.05 db larger than the expected sidelobe and the 99% confidence level is 7.58 db larger. For example, if the expected peak to sidelobe ratio is 30 db, the peak to rms sidelobe is 29 db, and the peak to 99% level is 22 db.

Format I involves a one-dimensional integration in the direction of film motion (x-direction) with the spatial carrier in the y-direction. The envelope data in the y-direction is simply imaged onto the output film. We have seen that random positional errors along the sweep should not exceed one part on 10^4. Systematic errors, which repeat identically on each trace can be much larger In order to estimate peak locations to 1/10'th of a range cell, the maximum systematic deviation in the range direction should not exceed one part in 800. Systematic variations in the film velocity have the same effect in both formats, and the analysis will be outlined in the following discussion of Format II.

c. Format II Errors

Format II involves a two-dimensional integration for each output cell. Ruse's paper[5] shows that in this case the factor R becomes

$$R = \left(\sum_{\ell=-L}^{L} \sum_{m=-M/2}^{M/2} \bar{x}_0^2{}_{m,\ell} \right) \Bigg/ \left(\sum_{\ell=-L}^{L} \sum_{m=-M/2}^{M/2} \sigma_{m,\ell} \right)^2 \alpha \frac{1}{M(2(L+1))} \qquad (21)$$

As was pointed out in the introduction to this section, the number M should be the number of statistically independent samples along the data trace. A pessimistic estimate is that an

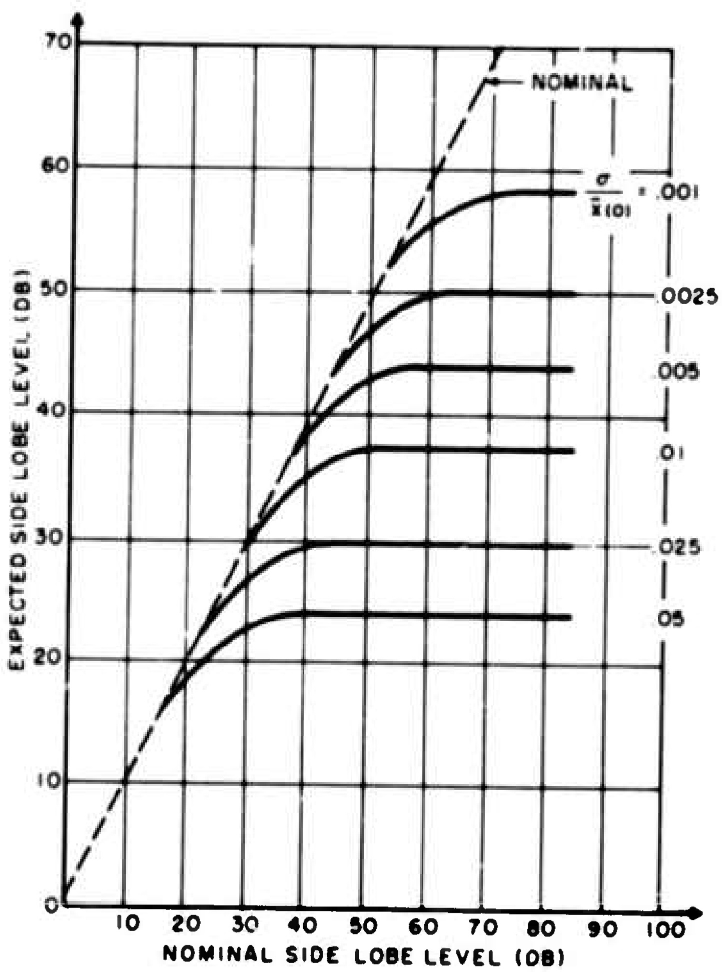


Figure 8. Sidelobe Level Vs Design Level

entire range cell (roughly 10 cycles of the spatial carrier) is correlated. The number of independent samples in a trace will be taken to be 100 (which is an order of magnitude smaller than the number of samples per sweep.

Using 100 pulses, as before, with 100 independent sweep samples, one finds R to be 10^{-4} instead of 10^{-2}. Thus, the random error tolerance, σ, can increase by a factor of 10 compared to Format I. For given tolerances, the added dimension of integration will raise the performance limit by 20 db. This can be seen in Figure 8 . It is therefore expected that random errors will degrade the output when Format II is used as severely as in Format I.

The systematic errors in Format II can create a problem. The repeatable non-linearities in the sweep are not simply imaged on the output film. Instead, the processor does a spectral analysis of the non-linearity and produces main lobe broadening and localized sidelobes accordingly.

One divides systematic non-linearities into two types - those without a repetitive structure and those with repetition or oscillation. The first are analyzed by representation with a few terms of a power series; the second are represented by a Fourier series. The power series type produces distortion of the main lobe and perhaps close-in sidelobes (for small errors). The oscillatory type gives rise to localized sidelobes having the appearance of diminished main lobes or "echoes".

Calculations for quadratic phase errors appear in "Theory and Design of Chirp Radars" by Klauder, Price, Darlington and Albersheim, Reference 4. A combination of quadratic and cubic phase errors is treated in Reference 2. When no spectrum taper is employed, appreciable degradation occurs for a maximum phase difference across the aperture of $\frac{1}{4}\pi$. For weighted spectra (say 25-30 db nominal sidelobes) total phase deviation of $\frac{\pi}{2}$ to π is tolerable. Actually, systematic error of deviation π will produce large main lobe broadening (a factor of 1.4 to 1.5) and peak reduction of 15%. A deviation of $\frac{\pi}{2}$ is much less severe, 5% reduction of peak and broadening by a factor of 1.1.

At a spatial carrier of 10 line pr mm, a deviation of π represents 1/20 mm. This represents (1/1000)th of the sweep length and is a more stringent than in Format I. In fact, Format II has comparable tolerances on random and systematic errors, about 1 part in 1000.

Errors of the oscillatory type are best represented by Fourier components. For example, an amplitude ripple, would modify the pattern as follows:

$$W(\tau) = \sum_{m=-M/2}^{M/2} a_m \left[1 + A\cos(mT+\alpha)\right] e^{im\tau} \tag{22}$$

$$= W_o(\tau) + \frac{1}{2} A e^{i\alpha} W_o(\tau+T) + \frac{1}{2} A e^{-i\alpha} W_o(\tau-T)$$

where W_o is the nominal pattern and the summation represents integration in the dimension having the oscillatory error. Thus, in addition to the nominal pattern, there are two "paired echoes" of relative amplitude (1/2)A, located $\pm T$ away from the central peak. If we require no paired echo (due to amplitude ripple) be greater than -26 db, A cannot exceed 0.1 or 10% ripple amplitude.

A similar analysis applies to repetitive phase errors. A single phase oscillation of the form B cos (mT $+\beta$) will be considered.

$$W(\tau) = \sum_{m=-M/2}^{M/2} a_m e^{iB\cos(mT+\beta)} e^{im\tau}$$

$$= \sum_{m=-M/2}^{M/2} a_m \left[J_0 + 2iJ_1(B)\cos(mT+\ )-2J_2(B)\cos(2mT+2\beta) + \ldots\right] e^{im\tau} \quad (23)$$

For small phase ripple, B < 0.4 radians, one uses the approximations

$J_0=1$, $J_1(B) = 1/2\ B$, $J_2(B) = 0$ etc. The expression above reduces to

$$W(\tau) = W_0(\tau) + 1/2\ iBe^{i\beta} W_0(\tau+T) + 1/2\ iBe^{-i\beta} W_0(\tau - T) \quad (24)$$

There are two "paired echoes" arising from the phase ripple, in quadrature with those associated with a similar amplitude ripple. If the criterion of -26 db suppression of these echoes is applied, one must keep the ripple coefficient B less than 0.1 radian (5.7°). Suppression of -20 db would require the phase ripple less than 0.2 radian (0.032 cycles).

The tolerance on the oscillatory part of the systematic phase error is quite severe. At a carrier of 10 line pr/mm the 0.1 radian tolerance (-26 db suppression) represents only (1/630) mm on the film, when in the sweep length. It should be emphasized that these effects are associated with a sinusoidal sweep or film velocity variation giving range lobes and cross-range lobes respectively. It does not represent the total systematic error which undoubtly has a non oscillatory part. Degradation caused by systematic errors in the sweep velocity have been observed in processed outputs with Format II recordings. Simusoidal drive speed variations would produce paired echoes in cross-range for both formats. This has not been observed in the outputs.

5. REFERENCES FOR SECTION II

1. "Synthetic Spectrum Modification Program for Radar Set AN/FPS-4 Interim Report for Phase II," Contract DA-36-034-ORD-3335RD, Westighouse Defense and Space Center, Oct. 1961.

2. "An Investigation of a Two-Dimensional High Resolution Radar Technique," Final Rept. Contract DA-36-034-ORD-3758Z, Westinghouse Defense and Space Center, Nov. 1963.

3. Helms and Thomas, Proc. IRE Vol 50 pp. 179-184 February 1962.

4. Klauder, Price, Darlington and Albersheim, "The Theory and Design of Chirp Radars," BSTJ, Vol. 39, pp. 745-808, July 1960.

SECTION III

EQUIPMENT DESIGN PARAMETERS

The film recorder was designed to provide a stable and accurate trace from which to record data on film. The recording on film is accomplished by repeating a one-line scan across the face of a CRT and moving the film past the image plane at a constant rate. The rate of film motion is set such that lines are written closely spaced with some overlapping.

1. CATHODE RAY TUBE ASSEMBLY

The resolution requirement was to provide a dynamically focused spot of less than 1.5 mils size (measured at the 70% points). Sweep accuracy was to limit spot jitter to 0.5 mils from sweep to sweep. Intensity compensation was provided to prevent long term threshold instability of the CRT trace.

The CRT Assembly contains all circuitry necessary to display video information by intensity modulating a single line trace on a 5CEP11 cathode ray tube. External signals necessary for proper operation of the film recorder include a start pulse, stop pulse, film drive frequency control, and video input.

The start and stop pulses provide the proper timing and voltage amplitudes required to sweep a four inch trace on the face of the CRT. The start pulse is reshaped by a logic differential receiver and expanded, via a monostable multivibrator. Reshaping and expanding of the stop pulse is also provided. Both pulses provide inputs to two nand gates connected in a flip flop configuration. The output of this flip flop determines the time that an electronic switch allows the integrating capacitor to charge. This output is then amplified and offset about zero volts since the final deflection amplifier is double ended (push-pull). To compensate for the fact that the 5CEP is a flat faced tube, the sweep voltage is predistorted prior to final amplification. This predistortion is accomplished thru a diode-resistor linearity matrix which changes the shape of the sweep ramp such that the trace velocity will be uniform across the face of the tube.

Variation in line width or spot growth from the center of the tube to the edge can be as great as 5:1 over the center specified spot size (1.5 mils in 5CEP). To prevent this growth, dynamic focusing must be accomplished. The ideal dynamic voltage waveshape is a 120 volt parabola which modulates the focus power supply. To generate this waveshape, a piece-wise linear approximation to the parabola from an operational amplifier modulates the focus power supply. A brightness control is provided so that proper operating bias may be obtained from tube to tube.

To insure that the design goals are set forth for the film recorder were met, the commercial power supplies were specified to have 0.05% regulation. The anode and focus power supplies were built in-house by Westinghouse and were 0.01% regulation. This tight regulation was to prevent random spot growth. An attempt was made in the design of the analog circuits to molecularize as much circuitry as possible.

2. CAMERA SYSTEM

a. Lens

The system camera was provided by Photogrammetry, Inc., of Rockville, Maryland. It was designed specifically for this application and is designed to record intensity modulated traces from a 5CEP11 CRT onto continuous 70 mm film at a 2:1 reduction. The film is continuously advanced by a frequency synchronized capstan during exposure. There is no shutter and exposure is controlled by sweep intensity and lens opening.

The lens is a Boyer-Beryl 110 mm f. l. Beryl lens having a maximum aperture of f/6.8. The lens mounting allows continuous focus adjustment over the desired range. A reflex mirror is so mounted that it may be rotated to reflect light collected by the lens onto a high resolution, Polacoat Lenscreen, which is set closely in the focal plane. The CRT sweep was focused on this screen and observed with a 120X microscope assuring optimum practical focus.

b. Film Drive

The camera drive system, manufactured by Sequential Electronics Sy .ms, Inc., consists of a film drive servo controlled capstan and a constant torque film tension take-up system. This may be seen in Figure 9 . The capstan servo system, completely eliminates the inherent limiting characteristics of sampled-data phase locking systems that derive feedback information from a pulse rate tachometer.

The infinite resolution Band-Scan Readout has the unique capability of electro-optically monitoring continuous speed/phase coordinates converting this information into the required digitized format at a constant conversion rate which is independent of shaft speed. It computes speed with zero error and position or phase with a maximum peak inaccuracy of 0.33 arc seconds, at a minimum settable conversion rate of 20 KHz. This action produces a rate/position control system with a constant closed-loop control bandwidth on the order of 3 KHz.

The reference input to the Band-Scan Reference Generator is a signal whose frequency directly corresponds to commanded motor speed and whose phase relates to drive shaft position. The standard speed-frequency conversion ratio is such that Motor RPM = K x f, where K is the conversion ratio selected for a given application and f is the reference frequency. The Reference Generator samples and stores speed/phase command information. Updating of the coordinates in storage is once per cycle of the reference. The interrogation rate of the Generator for information fed into the Band-Scan FPL is automatically made compatible with the conversion rate of the Band-Scan Readout.

The Band-Scan Computer compares the commanded speed/phase information produced by the Reference Generator, with the motor shaft speed/phase coordinates measured by the Band-Scan Readout, and produces a digitalized error signal. The Digital to Analog Converter converts this error signal into an analog voltage, with a conversion rate that is automatically made compatible with the conversion rate of the Band-Scan Readout. This conversion is performed with no significant time constant, and with virtually no ripple, which permits design of an extremely wideband, high gain control loop. The Stabilization Section contains stabilization circuitry which applies phase, velocity, and acceleration control in the proper ratio, and with the proper frequency characteristics, to optimize the desired closed loop performance of the control loop. The Driver unit supplies the correct excitations to the motor windings to maintain constant shaft velocity. The driver and motor windings are operated in a secondary, closed-loop manner, to linearize the torque output characteristics, and to virtually eliminate the electrical time constant of the motor winding.

The position sensitivity of the Readout is such that if the electronic gain were unity, less than 100 arc seconds of position error would correspond to a full applied torque command. Achievable electronic gains with the proper stabilizing elements are on the order of 40 db. This results in a total open-loop gain of 1,000,000 (120 db) radians/sec/radians; it produces an absolute closed-loop position control accuracy on the order of seconds of arc.

3. DATA RATE

The input to the SDS 910 is a tape written at a density of 556 bits per inch. On this tape are records of date (a record is one pulse of radar data which is also a single sweep across the face of the CRT) each of which is 1024 or 2048 words in length. There are nine bits per word. The highest possible input rate to the SDS 910 is 5 records per second, the lowest is

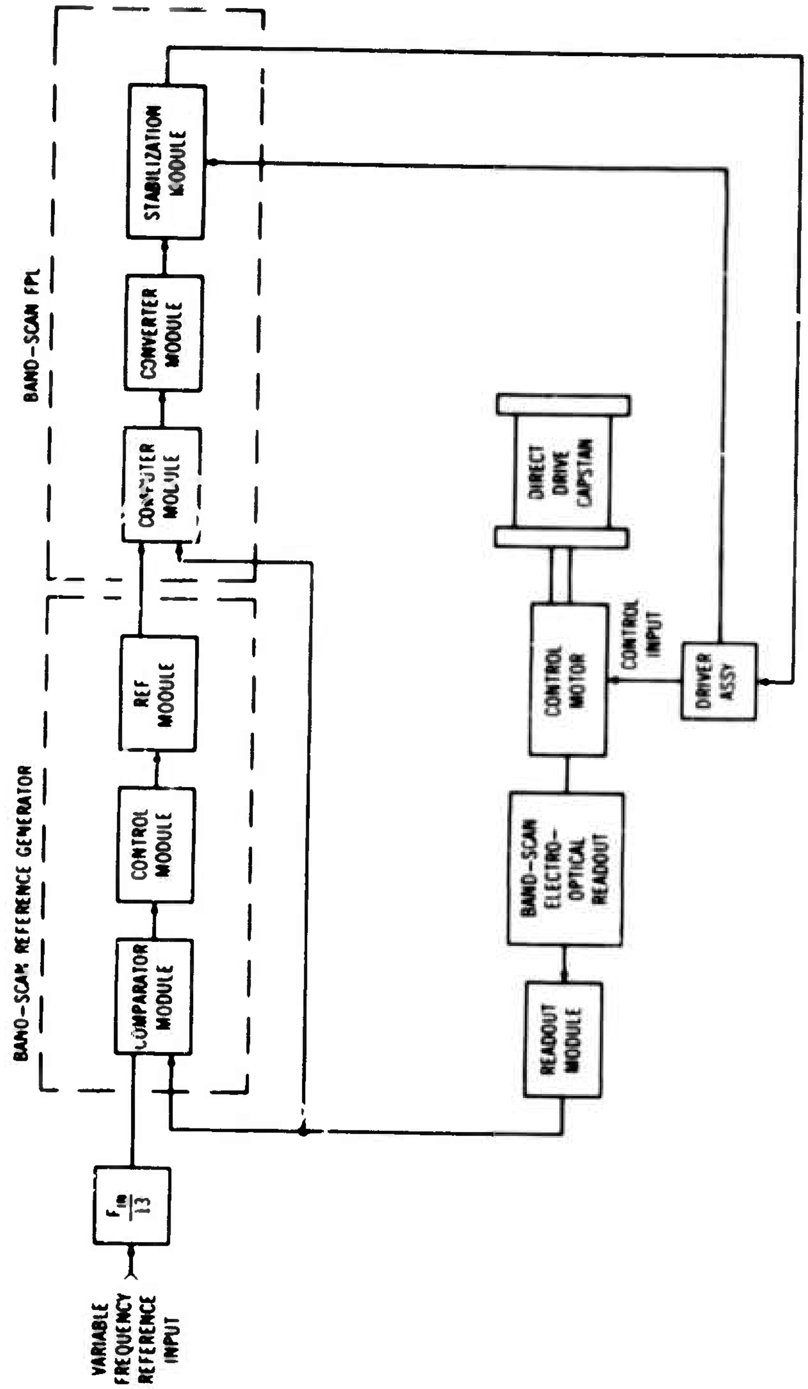


Figure 9. Block Diagram of Camera Servo Control System

2.5 records per second. This corresponds to 200 and 400 milliseconds between line start pulses, respectively. The highest data rate at the output of the SDS 910, therefore, is 92,160 bits per second and it occurs when using records of 2048 words and 200 milliseconds between lines start pulses (i.e. 92,160 bits/sec = 5 X 9 X 2048). The lowest usable data rate is 23,040 bits per second = 2.5 X 9 X 1024; this occurs when using records of 1024 words and 400 milliseconds between line start pulses. Normally, the film recorder is operated using the highest data rate.

It is necessary to know how many pulses of radar data can be written on a magnetic tape. Since one word requires 12 bits, at a tape density of 556 bits per inch, there are 556/2 words per inch. If the record length is 2048 words, then have (2/556) 2048 = 7.37 inches/pulse. Between each pulse there is a .75 inch record gap. Therefore, the number of inches per pulse is 7.37 + .75 = 8.12. The magnetic tapes we use are 2400 feet long. Therefore, the maximum number of pulses per tape is (2400/8.12) 12 = 3550. At a record length of 1024 words, there are 2(3550) or 7100 pulses/tape.

4. D/A CONVERTER

The D/A converter (SDS DX15, SX11) is used to convert the digital data from the SDS 910 to an analog voltage that is applied to the grid of the CRT of the film recorder. The block diagram below (Figure 10) shows how the D/A converter is used in conjunction with the SDS 910 and the film recorder (all numbers are SDS numbers).

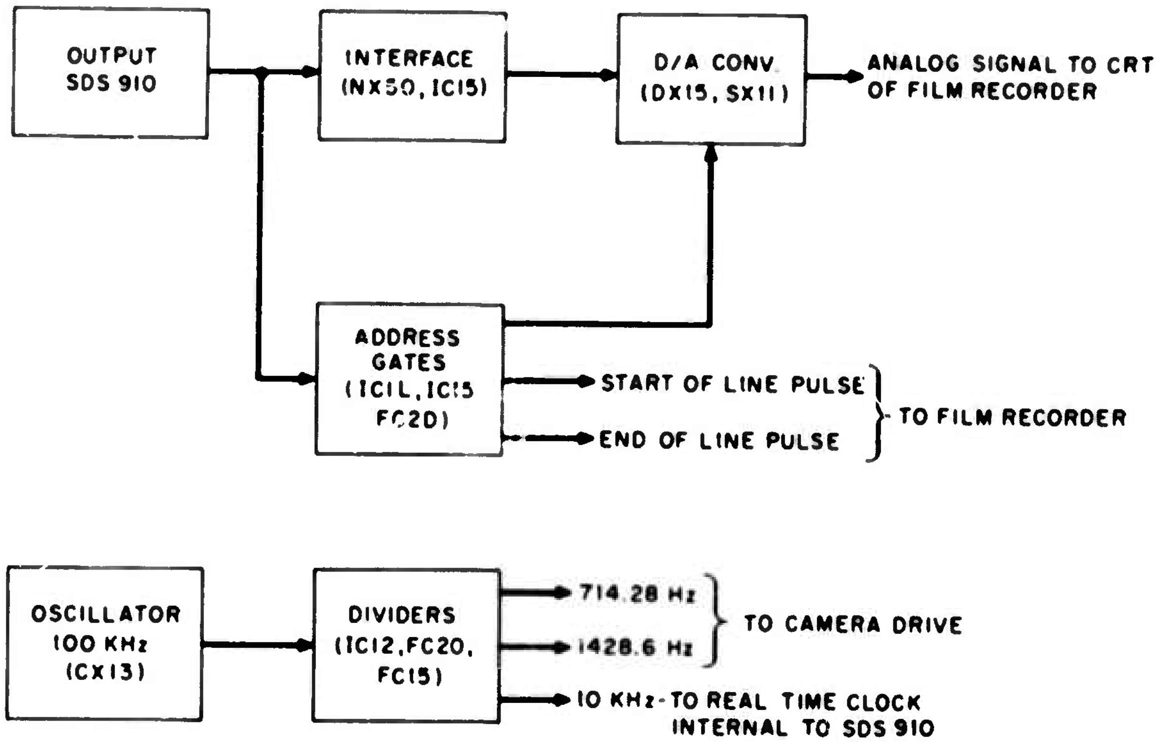


Figure 10 . Block Diagram of SDS and D/A Converter

The output of the SDS 910 are records of either 1024 or 2048 words each. Each word has an amplitude in the range -255 to 256. The D/A converts these digital amplitudes to analog voltages fanging from -10 to +10 volts. The actual signal supplied to the CRT is shown in the diagram below (figure 11) which assumes 2048 words per line.

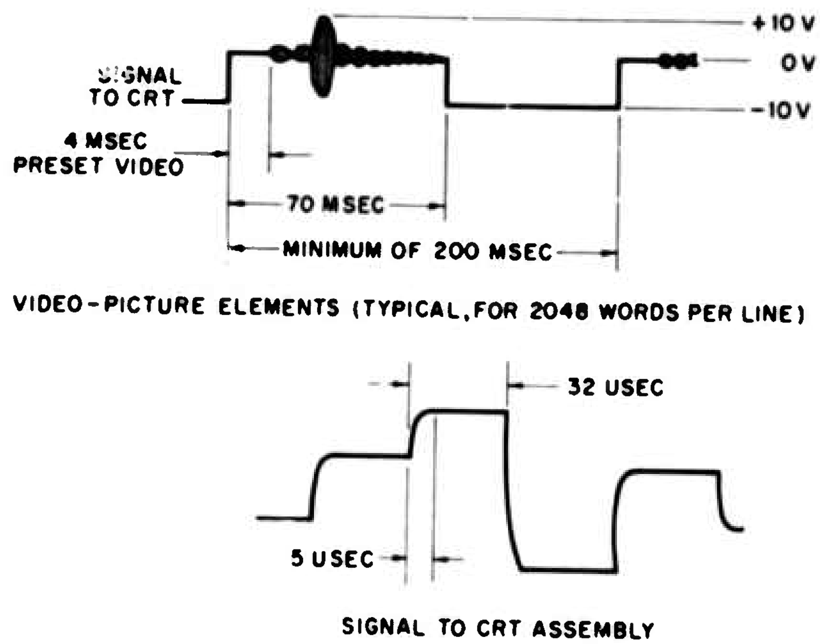
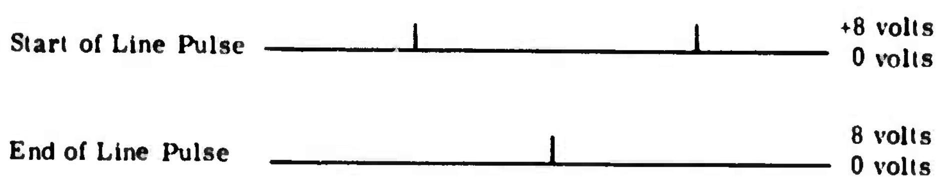


Figure 11 . DA Converter

Thus each computer word (amplitude) is replaced by a constant voltage of 32 usec. The higher the voltage supplied to the film recorder, the brighter the spot will be on the face of the CRT.

In addition, the following line start and end of line pulses are supplied to the film recorder: (See figure 12).



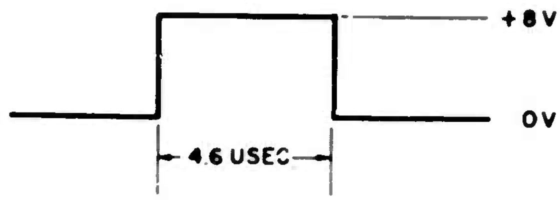


Figure 12 . Line Start and Stop Pulses

5. FILM DEVELOPMENT

Kodak 2490 Film may be processed and loaded in the camera in the presence of red light with wavelengths longer than 5750A°.

A special light source has been assembled which provides variable illumination without affecting film fog level.

Film development is done in a special dark tank which has a spool for 70mm film. Chemicals are poured into and from vents in the top at the cannister.

Steps in the development are listed in sequence.

(1) After film is loaded on spool and placed in cannister with cover secured, fill with developer and continuously agitate. Time in developer is taken from gamma vs. time curve. (Figure 13).

After elapsed time the developer is poured out. If close time control is desired it will be necessary to remove the cannister cover in red light and pour the solution out rapidly and proceed immediately to Step 2.

(2) Fill cannister with KODAK stop-bath, pour out after approximately 25 seconds. Because of the long time required to fill and empty the cannister with the top on it is recommended that this be done with top off in red light. The purpose of this step is to stop the action of the developer. Instead of KODAK Stop Bath the cannister may be flushed with water for approximately 10 seconds or longer and poured out.

(3) Fill cannister with FIXER Solution. The time for KODAK Fixer or KODAK Fixing Bath F-5 is 2-4 minutes. For KODAK Rapid Fixer allow 2-3 minutes. Pour out. The film is now no longer sensitive to room light.

(4) Flush cannister with 65-95°F water for approximately 30 seconds and pour out.

(5) Fill cannister with Hypo-cleaning Agent for 1-2 minutes. Pour out.

(6) Wash film in running water for 2-4 minutes.

(7) (Optional) To avoid water spots and streaks on the film the negative can be washed in photo-flo solution before drying.

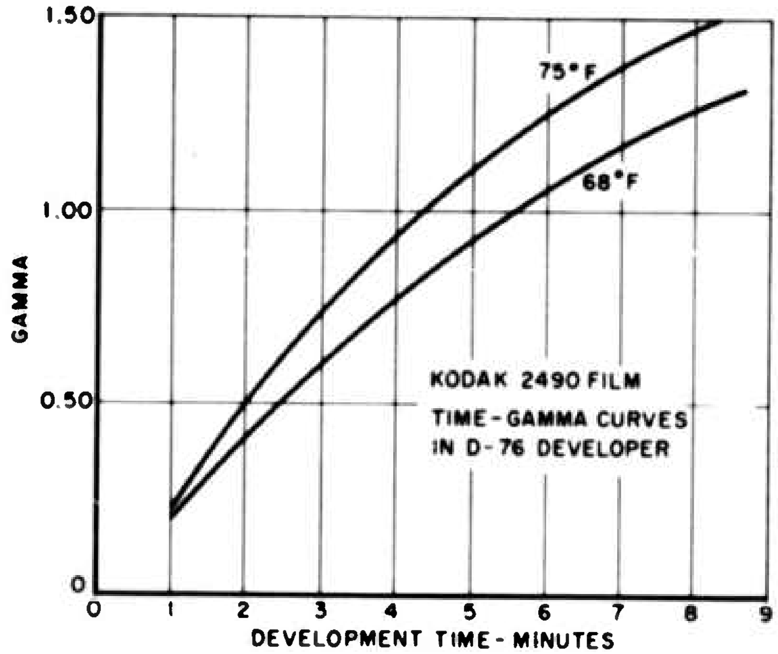


Figure 13 . Gamma vs Development Time

6. COMPUTER SOFTWARE

There are two main programs that are used to process calibrated radar data and put it in a form suitable for use by the film recorder and the optical processor. The first, the Optical Processor Program, is used on the UNIVAC 1108 computer. It accepts pulses of radar date, interpolates each pulse to produce 2048 or 1024 points, applies the spatial carrier with phase, and adds a bias level. This data is then written, a pulse at a time, on a tape which is in a format compatible with the SDS 910 computer. The second program is used by the SDS 910 computer. It accepts the tape produced by the Optical Processor Program, and alternately reads from the tape pulses of radar data (each pulse is a record of 2048 or 1024 points) and outputs them at a fixed rate to the D/A converter and then to the CRT. In addition, it also provides line start and line stop pulses and camera clock frequencies to the film recorder.

In addition there is a third program, DATA2X, which simulates radar data and outputs it onto tape in the calibrated radar data format. This program is used to test the UNIVAC 1108 software, the SDS 910 software, and the optical recording system when real radar data is not readily available. The program can simulate radar returns from a rotating target of rigidly connected scatterers. At the present time it is capable of simulating up to twenty scatterers each having an arbitrary amplitude, relative phase, and any position relative to center of rotation. These can be rotating at a described rotation rate, and recorded at a given pulse repetition frequency, for any number of pulses. This program also retains the maximum amplitude for any number of pulses. This maximum amplitude is used as a bias level for a frame of pulses by the Optical Processor Program.

More detail concerning all these programs can be found in Appendix III.

SECTION IV

RESULTS OF SUBSYSTEM TEST

1. FILM RECORDER

The film recorder console receives analog signal information which intensity modulates the beam of a CRT. This display is recorded by a continuous 70 mm servo-controlled moving film camera.

The display console receives from its signal source three signals, a pulse to start the CRT sweep, analog video information to be displayed during the sweep, and a sweep stop pulse. The time between sweep start pulses may be varied at will between limits but the video display time is fixed at approximately 68 msec.

A simplified block diagram of the display console is illustrated in figure 14 .

a. Circuit Operation

Upon receiving the sweep start pulse the logic circuitry generates a variety of pulses at different time positions to operate the sweep, unblanking and optical feedback circuits. These pulse waveforms are shown on the system block diagram with the times referenced against the externally provided sweep start pulse at t_o. .0 msec rectangular pulse causes a gate switch to unclamp a Miller integrator which then generates a linear ramp of 70 msec duration. This amplified ramp drives the CRT horizontal deflection coils providing the horizontal sweep trace. The logic circuitry provides a 2 msec pulse which provides switching pulses to operate the optical feedback loop. This loop serves to maintain a constant operating brightness of the CRT tracer by providing a negative feedback loop which has as its reference an input voltage set by the "calibrate intensity" control. This serves to maintain a constant brightness as CRT parameters such as cathode emission and phosphor efficiency are degraded with long term aging effects. At the beginning of the CRT trace a phototransistor views the first 2 msec of the sweep. This 2 msec sample is then held by a "hold" circuit for the duration of the sweep. The output of this sample and hold circuit provides the feedback voltage which is compared against the "calibrate intensity" input by their mutual summing into the input of the video amplifier as shown. The 2 msec display of the sweep viewed by the phototransistor is turned on by the unblanking amplifier which is driven by an output of the logic circuitry shown in the figure. The Brightness control varies the amplitude of the unblanking pulse but this has little effect on the trace brightness because of the intensity stabilization of the optical feedback and serves only to establish loop operating parameters.

The video input is amplified and applied to the CRT control grid. The dynamic range of the CRT intensity modulation is controlled by the video gain control.

Dynamic focussing is applied to the CRT to compensate for spot nonuniformity across the trace. Due to the geometry of the tube the focal length of the electrostatic lens must be varied as the electron beam is swept across the tube face to maintain optimum focus. This focus correction is a 120 volt parabolic pulse which is applied to the CRT in series with the 2.2 kv focus potential.

b. Transfer Curves

Curves are provided which enable the operator to determine the recorded film characteristics. Figure 15 , percent transmission versus calibrate intensity setting, gives a plot of '0' level operating points for various control settings. This curve is valid for 2490 film processed for unity gamma, and a pulse density of 16.4 line pairs/mm on the film.

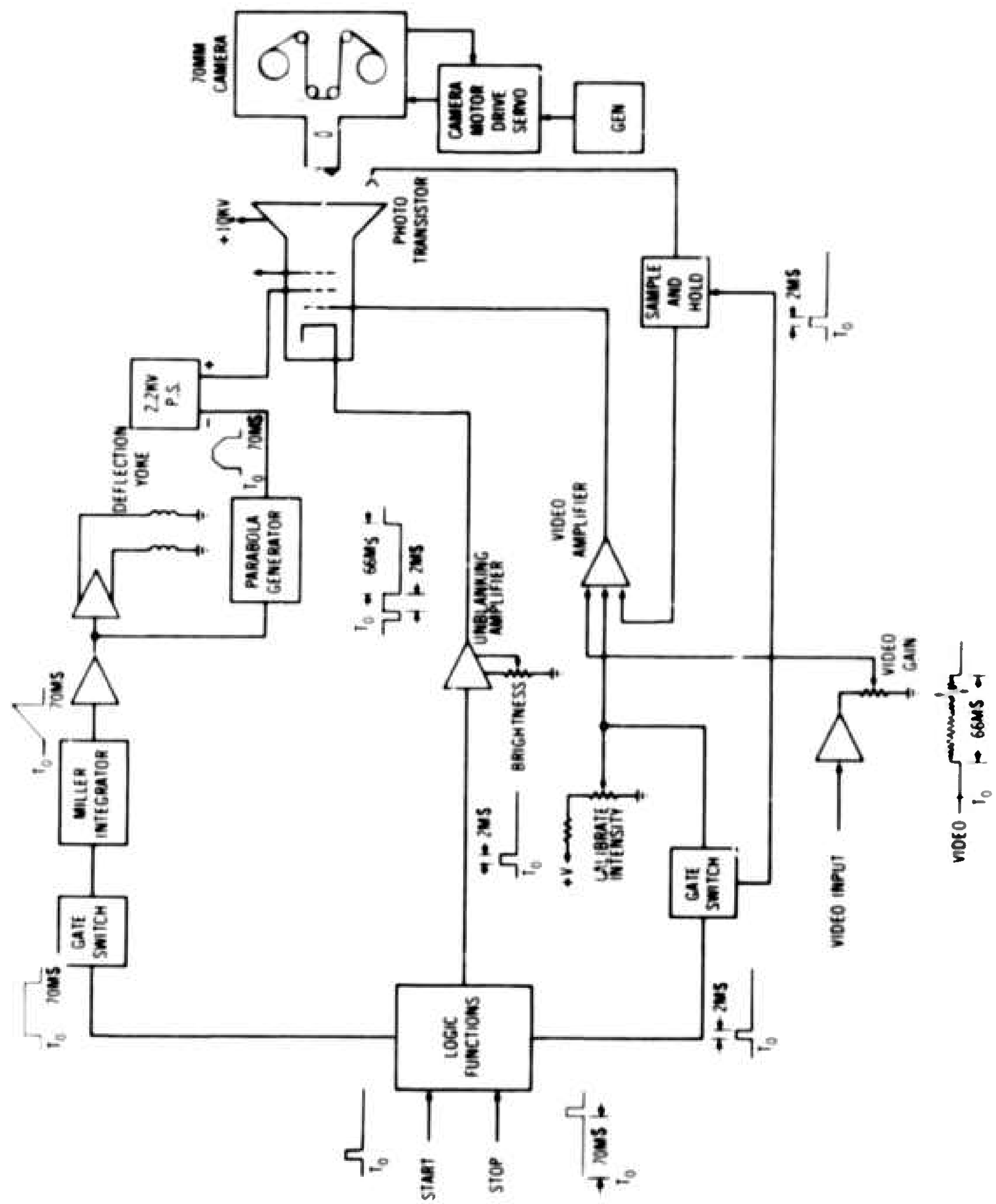


Figure 14. Film Recorder Block Diagram

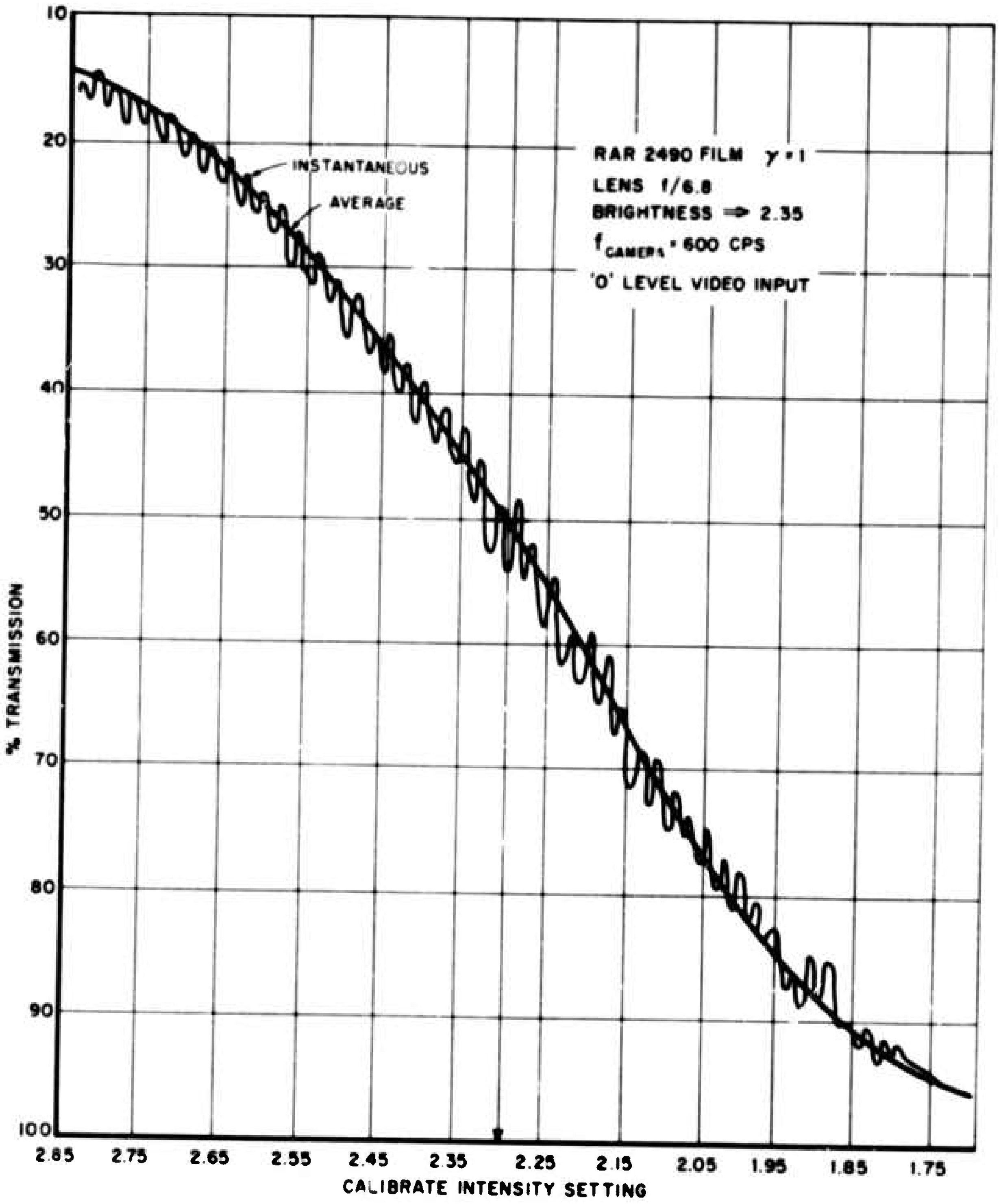


Figure 15 . Calibrated Intensity Setting

Figure 16 gives the dynamic swing of the video input signal in terms of calibrate intensity setting versus gain setting and may be read graphically. It should be pointed out that the operating conditions are subject to other effects such as CRT and film grain noise, resolution limits and line density on the film. At present the CRT spot has an astigmatic focus with approximately a 2:1 height to width ratio. With a 600 cps camera input frequency or a line density of 16.4 lines/mm there is line to line overlap which yield less than a 5% increase in light transmission between pulses on the film negative. This should effectively suppress the Raster effect and relieve the impression that there is insufficient exposure of the film. The line density recorded on the film is given by

$$\text{lines/mm} = \frac{1.96 \times 10^3}{\tau \times f_c}$$

where τ is the period between line start pulses in seconds and f_c is the camera input frequency in cps.

c. Spatial Resolution

There is some question as to the determination of the system spatial resolution along the sweep. Measurements indicate that the CRT spot size is approximately 1.3 mils and is within the expected limits. There is spot growth over the dynamic range of the system and thus the resolution limit imposed by the CRT spot size is a function of the instantaneous amplitude of the input video signal and the '0' level setting of the calibrate intensity control. With a modulation transfer of 0.5 the resolution on the film may be said to be approximately 55 lines/mm due to spot size alone.

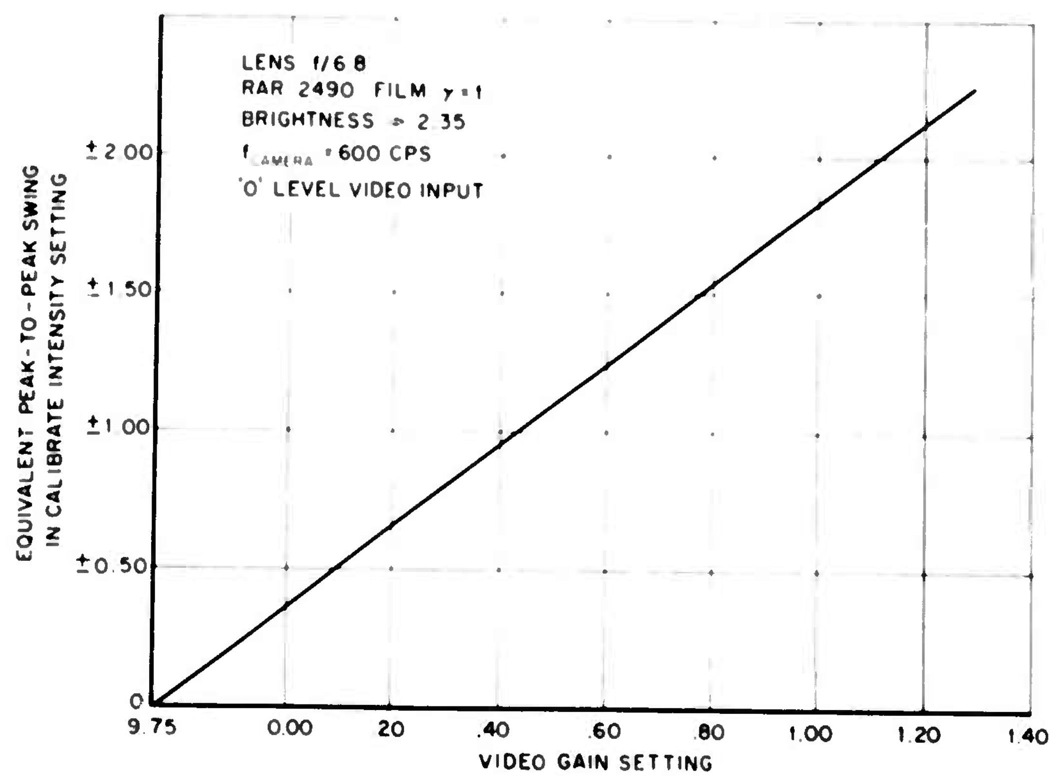


Figure 16 . Dynamic Swing of the Video Input Signal

The sample time of the D/A converter yield 4.9×10^{-2} mm/sample on the CRT face or 2.45×10^{-2} mm/sample on the film due to 2:1 reduction by camera optics. It is not meaningful to speak of line pr. resolution as limited directly by the spatial sample length but to find how many samples are required per unit length to yield the desired results in the optical processor output plane.

The combined f/6.8 camera lens and 2490 film with unity gamma has a modulation transfer function of 0.5 at approximately 20 lines/mm with the modulation transfer being defined as

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

where I is the transmitted light intensity through the film negative. The system spatial resolution is a function of all the above mentioned items.

d. Sweep Repeatability and Linearity

It is necessary to provide close control of the beam as it is swept across the CRT face. In order to minimize phase jitter of the special carrier, special care must be taken to reduce the electrical noise level in the deflection circuits. After evidence had shown that such disturbances in the film record could not be tolerated, special efforts were made to reduce circuit noise levels until there was no longer any observable phase disturbances recorded on the film. Acceptable noise levels would produce jitter in the trace which is on the order of one spatial sample of the D/A converter or less. This requires spatial noise of 5×10^{-2}mm or less. It was difficult to translate this to circuit noise levels because the voltage observed is related to the deflection coil current by

$$i_c = \frac{1}{L_{coil}} \int v\,dt$$

and, thus the duration of a noise pulse is as important as its magnitude.

Effective noise that appears in the video chain has been reduced below the gross fog level in the recorded film. Spurious noise, and ringing transients that are of short duration are effectively suppressed both by electrical filtering and by the effective low pass effect of the speed of the film emulsion.

SECTION V

RESULTS OF SIMULATED TARGET TESTS

1. BACKGROUND

In addition to sub-system and system tests which were performed to check out the equipment, a number of moving targets were simulated for subsequent processing in an optical correlator to demonstrate system operation. In these tests, the returns from point scatterers were simulated as the scatterers rotated relative to the radar line of sight. These returns were simulated once each 0.01° rotation, and written onto film. The films were then sent to the Institute of Science and Technology of the University of Michigan where they were processed in their experimental optical correlator. The processed film demonstrated the capability of the optical processor (both the film recorder and the optical correlator) and was useful in pointing out abnormalities in the recording process, as well as in demonstrating successful recording for processing

2. FILM FRAMES PRODUCED

A listing of the film frames provided to IST is given in Table 2, all of which used simulated data. Further explanation of the target configuration and orientation for which the simulated data was generated follows the table.

TABLE 2 FILM FRAMES FOR IST

FRAME NO.		TYPE OF FRAME	DATA TAPE REQUIRED
1	SPATIAL CARRIER ONLY	10 line pr/mm	One Record or sweep
2		12 line pr/mm	One Record or sweep
3		14 line pr/mm	One Record or sweep
4		16 line pr/mm	One Record or sweep
5	2 SCATTERERS FORMAT 1	CASE A FIGURE 17	One frame 511 pulses 5.11° rotation
6		CASE B FIGURE 17	One frame 511 pulses 5.11° rotation
7		CASE C FIGURE 17	One frame 511 pulses 5.11° rotation
8		CASE D FIGURE 17	One frame 511 pulses 5.11° rotation
9	2 SCATTERERS FORMAT I	CASE A FIGURE 17	One frame 511 pulses 5.11° rotation
10		CASE B FIGURE 17	One frame 511 pulses 5.11° rotation
11		CASE C FIGURE 17	One frame 511 pulses 5.11° rotation
12		CASE D FIGURE 17	One frame 511 pulses 5.11° rotation
13	2 SCATTERERS FORMAT II	CASE A FIGURE 17	One frame 511 pulses 5.11° rotation
14		CASE B FIGURE 17	One frame 511 pulses 5.11° rotation
15		CASE C FIGURE 17	One frame 511 pulses 5.11° rotation
16		CASE D FIGURE 17	One frame 511 pulses 5.11° rotation
17	FORMAT I	11-ELEMENT IST TARGET	6 frames continuous 5/500 pulses
18	FORMAT II	11-ELEMENT IST TARGET	6 frames continuous 5/500 pulses

Frames 1 through 4 were recordings of the spatial carrier only with the carrier being introduced in the sweep direction. A frame of about 50 mm length was recorded by reproducing this trace repetitively. For these frames several values of spatial frequency were used as indicated in the table.

Frames 5 through 16 were recorded using a target having two point scatterers separated by 20 feet. The orientation of this target was varied as shown in Figure 17 in order to get several types of recordings.

Four target orientations are shown in this figure and labeled Cases A, B, C and D. In all these, data was simulated as the target rotated through 5.11° (or approximately 0.1 radian). The orientations shown in the figure are for the center of the recording interval.

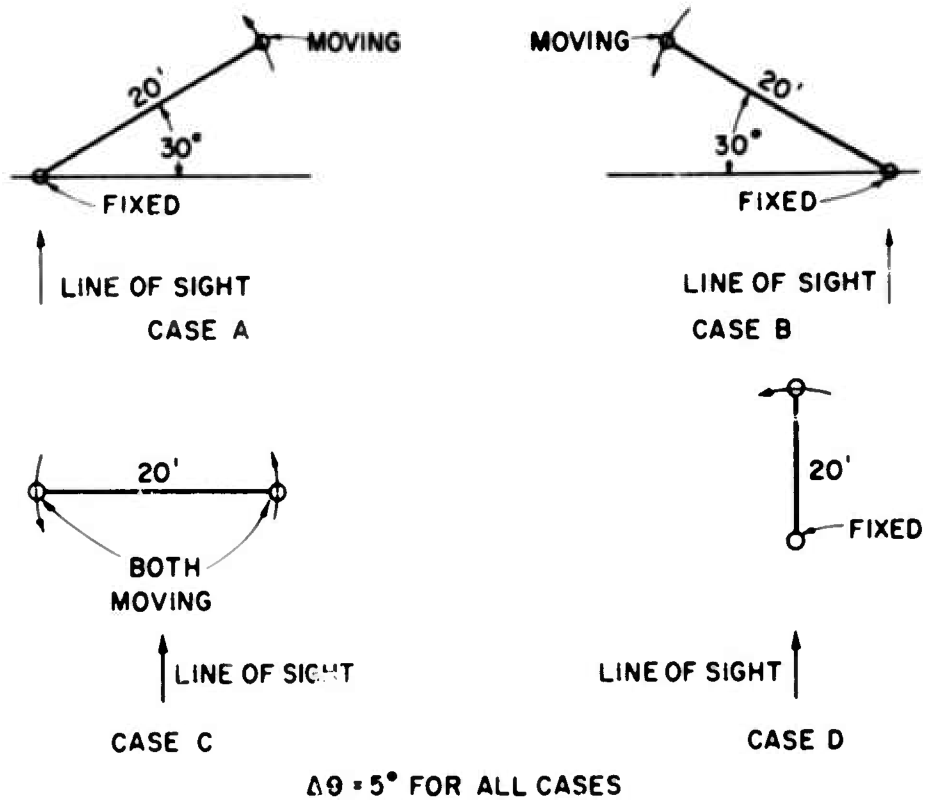


Figure 17. Target Orientation

Frames 9 through 12 duplicated frames 5 through 8. Processing of frames 5 through 8 revealed improper recording of data and the necessity for further equipment adjustments. After adjustments were made to the equipment, frames 9 through 12 were recorded. These frames had data recorded more densely than before, having a frame length of approximately 20 mm rather than the original 50 mm with the same number of pulses being used.

Frames 13 through 16 were also recorded using the simulated data for the target orientations shown in Figure 17, except here Format II is used. In Format II the Fourier transform of the range response coefficients is taken prior to recording. Processing of these frames revealed that there are excessive range sidelobes appearing in the correlator output, which preclude best processing in Format II. The source of the range sidelobes has been traced to a variation in the spatial frequency with distance along the trace (across the film). The cause of this problem has not been firmly established at the writing of this report. However, as may be seen in the processed output included later in this section, there is recognizable information in the output for the simple target configuration, although serious degradation in sidelobes is most evident.

Frames 17 and 18 were recorded using an approximate model of the Michigan 11-element target. The target model used is shown in Figure 18, in which the position of eleven corner reflectors from the center of rotation of the target is given by distance (in feet) and angle (in degrees). The simulation was performed assuming a rotational rate of 1.22 degrees/second. For the simulation, the target was rotated through 30° aspect change. Frame 17 was recorded using Format I; frame 18 was recorded using Format II.

Processing of frame 17 yielded no output. However, the cause of this is in the normalization procedure used. When this target is at the aspect shown (with one cross arm normal to the line-of-sight), seven of the corner reflectors are at the same range and a large return is observed at the radar. As the target moves, there is constructive and destructive interference and the amplitude of the return is much lower than for the above aspect. The ratio of the

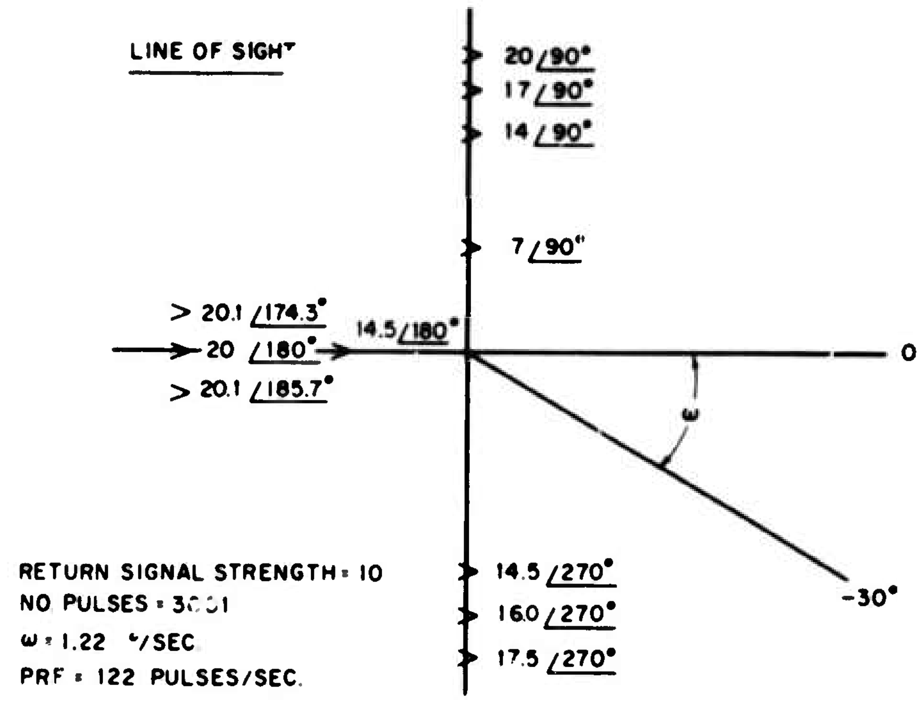


Figure 18. Michigan 11 - Element Target

maximum return to the peak return at other aspects is quite large. Since the d. c. bias is chosen according to the maximum return and the fluctuation about this value corresponds to the returns at other aspects, there is in this frame insufficient contrast in the recorded information to yield an output. A simple expedient to avoid this problem is to normalize the returns only over an interval of time corresponding to the integration interval. Then only those intervals containing large returns are examined with their corresponding high bias. Adjacent intervals containing lower return amplitudes are then processed with a different bias and outputs will result.

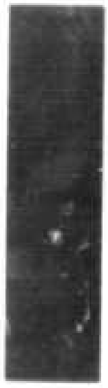
Frame 18, recorded in Format II, could be processed to yield a recognizable output. Although the high range sidelobes are apparent in this figure also, the target is clearly identifiable.

3. PROCESSED OUTPUTS

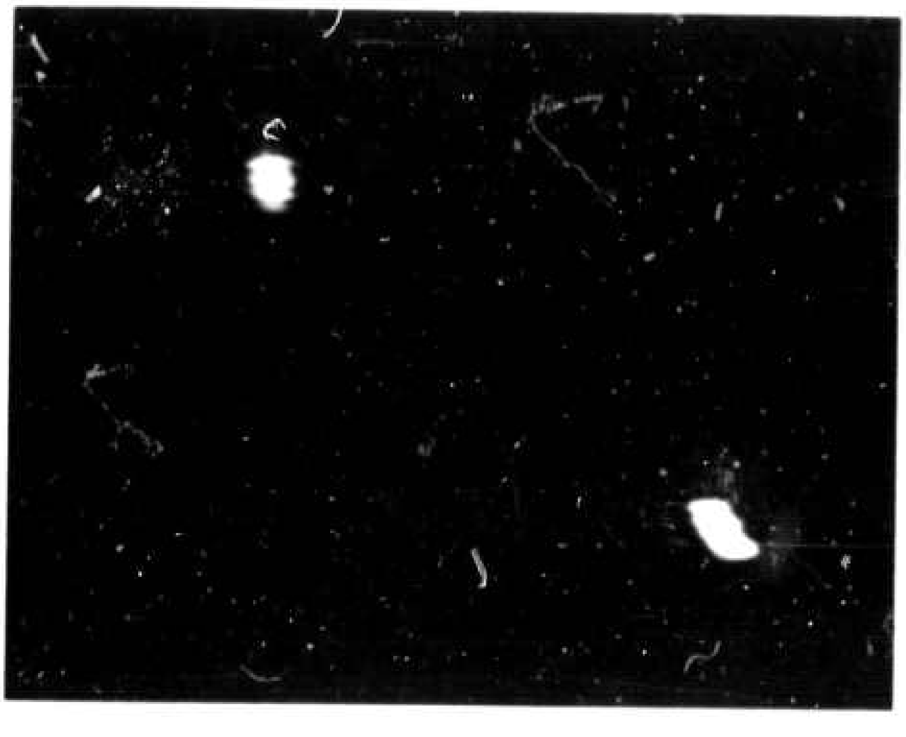
Three processed outputs are shown in this section, two for dumbbell configuration and one for the Michigan 11-element target. These are given in Figures 19, 20, and 21 . In each of these figures, a sample of the recorded film is shown and an enlargement of the processed output is shown. The output films were all made by the University of Michigan. The high range sidelobes due to recording in Format II are observed in the processed output of Figures 20 and 21 .

4. CONCLUSIONS

The results obtained with the simulated targets demonstrate conclusively the feasibility of optically processing synthetic spectrum data. Although adjustments need to be made to the present film recorder to optimize the recording of Format II data, the equipment can now be used to process data in both Formats I and II.

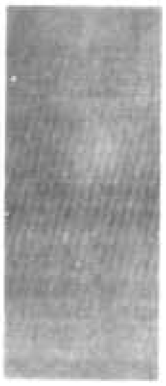


(a) Sample of Recorded Film Frame

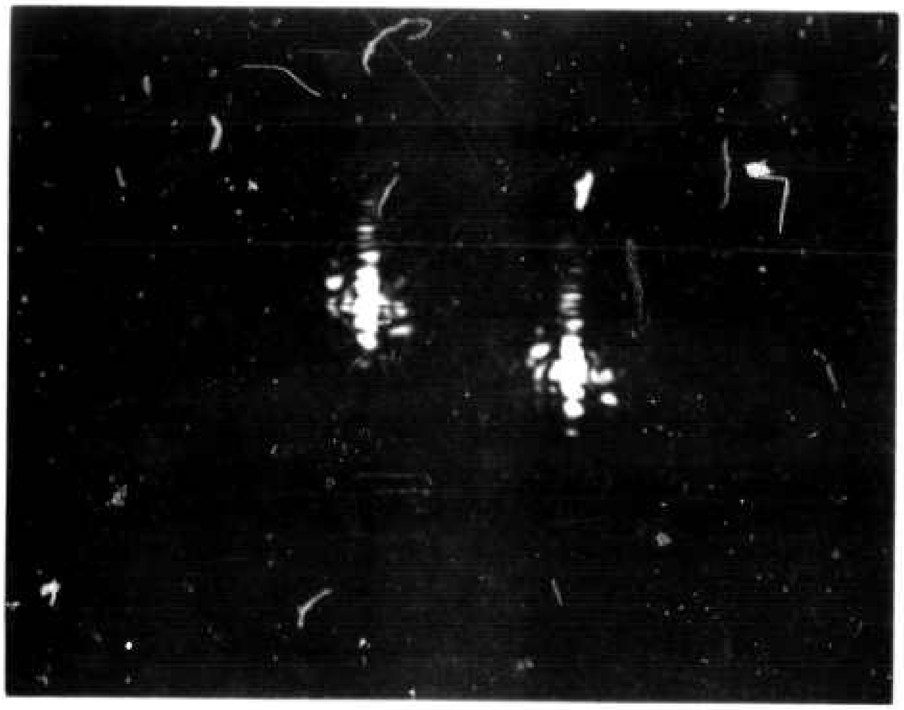


(b) Processed Output (ISI)

Figure 19. Frame 9, Format I

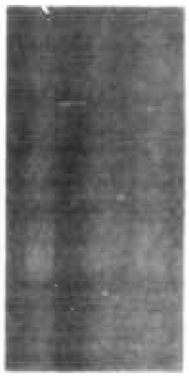


(a) Sample of Recorded Film Frame

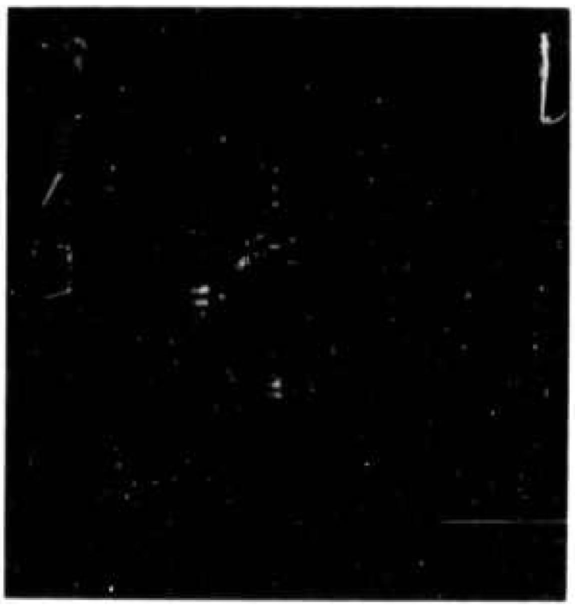


(b) Processed Output (ISI)

Figure 20. Frame 14, Format II



(a) Sample of Recorded Film Frame



(b) Processed Output (ISI)

Figure 21. Frame 18, Format II.

SECTION VI

CONCLUSIONS AND RECOMMENDATIONS

1. EQUIPMENT CAPABILITIES

The film recorder equipment built on this contract has met all design goals set forth at the beginning of the program and, in addition, it has been operated satisfactorily beyond the design specifications in order to improve the quality of the processed data. The equipment can write from 1024 to 2048 spots per line. These are recorded on a 50 mm width of film. It has been used to write up to approximately 25 line pair per millimeter along the length of the film. The film drive control frequency can be as low as 600 Hz (714 Hz was design goal) and still achieve reliable operation. The stability of the trace is regulated so that reliable recording of the data can be achieved. The equipment can be operated to record approximately 3550 pulses in a continuous strip (which, at a line writing density of 25 lines/mm, is a length of about 5 1/2 inches).

2. LIMITING FACTORS

The SDS-910 computer unit used with the film recorder has only one magnetic tape unit, which limits the number of pulses which can be recorded continuously on film. However, the film recorder has no other limitation in recording a continuous record up to the length of film on a spool, which is 60 feet. By use of more than one input tape unit these would be no difficulty in recording radar data even if the number of pulses on a run exceeded 3550.

At present, the quality of the recording in Format II is not sufficiently good to give adequate dynamic range for processing returns from complex targets. Similarly, a minor modification needs to be made to the software for Format I such that the maximum return amplitude is obtained for data blocks corresponding to the integration interval, rather than for a complete run. This can be easily accomplished. For processing of radar data, both of these limitations will have to be corrected to achieve best results in the two formats.

To write data more compactly than was originally the design goal, the speed of the film drive unit was decreased. This required that the control frequency to the film drive unit be lower than 714 Hz. In order to obtain a frequency lower than this, an external source was used. To continue recording at the lower speed the external source will be required. Modification could be made to the timing circuitry to obtain the lower control frequency if continued operation is desired here. Present and planned usage of the film recorder using the external source will not hinder recording and processing of radar data.

3. FOLLOW-ON USE OF EQUIPMENT

Whereas the basic feasibility of optically processing synthetic spectrum data has been demonstrated on this contract using simulated data, there are several uses of the equipment which have not been attempted as yet. Foremost, of course, is the processing of data recorded by the radar, which was not available during the period of this contract. Also a number of simple geometric shapes could be used in a simulation program to demonstrate not only the capabilities of the optical processing but also the capabilities of having a time history of the output from which the target configuration may be deduced.

It is recommended that additional experimentation be continued to study the two-dimensional imaging of geometric configurations of interest, especially with regard to the time history of the correlator output. In addition, the film recorder and optical correlator should be used to process radar data. Utilization of the equipment in this manner will provide important information on the quality of processed data available and how it may be used most efficiently. In addition, some systematic procedures for information sorting and interpretation should be gained by this approach.

APPENDIX I

EQUIPMENT OPERATION PROCEDURES

IA SDS-910 OPERATIONAL PROCEDURE

IB FILM RECORDER OPERATIONAL PROCEDURE

APPENDIX IA

SDS-910 OPERATIONAL PROCEDURE

A: Action C: Comment

A1. Turn computer power on and connect cables (See A1. 1 for details).

C1. Clock pulses for camera drive will be present anytime the computer power is on.

A2. With Magnetic Tape Transport off, mount reel of magnetic tape and thread tape. Turn transport power on and position tape at load point. (See A2. 1 for details).

A3. Set transport switches to
Unit Number - 3
Density - 556

and set AUTO mode.

A4. On computer, reset all breakpoint switches.

A5. Load Optical Processor Program into computer from paper tape using "Standard Fill Procedure". (See A5. 1 for details)

C5. Program is now in the idling mode and ready for operation. The mode of operation is determined by the breakpoint settings as described in next paragraph.

A6. 0 <u>Breakpoint Operation</u> (See Table 3.)

With all breakpoints in the reset position, the program is in the idling mode.

A6. 1 If the operator wants to change the timing between the Line Start pulses, he should - -

A6. 1. 1 Set and then reset breakpoint 1. - - -
The light on the typewriter will come on.

A6. 1. 2 Type a four digit number equal to the desired number of milliseconds. - -
The program will now automatically return to the idling mode.

C6. 1. 2 The number of milliseconds is initially set to 200 ms. The new number must be equal to or greater than this number.

A6. 2 With Breakpoint 2 Set, the operation is the same as in the normal operating mode except that the magnetic tape is not read. Line Start and Line Stop pulses are generated as in the normal mode. Whatever data is in the record storage area (e. g. the last record read from magnetic tape) will be output respectively.

A6. 3 Set breakpoint 3 when normal operation is desired. Normal operation will continue as long as breakpoint 3 is set or until the end of file mark on the magnetic tape is reached. When the end of file mark is reached, all outputs will cease until breakpoint 3 is reset and set again.

When the end of the reel of magnetic tape is reached the typewriter will type a message indicating same and the program will halt.

TABLE 3. BREAKPOINT OPERATION

Set Breakpoint No. *	Operation
3 (Normal)	Program alternately reads a block off of magnetic tape and then outputs it at a fixed rate to the CRT. Line start and line stop pulses are output at a preset rate. All outputs (except camera clock frequency) are inhibited at the end of a frame until Breakpoint 3 is reset and set again. An end of magnetic tape reel signal will inhibit all further operation.
2 (Test)	The magnetic tape is not read. The program repeatedly outputs the last block that was read from magnetic tape. All timing is exactly the same as when breakpoint 3 is set.
1 (Set and Reset immediately) (Change Timing)	A new number to preset the number of milliseconds between line start pulses is accepted at this time.

* Breakpoint 3 inhibits 2 and 1. Breakpoint 2 inhibits 1.

A2. 1 MAGNETIC TAPE

OPERATING INSTRUCTIONS

A2. 1. 1 GENERAL.

This section contains operating instructions for the Tape Transport and describes the location and function of electrical and mechanical front panel operating controls and tape threading procedures.

A2. 1. 2 OPERATING CONTROLS, INDICATORS, AND MECHANISMS.

All controls, indicators and mechanisms required for operation of the Tape Transport System are located on or adjacent to the front panel of the tape transport. These controls are listed in Table 3. Interface signals for remote operation are shown in Table 4

Note

When changing speed or direction manually, it is required that the STOP pushbutton be depressed before making a new selection.

TABLE 4. FRONT PANEL CONTROLS

Control Pushbutton	Description	Function
OFF (TRANSPORT OFF)	Momentary contact pushbutton switch with integral lamp	When the switch is operated and pushbutton is lit, tape transport operation is inhibited except for the tension arm retraction motor. OFF button may be operated only if machine is first placed in STANDBY condition.
STOP	Momentary contact pushbutton switch with integral lamp	When switch is operated and pushbutton is lit, tape transport is enables and arm retraction motor is inhibited. Any previously operated locking pushbutton switches are released.
FAST FORWARD	Locking pushbutton switch with integral lamp	Provides manual selection of tape motion in the forward direction at high speed.
FORWARD	Locking pushbutton switch with integral lamp	Provides manual selection of tape motion in the forward direction at normal speed.
REVERSE	Locking pushbutton switch with integral lamp	Provides manual selection of tape motion in the reverse direction at normal speed.

TABLE 4. FRONT PANEL CONTROLS (Cont)

Control Pushbutton	Description	Function
FAST REVERSE	Locking pushbutton switch with integral lamp	Provides manual selection of tape motion in the reverse direction at high speed.
AUTO	Locking pushbutton switch with integral lamp	Transfer all control and indicator functions to remote locations.

A2.1.3 TURNING POWER ON.

Before tape threading can take place, the circuits associated with automatic tension arm retraction must be energized.

Power is remotely controlled by computer power switch.

a. Threading Tape

When threading tape, it is recommended that only the end of the tape is handled. Avoid touching the tape if any other point.

The steps in paragraph 3-3 must be satisfied first before proceeding to load tape.

CAUTION

To prevent tape break at end of run, do NOT slip free end of tape into slot in take-up reel core or in any way fasten the free end to the take-up reel.

b. Load Point Marker. It is desired to begin operating at the marker, press the FORWARD pushbutton on control unit until marker reaches the take-up reel. Press STOP and then REVERSE pushbutton. Tape will reverse direction and stop on the LOAD POINT marker.

c. Tape Removal. When reflective markers are used, the tape will stop on these markers when they reach the sensor and the driving pinch roller will be released. The transport will be inhibited from driving tape passed the marker once it has been stopped by this marker.

CAUTION

When the tape has been stopped from a fast mode of operation, it is necessary to wait for two to five seconds before selecting another mode. This will allow time for the capstans to slow down to normal speed.

To remove tape, select a fast reverse or reverse mode until tape reaches LOAD POINT marker. If no marker is used, stop the transport when approximately ten feet of tape remains on the pay-out reel.

- Press OFF pushbutton on control unit
- Rotate reels by hand until tension arm rollers just pass the stationary rollers
- Open read/write head cover, vacuum buffer cover and press arm retraction button
- When tension arms stop, wind the remaining tape onto the take-up reel while turning the pay-out reel to avoid stretching the tape.
- Close read-write head cover, vacuum buffer cover and remove tape reel.

A5.1 LOADING OPTICAL PROCESSOR PROGRAM

A5.1.1 Standard Fill Procedure

a) Set Operating Switch to IDLE.
b) Press Start Button
c) Mount paper tape in photo-reader. *
d) Place Operating Switch in RUN position
e) Raise and release Fill Switch.

A5.1.2 Standard Restart Procedure

a) Set Operating Switch to IDLE.
b) Press Start Button
c) Move the Operating Switch to the STEP position and then to the RUN position.

A5.1.3 Standard Turn-Off Procedure

a) Set Operating Switch to IDLE
b) Press Start Button
c) Press POWER Button.

* Tape feeds from left to right, in the direction of the arrow on the tape. The "TOP" side of the tape should be up. The tape should lie flat in the reader under all guide posts. Knob on photoreader should be turned from LOAD to RUN after mounting tape.

APPENDIX IB

FILM RECORDER OPERATIONAL PROCEDURE

A1. Turn power on SDS-910 computer

C1. See operational procedure for computer

C1. 1 Obtain start-stop pulses from computer (set Break Point 2).

A2. Turn power on for cathode ray tube assembly and film drive power supplies.

C2. Allow a minimum of 5 minutes to obtain thermal stability in equipment.

A3. Turn on power for film drive frequency reference.

C3. This signal was previously supplied by SDS-910. However to obtain greater density in line spacing this modification was incorporated in the operating procedure.

A4. Set video gain, calibrate reference and brightness controls to desired values.

C4. Desired values are obtained from figures 15 and 16.

A5. Set film drive range, push film drive motor control and synchronize film drive to desired film speed.

C4. Film drive speed now controlled by film drive frequency reference.

A6. Remove start-stop pulses from computer after examination of video signal.

A7. Set breakpoint 3 on computer.

C7. 1 This will start recording of data on film.

C7. 2 Should a missing line stop pulse occur, reset computer as quickly as possible.

A8. When frame has been computed, push fast film advance.

C8. Allow to run for 5-10 seconds to give a trailing leader on the film.

A9. Push film drive motor control.

C9. This will stop film.

A10. Remove camera assembly and develop film.

APPENDIX II

FORMAT ERROR ANALYSIS

ERROR ANALYSIS FOR THE OPTICAL RECORDING AND PROCESSING OF SYNTHETIC SPECTRUM DATA

by T. C. Griskey

August 5, 1966

The work reported herein was performed on contract AF30(602)-4287, ARPA Order 837.

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ERROR ANALYSIS FOR THE OPTICAL RECORDING AND PROCESSING OF SYNTHETIC SPECTRUM DATA

I. INTRODUCTION

The purpose of this report is to determine the errors associated with the optical recording and processing of synthetic spectrum data. The sidelobe degradation due to the inaccuracies of the system are calculated for two data formats. The format that yields the best system performance based on its inherent sidelobe degradation is determined from this analysis. A brief mathematical sketch of the system is performed to describe the system. The notation of P. M. Woodward will be used throughout this analysis.[1] This notation is given in Table I.

II. FORMAT DESCRIPTION

1. GENERAL

A brief introduction to the general format of the recorded information will be helpful in understanding the developments which follow. Information obtained on one radar transmitted pulse is recorded on one line across the film. The number of data points per pulse and the number of 'spots' per line is selectable and is denoted by N. Information from two successive pulses is recorded on successive lines adjacent to one another along the length of the film. Variation of the line density is possible in the equipment.

Each line recorded on film is positioned to be perpendicular to the edge of the film by adjusting the yoke of the CRT. Information from each pulse is intensity-modulated onto the CRT phosphor on a line which is regulated quite precisely. Every pulse is written onto the same line on the CRT phosphor. Separation of adjacent lines on the film is controlled by having a uniform film drive speed and delaying the start of writing each line of information.

Information can be recorded on film using two formats. In Format I, the amplitude information from a pulse is used to intensity-modulate the CRT and expose the film. In Format II, the Fourier transform of the amplitude information from a pulse is used to intensity-modulate the CRT and expose the film. This memorandum considers the errors for these two formats:

2. FORMAT I

In Format I, the amplitude of a block of radar pulses on the negative transparency can be represented by:

$$(1) \qquad \tau(x, y) = \text{rect}\left(\frac{x}{Y} - \frac{1}{2}\right) \text{rect}\left(\frac{y}{Y} - \frac{1}{2}\right) \left[A - A_1 \sum_{m=1}^{M} \sum_{n=1}^{N} \left[P(x) \bullet I\left(x - \frac{mY}{M}, y\right) \bullet \text{rect}\, \frac{yN}{Y} \bullet \left\langle \left\{ A_o + A(y) \cos\left[2\pi f y + \phi(y)\right] \right\} \delta\left(y - \frac{nY}{N}\right) \right\rangle \right] \right]$$

TABLE 5. WOODWARD'S NOTATION[1]

Definitions:

$$u(t) = \int_{-\infty}^{\infty} u(\tau)\ \delta(t-\tau)\ d\tau = \int_{-\infty}^{\infty} U(f)\ e^{i2\pi tf}\ df$$

$$u * v = \int_{-\infty}^{\infty} u(\tau)\ v(t-\tau)\ d\tau$$

$$U(f) = \int_{-\infty}^{\infty} u(t)\ e^{-i2\pi ft}\ dt$$

$$\text{rep}_T\ u(t) = \sum_{-\infty}^{\infty} u(t-nT)$$

$$\text{comb}_F\ U(f) = \sum_{-\infty}^{\infty} U(nF)\ \delta(f-nF)$$

$$\text{rect}\ t = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$

$$\text{sinc}\ f = \frac{\sin \pi f}{\pi f}$$

Fourier Transform Pairs:

Waveform	Spectrum	Waveform	Spectrum
$u(t)$	$U(f)$	uv	$U*V$
$c_1u + c_2v$	$c_1U + c_2V$	$\text{rep}_T\ u$	$\lvert 1/T \rvert\ \text{comb}_{1/T}\ U$
$u(-t)$	$U(-f)$	$\text{comb}_T\ u$	$\lvert 1/T \rvert\ \text{rep}_{1/T}\ U$
$u^*(t)$	$U^*(f)$	$U(t)$	$u(-f)$
$u'(t)$	$i2\pi f\ U(f)$	$\delta(t)$	1
$-i2\pi t\ u(t)$	$U'(f)$	$\text{rect}\ t$	$\text{sinc}\ f$
$u(t-\tau)$	$U(f)\ \exp(-i2\pi\tau f)$	$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$u(t)\ \exp(i2\pi\phi t)$	$U(f-\phi)$		
$u(t/T)$	$\lvert T \rvert\ U(fT)$		
$u*v$	UV		

where:

$\tau(x, y)$ is the ratio of the light amplitude after being modulated by the film to the light amplitude before being modulated by the film

x is distance measured along the film

y is distance measured across the film (and is proportional to range)

Y is the width of the film frame being used in both the x and y directions

A is determined by the exposure of the background on the film

A_1 is determined by the exposure associated with the information on the film

M = number of pulses per film frame

N = number of data points per pulse

$P(x)$ = persistence of the CRT phosphor

$I(x, y)$ = amplitude distribution of a dot on the film due to a dot on the CRT

$A_o = |A(y)|_{max}$ = maximum pulse amplitude

$A(y)$ = amplitude information obtained from a pulse

$\emptyset(y)$ = phase shift between pulses due to doppler

f = carrier frequency.

The function, $\left[\text{rect}\left(\frac{x}{Y} - \frac{1}{2}\right) \text{rect}\left(\frac{x}{Y} - \frac{1}{2}\right)\right]$, is unity within the data frame and zero outside it. The delta function, $\delta\left(y - \frac{nY}{N}\right)$, represents sampling of the data, $A_o + A(y) \cos \left[2\pi f y + \emptyset(y)\right]$, at points separated by Y/N. The function, rect $\left(\frac{yN}{Y}\right)$ indicates that these sampled values exist on the film for a distance Y/N. The characteristics of the CRT are taken into account by convolving the persistence of the phosphor P(x) and the distribution of the intensity of a dot I(x, y) with this data.

A possible configuration of the (simplified) optical processor is shown in Figure 22. The Fourier transform of the amplitude information is performed in both the x and y directions. A one-dimensional transformation is then taken on one of the sidebands of the information. A sideband is used here to allow recovery of both amplitude and phase information and to eliminate some of the noise from the system.

The two dimensional Fourier transform of the data can be represented by

$$(2) \qquad E(v, u) = K_1 \int_{-\infty}^{\infty} \tau(x, y) \exp\left[-j2\pi(uy - vx)\right] dx\, dy$$

where:

$v = x_1/\lambda F$

$u = y_1/\lambda F$

λ = wavelength of the coherent light used in the processor

F = focal length of the transforming lens

x_1, y_1 = new dimensions in the transform plane.

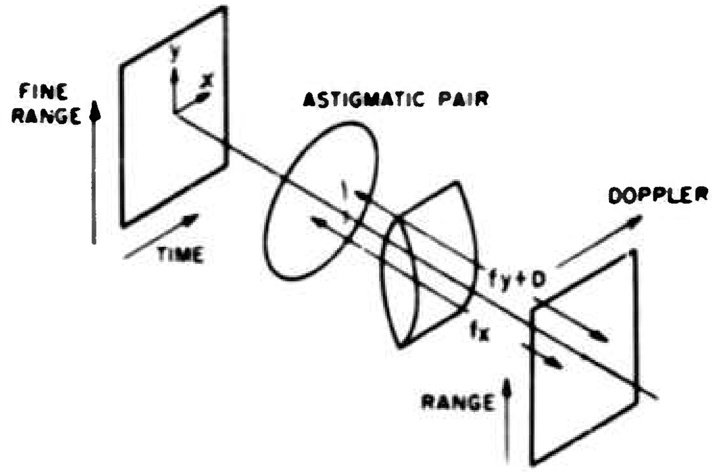

Figure 22. Simplified Processor Diagram

To illustrate the operation of the optical processor let us assume that there are two point targets, separated a distance T, being detected by the radar and that one is rotating about the other.

The amplitude transmissivity of the film for this particular case will be given by:

(3)
$$\tau(x,y) = \text{rect}\left(\frac{x}{Y} - \frac{1}{2}\right) \text{rect}\left(\frac{x}{Y} - \frac{1}{2}\right) \left[A - A_1 \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ P(x) \cdot I\left(x - \frac{mY}{M}, y\right) \cdot \text{rect}\left(\frac{yN}{Y}\right) \cdot \left\langle \left\{ A_o + \text{sinc}\left[\frac{2}{Z}\left(y - \frac{Y}{2} - T\right)\right] \cos(2\pi f y + m\phi) + \text{sinc}\left[\frac{2}{Z}\left(y - \frac{Y}{2}\right)\right] \cos(2\pi f y) \right\} \delta\left(y - \frac{nY}{N}\right) \right\rangle \right] \right]$$

This assumes that the amplitude information is of the form sinc $(\frac{2y}{Z})$ and a range cell is Z units long on the film. The general appearance of this information on the film is illustrated by Figure 23.

Carrying out the operation indicated by Equation (1) we have:

(4)
$$E(v,u) = \left[\text{sinc}\, Yv \exp(-j\pi Yv)\right] \cdot \left[\text{sinc}\, Yu \exp(-j\pi Yu)\right] \cdot \left[A\, \delta(u)\, \delta(v) - A_1 \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ P_1(v)\, I_1(v,u) \exp\left(-j2\pi v \frac{mY}{M}\right) \text{sinc}\left(\frac{Y}{N} u\right) \left\langle \left\{ A_o \delta(u)\, \delta(v) + \text{rect}\left(\frac{Zu}{2}\right) \exp\left[-j\pi(Yu + 2Tu)\right] \cdot \left[\frac{1}{2}\, \delta(u-f) \exp(j\, m\phi) + \frac{1}{2}\, \delta(u+f) \exp(-j\, m\phi)\right] + \text{rect}\left(\frac{Zu}{2}\right) \exp(-j\pi Yu) \cdot \left[\frac{1}{2}\, \delta(u+f) + \frac{1}{2}\, \delta(u-f)\right] \right\} \cdot \exp\left(-j2\pi u \frac{nY}{N}\right) \right\rangle \right] \right]$$

Performing the summation we have:

(5)
$$E(v,u) = K_1 \left[\text{sinc } Yv \exp(-j\pi Yv)\right] \cdot \left[\text{sinc } Yu \exp(-j\pi Yu)\right] \cdot$$
$$\Bigg[A\,\delta(u)\,\delta(v) - A_1 \Big\{ P_1(v)\, I_1(v,u) \,\text{sinc}\left(\frac{Yu}{N}\right) \Big\langle \Big\{ A_o\,\delta(u)\,\delta(v)$$
$$\frac{\sin(\pi Yv)}{\sin(\pi \frac{Y}{M} v)} \exp\left[-j(M+1)\pi \frac{Y}{M} v\right] + \text{rect}\left(\frac{Zu}{2}\right) \exp\left[-j\pi(Y+2T)u\right]$$
$$\cdot \Big\{ \frac{1}{2}\,\delta(u-f) \frac{\sin M\pi\left(\frac{\phi}{2\pi} - \frac{Y}{M}v\right)}{\sin \pi\left(\frac{\phi}{2\pi} - \frac{Y}{M}v\right)} \exp\left[j(M+1)\,\pi\left(\frac{\phi}{2\pi} - \frac{Y}{M}v\right)\right]$$
$$+ \frac{1}{2}\,\delta(u+f) \frac{\sin M\pi\left(\frac{\phi}{2\pi} + \frac{Y}{M}v\right)}{\sin \pi\left(\frac{\phi}{2\pi} + \frac{Y}{M}v\right)} \exp\left[-j(M+1)\,\pi\left(\frac{\phi}{2\pi} + \frac{Y}{M}v\right)\right]\Big\}$$
$$+ \text{rect}\left(\frac{Zu}{2}\right) \exp(-j\pi Yu) \frac{\sin(\pi Yv)}{\sin(\pi \frac{Y}{M} v)} \exp\left[-j(M+1)\,\pi \frac{Y}{M} v\right]$$
$$\cdot \left[\frac{1}{2}\,\delta(u-f) + \frac{1}{2}\,\delta(u+f)\right] \Big\} \cdot \frac{\sin(\pi Yu)}{\sin(\pi \frac{Y}{N} u)} \exp\left[j\pi(N+1)\frac{Y}{N} u\right] \Big\rangle \Big\} \Bigg]$$

The term $\sin(\pi Yu)/\sin(\pi \frac{Y}{N} u)$ causes the spectrum to be repeated in the u direction at integer multiples of N/Y. This is illustrated in Figure 23, where the DC term and the other information on the film appear as bright spots. An aperture can be placed about the lines at u = f or u = -f. In this illustration it is placed at u = -f. A cylindrical lens is now placed at the other side of this aperture and a one dimensional Fourier transform is taken in the u direction. The amplitude information on the other side of the aperture can be approximated by:

(6)
$$E(v,u) = K_2 \;\text{rect}\left(\frac{u-f}{X}\right) \left[\text{sinc } Yv \exp(-j\pi Yv)\right] \cdot \left[\text{sinc } Yu \exp(-j\pi Yu)\right]$$
$$\cdot \Big\{ P_1(v)\, I_1(v,u)\, \text{sinc}\left(\frac{Y}{N}u\right) \Big\langle \Big\{ \text{rect}\left[\frac{(u+f)Z}{2}\right] \exp\left[-j\pi(Y+2T)(u+f)\right]$$
$$\frac{\sin M\pi\left(\frac{\phi}{2\pi} + \frac{Y}{M}v\right)}{\sin \pi\left(\frac{\phi}{2\pi} + \frac{V}{M}v\right)} \exp\left[-j(M+1)\,\pi\left(\frac{\phi}{2\pi} + \frac{Y}{M}v\right)\right] + \text{rect}\left[\frac{Z(u+f)}{2}\right] \exp\left[-j\pi Y(u+f)\right]$$
$$\frac{\sin \pi Yv}{\sin \pi \frac{Y}{M} v} \exp\left[-j(M+1)\,\pi \frac{Y}{M} v\right]\Big\} \cdot \frac{\sin(\pi Yu)}{\sin(\pi \frac{Y}{N} u)} \exp\left[j(N+1)\pi \frac{Y}{N} u\right] \Big\rangle \Big\}$$

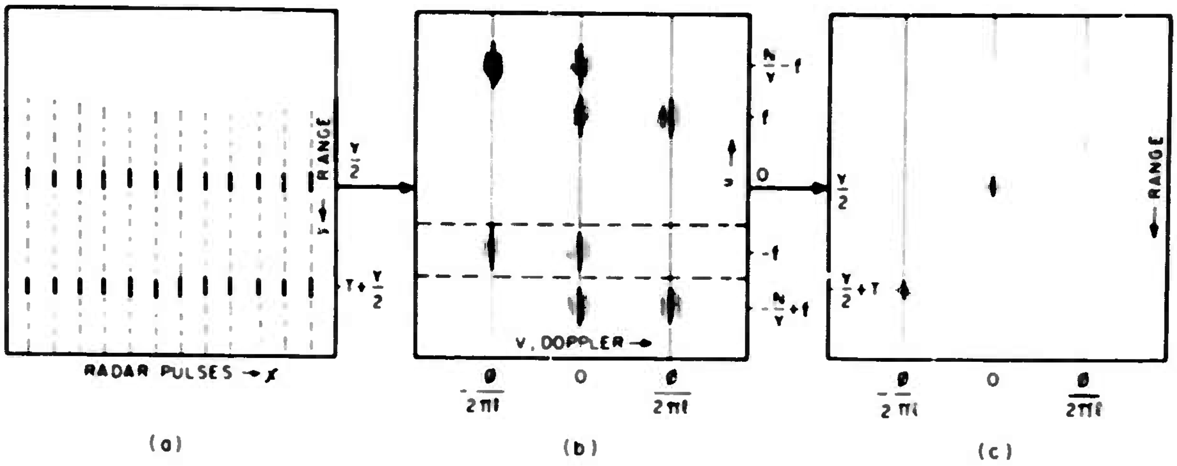

Figure 23. Optical Processing Using the Amplitude Information for Each Pulse

Now taking the one dimensional transformation in the u direction, we have:

$$(7) \qquad E(v,y') = K_3 \left[\mathrm{sinc}(Xy')\exp(-2\pi j y' f)\right] \cdot \left[\mathrm{sinc}(Yv)\exp(-j\pi Yv)\right] \cdot \mathrm{rect}\left(\frac{y'}{Y} - \frac{1}{2}\right)$$

$$\left[\left(P_1(v)\, 1_1(v) 1(y') \cdot \mathrm{rect}\left(\frac{Ny'}{Y}\right) \cdot \left\langle \left\{ \mathrm{sinc}\left[\frac{2}{Z}\left(y' - \frac{Y}{2} - T\right)\right] \exp(+j2\pi y' f) \right.\right.\right.\right.$$

$$\frac{\sin M\pi\left(\frac{\phi}{2\pi} + \frac{Y}{M}v\right)}{\sin \pi\left(\frac{\phi}{2\pi} + \frac{Y}{M}v\right)} \exp\left[-j(M+1)\pi\left(\frac{\phi}{2\pi} + \frac{Y}{M}v\right)\right] + \mathrm{sinc}\left[\frac{2}{Z}\left(y' - \frac{Y}{2}\right)\right]$$

$$\left.\left.\left.\exp(+j2\pi y' f)\, \frac{\sin \pi Yv}{\sin \pi \frac{Y}{M} v} \exp\left[-j(M+1)\pi \frac{Y}{M} v\right] \right\} \sum_{n=1}^{N} \delta\left(y' - \frac{nY}{N}\right) \right\rangle\right]$$

This is displayed on the film shown in Figure 23. It is seen that the two targets are now separated in range and doppler.

C. FORMAT II

The processing of the data using Format II is different from that using Format I in that the Fourier transform of the amplitude information is initially written on the film. The amplitude information on the film for a block of data can be represented by:

$$\tau(x,y) = \text{rect}\left(\frac{x}{Y} - \frac{1}{2}\right)\text{rect}\left(\frac{y}{Y} - \frac{1}{2}\right)\left[A - A_1 \sum_{m=1}^{M}\sum_{n=1}^{N}\left(P(x) * 1\left(x - \frac{mY}{M}, y\right) * \text{rect}\left(\frac{yN}{Y}\right) \cdot \left\langle\left\{A_o + \sum_{p=1}^{P} a_p(y) \cos\left[2\pi(f+T_p)y + n\phi_p\right]\right\} \delta\left(y - \frac{nY}{N}\right)\right\rangle\right)\right] \tag{8}$$

Where $a_p(y)$ is the Fourier transform of the amplitude information from the pth target, T_p represents the delay information from the pth target, and the other parameters are as defined in Equation (1).

The optical processor in this case is shown in Figure 24. Just one two-dimensional transformation is taken in this case.

Let us examine the example illustrated in the analysis of Format I. Assume that there are two point targets spaced a distance proportional to T and that one is moving with respect to the other such that there is a doppler shift of ϕ radians between each pulse. The information on the film can now be represented by:

$$\tau(x,y) = \text{rect}\left(\frac{x}{Y} - \frac{1}{2}\right)\text{rect}\left(\frac{y}{Y} - \frac{1}{2}\right)\left[A - A_1 \sum_{m=1}^{M}\sum_{n=1}^{N}\left(P(x) * I\left(x - \frac{mY}{M}, y\right) * \text{rect}\left(\frac{yN}{Y}\right) \cdot \left\langle\left\{A_o + \text{rect}\left(\frac{y}{Y} - \frac{1}{2}\right)\left[\cos 2\pi(f+T)y + n\phi + \cos 2\pi fy\right]\right\} A_{m,n}\, \delta\left(y - \frac{nY}{N}\right)\right\rangle\right)\right] \tag{9}$$

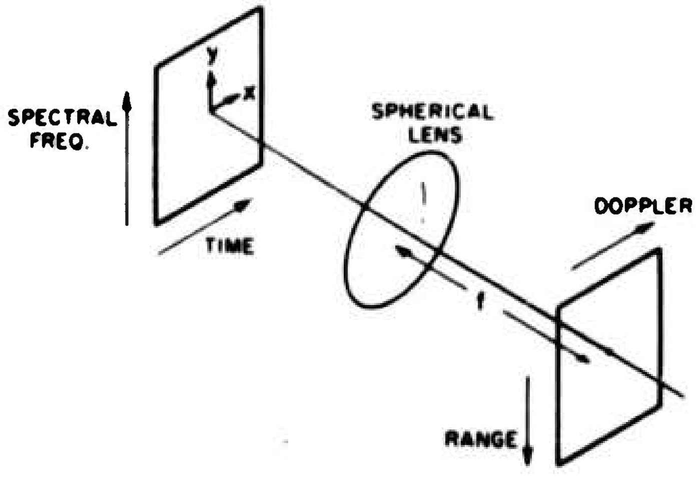

Figure 24. Simplified Processor Diagrams

This information is illustrated by Figure 25. Now taking the two dimensional Fourier transform indicated by Equation (2) we have:

(10)
$$E(v,u) = K_1 \left[\mathrm{sinc}(Yv)\exp(-j\pi Yv)\right] * \left[\mathrm{sinc}\,(Yu)\exp(-j\pi Yu)\right] *$$
$$\Bigg[A\,\delta(u)\,\delta(v) - A_1 \sum_{m=1}^{M}\sum_{n=1}^{N}\Big(P_1(v) I_1(v,u)\exp(-j2\pi \frac{mY}{M} v)\,\mathrm{sinc}\,(\frac{Yu}{N})$$
$$\Big\langle\Big\{A_o\,\delta(u)\,\delta(v) + \left[\mathrm{sinc}(Yu)\exp(-j\pi Yu)\right] * \Big[\frac{1}{2}\,\delta\,(u-f-T)\exp(jn\phi)$$
$$+\frac{1}{2}\,\delta\,(u+f+T)\exp(-jn\phi) + \frac{1}{2}\,\delta\,(u+f) + \frac{1}{2}\,\delta\,(u+f)\Big]\Big\} *$$
$$\exp\,(-j2\pi\frac{nY}{N}u)\Big\rangle\Big)\Bigg]$$

Carrying out the summation, we have:

(11)
$$E(v,u) = K_1 \left[\mathrm{sinc}(Yv)\exp(-j\pi Yv)\right] * \left[\mathrm{sinc}(Yu)\exp(-j\pi Yu)\right] *$$
$$\Bigg[A\,\delta(u)\,\delta(v) - A_1\Big(P_1(v)\,I_1(v,u)\,\mathrm{sinc}\,(\frac{Yu}{N})\Big\langle A_o\;\delta\,(u)$$
$$\frac{\sin\,(\pi Yv)}{\sin\,(\pi\frac{Y}{M}v)}\exp\left[-j\pi(M+1)\frac{Y}{M}v\right] + \mathrm{sinc}\;Y(u-f-T)\exp\left[-j\pi Y(n-f-T)\right]$$
$$\frac{\sin\frac{M}{2}\,(\pi\frac{Y}{M}v-\phi)}{\sin\frac{1}{2}\,(\pi\frac{Y}{M}v-\phi)}\exp\left[-j(\frac{M+1}{2})\,(2\pi\frac{Y}{M}v-\phi)\right] + \mathrm{sinc}\;Y\,(u+f+T)$$
$$\exp\left[-j\pi Y(n+f+T)\right]\frac{\sin\frac{M}{2}\,(2\pi\frac{Y}{M}v+\phi)}{\sin\frac{1}{2}\,(2\pi\frac{Y}{M}v+\phi)}\exp\left[-j\,(\frac{M+1}{2})\,(2\pi\frac{Y}{M}v+\phi)\right]$$
$$+\,\mathrm{sinc}\;Y(u-f)\,\frac{\sin\pi\,Yv}{\sin\pi\frac{Y}{M}v}\exp\left[-j\pi(M+1)\frac{Y}{M}v\right] + \mathrm{sinc}\;Y(u+f)$$
$$\frac{\sin\,(\pi Yv)}{\sin\,(\pi\frac{Y}{M}v)}\exp\left[-j\pi(M+1)\frac{Y}{M}v\right]\Big\rangle\Big)\Bigg] * \left\{\frac{\sin(\pi Yu)}{\sin(\pi\frac{Y}{N}u)}\exp\left[j\pi(N+1)\frac{Y}{N}u\right]\right\}$$

This information now appears as bright spots separated in range and doppler. This is illustrated in Figure 25. To remove the ambiguities, this information is now passed through an aperture located in the region of u = f to u = f + T and is imaged onto the film. The output film is shown in Figure 25.

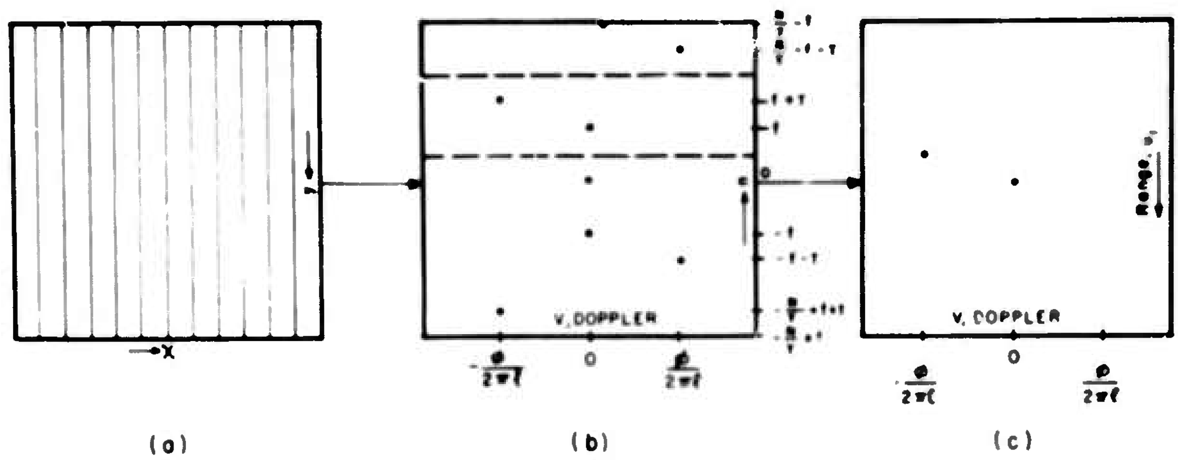

Figure 25. Optical Processing Using the Fourier Transform of the Amplitude Information for Each Pulse

III. ERROR ANALYSIS

1. FORMAT II

The system inaccuracies that will degrade the processed data will be mainly associated nonlinearities of the film drive and CRT. Using equation (9) it is possible to introduce errors in the positioning of the data points. The data on the film with errors can be represented by:

$$\tau(x,y) = \text{rect}\left(\frac{x}{Y} - \frac{1}{2}\right) \text{rect}\left(\frac{y}{Y} - \frac{1}{2}\right) \Bigg[A - A_1 \sum_{m=1}^{M} \sum_{n=1}^{N} \Big\{ P(x) * I\left(x - \frac{mY}{M} - \delta_m, y\right) * \text{rect}\left(\frac{y}{\frac{Y}{N} + \delta_n}\right) * \Big\langle \Big\{ 1 + \text{rect}\left(\frac{y}{Y} - \frac{1}{2}\right) \Big[\cos\Big\{ 2\pi(f+T)\frac{mY}{N} + m\phi \Big\} + \cos 2\pi f \frac{mY}{N} \Big] \Big\} \left(A_{m,n} + \Delta_{m,n} A_{m,n}\right) \delta\left(y - \frac{mY}{M} - \delta_{m,n}\right) \Big\rangle \Bigg] \tag{12}$$

where:

δ_m represents the error in positioning the m<u>th</u> pulse in the x dire..'on across the film

$\delta_{m,n}$ represents the error in positioning the n<u>th</u> data point of the m<u>th</u> pulse in the y direction along the film

$\Delta_{m,n}$ represents the fractional variation i.. the amplitude information on the film

Taking the two-dimensional Fourier transform of this film we have:

$$(13) \qquad E(v,u) = \left[\text{sinc } Yu \exp(-j\pi Yv)\right] * \left[\text{sinc } Yu \exp(-j\pi Yu)\right] * \left[A\,\delta(u)\,\delta(v)\right.$$

$$-A_1\left(P_1(v)\, I_1(u,v) \sum_{m=1}^{M} \sum_{n=1}^{N} \text{sinc}\left(\frac{Y}{N}+\delta_n\right)u \left\{\delta(u)\,\delta(v)\right.\right.$$

$$\left.+ \cos\left[2\pi(f+T)\frac{nY}{N} + m\phi\right] + \cos\left(2\pi f\frac{nY}{N}\right)\right\} \exp\left[-j2\pi v\left(\frac{mY}{M}+\delta_m\right)\right]$$

$$\left.\left.\cdot\left(A_{m,n} + \Delta_{m,n} A_{m,n}\right) \exp\left[-j2\pi u\left(\frac{nY}{N}+\delta_{m,n}\right)\right]\right)\right]$$

The functions sinc(Yv) exp(-jπYv) and sinc(Yu) exp(-jπYu) act like impulses being convolved with the remainder of the information. Hence a good approximation to this expression is:

$$(14) \qquad E(v,u) = A\delta(u)\,\delta(v) - A_1\left(P_1(v)\, I_1(v,u) \sum_{m=1}^{M} \sum_{n=1}^{N} \text{sinc}\left(\frac{Y}{N}+\delta_n\right)u\right.$$

$$\left\{\delta(u)\,\delta(v) + \cos\left[2\pi(f+T)\frac{nY}{N} + m\phi\right] + \cos\left(2\pi f\frac{nY}{N}\right)\right.$$

$$\exp\left[-j2\pi\left(\frac{mY}{M}+\delta_m\right)v\right] \left(A_{m,n} + \Delta_{m,n} A_{m,n}\right)$$

$$\left.\left.\exp\left[-j2\pi\left(\frac{nY}{N}+\delta_{m,n}\right)u\right]\right\}\right)$$

We are not concerned with the information at u = 0 and v = 0 (on the optic axis). Also, the functions $P_1(v)$, $I_1(v,u)$ and sinc $\left(\frac{Y}{N}+\delta_n\right)u$ act like wideband filters passing all the information of interest. Hence, these terms may be neglected such that we have:

$$(15) \qquad E(v,u) = K \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{\cos\left[2\pi(f+T)\frac{nY}{N} + m\phi\right] + \cos\left(2\pi f\frac{nY}{N}\right)\right\}$$

$$\cdot \exp\left[-j2\pi\left(\frac{mY}{M}+\delta_m\right)v\right] \left(A_{m,n} + \Delta_{m,n} A_{m,n}\right)$$

$$\cdot \exp\left[-j2\pi\left(\frac{nY}{N}+\delta_{m,n}\right)u\right]$$

Since the process of taking the two-dimensional transformation is a linear process we need only consider one of the targets here, keeping in mind that another target can be added at any time using the principle of superposition. Using the target that is delayed in range by T, we have:

(16) $$E(v,u) = K \sum_{m=1}^{M} \sum_{n=1}^{N} (A_{m,n} + \Delta_{m,n} A_{m,n}) \cos\left[2\pi(f+T)\frac{nY}{N} + m\phi\right]$$
$$\exp\left[-j2\pi\left(\frac{mY}{M} + \delta_m\right)v\right] \exp\left[-j2\pi\left(\frac{nY}{N} + \delta_{m,n}\right)u\right]$$

All the information concerning this target is located at $u = \pm (f+T)$. It is only necessary to consider one of these sidebands in order to determine the sidelobe degradation associated with the system inaccuracies. Using this, Equation (16) may be rewritten as:

(17) $$E(v,u) = K \sum_{m=1}^{M} \sum_{n=1}^{N} (A_{m,n} + \Delta_{m,n} A_{m,n}) \exp\left\{ j2\pi \left[(f+T-u)\frac{nY}{N} - \left(\frac{Yv}{M} - \phi\right) m - \delta_m v - \delta_{m,n} u\right]\right\}$$

We will be mainly concerned with the envelope of this function in the sidelobe region. Since it is practical to measure the sidelobes with respect to the peak at the main lobe, this function will be normalized with respect to its magnitude at $u = f+T$ and $v = -\frac{\phi M}{Y}$. We can approximate this by:

(18) $$E = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} (A_{m,n} + \Delta_{m,n} A_{m,n}) \exp\left\{ j2\pi \left[(f+T-u)\frac{nY}{N} - \left(\frac{Yv}{M} - \phi\right) m - \delta_m v - \delta_{m,n} u\right]\right\}}{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}}$$

It is assumed that the probability distribution of the random errors $\Delta_{m,n}$, δ_m and $\delta_{m,n}$ can be represented by $P(\Delta_{m,n})$, $P(\delta_m)$, and $P(\delta_{m,n})$ each having a mean of zero. This means that the film drive has a velocity that varies about the desired operating velocity in a random manner, the CRT has an intensity variation that varies about the desired intensity in a random manner, and the errors in the sweep on the CRT are mainly due to the sweep drive and not the triggers that start the sweep, thereby causing a random error in the displacement of the data points about their desired positions. Hence, the probability of a particular value of $\Delta_{m,n}$, δ_m, and $\delta_{m,n}$ is given by:

(19) $$P(\Delta_{m,n}, \delta_m, \delta_{m,n}) = P(\Delta_{m,n})\, P(\delta m)\, P(\delta_{m,n})$$

The standard deviation σ_E of the envelope $|E|$ can be represented by:

(20) $$\sigma_E^2 = \overline{|E|^2} - \overline{|E|}^2$$

where:

(21) $$\overline{|E|^2} = \iiint |E|^2 P(\Delta_{m,n}) P(\delta_m) P(\delta_{m,n}) \, d\Delta_{m,n} \, d\delta_m \, d\delta_{m,n}$$

(22) $$\overline{|E|} = \iiint |E| \, P(\Delta_{m,n}) P(\delta_m) P(\delta_{m,n}) \, d\Delta_{m,n} \, d\delta_m \, d\delta_{m,n}$$

In order to calculate the above expression it is necessary to examine the intensity of the information contained in equation (18). This is given by:

(23) $$|E|^2 = \left(\frac{\sum_{m=1}^{M} \sum_{n=1}^{N} (A_{m,n} + \Delta_{m,n} A_{m,n})}{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}} \cos\left\{ 2\pi \left[((f+T-u)\frac{nY}{N} - (\frac{Yv}{M} - \phi)m - \delta_m v - \delta_{m,n} u \right] \right\} \right)^2$$
$$+ \left(\frac{\sum_{m=1}^{M} \sum_{n=1}^{N} (A_{m,n} + \Delta_{m,n} A_{m,n})}{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}} \sin\left\{ 2\pi \left[((f+T-u)\frac{nY}{N} - (\frac{Yv}{M} - \phi) - \delta_m v - \delta_{m,n} u \right] \right\} \right)^2$$

Expanding the cosine and sine in this expression we have:

(24) $$|E|^2 = \left(\frac{\sum_{m=1}^{M} \sum_{n=1}^{N} (A_{m,n} + \Delta_{m,n} A_{m,n})}{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}} \left\{ \cos 2\pi \left[((f+T-u)\frac{nY}{N} - (\frac{Yv}{M} - \phi)m \right] \right. \right.$$
$$\left. \left. \cos\left[-2\pi(\delta_m v + \delta_{m,n} u) \right] + \sin 2\pi \left[((f+T-u)\frac{nY}{N} - (\frac{Yv}{M} - \phi)m \right] \sin\left[2\pi(\delta_m v + \delta_{m,n} u) \right] \right\} \right)^2$$
$$+ \left(\frac{\sum_{m=1}^{M} \sum_{n=1}^{N} (A_{m,n} + \Delta_{m,n} A_{m,n})}{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}} \left\{ \sin 2\pi \left[((f+T-u)\frac{nY}{N} - (\frac{Yv}{M} - \phi) m \right] \cos\left[-2\pi(\delta_m v + \delta_{m,n} u) \right] \right. \right.$$
$$\left. \left. + \cos 2\pi \left[((f+T-u)\frac{nY}{N} - (\frac{Yv}{M} - \phi)m \right] \sin\left[-2\pi(\delta_m v + \delta_{m,n}) \right] \right\} \right)^2$$

It is assumed that these errors are small and using a small angle approximation we have:

(25) $$|E|^2 = \left(\frac{\sum_{m=1}^{M} \sum_{n=1}^{N} (A_{m,n} + \Delta_{m,n} A_{m,n})}{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}} \left\{ \cos 2\pi \left[((f+T-u)\frac{nY}{N} - (\frac{Yv}{M} - \phi)m \right] \right. \right.$$

$$+ 2\pi(\delta_m v + \delta_{m,n} u) \sin 2\pi \left[(f+T-u)\frac{nY}{N} - \left(\frac{Yv}{M} - \phi\right)m \right] \Big\}\Big]^2$$

$$+ \left(\frac{\sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n})}{\sum_{m=1}^{M}\sum_{n=1}^{N}A_{m,n}} \left\{ \sin 2\pi \left[(f+T-u)\frac{nY}{N} - \left(\frac{Yv}{M}-\phi\right)m\right]\right.\right.$$

$$\left.\left. - 2\pi(\delta_m v+\delta_{m,n}u)\cos 2\pi\left[(f+T-u)\frac{nY}{N} - \left(\frac{Yv}{M}-\phi\right)m\right]\right\}\right)^2$$

Expanding this expression we have:

$$(26) \qquad |E|^2 = \frac{1}{\left(\sum_{m=1}^{M}\sum_{n=1}^{N}A_{m,n}\right)^2} \left(\left\{ \sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n}) \cos\left[2\pi(f+T-u)\frac{nY}{N}\right.\right.\right.$$

$$\left.\left. - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}^2$$

$$+ 2\left\{ \sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n}) \cos\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}$$

$$\left\{ \sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n}) \left[2\pi(\delta_m v+\delta_{m,n}u)\right] \sin\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}$$

$$+ \left\{ \sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n}) \left[2\pi(\delta_m v+\delta_{m,n}u)\right] \sin\left[2\pi(f+T-u)\frac{nY}{N}\right.\right.$$

$$\left.\left. - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}^2$$

$$+ \left\{ \sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n}) \sin\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}^2$$

$$+ 2\left\{ \sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n}) \sin\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}$$

$$\left\{ \sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n}) \left[-2\pi(\delta_m v+\delta_{m,n}u)\right] \cos\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)u\right]\right\}$$

$$+ \left\{ \sum_{m=1}^{M}\sum_{n=1}^{N}(A_{m,n}+\Delta_{m,n}A_{m,n}) \left[-2\pi(\delta_m v+\delta_{m,n}u) \quad \cos\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}^2\right)$$

Using equation (21) and carrying out the indicated integrations keeping in mind that the random parameters have a zero mean, we have:

$$(27) \qquad \overline{|E|^2} = \frac{1}{\left(\sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}\right)^2} \Bigg[\left\{ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n} \cos\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}^2$$

$$+ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2\, a_{m,n}^2 \cos^2\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]$$

$$+ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2\, 4\pi^2 \left(\sigma_m^2 v^2 + \sigma_{m,n}^2 u^2\right) \sin^2\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]$$

$$+ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2\, a_{m,n}^2\, 4\pi^2 \left(\sigma_m^2 v^2 + \sigma_{m,n}^2 u^2\right) \sin^2\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]$$

$$+ \left\{ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n} \sin\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}^2$$

$$+ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2\, a_{m,n}^2 \sin^2\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]$$

$$+ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2\, 4\pi^2 \left(\sigma_m^2 v^2 + \sigma_{m,n}^2 u^2\right) \cos^2\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]$$

$$+ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2\, a_{m,n}^2\, 4\pi^2 \left(\sigma_m^2 v^2 + \sigma_{m,n}^2 u^2\right) \cos^2\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right] \Bigg]$$

where $a_{m,n}$, $\sigma_{m,n}$ and σ_m represent the standard deviation of the inaccuracies in $\Delta_{m,n}$, $\delta_{m,n}$ and δ_m, respectively.

Rearranging these terms we have:

$$(28) \qquad \overline{|E|^2} = \frac{1}{\left(\sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}\right)^2} \Bigg[\left\{ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n} \cos\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}^2$$

$$+ \left\{ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n} \sin\left[2\pi(f+T-u)\frac{nY}{N} - 2\pi\left(\frac{Yv}{M}-\phi\right)m\right]\right\}^2$$

$$+ \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2\, 4\pi^2 \left(\sigma_m^2 v^2 + \sigma_{m,n}^2 u^2\right) + \sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2\, a_{m,n}^2$$

$$+ \sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}^2 \, a_{m,n}^2 \, 4\pi^2 (\sigma_m^2 v^2 + \sigma_{m,n}^2 u^2)$$

Since these errors have a zero mean it is obvious that the first two terms in the above expression can be approximated by $\overline{|E|}^2$. It is assumed that the standard deviations are small and the term containing the multiples of the squares of the standard deviations can be neglected. Doing this we have:

$$\overline{|E|^2} = \overline{|E|}^2 + \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}^2 (a_{m,n}^2 + 4\pi^2 \sigma_m^2 v^2 + 4\pi^2 \sigma_{m,n}^2 u^2)}{\left(\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n} \right)^2} \tag{29}$$

Now using equation (20) we have:

$$\sigma_E^2 = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}^2 (a_{m,n}^2 + 4\pi^2 \sigma_m^2 v^2 + 4\pi^2 \sigma_{m,n}^2 u^2)}{\left(\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n} \right)^2} \tag{30}$$

where

$A_{m,n}$ = represents the taper used to reduce sidelobes

$a_{m,n}$ = standard deviations of the fractional variation of the amplitude information

σ_m = standard deviation of the displacement of the radar pulses in the direction of motion of the film

$\sigma_{m,n}$ = standard deviations of the data points across the width of the film

$v = \frac{x_1}{\lambda F}$

$u = \frac{y_1}{\lambda F}$

λ = wavelength of the coherent light used in the processor

F = focal length of the processor

x_1, y_1 = new dimensions in the image plane

σ_E = standard deviation of the envelope

This equation is similar to that developed by Ruze[2] who showed that the variance of the envelope is related to the variance of the amplitude and phase errors for a phased array antenna.

The statistical behavior of the sidelobes, due to amplitude and phase errors, for this normalized envelope of equation (18) is described by the modified Rayleigh distribution.

The probability density function of the modified Rayleigh distribution is given by:

(31) $$p(r)dr = \frac{2r}{\sigma_E^2} \exp\left(-\frac{a^2 + r^2}{\sigma_E^2}\right) I_o\left(\frac{2ar}{\sigma_E^2}\right) dr$$

where

$p(r)dr$ = probability that the sidelobes of the envelope lies r and r + dr

σ_E^2 = mean-square value of r

I_o = modified Bessel function of the first kind

Good values to use for v and u in equation (30) for σ_E are those where the peak of the information is contained. This occurs at $v = \frac{\phi M}{2_\pi Y}$ & $u = f + T$, where ϕ is the phase shift from pulse to pulse due to doppler, $\frac{Y}{M}$ is the distance between pulses on the film, f is the spatial carrier at which the information is written, and T is the range delay of the object. Combining all of this information we have:

(32) $$\sigma_E^2 = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}^2}{\left(\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}\right)^2} \left[a_{m,n}^2 + 4\pi^2 (f + T)^2 \sigma_{m,n}^2 + \frac{\phi^2 M^2}{Y^2} \sigma_m^2\right]$$

A more detailed explanation is required to define the standard deviation of the error σ_m. This error contains velocity errors due to the velocity inaccuracies of the film drive and the CRT. The yoke of the tube is set such that the lines on the film are at a right angle to the edge of the film. The distance between the beginning and ending of the pulse along the direction of motion of the film is given by:

(33) $$x = v_v t - \frac{1}{2} v_s t \sin \theta$$

where

v_f = velocity of film

v_s = sweep velocity of the CRT

θ = angle at which the line is skewed on the CRT

The scan velocity of the CRT is divided by two since it undergoes a two to one reduction in size when it is imaged on the film. In equation (33) the angle θ is such that x = 0 when there are no errors in the velocity controls of the film drive or the sweep drive of the CRT. The sweep velocity of the CRT will be much greater than the film drive velocity. Hence, the function $\sin \theta$ can be approximated by: $\sin \theta = 2\, \bar{v}_f / \bar{v}_s$

where

$\bar{v}_f$ = mean velocity of the film

$\bar{v}_s$ = mean scan rate of the CRT.

Substituting this into equation (33) we have:

$$(34)\qquad x = \left(v_f - v_s \frac{\bar{v}_f}{\bar{v}_s} \right) t$$

Therefore an error in v_f of δv_f and an error in v_s of δv_s yields a position error of δ_x, which can be represented by:

$$(35)\qquad \delta x = \left(\delta v_f - \delta v_s \frac{\bar{v}_f}{\bar{v}_s} \right) t$$

If the time is set equal to the time between traces we have:

$$(36)\qquad \delta_x = \delta f - \delta_y \frac{\bar{v}_f}{\bar{v}_s}$$

where

$\delta f = \delta v_f t$ = displacement inaccuracy of the film drive from pulse to pulse

$\delta y = \delta v_s t$ = error in the repeatibility of the CRT from pulse to pulse

δx = error in spacing of the lines

Calculating the variance of equation (36) we have:

$$\sigma_m^2 = \sigma_f^2 + \sigma_{m,n}^2 \left(\frac{\bar{v}_f}{\bar{v}_s} \right)^2$$

where

σ_m = standard deviation of the displacement between pulses on the film

$\sigma_{m,n}$ = standard deviation of the repeatibility of the traces on the CRT

σ_f = standard deviation of the positioning inaccuracy of the film drive

The angular accuracy of the film drive is usually given in angular measurements. Using this fact and combining the above equation with our original expression for the variance of the sidelobes of the envelope in equation (32) we have:

$$(37)\qquad E^2 = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n}^2}{\left(\sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n} \right)^2} \left\{ a_{m,n}^2 + \frac{\beta^2 M^2}{Y^2} \left[R^2 \sigma_p^2 + Y^2 \sigma_R^2 \left(\frac{v_f}{v_s} \right)^2 \right] + 4\pi^2 (f+T)^2 Y^2 \sigma_R^2 \right\}$$

where

$A_{m,n}$ = represents the amplitude of the contribution from the m'th pulse of the processor and the n'th data point

$a_{m,n}$ = standard deviation of the normalized fractional amplitude on the film

f = spatial frequency of the data in cycles per millimeter

ϕ = phase shift from pulse to pulse due to doppler measured in radians

$\frac{Y}{M}$ = distance between pulses on film in millimeters

$\bar{v}_f$ = mean velocity of the film drive

$\bar{v}_s$ = mean scan rate for the CRT

Y = pulse length of the film, 50 millimeters

σ_R = standard deviation on the repeatibility of the information from pulse to pulse on the film, $(Y\sigma_R = \sigma_{m,n})$

R = radius of the capstan in the camera

σ_p = standard deviation of the positional error of the film drive, $(R\sigma_p = \sigma_f)$

The standard deviation of the repeatibility of the information from pulse to pulse σ_R is dependent on the repeatibility of the CRT and the errors associated with the film sliding back and forth in the y-direction.

In order to calculate the maximum standard deviation σ_Σ of the sidelobes of the envelope it is necessary to use the worst possible parameters in the above equation. Typical inaccuracies for this device are already known. However, it is not known how these "typical" inaccuracies are related to the standard deviation of the inaccuracy of the device. For the purposes of this calculation it is assumed that the "typical" inaccuracies are equal to the standard deviation of the inaccuracies of the system. This means that the inaccuracies are less than the "typical" inaccuracies for about 68.3% of the time.

Typical inaccuracies of the system are:

(a) repeatibility of the CRT of 1 part in 10,000

(b) instantaneous positional perturbation of the film drive is ±1 second of arc

(c) intensity deviation on the CRT is 5 percent

(d) the film will shift by 1 part in 5×10^6 in the y-direction per pulse.

We can now specify some of the other parameters associated with equation (37):

$a_{m,n}^2$ = 0.05

$f + T$ = a maximum of 20 cycles per millimeter

ϕ = a maximum of π radians

Y = 50 millimeters

σ_R = 10^{-}

R = a maximum of 1/2 inch or 12.7 mm

σ_p = 4.85×10^{-6} radians

Some additional explanation is required to specify the other parameters in this equation. The average sweep velocity of the CRT is equal to $\bar{v}_s = 100/32\ N \times 10^{-6}$ in millimeters per second. This was obtained from the fact that 100 millimeters of the tube surface will be used, there will be N data points displayed, and it takes 32 microseconds to write each data point on the film. The average film drive velocity is $\bar{v}_f = 50/83\ NM \times 10^{-6}$. This was obtained as follows. The pulses to be processed occupy 50 millimeters of film, it takes 83 milliseconds from the start of one pulse to the start of a second pulse at a thousand data points per pulse, and there are M pulses in the 50 millimeters of film. This means that the ratio of $\bar{v}_f$ to $\bar{v}_s$ is $\frac{0.193}{M}$, where M is the number of pulses along 50 millimeters of film.

Substituting these values into equation (37) we have:

$$\sigma_E^2 = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}^2}{\left(\sum_{m=1}^{M}\sum_{n=1}^{N} A_{m,n}\right)^2}\left[(0.05) + (1.5 \times M^2 \times 10^{-11} + 3.66 \times 10^{-9}) + 0.395\right]$$

Now we can investigate the factor involving amplitude information in the above equation. It is assumed that the amplitude information $A_{m,n}$ can be separated into two independent functions a_m dependent only on m and a_n dependent only on n. Hence $A_{m,n} = a_m a_n$, and the above equation becomes:

$$\sigma_E^2 = \frac{\sum_{m=1}^{M} a_m^2 \sum_{n=1}^{N} a_n^2}{\left(\sum_{m=1}^{M} a_m\right)^2\left(\sum_{n=1}^{N} a_n\right)^2}\left[0.445 + 1.5 \times M^2 \times 10^{-11}\right] \tag{38}$$

Now we can define the factor $\left(\sum_{m=1}^{M} a_m\right)^2 \Big/ M \sum_{m=1}^{M} a_m^2$. This factor is equivalent to the gain of an antenna array normalized with respect to the gain of a uniformly illuminated array. The optical processor will probably use an amplitude taper to maintain a low sidelobe level in the doppler dimension. It has been shown by D. Slepian[3] that, to concentrate this energy into a region of the film such that the sidelobe energy is minimized, it is necessary to produce an amplitude taper proportional to a prolate spheroidal wave function of zero order. This same criteria was also used for a discrete 20 element array and it is felt by the author that the results are approximately these that were obtained by Slepian. Using this work the variation of α with respect to sidelobe level is given by Figure 26. Now, using this in the above equation and the fact that there will probably be no taper in the y-direction (other than that already included in the radar waveform) we have:

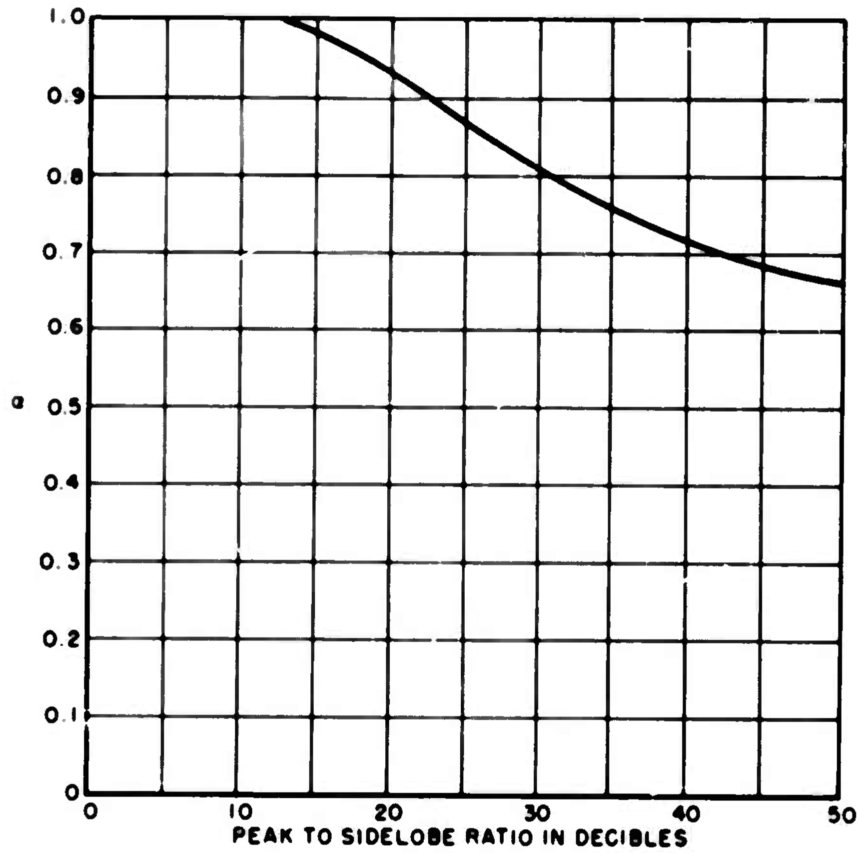

Figure 26. The Normalized Gain (Gain Of Array With Respect To A Uniform Array) Of An Optimum 20 Element Array Versus The Minimum Peak To Sidelobe Level

$$\sigma_E^{2} = \frac{0.445}{\alpha MN} + \frac{1.5 \times 10^{-11} \times M}{\alpha N}$$

N will probably take on the value of 2000. Using this fact and simplifying the above equation we have:

$$\sigma_E^{2} = \frac{2.22 \times 10^{-4}}{\alpha M} + \frac{7.5 \times 10^{-15} \times M}{\alpha}$$

Since M will range between 500 and 1000 pulses, it is seen that 500 is the worst case number to be used. This yields:

$$\sigma_E^{2} = \frac{0.444 \times 10^{-6}}{\alpha}$$

Now it is possible to calculate the degradation of the sidelobes using the Rayleigh distribution that describes the behavior of the sidelobes, see equation (31). Using reference 5 it was found that:

$$\frac{\sqrt{2}}{\sigma_E}(r - a) = 3.75$$

providing the sidelobe level of the envelope stays below r for 99.9% of the time using a nominal sidelobe level of a. Solving for the ratio of r/a using the above two equations we have:

$$\frac{r}{a} = \frac{1.77 \times 10^{-3}}{a\sqrt{\alpha}} + 1$$

Using this expression it is now possible to calculate the degradation of the sidelobe level for various nominal sidelobe levels. Figure 27displays this degradation as a function of nominal sidelobe levels.

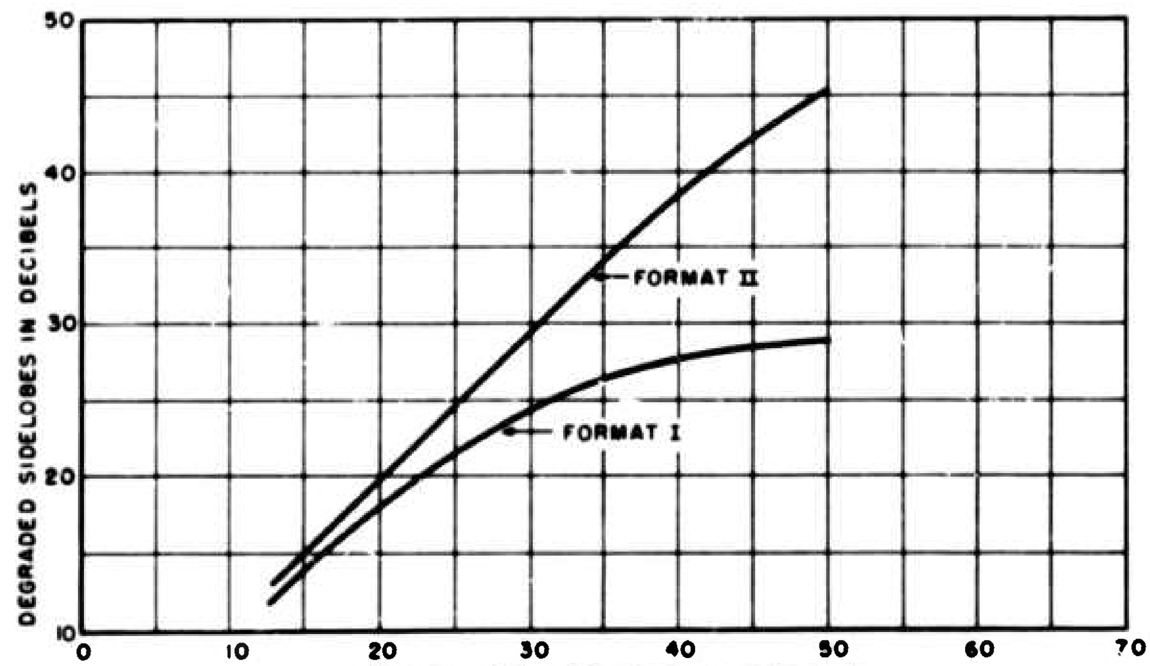

Figure 27. Sidelobes Will Be Below The Degraded Sidelobe Level for 99.9% Of The Time For A System Designed To Yield A Given Nominal Sidelobe Level

2. FORMAT I

Making the same assumptions that were made using format I, and introducing the appropriate positional errors in equation (3) we have

$$\tau(x,y) = \mathrm{rect}\left(\frac{x}{Y} - \frac{1}{2}\right)\mathrm{rect}\left(\frac{y}{Y} - \frac{1}{2}\right)\Bigg[A - A_1 \sum_{m=1}^{M} \sum_{n=1}^{N} (A_m + A_{m,n} A_m)$$

$$\left\langle P(x) * I\left(x - \frac{mY}{M} - \delta_{m,y}\right) * \mathrm{rect}\left(\frac{y}{\frac{Y}{N} + \delta_{m,n}}\right)\right\rangle *$$

$$\left\{ \left[A_o + \mathrm{sinc}\,\frac{2}{Z}\left(\frac{hY}{N} - \frac{Y}{2} - T\right)\right] \cos\left(2\pi f \frac{nY}{N} + m\phi\right)\right.$$

$$\left. + \mathrm{sinc}\,\frac{2}{Z}\left(\frac{nY}{N} - \frac{Y}{2}\right) \cos\left(2\pi f \frac{nY}{N}\right) \delta\left(y - \frac{nY}{N} - \delta_{m,n}\right)\right\}\Bigg\rangle\Bigg]$$

where

δ_m represents the error in positioning the m'th pulse in the x-direction on the film

$\delta_{m,n}$ represents the error in positioning the n'th data point of the m'th pulses in the y-direction along the film

$\Delta_{m,n}$ represents the fractional variation of the amplitude information on the film

Taking the two dimensional Fourier transform of the film, we have:

$$E(v,u) = K_1 \left[\mathrm{sinc}\, Y_v \exp(-jvY\pi)\right] * \left[\mathrm{sinc}\, Y_u \exp(-j\pi Yu)\right] *$$

$$\Bigg[A\,\delta(v)\,\delta(u) - A_1 \sum_{m=1}^{M} \sum_{n=1}^{N} (A_m + \Delta_{m,n} A_m)\left\langle P_1(v)\, I_1(v,u) \exp\left[-j2\pi v\left(\frac{mY}{M} + \delta_m\right)\right]\right.$$

$$\mathrm{sinc}\left(\frac{Y}{N} + \delta_{m,n}\right) u \left\{\left[A_o\,\delta(u)\,\delta(v) + \mathrm{sinc}\,\frac{2}{Z}\left(\frac{nY}{N} - \frac{Y}{2} - T\right) \cos\left(2\pi f \frac{hY}{N} + m\phi\right)\right.\right.$$

$$\left.\left. + \mathrm{sinc}\,\frac{2}{Z}\left(\frac{Y}{N} - \frac{Y}{2}\right) \cos\left(2\pi f \frac{nY}{N}\right)\right] \exp\left[-j2\pi u\left(\frac{hY}{N} + \delta_{m,n}\right)\right]\right\}\Bigg\rangle\Bigg]$$

At this point it is best to consider only one of these targets, keeping in mind that the other target may be added using the principal of superposition. The one sidelobe is now passed through another aperture located at U = f. The information at the second aperture can be represented by:

$$E(v,u) = K\,\mathrm{rect}\left(\frac{u-f}{X}\right) \Big(\left[\mathrm{sinc}\, Yv \exp(-j\pi Yv)\right] * \left[\mathrm{sinc}\, Yu \exp(-j\pi Yu)\right] *$$

$$\sum_{m=1}^{M} \sum_{n=1}^{N} (A_m + \Delta_{m,n} A_m) \Big\{ P_1(v)\, I_1(u,v)\, \text{sinc}\left(\frac{Y}{N} + \delta_{m,n}\right) u \exp\left[-j2\pi v \left(\frac{mY}{M} + \delta_m\right)\right]$$

$$\text{sinc}\, \frac{2}{Z}\left(\frac{nY}{N} - \frac{Y}{2} - T\right) \exp\left(j2\pi f \frac{nY}{N} + m\phi\right) \exp\left[-j2\pi\mu\left(\frac{nY}{N} \cdot \delta_{m,n}\right)\right]\Big\}$$

Taking the Fourier transform in the u direction we have

$$E(v,y') = K\left[\text{sinc}\, xy' \exp j2\pi f y'\right] * \left(\left[\left[\text{sinc}\, Yv \exp(-j\pi Yv)\right] * \text{rect}\left(\frac{y'}{Y} - \frac{1}{2}\right) \sum_{m=1}^{M} \sum_{h=1}^{N} (A_m + \Delta_{m,n} A_m) \left\langle P_1(v)\, I_1(v)\right.\right.\right.$$

$$I(y') * \text{rect}\left(\frac{y'}{\frac{Y}{N} + \delta_{m,n}}\right) * \Big\{\text{sinc}\, \frac{2}{Z}\left(\frac{hY}{N} - \frac{Y}{2} - T\right)$$

$$\exp\left[-j2\pi v\left(\frac{mY}{M} + \delta_m\right)\right] \exp\left(j2\pi f \frac{hY}{N} + m\phi\right)$$

$$\left.\left.\delta\left(y' - \frac{nY}{N} - \delta_{m,n}\right)\Big\}\right\rangle\right]$$

The terms $I(y')$, sinc Yv and sinc Xy' act like impulses which have no great effect on the system. The functions, $P_1(v)$, $I_1(v)$ and rect $\left(\frac{y'}{Y} - \frac{1}{2}\right)$ act like wideband filters which slightly attenuate the information. Using these facts this equation can be simplified to:

$$E(v,y') = K \sum_{m=1}^{M} \sum_{n=1}^{N} (A_m + \Delta_{m,n} A_m)\, \text{sinc}\, \frac{2}{Z}\left(\frac{hY}{N} - \frac{Y}{2} - T\right)$$

$$\text{rect}\left(\frac{2 - \frac{hY}{N} - \delta_{m,n}}{\frac{Y}{N} + \delta_{m,n}}\right) \exp\left[j2\pi\left(\frac{fnY}{N} + \frac{m\phi}{2\pi} - \frac{mYv}{M} - \delta_m v\right)\right]$$

However, the functions $I(y')$, sinc Yv and sinc Xy' cause the discrete sampled values to run together. This means that the variation in the rect$\left(\frac{y' - \frac{hY}{N} - \delta_{mn}}{\frac{Y}{N} - \delta_{m,n}}\right)$ can be neglected and this function can be simplified to rect$\left(\frac{2' - \frac{hY}{N}}{Y/N}\right)$. Rewriting this equation, we have:

(40) $$E(v,y') = K \sum_{m=1}^{M} \sum_{n=1}^{N} (A_m + \Delta_{m,n} A_m) \operatorname{sinc} \frac{2}{Z} \left(\frac{nY}{N} - \frac{Y}{2} - T\right)$$

$$\operatorname{rect}\left(\frac{y' - \frac{hY}{N}}{Y/N}\right) \exp j2\pi\left(\frac{fnY}{N} + \frac{m\phi}{2\pi} \quad \frac{mYv}{M} - \delta_m v\right)\Bigg]$$

This summation in n represents the envelope along the range dimension. This envelope has no sidelobes and can be ignored. The normalized envelope of interest is:

(41) $$|E| = \left| \frac{1}{\sum_{m=1}^{M} A_m} \left\{ \sum_{M=1}^{M} (A_m + \Delta_{m,n} A_m) \exp\left[j2\pi \frac{m\phi}{2\pi} - \frac{mYv}{M} - \delta_m v \right)\right]\right\} \right|$$

Carrying out the same analysis that was performed on equation (18) we have:

(42) $$\sigma_E^2 = \frac{\sum_{m=1}^{M} A_m^2 \left(a^2 + 4\pi^2 \sigma_m^2 v^2\right)}{\left(\sum_{m=1}^{M} A_m\right)^2}$$

We are interested in this equation at $v = \frac{\phi M}{2\pi Y}$

(43) $$\sigma_E^2 = \frac{\sum_{m=1}^{M} A_m^2 \left\{ a^2 + \frac{\phi^2 M^2}{Y^2} \left[R^2 \sigma_p^2 + Y^2 \sigma_R^2 \left(\frac{\bar{v}_f}{v_s}\right)^2 \right] \right\}}{\left(\sum_{m=1}^{M} A_m\right)^2}$$

The parameters in the above equation are defined in equation (37). This reduces to:

$$\sigma_E^2 = \frac{1}{\alpha M}\left[0.05 + M^2 \times 1.5 \times 10^{-11} + 3.66 \times 10^{-9}\right]$$

or

$$\sigma_E = \frac{10^{-2}}{\sqrt{\alpha}}$$

Using this the ratio of degraded sidelobe level to nominal sidelobe level is given by:

$$\frac{r}{a} = \frac{2.65 \times 10^{-2}}{a\sqrt{\alpha}} + 1.0$$

Calculated values of degraded sidelobe level vs. nominal level are shown in Figure 27, which also contains the curve for Format II.

IV. CONCLUSIONS

The signal power in the random error component sets a limit on attainable sidelobe suppression, with given system parameters. An inspection of the curves for Formats I and II (shown in Figure 27) indicates that with one-dimensional integration (Format I) the suppression level is limited at about 30 db. With two-dimensional integration, greater suppression of random errors is possible, with 45.5 db capability being indicated by the curve. It is anticipated that the poorer sidelobes in Format I will be concentrated in a ridge (at constant range) across the output plane. This is because the error power before integration is concentrated in a swath across the output plane about one radar range cell wide. Format II, on the other hand, has the error power distributed more or less uniformly over the input plane and should not have ridge-like doppler sidelobes. The structure with Format I has been observed with processed data from digital computer runs. The primary error sources in this case were in radar range lobe adjustments and in phase alignment of pulses to provide a coherent doppler reference.

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APPENDIX III

SOFTWARE

APPENDIX III

SOFTWARE

1. OPTICAL PROCESSOR PROGRAM

This is the main program in preparing radar data so that it may be used with the SDS 910 and the film recorder to produce photographic film which is optically processed. The input to this program is a magnetic tape which contains radar data in the form of 84 amplitude and 84 phase quantities per pulse. The output is another magnetic tape with records of 2048 or 1024 words. Each record represents a radar pulse that has been interpolated to produce more sample points, aligned (alignment is not necessary with simulated radar data produced by DATA2X), applied to a spatial carrier with phase, and then added to a bias level. This output tape is in a format compatible with the SDS 910.

a. Logic

This program begins by rewinding FORTRAN tape units 11 and 12. It then sets up the sin X/X table which is used to interpolate the radar data. In this table X varies from 0 to 85.5 by increments of $\pi/10$. The program now reads all the data cards (cards 1 thru 6). It also calls subroutines LSFIT4. This subroutine sets up a look up table to compensate for the overall film recorder nonlinearities. Then a table is set up for the spatial carrier and all the variables needed in calling any of the alignment subroutines are defined. Following this is the logic for input tape control. Here, according to the values of the variables on the data cards, the input tape can be advanced a number of pulses, backspaced, rewound, or given number of pulses can be skipped between each processed pulse. After a pulse is read in from tape, one of the three alignment subroutines (ALIGN1, ALIGN2, ALIGN3) is called depending on the value of NALIGN, an input variable. The align subroutines convert the 84 amplitudes and 84 phases to real and imaginary parts. These are then interpolated using sin X/X to give ten points between every two points of the original 84. This results in 831 real, and 831 imaginary values per pulse. Then the pulses are aligned. The method used depends on which align subroutine is called. Simulated radar data, produced by DATA2X, is not aligned.

The data now is in aligned and partially interpolated form. Linear interpolation between adjacent points is now used to form the necessary 1024 or 2048 points per radar pulse. Now the pulse number and the maximum amplitude for the present frame (a frame is a set of pulses which are recorded adjacent on film) is printed. The maximum amplitude for a frame was found by subroutine AMPMAX as the data was generated. If A (n) represents the n^{th} amplitude sample point of a pulse and $\phi(n)$ represents the n^{th} phase sample point (there are 1024 or 2048 sample points per pulse) and AMPMAX is the maximum amplitude of a given frame, then need to form the following function:

$$AMOD(n) = AMPMAX + A(n) \cos (2\pi f_c n + \phi(n)$$

where n varies from 1 to 1024 or 2048, f_c is the spatial carrier frequency. Since the data is not now in terms of amplitude and phase, but in the form of real and imaginary values, much computation time can be saved by not converting to amplitude and phases but by rewritting (1) as;

$$AMOD(n) = AMPMAX + A(n) \left[\cos (2\pi f_c n) \cos \phi(n) - \sin (2\pi f_c n) \sin \phi(n)\right]$$

or

$$AMOD(n) = AMPMAX + \cos (2\pi f_c n) \left[A(n) \cos \phi (n)\right] - \sin (2\pi f_c n) \left[A(n) \sin \phi(n)\right]$$

where the terms in brackets A(n) cos Ø(n) and A (n) sin Ø(n) are the real and imaginary values. The quantities cos ($2\pi f_c n$) and sin ($2\pi f_c n$) are the spatial carrier terms which are given in a look-up table formed in the beginning of this program, and which do not change from pulse to pulse in a given frame.

The values of AMOD(n) are now fixed, and scaled so that they are within the values -255 to +256, i.e. 9 bits. If an end of frame is now reached an end of file is written on the output tape. Here subroutine PACK is called. This subroutine checks the input data and makes certain each value is within -255 and +256. It also packs the data on the output magnetic tape in a format compatible with the SDS 910. Now the total time of execution of this program is written out, and end of file is written on the output tape, and the program pauses before exit so that tapes may be removed. See page 97 for flow chart.

b. Subroutines for Optical Processor Program

(1) ALIGNI - Prominent Point Align Subroutine

The prominent Point Align Subroutine aligns the radar pulse returns by making the phase at the prominent point zero. All other phase values are altered by the amount of this prominent point phase.

Logic

The routine determines a number which is the last desired channel to be processed. The routine multiplies the amplitude channel value by the cosine of the corresponding phase channel, and also by the sine of the phase channel. These two values are the real and the imaginary portions of the complex return. The above is repeated with each desired channel. The results of the above is a conversion from amplitude and phase to real imaginary arrays.

The routine takes these two arrays and fills-in between the sampling points. The interpolating is with SIN X/X values calculated in the main program. For each position, the routine multiplies each sample value by the correct value of SIN X/X; and sums all these products to give the correct weight at that position. The routine goes through the above with each position to give new real and imaginary arrays. These arrays are the arrays with nine values between each of the old values. For each desired position in the above arrays, the routine squares the real and the imaginary values; and adds these together. This is the square of the amplitude of the return at this position. Each squared amplitude over the desired region is calculated. The maximum value and the position of this value is found. The phase at this amplitude is subtracted from the phase of each position. The real and imaginary portions with the adjusted phases are determined. This routine outputs these arrays and the position of the maximum point in the arrays. For simulated radar data produced by DATA2X, alignment is not necessary.

The calling for this routine is

CALL ALIGNI (ICH, DATA, R, F, SXDX, NFA, NØC, MID, NDEL, INITL)

where the inputs are:

ICH - number of amplitude and phase channels in the pulse of data.

DATA - name of data array which is half amplitudes and half phases.

SXDX - the sin X/X array used for interpolation.

NFA - number of the first desired channel of amplitude data to be used.

NØC - number of amplitude channels to be used.

MID - number of the middle amplitude channel in the pulse.

NDEL - number of positions in the interpolated amplitude array to be searched each side of the middle position for the maximum amplitude.

INITL - control constant which is zero for the first pulse and non-zero for the rest of the pulses.

and the outputs are:

R - The complex radar return is converted to real and imaginary arrays, then these arrays are aligned. R is this aligned real array for the pulse.

F - The imaginary aligned array for the pulse.

MID - The position number in the interpolated array of the maximum amplitude.

(2) ALIGN2 - Cross-correlation Align Subroutine

ALIGN2 aligns the radar pulse returns by cross-correlating each return with a reference return. The maximum correlation is determined. The phase difference of the two returns at this maximum correlation is subtracted from each phase value in the return to be aligned. Also, the delay between the two returns at the maximum correlation is subtracted from the return to be aligned.

Logic

The program defines the midpoint of the arrays which are outputs from this routine. These arrays are the real and imaginary returns sampled at every 0.2 nanoseconds. The routine defines the number of the last desired channel to be processed, and then starts processing with the first desired channel. The cosine of the phase channel is multiplied by the value in the corresponding amplitude channel, and also the sine of the phase channel is multiplied by the value in the corresponding amplitude channel. These two products are the real and the imaginary portions of the complex radar return. Each amplitude - phase channel is converted to a real and an imaginary value in the above way.

The routine fills-in the above arrays using sin X/X interpolation. This interpolation converts the above arrays which are sampled at every 2.0 nanoseconds to arrays which are sampled at every 0.2 nanoseconds. This fill-in is accomplished for a point by summing the products of each 2.0 nanosecond sample and a sin X/X value. The sin X/X value used with a sample represents the distance of the sample from the point to be filled in. The sin X/X array has a zero crossover at every tenth position, thus, the 2.0 nanosecond samples which are at every tenth position on the new position scale are unchanged by this interpolation.

After this interpolation, the routine tests a quantity, which was set equal to zero in the main program, for being zero. For the first pulse this quantity is zero, thus the routine multiplies each element in the interpolation arrays of the first pulse return by the card input B. This modified return becomes the reference with which the second return is cross-correlated. After this first reference is established, the routine defines the quantity which had been zero to be equal to 77, and then the routine returns to the main program. For all pulse returns after the first to be used in a 2D map, the test on the above quantity allows the logic flow that gives the cross-correlation of the pulse return with the reference.

When two pulse returns differ only in their relative range and overall phase, cross-correlation may be used to find both quantities. If one function is shifted along the time axis while the other function is fixed, a set of correlation values is determined. If two functions have the same wave form the maximum correlation will occur when the two functions have no relative delay. When the two functions are complex, the complex conjugate of one function must be used in the cross-correlation integral. In this routine the complex return to be aligned is cross-correlated with the complex reference return. The routine defines the number of positions on the 0.2 nanosecond sampling scale that the return to be aligned is to be shifted relative to the reference return. The shift that gives the maximum correlation represents the relative delay between the two returns. The routine determines the cross-correlation for each shifted position and compares the correlation values to find the maximum. The shift that gave the maximum correlation, and the real and imaginary values of the correlation summation at the maximum correlation are used to align the return. Each real and imaginary value of the return is shifted by the relative delay determined in the above cross-correlation. The real and imaginary values of the above cross-correlation summation at the maximum correlation are converted to a phase which is the overall phase difference between the returns at the maximum correlation. This phase difference is subtracted from each phase of the return to be aligned. The result of all the above is to subtract out the range and gross phase changes between returns. The routine defines the real and the imaginary arrays representing this aligned return. The routine also forms a new reference return as: one minus B times the old reference, plus, B times the just aligned return.

The routine goes back to the main program with this aligned pulse return, and leaving an updated reference. No alignment is necessary for simulated data produced by DATA2X. The calling sequence for this routine is

```
CALL ALIGN2 (ICH, DATA, R, F, SXDX, NFA, NOC, LLMIN, LLMAX, INITL,
             MID, B)
```

where the inputs are:

ICH - number of amplitude and phase channels in the pulse of data

DATA - name of data array which is half amplitudes and half phases

SXDX - the sin X/X array used for interpolation

NFA - Number of the first desired channel of amplitude data to be used

NOC - number of amplitude channels to be used

LLMIN - number of positions below center to be shifted for correlation values

LLMAX - number of positions above center

INITL - control constant which is zero for the first pulse and non-zero for the rest of pulses making up a 2D map

MID - number of the middle amplitude channel in the radar pulse

B - smoothing constant used on reference pulse

and the outputs are:

R - The complex radar return is converted to real and imaginary arrays, then these arrays are aligned. R is this aligned real array for the pulse.

F - the imaginary aligned array for the pulse

MID - the position number in the interpolation array of the maximum amplitude.

(3) ALIGN3 - Smooth Track Align Subroutine

ALIGN3 aligns the radar returns by calculating from orbital parameters the range and phase that the return should have. The first return is used as a range reference and all other returns are corrected relative to this first return. The non-linear portion of this calculated equation is used to find the phase to be subtracted from each phase in the return.

Logic

The program defines the midpoint of the arrays which are outputs from this routine. These arrays are the real and imaginary returns sampled at every 0.2 nanoseconds. The routine defines the number of the last desired channel to be processed and then starts processing with the first desired channel. The cosine of the phase channel is multiplied by the value in the corresponding amplitude channel, and also the sine of the phase channel is multiplied by the value in the corresponding amplitude channel. These two products are the real and the imaginary portions of the complex radar return. Each amplitude-phase channel is converted to a real and an imaginary value in the above way.

The routine fills-in the above arrays using sin (X)/X interpolations. This interpolation converts the above arrays which are sampled at every 2.0 nanoseconds to arrays which are sampled at every 0.2 nanoseconds. This fill-in is accomplished for a point by summing the products of each 2.0 nanosecond sample and a sin (X)/X value. The sin X/X value used with a sample represents the distance of the sample from the point to be filled in. The sin (X)/X array has a zero crossover at every tenth position, thus, the 2.0 nanosecond samples which are at every tenth position on the new position scale are unchanged by this interpolation.

After this interpolation, the routine tests a quantity, which was set equal to zero in the main program, for being zero. For the first pulse return this quantity is zero, thus the routine follows the logic which establishes the reference range quantity to be used with all the remaining pulse returns. This range quantity "DELTA" is the recorded "finerange" minus the calculated time dependent range. This calculated time dependent range has the following equation:

$$R(t) = A_1 t + A_2 t^2 + A_3 t^3 + A_4 t^4 + A_5 t^5 + A_6 t^6$$

Where the A's are the inputs from the main program, and t is the relative time the pulse was predicted to reach the target. The first term ($A_1 t$) is subtracted from the above range equation leaving only the non-linear terms. This resultant non-linear value is converted into radians which represents the phase the return should have. The routine subtracts this phase from each value of phase in the 0.2 nanosecond sample, and returns to the main program with the above reference and with the real and imaginary phase corrected radar return.

For the second and all remaining radar returns to make-up a 2D map, the routine follows the above logic until after the test on the above quantity which had been zero for the first radar return. After the first radar return, this quantity was set equal to +77, thus, the test results in the routine following the logic which uses the range reference quantity DELTA with the radar return to be aligned. The routine determines the new time t, and uses it in the time dependent range equation in the same way as for the first pulse. The routine now determines the range shift (time difference) between the first pulse and the one to be aligned. The difference between the pulse to be aligned the first in nanoseconds is equal to the time dependent range for the pulse plus DELTA, minus the fine range of the pulse, all divided by ten times the speed of light in meters per nanosecond.

$$DR = (R(t) + DELTA - FRANGE)/10C$$

The number of positions the 0.2 nanosecond sampled pulse must be shifted to align it with the first is ten times DR. The phase of the pulse is determined in the same way as it was found for the first radar return. The real and the imaginary samples for the pulse are shifted 10*DR positions so as to align the pulses in range. The calculated phase is subtracted from each phase value, and the resultant aligned radar return is converted to real and imaginary values. The routine returns to the main program with these array defined. The calling sequence for this routine is

CALL ALIGN3 (ICH, DATA, R, F, SXDX, NFA, NOC, INITL, MID, FRUP, FRLO, NL, A1, A2, A3, A4, A5, A6, OMC, TTS, NPULSE, VEL)

where the inputs are:

ICH - number of amplitude and phase channels in the pulse of data

DATA - name of data array which is half amplitudes and half phases

SXDX - the sin X/X array used for interpolation

NFA - number of the first desired channel of amplitude data to be used

NOC - number of amplitude channels to be used

INITL - control constant which is zero for the first pulse and non-zero for rest of the pulses

MID - number of the middle amplitude channel in the radar pulse

FRUP - the 16 most significant bits of the 40 bit "fine range" number

FRLO - the 24 lower order bits of "fine range"

NL - number of pulses

A1 - modified first range derivative

A2 - modified second range derivative

A3 - modified third range derivative

A4 - modified fourth range derivative

A5 - modified fifth range derivative

A6 - modified sixth range derivative

OMC - center frequency of radar

TTS - time between processed pulses

NPULSE - number of pulse

VEL - speed of light

and the outputs are:

R - the complex radar return is converted to real and imaginary arrays, then these arrays are aligned. R is this aligned real array for the pulse.

F - the imaginary aligned array for the pulse

MID - the position number in the interpolated array of the maximum amplitude.

(4) LSFIT4 - Least Squares Fit Subroutine

This subroutine accepts N points defined by X(I), Y(I). It then fits a fourth order polynominal to these points by the "method of least squares". The output of this subroutine are the five coefficients that define the polynominal:

$$Y(x) = A + Bx + Cx^2 + Dx^3 + Ex^4 \tag{2}$$

This function y(x) will give a better approximation to the original points X(I), Y(I) as N increases: i.e. the subroutine 's given more points in a given interval.

It is with this function y(x) that the compensation look up table is formed. The inputs to the table are X values, the outputs are Y values. Therefore, it is necessary to choose the X(I), Y(I) so that the amplitude values after going through the compensation look up table, when applied to the grid of the CRT in the film recorder, result in a film whose amplitude transmission is linearly proportional to the uncompensated amplitude values.

The calling sequence for this routine is:

CALL LSFIT4 (X, Y, N, A, B, C, D, E)

where the inputs are;

X, Y - These are two arrays defined in the main program. They are read in from input cards 3 on, depending on N, i. e. if N is greater than 4 more than one card is required.

N - This is the number of points (defined by X, Y sets) to which a curve is fitted.

the outputs are;

A, B, C, D, E - These are the coefficients of the fourth order polynominal as defined by Equation (2).

(5) SIMEQ - Subroutine for Solution of Simultaneous Equations

This subroutine is used by LSFIT4 in solving a set of simultaneous equations needed in determining the coefficients A, B, C, D, E, by the method of least squares.

The calling sequence for this routine is:

CALL SIMEQ (A, NROWA, NCOLB, DET, MAXROW)

where the inputs are;

A - a matrix array containing the B vectors

NROWA - number of rows in matrix A

NCOLB - number of B vectors

MAXRON - maximum row dimension of A array.

the outputs are;

DET - value of determinate of A matrix

(The solutions to a set of equations are placed in first NCOLB columns of matrix A).

(6) PACK - Subroutine Which Writes Tape for SDS 910

The UNIVAC 1108 uses a 36 bit word. The SDS requires the 1024 or 2048 word arrays to be written on tape such that each word consists of 12 bits. These words are adjacent on the tape with no gaps except a record gap every 1024 or 2048 words. The first 3 bits of the 12 bit word are not used. The next 9 bits are used to represent amplitude values ranging from -255 to 256. Positive numbers are represented in the usual way. A positive number becomes negative by taking a two's compliment of the 9 bits.

This subroutine accepts the 36 bit UNIVAC 1108 word and packs it onto magnetic tape as described.

The calling sequence for this routine is;

CALL PACK (IWI, L)

the inputs are;

IWI - arrays containing values to be written on magnetic tape for SDS 910.

L - size of IWI array. After each L numbers a record gap is put on tape.

(The output is a magnetic tape).

d. Optical Processor Program Data Cards

Card 1. Format 8F10.2

AMX(1) to AMX(8) - Each number is the maximum amplitude for a given frame. This maximum for each frame is obtained from AMPMAX subroutine, not part of this program. AMX(1) is the maximum for the first frame, AMX(2) the second frame, etc.

Card 2. Format 14I5

NIPP - This is the number of interpolations per pulse and must be either 1024 or 2048.

N - Total number of points X, Y to be read in on the next set of data cards (number of cards depends on N).

NPPF - Number of pulses per frame. This is fixed for a given run.

Card 3. Format 8F10.3

X(1), Y(1) to X(N), Y(N) - These are points which are used by the program to form a look up table to compensate for the nonlinearities of the film recorder. The explanation of LSFIT4 defines how these points are determined. If N is greater than 4, more than one card is required.

Card 4. Format 5E14.6

B - This is the smoothing constant for the cross-correlation alignment subroutine (ALIGN2) where each pulse is aligned to give maximum correlation with

$$P_n = BP_n = (1 - B) P_{n-1}$$

where P_n is the immediately preceding aligned pulse and P_n is the weighted average of all aligned preceding pulses.

OMC - This is the center frequency of the radar in MHz.

FC - This is a quantity that determines the spatial carrier. For 10 line pair per millimeter FC = .293.

Card 5. Format 14I5

ITAP - This is the data tape rewind control. If ITAP ≠ 0, the data tape on Fortran IV tape unit 11 is rewound. If ITAP = 0 it is not rewound.

LF - This is the number of pulses skipped on the data tape before the first one used.

MODUL - If this pulse skipping control equals N, every nth pulse is skipped. If MODUL = I, every pulse is processed.

NALIGN - This determines which align subroutine is to be used. Simulated radar data produced by DATA2X is not aligned. If NALIGN = 1, ALIGN1 is used. If NALIGN = 2, ALIGN2 is used. If NALIGN=3 ALIGN3 is used.

NFC - This is the number of the first channel of the 84 to be processed. Normally NFC = 1.

NL - This is the total number of pulses to be processed by given run.

NOC - This is the number of channels, starting at NFC, actually used for each pulse.

Card 6. Format 6F12.6

RH01, RH02, ... RH06 - These six numbers are the coefficients for a power series representing range versus time used in ALIGN3.

2. DATA2X - RADAR DATA SIMULATION PROGRAM

This program produces a data tape in the calibrated radar data format. This tape can be used to test the optical recording system and software. The program simulates the radar returns from a rigid rotating system of point scatters. At the present time it is possible to simulate up to twenty point scatterers. Each point scatterer can have a arbitrary amplitude, position and relative phase. The rigid system of point scatterers can then be rotated at any desired rotation rate and recorded at a given pulse repetition frequency. There are two modes of operation of this program. In the first mode the return of each point scatterer of a given amplitude is of the form $\sin\sqrt{x^2-c^2}/\sqrt{x^2-c^2}$, where c is chosen to give a desired peak to side lobe ratio. This mode is designated as Format I. In the second mode, the return from each point scatterer is the Fourier transform in the range dimension of $\sin\sqrt{x^2-C^2}/\sqrt{x^2-C^2}$. This mode produces Format II data.

a. Logic

The program begins by calling subroutine SXØXS. This subroutine sets up a sin x/x table which is used by subroutine AMPMAX to find the maximum amplitude of a pulse. Now data cards 1 and 2 are read in. Here the variable MFORMAT is tested. If MFORMAT =1, Format I data is produced. This is done by calling subroutine RZERO. This subroutine sets up a table of $\sin\sqrt{x^2-C^2}/\sqrt{x^2-C^2}$ which is used in producing Format I data. If MFORMAT=2, subroutine BAMP is called. This produces a table which is the Fourier Transform of the function $\sin\sqrt{x^2-C^2}/\sqrt{x^2-C^2}$ in the range dimension. This is then used to produce Format II data. For this Format NSORT is set equal to 2, this determines how subroutines AMPMAX locates the maximum amplitude of a pulse. Now data card 3 is read in and various parameters are defined. The remaining data cards are now read. The number of cards depends on the number of point scatterers and on the number of different target configurations.

Each pulse of data consists of 84 amplitude and 84 phase channels. Therefore it is necessary to calculate the radar response of the point scatterer configuration at 84 points. This is done by assuming that the radar return from a point scatterer is given by the table computed by subroutine RZEROM for Format I, or by the table from subroutine BAMP for

Format II. In range, the axis of rotation of the point scatters is assumed to be at the center of the 84 range channels. The program now computes the relative phase and range of each point scatterer The function that was calculated by either RZEROM or BAMP is now multiplied by the amplitude of each point scatterer and the contribution from all the point scatterers is summed at every channel. This is done for both the real and imaginary components. Now the data is converted to amplitude and phase at each channel and written onto magnetic tape. Subroutine AMPMAX is called to find the maximum amplitude of this pulse. The target is now rotated through an angle, which depends on the pulse repetition frequency and the rotation rate, and the same computation is begun for a new pulse.

After all the pulses for a given target are completed, the maximum amplitude of these pulses is found from the maximum of each pulse. The program now reads in the value of IEND on the last card for a given target. If IEND = 0 a new set of target configuration data cards is read in. If IEND ≠ 0 an END OF FILE is placed on the tape and EXIT is called. See page 98 for flow chart.

b. Subroutines for DATA2X Program

(1) RZEROM - Subroutine for Format I response Pattern.

This routine sets up a table of the function $\sin\sqrt{x^2-C^2}/\sqrt{x^2-C^2}$. The table is the array RZ. RZ(1) is the value of the above function at X = 0. Then X is incremented by the amount DTAU until there are 855 values of RZ. The peak amplitude of the above function is $(e^c - e^{-c})/2c$ and the first zero crossing is at $\sqrt{\pi^2 + C^2}$. The calling sequence is:

CALL RZEROM (K, DTAU, C, OMB, TAU, RZ, IPT)

where the inputs are:

K	-	number of sample points is K + 1.
DTAU	-	increment of X.
C	-	determines the peak to sidelobe ratio of the response pattern.
OMB	-	This is the scale factor of the X values. With this scale factor the new variable U = (OMB)*X/2. Now the first zero crossing occurs at $X = (2 \cdot \sqrt{\pi^2 + C^2})/OMB$.
IPT	-	Print Control. If IPT = 0 there is no print out. If IPT ≠ 0 then the TAU and RZ arrays are printed out.

and the outputs are:

TAU	-	an array containing the K + 1 X values.
RZ	-	an array containing the K + 1 values of the Format I response pattern.

(2) BAMP - Subroutine for Format II Response Pattern.

This routine uses the modified Bessel function of order zero to calculate a table B which is an array containing the Fourier Transform of the table formed by subroutine RZEROM. This table is then used by DATA2X to produce Format II data. The calling sequence is

CALL BAMP (DDD, LLL, B, IPT)

where the inputs are:

DDD - the sidelobe suppression constant.

LLL - number of samples in the B array.

IPT - print control, if IPT = 0 there is no print out, if IPT ≠ 0 the B array is printed out.

The output is:

B - an array containing the LLL values of the Format II response pattern.

(3) AMPMAX - Subroutine For Finding The Maximum Amplitude of a Pulse

Since radar amplitudes for a pulse are sampled at only 84 points, the maximum of these may not be the true maximum. Interpolation of all 84 points to find the maximum is time consuming. This routine, depending on the variable NSORT, is used to accurately find the maximum amplitude of a pulse by two different rapid methods.

If NSORT = 1 the following method is used. First the maximum of the 84 amplitudes is found. Then another search is performed to find the relative maxima, these are amplitudes that are less than the maximum of the 84, and greater than this maximum minus DA. About the maximum of the 84 amplitudes, and about each relative maxima a sin X/X interpolation is performed. The number of interpolated points between each sample is NIPC. This interpolation is done in a region - IDX to IDX about each location. From these interpolations a maxima is found. This is the maximum amplitude for this pulse. Its value is stored in the array AMXT, and a return is made to the calling program.

If NSORT = 2 the region between channels NFT and NLT is interpolated by sin X/X. The number of interpolations between channel is given by NIPC. The maximum of these interpolated values is then the maximum for this pulse. It is stored in the array AMXT, and a return is made to the calling program.

The calling sequence is:

```
CALL AMPMAX (NIPC, NCH, DA, IDX, NPPF, AMP, SXDXS, NFT, NLT,
             NSORT, AMXT, INP)
```

where the inputs are:

NIPC - number of interpolations per channel.

NCH - number of channels

DA - used in finding relative maxima when NSORT = 1

IDX - used in interpolating about relative maxima when NSORT =1

NPPF - number of pulses per frame

AMP - this is the amplitude array that is searched for a maxima

SXDXS - Sinx/X array used in interpolation

NFT - number of first channel for interpolation when NSORT = 2

NLT - number of last channel for interpolation when NSORT = 2

INP - this variable must be one the first time AMPMAX is called, and incremented by one every following call.

the output is:

AMXT - an array containing the maximum of each pulse. The number of values in the array is equal to the number of times AMPMAX is called.

(4) SXØXS - Subroutine for producing sinX/X array.

This routine sets up the sinX/X table SXDXS. SXDXS(1) = 1, for sinX/X at X = 0. Then X is incremented by π/NIPC until there are (NCH)•(NIPC) + 10 values in the SXDXS array.

The calling sequence is

CALL SXØXS (SXDXS, NIPC, NCH).

where the inputs are:

NIPC - π/NIPC is the increment for X used in calculating the array of sinX/X.

NCA - There are NCH•NIPC + 10 values in the SXDXS array.

the output is:

SXDXS - an array of sinx/X values.

d. DATA2X Program Data Cards

Card 1. Format I10, F10.3, 2I10

NIPC - number of interpolations per channel, used by AMPMAX

DA - used by AMPMAX to find relative maximum when NSORT = 1

IDX - used by AMPMAX in interpolating about relative maximum when NSORT = 1

NSORT - determines which method is used by AMPMAX to find maximum amplitude of a pulse.

Card 2. Format 6I5, 4F10.3

MFORMT - If MFORMAT = 1, format I data is produced. If MFORMT = 2 format II data is produced.

K - there are K+1 values in the RZ array produced by subroutine RZEROM.

LLL - there are LLL values in the B array produced by subroutine BAMP.

IPT - print control. If IPT ≠ 0 the RZ and the B array produced by RZEROM and BAMP will be printed out. If IPT = 0 they will not be printed.

NFT - This is used by subroutine AMPMAX, and is the first channel to be interpolated when NSORT = 2

NLT - This is used by subroutine AMPMAX, and is the last channel to be interpolated when NSORT = 2.

DTAU - this is the increment for the X variable at which $\sin\sqrt{X^2 - C^2}/\sqrt{X^2 - C^2}$ is calculated by subroutine RZEROM.

C - determines the peak to sidelobe ratio of the response pattern produced by the RZEROM routine.

OMB - scale factor for the variable x in the RZEROM routine

DDD - sidelobe suppression constant for the BAMP subroutine.

Card 3. Format 2I5, 5F10.5

NPT - number of point scatterers

M - for each target 2M+1 pulses are generated.

AK - increment for the range, for 84 channels, AK = 1.

ROT - rotation rate of point scatterers in degrees per second.

PRF - pulse repetition frequency in Hertz.

DELX - actual range increment for 84 channels AK = 1

XSTART - this is the first range value relative to center of rotation. If the center of rotation of the target is to be in the center of the 84 channels then XSTART = -42 (i.e., will have -42 to +41 channel locations, the center of rotation at channel 0.)

Card 4. Format 8F10.3

R(1), R(2), ... R(NPT) - This is an array of the ranges of each point scatterer from the center of rotation. If NPT is greater than 8 more than one card is required.

Card 5. Format 8F10.3

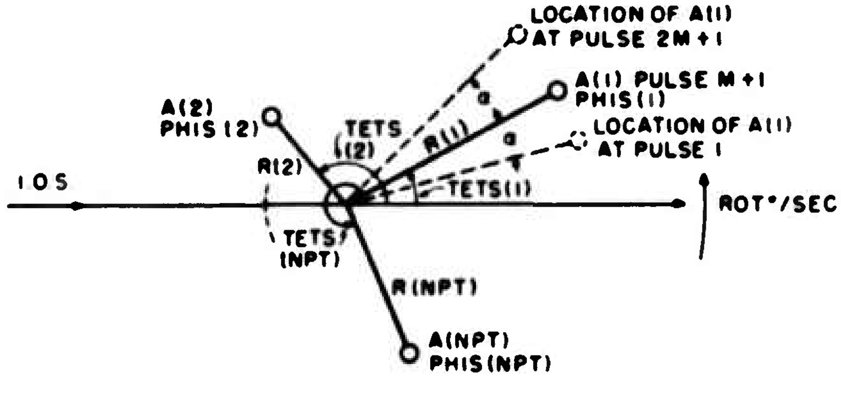
A(1), A(2), ... A(NPT) - This is an array of the amplitudes of each point scatterer. If NPT is greater than 8 more than one card is required.

Card 6. Format 8F10.3

PHIS(1), PHIS(2), ... PHIS(NPT) - This is array of the phase change, in degrees, on reflection from each point scatterer. If NPT is greater than 8 more than one card is required.

Card 7. Format 8F10.3

TETS(1), TETS(2), ... TETS(NPT) - This is the array of the angles of the lines drawn from each point scatterer to center of rotation of the target when it is at the M + 1 pulse (there are 2M+1 pulses for each target). Each angle is at the center of the rotation interval. There are M pulses to each side of TETS for each point scatter. The figure below shows the target configuration:



(The total angle through which the target rotates for 2M+1 pulses is $2\alpha = (2M+1)\,(ROT)\,(1/PRF)$ degrees)

Card 8. Format I3

IEND - If IEND ≤ 0 read another set of cards 3 through 7 for a new target and a new set of pulses. If IEND > 0 program writes an END FILE 11 on tape, pauses to remove tape then, exits.

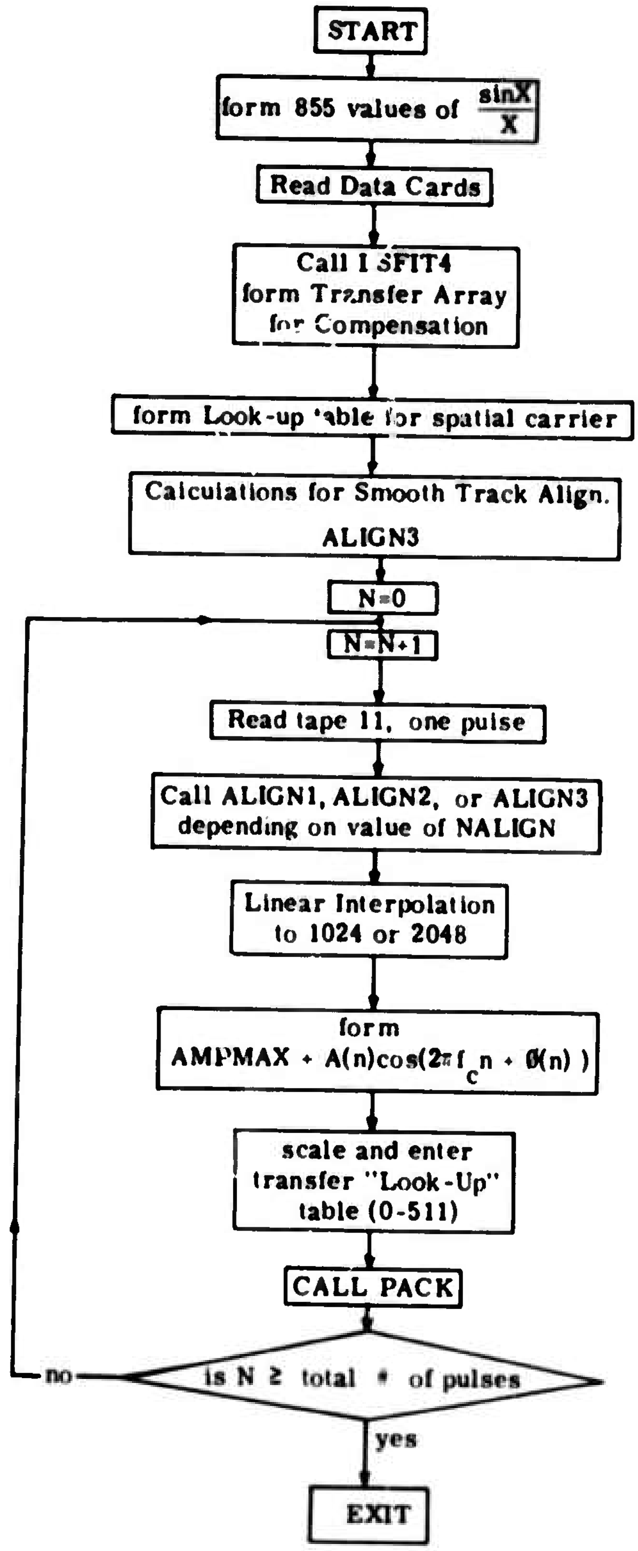
3. SDS 910 PROGRAM

The SDS 910 program accepts pulses of rada data from a magnetic tape produced on the UNIVAC 1108 by the Optical Processor Program. On this tape are records of either 2048 or 1024 words each. Each word represents a pulse width which is, in normal operation, a single sweep on the CRT. The SDS 910 reads these records into storage and then outputs them at a given rate to the D/A converter and the film recorder. In addition, line start pulses, line stop pulses, and camera clock frequencies are generated.

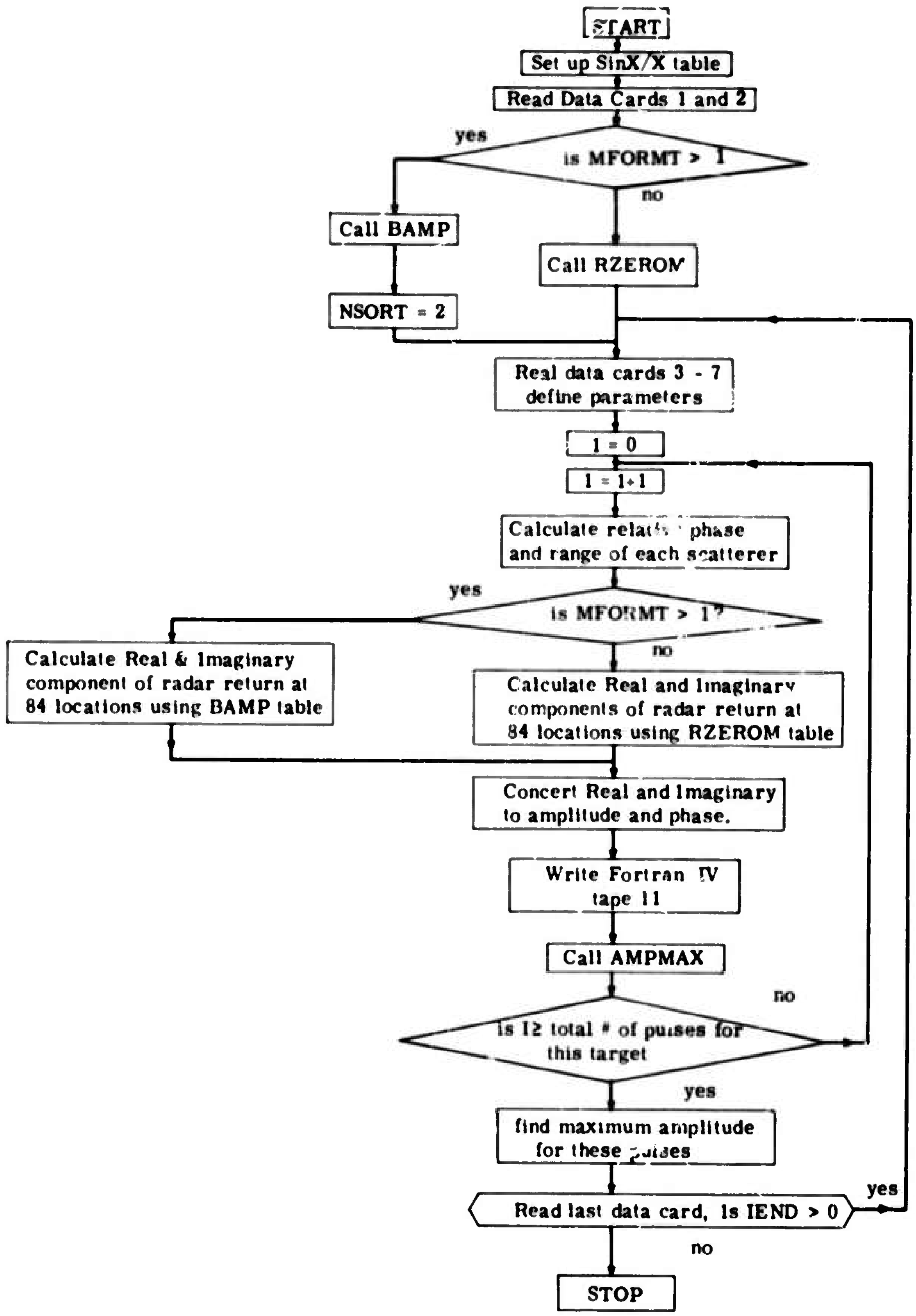
The SDS 910 can accept information from magnetic tape, paper tape, or an on-line typewriter. Also, control of a program can be initiated by means of breakpoint switches 1 through 4. The SDS 910 Program is read in from paper tape by the standard fill procedure (see section on operation of the film recorder for more detail on the program and operation of the SDS 910). All breakpoint switches should be reset. The program is now in the idling mode. It is necessary to provide time between Line Start pulses. This is done by setting and then resetting breakpoint 1, and then typing in a four digit number equal to the desired number of milliseconds. If this is not done the time between Line Start will be set to 200 milliseconds (i.e., 5 pulses per second). At the present time the number of milliseconds between line start, pulses must be greater than 200 and less than 400.

For normal operation break-point 3 is now set. The program now alternately reads a block off of magnetic tape and outputs it at the already fixed rate to the CRT. Line start and line stop pulses are also generated. At the end of each frame of pulses all outputs are inhibited, except the camera clock frequency. If breakpoint 3 is reset and 2 is set further magnetic tape is not read. The program repeatedly outputs the last block that was read from a magnetic tape. All timing is exactly the same as when breakpoint 3 was set.

Flow Chart for Optical Processor Program.



Flow Chart for DATA2X Program



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13 ABSTRACT

A film recorder for high resolution radar echoes has been designed and fabricated. It employs a modulated cathode ray tube trace which is imaged onto a moving film. Individual pulse echoes are written across one dimension of the film, corresponding to the direction of the CRT sweep. Successive echoes are accumulated side-by-side in the orthogonal direction. The data is recorded in a format suitable for subsequent processing in a coherent optical system. The purpose is to obtain doppler resolution by integration over a sequence of pulses. This is to be carried out simultaneously for each range resolution element, giving an output with two dimensions of resolution.

The specifications, design, fabrication, and check-out of the film recording equipment are described in this report. Simulated data has been employed to verify the overall performance of the device. A review is given of the data frames recorded and the test results obtained from them. The system can record data in two different formats, to allow some flexibility in implementing the coherent imaging processor.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
RADAR SYNTHETIC SPECTRUM HIGH RESOLUTION OPTICAL PROCESSING OPTICAL DATA PROCESSING						

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